O'Mega:
Optimal Monte-Carlo
Event Generation Amplitudes

Thorsten Ohl[*]

Institut für Theoretische Physik und Astrophysik
Julius-Maximilians-Universität Würzburg
Emil-Hilb-Weg 22, 97074 Würzburg, Germany

Jürgen Reuter[†]

DESY Theory Group, Notkestr. 85, 22603 Hamburg, Germany

Wolfgang Kilian[c,‡]

Theoretische Physik 1
Universität Siegen
Walter-Flex-Str. 3, 57068 Siegen, Germany

with contributions from Christian Speckner[d,§]
as well as Christian Schwinn et al.

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Abstract

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[*]ohl@physik.uni-wuerzburg.de, http://physik.uni-wuerzburg.de/ohl
[†]juergen.reuter@desy.de
[‡]kilian@physik.uni-siegen.de
[§]cnspeckn@googlemail.com

Revision Control

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INTRODUCTION

1.1 Complexity

There are

$$P(n) = \frac{2^n - 2}{2} - n = 2^{n-1} - n - 1 \qquad (1.1)$$

independent internal momenta in a n-particle scattering amplitude [1]. This grows much slower than the number

$$F(n) = (2n - 5)!! = (2n - 5) \cdot (2n - 7) \cdot \ldots \cdot 3 \cdot 1 \qquad (1.2)$$

of tree Feynman diagrams in vanilla ϕ^3 (see table 1.1). There are no known corresponding expressions for theories with more than one particle type. However, empirical evidence from numerical studies [1, 2] as well as explicit counting results from O'Mega suggest

$$P^*(n) \propto 10^{n/2} \qquad (1.3)$$

while he factorial growth of the number of Feynman diagrams remains unchecked, of course.

n	$P(n)$	$F(n)$
4	3	3
5	10	15
6	25	105
7	56	945
8	119	10395
9	246	135135
10	501	2027025
11	1012	34459425
12	2035	654729075
13	4082	13749310575
14	8177	316234143225
15	16368	7905853580625
16	32751	213458046676875

Table 1.1: The number of ϕ^3 Feynman diagrams $F(n)$ and independent poles $P(n)$.

The number of independent momenta in an amplitude is a better measure for the complexity of the amplitude than the number of Feynman diagrams, since there can be substantial cancellations among the latter. Therefore it should be possible to express the scattering amplitude more compactly than by a sum over Feynman diagrams.

1.2   Ancestors

Some of the ideas that O'Mega is based on can be traced back to HELAS [5]. HELAS builts Feynman amplitudes by recursively forming off-shell 'wave functions' from joining external lines with other external lines or off-shell 'wave functions'.

The program Madgraph [6] automatically generates Feynman diagrams and writes a Fortran program corresponding to their sum. The amplitudes are calculated by calls to HELAS [5]. Madgraph uses one straightforward optimization: no statement is written more than once. Since each statement corresponds to a collection of trees, this optimization is very effective for up to four particles in the final state. However, since the amplitudes are given as a sum of Feynman diagrams, this optimization can, by design, *not* remove the factorial growth and is substantially weaker than the algorithms of [1, 2] and the algorithm of O'Mega for more particles in the final state.

Then ALPHA [1] (see also the slightly modified variant [2]) provided a numerical algorithm for calculating scattering amplitudes and it could be shown empirically, that the calculational costs are rising with a power instead of factorially.

1.3   Architecture

1.3.1   General purpose libraries

Functions that are not specific to O'Mega and could be part of the O'Caml standard library

ThoList : (mostly) simple convenience functions for lists that are missing from the standard library module *List* (section G, p. 806)

Product : effcient tensor products for lists and sets (section L, p. 844)

Combinatorics : combinatorical formulae, sets of subsets, etc. (section O, p. 855)

1.3.2   O'Mega

The non-trivial algorithms that constitute O'Mega:

DAG : Directed Acyclical Graphs (section 4, p. 35)

Topology : unusual enumerations of unflavored tree diagrams (section 3, p. 18)

Momentum : finite sums of external momenta (section 5, p. 50)
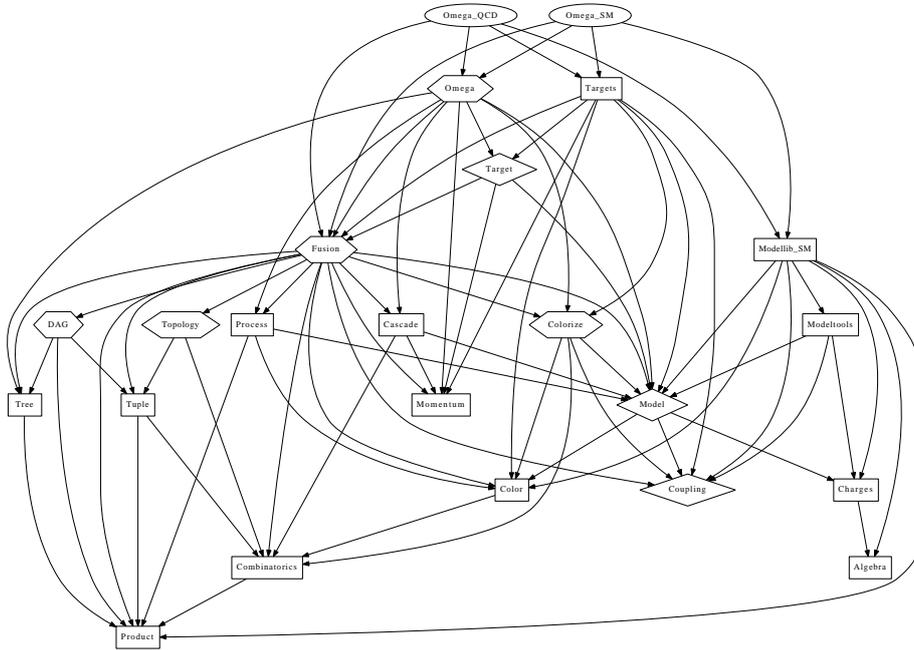
Figure 1.1: Module dependencies in O'Mega.

Fusion : off shell wave functions (section 8, p. 88)

Omega : functor constructing an application from a model and a target
(section 18, p. 773)

1.3.3   Abstract interfaces

The domains and co-domains of functors (section 9, p. 138)

Coupling : all possible couplings (not comprensive yet)

Model : physical models

Target : target programming languages

1.3.4   Models

(section 13, p. 225)

$Modellib_S M.QED$: Quantum Electrodynamics

$Modellib_S M.QCD$: Quantum Chromodynamics (not complete yet)

$Modellib_S M.SM$: Minimal Standard Model (not complete yet)

etc.

1.3.5   Targets

Any programming language that supports arithmetic and a textual representation of programs can be targeted by O'Caml. The implementations translate the abstract expressions derived by *Fusion* to expressions in the target (section 15, p. 611).

> *Targets.Fortran* : Fortran95 language implementation, calling subroutines

Other targets could come in the future: `C`, `C++`, O'Caml itself, symbolic manipulation languages, etc.

1.3.6   Applications

(section 18, p. 773)

1.4   The Big To Do Lists

1.4.1   Required

All features required for leading order physics applications are in place.

1.4.2   Useful

1. select allowed helicity combinations for massless fermions

2. Weyl-Van der Waerden spinors

3. speed up helicity sums by using discrete symmetries

4. general triple and quartic vector couplings

5. diagnostics: count corresponding Feynman diagrams more efficiently for more than ten external lines

6. recognize potential cascade decays (τ, b, etc.)

   - warn the user to add additional
   - kill fusions (at runtime), that contribute to a cascade

7. complete standard model in R_ξ-gauge

8. groves (the simple method of cloned generations works)

1.4.3   Future Features

1. investigate if unpolarized squared matrix elements can be calculated faster as traces of densitiy matrices. Unfortunately, the answer apears to be *no* for fermions and *up to a constant factor* for massive vectors. Since the number of fusions in the amplitude grows like $10^{n/2}$, the number of fusions in the squared matrix element grows like 10^n. On the other hand, there are $2^{\#\text{fermions}+\#\text{massless vectors}} \cdot 3^{\#\text{massive vectors}}$ terms in the helicity sum,

which grows *slower* than $10^{n/2}$. The constant factor is probably also not favorable. However, there will certainly be asymptotic gains for sums over gauge (and other) multiplets, like color sums.

2. compile Feynman rules from Lagrangians

3. evaluate amplitues in O'Caml by compiling it to three address code for a virtual machine

   type *mem* = *scalar array* × *spinor array* × *spinor array* × *vector array*
   type *instr* =
       — *VSS* of *int* × *int* × *int*
       — *SVS* of *int* × *int* × *int*
       — *AVA* of *int* × *int* × *int*
   . . .

   this could be as fast as [1] or [2].

4. a virtual machine will be useful for for other target as well, because native code appears to become to large for most compilers for more than ten external particles. Bytecode might even be faster due to improved cache locality.

5. use the virtual machine in O'Giga

1.4.4   Science Fiction

1. numerical and symbolical loop calculations with O'TERA: O'MEGA TOOL FOR EVALUATING RENORMALIZED AMPLITUDES

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Tuples and Polytuples

2.1 Interface of Tuple

The *Tuple.Poly* interface abstracts the notion of tuples with variable arity. Simple cases are binary polytuples, which are simply pairs and indefinite polytuples, which are nothing but lists. Another example is the union of pairs and triples. The interface is very similar to *List* from the O'Caml standard library, but the *Tuple.Poly* signature allows a more fine grained control of arities. The latter provides typesafe linking of models, targets and topologies.

module type *Mono* =
  sig
    type α *t*

    val *arity* : α *t* $\rightarrow$ *int*
    val *max_arity* : *int*

    val *compare* : $(\alpha \rightarrow \alpha \rightarrow int) \rightarrow \alpha\ t \rightarrow \alpha\ t \rightarrow int$

    val *for_all* : $(\alpha \rightarrow bool) \rightarrow \alpha\ t \rightarrow bool$

    val *map* : $(\alpha \rightarrow \beta) \rightarrow \alpha\ t \rightarrow \beta\ t$
    val *iter* : $(\alpha \rightarrow unit) \rightarrow \alpha\ t \rightarrow unit$
    val *fold_left* : $(\alpha \rightarrow \beta \rightarrow \alpha) \rightarrow \alpha \rightarrow \beta\ t \rightarrow \alpha$
    val *fold_right* : $(\alpha \rightarrow \beta \rightarrow \beta) \rightarrow \alpha\ t \rightarrow \beta \rightarrow \beta$

We have applications, where no sensible intial value can be defined:

    val *fold_left_internal* : $(\alpha \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha\ t \rightarrow \alpha$
    val *fold_right_internal* : $(\alpha \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha\ t \rightarrow \alpha$

    val *map2* : $(\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \alpha\ t \rightarrow \beta\ t \rightarrow \gamma\ t$

    val *split* : $(\alpha \times \beta)\ t \rightarrow \alpha\ t \times \beta\ t$

The distributive tensor product expands a tuple of lists into list of tuples, e. g. for binary tuples:

$$product\ ([x_1; x_2], [y_1; y_2]) = [(x_1, y_1); (x_1, y_2); (x_2, y_1); (x_2, y_2)] \qquad (2.1)$$

NB: *product_fold* is usually much more memory efficient than the combination of *product* and *List.fold_right* for large sets.

    val *product* : $\alpha\ list\ t \rightarrow \alpha\ t\ list$

$$\textsf{val } product\_fold \ : \ (\alpha \ t \ \to \ \beta \ \to \ \beta) \ \to \ \alpha \ list \ t \ \to \ \beta \ \to \ \beta$$

For homogeneous tuples the *power* function could trivially be built from *product*, e. g.:

$$power\,[x_1; x_2] = product\,([x_1; x_2], [x_1; x_2]) = [(x_1, x_1); (x_1, x_2); (x_2, x_1); (x_2, x_2)] \tag{2.2}$$

but it is also well defined for polytuples, e. g. for pairs and triples

$$power\,[x_1; x_2] = product\,([x_1; x_2], [x_1; x_2]) \cup product\,([x_1; x_2], [x_1; x_2], [x_1; x_2]) \tag{2.3}$$

For tuples and polytuples with bounded arity, the *power* and *power_fold* functions terminate. In polytuples with unbounded arity, the the *power* function always raises *No_termination*. *power_fold* also raises *No_termination*, but could be changed to run until the argument function raises an exception. However, if we need this behaviour, we should implemente *power_iter* instead.

$$\textsf{val } power \ : \ \alpha \ list \ \to \ \alpha \ t \ list$$
$$\textsf{val } power\_fold \ : \ (\alpha \ t \ \to \ \beta \ \to \ \beta) \ \to \ \alpha \ list \ \to \ \beta \ \to \ \beta$$

We can also identify all (poly)tuples with permuted elements and return only one representative, e. g.:

$$sym\_power\,[x_1; x_2] = [(x_1, x_1); (x_1, x_2); (x_2, x_2)] \tag{2.4}$$

NB: this function has not yet been implemented, because O'Mega only needs the more efficient special case *graded_sym_power*.

If a set X is graded (i. e. there is a map $\phi : X \to \mathbf{N}$, called *rank* below), the results of *power* or *sym_power* can canonically be filtered by requiring that the sum of the ranks in each (poly)tuple has one chosen value. Implementing such a function directly is much more efficient than constructing and subsequently disregarding many (poly)tuples. The elements of rank n are at offset $(n-1)$ in the array. The array is assumed to be *immutable*, even if O'Caml doesn't support immutable arrays. NB: *graded_power* has not yet been implemented, because O'Mega only needs *graded_sym_power*.

$$\textsf{type } \alpha \ graded \ = \ \alpha \ list \ array$$
$$\textsf{val } graded\_sym\_power \ : \ int \to \ \alpha \ graded \ \to \ \alpha \ t \ list$$
$$\textsf{val } graded\_sym\_power\_fold \ : \ int \to \ (\alpha \ t \ \to \ \beta \ \to \ \beta) \ \to \ \alpha \ graded \ \to$$
$$\beta \ \to \ \beta$$

We hope to be able to avoid the next one in the long run, because it mildly breaks typesafety for arities. Unfortunately, we're still working on it . . .

$$\textsf{val } to\_list \ : \ \alpha \ t \ \to \ \alpha \ list$$

The next one is only used for Fermi statistics below, but can not be implemented if there are no binary tuples. It must be retired as soon as possible.

$$\textsf{val } of2\_kludge \ : \ \alpha \ \to \ \alpha \ \to \ \alpha \ t$$

$$\textsf{val } rcs \ : \ RCS.t$$
$$\textsf{end}$$

```
module type Poly =
  sig
    include Mono
    exception Mismatched_arity
    exception No_termination
  end

module type Binary =
    sig
      include Poly (* should become Mono! *)
      val of2 : α → α → α t
    end
module Binary : Binary

module type Ternary =
    sig
      include Mono
      val of3 : α → α → α → α t
    end
module Ternary : Ternary

type α pair_or_triple = T2 of α × α | T3 of α × α × α

module type Mixed23 =
    sig
      include Poly
      val of2 : α → α → α t
      val of3 : α → α → α → α t
    end
module Mixed23 : Mixed23

module type Nary =
    sig
      include Poly
      val of2 : α → α → α t
      val of3 : α → α → α → α t
      val of_list : α list → α t
    end
module Unbounded_Nary : Nary

module type Bound = sig val max_arity : int end
module Nary (B : Bound) : Nary
```

⚠ For compleneteness sake, we could add most of the *List* signature

- val *length* : α t → int
- val *hd* : α t → α
- val *nth* : α t → int → α
- val *rev* : α t → α t
- val *rev_map* : (α → β) → α t → β t
- val *iter2* : (α → β → unit) → α t → β t → unit

- val *rev_map2* : $(\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \alpha\ t \rightarrow \beta\ t \rightarrow \gamma\ t$
- val *fold_left2* : $(\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \alpha) \rightarrow \alpha \rightarrow \beta\ t \rightarrow \gamma\ t \rightarrow \alpha$
- val *fold_right2* : $(\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \gamma) \rightarrow \alpha\ t \rightarrow \beta\ t \rightarrow \gamma \rightarrow \gamma$
- val *exists* : $(\alpha \rightarrow bool) \rightarrow \alpha\ t \rightarrow bool$
- val *for_all2* : $(\alpha \rightarrow \beta \rightarrow bool) \rightarrow \alpha\ t \rightarrow \beta\ t \rightarrow bool$
- val *exists2* : $(\alpha \rightarrow \beta \rightarrow bool) \rightarrow \alpha\ t \rightarrow \beta\ t \rightarrow bool$
- val *mem* : $\alpha \rightarrow \alpha\ t \rightarrow bool$
- val *memq* : $\alpha \rightarrow \alpha\ t \rightarrow bool$
- val *find* : $(\alpha \rightarrow bool) \rightarrow \alpha\ t \rightarrow \alpha$
- val *find_all* : $(\alpha \rightarrow bool) \rightarrow \alpha\ t \rightarrow \alpha\ list$
- val *assoc* : $\alpha \rightarrow (\alpha \times \beta)\ t \rightarrow \beta$
- val *assq* : $\alpha \rightarrow (\alpha \times \beta)\ t \rightarrow \beta$
- val *mem_assoc* : $\alpha \rightarrow (\alpha \times \beta)\ t \rightarrow bool$
- val *mem_assq* : $\alpha \rightarrow (\alpha \times \beta)\ t \rightarrow bool$
- val *combine* : $\alpha\ t \rightarrow \beta\ t \rightarrow (\alpha \times \beta)\ t$
- val *sort* : $(\alpha \rightarrow \alpha \rightarrow int) \rightarrow \alpha\ t \rightarrow \alpha\ t$
- val *stable_sort* : $(\alpha \rightarrow \alpha \rightarrow int) \rightarrow \alpha\ t \rightarrow \alpha\ t$

but only if we ever have too much time on our hand . . .

2.2 Implementation of Tuple

let *rcs_file* = *RCS.parse* "Tuple" ["Tuples␣of␣fixed␣and␣indefinite␣arity"]
  { *RCS.revision* = "\$Revision:␣6465␣\$";
    RCS.date = "\$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣\$";
    RCS.author = "\$Author:␣jr_reuter␣\$";
    RCS.source
       = "\$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s...

module type *Mono* =
  sig
    type $\alpha\ t$
    val *arity* : $\alpha\ t \rightarrow int$
    val *max_arity* : int
    val *compare* : $(\alpha \rightarrow \alpha \rightarrow int) \rightarrow \alpha\ t \rightarrow \alpha\ t \rightarrow int$
    val *for_all* : $(\alpha \rightarrow bool) \rightarrow \alpha\ t \rightarrow bool$
    val *map* : $(\alpha \rightarrow \beta) \rightarrow \alpha\ t \rightarrow \beta\ t$
    val *iter* : $(\alpha \rightarrow unit) \rightarrow \alpha\ t \rightarrow unit$
    val *fold_left* : $(\alpha \rightarrow \beta \rightarrow \alpha) \rightarrow \alpha \rightarrow \beta\ t \rightarrow \alpha$
    val *fold_right* : $(\alpha \rightarrow \beta \rightarrow \beta) \rightarrow \alpha\ t \rightarrow \beta \rightarrow \beta$
    val *fold_left_internal* : $(\alpha \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha\ t \rightarrow \alpha$
    val *fold_right_internal* : $(\alpha \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha\ t \rightarrow \alpha$
    val *map2* : $(\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \alpha\ t \rightarrow \beta\ t \rightarrow \gamma\ t$
    val *split* : $(\alpha \times \beta)\ t \rightarrow \alpha\ t \times \beta\ t$

```
    val product  :  α list t  →  α t list
    val product_fold  :  (α t  →  β  →  β)  →  α list t  →  β  →  β
    val power  :  α list →  α t list
    val power_fold  :  (α t  →  β  →  β)  →  α list →  β  →  β
    type α graded  =  α list array
    val graded_sym_power  :  int →  α graded  →  α t list
    val graded_sym_power_fold  :  int →  (α t  →  β  →  β)  →  α graded  →
        β  →  β
    val to_list  :  α t  →  α list
    val of2_kludge  :  α  →  α  →  α t
    val rcs  :  RCS.t
  end

module type Poly  =
  sig
    include Mono
    exception Mismatched_arity
    exception No_termination
  end
```

2.2.1   Typesafe Combinatorics

Wrap the combinatorical functions with varying arities into typesafe functions
with fixed arities. We could provide specialized implementations, but since
we *know* that *Impossible* is *never* raised, the present approach is just as good
(except for a tiny inefficiency).

```
exception Impossible of string
let impossible name  =  raise (Impossible name)

let choose2 set  =
    List.map (function [x; y]  →  (x, y)  |  _  →  impossible "choose2")
        (Combinatorics.choose 2 set)

let choose3 set  =
    List.map (function [x; y; z]  →  (x, y, z)  |  _  →  impossible "choose3")
        (Combinatorics.choose 3 set)
```

2.2.2   Pairs

```
module type Binary  =
    sig
        include Poly (∗ should become Mono! ∗)
        val of2  :  α  →  α  →  α t
    end

module Binary  =
  struct
    let rcs  =  RCS.rename rcs_file "Tuple.Binary" ["Pairs"]

    type α t  =  α  ×  α
```

```
let arity _ = 2
let max_arity = 2

let of2 x y = (x, y)

let compare cmp (x1, y1) (x2, y2) =
  let cx = cmp x1 x2 in
  if cx ≠ 0 then
    cx
  else
    cmp y1 y2

let for_all p (x, y) = p x ∧ p y

let map f (x, y) = (f x, f y)
let iter f (x, y) = f x; f y
let fold_left f init (x, y) = f (f init x) y
let fold_right f (x, y) init = f x (f y init)
let fold_left_internal f (x, y) = f x y
let fold_right_internal f (x, y) = f x y

exception Mismatched_arity
let map2 f (x1, y1) (x2, y2) = (f x1 x2, f y1 y2)

let split ((x1, x2), (y1, y2)) = ((x1, y1), (x2, y2))

let product (lx, ly) =
  Product.list2 (fun x y → (x, y)) lx ly
let product_fold f (lx, ly) init =
  Product.fold2 (fun x y → f (x, y)) lx ly init

let power l = product (l, l)
let power_fold f l = product_fold f (l, l)
```

In the special case of binary fusions, the implementation is very concise.

```
type α graded = α list array

let fuse2 f set (i, j) acc =
  if i = j then
    List.fold_right (fun (x, y) → f x y) (choose2 set.(pred i)) acc
  else
    Product.fold2 f set.(pred i) set.(pred j) acc

let graded_sym_power_fold rank f set acc =
  let max_rank = Array.length set in
  List.fold_right (fuse2 (fun x y → f (of2 x y)) set)
    (Partition.pairs rank 1 max_rank) acc

let graded_sym_power rank set =
  graded_sym_power_fold rank (fun pair acc → pair :: acc) set []

let to_list (x, y) = [x; y]
let of2_kludge = of2

exception No_termination
end
```

2.2.3   Triples

```
module type Ternary =
    sig
      include Mono
      val of3 : α → α → α → α t
    end

module Ternary =
  struct
    let rcs = RCS.rename rcs_file "Tuple.Ternary" ["Triples"]

    type α t = α × α × α

    let arity _ = 3
    let max_arity = 3

    let of3 x y z = (x, y, z)

    let compare cmp (x1, y1, z1) (x2, y2, z2) =
      let cx = cmp x1 x2 in
      if cx ≠ 0 then
        cx
      else
        let cy = cmp y1 y2 in
        if cy ≠ 0 then
          cy
        else
          cmp z1 z2

    let for_all p (x, y, z) = p x ∧ p y ∧ p z

    let map f (x, y, z) = (f x, f y, f z)
    let iter f (x, y, z) = f x; f y; f z
    let fold_left f init (x, y, z) = f (f (f init x) y) z
    let fold_right f (x, y, z) init = f x (f y (f z init))
    let fold_left_internal f (x, y, z) = f (f x y) z
    let fold_right_internal f (x, y, z) = f x (f y z)

    exception Mismatched_arity
    let map2 f (x1, y1, z1) (x2, y2, z2) = (f x1 x2, f y1 y2, f z1 z2)

    let split ((x1, x2), (y1, y2), (z1, z2)) = ((x1, y1, z1), (x2, y2, z2))

    let product (lx, ly, lz) =
      Product.list3 (fun x y z → (x, y, z)) lx ly lz
    let product_fold f (lx, ly, lz) init =
      Product.fold3 (fun x y z → f (x, y, z)) lx ly lz init

    let power l = product (l, l, l)
    let power_fold f l = product_fold f (l, l, l)

    type α graded = α list array

    let fuse3 f set (i, j, k) acc =
      if i = j then begin
        if j = k then
```

$List.fold\_right$ (fun $(x,\ y,\ z)\ \rightarrow\ f\ x\ y\ z$) ($choose3\ set.(pred\ i)$) acc
 else
  $Product.fold2$ (fun $(x,\ y)\ z\ \rightarrow\ f\ x\ y\ z$)
   ($choose2\ set.(pred\ i)$) $set.(pred\ k)\ acc$
 end else begin
  if $j\ =\ k$ then
   $Product.fold2$ (fun $x\ (y,\ z)\ \rightarrow\ f\ x\ y\ z$)
    $set.(pred\ i)$ ($choose2\ set.(pred\ j)$) acc
  else
   $Product.fold3$ (fun $x\ y\ z\ \rightarrow\ f\ x\ y\ z$)
    $set.(pred\ i)\ set.(pred\ j)\ set.(pred\ k)\ acc$
 end

let $graded\_sym\_power\_fold\ rank\ f\ set\ acc\ =$
 let $max\_rank\ =\ Array.length\ set$ in
 $List.fold\_right$ ($fuse3$ (fun $x\ y\ z\ \rightarrow\ f\ (of3\ x\ y\ z)$) set)
  ($Partition.triples\ rank\ 1\ max\_rank$) acc

let $graded\_sym\_power\ rank\ set\ =$
 $graded\_sym\_power\_fold\ rank$ (fun $pair\ acc\ \rightarrow\ pair\ ::\ acc$) $set\ [\,]$

let $of2\_kludge\ \_\ =\ failwith$ `"Tuple.Ternary.of2_kludge"`

let $to\_list\ (x,\ y,\ z)\ =\ [x;\ y;\ z]$
end

2.2.4   Pairs and Triples

type $\alpha\ pair\_or\_triple\ =\ T2$ of $\alpha\ \times\ \alpha\ \mid\ T3$ of $\alpha\ \times\ \alpha\ \times\alpha$
module type $Mixed23\ =$
 sig
  include $Poly$
  val $of2\ :\ \alpha\ \rightarrow\ \alpha\ \rightarrow\ \alpha\ t$
  val $of3\ :\ \alpha\ \rightarrow\ \alpha\ \rightarrow\ \alpha\ \rightarrow\ \alpha\ t$
 end
module $Mixed23\ =$
 struct
  let $rcs\ =\ RCS.rename\ rcs\_file$ `"Tuple.Mixed23"`
   [`"Mixed␣pairs␣and␣triples"`]

  type $\alpha\ t\ =\ \alpha\ pair\_or\_triple$

  let $arity\ =$ function
   | $T2\ \_\ \rightarrow\ 2$
   | $T3\ \_\ \rightarrow\ 3$
  let $max\_arity\ =\ 3$

  let $of2\ x\ y\ =\ T2\ (x,\ y)$
  let $of3\ x\ y\ z\ =\ T3\ (x,\ y,\ z)$

  let $compare\ cmp\ m1\ m2\ =$
   match $m1,\ m2$ with

```
    | T2 _,  T3 _  →  − 1
    | T3 _,  T2 _  →  1
    | T2 (x1,  y1),  T2 (x2,  y2)  →
        let cx  =  cmp x1 x2 in
        if cx  ≠  0 then
          cx
        else
          cmp y1 y2
    | T3 (x1,  y1,  z1 ),  T3 (x2,  y2,  z2 )  →
        let cx  =  cmp x1 x2 in
        if cx  ≠  0 then
          cx
        else
          let cy  =  cmp y1 y2 in
          if cy  ≠  0 then
            cy
          else
            cmp z1 z2

let for_all p  =  function
  | T2 (x, y)  →  p x  ∧  p y
  | T3 (x, y, z)  →  p x  ∧  p y  ∧  p z

let map f  =  function
  | T2 (x, y)  →  T2 (f x, f y)
  | T3 (x, y, z)  →  T3 (f x, f y, f z)

let iter f  =  function
  | T2 (x, y)  →  f x; f y
  | T3 (x, y, z)  →  f x; f y; f z

let fold_left f init  =  function
  | T2 (x, y)  →  f (f init x) y
  | T3 (x, y, z)  →  f (f (f init x) y) z

let fold_right f m init  =
  match m with
  | T2 (x, y)  →  f x (f y init)
  | T3 (x, y, z)  →  f x (f y (f z init))

let fold_left_internal f m  =
  match m with
  | T2 (x, y)  →  f x y
  | T3 (x, y, z)  →  f (f x y) z

let fold_right_internal f m  =
  match m with
  | T2 (x, y)  →  f x y
  | T3 (x, y, z)  →  f x (f y z)

exception Mismatched_arity
let map2 f m1 m2  =
  match m1, m2 with
  | T2 (x1, y1), T2 (x2, y2)  →  T2 (f x1 x2, f y1 y2)
```

```
        |  T3 (x1, y1, z1), T3 (x2, y2, z2)  →  T3 (f x1 x2, f y1 y2, f z1 z2)
        |  T2 _, T3 _  |  T3 _, T2 _  →  raise Mismatched_arity
    let split  =  function
        |  T2 ((x1, x2), (y1, y2))  →  (T2 (x1, y1), T2 (x2, y2))
        |  T3 ((x1, x2), (y1, y2), (z1, z2))  →  (T3 (x1, y1, z1), T3 (x2, y2, z2))

    let product  =  function
        |  T2 (lx, ly)  →  Product.list2 (fun x y  →  T2 (x, y)) lx ly
        |  T3 (lx, ly, lz)  →  Product.list3 (fun x y z  →  T3 (x, y, z)) lx ly lz
    let product_fold f m init  =
        match m with
        |  T2 (lx, ly)  →  Product.fold2 (fun x y  →  f (T2 (x, y))) lx ly init
        |  T3 (lx, ly, lz)  →
            Product.fold3 (fun x y z  →  f (T3 (x, y, z))) lx ly lz init

    exception No_termination

    let power_fold f l init  =
        product_fold f (T2 (l, l)) (product_fold f (T3 (l, l, l)) init)
    let power l  =
        power_fold (fun m acc  →  m :: acc) l []

    type α graded  =  α list array

    let graded_sym_power_fold rank f set acc  =
        let max_rank  =  Array.length set in
        List.fold_right (Binary.fuse2 (fun x y  →  f (of2 x y)) set)
            (Partition.pairs rank 1 max_rank)
            (List.fold_right (Ternary.fuse3 (fun x y z  →  f (of3 x y z)) set)
                (Partition.triples rank 1 max_rank) acc)

    let graded_sym_power rank set  =
        graded_sym_power_fold rank (fun pair acc  →  pair :: acc) set []

    let to_list  =  function
        |  T2 (x, y)  →  [x; y]
        |  T3 (x, y, z)  →  [x; y; z]

    let of2_kludge  =  of2

end
```

2.2.5   … and All The Rest

```
module type Nary  =
    sig
        include Poly
        val of2  :  α  →  α  →  α t
        val of3  :  α  →  α  →  α  →  α t
        val of_list  :  α list →  α t
    end
module Nary (A  :  sig val max_arity  :  int end)  =
    struct
```

```
let rcs = RCS.rename rcs_file "Tuple.Nary"
    ["Tupels␣of␣indefinite␣arity"]

type α t = α × α list

let arity (_, y) = succ (List.length y)
let max_arity = A.max_arity

let of2 x y = (x, [y])
let of3 x y z = (x, [y; z])

let of_list = function
  | x :: y → (x, y)
  | [] → invalid_arg "Tuple.Nary.of_list:␣empty"

let compare cmp (x1, y1) (x2, y2) =
  let c = cmp x1 x2 in
  if c ≠ 0 then
    c
  else
    ThoList.compare ˜cmp y1 y2

let for_all p (x, y) = p x ∧ List.for_all p y

let map f (x, y) = (f x, List.map f y)
let iter f (x, y) = f x; List.iter f y
let fold_left f init (x, y) = List.fold_left f (f init x) y
let fold_right f (x, y) init = f x (List.fold_right f y init)
let fold_left_internal f (x, y) = List.fold_left f x y
let fold_right_internal f (x, y) =
  match List.rev y with
  | [] → x
  | y0 :: y_sans_y0 →
      f x (List.fold_right f (List.rev y_sans_y0) y0)

exception Mismatched_arity
let map2 f (x1, y1) (x2, y2) =
  try (f x1 x2, List.map2 f y1 y2) with
  | Invalid_argument _ → raise Mismatched_arity

let split ((x1, x2), y12) =
  let y1, y2 = List.split y12 in
  ((x1, y1), (x2, y2))

let product (xl, yl) =
  Product.list (function
    | x :: y → (x, y)
    | [] → failwith "Tuple.Nary.product") (xl :: yl)
let product_fold f (xl, yl) init =
  Product.fold (function
    | x :: y → f (x, y)
    | [] → failwith "Tuple.Nary.product_fold") (xl :: yl) init

let bounded_power_fold f l init =
  List.fold_right (fun n → product_fold f (l, ThoList.clone (pred n) l))
    (ThoList.range 2 A.max_arity) init
```

```
let bounded_power l =
  bounded_power_fold (fun t acc → t :: acc) l []

exception No_termination
let unbounded_power_fold f l init = raise No_termination
let unbounded_power l = raise No_termination

let power_fold, power =
  if A.max_arity > 0 then
    (bounded_power_fold, bounded_power)
  else
    (unbounded_power_fold, unbounded_power)

type α graded = α list array

let fuse_n f set partition acc =
  let choose (n, r) =
    Printf.printf "chose:␣n=%d␣r=%d␣len=%d\n"
      n r (List.length set.(pred r));
    Combinatorics.choose n set.(pred r) in
  Product.fold (fun wfs → f (List.concat wfs))
    (List.map choose (ThoList.classify partition)) acc

let fuse_n f set partition acc =
  let choose (n, r) = Combinatorics.choose n set.(pred r) in
  Product.fold (fun wfs → f (List.concat wfs))
    (List.map choose (ThoList.classify partition)) acc
```

> ⚠ *graded_sym_power_fold* is well defined for unbounded arities as well: derive a reasonable replacement from *set*. The length of the flattened *set* is an upper limit, of course, but too pessimistic in most cases.

```
let graded_sym_power_fold rank f set acc =
  let max_rank = Array.length set in
  let degrees = ThoList.range 2 max_arity in
  let partitions =
    ThoList.flatmap
      (fun deg → Partition.tuples deg rank 1 max_rank) degrees in
  List.fold_right (fuse_n (fun wfs → f (of_list wfs)) set) partitions acc

let graded_sym_power rank set =
  graded_sym_power_fold rank (fun pair acc → pair :: acc) set []

let to_list (x, y) = x :: y
let of2_kludge = of2
```

end

```
module type Bound = sig val max_arity : int end
module Unbounded_Nary = Nary (struct let max_arity = −1 end)
```

—3—
Topologies

3.1   Interface of Topology

module type T =
  sig

partition is a collection of integers, with arity one larger than the arity of α *children* below. These arities can one fixed number corresponding to homogeneous tuples or a collection of tupes or lists.

    type *partition*

partitions n returns the union of all $[n_1; n_2; \ldots; n_d]$ with $1 \leq n_1 \leq n_2 \leq \ldots \leq n_d \leq \lfloor n/2 \rfloor$ and

$$\sum_{i=1}^{d} n_i = n \tag{3.1}$$

for d from 3 to $d_{\max}$, where $d_{\max}$ is a fixed number for each module implementating T. In particular, if type *partition* $=$ $int \times int \times int$, then *partitions* n returns all (n_1, n_2, n_3) with $n_1 \leq n_2 \leq n_3$ and $n_1 + n_2 + n_3 = n$.

    val *partitions* : $int \to$ *partition list*

A (poly)tuple as implemented by the modules in *Tuple*:

    type α *children*

keystones externals returns all keystones for the amplitude with external states *externals* in the vanilla scalar theory with a

$$\sum_{3 \leq k \leq d_{\max}} \lambda_k \phi^k \tag{3.2}$$

interaction. One factor of the products is factorized. In particular, if

    type α *children* $=$ α *Tuple.Binary.t* $=$ $\alpha \times \alpha$,

then *keystones externals* returns all keystones for the amplitude with external states *externals* in the vanilla scalar $\lambda\phi^3$-theory.

    val *keystones* : α *list* $\to$ (α *list* $\times$ α *list children list*) *list*

The maximal depth of subtrees for a given number of external lines.

     val *max_subtree* : *int* → *int*

Only for diagnostics:

     val *inspect_partition* : *partition* → *int list*
     val *rcs* : *RCS.t*
  end

module *Binary* : *T* with type α *children* = α *Tuple.Binary.t*
module *Ternary* : *T* with type α *children* = α *Tuple.Ternary.t*
module *Mixed23* : *T* with type α *children* = α *Tuple.Mixed23.t*
module *Nary* : functor (*B* : *Tuple.Bound*) →
  (*T* with type α *children* = α *Tuple.Nary(B).t*)

3.1.1 Diagnostics: Counting Diagrams and Factorizations for $\sum_n \lambda_n \phi^n$

The number of diagrams for many particles can easily exceed the range of native integers. Even if we can not calculate the corresponding amplitudes, we want to check combinatorical factors. Therefore we code a functor that can use arbitray implementations of integers.

module type *Integer* =
  sig
    type *t*
    val *zero* : *t*
    val *one* : *t*
    val (+) : *t* → *t* → *t*
    val (−) : *t* → *t* → *t*
    val (×) : *t* → *t* → *t*
    val (/) : *t* → *t* → *t*
    val *pred* : *t* → *t*
    val *succ* : *t* → *t*
    val (=) : *t* → *t* → *bool*
    val (≠) : *t* → *t* → *bool*
    val (<) : *t* → *t* → *bool*
    val (≤) : *t* → *t* → *bool*
    val (>) : *t* → *t* → *bool*
    val (≥) : *t* → *t* → *bool*
    val *of_int* : *int* → *t*
    val *to_int* : *t* → *int*
    val *to_string* : *t* → *string*
    val *compare* : *t* → *t* → *int*
    val *factorial* : *t* → *t*
  end

Of course, native integers will provide the fastest implementation:

module *Int* : *Integer*

module type *Count* =
  sig
    type *integer*

diagrams f d n returns the number of tree diagrams contributing to the *n*-point amplitude in vanilla scalar theory with

$$\sum_{3 \leq k \leq d \wedge f(k)} \lambda_k \phi^k \tag{3.3}$$

interaction. The default value of *f* returns true for all arguments.

val *diagrams* : ?*f* : (*integer* → *bool*) → *integer* → *integer* → *integer*
val *diagrams_via_keystones* : *integer* → *integer* → *integer*

$$\frac{1}{S(n_k, n - n_k)} \frac{1}{S(n_1, n_2, \ldots, n_k)} \binom{n_1 + n_2 + \ldots + n_k}{n_1, n_2, \ldots, n_k} \tag{3.4}$$

val *keystones* : *integer list* → *integer*

diagrams_via_keystones d n must produce the same results as *diagrams d n*. This is shown explicitly in tables 3.2, 3.3 and 3.4 for small values of *d* and *n*. The test program in appendix S can be used to verify this relation for larger values.

val *diagrams_per_keystone* : *integer* → *integer list* → *integer*

end

module *Count* : funct or (*I* : *Integer*) → *Count* with type *integer* = *I.t*

3.1.2   Emulating HELAC

We can also proceed á la [2].

module *Helac* : functor (*B* : *Tuple.Bound*) →
(*T* with type α *children* = α *Tuple.Nary*(*B*).*t*)

The following has never been tested, but it is no rocket science and should work anyway ...

module *Helac_Binary* : *T* with type α *children* = α *Tuple.Binary.t*

3.2   Implementation of Topology

let *rcs_file* = *RCS.parse* "Topology" ["Topologies"]
{ *RCS.revision* = "$Revision:␣6465␣$";
RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
RCS.author = "$Author:␣jr␣reuter␣$";
RCS.source
= "$URL:␣svn+ssh://jr␣reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s

module type *T* =
sig
type *partition*
val *partitions* : *int* → *partition list*

n	*partitions n*
4	(1,1,2)
5	(1,2,2)
6	(1,2,3), (2,2,2)
7	(1,3,3), (2,2,3)
8	(1,3,4), (2,2,4), (2,3,3)
9	(1,4,4), (2,3,4), (3,3,3)
10	(1,4,5), (2,3,5), (2,4,4), (3,3,4)
11	(1,5,5), (2,4,5), (3,3,5), (3,4,4)
12	(1,5,6), (2,4,6), (2,5,5), (3,3,6), (3,4,5), (4,4,4)
13	(1,6,6), (2,5,6), (3,4,6), (3,5,5), (4,4,5)
14	(1,6,7), (2,5,7), (2,6,6), (3,4,7), (3,5,6), (4,4,6), (4,5,5)
15	(1,7,7), (2,6,7), (3,5,7), (3,6,6), (4,4,7), (4,5,6), (5,5,5)
16	(1,7,8), (2,6,8), (2,7,7), (3,5,8), (3,6,7), (4,4,8), (4,5,7), (4,6,6), (5,5,6)

Table 3.1: *partitions n* for moderate values of n.

```
type α children
val keystones : α list → (α list × α list children list) list
val max_subtree : int → int
val inspect_partition : partition → int list
val rcs : RCS.t
end
```

3.2.1 Factorizing Diagrams for ϕ^3

```
module Binary =
  struct
    let rcs = RCS.rename rcs_file "Topology.Binary"
        ["phi**3␣topology"]

    type partition = int × int × int
    let inspect_partition (n1, n2, n3) = [n1; n2; n3]
```

One way [1] to lift the degeneracy is to select the vertex that is closest to the center (see table 3.1):

$$partitions : n \to \big\{(n_1, n_2, n_3) \,|\, n_1 + n_2 + n_3 = n \land n_1 \le n_2 \le n_3 \le \lfloor n/2 \rfloor \big\} \quad (3.5)$$

Other, less symmetric, approaches are possible. The simplest of these is: choose the vertex adjacent to a fixed external line [2]. They will be made available for comparison in the future.

An obvious consequence of $n_1 + n_2 + n_3 = n$ and $n_1 \le n_2 \le n_3$ is $n_1 \le \lfloor n/3 \rfloor$:

```
let rec partitions' n n1 =
  if n1 > n / 3 then
    []
  else
    List.map (fun (n2, n3) → (n1, n2, n3))
```
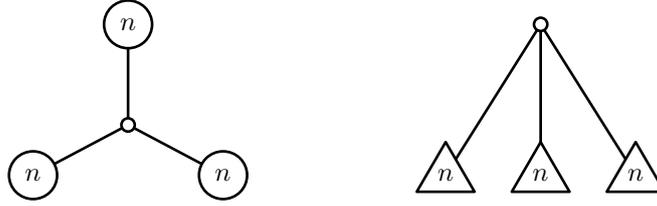
Figure 3.1: Topologies with a blatant three-fold permutation symmetry, if the number of external lines is a multiple of three
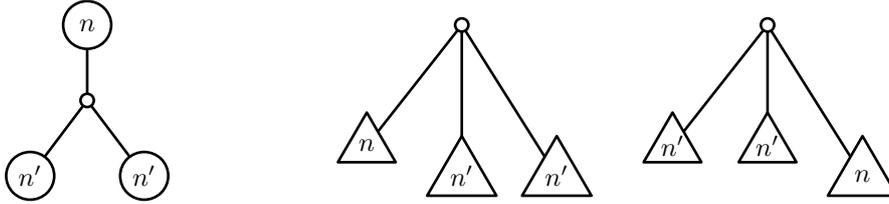
Figure 3.2: Topologies with a blatant two-fold symmetry.

$$(Partition.pairs\ (n\ -\ n1)\ n1\ (n\ /\ 2))\ @\ partitions'\ n\ (succ\ n1)$$

let $partitions\ n\ =\ partitions'\ n\ 1$

type $\alpha\ children\ =\ \alpha\ Tuple.Binary.t$

There remains one peculiar case, when the number of external lines is even and $n_3 = n_1 + n_2$ (cf. figure 3.3). Unfortunately, this reflection symmetry is not respected by the equivalence classes. E. g.

$$\{1\}\{2,3\}\{4,5,6\} \mapsto \big\{\{4\}\{5,6\}\{1,2,3\}; \{5\}\{4,6\}\{1,2,3\}; \{6\}\{4,5\}\{1,2,3\}\big\} \tag{3.6}$$

However, these reflections will always exchange the two halves and a representative can be chosen by requiring that one fixed momentum remains in one half. We choose to filter out the half of the partitions where the element p appears in the second half, i. e. the list of length $n3$.
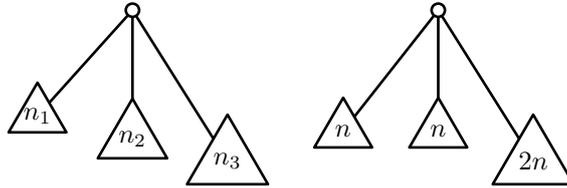
Figure 3.3: If $n_3 = n_1 + n_2$, the apparently asymmetric topologies on the left hand side have a non obvious two-fold symmetry, that exchanges the two halves. Therefore, the topologies on the right hand side have a four fold symmetry.

n	$(2n-5)!!$	$\sum N(n_1, n_2, n_3)$
4	3	$3 \cdot (1,1,2)$
5	15	$15 \cdot (1,2,2)$
6	105	$90 \cdot (1,2,3) + 15 \cdot (2,2,2)$
7	945	$630 \cdot (1,3,3) + 315 \cdot (2,2,3)$
8	10395	$6300 \cdot (1,3,4) + 1575 \cdot (2,2,4) + 2520 \cdot (2,3,3)$
9	135135	$70875 \cdot (1,4,4) + 56700 \cdot (2,3,4) + 7560 \cdot (3,3,3)$
10	2027025	$992250 \cdot (1,4,5) + 396900 \cdot (2,3,5)$
		$+\ 354375 \cdot (2,4,4) + 283500 \cdot (3,3,4)$
11	34459425	$15280650 \cdot (1,5,5) + 10914750 \cdot (2,4,5)$
		$+\ 4365900 \cdot (3,3,5) + 3898125 \cdot (3,4,4)$
12	654729075	$275051700 \cdot (1,5,6) + 98232750 \cdot (2,4,6)$
		$+\ 91683900 \cdot (2,5,5) + 39293100 \cdot (3,3,6)$
		$+\ 130977000 \cdot (3,4,5) + 19490625 \cdot (4,4,4)$

Table 3.2: Equation (3.9) for small values of n.

Finally, a closed expression for the number of Feynman diagrams in the equivalence class (n_1, n_2, n_3) is

$$N(n_1, n_2, n_3) = \frac{(n_1 + n_2 + n_3)!}{S(n_1, n_2, n_3)} \prod_{i=1}^{3} \frac{(2n_i - 3)!!}{n_i!} \qquad (3.7)$$

where the symmetry factor from the above arguments is

$$S(n_1, n_2, n_3) = \begin{cases} 3! & \text{for } n_1 = n_2 = n_3 \\ 2 \cdot 2 & \text{for } n_3 = 2n_1 = 2n_2 \\ 2 & \text{for } n_1 = n_2 \vee n_2 = n_3 \\ 2 & \text{for } n_1 + n_2 = n_3 \end{cases} \qquad (3.8)$$

Indeed, the sum of all Feynman diagrams

$$\sum_{\substack{n_1 + n_2 + n_3 = n \\ 1 \le n_1 \le n_2 \le n_3 \le \lfloor n/2 \rfloor}} N(n_1, n_2, n_3) = (2n - 5)!! \qquad (3.9)$$

can be checked numerically for large values of $n = n_1 + n_2 + n_3$, verifying the symmetry factor (see table 3.2).

P. M. claims to have seen similar formulae in the context of Young tableaux. That's a good occasion to read the new edition of Howard's book . . .

Return a list of all inequivalent partitions of the list *l* in three lists of length *n1*, *n2* and *n3*, respectively. Common first lists are factored. This is nothing more than a typedafe wrapper around *Combinatorics.factorized_keystones*.

```
exception Impossible of string
let tuple_of_list2 = function
  | [x1; x2] → Tuple.Binary.of2 x1 x2
  | _ → raise (Impossible "Topology.tuple_of_list")
```
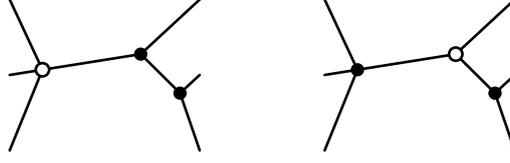
Figure 3.4: Degenerate $(1, 1, 1, 3)$ and $(1, 2, 3)$.

```
let keystone (n1, n2, n3) l =
  List.map (fun (p1, p23) → (p1, List.rev_map tuple_of_list2 p23))
    (Combinatorics.factorized_keystones [n1; n2; n3] l)

let keystones l =
  ThoList.flatmap (fun n123 → keystone n123 l) (partitions (List.length l))

let max_subtree n = n / 2

end
```

3.2.2   Factorizing Diagrams for $\sum_n \lambda_n \phi^n$

Mixed ϕ^n adds new degeneracies, as in figure 3.4. They appear if and only if one part takes exactly half of the external lines and can relate central vertices of different arity.

```
module Nary (B : Tuple.Bound) =
  struct
    let rcs = RCS.rename rcs_file "Topology.Nary"
        ["phi**n␣topology"]

    type partition = int list
    let inspect_partition p = p

    let partition d sum =
      Partition.tuples d sum 1 (sum / 2)

    let rec partitions′ d sum =
      if d < 3 then
        []
      else
        partition d sum @ partitions′ (pred d) sum

    let partitions sum = partitions′ (succ B.max_arity) sum


    module Tuple = Tuple.Nary(B)
    type α children = α Tuple.t

    let keystones′ l =
      let n = List.length l in
      ThoList.flatmap (fun p → Combinatorics.factorized_keystones p l)
```

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n	$\sum$	$\sum$
4	4	$1 \cdot (1,1,1,1) + 3 \cdot (1,1,2)$
5	25	$10 \cdot (1,1,1,2) + 15 \cdot (1,2,2)$
6	220	$40 \cdot (1,1,1,3) + 45 \cdot (1,1,2,2) + 120 \cdot (1,2,3) + 15 \cdot (2,2,2)$
7	2485	$840 \cdot (1,1,2,3) + 105 \cdot (1,2,2,2) + 1120 \cdot (1,3,3) + 420 \cdot (2,2,3)$
8	34300	$5250 \cdot (1,1,2,4) + 4480 \cdot (1,1,3,3) + 3360 \cdot (1,2,2,3)$ $+ 105 \cdot (2,2,2,2) + 14000 \cdot (1,3,4)$ $+ 2625 \cdot (2,2,4) + 4480 \cdot (2,3,3)$
9	559405	$126000 \cdot (1,1,3,4) + 47250 \cdot (1,2,2,4) + 40320 \cdot (1,2,3,3)$ $+ 5040 \cdot (2,2,2,3) + 196875 \cdot (1,4,4)$ $+ 126000 \cdot (2,3,4) + 17920 \cdot (3,3,3)$
10	10525900	$1108800 \cdot (1,1,3,5) + 984375 \cdot (1,1,4,4) + 415800 \cdot (1,2,2,5)$ $+ 1260000 \cdot (1,2,3,4) + 179200 \cdot (1,3,3,3) + 78750 \cdot (2,2,2,4)$ $+ 100800 \cdot (2,2,3,3) + 3465000 \cdot (1,4,5) + 1108800 \cdot (2,3,5)$ $+ 984375 \cdot (2,4,4) + 840000 \cdot (3,3,4)$

Table 3.3: $\mathcal{L} = \lambda_3 \phi^3 + \lambda_4 \phi^4$

n	$\sum$	$\sum$
4	4	$1 \cdot (1,1,1,1) + 3 \cdot (1,1,2)$
5	26	$1 \cdot (1,1,1,1,1) + 10 \cdot (1,1,1,2) + 15 \cdot (1,2,2)$
6	236	$1 \cdot (1,1,1,1,1,1) + 15 \cdot (1,1,1,1,2) + 40 \cdot (1,1,1,3)$ $+ 45 \cdot (1,1,2,2) + 120 \cdot (1,2,3) + 15 \cdot (2,2,2)$
7	2751	$21 \cdot (1,1,1,1,1,2) + 140 \cdot (1,1,1,1,3) + 105 \cdot (1,1,1,2,2)$ $+ 840 \cdot (1,1,2,3) + 105 \cdot (1,2,2,2) + 1120 \cdot (1,3,3) + 420 \cdot (2,2,3)$
8	39179	$224 \cdot (1,1,1,1,1,3) + 210 \cdot (1,1,1,1,2,2) + 910 \cdot (1,1,1,1,4)$ $+ 2240 \cdot (1,1,1,2,3) + 420 \cdot (1,1,2,2,2) + 5460 \cdot (1,1,2,4)$ $+ 4480 \cdot (1,1,3,3) + 3360 \cdot (1,2,2,3) + 105 \cdot (2,2,2,2)$ $+ 14560 \cdot (1,3,4) + 2730 \cdot (2,2,4) + 4480 \cdot (2,3,3)$

Table 3.4: $\mathcal{L} = \lambda_3 \phi^3 + \lambda_4 \phi^4 + \lambda_5 \phi^5 + \lambda_6 \phi^6$

```
          (partitions n)

      let keystones l =
        List.map (fun (bra, kets) → (bra, List.map Tuple.of_list kets))
          (keystones' l)

      let max_subtree n = n / 2

  end

module Nary4 = Nary (struct let max_arity = 3 end)
```

3.2.3   Factorizing Diagrams for ϕ^4

```
module Ternary =
  struct
      let rcs = RCS.rename rcs_file "Topology.Ternary"
          ["phi**4␣topology"]
      let rcs = rcs_file
      type partition = int × int × int × int
      let inspect_partition (n1, n2, n3, n4) = [n1; n2; n3; n4]
      type α children = α Tuple.Ternary.t
      let collect4 acc = function
        | [x; y; z; u] → (x, y, z, u) :: acc
        | _ → acc
      let partitions n =
        List.fold_left collect4 [] (Nary4.partitions n)
      let collect3 acc = function
        | [x; y; z] → Tuple.Ternary.of3 x y z :: acc
        | _ → acc
      let keystones l =
        List.map (fun (bra, kets) → (bra, List.fold_left collect3 [] kets))
          (Nary4.keystones' l)
      let max_subtree = Nary4.max_subtree
  end
```

3.2.4   Factorizing Diagrams for $\phi^3 + \phi^4$

```
module Mixed23 =
  struct
      let rcs = RCS.rename rcs_file "Topology.Mixed23"
          ["phi**3␣+␣phi**4␣topology"]
      type partition =
        | P3 of int × int × int
        | P4 of int × int × int × int
      let inspect_partition = function
        | P3 (n1, n2, n3) → [n1; n2; n3]
        | P4 (n1, n2, n3, n4) → [n1; n2; n3; n4]
      type α children = α Tuple.Mixed23.t
      let collect34 acc = function
```

```
      | [x; y; z]    → P3 (x, y, z) :: acc
      | [x; y; z; u] → P4 (x, y, z, u) :: acc
      | _           → acc
  let partitions n =
    List.fold_left collect34 [] (Nary4.partitions n)
  let collect23 acc = function
      | [x; y]    → Tuple.Mixed23.of2 x y :: acc
      | [x; y; z] → Tuple.Mixed23.of3 x y z :: acc
      | _         → acc
  let keystones l =
    List.map (fun (bra, kets) → (bra, List.fold_left collect23 [] kets))
      (Nary4.keystones′ l)
  let max_subtree = Nary4.max_subtree
end
```

3.2.5 Diagnostics: Counting Diagrams and Factorizations for $\sum_n \lambda_n \phi^n$

```
module type Integer =
  sig
    type t
    val zero : t
    val one  : t
    val ( + ) : t → t → t
    val ( − ) : t → t → t
    val ( × ) : t → t → t
    val ( / ) : t → t → t
    val pred : t → t
    val succ : t → t
    val ( = )  : t → t → bool
    val ( ≠ )  : t → t → bool
    val ( < )  : t → t → bool
    val ( ≤ )  : t → t → bool
    val ( > )  : t → t → bool
    val ( ≥ )  : t → t → bool
    val of_int : int → t
    val to_int : t → int
    val to_string : t → string
    val compare : t → t → int
    val factorial : t → t
  end
```

O'Caml's native integers suffice for all applications, but in appendix S, we want
to use big integers for numeric checks in high orders:

```
module Int : Integer =
  struct
    type t = int
    let zero = 0
    let one  = 1
```

```
    let ( + ) = ( + )
    let ( − ) = ( − )
    let ( × ) = ( × )
    let ( / ) = ( / )
    let pred = pred
    let succ = succ
    let ( = ) = ( = )
    let ( ≠ ) = ( ≠ )
    let ( < ) = ( < )
    let ( ≤ ) = ( ≤ )
    let ( > ) = ( > )
    let ( ≥ ) = ( ≥ )
    let of_int n = n
    let to_int n = n
    let to_string = string_of_int
    let compare = compare
    let factorial = Combinatorics.factorial
  end

module type Count =
  sig
    type integer
    val diagrams : ?f : (integer → bool) → integer → integer → integer
    val diagrams_via_keystones : integer → integer → integer
    val keystones : integer list → integer
    val diagrams_per_keystone : integer → integer list → integer
  end

module Count (I : Integer) =
  struct
    let rcs = rcs_file
    let description = ["(still␣inoperational)␣phi^n␣topology"]

    type integer = I.t
    open I
    let two = of_int 2
    let three = of_int 3
```

If $I.t$ is an abstract datatype, the polymorphic *Pervasives.min* can fail. Provide our own version using the specific comparison "$(\leq)$".

```
    let min x y =
      if x ≤ y then
        x
      else
        y
```

Counting Diagrams for $\sum_n \lambda_n \phi^n$

Classes of diagrams are defined by the number of vertices and their degrees. We could use fixed size arrays, but we will use a map instead. For efficiency, we also maintain the number of external lines and the total number of propagators.

module *IMap* $=$ *Map.Make* (struct type $t = integer$ let *compare* $=$ *compare* end)

type *diagram_class* $= \{$ *ext* $:$ *integer*; *prop* $:$ *integer*; *v* $:$ *integer IMap.t* $\}$

The numbers of external lines, propagators and vertices are determined by the degrees and multiplicities of vertices:

$$E(\{n_3, n_4, \ldots\}) = 2 + \sum_{d=3}^{\infty} (d-2)n_d \tag{3.10a}$$

$$P(\{n_3, n_4, \ldots\}) = \sum_{d=3}^{\infty} n_d - 1 = V(\{n_3, n_4, \ldots\}) - 1 \tag{3.10b}$$

$$V(\{n_3, n_4, \ldots\}) = \sum_{d=3}^{\infty} n_d \tag{3.10c}$$

let *num_ext* v $=$
  List.fold_left (fun *sum* (d, n) $\rightarrow$ *sum* $+$ $(d - two)$ $\times$ n) *two* v

let *num_prop* v $=$
  List.fold_left (fun *sum* $(\_, n)$ $\rightarrow$ *sum* $+$ n) $(zero - one)$ v

The sum of all vertex degrees must be equal to the number of propagator end points. This can be verified easily:

$$2P(\{n_3, n_4, \ldots\}) + E(\{n_3, n_4, \ldots\}) = \sum_{d=3}^{\infty} dn_d \tag{3.11}$$

let *add_degree map* (d, n) $=$
  if $d < three$ then
    invalid_arg `"add_degree:␣d␣<␣3"`
  else if $n < zero$ then
    invalid_arg `"add_degree:␣n␣<=␣0"`
  else if $n = zero$ then
    map
  else
    IMap.add d n *map*

let *create_class* v $=$
  $\{$ *ext* $=$ *num_ext* v;
    prop $=$ *num_prop* v;
    v $=$ *List.fold_left* *add_degree* *IMap.empty* v $\}$

let *multiplicity* cl d $=$
  if $d \geq three$ then
    try
      IMap.find d *cl.v*
    with
    $|$ *Not_found* $\rightarrow$ *zero*
  else
    invalid_arg `"multiplicity:␣d␣<␣3"`

Remove one vertex of degree d, maintaining the invariants. Raises *Zero* if all vertices of degree d are exhausted.

```
exception Zero

let remove cl d  =
  let n  =  pred (multiplicity cl d) in
  if n  <  zero then
    raise Zero
  else
    { ext  =  cl.ext  −  (d  −  two);
      prop  =  pred cl.prop;
      v  =  if n  =  zero then
        IMap.remove d cl.v
      else
        IMap.add d n cl.v }
```

Add one vertex of degree d, maintaining the invariants.

```
let add cl d  =
  { ext  =  cl.ext  +  (d  −  two);
    prop  =  succ cl.prop;
    v  =  IMap.add d (succ (multiplicity cl d)) cl.v }
```

Count the number of diagrams. Any diagram can be obtained recursively either from a diagram with one ternary vertex less by insertion if a ternary vertex in an internal or external propagator or from a diagram with a higher order vertex that has its degree reduced by one:

$$
\begin{aligned}
D(\{n_3, n_4, \ldots\}) = \\
(P(\{n_3 - 1, n_4, \ldots\}) + E(\{n_3 - 1, n_4, \ldots\})) \, D(\{n_3 - 1, n_4, \ldots\}) \\
+ \sum_{d=4}^{\infty} (n_{d-1} + 1) D(\{n_3, n_4, \ldots, n_{d-1} + 1, n_d - 1, \ldots\}) \quad (3.12)
\end{aligned}
$$

```
let rec class_size cl  =
  if cl.ext  =  two  ∨  cl.prop  =  zero then
    one
  else
    IMap.fold (fun d _ s  →  class_size_n cl d  +  s) cl.v (class_size_3 cl)
```

Purely ternary vertices recurse among themselves:

```
and class_size_3 cl  =
  try
    let d′  =  remove cl three in
    (d′.ext  +  d′.prop)  ×  class_size d′
  with
  | Zero  →  zero
```

Vertices of higher degree recurse one step towards lower degrees:

```
and class_size_n cl d  =
  if d  >  three then begin
    try
      let d′  =  pred d in
      let cl′  =  add (remove cl d) d′ in
```

```
        multiplicity cl′ d′  ×  class_size cl′
      with
      | Zero  →  zero
    end else
      zero
```

Find all $\{n_3, n_4, \ldots, n_d\}$ with

$$E(\{n_3, n_4, \ldots, n_d\}) - 2 = \sum_{i=3}^{c} l(i-2)n_i = sum \qquad (3.13)$$

The implementation is a variant of *tuples* above.

```
  let rec distribute_degrees′ d sum  =
    if d  <  three then
      invalid_arg "distribute_degrees"
    else if d  =  three then
      [[(d,  sum)]]
    else
      distribute_degrees″ d sum (sum / (d  −  two))

  and distribute_degrees″ d sum n  =
    if n  <  zero then
      []
    else
      List.fold_left (fun ll l  →  ((d,  n)  ::  l)  ::  ll)
        (distribute_degrees″ d sum (pred n))
        (distribute_degrees′ (pred d) (sum  −  (d  −  two)  ×  n))
```

Actually, we need to find all $\{n_3, n_4, \ldots, n_d\}$ with

$$E(\{n_3, n_4, \ldots, n_d\}) = sum \qquad (3.14)$$

```
  let distribute_degrees d sum  =  distribute_degrees′ d (sum  −  two)
```

Finally we can count all diagrams by adding all possible ways of splitting the degrees of vertices. We can also count diagrams where *all* degrees satisfy a predicate f:

```
  let diagrams ?(f  =  fun _  →  true) deg n  =
    List.fold_left (fun s d  →
      if List.for_all (fun (d′,  n′)  →  f d′  ∨  n′  =  zero) d then
        s  +  class_size (create_class d)
      else
        s)
      zero (distribute_degrees deg n)
```

The next two are duplicated from *ThoList* and *Combinatorics*, in order to use the specific comparison functions.

```
  let classify l  =
    let rec add_to_class a  =  function
      | []  →  [of_int 1,  a]
      | (n,  a′)  ::  rest  →
```

```
              if a = a' then
                (succ n, a) :: rest
              else
                (n, a') :: add_to_class a rest
        in
        let rec classify' cl = function
          | [] → cl
          | a :: rest → classify' (add_to_class a cl) rest
        in
        classify' [] l

    let permutation_symmetry l =
        List.fold_left (fun s (n, _) → factorial n × s) one (classify l)

    let symmetry l =
        let sum = List.fold_left (+) zero l in
        if List.exists (fun x → two × x = sum) l then
          two × permutation_symmetry l
        else
          permutation_symmetry l
```

The number of Feynman diagrams built of vertices with maximum degree $d_{\max}$ in a partition $N_{d,n} = \{n_1, n_2, \ldots, n_d\}$ with $n = n_1 + n_2 + \cdots + n_d$ and

$$\tilde{F}(d_{\max}, N_{d,n}) = \frac{n!}{|\mathcal{S}(N_{d,n})|\sigma(n_d, n)} \prod_{i=1}^{d} \frac{F(d_{\max}, n_i + 1)}{n_i!} \qquad (3.15)$$

with $|\mathcal{S}(N)|$ the size of the symmetric group of N, $\sigma(n, 2n) = 2$ and $\sigma(n, m) = 1$ otherwise.

```
    let keystones p =
        let sum = List.fold_left (+) zero p in
        List.fold_left (fun acc n → acc / (factorial n)) (factorial sum) p
          / symmetry p

    let diagrams_per_keystone deg p =
        List.fold_left (fun acc n → acc × diagrams deg (succ n)) one p
```

We must find

$$F(d_{\max}, n) = \sum_{d=3}^{d_{\max}} \sum_{\substack{N=\{n_1, n_2, \ldots, n_d\} \\ n_1 + n_2 + \cdots + n_d = n \\ 1 \le n_1 \le n_2 \le \cdots \le n_d \le \lfloor n/2 \rfloor}} \tilde{F}(d_{\max}, N) \qquad (3.16)$$

```
    let diagrams_via_keystones deg n =
        let module N = Nary (struct let max_arity = to_int (pred deg) end) in
        List.fold_left
          (fun acc p → acc + diagrams_per_keystone deg p × keystones p)
          zero (List.map (List.map of_int) (N.partitions (to_int n)))

end
```

3.2.6   Emulating HELAC

In [2], one leg is singled out:

```
module Helac (B : Tuple.Bound) =
  struct
    let rcs = RCS.rename rcs_file "Topology.Helac"
        ["phi**n␣topology,␣Helac␣style"]
    module Tuple = Tuple.Nary(B)

    type partition = int list
    let inspect_partition p = p

    let partition d sum =
      Partition.tuples d sum 1 (sum − d + 1)

    let rec partitions′ d sum =
      let d′ = pred d in
      if d′ < 2 then
        []
      else
        List.map (fun p → 1 :: p) (partition d′ (pred sum)) @ partitions′ d′ sum

    let partitions sum = partitions′ (succ B.max_arity) sum

    type α children = α Tuple.t

    let keystones′ l =
      match l with
      | [] → []
      | head :: tail →
          [([head],
              ThoList.flatmap (fun p → Combinatorics.partitions (List.tl p) tail)
                (partitions (List.length l)))]

    let keystones l =
      List.map (fun (bra, kets) → (bra, List.map Tuple.of_list kets))
        (keystones′ l)

    let max_subtree n = pred n
  end
```

The following is not tested, but it is no rocket science either . . .

```
module Helac_Binary =
  struct
    let rcs = RCS.rename rcs_file "Topology.Helac_Binary"
        ["phi**3␣topology,␣Helac␣style"]

    type partition = int × int × int
    let inspect_partition (n1, n2, n3) = [n1; n2; n3]

    let partitions sum =
      List.map (fun (n2, n3) → (1, n2, n3))
        (Partition.pairs (sum − 1) 1 (sum − 2))
```

```
type α children  =  α Tuple.Binary.t

let keystones′ l  =
  match l with
  | [] → []
  | head :: tail →
      [([head],
          ThoList.flatmap (fun (_, p2, _) → Combinatorics.split p2 tail)
            (partitions (List.length l)))]

let keystones l  =
  List.map (fun (bra, kets) →
    (bra, List.map (fun (x, y) → Tuple.Binary.of2 x y) kets))
    (keystones′ l)

let max_subtree n  =  pred n

end
```

—4—

DIRECTED ACYCLICAL GRAPHS

4.1 Interface of DAG

This datastructure describes large collections of trees with many shared nodes. The sharing of nodes is semantically irrelevant, but can turn a factorial complexity to exponential complexity. Note that DAG implements only a very specialized subset of Directed Acyclic Graphs (DAGs).

If $T(n, D)$ denotes the set of all binary trees with root n encoded in D, while

$$O(n, D) = \{(e_1, n_1, n_1'), \ldots, (e_k, n_k, n_k')\} \tag{4.1}$$

denotes the set of all *offspring* of n in D, and $\text{tree}(e, t, t')$ denotes the binary tree formed by joining the binary trees t and t' with the label e, then

$$T(n, D) = \big\{ \text{tree}(e_i, t_i, t_i') \,\big|\, (e_i, t_i, t_i') \in \{e_1\} \times T(n_1, D) \times T(n_1', D) \cup \ldots$$
$$\ldots \cup \{e_k\} \times T(n_k, D) \times T(n_k', D) \big\} \tag{4.2}$$

is the recursive definition of the binary trees encoded in D. It is obvious how this definitions translates to n-ary trees (including trees with mixed arity).

4.1.1 Forests

We require edges and nodes to be members of ordered sets. The sematics of *compare* are compatible with *Pervasives.compare*:

$$compare(x, y) = \begin{cases} -1 & \text{for } x < y \\ 0 & \text{for } x = y \\ 1 & \text{for } x > y \end{cases} \tag{4.3}$$

Note that this requirement does *not* exclude any trees. Even if we consider only topological equivalence classes with anonymous nodes, we can always construct a canonical labeling and order from the children of the nodes. However, if practical applications, we will often have more efficient labelings and orders at our disposal.

```
module type Ord  =
  sig
    type t
    val compare :  t  →  t  →  int
```

end

A forest F over a set of nodes and a set of edges is a map from the set of nodes N, to the direct product of the set of edges E and the power set 2^N of N augmented by a special element $\perp$ ("bottom").

$$F : N \rightarrow (E \times 2^N) \cup \{\perp\}$$
$$n \mapsto \begin{cases} (e, \{n'_1, n'_2, \ldots\}) \\ \perp \end{cases} \tag{4.4}$$

The nodes are ordered so that cycles can be detected

$$\forall n \in N : F(n) = (e, x) \Rightarrow \forall n' \in x : n > n' \tag{4.5}$$

A suitable function that exists for *all* forests is the depth of the tree beneath a node.

Nodes that are mapped to $\perp$ are called *leaf* nodes and nodes that do not appear in any $F(n)$ are called *root* nodes. There are as many trees in the forest as there are root nodes.

module type *Forest* $=$
  sig

    module *Nodes* : *Ord*
    type *node* $=$ *Nodes.t*
    type *edge*

A subset $X \subset 2^N$ of the powerset of the set of nodes. The members of X can be be characterized by a fixed number of members (e. g. two for binary trees, as in QED). We can also have mixed arities (e. g. two and three for QCD) or even arbitrary arities. However, in most cases, the members of X will have at least two members.

    type *children*

This type abbreviation and order allow to apply the *Set.Make* functor to $E \times X$.

    type t $=$ *edge* $\times$ *children*
    val *compare* : t $\rightarrow$ t $\rightarrow$ *int*

Test a predicate for *all* children.

    val *for_all* : (*node* $\rightarrow$ *bool*) $\rightarrow$ t $\rightarrow$ *bool*

fold f (_, *children*) *acc* will calculate

$$f(x_1, f(x_2, \cdots f(x_n, acc))) \tag{4.6}$$

where the *children* are $\{x_1, x_2, \ldots, x_n\}$. There are slightly more efficient alternatives for fixed arity (in particular binary), but we want to be general.

    val *fold* : (*node* $\rightarrow$ α $\rightarrow$ α) $\rightarrow$ t $\rightarrow$ α $\rightarrow$ α

  end

module *Forest* : functor (*PT* : *Tuple.Poly*) $\rightarrow$
  functor (*N* : *Ord*) $\rightarrow$ functor (*E* : *Ord*) $\rightarrow$
    Forest with module *Nodes* $=$ *N* and type *edge* $=$ *E.t*
    and type *node* $=$ *N.t* and type *children* $=$ *N.t PT.t*

4.1.2 DAGs

```
module type T =
  sig

    type node
    type edge
```

In the description of the function we assume for definiteness DAGs of binary trees with type *children* = *node* × *node*. However, we will also have implementations with type *children* = *node list* below.

Other possibilities include type *children* = *V3* of *node* × *node* | *V4* of *node* × *node* × *node*. There's probable never a need to use sets with logarithmic access, but it is easy to add.

```
    type children
    type t
```

The empty DAG.

```
    val empty : t
```

add_node n dag returns the DAG *dag* with the node *n*. If the node *n* already exists in *dag*, it is returned unchanged. Otherwise *n* is added without offspring.

```
    val add_node : node → t → t
```

add_offspring n (*e*, (*n1*, *n2*)) *dag* returns the DAG *dag* with the node *n* and its offspring *n1* and *n2* with edge label *e*. Each node can have an arbitrary number of offspring, but identical offspring are added only once. In order to prevent cycles, *add_offspring* requires both $n > n1$ and $n > n2$ in the given ordering. The nodes *n1* and *n2* are added as by *add_node*. NB: Adding all nodes *n1* and *n2*, even if they are sterile, is not strictly necessary for our applications. It even slows down the code by a few percent. But it is desirable for consistency and allows much more efficient *iter_nodes* and *fold_nodes* below.

```
    val add_offspring : node → edge × children → t → t
    exception Cycle
```

Just like *add_offspring*, but does not check for potential cycles.

```
    val add_offspring_unsafe : node → edge × children → t → t
```

is_node n dag returns true iff *n* is a node in *dag*.

```
    val is_node : node → t → bool
```

is_sterile n dag returns true iff *n* is a node in *dag* and boasts no offspring.

```
    val is_sterile : node → t → bool
```

is_offspring n (*e*, (*n1*, *n2*)) *dag* returns true iff *n1* and *n2* are offspring of *n* with label *e* in *dag*.

```
    val is_offspring : node → edge × children → t → bool
```

Note that the following functions can run into infinite recursion if the DAG given as argument contains cycles.

The usual functionals for processing all nodes (including sterile) . . .

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```
val iter_nodes  :  (node  →  unit)  →  t  →  unit
val map_nodes  :  (node  →  node)  →  t  →  t
val fold_nodes  :  (node  →  α  →  α)  →  t  →  α  →  α
```

... and all parent/offspring relations. Note that *map* requires *two* functions: one for the nodes and one for the edges and children. This is so because a change in the definition of node is *not* propagated automatically to where it is used as a child.

```
val iter  :  (node  →  edge  ×  children  →  unit)  →  t  →  unit
val map  :  (node  →  node)  →
    (node  →  edge  ×  children  →  edge  ×  children)  →  t  →  t
val fold  :  (node  →  edge  ×  children  →  α  →  α)  →  t  →  α  →  α
```

Note that in it's current incarnation, *fold add_offspring dag empty* copies *only* the fertile nodes, while *fold add_offspring dag (fold_nodes add_node dag empty)* includes sterile ones, as does *map* (fun $n → n$) (fun $n\ ec → ec$) *dag*.

Return the DAG as a list of lists.

```
val lists  :  t  →  (node  ×  (edge  ×  children)  list)  list
```

dependencies dag node returns a canonically sorted *Tree2.t* of all nodes reachable from *node*.

```
val dependencies  :  t  →  node  →  (node,  edge)  Tree2.t
```

harvest dag n roots returns the DAG *roots* enlarged by all nodes in *dag* reachable from *n*.

```
val harvest  :  t  →  node  →  t  →  t
```

harvest_list dag nlist returns the part of the DAG *dag* that is reachable from the nodes in *nlist*.

```
val harvest_list  :  t  →  node  list →  t
```

size dag returns the number of nodes in the DAG *dag*.

```
val size  :  t  →  int
```

eval f mul_edge mul_nodes add null unit root dag interprets the part of *dag* beneath *root* as an algebraic expression:

- each node is evaluated by $f :\ node\ →\ α$

- each set of children is evaluated by iterating the binary $mul\_nodes :\ α\ →\ γ\ →\ γ$ on the values of the nodes, starting from $unit : γ$

- each offspring relation $(node, (edge, children))$ is evaluated by applying $mul\_edge :\ node\ →\ edge\ →\ γ\ →\ δ$ to *node*, *edge* and the evaluation of *children*.

- all offspring relations of a *node* are combined by iterating the binary $add :\ δ\ →\ α\ →\ α$ starting from $null : α$

In our applications, we will always have $\alpha = \gamma = \delta$, but the more general type is useful for documenting the relationships. The memoizing variant *eval_memoized f mul_edge mul_nodes add null unit root dag* requires some overhead, but can be more efficient for complex operations.

> val *eval* : $(node \rightarrow \alpha) \rightarrow (node \rightarrow edge \rightarrow \gamma \rightarrow \delta) \rightarrow$
> $(\alpha \rightarrow \gamma \rightarrow \gamma) \rightarrow (\delta \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha \rightarrow \gamma \rightarrow node \rightarrow t \rightarrow \alpha$
> val *eval_memoized* : $(node \rightarrow \alpha) \rightarrow (node \rightarrow edge \rightarrow \gamma \rightarrow \delta) \rightarrow$
> $(\alpha \rightarrow \gamma \rightarrow \gamma) \rightarrow (\delta \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha \rightarrow \gamma \rightarrow node \rightarrow t \rightarrow \alpha$

forest root dag expands the *dag* beneath *root* into the equivalent list of trees *Tree.t*. *children* are represented as list of nodes.

A sterile node *n* is represented as *Tree.Leaf* $((n, \ None), \ n)$, cf. page 874. There might be a better way, but we need to change the interface and semantics of *Tree* for this.

> val *forest* : $node \rightarrow t \rightarrow (node \times edge \ option, node) \ Tree.t \ list$
> val *forest_memoized* : $node \rightarrow t \rightarrow (node \times edge \ option, node) \ Tree.t \ list$

count_trees n dag returns the number of trees with root *n* encoded in the DAG *dag*, i.e. $|T(n, D)|$. NB: the current implementation is very naive and can take a *very* long time for moderately sized DAGs that encode a large set of trees.

> val *count_trees* : $node \rightarrow t \rightarrow int$
>
> val *rcs* : $RCS.t$
> end

module *Make* $(F \ : \ Forest)$:
  T with type $node \ = \ F.node$ and type $edge \ = \ F.edge$
  and type $children \ = \ F.children$

4.1.3   Graded Sets, Forests & DAGs

A graded ordered[1] set is an ordered set with a map into another ordered set (often the non-negative integers). The grading does not necessarily respect the ordering.

module type $Graded\_Ord \ =$
  sig
    include Ord
    module $G \ : \ Ord$
    val *rank* : $t \rightarrow G.t$
  end

For all ordered sets, there are two canonical gradings: a *Chaotic* grading that assigns the same rank (e.g. *unit*) to all elements and the *Discrete* grading that uses the identity map as grading.

module type $Grader \ =$ functor $(O \ : \ Ord) \rightarrow Graded\_Ord$ with type $t \ = \ O.t$
module *Chaotic* : $Grader$
module *Discrete* : $Grader$

A graded forest is just a forest in which the nodes form a graded ordered set.

---

[1]We don't appear to have use for graded unordered sets.

There doesn't appear to be a nice syntax for avoiding the repetition here. Fortunately, the signature is short ...

```
module type Graded_Forest =
  sig
    module Nodes : Graded_Ord
    type node = Nodes.t
    type edge
    type children
    type t = edge × children
    val compare : t → t → int
    val for_all : (node → bool) → t → bool
    val fold : (node → α → α) → t → α → α
  end

module type Forest_Grader = functor (G : Grader) → functor (F : Forest) →

  Graded_Forest with type Nodes.t = F.node
  and type node = F.node
  and type edge = F.edge
  and type children = F.children
  and type t = F.t

module Grade_Forest : Forest_Grader
```

Finally, a graded DAG is a DAG in which the nodes form a graded ordered set and the subsets with a given rank can be accessed cheaply.

```
module type Graded =
  sig
    include T
    type rank
    val rank : node → rank
    val ranks : t → rank list
    val min_max_rank : t → rank × rank
    val ranked : rank → t → node list
  end

module Graded (F : Graded_Forest) :
    Graded with type node = F.node and type edge = F.edge
    and type children = F.children and type rank = F.Nodes.G.t
```

4.2   Implementation of DAG

```
let rcs_file = RCS.parse "DAG" ["Directed␣Acyclical␣Graph"]
    { RCS.revision = "$Revision:␣6465␣$";
      RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
      RCS.author = "$Author:␣jr_reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s
```

```
module type Ord =
```

```
sig
  type t
  val compare : t → t → int
end

module type Forest =
  sig
    module Nodes : Ord
    type node = Nodes.t
    type edge
    type children
    type t = edge × children
    val compare : t → t → int
    val for_all : (node → bool) → t → bool
    val fold : (node → α → α) → t → α → α
  end

module type T =
  sig
    type node
    type edge
    type children
    type t
    val empty : t
    val add_node : node → t → t
    val add_offspring : node → edge × children → t → t
    exception Cycle
    val add_offspring_unsafe : node → edge × children → t → t
    val is_node : node → t → bool
    val is_sterile : node → t → bool
    val is_offspring : node → edge × children → t → bool
    val iter_nodes : (node → unit) → t → unit
    val map_nodes : (node → node) → t → t
    val fold_nodes : (node → α → α) → t → α → α
    val iter : (node → edge × children → unit) → t → unit
    val map : (node → node) →
      (node → edge × children → edge × children) → t → t
    val fold : (node → edge × children → α → α) → t → α → α
    val lists : t → (node × (edge × children) list) list
    val dependencies : t → node → (node, edge) Tree2.t
    val harvest : t → node → t → t
    val harvest_list : t → node list → t
    val size : t → int
    val eval : (node → α) → (node → edge → γ → δ) →
      (α → γ → γ) → (δ → α → α) → α → γ → node → t → α
    val eval_memoized : (node → α) → (node → edge → γ → δ) →
      (α → γ → γ) → (δ → α → α) → α → γ → node → t → α
    val forest : node → t → (node × edge option, node) Tree.t list
    val forest_memoized : node → t → (node × edge option, node) Tree.t list
    val count_trees : node → t → int
    val rcs : RCS.t
```

```
    end

module type Graded_Ord =
  sig
    include Ord
    module G : Ord
    val rank : t → G.t
  end

module type Grader = functor (O : Ord) → Graded_Ord with type t = O.t

module type Graded_Forest =
  sig
    module Nodes : Graded_Ord
    type node = Nodes.t
    type edge
    type children
    type t = edge × children
    val compare : t → t → int
    val for_all : (node → bool) → t → bool
    val fold : (node → α → α) → t → α → α
  end

module type Forest_Grader = functor (G : Grader) → functor (F : Forest) →

  Graded_Forest with type Nodes.t = F.node
  and type node = F.node
  and type edge = F.edge
  and type children = F.children
  and type t = F.t
```

4.2.1   The Forest Functor

```
module Forest (PT : Tuple.Poly) (N : Ord) (E : Ord) :
    Forest with module Nodes = N and type edge = E.t
    and type node = N.t and type children = N.t PT.t =
  struct
    module Nodes = N
    type edge = E.t
    type node = N.t
    type children = node PT.t
    type t = edge × children

    let compare (e1, n1) (e2, n2) =
      let c = PT.compare N.compare n1 n2 in
      if c ≠ 0 then
        c
      else
        E.compare e1 e2

    let for_all f (_, nodes) = PT.for_all f nodes
    let fold f (_, nodes) acc = PT.fold_right f nodes acc
```

```
      end
```

4.2.2   Gradings

```
module Chaotic (O  :  Ord)  =
  struct
    include O
    module G  =
      struct
        type t  =  unit
        let compare _ _  =  0
      end
    let rank _  =  ()
  end
module Discrete (O  :  Ord)  =
  struct
    include O
    module G  =  O
    let rank x  =  x
  end
module Fake_Grading (O  :  Ord)  =
  struct
    include O
    exception Impossible of string
    module G  =
      struct
        type t  =  unit
        let compare _ _  =  raise (Impossible "G.compare")
      end
    let rank _  =  raise (Impossible "G.compare")
  end
module Grade_Forest (G  :  Grader) (F  :  Forest)  =
  struct
    module Nodes  =  G(F.Nodes)
    type node  =  Nodes.t
    type edge  =  F.edge
    type children  =  F.children
    type t  =  F.t
    let compare  =  F.compare
    let for_all  =  F.for_all
    let fold  =  F.fold
  end
```

The following can easily be extended to *Map.S* in its full glory, if we ever
need it.

```
module type Graded_Map  =
```

```
sig
    type key
    type rank
    type α t
    val empty : α t
    val add : key → α → α t → α t
    val find : key → α t → α
    val mem : key → α t → bool
    val iter : (key → α → unit) → α t → unit
    val fold : (key → α → β → β) → α t → β → β
    val ranks : α t → rank list
    val min_max_rank : α t → rank × rank
    val ranked : rank → α t → key list
end

module type Graded_Map_Maker = functor (O : Graded_Ord) →
    Graded_Map with type key = O.t and type rank = O.G.t

module Graded_Map (O : Graded_Ord) :
    Graded_Map with type key = O.t and type rank = O.G.t =
struct
    module M1 = Map.Make(O.G)
    module M2 = Map.Make(O)

    type key = O.t
    type rank = O.G.t

    type (+α) t = α M2.t M1.t

    let empty = M1.empty
    let add key data map1 =
        let rank = O.rank key in
        let map2 = try M1.find rank map1 with Not_found → M2.empty in
        M1.add rank (M2.add key data map2) map1
    let find key map = M2.find key (M1.find (O.rank key) map)
    let mem key map =
        M2.mem key (try M1.find (O.rank key) map with Not_found → M2.empty)
    let iter f map1 = M1.iter (fun rank → M2.iter f) map1
    let fold f map1 acc1 = M1.fold (fun rank → M2.fold f) map1 acc1
```

⚠ The set of ranks and its minimum and maximum should be maintained explicitly!

```
    module S1 = Set.Make(O.G)
    let ranks map = M1.fold (fun key data acc → key :: acc) map []
    let rank_set map = M1.fold (fun key data → S1.add key) map S1.empty
    let min_max_rank map =
        let s = rank_set map in
        (S1.min_elt s, S1.max_elt s)

    module S2 = Set.Make(O)
    let keys map = M2.fold (fun key data acc → key :: acc) map []
    let sorted_keys map =
```

```
        S2.elements (M2.fold (fun key data → S2.add key) map S2.empty)
      let ranked rank map =
        keys (try M1.find rank map with Not_found → M2.empty)
    end
```

4.2.3   The DAG Functor

```
module Maybe_Graded (GMM : Graded_Map_Maker) (F : Graded_Forest) =
  struct
      let rcs = RCS.rename rcs_file "DAG.Graded()"
          ["Graded␣directed␣Acyclical␣Graph␣";
           "representing␣binary␣or␣n-ary␣trees"]

      module G = F.Nodes.G

      type node = F.node
      type rank = G.t
      type edge = F.edge
      type children = F.children
```

If we get tired of graded DAGs, we just have to replace *Graded_Map* by *Map* here and remove *ranked* below and gain a tiny amount of simplicity and efficiency.

```
      module Parents = GMM(F.Nodes)
      module Offspring = Set.Make(F)

      type t = Offspring.t Parents.t

      let rank = F.Nodes.rank
      let ranks = Parents.ranks
      let min_max_rank = Parents.min_max_rank
      let ranked = Parents.ranked

      let empty = Parents.empty

      let add_node node dag =
        if Parents.mem node dag then
            dag
        else
            Parents.add node Offspring.empty dag

      let add_offspring_unsafe node offspring dag =
        let offsprings =
            try Parents.find node dag with Not_found → Offspring.empty in
        Parents.add node (Offspring.add offspring offsprings)
            (F.fold add_node offspring dag)

      exception Cycle

      let add_offspring node offspring dag =
        if F.for_all (fun n → F.Nodes.compare n node < 0) offspring then
            add_offspring_unsafe node offspring dag
        else
            raise Cycle
```

```
let is_node node dag  =
  Parents.mem node dag

let is_sterile node dag  =
  try
    Offspring.is_empty (Parents.find node dag)
  with
  | Not_found  →  false

let is_offspring node offspring dag  =
  try
    Offspring.mem offspring (Parents.find node dag)
  with
  | Not_found  →  false

let iter_nodes f dag  =
  Parents.iter (fun n _  →  f n) dag

let iter f dag  =
  Parents.iter (fun node  →  Offspring.iter (f node)) dag

let map_nodes f dag  =
  Parents.fold (fun n  →  Parents.add (f n)) dag Parents.empty

let map fn fo dag  =
  Parents.fold (fun node offspring  →
    Parents.add (fn node)
      (Offspring.fold (fun o  →  Offspring.add (fo node o))
          offspring Offspring.empty)) dag Parents.empty

let fold_nodes f dag acc  =
  Parents.fold (fun n _  →  f n) dag acc

let fold f dag acc  =
  Parents.fold (fun node  →  Offspring.fold (f node)) dag acc
```

⚠ Note that in it's current incarnation, *fold add_offspring dag empty* copies
only the fertile nodes, while *fold add_offspring dag (fold_nodes add_node dag empty)*
includes sterile ones, as does *map* (fun *n* → *n*) (fun *n ec* → *ec*) *dag*.

```
let dependencies dag node  =
  let rec dependencies' node'  =
    let offspring  =  Parents.find node' dag in
    if Offspring.is_empty offspring then
      Tree2.leaf node'
    else
      Tree2.cons
        (Offspring.fold
            (fun o acc  →
              (fst o,
               node',
               F.fold (fun wf acc'  →  dependencies' wf :: acc') o []) :: acc)
            offspring [])
  in
```

```
        dependencies′ node
    let lists dag  =
      Sort.list (fun (n1,  _) (n2,  _)  →  F.Nodes.compare n1 n2  ≤  0)
        (Parents.fold (fun node offspring l  →
           (node,  Offspring.elements offspring)  ::  l) dag [])

    let size dag  =
      Parents.fold (fun _ _ n  →  succ n) dag 0

    let rec harvest dag node roots  =
      Offspring.fold
        (fun offspring roots′  →
          if is_offspring node offspring roots′ then
            roots′
          else
            F.fold (harvest dag)
              offspring (add_offspring_unsafe node offspring roots′))
        (Parents.find node dag) (add_node node roots)

    let harvest_list dag nodes  =
      List.fold_left (fun roots node  →  harvest dag node roots) empty nodes
```

Build a closure once, so that we can recurse faster:

```
    let eval f mule muln add null unit node dag  =
      let rec eval′ n  =
        if is_sterile n dag then
          f n
        else
          Offspring.fold
            (fun (e,  _ as offspring) v0  →
              add (mule n e (F.fold muln′ offspring unit)) v0)
            (Parents.find n dag) null
      and muln′ n  =  muln (eval′ n) in
      eval′ node

    let count_trees node dag  =
      eval (fun _  →  1) (fun _ _ p  →  p) (  ×  ) (+) 0 1 node dag

    let build_forest evaluator node dag  =
      evaluator (fun n  →  [Tree.leaf (n,  None) n])
        (fun n e p  →  List.map (fun p′  →  Tree.cons (n,  Some e) p′) p)
        (fun p1 p2  →  Product.fold2 (fun n nl pl  →  (n  ::  nl)  ::  pl) p1 p2 [])
        (@) [] [[]] node dag

    let forest  =  build_forest eval
```

At least for *count_trees*, the memoizing variant *eval_memoized* is considerably slower than direct recursive evaluation with *eval*.

```
    let eval_offspring f mule muln add null unit dag values (node, offspring)  =
      let muln′ n  =  muln (Parents.find n values) in
      let v  =
        if is_sterile node dag then
          f node
```

```
          else
            Offspring.fold
              (fun (e, _ as offspring) v0 →
                add (mule node e (F.fold muln' offspring unit)) v0)
              offspring null
        in
        (v, Parents.add node v values)

    let eval_memoized' f mule muln add null unit dag =
      let result, _ =
        List.fold_left
          (fun (v, values) → eval_offspring f mule muln add null unit dag values)
          (null, Parents.empty)
          (Sort.list (fun (n1, _) (n2, _) → F.Nodes.compare n1 n2 ≤ 0)
            (Parents.fold
              (fun node offspring l → (node, offspring) :: l) dag [])) in
      result

    let eval_memoized f mule muln add null unit node dag =
      eval_memoized' f mule muln add null unit
        (harvest dag node empty)

    let forest_memoized = build_forest eval_memoized

  end

module type Graded =
  sig
    include T
    type rank
    val rank : node → rank
    val ranks : t → rank list
    val min_max_rank : t → rank × rank
    val ranked : rank → t → node list
  end

module Graded (F : Graded_Forest) = Maybe_Graded(Graded_Map)(F)
```

The following is not a graded map, obviously. But it can pass as one by the typechecker for constructing non-graded DAGs.

```
module Fake_Graded_Map (O : Graded_Ord) :
    Graded_Map with type key = O.t and type rank = O.G.t =
  struct
    module M = Map.Make(O)
    type key = O.t
    type (+α) t = α M.t
    let empty = M.empty
    let add = M.add
    let find = M.find
    let mem = M.mem
    let iter = M.iter
    let fold = M.fold
```

We make sure that the remaining three are never called inside DAG and are not visible outside.

```
    type rank  =  O.G.t
    exception Impossible of string
    let ranks _  =  raise (Impossible "ranks")
    let min_max_rank _  =  raise (Impossible "min_max_rank")
    let ranked _ _  =  raise (Impossible "ranked")
  end
```

We could also have used signature projection with a chaotic or discrete grading, but the *Graded_Map* can cost some efficiency. This is probably not the case for the current simple implementation, but future embellishment can change this. Therefore, the ungraded DAG uses *Map* directly, without overhead.

```
module Make (F : Forest)  =
  Maybe_Graded(Fake_Graded_Map)(Grade_Forest(Fake_Grading)(F))
```

If O'Caml had *polymorphic recursion*, we could think of even more elegant implementations unifying nodes and offspring (cf. the generalized tries in [4]).

<center>

—5—

Momenta

</center>

5.1 Interface of Momentum

Model the finite combinations

$$p = \sum_{n=1}^{k} c_k \bar{p}_n, \qquad \text{(with } c_k \in \{0,1\}) \tag{5.1}$$

of n_{in} incoming and $k - n_{\text{in}}$ outgoing momenta p_n

$$\bar{p}_n = \begin{cases} -p_n & \text{for } 1 \le n \le n_{\text{in}} \\ p_n & \text{for } n_{\text{in}} + 1 \le n \le k \end{cases} \tag{5.2}$$

where momentum is conserved

$$\sum_{n=1}^{k} \bar{p}_n = 0 \tag{5.3}$$

below, we need the notion of 'rank' and 'dimension':

$$dim(p) = k \tag{5.4a}$$

$$rank(p) = \sum_{n=1}^{k} c_k \tag{5.4b}$$

where 'dimension' is *not* the dimension of the underlying space-time, of course.

module type T =
  sig
    type t

Constructor: $(k, N) \to p = \sum_{n \in N} \bar{p}_n$ and $k = dim(p)$ is the *overall* number of independent momenta, while $rank(p) = |N|$ is the number of momenta in p. It would be possible to fix *dim* as a functor argument instead. This might be slightly faster and allow a few more compile time checks, but would be much more tedious to use, since the number of particles will be chosen at runtime.

val *of_ints* : *int* → *int list* → *t*

No two indices may be the same. Implementations of *of_ints* can either raise the exception *Duplicate* or ignore the duplicate, but implementations of *add* are required to raise *Duplicate*.

<center>50</center>

exception *Duplicate* of *int*

Raise *Range* iff $n > k$:

exception *Range* of *int*

Binary oparations require that both momenta have the same dimension. *Mismatch* is raised if this condition is violated.

exception *Mismatch* of *string* $\times$ *t* $\times$ *t*

Negative is raised if the result of *sub* is undefined.

exception *Negative*

The inverses of the constructor (we have $rank\ p\ =\ List.length\ (to\_ints\ p)$, but *rank* might be more efficient):

val *to_ints* : *t* $\rightarrow$ *int list*
val *dim* : *t* $\rightarrow$ *int*
val *rank* : *t* $\rightarrow$ *int*

Shortcuts: $singleton\ d\ p\ =\ of\_ints\ d\ [p]$ and $zero\ d\ =\ of\_ints\ d\ []$:

val *singleton* : *int* $\rightarrow$ *int* $\rightarrow$ *t*
val *zero* : *int* $\rightarrow$ *t*

An arbitrary total order, with the condition $rank(p_1) < rank(p_2) \Rightarrow p_1 < p_2$.

val *compare* : *t* $\rightarrow$ *t* $\rightarrow$ *int*

Use momentum conservation to construct the negative momentum with positive coefficients:

val *neg* : *t* $\rightarrow$ *t*

Return the momentum or its negative, whichever has the lower rank. NB: the present implementation does *not* guarantee that

$$\text{abs}p = \text{abs}q \Longleftrightarrow p = p \vee p = -q \tag{5.5}$$

for momenta with rank = dim/2.

val *abs* : *t* $\rightarrow$ *t*

Add and subtract momenta. This can fail, since the coefficients c_k must me either 0 or 1.

val *add* : *t* $\rightarrow$ *t* $\rightarrow$ *t*
val *sub* : *t* $\rightarrow$ *t* $\rightarrow$ *t*

Once more, but not raising exceptions this time:

val *try_add* : *t* $\rightarrow$ *t* $\rightarrow$ *t option*
val *try_sub* : *t* $\rightarrow$ *t* $\rightarrow$ *t option*

Not the total order provided by *compare*, but set inclusion of non-zero coefficients instead:

val *less* : *t* $\rightarrow$ *t* $\rightarrow$ *bool*
val *lesseq* : *t* $\rightarrow$ *t* $\rightarrow$ *bool*

$$p_1 + (\pm p_2) + (\pm p_3) = 0$$

   val *try_fusion* : $t \rightarrow t \rightarrow t \rightarrow (bool \times bool)$ *option*

A textual representation for debugging:

   val *to_string* : $t \rightarrow string$

split i n p splits $\bar{p}_i$ into n momenta $\bar{p}_i \rightarrow \bar{p}_i + \bar{p}_{i+1} + \ldots + \bar{p}_{i+n-1}$ and makes room via $\bar{p}_{j>i} \rightarrow \bar{p}_{j+n-1}$. This is used for implementating cascade decays, like combining

$$e^+(p_1)e^-(p_2) \rightarrow W^-(p_3)\nu_e(p_4)e^+(p_5) \tag{5.6a}$$

$$W^-(p_3) \rightarrow d(p'_3)\bar{u}(p'_4) \tag{5.6b}$$

to

$$e^+(p_1)e^-(p_2) \rightarrow d(p_3)\bar{u}(p_4)\nu_e(p_5)e^+(p_6) \tag{5.7}$$

in narrow width approximation for the W^-.

   val *split* : $int \rightarrow int \rightarrow t \rightarrow t$

5.1.1 Scattering Kinematics

From here on, we assume scattering kinematics $\{1, 2\} \rightarrow \{3, 4, \ldots\}$, i.e. $n_{in} = 2$.

Since functions like *timelike* can be used for decays as well (in which case they must *always* return true, the representation—and consequently the constructors—should be extended by a flag discriminating between the two cases!

   module *Scattering* :
      sig

Test if the momentum is an incoming one: $p = \bar{p}_1 \vee p = \bar{p}_2$

      val *incoming* : $t \rightarrow bool$

$p = \bar{p}_3 \vee p = \bar{p}_4 \vee \ldots$

      val *outgoing* : $t \rightarrow bool$

$p^2 \geq 0$. NB: *par abus de langange*, we report the incoming individual momenta as spacelike, instead as timelike. This will be useful for phasespace constructions below.

      val *timelike* : $t \rightarrow bool$

$p^2 \leq 0$. NB: the simple algebraic criterion can be violated for heavy initial state particles.

      val *spacelike* : $t \rightarrow bool$

$p = \bar{p}_1 + \bar{p}_2$

      val *s_channel_in* : $t \rightarrow bool$

$p = \bar{p}_3 + \bar{p}_4 + \ldots + \bar{p}_n$

val $s\_channel\_out$: $t \rightarrow bool$

$$p = \bar{p}_1 + \bar{p}_2 \vee p = \bar{p}_3 + \bar{p}_4 + \ldots + \bar{p}_n$$

val $s\_channel$: $t \rightarrow bool$

$$\bar{p}_1 + \bar{p}_2 \rightarrow \bar{p}_3 + \bar{p}_4 + \ldots + \bar{p}_n$$

val $flip\_s\_channel\_in$: $t \rightarrow t$

end

5.1.2 Decay Kinematics

module *Decay* :
    sig

Test if the momentum is an incoming one: $p = \bar{p}_1$

val *incoming* : $t \rightarrow bool$

$$p = \bar{p}_2 \vee p = \bar{p}_3 \vee \ldots$$

val *outgoing* : $t \rightarrow bool$

$p^2 \geq 0$. NB: here, we report the incoming individual momenta as timelike.

val *timelike* : $t \rightarrow bool$

$p^2 \leq 0$.

val *spacelike* : $t \rightarrow bool$

    end

val rcs : $RCS.t$
  end

module *Lists* : T
module *Bits* : T
module *Default* : T

Wolfgang's funny tree codes:

$$(2^n, 2^{n-1}) \rightarrow (1, 2, 4, \ldots, 2^{n-2}) \tag{5.8}$$

module type *Whizard* =
  sig
    type t
    val $of\_momentum$: $t \rightarrow int$
    val $to\_momentum$: $int \rightarrow int \rightarrow t$
  end

module *ListsW* : *Whizard* with type $t = Lists.t$
module *BitsW* : *Whizard* with type $t = Bits.t$
module *DefaultW* : *Whizard* with type $t = Default.t$

5.2   *Implementation of Momentum*

let *rcs_file* = *RCS.parse* "Momentum" ["Finite␣disjoint␣sums␣of␣momenta"]
   { *RCS.revision* = "$Revision:␣6465␣$";
     RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
     RCS.author = "$Author:␣jr␣reuter␣$";
     RCS.source
       = "$URL:␣svn+ssh://jr␣reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s

module type *T* =
  sig
    type *t*
    val *of_ints* : *int* → *int list* → *t*
    exception *Duplicate* of *int*
    exception *Range* of *int*
    exception *Mismatch* of *string* × *t* × *t*
    exception *Negative*
    val *to_ints* : *t* → *int list*
    val *dim* : *t* → *int*
    val *rank* : *t* → *int*
    val *singleton* : *int* → *int* → *t*
    val *zero* : *int* → *t*
    val *compare* : *t* → *t* → *int*
    val *neg* : *t* → *t*
    val *abs* : *t* → *t*
    val *add* : *t* → *t* → *t*
    val *sub* : *t* → *t* → *t*
    val *try_add* : *t* → *t* → *t option*
    val *try_sub* : *t* → *t* → *t option*
    val *less* : *t* → *t* → *bool*
    val *lesseq* : *t* → *t* → *bool*
    val *try_fusion* : *t* → *t* → *t* → (*bool* × *bool*) *option*
    val *to_string* : *t* → *string*
    val *split* : *int* → *int* → *t* → *t*
    module *Scattering* :
        sig
          val *incoming* : *t* → *bool*
          val *outgoing* : *t* → *bool*
          val *timelike* : *t* → *bool*
          val *spacelike* : *t* → *bool*
          val *s_channel_in* : *t* → *bool*
          val *s_channel_out* : *t* → *bool*
          val *s_channel* : *t* → *bool*
          val *flip_s_channel_in* : *t* → *t*
        end
    module *Decay* :
        sig
          val *incoming* : *t* → *bool*
          val *outgoing* : *t* → *bool*
          val *timelike* : *t* → *bool*

```
        val spacelike : t → bool
      end
    val rcs : RCS.t
  end
```

5.2.1 Lists of Integers

The first implementation (as part of *Fusion*) was based on sorted lists, because I did not want to preclude the use of more general indices that integers. However, there's probably not much use for this generality (the indices are typically generated automatically and integer are the most natural choice) and it is no longer supported. by the current signature. Thus one can also use the more efficient implementation based on bitvectors below.

```
module Lists =
  struct
    let rcs = RCS.rename rcs_file "Momentum.Lists()"
        (RCS.description rcs_file @
        ["using␣lists␣as␣representation."])

    type t = { d : int; r : int; p : int list }

    exception Range of int
    exception Duplicate of int

    let rec check d = function
      | p1 :: p2 :: _ when p2 ≤ p1 → raise (Duplicate p1)
      | p1 :: (p2 :: _ as rest) → check d rest
      | [p] when p < 1 ∨ p > d → raise (Range p)
      | [p] → ()
      | [] → ()

    let of_ints d p =
      let p′ = List.sort compare p in
      check d p′;
      { d = d; r = List.length p; p = p′ }

    let to_ints p = p.p
    let dim p = p.d
    let rank p = p.r
    let zero d = { d = d; r = 0; p = [] }
    let singleton d p = { d = d; r = 1; p = [p] }

    let to_string p =
      "[" ^ String.concat "," (List.map string_of_int p.p) ^
      "/" ^ string_of_int p.r ^ "/" ^ string_of_int p.d ^ "]"

    exception Mismatch of string × t × t
    let mismatch s p1 p2 = raise (Mismatch (s, p1, p2))

    let matching f s p1 p2 =
      if p1.d = p2.d then
        f p1 p2
      else
```

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$\qquad$ *mismatch s p1 p2*

let *compare p1 p2* =
  if *p1.d* = *p2.d* then begin
    let *c* = *compare p1.r p2.r* in
    if *c* $\neq$ 0 then
      c
    else
      compare p1.p p2.p
  end else
    mismatch "compare" *p1 p2*

let rec *neg$'$ d i* = function
  | [] $\rightarrow$
    if *i* $\leq$ *d* then
      i :: *neg$'$ d* (*succ i*) []
    else
      []
  | *i$'$* :: *rest* as *p* $\rightarrow$
    if *i$'$* > *d* then
      failwith "Integer_List.neg:␣internal␣error"
    else if *i$'$* = *i* then
      neg$'$ d (*succ i*) *rest*
    else
      i :: *neg$'$ d* (*succ i*) *p*

let *neg p* = { *d* = *p.d*; *r* = *p.d* − *p.r*; *p* = *neg$'$ p.d* 1 *p.p* }

let *abs p* =
  if 2 × *p.r* > *p.d* then
    neg p
  else
    p

let rec *add$'$ p1 p2* =
  match *p1*, *p2* with
  | [], *p* $\rightarrow$ *p*
  | *p*, [] $\rightarrow$ *p*
  | *x1* :: *p1$'$*, *x2* :: *p2$'$* $\rightarrow$
    if *x1* < *x2* then
      x1 :: *add$'$ p1$'$ p2*
    else if *x2* < *x1* then
      x2 :: *add$'$ p1 p2$'$*
    else
      raise (*Duplicate x1*)

let *add p1 p2* =
  if *p1.d* = *p2.d* then
    { *d* = *p1.d*; *r* = *p1.r* + *p2.r*; *p* = *add$'$ p1.p p2.p* }
  else
    mismatch "add" *p1 p2*

let rec *try_add$'$ d r acc p1 p2* =
  match *p1*, *p2* with

```
    | [], p → Some ({ d = d; r = r; p = List.rev_append acc p })
    | p, [] → Some ({ d = d; r = r; p = List.rev_append acc p })
    | x1 :: p1', x2 :: p2' →
        if x1 < x2 then
          try_add' d r (x1 :: acc) p1' p2
        else if x2 < x1 then
          try_add' d r (x2 :: acc) p1 p2'
        else
          None

let try_add p1 p2 =
  if p1.d = p2.d then
    try_add' p1.d (p1.r + p2.r) [] p1.p p2.p
  else
    mismatch "try_add" p1 p2

exception Negative

let rec sub' p1 p2 =
  match p1, p2 with
  | p, [] → p
  | [], _ → raise Negative
  | x1 :: p1', x2 :: p2' →
      if x1 < x2 then
        x1 :: sub' p1' p2
      else if x1 = x2 then
        sub' p1' p2'
      else
        raise Negative

let rec sub p1 p2 =
  if p1.d = p2.d then begin
    if p1.r ≥ p2.r then
      { d = p1.d; r = p1.r − p2.r; p = sub' p1.p p2.p }
    else
      neg (sub p2 p1)
  end else
    mismatch "sub" p1 p2

let rec try_sub' d r acc p1 p2 =
  match p1, p2 with
  | p, [] → Some ({ d = d; r = r; p = List.rev_append acc p })
  | [], _ → None
  | x1 :: p1', x2 :: p2' →
      if x1 < x2 then
        try_sub' d r (x1 :: acc) p1' p2
      else if x1 = x2 then
        try_sub' d r acc p1' p2'
      else
        None

let try_sub p1 p2 =
  if p1.d = p2.d then begin
```

```
    if p1.r ≥ p2.r then
      try_sub' p1.d (p1.r − p2.r) [] p1.p p2.p
    else
      match try_sub' p1.d (p2.r − p1.r) [] p2.p p1.p with
      | None → None
      | Some p → Some (neg p)
  end else
    mismatch "try_sub" p1 p2

let rec less' equal p1 p2 =
  match p1, p2 with
  | [], [] → ¬ equal
  | [], _ → true
  | x1 :: _ , [] → false
  | x1 :: p1', x2 :: p2' when x1 = x2 → less' equal p1' p2'
  | x1 :: p1', x2 :: p2' → less' false p1 p2'

let less p1 p2 =
  if p1.d = p2.d then
    less' true p1.p p2.p
  else
    mismatch "sub" p1 p2

let rec lesseq' p1 p2 =
  match p1, p2 with
  | [], _ → true
  | x1 :: _ , [] → false
  | x1 :: p1', x2 :: p2' when x1 = x2 → lesseq' p1' p2'
  | x1 :: p1', x2 :: p2' → lesseq' p1 p2'

let lesseq p1 p2 =
  if p1.d = p2.d then
    lesseq' p1.p p2.p
  else
    mismatch "lesseq" p1 p2

module Scattering =
  struct

    let incoming p =
      if p.r = 1 then
        match p.p with
        | [1] | [2] → true
        | _ → false
      else
        false

    let outgoing p =
      if p.r = 1 then
        match p.p with
        | [1] | [2] → false
        | _ → true
      else
        false
```

```
let s_channel_in p =
  match p.p with
  | [1; 2]  →  true
  | _  →  false

let rec s_channel_out' d i  =  function
  | []  →  i  =  succ d
  | i' :: p when i'  =  i  →  s_channel_out' d (succ i) p
  | _  →  false

let s_channel_out p =
  match p.p with
  | 3 :: p'  →  s_channel_out' p.d 4 p'
  | _  →  false

let s_channel p  =  s_channel_in p  ∨  s_channel_out p

let timelike p  =
  match p.p with
  | p1 :: p2 :: _  →  p1  >  2  ∨  (p1  =  1  ∧  p2  =  2)
  | p1 :: _  →  p1  >  2
  | []  →  false

let spacelike p  =  ¬ (timelike p)

let flip_s_channel_in p  =
  if s_channel_in p then
    neg (of_ints p.d [1; 2])
  else
    p

end

module Decay  =
  struct

    let incoming p  =
      if p.r  =  1 then
        match p.p with
        | [1]  →  true
        | _  →  false
      else
        false

    let outgoing p  =
      if p.r  =  1 then
        match p.p with
        | [1]  →  false
        | _  →  true
      else
        false

    let timelike p  =
      match p.p with
      | [1]  →  true
      | p1 :: _  →  p1  >  1
```

```
                | [] → false

            let spacelike p = ¬ (timelike p)

        end

    let test_sum p inv1 p1 inv2 p2 =
        if p.d = p1.d then begin
            if p.d = p2.d then begin
                match (if inv1 then try_add else try_sub) p p1 with
                | None → false
                | Some p' →
                    begin match (if inv2 then try_add else try_sub) p' p2 with
                    | None → false
                    | Some p'' → p''.r = 0 ∨ p''.r = p.d
                    end
            end else
                mismatch "test_sum" p p2
        end else
            mismatch "test_sum" p p1

    let try_fusion p p1 p2 =
        if test_sum p false p1 false p2 then
            Some (false, false)
        else if test_sum p true p1 false p2 then
            Some (true, false)
        else if test_sum p false p1 true p2 then
            Some (false, true)
        else if test_sum p true p1 true p2 then
            Some (true, true)
        else
            None

    let split i n p =
        let n' = n − 1 in
        let rec split' head = function
            | [] → (p.r, List.rev head)
            | i1 :: ilist →
                if i1 < i then
                    split' (i1 :: head) ilist
                else if i1 > i then
                    (p.r, List.rev_append head (List.map ((+) n') (i1 :: ilist)))
                else
                    (p.r + n',
                    List.rev_append head
                        ((ThoList.range i1 (i1 + n')) @ (List.map ((+) n') ilist))) in
        let r', p' = split' [] p.p in
        { d = p.d + n'; r = r'; p = p' }

end
```

5.2.2   Bit Fiddlings

Bit vectors are popular in Fortran based implementations [1, 2, 11] and can be more efficient. In particular, when all infomation is packed into a single integer, much of the memory overhead is reduced.

```
module Bits =
  struct
    let rcs = RCS.rename rcs_file "Momentum.Bits()"
        (RCS.description rcs_file @
         [ "using bitfields as representation." ])

    type t = int
```

Bits $1\ldots21$ are used as a bitvector, indicating whether a particular momentum is included. Bits $22\ldots26$ represent the numbers of bits set in bits $1\ldots21$ and bits $27\ldots31$ denote the maximum number of momenta.

```
    let mask n = (1 lsl n) − 1
    let mask2 = mask 2
    let mask5 = mask 5
    let mask21 = mask 21

    let maskd = mask5 lsl 26
    let maskr = mask5 lsl 21
    let maskb = mask21

    let dim0 p = p land maskd
    let rank0 p = p land maskr
    let bits0 p = p land maskb

    let dim p = (dim0 p) lsr 26
    let rank p = (rank0 p) lsr 21
    let bits p = bits0 p

    let drb0 d r b = d lor r lor b
    let drb d r b = d lsl 26 lor r lsl 21 lor b
```

For a 64-bit architecture, the corresponding sizes could be increased to $1\ldots51$, $52\ldots57$, and $58\ldots63$. However, the combinatorical complexity will have killed us long before we can reach these values.

```
    exception Range of int
    exception Duplicate of int

    exception Mismatch of string × t × t
    let mismatch s p1 p2 = raise (Mismatch (s, p1, p2))

    let of_ints d p =
      let r = List.length p in
      if d ≤ 21 ∧ r ≤ 21 then begin
        List.fold_left (fun b p′ →
          if p′ ≤ d then
            b lor (1 lsl (pred p′))
          else
            raise (Range p′)) (drb d r 0) p
```

```
    end else
      raise (Range r)

let zero d  =  drb d 0 0

let singleton d p  =  drb d 1 (1 lsl (pred p))

let rec to_ints' acc p b  =
  if b  =  0 then
    List.rev acc
  else if (b land 1)  =  1 then
    to_ints' (p  ::  acc) (succ p) (b lsr 1)
  else
    to_ints' acc (succ p) (b lsr 1)

let to_ints p  =  to_ints' [] 1 (bits p)

let to_string p  =
  "[" ^ String.concat "," (List.map string_of_int (to_ints p)) ^
  "/" ^ string_of_int (rank p) ^ "/" ^ string_of_int (dim p) ^ "]"

let compare p1 p2  =
  if dim0 p1  =  dim0 p2 then begin
    let c  =  compare (rank0 p1) (rank0 p2) in
    if c  ≠  0 then
      c
    else
      compare (bits p1) (bits p2)
  end else
    mismatch "compare" p1 p2

let neg p  =
  let d  =  dim p and r  =  rank p in
  drb d (d  −  r) ((mask d) land (lnot p))

let abs p  =
  if 2 × (rank p)  >  dim p then
    neg p
  else
    p

let add p1 p2  =
  let d1  =  dim0 p1 and d2  =  dim0 p2 in
  if d1  =  d2 then begin
    let b1  =  bits p1 and b2  =  bits p2 in
    if b1 land b2  =  0 then
      drb0 d1 (rank0 p1  +  rank0 p2) (b1 lor b2)
    else
      raise (Duplicate 0)
  end else
    mismatch "add" p1 p2

exception Negative

let rec sub p1 p2  =
  let d1  =  dim0 p1 and d2  =  dim0 p2 in
```

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```
  if d1 = d2 then begin
    let r1 = rank0 p1 and r2 = rank0 p2 in
    if r1 ≥ r2 then begin
      let b1 = bits p1 and b2 = bits p2 in
      if b1 lor b2 = b1 then
        drb0 d1 (r1 − r2) (b1 lxor b2)
      else
        raise Negative
    end else
      neg (sub p2 p1)
  end else
    mismatch "sub" p1 p2

let try_add p1 p2 =
  let d1 = dim0 p1 and d2 = dim0 p2 in
  if d1 = d2 then begin
    let b1 = bits p1 and b2 = bits p2 in
    if b1 land b2 = 0 then
      Some (drb0 d1 (rank0 p1 + rank0 p2) (b1 lor b2))
    else
      None
  end else
    mismatch "try_add" p1 p2

let rec try_sub p1 p2 =
  let d1 = dim0 p1 and d2 = dim0 p2 in
  if d1 = d2 then begin
    let r1 = rank0 p1 and r2 = rank0 p2 in
    if r1 ≥ r2 then begin
      let b1 = bits p1 and b2 = bits p2 in
      if b1 lor b2 = b1 then
        Some (drb0 d1 (r1 − r2) (b1 lxor b2))
      else
        None
    end else
      begin match try_sub p2 p1 with
      | Some p → Some (neg p)
      | None → None
      end
  end else
    mismatch "sub" p1 p2

let lesseq p1 p2 =
  let d1 = dim0 p1 and d2 = dim0 p2 in
  if d1 = d2 then begin
    let r1 = rank0 p1 and r2 = rank0 p2 in
    if r1 ≤ r2 then begin
      let b1 = bits p1 and b2 = bits p2 in
      b1 lor b2 = b2
    end else
      false
  end else
```

```
      mismatch "less" p1 p2

let less p1 p2  =  p1  ≠  p2  ∧  lesseq p1 p2

let mask_in1  =  1
let mask_in2  =  2
let mask_in  =  mask_in1 lor mask_in2

module Scattering  =
  struct

    let incoming p  =
      rank p  =  1  ∧  (mask_in land p  ≠  0)

    let outgoing p  =
      rank p  =  1  ∧  (mask_in land p  =  0)

    let timelike p  =
      (rank p  >  0  ∧  (mask_in land p  =  0))  ∨  (bits p  =  mask_in)

    let spacelike p  =
      (rank p  >  0)  ∧  ¬ (timelike p)

    let s_channel_in p  =
      bits p  =  mask_in

    let s_channel_out p  =
      rank p  >  0  ∧  (mask_in lxor p  =  0)

    let s_channel p  =
      s_channel_in p  ∨  s_channel_out p

    let flip_s_channel_in p  =
      if s_channel_in p then
        neg p
      else
        p

  end

module Decay  =
  struct

    let incoming p  =
      rank p  =  1  ∧  (mask_in1 land p  =  mask_in1)

    let outgoing p  =
      rank p  =  1  ∧  (mask_in1 land p  =  0)

    let timelike p  =
      incoming p  ∨  (rank p  >  0  ∧  mask_in1 land p  =  0)

    let spacelike p  =
      ¬ (timelike p)

  end

let test_sum p inv1 p1 inv2 p2  =
  let d  =  dim p in
  if d  =  dim p1 then begin
```

```
    if d = dim p2 then begin
      match (if inv1 then try_add else try_sub) p p1 with
      | None → false
      | Some p' →
          begin match (if inv2 then try_add else try_sub) p' p2 with
          | None → false
          | Some p'' →
              let r = rank p'' in
              r = 0 ∨ r = d
          end
    end else
      mismatch "test_sum" p p2
  end else
    mismatch "test_sum" p p1

let try_fusion p p1 p2 =
  if test_sum p false p1 false p2 then
    Some (false, false)
  else if test_sum p true p1 false p2 then
    Some (true, false)
  else if test_sum p false p1 true p2 then
    Some (false, true)
  else if test_sum p true p1 true p2 then
    Some (true, true)
  else
    None
```

First create a gap of size $n - 1$ and subsequently fill it if and only if the bit i was set.

```
let split i n p =
  let delta_d = n − 1
  and b = bits p in
  let mask_low = mask (pred i)
  and mask_i = 1 lsl (pred i)
  and mask_high = lnot (mask i) in
  let b_low = mask_low land b
  and b_med, delta_r =
    if mask_i land b ≠ 0 then
      ((mask n) lsl (pred i), delta_d)
    else
      (0, 0)
  and b_high =
    if delta_d > 0 then
      (mask_high land b) lsl delta_d
    else if delta_d = 0 then
      mask_high land b
    else
      (mask_high land b) lsr (−delta_d) in
  drb (dim p + delta_d) (rank p + delta_r) (b_low lor b_med lor b_high)

end
```

5.2.3   Whizard

```
module type Whizard =
  sig
    type t
    val of_momentum : t → int
    val to_momentum : int → int → t
  end

module BitsW =
  struct
    type t = Bits.t
    open Bits (∗ NB: this includes the internal functions not in T! ∗)

    let of_momentum p =
      let d = dim p in
      let bit_in1 = 1 land p
      and bit_in2 = 1 land (p lsr 1)
      and bits_out = ((mask d) land p) lsr 2 in
      bits_out lor (bit_in1 lsl (d − 1)) lor (bit_in2 lsl (d − 2))

    let rec count_non_zero' acc i last b =
      if i > last then
        acc
      else if (1 lsl (pred i)) land b = 0 then
        count_non_zero' acc (succ i) last b
      else
        count_non_zero' (succ acc) (succ i) last b

    let count_non_zero first last b =
      count_non_zero' 0 first last b

    let to_momentum d w =
      let bit_in1 = 1 land (w lsr (d − 1))
      and bit_in2 = 1 land (w lsr (d − 2))
      and bits_out = (mask (d − 2)) land w in
      let b = (bits_out lsl 2) lor bit_in1 lor (bit_in2 lsl 1) in
      drb d (count_non_zero 1 d b) b

  end
```

The following would be a tad more efficient, if coded directly, but there's no point in wasting effort on this.

```
module ListsW =
  struct
    type t = Lists.t
    let of_momentum p =
      BitsW.of_momentum (Bits.of_ints p.Lists.d p.Lists.p)
    let to_momentum d w =
      Lists.of_ints d (Bits.to_ints (BitsW.to_momentum d w))
  end
```

5.2.4 Suggesting a Default Implementation

Lists is better tested, but the more recent *Bits* appears to work as well and is *much* more efficient, resulting in a relative factor of better than 2. This performance ratio is larger than I had expected and we are not likely to reach its limit of 21 independent vectors anyway.

module *Default* = *Bits*
module *DefaultW* = *BitsW*

—6—
Cascades

6.1 Interface of Cascade_syntax

type (*'flavor*, *'p*) *t* =
  | *True*
  | *False*
  | *On_shell* of *'flavor list* × *'p*
  | *On_shell_not* of *'flavor list* × *'p*
  | *Off_shell* of *'flavor list* × *'p*
  | *Off_shell_not* of *'flavor list* × *'p*
  | *Gauss* of *'flavor list* × *'p*
  | *Gauss_not* of *'flavor list* × *'p*
  | *Any_flavor* of *'p*
  | *Or* of (*'flavor*, *'p*) *t list*
  | *And* of (*'flavor*, *'p*) *t list*

val *mk_true* : *unit* → (*'flavor*, *'p*) *t*
val *mk_false* : *unit* → (*'flavor*, *'p*) *t*
val *mk_on_shell* : *'flavor list* → *'p* → (*'flavor*, *'p*) *t*
val *mk_on_shell_not* : *'flavor list* → *'p* → (*'flavor*, *'p*) *t*
val *mk_off_shell* : *'flavor list* → *'p* → (*'flavor*, *'p*) *t*
val *mk_off_shell_not* : *'flavor list* → *'p* → (*'flavor*, *'p*) *t*
val *mk_gauss* : *'flavor list* → *'p* → (*'flavor*, *'p*) *t*
val *mk_gauss_not* : *'flavor list* → *'p* → (*'flavor*, *'p*) *t*
val *mk_any_flavor* : *'p* → (*'flavor*, *'p*) *t*
val *mk_or* : (*'flavor*, *'p*) *t* → (*'flavor*, *'p*) *t* → (*'flavor*, *'p*) *t*
val *mk_and* : (*'flavor*, *'p*) *t* → (*'flavor*, *'p*) *t* → (*'flavor*, *'p*) *t*

val *to_string* : (*'flavor* → *string*) → (*'p* → *string*) → (*'flavor*, *'p*) *t* →
  string

exception *Syntax_Error* of *string* × *int* × *int*

6.2 Implementation of Cascade_syntax

Concerning the Gaussian propagators, we admit the following: In principle, they
would allow for flavor sums like the off-shell lines, but for all practical purposes

they are used only for determining the significance of a specified intermediate state. So we select them in the same manner as on-shell states.

```
type ('flavor, 'p) t =
  | True
  | False
  | On_shell of 'flavor list × 'p
  | On_shell_not of 'flavor list × 'p
  | Off_shell of 'flavor list × 'p
  | Off_shell_not of 'flavor list × 'p
  | Gauss of 'flavor list × 'p
  | Gauss_not of 'flavor list × 'p
  | Any_flavor of 'p
  | Or of ('flavor, 'p) t list
  | And of ('flavor, 'p) t list

let mk_true () = True
let mk_false () = False
let mk_on_shell f p = On_shell (f, p)
let mk_on_shell_not f p = On_shell_not (f, p)
let mk_off_shell f p = Off_shell (f, p)
let mk_off_shell_not f p = Off_shell_not (f, p)
let mk_gauss f p = Gauss (f, p)
let mk_gauss_not f p = Gauss_not (f, p)
let mk_any_flavor p = Any_flavor p

let mk_or c1 c2 =
  match c1, c2 with
  | _, True | True, _ → True
  | c, False | False, c → c
  | Or cs, Or cs' → Or (cs @ cs')
  | Or cs, c | c, Or cs → Or (c :: cs)
  | c, c' → Or [c; c']

let mk_and c1 c2 =
  match c1, c2 with
  | c, True | True, c → c
  | c, False | False, c → False
  | And cs, And cs' → And (cs @ cs')
  | And cs, c | c, And cs → And (c :: cs)
  | c, c' → And [c; c']

let to_string flavor_to_string momentum_to_string cascades =
  let rec to_string' = function
    | True → "true"
    | False → "false"
    | On_shell (fs, p) →
        momentum_to_string p ^ "␣=␣" ^ (String.concat ":" (List.map flavor_to_string fs))
    | On_shell_not (fs, p) →
        momentum_to_string p ^ "␣=␣!" ^ (String.concat ":" (List.map flavor_to_string fs))
    | Off_shell (fs, p) →
        momentum_to_string p ^ "␣~␣" ^
        (String.concat ":" (List.map flavor_to_string fs))
```

   | *Off_shell_not (fs, p)* →
     momentum_to_string p ^ "␣˜␣!" ^
     (*String.concat* ":" (*List.map flavor_to_string fs*))
   | *Gauss (fs, p)* →
     momentum_to_string p ^ "␣#␣" ^ (*String.concat* ":" (*List.map flavor_to_string fs*))
   | *Gauss_not (fs, p)* →
     momentum_to_string p ^ "␣#␣!" ^ (*String.concat* ":" (*List.map flavor_to_string fs*))
   | *Any_flavor p* →
     momentum_to_string p ^ "␣˜␣?"
   | *Or cs* →
     String.concat "␣||␣" (*List.map* (fun *c* → "(" ^ *to_string′ c* ^ ")") *cs*)
   | *And cs* →
     String.concat "␣&&␣" (*List.map* (fun *c* → "(" ^ *to_string′ c* ^ ")") *cs*) in
  to_string′ cascades

let *int_list_to_string p* =
  String.concat "+" (*List.map string_of_int* (*Sort.list* (<) *p*))

exception *Syntax_Error* of *string* × *int* × *int*

6.3   Lexer

{
open *Cascade_parser*
let *unquote s* =
  String.sub s 1 (*String.length s* − 2)
}

let *digit* = ['0'–'9']
let *upper* = ['A'–'Z']
let *lower* = ['a'–'z']
let *char* = *upper* | *lower*
let *white* = [' ' '\t' '\n']

We use a very liberal definition of strings for flavor names.

rule *token* = parse
  white { *token lexbuf* } (∗ skip blanks ∗)
 | '%' [^'\n']* '\n'
     { *token lexbuf* } (∗ skip comments ∗)
 | *digit*⁺ { *INT* (*int_of_string* (*Lexing.lexeme lexbuf*)) }
 | '+' { *PLUS* }
 | ':' { *COLON* }
 | '˜' { *OFFSHELL* }
 | '=' { *ONSHELL* }
 | '#' { *GAUSS* }
 | '!' { *NOT* }
 | '&' '&'? { *AND* }
 | '|' '|'? { *OR* }
 | '(' { *LPAREN* }
 | ')' { *RPAREN* }

```
| char [^ ' ' '\t' '\n' '|' '&' '(' ')' ':']*
            { FLAVOR (Lexing.lexeme lexbuf) }
| '"' [^ '"']* '"'
            { FLAVOR (unquote (Lexing.lexeme lexbuf)) }
| eof { END }
```

6.4   Parser

Header

```
open Cascade_syntax
let parse_error msg =
  raise (Syntax_Error (msg, symbol_start (), symbol_end ()))
```

Token declarations

```
%token < string > FLAVOR
%token < int > INT
%token LPAREN RPAREN
%token AND OR PLUS COLON NOT
%token ONSHELL OFFSHELL GAUSS
%token END
%left OR
%left AND
%left PLUS COLON
%left NOT

%start main
%type < (string, int list) Cascade_syntax.t > main
```

Grammar rules

```
main ::=
    END { mk_true () }
  | cascades END { $1 }


cascades ::=
    cascade { $1 }
  | LPAREN cascades RPAREN { $2 }
  | cascades AND cascades { mk_and $1 $3 }
  | cascades OR cascades { mk_or $1 $3 }
```

cascade ::=
  momentum_list { *mk_any_flavor* $1 }
 | *momentum_list ONSHELL flavor_list*
         { *mk_on_shell* $3 $1 }
 | *momentum_list ONSHELL NOT flavor_list*
         { *mk_on_shell_not* $4 $1 }
 | *momentum_list OFFSHELL flavor_list*
         { *mk_off_shell* $3 $1 }
 | *momentum_list OFFSHELL NOT flavor_list*
         { *mk_off_shell_not* $4 $1 }
 | *momentum_list GAUSS flavor_list* { *mk_gauss* $3 $1 }
 | *momentum_list GAUSS NOT flavor_list*
         { *mk_gauss_not* $4 $1 }

momentum_list ::=
 | *momentum* { [$1] }
 | *momentum_list PLUS momentum* { $3 :: $1 }

momentum ::=
 INT { $1 }

flavor_list ::=
 FLAVOR { [$1] }
 | *flavor_list COLON FLAVOR* { $3 :: $1 }

6.5 Interface of *Cascade*

module type T =
 sig

  type *flavor*
  type p

  type t
  val *of_string_list* : *int* → *string list* → t
  val *to_string* : t → *string*

An opaque type that describes the set of all constraints on an amplitude and how to construct it from a cascade description.

  type *selectors*
  val *to_selectors* : t → *selectors*

Don't throw anything away:

  val *no_cascades* : *selectors*

select_wf s is_timelike f p ps returns true iff either

- the flavor f and momentum p match the selection s or

- *all* combinations of the momenta in *ps* are compatible, i. e. $\pm \sum p_i \leq q$.

The latter test is only required in theories with quartic or higher vertices, where *ps* will be the list of all incoming momenta in a fusion. *is_timelike* is required to determine, whether particles and anti-particles should be distinct.

val *select_wf* : *selectors* → (*p* → *bool*) → *flavor* → *p* → *p list* → *bool*

select_p s p ps same as *select_wf s f p ps*, but ignores the flavor *f*

val *select_p* : *selectors* → *p* → *p list* → *bool*

on_shell s p

val *on_shell* : *selectors* → *flavor* → *p* → *bool*

is_gauss s p

val *is_gauss* : *selectors* → *flavor* → *p* → *bool*

partition s returns a partition of the external particles that can not be reordered without violating the cascade constraints.

val *partition* : *selectors* → *int list list*

Diagnostics:

val *description* : *selectors* → *string option*

  end

module *Make* (*M* : *Model.T*) (*P* : *Momentum.T*) :
   T with type *flavor* = *M.flavor* and type *p* = *P.t*

6.6   *Implementation of Cascade*

module type *T* =
  sig

    type *flavor*
    type *p*

    type *t*
    val *of_string_list* : *int* → *string list* → *t*
    val *to_string* : *t* → *string*

    type *selectors*
    val *to_selectors* : *t* → *selectors*
    val *no_cascades* : *selectors*

    val *select_wf* : *selectors* → (*p* → *bool*) → *flavor* → *p* → *p list* → *bool*
    val *select_p* : *selectors* → *p* → *p list* → *bool*
    val *on_shell* : *selectors* → *flavor* → *p* → *bool*
    val *is_gauss* : *selectors* → *flavor* → *p* → *bool*

    val *partition* : *selectors* → *int list list*
    val *description* : *selectors* → *string option*

  end

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module *Make* (*M* : *Model.T*) (*P* : *Momentum.T*) :
   (*T* with type *flavor* = *M.flavor* and type *p* = *P.t*) =
  struct

    module *CS* = *Cascade_syntax*

    type *flavor* = *M.flavor*
    type *p* = *P.t*

Since we have

$$p \leq q \Longleftrightarrow (-q) \leq (-p) \tag{6.1}$$

also for $\leq$ as set inclusion *lesseq*, only four of the eight combinations are independent

$$
\begin{aligned}
p \leq q &\quad\Longleftrightarrow\quad (-q) \leq (-p) \\
q \leq p &\quad\Longleftrightarrow\quad (-p) \leq (-q) \\
p \leq (-q) &\quad\Longleftrightarrow\quad q \leq (-p) \\
(-q) \leq p &\quad\Longleftrightarrow\quad (-p) \leq q
\end{aligned}
\tag{6.2}
$$

    let *one_compatible p q* =
      let *neg_q* = *P.neg q* in
      P.lesseq p q ∨
      P.lesseq q p ∨
      P.lesseq p neg_q ∨
      P.lesseq neg_q p

'tis wasteful ... (at least by a factor of two, because every momentum combination is generated, including the negative ones.

    let *all_compatible p p_list q* =
      let *l* = *List.length p_list* in
      if *l* ≤ 2 then
        one_compatible p q
      else
        let *tuple_lengths* = *ThoList.range* 2 (*succ l* / 2) in
        let *tuples* = *ThoList.flatmap* (fun *n* → *Combinatorics.choose n p_list*) *tuple_lengths* in
        let *momenta* = *List.map* (*List.fold_left P.add* (*P.zero* (*P.dim q*))) *tuples* in
        List.for_all (*one_compatible q*) *momenta*

The following assumes that the *flavor list* is always very short. Otherwise one should use an efficient set implementation.

    type *t* =
      | *True*
      | *False*
      | *On_shell* of *flavor list* × *P.t*
      | *On_shell_not* of *flavor list* × *P.t*
      | *Off_shell* of *flavor list* × *P.t*
      | *Off_shell_not* of *flavor list* × *P.t*
      | *Gauss* of *flavor list* × *P.t*
      | *Gauss_not* of *flavor list* × *P.t*
      | *Any_flavor* of *P.t*
      | *And* of *t list*

```
let of_string s  =
    Cascade_parser.main Cascade_lexer.token (Lexing.from_string s)
```

⚠ If we knew that we're dealing with a scattering, we could apply *P.flip_s_channel_in* to all momenta, so that $1 + 2$ accepts the particle and not the antiparticle. Right now, we don't have this information.

```
let import dim cascades  =
    let rec import'  = function
      |  CS.True  →
            True
      |  CS.False  →
            False
      |  CS.On_shell (f, p)  →
            On_shell (List.map M.flavor_of_string f, P.of_ints dim p)
      |  CS.On_shell_not (f, p)  →
            On_shell_not (List.map M.flavor_of_string f, P.of_ints dim p)
      |  CS.Off_shell (fs, p)  →
            Off_shell (List.map M.flavor_of_string fs, P.of_ints dim p)
      |  CS.Off_shell_not (fs, p)  →
            Off_shell_not (List.map M.flavor_of_string fs, P.of_ints dim p)
      |  CS.Gauss (f, p)  →
            Gauss (List.map M.flavor_of_string f, P.of_ints dim p)
      |  CS.Gauss_not (f, p)  →
            Gauss (List.map M.flavor_of_string f, P.of_ints dim p)
      |  CS.Any_flavor p  →
            Any_flavor (P.of_ints dim p)
      |  CS.Or cs  →
            invalid_arg "Cascade:␣OR␣patterns␣(||)␣not␣supported␣in␣this␣version!"
      |  CS.And cs  →  And (List.map import' cs) in
    import' cascades

let of_string_list dim strings  =
    match List.map of_string strings with
    | []  →  True
    | first :: next  →
        import dim (List.fold_right CS.mk_and next first)

let flavors_to_string fs  =
    (String.concat ":" (List.map M.flavor_to_string fs))

let momentum_to_string p  =
    String.concat "+" (List.map string_of_int (P.to_ints p))

let rec to_string  = function
    | True  →
        "true"
    | False  →
        "false"
    | On_shell (fs, p)  →
        momentum_to_string p ^ "␣=␣" ^ flavors_to_string fs
    | On_shell_not (fs, p)  →
```

momentum_to_string p ˆ "␣=␣!" ˆ *flavors_to_string fs*
| *Off_shell (fs, p)* →
 momentum_to_string p ˆ "␣˜␣" ˆ *flavors_to_string fs*
| *Off_shell_not (fs, p)* →
 momentum_to_string p ˆ "␣˜␣!" ˆ *flavors_to_string fs*
| *Gauss (fs, p)* →
 momentum_to_string p ˆ "␣#␣" ˆ *flavors_to_string fs*
| *Gauss_not (fs, p)* →
 momentum_to_string p ˆ "␣#␣!" ˆ *flavors_to_string fs*
| *Any_flavor p* →
 momentum_to_string p ˆ "␣˜␣?"
| *And cs* →
 String.concat "␣&&␣" (*List.map* (fun *c* → "(" ˆ *to_string c* ˆ ")") *cs*)

type *selectors* =
 { *select_p* : *p* → *p list* → *bool*;
 select_wf : (*p* → *bool*) → *flavor* → *p* → *p list* → *bool*;
 on_shell : *flavor* → *p* → *bool*;
 is_gauss : *flavor* → *p* → *bool*;
 partition : *int list list*;
 description : *string option* }

let *no_cascades* =
 { *select_p* = (fun _ _ → true);
 select_wf = (fun _ _ _ _ → true);
 on_shell = (fun _ _ → false);
 is_gauss = (fun _ _ → false);
 partition = [];
 description = *None* }

let *select_p s* = *s.select_p*
let *select_wf s* = *s.select_wf*
let *on_shell s* = *s.on_shell*
let *is_gauss s* = *s.is_gauss*
let *partition s* = *s.partition*
let *description s* = *s.description*

let *to_select_p cascades p p_in* =
 let rec *to_select_p′* = function
  | *True* → true
  | *False* → false
  | *On_shell (_, momentum)* | *On_shell_not (_, momentum)*
  | *Off_shell (_, momentum)* | *Off_shell_not (_, momentum)*
  | *Gauss (_, momentum)* | *Gauss_not (_, momentum)*
  | *Any_flavor momentum* → *all_compatible p p_in momentum*
  | *And []* → false
  | *And cs* → *List.for_all to_select_p′ cs* in
 to_select_p′ cascades

let *to_select_wf cascades is_timelike f p p_in* =
 let *f′* = *M.conjugate f* in
 let rec *to_select_wf′* = function
  | *True* → true

```
    | False → false
    | Off_shell (flavors, momentum) →
        if p = momentum then
          List.mem f' flavors ∨ (if is_timelike p then false else List.mem f flavors)
        else if p = P.neg momentum then
          List.mem f flavors ∨ (if is_timelike p then false else List.mem f' flavors)
        else
          one_compatible p momentum ∧ all_compatible p p_in momentum
    | On_shell (flavors, momentum) | Gauss (flavors, momentum) →
        if is_timelike p then begin
          if p = momentum then
            List.mem f' flavors
          else if p = P.neg momentum then
            List.mem f flavors
          else
            one_compatible p momentum ∧ all_compatible p p_in momentum
        end else
          false
    | Off_shell_not (flavors, momentum) →
        if p = momentum then
          ¬ (List.mem f' flavors ∨ (if is_timelike p then false else List.mem f flavors))
        else if p = P.neg momentum then
          ¬ (List.mem f flavors ∨ (if is_timelike p then false else List.mem f' flavors))
        else
          one_compatible p momentum ∧ all_compatible p p_in momentum
    | On_shell_not (flavors, momentum) | Gauss_not (flavors, momentum) →

        if is_timelike p then begin
          if p = momentum then
            ¬ (List.mem f' flavors)
          else if p = P.neg momentum then
            ¬ (List.mem f flavors)
          else
            one_compatible p momentum ∧ all_compatible p p_in momentum
        end else
          false
    | Any_flavor momentum →
        one_compatible p momentum ∧ all_compatible p p_in momentum
    | And [] → false
    | And cs → List.for_all to_select_wf' cs in
  to_select_wf' cascades
```

In case you're wondering: *to_on_shell f p* and *is_gauss f p* only search for on shell conditions and are to be used in a target, not in *Fusion*!

```
let to_on_shell cascades f p =
  let f' = M.conjugate f in
  let rec to_on_shell' = function
    | True | False | Any_flavor _
    | Off_shell (_, _) | Off_shell_not (_, _)
    | Gauss (_, _) | Gauss_not (_, _) → false
```

```
            | On_shell (flavors, momentum) →
                (p = momentum ∨ p = P.neg momentum) ∧ (List.mem f flavors ∨
List.mem f' flavors)
            | On_shell_not (flavors, momentum) →
                (p = momentum ∨ p = P.neg momentum) ∧ ¬ (List.mem f flavors ∨
List.mem f' flavors)
            | And [] → false
            | And cs → List.for_all to_on_shell' cs in
        to_on_shell' cascades

    let to_gauss cascades f p =
        let f' = M.conjugate f in
        let rec to_gauss' = function
            | True | False | Any_flavor _
            | Off_shell (_, _) | Off_shell_not (_, _)
            | On_shell (_, _) | On_shell_not (_, _) → false
            | Gauss (flavors, momentum) →
                (p = momentum ∨ p = P.neg momentum) ∧ (List.mem f flavors ∨
List.mem f' flavors)
            | Gauss_not (flavors, momentum) →
                (p = momentum ∨ p = P.neg momentum) ∧ ¬ (List.mem f flavors ∨
List.mem f' flavors)
            | And [] → false
            | And cs → List.for_all to_gauss' cs in
        to_gauss' cascades
```

Not a working implementation yet, but it isn't used either . . .

```
    module IPowSet =
        PowSet.Make (struct type t = int let compare = compare let to_string = string_of_int end)

    let rec coarsest_partition' = function
        | True | False → IPowSet.empty
        | On_shell (_, momentum) | On_shell_not (_, momentum)
        | Off_shell (_, momentum) | Off_shell_not (_, momentum)
        | Gauss (_, momentum) | Gauss_not (_, momentum)
        | Any_flavor momentum → IPowSet.of_lists [P.to_ints momentum]
        | And [] → IPowSet.empty
        | And cs → IPowSet.basis (IPowSet.union (List.map coarsest_partition' cs))

    let coarsest_partition cascades =
        let p = coarsest_partition' cascades in
        if IPowSet.is_empty p then
            []
        else
            IPowSet.to_lists p

    let part_to_string part =
        "{" ^ String.concat "," (List.map string_of_int part) ^ "}"

    let partition_to_string = function
        | [] → ""
```

```
       |  parts  →
            "␣␣grouping␣{" ^ String.concat "," (List.map part_to_string parts) ^ "}"
  let to_selectors  =  function
     |  True  →  no_cascades
     |  c  →
          let partition  =  coarsest_partition c in
          { select_p  =  to_select_p c;
            select_wf  =  to_select_wf c;
            on_shell  =  to_on_shell c;
            is_gauss  =  to_gauss c;
            partition  =  partition;
            description  =  Some (to_string c ^ partition_to_string partition) }

end
```

—7—
COLOR

7.1 Interface of Color

7.1.1 Quantum Numbers

Color is not necessarily the SU(3) of QCD. Conceptually, it can be any *unbroken* symmetry (*broken* symmetries correspond to *Model.flavor*). In order to keep the group theory simple, we confine ourselves to the fundamental and adjoint representation of a single SU(N_C) for the moment. Therefore, particles are either color singlets or live in the defining representation of SU(N_C): $SUN(|N_C|)$, its conjugate $SUN(-|N_C|)$ or in the adjoint representation of SU(N_C): $AdjSUN(N_C)$.

type $t = Singlet \mid SUN$ of $int \mid AdjSUN$ of int

val $conjugate : t \to t$
val $compare : t \to t \to int$

7.1.2 Color Flows

module type $Flow =$
  sig

    type $color$
    type $t = color\ list \times color\ list$
    val $rank : t \to int$

    val $of\_list : int\ list \to color$
    val $ghost : unit \to color$
    val $to\_lists : t \to int\ list\ list$
    val $in\_to\_lists : t \to int\ list\ list$
    val $out\_to\_lists : t \to int\ list\ list$
    val $ghost\_flags : t \to bool\ list$
    val $in\_ghost\_flags : t \to bool\ list$
    val $out\_ghost\_flags : t \to bool\ list$

A factor is a list of powers

$$\sum_i \left(\frac{num_i}{den_i}\right)^{power_i} \tag{7.1}$$

```
   type power  =  { num  :  int; den  :  int; power  :  int }
   type factor  =  power list

   val factor  :  t  →  t  →  factor
   val zero  :  factor

 end

module Flow  :  Flow
```

7.2   Implementation of Color

7.2.1   Quantum Numbers

```
type t  =
  | Singlet
  | SUN of int
  | AdjSUN of int

let conjugate  =  function
  | Singlet  →  Singlet
  | SUN n  →  SUN (−n)
  | AdjSUN n  →  AdjSUN n

let compare c1  c2  =
  match c1,  c2 with
  | Singlet,  Singlet  →  0
  | Singlet,  _  →  − 1
  | _,  Singlet  →  1
  | SUN n,  SUN n′  →  compare n n′
  | SUN _,  AdjSUN _  →  − 1
  | AdjSUN _,  SUN _  →  1
  | AdjSUN n,  AdjSUN n′  →  compare n n′

module type Line  =
  sig
    type t
    val conj  :  t  →  t
    val equal  :  t  →  t  →  bool
    val to_string  :  t  →  string
  end

module type Cycles  =
  sig

    type line
    type t  =  (line  ×  line) list
```

Contract the graph by connecting lines and return the number of cycles together
with the contracted graph.

The semantics of the contracted graph is not yet 100%ly fixed.

```
    val contract  :  t  →  int × t
```

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The same as *contract*, but returns only the number of cycles and raises *Open_line* when not all lines are closed.

```
val count : t → int
exception Open_line
```

Mainly for debugging . . .

```
val to_string : t → string

end
```

```
module Cycles (L : Line) : Cycles with type line = L.t =
  struct

  type line = L.t
  type t = (line × line) list

  exception Open_line
```

NB: The following algorithm for counting the cycles is quadratic since it performs nested scans of the lists. If this was a serious problem one could replace the lists of pairs by a *Map* and replace one power by a logarithm.

```
let rec find_fst c_final c1 disc seen = function
  | [] → ((L.conj c_final, c1) :: disc, List.rev seen)
  | (c1', c2') as c12' :: rest →
      if L.equal c1 c1' then
        find_snd c_final (L.conj c2') disc [] (List.rev_append seen rest)
      else
        find_fst c_final c1 disc (c12' :: seen) rest

and find_snd c_final c2 disc seen = function
  | [] → ((L.conj c_final, L.conj c2) :: disc, List.rev seen)
  | (c1', c2') as c12' :: rest →
      if L.equal c2' c2 then begin
        if L.equal c1' c_final then
          (disc, List.rev_append seen rest)
        else
          find_fst c_final (L.conj c1') disc [] (List.rev_append seen rest)
      end else
        find_snd c_final c2 disc (c12' :: seen) rest

let consume = function
  | [] → ([], [])
  | (c1, c2) :: rest → find_snd (L.conj c1) (L.conj c2) [] [] rest

let contract lines =
  let rec contract' acc disc = function
    | [] → (acc, List.rev disc)
    | rest →
        begin match consume rest with
        | [], rest' → contract' (succ acc) disc rest'
        | disc', rest' → contract' acc (List.rev_append disc' disc) rest'
        end in
  contract' 0 [] lines
```

```
  let count lines  =
    match contract lines with
    | n, []  →  n
    | n, _  →  raise Open_line

  let to_string lines  =
    String.concat ""
      (List.map
        (fun (c1, c2)  →  "[" ˆ L.to_string c1 ˆ "," ˆ L.to_string c2 ˆ "]")
        lines)

end
```

7.2.2   Color Flows

```
module type Flow  =
  sig
    type color
    type t  =  color list × color list
    val rank  :  t  →  int
    val of_list  :  int list  →  color
    val ghost  :  unit  →  color
    val to_lists  :  t  →  int list list
    val in_to_lists  :  t  →  int list list
    val out_to_lists  :  t  →  int list list
    val ghost_flags  :  t  →  bool list
    val in_ghost_flags  :  t  →  bool list
    val out_ghost_flags  :  t  →  bool list
    type power  =  { num  :  int; den  :  int; power  :  int }
    type factor  =  power list
    val factor  :  t  →  t  →  factor
    val zero  :  factor
  end

module Flow (∗ : Flow ∗) =
  struct

    type color  =
      | Lines of int × int
      | Ghost

    type t  =  color list × color list

    let rank cflow  =
      2
```

Constructors

```
    let ghost ()  =
      Ghost

    let of_list  =  function
```

```
    | [c1; c2]  →  Lines (c1, c2)
    | _  →  invalid_arg "Color.Flow.of_list:␣num_lines␣!=␣2"
```

let *to_list* = function
   | *Lines* (*c1*, *c2*) → [*c1*; *c2*]
   | *Ghost* → [0; 0]

let *to_lists* (*cfin*, *cfout*) =
  (*List.map to_list cfin*) @ (*List.map to_list cfout*)

let *in_to_lists* (*cfin*, _) =
  List.map to_list cfin

let *out_to_lists* (_, *cfout*) =
  List.map to_list cfout

let *ghost_flag* = function
   | *Lines* _ → false
   | *Ghost* → true

let *ghost_flags* (*cfin*, *cfout*) =
  (*List.map ghost_flag cfin*) @ (*List.map ghost_flag cfout*)

let *in_ghost_flags* (*cfin*, _) =
  List.map ghost_flag cfin

let *out_ghost_flags* (_, *cfout*) =
  List.map ghost_flag cfout

Evaluation

type *power* = { *num* : *int*; *den* : *int*; *power* : *int* }
type *factor* = *power list*
let *zero* = [ ]

let *count_ghosts1 colors* =
  List.fold_left
    (fun *acc* → function *Ghost* → *succ acc* | _ → *acc*)
    0 *colors*

let *count_ghosts* (*fin*, *fout*) =
  count_ghosts1 fin + *count_ghosts1 fout*

type α *square* =
   | *Square* of α
   | *Mismatch*

let *conjugate* = function
   | *Lines* (*c1*, *c2*) → *Lines* (−*c2*, − *c1*)
   | *Ghost* → *Ghost*

let *cross_in* (*cin*, *cout*) =
  cin @ (*List.map conjugate cout*)

let *cross_out* (*cin*, *cout*) =
  (*List.map conjugate cin*) @ *cout*

```
module C  =  Cycles (struct
  type t  =  int
  let conj  =  (−)
  let equal  =  (=)
  let to_string  =  string_of_int
end)

let square f1 f2  =
  let rec square' acc f1' f2'  =
    match f1', f2' with
    | [], []  →  Square (List.rev acc)
    | _, [] | [], _  →  Mismatch
    | Ghost :: rest1, Ghost :: rest2  →
        square' acc rest1 rest2
    | Lines (0, 0) :: rest1, Lines (0, 0) :: rest2  →
        square' acc rest1 rest2
    | Lines (0, c1') :: rest1, Lines (0, c2') :: rest2  →
        square' ((c1', c2') :: acc) rest1 rest2
    | Lines (c1, 0) :: rest1, Lines (c2, 0) :: rest2  →
        square' ((c1, c2) :: acc) rest1 rest2
    | Lines (0, _) :: _, _ | _, Lines (0, _) :: _
    | Lines (_, 0) :: _, _ | _, Lines (_, 0) :: _  →  Mismatch
    | Lines (_, _) :: _, Ghost :: _ | Ghost :: _, Lines (_, _) :: _  →
Mismatch
    | Lines (c1, c1') :: rest1, Lines (c2, c2') :: rest2  →
        square' ((c1', c2') :: (c1, c2) :: acc) rest1 rest2 in
  square' [] (cross_out f1) (cross_out f2)
```

In addition to counting closed color loops, we also need to count closed gluon loops. Fortunately, we can use the same algorithm on a different data type, provided it doesn't require all lines to be closed.

```
module C2  =  Cycles (struct
  type t  =  int × int
  let conj (c1, c2)  =  (− c2,  − c1)
  let equal (c1, c2) (c1', c2')  =  c1 = c1' ∧ c2 = c2'
  let to_string (c1, c2)  =  "(" ^ string_of_int c1 ^ "," ^ string_of_int c2 ^ ")"
end)

let square2 f1 f2  =
  let rec square2' acc f1' f2'  =
    match f1', f2' with
    | [], []  →  Square (List.rev acc)
    | _, [] | [], _  →  Mismatch
    | Ghost :: rest1, Ghost :: rest2  →
        square2' acc rest1 rest2
    | Lines (0, 0) :: rest1, Lines (0, 0) :: rest2  →
        square2' acc rest1 rest2
    | Lines (0, _) :: rest1, Lines (0, _) :: rest2
    | Lines (_, 0) :: rest1, Lines (_, 0) :: rest2  →
        square2' acc rest1 rest2
    | Lines (0, _) :: _, _ | _, Lines (0, _) :: _
```

```
        | Lines (_, 0) :: _, _  |  _, Lines (_, 0) :: _ → Mismatch
        | Lines (_, _) :: _, Ghost :: _  |  Ghost :: _, Lines (_, _) :: _ →
Mismatch
        | Lines (c1, c1') :: rest1, Lines (c2, c2') :: rest2 →
            square2' (((c1, c1'), (c2, c2')) :: acc) rest1 rest2 in
      square2' [] (cross_out f1) (cross_out f2)
```

$int\_power : n\,p \to n^p$ for integers is missing from *Pervasives*!

```
    let int_power n p =
      let rec int_power' acc i =
        if i < 0 then
          invalid_arg "int_power"
        else if i = 0 then
          acc
        else
          int_power' (n × acc) (pred i) in
      int_power' 1 p
```

Instead of implementing a full fledged algebraic evaluator, let's simply expand the binomial by hand:

$$\left(\frac{N_C^2 - 2}{N_C^2}\right)^n = \sum_{i=0}^{n} \binom{n}{i} (-2)^i N_C^{-2i} \qquad (7.2)$$

NB: Any result of *square* other than *Mismatch* guarantees $count\_ghosts\ f1 = count\_ghosts\ f2$.

```
    let factor f1 f2 =
      match square f1 f2, square2 f1 f2 with
      | Mismatch, _  |  _, Mismatch → []
      | Square f12, Square f12' →
          let num_cycles = C.count f12
          and num_cycles2, disc = C2.contract f12'
          and num_ghosts = count_ghosts f1 in
          List.map
            (fun i →
              let parity = if num_ghosts mod 2 = 0 then 1 else -1
              and power = num_cycles − num_ghosts in
              let coeff = int_power (−2) i × Combinatorics.binomial num_cycles2 i
              and power2 = − 2 × i in
              { num = parity × coeff;
                den = 1;
                power = power + power2 })
            (ThoList.range 0 num_cycles2)
  end
```

later:

```
module General_Flow =
  struct

    type color =
      | Lines of int list
```

```
    |  Ghost of int
type t  =  color list × color list
let rank_default  =  2 (∗ Standard model ∗)
let rank cflow  =
   try
      begin match List.hd cflow with
      |  Lines lines  →  List.length lines
      |  Ghost n_lines  →  n_lines
      end
   with
   |  _  →  rank_default
end
```

—8—
FUSIONS

8.1 Interface of Fusion

module type T =
  sig

    val *options* : *Options.t*

Wavefunctions are an abstract data type, containing a momentum p and additional quantum numbers, collected in *flavor*.

    type *wf*
    val *conjugate* : *wf* → *wf*

Obviously, *flavor* is not restricted to the physical notion of flavor, but can carry spin, color, etc.

    type *flavor*
    val *flavor* : *wf* → *flavor*
    type *flavor_sans_color*
    val *flavor_sans_color* : *wf* → *flavor_sans_color*

Momenta are represented by an abstract datatype (defined in *Momentum*) that is optimized for performance. They can be accessed either abstractly or as lists of indices of the external momenta. These indices are assigned sequentially by *amplitude* below.

    type p
    val *momentum* : *wf* → p
    val *momentum_list* : *wf* → *int list*

At tree level, the wave functions are uniquely specified by *flavor* and momentum. If loops are included, we need to distinguish among orders. Also, if we build a result from an incomplete sum of diagrams, we need to add a distinguishing mark. At the moment, we assume that a *string* that can be attached to the symbol suffices.

    val *wf_tag* : *wf* → *string option*

Coupling constants

    type *constant*

and right hand sides of assignments. The latter are formed from a sign from Fermi statistics, a coupling (constand and Lorentz structure) and wave functions.

> type *coupling*
> type *rhs*
> type α *children*
> val *sign* : *rhs* $\rightarrow$ *int*
> val *coupling* : *rhs* $\rightarrow$ *constant Coupling.t*
>
> val *coupling_tag* : *rhs* $\rightarrow$ *string option*

In renormalized perturbation theory, couplings come in different orders of the loop expansion. Be prepared: val *order* : *rhs* $\rightarrow$ *int*

This is here only for the benefit of *Target* and shall become val *children* : *rhs* $\rightarrow$ *wf children* later . . .

> val *children* : *rhs* $\rightarrow$ *wf list*

Fusions come in two types: fusions of wave functions to off-shell wave functions:

$$\phi(p + q) = \phi(p)\phi(q)$$

> type *fusion*
> val *lhs* : *fusion* $\rightarrow$ *wf*
> val *rhs* : *fusion* $\rightarrow$ *rhs list*

and products at the keystones:

$$\phi(-p - q) \cdot \phi(p)\phi(q)$$

> type *braket*
> val *bra* : *braket* $\rightarrow$ *wf*
> val *ket* : *braket* $\rightarrow$ *rhs list*

amplitude goldstones incoming outgoing calculates the amplitude for scattering of *incoming* to *outgoing*. If *goldstones* is true, also non-propagating off-shell Goldstone amplitudes are included to allow the checking of Slavnov-Taylor identities.

> type *amplitude*
> type *selectors*
> val *amplitudes* : *bool* $\rightarrow$ *selectors* $\rightarrow$
>   *flavor_sans_color list* $\rightarrow$ *flavor_sans_color list* $\rightarrow$ *amplitude list*
>
> val *dependencies* : *amplitude* $\rightarrow$ *wf* $\rightarrow$ (*wf*, *coupling*) *Tree2.t*

We should be precise regarding the semantics of the following functions, since modules implementating *Target* must not make any mistakes interpreting the return values. Instead of calculating the amplitude

$$\langle f_3, p_3, f_4, p_4, \ldots | T | f_1, p_1, f_2, p_2 \rangle \tag{8.1a}$$

directly, O'Mega calculates the—equivalent, but more symmetrical—crossed amplitude

$$\langle \bar{f}_1, -p_1, \bar{f}_2, -p_2, f_3, p_3, f_4, p_4, \ldots | T | 0 \rangle \tag{8.1b}$$

Internally, all flavors are represented by their charge conjugates

$$A(f_1, -p_1, f_2, -p_2, \bar{f}_3, p_3, \bar{f}_4, p_4, \ldots) \tag{8.1c}$$

The correspondence of vertex and term in the lagrangian
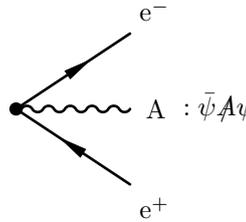
$$\text{A} \quad : \bar{\psi} A\!\!\!/\, \psi \tag{8.2}$$

suggests to denote the *outgoing* particle by the flavor of the *anti*particle and the *outgoing anti*particle by the flavor of the particle, since this choice allows to represent the vertex by a triple

$$\bar{\psi} A\!\!\!/\, \psi : (\mathrm{e}^+, A, \mathrm{e}^-) \tag{8.3}$$

which is more intuitive than the alternative $(\mathrm{e}^-, A, \mathrm{e}^+)$. Also, when thinking in terms of building wavefunctions from the outside in, the outgoing *antiparticle* is represented by a *particle* propagator and vice versa[1]. *incoming* and *outgoing* are the physical flavors as in (8.1a)

> val *incoming* : *amplitude* → *flavor list*
> val *outgoing* : *amplitude* → *flavor list*

externals are flavors and momenta as in (8.1c)

> val *externals* : *amplitude* → *wf list*

> val *variables* : *amplitude* → *wf list*
> val *fusions* : *amplitude* → *fusion list*
> val *brakets* : *amplitude* → *braket list*
> val *on_shell* : *amplitude* → (*wf* → *bool*)
> val *is_gauss* : *amplitude* → (*wf* → *bool*)
> val *constraints* : *amplitude* → *string option*
> val *symmetry* : *amplitude* → *int*

> val *allowed* : *amplitude* → *bool*

Performance Hacks

> val *initialize_cache* : *string* → *unit*
> val *set_cache_name* : *string* → *unit*

Diagnostics

> val *check_charges* : *unit* → *flavor_sans_color list list*
> val *count_fusions* : *amplitude* → *int*

---

[1]Even if this choice will appear slightly counter-intuitive on the *Target* side, one must keep in mind that much more people are expected to prepare *Model*s.

```
    val count_propagators  :  amplitude  →  int
    val count_diagrams  :  amplitude  →  int

    val forest  :  wf  →  amplitude  →  ((wf  ×  coupling option, wf) Tree.t) list
    val poles  :  amplitude  →  wf list list
    val s_channel  :  amplitude  →  wf list

    val tower_to_dot  :  out_channel  →  amplitude  →  unit
    val amplitude_to_dot  :  out_channel  →  amplitude  →  unit

    val rcs_list  :  RCS.t list
  end
```

There is more than one way to make fusions.

```
module type Maker  =
    functor (P  :  Momentum.T)  →  functor (M  :  Model.T)  →
      T with type p  =  P.t
      and type flavor  =  Colorize.It(M).flavor
      and type flavor_sans_color  =  M.flavor
      and type constant  =  M.constant
      and type selectors  =  Cascade.Make(M)(P).selectors
```

Straightforward Dirac fermions vs. slightly more complicated Majorana fermions:

```
module Binary  :  Maker
module Binary_Majorana  :  Maker

module Mixed23  :  Maker
module Mixed23_Majorana  :  Maker

module Nary  :  functor (B  :  Tuple.Bound)  →  Maker
module Nary_Majorana  :  functor (B  :  Tuple.Bound)  →  Maker
```

We can also proceed á la [2]. Empirically, this will use slightly ($O(10\%)$) fewer fusions than the symmetric factorization. Our implementation uses significantly ($O(50\%)$) fewer fusions than reported by [2]. Our pruning of the DAG might be responsible for this.

```
module Helac  :  functor (B  :  Tuple.Bound)  →  Maker
module Helac_Majorana  :  functor (B  :  Tuple.Bound)  →  Maker
```

8.1.1   Multiple Amplitudes

```
module type Multi  =
  sig
    exception Mismatch
    val options  :  Options.t

    type flavor
    type process  =  flavor list  ×  flavor list
    type amplitude
    type fusion
    type wf
    type selectors
```

type *amplitudes*

Construct all possible color flow amplitudes for a given process.

val *amplitudes* : *bool* → *int option* → *selectors* → *process list* → *amplitudes*
val *empty* : *amplitudes*

Precompute the vertex table cache.

val *initialize_cache* : *string* → *unit*
val *set_cache_name* : *string* → *unit*

The list of all combinations of incoming and outgoing particles with a nonvanishing scattering amplitude.

val *flavors* : *amplitudes* → *process list*

The list of all combinations of incoming and outgoing particles that don't lead to any color flow with non vanishing scattering amplitude.

val *vanishing_flavors* : *amplitudes* → *process list*

The list of all color flows with a nonvanishing scattering amplitude.

val *color_flows* : *amplitudes* → *Color.Flow.t list*

The list of all valid helicity combinations.

val *helicities* : *amplitudes* → (*int list* × *int list*) *list*

The list of all amplitudes.

val *processes* : *amplitudes* → *amplitude list*

(*process_table a*).(*f*).(*c*) returns the amplitude for the *f*th allowed flavor combination and the *c*th allowed color flow as an *amplitude option*.

val *process_table* : *amplitudes* → *amplitude option array array*

The list of all non redundant fusions together with the amplitudes they came from.

val *fusions* : *amplitudes* → (*fusion* × *amplitude*) *list*

If there's more than external flavor state, the wavefunctions are *not* uniquely specified by *flavor* and *Momentum.t*. This function can be used to determine how many variables must be allocated.

val *multiplicity* : *amplitudes* → *wf* → *int*

This function can be used to disambiguate wavefunctions with the same combination of *flavor* and *Momentum.t*.

val *dictionary* : *amplitudes* → *amplitude* → *wf* → *int*

(*color_factors a*).(*c1*).(*c2*) power of N_C for the given product of color flows.

val *color_factors* : *amplitudes* → *Color.Flow.factor array array*

A description of optional diagram selectors.

val *constraints* : *amplitudes* → *string option*

end

module type *Multi_Maker* = functor (*Fusion_Maker* : *Maker*) →
  functor (*P* : *Momentum.T*) →
    functor (*M* : *Model.T*) →
      Multi with type *flavor* = *M.flavor*
      and type *amplitude* = *Fusion_Maker(P)(M).amplitude*
      and type *fusion* = *Fusion_Maker(P)(M).fusion*
      and type *wf* = *Fusion_Maker(P)(M).wf*
      and type *selectors* = *Fusion_Maker(P)(M).selectors*

module *Multi* : *Multi_Maker*

8.1.2   Tags

It appears that there are useful applications for tagging couplings and wave
functions, e. g. skeleton expansion and diagram selections. We can abstract this
in a *Tags* signature:

module type *Tags* =
  sig
    type *wf*
    type *coupling*
    type *α children*
    val *null_wf* : *wf*
    val *null_coupling* : *coupling*
    val *fuse* : *coupling* → *wf children* → *wf*
    val *wf_to_string* : *wf* → *string option*
    val *coupling_to_string* : *coupling* → *string option*
  end

module type *Tagger* =
    functor (*PT* : *Tuple.Poly*) → *Tags* with type *α children* = *α PT.t*

module type *Tagged_Maker* =
    functor (*Tagger* : *Tagger*) →
      functor (*P* : *Momentum.T*) → functor (*M* : *Model.T*) →
        T with type *p* = *P.t*
        and type *flavor* = *Colorize.It(M).flavor*
        and type *flavor_sans_color* = *M.flavor*
        and type *constant* = *M.constant*

module *Tagged_Binary* : *Tagged_Maker*

8.2   Implementation of Fusion

let *rcs_file* = *RCS.parse* "Fusion" ["General␣Fusions"]
    { *RCS.revision* = "$Revision:␣7377␣$";
      RCS.date = "$Date:␣2015-11-20␣16:59:02␣+0100␣(Fri,␣20␣Nov␣2015)␣$";
      RCS.author = "$Author:␣jr_reuter␣$";
      RCS.source

= "\$URL:␣svn+ssh://jr␣reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s

module type T =
  sig
    val *options* : *Options.t*
    type *wf*
    val *conjugate* : *wf* → *wf*
    type *flavor*
    type *flavor␣sans␣color*
    val *flavor* : *wf* → *flavor*
    val *flavor␣sans␣color* : *wf* → *flavor␣sans␣color*
    type *p*
    val *momentum* : *wf* → *p*
    val *momentum␣list* : *wf* → *int list*
    val *wf␣tag* : *wf* → *string option*
    type *constant*
    type *coupling*
    type *rhs*
    type *α children*
    val *sign* : *rhs* → *int*
    val *coupling* : *rhs* → *constant Coupling.t*
    val *coupling␣tag* : *rhs* → *string option*
    val *children* : *rhs* → *wf list*
    type *fusion*
    val *lhs* : *fusion* → *wf*
    val *rhs* : *fusion* → *rhs list*
    type *braket*
    val *bra* : *braket* → *wf*
    val *ket* : *braket* → *rhs list*
    type *amplitude*
    type *selectors*
    val *amplitudes* : *bool* → *selectors* →
      flavor␣sans␣color list → *flavor␣sans␣color list* → *amplitude list*
    val *dependencies* : *amplitude* → *wf* → (*wf, coupling*) *Tree2.t*
    val *incoming* : *amplitude* → *flavor list*
    val *outgoing* : *amplitude* → *flavor list*
    val *externals* : *amplitude* → *wf list*
    val *variables* : *amplitude* → *wf list*
    val *fusions* : *amplitude* → *fusion list*
    val *brakets* : *amplitude* → *braket list*
    val *on␣shell* : *amplitude* → (*wf* → *bool*)
    val *is␣gauss* : *amplitude* → (*wf* → *bool*)
    val *constraints* : *amplitude* → *string option*
    val *symmetry* : *amplitude* → *int*
    val *allowed* : *amplitude* → *bool*
    val *initialize␣cache* : *string* → *unit*
    val *set␣cache␣name* : *string* → *unit*
    val *check␣charges* : *unit* → *flavor␣sans␣color list list*
    val *count␣fusions* : *amplitude* → *int*
    val *count␣propagators* : *amplitude* → *int*

```
    val count_diagrams : amplitude → int
    val forest : wf → amplitude → ((wf × coupling option, wf) Tree.t) list
    val poles : amplitude → wf list list
    val s_channel : amplitude → wf list
    val tower_to_dot : out_channel → amplitude → unit
    val amplitude_to_dot : out_channel → amplitude → unit
    val rcs_list : RCS.t list
  end

module type Maker =
    functor (P : Momentum.T) → functor (M : Model.T) →
      T with type p = P.t
      and type flavor = Colorize.It(M).flavor
      and type flavor_sans_color = M.flavor
      and type constant = M.constant
      and type selectors = Cascade.Make(M)(P).selectors
```

8.2.1 Fermi Statistics

```
module type Stat =
  sig
    type flavor
    type stat
    exception Impossible
    val stat : flavor → int → stat
    val stat_fuse : stat → stat → flavor → stat
    val stat_sign : stat → int
    val rcs : RCS.t
  end

module type Stat_Maker = functor (M : Model.T) →
  Stat with type flavor = M.flavor
```

8.2.2 Dirac Fermions

```
module Stat_Dirac (M : Model.T) : (Stat with type flavor = M.flavor) =
  struct
    let rcs = RCS.rename rcs_file "Fusion.Stat_Dirac()"
        [ "Fermi␣statistics␣for␣Dirac␣fermions"]

    type flavor = M.flavor
```

$$\gamma_\mu \psi(1)\, G^{\mu\nu}\, \bar{\psi}(2)\gamma_\nu\psi(3) - \gamma_\mu\psi(3)\, G^{\mu\nu}\, \bar{\psi}(2)\gamma_\nu\psi(1) \tag{8.4}$$

```
    type stat =
      | Fermion of int × (int option × int option) list
      | AntiFermion of int × (int option × int option) list
      | Boson of (int option × int option) list

    let stat f p =
```
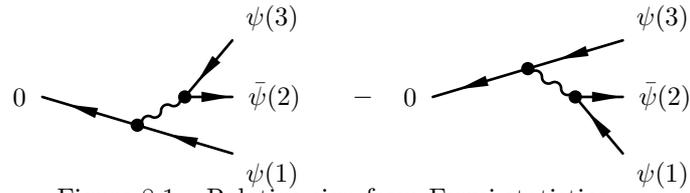
Figure 8.1: Relative sign from Fermi statistics.

```
let s = M.fermion f in
if s = 0 then
    Boson []
else if s < 0 then
    AntiFermion (p, [])
else (* if s > 0 then *)
    Fermion (p, [])
```

```
exception Impossible
```

```
let stat_fuse s1 s2 f =
  match s1, s2 with
  | Boson l1, Boson l2  →  Boson (l1 @ l2)
  | Boson l1, Fermion (p, l2)  →  Fermion (p, l1 @ l2)
  | Boson l1, AntiFermion (p, l2)  →  AntiFermion (p, l1 @ l2)
  | Fermion (p, l1), Boson l2  →  Fermion (p, l1 @ l2)
  | AntiFermion (p, l1), Boson l2  →  AntiFermion (p, l1 @ l2)
  | AntiFermion (pbar, l1), Fermion (p, l2)  →
      Boson ((Some pbar, Some p) :: l1 @ l2)
  | Fermion (p, l1), AntiFermion (pbar, l2)  →
      Boson ((Some pbar, Some p) :: l1 @ l2)
  | Fermion _, Fermion _ | AntiFermion _, AntiFermion _  →
      raise Impossible
```

$$\epsilon\left(\{(0,1),(2,3)\}\right) = -\epsilon\left(\{(0,3),(2,1)\}\right) \tag{8.5}$$

```
let permutation lines =
  let fout, fin = List.split lines in
  let eps_in, _ = Combinatorics.sort_signed compare fin
  and eps_out, _ = Combinatorics.sort_signed compare fout in
  (eps_in × eps_out)
```

This comparing of permutations of fermion lines is a bit tedious and takes a macroscopic fraction of time. However, it's less than 20 %, so we don't focus on improving on it yet.

```
let stat_sign = function
  | Boson lines  →  permutation lines
  | Fermion (p, lines)  →  permutation ((None, Some p) :: lines)
  | AntiFermion (pbar, lines)  →  permutation ((Some pbar, None) :: lines)
```

```
end
```

8.2.3   Tags

module type *Tags* =
  sig
    type *wf*
    type *coupling*
    type α *children*
    val *null_wf* : *wf*
    val *null_coupling* : *coupling*
    val *fuse* : *coupling* $\to$ *wf children* $\to$ *wf*
    val *wf_to_string* : *wf* $\to$ *string option*
    val *coupling_to_string* : *coupling* $\to$ *string option*
  end

module type *Tagger* =
    functor (*PT* : *Tuple.Poly*) $\to$ *Tags* with type α *children* = α *PT.t*

module type *Tagged_Maker* =
    functor (*Tagger* : *Tagger*) $\to$
      functor (*P* : *Momentum.T*) $\to$ functor (*M* : *Model.T*) $\to$
        T with type *p* = *P.t*
        and type *flavor* = *Colorize.It(M).flavor*
        and type *flavor_sans_color* = *M.flavor*
        and type *constant* = *M.constant*

No tags is one option for good tags . . .

module *No_Tags* (*PT* : *Tuple.Poly*) =
  struct
    type *wf* = *unit*
    type *coupling* = *unit*
    type α *children* = α *PT.t*
    let *null_wf* = ()
    let *null_coupling* = ()
    let *fuse* () _ = ()
    let *wf_to_string* () = *None*
    let *coupling_to_string* () = *None*
  end

Here's a simple additive tag that can grow into something useful for loop calculations.

module *Loop_Tags* (*PT* : *Tuple.Poly*) =
  struct
    type *wf* = *int*
    type *coupling* = *int*
    type α *children* = α *PT.t*
    let *null_wf* = 0
    let *null_coupling* = 0
    let *fuse c wfs* = *PT.fold_left* (+) *c wfs*
    let *wf_to_string n* = *Some* (*string_of_int n*)
    let *coupling_to_string n* = *Some* (*string_of_int n*)

end

module *Order_Tags* (*PT* : *Tuple.Poly*) =
  struct
    type *wf* = *int*
    type *coupling* = *int*
    type α *children* = α *PT.t*
    let *null_wf* = 0
    let *null_coupling* = 0
    let *fuse c wfs* = *PT.fold_left* (+) *c wfs*
    let *wf_to_string n* = *Some* (*string_of_int n*)
    let *coupling_to_string n* = *Some* (*string_of_int n*)
  end

8.2.4 Tagged, the Fusion.Make Functor

module *Tagged* (*Tagger* : *Tagger*) (*PT* : *Tuple.Poly*)
    (*Stat* : *Stat_Maker*) (*T* : *Topology.T* with type α *children* = α *PT.t*)
    (*P* : *Momentum.T*) (*M* : *Model.T*) =
  struct
    let *rcs* = *RCS.rename rcs_file* `"Fusion.Make()"`
       [`"Fusions␣for␣arbitrary␣topologies"`]

    type *cache_mode* = *Cache_Use* | *Cache_Ignore* | *Cache_Overwrite*
    let *cache_option* = *ref Cache_Use*
    type *qcd_order* =
      | *QCD_order* of *int*
    type *ew_order* =
      | *EW_order* of *int*
    let *qcd_order* = *ref* (*QCD_order* 99)
    let *ew_order* = *ref* (*EW_order* 99)

    let *options* = *Options.create*
      [`"ignore-cache"`, *Arg.Unit* (fun () $\rightarrow$ *cache_option* := *Cache_Ignore*),
        `"ignore␣cached␣model␣tables"`;
        `"overwrite-cache"`, *Arg.Unit* (fun () $\rightarrow$ *cache_option* := *Cache_Overwrite*),
        `"overwrite␣cached␣model␣tables"`;
        `"qcd"`, *Arg.Int* (fun *n* $\rightarrow$ *qcd_order* := *QCD_order n*),
        `"set␣QCD␣order␣n␣[>=␣0,␣default␣=␣99]"`;
        `"ew"`, *Arg.Int* (fun *n* $\rightarrow$ *ew_order* := *EW_order n*),
        `"set␣QCD␣order␣n␣[>=0,␣default␣=␣99]"`]

    exception *Negative_QCD_order*
    exception *Negative_EW_order*
    exception *Vanishing_couplings*
    exception *Negative_QCD_EW_orders*

    let *int_orders* =
      match !*qcd_order*, !*ew_order* with
        | *QCD_order n*, *EW_order n'* when $n < 0 \wedge n' \geq 0 \rightarrow$
          raise *Negative_QCD_order*
        | *QCD_order n*, *EW_order n'* when $n \geq 0 \wedge n' < 0 \rightarrow$

```
        raise Negative_EW_order
    |  QCD_order n,  EW_order n' when n < 0 ∧ n' < 0 →
        raise Negative_QCD_EW_orders
    |  QCD_order n,  EW_order n' → (n, n')
```

open *Coupling*

module $S = Stat(M)$

type $stat = S.stat$
let $stat = S.stat$
let $stat\_sign = S.stat\_sign$

⬙ This will do *something* for 4-, 6-, ...fermion vertices, but not necessarily
the right thing ...

let $stat\_fuse\ s\ f$ =
   $PT.fold\_right\_internal$ (fun $s'\ acc$ → $S.stat\_fuse\ s'\ acc\ f$) s

type $constant = M.constant$

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Wave Functions

</div>

⬙ The code below is not yet functional. Too often, we assign to *Tags.null_wf*
instead of calling *Tags.fuse*.

We will need two types of amplitudes: with color and without color. Since we
can build them using the same types with only *flavor* replaced, it pays to use a
functor to set up the scaffolding.

module $Tags = Tagger(PT)$

In the future, we might want to have *Coupling* among the functor arguments.
However, for the moment, *Coupling* is assumed to be comprehensive.

```
module type Tagged_Coupling =
  sig
    type sign = int
    type t =
        { sign : sign;
          coupling : constant Coupling.t;
          coupling_tag : Tags.coupling }
    val sign : t → sign
    val coupling : t → constant Coupling.t
    val coupling_tag : t → string option
  end
module Tagged_Coupling : Tagged_Coupling =
  struct
    type sign = int
    type t =
        { sign : sign;
          coupling : constant Coupling.t;
```

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</div>

```
            coupling_tag  :  Tags.coupling }
      let sign c  =  c.sign
      let coupling c  =  c.coupling
      let coupling_tag_raw c  =  c.coupling_tag
      let coupling_tag rhs  =  Tags.coupling_to_string (coupling_tag_raw rhs)
    end
```

Amplitudes: Monochrome and Colored

```
module type Amplitude  =
  sig

    module Tags  :  Tags

    type flavor
    type p

    type wf  =
        { flavor  :  flavor;
          momentum  :  p;
          wf_tag  :  Tags.wf }

    val flavor  :  wf  →  flavor
    val conjugate  :  wf  →  wf
    val momentum  :  wf  →  p
    val momentum_list  :  wf  →  int list
    val wf_tag  :  wf  →  string option
    val wf_tag_raw  :  wf  →  Tags.wf
    val order_wf  :  wf  →  wf  →  int
    val external_wfs  :  int →  (flavor  ×  int) list →  wf list

    type α children
    type coupling  =  Tagged_Coupling.t
    type rhs  =  coupling  ×  wf children
    val sign  :  rhs  →  int
    val coupling  :  rhs  →  constant Coupling.t
    val coupling_tag  :  rhs  →  string option
    val children  :  rhs  →  wf list

    type fusion  =  wf  ×  rhs list
    val lhs  :  fusion  →  wf
    val rhs  :  fusion  →  rhs list

    type braket  =  wf  ×  rhs list
    val bra  :  braket  →  wf
    val ket  :  braket  →  rhs list

    module D  :
        DAG.T with type node  =  wf and type edge  =  coupling and type children  =  wf children

    val wavefunctions  :  braket list →  wf list

    type amplitude  =
        { fusions  :  fusion list;
```

```
            brakets  :  braket list;
            on_shell  :  (wf  →  bool);
            is_gauss  :  (wf  →  bool);
            constraints  :  string option;
            incoming  :  flavor list;
            outgoing  :  flavor list;
            externals  :  wf list;
            symmetry  :  int;
            dependencies  :  (wf  →  (wf, coupling) Tree2.t);
            fusion_tower  :  D.t;
            fusion_dag  :  D.t }
```

val *incoming* : *amplitude* → *flavor list*
val *outgoing* : *amplitude* → *flavor list*
val *externals* : *amplitude* → *wf list*
val *variables* : *amplitude* → *wf list*
val *fusions* : *amplitude* → *fusion list*
val *brakets* : *amplitude* → *braket list*
val *on_shell* : *amplitude* → (*wf* → *bool*)
val *is_gauss* : *amplitude* → (*wf* → *bool*)
val *constraints* : *amplitude* → *string option*
val *symmetry* : *amplitude* → *int*
val *dependencies* : *amplitude* → *wf* → (*wf*, *coupling*) *Tree2.t*
val *fusion_dag* : *amplitude* → *D.t*

end

module *Amplitude* (*PT* : *Tuple.Poly*) (*P* : *Momentum.T*) (*M* : *Model.T*) :
    Amplitude
    with type *p* = *P.t*
    and type *flavor* = *M.flavor*
    and type *α children* = *α PT.t*
    and module *Tags* = *Tags* =
  struct

    type *flavor* = *M.flavor*
    type *p* = *P.t*

    module *Tags* = *Tags*

    type *wf* =
        { *flavor* : *flavor*;
          momentum : *p*;
          wf_tag : *Tags.wf* }

    let *flavor wf* = *wf.flavor*
    let *conjugate wf* = { *wf* with *flavor* = *M.conjugate wf.flavor* }
    let *momentum wf* = *wf.momentum*
    let *momentum_list wf* = *P.to_ints wf.momentum*
    let *wf_tag wf* = *Tags.wf_to_string wf.wf_tag*
    let *wf_tag_raw wf* = *wf.wf_tag*

    let *external_wfs rank particles* =
      List.map

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```
(fun (f, p) →
  { flavor = f;
    momentum = P.singleton rank p;
    wf_tag = Tags.null_wf })
particles
```

Order wavefunctions so that the external come first, then the pairs, etc. Also put possible Goldstone bosons *before* their gauge bosons.

```
let lorentz_ordering f =
  match M.lorentz f with
  | Coupling.Scalar → 0
  | Coupling.Spinor → 1
  | Coupling.ConjSpinor → 2
  | Coupling.Majorana → 3
  | Coupling.Vector → 4
  | Coupling.Massive_Vector → 5
  | Coupling.Tensor_2 → 6
  | Coupling.Tensor_1 → 7
  | Coupling.Vectorspinor → 8
  | Coupling.BRS Coupling.Scalar → 9
  | Coupling.BRS Coupling.Spinor → 10
  | Coupling.BRS Coupling.ConjSpinor → 11
  | Coupling.BRS Coupling.Majorana → 12
  | Coupling.BRS Coupling.Vector → 13
  | Coupling.BRS Coupling.Massive_Vector → 14
  | Coupling.BRS Coupling.Tensor_2 → 15
  | Coupling.BRS Coupling.Tensor_1 → 16
  | Coupling.BRS Coupling.Vectorspinor → 17
  | Coupling.BRS _ → invalid_arg "Fusion.lorentz_ordering:␣not␣needed"
  | Coupling.Maj_Ghost → 18

let order_flavor f1 f2 =
  let c = compare (lorentz_ordering f1) (lorentz_ordering f2) in
  if c ≠ 0 then
    c
  else
    compare f1 f2
```

Note that *Momentum().compare* guarantees that wavefunctions will be ordered according to *increasing Momentum().rank* of their momenta.

```
let order_wf wf1 wf2 =
  let c = P.compare wf1.momentum wf2.momentum in
  if c ≠ 0 then
    c
  else
    let c = order_flavor wf1.flavor wf2.flavor in
    if c ≠ 0 then
      c
    else
      compare wf1.wf_tag wf2.wf_tag
```

This *must* be a pair matching the *edge* × *node children* pairs of *DAG.Forest*!

```
type coupling = Tagged_Coupling.t
type α children = α PT.t
type rhs = coupling × wf children
let sign (c, _) = Tagged_Coupling.sign c
let coupling (c, _) = Tagged_Coupling.coupling c
let coupling_tag (c, _) = Tagged_Coupling.coupling_tag c
let children (_, wfs) = PT.to_list wfs

type fusion = wf × rhs list
let lhs (l, _) = l
let rhs (_, r) = r

type braket = wf × rhs list
let bra (b, _) = b
let ket (_, k) = k

module D = DAG.Make
    (DAG.Forest(PT)
        (struct type t = wf let compare = order_wf end)
        (struct type t = coupling let compare = compare end))

module WFSet =
  Set.Make (struct type t = wf let compare = order_wf end)

let wavefunctions brakets =
  WFSet.elements (List.fold_left (fun set (wf1, wf23) →
    WFSet.add wf1 (List.fold_left (fun set' (_, wfs) →
      PT.fold_right WFSet.add wfs set') set wf23)) WFSet.empty brakets)

type amplitude =
    { fusions : fusion list;
      brakets : braket list;
      on_shell : (wf → bool);
      is_gauss : (wf → bool);
      constraints : string option;
      incoming : flavor list;
      outgoing : flavor list;
      externals : wf list;
      symmetry : int;
      dependencies : (wf → (wf, coupling) Tree2.t);
      fusion_tower : D.t;
      fusion_dag : D.t }

let incoming a = a.incoming
let outgoing a = a.outgoing
let externals a = a.externals
let fusions a = a.fusions
let brakets a = a.brakets
let symmetry a = a.symmetry
let on_shell a = a.on_shell
let is_gauss a = a.is_gauss
let constraints a = a.constraints
```

```
      let variables a  =  List.map lhs a.fusions
      let dependencies a  =  a.dependencies
      let fusion_dag a  =  a.fusion_dag

   end

module A  =  Amplitude(PT)(P)(M)
```

Operator insertions can be fused only if they are external.

```
   let is_source wf  =
      match M.propagator wf.A.flavor with
      | Only_Insertion  →  P.rank wf.A.momentum  =  1
      | _  →  true
```

is_goldstone_of g v is true if and only if *g* is the Goldstone boson corresponding to the gauge particle *v*.

```
   let is_goldstone_of g v  =
      match M.goldstone v with
      | None  →  false
      | Some (g', _)  →  g  =  g'
```

In the end, *PT.to_list* should become redudant!

```
   let fuse_rhs rhs  =  M.fuse (PT.to_list rhs)
```

Vertices

Compute the set of all vertices in the model from the allowed fusions and the set of all flavors:

One could think of using *M.vertices* instead of *M.fuse2*, *M.fuse3* and *M.fuse* …

```
   module VSet  =  Map.Make(struct type t  =  A.flavor let compare  =  compare end)

   let add_vertices f rhs m  =
      VSet.add f (try rhs :: VSet.find f m with Not_found  →  [rhs]) m

   let collect_vertices rhs  =
      List.fold_right (fun (f1, c)  →  add_vertices (M.conjugate f1) (c, rhs))
         (fuse_rhs rhs)
```

The set of all vertices with common left fields factored.
I used to think that constant initializers are a good idea to allow compile time optimizations. The down side turned out to be that the constant initializers will be evaluated *every time* the functor is applied. *Relying on the fact that the functor will be called only once is not a good idea!*

```
   type vertices  =  (A.flavor × (constant Coupling.t × A.flavor PT.t) list) list

   let vertices_nocache max_degree flavors : vertices  =
      VSet.fold (fun f rhs v  →  (f, rhs) :: v)
         (PT.power_fold collect_vertices flavors VSet.empty) []
```

Performance hack:

```
type vertex_table =
            ((A.flavor × A.flavor × A.flavor) × constant Coupling.vertex3 ×
constant) list
          × ((A.flavor × A.flavor × A.flavor × A.flavor)
               × constant Coupling.vertex4 × constant) list
          × (A.flavor list × constant Coupling.vertexn × constant) list

module VCache =
    Cache.Make (struct type t = vertex_table end) (struct type t = RCS.t ×
vertices end)

let vertices_cache = ref None
let hash = VCache.hash (M.vertices ())
```

⚠ Can we do better than the executable name provided by *Config.cache_prefix*???
We need a better way to avoid collisions among the caches for different models in the same program.

```
let cache_name =
  ref (Config.cache_prefix ^ "." ^ Config.cache_suffix)

let set_cache_name name =
  cache_name := name

let initialize_cache dir =
  Printf.eprintf
    "␣>>>␣Initializing␣vertex␣table␣for␣model␣%s.␣␣This␣may␣take␣some␣time␣...␣"
    (RCS.name M.rcs);
  flush stderr;
  VCache.write_dir hash dir !cache_name
    (M.rcs, vertices_nocache (M.max_degree ()) (M.flavors()));
  Printf.eprintf "done.␣<<<␣\n"

let vertices max_degree flavors : vertices =
  match !vertices_cache with
  | None →
      begin match !cache_option with
      | Cache_Use →
          begin match VCache.maybe_read hash !cache_name with
          | VCache.Hit (rcs, result) →
              result
          | VCache.Miss →
              Printf.eprintf
                "␣>>>␣Initializing␣vertex␣table␣for␣model␣%s.␣␣This␣may␣take␣some␣time
                (RCS.name M.rcs);
              flush stderr;
              let result = vertices_nocache max_degree flavors in
              VCache.write hash !cache_name (M.rcs, result);
              vertices_cache := Some result;
              Printf.eprintf "done.␣<<<␣\n";
              flush stderr;
```

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```
                result
            |  VCache.Stale file  →
                Printf.eprintf
                  "␣>>>␣Re-initializing␣stale␣vertex␣table␣for␣model␣%s␣in␣file␣%s.␣␣"
                  (RCS.name M.rcs) file;
                Printf.eprintf "This␣may␣take␣some␣time␣...␣";
                flush stderr;
                let result  =  vertices_nocache max_degree flavors in
                VCache.write hash !cache_name (M.rcs, result);
                vertices_cache  :=  Some result;
                Printf.eprintf "done.␣<<<␣\n";
                flush stderr;
                result
          end
      |  Cache_Overwrite  →
          Printf.eprintf
            "␣>>>␣Overwriting␣vertex␣table␣for␣model␣%s.␣␣This␣may␣take␣some␣time␣...␣
            (RCS.name M.rcs);
          flush stderr;
          let result  =  vertices_nocache max_degree flavors in
          VCache.write hash !cache_name (M.rcs, result);
          vertices_cache  :=  Some result;
          Printf.eprintf "done.␣<<<␣\n";
          flush stderr;
          result
      |  Cache_Ignore  →
          Printf.eprintf
            "␣>>>␣Ignoring␣vertex␣table␣for␣model␣%s.␣␣This␣may␣take␣some␣time␣...␣"
            (RCS.name M.rcs);
          flush stderr;
          let result  =  vertices_nocache max_degree flavors in
          vertices_cache  :=  Some result;
          Printf.eprintf "done.␣<<<␣\n";
          flush stderr;
          result
      end
  |  Some result  →  result
```

<div align="center">

Partitions

</div>

Vertices that are not crossing invariant need special treatment so that they're only generated for the correct combinations of momenta.

NB: the *crossing* checks here are a bit redundant, because *CM.fuse* below will bring the killed vertices back to life and will have to filter once more. Nevertheless, we keep them here, for the unlikely case that anybody ever wants to use uncolored amplitudes directly.

NB: the analogous problem does not occur for *select_wf*, because this applies to momenta instead of vertices.

This approach worked before the colorize, but has become *futile*, because *CM.fuse* will bring the killed vertices back to life. We need to implement the same checks there again!!!

Using *PT.Mismatched_arity* is not really good style ...

Tho's approach doesn't work since he does not catch charge conjugated processes or crossed processes. Another very strange thing is that O'Mega seems always to run in the q2 q3 timelike case, but not in the other two. (Property of how the DAG is built?). For the *ZZZZ* vertex I add the same vertex again, but interchange 1 and 3 in the *crossing* vertex

```
let kmatrix_cuts c momenta  =
  match c with
  | V4 (Vector4_K_Matrix_tho (disc, _), fusion, _)
  | V4 (Vector4_K_Matrix_jr (disc, _), fusion, _) →
      let s12, s23, s13  =
        begin match PT.to_list momenta with
        | [q1; q2; q3] → (P.Scattering.timelike (P.add q1 q2),
                            P.Scattering.timelike (P.add q2 q3),
                            P.Scattering.timelike (P.add q1 q3))
        | _ → raise PT.Mismatched_arity
        end in
      begin match disc, s12, s23, s13, fusion with
      | 0, true, false, false, (F341 | F431 | F342 | F432 | F123 | F213 |
F124 | F214)
      | 0, false, true, false, (F134 | F143 | F234 | F243 | F312 | F321 |
F412 | F421)
      | 0, false, false, true, (F314 | F413 | F324 | F423 | F132 | F231 |
F142 | F241) →
            true
      | 1, true, false, false, (F341 | F431 | F342 | F432)
      | 1, false, true, false, (F134 | F143 | F234 | F243)
      | 1, false, false, true, (F314 | F413 | F324 | F423) →
            true
      | 2, true, false, false, (F123 | F213 | F124 | F214)
      | 2, false, true, false, (F312 | F321 | F412 | F421)
      | 2, false, false, true, (F132 | F231 | F142 | F241) →
            true
      | 3, true, false, false, (F143 | F413 | F142 | F412 | F321 | F231 |
F324 | F234)
      | 3, false, true, false, (F314 | F341 | F214 | F241 | F132 | F123 |
F432 | F423)
      | 3, false, false, true, (F134 | F431 | F124 | F421 | F312 | F213 |
F342 | F243) →
            true
      | _ → false
      end
  | V4 (DScalar2_Vector2_K_Matrix_ms (disc, _), fusion, _) →
      let s12, s23, s13  =
```

```
begin match PT.to_list momenta with
| [q1; q2; q3] → (P.Scattering.timelike (P.add q1 q2),
                  P.Scattering.timelike (P.add q2 q3),
                  P.Scattering.timelike (P.add q1 q3))
| _ → raise PT.Mismatched_arity
end in
begin match disc, s12, s23, s13, fusion with
| 0, true, false, false, (F341 | F431 | F342 | F432 | F123 | F213 |
F124 | F214)
| 0, false, true, false, (F134 | F143 | F234 | F243 | F312 | F321 |
F412 | F421)
| 0, false, false, true, (F314 | F413 | F324 | F423 | F132 | F231 |
F142 | F241) →
        true
| 1, true, false, false, (F341 | F432 | F123 | F214)
| 1, false, true, false, (F134 | F243 | F312 | F421)
| 1, false, false, true, (F314 | F423 | F132 | F241) →
        true
| 2, true, false, false, (F431 | F342 | F213 | F124)
| 2, false, true, false, (F143 | F234 | F321 | F412)
| 2, false, false, true, (F413 | F324 | F231 | F142) →
        true
| 3, true, false, false, (F143 | F413 | F142 | F412 | F321 | F231 |
F324 | F234)
| 3, false, true, false, (F314 | F341 | F214 | F241 | F132 | F123 |
F432 | F423)
| 3, false, false, true, (F134 | F431 | F124 | F421 | F312 | F213 |
F342 | F243) →
        true
| 4, true, false, false, (F142 | F413 | F231 | F324)
| 4, false, true, false, (F214 | F341 | F123 | F432)
| 4, false, false, true, (F124 | F431 | F213 | F342) →
        true
| 5, true, false, false, (F143 | F412 | F321 | F234)
| 5, false, true, false, (F314 | F241 | F132 | F423)
| 5, false, false, true, (F134 | F421 | F312 | F243) →
        true
| 6, true, false, false, (F134 | F132 | F314 | F312 | F241 | F243 |
F421 | F423)
| 6, false, true, false, (F213 | F413 | F231 | F431 | F124 | F324 |
F142 | F342)
| 6, false, false, true, (F143 | F123 | F341 | F321 | F412 | F214 |
F432 | F234) →
        true
| 7, true, false, false, (F134 | F312 | F421 | F243)
| 7, false, true, false, (F413 | F231 | F142 | F324)
| 7, false, false, true, (F143 | F321 | F412 | F432) →
        true
| 8, true, false, false, (F132 | F314 | F241 | F423)
| 8, false, true, false, (F213 | F431 | F124 | F342)
```

```
                  | 8, false, false, true, (F123 | F341 | F214 | F234) →
                        true
                  | _ → false
                  end
            | V4 (DScalar4_K_Matrix_ms (disc, _), fusion, _) →
                  let s12, s23, s13 =
                     begin match PT.to_list momenta with
                     | [q1; q2; q3] → (P.Scattering.timelike (P.add q1 q2),
                                          P.Scattering.timelike (P.add q2 q3),
                                          P.Scattering.timelike (P.add q1 q3))
                     | _ → raise PT.Mismatched_arity
                     end in
                  begin match disc, s12, s23, s13, fusion with
                  | 0, true, false, false, (F341 | F431 | F342 | F432 | F123 | F213 |
F124 | F214)
                  | 0, false, true, false, (F134 | F143 | F234 | F243 | F312 | F321 |
F412 | F421)
                  | 0, false, false, true, (F314 | F413 | F324 | F423 | F132 | F231 |
F142 | F241) →
                        true
                  | 3, true, false, false, (F143 | F413 | F142 | F412 | F321 | F231 |
F324 | F234)
                  | 3, false, true, false, (F314 | F341 | F214 | F241 | F132 | F123 |
F432 | F423)
                  | 3, false, false, true, (F134 | F431 | F124 | F421 | F312 | F213 |
F342 | F243) →
                        true
                  | 4, true, false, false, (F142 | F413 | F231 | F324)
                  | 4, false, true, false, (F214 | F341 | F123 | F432)
                  | 4, false, false, true, (F124 | F431 | F213 | F342) →
                        true
                  | 5, true, false, false, (F143 | F412 | F321 | F234)
                  | 5, false, true, false, (F314 | F241 | F132 | F423)
                  | 5, false, false, true, (F134 | F421 | F312 | F243) →
                        true
                  | 6, true, false, false, (F134 | F132 | F314 | F312 | F241 | F243 |
F421 | F423)
                  | 6, false, true, false, (F213 | F413 | F231 | F431 | F124 | F324 |
F142 | F342)
                  | 6, false, false, true, (F143 | F123 | F341 | F321 | F412 | F214 |
F432 | F234) →
                        true
                  | 7, true, false, false, (F134 | F312 | F421 | F243)
                  | 7, false, true, false, (F413 | F231 | F142 | F324)
                  | 7, false, false, true, (F143 | F321 | F412 | F432) →
                        true
                  | 8, true, false, false, (F132 | F314 | F241 | F423)
                  | 8, false, true, false, (F213 | F431 | F124 | F342)
                  | 8, false, false, true, (F123 | F341 | F214 | F234) →
                        true
```

```
      |  _  →  false
      end
 |  _  →  true
```

Counting QCD and EW orders.

```
let qcd_ew_check orders =
  if fst (orders) ≤ fst (int_orders) ∧
      snd (orders) ≤ snd (int_orders) then
    true
  else
    false
```

Match a set of flavors to a set of momenta. Form the direct product for the lists of momenta two and three with the list of couplings and flavors two and three.

```
let flavor_keystone select_p dim (f1, f23) (p1, p23) =
  ({ A.flavor  =  f1;
     A.momentum  =  P.of_ints dim p1;
     A.wf_tag  =  A.Tags.null_wf },
   Product.fold2 (fun (c, f) p acc →
     try
       let p' = PT.map (P.of_ints dim) p in
       if select_p (P.of_ints dim p1) (PT.to_list p') ∧ kmatrix_cuts c p' then
         (c, PT.map2 (fun f'' p'' → { A.flavor = f'';
                                       A.momentum = p'';
                                       A.wf_tag = A.Tags.null_wf }) f p') :: acc
       else
         acc
     with
     |  PT.Mismatched_arity  →  acc) f23 p23 [])
```

Produce all possible combinations of vertices (flavor keystones) and momenta by forming the direct product. The semantically equivalent *Product.list2 (flavor_keystone select_wf n) vertices k* with *subsequent* filtering would be a *very bad* idea, because a potentially huge intermediate list is built for large models. E.g. for the MSSM this would lead to non-termination by thrashing for $2 \to 4$ processes on most PCs.

```
let flavor_keystones filter select_p dim vertices keystones =
  Product.fold2 (fun v k acc →
    filter (flavor_keystone select_p dim v k) acc) vertices keystones []
```

Flatten the nested lists of vertices into a list of attached lines.

```
let flatten_keystones t =
  ThoList.flatmap (fun (p1, p23) →
    p1 :: (ThoList.flatmap (fun (_, rhs) → PT.to_list rhs) p23)) t
```

<center>*Subtrees*</center>

Fuse a tuple of wavefunctions, keeping track of Fermi statistics. Record only the the sign *relative* to the children. (The type annotation is only for documentation.)

```
let fuse select_wf wfss : (A.wf × stat × A.rhs) list =
```

```
    if PT.for_all (fun (wf, _) → is_source wf) wfss then
      try
        let wfs, ss = PT.split wfss in
        let flavors = PT.map A.flavor wfs
        and momenta = PT.map A.momentum wfs
        and wf_tags = PT.map A.wf_tag_raw wfs in
        let p = PT.fold_left_internal P.add momenta in
        List.fold_left
          (fun acc (f, c) →
            if select_wf f p (PT.to_list momenta) ∧ kmatrix_cuts c momenta then
              let s = stat_fuse ss f in
              let flip =
                PT.fold_left (fun acc s' → acc × stat_sign s') (stat_sign s) ss in
              ({ A.flavor = f;
                 A.momentum = p;
                 A.wf_tag = A.Tags.null_wf }, s,
               ({ Tagged_Coupling.sign = flip;
                  Tagged_Coupling.coupling = c;
                  Tagged_Coupling.coupling_tag = A.Tags.null_coupling }, wfs)) :: acc
            else
              acc)
          [] (fuse_rhs flavors)
      with
      | P.Duplicate _ | S.Impossible → []
    else
      []
```

Eventually, the pairs of *tower* and *dag* in *fusion_tower′* below could and should be replaced by a graded *DAG*. This will look like, but currently *tower* containts statistics information that is missing from *dag*:

```
Type node = flavor * p is not compatible with type wf * stat
```

This should be easy to fix. However, replacing type $t = wf$ with type $t = wf \times stat$ is *not* a good idea because the variable *stat* makes it impossible to test for the existance of a particular *wf* in a *DAG*.

In summary, it seems that $(wf \times stat)$ *list array* $\times$ *A.D.t* should be replaced by $(wf \rightarrow stat) \times$ *A.D.t*.

```
module GF =
  struct
    module Nodes =
      struct
        type t = A.wf
        module G = struct type t = int let compare = compare end
        let compare = A.order_wf
        let rank wf = P.rank wf.A.momentum
      end
    module Edges = struct type t = A.coupling let compare = compare end
```

```
      module F  =  DAG.Forest(PT)(Nodes)(Edges)
      type node  =  Nodes.t
      type edge  =  F.edge
      type children  =  F.children
      type t  =  F.t
      let compare  =  F.compare
      let for_all  =  F.for_all
      let fold  =  F.fold
    end

  module D'  =  DAG.Graded(GF)

  let tower_of_dag dag  =
    let _, max_rank  =  D'.min_max_rank dag in
    Array.init max_rank (fun n  →  D'.ranked n dag)
```

The function $fusion\_tower'$ recursively builds the tower of all fusions from bottom up to a chosen level. The argument *tower* is an array of lists, where the i-th sublist (counting from 0) represents all off shell wave functions depending on $i + 1$ momenta and their Fermistatistics.

$$
\begin{aligned}
\Big[& \{\phi_1(p_1), \phi_2(p_2), \phi_3(p_3), \ldots\}, \\
& \{\phi_{12}(p_1 + p_2), \phi'_{12}(p_1 + p_2), \ldots, \phi_{13}(p_1 + p_3), \ldots, \phi_{23}(p_2 + p_3), \ldots\}, \\
& \ldots \\
& \{\phi_{1\cdots n}(p_1 + \cdots + p_n), \phi'_{1\cdots n}(p_1 + \cdots + p_n), \ldots\} \Big]
\end{aligned} \tag{8.6}
$$

The argument *dag* is a DAG representing all the fusions calculated so far. NB: The outer array in *tower* is always very short, so we could also have accessed a list with *List.nth*. Appending of new members at the end brings no loss of performance. NB: the array is supposed to be immutable.

The towers must be sorted so that the combinatorical functions can make consistent selections.

Intuitively, this seems to be correct. However, one could have expected that no element appears twice and that this ordering is not necessary ...

```
  let grow select_wf tower  =
    let rank  =  succ (Array.length tower) in
    List.sort Pervasives.compare
      (PT.graded_sym_power_fold rank
          (fun wfs acc  →  fuse select_wf wfs @ acc) tower [])

  let add_offspring dag (wf, _, rhs)  =
    A.D.add_offspring wf rhs dag

  let filter_offspring fusions  =
    List.map (fun (wf, s, _)  →  (wf, s)) fusions

  let rec fusion_tower' n_max select_wf tower dag  :  (A.wf  ×  stat) list ar-
ray  ×  A.D.t  =
      if Array.length tower  ≥  n_max then
        (tower, dag)
```

```
  else
    let tower′ = grow select_wf tower in
    fusion_tower′ n_max select_wf
      (Array.append tower [|filter_offspring tower′|])
      (List.fold_left add_offspring dag tower′)
```

Discard the tower and return a map from wave functions to Fermistatistics together with the DAG.

```
  let make_external_dag wfs =
    List.fold_left (fun m (wf, _) → A.D.add_node wf m) A.D.empty wfs

  let mixed_fold_left f acc lists =
    Array.fold_left (List.fold_left f) acc lists

  module Stat_Map =
    Map.Make (struct type t = A.wf let compare = A.order_wf end)

  let fusion_tower height select_wf wfs : (A.wf → stat) × A.D.t =
    let tower, dag =
      fusion_tower′ height select_wf [|wfs|] (make_external_dag wfs) in
    let stats = mixed_fold_left
        (fun m (wf, s) → Stat_Map.add wf s m) Stat_Map.empty tower in
    ((fun wf → Stat_Map.find wf stats), dag)
```

Calculate the minimal tower of fusions that suffices for calculating the amplitude.

```
  let minimal_fusion_tower n select_wf wfs : (A.wf → stat) × A.D.t =
    fusion_tower (T.max_subtree n) select_wf wfs
```

Calculate the complete tower of fusions. It is much larger than required, but it allows a complete set of gauge checks.

```
  let complete_fusion_tower select_wf wfs : (A.wf → stat) × A.D.t =
    fusion_tower (List.length wfs − 1) select_wf wfs
```

There is a natural product of two DAGs using *fuse*. Can this be used in a replacement for *fusion_tower*? The hard part is to avoid double counting, of course. A straight forward solution could do a diagonal sum (in order to reject flipped offspring representing the same fusion) and rely on the uniqueness in *DAG* otherwise. However, this will (probably) slow down the procedure significanty, because most fusions (including Fermi signs!) will be calculated before being rejected by *DAG().add_offspring*.

Add to *dag* all Goldstone bosons defined in *tower* that correspond to gauge bosons in *dag*. This is only required for checking Slavnov-Taylor identities in unitarity gauge. Currently, it is not used, because we use the complete tower for gauge checking.

```
  let harvest_goldstones tower dag =
    A.D.fold_nodes (fun wf dag′ →
      match M.goldstone wf.A.flavor with
      | Some (g, _) →
        let wf′ = { wf with A.flavor = g } in
```

```
    if A.D.is_node wf' tower then begin
      A.D.harvest tower wf' dag'
    end else begin
      dag'
    end
| None → dag') dag dag
```

Calculate the sign from Fermi statistics that is not already included in the children.

⬦ The use of *PT.of2_kludge* is the largest skeleton on the cupboard of unified fusions. Currently, it is just another name for *PT.of2*, but the existence of the latter requires binary fusions. Of course, this is just a symptom for not fully supporting four fermion vertices ...

```
let stat_keystone stats wf1 wfs =
  let wf1' = stats wf1
  and wfs' = PT.map stats wfs in
  stat_sign
    (stat_fuse
       (PT.of2_kludge wf1' (stat_fuse wfs' (M.conjugate (A.flavor wf1))))
       (A.flavor wf1))
     × PT.fold_left (fun acc wf → acc × stat_sign wf) (stat_sign wf1') wfs'
```

Test all members of a list of wave functions are defined by the DAG simultaneously:

```
let test_rhs dag (_, wfs) =
  PT.for_all (fun wf → is_source wf ∧ A.D.is_node wf dag) wfs
```

Add the keystone $(wf1, pairs)$ to *acc* only if it is present in *dag* and calculate the statistical factor depending on *stats en passant*:

```
let filter_keystone stats dag (wf1, pairs) acc =
  if is_source wf1 ∧ A.D.is_node wf1 dag then
    match List.filter (test_rhs dag) pairs with
    | [] → acc
    | pairs' → (wf1, List.map (fun (c, wfs) →
        ({ Tagged_Coupling.sign = stat_keystone stats wf1 wfs;
           Tagged_Coupling.coupling = c;
           Tagged_Coupling.coupling_tag = A.Tags.null_coupling },
         wfs)) pairs') :: acc
  else
    acc
```

Amplitudes

```
module C = Cascade.Make(M)(P)
type selectors = C.selectors

let external_wfs n particles =
  List.map (fun (f, p) →
```
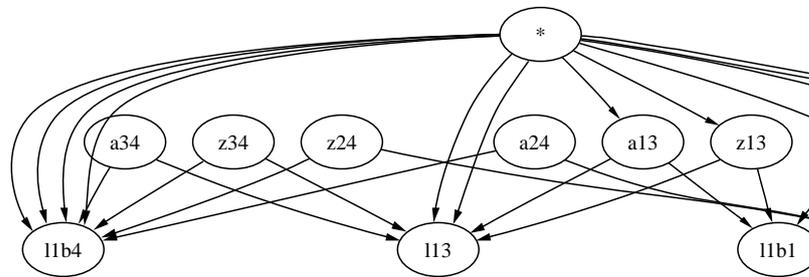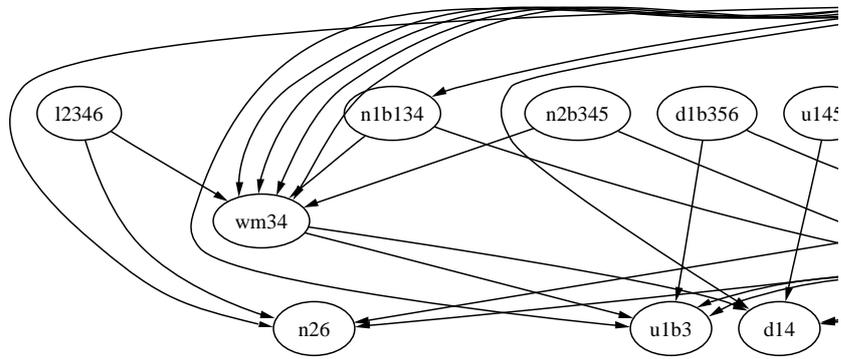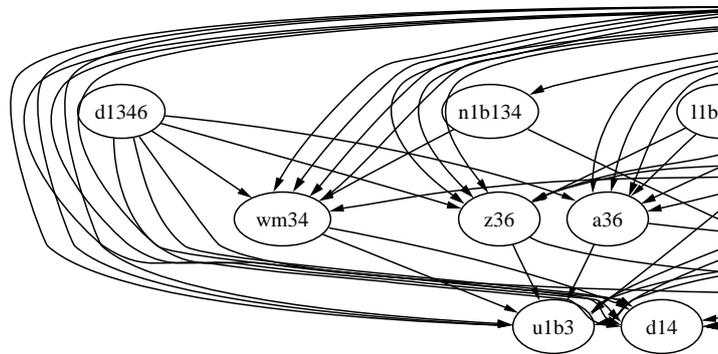
($\{$ *A.flavor* $=$ *f*;
  A.momentum $=$ *P.singleton n p*;
  A.wf_tag $=$ *A.Tags.null_wf* $\}$,
  stat f p)) *particles*

Main Function

module *WFMap* $=$ *Map.Make* (struct type t $=$ *A.wf* let *compare* $=$ *compare* end)

map_amplitude_wfs f a applies the function f : *wf* $\to$ *wf* to all wavefunctions
appearing in the amplitude a.

let *map_amplitude_wfs f a* $=$
  let *map_rhs* (*c*, *wfs*) $=$ (*c*, *PT.map f wfs*) in
  let *map_braket* (*wf*, *rhs*) $=$ (*f wf*, *List.map map_rhs rhs*)
  and *map_fusion* (*lhs*, *rhs*) $=$ (*f lhs*, *List.map map_rhs rhs*) in
  let *map_dag* $=$ *A.D.map f* (fun *node rhs* $\to$ *map_rhs rhs*) in
  let *tower* $=$ *map_dag a.A.fusion_tower*
  and *dag* $=$ *map_dag a.A.fusion_dag* in
  let *dependencies_map* $=$
    A.D.fold (fun *wf* _ $\to$ *WFMap.add wf* (*A.D.dependencies dag wf*)) *dag WFMap.empty* in
  $\{$ *A.fusions* $=$ *List.map map_fusion a.A.fusions*;
    A.brakets $=$ *List.map map_braket a.A.brakets*;
    A.on_shell $=$ *a.A.on_shell*;
    A.is_gauss $=$ *a.A.is_gauss*;
    A.constraints $=$ *a.A.constraints*;
    A.incoming $=$ *a.A.incoming*;
    A.outgoing $=$ *a.A.outgoing*;
    A.externals $=$ *List.map f a.A.externals*;
    A.symmetry $=$ *a.A.symmetry*;
    A.dependencies $=$ (fun *wf* $\to$ *WFMap.find wf dependencies_map*);
    A.fusion_tower $=$ *tower*;
    A.fusion_dag $=$ *dag* $\}$

This is the main function that constructs the amplitude for sets of incoming
and outgoing particles and returns the results in conveniently packaged pieces.

let *amplitude goldstones selectors fin fout* $=$

Set up external lines and match flavors with numbered momenta.

let *f* $=$ *fin* @ *List.map M.conjugate fout* in
let *nin*, *nout* $=$ *List.length fin*, *List.length fout* in
let *n* $=$ *nin* $+$ *nout* in
let *externals* $=$ *List.combine f* (*ThoList.range 1 n*) in
let *wfs* $=$ *external_wfs n externals* in
let *select_p* $=$ *C.select_p selectors* in
let *select_wf* $=$
  match *fin* with
  | [_] $\to$ *C.select_wf selectors P.Decay.timelike*
  | _ $\to$ *C.select_wf selectors P.Scattering.timelike* in

Build the full fusion tower (including nodes that are never needed in the amplitude).

> let *stats*, *tower* =
>
>> if *goldstones* then
>>> *complete_fusion_tower select_wf wfs*
>> else
>>> *minimal_fusion_tower n select_wf wfs* in

Find all vertices for which *all* off shell wavefunctions are defined by the tower.

> let *brakets* =
>> *flavor_keystones* (*filter_keystone stats tower*) *select_p n*
>>> (*vertices* (*M.max_degree* ()) (*M.flavors* ()))
>>> (*T.keystones* (*ThoList.range* 1 *n*)) in

Remove the part of the DAG that is never needed in the amplitude.

> let *dag* =
>> if *goldstones* then
>>> *tower*
>> else
>>> *A.D.harvest_list tower* (*A.wavefunctions brakets*) in

Remove the leaf nodes of the DAG, corresponding to external lines.

> let *fusions* =
>> *List.filter* (function (_, []) → false | _ → true) (*A.D.lists dag*) in

Calculate the symmetry factor for identical particles in the final state.

> let *symmetry* =
>> *Combinatorics.symmetry fout* in
>
> let *dependencies_map* =
>> *A.D.fold* (fun *wf* _ → *WFMap.add wf* (*A.D.dependencies dag wf*)) *dag WFMap.empty* in

Finally: package the results:

> { *A.fusions* = *fusions*;
>   *A.brakets* = *brakets*;
>   *A.on_shell* = (fun *wf* → *C.on_shell selectors* (*A.flavor wf*) *wf.A.momentum*);
>   *A.is_gauss* = (fun *wf* → *C.is_gauss selectors* (*A.flavor wf*) *wf.A.momentum*);
>   *A.constraints* = *C.description selectors*;
>   *A.incoming* = *fin*;
>   *A.outgoing* = *fout*;
>   *A.externals* = *List.map fst wfs*;
>   *A.symmetry* = *symmetry*;
>   *A.dependencies* = (fun *wf* → *WFMap.find wf dependencies_map*);
>   *A.fusion_tower* = *tower*;
>   *A.fusion_dag* = *dag* }

<div align="center">

Color

</div>

> module *CM* = *Colorize.It*(*M*)

module CA = $Amplitude(PT)(P)(CM)$

let $colorize\_wf$ $flavor$ wf =
  { $CA.flavor$ = $flavor$;
    $CA.momentum$ = $wf.A.momentum$;
    $CA.wf\_tag$ = $wf.A.wf\_tag$ }

let $uncolorize\_wf$ wf =
  { $A.flavor$ = $CM.flavor\_sans\_color$ $wf.CA.flavor$;
    $A.momentum$ = $wf.CA.momentum$;
    $A.wf\_tag$ = $wf.CA.wf\_tag$ }

At the end of the day, I shall want to have some sort of *fibered DAG* as abstract data type, with a projection of colored nodes to their uncolored counterparts.

module $CWFBundle$ = $Bundle.Make$
    (struct
       type elt = $CA.wf$
       let $compare\_elt$ = $compare$
       type $base$ = $A.wf$
       let $compare\_base$ = $compare$
       let pi wf =
         { $A.flavor$ = $CM.flavor\_sans\_color$ $wf.CA.flavor$;
           $A.momentum$ = $wf.CA.momentum$;
           $A.wf\_tag$ = $wf.CA.wf\_tag$ }
    end)

For now, we can live with simple aggregation:

type $fibered\_dag$ = { dag : $CA.D.t$; $bundle$: $CWFBundle.t$ }

Not yet(?) needed: module CS = $Stat$ (CM)

let $colorize\_sterile\_nodes$ dag f wf $fibered\_dag$ =
  if $A.D.is\_sterile$ wf dag then
    let wf', wf_bundle' = f wf $fibered\_dag$ in
    { dag = $CA.D.add\_node$ wf' $fibered\_dag.dag$;
      $bundle$ = wf_bundle' }
  else
    $fibered\_dag$

let $colorize\_nodes$ f wf rhs $fibered\_dag$ =
  let wf_rhs_list', wf_bundle' = f wf rhs $fibered\_dag$ in
  let dag' =
    $List.fold\_right$
      (fun (wf', rhs') → $CA.D.add\_offspring$ wf' rhs')
      wf_rhs_list' $fibered\_dag.dag$ in
  { dag = dag';
    $bundle$ = wf_bundle' }

O'Caml (correctly) infers the type val $colorize\_dag$: $(D.node$ → $D.edge$ × $D.children$ → $fibered\_dag$ → $(CA.D.node$ × $(CA.D.edge$ × $CA.D.children))$ *list* × $CWFBundle.t)$ →

$(D.node \ \rightarrow \ fibered\_dag \ \rightarrow \ CA.D.node \ \times \ CWFBundle.t) \ \rightarrow \ D.t \ \rightarrow$
$CWFBundle.t \ \rightarrow \ fibered\_dag.$

let *colorize_dag f_node f_ext dag wf_bundle* =
  A.D.fold (*colorize_nodes f_node*) *dag*
    (*A.D.fold_nodes* (*colorize_sterile_nodes dag f_ext*) *dag*
      { *dag* = *CA.D.empty*; *bundle* = *wf_bundle* })

let *colorize_external wf fibered_dag* =
  match *CWFBundle.inv_pi wf fibered_dag.bundle* with
  | [*c_wf*] $\rightarrow$ (*c_wf*, *fibered_dag.bundle*)
  | [] $\rightarrow$ *failwith* "colorize_external:␣not␣found"
  | _ $\rightarrow$ *failwith* "colorize_external:␣not␣unique"

let *fuse_c_wf rhs* =
  let *momenta* = *PT.map* (fun *wf* $\rightarrow$ *wf.CA.momentum*) *rhs* in
  List.filter
    (fun (_, *c*) $\rightarrow$ *kmatrix_cuts c momenta*)
    (*CM.fuse* (*List.map* (fun *wf* $\rightarrow$ *wf.CA.flavor*) (*PT.to_list rhs*)))

let *colorize_coupling c coupling* =
  { *coupling* with *Tagged_Coupling.coupling* = *c* }

let *colorize_fusion wf* (*coupling, children*) *fibered_dag* =
  let *match_flavor* (*f*, _) = (*CM.flavor_sans_color f* = *A.flavor wf*)
  and *find_colored wf′* = *CWFBundle.inv_pi wf′ fibered_dag.bundle* in
  let *fusions* =
    ThoList.flatmap
      (fun *c_children* $\rightarrow$
        List.map
          (fun (*f*, *c*) $\rightarrow$
            (*colorize_wf f wf*, (*colorize_coupling c coupling*, *c_children*))))
          (*List.filter match_flavor* (*fuse_c_wf c_children*)))
      (*PT.product* (*PT.map find_colored children*)) in
  let *bundle* =
    List.fold_right
      (fun (*c_wf*, _) $\rightarrow$ *CWFBundle.add c_wf*)
      fusions fibered_dag.bundle in
  (*fusions, bundle*)

let *colorize_braket1* (*wf*, (*coupling, children*)) *fibered_dag* =
  let *find_colored wf′* = *CWFBundle.inv_pi wf′ fibered_dag.bundle* in
  Product.fold2
    (fun *bra ket acc* $\rightarrow$
      List.fold_left
        (fun *brackets* (*f*, *c*) $\rightarrow$
          if *CM.conjugate bra.CA.flavor* = *f* then
            (*bra*, (*colorize_coupling c coupling*, *ket*)) :: *brackets*
          else
            brackets)
        acc (*fuse_c_wf ket*))
    (*find_colored wf*) (*PT.product* (*PT.map find_colored children*)) []

module *CWFMap* =
  Map.Make (struct type *t* = *CA.wf* let *compare* = *CA.order_wf* end)

module *CKetSet* =
  Set.Make (struct type *t* = *CA.rhs* let *compare* = *compare* end)

Find a set of kets in *map* that belong to *bra*. Return the empty set, if nothing is found.

let *lookup_ketset bra map* =
  try *CWFMap.find bra map* with *Not_found* → *CKetSet.empty*

Return the set of kets belonging to *bra* in *map*, augmented by *ket*.

let *addto_ketset bra ket map* =
  CKetSet.add ket (*lookup_ketset bra map*)

Augment or update *map* with a new (*bra*, *ket*) relation.

let *addto_ketset_map map* (*bra*, *ket*) =
  CWFMap.add bra (*addto_ketset bra ket map*) *map*

Take a list of (*bra*, *ket*) pairs and group the *ket*s according to *bra*. This is very similar to *ThoList.factorize* on page 807, but the latter keeps duplicate copies, while we keep only one, with equality determined by *CA.order_wf*.

Isn't *Bundle* M.1 the correct framework for this?

let *factorize_brakets brakets* =
  CWFMap.fold
    (fun *bra ket acc* → (*bra*, *CKetSet.elements ket*) :: *acc*)
    (*List.fold_left addto_ketset_map CWFMap.empty brakets*)
    [ ]

let *colorize_braket* (*wf*, *rhs_list*) *fibered_dag* =
  factorize_brakets
    (*ThoList.flatmap*
      (fun *rhs* → (*colorize_braket1* (*wf*, *rhs*) *fibered_dag*))
      rhs_list)

let *colorize_amplitude a fin fout* =
  let *f* = *fin* @ *List.map CM.conjugate fout* in
  let *nin*, *nout* = *List.length fin*, *List.length fout* in
  let *n* = *nin* + *nout* in
  let *externals* = *List.combine f* (*ThoList.range* 1 *n*) in
  let *external_wfs* = *CA.external_wfs n externals* in
  let *wf_bundle* = *CWFBundle.of_list external_wfs* in

  let *fibered_dag* =
    colorize_dag
      colorize_fusion colorize_external a.A.fusion_dag wf_bundle in

  let *brakets* =
    ThoList.flatmap
      (fun *braket* → *colorize_braket braket fibered_dag*)
      a.A.brakets in

let *dag* = *CA.D.harvest_list fibered_dag.dag* (*CA.wavefunctions brakets*) in

let *fusions* =
  List.filter (function (_, []) → false | _ → true) (*CA.D.lists dag*) in

let *dependencies_map* =
  CA.D.fold
    (fun *wf* _ → *CWFMap.add wf* (*CA.D.dependencies dag wf*))
    dag CWFMap.empty in

{ *CA.fusions* = *fusions*;
  CA.brakets = *brakets*;
  CA.constraints = *a.A.constraints*;
  CA.incoming = *fin*;
  CA.outgoing = *fout*;
  CA.externals = *external_wfs*;
  CA.fusion_dag = *dag*;
  CA.fusion_tower = *dag*;
  CA.symmetry = *a.A.symmetry*;
  CA.on_shell = (fun *wf* → *a.A.on_shell* (*uncolorize_wf wf*));
  CA.is_gauss = (fun *wf* → *a.A.is_gauss* (*uncolorize_wf wf*));
  CA.dependencies = (fun *wf* → *CWFMap.find wf dependencies_map*) }

let *allowed amplitude* =
  match *amplitude.CA.brakets* with
  | [] → false
  | _ → true

let *colorize_amplitudes a* =
  List.fold_left
    (fun *amps* (*fin*, *fout*) →
      let *amp* = *colorize_amplitude a fin fout* in
      if *allowed amp* then
        amp :: *amps*
      else
        amps)
    [] (*CM.amplitude a.A.incoming a.A.outgoing*)

let *amplitudes goldstones selectors fin fout* =
  colorize_amplitudes (*amplitude goldstones selectors fin fout*)

type *flavor* = *CA.flavor*
type *flavor_sans_color* = *A.flavor*
type *p* = *A.p*
type *wf* = *CA.wf*
let *conjugate* = *CA.conjugate*
let *flavor* = *CA.flavor*
let *flavor_sans_color wf* = *CM.flavor_sans_color* (*CA.flavor wf*)
let *momentum* = *CA.momentum*
let *momentum_list* = *CA.momentum_list*
let *wf_tag* = *CA.wf_tag*

type *coupling* = *CA.coupling*

```
let sign  =  CA.sign
let coupling  =  CA.coupling
let coupling_tag  =  CA.coupling_tag

type α children  =  α CA.children
type rhs  =  CA.rhs
let children  =  CA.children

type fusion  =  CA.fusion
let lhs  =  CA.lhs
let rhs  =  CA.rhs

type braket  =  CA.braket
let bra  =  CA.bra
let ket  =  CA.ket

type amplitude  =  CA.amplitude
let incoming  =  CA.incoming
let outgoing  =  CA.outgoing
let externals  =  CA.externals
let fusions  =  CA.fusions
let brakets  =  CA.brakets
let symmetry  =  CA.symmetry
let on_shell  =  CA.on_shell
let is_gauss  =  CA.is_gauss
let constraints  =  CA.constraints
let variables a  =  List.map lhs (fusions a)
let dependencies  =  CA.dependencies
```

Checking Conservation Laws

```
let check_charges ()  =
  let vlist3, vlist4, vlistn  =  M.vertices () in
  List.filter
    (fun flist → ¬ (M.Ch.is_null (M.Ch.sum (List.map M.charges flist))))
    (List.map (fun ((f1, f2, f3), _, _) → [f1; f2; f3]) vlist3
     @ List.map (fun ((f1, f2, f3, f4), _, _) → [f1; f2; f3; f4]) vlist4
     @ List.map (fun (flist, _, _) → flist) vlistn)
```

Diagnostics

```
let count_propagators a  =
  List.length a.CA.fusions

let count_fusions a  =
  List.fold_left (fun n (_, a) → n + List.length a) 0 a.CA.fusions
    + List.fold_left (fun n (_, t) → n + List.length t) 0 a.CA.brakets
    + List.length a.CA.brakets
```

This brute force approach blows up for more than ten particles. Find a smarter algorithm.

```
let count_diagrams a =
  List.fold_left (fun n (wf1, wf23) →
    n + CA.D.count_trees wf1 a.CA.fusion_dag ×
      (List.fold_left (fun n' (_, wfs) →
        n' + PT.fold_left (fun n'' wf →
          n'' × CA.D.count_trees wf a.CA.fusion_dag) 1 wfs) 0 wf23))
    0 a.CA.brakets

exception Impossible

let forest' a =
  let below wf = CA.D.forest_memoized wf a.CA.fusion_dag in
  ThoList.flatmap
    (fun (bra, ket) →
      (Product.list2 (fun bra' ket' → bra' :: ket')
        (below bra)
        (ThoList.flatmap
          (fun (_, wfs) →
            Product.list (fun w → w) (PT.to_list (PT.map below wfs)))
          ket)))
    a.CA.brakets

let cross wf =
  { CA.flavor = CM.conjugate wf.CA.flavor;
    CA.momentum = P.neg wf.CA.momentum;
    CA.wf_tag = wf.CA.wf_tag }

let fuse_trees wf ts =
  Tree.fuse (fun (wf', e) → (cross wf', e))
    wf (fun t → List.mem wf (Tree.leafs t)) ts

let forest wf a =
  List.map (fuse_trees wf) (forest' a)

let poles_beneath wf dag =
  CA.D.eval_memoized (fun wf' → [[]])
    (fun wf' _ p → List.map (fun p' → wf' :: p') p)
    (fun wf1 wf2 →
      Product.fold2 (fun wf' wfs' wfs'' → (wf' @ wfs') :: wfs'') wf1 wf2 [])
    (@) [[]] [[]] wf dag

let poles a =
  ThoList.flatmap (fun (wf1, wf23) →
    let poles_wf1 = poles_beneath wf1 a.CA.fusion_dag in
    (ThoList.flatmap (fun (_, wfs) →
      Product.list List.flatten
        (PT.to_list (PT.map (fun wf →
          poles_wf1 @ poles_beneath wf a.CA.fusion_dag) wfs)))
      wf23))
    a.CA.brakets

module WFSet =
  Set.Make (struct type t = CA.wf let compare = CA.order_wf end)
```

```
let s_channel a =
  WFSet.elements
    (ThoList.fold_right2
      (fun wf wfs →
        if P.Scattering.timelike wf.CA.momentum then
          WFSet.add wf wfs
        else
          wfs) (poles a) WFSet.empty)
```

This should be much faster! Is it correct? Is it faster indeed?

```
let poles' a =
  List.map CA.lhs a.CA.fusions

let s_channel a =
  WFSet.elements
    (List.fold_right
      (fun wf wfs →
        if P.Scattering.timelike wf.CA.momentum then
          WFSet.add wf wfs
        else
          wfs) (poles' a) WFSet.empty)
```

Pictures

Export the DAG in the `dot(1)` file format so that we can draw pretty pictures
to impress audiences . . .

```
let p2s p =
  if p ≥ 0 ∧ p ≤ 9 then
    string_of_int p
  else if p ≤ 36 then
    String.make 1 (Char.chr (Char.code 'A' + p − 10))
  else
    "_"

let variable wf =
  CM.flavor_symbol wf.CA.flavor ^
  String.concat "" (List.map p2s (P.to_ints wf.CA.momentum))

module Int = Map.Make (struct type t = int let compare = compare end)

let add_to_list i n m =
  Int.add i (n :: try Int.find i m with Not_found → []) m

let classify_nodes dag =
  Int.fold (fun i n acc → (i, n) :: acc)
    (CA.D.fold_nodes (fun wf → add_to_list (P.rank wf.CA.momentum) wf)
      dag Int.empty) []

let dag_to_dot ch brakets dag =
  Printf.fprintf ch "digraph␣OMEGA␣{\n";
  CA.D.iter_nodes (fun wf →
```

```
        Printf.fprintf ch "␣␣\"%s\"␣[␣label␣=␣\"%s\"␣];\n"
          (variable wf) (variable wf)) dag;
      List.iter (fun (_, wfs) →
        Printf.fprintf ch "␣␣{␣rank␣=␣same;";
        List.iter (fun n →
          Printf.fprintf ch "␣\"%s\";" (variable n)) wfs;
        Printf.fprintf ch "␣};\n") (classify_nodes dag);
      List.iter (fun n →
        Printf.fprintf ch "␣\"*\"␣->␣\"%s\";\n" (variable n))
        (flatten_keystones brakets);
      CA.D.iter (fun n (_, ns) →
        let p = variable n in
        PT.iter (fun n' →
          Printf.fprintf ch "␣␣\"%s\"␣->␣\"%s\";\n" p (variable n')) ns) dag;
      Printf.fprintf ch "}\n"

  let tower_to_dot ch a =
    dag_to_dot ch a.CA.brakets a.CA.fusion_tower

  let amplitude_to_dot ch a =
    dag_to_dot ch a.CA.brakets a.CA.fusion_dag

  let rcs_list = [CA.D.rcs; T.rcs; P.rcs; CM.rcs; rcs]

  end

module Make = Tagged(Order_Tags)

module Binary = Make(Tuple.Binary)(Stat_Dirac)(Topology.Binary)
module Tagged_Binary (T : Tagger) =
  Tagged(T)(Tuple.Binary)(Stat_Dirac)(Topology.Binary)
```

8.2.5   Fusions with Majorana Fermions

```
module Stat_Majorana (M : Model.T) : (Stat with type flavor = M.flavor) =
  struct
    let rcs = RCS.rename rcs_file "Fusion.Stat_Majorana()"
        [ "Fermi␣statistics␣with␣fermion␣number␣violation"]

    type flavor = M.flavor

    type stat =
      | Fermion of int × int list
      | AntiFermion of int × int list
      | Boson of int list
      | Majorana of int × int list

    let stat f p =
      let s = M.fermion f in
      if s = 0 then
        Boson []
      else if s < 0 then
        AntiFermion (p, [])
      else if s = 1 then (* if s = 1 then *)
```

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```
    Fermion (p, [])
  else (∗ if s > 1 then ∗)
    Majorana (p, [])
```

JR sez' (regarding the Majorana Feynman rules): In the formalism of [7], it does not matter to distinguish spinors and conjugate spinors, it is only important to know in which direction a fermion line is calculated. So the sign is made by the calculation together with an aditional one due to the permuation of the pairs of endpoints of fermion lines in the direction they are calculated. We propose a "canonical" direction from the right to the left child at a fusion point so we only have to keep in mind which external particle hangs at each side. Therefore we need not to have a list of pairs of conjugate spinors and spinors but just a list in which the pairs are right-left-right-left and so on. Unfortunately it is unavoidable to have couplings with clashing arrows in supersymmetric theories so we need transmutations from fermions in antifermions and vice versa as well. *(JR's probably right, but I need to check myself . . .)*

```
exception Impossible

let stat_fuse s1 s2 f =
  match s1, s2, M.lorentz f with
  | Boson l1, Fermion (p, l2), Coupling.Majorana
  | Boson l1, AntiFermion (p, l2), Coupling.Majorana
  | Fermion (p, l1), Boson l2, Coupling.Majorana
  | AntiFermion (p, l1), Boson l2, Coupling.Majorana
  | Majorana (p, l1), Boson l2, Coupling.Majorana
  | Boson l1, Majorana (p, l2), Coupling.Majorana →
      Majorana (p, l1 @ l2)
  | Boson l1, Fermion (p, l2), Coupling.Spinor
  | Boson l1, AntiFermion (p, l2), Coupling.Spinor
  | Fermion (p, l1), Boson l2, Coupling.Spinor
  | AntiFermion (p, l1), Boson l2, Coupling.Spinor
  | Majorana (p, l1), Boson l2, Coupling.Spinor
  | Boson l1, Majorana (p, l2), Coupling.Spinor →
      Fermion (p, l1 @ l2)
  | Boson l1, Fermion (p, l2), Coupling.ConjSpinor
  | Boson l1, AntiFermion (p, l2), Coupling.ConjSpinor
  | Fermion (p, l1), Boson l2, Coupling.ConjSpinor
  | AntiFermion (p, l1), Boson l2, Coupling.ConjSpinor
  | Majorana (p, l1), Boson l2, Coupling.ConjSpinor
  | Boson l1, Majorana (p, l2), Coupling.ConjSpinor →
      AntiFermion (p, l1 @ l2)
  | Boson l1, Fermion (p, l2), Coupling.Vectorspinor
  | Boson l1, AntiFermion (p, l2), Coupling.Vectorspinor
  | Fermion (p, l1), Boson l2, Coupling.Vectorspinor
  | AntiFermion (p, l1), Boson l2, Coupling.Vectorspinor
  | Majorana (p, l1), Boson l2, Coupling.Vectorspinor
  | Boson l1, Majorana (p, l2), Coupling.Vectorspinor →
      Majorana (p, l1 @ l2)
```

```
      |  Boson l1, Boson l2, _  →  Boson (l1 @ l2)
      |  AntiFermion (p1, l1), Fermion (p2, l2), _
      |  Fermion (p1, l1), AntiFermion (p2, l2), _
      |  Fermion (p1, l1), Fermion (p2, l2), _
      |  AntiFermion (p1, l1), AntiFermion (p2, l2), _
      |  Fermion (p1, l1), Majorana (p2, l2), _
      |  Majorana (p1, l1), Fermion (p2, l2), _
      |  AntiFermion (p1, l1), Majorana (p2, l2), _
      |  Majorana (p1, l1), AntiFermion (p2, l2), _
      |  Majorana (p1, l1), Majorana (p2, l2), _  →
            Boson ([p2; p1] @ l1 @ l2)
      |  Boson l1, Majorana (p, l2), _  →  Majorana (p, l1 @ l2)
      |  Boson l1, Fermion (p, l2), _  →  Fermion (p, l1 @ l2)
      |  Boson l1, AntiFermion (p, l2), _  →  AntiFermion (p, l1 @ l2)
      |  Fermion (p, l1), Boson l2, _  →  Fermion (p, l1 @ l2)
      |  AntiFermion (p, l1), Boson l2, _  →  AntiFermion (p, l1 @ l2)
      |  Majorana (p, l1), Boson l2, _  →  Majorana (p, l1 @ l2)

    let permutation lines  =  fst(Combinatorics.sort_signed compare lines)

    let stat_sign  =  function
      |  Boson lines  →  permutation lines
      |  Fermion (p, lines)  →  permutation (p :: lines)
      |  AntiFermion (pbar, lines)  →  permutation (pbar :: lines)
      |  Majorana (pm, lines)  →  permutation (pm :: lines)

  end
```

```
module Binary_Majorana  =
  Make(Tuple.Binary)(Stat_Majorana)(Topology.Binary)

module Nary (B : Tuple.Bound)  =
  Make(Tuple.Nary(B))(Stat_Dirac)(Topology.Nary(B))
module Nary_Majorana (B : Tuple.Bound)  =
  Make(Tuple.Nary(B))(Stat_Majorana)(Topology.Nary(B))

module Mixed23  =
  Make(Tuple.Mixed23)(Stat_Dirac)(Topology.Mixed23)
module Mixed23_Majorana  =
  Make(Tuple.Mixed23)(Stat_Majorana)(Topology.Mixed23)

module Helac (B : Tuple.Bound)  =
  Make(Tuple.Nary(B))(Stat_Dirac)(Topology.Helac(B))
module Helac_Majorana (B : Tuple.Bound)  =
  Make(Tuple.Nary(B))(Stat_Majorana)(Topology.Helac(B))
```

8.2.6 Multiple Amplitudes

```
module type Multi  =
  sig
    exception Mismatch
    val options  :  Options.t
    type flavor
```

```
    type process = flavor list × flavor list
    type amplitude
    type fusion
    type wf
    type selectors
    type amplitudes
    val amplitudes : bool →  int option →  selectors  →  process list →
amplitudes
    val empty : amplitudes
    val initialize_cache : string →  unit
    val set_cache_name : string →  unit
    val flavors : amplitudes  →  process list
    val vanishing_flavors : amplitudes  →  process list
    val color_flows : amplitudes  →  Color.Flow.t list
    val helicities : amplitudes  →  (int list × int list) list
    val processes : amplitudes  →  amplitude list
    val process_table : amplitudes  →  amplitude option array array
    val fusions : amplitudes  →  (fusion  ×  amplitude) list
    val multiplicity : amplitudes  →  wf  →  int
    val dictionary : amplitudes  →  amplitude  →  wf  →  int
    val color_factors : amplitudes  →  Color.Flow.factor array array
    val constraints : amplitudes  →  string option
  end

module type Multi_Maker = functor (Fusion_Maker : Maker) →
  functor (P : Momentum.T) →
    functor (M : Model.T) →
      Multi with type flavor = M.flavor
      and type amplitude = Fusion_Maker(P)(M).amplitude
      and type fusion = Fusion_Maker(P)(M).fusion
      and type wf = Fusion_Maker(P)(M).wf
      and type selectors = Fusion_Maker(P)(M).selectors

module Multi (Fusion_Maker : Maker) (P : Momentum.T) (M : Model.T) =
  struct

    exception Mismatch

    type progress_mode =
      | Quiet
      | Channel of out_channel
      | File of string

    let progress_option = ref Quiet

    module CM = Colorize.It(M)
    module F = Fusion_Maker(P)(M)
    module C = Cascade.Make(M)(P)
```

⧄ A kludge, at best . . .

```
    let options = Options.extend F.options
        ["progress", Arg.Unit (fun () → progress_option := Channel stderr),
```

```
let process_to_string fin fout =
  String.concat "␣" (List.map M.flavor_to_string fin)
  ^ "␣->␣" ^ String.concat "␣" (List.map M.flavor_to_string fout)

let count_processes colored_processes =
  List.length colored_processes

module FMap =
  Map.Make (struct type t = process let compare = compare end)

module CMap =
  Map.Make (struct type t = Color.Flow.t let compare = compare end)
```

Recently *Product.list* began to guarantee lexicographic order for sorted arguments. Anyway, we still force a lexicographic order.

```
let rec order_spin_table1 s1 s2 =
  match s1, s2 with
  | h1 :: t1, h2 :: t2 →
      let c = compare h1 h2 in
      if c ≠ 0 then
        c
      else
        order_spin_table1 t1 t2
  | [], [] → 0
  | _ → invalid_arg "order_spin_table:␣inconsistent␣lengths"

let order_spin_table (s1_in, s1_out) (s2_in, s2_out) =
  let c = compare s1_in s2_in in
  if c ≠ 0 then
    c
  else
    order_spin_table1 s1_out s2_out

let sort_spin_table table =
  List.sort order_spin_table table

let id x = x

let pair x y = (x, y)
```

Improve support for on shell Ward identities: *Coupling.Vector* → [4] for one and only one external vector.

```
let rec hs_of_lorentz = function
  | Coupling.Scalar → [0]
  | Coupling.Spinor | Coupling.ConjSpinor
  | Coupling.Majorana | Coupling.Maj_Ghost → [−1; 1]
  | Coupling.Vector → [−1; 1]
  | Coupling.Massive_Vector → [−1; 0; 1]
  | Coupling.Tensor_1 → [−1; 0; 1]
  | Coupling.Vectorspinor → [−2; −1; 1; 2]
  | Coupling.Tensor_2 → [−2; −1; 0; 1; 2]
  | Coupling.BRS f → hs_of_lorentz f
```

let *hs_of_flavor f* =
　hs_of_lorentz (*M.lorentz f*)

let *hs_of_flavors* (*fin*, *fout*) =
　(*List.map hs_of_flavor fin*, *List.map hs_of_flavor fout*)

let rec *unphysical_of_lorentz* = function
　| *Coupling.Vector* → [4]
　| *Coupling.Massive_Vector* → [4]
　| _ → *invalid_arg* "unphysical_of_lorentz:␣not␣a␣vector␣particle"

let *unphysical_of_flavor f* =
　unphysical_of_lorentz (*M.lorentz f*)

let *unphysical_of_flavors1 n f_list* =
　ThoList.mapi
　　(fun *i f* → if *i* = *n* then *unphysical_of_flavor f* else *hs_of_flavor f*)
　　1 *f_list*

let *unphysical_of_flavors n* (*fin*, *fout*) =
　(*unphysical_of_flavors1 n fin*, *unphysical_of_flavors1* (*n* − *List.length fin*) *fout*)

let *helicity_table unphysical flavors* =
　let *hs* =
　　begin match *unphysical* with
　　| *None* → *List.map hs_of_flavors flavors*
　　| *Some n* → *List.map* (*unphysical_of_flavors n*) *flavors*
　　end in
　if ¬ (*ThoList.homogeneous hs*) then
　　invalid_arg "Fusion.helicity_table:␣not␣all␣flavors␣have␣the␣same␣helicity␣states!"
　else
　　match *hs* with
　　| [] → []
　　| (*hs_in*, *hs_out*) :: _ →
　　　sort_spin_table (*Product.list2 pair* (*Product.list id hs_in*) (*Product.list id hs_out*))

module *Proc* = *Process.Make*(*M*)

module *WFMap* = *Map.Make* (struct type *t* = *F.wf* let *compare* = *compare* end)
module *WFSet2* =
　Set.Make (struct type *t* = *F.wf* × (*F.wf*, *F.coupling*) *Tree2.t* let *compare* = *compare* end)
module *WFMap2* =
　Map.Make (struct type *t* = *F.wf* × (*F.wf*, *F.coupling*) *Tree2.t* let *compare* = *compare* end)
module *WFTSet* =
　Set.Make (struct type *t* = (*F.wf*, *F.coupling*) *Tree2.t* let *compare* = *compare* end)

All wavefunctions are unique per amplitude. So we can use per-amplitude dependency trees without additional *internal* tags to identify identical wave functions.

NB: we miss potential optimizations, because we assume all coupling to be different, while in fact we have horizontal/family symmetries and non abelian gauge couplings are universal anyway.

let *disambiguate_fusions amplitudes* =
　let *fusions* =

```
        ThoList.flatmap (fun amplitude →
          List.map
            (fun fusion → (fusion, F.dependencies amplitude (F.lhs fusion)))
            (F.fusions amplitude))
          amplitudes in
    let duplicates =
      List.fold_left
        (fun map (fusion, dependencies) →
          let wf = F.lhs fusion in
          let set = try WFMap.find wf map with Not_found → WFTSet.empty in
          WFMap.add wf (WFTSet.add dependencies set) map)
        WFMap.empty fusions in
    let multiplicity_map =
      WFMap.fold (fun wf dependencies acc →
        let cardinal = WFTSet.cardinal dependencies in
        if cardinal ≤ 1 then
          acc
        else
          WFMap.add wf cardinal acc)
        duplicates WFMap.empty
    and dictionary_map =
      WFMap.fold (fun wf dependencies acc →
        let cardinal = WFTSet.cardinal dependencies in
        if cardinal ≤ 1 then
          acc
        else
          snd (WFTSet.fold
                 (fun dependency (i', acc') →
                    (succ i', WFMap2.add (wf, dependency) i' acc'))
                 dependencies (1, acc)))
        duplicates WFMap2.empty in
    let multiplicity wf =
      WFMap.find wf multiplicity_map
    and dictionary amplitude wf =
      WFMap2.find (wf, F.dependencies amplitude wf) dictionary_map in
    (multiplicity, dictionary)

let eliminate_common_fusions1 seen_wfs amplitude =
  List.fold_left
    (fun (seen, acc) f →
       let wf = F.lhs f in
       let dependencies = F.dependencies amplitude wf in
       if WFSet2.mem (wf, dependencies) seen then
         (seen, acc)
       else
         (WFSet2.add (wf, dependencies) seen, (f, amplitude) :: acc))
    seen_wfs (F.fusions amplitude)

let eliminate_common_fusions processes =
  let _, rev_fusions =
    List.fold_left
```

     eliminate_common_fusions1
      (*WFSet2.empty*, []) *processes* in
  List.rev rev_fusions

<div align="center">

Calculate All The Amplitudes

</div>

  let *amplitudes goldstones unphysical select_wf processes* =

Eventually, we might want to support inhomogeneous helicities. However,
this makes little physics sense for external particles on the mass shell, unless
we have a model with degenerate massive fermions and bosons.

    if ¬ (*ThoList.homogeneous* (*List.map hs_of_flavors processes*)) then
      invalid_arg "Fusion.Multi.amplitudes:␣incompatible␣helicities";

    let *unique_uncolored_processes* =
      Proc.remove_duplicate_final_states (*C.partition select_wf*) *processes* in

    let *progress* =
      match !*progress_option* with
      | *Quiet* → *Progress.dummy*
      | *Channel oc* → *Progress.channel oc* (*count_processes unique_uncolored_processes*)
      | *File name* → *Progress.file name* (*count_processes unique_uncolored_processes*) in

    let *allowed* =
      ThoList.flatmap
        (fun (*fi*, *fo*) →
         Progress.begin_step progress (*process_to_string fi fo*);
         let *amps* = *F.amplitudes goldstones select_wf fi fo* in
         begin match *amps* with
         | [] → *Progress.end_step progress* "forbidden"
         | _ → *Progress.end_step progress* "allowed"
         end;
         amps) *unique_uncolored_processes* in

    Progress.summary progress "all␣processes␣done";

    let *color_flows* =
      ThoList.uniq (*List.sort compare* (*List.map color_flow allowed*))
    and *flavors* =
      ThoList.uniq (*List.sort compare* (*List.map process_sans_color allowed*)) in

    let *vanishing_flavors* =
      Proc.diff processes flavors in

    let *helicities* =
      helicity_table unphysical flavors in

    let *f_index* =
      fst (*List.fold_left*
           (fun (*m*, *i*) *f* → (*FMap.add f i m*, *succ i*))
           (*FMap.empty*, 0) *flavors*)
    and *c_index* =

<div align="center">

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</div>

```
      fst (List.fold_left
              (fun (m, i) c → (CMap.add c i m, succ i))
              (CMap.empty, 0) color_flows) in
  let table =
    Array.make_matrix (List.length flavors) (List.length color_flows) None in
  List.iter
    (fun a →
       let f = FMap.find (process_sans_color a) f_index
       and c = CMap.find (color_flow a) c_index in
       table.(f).(c) ← Some (a))
    allowed;

  let cf_array = Array.of_list color_flows in
  let ncf = Array.length cf_array in
  let color_factor_table = Array.make_matrix ncf ncf Color.Flow.zero in

  for i = 0 to pred ncf do
    for j = 0 to i do
      color_factor_table.(i).(j) ←
         Color.Flow.factor cf_array.(i) cf_array.(j);
      color_factor_table.(j).(i) ←
         color_factor_table.(i).(j)
    done
  done;

  let fusions = eliminate_common_fusions allowed
  and multiplicity, dictionary = disambiguate_fusions allowed in

  { flavors = flavors;
    vanishing_flavors = vanishing_flavors;
    color_flows = color_flows;
    helicities = helicities;
    processes = allowed;
    process_table = table;
    fusions = fusions;
    multiplicity = multiplicity;
    dictionary = dictionary;
    color_factors = color_factor_table;
    constraints = C.description select_wf }

let initialize_cache = F.initialize_cache
let set_cache_name = F.set_cache_name

let empty =
  { flavors = [];
    vanishing_flavors = [];
    color_flows = [];
    helicities = [];
    processes = [];
    process_table = Array.make_matrix 0 0 None;
    fusions = [];
    multiplicity = (fun _ → 1);
    dictionary = (fun _ _ → 1);
```

$color\_factors \ = \ Array.make\_matrix \ 0 \ 0 \ Color.Flow.zero$;
$constraints \ = \ None$ }

end

—9—
Lorentz Representations, Couplings, Models and Targets

9.1   Interface of Coupling

The enumeration types used for communication from *Models* to *Targets*. On the physics side, the modules in *Models* must implement the Feynman rules according to the conventions set up here. On the numerics side, the modules in *Targets* must handle all cases according to the same conventions.

9.1.1   Propagators

The Lorentz representation of the particle. NB: O'Mega treats all lines as *outgoing* and particles are therefore transforming as *ConjSpinor* and antiparticles as *Spinor*.

type *lorentz* =
   | *Scalar*
   | *Spinor* (∗ ψ ∗)
   | *ConjSpinor* (∗ $\bar{\psi}$ ∗)
   | *Majorana* (∗ χ ∗)
   | *Maj_Ghost* (∗ SUSY ghosts ∗)
   | *Vector*
   | *Massive_Vector*
   | *Vectorspinor* (∗ supersymmetric currents and gravitinos ∗)
   | *Tensor_1*
   | *Tensor_2* (∗ massive gravitons (large extra dimensions) ∗)
   | *BRS* of *lorentz*

 If there were no vectors or auxiliary fields, we could deduce the propagator from the Lorentz representation. While we're at it, we can introduce "propagators" for the contact interactions of auxiliary fields as well. *Prop_Gauge* and *Prop_Feynman* are redundant as special cases of *Prop_Rxi*.

    The special case *Only_Insertion* corresponds to operator insertions that do not correspond to a propagating field all. These are used for checking Slavnov-Taylor identities

$$\partial_\mu \langle \text{out}|W^\mu(x)|\text{in}\rangle = m_W \langle \text{out}|\phi(x)|\text{in}\rangle \tag{9.1}$$

	only Dirac fermions	incl. Majorana fermions
Prop_Scalar	$\phi(p) \leftarrow \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma}\phi(p)$	
Prop_Spinor	$\psi(p) \leftarrow \dfrac{\mathrm{i}(-\not{p} + m)}{p^2 - m^2 + \mathrm{i}m\Gamma}\psi(p)$	$\psi(p) \leftarrow \dfrac{\mathrm{i}(-\not{p} + m)}{p^2 - m^2 + \mathrm{i}m\Gamma}\psi(p)$
Prop_ConjSpinor	$\bar{\psi}(p) \leftarrow \bar{\psi}(p)\dfrac{\mathrm{i}(\not{p} + m)}{p^2 - m^2 + \mathrm{i}m\Gamma}$	$\psi(p) \leftarrow \dfrac{\mathrm{i}(-\not{p} + m)}{p^2 - m^2 + \mathrm{i}m\Gamma}\psi(p)$
Prop_Majorana	N/A	$\chi(p) \leftarrow \dfrac{\mathrm{i}(-\not{p} + m)}{p^2 - m^2 + \mathrm{i}m\Gamma}\chi(p)$
Prop_Unitarity	$\epsilon_\mu(p) \leftarrow \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma}\left(-g_{\mu\nu} + \dfrac{p_\mu p_\nu}{m^2}\right)\epsilon^\nu(p)$	
Prop_Feynman	$\epsilon^\nu(p) \leftarrow \dfrac{-\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma}\epsilon^\nu(p)$	
Prop_Gauge	$\epsilon_\mu(p) \leftarrow \dfrac{\mathrm{i}}{p^2}\left(-g_{\mu\nu} + (1 - \xi)\dfrac{p_\mu p_\nu}{p^2}\right)\epsilon^\nu(p)$	
Prop_Rxi	$\epsilon_\mu(p) \leftarrow \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma}\left(-g_{\mu\nu} + (1 - \xi)\dfrac{p_\mu p_\nu}{p^2 - \xi m^2}\right)\epsilon^\nu(p)$	

Table 9.1: Propagators. NB: The sign of the momenta in the spinor propagators comes about because O'Mega treats all momenta as *outgoing* and the charge flow for *Spinor* is therefore opposite to the momentum, while the charge flow for *ConjSpinor* is parallel to the momentum.

Aux_Scalar	$\phi(p) \leftarrow \mathrm{i}\phi(p)$
Aux_Spinor	$\psi(p) \leftarrow \mathrm{i}\psi(p)$
Aux_ConjSpinor	$\bar{\psi}(p) \leftarrow \mathrm{i}\bar{\psi}(p)$
Aux_Vector	$\epsilon^\mu(p) \leftarrow \mathrm{i}\epsilon^\mu(p)$
Aux_Tensor_1	$T^{\mu\nu}(p) \leftarrow \mathrm{i}T^{\mu\nu}(p)$
Only_Insertion	N/A

Table 9.2: Auxiliary and non propagating fields

of gauge theories in unitarity gauge where the Goldstone bosons are not propagating. Numerically, it would suffice to use a vanishing propagator, but then superflous fusions would be calculated in production code in which the Slavnov-Taylor identities are not tested.

type α *propagator* =
   | *Prop_Scalar* | *Prop_Ghost*
   | *Prop_Spinor* | *Prop_ConjSpinor* | *Prop_Majorana*
   | *Prop_Unitarity* | *Prop_Feynman* | *Prop_Gauge* of α | *Prop_Rxi* of α
   | *Prop_Tensor_2* | *Prop_Tensor_pure* | *Prop_Vector_pure*
   | *Prop_Vectorspinor*
   | *Prop_Col_Scalar* | *Prop_Col_Feynman* | *Prop_Col_Majorana*
   | *Prop_Col_Unitarity*
   | *Aux_Scalar* | *Aux_Vector* | *Aux_Tensor_1*
   | *Aux_Col_Scalar* | *Aux_Col_Vector* | *Aux_Col_Tensor_1*
   | *Aux_Spinor* | *Aux_ConjSpinor* | *Aux_Majorana*
   | *Only_Insertion*

JR sez' (regarding the Majorana Feynman rules): We don't need different fermionic propagators as supposed by the variable names *Prop_Spinor*, *Prop_ConjSpinor* or *Prop_Majorana*. The propagator in all cases has to be multiplied on the left hand side of the spinor out of which a new one should be built. All momenta are treated as *outgoing*, so for the propagation of the different fermions the following table arises, in which the momentum direction is always downwards and the arrows show whether the momentum and the fermion line, respectively are parallel or antiparallel to the direction of calculation:

Fermion type	fermion arrow	mom.		calc.		sign
Dirac fermion	↑	↑	↓	↑	↑	negative
Dirac antifermion	↓	↓	↓	↑	↓	negative
Majorana fermion	-	↑	↓	-		negative

So the sign of the momentum is always negative and no further distinction is needed. *(JR's probably right, but I need to check myself . . .)*

type *width* =
   | *Vanishing*
   | *Constant*
   | *Timelike*
   | *Running*
   | *Fudged*
   | *Custom* of *string*

9.1.2 Vertices

The combined $S - P$ and $V - A$ couplings (see tables 9.5, 9.6, 9.8 and 9.12) are redundant, of course, but they allow some targets to create more efficient

numerical code.[1] Choosing VA2 over VA will cause the FORTRAN backend to pass the coupling as a whole array

type *fermion = Psi | Chi | Grav*
type *fermionbar = Psibar | Chibar | Gravbar*
type *boson =*
  | SP | SPM | S | P | SL | SR | SLR | VA | V | A | VL | VR |
VLR | VLRM | VAM
  | TVA | TLR | TRL | TVAM | TLRM | TRLM
  | POT | MOM | MOM5 | MOML | MOMR | LMOM | RMOM |
VMOM | VA2 | VA3 | VA3M
type *boson2 = S2 | P2 | S2P | S2L | S2R | S2LR*
  | SV | PV | SLV | SRV | SLRV | V2 | V2LR

The integer is an additional coefficient that multiplies the respective coupling constant. This allows to reduce the number of required coupling constants in manifestly symmetrc cases. Most of times it will be equal unity, though.

The two vertex types *PBP* and *BBB* for the couplings of two fermions or two antifermions ("clashing arrows") is unavoidable in supersymmetric theories.

⚠ ... tho doesn't like the names and has promised to find a better mnemonics!

type α *vertex3 =*
  | FBF of *int × fermionbar × boson × fermion*
  | PBP of *int × fermion × boson × fermion*
  | BBB of *int × fermionbar × boson × fermionbar*
  | GBG of *int × fermionbar × boson × fermion* (* gravitino-boson-fermion
*)
  | Gauge_Gauge_Gauge of *int | Aux_Gauge_Gauge* of *int*
  | Scalar_Vector_Vector of *int*
  | Aux_Vector_Vector of *int | Aux_Scalar_Vector* of *int*
  | Scalar_Scalar_Scalar of *int | Aux_Scalar_Scalar* of *int*
  | Vector_Scalar_Scalar of *int*
  | Graviton_Scalar_Scalar of *int*
  | Graviton_Vector_Vector of *int*
  | Graviton_Spinor_Spinor of *int*
  | Dim4_Vector_Vector_Vector_T of *int*
  | Dim4_Vector_Vector_Vector_L of *int*
  | Dim4_Vector_Vector_Vector_T5 of *int*
  | Dim4_Vector_Vector_Vector_L5 of *int*
  | Dim6_Gauge_Gauge_Gauge of *int*
  | Dim6_Gauge_Gauge_Gauge_5 of *int*
  | Aux_DScalar_DScalar of *int | Aux_Vector_DScalar* of *int*
  | Dim5_Scalar_Gauge2 of *int* (* $\frac{1}{2}\phi F_{1,\mu\nu} F_2^{\mu\nu} = -\frac{1}{2}\phi(\mathrm{i}\partial_{[\mu},V_{1,\nu]})(\mathrm{i}\partial^{[\mu},V_2^{\nu]})$ *)
  | Dim5_Scalar_Gauge2_Skew of *int*
    (* $\frac{1}{2}\phi F_{1,\mu\nu} \tilde{F}_2^{\mu\nu} = -\phi(\mathrm{i}\partial_\mu V_{1,\nu})(\mathrm{i}\partial_\rho V_{2,\sigma})\epsilon^{\mu\nu\rho\sigma}$ *)
  | Dim5_Scalar_Scalar2 of *int* (* $\phi_1\partial_\mu\phi_2\partial^\mu\phi_3$ *)
  | Dim5_Scalar_Vector_Vector_T of *int* (* $\phi(\mathrm{i}\partial_\mu V_1^\nu)(\mathrm{i}\partial_\nu V_2^\mu)$ *)
  | Dim5_Scalar_Vector_Vector_TU of *int* (* $(\mathrm{i}\partial_\nu\phi)(\mathrm{i}\partial_\mu V_1^\nu)V_2^\mu$ *)

---

[1]An additional benefit is that the counting of Feynman diagrams is not upset by a splitting of the vectorial and axial pieces of gauge bosons.

| *Dim5_Scalar_Vector_Vector_U* of *int* $(* (\mathrm{i}\partial_\nu\phi)(\mathrm{i}\partial_\mu V^\nu)V^\mu *)$
| *Scalar_Vector_Vector_t* of *int* $(* (\partial_\mu V_\nu - \partial_\nu V_\mu)^2 *)$
| *Dim6_Vector_Vector_Vector_T* of *int* $(* V_1^\mu((\mathrm{i}\partial_\nu V_2^\rho)\mathrm{i}\overleftrightarrow{\partial}_\mu(\mathrm{i}\partial_\rho V_3^\nu)) *)$
| *Tensor_2_Vector_Vector* of *int* $(* T^{\mu\nu}(V_{1,\mu}V_{2,\nu} + V_{1,\nu}V_{2,\mu}) *)$
| *Tensor_2_Vector_Vector_1* of *int* $(* T^{\mu\nu}(V_{1,\mu}V_{2,\nu} + V_{1,\nu}V_{2,\mu} - g_{\mu,\nu}V_1^\rho V_{2,\rho})$
$*)$
| *Tensor_2_Vector_Vector_cf* of *int* $(* T^{\mu\nu}(-\frac{c_f}{2}g_{\mu,\nu}V_1^\rho V_{2,\rho}) *)$
| *Tensor_2_Scalar_Scalar* of *int* $(* T^{\mu\nu}(\partial_\mu\phi_1\partial_\nu\phi_2 + \partial_\nu\phi_1\partial_\mu\phi_2) *)$
| *Tensor_2_Scalar_Scalar_cf* of *int* $(* T^{\mu\nu}(-\frac{c_f}{2}g_{\mu,\nu}\partial_\rho\phi_1\partial_\rho\phi_2) *)$
| *Tensor_2_Vector_Vector_t* of *int* $(* T^{\mu\nu}(V_{1,\mu}V_{2,\nu} + V_{1,\nu}V_{2,\mu} - g_{\mu,\nu}V_1^\rho V_{2,\rho})$
$*)$
| *Dim5_Tensor_2_Vector_Vector_1* of *int* $(* T^{\alpha\beta}(V_1^\mu\mathrm{i}\overleftrightarrow{\partial}_\alpha\mathrm{i}\overleftrightarrow{\partial}_\beta V_{2,\mu} *)$
| *Dim5_Tensor_2_Vector_Vector_2* of *int*
  $(* T^{\alpha\beta}(V_1^\mu\mathrm{i}\overleftrightarrow{\partial}_\beta(\mathrm{i}\partial_\mu V_{2,\alpha}) + V_1^\mu\mathrm{i}\overleftrightarrow{\partial}_\alpha(\mathrm{i}\partial_\mu V_{2,\beta})) *)$
| *Dim7_Tensor_2_Vector_Vector_T* of *int* $(* T^{\alpha\beta}((\mathrm{i}\partial^\mu V_1^\nu)\mathrm{i}\overleftrightarrow{\partial}_\alpha\mathrm{i}\overleftrightarrow{\partial}_\beta(\mathrm{i}\partial_\nu V_{2,\mu}))$
$*)$
| *Dim6_Scalar_Vector_Vector_D* of *int*
  $(* \mathrm{i}\phi(-(\partial^\mu\partial^\nu W_\mu^-)W_\nu^+ - (\partial^\mu\partial^\nu W_\nu^+)W_\mu^-$
  $+ ((\partial^\rho\partial_\rho W_\mu^-)W_\nu^+ + (\partial^\rho\partial_\rho W_\nu^+)W_\mu^-)g^{\mu\nu}) *)$
| *Dim6_Scalar_Vector_Vector_DP* of *int*
  $(* \mathrm{i}((\partial^\mu H)(\partial^\nu W_\mu^-)W_\nu^+ + (\partial^\nu H)(\partial^\mu W_\nu^+)W_\mu^-$
  $- ((\partial^\rho H)(\partial_\rho W_\mu^-)W_\nu^+(\partial^\rho H)(\partial^\rho W_\nu^+)W_\mu^-)g^{\mu\nu})*)$
| *Dim6_HAZ_D* of *int* $(* \mathrm{i}((\partial^\mu\partial^\nu A_\mu)Z_\nu + (\partial^\rho\partial_\rho A_\mu)Z_\nu g^{\mu\nu}) *)$
| *Dim6_HAZ_DP* of *int* $(* \mathrm{i}((\partial^\nu A_\mu)(\partial^\mu H)Z_\nu - (\partial^\rho A_\mu)(\partial_\rho H)Z_\nu g^{\mu\nu}) *)$
| *Dim6_AWW_DP* of *int* $(* \mathrm{i}((\partial^\rho A_\mu)W_\nu^- W_\rho^+ g^{\mu\nu} - (\partial^\nu A_\mu)W_\nu^- W_\rho^+ g^{\mu\rho}) *)$
| *Dim6_AWW_DW* of *int*
  $(*\mathrm{i}[(3(\partial^\rho A_\mu)W_\nu^- W_\rho^+ - (\partial^\rho W_\nu^-)A_\mu W_\rho^+ + (\partial^\rho W_\rho^+)A_\mu W_\nu^-)g^{\mu\nu}$
  $+ (-3(\partial^\nu A_\mu)W_\nu^- W_\rho^+ - (\partial^\nu W_\nu^-)A_\mu W_\rho^+ + (\partial^\nu W_\rho^+)A_\mu W_\nu^-)g^{\mu\rho}$
  $+ (2(\partial^\mu W_\nu^-)A_\mu W_\rho^+ - 2(\partial^\mu W_\rho^+)A_\mu W_\nu^-)g^{\nu\rho}] *)$
| *Dim6_HHH* of *int* $(*\mathrm{i}(-(\partial^\mu H_1)(\partial_\mu H_2)H_3 - (\partial^\mu H_1)H_2(\partial_\mu H_3) - H_1(\partial^\mu H_2)(\partial_\mu H_3))$
$*)$
| *Dim6_Gauge_Gauge_Gauge_i* of *int*
  $(*\mathrm{i}(-(\partial^\nu V_\mu)(\partial^\rho V_\nu)(\partial^\mu V_\rho) + (\partial^\rho V_\mu)(\partial^\mu V_\nu)(\partial^\nu V_\rho)$
  $+ (-\partial^\nu V_\rho g^{\mu\rho} + \partial^\mu V_\rho g^{\nu\rho})(\partial^\sigma V_\mu)(\partial_\sigma V_\nu) + (\partial^\rho V_\nu g^{\mu\nu} - \partial^\mu V_\nu g^{\nu\rho})(\partial^\sigma V_\mu)(\partial_\sigma V_\rho)$
  $+ (-\partial^\rho V_\mu g^{\mu\nu} + \partial^\mu V_\mu g^{\mu\rho})(\partial^\sigma V_\nu)(\partial_\sigma V_\rho)) *)$
| *Gauge_Gauge_Gauge_i* of *int*
| *Dim6_GGG* of *int*
| *Dim6_WWZ_DPWDW* of *int*
  $(* \mathrm{i}(((\partial^\rho V_\mu)V_\nu V_\rho - (\partial^\rho V_\nu)V_\mu V_\rho)g^{\mu\nu} - (\partial^\nu V_\mu)V_\nu V_\rho g^{\mu\rho} + (\partial^\mu V_\nu)V_\mu V_\rho)g^{\rho\nu}) *)$
| *Dim6_WWZ_DW* of *int*
  $(* \mathrm{i}(((\partial^\mu V_\mu)V_\nu V_\rho + V_\mu(\partial^\mu V_\nu)V_\rho)g^{\nu\rho} - ((\partial^\nu V_\mu)V_\nu V_\rho + V_\mu(\partial^\nu V_\nu)V_\rho)g^{\mu\rho}) *)$
| *Dim6_WWZ_D* of *int* $(* \mathrm{i}(V_\mu)V_\nu(\partial^\nu V_\rho)g^{\mu\rho} + V_\mu V_\nu(\partial^\mu V_\rho)g^{\nu\rho}) *)$
| *TensorVector_Vector_Vector* of *int*
| *TensorVector_Vector_Vector_cf* of *int*
| *TensorVector_Scalar_Scalar* of *int*
| *TensorVector_Scalar_Scalar_cf* of *int*
| *TensorScalar_Vector_Vector* of *int*
| *TensorScalar_Vector_Vector_cf* of *int*
| *TensorScalar_Scalar_Scalar* of *int*

  | *TensorScalar_Scalar_Scalar_cf* of *int*

As long as we stick to renormalizable couplings, there are only three types of quartic couplings: *Scalar4*, *Scalar2_Vector2* and *Vector4*. However, there are three inequivalent contractions for the latter and the general vertex will be a linear combination with integer coefficients:

$$Scalar4\ 1: \quad \phi_1\phi_2\phi_3\phi_4 \tag{9.2a}$$

$$Scalar2\_Vector2\ 1: \quad \phi_1\phi_2 V_3^\mu V_{4,\mu} \tag{9.2b}$$

$$Vector4\ [1, C\_12\_34]: \quad V_1^\mu V_{2,\mu} V_3^\nu V_{4,\nu} \tag{9.2c}$$

$$Vector4\ [1, C\_13\_42]: \quad V_1^\mu V_2^\nu V_{3,\mu} V_{4,\nu} \tag{9.2d}$$

$$Vector4\ [1, C\_14\_23]: \quad V_1^\mu V_2^\nu V_{3,\nu} V_{4,\mu} \tag{9.2e}$$

type *contract4* = *C_12_34* | *C_13_42* | *C_14_23*

type α *vertex4* =
  | *Scalar4* of *int*
  | *Scalar2_Vector2* of *int*
  | *Vector4* of (*int* × *contract4*) *list*
  | *DScalar4* of (*int* × *contract4*) *list*
  | *DScalar2_Vector2* of (*int* × *contract4*) *list*
  | *Dim8_Scalar2_Vector2_1* of *int*
  | *Dim8_Scalar2_Vector2_2* of *int*
  | *Dim8_Scalar4* of *int*
  | *GBBG* of *int* × *fermionbar* × *boson2* × *fermion*

In some applications, we have to allow for contributions outside of perturbation theory. The most prominent example is heavy gauge boson scattering at very high energies, where the perturbative expression violates unitarity.

One solution is the '*K*-matrix' ansatz. Such unitarizations typically introduce effective propagators and/or vertices that violate crossing symmetry and vanish in the *t*-channel. This can be taken care of in *Fusion* by filtering out vertices that have the wrong momenta.

In this case the ordering of the fields in a vertex of the Feynman rules becomes significant. In particular, we assume that (V_1, V_2, V_3, V_4) implies

$$\tag{9.3}$$

The list of pairs of parameters denotes the location and strengths of the poles in the *K*-matrix ansatz:

$$(c_1, a_1, c_2, a_2, \ldots, c_n, a_n) \Longrightarrow f(s) = \sum_{i=1}^n \frac{c_i}{s - a_i} \tag{9.4}$$

  | *Vector4_K_Matrix_tho* of *int* × (α × α) *list*

   | *Vector4_K_Matrix_jr* of *int* × (*int* × *contract4*) *list*
   | *DScalar2_Vector2_K_Matrix_ms* of *int* × (*int* × *contract4*) *list*
   | *DScalar4_K_Matrix_ms* of *int* × (*int* × *contract4*) *list*
   | *Dim6_H4_P2* of *int*
     (∗ i(−(∂^μ H_1)(∂_μ H_2)H_3 H_4 − (∂^μ H_1)H_2(∂_μ H_3)H_4 − (∂^μ H_1)H_2 H_3(∂_{mu}H_4)
$$- H_1(\partial^\mu H_2)(\partial_\mu H_3)H_4 - H_1(\partial^\mu H_2)H_3(\partial_\mu H_4) - H_1 H_2(\partial^\mu H_3)(\partial_\mu H_4)) \ *)$$
   | *Dim6_AHWW_DPB* of *int* (∗ i$H((\partial^\rho A_\mu)W_\nu W_\rho g^{\mu\nu} - (\partial^\nu A_\mu)W_\nu W_\rho g^{\mu\rho})$
∗)
   | *Dim6_AHWW_DPW* of *int*
     (∗ i$(((\partial^\rho A_\mu)W_\nu W_\rho - (\partial^\rho H)A_\mu W_\nu W_\rho)g^{\mu\nu}$
     $(-(\partial^\nu A_\mu)W_\nu W_\rho + (\partial^\nu H)A_\mu W_\nu W_\rho)g^{\mu\rho}) \ *)$
   | *Dim6_AHWW_DW* of *int*
     (∗ i$H((3(\partial^\rho A_\mu)W_\nu W_\rho - A_\mu(\partial^\rho W_\nu)W_\rho + A_\mu W_\nu(\partial^\rho W_\rho))g^{\mu\nu}$
     $+ (-3(\partial^\nu A_\mu)W_\nu W_\rho - A_\mu(\partial^\nu W_\nu)W_\rho + A_\mu W_\nu(\partial^\nu W_\rho))g^{\mu\rho}$
     $+ 2(A_\mu(\partial^\mu W_\nu)W_\rho + A_\mu W_\nu(\partial^\mu W_\rho)))g^{\nu\rho}) \ *)$
   | *Dim6_Vector4_DW* of *int* (∗i$(-V_{1,\mu}V_{2,\nu}V^{3,\nu}V^{4,\mu} - V_{1,\mu}V_{2,\nu}V^{3,\mu}V^{4,\nu}$
     $+ 2V_{1,\mu}V^{2,\mu}V_{3,\nu}V^{4,\nu} \ *)$
   | *Dim6_Vector4_W* of *int*
     (∗ i$(((\partial^\rho V_{1,\mu})V_2^\mu(\partial^\sigma V_{3,\rho})V_{4,\sigma} + V_{1,\mu}(\partial^\rho V_2^\mu)(\partial^\sigma V_{3,\rho})V_{4,\sigma}$
     $+ (\partial^\sigma V_{1,\mu})V_2^\mu V_{3,\rho}(\partial^\rho V_{4,\sigma}) + V_{1,\mu}(\partial^\sigma V_2^\mu)V_{3,\rho}(\partial^\rho V_{4,\sigma}))$
     $+ ((\partial^\sigma V_{1,\mu})V_{2,\nu}(\partial^\nu V_3^\mu)V_{4,\sigma} - V_{1,\mu}(\partial^\sigma V_{2,\nu})(\partial^\nu V_3^\mu)V_{4,\sigma}$
     $- (\partial^\nu V_1^\mu)V_{2,\nu}(\partial^\sigma V_{3,\mu})V_{4,\sigma} - (\partial^\sigma V_{1,\mu})V_{2,\nu}V_3^\mu(\partial^\nu V_{4,\sigma}))$
     $+ (-(\partial^\rho V_{1,\mu})V_{2,\nu}(\partial^\nu V_{3,\rho})V_4^\mu + (\partial^\rho V_{1,\mu})V_{2,\nu}V_{3,\rho}(\partial^\nu V_4^\mu)$
     $- V_{1,\mu}(\partial^\rho V_{2,\nu})V_{3,\rho}(\partial^\nu V_4^\mu) - (\partial^\nu V_{1,\mu})V_{2,\nu}V_{3,\rho}(\partial^\rho V_4^\mu))$
     $+ (-(\partial^\sigma V_{1,\mu})V_{2,\nu}(\partial^\mu V_3^\nu)V_{4,\sigma} + V_{1,\mu}(\partial^\sigma V_{2,\nu})(\partial^\mu V_3^\nu)V_{4,\sigma}$
     $- V_{1,\mu}(\partial^\mu V_{2,\nu})(\partial^\sigma V_3^\nu)V_{4,\sigma} - V_{1,\mu}(\partial^\sigma V_{2,\nu})V_3^\nu(\partial^\mu V_{4,\sigma}))$
     $+ (-V_{1,\mu}(\partial^\rho V_{2,\nu})(\partial^\mu V_{3,\rho})V_4^\nu - (\partial^\rho V_{1,\mu})V_{2,\nu}V_{3,\rho}(\partial^\mu V_4^\nu)$
     $+ V_{1,\mu}(\partial^\rho V_{2,\nu})V_{3,\rho}(\partial^\mu V_4^\nu) - V_{1,\mu}(\partial^\mu V_{2,\nu})V_{3,\rho}(\partial^\rho V_4^\nu))$
     $+ ((\partial^\nu V_{1,\mu})V_{2,\nu}(\partial^\mu V_{3,\rho})V_4^\rho + V_{1,\mu}(\partial^\mu V_{2,\nu})(\partial^\nu V_{3,\rho})V_4^\rho$
     $+ (\partial^\nu V_{1,\mu})V_{2,\nu}V_{3,\rho}(\partial^\mu V_4^\rho) + V_{1,\mu}(\partial^\mu V_{2,\nu})V_{3,\rho}(\partial^\nu V_4^\rho))$
     $+ (\partial^\rho V_{1,\mu})V_{2,\nu}V_3^\mu(\partial_\rho V_4^\nu) - (\partial^\rho V_{1,\mu})V_2^\mu V_{3,\nu}(\partial_\rho V_4^\nu)$
     $+ V_{1,\mu}(\partial^\rho V_{2,\nu})(\partial_\rho V_3^\mu)V_4^\nu - V_{1,\mu}(\partial^\rho V_2^\mu)(\partial_\rho V_{3,\nu})V_4^\nu$
     $+ (\partial^\rho V_{1,\mu})V_{2,\nu}(\partial_\rho V_3^\nu)V_4^\mu - (\partial^\rho V_{1,\mu})V_2^\mu(\partial_\rho V_{3,\nu})V_4^\nu$
     $+ V_{1,\mu}(\partial^\rho V_{2,\nu})V_3^\nu(\partial_\rho V_4^\mu) - V_{1,\mu}(\partial^\rho V_2^\mu)V_{3,\nu}(\partial_\rho V_4^\nu)) \ *)$
   | *Dim6_Scalar2_Vector2_D* of *int*
     (∗i$H_1 H_2(-(\partial^\mu \partial^\nu V_{3,\mu})V_{4,\nu} + (\partial^\mu \partial_\mu V_{3,\nu})V_4^\nu$
     $- V_{3,\mu}(\partial^\mu \partial^\nu V_{4,\nu}) + V_{3,\mu}(\partial^\nu \partial_\nu V_4^\mu)) \ *)$
   | *Dim6_Scalar2_Vector2_DP* of *int*
     (∗i$((\partial^\mu H_1)H_2(\partial^\nu V_{3,\mu})V_{4,\nu} - (\partial^\nu H_1)H_2(\partial_\nu V_{3,\mu})V^{4,\mu} + H_1(\partial^\mu H_2)(\partial^\nu V_{3,\mu})V_{4,\nu}$
     $- H_1(\partial^\nu H_2)(\partial_\nu V_{3,\mu})V^{4,\mu} + (\partial^\nu H_1)H_2 V_{3,\mu}(\partial^\mu V_{4,\nu}) - (\partial^\nu H_1)H_2 V_{3,\mu}(\partial_\nu V^{4,\mu})$
     $+ H_1(\partial^\nu H_2)V_{3,\mu}(\partial^\mu V_{4,\nu}) - H_1(\partial^\nu H_2)V_{3,\mu}(\partial_\nu V^{4,\mu})) \ *)$
   | *Dim6_Scalar2_Vector2_PB* of *int*
     (∗i$(H_1 H_2(\partial^\nu V_{3,\mu})(\partial^\mu V_{4,\nu}) - H_1 H_2(\partial^\nu V_{3,\mu})(\partial_\nu V^{4,\mu})) \ *)$
   | *Dim6_HHZZ_T* of *int* (∗i$H_1 H_2 V_{3,\mu}V^{4,\mu} \ *)$
   | *Dim6_HWWZ_DW* of *int*
     (∗ i$(H_1(\partial^\rho W_{2,\mu})W^{3,\mu}Z_{4,\rho} - H_1 W_{2,\mu}(\partial^\rho W^{3,\mu})Z_{4,\rho} - 2H_1(\partial^\nu W_{2,\mu})W_{3,\nu}Z^{4,\mu}$
     $- H_1 W_{2,\mu}(\partial^\nu W_{3,\nu})Z^{4,\mu} + H_1(\partial^\mu W_{2,\mu})W_{3,\nu}Z^{4,\nu} + 2H_1 W_{2,\mu}(\partial^\mu W_{3,\nu})Z^{4,\nu})$
∗)
   | *Dim6_HWWZ_DPB* of *int*
     (∗ i$(-H_1 W_{2,\mu}W_{3,\nu}(\partial^\nu Z^{4,\mu}) + H_1 W_{2,\mu}W_{3,\nu}(\partial^\mu Z^{4,\nu})) \ *)$

| *Dim6_HWWZ_DDPW* of *int*
  $(* \; \mathrm{i}(H_1(\partial^\nu W_{2,\mu})W^{3,\mu}Z_{4,\nu} - H_1 W_{2,\mu}(\partial^\nu W^{3,\mu})Z_{4,\nu} - H_1(\partial^\nu W_{2,\mu})W_{3,\nu}Z^{4,\mu}$
  $+ H_1 W_{2,\mu} W_{3,\nu}(\partial^\nu Z^{4,\mu}) + H_1 W_{2,\mu}(\partial^\mu W_{3,\nu})Z^{4,\nu} - H_1 W_{2,\mu} W_{3,\nu}(\partial^\mu Z^{4,\nu})) \; *)$
| *Dim6_HWWZ_DPW* of *int*
  $(* \; \mathrm{i}(H_1(\partial^\nu W_{2,\mu})W^{3,\mu}Z_{4,\nu} - H_1 W_{2,\mu}(\partial^\nu W^{3,\mu})Z_{4,\nu} + (\partial^\nu H_1)W_{2,\mu}W_{3,\nu}Z^{4,\mu}$
  $- H_1(\partial^\nu W_{2,\mu})W_{3,\nu}Z^{4,\mu} - (\partial^\mu H_1)W_{2,\mu}W_{3,\nu}Z^{4,\nu} + H_1 W_{2,\mu}(\partial^\mu W_{3,\nu})Z^{4,\nu}) \; *)$
| *Dim6_AHHZ_D* of *int*
  $(* \; \mathrm{i}(H_1 H_2(\partial^\mu \partial^\nu A_\mu)Z_\nu - H_1 H_2(\partial^\nu \partial_\nu A_\mu)Z^\mu) \; *)$
| *Dim6_AHHZ_DP* of *int*
  $(* \; \mathrm{i}((\partial^\mu H_1)H_2(\partial^\nu A_\mu)Z_\nu + H_1(\partial^\mu H_2)(\partial^\nu A_\mu)Z_\nu$
  $- (\partial^\nu H_1)H_2(\partial_\nu A_\mu)Z^\mu - H_1(\partial^\nu H_2)(\partial_\nu A_\mu)Z^\mu) \; *)$
| *Dim6_AHHZ_PB* of *int*
  $(* \; \mathrm{i}(H_1 H_2(\partial^\nu A_\mu)(\partial_\nu Z^\mu) - H_1 H_2(\partial^\nu A_\mu)(\partial^\mu Z_\nu)) \; *)$

type α *vertexn* $=$ *unit*

An obvious candidate for addition to *boson* is T, of course.

> This list is sufficient for the minimal standard model, but not comprehensive enough for most of its extensions, supersymmetric or otherwise. In particular, we need a *general* parameterization for all trilinear vertices. One straightforward possibility are polynomials in the momenta for each combination of fields.

> *JR sez' (regarding the Majorana Feynman rules):* Here we use the rules which can be found in [7] and are more properly described in *Targets* where the performing of the fusion rules in analytical expressions is encoded. *(JR's probably right, but I need to check myself ...)*

Signify which two of three fields are fused:

type *fuse2* $=$ *F23* | *F32* | *F31* | *F13* | *F12* | *F21*

Signify which three of four fields are fused:

type *fuse3* $=$
  | *F123* | *F231* | *F312* | *F132* | *F321* | *F213*
  | *F124* | *F241* | *F412* | *F142* | *F421* | *F214*
  | *F134* | *F341* | *F413* | *F143* | *F431* | *F314*
  | *F234* | *F342* | *F423* | *F243* | *F432* | *F324*

Explicit enumeration types make no sense for higher degrees.

type *fusen* $=$ *int list*

The third member of the triplet will contain the coupling constant:

type α *t* $=$
  | *V3* of α *vertex3* $\times$ *fuse2* $\times$ α
  | *V4* of α *vertex4* $\times$ *fuse3* $\times$ α
  | *Vn* of α *vertexn* $\times$ *fusen* $\times$ α

	only Dirac fermions	incl. Majorana fermions
FBF (*Psibar*, *S*, *Psi*): $\mathcal{L}_I = g_S \bar{\psi}_1 S \psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_S \bar{\psi}_1 S$	$\psi_2 \leftarrow \mathrm{i} \cdot g_S \psi_1 S$
F21	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_S S \bar{\psi}_1$	$\psi_2 \leftarrow \mathrm{i} \cdot g_S S \psi_1$
F13	$S \leftarrow \mathrm{i} \cdot g_S \bar{\psi}_1 \psi_2$	$S \leftarrow \mathrm{i} \cdot g_S \psi_1^T \mathrm{C} \psi_2$
F31	$S \leftarrow \mathrm{i} \cdot g_S \psi_{2,\alpha} \bar{\psi}_{1,\alpha}$	$S \leftarrow \mathrm{i} \cdot g_S \psi_2^T \mathrm{C} \psi_1$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_S S \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot g_S S \psi_2$
F32	$\psi_1 \leftarrow \mathrm{i} \cdot g_S \psi_2 S$	$\psi_1 \leftarrow \mathrm{i} \cdot g_S \psi_2 S$
FBF (*Psibar*, *P*, *Psi*): $\mathcal{L}_I = g_P \bar{\psi}_1 P \gamma_5 \psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_P \bar{\psi}_1 \gamma_5 P$	$\psi_2 \leftarrow \mathrm{i} \cdot g_P \gamma_5 \psi_1 P$
F21	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_P P \bar{\psi}_1 \gamma_5$	$\psi_2 \leftarrow \mathrm{i} \cdot g_P P \gamma_5 \psi_1$
F13	$P \leftarrow \mathrm{i} \cdot g_P \bar{\psi}_1 \gamma_5 \psi_2$	$P \leftarrow \mathrm{i} \cdot g_P \psi_1^T \mathrm{C} \gamma_5 \psi_2$
F31	$P \leftarrow \mathrm{i} \cdot g_P [\gamma_5 \psi_2]_\alpha \bar{\psi}_{1,\alpha}$	$P \leftarrow \mathrm{i} \cdot g_P \psi_2^T \mathrm{C} \gamma_5 \psi_1$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_P P \gamma_5 \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot g_P P \gamma_5 \psi_2$
F32	$\psi_1 \leftarrow \mathrm{i} \cdot g_P \gamma_5 \psi_2 P$	$\psi_1 \leftarrow \mathrm{i} \cdot g_P \gamma_5 \psi_2 P$
FBF (*Psibar*, *V*, *Psi*): $\mathcal{L}_I = g_V \bar{\psi}_1 \slashed{V} \psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_V \bar{\psi}_1 \slashed{V}$	$\psi_{2,\alpha} \leftarrow \mathrm{i} \cdot (-g_V) \psi_{1,\beta} \slashed{V}_{\alpha\beta}$
F21	$\bar{\psi}_{2,\beta} \leftarrow \mathrm{i} \cdot g_V \slashed{V}_{\alpha\beta} \bar{\psi}_{1,\alpha}$	$\psi_2 \leftarrow \mathrm{i} \cdot (-g_V) \slashed{V} \psi_1$
F13	$V_\mu \leftarrow \mathrm{i} \cdot g_V \bar{\psi}_1 \gamma_\mu \psi_2$	$V_\mu \leftarrow \mathrm{i} \cdot g_V (\psi_1)^T \mathrm{C} \gamma_\mu \psi_2$
F31	$V_\mu \leftarrow \mathrm{i} \cdot g_V [\gamma_\mu \psi_2]_\alpha \bar{\psi}_{1,\alpha}$	$V_\mu \leftarrow \mathrm{i} \cdot (-g_V)(\psi_2)^T \mathrm{C} \gamma_\mu \psi_1$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_V \slashed{V} \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot g_V \slashed{V} \psi_2$
F32	$\psi_{1,\alpha} \leftarrow \mathrm{i} \cdot g_V \psi_{2,\beta} \slashed{V}_{\alpha\beta}$	$\psi_{1,\alpha} \leftarrow \mathrm{i} \cdot g_V \psi_{2,\beta} \slashed{V}_{\alpha\beta}$
FBF (*Psibar*, *A*, *Psi*): $\mathcal{L}_I = g_A \bar{\psi}_1 \gamma_5 \slashed{A} \psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_A \bar{\psi}_1 \gamma_5 \slashed{A}$	$\psi_{2,\alpha} \leftarrow \mathrm{i} \cdot g_A \psi_\beta [\gamma_5 \slashed{A}]_{\alpha\beta}$
F21	$\bar{\psi}_{2,\beta} \leftarrow \mathrm{i} \cdot g_A [\gamma_5 \slashed{A}]_{\alpha\beta} \bar{\psi}_{1,\alpha}$	$\psi_2 \leftarrow \mathrm{i} \cdot g_A \gamma_5 \slashed{A} \psi$
F13	$A_\mu \leftarrow \mathrm{i} \cdot g_A \bar{\psi}_1 \gamma_5 \gamma_\mu \psi_2$	$A_\mu \leftarrow \mathrm{i} \cdot g_A \psi_1^T \mathrm{C} \gamma_5 \gamma_\mu \psi_2$
F31	$A_\mu \leftarrow \mathrm{i} \cdot g_A [\gamma_5 \gamma_\mu \psi_2]_\alpha \bar{\psi}_{1,\alpha}$	$A_\mu \leftarrow \mathrm{i} \cdot g_A \psi_2^T \mathrm{C} \gamma_5 \gamma_\mu \psi_1$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_A \gamma_5 \slashed{A} \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot g_A \gamma_5 \slashed{A} \psi_2$
F32	$\psi_{1,\alpha} \leftarrow \mathrm{i} \cdot g_A \psi_{2,\beta} [\gamma_5 \slashed{A}]_{\alpha\beta}$	$\psi_{1,\alpha} \leftarrow \mathrm{i} \cdot g_A \psi_{2,\beta} [\gamma_5 \slashed{A}]_{\alpha\beta}$

Table 9.3: Dimension-4 trilinear fermionic couplings. The momenta are unambiguous, because there are no derivative couplings and all participating fields are different.

	only Dirac fermions	incl. Majorana fermions
FBF (_Psibar_, _T_, _Psi_): $\mathcal{L}_I = g_T T_{\mu\nu} \bar{\psi}_1 [\gamma^\mu, \gamma^\nu]_- \psi_2$		
_F12_	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_T \bar{\psi}_1 [\gamma^\mu, \gamma^\nu]_- T_{\mu\nu}$	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_T \cdots$
_F21_	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_T T_{\mu\nu} \bar{\psi}_1 [\gamma^\mu, \gamma^\nu]_-$	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_T \cdots$
_F13_	$T_{\mu\nu} \leftarrow \mathrm{i} \cdot g_T \bar{\psi}_1 [\gamma_\mu, \gamma_\nu]_- \psi_2$	$T_{\mu\nu} \leftarrow \mathrm{i} \cdot g_T \cdots$
_F31_	$T_{\mu\nu} \leftarrow \mathrm{i} \cdot g_T [[\gamma_\mu, \gamma_\nu]_- \psi_2]_\alpha \bar{\psi}_{1,\alpha}$	$T_{\mu\nu} \leftarrow \mathrm{i} \cdot g_T \cdots$
_F23_	$\psi_1 \leftarrow \mathrm{i} \cdot g_T T_{\mu\nu} [\gamma^\mu, \gamma^\nu]_- \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot g_T \cdots$
_F32_	$\psi_1 \leftarrow \mathrm{i} \cdot g_T [\gamma^\mu, \gamma^\nu]_- \psi_2 T_{\mu\nu}$	$\psi_1 \leftarrow \mathrm{i} \cdot g_T \cdots$

Table 9.4:   Dimension-5 trilinear fermionic couplings (NB: the coefficients and signs are not fixed yet). The momenta are unambiguous, because there are no derivative couplings and all participating fields are different.

9.1.3   Gauge Couplings

Dimension-4 trilinear vector boson couplings

$$
\begin{aligned}
f_{abc} \partial^\mu A^{a,\nu} A^b_\mu A^c_\nu &\rightarrow \mathrm{i} f_{abc} k_1^\mu A^{a,\nu}(k_1) A^b_\mu(k_2) A^c_\nu(k_3) \\
&= -\frac{\mathrm{i}}{3!} f_{a_1 a_2 a_3} C^{\mu_1 \mu_2 \mu_3}(k_1, k_2, k_3) A^{a_1}_{\mu_1}(k_1) A^{a_2}_{\mu_2}(k_2) A^{a_3}_{\mu_3}(k_3) \quad (9.5\mathrm{a})
\end{aligned}
$$

with the totally antisymmetric tensor (under simultaneous permutations of all quantum numbers μ_i and k_i) and all momenta _outgoing_

$$
C^{\mu_1 \mu_2 \mu_3}(k_1, k_2, k_3) = (g^{\mu_1 \mu_2}(k_1^{\mu_3} - k_2^{\mu_3}) + g^{\mu_2 \mu_3}(k_2^{\mu_1} - k_3^{\mu_1}) + g^{\mu_3 \mu_1}(k_3^{\mu_2} - k_1^{\mu_2}))
$$
(9.5b)

Since $f_{a_1 a_2 a_3} C^{\mu_1 \mu_2 \mu_3}(k_1, k_2, k_3)$ is totally symmetric (under simultaneous permutations of all quantum numbers a_i, μ_i and k_i), it is easy to take the partial derivative

$$
A^{a,\mu}(k_2 + k_3) = -\frac{\mathrm{i}}{2!} f_{abc} C^{\mu\rho\sigma}(-k_2 - k_3, k_2, k_3) A^b_\rho(k_2) A^c_\sigma(k_3) \qquad (9.6\mathrm{a})
$$

with

$$
C^{\mu\rho\sigma}(-k_2 - k_3, k_2, k_3) = (g^{\rho\sigma}(k_2^\mu - k_3^\mu) + g^{\mu\sigma}(2k_3^\rho + k_2^\rho) - g^{\mu\rho}(2k_2^\sigma + k_3^\sigma)) \quad (9.6\mathrm{b})
$$

i. e.

$$
\begin{aligned}
A^{a,\mu}(k_2 + k_3) = -\frac{\mathrm{i}}{2!} f_{abc} &\big((k_2^\mu - k_3^\mu) A^b(k_2) \cdot A^c(k_3) \\
&+ (2k_3 + k_2) \cdot A^b(k_2) A^{c,\mu}(k_3) - A^{b,\mu}(k_2) A^c(k_3) \cdot (2k_2 + k_3) \big) \quad (9.6\mathrm{c})
\end{aligned}
$$

Investigate the rearrangements proposed in [5] for improved numerical stability.

	only Dirac fermions	incl. Majorana fermions
FBF (*Psibar*, *SP*, *Psi*): $\mathcal{L}_I = \bar{\psi}_1 \phi (g_S + g_P \gamma_5) \psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot \bar{\psi}_1 (g_S + g_P \gamma_5) \phi$	$\psi_2 \leftarrow \mathrm{i} \cdot \cdots$
F21	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot \phi \bar{\psi}_1 (g_S + g_P \gamma_5)$	$\psi_2 \leftarrow \mathrm{i} \cdot \cdots$
F13	$\phi \leftarrow \mathrm{i} \cdot \bar{\psi}_1 (g_S + g_P \gamma_5) \psi_2$	$\phi \leftarrow \mathrm{i} \cdot \cdots$
F31	$\phi \leftarrow \mathrm{i} \cdot [(g_S + g_P \gamma_5) \psi_2]_\alpha \bar{\psi}_{1,\alpha}$	$\phi \leftarrow \mathrm{i} \cdot \cdots$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot \phi (g_S + g_P \gamma_5) \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot \cdots$
F32	$\psi_1 \leftarrow \mathrm{i} \cdot (g_S + g_P \gamma_5) \psi_2 \phi$	$\psi_1 \leftarrow \mathrm{i} \cdot \cdots$
FBF (*Psibar*, *SL*, *Psi*): $\mathcal{L}_I = g_L \bar{\psi}_1 \phi (1 - \gamma_5) \psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_L \bar{\psi}_1 (1 - \gamma_5) \phi$	$\psi_2 \leftarrow \mathrm{i} \cdot \cdots$
F21	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_L \phi \bar{\psi}_1 (1 - \gamma_5)$	$\psi_2 \leftarrow \mathrm{i} \cdot \cdots$
F13	$\phi \leftarrow \mathrm{i} \cdot g_L \bar{\psi}_1 (1 - \gamma_5) \psi_2$	$\phi \leftarrow \mathrm{i} \cdot \cdots$
F31	$\phi \leftarrow \mathrm{i} \cdot g_L [(1 - \gamma_5) \psi_2]_\alpha \bar{\psi}_{1,\alpha}$	$\phi \leftarrow \mathrm{i} \cdot \cdots$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_L \phi (1 - \gamma_5) \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot \cdots$
F32	$\psi_1 \leftarrow \mathrm{i} \cdot g_L (1 - \gamma_5) \psi_2 \phi$	$\psi_1 \leftarrow \mathrm{i} \cdot \cdots$
FBF (*Psibar*, *SR*, *Psi*): $\mathcal{L}_I = g_R \bar{\psi}_1 \phi (1 + \gamma_5) \psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_R \bar{\psi}_1 (1 + \gamma_5) \phi$	$\psi_2 \leftarrow \mathrm{i} \cdot \cdots$
F21	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_R \phi \bar{\psi}_1 (1 + \gamma_5)$	$\psi_2 \leftarrow \mathrm{i} \cdot \cdots$
F13	$\phi \leftarrow \mathrm{i} \cdot g_R \bar{\psi}_1 (1 + \gamma_5) \psi_2$	$\phi \leftarrow \mathrm{i} \cdot \cdots$
F31	$\phi \leftarrow \mathrm{i} \cdot g_R [(1 + \gamma_5) \psi_2]_\alpha \bar{\psi}_{1,\alpha}$	$\phi \leftarrow \mathrm{i} \cdot \cdots$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_R \phi (1 + \gamma_5) \psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot \cdots$
F32	$\psi_1 \leftarrow \mathrm{i} \cdot g_R (1 + \gamma_5) \psi_2 \phi$	$\psi_1 \leftarrow \mathrm{i} \cdot \cdots$
FBF (*Psibar*, *SLR*, *Psi*): $\mathcal{L}_I = g_L \bar{\psi}_1 \phi (1 - \gamma_5) \psi_2 + g_R \bar{\psi}_1 \phi (1 + \gamma_5) \psi_2$		

Table 9.5:   Combined dimension-4 trilinear fermionic couplings.

	only Dirac fermions	incl. Majorana fermions
FBF (*Psibar*, *VA*, *Psi*): $\mathcal{L}_I = \bar{\psi}_1 \slashed{Z}(g_V - g_A\gamma_5)\psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot \bar{\psi}_1 \slashed{Z}(g_V - g_A\gamma_5)$	$\psi_2 \leftarrow \mathrm{i} \cdot \dots$
F21	$\bar{\psi}_{2,\beta} \leftarrow \mathrm{i} \cdot [\slashed{Z}(g_V - g_A\gamma_5)]_{\alpha\beta}\bar{\psi}_{1,\alpha}$	$\psi_2 \leftarrow \mathrm{i} \cdot \dots$
F13	$Z_\mu \leftarrow \mathrm{i} \cdot \bar{\psi}_1\gamma_\mu(g_V - g_A\gamma_5)\psi_2$	$Z_\mu \leftarrow \mathrm{i} \cdot \dots$
F31	$Z_\mu \leftarrow \mathrm{i} \cdot [\gamma_\mu(g_V - g_A\gamma_5)\psi_2]_\alpha\bar{\psi}_{1,\alpha}$	$Z_\mu \leftarrow \mathrm{i} \cdot \dots$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot \slashed{Z}(g_V - g_A\gamma_5)\psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot \dots$
F32	$\psi_{1,\alpha} \leftarrow \mathrm{i} \cdot \psi_{2,\beta}[\slashed{Z}(g_V - g_A\gamma_5)]_{\alpha\beta}$	$\psi_1 \leftarrow \mathrm{i} \cdot \dots$
FBF (*Psibar*, *VL*, *Psi*): $\mathcal{L}_I = g_L\bar{\psi}_1 \slashed{Z}(1 - \gamma_5)\psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_L\bar{\psi}_1 \slashed{Z}(1 - \gamma_5)$	$\psi_2 \leftarrow \mathrm{i} \cdot \dots$
F21	$\bar{\psi}_{2,\beta} \leftarrow \mathrm{i} \cdot g_L[\slashed{Z}(1 - \gamma_5)]_{\alpha\beta}\bar{\psi}_{1,\alpha}$	$\psi_2 \leftarrow \mathrm{i} \cdot \dots$
F13	$Z_\mu \leftarrow \mathrm{i} \cdot g_L\bar{\psi}_1\gamma_\mu(1 - \gamma_5)\psi_2$	$Z_\mu \leftarrow \mathrm{i} \cdot \dots$
F31	$Z_\mu \leftarrow \mathrm{i} \cdot g_L[\gamma_\mu(1 - \gamma_5)\psi_2]_\alpha\bar{\psi}_{1,\alpha}$	$Z_\mu \leftarrow \mathrm{i} \cdot \dots$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_L\slashed{Z}(1 - \gamma_5)\psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot \dots$
F32	$\psi_{1,\alpha} \leftarrow \mathrm{i} \cdot g_L\psi_{2,\beta}[\slashed{Z}(1 - \gamma_5)]_{\alpha\beta}$	$\psi_1 \leftarrow \mathrm{i} \cdot \dots$
FBF (*Psibar*, *VR*, *Psi*): $\mathcal{L}_I = g_R\bar{\psi}_1 \slashed{Z}(1 + \gamma_5)\psi_2$		
F12	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot g_R\bar{\psi}_1 \slashed{Z}(1 + \gamma_5)$	$\psi_2 \leftarrow \mathrm{i} \cdot \dots$
F21	$\bar{\psi}_{2,\beta} \leftarrow \mathrm{i} \cdot g_R[\slashed{Z}(1 + \gamma_5)]_{\alpha\beta}\bar{\psi}_{1,\alpha}$	$\psi_2 \leftarrow \mathrm{i} \cdot \dots$
F13	$Z_\mu \leftarrow \mathrm{i} \cdot g_R\bar{\psi}_1\gamma_\mu(1 + \gamma_5)\psi_2$	$Z_\mu \leftarrow \mathrm{i} \cdot \dots$
F31	$Z_\mu \leftarrow \mathrm{i} \cdot g_R[\gamma_\mu(1 + \gamma_5)\psi_2]_\alpha\bar{\psi}_{1,\alpha}$	$Z_\mu \leftarrow \mathrm{i} \cdot \dots$
F23	$\psi_1 \leftarrow \mathrm{i} \cdot g_R\slashed{Z}(1 + \gamma_5)\psi_2$	$\psi_1 \leftarrow \mathrm{i} \cdot \dots$
F32	$\psi_{1,\alpha} \leftarrow \mathrm{i} \cdot g_R\psi_{2,\beta}[\slashed{Z}(1 + \gamma_5)]_{\alpha\beta}$	$\psi_1 \leftarrow \mathrm{i} \cdot \dots$
FBF (*Psibar*, *VLR*, *Psi*): $\mathcal{L}_I = g_L\bar{\psi}_1 \slashed{Z}(1 - \gamma_5)\psi_2 + g_R\bar{\psi}_1 \slashed{Z}(1 + \gamma_5)\psi_2$		

Table 9.6: Combined dimension-4 trilinear fermionic couplings continued.

FBF (Psibar, S, Chi): $\bar{\psi}S\chi$	
_F12:_ $\chi \leftarrow \psi S$	_F21:_ $\chi \leftarrow S\psi$
_F13:_ $S \leftarrow \psi^T C\chi$	_F31:_ $S \leftarrow \chi^T C\psi$
_F23:_ $\psi \leftarrow S\chi$	_F32:_ $\psi \leftarrow \chi S$
FBF (Psibar, P, Chi): $\bar{\psi}P\gamma_5\chi$	
_F12:_ $\chi \leftarrow \gamma_5\psi P$	_F21:_ $\chi \leftarrow P\gamma_5\psi$
_F13:_ $P \leftarrow \psi^T C\gamma_5\chi$	_F31:_ $P \leftarrow \chi^T C\gamma_5\psi$
_F23:_ $\psi \leftarrow P\gamma_5\chi$	_F32:_ $\psi \leftarrow \gamma_5\chi P$
FBF (Psibar, V, Chi): $\bar{\psi}\slashed{V}\chi$	
_F12:_ $\chi_\alpha \leftarrow -\psi_\beta\slashed{V}_{\alpha\beta}$	_F21:_ $\chi \leftarrow -\slashed{V}\psi$
_F13:_ $V_\mu \leftarrow \psi^T C\gamma_\mu\chi$	_F31:_ $V_\mu \leftarrow \chi^T C(-\gamma_\mu\psi)$
_F23:_ $\psi \leftarrow \slashed{V}\chi$	_F32:_ $\psi_\alpha \leftarrow \chi_\beta\slashed{V}_{\alpha\beta}$
FBF (Psibar, A, Chi): $\bar{\psi}\gamma^5\slashed{A}\chi$	
_F12:_ $\chi_\alpha \leftarrow \psi_\beta[\gamma^5\slashed{A}]_{\alpha\beta}$	_F21:_ $\chi \leftarrow \gamma^5\slashed{A}\psi$
_F13:_ $A_\mu \leftarrow \psi^T C\gamma^5\gamma_\mu\chi$	_F31:_ $A_\mu \leftarrow \chi^T C(\gamma^5\gamma_\mu\psi)$
_F23:_ $\psi \leftarrow \gamma^5\slashed{A}\chi$	_F32:_ $\psi_\alpha \leftarrow \chi_\beta[\gamma^5\slashed{A}]_{\alpha\beta}$

Table 9.7: Dimension-4 trilinear couplings including one Dirac and one Majorana fermion

FBF (Psibar, SP, Chi): $\bar{\psi}\phi(g_S + g_P\gamma_5)\chi$	
_F12:_ $\chi \leftarrow (g_S + g_P\gamma_5)\psi\phi$	_F21:_ $\chi \leftarrow \phi(g_S + g_P\gamma_5)\psi$
_F13:_ $\phi \leftarrow \psi^T C(g_S + g_P\gamma_5)\chi$	_F31:_ $\phi \leftarrow \chi^T C(g_S + g_P\gamma_5)\chi$
_F23:_ $\psi \leftarrow \phi(g_S + g_P\gamma_5)\chi$	_F32:_ $\psi \leftarrow (g_S + g_P\gamma_5)\chi\phi$
FBF (Psibar, VA, Chi): $\bar{\psi}\slashed{Z}(g_V - g_A\gamma_5)\chi$	
_F12:_ $\chi_\alpha \leftarrow \psi_\beta[\slashed{Z}(-g_V - g_A\gamma_5)]_{\alpha\beta}$	_F21:_ $\chi \leftarrow \slashed{Z}(-g_V - g_A\gamma_5)]\psi$
_F13:_ $Z_\mu \leftarrow \psi^T C\gamma_\mu(g_V - g_A\gamma_5)\chi$	_F31:_ $Z_\mu \leftarrow \chi^T C\gamma_\mu(-g_V - g_A\gamma_5)\psi$
_F23:_ $\psi \leftarrow \slashed{Z}(g_V - g_A\gamma_5)\chi$	_F32:_ $\psi_\alpha \leftarrow \chi_\beta[\slashed{Z}(g_V - g_A\gamma_5)]_{\alpha\beta}$

Table 9.8: Combined dimension-4 trilinear fermionic couplings including one Dirac and one Majorana fermion.

FBF (Chibar, S, Psi): $\bar{\chi}S\psi$			
F12:	$\psi \leftarrow \chi S$	*F21:*	$\psi \leftarrow S\chi$
F13:	$S \leftarrow \chi^T \mathrm{C}\psi$	*F31:*	$S \leftarrow \psi^T \mathrm{C}\chi$
F23:	$\chi \leftarrow S\psi$	*F32:*	$\chi \leftarrow \psi S$
FBF (Chibar, P, Psi): $\bar{\chi}P\gamma_5\psi$			
F12:	$\psi \leftarrow \gamma_5\chi P$	*F21:*	$\psi \leftarrow P\gamma_5\chi$
F13:	$P \leftarrow \chi^T \mathrm{C}\gamma_5\psi$	*F31:*	$P \leftarrow \psi^T \mathrm{C}\gamma_5\chi$
F23:	$\chi \leftarrow P\gamma_5\psi$	*F32:*	$\chi \leftarrow \gamma_5\psi P$
FBF (Chibar, V, Psi): $\bar{\chi}\slashed{V}\psi$			
F12:	$\psi_\alpha \leftarrow -\chi_\beta \slashed{V}_{\alpha\beta}$	*F21:*	$\psi \leftarrow -\slashed{V}\chi$
F13:	$V_\mu \leftarrow \chi^T \mathrm{C}\gamma_\mu\psi$	*F31:*	$V_\mu \leftarrow \psi^T \mathrm{C}(-\gamma_\mu\chi)$
F23:	$\chi \leftarrow \slashed{V}\psi$	*F32:*	$\chi_\alpha \leftarrow \psi_\beta \slashed{V}_{\alpha\beta}$
FBF (Chibar, A, Psi): $\bar{\chi}\gamma^5\slashed{A}\psi$			
F12:	$\psi_\alpha \leftarrow \chi_\beta[\gamma^5\slashed{A}]_{\alpha\beta}$	*F21:*	$\psi \leftarrow \gamma^5\slashed{A}\chi$
F13:	$A_\mu \leftarrow \chi^T \mathrm{C}(\gamma^5\gamma_\mu\psi)$	*F31:*	$A_\mu \leftarrow \psi^T \mathrm{C}\gamma^5\gamma_\mu\chi$
F23:	$\chi \leftarrow \gamma^5\slashed{A}\psi$	*F32:*	$\chi_\alpha \leftarrow \psi_\beta[\gamma^5\slashed{A}]_{\alpha\beta}$

Table 9.9: Dimension-4 trilinear couplings including one Dirac and one Majorana fermion

FBF (Chibar, SP, Psi): $\bar{\chi}\phi(g_S + g_P\gamma_5)\psi$			
F12:	$\psi \leftarrow (g_S + g_P\gamma_5)\chi\phi$	*F21:*	$\psi \leftarrow \phi(g_S + g_P\gamma_5)\chi$
F13:	$\phi \leftarrow \chi^T \mathrm{C}(g_S + g_P\gamma_5)\psi$	*F31:*	$\phi \leftarrow \psi^T \mathrm{C}(g_S + g_P\gamma_5)\chi$
F23:	$\chi \leftarrow \phi(g_S + g_P\gamma_5)\psi$	*F32:*	$\chi \leftarrow (g_S + g_P\gamma_5)\psi\phi$
FBF (Chibar, VA, Psi): $\bar{\chi}\slashed{Z}(g_V - g_A\gamma_5)\psi$			
F12:	$\psi_\alpha \leftarrow \chi_\beta[\slashed{Z}(-g_V - g_A\gamma_5)]_{\alpha\beta}$	*F21:*	$\psi \leftarrow \slashed{Z}(-g_V - g_A\gamma_5)\chi$
F13:	$Z_\mu \leftarrow \chi^T \mathrm{C}\gamma_\mu(g_V - g_A\gamma_5)\psi$	*F31:*	$Z_\mu \leftarrow \psi^T \mathrm{C}\gamma_\mu(-g_V - g_A\gamma_5)\chi$
F23:	$\chi \leftarrow \slashed{Z}(g_V - g_A\gamma_5)]\psi$	*F32:*	$\chi_\alpha \leftarrow \psi_\beta[\slashed{Z}(g_V - g_A\gamma_5)]_{\alpha\beta}$

Table 9.10: Combined dimension-4 trilinear fermionic couplings including one Dirac and one Majorana fermion.

FBF (*Chibar, S, Chi*): $\bar\chi_a S \chi_b$	
F12: $\quad \chi_b \leftarrow \chi_a S$	F21: $\quad \chi_b \leftarrow S\chi_a$
F13: $\quad S \leftarrow \chi_a^T \mathrm{C}\chi_b$	F31: $\quad S \leftarrow \chi_b^T \mathrm{C}\chi_a$
F23: $\quad \chi_a \leftarrow S\chi_b$	F32: $\quad \chi_a \leftarrow \chi S_b$
FBF (*Chibar, P, Chi*): $\bar\chi_a P \gamma_5 \psi_b$	
F12: $\quad \chi_b \leftarrow \gamma_5 \chi_a P$	F21: $\quad \chi_b \leftarrow P\gamma_5 \chi_a$
F13: $\quad P \leftarrow \chi_a^T \mathrm{C}\gamma_5 \chi_b$	F31: $\quad P \leftarrow \chi_b^T \mathrm{C}\gamma_5 \chi_a$
F23: $\quad \chi_a \leftarrow P \gamma_5 \chi_b$	F32: $\quad \chi_a \leftarrow \gamma_5 \chi_b P$
FBF (*Chibar, V, Chi*): $\bar\chi_a \slashed{V} \chi_b$	
F12: $\quad \chi_{b,\alpha} \leftarrow -\chi_{a,\beta}\slashed{V}_{\alpha\beta}$	F21: $\quad \chi_b \leftarrow -\slashed{V}\chi_a$
F13: $\quad V_\mu \leftarrow \chi_a^T \mathrm{C}\gamma_\mu \chi_b$	F31: $\quad V_\mu \leftarrow -\chi_b^T \mathrm{C}\gamma_\mu \chi_a$
F23: $\quad \chi_a \leftarrow \slashed{V}\chi_b$	F32: $\quad \chi_{a,\alpha} \leftarrow \chi_{b,\beta}\slashed{V}_{\alpha\beta}$
FBF (*Chibar, A, Chi*): $\bar\chi_a \gamma^5 \slashed{A}\chi_b$	
F12: $\quad \chi_{b,\alpha} \leftarrow \chi_{a,\beta}[\gamma^5\slashed{A}]_{\alpha\beta}$	F21: $\quad \chi_b \leftarrow \gamma^5\slashed{A}\chi_a$
F13: $\quad A_\mu \leftarrow \chi_a^T \mathrm{C}\gamma^5\gamma_\mu \chi_b$	F31: $\quad A_\mu \leftarrow \chi_b^T \mathrm{C}(\gamma^5\gamma_\mu\chi_a)$
F23: $\quad \chi_a \leftarrow \gamma^5\slashed{A}\chi_b$	F32: $\quad \chi_{a,\alpha} \leftarrow \chi_{b,\beta}[\gamma^5\slashed{A}]_{\alpha\beta}$

Table 9.11: Dimension-4 trilinear couplings of two Majorana fermions

FBF (*Chibar, SP, Chi*): $\bar\chi \phi_a (g_S + g_P\gamma_5)\chi_b$	
F12: $\quad \chi_b \leftarrow (g_S + g_P\gamma_5)\chi_a\phi$	F21: $\quad \chi_b \leftarrow \phi(g_S+g_P\gamma_5)\chi_a$
F13: $\quad \phi \leftarrow \chi_a^T \mathrm{C}(g_S+g_P\gamma_5)\chi_b$	F31: $\quad \phi \leftarrow \chi_b^T \mathrm{C}(g_S+g_P\gamma_5)\chi_a$
F23: $\quad \chi_a \leftarrow \phi(g_S+g_P\gamma_5)\chi_b$	F32: $\quad \chi_a \leftarrow (g_S+g_P\gamma_5)\chi_b\phi$
FBF (*Chibar, VA, Chi*): $\bar\chi_a\slashed{Z}(g_V - g_A\gamma_5)\chi_b$	
F12: $\quad \chi_{b,\alpha} \leftarrow \chi_{a,\beta}[\slashed{Z}(-g_V-g_A\gamma_5)]_{\alpha\beta}$	F21: $\quad \chi_b \leftarrow \slashed{Z}(-g_V-g_A\gamma_5)]\chi_a$
F13: $\quad Z_\mu \leftarrow \chi_a^T \mathrm{C}\gamma_\mu(g_V-g_A\gamma_5)\chi_b$	F31: $\quad Z_\mu \leftarrow \chi_b^T \mathrm{C}\gamma_\mu(-g_V-g_A\gamma_5)\chi_a$
F23: $\quad \chi_a \leftarrow \slashed{Z}(g_V-g_A\gamma_5)\chi_b$	F32: $\quad \chi_{a,\alpha} \leftarrow \chi_{b,\beta}[\slashed{Z}(g_V-g_A\gamma_5)]_{\alpha\beta}$

Table 9.12: Combined dimension-4 trilinear fermionic couplings of two Majorana fermions.

Gauge_Gauge_Gauge: $\mathcal{L}_I = g f_{abc} A_a^\mu A_b^\nu \partial_\mu A_{c,\nu}$
_: $A_a^\mu \leftarrow \mathrm{i} \cdot (-\mathrm{i}g/2) \cdot C_{abc}^{\mu\rho\sigma}(-k_2 - k_3, k_2, k_3) A_\rho^b A_\sigma^c$
Aux_Gauge_Gauge: $\mathcal{L}_I = g f_{abc} X_{a,\mu\nu}(k_1)(A_b^\mu(k_2) A_c^\nu(k_3) - A_b^\nu(k_2) A_c^\mu(k_3))$
F23∨*F32*: $X_a^{\mu\nu}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g f_{abc}(A_b^\mu(k_2) A_c^\nu(k_3) - A_b^\nu(k_2) A_c^\mu(k_3))$
F12∨*F13*: $A_{a,\mu}(k_1 + k_{2/3}) \leftarrow \mathrm{i} \cdot g f_{abc} X_{b,\nu\mu}(k_1) A_c^\nu(k_{2/3})$
F21∨*F31*: $A_{a,\mu}(k_{2/3} + k_1) \leftarrow \mathrm{i} \cdot g f_{abc} A_b^\nu(k_{2/3}) X_{c,\mu\nu}(k_1)$

Table 9.13: Dimension-4 Vector Boson couplings with *outgoing* momenta. See (11.1b) and (9.6b) for the definition of the antisymmetric tensor $C^{\mu_1\mu_2\mu_3}(k_1, k_2, k_3)$.

Scalar_Vector_Vector: $\mathcal{L}_I = g\phi V_1^\mu V_{2,\mu}$			
F13:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F31*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F12:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F21*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F23:	$\phi \leftarrow \mathrm{i} \cdot g V_1^\mu V_{2,\mu}$	*F32*:	$\phi \leftarrow \mathrm{i} \cdot g V_{2,\mu} V_1^\mu$
Aux_Vector_Vector: $\mathcal{L}_I = g X V_1^\mu V_{2,\mu}$			
F13:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F31*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F12:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F21*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F23:	$X \leftarrow \mathrm{i} \cdot g V_1^\mu V_{2,\mu}$	*F32*:	$X \leftarrow \mathrm{i} \cdot g V_{2,\mu} V_1^\mu$
Aux_Scalar_Vector: $\mathcal{L}_I = g X^\mu \phi V_\mu$			
F13:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F31*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F12:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F21*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F23:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F32*:	$\leftarrow \mathrm{i} \cdot g \cdots$

Table 9.14: …

Scalar_Scalar_Scalar: $\mathcal{L}_I = g\phi_1\phi_2\phi_3$			
F13:	$\phi_2 \leftarrow \mathrm{i} \cdot g\phi_1\phi_3$	*F31*:	$\phi_2 \leftarrow \mathrm{i} \cdot g\phi_3\phi_1$
F12:	$\phi_3 \leftarrow \mathrm{i} \cdot g\phi_1\phi_2$	*F21*:	$\phi_3 \leftarrow \mathrm{i} \cdot g\phi_2\phi_1$
F23:	$\phi_1 \leftarrow \mathrm{i} \cdot g\phi_2\phi_3$	*F32*:	$\phi_1 \leftarrow \mathrm{i} \cdot g\phi_3\phi_2$
Aux_Scalar_Scalar: $\mathcal{L}_I = g X \phi_1\phi_2$			
F13:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F31*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F12:	$\leftarrow \mathrm{i} \cdot g \cdots$	*F21*:	$\leftarrow \mathrm{i} \cdot g \cdots$
F23:	$X \leftarrow \mathrm{i} \cdot g\phi_1\phi_2$	*F32*:	$X \leftarrow \mathrm{i} \cdot g\phi_2\phi_1$

Table 9.15: …

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Vector_Scalar_Scalar: $\mathcal{L}_I = gV^\mu\phi_1 \mathrm{i}\overleftrightarrow{\partial_\mu}\phi_2$	
F23:	$V^\mu(k_2+k_3) \leftarrow \mathrm{i}\cdot g(k_2^\mu - k_3^\mu)\phi_1(k_2)\phi_2(k_3)$
F32:	$V^\mu(k_2+k_3) \leftarrow \mathrm{i}\cdot g(k_2^\mu - k_3^\mu)\phi_2(k_3)\phi_1(k_2)$
F12:	$\phi_2(k_1+k_2) \leftarrow \mathrm{i}\cdot g(k_1^\mu + 2k_2^\mu)V_\mu(k_1)\phi_1(k_2)$
F21:	$\phi_2(k_1+k_2) \leftarrow \mathrm{i}\cdot g(k_1^\mu + 2k_2^\mu)\phi_1(k_2)V_\mu(k_1)$
F13:	$\phi_1(k_1+k_3) \leftarrow \mathrm{i}\cdot g(-k_1^\mu - 2k_3^\mu)V_\mu(k_1)\phi_2(k_3)$
F31:	$\phi_1(k_1+k_3) \leftarrow \mathrm{i}\cdot g(-k_1^\mu - 2k_3^\mu)\phi_2(k_3)V_\mu(k_1)$

Table 9.16: ...

Aux_DScalar_DScalar: $\mathcal{L}_I = g\chi(\mathrm{i}\partial_\mu\phi_1)(\mathrm{i}\partial^\mu\phi_2)$	
F23:	$\chi(k_2+k_3) \leftarrow \mathrm{i}\cdot g(k_2\cdot k_3)\phi_1(k_2)\phi_2(k_3)$
F32:	$\chi(k_2+k_3) \leftarrow \mathrm{i}\cdot g(k_3\cdot k_2)\phi_2(k_3)\phi_1(k_2)$
F12:	$\phi_2(k_1+k_2) \leftarrow \mathrm{i}\cdot g((-k_1-k_2)\cdot k_2)\chi(k_1)\phi_1(k_2)$
F21:	$\phi_2(k_1+k_2) \leftarrow \mathrm{i}\cdot g(k_2\cdot(-k_1-k_2))\phi_1(k_2)\chi(k_1)$
F13:	$\phi_1(k_1+k_3) \leftarrow \mathrm{i}\cdot g((-k_1-k_3)\cdot k_3)\chi(k_1)\phi_2(k_3)$
F31:	$\phi_1(k_1+k_3) \leftarrow \mathrm{i}\cdot g(k_3\cdot(-k_1-k_3))\phi_2(k_3)\chi(k_1)$

Table 9.17: ...

Aux_Vector_DScalar: $\mathcal{L}_I = g\chi V_\mu(\mathrm{i}\partial^\mu\phi)$	
F23:	$\chi(k_2+k_3) \leftarrow \mathrm{i}\cdot gk_3^\mu V_\mu(k_2)\phi(k_3)$
F32:	$\chi(k_2+k_3) \leftarrow \mathrm{i}\cdot g\phi(k_3)k_3^\mu V_\mu(k_2)$
F12:	$\phi(k_1+k_2) \leftarrow \mathrm{i}\cdot g\chi(k_1)(-k_1-k_2)^\mu V_\mu(k_2)$
F21:	$\phi(k_1+k_2) \leftarrow \mathrm{i}\cdot g(-k_1-k_2)^\mu V_\mu(k_2)\chi(k_1)$
F13:	$V_\mu(k_1+k_3) \leftarrow \mathrm{i}\cdot g(-k_1-k_3)_\mu\chi(k_1)\phi(k_3)$
F31:	$V_\mu(k_1+k_3) \leftarrow \mathrm{i}\cdot g(-k_1-k_3)_\mu\phi(k_3)\chi(k_1)$

Table 9.18: ...

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Non-Gauge Vector Couplings

As a basis for the dimension-4 couplings of three vector bosons, we choose "transversal" and "longitudinal" (with respect to the first vector field) tensors that are odd and even under permutation of the second and third argument

$$\mathcal{L}_T(V_1, V_2, V_3) = V_1^\mu (V_{2,\nu} \mathrm{i} \overleftrightarrow{\partial}_\mu V_3^\nu) = -\mathcal{L}_T(V_1, V_3, V_2) \tag{9.7a}$$

$$\mathcal{L}_L(V_1, V_2, V_3) = (\mathrm{i}\partial_\mu V_1^\mu) V_{2,\nu} V_3^\nu = \mathcal{L}_L(V_1, V_3, V_2) \tag{9.7b}$$

Using partial integration in $\mathcal{L}_L$, we find the convenient combinations

$$\mathcal{L}_T(V_1, V_2, V_3) + \mathcal{L}_L(V_1, V_2, V_3) = -2V_1^\mu \mathrm{i}\partial_\mu V_{2,\nu} V_3^\nu \tag{9.8a}$$

$$\mathcal{L}_T(V_1, V_2, V_3) - \mathcal{L}_L(V_1, V_2, V_3) = 2V_1^\mu V_{2,\nu} \mathrm{i}\partial_\mu V_3^\nu \tag{9.8b}$$

As an important example, we can rewrite the dimension-4 "anomalous" triple gauge couplings

$$\mathrm{i}\mathcal{L}_{\mathrm{TGC}}(g_1, \kappa, g_4)/g_{VWW} = g_1 V^\mu (W_{\mu\nu}^- W^{+,\nu} - W_{\mu\nu}^+ W^{-,\nu})$$
$$+ \kappa W_\mu^+ W_\nu^- V^{\mu\nu} + g_4 W_\mu^+ W_\nu^- (\partial^\mu V^\nu + \partial^\nu V^\mu) \tag{9.9}$$

as

$$\mathcal{L}_{\mathrm{TGC}}(g_1, \kappa, g_4) = g_1 \mathcal{L}_T(V, W^-, W^+)$$
$$- \frac{\kappa + g_1 - g_4}{2} \mathcal{L}_T(W^-, V, W^+) + \frac{\kappa + g_1 + g_4}{2} \mathcal{L}_T(W^+, V, W^-)$$
$$- \frac{\kappa - g_1 - g_4}{2} \mathcal{L}_L(W^-, V, W^+) + \frac{\kappa - g_1 + g_4}{2} \mathcal{L}_L(W^+, V, W^-) \tag{9.10}$$

CP Violation

$$\mathcal{L}_{\tilde{T}}(V_1, V_2, V_3) = V_{1,\mu}(V_{2,\rho} \mathrm{i} \overleftrightarrow{\partial}_\nu V_{3,\sigma}) \epsilon^{\mu\nu\rho\sigma} = +\mathcal{L}_T(V_1, V_3, V_2) \tag{9.11a}$$

$$\mathcal{L}_{\tilde{L}}(V_1, V_2, V_3) = (\mathrm{i}\partial_\mu V_{1,\nu}) V_{2,\rho} V_{3,\sigma} \epsilon^{\mu\nu\rho\sigma} = -\mathcal{L}_L(V_1, V_3, V_2) \tag{9.11b}$$

Here the notations $\tilde{T}$ and $\tilde{L}$ are clearly _abuse de langage_, because $\mathcal{L}_{\tilde{L}}(V_1, V_2, V_3)$ is actually the transversal combination, due to the antisymmetry of ϵ. Using partial integration in $\mathcal{L}_{\tilde{L}}$, we could again find combinations

$$\mathcal{L}_{\tilde{T}}(V_1, V_2, V_3) + \mathcal{L}_{\tilde{L}}(V_1, V_2, V_3) = -2V_{1,\mu} V_{2,\nu} \mathrm{i}\partial_\rho V_{3,\sigma} \epsilon^{\mu\nu\rho\sigma} \tag{9.12a}$$

$$\mathcal{L}_{\tilde{T}}(V_1, V_2, V_3) - \mathcal{L}_{\tilde{L}}(V_1, V_2, V_3) = -2V_{1,\mu} \mathrm{i}\partial_\nu V_{2,\rho} V_{3,\sigma} \epsilon^{\mu\nu\rho\sigma} \tag{9.12b}$$

but we don't need them, since

$$\mathrm{i}\mathcal{L}_{\mathrm{TGC}}(g_5, \tilde{\kappa})/g_{VWW} = g_5 \epsilon_{\mu\nu\rho\sigma}(W^{+,\mu} \mathrm{i} \overleftrightarrow{\partial}^\rho W^{-,\nu}) V^\sigma$$
$$- \frac{\tilde{\kappa}_V}{2} W_\mu^- W_\nu^+ \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} \tag{9.13}$$

is immediately recognizable as

$$\mathcal{L}_{\mathrm{TGC}}(g_5, \tilde{\kappa})/g_{VWW} = -\mathrm{i}g_5 \mathcal{L}_{\tilde{L}}(V, W^-, W^+) + \tilde{\kappa}\mathcal{L}_{\tilde{T}}(V, W^-, W^+) \tag{9.14}$$

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$Dim4\_Vector\_Vector\_Vector\_T$: $\mathcal{L}_I = gV_1^\mu V_{2,\nu} \mathrm{i}\overset{\leftrightarrow}{\partial_\mu} V_3^\nu$
$F23$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\mu - k_3^\mu)V_{2,\nu}(k_2)V_3^\nu(k_3)$
$F32$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\mu - k_3^\mu)V_3^\nu(k_3)V_{2,\nu}(k_2)$
$F12$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(2k_2^\nu + k_1^\nu)V_{1,\nu}(k_1)V_2^\mu(k_2)$
$F21$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(2k_2^\nu + k_1^\nu)V_2^\mu(k_2)V_{1,\nu}(k_1)$
$F13$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(-k_1^\nu - 2k_3^\nu)V_1^\nu(k_1)V_3^\mu(k_3)$
$F31$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(-k_1^\nu - 2k_3^\nu)V_3^\mu(k_3)V_1^\nu(k_1)$
$Dim4\_Vector\_Vector\_Vector\_L$: $\mathcal{L}_I = g\mathrm{i}\partial_\mu V_1^\mu V_{2,\nu} V_3^\nu$
$F23$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\mu + k_3^\mu)V_{2,\nu}(k_2)V_3^\nu(k_3)$
$F32$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\mu + k_3^\mu)V_3^\nu(k_3)V_{2,\nu}(k_2)$
$F12$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(-k_1^\nu)V_{1,\nu}(k_1)V_2^\mu(k_2)$
$F21$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(-k_1^\nu)V_2^\mu(k_2)V_{1,\nu}(k_1)$
$F13$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(-k_1^\nu)V_1^\nu(k_1)V_3^\mu(k_3)$
$F31$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(-k_1^\nu)V_3^\mu(k_3)V_1^\nu(k_1)$

Table 9.19: . . .

$Dim4\_Vector\_Vector\_Vector\_T5$: $\mathcal{L}_I = gV_{1,\mu}V_{2,\rho}\mathrm{i}\overset{\leftrightarrow}{\partial_\nu}V_{3,\sigma}\epsilon^{\mu\nu\rho\sigma}$
$F23$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(k_{2,\nu} - k_{3,\nu})V_{2,\rho}(k_2)V_{3,\sigma}(k_3)$
$F32$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(k_{2,\nu} - k_{3,\nu})V_{3,\sigma}(k_3)V_{2,\rho}(k_2)$
$F12$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(2k_{2,\nu} + k_{1,\nu})V_{1,\rho}(k_1)V_{2,\sigma}(k_2)$
$F21$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(2k_{2,\nu} + k_{1,\nu})V_{2,\sigma}(k_2)V_{1,\rho}(k_1)$
$F13$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(-k_{1,\nu} - 2k_{3,\nu})V_{1,\rho}(k_1)V_{3,\sigma}(k_3)$
$F31$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(-k_{1,\nu} - 2k_{3,\nu})V_{3,\sigma}(k_3)V_{1,\rho}(k_1)$
$Dim4\_Vector\_Vector\_Vector\_L5$: $\mathcal{L}_I = g\mathrm{i}\partial_\mu V_{1,\nu}V_{2,\nu}V_{3,\sigma}\epsilon^{\mu\nu\rho\sigma}$
$F23$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(k_{2,\nu} + k_{3,\nu})V_{2,\rho}(k_2)V_{3,\sigma}(k_3)$
$F32$: $V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(k_{2,\nu} + k_{3,\nu})V_{2,\rho}(k_2)V_{3,\sigma}(k_3)$
$F12$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(-k_{1,\nu})V_{1,\rho}(k_1)V_{2,\sigma}(k_2)$
$F21$: $V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(-k_{1,\nu})V_{2,\sigma}(k_2)V_{1,\rho}(k_1)$
$F13$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(-k_{1,\nu})V_{1,\rho}(k_1)V_{3,\sigma}(k_3)$
$F31$: $V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g\epsilon^{\mu\nu\rho\sigma}(-k_{1,\nu})V_{3,\sigma}(k_3)V_{1,\rho}(k_1)$

Table 9.20: . . .

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Dim6_Gauge_Gauge_Gauge: $\mathcal{L}_I = g F_1^{\mu\nu} F_{2,\nu\rho} F_{3,\ \mu}^{\ \ \rho}$	
_: $A_1^{\mu}(k_2+k_3) \leftarrow -\mathrm{i} \cdot \Lambda^{\mu\rho\sigma}(-k_2-k_3,k_2,k_3)A_{2,\rho}A_{c,\sigma}$	

Table 9.21: ...

Dim6_Gauge_Gauge_Gauge_5: $\mathcal{L}_I = g/2 \cdot \epsilon^{\mu\nu\lambda\tau} F_{1,\mu\nu} F_{2,\tau\rho} F_{3,\ \lambda}^{\ \ \rho}$	
F23: $A_1^{\mu}(k_2+k_3) \leftarrow -\mathrm{i} \cdot \Lambda_5^{\mu\rho\sigma}(-k_2-k_3,k_2,k_3)A_{2,\rho}A_{3,\sigma}$
F32: $A_1^{\mu}(k_2+k_3) \leftarrow -\mathrm{i} \cdot \Lambda_5^{\mu\rho\sigma}(-k_2-k_3,k_2,k_3)A_{3,\sigma}A_{2,\rho}$
F12: $A_3^{\mu}(k_1+k_2) \leftarrow -\mathrm{i} \cdot$
F21: $A_3^{\mu}(k_1+k_2) \leftarrow -\mathrm{i} \cdot$
F13: $A_2^{\mu}(k_1+k_3) \leftarrow -\mathrm{i} \cdot$
F31: $A_2^{\mu}(k_1+k_3) \leftarrow -\mathrm{i} \cdot$

Table 9.22: ...

9.1.4 *SU(2) Gauge Bosons*

An important special case for table 9.13 are the two usual coordinates of SU(2)

$$W_{\pm} = \frac{1}{\sqrt{2}} \left(W_1 \mp \mathrm{i} W_2 \right) \tag{9.15}$$

i. e.

$$W_1 = \frac{1}{\sqrt{2}} \left(W_+ + W_- \right) \tag{9.16a}$$

$$W_2 = \frac{\mathrm{i}}{\sqrt{2}} \left(W_+ - W_- \right) \tag{9.16b}$$

and

$$W_1^{\mu} W_2^{\nu} - W_2^{\mu} W_1^{\nu} = \mathrm{i} \left(W_-^{\mu} W_+^{\nu} - W_+^{\mu} W_-^{\nu} \right) \tag{9.17}$$

Thus the symmtry remains after the change of basis:

$$\begin{aligned}
\epsilon^{abc} W_a^{\mu_1} W_b^{\mu_2} W_c^{\mu_3} = &\ \mathrm{i} W_-^{\mu_1} (W_+^{\mu_2} W_3^{\mu_3} - W_3^{\mu_2} W_+^{\mu_3}) \\
&+ \mathrm{i} W_+^{\mu_1} (W_3^{\mu_2} W_-^{\mu_3} - W_-^{\mu_2} W_3^{\mu_3}) + \mathrm{i} W_3^{\mu_1} (W_-^{\mu_2} W_+^{\mu_3} - W_+^{\mu_2} W_-^{\mu_3})
\end{aligned} \tag{9.18}$$

9.1.5 *Quartic Couplings and Auxiliary Fields*

Quartic couplings can be replaced by cubic couplings to a non-propagating auxiliary field. The quartic term should get a negative sign so that it the energy is bounded from below for identical fields. In the language of functional integrals

$$\mathcal{L}_{\phi^4} = -g^2 \phi_1 \phi_2 \phi_3 \phi_4 \Longrightarrow$$
$$\mathcal{L}_{X\phi^2} = X^* X \pm g X \phi_1 \phi_2 \pm g X^* \phi_3 \phi_4 = (X^* \pm g \phi_1 \phi_2)(X \pm g \phi_3 \phi_4) - g^2 \phi_1 \phi_2 \phi_3 \phi_4 \tag{9.19a}$$

and in the language of Feynman diagrams

$$\qquad\qquad (9.19\text{b})$$

The other choice of signs

$$\mathcal{L}'_{X\phi^2} = -X^*X \pm gX\phi_1\phi_2 \mp gX^*\phi_3\phi_4 = -(X^* \pm g\phi_1\phi_2)(X \mp g\phi_3\phi_4) - g^2\phi_1\phi_2\phi_3\phi_4 \tag{9.20}$$

can not be extended easily to identical particles and is therefore not used. For identical particles we have

$$\mathcal{L}_{\phi^4} = -\frac{g^2}{4!}\phi^4 \Longrightarrow$$

$$\mathcal{L}_{X\phi^2} = \frac{1}{2}X^2 \pm \frac{g}{2}X\phi^2 \pm \frac{g}{2}X\phi^2 = \frac{1}{2}\left(X \pm \frac{g}{2}\phi^2\right)\left(X \pm \frac{g}{2}\phi^2\right) - \frac{g^2}{4!}\phi^4 \tag{9.21}$$

Explain the factor 1/3 in the functional setting and its relation to the three diagrams in the graphical setting?

Quartic Gauge Couplings

The three crossed versions of figure 9.2 reproduces the quartic coupling in figure 9.1, because

$$-\,\mathrm{i}g^2 f_{a_1 a_2 b} f_{a_3 a_4 b}(g_{\mu_1\mu_3}g_{\mu_4\mu_2} - g_{\mu_1\mu_4}g_{\mu_2\mu_3})$$

$$= (\mathrm{i}g f_{a_1 a_2 b} T_{\mu_1\mu_2,\nu_1\nu_2})\left(\frac{\mathrm{i}g^{\nu_1\nu_3}g^{\nu_2\nu_4}}{2}\right)(\mathrm{i}g f_{a_3 a_4 b} T_{\mu_3\mu_4,\nu_3\nu_4}) \tag{9.24}$$

with $T_{\mu_1\mu_2,\mu_3\mu_4} = g_{\mu_1\mu_3}g_{\mu_4\mu_2} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}$.

9.1.6   *Gravitinos and supersymmetric currents*

In supergravity theories there is a fermionic partner of the graviton, the gravitino. Therefore we have introduced the Lorentz type *Vectorspinor*.

9.1.7   *Perturbative Quantum Gravity and Kaluza-Klein Interactions*

The gravitational coupling constant and the relative strength of the dilaton coupling are abbreviated as

$$\kappa = \sqrt{16\pi G_N} \tag{9.25a}$$

$$\omega = \sqrt{\frac{2}{3(n+2)}} = \sqrt{\frac{2}{3(d-2)}}\,, \tag{9.25b}$$

where $n = d - 4$ is the number of extra space dimensions.

Figure 9.1: Gauge couplings. See (11.1b) for the definition of the antisymmetric tensor $C^{\mu_1\mu_2\mu_3}(k_1, k_2, k_3)$.
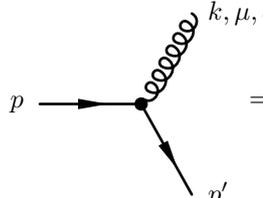
$$+\mathrm{i}g\gamma_\mu T_a \quad (9.22a)$$
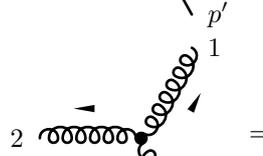
$$gf_{a_1a_2a_3}C^{\mu_1\mu_2\mu_3}(k_1, k_2, k_3) \quad (9.22b)$$

$$\begin{aligned} &-\mathrm{i}g^2 f_{a_1a_2b}f_{a_3a_4b}(g_{\mu_1\mu_3}g_{\mu_4\mu_2} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}) \\ = &-\mathrm{i}g^2 f_{a_1a_3b}f_{a_4a_2b}(g_{\mu_1\mu_4}g_{\mu_2\mu_3} - g_{\mu_1\mu_2}g_{\mu_3\mu_4}) \quad (9.22c) \\ &-\mathrm{i}g^2 f_{a_1a_4b}f_{a_2a_3b}(g_{\mu_1\mu_2}g_{\mu_3\mu_4} - g_{\mu_1\mu_3}g_{\mu_4\mu_2}) \end{aligned}$$
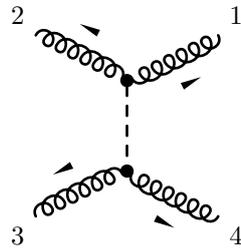
$$= -\,\mathrm{i}g^2 f_{a_1a_2b}f_{a_3a_4b}(g_{\mu_1\mu_3}g_{\mu_4\mu_2} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}) \quad (9.23)$$

Figure 9.2: Gauge couplings.

GBG (Fermbar, MOM, Ferm): $\bar{\psi}_1(\mathrm{i}\partial\!\!\!/ \pm m)\phi\psi_2$	
F12: $\psi_2 \leftarrow -(\not{k} \mp m)\psi_1 S$	*F21*: $\psi_2 \leftarrow -S(\not{k} \mp m)\psi_1$
F13: $S \leftarrow \psi_1^T \mathrm{C}(\not{k} \pm m)\psi_2$	*F31*: $S \leftarrow \psi_2^T \mathrm{C}(-(\not{k} \mp m)\psi_1)$
F23: $\psi_1 \leftarrow S(\not{k} \pm m)\psi_2$	*F32*: $\psi_1 \leftarrow (\not{k} \pm m)\psi_2 S$
GBG (Fermbar, MOM5, Ferm): $\bar{\psi}_1(\mathrm{i}\partial\!\!\!/ \pm m)\phi\gamma^5\psi_2$	
F12: $\psi_2 \leftarrow (\not{k} \pm m)\gamma^5\psi_1 P$	*F21*: $\psi_2 \leftarrow P(\not{k} \pm m)\gamma^5\psi_1$
F13: $P \leftarrow \psi_1^T \mathrm{C}(\not{k} \pm m)\gamma^5\psi_2$	*F31*: $P \leftarrow \psi_2^T \mathrm{C}(\not{k} \pm m)\gamma^5\psi_1$
F23: $\psi_1 \leftarrow P(\not{k} \pm m)\gamma^5\psi_2$	*F32*: $\psi_1 \leftarrow (\not{k} \pm m)\gamma^5\psi_2 P$
GBG (Fermbar, MOML, Ferm): $\bar{\psi}_1(\mathrm{i}\partial\!\!\!/ \pm m)\phi(1 - \gamma^5)\psi_2$	
F12: $\psi_2 \leftarrow -(1 - \gamma^5)(\not{k} \mp m)\psi_1\phi$	*F21*: $\psi_2 \leftarrow -\phi(1 - \gamma^5)(\not{k} \mp m)\psi_1$
F13: $\phi \leftarrow \psi_1^T \mathrm{C}(\not{k} \pm m)(1 - \gamma^5)\psi_2$	*F31*: $\phi \leftarrow \psi_2^T \mathrm{C}(1 - \gamma^5)(-(\not{k} \mp m)\psi_1)$
F23: $\psi_1 \leftarrow \phi(\not{k} \pm m)(1 - \gamma^5)\psi_2$	*F32*: $\psi_1 \leftarrow (\not{k} \pm m)(1 - \gamma^5)\psi_2\phi$
GBG (Fermbar, LMOM, Ferm): $\bar{\psi}_1\phi(1 - \gamma^5)(\mathrm{i}\partial\!\!\!/ \pm m)\psi_2$	
F12: $\psi_2 \leftarrow -(\not{k} \mp m)\psi_1(1 - \gamma^5)\phi$	*F21*: $\psi_2 \leftarrow -\phi(\not{k} \mp m)(1 - \gamma^5)\psi_1$
F13: $\phi \leftarrow \psi_1^T \mathrm{C}(1 - \gamma^5)(\not{k} \pm m)\psi_2$	*F31*: $\phi \leftarrow \psi_2^T \mathrm{C}(-(\not{k} \mp m)(1 - \gamma^5)\psi_1)$
F23: $\psi_1 \leftarrow \phi(1 - \gamma^5)(\not{k} \pm m)\psi_2$	*F32*: $\psi_1 \leftarrow (1 - \gamma^5)(\not{k} \pm m)\psi_2\phi$
GBG (Fermbar, VMOM, Ferm): $\bar{\psi}_1\mathrm{i}\partial\!\!\!/_\alpha V_\beta[\gamma^\alpha, \gamma^\beta]\psi_2$	
F12: $\psi_2 \leftarrow -[\not{k}, \gamma^\alpha]\psi_1 V_\alpha$	*F21*: $\psi_2 \leftarrow -[\not{k}, \not{V}]\psi_1$
F13: $V_\alpha \leftarrow \psi_1^T \mathrm{C}[\not{k}, \gamma_\alpha]\psi_2$	*F31*: $V_\alpha \leftarrow \psi_2^T \mathrm{C}(-[\not{k}, \gamma_\alpha]\psi_1)$
F23: $\psi_1 \leftarrow][\not{k}, \not{V}]\psi_2$	*F32*: $\psi_1 \leftarrow [\not{k}, \gamma^\alpha]\psi_2 V_\alpha$

Table 9.23: Combined dimension-4 trilinear fermionic couplings including a momentum. *Ferm* stands for *Psi* and *Chi*. The case of *MOMR* is identical to *MOML* if one substitutes $1 + \gamma^5$ for $1 - \gamma^5$, as well as for *LMOM* and *RMOM*. The mass term forces us to keep the chiral projector always on the left after "inverting the line" for *MOML* while on the right for *LMOM*.

GBBG (Fermbar, S2LR, Ferm): $\bar{\psi}_1 S_1 S_2 (g_L P_L + g_R P_R)\psi_2$	
F123 F213 F132 F231 F312 F321:	$\psi_2 \leftarrow S_1 S_2 (g_R P_L + g_L P_R)\psi_1$
F423 F243 F432 F234 F342 F324:	$\psi_1 \leftarrow S_1 S_2 (g_L P_L + g_R P_R)\psi_2$
F134 F143 F314:	$S_1 \leftarrow \psi_1^T C S_2 (g_L P_L + g_R P_R)\psi_2$
F124 F142 F214:	$S_2 \leftarrow \psi_1^T C S_1 (g_L P_L + g_R P_R)\psi_2$
F413 F431 F341:	$S_1 \leftarrow \psi_2^T C S_2 (g_R P_L + g_L P_R)\psi_1$
F412 F421 F241:	$S_2 \leftarrow \psi_2^T C S_1 (g_R P_L + g_L P_R)\psi_1$
GBBG (Fermbar, S2, Ferm): $\bar{\psi}_1 S_1 S_2 \gamma^5 \psi_2$	
F123 F213 F132 F231 F312 F321:	$\psi_2 \leftarrow S_1 S_2 \gamma^5 \psi_1$
F423 F243 F432 F234 F342 F324:	$\psi_1 \leftarrow S_1 S_2 \gamma^5 \psi_2$
F134 F143 F314:	$S_1 \leftarrow \psi_1^T C S_2 \gamma^5 \psi_2$
F124 F142 F214:	$S_2 \leftarrow \psi_1^T C S_1 \gamma^5 \psi_2$
F413 F431 F341:	$S_1 \leftarrow \psi_2^T C S_2 \gamma^5 \psi_1$
F412 F421 F241:	$S_2 \leftarrow \psi_2^T C S_1 \gamma^5 \psi_1$
GBBG (Fermbar, V2, Ferm): $\bar{\psi}_1 [\slashed{V}_1, \slashed{V}_2]\psi_2$	
F123 F213 F132 F231 F312 F321:	$\psi_2 \leftarrow -[\slashed{V}_1, \slashed{V}_2]\psi_1$
F423 F243 F432 F234 F342 F324:	$\psi_1 \leftarrow [\slashed{V}_1, \slashed{V}_2]\psi_2$
F134 F143 F314:	$V_{1\,\alpha} \leftarrow \psi_1^T C [\gamma_\alpha, \slashed{V}_2]\psi_2$
F124 F142 F214:	$V_{2\,\alpha} \leftarrow \psi_1^T C (-[\gamma_\alpha, \slashed{V}_1])\psi_2$
F413 F431 F341:	$V_{1\,\alpha} \leftarrow \psi_2^T C (-[\gamma_\alpha, \slashed{V}_2])\psi_1$
F412 F421 F241:	$V_{2\,\alpha} \leftarrow \psi_2^T C [\gamma_\alpha, \slashed{V}_1]\psi_1$

Table 9.24: Vertices with two fermions (*Ferm* stands for *Psi* and *Chi*, but not for *Grav*) and two bosons (two scalars, scalar/vector, two vectors) for the BRST transformations. Part I

GBBG (Fermbar, SV, Ferm): $\bar{\psi}_1 V\!\!\!/ S \psi_2$
F123 F213 F132 F231 F312 F321: $\quad \psi_2 \leftarrow -V\!\!\!/ S \psi_1$
F423 F243 F432 F234 F342 F324: $\quad \psi_1 \leftarrow V\!\!\!/ S \psi_2$
F134 F143 F314: $\quad V_\alpha \leftarrow \psi_1^T C \gamma_\alpha S \psi_2$
F124 F142 F214: $\quad S \leftarrow \psi_1^T C V\!\!\!/ \psi_2$
F413 F431 F341: $\quad V_\alpha \leftarrow \psi_2^T C(-\gamma_\alpha S \psi_1)$
F412 F421 F241: $\quad S \leftarrow \psi_2^T C(-V\!\!\!/ \psi_1)$
GBBG (Fermbar, PV, Ferm): $\bar{\psi}_1 V\!\!\!/ \gamma^5 P \psi_2$
F123 F213 F132 F231 F312 F321: $\quad \psi_2 \leftarrow V\!\!\!/ \gamma^5 P \psi_1$
F423 F243 F432 F234 F342 F324: $\quad \psi_1 \leftarrow V\!\!\!/ \gamma^5 P \psi_2$
F134 F143 F314: $\quad V_\alpha \leftarrow \psi_1^T C \gamma_\alpha \gamma^5 P \psi_2$
F124 F142 F214: $\quad P \leftarrow \psi_1^T C V\!\!\!/ \gamma^5 \psi_2$
F413 F431 F341: $\quad V_\alpha \leftarrow \psi_2^T C \gamma_\alpha \gamma^5 P \psi_1$
F412 F421 F241: $\quad P \leftarrow \psi_2^T C V\!\!\!/ \gamma^5 \psi_1$
GBBG (Fermbar, S(L/R)V, Ferm): $\bar{\psi}_1 V\!\!\!/ (1 \mp \gamma^5) \phi \psi_2$
F123 F213 F132 F231 F312 F321: $\quad \psi_2 \leftarrow -V\!\!\!/ (1 \pm \gamma^5) \phi \psi_1$
F423 F243 F432 F234 F342 F324: $\quad \psi_1 \leftarrow V\!\!\!/ (1 \mp \gamma^5) \phi \psi_2$
F134 F143 F314: $\quad V_\alpha \leftarrow \psi_1^T C \gamma_\alpha (1 \mp \gamma^5) \phi \psi_2$
F124 F142 F214: $\quad \phi \leftarrow \psi_1^T C V\!\!\!/ (1 \mp \gamma^5) \psi_2$
F413 F431 F341: $\quad V_\alpha \leftarrow \psi_2^T C \gamma_\alpha (-(1 \pm \gamma^5) \phi \psi_1)$
F412 F421 F241: $\quad \phi \leftarrow \psi_2^T C V\!\!\!/ (-(1 \pm \gamma^5) \psi_1)$

Table 9.25: Vertices with two fermions (*Ferm* stands for *Psi* and *Chi*, but not for *Grav*) and two bosons (two scalars, scalar/vector, two vectors) for the BRST transformations. Part II

GBG (*Gravbar*, *POT*, *Psi*): $\bar{\psi}_\mu S\gamma^\mu\psi$	
F12: $\psi \leftarrow -\gamma^\mu\psi_\mu S$	F21: $\psi \leftarrow -S\gamma^\mu\psi_\mu$
F13: $S \leftarrow \psi_\mu^T \mathrm{C}\gamma^\mu\psi$	F31: $S \leftarrow \psi^T \mathrm{C}(-\gamma^\mu)\psi_\mu$
F23: $\psi_\mu \leftarrow S\gamma_\mu\psi$	F32: $\psi_\mu \leftarrow \gamma_\mu\psi S$
GBG (*Gravbar*, *S*, *Psi*): $\bar{\psi}_\mu \not{k}_S S\gamma^\mu\psi$	
F12: $\psi \leftarrow \gamma^\mu \not{k}_S \psi_\mu S$	F21: $\psi \leftarrow S\gamma^\mu \not{k}_S \psi_\mu$
F13: $S \leftarrow \psi_\mu^T \mathrm{C}\not{k}_S\gamma^\mu\psi$	F31: $S \leftarrow \psi^T \mathrm{C}\gamma^\mu \not{k}_S \psi_\mu$
F23: $\psi_\mu \leftarrow S\not{k}_S\gamma_\mu\psi$	F32: $\psi_\mu \leftarrow \not{k}_S\gamma_\mu\psi S$
GBG (*Gravbar*, *P*, *Psi*): $\bar{\psi}_\mu \not{k}_P P\gamma^\mu\gamma_5\psi$	
F12: $\psi \leftarrow \gamma^\mu \not{k}_P \gamma_5\psi_\mu P$	F21: $\psi \leftarrow P\gamma^\mu \not{k}_P \gamma_5\psi_\mu$
F13: $P \leftarrow \psi_\mu^T \mathrm{C}\not{k}_P\gamma^\mu\gamma_5\psi$	F31: $P \leftarrow \psi^T \mathrm{C}\gamma^\mu \not{k}_P \gamma_5\psi_\mu$
F23: $\psi_\mu \leftarrow P\not{k}_P\gamma_\mu\gamma_5\psi$	F32: $\psi_\mu \leftarrow \not{k}_P\gamma_\mu\gamma_5\psi P$
GBG (*Gravbar*, *V*, *Psi*): $\bar{\psi}_\mu [\not{k}_V, \not{V}]\gamma^\mu\gamma^5\psi$	
F12: $\psi \leftarrow \gamma^5\gamma^\mu[\not{k}_V, \gamma^\alpha]\psi_\mu V_\alpha$	F21: $\psi \leftarrow \gamma^5\gamma^\mu[\not{k}_V, \not{V}]\psi_\mu$
F13: $V_\mu \leftarrow \psi_\rho^T \mathrm{C}[\not{k}_V, \gamma_\mu]\gamma^\rho\gamma^5\psi$	F31: $V_\mu \leftarrow \psi^T \mathrm{C}\gamma^5\gamma^\rho[\not{k}_V, \gamma_\mu]\psi_\rho$
F23: $\psi_\mu \leftarrow [\not{k}_V, \not{V}]\gamma_\mu\gamma^5\psi$	F32: $\psi_\mu \leftarrow [\not{k}_V, \gamma^\alpha]\gamma_\mu\gamma^5\psi V_\alpha$

Table 9.26: Dimension-5 trilinear couplings including one Dirac, one Gravitino fermion and one additional particle.The option *POT* is for the coupling of the supersymmetric current to the derivative of the quadratic terms in the superpotential.

GBG (*Psibar, POT, Grav*): $\bar{\psi}\gamma^\mu S\psi_\mu$	
F12: $\psi_\mu \leftarrow -\gamma_\mu\psi S$	F21: $\psi_\mu \leftarrow -S\gamma_\mu\psi$
F13: $S \leftarrow \psi^T C\gamma^\mu\psi_\mu$	F31: $S \leftarrow \psi_\mu^T C(-\gamma^\mu)\psi$
F23: $\psi \leftarrow S\gamma^\mu\psi_\mu$	F32: $\psi \leftarrow \gamma^\mu\psi_\mu S$
GBG (*Psibar, S, Grav*): $\bar{\psi}\gamma^\mu\not{k}_S S\psi_\mu$	
F12: $\psi_\mu \leftarrow \not{k}_S\gamma_\mu\psi S$	F21: $\psi_\mu \leftarrow S\not{k}_S\gamma_\mu\psi$
F13: $S \leftarrow \psi^T C\gamma^\mu\not{k}_S\psi_\mu$	F31: $S \leftarrow \psi_\mu^T C\not{k}_S\gamma^\mu\psi$
F23: $\psi \leftarrow S\gamma^\mu\not{k}_S\psi_\mu$	F32: $\psi \leftarrow \gamma^\mu\not{k}_S\psi_\mu S$
GBG (*Psibar, P, Grav*): $\bar{\psi}\gamma^\mu\gamma^5 P\not{k}_P\psi_\mu$	
F12: $\psi_\mu \leftarrow -\not{k}_P\gamma_\mu\gamma^5\psi P$	F21: $\psi_\mu \leftarrow -P\not{k}_P\gamma_\mu\gamma^5\psi$
F13: $P \leftarrow \psi^T C\gamma^\mu\gamma^5\not{k}_P\psi_\mu$	F31: $P \leftarrow -\psi_\mu^T C\not{k}_P\gamma^\mu\gamma_5\psi$
F23: $\psi \leftarrow P\gamma^\mu\gamma^5\not{k}_P\psi_\mu$	F32: $\psi \leftarrow \gamma^\mu\gamma^5\not{k}_P\psi_\mu P$
GBG (*Psibar, V, Grav*): $\bar{\psi}\gamma^5\gamma^\mu[\not{k}_V, \not{V}]\psi_\mu$	
F12: $\psi_\mu \leftarrow [\not{k}_V, \gamma^\alpha]\gamma_\mu\gamma^5\psi V_\alpha$	F21: $\psi_\mu \leftarrow [\not{k}_V, \not{V}]\gamma_\mu\gamma^5\psi$
F13: $V_\mu \leftarrow \psi^T C\gamma^5\gamma^\rho[\not{k}_V, \gamma_\mu]\psi_\rho$	F31: $V_\mu \leftarrow \psi_\rho^T C[\not{k}_V, \gamma_\mu]\gamma^\rho\gamma^5\psi$
F23: $\psi \leftarrow \gamma^5\gamma^\mu[\not{k}_V, \not{V}]\psi_\mu$	F32: $\psi \leftarrow \gamma^5\gamma^\mu[\not{k}_V, \gamma^\alpha]\psi_\mu V_\alpha$

Table 9.27:   Dimension-5 trilinear couplings including one conjugated Dirac, one Gravitino fermion and one additional particle.

GBG (Gravbar, POT, Chi): $\bar{\psi}_\mu S \gamma^\mu \chi$	
F12: $\chi \leftarrow -\gamma^\mu \psi_\mu S$	*F21*: $\chi \leftarrow -S\gamma^\mu \psi_\mu$
F13: $S \leftarrow \psi_\mu^T \mathrm{C}\gamma^\mu \chi$	*F31*: $S \leftarrow \chi^T \mathrm{C}(-\gamma^\mu)\psi_\mu$
F23: $\psi_\mu \leftarrow S\gamma_\mu \chi$	*F32*: $\psi_\mu \leftarrow \gamma_\mu \chi S$
GBG (Gravbar, S, Chi): $\bar{\psi}_\mu \not{k}_S S \gamma^\mu \chi$	
F12: $\chi \leftarrow \gamma^\mu \not{k}_S \psi_\mu S$	*F21*: $\chi \leftarrow S\gamma^\mu \not{k}_S \psi_\mu$
F13: $S \leftarrow \psi_\mu^T \mathrm{C}\not{k}_S \gamma^\mu \chi$	*F31*: $S \leftarrow \chi^T \mathrm{C}\gamma^\mu \not{k}_S \psi_\mu$
F23: $\psi_\mu \leftarrow S\not{k}_S \gamma_\mu \chi$	*F32*: $\psi_\mu \leftarrow \not{k}_S \gamma_\mu \chi S$
GBG (Gravbar, P, Chi): $\bar{\psi}_\mu \not{k}_P P \gamma^\mu \gamma_5 \chi$	
F12: $\chi \leftarrow \gamma^\mu \not{k}_P \gamma_5 \psi_\mu P$	*F21*: $\chi \leftarrow P\gamma^\mu \not{k}_P \gamma_5 \psi_\mu$
F13: $P \leftarrow \psi_\mu^T \mathrm{C}\not{k}_P \gamma^\mu \gamma_5 \chi$	*F31*: $P \leftarrow \chi^T \mathrm{C}\gamma^\mu \not{k}_P \gamma_5 \psi_\mu$
F23: $\psi_\mu \leftarrow P\not{k}_P \gamma_\mu \gamma_5 \chi$	*F32*: $\psi_\mu \leftarrow \not{k}_P \gamma_\mu \gamma_5 \chi P$
GBG (Gravbar, V, Chi): $\bar{\psi}_\mu [\not{k}_V, \not{V}]\gamma^\mu \gamma^5 \chi$	
F12: $\chi \leftarrow \gamma^5 \gamma^\mu [\not{k}_V, \gamma^\alpha]\psi_\mu V_\alpha$	*F21*: $\chi \leftarrow \gamma^5 \gamma^\mu [\not{k}_V, \not{V}]\psi_\mu$
F13: $V_\mu \leftarrow \psi_\rho^T \mathrm{C}[\not{k}_V, \gamma_\mu]\gamma^\rho \gamma^5 \chi$	*F31*: $V_\mu \leftarrow \chi^T \mathrm{C}\gamma^5 \gamma^\rho [\not{k}_V, \gamma_\mu]\psi_\rho$
F23: $\psi_\mu \leftarrow [\not{k}_V, \not{V}]\gamma_\mu \gamma^5 \chi$	*F32*: $\psi_\mu \leftarrow [\not{k}_V, \gamma^\alpha]\gamma_\mu \gamma^5 \chi V_\alpha$

Table 9.28: Dimension-5 trilinear couplings including one Majorana, one Gravitino fermion and one additional particle. The table is essentially the same as the one with the Dirac fermion and only written for the sake of completeness.

GBG (*Chibar, POT, Grav*): $\bar{\chi}\gamma^\mu S\psi_\mu$		
F12: $\psi_\mu \leftarrow -\gamma_\mu\chi S$	*F21*:	$\psi_\mu \leftarrow -S\gamma_\mu\chi$
F13: $S \leftarrow \chi^T\mathrm{C}\gamma^\mu\psi_\mu$	*F31*:	$S \leftarrow \psi_\mu^T\mathrm{C}(-\gamma^\mu)\chi$
F23: $\chi \leftarrow S\gamma^\mu\psi_\mu$	*F32*:	$\chi \leftarrow \gamma^\mu\psi_\mu S$
GBG (*Chibar, S, Grav*): $\bar{\chi}\gamma^\mu \not{k}_S S\psi_\mu$		
F12: $\psi_\mu \leftarrow \not{k}_S\gamma_\mu\chi S$	*F21*:	$\psi_\mu \leftarrow S\not{k}_S\gamma_\mu\chi$
F13: $S \leftarrow \chi^T\mathrm{C}\gamma^\mu\not{k}_S\psi_\mu$	*F31*:	$S \leftarrow \psi_\mu^T\mathrm{C}\not{k}_S\gamma^\mu\chi$
F23: $\chi \leftarrow S\gamma^\mu\not{k}_S\psi_\mu$	*F32*:	$\chi \leftarrow \gamma^\mu\not{k}_S\psi_\mu S$
GBG (*Chibar, P, Grav*): $\bar{\chi}\gamma^\mu\gamma^5 P\not{k}_P\psi_\mu$		
F12: $\psi_\mu \leftarrow -\not{k}_P\gamma_\mu\gamma^5\chi P$	*F21*:	$\psi_\mu \leftarrow -P\not{k}_P\gamma_\mu\gamma^5\chi$
F13: $P \leftarrow \chi^T\mathrm{C}\gamma^\mu\gamma^5\not{k}_P\psi_\mu$	*F31*:	$P \leftarrow -\psi_\mu^T\mathrm{C}\not{k}_P\gamma^\mu\gamma_5\chi$
F23: $\chi \leftarrow P\gamma^\mu\gamma^5\not{k}_P\psi_\mu$	*F32*:	$\chi \leftarrow \gamma^\mu\gamma^5\not{k}_P\psi_\mu P$
GBG (*Chibar, V, Grav*): $\bar{\chi}\gamma^5\gamma^\mu[\not{k}_V, \not{V}]\psi_\mu$		
F12: $\psi_\mu \leftarrow [\not{k}_V, \gamma^\alpha]\gamma_\mu\gamma^5\chi V_\alpha$	*F21*:	$\psi_\mu \leftarrow [\not{k}_V, \not{V}]\gamma_\mu\gamma^5\chi$
F13: $V_\mu \leftarrow \chi^T\mathrm{C}\gamma^5\gamma^\rho[\not{k}_V, \gamma_\mu]\psi_\rho$	*F31*:	$V_\mu \leftarrow \psi_\rho^T\mathrm{C}[\not{k}_V, \gamma_\mu]\gamma^\rho\gamma^5\chi$
F23: $\chi \leftarrow \gamma^5\gamma^\mu[\not{k}_V, \not{V}]\psi_\mu$	*F32*:	$\chi \leftarrow \gamma^5\gamma^\mu[\not{k}_V, \gamma^\alpha]\psi_\mu V_\alpha$

Table 9.29: Dimension-5 trilinear couplings including one conjugated Majorana, one Gravitino fermion and one additional particle. This table is not only the same as the one with the conjugated Dirac fermion but also the same part of the Lagrangian density as the one with the Majorana particle on the right of the gravitino.

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GBBG (*Gravbar*, *S2*, *Psi*): $\bar{\psi}_\mu S_1 S_2 \gamma^\mu \psi$	
F123 F213 F132 F231 F312 F321:	$\psi \leftarrow -\gamma^\mu S_1 S_2 \psi_\mu$
F423 F243 F432 F234 F342 F324:	$\psi_\mu \leftarrow \gamma_\mu S_1 S_2 \psi$
F134 F143 F314:	$S_1 \leftarrow \psi_\mu^T C S_2 \gamma^\mu \psi$
F124 F142 F214:	$S_2 \leftarrow \psi_\mu^T C S_1 \gamma^\mu \psi$
F413 F431 F341:	$S_1 \leftarrow -\psi^T C S_2 \gamma^\mu \psi_\mu$
F412 F421 F241:	$S_2 \leftarrow -\psi^T C S_1 \gamma^\mu \psi_\mu$
GBBG (*Gravbar*, *SV*, *Psi*): $\bar{\psi}_\mu S \slashed{V} \gamma^\mu \gamma^5 \psi$	
F123 F213 F132 F231 F312 F321:	$\psi \leftarrow \gamma^5 \gamma^\mu S \slashed{V} \psi_\mu$
F423 F243 F432 F234 F342 F324:	$\psi_\mu \leftarrow \slashed{V} S \gamma_\mu \gamma^5 \psi$
F134 F143 F314:	$S \leftarrow \psi_\mu^T C \slashed{V} \gamma^\mu \gamma^5 \psi$
F124 F142 F214:	$V_\mu \leftarrow \psi_\rho^T C S \gamma_\mu \gamma^\rho \gamma^5 \psi$
F413 F431 F341:	$S \leftarrow \psi^T C \gamma^5 \gamma^\mu \slashed{V} \psi_\mu$
F412 F421 F241:	$V_\mu \leftarrow \psi^T C S \gamma^5 \gamma^\rho \gamma_\mu \psi_\rho$
GBBG (*Gravbar*, *PV*, *Psi*): $\bar{\psi}_\mu P \slashed{V} \gamma^\mu \psi$	
F123 F213 F132 F231 F312 F321:	$\psi \leftarrow \gamma^\mu P \slashed{V} \psi_\mu$
F423 F243 F432 F234 F342 F324:	$\psi_\mu \leftarrow \slashed{V} P \gamma_\mu \psi$
F134 F143 F314:	$P \leftarrow \psi_\mu^T C \slashed{V} \gamma^\mu \psi$
F124 F142 F214:	$V_\mu \leftarrow \psi_\rho^T C P \gamma_\mu \gamma^\rho \psi$
F413 F431 F341:	$P \leftarrow \psi^T C \gamma^\mu \slashed{V} \psi_\mu$
F412 F421 F241:	$V_\mu \leftarrow \psi^T C P \gamma^\rho \gamma_\mu \psi_\rho$
GBBG (*Gravbar*, *V2*, *Psi*): $\bar{\psi}_\mu f_{abc}[\slashed{V}^a, \slashed{V}^b]\gamma^\mu \gamma^5 \psi$	
F123 F213 F132 F231 F312 F321:	$\psi \leftarrow f_{abc}\gamma^5 \gamma^\mu [\slashed{V}^a, \slashed{V}^b]\psi_\mu$
F423 F243 F432 F234 F342 F324:	$\psi_\mu \leftarrow f_{abc}[\slashed{V}^a, \slashed{V}^b]\gamma_\mu \gamma^5 \psi$
F134 F143 F314 F124 F142 F214:	$V_\mu^a \leftarrow \psi_\rho^T C f_{abc}[\gamma_\mu, \slashed{V}^b]\gamma^\rho \gamma^5 \psi$
F413 F431 F341 F412 F421 F241:	$V_\mu^a \leftarrow \psi^T C f_{abc}\gamma^5 \gamma^\rho [\gamma_\mu, \slashed{V}^b]\psi_\rho$

Table 9.30: Dimension-5 trilinear couplings including one Dirac, one Gravitino fermion and two additional bosons. In each lines we list the fusion possibilities with the same order of the fermions, but the order of the bosons is arbitrary (of course, one has to take care of this order in the mapping of the wave functions in *fusion*).

GBBG (*Psibar, S2, Grav*): $\bar{\psi} S_1 S_2 \gamma^\mu \psi_\mu$	
F123 F213 F132 F231 F312 F321:	$\psi_\mu \leftarrow -\gamma_\mu S_1 S_2 \psi$
F423 F243 F432 F234 F342 F324:	$\psi \leftarrow \gamma^\mu S_1 S_2 \psi_\mu$
F134 F143 F314:	$S_1 \leftarrow \psi^T C S_2 \gamma^\mu \psi_\mu$
F124 F142 F214:	$S_2 \leftarrow \psi^T C S_1 \gamma^\mu \psi_\mu$
F413 F431 F341:	$S_1 \leftarrow -\psi_\mu^T C S_2 \gamma^\mu \psi$
F412 F421 F241:	$S_2 \leftarrow -\psi_\mu^T C S_1 \gamma^\mu \psi$
GBBG (*Psibar, SV, Grav*): $\bar{\psi} S \gamma^\mu \gamma^5 \slashed{V} \psi_\mu$	
F123 F213 F132 F231 F312 F321:	$\psi_\mu \leftarrow \slashed{V} S \gamma^5 \gamma^\mu \psi$
F423 F243 F432 F234 F342 F324:	$\psi \leftarrow \gamma^\mu \gamma^5 S \slashed{V} \psi_\mu$
F134 F143 F314:	$S \leftarrow \psi^T C \gamma^\mu \gamma^5 \slashed{V} \psi$
F124 F142 F214:	$V_\mu \leftarrow \psi^T C \gamma^\rho \gamma^5 S \gamma_\mu \psi_\rho$
F413 F431 F341:	$S \leftarrow \psi_\mu^T C \slashed{V} \gamma^5 \gamma^\mu \psi$
F412 F421 F241:	$V_\mu \leftarrow \psi_\rho^T C S \gamma_\mu \gamma^5 \gamma^\rho \psi$
GBBG (*Psibar, PV, Grav*): $\bar{\psi} P \gamma^\mu \slashed{V} \psi_\mu$	
F123 F213 F132 F231 F312 F321:	$\psi_\mu \leftarrow \slashed{V} \gamma_\mu P \psi$
F423 F243 F432 F234 F342 F324:	$\psi \leftarrow \gamma^\mu \slashed{V} P \psi_\mu$
F134 F143 F314:	$P \leftarrow \psi^T C \gamma^\mu \slashed{V} \psi_\mu$
F124 F142 F214:	$V_\mu \leftarrow \psi^T C P \gamma^\rho \gamma_\mu \psi_\rho$
F413 F431 F341:	$P \leftarrow \psi_\mu^T C \slashed{V} \gamma^\mu \psi$
F412 F421 F241:	$V_\mu \leftarrow \psi_\rho^T C P \gamma_\mu \gamma^\rho \psi$
GBBG (*Psibar, V2, Grav*): $\bar{\psi} f_{abc} \gamma^5 \gamma^\mu [\slashed{V}^a, \slashed{V}^b] \psi_\mu$	
F123 F213 F132 F231 F312 F321:	$\psi_\mu \leftarrow f_{abc} [\slashed{V}^a, \slashed{V}^b] \gamma_\mu \gamma^5 \psi$
F423 F243 F432 F234 F342 F324:	$\psi \leftarrow f_{abc} \gamma^5 \gamma^\mu [\slashed{V}^a, \slashed{V}^b] \psi_\mu$
F134 F143 F314 F124 F142 F214:	$V_\mu^a \leftarrow \psi^T C f_{abc} \gamma^5 \gamma^\rho [\gamma_\mu, \slashed{V}^b] \psi_\rho$
F413 F431 F341 F412 F421 F241:	$V_\mu^a \leftarrow \psi_\rho^T C f_{abc} [\gamma_\mu, \slashed{V}^b] \gamma^\rho \gamma^5 \psi$

Table 9.31: Dimension-5 trilinear couplings including one conjugated Dirac, one Gravitino fermion and two additional bosons. The couplings of Majorana fermions to the gravitino and two bosons are essentially the same as for Dirac fermions and they are omitted here.
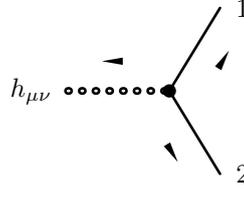
$$-\mathrm{i}\frac{\kappa}{2}g_{\mu\nu}m^2 + \mathrm{i}\frac{\kappa}{2}C_{\mu\nu,\mu_1\mu_2}k_1^{\mu_1}k_2^{\mu_2}$$

(9.27a)
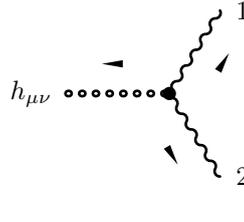
$$-\mathrm{i}\frac{\kappa}{2}m^2 C_{\mu\nu,\mu_1\mu_2} - \mathrm{i}\frac{\kappa}{2}(k_1 k_2 C_{\mu\nu,\mu_1\mu_2}$$
$$+ D_{\mu\nu,\mu_1\mu_2}(k_1,k_2)$$
$$+ \xi^{-1}E_{\mu\nu,\mu_1\mu_2}(k_1,k_2))$$

(9.27b)
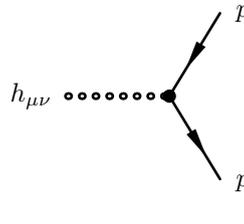
$$-\mathrm{i}\frac{\kappa}{2}mg_{\mu\nu} - \mathrm{i}\frac{\kappa}{8}(\gamma_\mu(p+p')_\nu + \gamma_\nu(p+p')_\mu$$
$$- 2g_{\mu\nu}(\not{p}+\not{p}'))$$

(9.27c)

Figure 9.3: Three-point graviton couplings.

In (9.27-9.34), we use the notation of [13]:

$$C_{\mu\nu,\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma} \tag{9.26a}$$

$$D_{\mu\nu,\rho\sigma}(k_1,k_2) = g_{\mu\nu}k_{1,\sigma}k_{2,\rho}$$
$$- (g_{\mu\sigma}k_{1,\nu}k_{2,\rho} + g_{\mu\rho}k_{1,\sigma}k_{2,\nu} - g_{\rho\sigma}k_{1,\mu}k_{2,\nu} + (\mu \leftrightarrow \nu)) \tag{9.26b}$$

$$E_{\mu\nu,\rho\sigma}(k_1,k_2) = g_{\mu\nu}(k_{1,\rho}k_{1,\sigma} + k_{2,\rho}k_{2,\sigma} + k_{1,\rho}k_{2,\sigma})$$
$$- (g_{\nu\sigma}k_{1,\mu}k_{1,\rho} + g_{\nu\rho}k_{2,\mu}k_{2,\sigma} + (\mu \leftrightarrow \nu)) \tag{9.26c}$$

$$F_{\mu\nu,\rho\sigma\lambda}(k_1,k_2,k_3) =$$
$$g_{\mu\rho}g_{\sigma\lambda}(k_2 - k_3)_\nu + g_{\mu\sigma}g_{\lambda\rho}(k_3 - k_1)_\nu + g_{\mu\lambda}g_{\rho\sigma}(k_1 - k_2)_\nu + (\mu \leftrightarrow \nu) \tag{9.26d}$$

$$G_{\mu\nu,\rho\sigma\lambda\delta} = g_{\mu\nu}(g_{\rho\sigma}g_{\lambda\delta} - g_{\rho\delta}g_{\lambda\sigma})$$
$$+ (g_{\mu\rho}g_{\nu\delta}g_{\lambda\sigma} + g_{\mu\lambda}g_{\nu\sigma}g_{\rho\delta} - g_{\mu\rho}g_{\nu\sigma}g_{\lambda\delta} - g_{\mu\lambda}g_{\nu\delta}g_{\rho\sigma} + (\mu \leftrightarrow \nu)) \tag{9.26e}$$

Derivation of (9.27a)

$$L = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{m^2}{2}\phi^2 \tag{9.28a}$$

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Graviton_Scalar_Scalar: $h_{\mu\nu} C_0^{\mu\nu}(k_1, k_2) \phi_1 \phi_2$
F12 | *F21*: $\phi_2 \leftarrow \mathrm{i} \cdot h_{\mu\nu} C_0^{\mu\nu}(k_1, -k - k_1) \phi_1$
F13 | *F31*: $\phi_1 \leftarrow \mathrm{i} \cdot h_{\mu\nu} C_0^{\mu\nu}(-k - k_2, k_2) \phi_2$
F23 | *F32*: $h^{\mu\nu} \leftarrow \mathrm{i} \cdot C_0^{\mu\nu}(k_1, k_2) \phi_1 \phi_2$
Graviton_Vector_Vector: $h_{\mu\nu} C_1^{\mu\nu,\mu_1\mu_2}(k_1, k_2, \xi) V_{\mu_1} V_{\mu_2}$
F12 | *F21*: $V_2^\mu \leftarrow \mathrm{i} \cdot h_{\kappa\lambda} C_1^{\kappa\lambda,\mu\nu}(-k - k_1, k_1\xi) V_{1,\nu}$
F13 | *F31*: $V_1^\mu \leftarrow \mathrm{i} \cdot h_{\kappa\lambda} C_1^{\kappa\lambda,\mu\nu}(-k - k_2, k_2, \xi) V_{2,\nu}$
F23 | *F32*: $h^{\mu\nu} \leftarrow \mathrm{i} \cdot C_1^{\mu\nu,\mu_1\mu_2}(k_1, k_2, \xi) V_{1,\mu_1} V_{2,\mu_2}$
Graviton_Spinor_Spinor: $h_{\mu\nu} \bar{\psi}_1 C_{\frac{1}{2}}^{\mu\nu}(k_1, k_2) \psi_2$
F12: $\bar{\psi}_2 \leftarrow \mathrm{i} \cdot h_{\mu\nu} \bar{\psi}_1 C_{\frac{1}{2}}^{\mu\nu}(k_1, -k - k_1)$
F21: $\bar{\psi}_2 \leftarrow \mathrm{i} \cdot \ldots$
F13: $\psi_1 \leftarrow \mathrm{i} \cdot h_{\mu\nu} C_{\frac{1}{2}}^{\mu\nu}(-k - k_2, k_2) \psi_2$
F31: $\psi_1 \leftarrow \mathrm{i} \cdot \ldots$
F23: $h^{\mu\nu} \leftarrow \mathrm{i} \cdot \bar{\psi}_1 C_{\frac{1}{2}}^{\mu\nu}(k_1, k_2) \psi_2$
F32: $h^{\mu\nu} \leftarrow \mathrm{i} \cdot \ldots$

Table 9.32: ...

$$\phi(k) \cdots \cdots \cdots \quad = -\mathrm{i}\omega\kappa 2m^2 - \mathrm{i}\omega\kappa k_1 k_2 \tag{9.30a}$$

$$\phi(k) \cdots \cdots \cdots \quad = -\mathrm{i}\omega\kappa g_{\mu_1\mu_2} m^2 - \mathrm{i}\omega\kappa\xi^{-1}(k_{1,\mu_1}k_{\mu_2} + k_{2,\mu_2}k_{\mu_1}) \tag{9.30b}$$

$$\phi(k) \cdots \cdots \cdots \quad = -\mathrm{i}\omega\kappa 2m + \mathrm{i}\omega\kappa\frac{3}{4}(\not{p} + \not{p}') \tag{9.30c}$$

Figure 9.4: Three-point dilaton couplings.

$$(\partial_\mu\phi)\frac{\partial L}{\partial(\partial^\nu\phi)} = (\partial_\mu\phi)(\partial_\nu\phi) \tag{9.28b}$$

$$T_{\mu\nu} = -g_{\mu\nu}L + (\partial_\mu\phi)\frac{\partial L}{\partial(\partial^\nu\phi)} + \tag{9.28c}$$

$$C_0^{\mu\nu}(k_1, k_2) = C^{\mu\nu,\mu_1\mu_2}k_{1,\mu_1}k_{2,\mu_2} \tag{9.29a}$$

$$C_1^{\mu\nu,\mu_1\mu_2}(k_1, k_2, \xi) = k_1 k_2 C^{\mu\nu,\mu_1\mu_2} + D^{\mu\nu,\mu_1\mu_2}(k_1, k_2) + \xi^{-1}E^{\mu\nu,\mu_1\mu_2}(k_1, k_2) \tag{9.29b}$$

$$C_{\frac{1}{2},\alpha\beta}^{\mu\nu}(p, p') = \gamma_{\alpha\beta}^{\mu}(p + p')^{\nu} + \gamma_{\alpha\beta}^{\nu}(p + p')^{\mu} - 2g^{\mu\nu}(\not{p} + \not{p}')_{\alpha\beta} \tag{9.29c}$$

9.1.8 Dependent Parameters

This is a simple abstract syntax for parameter dependencies. Later, there will
be a parser for a convenient concrete syntax as a part of a concrete syntax for
models. There is no intention to do *any* symbolic manipulation with this. The
expressions will be translated directly by *Targets* to the target language.

type α *expr* =
 | *I* | *Const* of *int*

Dilaton_Scalar_Scalar: $\phi \ldots k_1 k_2 \phi_1 \phi_2$	
F12 | *F21*:	$\phi_2 \leftarrow \mathrm{i} \cdot k_1(-k - k_1)\phi\phi_1$
F13 | *F31*:	$\phi_1 \leftarrow \mathrm{i} \cdot (-k - k_2)k_2\phi\phi_2$
F23 | *F32*:	$\phi \leftarrow \mathrm{i} \cdot k_1 k_2 \phi_1 \phi_2$
Dilaton_Vector_Vector: $\phi \ldots$	
F12:	$V_{2,\mu} \leftarrow \mathrm{i} \cdot \ldots$
F21:	$V_{2,\mu} \leftarrow \mathrm{i} \cdot \ldots$
F13:	$V_{1,\mu} \leftarrow \mathrm{i} \cdot \ldots$
F31:	$V_{1,\mu} \leftarrow \mathrm{i} \cdot \ldots$
F23:	$\phi \leftarrow \mathrm{i} \cdot \ldots$
F32:	$\phi \leftarrow \mathrm{i} \cdot \ldots$
Dilaton_Spinor_Spinor: $\phi \ldots$	
F12:	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot \ldots$
F21:	$\bar{\psi}_2 \leftarrow \mathrm{i} \cdot \ldots$
F13:	$\psi_1 \leftarrow \mathrm{i} \cdot \ldots$
F31:	$\psi_1 \leftarrow \mathrm{i} \cdot \ldots$
F23:	$\phi \leftarrow \mathrm{i} \cdot \ldots$
F32:	$\phi \leftarrow \mathrm{i} \cdot \ldots$

Table 9.33: ...

<figure>

$$= \quad \text{???} \quad (9.31\text{a})$$

$$= \quad -\mathrm{i}g\frac{\kappa}{2}C_{\mu\nu,\mu_3\rho}(k_1-k_2)^\rho T^{a_3}_{n_2 n_1} \quad (9.31\text{b})$$

$$= \quad \text{???} \quad (9.31\text{c})$$

$$= \quad \begin{aligned} -\,g\frac{\kappa}{2}f^{a_1 a_2 a_3}(&C_{\mu\nu,\mu_1\mu_2}(k_1-k_2)_{\mu_3} \\ &+ C_{\mu\nu,\mu_2\mu_3}(k_2-k_3)_{\mu_1} \\ &+ C_{\mu\nu,\mu_3\mu_1}(k_3-k_1)_{\mu_2} \\ &+ F_{\mu\nu,\mu_1\mu_2\mu_3}(k_1,k_2,k_3)) \end{aligned} \quad (9.31\text{d})$$

$$= \quad \text{???} \quad (9.31\text{e})$$

$$= \quad \mathrm{i}g\frac{\kappa}{4}(C_{\mu\nu,\mu_3\rho}-g_{\mu\nu}g_{\mu_3\rho})\gamma^\rho T^{a_3}_{n_2 n_1} \quad (9.31\text{f})$$

</figure>
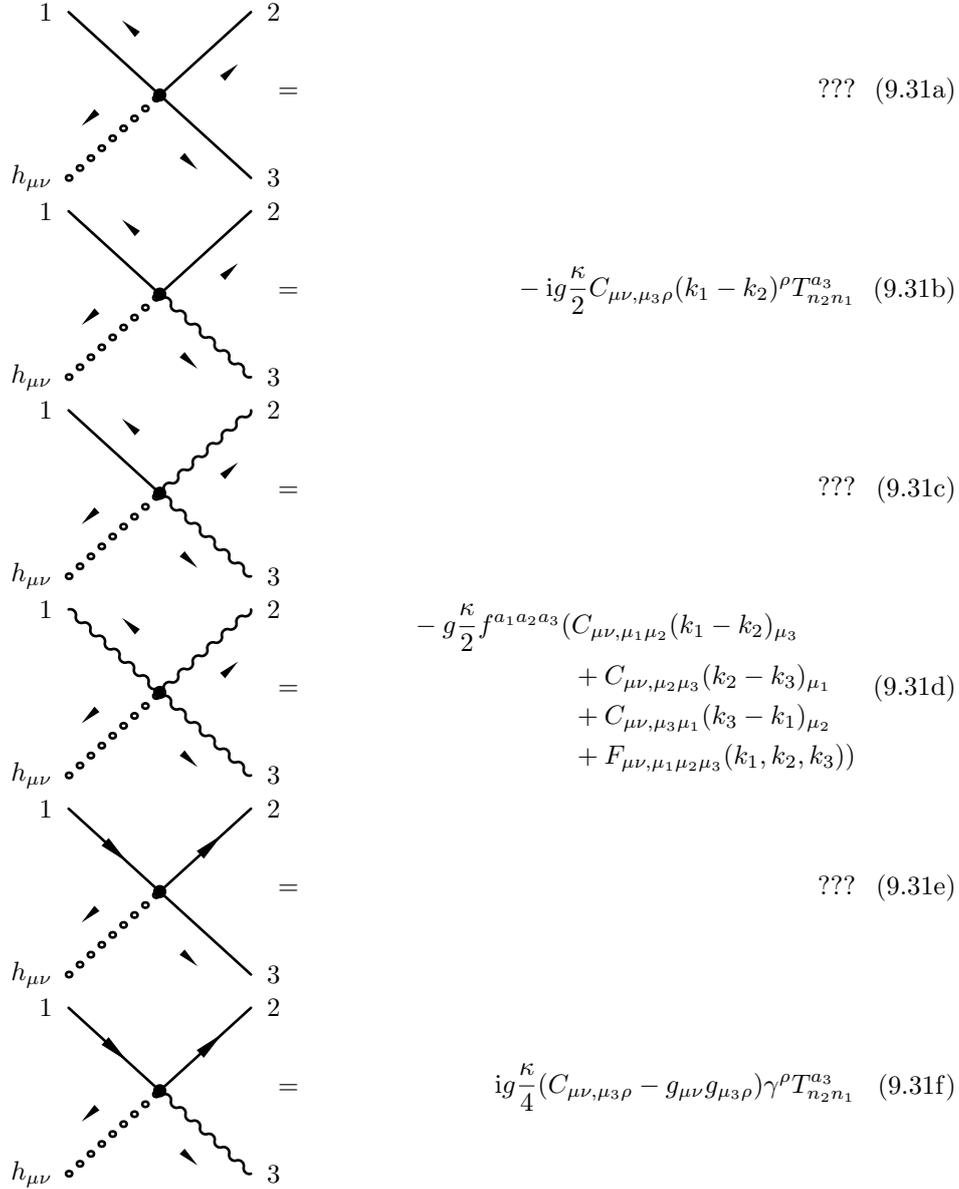
Figure 9.5: Four-point graviton couplings. (9.31a), (9.31c), and (**??** are missing in [13], but should be generated by standard model Higgs selfcouplings, Higgs-gaugeboson couplings, and Yukawa couplings.

$$=??? \tag{9.32a}$$

$$= -\mathrm{i}\omega\kappa(k_1 + k_2)_{\mu_3} T^{a_3}_{n_1,n_2} \tag{9.32b}$$

$$=??? \tag{9.32c}$$

$$= 0 \tag{9.32d}$$

$$=??? \tag{9.32e}$$

$$= -\mathrm{i}\frac{3}{2}\omega g\kappa\gamma_{\mu_3} T^{a_3}_{n_1 n_2} \tag{9.32f}$$
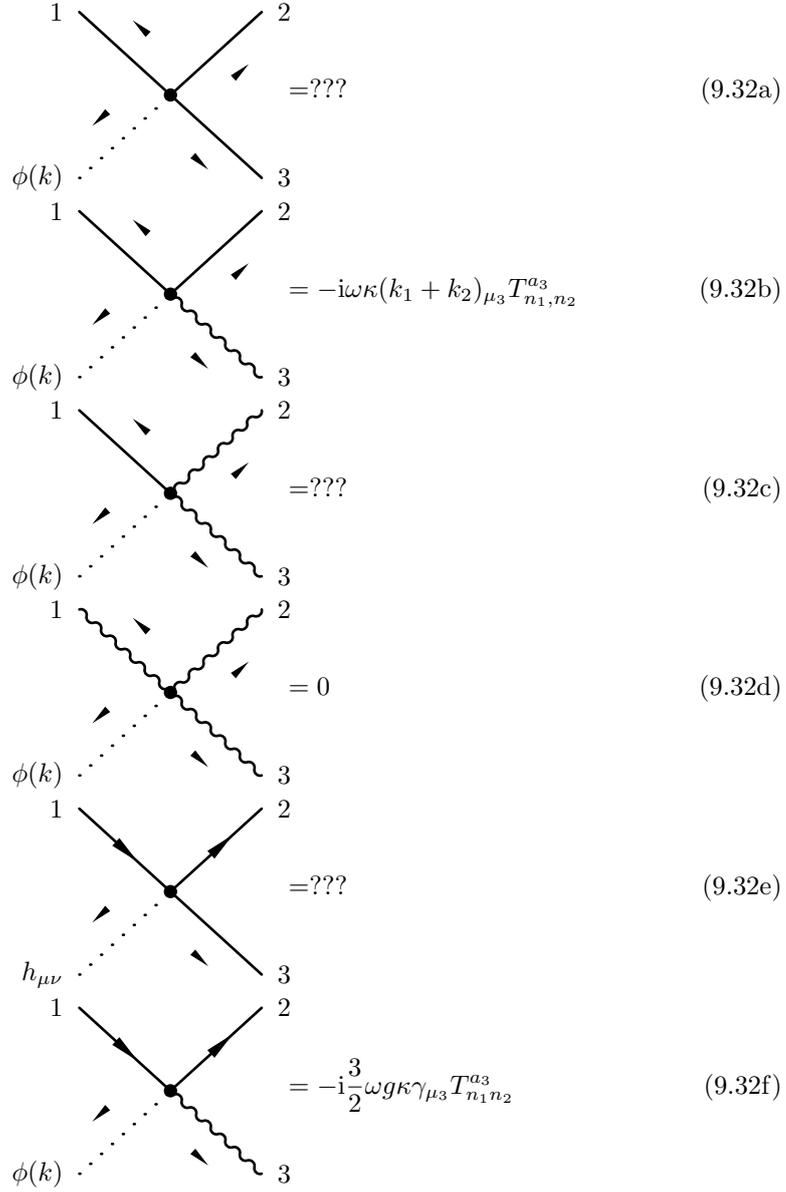
Figure 9.6: Four-point dilaton couplings. (9.32a), (9.32c) and (9.32e) are missing in [13], but could be generated by standard model Higgs selfcouplings, Higgs-gaugeboson couplings, and Yukawa couplings.

$$???$$
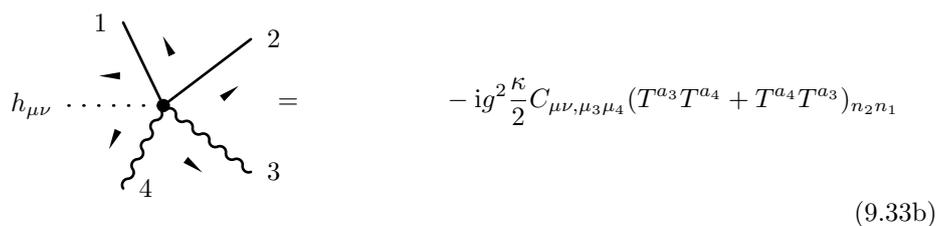
$$(9.33a)$$

$$-\mathrm{i}g^2\frac{\kappa}{2}C_{\mu\nu,\mu_3\mu_4}(T^{a_3}T^{a_4}+T^{a_4}T^{a_3})_{n_2n_1}$$

$$(9.33b)$$
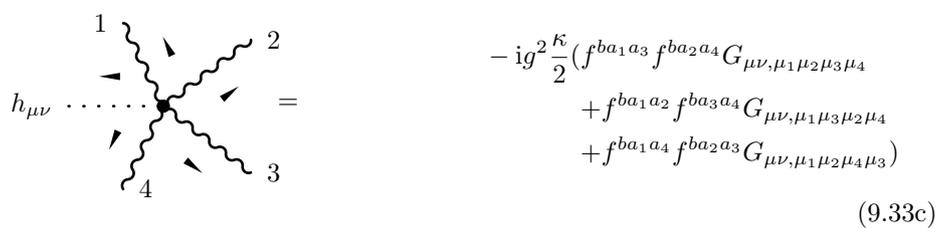
$$-\mathrm{i}g^2\frac{\kappa}{2}(f^{ba_1a_3}f^{ba_2a_4}G_{\mu\nu,\mu_1\mu_2\mu_3\mu_4}$$
$$+f^{ba_1a_2}f^{ba_3a_4}G_{\mu\nu,\mu_1\mu_3\mu_2\mu_4}$$
$$+f^{ba_1a_4}f^{ba_2a_3}G_{\mu\nu,\mu_1\mu_2\mu_4\mu_3})$$

$$(9.33c)$$

Figure 9.7: Five-point graviton couplings. (9.33a) is missing in [13], but should be generated by standard model Higgs selfcouplings.

$$\phi(k) \cdots\cdots \quad\begin{array}{c}1\\2\\3\\4\end{array}\quad =??? \tag{9.34a}$$

$$\phi(k) \cdots\cdots \quad\begin{array}{c}1\\2\\3\\4\end{array}\quad = \mathrm{i}\omega g^2 \kappa g_{\mu_3\mu_4}(T^{a_3}T^{a_4} + T^{a_4}T^{a_3})_{n_2 n_1} \tag{9.34b}$$

$$\phi(k) \cdots\cdots \quad\begin{array}{c}1\\2\\3\\4\end{array}\quad = 0 \tag{9.34c}$$

Figure 9.8: Five-point dilaton couplings. (9.34a) is missing in [13], but could be generated by standard model Higgs selfcouplings.

$$
\begin{array}{l}
| \ \textit{Atom} \text{ of } \alpha \\
| \ \textit{Sum} \text{ of } \alpha \textit{ expr list} \\
| \ \textit{Diff} \text{ of } \alpha \textit{ expr } \times \textit{ } \alpha \textit{ expr} \\
| \ \textit{Neg} \text{ of } \alpha \textit{ expr} \\
| \ \textit{Prod} \text{ of } \alpha \textit{ expr list} \\
| \ \textit{Quot} \text{ of } \alpha \textit{ expr } \times \textit{ } \alpha \textit{ expr} \\
| \ \textit{Rec} \text{ of } \alpha \textit{ expr} \\
| \ \textit{Pow} \text{ of } \alpha \textit{ expr } \times \textit{ int} \\
| \ \textit{Sqrt} \text{ of } \alpha \textit{ expr} \\
| \ \textit{Sin} \text{ of } \alpha \textit{ expr} \\
| \ \textit{Cos} \text{ of } \alpha \textit{ expr} \\
| \ \textit{Tan} \text{ of } \alpha \textit{ expr} \\
| \ \textit{Cot} \text{ of } \alpha \textit{ expr} \\
| \ \textit{Atan2} \text{ of } \alpha \textit{ expr } \times \textit{ } \alpha \textit{ expr} \\
| \ \textit{Conj} \text{ of } \alpha \textit{ expr}
\end{array}
$$

type α *variable* = *Real* of α | *Complex* of α
type α *variable_array* = *Real_Array* of α | *Complex_Array* of α

type α *parameters* =
  { *input* : ($\alpha \times$ *float*) *list*;
    derived : (α *variable* $\times$ α *expr*) *list*;
    derived_arrays : (α *variable_array* $\times$ α *expr list*) *list* }

$Dim5\_Scalar\_Vector\_Vector\_T$: $\mathcal{L}_I = g\phi(\mathrm{i}\partial_\mu V_1^\nu)(\mathrm{i}\partial_\nu V_2^\mu)$
F23: $\quad \phi(k_2 + k_3) \leftarrow \mathrm{i} \cdot g k_3^\mu V_{1,\mu}(k_2) k_2^\nu V_{2,\nu}(k_3)$
F32: $\quad \phi(k_2 + k_3) \leftarrow \mathrm{i} \cdot g k_2^\mu V_{2,\mu}(k_3) k_3^\nu V_{1,\nu}(k_2)$
F12: $\quad V_2^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g k_2^\mu \phi(k_1)(-k_1^\nu - k_2^\nu) V_{1,\nu}(k_2)$
F21: $\quad V_2^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g k_2^\mu (-k_1^\nu - k_2^\nu) V_{1,\nu}(k_2) \phi(k_1)$
F13: $\quad V_1^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g k_3^\mu \phi(k_1)(-k_1^\nu - k_3^\nu) V_{2,\nu}(k_3)$
F31: $\quad V_1^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g k_3^\mu (-k_1^\nu - k_3^\nu) V_{2,\nu}(k_3) \phi(k_1)$

Table 9.34: ...

$Dim6\_Vector\_Vector\_Vector\_T$: $\mathcal{L}_I = g V_1^\mu((\mathrm{i}\partial_\nu V_2^\rho) \mathrm{i} \overleftrightarrow{\partial_\mu} (\mathrm{i}\partial_\rho V_3^\nu))$
F23: $\quad V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\mu - k_3^\mu) k_3^\nu V_{2,\nu}(k_2) k_2^\rho V_{3,\rho}(k_3)$
F32: $\quad V_1^\mu(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\mu - k_3^\mu) k_2^\nu V_{3,\nu}(k_3) k_3^\rho V_{2,\rho}(k_2)$
F12: $\quad V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g k_2^\mu(k_1^\nu + 2k_2^\nu) V_{1,\nu}(k_1)(-k_1^\rho - k_2^\rho) V_{2,\rho}(k_2)$
F21: $\quad V_3^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g k_2^\mu(-k_1^\rho - k_2^\rho) V_{2,\rho}(k_2)(k_1^\nu + 2k_2^\nu) V_{1,\nu}(k_1)$
F13: $\quad V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g k_3^\mu(k_1^\nu + 2k_3^\nu) V_{1,\nu}(k_1)(-k_1^\rho - k_3^\rho) V_{3,\rho}(k_3)$
F31: $\quad V_2^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g k_3^\mu(-k_1^\rho - k_3^\rho) V_{3,\rho}(k_3)(k_1^\nu + 2k_3^\nu) V_{1,\nu}(k_1)$

Table 9.35: ...

9.1.9   More Exotic Couplings

9.2   Interface of Model

9.2.1   General Quantum Field Theories

module type T =
  sig

flavor abstractly encodes all quantum numbers.

   type *flavor*

$Color.t$ encodes the $(\mathrm{SU}(N))$ color representation.

   val *color* : *flavor* $\rightarrow$ $Color.t$

The set of conserved charges.

   module Ch : $Charges.T$
   val *charges* : *flavor* $\rightarrow$ $Ch.t$

The PDG particle code for interfacing with Monte Carlos.

   val *pdg* : *flavor* $\rightarrow$ *int*

The Lorentz representation of the particle.

$Tensor\_2\_Vector\_Vector$: $\mathcal{L}_I = gT^{\mu\nu}(V_{1,\mu}V_{2,\nu} + V_{1,\nu}V_{2,\mu})$	
F23:	$T^{\mu\nu}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(V_{1,\mu}(k_2)V_{2,\nu}(k_3) + V_{1,\nu}(k_2)V_{2,\mu}(k_3))$
F32:	$T^{\mu\nu}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(V_{2,\nu}(k_3)V_{1,\mu}(k_2) + V_{2,\mu}(k_3)V_{1,\nu}(k_2))$
F12:	$V_2^{\mu}(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(T^{\mu\nu}(k_1) + T^{\nu\mu}(k_1))V_{1,\nu}(k_2)$
F21:	$V_2^{\mu}(k_1 + k_2) \leftarrow \mathrm{i} \cdot gV_{1,\nu}(k_2)(T^{\mu\nu}(k_1) + T^{\nu\mu}(k_1))$
F13:	$V_1^{\mu}(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(T^{\mu\nu}(k_1) + T^{\nu\mu}(k_1))V_{2,\nu}(k_3)$
F31:	$V_1^{\mu}(k_1 + k_3) \leftarrow \mathrm{i} \cdot gV_{2,\nu}(k_3)(T^{\mu\nu}(k_1) + T^{\nu\mu}(k_1))$

Table 9.36:   . . .

$Dim5\_Tensor\_2\_Vector\_Vector\_1$: $\mathcal{L}_I = gT^{\alpha\beta}(V_1^{\mu}\mathrm{i}\overleftrightarrow{\partial}_\alpha\mathrm{i}\overleftrightarrow{\partial}_\beta V_{2,\mu})$	
F23:	$T^{\alpha\beta}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\alpha - k_3^\alpha)(k_2^\beta - k_3^\beta)V_1^\mu(k_2)V_{2,\mu}(k_3)$
F32:	$T^{\alpha\beta}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\alpha - k_3^\alpha)(k_2^\beta - k_3^\beta)V_{2,\mu}(k_3)V_1^\mu(k_2)$
F12:	$V_2^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(k_1^\alpha + 2k_2^\alpha)(k_1^\beta + 2k_2^\beta)T_{\alpha\beta}(k_1)V_1^\mu(k_2)$
F21:	$V_2^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(k_1^\alpha + 2k_2^\alpha)(k_1^\beta + 2k_2^\beta)V_1^\mu(k_2)T_{\alpha\beta}(k_1)$
F13:	$V_1^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(k_1^\alpha + 2k_3^\alpha)(k_1^\beta + 2k_3^\beta)T_{\alpha\beta}(k_1)V_2^\mu(k_3)$
F31:	$V_1^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(k_1^\alpha + 2k_3^\alpha)(k_1^\beta + 2k_3^\beta)V_2^\mu(k_3)T_{\alpha\beta}(k_1)$

Table 9.37:   . . .

$Dim5\_Tensor\_2\_Vector\_Vector\_2$: $\mathcal{L}_I = gT^{\alpha\beta}(V_1^{\mu}\mathrm{i}\overleftrightarrow{\partial}_\beta(\mathrm{i}\partial_\mu V_{2,\alpha}) + V_1^{\mu}\mathrm{i}\overleftrightarrow{\partial}_\alpha(\mathrm{i}\partial_\mu V_{2,\beta}))$	
F23:	$T^{\alpha\beta}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_3^\beta - k_2^\beta)k_3^\mu V_{1,\mu}(k_2)V_2^\alpha(k_3) + (\alpha \leftrightarrow \beta)$
F32:	$T^{\alpha\beta}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_3^\beta - k_2^\beta)V_2^\alpha(k_3)k_3^\mu V_{1,\mu}(k_2) + (\alpha \leftrightarrow \beta)$
F12:	$V_2^\alpha(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(k_1^\beta + 2k_2^\beta)(T^{\alpha\beta}(k_1) + T^{\beta\alpha}(k_1))(k_1^\mu + k_2^\mu)V_{1,\mu}(k_2)$
F21:	$V_2^\alpha(k_1 + k_2) \leftarrow \mathrm{i} \cdot g(k_1^\mu + k_2^\mu)V_{1,\mu}(k_2)(k_1^\beta + 2k_2^\beta)(T^{\alpha\beta}(k_1) + T^{\beta\alpha}(k_1))$
F13:	$V_1^\alpha(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(k_1^\beta + 2k_3^\beta)(T^{\alpha\beta}(k_1) + T^{\beta\alpha}(k_1))(k_1^\mu + k_3^\mu)V_{2,\mu}(k_3)$
F31:	$V_1^\alpha(k_1 + k_3) \leftarrow \mathrm{i} \cdot g(k_1^\mu + k_3^\mu)V_{2,\mu}(k_3)(k_1^\beta + 2k_3^\beta)(T^{\alpha\beta}(k_1) + T^{\beta\alpha}(k_1))$

Table 9.38:   . . .

$Dim7\_Tensor\_2\_Vector\_Vector\_T$: $\mathcal{L}_I = gT^{\alpha\beta}((\mathrm{i}\partial^\mu V_1^\nu)\mathrm{i}\overset{\leftrightarrow}{\partial}{}_\alpha\mathrm{i}\overset{\leftrightarrow}{\partial}{}_\beta(\mathrm{i}\partial_\nu V_{2,\mu}))$

F23:	$T^{\alpha\beta}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\alpha - k_3^\alpha)(k_2^\beta - k_3^\beta)k_3^\mu V_{1,\mu}(k_2)k_2^\nu V_{2,\nu}(k_3)$
F32:	$T^{\alpha\beta}(k_2 + k_3) \leftarrow \mathrm{i} \cdot g(k_2^\alpha - k_3^\alpha)(k_2^\beta - k_3^\beta)k_2^\nu V_{2,\nu}(k_3)k_3^\mu V_{1,\mu}(k_2)$
F12:	$V_2^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot gk_2^\mu(k_1^\alpha + 2k_2^\alpha)(k_1^\beta + 2k_2^\beta)T_{\alpha\beta}(k_1)(-k_1^\nu - k_2^\nu)V_{1,\nu}(k_2)$
F21:	$V_2^\mu(k_1 + k_2) \leftarrow \mathrm{i} \cdot gk_2^\mu(-k_1^\nu - k_2^\nu)V_{1,\nu}(k_2)(k_1^\alpha + 2k_2^\alpha)(k_1^\beta + 2k_2^\beta)T_{\alpha\beta}(k_1)$
F13:	$V_1^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot gk_3^\mu(k_1^\alpha + 2k_3^\alpha)(k_1^\beta + 2k_3^\beta)T_{\alpha\beta}(k_1)(-k_1^\nu - k_3^\nu)V_{2,\nu}(k_3)$
F31:	$V_1^\mu(k_1 + k_3) \leftarrow \mathrm{i} \cdot gk_3^\mu(-k_1^\nu - k_3^\nu)V_{2,\nu}(k_3)(k_1^\alpha + 2k_3^\alpha)(k_1^\beta + 2k_3^\beta)T_{\alpha\beta}(k_1)$

Table 9.39: . . .

val *lorentz* : *flavor* $\rightarrow$ *Coupling.lorentz*

The propagator for the particle, which *can* depend on a gauge parameter.

type *gauge*
val *propagator* : *flavor* $\rightarrow$ *gauge Coupling.propagator*

Not the symbol for the numerical value, but the scheme or strategy.

val *width* : *flavor* $\rightarrow$ *Coupling.width*

Charge conjugation, with and without color.

val *conjugate* : *flavor* $\rightarrow$ *flavor*

Returns 1 for fermions, -1 for anti-fermions, 2 for Majoranas and 0 otherwise.

val *fermion* : *flavor* $\rightarrow$ *int*

The Feynman rules. *vertices* and (*fuse2*, *fuse3*, *fusen*) are redundant, of course. However, *vertices* is required for building functors for models and *vertices* can be recovered from (*fuse2*, *fuse3*, *fusen*) only at great cost.

Nevertheless: *vertices* is a candidate for removal, b/c we can build a smarter *Colorize* functor acting on (*fuse2*, *fuse3*, *fusen*). It can support an arbitrary numer of color lines. But we have to test whether it is efficient enough.

type *constant*

Later: type *orders* to count orders of couplings

val *max_degree* : *unit* $\rightarrow$ *int*
val *vertices* : *unit* $\rightarrow$
   ((((*flavor* $\times$ *flavor* $\times$ *flavor*) $\times$ *constant Coupling.vertex3* $\times$ *constant*) *list*)
      $\times$ (((*flavor* $\times$ *flavor* $\times$ *flavor* $\times$ *flavor*) $\times$ *constant Coupling.vertex4* $\times$
constant) *list*)
      $\times$ (((*flavor list*) $\times$ *constant Coupling.vertexn* $\times$ *constant*) *list*))
val *fuse2* : *flavor* $\rightarrow$ *flavor* $\rightarrow$ (*flavor* $\times$ *constant Coupling.t*) *list*
val *fuse3* : *flavor* $\rightarrow$ *flavor* $\rightarrow$ *flavor* $\rightarrow$ (*flavor* $\times$ *constant Coupling.t*) *list*
val *fuse* : *flavor list* $\rightarrow$ (*flavor* $\times$ *constant Coupling.t*) *list*

Later: val *orders* : *constant* $\rightarrow$ *orders* counting orders of couplings

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The list of all known flavors.

> val *flavors* : *unit* → *flavor list*

The flavors that can appear in incoming or outgoing states, grouped in a way that is useful for user interfaces.

> val *external_flavors* : *unit* → (*string* × *flavor list*) *list*

The Goldstone bosons corresponding to a gauge field, if any.

> val *goldstone* : *flavor* → (*flavor* × *constant Coupling.expr*) *option*

The dependent parameters.

> val *parameters* : *unit* → *constant Coupling.parameters*

Translate from and to convenient textual representations of flavors.

> val *flavor_of_string* : *string* → *flavor*
> val *flavor_to_string* : *flavor* → *string*

TₑX and LᴬTₑX

> val *flavor_to_TeX* : *flavor* → *string*

The following must return unique symbols that are acceptable as symbols in all programming languages under consideration as targets. Strings of alphanumeric characters (starting with a letter) should be safe. Underscores are also usable, but would violate strict Fortran77.

> val *flavor_symbol* : *flavor* → *string*
> val *gauge_symbol* : *gauge* → *string*
> val *mass_symbol* : *flavor* → *string*
> val *width_symbol* : *flavor* → *string*
> val *constant_symbol* : *constant* → *string*

Model specific options.

> val *options* : *Options.t*

Revision control information.

> val *rcs* : *RCS.t*

> end

In addition to hardcoded models, we can have models that are initialized at run time.

9.2.2   Mutable Quantum Field Theories

module type *Mutable* =
  sig
    include *T*

Export only one big initialization function to discourage partial initializations. Labels make this usable.

> val *setup* :

  color : (*flavor* → *Color.t*) →
  pdg : (*flavor* → *int*) →
  lorentz : (*flavor* → *Coupling.lorentz*) →
  propagator : (*flavor* → *gauge Coupling.propagator*) →
  width : (*flavor* → *Coupling.width*) →
  goldstone : (*flavor* → (*flavor* × *constant Coupling.expr*) *option*) →
  conjugate : (*flavor* → *flavor*) →
  fermion : (*flavor* → *int*) →
  max_degree :*int* →
  vertices : (*unit* →
   ((((*flavor* × *flavor* × *flavor*) × *constant Coupling.vertex3* ×
constant) *list*)
     × (((*flavor* × *flavor* × *flavor* × *flavor*) × *constant Coupling.vertex4* ×
constant) *list*)
     × (((*flavor list*) × *constant Coupling.vertexn* × *constant*) *list*))) →

  fuse : ((*flavor* → *flavor* → (*flavor* × *constant Coupling.t*) *list*)
    × (*flavor* → *flavor* → *flavor* →
     (*flavor* × *constant Coupling.t*) *list*)
    × (*flavor list* → (*flavor* × *constant Coupling.t*) *list*)) →
  flavors : ((*string* × *flavor list*) *list*) →
  parameters : (*unit* → *constant Coupling.parameters*) →
  flavor_of_string : (*string* → *flavor*) →
  flavor_to_string : (*flavor* → *string*) →
  flavor_to_TeX : (*flavor* → *string*) →
  flavor_symbol : (*flavor* → *string*) →
  gauge_symbol : (*gauge* → *string*) →
  mass_symbol : (*flavor* → *string*) →
  width_symbol : (*flavor* → *string*) →
  constant_symbol : (*constant* → *string*) →
  unit
 end

9.2.3 Gauge Field Theories

The following signatures are used only for model building. The diagrammatics and numerics is supposed to be completely ignorant about the detail of the models and expected to rely on the interface *T* exclusively.

⚠ In the end, we might have functors (*M* : *T*) → *Gauge*, but we will need to add the quantum numbers to *T*.

module type *Gauge* =
 sig
  include *T*

Matter field carry conserved quantum numbers and can be replicated in generations without changing the gauge sector.

 type *matter_field*

Gauge bosons proper.

    type *gauge_boson*

Higgses, Goldstones and all the rest:

    type *other*

We can query the kind of field

    type *field* =
      | *Matter* of *matter_field*
      | *Gauge* of *gauge_boson*
      | *Other* of *other*
    val *field* : *flavor* → *field*

and we can build new fields of a given kind:

    val *matter_field* : *matter_field* → *flavor*
    val *gauge_boson* : *gauge_boson* → *flavor*
    val *other* : *other* → *flavor*
  end

9.2.4   Gauge Field Theories with Broken Gauge Symmetries

Both are carefully crafted as subtypes of *Gauge* so that they can be used in place of *Gauge* and *T* everywhere:

module type *Broken_Gauge* =
  sig
    include *Gauge*

    type *massless*
    type *massive*
    type *goldstone*

    type *kind* =
      | *Massless* of *massless*
      | *Massive* of *massive*
      | *Goldstone* of *goldstone*
    val *kind* : *gauge_boson* → *kind*

    val *massless* : *massive* → *gauge_boson*
    val *massive* : *massive* → *gauge_boson*
    val *goldstone* : *goldstone* → *gauge_boson*

  end

module type *Unitarity_Gauge* =
  sig
    include *Gauge*

    type *massless*
    type *massive*

    type *kind* =
      | *Massless* of *massless*
      | *Massive* of *massive*
    val *kind* : *gauge_boson* → *kind*

```
    val massless : massive → gauge_boson
    val massive : massive → gauge_boson

  end

module type Colorized =
  sig

    include T

    type flavor_sans_color
    val flavor_sans_color : flavor → flavor_sans_color
    val conjugate_sans_color : flavor_sans_color → flavor_sans_color

    val nc : unit → int
    val amplitude : flavor_sans_color list → flavor_sans_color list →
      (flavor list × flavor list) list
    val flow : flavor list → flavor list → Color.Flow.t

  end

module type Colorized_Gauge =
  sig

    include Gauge

    type flavor_sans_color
    val flavor_sans_color : flavor → flavor_sans_color
    val conjugate_sans_color : flavor_sans_color → flavor_sans_color

    val nc : unit → int
    val amplitude : flavor_sans_color list → flavor_sans_color list →
      (flavor list × flavor list) list
    val flow : flavor list → flavor list → Color.Flow.t

  end
```

⬙ *Interface* `vertex.mli` *unavailable!*

⬙ *Implementation* `vertex.ml` *unavailable!*

9.3 Interface of Target

```
module type T =
  sig
    type amplitudes

    val options : Options.t
    type diagnostic = All | Arguments | Momenta | Gauge
```

Format the amplitudes as a sequence of strings.

```
    val amplitudes_to_channel : string → out_channel →
      (diagnostic × bool) list → amplitudes → unit

    val parameters_to_channel : out_channel → unit
```

val *rcs_list* : *RCS.t list*
  end

module type *Maker* =
    functor (*F* : *Fusion.Maker*) →
      functor (*P* : *Momentum.T*) → functor (*M* : *Model.T*) →
        T with type *amplitudes* = *Fusion.Multi(F)(P)(M).amplitudes*

—10—
Conserved Quantum Numbers

10.1 Interface of Charges

10.1.1 Abstract Type

module type T =
  sig

The abstract type of the set of conserved charges or additive quantum numbers.

    type t

Add the quantum numbers of a pair or a list of particles.

    val add : $t \rightarrow t \rightarrow t$
    val sum : $t\ list \rightarrow t$

Test the charge conservation.

    val is_null : $t \rightarrow bool$

  end

10.1.2 Trivial Realisation

module $Null$: T with type t = $unit$

10.1.3 Nontrivial Realisations

Z

module Z : T with type t = int

$\mathbf{Z} \times \mathbf{Z} \times \cdots \times \mathbf{Z}$

module ZZ : T with type t = $int\ list$

Q

module Q : T with type t = $Algebra.Small\_Rational.t$

$$\mathbf{Q} \times \mathbf{Q} \times \cdots \times \mathbf{Q}$$

module QQ : T with type t = $Algebra.Small\_Rational.t\ list$

10.2   Implementation of Charges

module type T =
  sig
    type t
    val add : $t \to t \to t$
    val sum : $t\ list \to t$
    val is_null : $t \to bool$
  end

module $Null$: T with type t = $unit$ =
  struct
    type t = $unit$
    let add () () = ()
    let sum _ = ()
    let is_null _ = true
  end

module Z : T with type t = int =
  struct
    type t = int
    let add = (+)
    let sum = $List.fold\_left\ add$ 0
    let $is\_null\ n$ = (n = 0)
  end

module ZZ : T with type t = $int\ list$ =
  struct
    type t = $int\ list$
    let add = $List.map2$ (+)
    let sum = function
      | [] $\to$ []
      | [$charges$] $\to charges$
      | $charges$ :: $rest$ $\to List.fold\_left\ add\ charges\ rest$
    let is_null = $List.for\_all$ (fun $n \to n$ = 0)
  end

module Rat = $Algebra.Small\_Rational$

module Q : T with type t = $Rat.t$ =
  struct
    type t = $Rat.t$
    let add = $Rat.add$

```
      let sum  =  List.fold_left Rat.add Rat.null
      let is_null  =  Rat.is_null
    end

module QQ  :  T with type t  =  Rat.t list  =
    struct
      type t  =  Rat.t list
      let add  =  List.map2 Rat.add
      let sum  =  function
        | []  →  []
        | [charges]  →  charges
        | charges  ::  rest  →  List.fold_left add charges rest
      let is_null  =  List.for_all Rat.is_null
    end
```

—11—
Colorization

11.1 Interface of Colorize

11.1.1 . . .

module *It* (*M* : *Model.T*) :
  Model.Colorized with type *flavor_sans_color* = *M.flavor*
  and type *constant* = *M.constant*

module *Gauge* (*M* : *Model.Gauge*) :
  Model.Colorized_Gauge with type *flavor_sans_color* = *M.flavor*
  and type *constant* = *M.constant*

11.2 Implementation of Colorize

let *rcs_file* = *RCS.parse* "Colorize" ["Colorizing␣Monochrome␣Models"]
  { *RCS.revision* = "$Revision:␣7372␣$";
    RCS.date = "$Date:␣2015-11-18␣19:07:59␣+0100␣(Wed,␣18␣Nov␣2015)␣$";
    RCS.author = "$Author:␣jr_reuter␣$";
    RCS.source
      = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s

11.2.1 Colorizing a Monochrome Model

module *It* (*M* : *Model.T*) =
  struct

    let *rcs* = *RCS.rename rcs_file* "Colorize.It()"
      ["Colorizing␣Generic␣Monochrome␣Models"]

    open *Coupling*

    module *C* = *Color*

    let *incomplete s* =
      failwith ("Colorize.It()." ˆ *s* ˆ "␣not␣done␣yet!")

    let *invalid s* =
      invalid_arg ("Colorize.It()." ˆ *s* ˆ "␣must␣not␣be␣evaluated!")

```
let impossible s =
  invalid_arg ("Colorize.It()." ^ s ^ "␣can't␣happen!␣(but␣just␣did␣...)")

let su0 s =
  invalid_arg ("Colorize.It()." ^ s ^ ":␣found␣SU(0)!")

let colored_vertex s =
  invalid_arg ("Colorize.It()." ^ s ^ ":␣colored␣vertex!")

let baryonic_vertex s =
  invalid_arg ("Colorize.It()." ^ s ^
               ":␣baryonic␣(i.e.␣eps_ijk)␣vertices␣not␣supported␣yet!")

let color_flow_ambiguous s =
  invalid_arg ("Colorize.It()." ^ s ^ ":␣ambiguous␣color␣flow!")

let color_flow_of_string s =
  let c = int_of_string s in
  if c < 1 then
    invalid_arg ("Colorize.It()." ^ s ^ ":␣color␣flow␣#␣<␣1!")
  else
    c

type cf_in = int
type cf_out = int

type flavor =
  | White of M.flavor
  | CF_in of M.flavor × cf_in
  | CF_out of M.flavor × cf_out
  | CF_io of M.flavor × cf_in × cf_out
  | CF_aux of M.flavor

type flavor_sans_color = M.flavor

let flavor_sans_color = function
  | White f → f
  | CF_in (f, _) → f
  | CF_out (f, _) → f
  | CF_io (f, _, _) → f
  | CF_aux f → f

let pullback f arg1 =
  f (flavor_sans_color arg1)

type gauge = M.gauge
type constant = M.constant
let options = M.options

let color = pullback M.color
let pdg = pullback M.pdg
let lorentz = pullback M.lorentz

module Ch = M.Ch
let charges = pullback M.charges
```

For the propagator we cannot use pullback because we have to add the case of
the color singlet propagator by hand.

```
let cf_aux_propagator = function
  | Prop_Scalar → Prop_Col_Scalar (* Spin 0 octets. *)
  | Prop_Majorana → Prop_Col_Majorana (* Spin 1/2 octets. *)
  | Prop_Feynman → Prop_Col_Feynman (* Spin 1 states, massless. *)
  | Prop_Unitarity → Prop_Col_Unitarity (* Spin 1 states, massive. *)
  | Aux_Scalar → Aux_Col_Scalar (* constant colored scalar propagator
*)
  | Aux_Vector → Aux_Col_Vector (* constant colored vector propaga-
tor *)
  | Aux_Tensor_1 → Aux_Col_Tensor_1 (* constant colored tensor
propagator *)
  | Prop_Col_Scalar | Prop_Col_Feynman
  | Prop_Col_Majorana | Prop_Col_Unitarity
  | Aux_Col_Scalar | Aux_Col_Vector | Aux_Col_Tensor_1
    → failwith ("Colorize.It().colorize_propagator:␣already␣colored␣particle!")
  | _ → failwith ("Colorize.It().colorize_propagator:␣impossible!")

let propagator = function
  | CF_aux f → cf_aux_propagator (M.propagator f)
  | White f → M.propagator f
  | CF_in (f, _) → M.propagator f
  | CF_out (f, _) → M.propagator f
  | CF_io (f, _, _) → M.propagator f

let width = pullback M.width

let goldstone = function
  | White f →
      begin match M.goldstone f with
      | None → None
      | Some (f', g) → Some (White f', g)
      end
  | CF_in (f, c) →
      begin match M.goldstone f with
      | None → None
      | Some (f', g) → Some (CF_in (f', c), g)
      end
  | CF_out (f, c) →
      begin match M.goldstone f with
      | None → None
      | Some (f', g) → Some (CF_out (f', c), g)
      end
  | CF_io (f, c1, c2) →
      begin match M.goldstone f with
      | None → None
      | Some (f', g) → Some (CF_io (f', c1, c2), g)
      end
  | CF_aux f →
      begin match M.goldstone f with
      | None → None
      | Some (f', g) → Some (CF_aux f', g)
      end
```

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let *conjugate* = function
  | *White f* → *White* (*M.conjugate f*)
  | *CF_in* (*f, c*) → *CF_out* (*M.conjugate f, c*)
  | *CF_out* (*f, c*) → *CF_in* (*M.conjugate f, c*)
  | *CF_io* (*f, c1, c2*) → *CF_io* (*M.conjugate f, c2, c1*)
  | *CF_aux f* → *CF_aux* (*M.conjugate f*)

let *conjugate_sans_color* = *M.conjugate*

let *fermion* = *pullback M.fermion*

let *max_degree* = *M.max_degree*

let *flavors* () =
  invalid "flavors"

let *external_flavors* () =
  invalid "external_flavors"

let *parameters* = *M.parameters*

module *ISet* = *Set.Make* (struct type *t* = *int* let *compare* = *compare* end)

let *nc_value* =
  let *nc_set* =
    List.fold_left
      (fun *nc_set f* →
        match *M.color f* with
        | *C.Singlet* → *nc_set*
        | *C.SUN nc* → *ISet.add* (*abs nc*) *nc_set*
        | *C.AdjSUN nc* → *ISet.add* (*abs nc*) *nc_set*)
      ISet.empty (*M.flavors* ()) in
  match *ISet.elements nc_set* with
  | [] → 0
  | [*n*] → *n*
  | *nc_list* →
    invalid_arg
      ("Colorize.It():␣more␣than␣one␣value␣of␣N_C:␣" ^
       String.concat ",␣" (*List.map string_of_int nc_list*))

let *nc* () =
  nc_value

let *split_color_string s* =
  try
    let *i1* = *String.index s* '/' in
    let *i2* = *String.index_from s* (*succ i1*) '/' in
    let *sf* = *String.sub s* 0 *i1*
    and *sc1* = *String.sub s* (*succ i1*) (*i2* − *i1* − 1)
    and *sc2* = *String.sub s* (*succ i2*) (*String.length s* − *i2* − 1) in
    (*sf, sc1, sc2*)
  with
  | *Not_found* → (*s*, "", "")

let *flavor_of_string s* =
  try

```
      let sf, sc1, sc2 = split_color_string s in
      let f = M.flavor_of_string sf in
      match M.color f with
      | C.Singlet → White f
      | C.SUN nc →
           if nc > 0 then
             CF_in (f, color_flow_of_string sc1)
           else
             CF_out (f, color_flow_of_string sc2)
      | C.AdjSUN _ →
           begin match sc1, sc2 with
           | "", "" → CF_aux f
           | _, _ → CF_io (f, color_flow_of_string sc1, color_flow_of_string sc2)
           end
  with
  | Failure "int_of_string" →
       invalid_arg "Colorize().flavor_of_string:␣expecting␣integer"

let flavor_to_string = function
  | White f →
       M.flavor_to_string f
  | CF_in (f, c) →
       M.flavor_to_string f ^ "/" ^ string_of_int c ^ "/"
  | CF_out (f, c) →
       M.flavor_to_string f ^ "//" ^ string_of_int c
  | CF_io (f, c1, c2) →
       M.flavor_to_string f ^ "/" ^ string_of_int c1 ^ "/" ^ string_of_int c2
  | CF_aux f →
       M.flavor_to_string f ^ "//"

let flavor_to_TeX = function
  | White f →
       M.flavor_to_TeX f
  | CF_in (f, c) →
       "{" ^ M.flavor_to_TeX f ^ "}_{\\mathstrut␣" ^ string_of_int c ^ "}"
  | CF_out (f, c) →
       "{" ^ M.flavor_to_TeX f ^ "}_{\\mathstrut\\overline{" ^
       string_of_int c ^ "}}"
  | CF_io (f, c1, c2) →
       "{" ^ M.flavor_to_TeX f ^ "}_{\\mathstrut␣" ^
       string_of_int c1 ^ "\\overline{" ^ string_of_int c2 ^ "}}"
  | CF_aux f →
       "{" ^ M.flavor_to_TeX f ^ "}_{\\mathstrut␣0}"

let flavor_symbol = function
  | White f →
       M.flavor_symbol f
  | CF_in (f, c) →
       M.flavor_symbol f ^ "_" ^ string_of_int c ^ "_"
  | CF_out (f, c) →
       M.flavor_symbol f ^ "__" ^ string_of_int c
  | CF_io (f, c1, c2) →
```

$\qquad M.flavor\_symbol\ f$ ˆ "_" ˆ $string\_of\_int\ c1$ ˆ "_" ˆ $string\_of\_int\ c2$
$\quad |\ CF\_aux\ f\ \rightarrow$
$\qquad M.flavor\_symbol\ f$ ˆ "__"

let $gauge\_symbol\ =\ M.gauge\_symbol$

Masses and widths must not depend on the colors anyway!

let $mass\_symbol\ =\ pullback\ M.mass\_symbol$
let $width\_symbol\ =\ pullback\ M.width\_symbol$

let $constant\_symbol\ =\ M.constant\_symbol$

Vertices

Auxiliary functions

let $mult\_vertex3\ x\ =$ function
$\quad |\ FBF\ (c,\ fb,\ coup,\ f)\ \rightarrow$
$\qquad FBF\ ((x\ \times\ c),\ fb,\ coup,\ f)$
$\quad |\ PBP\ (c,\ fb,\ coup,\ f)\ \rightarrow$
$\qquad PBP\ ((x\ \times\ c),\ fb,\ coup,\ f)$
$\quad |\ BBB\ (c,\ fb,\ coup,\ f)\ \rightarrow$
$\qquad BBB\ ((x\ \times\ c),\ fb,\ coup,\ f)$
$\quad |\ GBG\ (c,\ fb,\ coup,\ f)\ \rightarrow$
$\qquad GBG\ ((x\ \times\ c),\ fb,\ coup,\ f)$
$\quad |\ Gauge\_Gauge\_Gauge\ c\ \rightarrow$
$\qquad Gauge\_Gauge\_Gauge\ (x\ \times\ c)$
$\quad |\ Aux\_Gauge\_Gauge\ c\ \rightarrow$
$\qquad Aux\_Gauge\_Gauge\ (x\ \times\ c)$
$\quad |\ Scalar\_Vector\_Vector\ c\ \rightarrow$
$\qquad Scalar\_Vector\_Vector\ (x\ \times\ c)$
$\quad |\ Aux\_Vector\_Vector\ c\ \rightarrow$
$\qquad Aux\_Vector\_Vector\ (x\ \times\ c)$
$\quad |\ Aux\_Scalar\_Vector\ c\ \rightarrow$
$\qquad Aux\_Scalar\_Vector\ (x\ \times\ c)$
$\quad |\ Scalar\_Scalar\_Scalar\ c\ \rightarrow$
$\qquad Scalar\_Scalar\_Scalar\ (x\ \times\ c)$
$\quad |\ Aux\_Scalar\_Scalar\ c\ \rightarrow$
$\qquad Aux\_Scalar\_Scalar\ (x\ \times\ c)$
$\quad |\ Vector\_Scalar\_Scalar\ c\ \rightarrow$
$\qquad Vector\_Scalar\_Scalar\ (x\ \times\ c)$
$\quad |\ Graviton\_Scalar\_Scalar\ c\ \rightarrow$
$\qquad Graviton\_Scalar\_Scalar\ (x\ \times\ c)$
$\quad |\ Graviton\_Vector\_Vector\ c\ \rightarrow$
$\qquad Graviton\_Vector\_Vector\ (x\ \times\ c)$
$\quad |\ Graviton\_Spinor\_Spinor\ c\ \rightarrow$
$\qquad Graviton\_Spinor\_Spinor\ (x\ \times\ c)$
$\quad |\ Dim4\_Vector\_Vector\_Vector\_T\ c\ \rightarrow$
$\qquad Dim4\_Vector\_Vector\_Vector\_T\ (x\ \times\ c)$

```
    | Dim4_Vector_Vector_Vector_L c →
        Dim4_Vector_Vector_Vector_L (x × c)
    | Dim4_Vector_Vector_Vector_T5 c →
        Dim4_Vector_Vector_Vector_T5 (x × c)
    | Dim4_Vector_Vector_Vector_L5 c →
        Dim4_Vector_Vector_Vector_L5 (x × c)
    | Dim6_Gauge_Gauge_Gauge c →
        Dim6_Gauge_Gauge_Gauge (x × c)
    | Dim6_Gauge_Gauge_Gauge_5 c →
        Dim6_Gauge_Gauge_Gauge_5 (x × c)
    | Aux_DScalar_DScalar c →
        Aux_DScalar_DScalar (x × c)
    | Aux_Vector_DScalar c →
        Aux_Vector_DScalar (x × c)
    | Dim5_Scalar_Gauge2 c →
        Dim5_Scalar_Gauge2 (x × c)
    | Dim5_Scalar_Gauge2_Skew c →
        Dim5_Scalar_Gauge2_Skew (x × c)
    | Dim5_Scalar_Vector_Vector_T c →
        Dim5_Scalar_Vector_Vector_T (x × c)
    | Dim5_Scalar_Vector_Vector_U c →
        Dim5_Scalar_Vector_Vector_U (x × c)
    | Dim5_Scalar_Vector_Vector_TU c →
        Dim5_Scalar_Vector_Vector_TU (x × c)
    | Dim5_Scalar_Scalar2 c →
        Dim5_Scalar_Scalar2 (x × c)
    | Scalar_Vector_Vector_t c →
        Scalar_Vector_Vector_t (x × c)
    | Dim6_Vector_Vector_Vector_T c →
        Dim6_Vector_Vector_Vector_T (x × c)
    | Tensor_2_Vector_Vector c →
        Tensor_2_Vector_Vector (x × c)
    | Tensor_2_Vector_Vector_cf c →
        Tensor_2_Vector_Vector_cf (x × c)
    | Tensor_2_Scalar_Scalar c →
        Tensor_2_Scalar_Scalar (x × c)
    | Tensor_2_Scalar_Scalar_cf c →
        Tensor_2_Scalar_Scalar_cf (x × c)
    | Tensor_2_Vector_Vector_1 c →
        Tensor_2_Vector_Vector_1 (x × c)
    | Tensor_2_Vector_Vector_t c →
        Tensor_2_Vector_Vector_t (x × c)
    | Dim5_Tensor_2_Vector_Vector_1 c →
        Dim5_Tensor_2_Vector_Vector_1 (x × c)
    | Dim5_Tensor_2_Vector_Vector_2 c →
        Dim5_Tensor_2_Vector_Vector_2 (x × c)
    | TensorVector_Vector_Vector c →
        TensorVector_Vector_Vector (x × c)
    | TensorVector_Vector_Vector_cf c →
        TensorVector_Vector_Vector_cf (x × c)
```

```
      | TensorVector_Scalar_Scalar c →
          TensorVector_Scalar_Scalar (x × c)
      | TensorVector_Scalar_Scalar_cf c →
          TensorVector_Scalar_Scalar_cf (x × c)
      | TensorScalar_Vector_Vector c →
          TensorScalar_Vector_Vector (x × c)
      | TensorScalar_Vector_Vector_cf c →
          TensorScalar_Vector_Vector_cf (x × c)
      | TensorScalar_Scalar_Scalar c →
          TensorScalar_Scalar_Scalar (x × c)
      | TensorScalar_Scalar_Scalar_cf c →
          TensorScalar_Scalar_Scalar_cf (x × c)
      | Dim7_Tensor_2_Vector_Vector_T c →
          Dim7_Tensor_2_Vector_Vector_T (x × c)
      | Dim6_Scalar_Vector_Vector_D c →
          Dim6_Scalar_Vector_Vector_D (x × c)
      | Dim6_Scalar_Vector_Vector_DP c →
          Dim6_Scalar_Vector_Vector_DP (x × c)
      | Dim6_HAZ_D c →
          Dim6_HAZ_D (x × c)
      | Dim6_HAZ_DP c →
          Dim6_HAZ_DP (x × c)
      | Gauge_Gauge_Gauge_i c →
          Gauge_Gauge_Gauge_i (x × c)
      | Dim6_GGG c →
          Dim6_GGG (x × c)
      | Dim6_AWW_DP c →
          Dim6_AWW_DP (x × c)
      | Dim6_AWW_DW c →
          Dim6_AWW_DW (x × c)
      | Dim6_Gauge_Gauge_Gauge_i c →
          Dim6_Gauge_Gauge_Gauge_i (x × c)
      | Dim6_HHH c →
          Dim6_HHH (x × c)
      | Dim6_WWZ_DPWDW c →
          Dim6_WWZ_DPWDW (x × c)
      | Dim6_WWZ_DW c →
          Dim6_WWZ_DW (x × c)
      | Dim6_WWZ_D c →
          Dim6_WWZ_D (x × c)

let mult_vertex4 x = function
  | Scalar4 c →
      Scalar4 (x × c)
  | Scalar2_Vector2 c →
      Scalar2_Vector2 (x × c)
  | Vector4 ic4_list →
      Vector4 (List.map (fun (c, icl) → (x × c, icl)) ic4_list)
  | DScalar4 ic4_list →
      DScalar4 (List.map (fun (c, icl) → (x × c, icl)) ic4_list)
```

| $DScalar2\_Vector2\ ic4\_list\ \rightarrow$
  $DScalar2\_Vector2\ (List.map\ (\mathsf{fun}\ (c,\ icl)\ \rightarrow\ (x\ \times\ c,\ icl))\ ic4\_list)$
| $GBBG\ (c,\ fb,\ b2,\ f)\ \rightarrow$
  $GBBG\ ((x\ \times\ c),\ fb,\ b2,\ f)$
| $Vector4\_K\_Matrix\_tho\ (c,\ ic4\_list)\ \rightarrow$
  $Vector4\_K\_Matrix\_tho\ ((x\ \times\ c),\ ic4\_list)$
| $Vector4\_K\_Matrix\_jr\ (c,\ ch2\_list)\ \rightarrow$
  $Vector4\_K\_Matrix\_jr\ ((x\ \times\ c),\ ch2\_list)$
| $DScalar2\_Vector2\_K\_Matrix\_ms\ (c,\ ch2\_list)\ \rightarrow$
  $DScalar2\_Vector2\_K\_Matrix\_ms\ ((x\ \times\ c),\ ch2\_list)$
| $DScalar4\_K\_Matrix\_ms\ (c,\ ch2\_list)\ \rightarrow$
  $DScalar4\_K\_Matrix\_ms\ ((x\ \times\ c),\ ch2\_list)$
| $Dim8\_Scalar2\_Vector2\_1\ c\ \rightarrow$
  $Dim8\_Scalar2\_Vector2\_1\ (x\ \times\ c)$
| $Dim8\_Scalar2\_Vector2\_2\ c\ \rightarrow$
  $Dim8\_Scalar2\_Vector2\_1\ (x\ \times\ c)$
| $Dim8\_Scalar4\ c\ \rightarrow$
  $Dim8\_Scalar4\ (x\ \times\ c)$
| $Dim6\_H4\_P2\ c\ \rightarrow$
  $Dim6\_H4\_P2\ (x\ \times\ c)$
| $Dim6\_AHWW\_DPB\ c\ \rightarrow$
  $Dim6\_AHWW\_DPB\ (x\ \times\ c)$
| $Dim6\_AHWW\_DPW\ c\ \rightarrow$
  $Dim6\_AHWW\_DPW\ (x\ \times\ c)$
| $Dim6\_AHWW\_DW\ c\ \rightarrow$
  $Dim6\_AHWW\_DW\ (x\ \times\ c)$
| $Dim6\_Vector4\_DW\ c\ \rightarrow$
  $Dim6\_Vector4\_DW\ (x\ \times\ c)$
| $Dim6\_Vector4\_W\ c\ \rightarrow$
  $Dim6\_Vector4\_W\ (x\ \times\ c)$
| $Dim6\_Scalar2\_Vector2\_PB\ c\ \rightarrow$
  $Dim6\_Scalar2\_Vector2\_PB\ (x\ \times\ c)$
| $Dim6\_Scalar2\_Vector2\_D\ c\ \rightarrow$
  $Dim6\_Scalar2\_Vector2\_D\ (x\ \times\ c)$
| $Dim6\_Scalar2\_Vector2\_DP\ c\ \rightarrow$
  $Dim6\_Scalar2\_Vector2\_DP\ (x\ \times\ c)$
| $Dim6\_HHZZ\_T\ c\ \rightarrow$
  $Dim6\_HHZZ\_T\ (x\ \times\ c)$
| $Dim6\_HWWZ\_DW\ c\ \rightarrow$
  $Dim6\_HWWZ\_DW\ (x\ \times\ c)$
| $Dim6\_HWWZ\_DPB\ c\ \rightarrow$
  $Dim6\_HWWZ\_DPB\ (x\ \times\ c)$
| $Dim6\_HWWZ\_DDPW\ c\ \rightarrow$
  $Dim6\_HWWZ\_DDPW\ (x\ \times\ c)$
| $Dim6\_HWWZ\_DPW\ c\ \rightarrow$
  $Dim6\_HWWZ\_DPW\ (x\ \times\ c)$
| $Dim6\_AHHZ\_D\ c\ \rightarrow$
  $Dim6\_AHHZ\_D\ (x\ \times\ c)$
| $Dim6\_AHHZ\_DP\ c\ \rightarrow$
  $Dim6\_AHHZ\_DP\ (x\ \times\ c)$

```
      |  Dim6_AHHZ_PB  c  →
            Dim6_AHHZ_PB  (x  ×  c)

    let mult_vertexn x  =  function
      |  foo  →  ignore (incomplete "mult_vertexn"); foo

    let mult_vertex x  =  function
      |  V3  (v,  fuse,  c)  →  V3  (mult_vertex3  x  v,  fuse,  c)
      |  V4  (v,  fuse,  c)  →  V4  (mult_vertex4  x  v,  fuse,  c)
      |  Vn  (v,  fuse,  c)  →  Vn  (mult_vertexn  x  v,  fuse,  c)
```

Below, we will need to permute Lorentz structures. The following permutes the three possible contractions of four vectors. We permute the first three indices, as they correspond to the particles entering the fusion.

```
    type permutation4  =
      |  P123  |  P231  |  P312
      |  P213  |  P321  |  P132

    let permute_contract4  =  function
      |  P123  →
            begin function
              |  C_12_34  →  C_12_34
              |  C_13_42  →  C_13_42
              |  C_14_23  →  C_14_23
            end
      |  P231  →
            begin function
              |  C_12_34  →  C_14_23
              |  C_13_42  →  C_12_34
              |  C_14_23  →  C_13_42
            end
      |  P312  →
            begin function
              |  C_12_34  →  C_13_42
              |  C_13_42  →  C_14_23
              |  C_14_23  →  C_12_34
            end
      |  P213  →
            begin function
              |  C_12_34  →  C_12_34
              |  C_13_42  →  C_14_23
              |  C_14_23  →  C_13_42
            end
      |  P321  →
            begin function
              |  C_12_34  →  C_14_23
              |  C_13_42  →  C_13_42
              |  C_14_23  →  C_12_34
            end
      |  P132  →
            begin function
              |  C_12_34  →  C_13_42
```

```
              |  C_13_42  →  C_12_34
              |  C_14_23  →  C_14_23
          end

let permute_contract4_list perm ic4_list =
    List.map (fun (i, c4) → (i, permute_contract4 perm c4)) ic4_list

let permute_vertex4' perm = function
    |  Scalar4 c  →
          Scalar4 c
    |  Vector4 ic4_list  →
          Vector4 (permute_contract4_list perm ic4_list)
    |  Vector4_K_Matrix_jr (c, ic4_list)  →
          Vector4_K_Matrix_jr (c, permute_contract4_list perm ic4_list)
    |  DScalar2_Vector2_K_Matrix_ms (c, ic4_list)  →
          DScalar2_Vector2_K_Matrix_ms (c, permute_contract4_list perm ic4_list)
    |  DScalar4_K_Matrix_ms (c, ic4_list)  →
          DScalar4_K_Matrix_ms (c, permute_contract4_list perm ic4_list)
    |  Scalar2_Vector2 c  →
          incomplete "permute_vertex4'␣Scalar2_Vector2"
    |  DScalar4 ic4_list  →
          incomplete "permute_vertex4'␣DScalar4"
    |  DScalar2_Vector2 ic4_list  →
          incomplete "permute_vertex4'␣DScalar2_Vector2"
    |  GBBG (c, fb, b2, f)  →
          incomplete "permute_vertex4'␣GBBG"
    |  Vector4_K_Matrix_tho (c, ch2_list)  →
          incomplete "permute_vertex4'␣Vector4_K_Matrix_tho"
    |  Dim8_Scalar2_Vector2_1 ic4_list  →
          incomplete "permute_vertex4'␣Dim8_Scalar2_Vector2_1"
    |  Dim8_Scalar2_Vector2_2 ic4_list  →
          incomplete "permute_vertex4'␣Dim8_Scalar2_Vector2_2"
    |  Dim8_Scalar4 ic4_list  →
          incomplete "permute_vertex4'␣Dim8_Scalar4"
    |  Dim6_H4_P2 ic4_list  →
          incomplete "permute_vertex4'␣Dim6_H4_P2"
    |  Dim6_AHWW_DPB ic4_list  →
          incomplete "permute_vertex4'␣Dim6_AHWW_DPB"
    |  Dim6_AHWW_DPW ic4_list  →
          incomplete "permute_vertex4'␣Dim6_AHWW_DPW"
    |  Dim6_AHWW_DW ic4_list  →
          incomplete "permute_vertex4'␣Dim6_AHWW_DW"
    |  Dim6_Vector4_DW ic4_list  →
          incomplete "permute_vertex4'␣Dim6_Vector4_DW"
    |  Dim6_Vector4_W ic4_list  →
          incomplete "permute_vertex4'␣Dim6_Vector4_W"
    |  Dim6_Scalar2_Vector2_D ic4_list  →
          incomplete "permute_vertex4'␣Dim6_Scalar2_Vector2_D"
    |  Dim6_Scalar2_Vector2_DP ic4_list  →
          incomplete "permute_vertex4'␣Dim6_Scalar2_Vector2_DP"
    |  Dim6_Scalar2_Vector2_PB ic4_list  →
```

```
            incomplete "permute_vertex4'␣Dim6_Scalar2_Vector2_PB"
      |  Dim6_HHZZ_T  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_HHZZ_T"
      |  Dim6_HWWZ_DW  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_HWWZ_DW"
      |  Dim6_HWWZ_DPB  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_HWWZ_DPB"
      |  Dim6_HWWZ_DDPW  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_HWWZ_DDPW"
      |  Dim6_HWWZ_DPW  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_HWWZ_DPW"
      |  Dim6_AHHZ_D  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_AHHZ_D"
      |  Dim6_AHHZ_DP  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_AHHZ_DP"
      |  Dim6_AHHZ_PB  ic4_list  →
            incomplete "permute_vertex4'␣Dim6_AHHZ_PB"

   let permute_vertex4 perm  = function
      |  V3 (v, fuse, c)  →  V3 (v, fuse, c)
      |  V4 (v, fuse, c)  →  V4 (permute_vertex4' perm v, fuse, c)
      |  Vn (v, fuse, c)  →  Vn (v, fuse, c)
```

vertices are *only* used by functor applications and for indexing a cache of pre-computed fusion rules, which is not used for colorized models.

```
   let vertices ()  =
      invalid "vertices"
```

Cubic Vertices

⚠ The following pattern matches could eventually become quite long. The O'Caml compiler will (hopefully) optimize them aggressively (http://pauillac.inria.fr/~maranget/papers/opat/).

```
   let colorize_fusion2 f1 f2 (f, v)  =
      match M.color f with

      |  C.Singlet  →
            begin match f1, f2 with

            |  White _, White _  →
               [White f, v]

            |  CF_in (_, c1), CF_out (_, c2')
            |  CF_out (_, c1), CF_in (_, c2')  →
               if c1  =  c2' then
                  [White f, v]
               else
                  []

            |  CF_io (f1, c1, c1'), CF_io (f2, c2, c2')  →
               if c1  =  c2'  ∧  c2  =  c1' then
```

```
          [White f, v]
        else
          []

  |  CF_aux f1,  CF_aux f2  →
       [White f,  mult_vertex (− (nc ())) v]

  |  CF_aux _,  CF_io _  |  CF_io _,  CF_aux _  →
       []

  |  (CF_in _  |  CF_out _  |  CF_io _  |  CF_aux _),  White _
  |  White _,  (CF_in _  |  CF_out _  |  CF_io _  |  CF_aux _)
  |  (CF_io _  |  CF_aux _),  (CF_in _  |  CF_out _)
  |  (CF_in _  |  CF_out _),  (CF_io _  |  CF_aux _)
  |  CF_in _,  CF_in _  |  CF_out _,  CF_out _  →
       colored_vertex "colorize_fusion2"

  end

|  C.SUN nc1  →
     begin match f1,  f2 with

  |  CF_in (_, c1),  (White _  |  CF_aux _)
  |  (White _  |  CF_aux _),  CF_in (_, c1)  →
       if nc1 > 0 then
         [CF_in (f, c1), v]
       else
         colored_vertex "colorize_fusion2"

  |  CF_out (_, c1′),  (White _  |  CF_aux _)
  |  (White _  |  CF_aux _),  CF_out (_, c1′)  →
       if nc1 < 0 then
         [CF_out (f, c1′), v]
       else
         colored_vertex "colorize_fusion2"

  |  CF_in (_, c1),  CF_io (_, c2, c2′)
  |  CF_io (_, c2, c2′),  CF_in (_, c1)  →
       if nc1 > 0 then begin
         if c1 = c2′ then
           [CF_in (f, c2), v]
         else
           []
       end else
         colored_vertex "colorize_fusion2"

  |  CF_out (_, c1′),  CF_io (_, c2, c2′)
  |  CF_io (_, c2, c2′),  CF_out (_, c1′)  →
       if nc1 < 0 then begin
         if c1′ = c2 then
           [CF_out (f, c2′), v]
         else
           []
       end else
         colored_vertex "colorize_fusion2"
```

```
            |  CF_in _,  CF_in _  →
                  if nc1  >  0 then
                      baryonic_vertex "colorize_fusion2"
                  else
                      colored_vertex "colorize_fusion2"

            |  CF_out _,  CF_out _  →
                  if nc1  <  0 then
                      baryonic_vertex "colorize_fusion2"
                  else
                      colored_vertex "colorize_fusion2"

            |  CF_in _,  CF_out _  |  CF_out _,  CF_in _
            |  (White _  |  CF_io _  |  CF_aux _),
                  (White _  |  CF_io _  |  CF_aux _)  →
                  colored_vertex "colorize_fusion2"

         end

   |  C.AdjSUN _  →
         begin match f1, f2 with

         |  White _,  CF_io (_, c1, c2′)  |  CF_io (_, c1, c2′),  White _  →
              [CF_io (f, c1, c2′), v]

         |  White _,  CF_aux _  |  CF_aux _,  White _  →
              [CF_aux f, mult_vertex (− (nc ())) v]

         |  CF_in (_, c1),  CF_out (_, c2′)
         |  CF_out (_, c2′),  CF_in (_, c1)  →
              if c1  ≠  c2′ then
                  [CF_io (f, c1, c2′), v]
              else
                  [CF_aux f, v]
```

In the adjoint representation

$$
= g f_{a_1 a_2 a_3} C^{\mu_1 \mu_2 \mu_3}(k_1, k_2, k_3) \tag{11.1a}
$$

with

$$
C^{\mu_1 \mu_2 \mu_3}(k_1, k_2, k_3) = \\
(g^{\mu_1 \mu_2}(k_1^{\mu_3} - k_2^{\mu_3}) + g^{\mu_2 \mu_3}(k_2^{\mu_1} - k_3^{\mu_1}) + g^{\mu_3 \mu_1}(k_3^{\mu_2} - k_1^{\mu_2})) \tag{11.1b}
$$

while in the color flow basis find from

$$
\mathrm{i} f_{a_1 a_2 a_3} = \operatorname{tr}\left(T_{a_1}\left[T_{a_2}, T_{a_3}\right]\right) = \operatorname{tr}\left(T_{a_1} T_{a_2} T_{a_3}\right) - \operatorname{tr}\left(T_{a_1} T_{a_3} T_{a_2}\right) \tag{11.2}
$$

the decomposition

$$
\mathrm{i} f_{a_1 a_2 a_3} T_{a_1}^{i_1 j_1} T_{a_2}^{i_2 j_2} T_{a_3}^{i_3 j_3} = \delta^{i_1 j_2} \delta^{i_2 j_3} \delta^{i_3 j_1} - \delta^{i_1 j_3} \delta^{i_3 j_2} \delta^{i_2 j_1}. \tag{11.3}
$$

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The resulting Feynman rule is
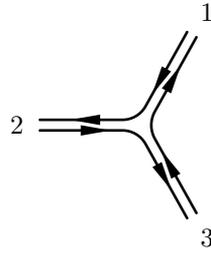
$$= \mathrm{i} g \left(\delta^{i_1 j_3} \delta^{i_2 j_1} \delta^{i_3 j_2} - \delta^{i_1 j_2} \delta^{i_2 j_3} \delta^{i_3 j_1} \right) C^{\mu_1 \mu_2 \mu_3}(k_1, k_2, k_3)$$

(11.4)

We have to generalize this for cases of three particles in the adjoint that are not all gluons (gluinos, scalar octets):

- scalar-scalar-scalar
- scalar-scalar-vector
- scalar-vector-vector
- scalar-fermion-fermion
- vector-fermion-fermion

We could use a better understanding of the signs for the gaugino-gaugino-gaugeboson couplings!!!

```
| CF_io (f1, c1, c1'), CF_io (f2, c2, c2') →
    let sign =
      begin match v with
      | V3 (Gauge_Gauge_Gauge _, _, _)
      | V3 (Aux_Gauge_Gauge _, _, _) → 1
      | V3 (FBF (_, _, _, _), fuse2, _) →
          begin match fuse2 with
          | F12 → 1 (* works, but needs theoretical underpinning
*)
          | F21 → −1 (* dto. *)
          | F31 → 1 (* dto. *)
          | F32 → −1 (* transposition of F12 (no testcase) *)
          | F23 → 1 (* transposition of F21 (no testcase) *)
          | F13 → −1 (* transposition of F12 (no testcase) *)
          end
      | V3 _ → incomplete "colorize_fusion2␣(V3␣_)"
      | V4 _ → impossible "colorize_fusion2␣(V4␣_)"
      | Vn _ → impossible "colorize_fusion2␣(Vn␣_)"
      end in
    if c1' = c2 then
      [CF_io (f, c1, c2'), mult_vertex (−sign) v]
    else if c2' = c1 then
      [CF_io (f, c2, c1'), mult_vertex ( sign) v]
    else
      []
| CF_aux _ , CF_io _
```

```
        |  CF_io _ ,  CF_aux _
        |  CF_aux _ ,  CF_aux _  →
              []

        |  White _,  White _
        |  (White _  |  CF_io _  |  CF_aux _),  (CF_in _  |  CF_out _)
        |  (CF_in _  |  CF_out _),  (White _  |  CF_io _  |  CF_aux _)
        |  CF_in _,  CF_in _  |  CF_out _,  CF_out _  →
              colored_vertex "colorize_fusion2"

        end
```

<div align="center">*Quartic Vertices*</div>

```
let colorize_fusion3 f1 f2 f3 (f,  v)  =
   match M.color f with

   |  C.Singlet  →
        begin match f1,  f2,  f3 with

        |  White _,  White _,  White _  →
             [White f,  v]

        |  (White _  |  CF_aux _),  CF_in (_, c1),  CF_out (_, c2′)
        |  (White _  |  CF_aux _),  CF_out (_, c1),  CF_in (_, c2′)
        |  CF_in (_, c1),  (White _  |  CF_aux _),  CF_out (_, c2′)
        |  CF_out (_, c1),  (White _  |  CF_aux _),  CF_in (_, c2′)
        |  CF_in (_, c1),  CF_out (_, c2′),  (White _  |  CF_aux _)
        |  CF_out (_, c1),  CF_in (_, c2′),  (White _  |  CF_aux _)  →
              if c1  =  c2′ then
                [White f,  v]
              else
                []

        |  White _,  CF_io (_, c1, c1′),  CF_io (_, c2, c2′)
        |  CF_io (_, c1, c1′),  White _,  CF_io (_, c2, c2′)
        |  CF_io (_, c1, c1′),  CF_io (_, c2, c2′),  White _  →
              if c1  =  c2′  ∧  c2  =  c1′ then
                [White f,  v]
              else
                []

        |  White _,  CF_aux _,  CF_aux _
        |  CF_aux _,  White _,  CF_aux _
        |  CF_aux _,  CF_aux _,  White _  →
              [White f,  mult_vertex (− (nc ())) v]

        |  White _,  CF_io _,  CF_aux _
        |  White _,  CF_aux _,  CF_io _
        |  CF_io _,  White _,  CF_aux _
        |  CF_aux _,  White _,  CF_io _
        |  CF_io _,  CF_aux _,  White _
        |  CF_aux _,  CF_io _,  White _  →
```

<div align="center">203</div>

[]

| CF_io (_, $c1$, $c1'$), CF_in (_, $c2$), CF_out (_, $c3'$)
| CF_io (_, $c1$, $c1'$), CF_out (_, $c3'$), CF_in (_, $c2$)
| CF_in (_, $c2$), CF_io (_, $c1$, $c1'$), CF_out (_, $c3'$)
| CF_out (_, $c3'$), CF_io (_, $c1$, $c1'$), CF_in (_, $c2$)
| CF_in (_, $c2$), CF_out (_, $c3'$), CF_io (_, $c1$, $c1'$)
| CF_out (_, $c3'$), CF_in (_, $c2$), CF_io (_, $c1$, $c1'$) $\rightarrow$
   if $c1 = c3' \wedge c1' = c2$ then
    [*White* f, v]
   else
    []

| CF_io (_, $c1$, $c1'$), CF_io (_, $c2$, $c2'$), CF_io (_, $c3$, $c3'$) $\rightarrow$
   if $c1' = c2 \wedge c2' = c3 \wedge c3' = c1$ then
    [*White* f, $mult\_vertex$ (-1) v]
   else if $c1' = c3 \wedge c2' = c1 \wedge c3' = c2$ then
    [*White* f, $mult\_vertex$ (1) v]
   else
    []

| CF_io _, CF_io _, CF_aux _
| CF_io _, CF_aux _, CF_io _
| CF_aux _, CF_io _, CF_io _
| CF_io _, CF_aux _, CF_aux _
| CF_aux _, CF_io _, CF_aux _
| CF_aux _, CF_aux _, CF_io _
| CF_aux _, CF_aux _, CF_aux _ $\rightarrow$
   []

| CF_in _, CF_in _, CF_in _
| CF_out _, CF_out _, CF_out _ $\rightarrow$
  $baryonic\_vertex$ "colorize_fusion3"

| CF_in _, CF_in _, CF_out _
| CF_in _, CF_out _, CF_in _
| CF_out _, CF_in _, CF_in _
| CF_in _, CF_out _, CF_out _
| CF_out _, CF_in _, CF_out _
| CF_out _, CF_out _, CF_in _

| *White* _, *White* _, (CF_io _ | CF_aux _)
| *White* _, (CF_io _ | CF_aux _), *White* _
| (CF_io _ | CF_aux _), *White* _, *White* _

| (*White* _ | CF_io _ | CF_aux _), CF_in _, CF_in _
| CF_in _, (*White* _ | CF_io _ | CF_aux _), CF_in _
| CF_in _, CF_in _, (*White* _ | CF_io _ | CF_aux _)

| (*White* _ | CF_io _ | CF_aux _), CF_out _, CF_out _
| CF_out _, (*White* _ | CF_io _ | CF_aux _), CF_out _
| CF_out _, CF_out _, (*White* _ | CF_io _ | CF_aux _)

| (CF_in _ | CF_out _),
  (*White* _ | CF_io _ | CF_aux _),

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```
                (White _  |  CF_io _  |  CF_aux _)
           |  (White _  |  CF_io _  |  CF_aux _),
                (CF_in _  |  CF_out _),
                (White _  |  CF_io _  |  CF_aux _)
           |  (White _  |  CF_io _  |  CF_aux _),
                (White _  |  CF_io _  |  CF_aux _),
                (CF_in _  |  CF_out _)  →
                    colored_vertex "colorize_fusion3"

           end

|  C.SUN nc1  →
     begin match f1, f2, f3 with

     |  CF_in (_, c1), CF_io (_, c2, c2'), CF_io (_, c3, c3')
     |  CF_io (_, c2, c2'), CF_in (_, c1), CF_io (_, c3, c3')
     |  CF_io (_, c2, c2'), CF_io (_, c3, c3'), CF_in (_, c1)  →
          if nc1 > 0 then
            if c1 = c2' ∧ c2 = c3' then
              [CF_in (f, c3), v]
            else if c1 = c3' ∧ c3 = c2' then
              [CF_in (f, c2), v]
            else
              []
          else
            colored_vertex "colorize_fusion3"

     |  CF_out (_, c1'), CF_io (_, c2, c2'), CF_io (_, c3, c3')
     |  CF_io (_, c2, c2'), CF_out (_, c1'), CF_io (_, c3, c3')
     |  CF_io (_, c2, c2'), CF_io (_, c3, c3'), CF_out (_, c1')  →
          if nc1 < 0 then
            if c1' = c2 ∧ c2' = c3 then
              [CF_out (f, c3'), v]
            else if c1' = c3 ∧ c3' = c2 then
              [CF_out (f, c2'), v]
            else
              []
          else
            colored_vertex "colorize_fusion3"

     |  CF_aux _, CF_in (_, c1), CF_io (_, c2, c2')
     |  CF_aux _, CF_io (_, c2, c2'), CF_in (_, c1)
     |  CF_in (_, c1), CF_aux _, CF_io (_, c2, c2')
     |  CF_io (_, c2, c2'), CF_aux _, CF_in (_, c1)
     |  CF_in (_, c1), CF_io (_, c2, c2'), CF_aux _
     |  CF_io (_, c2, c2'), CF_in (_, c1), CF_aux _  →
          if nc1 > 0 then
            if c1 = c2' then
              [CF_in (f, c2), mult_vertex ( 2) v]
            else
              []
          else
            colored_vertex "colorize_fusion3"
```

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```
| CF_aux _, CF_out (_, c1'), CF_io (_, c2, c2')
| CF_aux _, CF_io (_, c2, c2'), CF_out (_, c1')
| CF_out (_, c1'), CF_aux _, CF_io (_, c2, c2')
| CF_io (_, c2, c2'), CF_aux _, CF_out (_, c1')
| CF_out (_, c1'), CF_io (_, c2, c2'), CF_aux _
| CF_io (_, c2, c2'), CF_out (_, c1'), CF_aux _  →
      if nc1  <  0 then
        if c1'  =  c2 then
          [CF_out (f, c2'), mult_vertex ( 2) v]
        else
          []
      else
        colored_vertex "colorize_fusion3"

| White _, CF_in (_, c1), CF_io (_, c2, c2')
| White _, CF_io (_, c2, c2'), CF_in (_, c1)
| CF_in (_, c1), White _, CF_io (_, c2, c2')
| CF_io (_, c2, c2'), White _, CF_in (_, c1)
| CF_in (_, c1), CF_io (_, c2, c2'), White _
| CF_io (_, c2, c2'), CF_in (_, c1), White _  →
      if nc1  >  0 then
        if c1  =  c2' then
          [CF_in (f, c2), v]
        else
          []
      else
        colored_vertex "colorize_fusion3"

| White _, CF_out (_, c1'), CF_io (_, c2, c2')
| White _, CF_io (_, c2, c2'), CF_out (_, c1')
| CF_out (_, c1'), White _, CF_io (_, c2, c2')
| CF_io (_, c2, c2'), White _, CF_out (_, c1')
| CF_out (_, c1'), CF_io (_, c2, c2'), White _
| CF_io (_, c2, c2'), CF_out (_, c1'), White _  →
      if nc1  <  0 then
        if c2  =  c1' then
          [CF_out (f, c2'), v]
        else
          []
      else
        colored_vertex "colorize_fusion3"

| CF_in (_, c1), CF_aux _, CF_aux _
| CF_aux _, CF_in (_, c1), CF_aux _
| CF_aux _, CF_aux _, CF_in (_, c1)  →
      if nc1  >  0 then
        [CF_in (f, c1), mult_vertex ( 2) v]
      else
        colored_vertex "colorize_fusion3"

| CF_in (_, c1), CF_aux _, White _
| CF_in (_, c1), White _, CF_aux _
```

```
| CF_in (_, c1),  White _,  White _
| CF_aux _,  CF_in (_, c1),  White _
| White _,  CF_in (_, c1),  CF_aux _
| White _,  CF_in (_, c1),  White _
| CF_aux _,  White _,  CF_in (_, c1)
| White _,  CF_aux _,  CF_in (_, c1)
| White _,  White _,  CF_in (_, c1)  →
    if nc1  >  0 then
      [CF_in (f, c1), v]
    else
      colored_vertex "colorize_fusion3"

| CF_out (_, c1'),  CF_aux _,  CF_aux _
| CF_aux _,  CF_out (_, c1'),  CF_aux _
| CF_aux _,  CF_aux _,  CF_out (_, c1')  →
    if nc1  <  0 then
      [CF_out (f, c1'), mult_vertex ( 2) v]
    else
      colored_vertex "colorize_fusion3"

| CF_out (_, c1'),  CF_aux _,  White _
| CF_out (_, c1'),  White _,  CF_aux _
| CF_out (_, c1'),  White _,  White _
| CF_aux _,  CF_out (_, c1'),  White _
| White _,  CF_out (_, c1'),  CF_aux _
| White _,  CF_out (_, c1'),  White _
| CF_aux _,  White _,  CF_out (_, c1')
| White _,  CF_aux _,  CF_out (_, c1')
| White _,  White _,  CF_out (_, c1')  →
    if nc1  <  0 then
      [CF_out (f, c1'), v]
    else
      colored_vertex "colorize_fusion3"

| CF_in _,  CF_in _,  CF_out _
| CF_in _,  CF_out _,  CF_in _
| CF_out _,  CF_in _,  CF_in _  →
    if nc1  >  0 then
      color_flow_ambiguous "colorize_fusion3"
    else
      colored_vertex "colorize_fusion3"

| CF_in _,  CF_out _,  CF_out _
| CF_out _,  CF_in _,  CF_out _
| CF_out _,  CF_out _,  CF_in _  →
    if nc1  <  0 then
      color_flow_ambiguous "colorize_fusion3"
    else
      colored_vertex "colorize_fusion3"

| CF_in _,  CF_in _,  CF_in _
| CF_out _,  CF_out _,  CF_out _
```

```
        | (White _  |  CF_io _  |  CF_aux _),
          (White _  |  CF_io _  |  CF_aux _),
          (White _  |  CF_io _  |  CF_aux _)

        | (CF_in _  |  CF_out _),
            (CF_in _  |  CF_out _),
            (White _  |  CF_io _  |  CF_aux _)
        | (CF_in _  |  CF_out _),
            (White _  |  CF_io _  |  CF_aux _),
            (CF_in _  |  CF_out _)
        | (White _  |  CF_io _  |  CF_aux _),
            (CF_in _  |  CF_out _),
            (CF_in _  |  CF_out _)  →
            colored_vertex "colorize_fusion3"

        end

| C.AdjSUN nc  →
    begin match f1, f2, f3 with

    | CF_in (_, c1), CF_out (_, c1′), White _
    | CF_out (_, c1′), CF_in (_, c1), White _
    | CF_in (_, c1), White _, CF_out (_, c1′)
    | CF_out (_, c1′), White _, CF_in (_, c1)
    | White _, CF_in (_, c1), CF_out (_, c1′)
    | White _, CF_out (_, c1′), CF_in (_, c1)  →
        if c1 ≠ c1′ then
          [CF_io (f, c1, c1′), v]
        else
          [CF_aux f, v]

    | CF_in (_, c1), CF_out (_, c1′), CF_aux _
    | CF_out (_, c1′), CF_in (_, c1), CF_aux _
    | CF_in (_, c1), CF_aux _, CF_out (_, c1′)
    | CF_out (_, c1′), CF_aux _, CF_in (_, c1)
    | CF_aux _, CF_in (_, c1), CF_out (_, c1′)
    | CF_aux _, CF_out (_, c1′), CF_in (_, c1)  →
        if c1 ≠ c1′ then
          [CF_io (f, c1, c1′), mult_vertex ( 2) v]
        else
          [CF_aux f, mult_vertex ( 2) v]

    | CF_in (_, c1), CF_out (_, c1′), CF_io (_, c2, c2′)
    | CF_out (_, c1′), CF_in (_, c1), CF_io (_, c2, c2′)
    | CF_in (_, c1), CF_io (_, c2, c2′), CF_out (_, c1′)
    | CF_out (_, c1′), CF_io (_, c2, c2′), CF_in (_, c1)
    | CF_io (_, c2, c2′), CF_in (_, c1), CF_out (_, c1′)
    | CF_io (_, c2, c2′), CF_out (_, c1′), CF_in (_, c1)  →
        if c1 = c2′ ∧ c2 = c1′ then
          [CF_aux f, mult_vertex ( 2) v]
        else if c1 = c2′ then
          [CF_io (f, c2, c1′), v]
        else if c2 = c1′ then
```

$$[CF\_io\ (f,\ c1,\ c2'),\ v]$$
else
$$[\,]$$
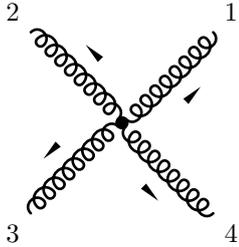
$$
\begin{aligned}
= \quad & -\mathrm{i}g^2 f_{a_1 a_2 b} f_{a_3 a_4 b}(g_{\mu_1 \mu_3} g_{\mu_4 \mu_2} - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3}) \\
& -\mathrm{i}g^2 f_{a_1 a_3 b} f_{a_4 a_2 b}(g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4}) \\
& -\mathrm{i}g^2 f_{a_1 a_4 b} f_{a_2 a_3 b}(g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} - g_{\mu_1 \mu_3} g_{\mu_4 \mu_2})
\end{aligned}
\tag{11.5}
$$

Using

$$
\mathcal{P}_4 = \{\{1,2,3,4\},\{1,3,4,2\},\{1,4,2,3\},\{1,2,4,3\},\{1,4,3,2\},\{1,3,2,4\}\}
\tag{11.6}
$$

as the set of permutations of $\{1,2,3,4\}$ with the cyclic permutations factored out, we have:
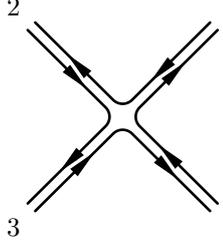
$$
= \quad \mathrm{i}g^2 \sum_{\{\alpha_k\}_{k=1,2,3,4} \in \mathcal{P}_4} \delta^{i_{\alpha_1} j_{\alpha_2}} \delta^{i_{\alpha_2} j_{\alpha_3}} \delta^{i_{\alpha_3} j_{\alpha_4}} \delta^{i_{\alpha_4} j_{\alpha_1}}
$$
$$
\left(2 g_{\mu_{\alpha_1} \mu_{\alpha_3}} g_{\mu_{\alpha_4} \mu_{\alpha_2}} - g_{\mu_{\alpha_1} \mu_{\alpha_4}} g_{\mu_{\alpha_2} \mu_{\alpha_3}} - g_{\mu_{\alpha_1} \mu_{\alpha_2}} g_{\mu_{\alpha_3} \mu_{\alpha_4}} \right)
\tag{11.7}
$$

The different color connections correspond to permutations of the particles entering the fusion and have to be matched by a corresponding permutation of the Lorentz structure:

We have to generalize this for cases of four particles in the adjoint that are not all gluons:

- scalar-scalar-scalar-scalar
- scalar-scalar-vector-vector

and even ones including fermions (gluinos) if higher dimensional operators are involved.

```
| CF_io (_, c1, c1'), CF_io (_, c2, c2'), CF_io (_, c3, c3') →
    if c1' = c2 ∧ c2' = c3 then
       [CF_io (f, c1, c3'), permute_vertex4 P123 v]
    else if c1' = c3 ∧ c3' = c2 then
       [CF_io (f, c1, c2'), permute_vertex4 P132 v]
    else if c2' = c3 ∧ c3' = c1 then
       [CF_io (f, c2, c1'), permute_vertex4 P231 v]
    else if c2' = c1 ∧ c1' = c3 then
       [CF_io (f, c2, c3'), permute_vertex4 P213 v]
    else if c3' = c1 ∧ c1' = c2 then
       [CF_io (f, c3, c2'), permute_vertex4 P312 v]
```

```
          else if c3′ = c2 ∧ c2′ = c1 then
            [CF_io (f, c3, c1′), permute_vertex4 P321 v]
          else
            []
  | CF_io _,  CF_io _,  CF_aux _
  | CF_io _,  CF_aux _,  CF_io _
  | CF_aux _, CF_io _,  CF_io _
  | CF_io _,  CF_aux _,  CF_aux _
  | CF_aux _, CF_aux _, CF_io _
  | CF_aux _, CF_io _,  CF_aux _
  | CF_aux _, CF_aux _, CF_aux _ →
      []

  | CF_io (_, c1, c1′), CF_io (_, c2, c2′), White _
  | CF_io (_, c1, c1′), White _, CF_io (_, c2, c2′)
  | White _, CF_io (_, c1, c1′), CF_io (_, c2, c2′) →
      if c1′ = c2 then
        [CF_io (f, c1, c2′), mult_vertex (−1) v]
      else if c2′ = c1 then
        [CF_io (f, c2, c1′), mult_vertex ( 1) v]
      else
        []

  | CF_io (_, c1, c1′), CF_aux _, White _
  | CF_aux _, CF_io (_, c1, c1′), White _
  | CF_io (_, c1, c1′), White _, CF_aux _
  | CF_aux _, White _, CF_io (_, c1, c1′)
  | White _, CF_io (_, c1, c1′), CF_aux _
  | White _, CF_aux _, CF_io (_, c1, c1′) →
      []

  | CF_aux _, CF_aux _, White _
  | CF_aux _, White _, CF_aux _
  | White _, CF_aux _, CF_aux _ →
      []

  | White _, White _, CF_io (_, c1, c1′)
  | White _, CF_io (_, c1, c1′), White _
  | CF_io (_, c1, c1′), White _, White _ →
      [CF_io (f, c1, c1′), v]

  | White _, White _, CF_aux _
  | White _, CF_aux _, White _
  | CF_aux _, White _, White _ →
      []

  | White _, White _, White _

  | (White _ | CF_io _ | CF_aux _),
      (White _ | CF_io _ | CF_aux _),
      (CF_in _ | CF_out _)
  | (White _ | CF_io _ | CF_aux _),
      (CF_in _ | CF_out _),
      (White _ | CF_io _ | CF_aux _)
```

```
      | (CF_in _ | CF_out _),
          (White _ | CF_io _ | CF_aux _),
          (White _ | CF_io _ | CF_aux _)

      | CF_in _, CF_in _, (White _ | CF_io _ | CF_aux _)
      | CF_in _, (White _ | CF_io _ | CF_aux _), CF_in _
      | (White _ | CF_io _ | CF_aux _), CF_in _, CF_in _

      | CF_out _, CF_out _, (White _ | CF_io _ | CF_aux _)
      | CF_out _, (White _ | CF_io _ | CF_aux _), CF_out _
      | (White _ | CF_io _ | CF_aux _), CF_out _, CF_out _

      | (CF_in _ | CF_out _),
          (CF_in _ | CF_out _),
          (CF_in _ | CF_out _) →
          colored_vertex "colorize_fusion3"

    end
```

Quintic and Higher Vertices

```
let is_white  = function
  | White _ → true
  | _ → false

let colorize_fusionn flist (f, v)  =
  let incomplete_match ()  =
    incomplete
      ("colorize_fusionn␣{␣" ^
       String.concat ",␣" (List.map flavor_to_string flist) ^
       "␣}␣->␣" ^ M.flavor_to_string f) in
  match M.color f with
  | C.Singlet →
      if List.for_all is_white flist then
        [White f, v]
      else
        incomplete_match ()
  | C.SUN _ →
      if List.for_all is_white flist then
        colored_vertex "colorize_fusionn"
      else
        incomplete_match ()
  | C.AdjSUN _ →
      if List.for_all is_white flist then
        colored_vertex "colorize_fusionn"
      else
        incomplete_match ()

let fuse2 f1 f2  =
  ThoList.flatmap
    (colorize_fusion2 f1 f2)
    (M.fuse2 (flavor_sans_color f1) (flavor_sans_color f2))
```

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```
let fuse3 f1 f2 f3 =
  ThoList.flatmap
    (colorize_fusion3 f1 f2 f3)
    (M.fuse3 (flavor_sans_color f1) (flavor_sans_color f2) (flavor_sans_color f3))

let fuse_list flist =
  ThoList.flatmap
    (colorize_fusionn flist)
    (M.fuse (List.map flavor_sans_color flist))

let fuse = function
  | [] | [_] → invalid_arg "Colorize.It().fuse"
  | [f1; f2] → fuse2 f1 f2
  | [f1; f2; f3] → fuse3 f1 f2 f3
  | flist → fuse_list flist

let max_degree = M.max_degree
```

Adding Color to External Particles

```
let count_color_strings f_list =
  let rec count_color_strings' n_in n_out n_glue = function
    | f :: rest →
        begin match M.color f with
        | C.Singlet → count_color_strings' n_in n_out n_glue rest
        | C.SUN nc →
            if nc > 0 then
              count_color_strings' (succ n_in) n_out n_glue rest
            else if nc < 0 then
              count_color_strings' n_in (succ n_out) n_glue rest
            else
              su0 "count_color_strings"
        | C.AdjSUN _ →
            count_color_strings' (succ n_in) (succ n_out) (succ n_glue) rest
        end
    | [] → (n_in, n_out, n_glue)
  in
  count_color_strings' 0 0 0 f_list

let external_color_flows f_list =
  let n_in, n_out, n_glue = count_color_strings f_list in
  if n_in ≠ n_out then
    []
  else
    let color_strings = ThoList.range 1 n_in in
    List.rev_map
      (fun permutation → (color_strings, permutation))
      (Combinatorics.permute color_strings)
```

If there are only adjoints *and* there are no couplings of adjoints to singlets, we
can ignore the U(1)-ghosts.

```
let pure_adjoints f_list =
```

```
      List.for_all (fun f  →  match M.color f with C.AdjSUN _  →  true |
_  →  false) f_list

    let two_adjoints_couple_to_singlets () =
      let vertices3, vertices4, verticesn = M.vertices () in
      List.exists (fun ((f1, f2, f3), _, _) →
        match M.color f1, M.color f2, M.color f3 with
        | C.AdjSUN _, C.AdjSUN _, C.Singlet
        | C.AdjSUN _, C.Singlet, C.AdjSUN _
        | C.Singlet, C.AdjSUN _, C.AdjSUN _ → true
        | _ → false) vertices3 ∨
      List.exists (fun ((f1, f2, f3, f4), _, _) →
        match M.color f1, M.color f2, M.color f3, M.color f4 with
        | C.AdjSUN _, C.AdjSUN _, C.Singlet, C.Singlet
        | C.AdjSUN _, C.Singlet, C.AdjSUN _, C.Singlet
        | C.Singlet, C.AdjSUN _, C.AdjSUN _, C.Singlet
        | C.AdjSUN _, C.Singlet, C.Singlet, C.AdjSUN _
        | C.Singlet, C.AdjSUN _, C.Singlet, C.AdjSUN _
        | C.Singlet, C.Singlet, C.AdjSUN _, C.AdjSUN _ → true
        | _ → false) vertices4 ∨
      List.exists (fun (flist, _, g) → true) verticesn

    let external_ghosts f_list =
      if pure_adjoints f_list then
        two_adjoints_couple_to_singlets ()
      else
        true
```

We use *List.hd* and *List.tl* instead of pattern matching, because we consume
ecf_in and *ecf_out* at a different pace.

```
    let tail_opt = function
      | [] → []
      | _ :: tail → tail

    let head_req = function
      | [] →
          invalid_arg "Colorize.It().colorize_crossed_amplitude1:␣insufficient␣flows"
      | x :: _ → x

    let rec colorize_crossed_amplitude1 ghosts acc f_list (ecf_in, ecf_out) =
      match f_list, ecf_in, ecf_out with
      | [], [], [] → [List.rev acc]
      | [], _, _ →
          invalid_arg "Colorize.It().colorize_crossed_amplitude1:␣leftover␣flows"
      | f :: rest, _, _ →
          begin match M.color f with
          | C.Singlet →
              colorize_crossed_amplitude1 ghosts
                (White f :: acc)
                rest (ecf_in, ecf_out)
          | C.SUN nc →
              if nc > 0 then
```

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```
                    colorize_crossed_amplitude1 ghosts
                      (CF_in (f, head_req ecf_in) :: acc)
                      rest (tail_opt ecf_in, ecf_out)
                  else if nc < 0 then
                    colorize_crossed_amplitude1 ghosts
                      (CF_out (f, head_req ecf_out) :: acc)
                      rest (ecf_in, tail_opt ecf_out)
                  else
                    su0 "colorize_flavor"
              | C.AdjSUN _ →
                  let ecf_in' = head_req ecf_in
                  and ecf_out' = head_req ecf_out in
                  if ecf_in' = ecf_out' then begin
                    if ghosts then
                      colorize_crossed_amplitude1 ghosts
                        (CF_aux f :: acc)
                        rest (tail_opt ecf_in, tail_opt ecf_out)
                    else
                      []
                  end else
                    colorize_crossed_amplitude1 ghosts
                      (CF_io (f, ecf_in', ecf_out') :: acc)
                      rest (tail_opt ecf_in, tail_opt ecf_out)
              end

let colorize_crossed_amplitude1 ghosts f_list (ecf_in, ecf_out) =
  colorize_crossed_amplitude1 ghosts [] f_list (ecf_in, ecf_out)

let colorize_crossed_amplitude f_list =
  ThoList.rev_flatmap
    (colorize_crossed_amplitude1 (external_ghosts f_list) f_list)
    (external_color_flows f_list)

let cross_uncolored p_in p_out =
  (List.map M.conjugate p_in) @ p_out

let uncross_colored n_in p_lists_colorized =
  let p_in_out_colorized = List.map (ThoList.splitn n_in) p_lists_colorized in
  List.map
    (fun (p_in_colored, p_out_colored) →
      (List.map conjugate p_in_colored, p_out_colored))
    p_in_out_colorized

let amplitude p_in p_out =
  uncross_colored
    (List.length p_in)
    (colorize_crossed_amplitude (cross_uncolored p_in p_out))
```

The −-sign in the second component is redundant, but a Whizard convention.

```
let indices = function
  | White _ → Color.Flow.of_list [0; 0]
  | CF_in (_, c) → Color.Flow.of_list [c; 0]
  | CF_out (_, c) → Color.Flow.of_list [0; − c]
```

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```
        |  CF_io (_, c1, c2)  →  Color.Flow.of_list [c1; − c2]
        |  CF_aux f  →  Color.Flow.ghost ()

    let flow p_in p_out  =
        (List.map indices p_in, List.map indices p_out)

  end
```

11.2.2   Colorizing a Monochrome Gauge Model

```
module Gauge (M : Model.Gauge)  =
  struct

      let rcs  =  RCS.rename rcs_file "Colorize.Gauge()"
          [ "Colorizing␣Monochrome␣Gauge␣Models"]

      module CM  =  It(M)

      type flavor  =  CM.flavor
      type flavor_sans_color  =  CM.flavor_sans_color
      type gauge  =  CM.gauge
      type constant  =  CM.constant
      module Ch  =  CM.Ch
      let charges  =  CM.charges
      let flavor_sans_color  =  CM.flavor_sans_color
      let color  =  CM.color
      let pdg  =  CM.pdg
      let lorentz  =  CM.lorentz
      let propagator  =  CM.propagator
      let width  =  CM.width
      let conjugate  =  CM.conjugate
      let conjugate_sans_color  =  CM.conjugate_sans_color
      let fermion  =  CM.fermion
      let max_degree  =  CM.max_degree
      let vertices  =  CM.vertices
      let fuse2  =  CM.fuse2
      let fuse3  =  CM.fuse3
      let fuse  =  CM.fuse
      let flavors  =  CM.flavors
      let nc  =  CM.nc
      let external_flavors  =  CM.external_flavors
      let goldstone  =  CM.goldstone
      let parameters  =  CM.parameters
      let flavor_of_string  =  CM.flavor_of_string
      let flavor_to_string  =  CM.flavor_to_string
      let flavor_to_TeX  =  CM.flavor_to_TeX
      let flavor_symbol  =  CM.flavor_symbol
      let gauge_symbol  =  CM.gauge_symbol
      let mass_symbol  =  CM.mass_symbol
      let width_symbol  =  CM.width_symbol
      let constant_symbol  =  CM.constant_symbol
      let options  =  CM.options
```

```
  let incomplete s =
    failwith ("Colorize.Gauge()." ^ s ^ "␣not␣done␣yet!")

type matter_field  =  M.matter_field
type gauge_boson  =  M.gauge_boson
type other  =  M.other

type field  =
  | Matter of matter_field
  | Gauge of gauge_boson
  | Other of other

let field f  =
  incomplete "field"

let matter_field f  =
  incomplete "matter_field"

let gauge_boson f  =
  incomplete "gauge_boson"

let other f  =
  incomplete "other"

let amplitude  =  CM.amplitude

let flow  =  CM.flow

end
```

—12—
Processes

12.1 Interface of Process

```
module type T =
  sig
    type flavor
```

Eventually this should become an abstract type:

```
    type t  =  flavor list × flavor list

    val incoming  :  t  →  flavor list
    val outgoing  :  t  →  flavor list
```

parse_decay s decodes a decay description `"a␣->␣b␣c␣..."`, where each word is split into a bag of flavors separated by `':'`s.

```
    type decay
    val parse_decay  :  string →  decay
    val expand_decays  :  decay list →  t list
```

parse_scattering s decodes a scattering description `"a␣b␣->␣c␣d␣..."`, where each word is split into a bag of flavors separated by `':'`s.

```
    type scattering
    val parse_scattering  :  string →  scattering
    val expand_scatterings  :  scattering list →  t list
```

parse_process s decodes process descriptions

$$\texttt{"a b c d"} \Rightarrow Any\ [a;\ b;\ c;\ d] \tag{12.1a}$$

$$\texttt{"a -> b c d"} \Rightarrow Decay\ (a,\ [b;\ c;\ d]) \tag{12.1b}$$

$$\texttt{"a b -> c d"} \Rightarrow Scattering\ (a,\ b,\ [c;\ d]) \tag{12.1c}$$

where each word is split into a bag of flavors separated by `':'`s.

```
    type any
    type process  =  Any of any  |  Decay of decay  |  Scattering of scattering
    val parse_process  :  string →  process
```

remove_duplicate_final_states partition processes removes duplicates from *processes*, which differ only by a permutation of final state particles. The permutation must respect the partitioning given by the offset 1 integers in *partition*.

val *remove_duplicate_final_states* : *int list list* → *t list* → *t list*

diff set1 set2 returns the processes in *set1* with the processes in *set2* removed. *set2* does not need to be a subset of *set1*.

val *diff* : *t list* → *t list* → *t list*

Not functional yet. Interface subject to change. Should be moved to *Fusion.Multi*, because we will want to cross *colored* matrix elements.

Factor amplitudes that are related by crossing symmetry.

val *crossing* : *t list* → (*flavor list* × *int list* × *t*) *list*

end

module *Make* (*M* : *Model.T*) : *T* with type *flavor* = *M.flavor*

12.2 *Implementation of Process*

module type *T* =
  sig
    type *flavor*
    type *t* = *flavor list* × *flavor list*
    val *incoming* : *t* → *flavor list*
    val *outgoing* : *t* → *flavor list*
    type *decay*
    val *parse_decay* : *string* → *decay*
    val *expand_decays* : *decay list* → *t list*
    type *scattering*
    val *parse_scattering* : *string* → *scattering*
    val *expand_scatterings* : *scattering list* → *t list*
    type *any*
    type *process* = *Any* of *any* | *Decay* of *decay* | *Scattering* of *scattering*
    val *parse_process* : *string* → *process*
    val *remove_duplicate_final_states* : *int list list* → *t list* → *t list*
    val *diff* : *t list* → *t list* → *t list*
    val *crossing* : *t list* → (*flavor list* × *int list* × *t*) *list*
  end

module *Make* (*M* : *Model.T*) =
  struct

    type *flavor* = *M.flavor*

    type *t* = *flavor list* × *flavor list*

    let *incoming* (*fin*, _) = *fin*
    let *outgoing* (_, *fout*) = *fout*

12.2.1 Select Charge Conserving Processes

let *allowed* (*fin*, *fout*) =
  M.Ch.is_null (*M.Ch.sum* (*List.map M.charges* (*List.map M.conjugate fin* @ *fout*)))

12.2.2 Parsing Process Descriptions

type α *bag* = α *list*

type *any* = *flavor bag list*
type *decay* = *flavor bag* $\times$ *flavor bag list*
type *scattering* = *flavor bag* $\times$ *flavor bag* $\times$ *flavor bag list*

type *process* =
  | *Any* of *any*
  | *Decay* of *decay*
  | *Scattering* of *scattering*

let *unique_flavors f_bags* =
  List.for_all (function [*f*] $\rightarrow$ true | _ $\rightarrow$ false) *f_bags*

let *unique_final_state* = function
  | *Any fs* $\rightarrow$ *unique_flavors fs*
  | *Decay* (_, *fs*) $\rightarrow$ *unique_flavors fs*
  | *Scattering* (_, _, *fs*) $\rightarrow$ *unique_flavors fs*

let *parse_process process* =
  let *last* = *String.length process* $-$ 1
  and *flavor off len* = *M.flavor_of_string* (*String.sub process off len*) in

  let *add_flavors flavors* = function
    | *Any l* $\rightarrow$ *Any* (*List.rev flavors* :: *l*)
    | *Decay* (*i*, *f*) $\rightarrow$ *Decay* (*i*, *List.rev flavors* :: *f*)
    | *Scattering* (*i1*, *i2*, *f*) $\rightarrow$ *Scattering* (*i1*, *i2*, *List.rev flavors* :: *f*) in

  let rec *scan_list so_far n* =
    if *n* $>$ *last* then
      so_far
    else
      let *n'* = *succ n* in
      match *process*.[*n*] with
      | ' ' | '\n' $\rightarrow$ *scan_list so_far n'*
      | '-' $\rightarrow$ *scan_gtr so_far n'*
      | *c* $\rightarrow$ *scan_flavors so_far* [] *n n'*

  and *scan_flavors so_far flavors w n* =
    if *n* $>$ *last* then
      add_flavors (*flavor w* (*last* $-$ *w* + 1) :: *flavors*) *so_far*
    else
      let *n'* = *succ n* in
      match *process*.[*n*] with
      | ' ' | '\n' $\rightarrow$
          scan_list (*add_flavors* (*flavor w* (*n* $-$ *w*) :: *flavors*) *so_far*) *n'*

```
      | ':' → scan_flavors so_far (flavor w (n − w) :: flavors) n′ n′
      | _ → scan_flavors so_far flavors w n′

  and scan_gtr so_far n =
    if n > last then
      invalid_arg "expecting␣'>'"
    else
      let n′ = succ n in
      match process.[n] with
      | '>' →
          begin match so_far with
          | Any [i] → scan_list (Decay (i, [])) n′
          | Any [i2; i1] → scan_list (Scattering (i1, i2, [])) n′
          | Any _ → invalid_arg "only␣1␣or␣2␣particles␣in␣|in>"
          | _ → invalid_arg "too␣many␣'->'s"
          end
      | _ → invalid_arg "expecting␣'>'" in

  match scan_list (Any []) 0 with
  | Any l → Any (List.rev l)
  | Decay (i, f) → Decay (i, List.rev f)
  | Scattering (i1, i2, f) → Scattering (i1, i2, List.rev f)

let parse_decay process =
  match parse_process process with
  | Any (i :: f) →
      prerr_endline "missing␣'->'␣in␣process␣description,␣assuming␣decay.";
      (i, f)
  | Decay (i, f) → (i, f)
  | _ → invalid_arg "expecting␣decay␣description:␣got␣scattering"

let parse_scattering process =
  match parse_process process with
  | Any (i1 :: i2 :: f) →
      prerr_endline "missing␣'->'␣in␣process␣description,␣assuming␣scattering.";
      (i1, i2, f)
  | Scattering (i1, i2, f) → (i1, i2, f)
  | _ → invalid_arg "expecting␣scattering␣description:␣got␣decay"

let expand_scatterings scatterings =
  ThoList.flatmap
    (function (fin1, fin2, fout) →
      Product.fold
        (fun flist acc →
          match flist with
          | fin1′ :: fin2′ :: fout′ →
              let fin_fout′ = ([fin1′; fin2′], fout′) in
              if allowed fin_fout′ then
                fin_fout′ :: acc
              else
                acc
          | [_] | [] → failwith "Omega.expand_scatterings:␣can't␣happen")
        (fin1 :: fin2 :: fout) []) scatterings
```

```
let expand_decays decays =
  ThoList.flatmap
    (function (fin, fout) →
      Product.fold
        (fun flist acc →
          match flist with
          | fin' :: fout' →
              let fin_fout' = ([fin'], fout') in
              if allowed fin_fout' then
                fin_fout' :: acc
              else
                acc
          | [] → failwith "Omega.expand_decays:␣can't␣happen")
        (fin :: fout) []) decays
```

12.2.3   Remove Duplicate Final States

Test if all final states are the same. Identical to *ThoList.homogeneous* ∘ (*List.map snd*).

```
let rec homogeneous_final_state = function
  | [] | [_] → true
  | (_, fs1) :: ((_, fs2) :: _ as rest) →
      if fs1 ≠ fs2 then
        false
      else
        homogeneous_final_state rest

let by_color f1 f2 =
  let c = Color.compare (M.color f1) (M.color f2) in
  if c ≠ 0 then
    c
  else
    compare f1 f2

module Pre_Bundle =
  struct

    type elt = t
    type base = elt

    let compare_elt (fin1, fout1) (fin2, fout2) =
      let c = ThoList.compare ˜cmp : by_color fin1 fin2 in
      if c ≠ 0 then
        c
      else
        ThoList.compare ˜cmp : by_color fout1 fout2

    let compare_base b1 b2 = compare_elt b2 b1

  end

module Process_Bundle = Bundle.Dyn (Pre_Bundle)

let to_string (fin, fout) =
```

```
    String.concat "␣" (List.map M.flavor_to_string fin)
    ˆ "␣->␣" ˆ String.concat "␣" (List.map M.flavor_to_string fout)

  let fiber_to_string (base, fiber) =
    (to_string base) ˆ "␣->␣[" ˆ
    (String.concat ",␣" (List.map to_string fiber)) ˆ "]"

  let bundle_to_strings list =
    List.map fiber_to_string list
```

Subtract $n + 1$ from each element in *index_set* and drop all negative numbers from the result.

```
  let shift_left_pred' n index_set =
    List.fold_right
      (fun i acc → let i' = i − n − 1 in if i' < 0 then acc else i' :: acc)
      index_set []
```

Convert 1-based indices for initial and final state to 0-based indices for the final state only. (NB: *ThoList.partitioned_sort* expects 0-based indices.)

```
  let shift_left_pred fin index_sets =
    let n = match fin with [_] → 1 | [_; _] → 2 | _ → 0 in
    List.fold_right
      (fun iset acc →
        match shift_left_pred' n iset with
        | [] → acc
        | iset' → iset' :: acc)
      index_sets []

  module FSet = Set.Make (struct type t = flavor let compare = compare end)
```

Take a list of final states and return a list of sets of flavors appearing in each slot.

```
  let flavors = function
    | [] → []
    | fs :: fs_list →
        List.fold_right (List.map2 FSet.add) fs_list (List.map FSet.singleton fs)

  let flavor_sums flavor_sets =
    let _, result =
      List.fold_left
        (fun (n, acc) flavors →
          if FSet.cardinal flavors = 1 then
            (succ n, acc)
          else
            (succ n, (n, flavors) :: acc))
        (0, []) flavor_sets in
    List.rev result

  let overlapping s1 s2 =
    ¬ (FSet.is_empty (FSet.inter s1 s2))

  let rec merge_overlapping (n, flavors) = function
    | [] → [([n], flavors)]
```

```
      | (n_list, flavor_set) :: rest →
          if overlapping flavors flavor_set then
            (n :: n_list, FSet.union flavors flavor_set) :: rest
          else
            (n_list, flavor_set) :: merge_overlapping (n, flavors) rest
```

let *overlapping_flavor_sums flavor_sums* =
  List.rev_map
    (fun (*n_list, flavor_set*) → (*n_list, FSet.elements flavor_set*))
    (*List.fold_right merge_overlapping flavor_sums* [])

module *ISet* = *Set.Make* (struct type *t* = *int* let *compare* = *compare* end)

let *integer_range n1 n2* =
  let rec *integer_range′ acc n′* =
    if *n′* < *n1* then
      acc
    else
      integer_range′ (*ISet.add n′ acc*) (*pred n′*) in
  integer_range′ ISet.empty n2

let *coarsest_partition* = function
  | [] → *invalid_arg* "coarsest_partition:␣empty␣process␣list"
  | ((_, *fs*) :: _) as *proc_list* →
      let *fs_list* = *List.map snd proc_list* in
      let *overlaps* =
        List.map fst (*overlapping_flavor_sums* (*flavor_sums* (*flavors fs_list*))) in
      let *singletons* =
        ISet.elements
          (*List.fold_right ISet.remove*
            (*List.concat overlaps*) (*integer_range* 0 (*pred* (*List.length fs*)))) in
      List.map (fun *n* → [*n*]) *singletons* @ *overlaps*

module *IPowSet* =
  PowSet.Make (struct type *t* = *int* let *compare* = *compare* let *to_string* = *string_of_int* end)

let *merge_partitions p_list* =
  IPowSet.to_lists (*IPowSet.basis* (*IPowSet.union* (*List.map IPowSet.of_lists p_list*)))

let *remove_duplicate_final_states cascade_partition* = function
  | [] → []
  | [*process*] → [*process*]
  | *list* →
      if *homogeneous_final_state list* then
        list
      else
        let *partition* = *coarsest_partition list* in
        let *pi* (*fin, fout*) =
          let *partition′* =
            merge_partitions [*partition; shift_left_pred fin cascade_partition*] in
          (*fin, ThoList.partitioned_sort by_color partition′ fout*) in
        Process_Bundle.base (*Process_Bundle.of_list pi list*)

type *t′* = *t*

module *PSet* = *Set.Make* (struct type t = t' let *compare* = *compare* end)

let *set list* =
  List.fold_right PSet.add list PSet.empty

let *diff list1 list2* =
  PSet.elements (*PSet.diff* (*set list1*) (*set list2*))

◈ Not functional yet.

module *Crossing_Projection* =
  struct

    type *elt* = t
    type *base* = *flavor list* × *int list* × t

    let *compare_elt* (*fin1*, *fout1*) (*fin2*, *fout2*) =
      let c = *ThoList.compare* ~*cmp* : *by_color fin1 fin2* in
      if c ≠ 0 then
        c
      else
        ThoList.compare ~*cmp* : *by_color fout1 fout2*

    let *compare_base* (*f1*, _, _) (*f2*, _, _) =
      ThoList.compare ~*cmp* : *by_color f1 f2*

    let *pi* (*fin*, *fout* as *process*) =
      let *flist*, *indices* =
        ThoList.ariadne_sort ~*cmp* : *by_color* (*List.map M.conjugate fin* @ *fout*) in
      (*flist*, *indices*, *process*)

    end

module *Crossing_Bundle* = *Bundle.Make* (*Crossing_Projection*)

let *crossing processes* =
  List.map
    (fun (*fin*, *fout* as *process*) →
      (*List.map M.conjugate fin* @ *fout*, [], *process*))
    processes

end

—13—
Hardcoded Models

13.1   Interface of Modeltools

13.1.1   Compilation

```
module type Flavor  =
  sig
    type f
    type c
    val compare  :  f  →  f  →  int
    val conjugate  :  f  →  f
  end
module type Fusions  =
  sig
    type t
    type f
    type c
    val fuse2  :  t  →  f  →  f  →  (f  ×  c Coupling.t) list
    val fuse3  :  t  →  f  →  f  →  f  →  (f  ×  c Coupling.t) list
    val fuse  :  t  →  f list →  (f  ×  c Coupling.t) list
    val of_vertices  :
        (((f  ×  f  ×  f)  ×  c Coupling.vertex3  ×  c) list
            ×  ((f  ×  f  ×  f  ×  f)  ×  c Coupling.vertex4  ×  c) list
            ×  (f  list ×  c Coupling.vertexn  ×  c) list)  →  t
  end
module Fusions  :  functor (F  :  Flavor)  →
  Fusions with type f  =  F.f and type c  =  F.c
```

13.1.2   Mutable Models

```
module Mutable  :  functor (FGC  :  sig type f and g and c end)  →
  Model.Mutable with type flavor  =  FGC.f and type gauge  =  FGC.g
  and type constant  =  FGC.c
```

13.2   Implementation of Modeltools

```
let rcs_file = RCS.parse "Modeltools" ["Lagragians"]
    { RCS.revision = "$Revision:␣6465␣$";
      RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
      RCS.author = "$Author:␣jr␣reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr␣reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr
```

13.2.1   Compilation

Flavors and coupling constants: flavors can be tested for equality and charge conjugation is defined.

```
module type Flavor =
  sig
    type f
    type c
    val compare : f → f → int
    val conjugate : f → f
  end
```

Compiling fusions from a list of vertices:

```
module type Fusions =
  sig
    type t
    type f
    type c
    val fuse2 : t → f → f → (f × c Coupling.t) list
    val fuse3 : t → f → f → f → (f × c Coupling.t) list
    val fuse : t → f list → (f × c Coupling.t) list
    val of_vertices :
        (((f × f × f) × c Coupling.vertex3 × c) list
            × ((f × f × f × f) × c Coupling.vertex4 × c) list
            × (f list × c Coupling.vertexn × c) list) → t
  end

module Fusions (F : Flavor) : Fusions with type f = F.f and type c = F.c =
  struct

    type f = F.f
    type c = F.c

    module F2 =
      struct
        type t = f × f
        let hash = Hashtbl.hash
        let compare (f1, f2) (f1', f2') =
          let c1 = F.compare f1 f1' in
          if c1 ≠ 0 then
            c1
```

```
        else
            F.compare f2 f2'
        let equal f f' = compare f f' = 0
    end

module F3 =
    struct
        type t = f × f × f
        let hash = Hashtbl.hash
        let compare (f1, f2, f3) (f1', f2', f3') =
            let c1 = F.compare f1 f1' in
            if c1 ≠ 0 then
                c1
            else
                let c2 = F.compare f2 f2' in
                if c2 ≠ 0 then
                    c2
                else
                    F.compare f3 f3'
        let equal f f' = compare f f' = 0
    end

module Fn =
    struct
        type t = f list
        let hash = Hashtbl.hash
        let compare f f' = ThoList.compare ~cmp:F.compare f f'
        let equal f f' = compare f f' = 0
    end

module H2 = Hashtbl.Make (F2)
module H3 = Hashtbl.Make (F3)
module Hn = Hashtbl.Make (Fn)

type t =
    { v3 : (f × c Coupling.t) list H2.t;
      v4 : (f × c Coupling.t) list H3.t;
      vn : (f × c Coupling.t) list Hn.t }

let fuse2 table f1 f2 =
    try
        H2.find table.v3 (f1, f2)
    with
    | Not_found → []

let fuse3 table f1 f2 f3 =
    try
        H3.find table.v4 (f1, f2, f3)
    with
    | Not_found → []

let fusen table f =
    try
        Hn.find table.vn f
```

```
    with
    | Not_found → []
  let fuse table = function
    | [] | [_] → invalid_arg "Fusions().fuse"
    | [f1; f2] → fuse2 table f1 f2
    | [f1; f2; f3] → fuse3 table f1 f2 f3
    | f → fusen table f
```

Note that a pair or a triplet can appear more than once (e.g. $e^+e^- \to \gamma$ and $e^+e^- \to Z$). Therefore don't replace the entry, but augment it instead.

```
  let add_fusion2 table f1 f2 fusions =
    H2.add table.v3 (f1, f2) (fusions :: fuse2 table f1 f2)

  let add_fusion3 table f1 f2 f3 fusions =
    H3.add table.v4 (f1, f2, f3) (fusions :: fuse3 table f1 f2 f3)

  let add_fusionn table f fusions =
    Hn.add table.vn f (fusions :: fusen table f)
```

⚡ Do we need to take into account the charge conjugation of the coupling constants here?

If some flavors are identical, we must not introduce the same vertex more than once:

```
  open Coupling

  let permute3 (f1, f2, f3) =
    [ (f1, f2), F.conjugate f3, F12;
      (f2, f1), F.conjugate f3, F21;
      (f2, f3), F.conjugate f1, F23;
      (f3, f2), F.conjugate f1, F32;
      (f3, f1), F.conjugate f2, F31;
      (f1, f3), F.conjugate f2, F13 ]
```

Here we add identical permutations of pairs only once:

```
  module F2' = Set.Make (F2)

  let add_permute3 table v c set ((f1, f2 as f12), f, p) =
    if F2'.mem f12 set then
      set
    else begin
      add_fusion2 table f1 f2 (f, V3 (v, p, c));
      F2'.add f12 set
    end

  let add_vertex3 table (f123, v, c) =
    ignore (List.fold_left (fun set f → add_permute3 table v c set f)
            F2'.empty (permute3 f123))
```

⚡ Handling all the cases explicitly is OK for cubic vertices, but starts to become questionable already for quartic couplings. The advantage remains that we can check completeness in *Targets*.

```
let permute4 (f1, f2, f3, f4) =
  [ (f1, f2, f3), F.conjugate f4, F123;
    (f2, f3, f1), F.conjugate f4, F231;
    (f3, f1, f2), F.conjugate f4, F312;
    (f2, f1, f3), F.conjugate f4, F213;
    (f3, f2, f1), F.conjugate f4, F321;
    (f1, f3, f2), F.conjugate f4, F132;
    (f1, f2, f4), F.conjugate f3, F124;
    (f2, f4, f1), F.conjugate f3, F241;
    (f4, f1, f2), F.conjugate f3, F412;
    (f2, f1, f4), F.conjugate f3, F214;
    (f4, f2, f1), F.conjugate f3, F421;
    (f1, f4, f2), F.conjugate f3, F142;
    (f1, f3, f4), F.conjugate f2, F134;
    (f3, f4, f1), F.conjugate f2, F341;
    (f4, f1, f3), F.conjugate f2, F413;
    (f3, f1, f4), F.conjugate f2, F314;
    (f4, f3, f1), F.conjugate f2, F431;
    (f1, f4, f3), F.conjugate f2, F143;
    (f2, f3, f4), F.conjugate f1, F234;
    (f3, f4, f2), F.conjugate f1, F342;
    (f4, f2, f3), F.conjugate f1, F423;
    (f3, f2, f4), F.conjugate f1, F324;
    (f4, f3, f2), F.conjugate f1, F432;
    (f2, f4, f3), F.conjugate f1, F243 ]
```

Add identical permutations of triplets only once:

```
module F3' = Set.Make (F3)

let add_permute4 table v c set ((f1, f2, f3 as f123), f, p) =
  if F3'.mem f123 set then
    set
  else begin
    add_fusion3 table f1 f2 f3 (f, V4 (v, p, c));
    F3'.add f123 set
  end

let add_vertex4 table (f1234, v, c) =
  ignore (List.fold_left (fun set f → add_permute4 table v c set f)
            F3'.empty (permute4 f1234))

let of_vertices (vlist3, vlist4, vlistn) =
  match vlistn with
  | [] →
      let table =
        { v3 = H2.create 37; v4 = H3.create 37; vn = Hn.create 37 } in
      List.iter (add_vertex3 table) vlist3;
      List.iter (add_vertex4 table) vlist4;
      table
  | _ → failwith "Models.Fusions.of_vertices:⎵incomplete"

end
```

13.2.2   Mutable Models

```
module Mutable (FGC : sig type f and g and c end) =
  struct
    type flavor = FGC.f
    type gauge = FGC.g
    type constant = FGC.c

    let options = Options.empty

    module Ch = Charges.Null
    let charges _ = ()

    exception Uninitialized of string
    let unitialized name =
      raise (Uninitialized name)
```

Note that *lookup* works, by the magic of currying, for any arity. But we need to supply one argument to delay evaluation.

Also note that the references are *not* shared among results of functor applications. Simple module renaming causes sharing.

```
    let declare template =
      let reference = ref template in
      let update fct = reference := fct
      and lookup arg = !reference arg in
      (update, lookup)

    let set_color, color =
      declare (fun f → unitialized "color")
    let set_pdg, pdg =
      declare (fun f → unitialized "pdg")
    let set_lorentz, lorentz =
      declare (fun f → unitialized "lorentz")
    let set_propagator, propagator =
      declare (fun f → unitialized "propagator")
    let set_width, width =
      declare (fun f → unitialized "width")
    let set_goldstone, goldstone =
      declare (fun f → unitialized "goldstone")
    let set_conjugate, conjugate =
      declare (fun f → unitialized "conjugate")
    let set_fermion, fermion =
      declare (fun f → unitialized "fermion")
    let set_max_degree, max_degree =
      declare (fun () → unitialized "max_degree")
    let set_vertices, vertices =
      declare (fun () → unitialized "vertices")
    let set_fuse2, fuse2 =
      declare (fun f1 f2 → unitialized "fuse2")
    let set_fuse3, fuse3 =
      declare (fun f1 f2 f3 → unitialized "fuse3")
    let set_fuse, fuse =
```

```
      declare (fun f  →  unitialized "fuse")
let set_flavors, flavors  =
   declare (fun ()  →  [])
let set_external_flavors, external_flavors  =
   declare (fun ()  →  [("unitialized", [])])
let set_parameters, parameters  =
   declare (fun f  →  unitialized "parameters")
let set_flavor_of_string, flavor_of_string  =
   declare (fun f  →  unitialized "flavor_of_string")
let set_flavor_to_string, flavor_to_string  =
   declare (fun f  →  unitialized "flavor_to_string")
let set_flavor_to_TeX, flavor_to_TeX  =
   declare (fun f  →  unitialized "flavor_to_TeX")
let set_flavor_symbol, flavor_symbol  =
   declare (fun f  →  unitialized "flavor_symbol")
let set_gauge_symbol, gauge_symbol  =
   declare (fun f  →  unitialized "gauge_symbol")
let set_mass_symbol, mass_symbol  =
   declare (fun f  →  unitialized "mass_symbol")
let set_width_symbol, width_symbol  =
   declare (fun f  →  unitialized "width_symbol")
let set_constant_symbol, constant_symbol  =
   declare (fun f  →  unitialized "constant_symbol")

let setup ˜color ˜pdg ˜lorentz ˜propagator ˜width ˜goldstone
      ˜conjugate ˜fermion ˜max_degree ˜vertices
      ˜fuse : (fuse2, fuse3, fusen)
      ˜flavors ˜parameters ˜flavor_of_string ˜flavor_to_string
      ˜flavor_to_TeX ˜flavor_symbol
      ˜gauge_symbol ˜mass_symbol ˜width_symbol ˜constant_symbol  =
   set_color color;
   set_pdg pdg;
   set_lorentz lorentz;
   set_propagator propagator;
   set_width width;
   set_goldstone goldstone;
   set_conjugate conjugate;
   set_fermion fermion;
   set_max_degree (fun ()  →  max_degree);
   set_vertices vertices;
   set_fuse2 fuse2;
   set_fuse3 fuse3;
   set_fuse fusen;
   set_external_flavors (fun f  →  flavors);
   let flavors  =  ThoList.flatmap snd flavors in
   set_flavors (fun f  →  flavors);
   set_parameters parameters;
   set_flavor_of_string flavor_of_string;
   set_flavor_to_string flavor_to_string;
   set_flavor_to_TeX flavor_to_TeX;
```

$set\_flavor\_symbol\ flavor\_symbol;$
$set\_gauge\_symbol\ gauge\_symbol;$
$set\_mass\_symbol\ mass\_symbol;$
$set\_width\_symbol\ width\_symbol;$
$set\_constant\_symbol\ constant\_symbol$

let $rcs\ =\ RCS.rename\ rcs\_file$ "Models.Mutable" ["Mutable␣Model"]
end

13.3   Interface of Modellib_SM

13.3.1   Hardcoded Models

module $Phi3$: $Model.T$ with module $Ch\ =\ Charges.Null$
module $Phi4$: $Model.T$ with module $Ch\ =\ Charges.Null$
module QED : $Model.T$ with module $Ch\ =\ Charges.ZZ$
module QCD : $Model.T$ with module $Ch\ =\ Charges.ZZ$

module type $SM\_flags\ =$
  sig
    val $higgs\_triangle$: $bool$ $(*\ H\gamma\gamma,\ Hg\gamma$ and $Hggcouplings*)$
    val $higgs\_hmm$: $bool$
    val $triple\_anom$: $bool$
    val $quartic\_anom$: $bool$
    val $higgs\_anom$: $bool$
    val $dim6$: $bool$
    val k_matrix : $bool$
    val $ckm\_present$: $bool$
    val top_anom : $bool$
    val top_anom_4f : $bool$
    val $tt\_threshold$: $bool$
  end

module $SM\_no\_anomalous$: SM_flags
module $SM\_anomalous$: SM_flags
module SM_k_matrix : SM_flags
module $SM\_no\_anomalous\_ckm$: SM_flags
module $SM\_anomalous\_ckm$: SM_flags
module SM_Higgs : SM_flags
module $SM\_anomalous\_top$: SM_flags
module $SM\_tt\_threshold$: SM_flags
module SM_dim6 : SM_flags

module SM : functor $(F\ :\ SM\_flags)\ \to\ Model.Gauge$ with module $Ch\ =\ Charges.QQ$

module SM_Rxi : $Model.T$ with module $Ch\ =\ Charges.QQ$

module $Groves$: functor $(M\ :\ Model.Gauge)\ \to\ Model.Gauge$ with module $Ch\ =\ M.Ch$
module SM_clones : $Model.Gauge$ with module $Ch\ =\ Charges.QQ$

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13.4   Implementation of Modellib_SM

```
let rcs_file  =  RCS.parse "Modellib_SM" ["Lagragians"]
    { RCS.revision  =  "$Revision:␣7378␣$";
      RCS.date  =  "$Date:␣2015-11-20␣17:08:49␣+0100␣(Fri,␣20␣Nov␣2015)␣$";
      RCS.author  =  "$Author:␣jr_reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s
```

13.4.1 ϕ^3

```
module Phi3  =
  struct
    let rcs  =  RCS.rename rcs_file "Modellib.Phi3"
        ["phi**3␣with␣a␣single␣flavor"]

    open Coupling

    let options  =  Options.empty

    type flavor  =  Phi
    let external_flavors ()  =  [ "", [Phi]]
    let flavors ()  =  ThoList.flatmap snd (external_flavors ())

    type gauge  =  unit
    type constant  =  G

    type orders  =  unit
    let orders  = function
      | _  →  ()

    let lorentz _  =  Scalar
    let color _  =  Color.Singlet
    let propagator _  =  Prop_Scalar
    let width _  =  Timelike
    let goldstone _  =  None
    let conjugate f  =  f
    let fermion _  =  0

    module Ch  =  Charges.Null
    let charges _  =  ()

    module F  =  Modeltools.Fusions (struct
      type f  =  flavor
      type c  =  constant
      let compare  =  compare
      let conjugate  =  conjugate
    end)

    let vertices ()  =
      ([(Phi,  Phi,  Phi),  Scalar_Scalar_Scalar 1,  G], [], [])

    let table  =  F.of_vertices (vertices ())
    let fuse2  =  F.fuse2 table
```

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```
let fuse3 = F.fuse3 table
let fuse = F.fuse table
let max_degree () = 3
let parameters () = { input = [G, 1.0]; derived = []; derived_arrays = [] }

let flavor_of_string = function
   | "p" → Phi
   | _ → invalid_arg "Modellib.Phi3.flavor_of_string"

let flavor_to_string Phi = "phi"
let flavor_to_TeX Phi = "\\phi"
let flavor_symbol Phi = "phi"

let gauge_symbol () =
   failwith "Modellib.Phi3.gauge_symbol:␣internal␣error"

let pdg _ = 1
let mass_symbol _ = "m"
let width_symbol _ = "w"
let constant_symbol G = "g"
```

end

13.4.2 $\lambda_3\phi^3 + \lambda_4\phi^4$

```
module Phi4 =
  struct
    let rcs = RCS.rename rcs_file "Modellib.Phi4"
        ["phi**4␣with␣a␣single␣flavor"]

    open Coupling

    let options = Options.empty

    type flavor = Phi
    let external_flavors () = [ "", [Phi]]
    let flavors () = ThoList.flatmap snd (external_flavors ())

    type gauge = unit
    type constant = G3 | G4

    type orders = unit
    let orders = function
       | _ → ()

    let lorentz _ = Scalar
    let color _ = Color.Singlet
    let propagator _ = Prop_Scalar
    let width _ = Timelike
    let goldstone _ = None
    let conjugate f = f
    let fermion _ = 0

    module Ch = Charges.Null
    let charges _ = ()
```

```
module F = Modeltools.Fusions (struct
  type f = flavor
  type c = constant
  let compare = compare
  let conjugate = conjugate
end)

let vertices () =
  ([[(Phi, Phi, Phi), Scalar_Scalar_Scalar 1, G3],
    [(Phi, Phi, Phi, Phi), Scalar4 1, G4], [])

let fuse2 _ = failwith "Modellib.Phi4.fuse2"
let fuse3 _ = failwith "Modellib.Phi4.fuse3"
let fuse = function
  | [] | [_] → invalid_arg "Modellib.Phi4.fuse"
  | [_; _] → [Phi, V3 (Scalar_Scalar_Scalar 1, F23, G3)]
  | [_; _; _] → [Phi, V4 (Scalar4 1, F234, G4)]
  | _ → []
let max_degree () = 4
let parameters () =
  { input = [G3, 1.0; G4, 1.0]; derived = []; derived_arrays = [] }

let flavor_of_string = function
  | "p" → Phi
  | _ → invalid_arg "Modellib.Phi4.flavor_of_string"

let flavor_to_string Phi = "phi"
let flavor_to_TeX Phi = "\\phi"
let flavor_symbol Phi = "phi"

let gauge_symbol () =
  failwith "Modellib.Phi4.gauge_symbol:␣internal␣error"

let pdg _ = 1
let mass_symbol _ = "m"
let width_symbol _ = "w"
let constant_symbol = function
  | G3 → "g3"
  | G4 → "g4"
end
```

13.4.3 Quantum Electro Dynamics

```
module QED =
  struct
    let rcs = RCS.rename rcs_file "Modellib.QED"
        ["QED␣with␣two␣leptonic␣flavors"]

    open Coupling

    let options = Options.empty

    type flavor =
```

```
    |  Electron  |  Positron
    |  Muon  |  AntiMuon
    |  Tau  |  AntiTau
    |  Photon
let external_flavors () =
  [ "Leptons", [Electron; Positron; Muon; AntiMuon; Tau; AntiTau];
    "Gauge␣Bosons", [Photon] ]
let flavors () = ThoList.flatmap snd (external_flavors ())

type gauge = unit
type constant = Q

type orders = unit
let orders = function
  | _ → ()

let lorentz = function
  | Electron | Muon | Tau → Spinor
  | Positron | AntiMuon | AntiTau → ConjSpinor
  | Photon → Vector

let color _ = Color.Singlet

let propagator = function
  | Electron | Muon | Tau → Prop_Spinor
  | Positron | AntiMuon | AntiTau → Prop_ConjSpinor
  | Photon → Prop_Feynman

let width _ = Timelike

let goldstone _ =
  None

let conjugate = function
  | Electron → Positron | Positron → Electron
  | Muon → AntiMuon | AntiMuon → Muon
  | Tau → AntiTau | AntiTau → Tau
  | Photon → Photon

let fermion = function
  | Electron | Muon | Tau → 1
  | Positron | AntiMuon | AntiTau → −1
  | Photon → 0
```

Taking generation numbers makes electric charge redundant.

```
module Ch = Charges.ZZ
let charges = function
  | Electron → [1; 0; 0]
  | Muon → [0; 1; 0]
  | Tau → [0; 0; 1]
  | Positron → [−1; 0; 0]
  | AntiMuon → [0; −1; 0]
  | AntiTau → [0; 0; −1]
  | Photon → [0; 0; 0]
```

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```
module F  =  Modeltools.Fusions (struct
  type f  =  flavor
  type c  =  constant
  let compare  =  compare
  let conjugate  =  conjugate
end)
```

let *vertices* () =
  ([[(*Positron*, *Photon*, *Electron*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q*;
     (*AntiMuon*, *Photon*, *Muon*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q*;
     (*AntiTau*, *Photon*, *Tau*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q*], [], [])

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 3

let *parameters* () = { *input* = [*Q*, 1.0]; *derived* = []; *derived_arrays* = [] }

let *flavor_of_string* = function
  | `"e-"` → *Electron* | `"e+"` → *Positron*
  | `"m-"` → *Muon* | `"m+"` → *AntiMuon*
  | `"t-"` → *Tau* | `"t+"` → *AntiTau*
  | `"A"` → *Photon*
  | _ → *invalid_arg* `"Modellib.QED.flavor_of_string"`

let *flavor_to_string* = function
  | *Electron* → `"e-"` | *Positron* → `"e+"`
  | *Muon* → `"m-"` | *AntiMuon* → `"m+"`
  | *Tau* → `"t-"` | *AntiTau* → `"t+"`
  | *Photon* → `"A"`

let *flavor_to_TeX* = function
  | *Electron* → `"e^-"` | *Positron* → `"e^+"`
  | *Muon* → `"\\mu^-"` | *AntiMuon* → `"\\mu^+"`
  | *Tau* → `"^\\tau^-"` | *AntiTau* → `"\\tau+^"`
  | *Photon* → `"\\gamma"`

let *flavor_symbol* = function
  | *Electron* → `"ele"` | *Positron* → `"pos"`
  | *Muon* → `"muo"` | *AntiMuon* → `"amu"`
  | *Tau* → `"tau"` | *AntiTau* → `"ata"`
  | *Photon* → `"gam"`

let *gauge_symbol* () =
  failwith `"Modellib.QED.gauge_symbol:␣internal␣error"`

let *pdg* = function
  | *Electron* → 11 | *Positron* → − 11
  | *Muon* → 13 | *AntiMuon* → − 13
  | *Tau* → 15 | *AntiTau* → − 15
  | *Photon* → 22

let *mass_symbol f* =

```
      "mass(" ^ string_of_int (abs (pdg f)) ^ ")"

    let width_symbol f  =
      "width(" ^ string_of_int (abs (pdg f)) ^ ")"

    let constant_symbol = function
      | Q → "qlep"
  end
```

13.4.4   Quantum Chromo Dynamics

```
module QCD  =
  struct
    let rcs = RCS.rename rcs_file "Modellib.QCD"
        ["QCD"]

    open Coupling

    let options  =  Options.empty

    type flavor  =
      | U  | Ubar  | D  | Dbar
      | C  | Cbar  | S  | Sbar
      | T  | Tbar  | B  | Bbar
      | Gl

    let external_flavors () =
      [ "Quarks", [U; D; C; S; T; B; Ubar; Dbar; Cbar; Sbar; Tbar; Bbar];
        "Gauge␣Bosons", [Gl]]
    let flavors () = ThoList.flatmap snd (external_flavors ())

    type gauge  =  unit
    type constant  =  Gs  |  G2  |  I_Gs

    type orders  =  unit
    let orders  = function
      | _ → ()

    let lorentz  = function
      | U  | D  | C  | S  | T  | B → Spinor
      | Ubar  | Dbar  | Cbar  | Sbar  | Tbar  | Bbar → ConjSpinor
      | Gl → Vector

    let color  = function
      | U  | D  | C  | S  | T  | B → Color.SUN 3
      | Ubar  | Dbar  | Cbar  | Sbar  | Tbar  | Bbar → Color.SUN (−3)
      | Gl → Color.AdjSUN 3

    let propagator  = function
      | U  | D  | C  | S  | T  | B → Prop_Spinor
      | Ubar  | Dbar  | Cbar  | Sbar  | Tbar  | Bbar → Prop_ConjSpinor
      | Gl → Prop_Feynman

    let width _  =  Timelike

    let goldstone _  =
```

None

```
let conjugate = function
  | U    →  Ubar
  | D    →  Dbar
  | C    →  Cbar
  | S    →  Sbar
  | T    →  Tbar
  | B    →  Bbar
  | Ubar →  U
  | Dbar →  D
  | Cbar →  C
  | Sbar →  S
  | Tbar →  T
  | Bbar →  B
  | Gl   →  Gl

let fermion = function
  | U  | D  | C  | S  | T  | B  →  1
  | Ubar | Dbar | Cbar | Sbar | Tbar | Bbar  →  − 1
  | Gl  →  0

module Ch  =  Charges.ZZ
let charges = function
  | D    →  [1; 0; 0; 0; 0; 0]
  | U    →  [0; 1; 0; 0; 0; 0]
  | S    →  [0; 0; 1; 0; 0; 0]
  | C    →  [0; 0; 0; 1; 0; 0]
  | B    →  [0; 0; 0; 0; 1; 0]
  | T    →  [0; 0; 0; 0; 0; 1]
  | Dbar →  [−1; 0; 0; 0; 0; 0]
  | Ubar →  [0; − 1; 0; 0; 0; 0]
  | Sbar →  [0; 0; − 1; 0; 0; 0]
  | Cbar →  [0; 0; 0; − 1; 0; 0]
  | Bbar →  [0; 0; 0; 0; − 1; 0]
  | Tbar →  [0; 0; 0; 0; 0; − 1]
  | Gl   →  [0; 0; 0; 0; 0; 0]

module F  =  Modeltools.Fusions (struct
  type f  =  flavor
  type c  =  constant
  let compare  =  compare
  let conjugate  =  conjugate
end)
```

This is compatible with CD+.

```
let color_current =
  [ ((Dbar, Gl, D), FBF ((−1), Psibar, V, Psi), Gs);
    ((Ubar, Gl, U), FBF ((−1), Psibar, V, Psi), Gs);
    ((Cbar, Gl, C), FBF ((−1), Psibar, V, Psi), Gs);
    ((Sbar, Gl, S), FBF ((−1), Psibar, V, Psi), Gs);
    ((Tbar, Gl, T), FBF ((−1), Psibar, V, Psi), Gs);
```

$((Bbar,\ Gl,\ B),\ FBF\ ((-1),\ Psibar,\ V,\ Psi),\ Gs)]$

let *three_gluon* =
  $[\ ((Gl,\ Gl,\ Gl),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Gs)]$

let *gauge4* = *Vector4* $[(2,\ C\_13\_42);\ (-1,\ C\_12\_34);\ (-1,\ C\_14\_23)]$

let *four_gluon* =
  $[\ ((Gl,\ Gl,\ Gl,\ Gl),\ gauge4,\ G2)]$

let *vertices3* =
  (*color_current* @ *three_gluon*)

let *vertices4* = *four_gluon*

let *vertices* () =
  (*vertices3*, *vertices4*, [])

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *parameters* () = { *input* = [*Gs*, 1.0]; *derived* = []; *derived_arrays* = [ ] }

let *flavor_of_string* = function
  | `"u"` $\rightarrow$ *U*
  | `"d"` $\rightarrow$ *D*
  | `"c"` $\rightarrow$ *C*
  | `"s"` $\rightarrow$ *S*
  | `"t"` $\rightarrow$ *T*
  | `"b"` $\rightarrow$ *B*
  | `"ubar"` $\rightarrow$ *Ubar*
  | `"dbar"` $\rightarrow$ *Dbar*
  | `"cbar"` $\rightarrow$ *Cbar*
  | `"sbar"` $\rightarrow$ *Sbar*
  | `"tbar"` $\rightarrow$ *Tbar*
  | `"bbar"` $\rightarrow$ *Bbar*
  | `"gl"` $\rightarrow$ *Gl*
  | _ $\rightarrow$ *invalid_arg* `"Modellib.QCD.flavor_of_string"`

let *flavor_to_string* = function
  | *U* $\rightarrow$ `"u"`
  | *Ubar* $\rightarrow$ `"ubar"`
  | *D* $\rightarrow$ `"d"`
  | *Dbar* $\rightarrow$ `"dbar"`
  | *C* $\rightarrow$ `"c"`
  | *Cbar* $\rightarrow$ `"cbar"`
  | *S* $\rightarrow$ `"s"`
  | *Sbar* $\rightarrow$ `"sbar"`
  | *T* $\rightarrow$ `"t"`
  | *Tbar* $\rightarrow$ `"tbar"`
  | *B* $\rightarrow$ `"b"`
  | *Bbar* $\rightarrow$ `"bbar"`

```
  | Gl → "gl"
let flavor_to_TeX = function
  | U → "u"
  | Ubar → "\\bar{u}"
  | D → "d"
  | Dbar → "\\bar{d}"
  | C → "c"
  | Cbar → "\\bar{c}"
  | S → "s"
  | Sbar → "\\bar{s}"
  | T → "t"
  | Tbar → "\\bar{t}"
  | B → "b"
  | Bbar → "\\bar{b}"
  | Gl → "g"
let flavor_symbol = function
  | U → "u"
  | Ubar → "ubar"
  | D → "d"
  | Dbar → "dbar"
  | C → "c"
  | Cbar → "cbar"
  | S → "s"
  | Sbar → "sbar"
  | T → "t"
  | Tbar → "tbar"
  | B → "b"
  | Bbar → "bbar"
  | Gl → "gl"
let gauge_symbol () =
  failwith "Modellib.QCD.gauge_symbol:␣internal␣error"
let pdg = function
  | D → 1 | Dbar → −1
  | U → 2 | Ubar → −2
  | S → 3 | Sbar → −3
  | C → 4 | Cbar → −4
  | B → 5 | Bbar → −5
  | T → 6 | Tbar → −6
  | Gl → 21
let mass_symbol f =
  "mass(" ^ string_of_int (abs (pdg f)) ^ ")"
let width_symbol f =
  "width(" ^ string_of_int (abs (pdg f)) ^ ")"
let constant_symbol = function
  | I_Gs → "(0,1)*gs"
  | Gs → "gs"
  | G2 → "gs**2"
```

end

13.4.5 Complete Minimal Standard Model (Unitarity Gauge)

```
module type SM_flags =
  sig
    val higgs_triangle : bool (* Hγγ, Hgγ and Hgg couplings *)
    val higgs_hmm : bool
    val triple_anom : bool
    val quartic_anom : bool
    val higgs_anom : bool
    val dim6 : bool
    val k_matrix : bool
    val ckm_present : bool
    val top_anom : bool
    val top_anom_4f : bool
    val tt_threshold : bool
  end
module SM_no_anomalous : SM_flags =
  struct
    let higgs_triangle = false
    let higgs_hmm = false
    let triple_anom = false
    let quartic_anom = false
    let higgs_anom = false
    let dim6 = false
    let k_matrix = false
    let ckm_present = false
    let top_anom = false
    let top_anom_4f = false
    let tt_threshold = false
  end
module SM_no_anomalous_ckm : SM_flags =
  struct
    let higgs_triangle = false
    let higgs_hmm = false
    let triple_anom = false
    let quartic_anom = false
    let higgs_anom = false
    let dim6 = false
    let k_matrix = false
    let ckm_present = true
    let top_anom = false
    let top_anom_4f = false
    let tt_threshold = false
  end
module SM_anomalous : SM_flags =
  struct
```

```
    let higgs_triangle  =  false
    let higgs_hmm  =  false
    let triple_anom  =  true
    let quartic_anom  =  true
    let higgs_anom  =  true
    let dim6  =  false
    let k_matrix  =  false
    let ckm_present  =  false
    let top_anom  =  false
    let top_anom_4f  =  false
    let tt_threshold  =  false
  end
module SM_anomalous_ckm  :  SM_flags  =
  struct
    let higgs_triangle  =  false
    let higgs_hmm  =  false
    let triple_anom  =  true
    let quartic_anom  =  true
    let higgs_anom  =  true
    let dim6  =  false
    let k_matrix  =  false
    let ckm_present  =  true
    let top_anom  =  false
    let top_anom_4f  =  false
    let tt_threshold  =  false
  end
module SM_k_matrix  :  SM_flags  =
  struct
    let higgs_triangle  =  false
    let higgs_hmm  =  false
    let triple_anom  =  false
    let quartic_anom  =  true
    let higgs_anom  =  false
    let dim6  =  false
    let k_matrix  =  true
    let ckm_present  =  false
    let top_anom  =  false
    let top_anom_4f  =  false
    let tt_threshold  =  false
  end
module SM_Higgs  :  SM_flags  =
  struct
    let higgs_triangle  =  true
    let higgs_hmm  =  true
    let triple_anom  =  false
    let quartic_anom  =  false
    let higgs_anom  =  false
    let dim6  =  false
    let k_matrix  =  false
```

```
    let ckm_present  =  false
    let top_anom  =  false
    let top_anom_4f  =  false
    let tt_threshold  =  false
  end

module SM_anomalous_top  :  SM_flags  =
  struct
    let higgs_triangle  =  false
    let higgs_hmm  =  false
    let triple_anom  =  false
    let quartic_anom  =  false
    let higgs_anom  =  false
    let dim6  =  false
    let k_matrix  =  false
    let ckm_present  =  false
    let top_anom  =  true
    let top_anom_4f  =  true
    let tt_threshold  =  false
  end

module SM_tt_threshold  :  SM_flags  =
  struct
    let higgs_triangle  =  false
    let higgs_hmm  =  false
    let triple_anom  =  false
    let quartic_anom  =  false
    let higgs_anom  =  false
    let dim6  =  false
    let k_matrix  =  false
    let ckm_present  =  true
    let top_anom  =  false
    let top_anom_4f  =  false
    let tt_threshold  =  true
  end

module SM_dim6  :  SM_flags  =
  struct
    let higgs_triangle  =  false
    let higgs_hmm  =  false
    let triple_anom  =  false
    let quartic_anom  =  false
    let higgs_anom  =  false
    let dim6  =  true
    let k_matrix  =  false
    let ckm_present  =  false
    let top_anom  =  false
    let top_anom_4f  =  false
    let tt_threshold  =  false
  end
```

13.4.6 Complete Minimal Standard Model (including some extensions)

```
module SM (Flags : SM_flags) =
  struct
    let rcs = RCS.rename rcs_file "Modellib.SM"
        [ "minimal␣electroweak␣standard␣model␣in␣unitarity␣gauge"]

    open Coupling

    let default_width = ref Timelike
    let use_fudged_width = ref false

    let options = Options.create
      [ "constant_width", Arg.Unit (fun () → default_width := Constant),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", Arg.Set use_fudged_width,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", Arg.String (fun f → default_width := Custom f),
        "use␣custom␣width";
        "cancel_widths", Arg.Unit (fun () → default_width := Vanishing),
        "use␣vanishing␣width"]

    type f_aux_top = TTGG | TBWA | TBWZ | TTWW | BBWW
                    | TCGG | QGUG | QBUB | QW | DL | DR
                    | QUQD1L | QUQD1R | QUQD8L | QUQD8R

    type matter_field = L of int | N of int | U of int | D of int
    type gauge_boson = Ga | Wp | Wm | Z | Gl
    type other = Phip | Phim | Phi0 | H
                    | Aux_top of int×int×int×bool×f_aux_top type flavor = M of matter_field |
G of gauge_boson | O of other

    let matter_field f = M f
    let gauge_boson f = G f
    let other f = O f

    type field =
       | Matter of matter_field
       | Gauge of gauge_boson
       | Other of other

    let field = function
       | M f → Matter f
       | G f → Gauge f
       | O f → Other f

    type gauge = unit

    let gauge_symbol () =
       failwith "Modellib.SM.gauge_symbol:␣internal␣error"

    let family n = List.map matter_field [ L n; N n; U n; D n ]

    let rec aux_top_flavors (f, l, co, ch) = List.append
       ( List.map other [ Aux_top (l, co, ch/2,true,f);
```

$$Aux\_top \ (l, co, ch/2, \mathsf{false}, f) \ ] \ )$$
(if $ch > 1$ then $List.append$
 ($List.map \ other \ [\ Aux\_top \ (l, co, -ch/2, \mathsf{true}, f);$
       $Aux\_top \ (l, co, -ch/2, \mathsf{false}, f) \ ] \ )$
 ($aux\_top\_flavors \ (f, l, co, (ch - 2)) \ )$
 else $[\ ] \ )$

let *external_flavors* () =
 ["1st␣Generation", *ThoList.flatmap family* [1; −1];
  "2nd␣Generation", *ThoList.flatmap family* [2; −2];
  "3rd␣Generation", *ThoList.flatmap family* [3; −3];
  "Gauge␣Bosons", *List.map gauge_boson* [*Ga*; *Z*; *Wp*; *Wm*; *Gl*];
  "Higgs", *List.map other* [*H*];
  "Goldstone␣Bosons", *List.map other* [*Phip*; *Phim*; *Phi0*]]

let *flavors* () = *List.append*
 (*ThoList.flatmap snd* (*external_flavors* ()))
 (*ThoList.flatmap aux_top_flavors*
  [(*TTGG*, 2, 1, 1); (*TCGG*, 2, 1, 1); (*TBWA*, 2, 0, 2); (*TBWZ*, 2, 0, 2);
   (*TTWW*, 2, 0, 1); (*BBWW*, 2, 0, 1);
   (*QGUG*, 1, 1, 1); (*QBUB*, 1, 0, 1); (*QW*, 1, 0, 3); (*DL*, 0, 0, 3); (*DR*, 0, 0, 3);
   (*QUQD1L*, 0, 0, 3); (*QUQD1R*, 0, 0, 3); (*QUQD8L*, 0, 1, 3); (*QUQD8R*, 0, 1, 3)])

let *spinor* n =
 if $n \geq 0$ then
  Spinor
 else
  ConjSpinor

let *lorentz_aux* = function
 | 2 → *Tensor_1*
 | 1 → *Vector*
 | 0 → *Scalar*
 | _ → *invalid_arg* ("SM.lorentz_aux:␣wrong␣value")

let *lorentz* = function
 | *M f* →
  begin match f with
  | *L n* → *spinor n* | *N n* → *spinor n*
  | *U n* → *spinor n* | *D n* → *spinor n*
  end
 | *G f* →
  begin match f with
  | *Ga* | *Gl* → *Vector*
  | *Wp* | *Wm* | *Z* → *Massive_Vector*
  end
 | *O f* →
  begin match f with
  | *Aux_top* (*l*, _, _, _, _) → *lorentz_aux l*
  | _ → *Scalar*
  end

let *color* = function

```
    |  M (U n)  →  Color.SUN (if n > 0 then 3 else -3)
    |  M (D n)  →  Color.SUN (if n > 0 then 3 else -3)
    |  G Gl  →  Color.AdjSUN 3
    |  O (Aux_top (_, co, _, _, _))  →  if co ≡ 0 then Color.Singlet else Color.AdjSUN 3
    |  _  →  Color.Singlet

let prop_spinor n  =
  if n ≥ 0 then
    Prop_Spinor
  else
    Prop_ConjSpinor

let prop_aux  = function
  | 2  →  Aux_Tensor_1
  | 1  →  Aux_Vector
  | 0  →  Aux_Scalar
  | _  →  invalid_arg ("SM.prop_aux:␣wrong␣value")

let propagator  = function
  | M f  →
      begin match f with
      |  L n  →  prop_spinor n  |  N n  →  prop_spinor n
      |  U n  →  prop_spinor n  |  D n  →  prop_spinor n
      end
  | G f  →
      begin match f with
      |  Ga  |  Gl  →  Prop_Feynman
      |  Wp  |  Wm  |  Z  →  Prop_Unitarity
      end
  | O f  →
      begin match f with
      |  Phip  |  Phim  |  Phi0  →  Only_Insertion
      |  H  →  Prop_Scalar
      |  Aux_top (l, _, _, _, _)  →  prop_aux l
      end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles.
Currently, this only applies to $W^\pm$ and top.

```
let width f  =
  if !use_fudged_width then
    match f with
    |  G Wp  |  G Wm  |  M (U 3)  |  M (U (−3))  →  Fudged
    |  _  →  !default_width
  else
    !default_width

let goldstone  = function
  | G f  →
      begin match f with
      |  Wp  →  Some (O Phip, Coupling.Const 1)
      |  Wm  →  Some (O Phim, Coupling.Const 1)
      |  Z  →  Some (O Phi0, Coupling.Const 1)
```

```
        |  _  →  None
        end
    |  _  →  None
let conjugate  =  function
    |  M f  →
        M (begin match f with
        |  L n  →  L (−n)  |  N n  →  N (−n)
        |  U n  →  U (−n)  |  D n  →  D (−n)
        end)
    |  G f  →
        G (begin match f with
        |  Gl  →  Gl  |  Ga  →  Ga  |  Z  →  Z
        |  Wp  →  Wm  |  Wm  →  Wp
        end)
    |  O f  →
        O (begin match f with
        |  Phip  →  Phim  |  Phim  →  Phip  |  Phi0  →  Phi0
        |  H  →  H
        |  Aux_top (l, co, ch, n, f)  →  Aux_top (l, co, (−ch), (¬ n), f)
        end)
let fermion  =  function
    |  M f  →
        begin match f with
        |  L n  →  if n > 0 then 1 else -1
        |  N n  →  if n > 0 then 1 else -1
        |  U n  →  if n > 0 then 1 else -1
        |  D n  →  if n > 0 then 1 else -1
        end
    |  G f  →
        begin match f with
        |  Gl  |  Ga  |  Z  |  Wp  |  Wm  →  0
        end
    |  O _  →  0
```

Electrical charge, lepton number, baryon number. We could avoid the rationals altogether by multiplying the first and last by 3 . . .

```
module Ch  =  Charges.QQ
let ( // )  =  Algebra.Small_Rational.make

let generation′  =  function
    |  1  →  [ 1//1; 0//1; 0//1]
    |  2  →  [ 0//1; 1//1; 0//1]
    |  3  →  [ 0//1; 0//1; 1//1]
    |  − 1  →  [−1//1; 0//1; 0//1]
    |  − 2  →  [ 0//1; −1//1; 0//1]
    |  − 3  →  [ 0//1; 0//1; −1//1]
    |  n  →  invalid_arg ("SM.generation':␣" ^ string_of_int n)
```

Generation is not a good quantum number for models with flavor mixing, i.e. if CKM mixing is present. Also, for the FCNC vertices implemented in the SM variant with anomalous top couplings it is not a valid symmetry.

```
let generation f  =
  if (Flags.ckm_present  ∨  Flags.top_anom) then
      [ ]
  else
      match f with
      |  M (L n  |  N n  |  U n  |  D n)  →  generation′ n
      |  G _  |  O _  →  [0//1; 0//1; 0//1]
let charge  =  function
  |  M f  →
         begin match f with
         |  L n  →  if n  >  0 then -1//1 else 1//1
         |  N n  →  0//1
         |  U n  →  if n  >  0 then 2//3 else -2//3
         |  D n  →  if n  >  0 then -1//3 else 1//3
         end
  |  G f  →
         begin match f with
         |  Gl  |  Ga  |  Z  →  0//1
         |  Wp  →  1//1
         |  Wm  →  − 1//1
         end
  |  O f  →
         begin match f with
         |  H  |  Phi0  →  0//1
         |  Phip  →  1//1
         |  Phim  →  − 1//1
         |  Aux_top (_, _, ch, _, _)  →  ch//1
         end
let lepton  =  function
  |  M f  →
         begin match f with
         |  L n  |  N n  →  if n  >  0 then 1//1 else -1//1
         |  U _  |  D _  →  0//1
         end
  |  G _  |  O _  →  0//1
let baryon  =  function
  |  M f  →
         begin match f with
         |  L _  |  N _  →  0//1
         |  U n  |  D n  →  if n  >  0 then 1//1 else -1//1
         end
  |  G _  |  O _  →  0//1
let charges f  =
  [ charge f;  lepton f;  baryon f] @ generation f
type constant  =
  |  Unit  |  Half  |  Pi  |  Alpha_QED  |  Sin2thw
  |  Sinthw  |  Costhw  |  E  |  G_weak  |  I_G_weak  |  Vev
  |  Q_lepton  |  Q_up  |  Q_down  |  G_CC  |  G_CCQ of int×int
```

| *G_NC_neutrino* | *G_NC_lepton* | *G_NC_up* | *G_NC_down*
| *G_TVA_ttA* | *G_TVA_bbA* | *G_TVA_tcA* | *G_TVA_tcZ*
| *G_VLR_ttZ* | *G_TVA_ttZ* | *G_VLR_tcZ* | *G_TVA_bbZ*
| *VA_ILC_ttA* | *VA_ILC_ttZ*
| *G_VLR_btW* | *G_VLR_tbW*
| *G_TLR_btW* | *G_TRL_tbW*
| *G_TLR_btWZ* | *G_TRL_tbWZ*
| *G_TLR_btWA* | *G_TRL_tbWA*
| *G_TVA_ttWW* | *G_TVA_bbWW*
| *G_TVA_ttG* | *G_TVA_ttGG* | *G_TVA_tcG* | *G_TVA_tcGG*
| *G_SP_ttH*
| *G_VLR_qGuG* | *G_VLR_qBuB*
| *G_VLR_qBuB_u* | *G_VLR_qBuB_d* | *G_VLR_qBuB_e* | *G_VL_qBuB_n*
| *G_VL_qW* | *G_VL_qW_u* | *G_VL_qW_d*
| *G_SL_DttR* | *G_SR_DttR* | *G_SL_DttL* | *G_SLR_DbtR* |
G_SL_DbtL
| *C_quqd1R_bt* | *C_quqd1R_tb* | *C_quqd1L_bt* | *C_quqd1L_tb*
| *C_quqd8R_bt* | *C_quqd8R_tb* | *C_quqd8L_bt* | *C_quqd8L_tb*
| *I_Q_W* | *I_G_ZWW*
| *G_WWWW* | *G_ZZWW* | *G_AZWW* | *G_AAWW*
| *I_G1_AWW* | *I_G1_ZWW*
| *I_G1_plus_kappa_plus_G4_AWW*
| *I_G1_plus_kappa_plus_G4_ZWW*
| *I_G1_plus_kappa_minus_G4_AWW*
| *I_G1_plus_kappa_minus_G4_ZWW*
| *I_G1_minus_kappa_plus_G4_AWW*
| *I_G1_minus_kappa_plus_G4_ZWW*
| *I_G1_minus_kappa_minus_G4_AWW*
| *I_G1_minus_kappa_minus_G4_ZWW*
| *I_lambda_AWW* | *I_lambda_ZWW*
| *G5_AWW* | *G5_ZWW*
| *I_kappa5_AWW* | *I_kappa5_ZWW*
| *I_lambda5_AWW* | *I_lambda5_ZWW*
| *Alpha_WWWW0* | *Alpha_ZZWW1* | *Alpha_WWWW2*
| *Alpha_ZZWW0* | *Alpha_ZZZZ*
| *D_Alpha_ZZWW0_S* | *D_Alpha_ZZWW0_T* | *D_Alpha_ZZWW1_S*
| *D_Alpha_ZZWW1_T* | *D_Alpha_ZZWW1_U* | *D_Alpha_WWWW0_S*
| *D_Alpha_WWWW0_T* | *D_Alpha_WWWW0_U* | *D_Alpha_WWWW2_S*
| *D_Alpha_WWWW2_T* | *D_Alpha_ZZZZ_S* | *D_Alpha_ZZZZ_T*
| *G_HWW* | *G_HHWW* | *G_HZZ* | *G_HHZZ*
| *G_Htt* | *G_Hbb* | *G_Hcc* | *G_Hmm* | *G_Htautau* | *G_H3* |
G_H4
| *G_HGaZ* | *G_HGaGa* | *G_Hgg*
| *G_HGaZ_anom* | *G_HGaGa_anom* | *G_HZZ_anom* | *G_HWW_anom*
| *G_HGaZ_u* | *G_HZZ_u* | *G_HWW_u*
| *Gs* | *I_Gs* | *G2*
| *Mass* of *flavor* | *Width* of *flavor*
| *K_Matrix_Coeff* of *int* | *K_Matrix_Pole* of *int*
| *I_Dim6_AWW_Gauge* | *I_Dim6_AWW_GGG* | *I_Dim6_AWW_DP* |
I_Dim6_AWW_DW

    | *I_Dim6_WWZ_W* | *I_Dim6_WWZ_DPWDW* | *I_Dim6_WWZ_DW* |
I_Dim6_WWZ_D
    | *G_HZZ6_V3* | *G_HZZ6_D* | *G_HZZ6_DP* | *G_HZZ6_PB*
    | *G_HWW_6_D* | *G_HWW_6_DP*
    | *G_HGaZ6_D* | *G_HGaZ6_DP* | *G_HGaZ6_PB*
    | *G_HGaGa6*
    | *Dim6_vev3* | *Dim6_Cphi* | *Anom_Dim6_AAWW_DW* | *Anom_Dim6_AAWW_W*
    | *Anom_Dim6_H4_v2* | *Anom_Dim6_H4_P2*
    | *Anom_Dim6_AHWW_DPB* | *Anom_Dim6_AHWW_DPW* | *Anom_Dim6_AHWW_DW*
    | *Anom_Dim6_HHWW_DW* | *Anom_Dim6_HHWW_DPW*
    | *Anom_Dim6_HWWZ_DW* | *Anom_Dim6_HWWZ_DDPW* | *Anom_Dim6_HWWZ_DPW*
    | *Anom_Dim6_HWWZ_DPB*
    | *Anom_Dim6_AHHZ_D* | *Anom_Dim6_AHHZ_DP* | *Anom_Dim6_AHHZ_PB*
    | *Anom_Dim6_AZWW_W* | *Anom_Dim6_AZWW_DWDPW*
    | *Anom_Dim6_WWWW_W* | *Anom_Dim6_WWWW_DWDPW* |
Anom_Dim6_WWZZ_W
    | *Anom_Dim6_WWZZ_DWDPW*
    | *Anom_Dim6_HHAA* | *Anom_Dim6_HHZZ_D* | *Anom_Dim6_HHZZ_DP*
    | *Anom_Dim6_HHZZ_PB* | *Anom_Dim6_HHZZ_T*

Two integer counters for the QCD and EW order of the couplings.

    type *orders* = *int* × *int*

    let *orders* = function
    | *Q_lepton* | *Q_up* | *Q_down* | *G_NC_lepton* | *G_NC_neutrino*
    | *G_NC_up* | *G_NC_down* | *G_CC* | *G_CCQ _* | *G_Htt* | *G_H3*
    | *G_Hbb* | *G_Hcc* | *G_Htautau* | *G_Hmm* | *I_Q_W*
    | *I_G_ZWW* | *I_G1_AWW* | *I_G1_ZWW* | *I_G_weak*
    | *G_HWW* | *G_HZZ* | *G_HWW_u* | *G_HZZ_u* | *G_HGaZ_u*
    | *G_HWW_anom* | *G_HZZ_anom* | *G_HGaZ* | *G_HGaGa* |
G_HGaZ_anom
    | *G_HGaGa_anom* | *Half* | *Unit*
    | *I_G1_plus_kappa_plus_G4_AWW*
    | *I_G1_plus_kappa_plus_G4_ZWW*
    | *I_G1_minus_kappa_plus_G4_AWW*
    | *I_G1_minus_kappa_plus_G4_ZWW*
    | *I_G1_plus_kappa_minus_G4_AWW*
    | *I_G1_plus_kappa_minus_G4_ZWW*
    | *I_G1_minus_kappa_minus_G4_AWW*
    | *I_G1_minus_kappa_minus_G4_ZWW* | *I_kappa5_AWW*
    | *I_kappa5_ZWW* | *G5_AWW* | *G5_ZWW*
    | *I_lambda_AWW* | *I_lambda_ZWW* | *I_lambda5_AWW*
    | *I_lambda5_ZWW* | *G_TVA_ttA* | *G_TVA_bbA* | *G_TVA_tcA*
    | *G_VLR_ttZ* | *G_TVA_ttZ* | *G_VLR_tcZ* | *G_TVA_tcZ* |
G_TVA_bbZ
    | *VA_ILC_ttA* | *VA_ILC_ttZ*
    | *G_VLR_btW* | *G_VLR_tbW* | *G_TLR_btW* | *G_TRL_tbW*
    | *G_TLR_btWA* | *G_TRL_tbWA* | *G_TLR_btWZ* | *G_TRL_tbWZ*
    | *G_VLR_qBuB* | *G_VLR_qBuB_u* | *G_VLR_qBuB_d*
    | *G_VLR_qBuB_e* | *G_VL_qBuB_n* | *G_VL_qW* | *G_VL_qW_u* |
G_VL_qW_d

     | *G_SL_DttR* | *G_SR_DttR* | *G_SL_DttL* | *G_SLR_DbtR* |
G_SL_DbtL
     | *G_HZZ6_V3* | *G_HZZ6_D* | *G_HZZ6_DP* | *G_HZZ6_PB*
     | *G_HGaZ6_D* | *G_HGaZ6_DP* | *G_HGaZ6_PB*
     | *G_HWW_6_D* | *G_HWW_6_DP*
     | *G_HGaGa6*
     | *I_Dim6_AWW_Gauge* | *I_Dim6_AWW_GGG* | *I_Dim6_AWW_DP* |
I_Dim6_AWW_DW
     | *I_Dim6_WWZ_W* | *I_Dim6_WWZ_DPWDW* | *I_Dim6_WWZ_DW* |
I_Dim6_WWZ_D
     | *Dim6_vev3* | *Dim6_Cphi*
     | *Anom_Dim6_H4_v2* | *Anom_Dim6_H4_P2* | *Anom_Dim6_AAWW_DW*
     | *Anom_Dim6_AAWW_W*
     | *Anom_Dim6_AHWW_DPB* | *Anom_Dim6_AHWW_DPW* | *Anom_Dim6_AHWW_DW*
     | *Anom_Dim6_HHWW_DW* | *Anom_Dim6_HHWW_DPW*
     | *Anom_Dim6_HWWZ_DW* | *Anom_Dim6_HWWZ_DDPW* | *Anom_Dim6_HWWZ_DPW*
     | *Anom_Dim6_HWWZ_DPB*
     | *Anom_Dim6_AHHZ_D* | *Anom_Dim6_AHHZ_DP* | *Anom_Dim6_AHHZ_PB*
     | *Anom_Dim6_AZWW_W* | *Anom_Dim6_AZWW_DWDPW*
     | *Anom_Dim6_WWWW_W* | *Anom_Dim6_WWWW_DWDPW* |
Anom_Dim6_WWZZ_W
     | *Anom_Dim6_WWZZ_DWDPW*
     | *Anom_Dim6_HHAA* | *Anom_Dim6_HHZZ_D* | *Anom_Dim6_HHZZ_DP*
     | *Anom_Dim6_HHZZ_PB* | *Anom_Dim6_HHZZ_T*
     | *G_TVA_ttWW* | *G_TVA_bbWW* | *G_SP_ttH* $\rightarrow$ $(0,1)$
     | *G_HHWW* | *G_HHZZ* | *G_H4*
     | *G_WWWW* | *G_ZZWW* | *G_AZWW* | *G_AAWW*
     | *Alpha_WWWW0* | *Alpha_WWWW2* | *Alpha_ZZWW0*
     | *Alpha_ZZWW1* | *Alpha_ZZZZ*
     | *D_Alpha_WWWW0_S* | *D_Alpha_WWWW0_T* | *D_Alpha_WWWW0_U*
     | *D_Alpha_WWWW2_S* | *D_Alpha_WWWW2_T* | *D_Alpha_ZZWW0_S*
     | *D_Alpha_ZZWW0_T* | *D_Alpha_ZZWW1_S* | *D_Alpha_ZZWW1_T*
     | *D_Alpha_ZZWW1_U* | *D_Alpha_ZZZZ_S* | *D_Alpha_ZZZZ_T* $\rightarrow$
$(0,2)$
     | *Gs* | *I_Gs* | *G_TVA_ttG* | *G_TVA_ttGG* | *G_TVA_tcG* |
G_TVA_tcGG
     | *G_VLR_qGuG*
     | *C_quqd1R_bt* | *C_quqd1R_tb* | *C_quqd1L_bt* | *C_quqd1L_tb*
     | *C_quqd8R_bt* | *C_quqd8R_tb* | *C_quqd8L_bt* | *C_quqd8L_tb* $\rightarrow$
$(1,0)$
     | *G2* | *G_Hgg* $\rightarrow$ $(2,0)$
     (∗ These constants are not used, hence initialized to zero. ∗)
     | *Sinthw* | *Sin2thw* | *Costhw* | *Pi*
     | *Alpha_QED* | *G_weak* | *K_Matrix_Coeff* _
     | *K_Matrix_Pole* _ | *Mass* _ | *Width* _ | *Vev* | *E* $\rightarrow$ $(0,0)$

⚠ The current abstract syntax for parameter dependencies is admittedly te-
dious. Later, there will be a parser for a convenient concrete syntax as a
part of a concrete syntax for models. But as these examples show, it should
include simple functions.

$$\alpha_{\text{QED}} = \frac{1}{137.0359895} \tag{13.1a}$$

$$\sin^2\theta_w = 0.23124 \tag{13.1b}$$

let *input_parameters* =
[*Alpha_QED*, 1. /. 137.0359895;
  Sin2thw, 0.23124;
  Mass (*G Z*), 91.187;
  Mass (*M* (*N* 1)), 0.0; *Mass* (*M* (*L* 1)), 0.51099907 · 10⁻³;
  Mass (*M* (*N* 2)), 0.0; *Mass* (*M* (*L* 2)), 0.105658389;
  Mass (*M* (*N* 3)), 0.0; *Mass* (*M* (*L* 3)), 1.77705;
  Mass (*M* (*U* 1)), 5.0 · 10⁻³; *Mass* (*M* (*D* 1)), 3.0 · 10⁻³;
  Mass (*M* (*U* 2)), 1.2; *Mass* (*M* (*D* 2)), 0.1;
  Mass (*M* (*U* 3)), 174.0; *Mass* (*M* (*D* 3)), 4.2]

$$e = \sqrt{4\pi\alpha} \tag{13.2a}$$

$$\sin\theta_w = \sqrt{\sin^2\theta_w} \tag{13.2b}$$

$$\cos\theta_w = \sqrt{1 - \sin^2\theta_w} \tag{13.2c}$$

$$g = \frac{e}{\sin\theta_w} \tag{13.2d}$$

$$m_W = \cos\theta_w m_Z \tag{13.2e}$$

$$v = \frac{2m_W}{g} \tag{13.2f}$$

$$g_{CC} = -\frac{g}{2\sqrt{2}} = -\frac{e}{2\sqrt{2}\sin\theta_w} \tag{13.2g}$$

$$Q_{\text{lepton}} = -q_{\text{lepton}}e = e \tag{13.2h}$$

$$Q_{\text{up}} = -q_{\text{up}}e = -\frac{2}{3}e \tag{13.2i}$$

$$Q_{\text{down}} = -q_{\text{down}}e = \frac{1}{3}e \tag{13.2j}$$

$$iq_W e = ig_{\gamma WW} = ie \tag{13.2k}$$

$$ig_{ZWW} = ig\cos\theta_w \tag{13.2l}$$

$$ig_{WWW} = ig \tag{13.2m}$$

... to be continued ... The quartic couplings can't be correct, because the dimensions are wrong!

$$g_{HWW} = gm_W = 2\frac{m_W^2}{v} \tag{13.3a}$$

$$g_{HHWW} = 2\frac{m_W^2}{v^2} = \frac{g^2}{2} \tag{13.3b}$$

$$g_{HZZ} = \frac{g}{\cos\theta_w}m_Z \tag{13.3c}$$

$$g_{HHZZ} = 2\frac{m_Z^2}{v^2} = \frac{g^2}{2\cos\theta_w} \tag{13.3d}$$

$$g_{Htt} = \lambda_t \tag{13.3e}$$

$$g_{Hbb} = \lambda_b = \frac{m_b}{m_t}\lambda_t \tag{13.3f}$$

$$g_{H^3} = -\frac{3g}{2}\frac{m_H^2}{m_W} = -3\frac{m_H^2}{v}g_{H^4} \quad = -\frac{3g^2}{4}\frac{m_W^2}{v^2} = -3\frac{m_H^2}{v^2} \tag{13.3g}$$

let *derived_parameters* =
  [*Real E*, *Sqrt* (*Prod* [*Const 4*; *Atom Pi*; *Atom Alpha_QED*]);
    Real Sinthw, *Sqrt* (*Atom Sin2thw*);
    Real Costhw, *Sqrt* (*Diff* (*Const 1*, *Atom Sin2thw*));
    Real G_weak, *Quot* (*Atom E*, *Atom Sinthw*);
    Real (*Mass* (*G Wp*)), *Prod* [*Atom Costhw*; *Atom* (*Mass* (*G Z*))];
    Real Vev, *Quot* (*Prod* [*Const 2*; *Atom* (*Mass* (*G Wp*))], *Atom G_weak*);
    Real Q_lepton, *Atom E*;
    Real Q_up, *Prod* [*Quot* (*Const* (−2), *Const 3*); *Atom E*];
    Real Q_down, *Prod* [*Quot* (*Const 1*, *Const 3*); *Atom E*];
    Real G_CC, *Neg* (*Quot* (*Atom G_weak*, *Prod* [*Const 2*; *Sqrt* (*Const 2*)]));
    Complex I_Q_W, *Prod* [*I*; *Atom E*];
    Complex I_G_weak, *Prod* [*I*; *Atom G_weak*];
    Complex I_G_ZWW, *Prod* [*I*; *Atom G_weak*; *Atom Costhw*]]

$$-\frac{g}{2\cos\theta_w} \tag{13.4}$$

let *g_over_2_costh* =
  Quot (*Neg* (*Atom G_weak*), *Prod* [*Const 2*; *Atom Costhw*])

$$-\frac{g}{2\cos\theta_w}g_V = -\frac{g}{2\cos\theta_w}(T_3 - 2q\sin^2\theta_w) \tag{13.5a}$$

$$-\frac{g}{2\cos\theta_w}g_A = -\frac{g}{2\cos\theta_w}T_3 \tag{13.5b}$$

let *nc_coupling c t3 q* =
  (*Real_Array c*,
   [*Prod* [*g_over_2_costh*; *Diff* (*t3*, *Prod* [*Const 2*; *q*; *Atom Sin2thw*])];
     Prod [*g_over_2_costh*; *t3*]])

let *half* = *Quot* (*Const 1*, *Const 2*)

let *derived_parameter_arrays* =
  [*nc_coupling G_NC_neutrino half* (*Const 0*);
    nc_coupling G_NC_lepton (*Neg half*) (*Const* (−1));
    nc_coupling G_NC_up half (*Quot* (*Const 2*, *Const 3*));
    nc_coupling G_NC_down (*Neg half*) (*Quot* (*Const* (−1), *Const 3*))]

let *parameters* () =
  { *input* = *input_parameters*;
    derived = *derived_parameters*;

    derived_arrays = *derived_parameter_arrays* }

module *F* = *Modeltools.Fusions* (struct
  type *f* = *flavor*
  type *c* = *constant*
  let *compare* = *compare*
  let *conjugate* = *conjugate*
end)

$$\mathcal{L}_{\text{EM}} = -e \sum_i q_i \bar{\psi}_i A\!\!\!/ \psi_i \tag{13.6}$$

let *mgm* ((*m1*, *g*, *m2*), *fbf*, *c*) = ((*M m1*, *G g*, *M m2*), *fbf*, *c*)
let *mom* ((*m1*, *o*, *m2*), *fbf*, *c*) = ((*M m1*, *O o*, *M m2*), *fbf*, *c*)

let *electromagnetic_currents* *n* =
  List.map mgm
   [((*L* (−*n*), *Ga*, *L n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_lepton*);
    ((*U* (−*n*), *Ga*, *U n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
    ((*D* (−*n*), *Ga*, *D n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

let *color_currents* *n* =
  List.map mgm
   [((*U* (−*n*), *Gl*, *U n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*);
    ((*D* (−*n*), *Gl*, *D n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*)]

$$\mathcal{L}_{\text{NC}} = -\frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i Z\!\!\!/ (g_V^i - g_A^i \gamma_5)\psi_i \tag{13.7}$$

let *neutral_currents* *n* =
  List.map mgm
   [((*L* (−*n*), *Z*, *L n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_lepton*);
    ((*N* (−*n*), *Z*, *N n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_neutrino*);
    ((*U* (−*n*), *Z*, *U n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_up*);
    ((*D* (−*n*), *Z*, *D n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_down*)]

$$\mathcal{L}_{\text{CC}} = -\frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i (T^+ W\!\!\!/^+ + T^- W\!\!\!/^-)(1-\gamma_5)\psi_i \tag{13.8}$$

let *charged_currents′* *n* =
  List.map mgm
   [((*L* (−*n*), *Wm*, *N n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
    ((*N* (−*n*), *Wp*, *L n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*)]

let *charged_currents″* *n* =
  List.map mgm
   [((*D* (−*n*), *Wm*, *U n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
    ((*U* (−*n*), *Wp*, *D n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*)]

let *charged_currents_triv* =
  ThoList.flatmap charged_currents′ [1; 2; 3] @
  ThoList.flatmap charged_currents″ [1; 2; 3]

```
let charged_currents_ckm =
  let charged_currents_2 n1 n2 =
    List.map mgm
      [ ((D (−n1), Wm, U n2), FBF (1, Psibar, VL, Psi), G_CCQ (n2, n1));
        ((U (−n1), Wp, D n2), FBF (1, Psibar, VL, Psi), G_CCQ (n1, n2)) ] in
  ThoList.flatmap charged_currents' [1; 2; 3] @
  List.flatten (Product.list2 charged_currents_2 [1; 2; 3] [1; 2; 3])

let yukawa =
  [ ((M (U (−3)), O H, M (U 3)), FBF (1, Psibar, S, Psi), G_Htt);
    ((M (D (−3)), O H, M (D 3)), FBF (1, Psibar, S, Psi), G_Hbb);
    ((M (U (−2)), O H, M (U 2)), FBF (1, Psibar, S, Psi), G_Hcc);
    ((M (L (−3)), O H, M (L 3)), FBF (1, Psibar, S, Psi), G_Htautau) ] @
  if Flags.higgs_hmm then
  [ ((M (L (−2)), O H, M (L 2)), FBF (1, Psibar, S, Psi), G_Hmm)]
      else
  [ ]
```

$$\mathcal{L}_{\text{TGC}} = -e\partial_\mu A_\nu W_+^\mu W_-^\nu + \ldots - e\cot\theta_w \partial_\mu Z_\nu W_+^\mu W_-^\nu + \ldots \qquad (13.9)$$

```
let tgc ((g1, g2, g3), t, c) = ((G g1, G g2, G g3), t, c)

let standard_triple_gauge =
  List.map tgc
    [ ((Ga, Wm, Wp), Gauge_Gauge_Gauge 1, I_Q_W);
      ((Z, Wm, Wp), Gauge_Gauge_Gauge 1, I_G_ZWW);
      ((Gl, Gl, Gl), Gauge_Gauge_Gauge 1, I_Gs)]
```

$$
\begin{aligned}
\mathcal{L}_{\text{TGC}}(g_1, \kappa) = & \, g_1 \mathcal{L}_T(V, W^+, W^-) \\
& + \frac{\kappa + g_1}{2}\Big(\mathcal{L}_T(W^-, V, W^+) - \mathcal{L}_T(W^+, V, W^-)\Big) \\
& + \frac{\kappa - g_1}{2}\Big(\mathcal{L}_L(W^-, V, W^+) - \mathcal{L}_T(W^+, V, W^-)\Big) \quad (13.10)
\end{aligned}
$$

The whole thing in the LEP2 workshop notation:

$$
\begin{aligned}
\mathrm{i}\mathcal{L}_{\text{TGC},V}/g_{WWV} = & \\
g_1^V V^\mu (W_{\mu\nu}^- W^{+,\nu} & - W_{\mu\nu}^+ W^{-,\nu}) + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{m_W^2} V_{\mu\nu} W_{\rho\mu}^- W^{+,\rho}_\nu \\
& + \mathrm{i}g_5^V \epsilon_{\mu\nu\rho\sigma}\big((\partial^\rho W^{-,\mu})W^{+,\nu} - W^{-,\mu}(\partial^\rho W^{+,\nu})\big)V^\sigma \\
+\mathrm{i}g_4^V W_\mu^- W_\nu^+ (\partial^\mu V^\nu + \partial^\nu V^\mu) & - \frac{\tilde{\kappa}_V}{2} W_\mu^- W_\nu^+ \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} - \frac{\tilde{\lambda}_V}{2m_W^2} W_{\rho\mu}^- W^{+,\mu}_\nu \epsilon^{\nu\rho\alpha\beta} V_{\alpha\beta}
\end{aligned}
$$
$$(13.11)$$

using the conventions of Itzykson and Zuber with $\epsilon^{0123} = +1$.

This is equivalent to the notation of Hagiwara et al. [**?**], if we remember that they have opposite signs for g_{WWV}:

$\mathcal{L}_{WWV}/(-g_{WWV}) =$

$\mathrm{i}g_1^V \left(W_{\mu\nu}^\dagger W^\mu - W_\mu^\dagger W^\mu{}_\nu\right) V^\nu + \mathrm{i}\kappa_V W_\mu^\dagger W_\nu V^{\mu\nu} + \mathrm{i}\dfrac{\lambda_V}{m_W^2} W_{\lambda\mu}^\dagger W^\mu{}_\nu V^{\nu\lambda}$

$- g_4^V W_\mu^\dagger W_\nu \left(\partial^\mu V^\nu + \partial^\nu V^\mu\right) + g_5^V \epsilon^{\mu\nu\lambda\sigma} \left(W_\mu^\dagger \overset{\leftrightarrow}{\partial_\lambda} W_\nu\right) V_\sigma$

$+ \mathrm{i}\tilde{\kappa}_V W_\mu^\dagger W_\nu \tilde{V}^{\mu\nu} + \mathrm{i}\dfrac{\tilde{\lambda}_V}{m_W^2} W_{\lambda\mu}^\dagger W^\mu{}_\nu \tilde{V}^{\nu\lambda}$ (13.12)

Here V^μ stands for either the photon or the Z field, W^μ is the W^- field, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and $\tilde{V}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}V^{\lambda\sigma}$.

```
let anomalous_triple_gauge  =
  List.map tgc
    [ ((Ga,  Wm,  Wp),  Dim4_Vector_Vector_Vector_T (−1),
       I_G1_AWW);
      ((Z,  Wm,  Wp),  Dim4_Vector_Vector_Vector_T (−1),
       I_G1_ZWW);
      ((Wm,  Ga,  Wp),  Dim4_Vector_Vector_Vector_T 1,
       I_G1_plus_kappa_minus_G4_AWW);
      ((Wm,  Z,  Wp),  Dim4_Vector_Vector_Vector_T 1,
       I_G1_plus_kappa_minus_G4_ZWW);
      ((Wp,  Ga,  Wm),  Dim4_Vector_Vector_Vector_T (−1),
       I_G1_plus_kappa_plus_G4_AWW);
      ((Wp,  Z,  Wm),  Dim4_Vector_Vector_Vector_T (−1),
       I_G1_plus_kappa_plus_G4_ZWW);
      ((Wm,  Ga,  Wp),  Dim4_Vector_Vector_Vector_L (−1),
       I_G1_minus_kappa_plus_G4_AWW);
      ((Wm,  Z,  Wp),  Dim4_Vector_Vector_Vector_L (−1),
       I_G1_minus_kappa_plus_G4_ZWW);
      ((Wp,  Ga,  Wm),  Dim4_Vector_Vector_Vector_L 1,
       I_G1_minus_kappa_minus_G4_AWW);
      ((Wp,  Z,  Wm),  Dim4_Vector_Vector_Vector_L 1,
       I_G1_minus_kappa_minus_G4_ZWW);
      ((Ga,  Wm,  Wp),  Dim4_Vector_Vector_Vector_L5 (−1),
       I_kappa5_AWW);
      ((Z,  Wm,  Wp),  Dim4_Vector_Vector_Vector_L5 (−1),
       I_kappa5_ZWW);
      ((Ga,  Wm,  Wp),  Dim4_Vector_Vector_Vector_T5 (−1),
       G5_AWW);
      ((Z,  Wm,  Wp),  Dim4_Vector_Vector_Vector_T5 (−1),
       G5_ZWW);
      ((Ga,  Wp,  Wm),  Dim6_Gauge_Gauge_Gauge (−1),
       I_lambda_AWW);
      ((Z,  Wp,  Wm),  Dim6_Gauge_Gauge_Gauge (−1),
       I_lambda_ZWW);
      ((Ga,  Wp,  Wm),  Dim6_Gauge_Gauge_Gauge_5 (−1),
       I_lambda5_AWW);
      ((Z,  Wp,  Wm),  Dim6_Gauge_Gauge_Gauge_5 (−1),
       I_lambda5_ZWW) ]
```

let *anomalous_dim6_triple_gauge* =
  List.map tgc
    [((*Ga*, *Wm*, *Wp*), *Dim6_Gauge_Gauge_Gauge_i* 1,
      I_Dim6_AWW_GGG);
    ((*Ga*, *Wm*, *Wp*), *Dim6_AWW_DP* 1,
      I_Dim6_AWW_DP);
    ((*Ga*, *Wm*, *Wp*), *Dim6_AWW_DW* 1,
      I_Dim6_AWW_DW);
    ((*Wm*, *Wp*, *Z*), *Dim6_Gauge_Gauge_Gauge_i* 1,
      I_Dim6_WWZ_W);
    ((*Wm*, *Wp*, *Z*), *Dim6_WWZ_DPWDW* 1,
      I_Dim6_WWZ_DPWDW);
    ((*Wm*, *Wp*, *Z*), *Dim6_WWZ_DW* 1,
      I_Dim6_WWZ_DW);
    ((*Wm*, *Wp*, *Z*), *Dim6_WWZ_D* 1,
      I_Dim6_WWZ_D)]

let *triple_gauge* =
  if *Flags.triple_anom* then
    anomalous_triple_gauge
  else if *Flags.dim6* then
    standard_triple_gauge @ *anomalous_dim6_triple_gauge*
  else
    standard_triple_gauge

$$\mathcal{L}_{\mathrm{QGC}} = -g^2 W_{+,\mu} W_{-,\nu} W_+^\mu W_-^\nu + \ldots \tag{13.13}$$

Actually, quartic gauge couplings are a little bit more straightforward using auxiliary fields. Here we have to impose the antisymmetry manually:

$$(W_1^{+,\mu} W_2^{-,\nu} - W_1^{+,\nu} W_2^{-,\mu})(W_{3,\mu}^+ W_{4,\nu}^- - W_{3,\nu}^+ W_{4,\mu}^-)$$
$$= 2(W_1^+ W_3^+)(W_2^- W_4^-) - 2(W_1^+ W_4^-)(W_2^- W_3^+) \tag{13.14a}$$

also (*V* can be *A* or *Z*)

$$(W_1^{+,\mu} V_2^\nu - W_1^{+,\nu} V_2^\mu)(W_{3,\mu}^- V_{4,\nu} - W_{3,\nu}^- V_{4,\mu})$$
$$= 2(W_1^+ W_3^-)(V_2 V_4) - 2(W_1^+ V_4)(V_2 W_3^-) \tag{13.14b}$$

$$W^{+,\mu} W^{-,\nu} W_\mu^+ W_\nu^- \tag{13.15a}$$

let *qgc* ((*g1*, *g2*, *g3*, *g4*), *t*, *c*) = ((*G g1*, *G g2*, *G g3*, *G g4*), *t*, *c*)

let *gauge4* = *Vector4* [(2, *C_13_42*); (−1, *C_12_34*); (−1, *C_14_23*)]
let *minus_gauge4* = *Vector4* [(−2, *C_13_42*); (1, *C_12_34*); (1, *C_14_23*)]
let *standard_quartic_gauge* =
  List.map qgc
    [(*Wm*, *Wp*, *Wm*, *Wp*), *gauge4*, *G_WWWW*;
    (*Wm*, *Z*, *Wp*, *Z*), *minus_gauge4*, *G_ZZWW*;

$(Wm, \; Z, \; Wp, \; Ga), \; minus\_gauge4, \; G\_AZWW;$
$(Wm, \; Ga, \; Wp, \; Ga), \; minus\_gauge4, \; G\_AAWW;$
$(Gl, \; Gl, \; Gl, \; Gl), \; gauge4, \; G2 \;]$

$$\mathcal{L}_4 = \alpha_4 \left(\frac{g^4}{2} \left((W_\mu^+ W^{-,\mu})^2 + W_\mu^+ W^{+,\mu} W_\mu^- W^{-,\mu} \right) \right.$$
$$\left. + \frac{g^4}{\cos^2 \theta_w} W_\mu^+ Z^\mu W_\nu^- Z^\nu + \frac{g^4}{4 \cos^4 \theta_w} (Z_\mu Z^\mu)^2 \right) \qquad (13.16a)$$

$$\mathcal{L}_5 = \alpha_5 \left(g^4 (W_\mu^+ W^{-,\mu})^2 + \frac{g^4}{\cos^2 \theta_w} W_\mu^+ W^{-,\mu} Z_\nu Z^\nu + \frac{g^4}{4 \cos^4 \theta_w} (Z_\mu Z^\mu)^2 \right)$$
$$(13.16b)$$

or

$$\mathcal{L}_4 + \mathcal{L}_5 = (\alpha_4 + 2\alpha_5) g^4 \frac{1}{2} (W_\mu^+ W^{-,\mu})^2$$
$$+ 2\alpha_4 g^4 \frac{1}{4} W_\mu^+ W^{+,\mu} W_\mu^- W^{-,\mu} + \alpha_4 \frac{g^4}{\cos^2 \theta_w} W_\mu^+ Z^\mu W_\nu^- Z^\nu$$
$$+ 2\alpha_5 \frac{g^4}{\cos^2 \theta_w} \frac{1}{2} W_\mu^+ W^{-,\mu} Z_\nu Z^\nu + (2\alpha_4 + 2\alpha_5) \frac{g^4}{\cos^4 \theta_w} \frac{1}{8} (Z_\mu Z^\mu)^2 \quad (13.17)$$

and therefore

$$\alpha_{(WW)_0} = (\alpha_4 + 2\alpha_5) g^4 \qquad (13.18a)$$

$$\alpha_{(WW)_2} = 2\alpha_4 g^4 \qquad (13.18b)$$

$$\alpha_{(WZ)_0} = 2\alpha_5 \frac{g^4}{\cos^2 \theta_w} \qquad (13.18c)$$

$$\alpha_{(WZ)_1} = \alpha_4 \frac{g^4}{\cos^2 \theta_w} \qquad (13.18d)$$

$$\alpha_{ZZ} = (2\alpha_4 + 2\alpha_5) \frac{g^4}{\cos^4 \theta_w} \qquad (13.18e)$$

```
let anomalous_quartic_gauge =
  if Flags.quartic_anom then
    List.map qgc
      [ ((Wm, Wm, Wp, Wp),
         Vector4 [(1, C_13_42); (1, C_14_23)], Alpha_WWWW0);
        ((Wm, Wm, Wp, Wp),
         Vector4 [1, C_12_34], Alpha_WWWW2);
        ((Wm, Wp, Z, Z),
         Vector4 [1, C_12_34], Alpha_ZZWW0);
        ((Wm, Wp, Z, Z),
         Vector4 [(1, C_13_42); (1, C_14_23)], Alpha_ZZWW1);
        ((Z, Z, Z, Z),
         Vector4 [(1, C_12_34); (1, C_13_42); (1, C_14_23)], Alpha_ZZZZ) ]
  else
    []
```

let *anomalous_dim6_quartic_gauge* =
  if *Flags.dim6* then
    List.map qgc
      [((*Ga, Ga, Wm, Wp*),
          Dim6_Vector4_DW 1, *Anom_Dim6_AAWW_DW*);
        ((*Ga, Ga, Wm, Wp*),
          Dim6_Vector4_W 1, *Anom_Dim6_AAWW_W*);
        ((*Ga, Z, Wm, Wp*),
          Dim6_Vector4_W 1, *Anom_Dim6_AZWW_W*);
        ((*Ga, Z, Wm, Wp*),
          Dim6_Vector4_DW 1, *Anom_Dim6_AZWW_DWDPW*);
        ((*Wm, Wp, Wm, Wp*),
          Dim6_Vector4_W 1, *Anom_Dim6_WWWW_W*);
        ((*Wm, Wp, Wm, Wp*),
          Dim6_Vector4_DW 1, *Anom_Dim6_WWWW_DWDPW*);
        ((*Z, Z, Wm, Wp*),
          Dim6_Vector4_W 1, *Anom_Dim6_WWZZ_W*);
        ((*Z, Z, Wm, Wp*),
          Dim6_Vector4_DW 1, *Anom_Dim6_WWZZ_DWDPW*)
      ]
    else
      []

In any diagonal channel χ, the scattering amplitude $a_\chi(s)$ is unitary iff[1]

$$\mathrm{Im}\left(\frac{1}{a_\chi(s)}\right) = -1 \qquad (13.20)$$

For a real perturbative scattering amplitude $r_\chi(s)$ this can be enforced easily–and arbitrarily–by

$$\frac{1}{a_\chi(s)} = \frac{1}{r_\chi(s)} - \mathrm{i} \qquad (13.21)$$

let *k_matrix_quartic_gauge* =
  if *Flags.k_matrix* then
    List.map qgc
      [((*Wm, Wp, Wm, Wp*), *Vector4_K_Matrix_jr* (0,
              [(1, *C_12_34*)]), *D_Alpha_WWWW0_S*);
        ((*Wm, Wp, Wm, Wp*), *Vector4_K_Matrix_jr* (0,
              [(1, *C_14_23*)]), *D_Alpha_WWWW0_T*);
        ((*Wm, Wp, Wm, Wp*), *Vector4_K_Matrix_jr* (0,
              [(1, *C_13_42*)]), *D_Alpha_WWWW0_U*);
        ((*Wp, Wm, Wp, Wm*), *Vector4_K_Matrix_jr* (0,
              [(1, *C_12_34*)]), *D_Alpha_WWWW0_S*);
        ((*Wp, Wm, Wp, Wm*), *Vector4_K_Matrix_jr* (0,
              [(1, *C_14_23*)]), *D_Alpha_WWWW0_T*);

_____

[1]Trivial proof:

$$-1 = \mathrm{Im}\left(\frac{1}{a_\chi(s)}\right) = \frac{\mathrm{Im}(a_\chi^*(s))}{|a_\chi(s)|^2} = -\frac{\mathrm{Im}(a_\chi(s))}{|a_\chi(s)|^2} \qquad (13.19)$$

i.e. $\mathrm{Im}(a_\chi(s)) = |a_\chi(s)|^2$.

```
          ((Wp, Wm, Wp, Wm), Vector4_K_Matrix_jr (0,
                  [(1, C_13_42)]), D_Alpha_WWWW0_U);
          ((Wm, Wm, Wp, Wp), Vector4_K_Matrix_jr (0,
                  [(1, C_12_34)]), D_Alpha_WWWW2_S);
          ((Wm, Wm, Wp, Wp), Vector4_K_Matrix_jr (0,
                  [(1, C_13_42); (1, C_14_23)]), D_Alpha_WWWW2_T);
          ((Wm, Wp, Z, Z), Vector4_K_Matrix_jr (0,
                  [(1, C_12_34)]), D_Alpha_ZZWW0_S);
          ((Wm, Wp, Z, Z), Vector4_K_Matrix_jr (0,
                  [(1, C_13_42); (1, C_14_23)]), D_Alpha_ZZWW0_T);
          ((Wm, Z, Wp, Z), Vector4_K_Matrix_jr (0,
                  [(1, C_12_34)]), D_Alpha_ZZWW1_S);
          ((Wm, Z, Wp, Z), Vector4_K_Matrix_jr (0,
                  [(1, C_13_42)]), D_Alpha_ZZWW1_T);
          ((Wm, Z, Wp, Z), Vector4_K_Matrix_jr (0,
                  [(1, C_14_23)]), D_Alpha_ZZWW1_U);
          ((Wp, Z, Z, Wm), Vector4_K_Matrix_jr (1,
                  [(1, C_12_34)]), D_Alpha_ZZWW1_S);
          ((Wp, Z, Z, Wm), Vector4_K_Matrix_jr (1,
                  [(1, C_13_42)]), D_Alpha_ZZWW1_U);
          ((Wp, Z, Z, Wm), Vector4_K_Matrix_jr (1,
                  [(1, C_14_23)]), D_Alpha_ZZWW1_T);
          ((Z, Wp, Wm, Z), Vector4_K_Matrix_jr (2,
                  [(1, C_12_34)]), D_Alpha_ZZWW1_S);
          ((Z, Wp, Wm, Z), Vector4_K_Matrix_jr (2,
                  [(1, C_13_42)]), D_Alpha_ZZWW1_U);
          ((Z, Wp, Wm, Z), Vector4_K_Matrix_jr (2,
                  [(1, C_14_23)]), D_Alpha_ZZWW1_T);
          ((Z, Z, Z, Z), Vector4_K_Matrix_jr (0,
                  [(1, C_12_34)]), D_Alpha_ZZZZ_S);
          ((Z, Z, Z, Z), Vector4_K_Matrix_jr (0,
                  [(1, C_13_42); (1, C_14_23)]), D_Alpha_ZZZZ_T);
          ((Z, Z, Z, Z), Vector4_K_Matrix_jr (3,
                  [(1, C_14_23)]), D_Alpha_ZZZZ_S);
          ((Z, Z, Z, Z), Vector4_K_Matrix_jr (3,
                  [(1, C_13_42); (1, C_12_34)]), D_Alpha_ZZZZ_T)]
    else
      []

let quartic_gauge =
  standard_quartic_gauge @ anomalous_quartic_gauge @
    anomalous_dim6_quartic_gauge @ k_matrix_quartic_gauge

let standard_gauge_higgs =
  [ ((O H, G Wp, G Wm), Scalar_Vector_Vector 1, G_HWW);
    ((O H, G Z, G Z), Scalar_Vector_Vector 1, G_HZZ) ]

let standard_gauge_higgs4 =
  [ (O H, O H, G Wp, G Wm), Scalar2_Vector2 1, G_HHWW;
    (O H, O H, G Z, G Z), Scalar2_Vector2 1, G_HHZZ ]

let standard_higgs =
```

[(*O H*, *O H*, *O H*), *Scalar_Scalar_Scalar* 1, *G_H3*]

let *standard_higgs4* =
[(*O H*, *O H*, *O H*, *O H*), *Scalar4* 1, *G_H4*]

WK's couplings (apparently, he still intends to divide by $\Lambda_{\text{EWSB}}^2 = 16\pi^2 v_{\text{F}}^2$):

$$\mathcal{L}_4^\tau = \left[(\partial_\mu H)(\partial^\mu H) + \frac{g^2 v_{\text{F}}^2}{4} V_\mu V^\mu \right]^2 \tag{13.22a}$$

$$\mathcal{L}_5^\tau = \left[(\partial_\mu H)(\partial_\nu H) + \frac{g^2 v_{\text{F}}^2}{4} V_\mu V_\nu \right]^2 \tag{13.22b}$$

with

$$V_\mu V_\nu = \frac{1}{2} \left(W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^- \right) + \frac{1}{2\cos^2\theta_w} Z_\mu Z_\nu \tag{13.23}$$

(note the symmetrization!), i. e.

$$\mathcal{L}_4 = \alpha_4 \frac{g^4 v_{\text{F}}^4}{16} (V_\mu V_\nu)^2 \tag{13.24a}$$

$$\mathcal{L}_5 = \alpha_5 \frac{g^4 v_{\text{F}}^4}{16} (V_\mu V^\mu)^2 \tag{13.24b}$$

Breaking thinks up

$$\mathcal{L}_4^{\tau, H^4} = [(\partial_\mu H)(\partial^\mu H)]^2 \tag{13.25a}$$

$$\mathcal{L}_5^{\tau, H^4} = [(\partial_\mu H)(\partial^\mu H)]^2 \tag{13.25b}$$

and

$$\mathcal{L}_4^{\tau, H^2 V^2} = \frac{g^2 v_{\text{F}}^2}{2} (\partial_\mu H)(\partial^\mu H) V_\mu V^\mu \tag{13.26a}$$

$$\mathcal{L}_5^{\tau, H^2 V^2} = \frac{g^2 v_{\text{F}}^2}{2} (\partial_\mu H)(\partial_\nu H) V_\mu V_\nu \tag{13.26b}$$

i. e.

$$\mathcal{L}_4^{\tau, H^2 V^2} = \frac{g^2 v_{\text{F}}^2}{2} \left[(\partial_\mu H)(\partial^\mu H) W_\nu^+ W^{-,\nu} + \frac{1}{2\cos^2\theta_w} (\partial_\mu H)(\partial^\mu H) Z_\nu Z^\nu \right]$$
$$\tag{13.27a}$$

$$\mathcal{L}_5^{\tau, H^2 V^2} = \frac{g^2 v_{\text{F}}^2}{2} \left[(W^{+,\mu}\partial_\mu H)(W^{-,\nu}\partial_\nu H) + \frac{1}{2\cos^2\theta_w} (Z^\mu \partial_\mu H)(Z^\nu \partial_\nu H) \right]$$
$$\tag{13.27b}$$

$$\tau_8^4 \mathcal{L}_4^{\tau, H^2 V^2} + \tau_8^5 \mathcal{L}_5^{\tau, H^2 V^2} =$$
$$- \frac{g^2 v_{\text{F}}^2}{2} \left[2\tau_8^4 \frac{1}{2} (\mathrm{i}\partial_\mu H)(\mathrm{i}\partial^\mu H) W_\nu^+ W^{-,\nu} + \tau_8^5 (W^{+,\mu}\mathrm{i}\partial_\mu H)(W^{-,\nu}\mathrm{i}\partial_\nu H) \right.$$
$$\left. + \frac{2\tau_8^4}{\cos^2\theta_w} \frac{1}{4} (\mathrm{i}\partial_\mu H)(\mathrm{i}\partial^\mu H) Z_\nu Z^\nu + \frac{\tau_8^5}{\cos^2\theta_w} \frac{1}{2} (Z^\mu \mathrm{i}\partial_\mu H)(Z^\nu \mathrm{i}\partial_\nu H) \right] \tag{13.28}$$

where the two powers of i make the sign conveniently negative, i. e.

$$\alpha^2_{(\partial H)^2 W^2} = \tau_8^4 g^2 v_{\mathrm{F}}^2 \tag{13.29a}$$

$$\alpha^2_{(\partial H W)^2} = \frac{\tau_8^5 g^2 v_{\mathrm{F}}^2}{2} \tag{13.29b}$$

$$\alpha^2_{(\partial H)^2 Z^2} = \frac{\tau_8^4 g^2 v_{\mathrm{F}}^2}{\cos^2 \theta_w} \tag{13.29c}$$

$$\alpha^2_{(\partial H Z)^2} = \frac{\tau_8^5 g^2 v_{\mathrm{F}}^2}{2\cos^2 \theta_w} \tag{13.29d}$$

let *anomalous_gauge_higgs* =
  [(*O H*, *G Ga*, *G Ga*), *Dim5_Scalar_Gauge2* 1, *G_HGaGa_anom*;
    (*O H*, *G Ga*, *G Z*), *Dim5_Scalar_Gauge2* 1, *G_HGaZ_anom*;
    (*O H*, *G Z*, *G Z*), *Dim5_Scalar_Gauge2* 1, *G_HZZ_anom*;
    (*O H*, *G Wp*, *G Wm*), *Dim5_Scalar_Gauge2* 1, *G_HWW_anom*;
    (*O H*, *G Ga*, *G Z*), *Dim5_Scalar_Vector_Vector_TU* 1, *G_HGaZ_u*;
    (*O H*, *G Z*, *G Z*), *Dim5_Scalar_Vector_Vector_U* 1, *G_HZZ_u*;
    (*O H*, *G Wp*, *G Wm*), *Dim5_Scalar_Vector_Vector_U* 1, *G_HWW_u*
  ]

let *anomalous_dim6_gauge_higgs* =
  [(*O H*, *G Z*, *G Z*), *Scalar_Vector_Vector* 1, *G_HZZ6_V3*;
    (*O H*, *G Z*, *G Z*), *Dim6_Scalar_Vector_Vector_D* 1, *G_HZZ6_D*;
    (*O H*, *G Z*, *G Z*), *Dim6_Scalar_Vector_Vector_DP* 1, *G_HZZ6_DP*;
    (*O H*, *G Z*, *G Z*), *Scalar_Vector_Vector_t* 1, *G_HZZ6_PB*;
    (*O H*, *G Ga*, *G Z*), *Dim6_HAZ_D* 1, *G_HGaZ6_D*;
    (*O H*, *G Ga*, *G Z*), *Dim6_HAZ_DP* 1, *G_HGaZ6_DP*;
    (*O H*, *G Ga*, *G Z*), *Scalar_Vector_Vector_t* 1, *G_HGaZ6_PB*;
    (*O H*, *G Ga*, *G Ga*), *Scalar_Vector_Vector_t* 1, *G_HGaGa6*;
    (*O H*, *G Wm*, *G Wp*), *Dim6_Scalar_Vector_Vector_D* 1, *G_HWW_6_D*;
    (*O H*, *G Wm*, *G Wp*), *Dim6_Scalar_Vector_Vector_DP* 1, *G_HWW_6_DP*
  ]

let *anomalous_gauge_higgs4* =
  []

let *anomalous_dim6_gauge_higgs4* =
  [(*G Ga*, *O H*, *G Wm*, *G Wp*), *Dim6_AHWW_DPB* 1, *Anom_Dim6_AHWW_DPB*;
   (*G Ga*, *O H*, *G Wm*, *G Wp*), *Dim6_AHWW_DPW* 1, *Anom_Dim6_AHWW_DPW*;
   (*G Ga*, *O H*, *G Wm*, *G Wp*), *Dim6_AHWW_DW* 1, *Anom_Dim6_AHWW_DW*;
   (*O H*, *G Wm*, *G Wp*, *G Z*), *Dim6_HWWZ_DW* 1, *Anom_Dim6_HWWZ_DW*;
   (*O H*, *G Wm*, *G Wp*, *G Z*), *Dim6_HWWZ_DDPW* 1, *Anom_Dim6_HWWZ_DDPW*;
   (*O H*, *G Wm*, *G Wp*, *G Z*), *Dim6_HWWZ_DPW* 1, *Anom_Dim6_HWWZ_DPW*;
   (*O H*, *G Wm*, *G Wp*, *G Z*), *Dim6_HWWZ_DPB* 1, *Anom_Dim6_HWWZ_DPB*;
   (*G Ga*, *O H*, *O H*, *G Z*), *Dim6_AHHZ_D* 1, *Anom_Dim6_AHHZ_D*;
   (*G Ga*, *O H*, *O H*, *G Z*), *Dim6_AHHZ_DP* 1, *Anom_Dim6_AHHZ_DP*;
   (*G Ga*, *O H*, *O H*, *G Z*), *Dim6_AHHZ_PB* 1, *Anom_Dim6_AHHZ_PB*;
   (*O H*, *O H*, *G Ga*, *G Ga*), *Dim6_Scalar2_Vector2_PB* 1, *Anom_Dim6_HHAA*;
   (*O H*, *O H*, *G Wm*, *G Wp*), *Dim6_Scalar2_Vector2_D* 1, *Anom_Dim6_HHWW_DW*;
   (*O H*, *O H*, *G Wm*, *G Wp*), *Dim6_Scalar2_Vector2_DP* 1, *Anom_Dim6_HHWW_DPW*;
   (*O H*, *O H*, *G Z*, *G Z*), *Dim6_HHZZ_T* 1, *Anom_Dim6_HHZZ_T*;

```
      (O H, O H, G Z, G Z), Dim6_Scalar2_Vector2_D 1, Anom_Dim6_HHZZ_D;
      (O H, O H, G Z, G Z), Dim6_Scalar2_Vector2_DP 1, Anom_Dim6_HHZZ_DP;
      (O H, O H, G Z, G Z), Dim6_Scalar2_Vector2_PB 1, Anom_Dim6_HHZZ_PB
    ]

let anomalous_higgs  =
  []

let anomalous_dim6_higgs  =
  [(O H, O H, O H), Scalar_Scalar_Scalar 1, Dim6_vev3;
   (O H, O H, O H), Dim6_HHH 1, Dim6_Cphi ]

let higgs_triangle_vertices  =
  if Flags.higgs_triangle then
    [ (O H, G Ga, G Ga), Dim5_Scalar_Gauge2 1, G_HGaGa;
      (O H, G Ga, G Z), Dim5_Scalar_Gauge2 1, G_HGaZ;
      (O H, G Gl, G Gl), Dim5_Scalar_Gauge2 1, G_Hgg ]
  else
    []

let anomalous_higgs4  =
  []

let anomalous_dim6_higgs4  =
  [(O H, O H, O H, O H), Scalar4 1, Anom_Dim6_H4_v2;
   (O H, O H, O H, O H), Dim6_H4_P2 1, Anom_Dim6_H4_P2]

let gauge_higgs  =
  if Flags.higgs_anom then
    standard_gauge_higgs @ anomalous_gauge_higgs
  else if Flags.dim6 then
    standard_gauge_higgs @ anomalous_dim6_gauge_higgs
  else
    standard_gauge_higgs

let gauge_higgs4  =
  if Flags.higgs_anom then
    standard_gauge_higgs4 @ anomalous_gauge_higgs4
  else if Flags.dim6 then
    standard_gauge_higgs4 @ anomalous_dim6_gauge_higgs4
  else
    standard_gauge_higgs4

let higgs  =
  if Flags.higgs_anom then
    standard_higgs @ anomalous_higgs
  else if Flags.dim6 then
    standard_higgs @ anomalous_dim6_higgs
  else
    standard_higgs

let higgs4  =
  if Flags.higgs_anom then
    standard_higgs4 @ anomalous_higgs4
  else if Flags.dim6 then
```

   _standard_higgs4_ @ _anomalous_dim6_higgs4_
  else
   _standard_higgs4_

 let _goldstone_vertices_ =
  [((_O Phi0_, _G Wm_, _G Wp_), _Scalar_Vector_Vector_ 1, _I_G_ZWW_);
   ((_O Phip_, _G Ga_, _G Wm_), _Scalar_Vector_Vector_ 1, _I_Q_W_);
   ((_O Phip_, _G Z_, _G Wm_), _Scalar_Vector_Vector_ 1, _I_G_ZWW_);
   ((_O Phim_, _G Wp_, _G Ga_), _Scalar_Vector_Vector_ 1, _I_Q_W_);
   ((_O Phim_, _G Wp_, _G Z_), _Scalar_Vector_Vector_ 1, _I_G_ZWW_)]

Anomalous trilinear interactions $f_i f_j V$ and ttH:

$$\Delta\mathcal{L}_{tt\gamma} = -e\frac{\upsilon}{\Lambda^2}\bar{t}i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)tA_\mu \qquad (13.30)$$

$$\Delta\mathcal{L}_{tc\gamma} = -e\frac{\upsilon}{\Lambda^2}\bar{t}i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)cA_\mu + \text{h.c.} \qquad (13.31)$$

 let _anomalous_ttA_ =
  if _Flags.top_anom_ then
   [((_M_ (_U_ (−3))), _G Ga_, _M_ (_U_ 3)), _FBF_ (1, _Psibar_, _TVAM_, _Psi_), _G_TVA_ttA_);
    ((_M_ (_U_ (−3))), _G Ga_, _M_ (_U_ 2)), _FBF_ (1, _Psibar_, _TVAM_, _Psi_), _G_TVA_tcA_);
    ((_M_ (_U_ (−2))), _G Ga_, _M_ (_U_ 3)), _FBF_ (1, _Psibar_, _TVAM_, _Psi_), _G_TVA_tcA_)]
  else
   [ ]

 let _tt_threshold_ttA_ =
  if _Flags.tt_threshold_ then
   [((_M_ (_U_ (−3))), _G Ga_, _M_ (_U_ 3)), _FBF_ (1, _Psibar_, _VAM_, _Psi_), _VA_ILC_ttA_)]
  else
   [ ]

$$\Delta\mathcal{L}_{bb\gamma} = -e\frac{\upsilon}{\Lambda^2}\bar{b}i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)bA_\mu \qquad (13.32)$$

 let _anomalous_bbA_ =
  if _Flags.top_anom_ then
   [((_M_ (_D_ (−3))), _G Ga_, _M_ (_D_ 3)), _FBF_ (1, _Psibar_, _TVAM_, _Psi_), _G_TVA_bbA_)]
  else
   [ ]

$$\Delta\mathcal{L}_{ttg} = -g_s\frac{\upsilon}{\Lambda^2}\bar{t}\lambda^a i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)tG_\mu^a \qquad (13.33)$$

$$\Delta\mathcal{L}_{tcg} = -g_s\frac{\upsilon}{\Lambda^2}\bar{t}\lambda^a i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)cG_\mu^a + \text{h.c.} \qquad (13.34)$$

 let _anomalous_ttG_ =
  if _Flags.top_anom_ then
   [((_M_ (_U_ (−3))), _G Gl_, _M_ (_U_ 3)), _FBF_ (1, _Psibar_, _TVAM_, _Psi_), _G_TVA_ttG_);
    ((_M_ (_U_ (−3))), _G Gl_, _M_ (_U_ 2)), _FBF_ (1, _Psibar_, _TVAM_, _Psi_), _G_TVA_tcG_);
    ((_M_ (_U_ (−2))), _G Gl_, _M_ (_U_ 3)), _FBF_ (1, _Psibar_, _TVAM_, _Psi_), _G_TVA_tcG_)]

```
else
   []
```

$$\Delta\mathcal{L}_{ttZ} = -\frac{g}{2c_W}\frac{v^2}{\Lambda^2}\left[\bar{t}\slashed{Z}(X_L(k^2)P_L + X_R(k^2)P_R)t + \bar{t}\frac{i\sigma^{\mu\nu}k_\nu}{m_Z}(d_V(k^2) + id_A(k^2)\gamma_5)tZ_\mu\right]$$
(13.35)

$$\Delta\mathcal{L}_{tcZ} = -\frac{g}{2c_W}\frac{v^2}{\Lambda^2}\left[\bar{t}\slashed{Z}(X_L(k^2)P_L + X_R(k^2)P_R)c + \bar{t}\frac{i\sigma^{\mu\nu}k_\nu}{m_Z}(d_V(k^2) + id_A(k^2)\gamma_5)cZ_\mu\right] + \text{h.c.}$$
(13.36)

```
let anomalous_ttZ =
  if Flags.top_anom then
    [ ((M (U (−3)), G Z, M (U 3)), FBF (1, Psibar, VLRM, Psi), G_VLR_ttZ);
      ((M (U (−3)), G Z, M (U 2)), FBF (1, Psibar, VLRM, Psi), G_VLR_tcZ);
      ((M (U (−2)), G Z, M (U 3)), FBF (1, Psibar, VLRM, Psi), G_VLR_tcZ);
      ((M (U (−3)), G Z, M (U 3)), FBF (1, Psibar, TVAM, Psi), G_TVA_ttZ);
      ((M (U (−2)), G Z, M (U 3)), FBF (1, Psibar, TVAM, Psi), G_TVA_tcZ);
      ((M (U (−3)), G Z, M (U 2)), FBF (1, Psibar, TVAM, Psi), G_TVA_tcZ) ]
  else
    []

let tt_threshold_ttZ =
  if Flags.tt_threshold then
    [ ((M (U (−3)), G Z, M (U 3)), FBF (1, Psibar, VAM, Psi), VA_ILC_ttZ) ]
  else
    []
```

$$\Delta\mathcal{L}_{bbZ} = -\frac{g}{2c_W}\frac{v^2}{\Lambda^2}\bar{b}\frac{i\sigma^{\mu\nu}k_\nu}{m_Z}(d_V(k^2) + id_A(k^2)\gamma_5)bZ_\mu$$
(13.37)

```
let anomalous_bbZ =
  if Flags.top_anom then
    [ ((M (D (−3)), G Z, M (D 3)), FBF (1, Psibar, TVAM, Psi), G_TVA_bbZ) ]
  else
    []
```

$$\Delta\mathcal{L}_{tbW} = -\frac{g}{\sqrt{2}}\frac{v^2}{\Lambda^2}\left[\bar{b}\slashed{W}^-(V_L(k^2)P_L + V_R(k^2)P_R)t + \bar{b}\frac{i\sigma^{\mu\nu}k_\nu}{m_W}(g_L(k^2)P_L + g_R(k^2)P_R)tW_\mu^-\right] + \text{h.c.}$$
(13.38)

```
let anomalous_tbW =
  if Flags.top_anom then
    [ ((M (D (−3)), G Wm, M (U 3)), FBF (1, Psibar, VLRM, Psi), G_VLR_btW);
      ((M (U (−3)), G Wp, M (D 3)), FBF (1, Psibar, VLRM, Psi), G_VLR_tbW);
      ((M (D (−3)), G Wm, M (U 3)), FBF (1, Psibar, TLRM, Psi), G_TLR_btW);
      ((M (U (−3)), G Wp, M (D 3)), FBF (1, Psibar, TRLM, Psi), G_TRL_tbW) ]
  else
```

[]

$$\Delta \mathcal{L}_{ttH} = -\frac{1}{\sqrt{2}} \bar{t}(Y_V(k^2) + iY_A(k^2)\gamma_5)tH \qquad (13.39)$$

let *anomalous_ttH* =
  if *Flags.top_anom* then
    [((M (U (−3)), O H, M (U 3)), FBF (1, Psibar, SPM, Psi), G_SP_ttH)]
  else
    []

quartic fermion-gauge interactions $f_i f_j V_1 V_2$ emerging from gauge-invariant effective operators:

$$\Delta \mathcal{L}_{ttgg} = -\frac{g_s^2}{2} f_{abc} \frac{v}{\Lambda^2} \bar{t}\lambda^a \sigma^{\mu\nu}(d_V(k^2) + id_A(k^2)\gamma_5)tG_\mu^b G_\nu^c \qquad (13.40)$$

$$\Delta \mathcal{L}_{tcgg} = -\frac{g_s^2}{2} f_{abc} \frac{v}{\Lambda^2} \bar{t}\lambda^a \sigma^{\mu\nu}(d_V(k^2) + id_A(k^2)\gamma_5)cG_\mu^b G_\nu^c + \text{h.c.} \qquad (13.41)$$

let *anomalous_ttGG* =
  if *Flags.top_anom* then
    [((M (U (−3)), O (Aux_top (2, 1, 0,true,TTGG)), M (U 3)),
        FBF (1, Psibar, TVA, Psi), G_TVA_ttGG);
      ((M (U (−3)), O (Aux_top (2, 1, 0,true,TCGG)), M (U 2)),
        FBF (1, Psibar, TVA, Psi), G_TVA_tcGG);
      ((M (U (−2)), O (Aux_top (2, 1, 0,true,TCGG)), M (U 3)),
        FBF (1, Psibar, TVA, Psi), G_TVA_tcGG);
      ((O (Aux_top (2, 1, 0,false,TTGG)), G Gl, G Gl),
        Aux_Gauge_Gauge 1, I_Gs);
      ((O (Aux_top (2, 1, 0,false,TCGG)), G Gl, G Gl),
        Aux_Gauge_Gauge 1, I_Gs)]
  else
    []

$$\Delta \mathcal{L}_{tbWA} = -i\sin\theta_w \frac{g^2}{2\sqrt{2}} \frac{v^2}{\Lambda^2} \left[\bar{b}\frac{\sigma^{\mu\nu}}{m_W}(g_L(k^2)P_L + g_R(k^2)P_R)tA_\mu W_\nu^- \right] + \text{h.c.}$$
$$(13.42)$$

let *anomalous_tbWA* =
  if *Flags.top_anom* then
    [((M (D (−3)), O (Aux_top (2, 0, −1,true,TBWA)), M (U 3)), FBF (1, Psibar, TLR, Psi), G_TL
      ((O (Aux_top (2, 0, 1,false,TBWA)), G Ga, G Wm), Aux_Gauge_Gauge 1, I_G_weak);
      ((M (U (−3)), O (Aux_top (2, 0, 1,true,TBWA)), M (D 3)), FBF (1, Psibar, TRL, Psi), G_TRI
      ((O (Aux_top (2, 0, −1,false,TBWA)), G Wp, G Ga), Aux_Gauge_Gauge 1, I_G_weak)]
  else
    []

$$\Delta \mathcal{L}_{tbWZ} = -i\cos\theta_w \frac{g^2}{2\sqrt{2}} \frac{v^2}{\Lambda^2} \left[\bar{b}\frac{\sigma^{\mu\nu}}{m_W}(g_L(k^2)P_L + g_R(k^2)P_R)tZ_\mu W_\nu^- \right] + \text{h.c.}$$
$$(13.43)$$

let *anomalous_tbWZ* =
  if *Flags.top_anom* then
    [$((M\ (D\ (-3)),\ O\ (Aux\_top\ (2,0,-1,\text{true},TBWZ)),\ M\ (U\ 3)),$
        $FBF\ (1,\ Psibar,\ TLR,\ Psi),\ G\_TLR\_btWZ);$
      $((O\ (Aux\_top\ (2,0,1,\text{false},TBWZ)),\ G\ Z,\ G\ Wm),$
        $Aux\_Gauge\_Gauge\ 1,\ I\_G\_weak);$
      $((M\ (U\ (-3)),\ O\ (Aux\_top\ (2,0,1,\text{true},TBWZ)),\ M\ (D\ 3)),$
        $FBF\ (1,\ Psibar,\ TRL,\ Psi),\ G\_TRL\_tbWZ);$
      $((O\ (Aux\_top\ (2,0,-1,\text{false},TBWZ)),\ G\ Wp,\ G\ Z),$
        $Aux\_Gauge\_Gauge\ 1,\ I\_G\_weak)\ ]$
  else
    [ ]

$$\Delta\mathcal{L}_{ttWW} = -i\frac{g^2}{2}\frac{v^2}{\Lambda^2}\bar{t}\frac{\sigma^{\mu\nu}}{m_W}(d_V(k^2) + id_A(k^2)\gamma_5)tW_\mu^- W_\nu^+ \qquad (13.44)$$

let *anomalous_ttWW* =
  if *Flags.top_anom* then
    [$((M\ (U\ (-3)),\ O\ (Aux\_top\ (2,0,0,\text{true},TTWW)),\ M\ (U\ 3)),\ FBF\ (1,\ Psibar,\ TVA,\ Psi),\ G\_TV$
      $((O\ (Aux\_top\ (2,0,0,\text{false},TTWW)),\ G\ Wm,\ G\ Wp),\ Aux\_Gauge\_Gauge\ 1,\ I\_G\_weak)\ ]$
  else
    [ ]

$$\Delta\mathcal{L}_{bbWW} = -i\frac{g^2}{2}\frac{v^2}{\Lambda^2}\bar{b}\frac{\sigma^{\mu\nu}}{m_W}(d_V(k^2) + id_A(k^2)\gamma_5)bW_\mu^- W_\nu^+ \qquad (13.45)$$

let *anomalous_bbWW* =
  if *Flags.top_anom* then
    [$((M\ (D\ (-3)),\ O\ (Aux\_top\ (2,0,0,\text{true},BBWW)),\ M\ (D\ 3)),\ FBF\ (1,\ Psibar,\ TVA,\ Psi),\ G\_TV$
      $((O\ (Aux\_top\ (2,0,0,\text{false},BBWW)),\ G\ Wm,\ G\ Wp),\ Aux\_Gauge\_Gauge\ 1,\ I\_G\_weak)\ ]$
  else
    [ ]

4-fermion contact terms emerging from operator rewriting:

let *anomalous_top_qGuG_tt* =
  [$((M\ (U\ (-3)),\ O\ (Aux\_top\ (1,1,0,\text{true},QGUG)),\ M\ (U\ 3)),\ FBF\ (1,\ Psibar,\ VLR,\ Psi),\ G\_VLR\_q$

let *anomalous_top_qGuG_ff* n =
  List.map mom
    [$((U\ (-n),\ Aux\_top\ (1,1,0,\text{false},QGUG),\ U\ n),\ FBF\ (1,\ Psibar,\ V,\ Psi),\ Unit);$
      $((D\ (-n),\ Aux\_top\ (1,1,0,\text{false},QGUG),\ D\ n),\ FBF\ (1,\ Psibar,\ V,\ Psi),\ Unit)\ ]$

let *anomalous_top_qGuG* =
  if *Flags.top_anom_4f* then
    anomalous_top_qGuG_tt @ *ThoList.flatmap anomalous_top_qGuG_ff* $[1;2;3]$
  else
    [ ]

let *anomalous_top_qBuB_tt* =
  [$((M\ (U\ (-3)),\ O\ (Aux\_top\ (1,0,0,\text{true},QBUB)),\ M\ (U\ 3)),\ FBF\ (1,\ Psibar,\ VLR,\ Psi),\ G\_VLR\_q$

let *anomalous_top_qBuB_ff* n =

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List.map mom
  [((U ($-n$), *Aux_top* (1, 0, 0,false,*QBUB*), U n), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_u*);
    ((D ($-n$), *Aux_top* (1, 0, 0,false,*QBUB*), D n), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_d*);
    ((L ($-n$), *Aux_top* (1, 0, 0,false,*QBUB*), L n), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_e*);
    ((N ($-n$), *Aux_top* (1, 0, 0,false,*QBUB*), N n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qBuB_n*)]

let *anomalous_top_qBuB* =
  if *Flags.top_anom_4f* then
    anomalous_top_qBuB_tt @ *ThoList.flatmap anomalous_top_qBuB_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_qW_tq* =
  [((M (U (-3)), O (*Aux_top* (1, 0, 0,true,*QW*)), M (U 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*);
    ((M (D (-3)), O (*Aux_top* (1, 0, -1,true,*QW*)), M (U 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*);
    ((M (U (-3)), O (*Aux_top* (1, 0, 1,true,*QW*)), M (D 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*)]

let *anomalous_top_qW_ff* n =
  List.map mom
  [((U ($-n$), *Aux_top* (1, 0, 0,false,*QW*), U n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_u*);
    ((D ($-n$), *Aux_top* (1, 0, 0,false,*QW*), D n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_d*);
    ((N ($-n$), *Aux_top* (1, 0, 0,false,*QW*), N n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_u*);
    ((L ($-n$), *Aux_top* (1, 0, 0,false,*QW*), L n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_d*);
    ((D ($-n$), *Aux_top* (1, 0, -1,false,*QW*), U n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
    ((U ($-n$), *Aux_top* (1, 0, 1,false,*QW*), D n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
    ((L ($-n$), *Aux_top* (1, 0, -1,false,*QW*), N n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
    ((N ($-n$), *Aux_top* (1, 0, 1,false,*QW*), L n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*)]

let *anomalous_top_qW* =
  if *Flags.top_anom_4f* then
    anomalous_top_qW_tq @ *ThoList.flatmap anomalous_top_qW_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_DuDd* =
  if *Flags.top_anom_4f* then
    [((M (U (-3)), O (*Aux_top* (0, 0, 0,true,*DR*)), M (U 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *Half*);
      ((M (U (-3)), O (*Aux_top* (0, 0, 0,false,*DR*)), M (U 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *G_SL_DttR*;
      ((M (D (-3)), O (*Aux_top* (0, 0, 0,false,*DR*)), M (D 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *G_SR_DttR*;
      ((M (U (-3)), O (*Aux_top* (0, 0, 0,true,*DL*)), M (U 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *Half*);
      ((M (D (-3)), O (*Aux_top* (0, 0, 0,false,*DL*)), M (D 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *G_SL_DttL*);
      ((M (D (-3)), O (*Aux_top* (0, 0, -1,true,*DR*)), M (U 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *Half*);
      ((M (U (-3)), O (*Aux_top* (0, 0, 1,false,*DR*)), M (D 3)), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_SLR_D*;
      ((M (D (-3)), O (*Aux_top* (0, 0, -1,true,*DL*)), M (U 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *Half*);
      ((M (U (-3)), O (*Aux_top* (0, 0, 1,false,*DL*)), M (D 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *G_SL_DbtL*)
  else
    [ ]

let *anomalous_top_quqd1_tq* =
  [((M (D (-3)), O (*Aux_top* (0, 0, -1,true,*QUQD1R*)), M (U 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *C_quqd*;
    ((M (U (-3)), O (*Aux_top* (0, 0, 1,true,*QUQD1R*)), M (D 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *C_quqd*;
    ((M (D (-3)), O (*Aux_top* (0, 0, -1,true,*QUQD1L*)), M (U 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *C_quqd*;
    ((M (U (-3)), O (*Aux_top* (0, 0, 1,true,*QUQD1L*)), M (D 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *C_quqd*;

```
let anomalous_top_quqd1_ff n =
  List.map mom
    [ ((U (−n), Aux_top (0, 0, 1,false,QUQD1R), D n), FBF (1, Psibar, SR, Psi), Half);
      ((D (−n), Aux_top (0, 0, −1,false,QUQD1R), U n), FBF (1, Psibar, SL, Psi), Half);
      ((U (−n), Aux_top (0, 0, 1,false,QUQD1L), D n), FBF (1, Psibar, SL, Psi), Half);
      ((D (−n), Aux_top (0, 0, −1,false,QUQD1L), U n), FBF (1, Psibar, SR, Psi), Half) ]

let anomalous_top_quqd1 =
  if Flags.top_anom_4f then
    anomalous_top_quqd1_tq @ ThoList.flatmap anomalous_top_quqd1_ff [1; 2; 3]
  else
    []

let anomalous_top_quqd8_tq =
  [ ((M (D (−3)), O (Aux_top (0, 1, −1,true,QUQD8R)), M (U 3)), FBF (1, Psibar, SR, Psi), C_quqd
    ((M (U (−3)), O (Aux_top (0, 1, 1,true,QUQD8R)), M (D 3)), FBF (1, Psibar, SL, Psi), C_quqd8
    ((M (D (−3)), O (Aux_top (0, 1, −1,true,QUQD8L)), M (U 3)), FBF (1, Psibar, SL, Psi), C_quq
    ((M (U (−3)), O (Aux_top (0, 1, 1,true,QUQD8L)), M (D 3)), FBF (1, Psibar, SR, Psi), C_quqd

let anomalous_top_quqd8_ff n =
  List.map mom
    [ ((U (−n), Aux_top (0, 1, 1,false,QUQD8R), D n), FBF (1, Psibar, SR, Psi), Half);
      ((D (−n), Aux_top (0, 1, −1,false,QUQD8R), U n), FBF (1, Psibar, SL, Psi), Half);
      ((U (−n), Aux_top (0, 1, 1,false,QUQD8L), D n), FBF (1, Psibar, SL, Psi), Half);
      ((D (−n), Aux_top (0, 1, −1,false,QUQD8L), U n), FBF (1, Psibar, SR, Psi), Half) ]

let anomalous_top_quqd8 =
  if Flags.top_anom_4f then
    anomalous_top_quqd8_tq @ ThoList.flatmap anomalous_top_quqd8_ff [1; 2; 3]
  else
    []

let vertices3 =
  (ThoList.flatmap electromagnetic_currents [1; 2; 3] @
   ThoList.flatmap color_currents [1; 2; 3] @
   ThoList.flatmap neutral_currents [1; 2; 3] @
   (if Flags.ckm_present then
      charged_currents_ckm
    else
      charged_currents_triv) @
   yukawa @ triple_gauge @
   gauge_higgs @ higgs @ higgs_triangle_vertices
   @ goldstone_vertices @
   tt_threshold_ttA @ tt_threshold_ttZ @
   anomalous_ttA @ anomalous_bbA @
   anomalous_ttZ @ anomalous_bbZ @
   anomalous_tbW @ anomalous_tbWA @ anomalous_tbWZ @
   anomalous_ttWW @ anomalous_bbWW @
   anomalous_ttG @ anomalous_ttGG @
   anomalous_ttH @
   anomalous_top_qGuG @ anomalous_top_qBuB @
   anomalous_top_qW @ anomalous_top_DuDd @
   anomalous_top_quqd1 @ anomalous_top_quqd8)
```

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let *vertices4* =
  quartic_gauge @ *gauge_higgs4* @ *higgs4*

let *vertices* () = (*vertices3*, *vertices4*, [])

For efficiency, make sure that *F.of_vertices vertices* is evaluated only once.

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *flavor_of_string* = function
  | "e-" → M (L 1) | "e+" → M (L (−1))
  | "mu-" → M (L 2) | "mu+" → M (L (−2))
  | "tau-" → M (L 3) | "tau+" → M (L (−3))
  | "nue" → M (N 1) | "nuebar" → M (N (−1))
  | "numu" → M (N 2) | "numubar" → M (N (−2))
  | "nutau" → M (N 3) | "nutaubar" → M (N (−3))
  | "u" → M (U 1) | "ubar" → M (U (−1))
  | "c" → M (U 2) | "cbar" → M (U (−2))
  | "t" → M (U 3) | "tbar" → M (U (−3))
  | "d" → M (D 1) | "dbar" → M (D (−1))
  | "s" → M (D 2) | "sbar" → M (D (−2))
  | "b" → M (D 3) | "bbar" → M (D (−3))
  | "g" | "gl" → G Gl
  | "A" → G Ga | "Z" | "Z0" → G Z
  | "W+" → G Wp | "W-" → G Wm
  | "H" → O H
  | "Aux_t_ttGG0" → O (Aux_top (2, 1, 0,true,$TTGG$))
  | "Aux_ttGG0" → O (Aux_top (2, 1, 0,false,$TTGG$))
  | "Aux_t_tcGG0" → O (Aux_top (2, 1, 0,true,$TCGG$))
  | "Aux_tcGG0" → O (Aux_top (2, 1, 0,false,$TCGG$))
  | "Aux_t_tbWA+" → O (Aux_top (2, 0, 1,true,$TBWA$))
  | "Aux_tbWA+" → O (Aux_top (2, 0, 1,false,$TBWA$))
  | "Aux_t_tbWA-" → O (Aux_top (2, 0, −1,true,$TBWA$))
  | "Aux_tbWA-" → O (Aux_top (2, 0, −1,false,$TBWA$))
  | "Aux_t_tbWZ+" → O (Aux_top (2, 0, 1,true,$TBWZ$))
  | "Aux_tbWZ+" → O (Aux_top (2, 0, 1,false,$TBWZ$))
  | "Aux_t_tbWZ-" → O (Aux_top (2, 0, −1,true,$TBWZ$))
  | "Aux_tbWZ-" → O (Aux_top (2, 0, −1,false,$TBWZ$))
  | "Aux_t_ttWW0" → O (Aux_top (2, 0, 0,true,$TTWW$))
  | "Aux_ttWW0" → O (Aux_top (2, 0, 0,false,$TTWW$))
  | "Aux_t_bbWW0" → O (Aux_top (2, 0, 0,true,$BBWW$))
  | "Aux_bbWW0" → O (Aux_top (2, 0, 0,false,$BBWW$))
  | "Aux_t_qGuG0" → O (Aux_top (1, 1, 0,true,$QGUG$))
  | "Aux_qGuG0" → O (Aux_top (1, 1, 0,false,$QGUG$))
  | "Aux_t_qBuB0" → O (Aux_top (1, 0, 0,true,$QBUB$))
  | "Aux_qBuB0" → O (Aux_top (1, 0, 0,false,$QBUB$))
  | "Aux_t_qW0" → O (Aux_top (1, 0, 0,true,QW))
  | "Aux_qW0" → O (Aux_top (1, 0, 0,false,QW))

```
| "Aux_t_qW+" →  O (Aux_top (1, 0,  1,true,QW))
| "Aux_qW+" →  O (Aux_top (1, 0,  1,false,QW))
| "Aux_t_qW-" →  O (Aux_top (1, 0, −1,true,QW))
| "Aux_qW-" →  O (Aux_top (1, 0, −1,false,QW))
| "Aux_t_dL0" →  O (Aux_top (0, 0,  0,true,DL))
| "Aux_dL0" →  O (Aux_top (0, 0,  0,false,DL))
| "Aux_t_dL+" →  O (Aux_top (0, 0,  1,true,DL))
| "Aux_dL+" →  O (Aux_top (0, 0,  1,false,DL))
| "Aux_t_dL-" →  O (Aux_top (0, 0, −1,true,DL))
| "Aux_dL-" →  O (Aux_top (0, 0, −1,false,DL))
| "Aux_t_dR0" →  O (Aux_top (0, 0,  0,true,DR))
| "Aux_dR0" →  O (Aux_top (0, 0,  0,false,DR))
| "Aux_t_dR+" →  O (Aux_top (0, 0,  1,true,DR))
| "Aux_dR+" →  O (Aux_top (0, 0,  1,false,DR))
| "Aux_t_dR-" →  O (Aux_top (0, 0, −1,true,DR))
| "Aux_dR-" →  O (Aux_top (0, 0, −1,false,DR))
| "Aux_t_quqd1L+" →  O (Aux_top (0, 0,  1,true,QUQD1L))
| "Aux_quqd1L+" →  O (Aux_top (0, 0,  1,false,QUQD1L))
| "Aux_t_quqd1L-" →  O (Aux_top (0, 0, −1,true,QUQD1L))
| "Aux_quqd1L-" →  O (Aux_top (0, 0, −1,false,QUQD1L))
| "Aux_t_quqd1R+" →  O (Aux_top (0, 0,  1,true,QUQD1R))
| "Aux_quqd1R+" →  O (Aux_top (0, 0,  1,false,QUQD1R))
| "Aux_t_quqd1R-" →  O (Aux_top (0, 0, −1,true,QUQD1R))
| "Aux_quqd1R-" →  O (Aux_top (0, 0, −1,false,QUQD1R))
| "Aux_t_quqd8L+" →  O (Aux_top (0, 1,  1,true,QUQD8L))
| "Aux_quqd8L+" →  O (Aux_top (0, 1,  1,false,QUQD8L))
| "Aux_t_quqd8L-" →  O (Aux_top (0, 1, −1,true,QUQD8L))
| "Aux_quqd8L-" →  O (Aux_top (0, 1, −1,false,QUQD8L))
| "Aux_t_quqd8R+" →  O (Aux_top (0, 1,  1,true,QUQD8R))
| "Aux_quqd8R+" →  O (Aux_top (0, 1,  1,false,QUQD8R))
| "Aux_t_quqd8R-" →  O (Aux_top (0, 1, −1,true,QUQD8R))
| "Aux_quqd8R-" →  O (Aux_top (0, 1, −1,false,QUQD8R))
| _ →  invalid_arg "Modellib.SM.flavor_of_string"

let flavor_to_string  = function
  | M f  →
      begin match f with
      | L 1 →  "e-" | L (−1) →  "e+"
      | L 2 →  "mu-" | L (−2) →  "mu+"
      | L 3 →  "tau-" | L (−3) →  "tau+"
      | L _ →  invalid_arg
          "Modellib.SM.flavor_to_string:␣invalid␣lepton"
      | N 1 →  "nue" | N (−1) →  "nuebar"
      | N 2 →  "numu" | N (−2) →  "numubar"
      | N 3 →  "nutau" | N (−3) →  "nutaubar"
      | N _ →  invalid_arg
          "Modellib.SM.flavor_to_string:␣invalid␣neutrino"
      | U 1 →  "u" | U (−1) →  "ubar"
      | U 2 →  "c" | U (−2) →  "cbar"
      | U 3 →  "t" | U (−3) →  "tbar"
```

```
            | U _  →  invalid_arg
                  "Modellib.SM.flavor_to_string:␣invalid␣up␣type␣quark"
            | D 1 → "d" | D (−1) → "dbar"
            | D 2 → "s" | D (−2) → "sbar"
            | D 3 → "b" | D (−3) → "bbar"
            | D _  →  invalid_arg
                  "Modellib.SM.flavor_to_string:␣invalid␣down␣type␣quark"
          end
      | G f  →
          begin match f with
          | Gl  →  "gl"
          | Ga  →  "A" | Z  →  "Z"
          | Wp  →  "W+" | Wm  →  "W-"
          end
      | O f  →
          begin match f with
          | Phip  →  "phi+" | Phim  →  "phi-" | Phi0  →  "phi0"
          | H  →  "H"
          | Aux_top (_, _, ch, n, v)  →  "Aux_" ^ (if n then "t_" else "") ^ (
              begin match v with
              | TTGG  →  "ttGG" | TBWA  →  "tbWA" | TBWZ  →
"tbWZ"
              | TTWW  →  "ttWW" | BBWW  →  "bbWW" | TCGG  →
"tcgg"
              | QGUG  →  "qGuG" | QBUB  →  "qBuB"
              | QW  →  "qW" | DL  →  "dL" | DR  →  "dR"
              | QUQD1L  →  "quqd1L" | QUQD1R  →  "quqd1R"
              | QUQD8L  →  "quqd8L" | QUQD8R  →  "quqd8R"
              end ) ^ ( if ch > 0 then "+" else if ch < 0 then "-" else "0" )
          end
  let flavor_to_TeX = function
    | M f  →
        begin match f with
        | L 1 → "e^-" | L (−1) → "e^+"
        | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
        | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
        | L _  →  invalid_arg
              "Modellib.SM.flavor_to_TeX:␣invalid␣lepton"
        | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
        | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
        | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
        | N _  →  invalid_arg
              "Modellib.SM.flavor_to_TeX:␣invalid␣neutrino"
        | U 1 → "u" | U (−1) → "\\bar{u}"
        | U 2 → "c" | U (−2) → "\\bar{c}"
        | U 3 → "t" | U (−3) → "\\bar{t}"
        | U _  →  invalid_arg
              "Modellib.SM.flavor_to_TeX:␣invalid␣up␣type␣quark"
        | D 1 → "d" | D (−1) → "\\bar{d}"
```

```
            | D 2 → "s" | D (−2) → "\\bar{s}"
            | D 3 → "b" | D (−3) → "\\bar{b}"
            | D _ → invalid_arg
                  "Modellib.SM.flavor_to_TeX:␣invalid␣down␣type␣quark"
          end
      | G f →
          begin match f with
          | Gl → "g"
          | Ga → "\\gamma" | Z → "Z"
          | Wp → "W^+" | Wm → "W^-"
          end
      | O f →
          begin match f with
          | Phip → "\\phi^+" | Phim → "\\phi^-" | Phi0 →
"\\phi^0"
          | H → "H"
          | Aux_top (_, _, ch, n, v) →
                "\\textnormal{Aux_" ^ (if n then "t_" else "") ^ (
                begin match v with
                | TTGG → "ttGG" | TBWA → "tbWA" | TBWZ →
"tbWZ"
                | TTWW → "ttWW" | BBWW → "bbWW" | TCGG →
"tcgg"
                | QGUG → "qGuG" | QBUB → "qBuB"
                | QW → "qW" | DL → "dL" | DR → "dR"
                | QUQD1L → "quqd1L" | QUQD1R → "quqd1R"
                | QUQD8L → "quqd8L" | QUQD8R → "quqd8R"
                end ) ^
                ( if ch > 0 then "^+" else if ch < 0 then
                    "^-" else "^0" ) ^ "}"
          end

    let flavor_symbol = function
      | M f →
          begin match f with
          | L n when n > 0 → "l" ^ string_of_int n
          | L n → "l" ^ string_of_int (abs n) ^ "b"
          | N n when n > 0 → "n" ^ string_of_int n
          | N n → "n" ^ string_of_int (abs n) ^ "b"
          | U n when n > 0 → "u" ^ string_of_int n
          | U n → "u" ^ string_of_int (abs n) ^ "b"
          | D n when n > 0 → "d" ^ string_of_int n
          | D n → "d" ^ string_of_int (abs n) ^ "b"
          end
      | G f →
          begin match f with
          | Gl → "gl"
          | Ga → "a" | Z → "z"
          | Wp → "wp" | Wm → "wm"
          end
```

```
      | O f →
          begin match f with
          | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
          | H → "h"
          | Aux_top (_, _, ch, n, v) → "aux_" ^ (if n then "t_" else "") ^ (
              begin match v with
              | TTGG → "ttgg" | TBWA → "tbwa" | TBWZ →
"tbwz"
              | TTWW → "ttww" | BBWW → "bbww" | TCGG →
"tcgg"
              | QGUG → "qgug" | QBUB → "qbub"
              | QW → "qw" | DL → "dl" | DR → "dr"
              | QUQD1L → "quqd1l" | QUQD1R → "quqd1r"
              | QUQD8L → "quqd8l" | QUQD8R → "quqd8r"
              end ) ^ "_" ^ ( if ch > 0 then "p" else
                  if ch < 0 then "m" else "0" )
          end
    let pdg = function
      | M f →
          begin match f with
          | L n when n > 0 → 9 + 2 × n
          | L n → − 9 + 2 × n
          | N n when n > 0 → 10 + 2 × n
          | N n → − 10 + 2 × n
          | U n when n > 0 → 2 × n
          | U n → 2 × n
          | D n when n > 0 → − 1 + 2 × n
          | D n → 1 + 2 × n
          end
      | G f →
          begin match f with
          | Gl → 21
          | Ga → 22 | Z → 23
          | Wp → 24 | Wm → (−24)
          end
      | O f →
          begin match f with
          | Phip | Phim → 27 | Phi0 → 26
          | H → 25
          | Aux_top (_, _, ch, t, f) → let n =
            begin match f with
            | QW → 0
            | QUQD1R → 1 | QUQD1L → 2
            | QUQD8R → 3 | QUQD8L → 4
            | _ → 5
            end
            in (602 + 3 × n − ch) × ( if t then (1) else (−1) )
          end
    let mass_symbol f =
```

```
    if ( Flags.tt_threshold ∧ (abs (pdg f)) ≡ 6 ) then
      "ttv_mtpole(p12*p12)"
    else
      "mass(" ^ string_of_int (abs (pdg f)) ^ ")"

  let width_symbol f =
    "width(" ^ string_of_int (abs (pdg f)) ^ ")"

  let constant_symbol = function
    | Unit → "unit" | Half → "half" | Pi → "PI"
    | Alpha_QED → "alpha" | E → "e" | G_weak → "g" | Vev →
"vev"
    | I_G_weak → "ig"
    | Sin2thw → "sin2thw" | Sinthw → "sinthw" | Costhw →
"costhw"
    | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
    | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
    | G_NC_up → "gncup" | G_NC_down → "gncdwn"
    | G_TVA_ttA → "gtva_tta" | G_TVA_bbA → "gtva_bba"
    | G_VLR_ttZ → "gvlr_ttz" | G_TVA_ttZ → "gtva_ttz"
    | G_VLR_tcZ → "gvlr_tcz" | G_TVA_tcZ → "gtva_tcz"
    | G_TVA_bbZ → "gtva_bbz" | G_TVA_tcA → "gtva_tca"
    | VA_ILC_ttA → "va_ilc_tta" | VA_ILC_ttZ → "va_ilc_ttz"
    | G_VLR_btW → "gvlr_btw" | G_VLR_tbW → "gvlr_tbw"
    | G_TLR_btW → "gtlr_btw" | G_TRL_tbW → "gtrl_tbw"
    | G_TLR_btWA → "gtlr_btwa" | G_TRL_tbWA → "gtrl_tbwa"
    | G_TLR_btWZ → "gtlr_btwz" | G_TRL_tbWZ → "gtrl_tbwz"
    | G_TVA_ttWW → "gtva_ttww" | G_TVA_bbWW → "gtva_bbww"
    | G_TVA_ttG → "gtva_ttg" | G_TVA_ttGG → "gtva_ttgg"
    | G_TVA_tcG → "gtva_tcg" | G_TVA_tcGG → "gtva_tcgg"
    | G_SP_ttH → "gsp_tth"
    | G_VLR_qGuG → "gvlr_qgug"
    | G_VLR_qBuB → "gvlr_qbub"
    | G_VLR_qBuB_u → "gvlr_qbub_u" | G_VLR_qBuB_d → "gvlr_qbub_d"
    | G_VLR_qBuB_e → "gvlr_qbub_e" | G_VL_qBuB_n → "gvl_qbub_n"
    | G_VL_qW → "gvl_qw"
    | G_VL_qW_u → "gvl_qw_u" | G_VL_qW_d → "gvl_qw_d"
    | G_SL_DttR → "gsl_dttr" | G_SR_DttR → "gsr_dttr"
    | G_SL_DttL → "gsl_dttl"
    | G_SLR_DbtR → "gslr_dbtr" | G_SL_DbtL → "gsl_dbtl"
    | C_quqd1R_bt → "c_quqd1_1" | C_quqd1R_tb → "conjg(c_quqd1_1)"
    | C_quqd1L_bt → "conjg(c_quqd1_2)" | C_quqd1L_tb → "c_quqd1_2"
    | C_quqd8R_bt → "c_quqd8_1" | C_quqd8R_tb → "conjg(c_quqd8_1)"
    | C_quqd8L_bt → "conjg(c_quqd8_2)" | C_quqd8L_tb → "c_quqd8_2"
    | G_CC → "gcc"
    | G_CCQ (n1, n2) → "gccq" ^ string_of_int n1 ^ string_of_int n2
    | I_Q_W → "iqw" | I_G_ZWW → "igzww"
    | G_WWWW → "gw4" | G_ZZWW → "gzzww"
    | G_AZWW → "gazww" | G_AAWW → "gaaww"
    | I_G1_AWW → "ig1a" | I_G1_ZWW → "ig1z"
    | I_G1_plus_kappa_plus_G4_AWW → "ig1pkpg4a"
```

```
| I_G1_plus_kappa_plus_G4_ZWW  → "ig1pkpg4z"
| I_G1_plus_kappa_minus_G4_AWW  → "ig1pkmg4a"
| I_G1_plus_kappa_minus_G4_ZWW  → "ig1pkmg4z"
| I_G1_minus_kappa_plus_G4_AWW  → "ig1mkpg4a"
| I_G1_minus_kappa_plus_G4_ZWW  → "ig1mkpg4z"
| I_G1_minus_kappa_minus_G4_AWW  → "ig1mkmg4a"
| I_G1_minus_kappa_minus_G4_ZWW  → "ig1mkmg4z"
| I_lambda_AWW  → "ila"
| I_lambda_ZWW  → "ilz"
| G5_AWW  → "rg5a"
| G5_ZWW  → "rg5z"
| I_kappa5_AWW  → "ik5a"
| I_kappa5_ZWW  → "ik5z"
| I_lambda5_AWW  → "il5a" | I_lambda5_ZWW  → "il5z"
| Alpha_WWWW0  → "alww0" | Alpha_WWWW2  → "alww2"
| Alpha_ZZWW0  → "alzw0" | Alpha_ZZWW1  → "alzw1"
| Alpha_ZZZZ  → "alzz"
| D_Alpha_ZZWW0_S  → "dalzz0_s(gkm,mkm,"
| D_Alpha_ZZWW0_T  → "dalzz0_t(gkm,mkm,"
| D_Alpha_ZZWW1_S  → "dalzz1_s(gkm,mkm,"
| D_Alpha_ZZWW1_T  → "dalzz1_t(gkm,mkm,"
| D_Alpha_ZZWW1_U  → "dalzz1_u(gkm,mkm,"
| D_Alpha_WWWW0_S  → "dalww0_s(gkm,mkm,"
| D_Alpha_WWWW0_T  → "dalww0_t(gkm,mkm,"
| D_Alpha_WWWW0_U  → "dalww0_u(gkm,mkm,"
| D_Alpha_WWWW2_S  → "dalww2_s(gkm,mkm,"
| D_Alpha_WWWW2_T  → "dalww2_t(gkm,mkm,"
| D_Alpha_ZZZZ_S  → "dalz4_s(gkm,mkm,"
| D_Alpha_ZZZZ_T  → "dalz4_t(gkm,mkm,"
| G_HWW  → "ghww" | G_HZZ  → "ghzz"
| G_HHWW  → "ghhww" | G_HHZZ  → "ghhzz"
| G_Htt  → "ghtt" | G_Hbb  → "ghbb"
| G_Htautau  → "ghtautau" | G_Hcc  → "ghcc" | G_Hmm  →
"ghmm"
| G_HGaZ  → "ghgaz" | G_HGaGa  → "ghgaga" | G_Hgg  →
"ghgg"
| G_HGaGa_anom  → "ghgaga_ac" | G_HGaZ_anom  → "ghgaz_ac"
| G_HZZ_anom  → "ghzz_ac" | G_HWW_anom  → "ghww_ac"
| G_HGaZ_u  → "ghgaz_u" | G_HZZ_u  → "ghzz_u"
| G_HWW_u  → "ghww_u"
| G_H3  → "gh3" | G_H4  → "gh4"
| Gs  → "gs" | I_Gs  → "igs" | G2  → "gs**2"
| Mass f  → "mass" ˆ flavor_symbol f
| Width f  → "width" ˆ flavor_symbol f
| K_Matrix_Coeff i  → "kc" ˆ string_of_int i
| K_Matrix_Pole i  → "kp" ˆ string_of_int i
| G_HZZ6_V3  → "ghzz6v3" | G_HZZ6_D  → "ghzz6d"
| G_HZZ6_DP  → "ghzz6dp" | G_HZZ6_PB  → "ghzz6pb"
| G_HGaZ6_D  → "ghaz6d" | G_HGaZ6_DP  → "ghaz6dp"
| G_HGaZ6_PB  → "ghaz6pb" | G_HGaGa6  → "ghgaga6"
```

```
      |  G_HWW_6_D  →  "ghww6d" |  G_HWW_6_DP  →  "ghww6dp"
      |  I_Dim6_AWW_Gauge  →  "dim6awwgauge" |  I_Dim6_AWW_GGG  →
"dim6awwggg"
      |  I_Dim6_AWW_DP   →   "dim6awwdp" |  I_Dim6_AWW_DW   →
"dim6awwdw"
      |  I_Dim6_WWZ_W  →  "dim6wwzw" |  I_Dim6_WWZ_DPWDW  →
"dim6wwzdpwdw"
      |  I_Dim6_WWZ_DW   →   "dim6wwzdw" |   I_Dim6_WWZ_D   →
"dim6wwzd"
      |  Dim6_vev3  →  "dim6vev3" |  Dim6_Cphi  →  "dim6cphi"
      |  Anom_Dim6_H4_v2  →  "adim6h4v2" |  Anom_Dim6_H4_P2  →
"adim6h4p2"
      |  Anom_Dim6_AHWW_DPB  →  "adim6ahwwdpb"
      |  Anom_Dim6_AHWW_DPW  →  "adim6ahwwdpw"
      |  Anom_Dim6_AHWW_DW  →  "adim6ahwwdw"
      |  Anom_Dim6_AAWW_DW  →  "adim6aawwdw" |  Anom_Dim6_AAWW_W  →
"adim6aawww"
      |  Anom_Dim6_HHWW_DW  →  "adim6hhwwdw"
      |  Anom_Dim6_HHWW_DPW  →  "adim6hhwwdpw"
      |  Anom_Dim6_HWWZ_DW  →  "adim6hwwzdw"
      |  Anom_Dim6_HWWZ_DDPW  →  "adim6hwwzddpw"
      |  Anom_Dim6_HWWZ_DPW  →  "adim6hwwzdpw"
      |  Anom_Dim6_HWWZ_DPB  →  "adim6hwwzdpb"
      |  Anom_Dim6_AHHZ_D  →  "adim6ahhzd" |  Anom_Dim6_AHHZ_DP  →
"adim6ahhzdp"
      |  Anom_Dim6_AHHZ_PB  →  "adim6ahhzpb"
      |  Anom_Dim6_AZWW_W  →  "adim6azwww"
      |  Anom_Dim6_AZWW_DWDPW  →  "adim6azwwdwdpw"
      |  Anom_Dim6_WWWW_W  →  "adim6wwwww"
      |  Anom_Dim6_WWWW_DWDPW  →  "adim6wwwwdwdpw"
      |  Anom_Dim6_WWZZ_W  →  "adim6wwzzw"
      |  Anom_Dim6_WWZZ_DWDPW  →  "adim6wwzzdwdpw"
      |  Anom_Dim6_HHAA  →  "adim6hhaa"
      |  Anom_Dim6_HHZZ_D  →  "adim6hhzzd" |  Anom_Dim6_HHZZ_DP  →
"adim6hhzzdp"
      |  Anom_Dim6_HHZZ_PB  →  "adim6hhzzpb" |  Anom_Dim6_HHZZ_T  →
"adim6hhzzt"

   end
```

13.4.7 Incomplete Standard Model in R_ξ Gauge

At the end of the day, we want a functor mapping from gauge models in unitarity gauge to R_ξ gauge and vice versa. For this, we will need a more abstract implementation of (spontaneously broken) gauge theories.

```
module SM_Rxi =
  struct
    let rcs = RCS.rename rcs_file "Modellib.SM_Rxi"
```

```
[ "minimal␣electroweak␣standard␣model␣in␣R-xi␣gauge";
    "NB:␣very␣incomplete␣still!,␣no␣CKM␣matrix" ]
```

open *Coupling*

module $SM = SM(SM\_no\_anomalous)$
let $options = SM.options$
type $flavor = SM.flavor$
let $flavors = SM.flavors$
let $external\_flavors = SM.external\_flavors$
(∗ Later: type $orders = SM.orders$ ∗)
type $constant = SM.constant$
(∗ Later: let $orders = SM.orders$ ∗)
let $lorentz = SM.lorentz$
let $color = SM.color$
let $goldstone = SM.goldstone$
let $conjugate = SM.conjugate$
let $fermion = SM.fermion$

Check if it makes sense to have separate gauge fixing parameters for each vector boson. There's probably only one independent parameter for each group factor.

type $gauge =$
  | XiA | XiZ | XiW

let $gauge\_symbol =$ function
  | $XiA \rightarrow$ "xia" | $XiZ \rightarrow$ "xi0" | $XiW \rightarrow$ "xipm"

Change the gauge boson propagators and make the Goldstone bosons propagating.

let $propagator =$ function
  | $SM.G\ SM.Ga \rightarrow Prop\_Gauge\ XiA$
  | $SM.G\ SM.Z \rightarrow Prop\_Rxi\ XiZ$
  | $SM.G\ SM.Wp$ | $SM.G\ SM.Wm \rightarrow Prop\_Rxi\ XiW$
  | $SM.O\ SM.Phip$ | $SM.O\ SM.Phim$ | $SM.O\ SM.Phi0 \rightarrow Prop\_Scalar$
  | $f \rightarrow SM.propagator\ f$

let $width = SM.width$

module $Ch = Charges.QQ$
let $charges = SM.charges$

module $F = Modeltools.Fusions$ (struct
  type $f = flavor$
  type $c = constant$
  let $compare = compare$
  let $conjugate = conjugate$
end)

let $vertices = SM.vertices$

let $table = F.of\_vertices\ (vertices\ ())$
let $fuse2 = F.fuse2\ table$
```

```
let fuse3 = F.fuse3 table
let fuse = F.fuse table
let max_degree () = 3

let parameters = SM.parameters
let flavor_of_string = SM.flavor_of_string
let flavor_to_string = SM.flavor_to_string
let flavor_to_TeX = SM.flavor_to_TeX
let flavor_symbol = SM.flavor_symbol
let pdg = SM.pdg
let mass_symbol = SM.mass_symbol
let width_symbol = SM.width_symbol
let constant_symbol = SM.constant_symbol
```

end

13.4.8 Groves

module *Groves* (*M* : *Model.Gauge*) : *Model.Gauge* with module *Ch* = *M.Ch* =
  struct
```
    let max_generations = 5
    let rcs = RCS.rename M.rcs
        ("Modellib.Groves(" ^ (RCS.name M.rcs) ^ ")")
        ([ "experimental␣Groves␣functor";
            Printf.sprintf "for␣maximally␣%d␣flavored␣legs"
              (2 × max_generations) ] @
          RCS.description M.rcs)

    let options = M.options

    type matter_field = M.matter_field × int
    type gauge_boson = M.gauge_boson
    type other = M.other
    type field =
      | Matter of matter_field
      | Gauge of gauge_boson
      | Other of other
    type flavor = M of matter_field | G of gauge_boson | O of other
    let matter_field (f, g) = M (f, g)
    let gauge_boson f = G f
    let other f = O f
    let field = function
      | M f → Matter f
      | G f → Gauge f
      | O f → Other f
    let project = function
      | M (f, _) → M.matter_field f
      | G f → M.gauge_boson f
      | O f → M.other f
    let inject g f =
      match M.field f with
```
```

```
    | M.Matter f  →  M (f, g)
    | M.Gauge f  →  G f
    | M.Other f  →  O f
type gauge  =  M.gauge
let gauge_symbol  =  M.gauge_symbol
let color f  =  M.color (project f)
let pdg f  =  M.pdg (project f)
let lorentz f  =  M.lorentz (project f)
let propagator f  =  M.propagator (project f)
let fermion f  =  M.fermion (project f)
let width f  =  M.width (project f)
let mass_symbol f  =  M.mass_symbol (project f)
let width_symbol f  =  M.width_symbol (project f)
let flavor_symbol f  =  M.flavor_symbol (project f)

type constant  =  M.constant
(∗ Later: type orders  =  M.orders ∗)
let constant_symbol  =  M.constant_symbol
let max_degree  =  M.max_degree
let parameters  =  M.parameters
(∗ Later: let orders  =  M.orders ∗)

let conjugate  =  function
  | M (_, g) as f  →  inject g (M.conjugate (project f))
  | f  →  inject 0 (M.conjugate (project f))

let read_generation s  =
  try
    let offset  =  String.index s '/' in
    (int_of_string
        (String.sub s (succ offset) (String.length s − offset − 1)),
      String.sub s 0 offset)
  with
  | Not_found  →  (1, s)

let format_generation c s  =
  s ^ "/" ^ string_of_int c

let flavor_of_string s  =
  let g, s  =  read_generation s in
  inject g (M.flavor_of_string s)

let flavor_to_string  =  function
  | M (_, g) as f  →  format_generation g (M.flavor_to_string (project f))
  | f  →  M.flavor_to_string (project f)

let flavor_to_TeX  =  function
  | M (_, g) as f  →  format_generation g (M.flavor_to_TeX (project f))
  | f  →  M.flavor_to_TeX (project f)

let goldstone  =  function
  | G _ as f  →
        begin match M.goldstone (project f) with
        | None  →  None
```

```
          |  Some (f, c)  →  Some (inject 0 f, c)
          end
  |  M _  |  O _  →  None
let clone generations flavor  =
  match M.field flavor with
  |  M.Matter f  →  List.map (fun g  →  M (f, g)) generations
  |  M.Gauge f  →  [G f]
  |  M.Other f  →  [O f]

let generations  =  ThoList.range 1 max_generations

let flavors ()  =
  ThoList.flatmap (clone generations) (M.flavors ())

let external_flavors ()  =
  List.map (fun (s, fl)  →  (s, ThoList.flatmap (clone generations) fl))
    (M.external_flavors ())

module Ch  =  M.Ch
let charges f  =  M.charges (project f)

module F  =  Modeltools.Fusions (struct
  type f  =  flavor
  type c  =  constant
  let compare  =  compare
  let conjugate  =  conjugate
end)
```

In the following functions, we might replace _ by (*M.Gauge* _ | *M.Other* _), in order to allow the compiler to check completeness. However, this makes the code much less readable.

```
let clone3 ((f1, f2, f3), v, c)  =
  match M.field f1, M.field f2, M.field f3 with
  |  M.Matter _, M.Matter _, M.Matter _  →
      invalid_arg "Modellib.Groves().vertices:␣three␣matter␣fields!"
  |  M.Matter f1′, M.Matter f2′, _  →
      List.map (fun g  →  ((M (f1′, g), M (f2′, g), inject 0 f3), v, c))
        generations
  |  M.Matter f1′, _, M.Matter f3′  →
      List.map (fun g  →  ((M (f1′, g), inject 0 f2, M (f3′, g)), v, c))
        generations
  |  _, M.Matter f2′, M.Matter f3′  →
      List.map (fun g  →  ((inject 0 f1, M (f2′, g), M (f3′, g)), v, c))
        generations
  |  M.Matter _, _, _  |  _, M.Matter _, _  |  _, _, M.Matter _  →
      invalid_arg "Modellib.Groves().vertices:␣lone␣matter␣field!"
  |  _, _, _  →
      [(inject 0 f1, inject 0 f2, inject 0 f3), v, c]

let clone4 ((f1, f2, f3, f4), v, c)  =
  match M.field f1, M.field f2, M.field f3, M.field f4 with
  |  M.Matter _, M.Matter _, M.Matter _, M.Matter _  →
      invalid_arg "Modellib.Groves().vertices:␣four␣matter␣fields!"
```

```
      |  M.Matter _,  M.Matter _,  M.Matter _,  _
      |  M.Matter _,  M.Matter _,  _,  M.Matter _
      |  M.Matter _,  _,  M.Matter _,  M.Matter _
      |  _,  M.Matter _,  M.Matter _,  M.Matter _  →
          invalid_arg "Modellib.Groves().vertices:␣three␣matter␣fields!"
      |  M.Matter f1′,  M.Matter f2′,  _,  _  →
          List.map (fun g  →
            ((M (f1′, g), M (f2′, g), inject 0 f3, inject 0 f4), v, c))
            generations
      |  M.Matter f1′,  _,  M.Matter f3′,  _  →
          List.map (fun g  →
            ((M (f1′, g), inject 0 f2, M (f3′, g), inject 0 f4), v, c))
            generations
      |  M.Matter f1′,  _,  _,  M.Matter f4′  →
          List.map (fun g  →
            ((M (f1′, g), inject 0 f2, inject 0 f3, M (f4′, g)), v, c))
            generations
      |  _,  M.Matter f2′,  M.Matter f3′,  _  →
          List.map (fun g  →
            ((inject 0 f1, M (f2′, g), M (f3′, g), inject 0 f4), v, c))
            generations
      |  _,  M.Matter f2′,  _,  M.Matter f4′  →
          List.map (fun g  →
            ((inject 0 f1, M (f2′, g), inject 0 f3, M (f4′, g)), v, c))
            generations
      |  _,  _,  M.Matter f3′,  M.Matter f4′  →
          List.map (fun g  →
            ((inject 0 f1, inject 0 f2, M (f3′, g), M (f4′, g)), v, c))
            generations
      |  M.Matter _,  _,  _,  _  |  _,  M.Matter _,  _,  _
      |  _,  _,  M.Matter _,  _  |  _,  _,  _,  M.Matter _  →
          invalid_arg "Modellib.Groves().vertices:␣lone␣matter␣field!"
      |  _,  _,  _,  _  →
          [(inject 0 f1, inject 0 f2, inject 0 f3, inject 0 f4), v, c]
  let clonen (fl, v, c)  =
    match List.map M.field fl with
    |  _  →  failwith "Modellib.Groves().vertices:␣incomplete"

  let vertices ()  =
    let vertices3, vertices4, verticesn  =  M.vertices () in
    (ThoList.flatmap clone3 vertices3,
      ThoList.flatmap clone4 vertices4,
      ThoList.flatmap clonen verticesn)

  let table  =  F.of_vertices (vertices ())
  let fuse2  =  F.fuse2 table
  let fuse3  =  F.fuse3 table
  let fuse  =  F.fuse table
```

The following (incomplete) alternative implementations are included for il-
lustrative purposes only:

```
let injectl g fcl  =
  List.map (fun (f, c)  →  (inject g f, c)) fcl

let alt_fuse2 f1 f2  =
  match f1, f2 with
  |  M (f1', g1'), M (f2', g2')  →
       if g1'  =  g2' then
         injectl 0 (M.fuse2 (M.matter_field f1') (M.matter_field f2'))
       else
         []
  |  M (f1', g'), _  →  injectl g' (M.fuse2 (M.matter_field f1') (project f2))
  |  _, M (f2', g')  →  injectl g' (M.fuse2 (project f1) (M.matter_field f2'))
  |  _, _  →  injectl 0 (M.fuse2 (project f1) (project f2))

let alt_fuse3 f1 f2 f3  =
  match f1, f2, f3 with
  |  M (f1', g1'), M (f2', g2'), M (f3', g3')  →
       invalid_arg "Modellib.Groves().fuse3:␣three␣matter␣fields!"
  |  M (f1', g1'), M (f2', g2'), _  →
       if g1'  =  g2' then
         injectl 0
           (M.fuse3 (M.matter_field f1') (M.matter_field f2') (project f3))
       else
         []
  |  M (f1', g1'), _, M (f3', g3')  →
       if g1'  =  g3' then
         injectl 0
           (M.fuse3 (M.matter_field f1') (project f2) (M.matter_field f3'))
       else
         []
  |  _, M (f2', g2'), M (f3', g3')  →
       if g2'  =  g3' then
         injectl 0
           (M.fuse3 (project f1) (M.matter_field f2') (M.matter_field f3'))
       else
         []
  |  M (f1', g'), _, _  →
       injectl g' (M.fuse3 (M.matter_field f1') (project f2) (project f3))
  |  _, M (f2', g'), _  →
       injectl g' (M.fuse3 (project f1) (M.matter_field f2') (project f3))
  |  _, _, M (f3', g')  →
       injectl g' (M.fuse3 (project f1) (project f2) (M.matter_field f3'))
  |  _, _, _  →  injectl 0 (M.fuse3 (project f1) (project f2) (project f3))

end
```

13.4.9 MSM With Cloned Families

```
module SM_clones  =  Groves(SM(SM_no_anomalous))
```

13.5   Interface of Modellib_BSM

13.5.1   More Hardcoded BSM Models

module type BSM_flags =
  sig
    val $u1\_gauged$: $bool$
    val $anom\_ferm\_ass$: $bool$
  end

module type $THDM\_flags$ =
  sig
    val $ckm\_present$: $bool$
  end

module BSM_bsm : BSM_flags
module $BSM\_ungauged$: BSM_flags
module BSM_anom : BSM_flags
module $Littlest$: functor (F : BSM_flags) → $Model.Gauge$ with module Ch = $Charges.QQ$
module $Littlest\_Tpar$: functor (F : BSM_flags) → $Model.T$ with module Ch = $Charges.QQ$
module $Simplest$: functor (F : BSM_flags) → $Model.T$ with module Ch = $Charges.QQ$
module $Xdim$: functor (F : BSM_flags) → $Model.Gauge$ with module Ch = $Charges.QQ$
module UED : functor (F : BSM_flags) → $Model.Gauge$ with module Ch = $Charges.QQ$
module $GravTest$: functor (F : BSM_flags) → $Model.Gauge$ with module Ch = $Charges.QQ$
module $THDM$: $THDM\_flags$
module $THDM\_CKM$: $THDM\_flags$
module $TwoHiggsDoublet$: functor (F : $THDM\_flags$) → $Model.Gauge$ with module Ch = $Charges.QQ$
module $Template$: functor (F : BSM_flags) → $Model.Gauge$ with module Ch = $Charges.QQ$
module $HSExt$: functor (F : BSM_flags) → $Model.Gauge$ with module Ch = $Charges.QQ$

module type $Threeshl\_options$ =
  sig
    val $include\_ckm$: $bool$
    val $include\_hf$: $bool$
    val $diet$: $bool$
  end

module $Threeshl\_no\_ckm$: $Threeshl\_options$
module $Threeshl\_ckm$: $Threeshl\_options$
module $Threeshl\_no\_ckm\_no\_hf$: $Threeshl\_options$
module $Threeshl\_ckm\_no\_hf$: $Threeshl\_options$
module $Threeshl\_diet\_no\_hf$: $Threeshl\_options$
module $Threeshl\_diet$: $Threeshl\_options$
module $Threeshl$: functor ($Module\_options$: $Threeshl\_options$) →

Model.T with module *Ch* = *Charges.QQ*

module type *SSC_flags* =
  sig
    val *higgs_triangle* : *bool* (∗ *Hγγ*, *Hgγ* and *Hgg* couplings ∗)
    val *higgs_hmm* : *bool*
    val *triple_anom* : *bool*
    val *quartic_anom* : *bool*
    val *higgs_anom* : *bool*
    val *k_matrix* : *bool*
    val *ckm_present* : *bool*
    val *top_anom* : *bool*
    val *top_anom_4f* : *bool*
    val *cf_arbitrary* : *bool*
    val *higgs_matrix* : *bool*
  end

module *SSC_kmatrix* : *SSC_flags*

module *SSC_kmatrix_2* : *SSC_flags*

module *SSC* : functor (*F* : *SSC_flags*) → *Model.Gauge* with module *Ch* = *Charges.QQ*

module *SSC_AltT* : functor (*F* : *SSC_flags*) → *Model.Gauge* with module *Ch* = *Charges.QQ*

13.6 Implementation of Modellib_BSM

let *rcs_file* = *RCS.parse* "Modellib␣BSM" ["BSM␣Models"]
  { *RCS.revision* = "$Revision:␣7367␣$";
   RCS.date = "$Date:␣2015-11-13␣18:28:39␣+0100␣(Fri,␣13␣Nov␣2015)␣$";
   RCS.author = "$Author:␣jr␣reuter␣$";
   RCS.source
    = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr

13.6.1 Littlest Higgs Model

module type *BSM_flags* =
  sig
    val *u1_gauged* : *bool*
    val *anom_ferm_ass* : *bool*
  end

module *BSM_bsm* : *BSM_flags* =
  struct
    let *u1_gauged* = true
    let *anom_ferm_ass* = false
  end

module *BSM_ungauged* : *BSM_flags* =
  struct

```
    let u1_gauged = false
    let anom_ferm_ass = false
  end

module BSM_anom : BSM_flags =
  struct
    let u1_gauged = false
    let anom_ferm_ass = true
  end

module Littlest (Flags : BSM_flags) =
  struct
    let rcs = rcs_file

    open Coupling

    let default_width = ref Timelike
    let use_fudged_width = ref false

    let options = Options.create
      [ "constant_width", Arg.Unit (fun () → default_width := Constant),
          "use constant width (also in t-channel)";
        "fudged_width", Arg.Set use_fudged_width,
          "use fudge factor for charge particle width";
        "custom_width", Arg.String (fun f → default_width := Custom f),
          "use custom width";
        "cancel_widths", Arg.Unit (fun () → default_width := Vanishing),
          "use vanishing width" ]

    let gauge_symbol () =
      failwith "Modellib_BSM.Littlest.gauge_symbol: internal error"

    type matter_field = L of int | N of int | U of int | D of int
        | TopH | TopHb
    type gauge_boson = Ga | Wp | Wm | Z | Gl | WHp | WHm
        | ZH | AH
    type other = Phip | Phim | Phi0 | H | Eta | Psi0
        | Psi1 | Psip | Psim | Psipp | Psimm

    type flavor = M of matter_field | G of gauge_boson | O of other

    let matter_field f = M f
    let gauge_boson f = G f
    let other f = O f

    type field =
      | Matter of matter_field
      | Gauge of gauge_boson
      | Other of other

    let field = function
      | M f → Matter f
      | G f → Gauge f
      | O f → Other f

    type gauge = unit
```

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```
let gauge_symbol () =
  failwith "Modellib_BSM.Littlest.gauge_symbol:␣internal␣error"

let family n = List.map matter_field [ L n; N n; U n; D n ]
```

Since *Phi* already belongs to the EW Goldstone bosons we use *Psi* for the TeV scale complex triplet.

```
let external_flavors () =
  [ "1st␣Generation", ThoList.flatmap family [1; − 1];
    "2nd␣Generation", ThoList.flatmap family [2; − 2];
    "3rd␣Generation", ThoList.flatmap family [3; − 3];
    "Heavy␣Quarks", List.map matter_field [TopH; TopHb];
    "Heavy␣Scalars", List.map other
      [Psi0; Psi1; Psip; Psim; Psipp; Psimm];
    "Gauge␣Bosons", List.map gauge_boson
      (if Flags.u1_gauged then
        [Ga; Z; Wp; Wm; Gl; WHp; WHm; ZH; AH]
          else
        [Ga; Z; Wp; Wm; Gl; WHp; WHm; ZH]);
    "Higgs", List.map other
      (if Flags.u1_gauged then [H]
          else [H; Eta]);
    "Goldstone␣Bosons", List.map other [Phip; Phim; Phi0] ]

let flavors () = ThoList.flatmap snd (external_flavors ())

let spinor n =
  if n ≥ 0 then
    Spinor
  else
    ConjSpinor

let lorentz = function
  | M f →
      begin match f with
      | L n → spinor n | N n → spinor n
      | U n → spinor n | D n → spinor n
      | TopH → Spinor | TopHb → ConjSpinor
      end
  | G f →
      begin match f with
      | Ga | Gl → Vector
      | Wp | Wm | Z | WHp | WHm | ZH | AH →
Massive_Vector
      end
  | O f →
      begin match f with
      | Phip | Phim | Phi0 | H | Eta | Psi0
      | Psi1 | Psip | Psim | Psipp | Psimm → Scalar
      end

let color = function
  | M (U n) → Color.SUN (if n > 0 then 3 else -3)
```

```
    | M (D n) → Color.SUN (if n > 0 then 3 else -3)
    | M TopH → Color.SUN 3 | M TopHb → Color.SUN (−3)
    | G Gl → Color.AdjSUN 3
    | _ → Color.Singlet

let prop_spinor n =
  if n ≥ 0 then
    Prop_Spinor
  else
    Prop_ConjSpinor

let propagator = function
  | M f →
      begin match f with
      | L n → prop_spinor n | N n → prop_spinor n
      | U n → prop_spinor n | D n → prop_spinor n
      | TopH → Prop_Spinor | TopHb → Prop_ConjSpinor
      end
  | G f →
      begin match f with
      | Ga | Gl → Prop_Feynman
      | Wp | Wm | Z | WHp | WHm | ZH | AH →
Prop_Unitarity
      end
  | O f →
      begin match f with
      | Phip | Phim | Phi0 → Only_Insertion
      | H | Eta | Psi0 | Psi1 | Psip | Psim
      | Psipp | Psimm → Prop_Scalar
      end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

```
let width f =
  if !use_fudged_width then
    match f with
    | G Wp | G Wm | M (U 3) | M (U (−3))
    | G WHp | G WHm | G ZH | G AH
    | M TopH | M TopHb → Fudged
    | _ → !default_width
  else
    !default_width

let goldstone = function
  | G f →
      begin match f with
      | Wp → Some (O Phip, Coupling.Const 1)
      | Wm → Some (O Phim, Coupling.Const 1)
      | Z → Some (O Phi0, Coupling.Const 1)
      | _ → None
      end
  | _ → None
```

```
let conjugate = function
  | M f →
      M (begin match f with
      | L n → L (−n) | N n → N (−n)
      | U n → U (−n) | D n → D (−n)
      | TopH → TopHb | TopHb → TopH
      end)
  | G f →
      G (begin match f with
      | Gl → Gl | Ga → Ga | Z → Z
      | Wp → Wm | Wm → Wp | WHm → WHp
      | WHp → WHm | ZH → ZH | AH → AH
      end)
  | O f →
      O (begin match f with
      | Psi0 → Psi0 | Psi1 → Psi1 | Psip → Psim
      | Psim → Psip | Psipp → Psimm | Psimm → Psipp
      | Phip → Phim | Phim → Phip | Phi0 → Phi0
      | H → H | Eta → Eta
      end)

let fermion = function
  | M f →
      begin match f with
      | L n → if n > 0 then 1 else -1
      | N n → if n > 0 then 1 else -1
      | U n → if n > 0 then 1 else -1
      | D n → if n > 0 then 1 else -1
      | TopH → 1 | TopHb → −1
      end
  | G f →
      begin match f with
      | Gl | Ga | Z | Wp | Wm | WHp
      | WHm | AH | ZH → 0
      end
  | O f →
      begin match f with
      | Psi0 | Psi1 | Psip | Psim | Psipp | Psimm
      | Phip | Phim | Phi0 | H | Eta → 0
      end
```

This model does NOT have a conserved generation charge even in absence of CKM mixing because of the heavy top admixture.

```
module Ch = Charges.QQ
let ( // ) = Algebra.Small_Rational.make

let charge = function
  | M f →
      begin match f with
      | L n → if n > 0 then -1//1 else 1//1
      | N n → 0//1
```

```
            | U n → if n > 0 then 2//3 else -2//3
            | D n → if n > 0 then -1//3 else 1//3
            | TopH → 2//3
            | TopHb → − 2//3
            end
        | G f →
            begin match f with
            | Gl | Ga | Z | AH | ZH → 0//1
            | Wp | WHp → 1//1
            | Wm | WHm → − 1//1
            end
        | O f →
            begin match f with
            | H | Phi0 | Eta | Psi1 | Psi0 → 0//1
            | Phip | Psip → 1//1
            | Phim | Psim → − 1//1
            | Psipp → 2//1
            | Psimm → − 2//1
            end

    let lepton = function
        | M f →
            begin match f with
            | L n | N n → if n > 0 then 1//1 else -1//1
            | U _ | D _ | _ → 0//1
            end
        | G _ | O _ → 0//1

    let baryon = function
        | M f →
            begin match f with
            | L _ | N _ → 0//1
            | U n | D n → if n > 0 then 1//1 else -1//1
            | TopH → 1//1
            | TopHb → − 1//1
            end
        | G _ | O _ → 0//1

    let charges f =
        [ charge f; lepton f; baryon f]

    type constant =
        | Unit | Pi | Alpha_QED | Sin2thw
        | Sinthw | Costhw | E | G_weak | Vev | VHeavy
        | Supp | Supp2
        | Sinpsi | Cospsi | Atpsi | Sccs (∗ Mixing angles of SU(2) ∗)
        | Q_lepton | Q_up | Q_down | Q_Z_up | G_CC | G_CCtop
        | G_NC_neutrino | G_NC_lepton | G_NC_up | G_NC_down |
G_NC_heavy
        | G_NC_h_neutrino | G_NC_h_lepton | G_NC_h_up | G_NC_h_down
        | G_CC_heavy | G_ZHTHT | G_ZTHT | G_AHTHTH |
G_AHTHT | G_AHTT
```

```
    | G_CC_WH | G_CC_W
    | I_Q_W | I_G_ZWW | I_G_WWW
    | I_G_AHWW | I_G_ZHWW | I_G_ZWHW | I_G_AHWHWH |
I_G_ZHWHWH
    | I_G_AHWHW | I_Q_H
    | G_WWWW | G_ZZWW | G_AZWW | G_AAWW
    | G_WH4 | G_WHWHWW | G_WHWWW | G_WH3W
    | G_WWAAH | G_WWAZH | G_WWZZH | G_WWZAH |
G_WHWHAAH
    | G_WHWHAZH | G_WHWHZZH | G_WHWHZAH | G_WWZHAH
    | G_WHWHZHAH | G_WHWZZ | G_WHWAZ | G_WHWAAH |
G_WHWZAH
    | G_WHWZHZH | G_WHWZHAH | G_WHWAZH | G_WHWZZH
    | G_HWW | G_HHWW | G_HZZ | G_HHZZ
    | G_PsiWW | G_PsiWHW | G_PsiZZ | G_PsiZHZH
    | G_PsiZHZ | G_PsiZAH | G_PsiZHAH | G_PsiAHAH
    | G_PsiZW | G_PsiZWH | G_PsiAHW | G_PsiAHWH
    | G_PsiZHW | G_PsiZHWH
    | G_PsippWW | G_PsippWHW | G_PsippWHWH
    | G_PsiHW | G_PsiHWH | G_Psi0W | G_Psi0WH
    | G_Psi1W | G_Psi1WH | G_PsiPPW | G_PsiPPWH
    | G_Psi1HAH | G_Psi01AH | G_AHPsip | G_Psi1HZ
    | G_Psi1HZH | G_Psi01Z | G_Psi01ZH | G_ZPsip | G_ZPsipp |
G_ZHPsipp
    | G_HHAA | G_HHWHW | G_HHZHZH | G_HHAHZ | G_HHZHAH
    | G_HPsi0WW | G_HPsi0WHW | G_HPsi0ZZ
    | G_HPsi0ZHZH | G_HPsi0ZHZ | G_HPsi0AHAH | G_HPsi0ZAH |
G_HPsi0ZHAH
    | G_HPsipWA | G_HPsipWHA | G_HPsipWZ | G_HPsipWHZ |
G_HPsipWAH
    | G_HPsipWHAH | G_HPsipWZH | G_HPsipWHZH | G_HPsippWW |
G_HPsippWHWH
    | G_HPsippWHW | G_Psi00ZH | G_Psi00AH | G_Psi00ZHAH
    | G_Psi0pWA | G_Psi0pWHA | G_Psi0pWZ | G_Psi0pWHZ |
G_Psi0pWAH
    | G_Psi0pWHAH | G_Psi0pWZH | G_Psi0pWHZH | G_Psi0ppWW |
G_Psi0ppWHWH
    | G_Psi0ppWHW | I_G_Psi0pWA | I_G_Psi0pWHA | I_G_Psi0pWZ |
I_G_Psi0pWHZ
    | I_G_Psi0pWAH | I_G_Psi0pWHAH | I_G_Psi0pWZH | I_G_Psi0pWHZH
    | I_G_Psi0ppWW | I_G_Psi0ppWHWH | I_G_Psi0ppWHW
    | G_PsippZZ | G_PsippZHZH | G_PsippAZ | G_PsippAAH |
G_PsippZAH
    | G_PsippWA | G_PsippWHA | G_PsippWZ | G_PsippWHZ |
G_PsippWAH
    | G_PsippWHAH | G_PsippWZH | G_PsippWHZH
    | G_PsiccZZ | G_PsiccAZ | G_PsiccAAH | G_PsiccZZH |
G_PsiccAZH
    | G_PsiccZAH
    | G_Htt | G_Hbb | G_Hcc | G_Htautau | G_H3 | G_H4
```

| *G_Hthth* | *G_Htht* | *G_Ethth* | *G_Etht* | *G_Ett*
| *G_HHtt* | *G_HHthth* | *G_HHtht*
| *G_Psi0tt* | *G_Psi0bb* | *G_Psi0cc* | *G_Psi0tautau*
| *G_Psi1tt* | *G_Psi1bb* | *G_Psi1cc* | *G_Psi1tautau*
| *G_Psipq3* | *G_Psipq2* | *G_Psipl3* | *G_Psi0tth* | *G_Psi1tth*
| *G_Psipbth* | *G_Ebb*
| *G_HGaGa* | *G_HGaZ* | *G_EGaGa* | *G_EGaZ* | *G_EGlGl*
| *Gs* | *I_Gs* | *G2*
| *G_HWHW* | *G_HWHWH* | *G_HAHAH* | *G_HZHZ* | *G_HZHAH* |
G_HAHZ
| *Mass* of *flavor* | *Width* of *flavor*

Two integer counters for the QCD and EW order of the couplings.

type *orders* $=$ *int* $\times$ *int*

let *orders* $=$ function
| _ $\rightarrow$ $(0, 0)$

let *input_parameters* $=$
[]

let *derived_parameters* $=$
[]

let *g_over_2_costh* $=$
Quot (*Neg* (*Atom* *G_weak*), *Prod* [*Const* 2; *Atom* *Costhw*])

let *nc_coupling* *c* *t3* *q* $=$
(*Real_Array* *c*,
[*Prod* [*g_over_2_costh*; *Diff* (*t3*, *Prod* [*Const* 2; *q*; *Atom* *Sin2thw*])];
Prod [*g_over_2_costh*; *t3*]])

let *half* $=$ *Quot* (*Const* 1, *Const* 2)

let *derived_parameter_arrays* $=$
[*nc_coupling* *G_NC_neutrino* *half* (*Const* 0);
nc_coupling *G_NC_lepton* (*Neg* *half*) (*Const* (−1));
nc_coupling *G_NC_up* *half* (*Quot* (*Const* 2, *Const* 3));
nc_coupling *G_NC_down* (*Neg* *half*) (*Quot* (*Const* (−1), *Const* 3));
nc_coupling *G_NC_h_neutrino* *half* (*Const* 0);
nc_coupling *G_NC_h_lepton* (*Neg* *half*) (*Const* (−1));
nc_coupling *G_NC_h_up* *half* (*Quot* (*Const* 2, *Const* 3));
nc_coupling *G_NC_h_down* (*Neg* *half*) (*Quot* (*Const* (−1), *Const* 3))]

let *parameters* () $=$
{ *input* $=$ *input_parameters*;
derived $=$ *derived_parameters*;
derived_arrays $=$ *derived_parameter_arrays* }

module *F* $=$ *Modeltools.Fusions* (struct
type *f* $=$ *flavor*
type *c* $=$ *constant*
let *compare* $=$ *compare*
let *conjugate* $=$ *conjugate*
end)

let *mgm* ((*m1*, *g*, *m2*), *fbf*, *c*) = ((*M m1*, *G g*, *M m2*), *fbf*, *c*)
let *mhm* ((*m1*, *h*, *m2*), *fbf*, *c*) = ((*M m1*, *O h*, *M m2*), *fbf*, *c*)
let *tgc* ((*g1*, *g2*, *g3*), *t*, *c*) = ((*G g1*, *G g2*, *G g3*), *t*, *c*)
let *qgc* ((*g1*, *g2*, *g3*, *g4*), *t*, *c*) = ((*G g1*, *G g2*, *G g3*, *G g4*), *t*, *c*)
let *hgg* ((*h*, *g1*, *g2*), *coup*, *c*) = ((*O h*, *G g1*, *G g2*), *coup*, *c*)
let *ghh* ((*g*, *h1*, *h2*), *coup*, *c*) = ((*G g*, *O h1*, *O h2*), *coup*, *c*)
let *hhgg* ((*h1*, *h2*, *g1*, *g2*), *coup*, *c*) = ((*O h1*, *O h2*, *G g1*, *G g2*), *coup*, *c*)

let *electromagnetic_currents n* =
  List.map mgm
    [((*L* (−*n*), *Ga*, *L n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_lepton*);
      ((*U* (−*n*), *Ga*, *U n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
      ((*D* (−*n*), *Ga*, *D n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

let *neutral_currents n* =
  List.map mgm
    [((*L* (−*n*), *Z*, *L n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_lepton*);
      ((*N* (−*n*), *Z*, *N n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_neutrino*);
      ((*U* (−*n*), *Z*, *U n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_up*);
      ((*D* (−*n*), *Z*, *D n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_down*)]

The sign of this coupling is just the one of the T3, being -(1/2) for leptons and down quarks, and +(1/2) for neutrinos and up quarks.

let *neutral_heavy_currents n* =
  List.map mgm
    ([((*L* (−*n*), *ZH*, *L n*), *FBF* ((−1), *Psibar*, *VL*, *Psi*), *G_NC_heavy*);
       ((*N* (−*n*), *ZH*, *N n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_NC_heavy*);
       ((*U* (−*n*), *ZH*, *U n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_NC_heavy*);
       ((*D* (−*n*), *ZH*, *D n*), *FBF* ((−1), *Psibar*, *VL*, *Psi*), *G_NC_heavy*)]
    @
      (if *Flags.u1_gauged* then
    [((*L* (−*n*), *AH*, *L n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_h_lepton*);
      ((*N* (−*n*), *AH*, *N n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_h_neutrino*);
      ((*D* (−*n*), *AH*, *D n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_h_down*)]
      else
        []))

let *color_currents n* =
  List.map mgm
    [((*D* (−*n*), *Gl*, *D n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*);
      ((*U* (−*n*), *Gl*, *U n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*)]

let *heavy_top_currents* =
  List.map mgm
    ([((*TopHb*, *Ga*, *TopH*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
       ((*TopHb*, *Z*, *TopH*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_Z_up*);
       ((*TopHb*, *Z*, *U* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZTHT*);
       ((*U* (−3), *Z*, *TopH*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZTHT*);
       ((*TopHb*, *ZH*, *U* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZHTHT*);
       ((*U* (−3), *ZH*, *TopH*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZHTHT*);
       ((*U* (−3), *Wp*, *D* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CCtop*);
       ((*D* (−3), *Wm*, *U* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CCtop*);

$((\text{\textit{TopHb}, \textit{WHp}, D 3), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_WH});$

$((D (-3), \textit{WHm}, \textit{TopH}), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_WH});$

$((\textit{TopHb}, \textit{Wp}, D 3), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_W});$

$((D (-3), \textit{Wm}, \textit{TopH}), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_W)]$

@

(if *Flags.u1_gauged* then

[$((U (-3), AH, U 3), \textit{FBF} (1, \textit{Psibar}, \textit{VA}, \textit{Psi}), G\_AHTT});$

$((\textit{TopHb}, AH, \textit{TopH}), \textit{FBF} (1, \textit{Psibar}, \textit{VA}, \textit{Psi}), G\_AHTHTH});$

$((\textit{TopHb}, AH, U 3), \textit{FBF} (1, \textit{Psibar}, \textit{VR}, \textit{Psi}), G\_AHTHT});$

$((U (-3), AH, \textit{TopH}), \textit{FBF} (1, \textit{Psibar}, \textit{VR}, \textit{Psi}), G\_AHTHT)]$

else

[]))

$$\mathcal{L}_{\text{CC}} = -\frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i (T^+ W\!\!\!/^+ + T^- W\!\!\!/^-)(1 - \gamma_5)\psi_i \qquad (13.46)$$

let *charged_currents* n =

List.map *mgm*

[$((L (-n), \textit{Wm}, N \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC});$

$((N (-n), \textit{Wp}, L \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC});$

$((D (-n), \textit{Wm}, U \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC});$

$((U (-n), \textit{Wp}, D \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC})]$

let *charged_heavy_currents* n =

List.map *mgm*

([$((L (-n), \textit{WHm}, N \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_heavy});$

$((N (-n), \textit{WHp}, L \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_heavy});$

$((D (-n), \textit{WHm}, U \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_heavy});$

$((U (-n), \textit{WHp}, D \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VL}, \textit{Psi}), G\_CC\_heavy})]$

@

(if *Flags.u1_gauged* then

[$((U (-n), AH, U \; n), \textit{FBF} (1, \textit{Psibar}, \textit{VA}, \textit{Psi}), G\_NC\_h\_up)]$

else

[]))

We specialize the third generation since there is an additional shift coming from the admixture of the heavy top quark. The universal shift, coming from the mixing in the non-Abelian gauge boson sector is unobservable. (Redefinition of coupling constants by measured ones.

let *yukawa* =

List.map *mhm*

[$((U (-3), H, U 3), \textit{FBF} (1, \textit{Psibar}, S, \textit{Psi}), G\_Htt});$

$((D (-3), H, D 3), \textit{FBF} (1, \textit{Psibar}, S, \textit{Psi}), G\_Hbb});$

$((U (-2), H, U 2), \textit{FBF} (1, \textit{Psibar}, S, \textit{Psi}), G\_Hcc});$

$((L (-3), H, L 3), \textit{FBF} (1, \textit{Psibar}, S, \textit{Psi}), G\_Htautau)]$

let *yukawa_add'* =

List.map *mhm*

[$((\textit{TopHb}, H, \textit{TopH}), \textit{FBF} (1, \textit{Psibar}, S, \textit{Psi}), G\_Hthth});$

$((\textit{TopHb}, H, U 3), \textit{FBF} (1, \textit{Psibar}, \textit{SLR}, \textit{Psi}), G\_Htht});$

$((U (-3), H, \textit{TopH}), \textit{FBF} (1, \textit{Psibar}, \textit{SLR}, \textit{Psi}), G\_Htht});$

$((U\ (-3),\ Psi0,\ U\ 3),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Psi0tt);$
$((D\ (-3),\ Psi0,\ D\ 3),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Psi0bb);$
$((U\ (-2),\ Psi0,\ U\ 2),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Psi0cc);$
$((L\ (-3),\ Psi0,\ L\ 3),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Psi0tautau);$
$((U\ (-3),\ Psi1,\ U\ 3),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Psi1tt);$
$((D\ (-3),\ Psi1,\ D\ 3),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Psi1bb);$
$((U\ (-2),\ Psi1,\ U\ 2),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Psi1cc);$
$((L\ (-3),\ Psi1,\ L\ 3),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Psi1tautau);$
$((U\ (-3),\ Psip,\ D\ 3),\ FBF\ (1,\ Psibar,\ SLR,\ Psi),\ G\_Psipq3);$
$((U\ (-2),\ Psip,\ D\ 2),\ FBF\ (1,\ Psibar,\ SLR,\ Psi),\ G\_Psipq2);$
$((N\ (-3),\ Psip,\ L\ 3),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Psipl3);$
$((D\ (-3),\ Psim,\ U\ 3),\ FBF\ (1,\ Psibar,\ SLR,\ Psi),\ G\_Psipq3);$
$((D\ (-2),\ Psim,\ U\ 2),\ FBF\ (1,\ Psibar,\ SLR,\ Psi),\ G\_Psipq2);$
$((L\ (-3),\ Psim,\ N\ 3),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Psipl3);$
$((TopHb,\ Psi0,\ U\ 3),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Psi0tth);$
$((U\ (-3),\ Psi0,\ TopH),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Psi0tth);$
$((TopHb,\ Psi1,\ U\ 3),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Psi1tth);$
$((U\ (-3),\ Psi1,\ TopH),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Psi1tth);$
$((TopHb,\ Psip,\ D\ 3),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Psipbth);$
$((D\ (-3),\ Psim,\ TopH),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Psipbth)]$

let *yukawa_add* =
   if *Flags.u1_gauged* then
    yukawa_add'
   else
    yukawa_add' @
    List.map mhm
    $[\ ((U\ (-3),\ Eta,\ U\ 3),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Ett);$
     $((TopHb,\ Eta,\ U\ 3),\ FBF\ (1,\ Psibar,\ SLR,\ Psi),\ G\_Etht);$
     $((D\ (-3),\ Eta,\ D\ 3),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Ebb);$
     $((U\ (-3),\ Eta,\ TopH),\ FBF\ (1,\ Psibar,\ SLR,\ Psi),\ G\_Etht)]$

$$\mathcal{L}_{\text{TGC}} = -e\partial_\mu A_\nu W_+^\mu W_-^\nu + \ldots - e\cot\theta_w \partial_\mu Z_\nu W_+^\mu W_-^\nu + \ldots \qquad (13.47)$$

let *standard_triple_gauge* =
  List.map tgc
   $[\ ((Ga,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_W);$
    $((Z,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWW);$
    $((Gl,\ Gl,\ Gl),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Gs)\ ]$

let *heavy_triple_gauge* =
  List.map tgc
  $([\ ((Ga,\ WHm,\ WHp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_W);$
   $((Z,\ WHm,\ WHp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWW);$
   $((ZH,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZHWW);$
   $((Z,\ WHm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWHW);$
   $((Z,\ Wm,\ WHp),\ Gauge\_Gauge\_Gauge\ (-1),\ I\_G\_ZWHW);$
   $((ZH,\ WHm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_WWW);$
   $((ZH,\ Wm,\ WHp),\ Gauge\_Gauge\_Gauge\ (-1),\ I\_G\_WWW);$
   $((ZH,\ WHm,\ WHp),\ Gauge\_Gauge\_Gauge\ (-1),\ I\_G\_ZHWHWH)]$

```
       @
         (if Flags.u1_gauged then
      [ ((AH, Wm, Wp), Gauge_Gauge_Gauge 1, I_G_AHWW);
        ((AH, WHm, Wp), Gauge_Gauge_Gauge 1, I_G_AHWHW);
        ((AH, Wm, WHp), Gauge_Gauge_Gauge (−1), I_G_AHWHW);
        ((AH, WHm, WHp), Gauge_Gauge_Gauge 1, I_G_AHWHWH)]
         else
           []))
let triple_gauge =
    standard_triple_gauge @ heavy_triple_gauge

let gauge4 = Vector4 [(2, C_13_42); (−1, C_12_34); (−1, C_14_23)]
let minus_gauge4 = Vector4 [(−2, C_13_42); (1, C_12_34); (1, C_14_23)]
let standard_quartic_gauge =
  List.map qgc
    [ (Wm, Wp, Wm, Wp), gauge4, G_WWWW;
      (Wm, Z, Wp, Z), minus_gauge4, G_ZZWW;
      (Wm, Z, Wp, Ga), minus_gauge4, G_AZWW;
      (Wm, Ga, Wp, Ga), minus_gauge4, G_AAWW;
      (Gl, Gl, Gl, Gl), gauge4, G2 ]

let heavy_quartic_gauge =
  List.map qgc
   ([ (WHm, Wp, WHm, Wp), gauge4, G_WWWW;
      (Wm, WHp, Wm, WHp), gauge4, G_WWWW;
      (WHm, WHp, WHm, WHp), gauge4, G_WH4;
      (Wm, Wp, WHm, WHp), gauge4, G_WHWHWW;
      (Wm, Wp, Wm, WHp), gauge4, G_WHWWW;
      (Wm, Wp, WHm, Wp), gauge4, G_WHWWW;
      (WHm, WHp, Wm, WHp), gauge4, G_WH3W;
      (WHm, WHp, WHm, Wp), gauge4, G_WH3W;
      (WHm, Z, WHp, Z), minus_gauge4, G_ZZWW;
      (WHm, Z, WHp, Ga), minus_gauge4, G_AZWW;
      (WHm, Ga, WHp, ZH), minus_gauge4, G_AAWW;
      (WHm, Z, WHp, ZH), minus_gauge4, G_ZZWW;
      (Wm, ZH, Wp, ZH), minus_gauge4, G_WWWW;
      (Wm, Ga, Wp, ZH), minus_gauge4, G_WWAZH;
      (Wm, Z, Wp, ZH), minus_gauge4, G_WWZZH;
      (WHm, Ga, WHp, ZH), minus_gauge4, G_WHWHAZH;
      (WHm, Z, WHp, ZH), minus_gauge4, G_WHWHZZH;
      (WHm, ZH, WHp, ZH), minus_gauge4, G_WH4;
      (WHm, Z, Wp, Z), minus_gauge4, G_WHWZZ;
      (Wm, Z, WHp, Z), minus_gauge4, G_WHWZZ;
      (WHm, Ga, Wp, Z), minus_gauge4, G_WHWAZ;
      (Wm, Ga, WHp, Z), minus_gauge4, G_WHWAZ;
      (WHm, ZH, Wp, ZH), minus_gauge4, G_WHWZHZH;
      (Wm, ZH, WHp, ZH), minus_gauge4, G_WHWZHZH;
      (WHm, Ga, Wp, ZH), minus_gauge4, G_WHWAZH;
      (Wm, Ga, WHp, ZH), minus_gauge4, G_WHWAZH;
      (WHm, Z, Wp, ZH), minus_gauge4, G_WHWZZH;
      (Wm, Z, WHp, ZH), minus_gauge4, G_WHWZZH]
```

```
    @
      (if Flags.u1_gauged then
        [ (Wm,  Ga,  Wp,  AH),  minus_gauge4,  G_WWAAH;
          (Wm,  Z,   Wp,  AH),  minus_gauge4,  G_WWZAH;
          (WHm, Ga,  WHp, AH),  minus_gauge4,  G_WHWHAAH;
          (WHm, Z,   WHp, AH),  minus_gauge4,  G_WHWHZAH;
          (Wm,  ZH,  Wp,  AH),  minus_gauge4,  G_WWZHAH;
          (WHm, ZH,  WHp, AH),  minus_gauge4,  G_WHWHZHAH;
          (WHm, Ga,  Wp,  AH),  minus_gauge4,  G_WHWAAH;
          (Wm,  Ga,  WHp, AH),  minus_gauge4,  G_WHWAAH;
          (WHm, Z,   Wp,  AH),  minus_gauge4,  G_WHWZAH;
          (Wm,  Z,   WHp, AH),  minus_gauge4,  G_WHWZAH;
          (WHm, ZH,  Wp,  AH),  minus_gauge4,  G_WHWZHAH;
          (Wm,  ZH,  WHp, AH),  minus_gauge4,  G_WHWZHAH]
      else
        []))

let quartic_gauge =
  standard_quartic_gauge @ heavy_quartic_gauge

let standard_gauge_higgs' =
  List.map hgg
    [ ((H,  Wp,  Wm),  Scalar_Vector_Vector 1,  G_HWW);
      ((H,  Z,   Z),   Scalar_Vector_Vector 1,  G_HZZ) ]

let heavy_gauge_higgs =
  List.map hgg
   ([ ((H,  Wp,  WHm), Scalar_Vector_Vector 1,  G_HWHW);
      ((H,  WHp, Wm),  Scalar_Vector_Vector 1,  G_HWHW);
      ((H,  WHp, WHm), Scalar_Vector_Vector 1,  G_HWHWH);
      ((H,  ZH,  ZH),  Scalar_Vector_Vector 1,  G_HWHWH);
      ((H,  ZH,  Z),   Scalar_Vector_Vector 1,  G_HZHZ);
      ((H,  Wp,  Wm),  Scalar_Vector_Vector 1,  G_HZHAH)]
    @
      (if Flags.u1_gauged then
        [((H,  AH,  AH),  Scalar_Vector_Vector 1,  G_HAHAH);
         ((H,  Z,   AH),  Scalar_Vector_Vector 1,  G_HAHZ)]
      else
        []))

let triplet_gauge_higgs =
  List.map hgg
   ([ ((Psi0, Wp,  Wm),  Scalar_Vector_Vector 1,    G_PsiWW);
      ((Psi0, WHp, WHm), Scalar_Vector_Vector (−1), G_PsiWW);
      ((Psi0, WHp, Wm),  Scalar_Vector_Vector 1,    G_PsiWHW);
      ((Psi0, WHm, Wp),  Scalar_Vector_Vector 1,    G_PsiWHW);
      ((Psi0, Z,   Z),   Scalar_Vector_Vector 1,    G_PsiZZ);
      ((Psi0, ZH,  ZH),  Scalar_Vector_Vector 1,    G_PsiZHZH);
      ((Psi0, ZH,  Z),   Scalar_Vector_Vector 1,    G_PsiZHZ);
      ((Psim, Wp,  Z),   Scalar_Vector_Vector 1,    G_PsiZW);
      ((Psip, Wm,  Z),   Scalar_Vector_Vector 1,    G_PsiZW);
      ((Psim, WHp, Z),   Scalar_Vector_Vector 1,    G_PsiZWH);
```

$((Psip,\ WHm,\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiZWH)$;
$((Psim,\ Wp,\ ZH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiZHW)$;
$((Psip,\ Wm,\ ZH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiZHW)$;
$((Psim,\ WHp,\ ZH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiZHWH)$;
$((Psip,\ WHm,\ ZH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiZHWH)$;
$((Psimm,\ Wp,\ Wp),\ Scalar\_Vector\_Vector\ 1,\ G\_PsippWW)$;
$((Psipp,\ Wm,\ Wm),\ Scalar\_Vector\_Vector\ 1,\ G\_PsippWW)$;
$((Psimm,\ WHp,\ Wp),\ Scalar\_Vector\_Vector\ 1,\ G\_PsippWHW)$;
$((Psipp,\ WHm,\ Wm),\ Scalar\_Vector\_Vector\ 1,\ G\_PsippWHW)$;
$((Psimm,\ WHp,\ WHp),\ Scalar\_Vector\_Vector\ 1,\ G\_PsippWHWH)$;
$((Psipp,\ WHm,\ WHm),\ Scalar\_Vector\_Vector\ 1,\ G\_PsippWHWH)]$

@

(if *Flags.u1_gauged* then
  $[((Psi0,\ AH,\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiZAH)$;
  $((Psi0,\ AH,\ ZH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiZHAH)$;
  $((Psi0,\ AH,\ AH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiAHAH)$;
  $((Psim,\ Wp,\ AH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiAHW)$;
  $((Psip,\ Wm,\ AH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiAHW)$;
  $((Psim,\ WHp,\ AH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiAHWH)$;
  $((Psip,\ WHm,\ AH),\ Scalar\_Vector\_Vector\ 1,\ G\_PsiAHWH)]$
else
  $[\ ]))$

let *triplet_gauge2_higgs* =
  List.map ghh
  $([\ ((Wp,\ H,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiHW)$;
  $((Wm,\ H,\ Psip),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiHW)$;
  $((WHp,\ H,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiHWH)$;
  $((WHm,\ H,\ Psip),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiHWH)$;
  $((Wp,\ Psi0,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi0W)$;
  $((Wm,\ Psi0,\ Psip),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi0W)$;
  $((WHp,\ Psi0,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi0WH)$;
  $((WHm,\ Psi0,\ Psip),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi0WH)$;
  $((Wp,\ Psi1,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi1W)$;
  $((Wm,\ Psi1,\ Psip),\ Vector\_Scalar\_Scalar\ (-1),\ G\_Psi1W)$;
  $((WHp,\ Psi1,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi1WH)$;
  $((WHm,\ Psi1,\ Psip),\ Vector\_Scalar\_Scalar\ (-1),\ G\_Psi1WH)$;
  $((Wp,\ Psip,\ Psimm),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiPPW)$;
  $((Wm,\ Psim,\ Psipp),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiPPW)$;
  $((WHp,\ Psip,\ Psimm),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiPPWH)$;
  $((WHm,\ Psim,\ Psipp),\ Vector\_Scalar\_Scalar\ 1,\ G\_PsiPPWH)$;
  $((Ga,\ Psip,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ Q\_lepton)$;
  $((Ga,\ Psipp,\ Psimm),\ Vector\_Scalar\_Scalar\ 2,\ Q\_lepton)$;
  $((Z,\ H,\ Psi1),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi1HZ)$;
  $((ZH,\ H,\ Psi1),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi1HZH)$;
  $((Z,\ Psi0,\ Psi1),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi01Z)$;
  $((ZH,\ Psi0,\ Psi1),\ Vector\_Scalar\_Scalar\ 1,\ G\_Psi01ZH)$;
  $((Z,\ Psip,\ Psim),\ Vector\_Scalar\_Scalar\ 1,\ G\_ZPsip)$;
  $((Z,\ Psipp,\ Psimm),\ Vector\_Scalar\_Scalar\ 2,\ G\_ZPsipp)$;
  $((ZH,\ Psipp,\ Psimm),\ Vector\_Scalar\_Scalar\ 2,\ G\_ZHPsipp)]$

```
      @
        (if Flags.u1_gauged then
          [((AH, H, Psi1), Vector_Scalar_Scalar 1, G_Psi1HAH);
           ((AH, Psi0, Psi1), Vector_Scalar_Scalar 1, G_Psi01AH);
           ((AH, Psip, Psim), Vector_Scalar_Scalar 1, G_AHPsip);
           ((AH, Psipp, Psimm), Vector_Scalar_Scalar 2, G_AHPsip)]
         else []))
```

let *standard_gauge_higgs* =
  standard_gauge_higgs' @ *heavy_gauge_higgs* @ *triplet_gauge_higgs* @
  triplet_gauge2_higgs

let *standard_gauge_higgs4* =
  List.map hhgg
  [(H, H, Wp, Wm), Scalar2_Vector2 1, G_HHWW;
    (H, H, Z, Z), Scalar2_Vector2 1, G_HHZZ]

let *littlest_gauge_higgs4* =
  List.map hhgg
  ([(H, H, WHp, WHm), Scalar2_Vector2 (−1), G_HHWW;
     (H, H, ZH, ZH), Scalar2_Vector2 (−1), G_HHWW;
     (H, H, Wp, WHm), Scalar2_Vector2 1, G_HHWHW;
     (H, H, WHp, Wm), Scalar2_Vector2 1, G_HHWHW;
     (H, H, ZH, Z), Scalar2_Vector2 (−1), G_HHZHZ;
     (H, Psi0, Wp, Wm), Scalar2_Vector2 1, G_HPsi0WW;
     (H, Psi0, WHp, WHm), Scalar2_Vector2 (−1), G_HPsi0WW;
     (H, Psi0, WHp, Wm), Scalar2_Vector2 1, G_HPsi0WHW;
     (H, Psi0, Wp, WHm), Scalar2_Vector2 1, G_HPsi0WHW;
     (H, Psi0, Z, Z), Scalar2_Vector2 1, G_HPsi0ZZ;
     (H, Psi0, ZH, ZH), Scalar2_Vector2 1, G_HPsi0ZHZH;
     (H, Psi0, ZH, Z), Scalar2_Vector2 1, G_HPsi0ZHZ;
     (H, Psim, Wp, Ga), Scalar2_Vector2 1, G_HPsipWA;
     (H, Psip, Wm, Ga), Scalar2_Vector2 1, G_HPsipWA;
     (H, Psim, WHp, Ga), Scalar2_Vector2 1, G_HPsipWHA;
     (H, Psip, WHm, Ga), Scalar2_Vector2 1, G_HPsipWHA;
     (H, Psim, Wp, Z), Scalar2_Vector2 1, G_HPsipWZ;
     (H, Psip, Wm, Z), Scalar2_Vector2 1, G_HPsipWZ;
     (H, Psim, WHp, Z), Scalar2_Vector2 1, G_HPsipWHZ;
     (H, Psip, WHm, Z), Scalar2_Vector2 1, G_HPsipWHZ;
     (H, Psim, Wp, ZH), Scalar2_Vector2 1, G_HPsipWZH;
     (H, Psip, Wm, ZH), Scalar2_Vector2 1, G_HPsipWZH;
     (H, Psim, WHp, ZH), Scalar2_Vector2 1, G_HPsipWHZH;
     (H, Psip, WHm, ZH), Scalar2_Vector2 1, G_HPsipWHZH;
     (H, Psimm, Wp, Wp), Scalar2_Vector2 1, G_HPsippWW;
     (H, Psipp, Wm, Wm), Scalar2_Vector2 1, G_HPsippWW;
     (H, Psimm, WHp, WHp), Scalar2_Vector2 1, G_HPsippWHWH;
     (H, Psipp, WHm, WHm), Scalar2_Vector2 1, G_HPsippWHWH;
     (H, Psimm, WHp, Wp), Scalar2_Vector2 1, G_HPsippWHW;
     (H, Psipp, WHm, Wm), Scalar2_Vector2 1, G_HPsippWHW;
     (Psi0, Psi0, Wp, Wm), Scalar2_Vector2 2, G_HHWW;
     (Psi0, Psi0, WHp, WHm), Scalar2_Vector2 (−2), G_HHWW;
     (Psi0, Psi0, Z, Z), Scalar2_Vector2 4, G_HHZZ;
```

($Psi0$, $Psi0$, $ZH$, $ZH$), $Scalar2\_Vector2$ 1, $G\_Psi00ZH$;
($Psi0$, $Psi0$, $WHp$, $Wm$), $Scalar2\_Vector2$ 2, $G\_HHWHW$;
($Psi0$, $Psi0$, $Wp$, $WHm$), $Scalar2\_Vector2$ 2, $G\_HHWHW$;
($Psi0$, $Psi0$, $Z$, $ZH$), $Scalar2\_Vector2$ 4, $G\_HHZHZ$;
($Psi0$, $Psim$, $Wp$, $Ga$), $Scalar2\_Vector2$ 1, $G\_Psi0pWA$;
($Psi0$, $Psip$, $Wm$, $Ga$), $Scalar2\_Vector2$ 1, $G\_Psi0pWA$;
($Psi0$, $Psim$, $WHp$, $Ga$), $Scalar2\_Vector2$ 1, $G\_Psi0pWHA$;
($Psi0$, $Psip$, $WHm$, $Ga$), $Scalar2\_Vector2$ 1, $G\_Psi0pWHA$;
($Psi0$, $Psim$, $Wp$, $Z$), $Scalar2\_Vector2$ 1, $G\_Psi0pWZ$;
($Psi0$, $Psip$, $Wm$, $Z$), $Scalar2\_Vector2$ 1, $G\_Psi0pWZ$;
($Psi0$, $Psim$, $WHp$, $Z$), $Scalar2\_Vector2$ 1, $G\_Psi0pWHZ$;
($Psi0$, $Psip$, $WHm$, $Z$), $Scalar2\_Vector2$ 1, $G\_Psi0pWHZ$;
($Psi0$, $Psim$, $Wp$, $ZH$), $Scalar2\_Vector2$ 1, $G\_Psi0pWZH$;
($Psi0$, $Psip$, $Wm$, $ZH$), $Scalar2\_Vector2$ 1, $G\_Psi0pWZH$;
($Psi0$, $Psim$, $WHp$, $ZH$), $Scalar2\_Vector2$ 1, $G\_Psi0pWHZH$;
($Psi0$, $Psip$, $WHm$, $ZH$), $Scalar2\_Vector2$ 1, $G\_Psi0pWHZH$;
($Psi0$, $Psimm$, $Wp$, $Wp$), $Scalar2\_Vector2$ 1, $G\_Psi0ppWW$;
($Psi0$, $Psipp$, $Wm$, $Wm$), $Scalar2\_Vector2$ 1, $G\_Psi0ppWW$;
($Psi0$, $Psimm$, $WHp$, $WHp$), $Scalar2\_Vector2$ 1, $G\_Psi0ppWHWH$;
($Psi0$, $Psipp$, $WHm$, $WHm$), $Scalar2\_Vector2$ 1, $G\_Psi0ppWHWH$;
($Psi0$, $Psimm$, $WHp$, $Wp$), $Scalar2\_Vector2$ 1, $G\_Psi0ppWHW$;
($Psi0$, $Psipp$, $WHm$, $Wm$), $Scalar2\_Vector2$ 1, $G\_Psi0ppWHW$;
($Psi1$, $Psi1$, $Wp$, $Wm$), $Scalar2\_Vector2$ 2, $G\_HHWW$;
($Psi1$, $Psi1$, $WHp$, $WHm$), $Scalar2\_Vector2$ ($-2$), $G\_HHWW$;
($Psi1$, $Psi1$, $Z$, $Z$), $Scalar2\_Vector2$ 4, $G\_HHZZ$;
($Psi1$, $Psi1$, $ZH$, $ZH$), $Scalar2\_Vector2$ 1, $G\_Psi00ZH$;
($Psi1$, $Psi1$, $WHp$, $Wm$), $Scalar2\_Vector2$ 2, $G\_HHWHW$;
($Psi1$, $Psi1$, $Wp$, $WHm$), $Scalar2\_Vector2$ 2, $G\_HHWHW$;
($Psi1$, $Psi1$, $Z$, $ZH$), $Scalar2\_Vector2$ 4, $G\_HHZHZ$;
($Psi1$, $Psim$, $Wp$, $Ga$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0pWA$;
($Psi1$, $Psip$, $Wm$, $Ga$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0pWA$;
($Psi1$, $Psim$, $WHp$, $Ga$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0pWHA$;
($Psi1$, $Psip$, $WHm$, $Ga$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0pWHA$;
($Psi1$, $Psim$, $Wp$, $Z$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0pWZ$;
($Psi1$, $Psip$, $Wm$, $Z$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0pWZ$;
($Psi1$, $Psim$, $WHp$, $Z$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0pWHZ$;
($Psi1$, $Psip$, $WHm$, $Z$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0pWHZ$;
($Psi1$, $Psim$, $Wp$, $ZH$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0pWZH$;
($Psi1$, $Psip$, $Wm$, $ZH$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0pWZH$;
($Psi1$, $Psim$, $WHp$, $ZH$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0pWHZH$;
($Psi1$, $Psip$, $WHm$, $ZH$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0pWHZH$;
($Psi1$, $Psimm$, $Wp$, $Wp$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0ppWW$;
($Psi1$, $Psipp$, $Wm$, $Wm$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0ppWW$;
($Psi1$, $Psimm$, $WHp$, $WHp$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0ppWHWH$;
($Psi1$, $Psipp$, $WHm$, $WHm$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0ppWHWH$;
($Psi1$, $Psimm$, $WHp$, $Wp$), $Scalar2\_Vector2$ 1, $I\_G\_Psi0ppWHW$;
($Psi1$, $Psipp$, $WHm$, $Wm$), $Scalar2\_Vector2$ ($-1$), $I\_G\_Psi0ppWHW$;
($Psip$, $Psim$, $Wp$, $Wm$), $Scalar2\_Vector2$ 4, $G\_HHWW$;
($Psip$, $Psim$, $WHp$, $WHm$), $Scalar2\_Vector2$ 1, $G\_Psi00ZH$;
($Psip$, $Psim$, $WHp$, $Wm$), $Scalar2\_Vector2$ 4, $G\_HHWHW$;

$(Psip,\ Psim,\ Wp,\ WHm),\ Scalar2\_Vector2\ 4,\ G\_HHWHW;$
$(Psip,\ Psim,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_PsippZZ;$
$(Psip,\ Psim,\ Ga,\ Ga),\ Scalar2\_Vector2\ 2,\ G\_AAWW;$
$(Psip,\ Psim,\ ZH,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_PsippZHZH;$
$(Psip,\ Psim,\ Ga,\ Z),\ Scalar2\_Vector2\ 4,\ G\_PsippAZ;$
$(Psip,\ Psimm,\ Wp,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_PsippWA;$
$(Psim,\ Psipp,\ Wm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_PsippWA;$
$(Psip,\ Psimm,\ WHp,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_PsippWHA;$
$(Psim,\ Psipp,\ WHm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_PsippWHA;$
$(Psip,\ Psimm,\ Wp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_PsippWZ;$
$(Psim,\ Psipp,\ Wm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_PsippWZ;$
$(Psip,\ Psimm,\ WHp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_PsippWHZ;$
$(Psim,\ Psipp,\ WHm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_PsippWHZ;$
$(Psip,\ Psimm,\ Wp,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_PsippWZH;$
$(Psim,\ Psipp,\ Wm,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_PsippWZH;$
$(Psip,\ Psimm,\ WHp,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_PsippWHZH;$
$(Psim,\ Psipp,\ WHm,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_PsippWHZH;$
$(Psipp,\ Psimm,\ Wp,\ Wm),\ Scalar2\_Vector2\ 2,\ G\_HHWW;$
$(Psipp,\ Psimm,\ WHp,\ WHm),\ Scalar2\_Vector2\ (-2),\ G\_HHWW;$
$(Psipp,\ Psimm,\ WHp,\ Wm),\ Scalar2\_Vector2\ 2,\ G\_HHWHW;$
$(Psipp,\ Psimm,\ Wp,\ WHm),\ Scalar2\_Vector2\ 2,\ G\_HHWHW;$
$(Psipp,\ Psimm,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_PsiccZZ;$
$(Psipp,\ Psimm,\ Ga,\ Ga),\ Scalar2\_Vector2\ 8,\ G\_AAWW;$
$(Psipp,\ Psimm,\ ZH,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi00ZH;$
$(Psipp,\ Psimm,\ Ga,\ Z),\ Scalar2\_Vector2\ 1,\ G\_PsiccAZ;$
$(Psipp,\ Psimm,\ Z,\ ZH),\ Scalar2\_Vector2\ 4,\ G\_PsiccZZH;$
$(Psipp,\ Psimm,\ Ga,\ ZH),\ Scalar2\_Vector2\ 4,\ G\_PsiccAZH]$
$@$
$(\text{if } Flags.u1\_gauged \text{ then}$
$\quad [(H,\ H,\ AH,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HHAA;$
$\quad (H,\ H,\ AH,\ Z),\ Scalar2\_Vector2\ (-1),\ G\_HHAHZ;$
$\quad (H,\ H,\ ZH,\ AH),\ Scalar2\_Vector2\ (-1),\ G\_HHZHAH;$
$\quad (H,\ Psi0,\ AH,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HPsi0AHAH;$
$\quad (H,\ Psi0,\ Z,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HPsi0ZAH;$
$\quad (H,\ Psi0,\ ZH,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HPsi0ZHAH;$
$\quad (H,\ Psim,\ Wp,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWAH;$
$\quad (H,\ Psip,\ Wm,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWAH;$
$\quad (H,\ Psim,\ WHp,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHAH;$
$\quad (H,\ Psip,\ WHm,\ AH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHAH;$
$\quad (Psi0,\ Psi0,\ AH,\ AH),\ Scalar2\_Vector2\ 1,\ G\_Psi00AH;$
$\quad (Psi0,\ Psi0,\ Z,\ AH),\ Scalar2\_Vector2\ 4,\ G\_HHAHZ;$
$\quad (Psi0,\ Psi0,\ AH,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi00ZHAH;$
$\quad (Psi0,\ Psim,\ Wp,\ AH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWAH;$
$\quad (Psi0,\ Psip,\ Wm,\ AH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWAH;$
$\quad (Psi0,\ Psim,\ WHp,\ AH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHAH;$
$\quad (Psi0,\ Psip,\ WHm,\ AH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHAH;$
$\quad (Psi1,\ Psi1,\ AH,\ AH),\ Scalar2\_Vector2\ 1,\ G\_Psi00AH;$
$\quad (Psi1,\ Psi1,\ Z,\ AH),\ Scalar2\_Vector2\ 4,\ G\_HHAHZ;$
$\quad (Psi1,\ Psi1,\ AH,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi00ZHAH;$
$\quad (Psi1,\ Psim,\ Wp,\ AH),\ Scalar2\_Vector2\ 1,\ I\_G\_Psi0pWAH;$

$\qquad$ (*Psi1*, *Psip*, *Wm*, *AH*), *Scalar2_Vector2* (−1), *I_G_Psi0pWAH*;
$\qquad$ (*Psi1*, *Psim*, *WHp*, *AH*), *Scalar2_Vector2* 1, *I_G_Psi0pWHAH*;
$\qquad$ (*Psi1*, *Psip*, *WHm*, *AH*), *Scalar2_Vector2* (−1), *I_G_Psi0pWHAH*;
$\qquad$ (*Psip*, *Psim*, *AH*, *AH*), *Scalar2_Vector2* 1, *G_Psi00AH*;
$\qquad$ (*Psip*, *Psim*, *Ga*, *AH*), *Scalar2_Vector2* 4, *G_PsippAAH*;
$\qquad$ (*Psip*, *Psim*, *Z*, *AH*), *Scalar2_Vector2* 4, *G_PsippZAH*;
$\qquad$ (*Psip*, *Psimm*, *Wp*, *AH*), *Scalar2_Vector2* 1, *G_PsippWAH*;
$\qquad$ (*Psim*, *Psipp*, *Wm*, *AH*), *Scalar2_Vector2* 1, *G_PsippWAH*;
$\qquad$ (*Psip*, *Psimm*, *WHp*, *AH*), *Scalar2_Vector2* 1, *G_PsippWHAH*;
$\qquad$ (*Psim*, *Psipp*, *WHm*, *AH*), *Scalar2_Vector2* 1, *G_PsippWHAH*;
$\qquad$ (*Psipp*, *Psimm*, *AH*, *AH*), *Scalar2_Vector2* 1, *G_Psi00AH*;
$\qquad$ (*Psipp*, *Psimm*, *AH*, *ZH*), *Scalar2_Vector2* (−1), *G_Psi00ZHAH*;
$\qquad$ (*Psipp*, *Psimm*, *Ga*, *AH*), *Scalar2_Vector2* 4, *G_PsiccAAH*;
$\qquad$ (*Psipp*, *Psimm*, *Z*, *AH*), *Scalar2_Vector2* 4, *G_PsiccZAH*]
$\qquad$ else [ ]))

$\quad$ let *standard_higgs* =
$\qquad$ [ (*O H*, *O H*, *O H*), *Scalar_Scalar_Scalar* 1, *G_H3* ]

$\quad$ let *anomaly_higgs* =
$\qquad$ *List.map hgg*
$\qquad$ [ (*Eta*, *Gl*, *Gl*), *Dim5_Scalar_Gauge2_Skew* 1, *G_EGlGl*;
$\qquad\quad$ (*Eta*, *Ga*, *Ga*), *Dim5_Scalar_Gauge2_Skew* 1, *G_EGaGa*;
$\qquad\quad$ (*Eta*, *Ga*, *Z*), *Dim5_Scalar_Gauge2_Skew* 1, *G_EGaZ*]
(* @ (*H*, *Ga*, *Ga*), *Dim5_Scalar_Gauge2* 1, *G_HGaGa*; (*H*, *Ga*, *Z*), *Dim5_Scalar_Gauge2* 1, *G_HGaZ*
*)

$\quad$ let *standard_higgs4* =
$\qquad$ [ (*O H*, *O H*, *O H*, *O H*), *Scalar4* 1, *G_H4* ]

$\quad$ let *gauge_higgs* =
$\qquad$ *standard_gauge_higgs*

$\quad$ let *gauge_higgs4* =
$\qquad$ *standard_gauge_higgs4*

$\quad$ let *higgs* =
$\qquad$ *standard_higgs*

$\quad$ let *higgs4* =
$\qquad$ *standard_higgs4*

$\quad$ let *top_quartic* =
$\qquad$ [ ((*M (U (−3))*, *O H*, *O H*, *M (U 3)*), *GBBG* (1, *Psibar*, *S2*, *Psi*), *G_HHtt*);
((*M (TopHb)*, *O H*, *O H*, *M TopH*), *GBBG* (1, *Psibar*, *S2*, *Psi*), *G_HHthth*);
((*M (U (−3))*, *O H*, *O H*, *M TopH*), *GBBG* (1, *Psibar*, *S2LR*, *Psi*), *G_HHtht*);
((*M (TopHb)*, *O H*, *O H*, *M (U 3)*), *GBBG* (1, *Psibar*, *S2LR*, *Psi*), *G_HHtht*)]

$\quad$ let *goldstone_vertices* =
$\qquad$ *List.map hgg*
$\qquad$ [ ((*Phi0*, *Wm*, *Wp*), *Scalar_Vector_Vector* 1, *I_G_ZWW*);
$\qquad\quad$ ((*Phip*, *Ga*, *Wm*), *Scalar_Vector_Vector* 1, *I_Q_W*);
$\qquad\quad$ ((*Phip*, *Z*, *Wm*), *Scalar_Vector_Vector* 1, *I_G_ZWW*);
$\qquad\quad$ ((*Phim*, *Wp*, *Ga*), *Scalar_Vector_Vector* 1, *I_Q_W*);
$\qquad\quad$ ((*Phim*, *Wp*, *Z*), *Scalar_Vector_Vector* 1, *I_G_ZWW*) ]

let *vertices3* =
  (*ThoList.flatmap electromagnetic_currents* [1; 2; 3] @
   *ThoList.flatmap color_currents* [1; 2; 3] @
   *ThoList.flatmap neutral_currents* [1; 2; 3] @
   *ThoList.flatmap neutral_heavy_currents* [1; 2; 3] @
   *ThoList.flatmap charged_currents* [1; 2; 3] @
   *ThoList.flatmap charged_heavy_currents* [1; 2; 3] @
   *heavy_top_currents* @
   (if *Flags.u1_gauged* then [ ]
       else *anomaly_higgs*) @
   *yukawa* @ *yukawa_add* @ *triple_gauge* @
   *gauge_higgs* @ *higgs* @ *goldstone_vertices*)

let *vertices4* =
  *quartic_gauge* @ *gauge_higgs4* @ *higgs4* @ *top_quartic*

let *vertices* () = (*vertices3*, *vertices4*, [ ])

For efficiency, make sure that *F.of_vertices vertices* is evaluated only once.

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *flavor_of_string* = function
  | "e-" → $M\ (L\ 1)$ | "e+" → $M\ (L\ (-1))$
  | "mu-" → $M\ (L\ 2)$ | "mu+" → $M\ (L\ (-2))$
  | "tau-" → $M\ (L\ 3)$ | "tau+" → $M\ (L\ (-3))$
  | "nue" → $M\ (N\ 1)$ | "nuebar" → $M\ (N\ (-1))$
  | "numu" → $M\ (N\ 2)$ | "numubar" → $M\ (N\ (-2))$
  | "nutau" → $M\ (N\ 3)$ | "nutaubar" → $M\ (N\ (-3))$
  | "u" → $M\ (U\ 1)$ | "ubar" → $M\ (U\ (-1))$
  | "c" → $M\ (U\ 2)$ | "cbar" → $M\ (U\ (-2))$
  | "t" → $M\ (U\ 3)$ | "tbar" → $M\ (U\ (-3))$
  | "d" → $M\ (D\ 1)$ | "dbar" → $M\ (D\ (-1))$
  | "s" → $M\ (D\ 2)$ | "sbar" → $M\ (D\ (-2))$
  | "b" → $M\ (D\ 3)$ | "bbar" → $M\ (D\ (-3))$
  | "th" → $M\ TopH$ | "thbar" → $M\ TopHb$
  | "g" | "gl" → $G\ Gl$
  | "A" → $G\ Ga$ | "Z" | "Z0" → $G\ Z$
  | "AH" | "AH0" | "Ah" | "Ah0" → $G\ AH$
  | "ZH" | "ZH0" | "Zh" | "Zh0" → $G\ ZH$
  | "W+" → $G\ Wp$ | "W-" → $G\ Wm$
  | "WH+" → $G\ WHp$ | "WH-" → $G\ WHm$
  | "H" | "h" → $O\ H$ | "eta" | "Eta" → $O\ Eta$
  | "Psi" | "Psi0" | "psi" | "psi0" → $O\ Psi0$
  | "Psi1" | "psi1" → $O\ Psi1$
  | "Psi+" | "psi+" | "Psip" | "psip" → $O\ Psip$
  | "Psi-" | "psi-" | "Psim" | "psim" → $O\ Psim$
  | "Psi++" | "psi++" | "Psipp" | "psipp" → $O\ Psipp$
  | "Psi--" | "psi--" | "Psimm" | "psimm" → $O\ Psimm$

```
    | _ → invalid_arg "Modellib_BSM.Littlest.flavor_of_string"
let flavor_to_string = function
  | M f →
      begin match f with
      | L 1 → "e-" | L (−1) → "e+"
      | L 2 → "mu-" | L (−2) → "mu+"
      | L 3 → "tau-" | L (−3) → "tau+"
      | L _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_string"
      | N 1 → "nue" | N (−1) → "nuebar"
      | N 2 → "numu" | N (−2) → "numubar"
      | N 3 → "nutau" | N (−3) → "nutaubar"
      | N _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_string"
      | U 1 → "u" | U (−1) → "ubar"
      | U 2 → "c" | U (−2) → "cbar"
      | U 3 → "t" | U (−3) → "tbar"
      | U _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_string"
      | D 1 → "d" | D (−1) → "dbar"
      | D 2 → "s" | D (−2) → "sbar"
      | D 3 → "b" | D (−3) → "bbar"
      | D _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_string"
      | TopH → "th" | TopHb → "thbar"
      end
  | G f →
      begin match f with
      | Gl → "g"
      | Ga → "A" | Z → "Z"
      | Wp → "W+" | Wm → "W-"
      | ZH → "ZH" | AH → "AH" | WHp → "WHp" | WHm → "WHm"
      end
  | O f →
      begin match f with
      | Phip → "phi+" | Phim → "phi-" | Phi0 → "phi0"
      | H → "H" | Eta → "Eta"
      | Psi0 → "Psi0" | Psi1 → "Psi1" | Psip → "Psi+"
      | Psim → "Psi-" | Psipp → "Psi++" | Psimm → "Psi--"
      end
let flavor_to_TeX = function
  | M f →
      begin match f with
      | L 1 → "e^-" | L (−1) → "e^+"
      | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
      | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
      | L _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_TeX"
      | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
      | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
      | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
      | N _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_TeX"
      | U 1 → "u" | U (−1) → "\\bar{u}"
      | U 2 → "c" | U (−2) → "\\bar{c}"
```

```
          | U 3 → "t" | U (−3) → "\\bar{t}"
          | U _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_TeX"
          | D 1 → "d" | D (−1) → "\\bar{d}"
          | D 2 → "s" | D (−2) → "\\bar{s}"
          | D 3 → "b" | D (−3) → "\\bar{b}"
          | D _ → invalid_arg "Modellib_BSM.Littlest.flavor_to_TeX"
          | TopH → "T" | TopHb → "\\bar{T}"
          end
      | G f →
          begin match f with
          | Gl → "g"
          | Ga → "\\gamma" | Z → "Z"
          | Wp → "W^+" | Wm → "W^-"
          | ZH → "Z_H" | AH → "\\gamma_H" | WHp → "W_H^+" |
WHm → "W_H^-"
          end
      | O f →
          begin match f with
          | Phip → "\\Phi^+" | Phim → "\\Phi^-" | Phi0 →
"\\Phi^0"
          | H → "H" | Eta → "\\eta"
          | Psi0 → "\\Psi_S" | Psi1 → "\\Psi_P" | Psip → "\\Psi^+"
          | Psim → "\\Psi^-" | Psipp → "\\Psi^{++}" | Psimm →
"\\Psi^{--}"
          end

    let flavor_symbol = function
      | M f →
          begin match f with
          | L n when n > 0 → "l" ^ string_of_int n
          | L n → "l" ^ string_of_int (abs n) ^ "b"
          | N n when n > 0 → "n" ^ string_of_int n
          | N n → "n" ^ string_of_int (abs n) ^ "b"
          | U n when n > 0 → "u" ^ string_of_int n
          | U n → "u" ^ string_of_int (abs n) ^ "b"
          | D n when n > 0 → "d" ^ string_of_int n
          | D n → "d" ^ string_of_int (abs n) ^ "b"
          | TopH → "th" | TopHb → "thb"
          end
      | G f →
          begin match f with
          | Gl → "gl"
          | Ga → "a" | Z → "z"
          | Wp → "wp" | Wm → "wm"
          | ZH → "zh" | AH → "ah" | WHp → "whp" | WHm → "whm"
          end
      | O f →
          begin match f with
          | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
          | H → "h" | Eta → "eta"
```

```
      | Psi0 → "psi0" | Psi1 → "psi1" | Psip → "psip"
      | Psim → "psim" | Psipp → "psipp" | Psimm → "psimm"
      end
```

There are PDG numbers for Z', Z", W', 32-34, respectively. We just introduce a number 38 for Y0 as a Z"'. As well, there is the number 8 for a t'. But we cheat a little bit and take the number 35 which is reserved for a heavy scalar Higgs for the Eta scalar. For the heavy Higgs states we take 35 and 36 for the neutral ones, 37 for the charged and 38 for the doubly-charged. The pseudoscalar gets the 39.

```
    let pdg = function
      | M f →
          begin match f with
          | L n when n > 0 → 9 + 2 × n
          | L n → − 9 + 2 × n
          | N n when n > 0 → 10 + 2 × n
          | N n → − 10 + 2 × n
          | U n when n > 0 → 2 × n
          | U n → 2 × n
          | D n when n > 0 → − 1 + 2 × n
          | D n → 1 + 2 × n
          | TopH → 8 | TopHb → (−8)
          end
      | G f →
          begin match f with
          | Gl → 21
          | Ga → 22 | Z → 23
          | Wp → 24 | Wm → (−24)
          | AH → 32 | ZH → 33 | WHp → 34 | WHm → (−34)
          end
      | O f →
          begin match f with
          | Phip | Phim → 27 | Phi0 → 26
          | Psi0 → 35 | Psi1 → 36 | Psip → 37 | Psim → (−37)
          | Psipp → 38 | Psimm → (−38)
          | H → 25 | Eta → 39
          end

    let mass_symbol f =
      "mass(" ^ string_of_int (abs (pdg f)) ^ ")"

    let width_symbol f =
      "width(" ^ string_of_int (abs (pdg f)) ^ ")"

    let constant_symbol = function
      | Unit → "unit" | Pi → "PI" | VHeavy → "vheavy"
      | Alpha_QED → "alpha" | E → "e" | G_weak → "g" | Vev →
"vev"
      | Sin2thw → "sin2thw" | Sinthw → "sinthw" | Costhw →
"costhw"
      | Sinpsi → "sinpsi" | Cospsi → "cospsi"
      | Atpsi → "atpsi" | Sccs → "sccs"
```

 | *Supp* → "vF" | *Supp2* → "v2F2"
 | *Q_lepton* → "qlep" | *Q_up* → "qup" | *Q_down* → "qdwn"
 | *Q_Z_up* → "qzup"
 | *G_ZHTHT* → "gzhtht" | *G_ZTHT* → "gzhtht"
 | *G_AHTHTH* → "gahthth" | *G_AHTHT* → "gahtht" | *G_AHTT* →
"gahtt"
 | *G_NC_lepton* → "gnclep" | *G_NC_neutrino* → "gncneu"
 | *G_NC_up* → "gncup" | *G_NC_down* → "gncdwn"
 | *G_CC* → "gcc" | *G_CCtop* → "gcctop" | *G_CC_heavy* →
"gcch"
 | *G_CC_WH* → "gccwh" | *G_CC_W* → "gccw"
 | *G_NC_h_lepton* → "gnchlep" | *G_NC_h_neutrino* → "gnchneu"
 | *G_NC_h_up* → "gnchup" | *G_NC_h_down* → "gnchdwn"
 | *G_NC_heavy* → "gnch"
 | *I_Q_W* → "iqw" | *I_G_ZWW* → "igzww" | *I_G_WWW* →
"igwww"
 | *I_G_AHWW* → "igahww" | *I_G_ZHWW* → "igzhww" | *I_G_ZWHW* →
"igzwhw"
 | *I_G_AHWHWH* → "igahwhwh" | *I_G_ZHWHWH* → "igzhwhwh"
 | *I_G_AHWHW* → "igahwhw"
 | *I_Q_H* → "iqh"
 | *G_WWWW* → "gw4" | *G_ZZWW* → "gzzww"
 | *G_AZWW* → "gazww" | *G_AAWW* → "gaaww"
 | *G_WH4* → "gwh4" | *G_WHWHWW* → "gwhwhww" | *G_WHWWW* →
"gwhwww"
 | *G_WH3W* → "gwh3w"
 | *G_WWAAH* → "gwwaah" | *G_WWAZH* → "gwwazh" | *G_WWZZH* →
"gwwzzh"
 | *G_WWZAH* → "gwwzah" | *G_WHWHAAH* → "gwhwhaah"
 | *G_WHWHAZH* → "gwhwhazh" | *G_WHWHZZH* → "gwhwhzzh"
 | *G_WHWHZAH* → "gwhwhzah"
 | *G_WWZHAH* → "gwwzhah" | *G_WHWHZHAH* → "gwhwhzhah"
 | *G_WHWZZ* → "gwhwzz" | *G_WHWAZ* → "gwhwaz"
 | *G_WHWAAH* → "gwhwaah" | *G_WHWZAH* → "gwhwzah"
 | *G_WHWZHZH* → "gwhwzhzh" | *G_WHWZHAH* → "gwhwzhah"
 | *G_WHWAZH* → "gwhwazh" | *G_WHWZZH* → "gwhwzzh"
 | *G_HWW* → "ghww" | *G_HZZ* → "ghzz"
 | *G_HHWW* → "ghhww" | *G_HHZZ* → "ghhzz"
 | *G_HWHW* → "ghwhw" | *G_HWHWH* → "ghwhwh" | *G_HAHAH* →
"ghahah"
 | *G_HZHZ* → "ghzhz" | *G_HZHAH* → "ghzhah"
 | *G_HAHZ* → "ghahz"
 | *G_Htt* → "ghtt" | *G_Hbb* → "ghbb"
 | *G_Htautau* → "ghtautau" | *G_Hcc* → "ghcc"
 | *G_Hthth* → "ghthth" | *G_Htht* → "ghtht"
 | *G_HHtt* → "ghhtt" | *G_HHthth* → "ghhthth" | *G_HHtht* →
"ghhtht"
 | *G_Psi0tt* → "gpsi0tt" | *G_Psi0bb* → "gpsi0bb"
 | *G_Psi0cc* → "gpsi0cc" | *G_Psi0tautau* → "gpsi0tautau"
 | *G_Psi1tt* → "gpsi1tt" | *G_Psi1bb* → "gpsi1bb"

$\mid G\_Psi1cc \rightarrow$ "gpsi1cc" $\mid G\_Psi1tautau \rightarrow$ "gpsi1tautau"
$\mid G\_Psipq3 \rightarrow$ "gpsipq3" $\mid G\_Psipq2 \rightarrow$ "gpsipq2" $\mid G\_Psipl3 \rightarrow$
"gpsil3"
$\mid G\_Psi0tth \rightarrow$ "gpsi0tth" $\mid G\_Psi1tth \rightarrow$ "gpsi1tth"
$\mid G\_Psipbth \rightarrow$ "gpsipbth"
$\mid G\_Ethth \rightarrow$ "gethth" $\mid G\_Etht \rightarrow$ "getht"
$\mid G\_Ett \rightarrow$ "gett" $\mid G\_Ebb \rightarrow$ "gebb"
$\mid G\_HGaGa \rightarrow$ "ghgaga" $\mid G\_HGaZ \rightarrow$ "ghgaz"
$\mid G\_EGaGa \rightarrow$ "geaa" $\mid G\_EGaZ \rightarrow$ "geaz" $\mid G\_EGlGl \rightarrow$ "gegg"
$\mid G\_H3 \rightarrow$ "gh3" $\mid G\_H4 \rightarrow$ "gh4"
$\mid G\_PsiWW \rightarrow$ "gpsiww" $\mid G\_PsiWHW \rightarrow$ "gpsiwhw"
$\mid G\_PsiZZ \rightarrow$ "gpsizz" $\mid G\_PsiZHZH \rightarrow$ "gpsizhzh"
$\mid G\_PsiZHZ \rightarrow$ "gpsizhz" $\mid G\_PsiZAH \rightarrow$ "gpsizah"
$\mid G\_PsiZHAH \rightarrow$ "gpsizhah" $\mid G\_PsiAHAH \rightarrow$ "gpsiahah"
$\mid G\_PsiZW \rightarrow$ "gpsizw" $\mid G\_PsiZWH \rightarrow$ "gpsizwh" $\mid G\_PsiAHW \rightarrow$
"gpsiahw"
$\mid G\_PsiAHWH \rightarrow$ "gpsiahwh" $\mid G\_PsiZHW \rightarrow$ "gpsizhw"
$\mid G\_PsiZHWH \rightarrow$ "gpsizhwh"
$\mid G\_PsippWW \rightarrow$ "gpsippww" $\mid G\_PsippWHW \rightarrow$ "gpsippwhw"
$\mid G\_PsippWHWH \rightarrow$ "gpsippwhwh"
$\mid Gs \rightarrow$ "gs" $\mid G2 \rightarrow$ "gs**2" $\mid I\_Gs \rightarrow$ "igs"
$\mid G\_PsiHW \rightarrow$ "gpsihw" $\mid G\_PsiHWH \rightarrow$ "gpsihwh"
$\mid G\_Psi0W \rightarrow$ "gpsi0w" $\mid G\_Psi0WH \rightarrow$ "gpsi0wh"
$\mid G\_Psi1W \rightarrow$ "gpsi1w" $\mid G\_Psi1WH \rightarrow$ "gpsi1wh"
$\mid G\_PsiPPW \rightarrow$ "gpsippw" $\mid G\_PsiPPWH \rightarrow$ "gpsippwh"
$\mid G\_Psi1HAH \rightarrow$ "gpsihah" $\mid G\_Psi01AH \rightarrow$ "gpsi0ah"
$\mid G\_AHPsip \rightarrow$ "gahpsip" $\mid G\_Psi1HZ \rightarrow$ "gpsi1hz"
$\mid G\_Psi1HZH \rightarrow$ "gpsi1hzh" $\mid G\_Psi01Z \rightarrow$ "gpsi01z"
$\mid G\_Psi01ZH \rightarrow$ "gpsi01zh" $\mid G\_ZPsip \rightarrow$ "gzpsip"
$\mid G\_ZPsipp \rightarrow$ "gzpsipp" $\mid G\_ZHPsipp \rightarrow$ "gzhpsipp"
$\mid G\_HHAA \rightarrow$ "ghhaa" $\mid G\_HHWHW \rightarrow$ "ghhwhw" $\mid G\_HHZHZ \rightarrow$
"ghhzhz"
$\mid G\_HHAHZ \rightarrow$ "ghhahz" $\mid G\_HHZHAH \rightarrow$ "ghhzhah"
$\mid G\_HPsi0WW \rightarrow$ "ghpsi0ww" $\mid G\_HPsi0WHW \rightarrow$ "ghpsi0whw"
$\mid G\_HPsi0ZZ \rightarrow$ "ghpsi0zz" $\mid G\_HPsi0ZHZH \rightarrow$ "ghpsi0zhzh"
$\mid G\_HPsi0ZHZ \rightarrow$ "ghpsi0zhz" $\mid G\_HPsi0AHAH \rightarrow$ "ghpsi0ahah"
$\mid G\_HPsi0ZAH \rightarrow$ "ghpsi0zah" $\mid G\_HPsi0ZHAH \rightarrow$ "ghpsi0zhah"
$\mid G\_HPsipWA \rightarrow$ "ghpsipwa" $\mid G\_HPsipWHA \rightarrow$ "ghpsipwha"
$\mid G\_HPsipWZ \rightarrow$ "ghpsipwz" $\mid G\_HPsipWHZ \rightarrow$ "ghpsiwhz"
$\mid G\_HPsipWAH \rightarrow$ "ghpsipwah" $\mid G\_HPsipWHAH \rightarrow$ "ghpsipwhah"
$\mid G\_HPsipWZH \rightarrow$ "ghpsipwzh" $\mid G\_HPsipWHZH \rightarrow$ "ghpsipwhzh"
$\mid G\_HPsippWW \rightarrow$ "ghpsippww" $\mid G\_HPsippWHWH \rightarrow$ "ghpsippwhwh"
$\mid G\_HPsippWHW \rightarrow$ "ghpsippwhw" $\mid G\_Psi00ZH \rightarrow$ "gpsi00zh"
$\mid G\_Psi00AH \rightarrow$ "gpsi00ah" $\mid G\_Psi00ZHAH \rightarrow$ "gpsi00zhah"
$\mid G\_Psi0pWA \rightarrow$ "gpsi0pwa" $\mid G\_Psi0pWHA \rightarrow$ "gpsi0pwha"
$\mid G\_Psi0pWZ \rightarrow$ "gpsi0pwz" $\mid G\_Psi0pWHZ \rightarrow$ "gpsi0pwhz"
$\mid G\_Psi0pWAH \rightarrow$ "gpsi0pwah" $\mid G\_Psi0pWHAH \rightarrow$ "gpsi0pwhah"
$\mid G\_Psi0pWZH \rightarrow$ "gpsi0pwzh" $\mid G\_Psi0pWHZH \rightarrow$ "gpsi0pwhzh"
$\mid G\_Psi0ppWW \rightarrow$ "gpsi0ppww" $\mid G\_Psi0ppWHWH \rightarrow$ "gpsi0ppwhwh"
$\mid G\_Psi0ppWHW \rightarrow$ "gpsi0ppwhw"

  | *I_G_Psi0pWA* → "i_gpsi0pwa" | *I_G_Psi0pWHA* → "i_gpsi0pwha"
  | *I_G_Psi0pWZ* → "i_gpsi0pwz" | *I_G_Psi0pWHZ* → "i_gpsi0pwhz"
  | *I_G_Psi0pWAH* → "i_gpsi0pwah" | *I_G_Psi0pWHAH* → "i_gpsi0pwhah"
  | *I_G_Psi0pWZH* → "i_gpsi0pwzh" | *I_G_Psi0pWHZH* → "i_gpsi0pwhzh"
  | *I_G_Psi0ppWW* → "i_gpsi0ppww" | *I_G_Psi0ppWHWH* →
"i_gpsi0ppwhwh"
  | *I_G_Psi0ppWHW* → "i_gpsi0ppwhw"
  | *G_PsippZZ* → "gpsippzz" | *G_PsippZHZH* → "gpsippzhzh"
  | *G_PsippAZ* → "gpsippaz" | *G_PsippAAH* → "gpsippaah"
  | *G_PsippZAH* → "gpsippzah"
  | *G_PsippWA* → "gpsippwa" | *G_PsippWHA* → "gpsippwha"
  | *G_PsippWZ* → "gpsippwz" | *G_PsippWHZ* → "gpsippwhz"
  | *G_PsippWAH* → "gpsippwah" | *G_PsippWHAH* → "gpsippwhah"
  | *G_PsippWZH* → "gpsippwzh" | *G_PsippWHZH* → "gpsippwhzh"
  | *G_PsiccZZ* → "gpsicczz" | *G_PsiccAZ* → "gpsiccaz"
  | *G_PsiccAAH* → "gpsiccaah" | *G_PsiccZZH* → "gpsicczzh"
  | *G_PsiccAZH* → "gpsiccazh" | *G_PsiccZAH* → "gpsicczah"
  | *Mass f* → "mass" ˆ *flavor_symbol f*
  | *Width f* → "width" ˆ *flavor_symbol f*

 end

module *Littlest_Tpar* (*Flags* : *BSM_flags*) =
 struct
  let *rcs* = *rcs_file*

  open *Coupling*

  let *default_width* = *ref Timelike*
  let *use_fudged_width* = *ref* false

  let *options* = *Options.create*
   ["constant_width", *Arg.Unit* (fun () → *default_width* := *Constant*),
    "use␣constant␣width␣(also␣in␣t-channel)";
    "fudged_width", *Arg.Set use_fudged_width*,
    "use␣fudge␣factor␣for␣charge␣particle␣width";
    "custom_width", *Arg.String* (fun *f* → *default_width* := *Custom f*),
    "use␣custom␣width";
    "cancel_widths", *Arg.Unit* (fun () → *default_width* := *Vanishing*),
    "use␣vanishing␣width"]

  type *flavor* = *L* of *int* | *N* of *int* | *U* of *int* | *D* of *int*
   | *Topp* | *Toppb*
   | *Ga* | *Wp* | *Wm* | *Z* | *Gl* | *Lodd* of *int* | *Nodd* of *int*
   | *Uodd* of *int* | *Dodd* of *int*
   | *WHp* | *WHm* | *ZH* | *AH* | *Phip* | *Phim* | *Phi0* | *H* | *Eta* |
Psi0
   | *Psi1* | *Psip* | *Psim* | *Psipp* | *Psimm*

  type *gauge* = *unit*

  let *gauge_symbol* () =
   failwith "Modellib_BSM.Littlest_Tpar.gauge_symbol:␣internal␣error"

let *family n* = [*L n*; *N n*; *U n*; *D n*; *Dodd n*; *Nodd n*; *Lodd n*; *Uodd n*]

Since *Phi* already belongs to the EW Goldstone bosons we use *Psi* for the TeV scale complex triplet.

We use the notation Todd1 = Uodd 3, Todd2 = Uodd 4.

let *external_flavors* () =
  ["1st␣Generation", *ThoList.flatmap family* [1; − 1];
    "2nd␣Generation", *ThoList.flatmap family* [2; − 2];
    "3rd␣Generation", *ThoList.flatmap family* [3; − 3];
    "Heavy␣Quarks", [*Topp*; *Toppb*; *Uodd* 4; *Uodd* (−4)];
    "Heavy␣Scalars", [*Psi0*; *Psi1*; *Psip*; *Psim*; *Psipp*; *Psimm*];
    "Gauge␣Bosons", if *Flags.u1_gauged* then
      [*Ga*; *Z*; *Wp*; *Wm*; *Gl*; *WHp*; *WHm*; *ZH*; *AH*]
        else
      [*Ga*; *Z*; *Wp*; *Wm*; *Gl*; *WHp*; *WHm*; *ZH*];
    "Higgs", if *Flags.u1_gauged* then [*H*]
    else [*H*; *Eta*];
    "Goldstone␣Bosons", [*Phip*; *Phim*; *Phi0*]]

let *flavors* () = *ThoList.flatmap snd* (*external_flavors* ())

let *spinor n* =
  if *n* ≥ 0 then
    Spinor
  else
    ConjSpinor

let *lorentz* = function
  | *L n* → *spinor n* | *N n* → *spinor n*
  | *U n* → *spinor n* | *D n* → *spinor n*
  | *Topp* → *Spinor* | *Toppb* → *ConjSpinor*
  | *Ga* | *Gl* → *Vector*
  | *Wp* | *Wm* | *Z* | *WHp* | *WHm* | *ZH* | *AH* → *Massive_Vector*
  | _ → *Scalar*

let *color* = function
  | *U n* → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *Uodd n* → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *D n* → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *Dodd n* → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *Topp* → *Color.SUN* 3 | *Toppb* → *Color.SUN* (−3)
  | *Gl* → *Color.AdjSUN* 3
  | _ → *Color.Singlet*

let *prop_spinor n* =
  if *n* ≥ 0 then
    Prop_Spinor
  else
    Prop_ConjSpinor

let *propagator* = function
  | *L n* → *prop_spinor n* | *N n* → *prop_spinor n*
  | *Lodd n* → *prop_spinor n* | *Nodd n* → *prop_spinor n*

```
      | U n → prop_spinor n | D n → prop_spinor n
      | Uodd n → prop_spinor n | Dodd n → prop_spinor n
      | Topp → Prop_Spinor | Toppb → Prop_ConjSpinor
      | Ga | Gl → Prop_Feynman
      | Wp | Wm | Z | WHp | WHm | ZH | AH → Prop_Unitarity
      | Phip | Phim | Phi0 → Only_Insertion
      | H | Eta | Psi0 | Psi1 | Psip | Psim | Psipp | Psimm →
   Prop_Scalar
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

```
   let width f =
     if !use_fudged_width then
       match f with
       | Wp | Wm | U 3 | U (−3)
       | WHp | WHm | ZH | AH
       | Uodd _ | Dodd _ | Nodd _ | Lodd _
       | Topp | Toppb → Fudged
       | _ → !default_width
     else
       !default_width

   let goldstone = function
     | Wp → Some (Phip, Coupling.Const 1)
     | Wm → Some (Phim, Coupling.Const 1)
     | Z → Some (Phi0, Coupling.Const 1)
     | _ → None

   let conjugate = function
     | L n → L (−n) | N n → N (−n)
     | Lodd n → L (−n) | Nodd n → N (−n)
     | U n → U (−n) | D n → D (−n)
     | Uodd n → U (−n) | Dodd n → D (−n)
     | Topp → Toppb | Toppb → Topp
     | Gl → Gl | Ga → Ga | Z → Z
     | Wp → Wm | Wm → Wp | WHm → WHp
     | WHp → WHm | ZH → ZH | AH → AH
     | Psi0 → Psi0 | Psi1 → Psi1 | Psip → Psim
     | Psim → Psip | Psipp → Psimm | Psimm → Psipp
     | Phip → Phim | Phim → Phip | Phi0 → Phi0
     | H → H | Eta → Eta

   let fermion = function
     | L n → if n > 0 then 1 else -1
     | N n → if n > 0 then 1 else -1
     | U n → if n > 0 then 1 else -1
     | D n → if n > 0 then 1 else -1
     | Lodd n → if n > 0 then 1 else -1
     | Nodd n → if n > 0 then 1 else -1
     | Uodd n → if n > 0 then 1 else -1
     | Dodd n → if n > 0 then 1 else -1
     | Topp → 1 | Toppb → − 1
```

```
        |  Gl  |  Ga  |  Z  |  Wp  |  Wm  |  WHp  |  WHm  |  AH  |  ZH  →  0
        |  _  →  0
    module Ch  =  Charges.QQ
    let ( // )  =  Algebra.Small_Rational.make

    let charge  = function
        |  L n  |  Lodd n  →  if n  >  0 then -1//1 else 1//1
        |  N n  |  Nodd n  →  0//1
        |  U n  |  Uodd n  →  if n  >  0 then 2//3 else -2//3
        |  D n  |  Dodd n  →  if n  >  0 then -1//3 else 1//3
        |  Topp  →  2//3
        |  Toppb  →  − 2//3
        |  Gl  |  Ga  |  Z  |  AH  |  ZH  →  0//1
        |  Wp  |  WHp  →  1//1
        |  Wm  |  WHm  →  − 1//1
        |  H  |  Phi0  |  Eta  |  Psi1  |  Psi0  →  0//1
        |  Phip  |  Psip  →  1//1
        |  Phim  |  Psim  →  − 1//1
        |  Psipp  →  2//1
        |  Psimm  →  − 2//1

    let lepton  = function
        |  L n  |  N n  |  Lodd n  |  Nodd n
            →  if n  >  0 then 1//1 else -1//1
        |  U _  |  D _  |  _  →  0//1

    let baryon  = function
        |  L _  |  N _  →  0//1
        |  U n  |  D n  |  Uodd n  |  Dodd n
            →  if n  >  0 then 1//1 else -1//1
        |  Topp  →  1//1
        |  Toppb  →  − 1//1
        |  _  →  0//1

    let charges f  =
      [ charge f; lepton f; baryon f]

    type constant  =
        |  Unit  |  Pi  |  Alpha_QED  |  Sin2thw
        |  Sinthw  |  Costhw  |  E  |  G_weak  |  Vev  |  VHeavy
        |  Supp  |  Supp2
        |  Sinpsi  |  Cospsi  |  Atpsi  |  Sccs (∗ Mixing angles of SU(2) ∗)
        |  Q_lepton  |  Q_up  |  Q_down  |  Q_Z_up  |  G_CC  |  G_CCtop
        |  G_NC_neutrino  |  G_NC_lepton  |  G_NC_up  |  G_NC_down  |
G_NC_heavy
        |  G_NC_h_neutrino | G_NC_h_lepton | G_NC_h_up | G_NC_h_down
        |  G_CC_heavy  |  G_ZHTHT  |  G_ZTHT  |  G_AHTHTH  |
G_AHTHT  |  G_AHTT
        |  G_CC_WH  |  G_CC_W
        |  Gs  |  I_Gs  |  G2
        |  I_Q_W  |  I_G_ZWW  |  I_G_WWW
        |  I_G_AHWW  |  I_G_ZHWW  |  I_G_ZWHW  |  I_G_AHWHWH  |
I_G_ZHWHWH
```

| I_G_AHWHW | I_Q_H
| G_WWWW | G_ZZWW | G_AZWW | G_AAWW
| G_WH4 | G_WHWHWW | G_WHWWW | G_WH3W
| G_WWAAH | G_WWAZH | G_WWZZH | G_WWZAH |
G_WHWHAAH
| G_WHWHAZH | G_WHWHZZH | G_WHWHZAH | G_WWZHAH
| G_WHWHZHAH | G_WHWZZ | G_WHWAZ | G_WHWAAH |
G_WHWZAH
| G_WHWZHZH | G_WHWZHAH | G_WHWAZH | G_WHWZZH
| G_HWW | G_HHWW | G_HZZ | G_HHZZ
| G_PsiWW | G_PsiWHW | G_PsiZZ | G_PsiZHZH
| G_PsiZHZ | G_PsiZAH | G_PsiZHAH | G_PsiAHAH
| G_PsiZW | G_PsiZWH | G_PsiAHW | G_PsiAHWH
| G_PsiZHW | G_PsiZHWH
| G_PsippWW | G_PsippWHW | G_PsippWHWH
| G_PsiHW | G_PsiHWH | G_Psi0W | G_Psi0WH
| G_Psi1W | G_Psi1WH | G_PsiPPW | G_PsiPPWH
| G_Psi1HAH | G_Psi01AH | G_AHPsip | G_Psi1HZ
| G_Psi1HZH | G_Psi01Z | G_Psi01ZH | G_ZPsip | G_ZPsipp |
G_ZHPsipp
| G_HHAA | G_HHWHW | G_HHZHZ | G_HHAHZ | G_HHZHAH
| G_HPsi0WW | G_HPsi0WHW | G_HPsi0ZZ
| G_HPsi0ZHZH | G_HPsi0ZHZ | G_HPsi0AHAH | G_HPsi0ZAH |
G_HPsi0ZHAH
| G_HPsipWA | G_HPsipWHA | G_HPsipWZ | G_HPsipWHZ |
G_HPsipWAH
| G_HPsipWHAH | G_HPsipWZH | G_HPsipWHZH | G_HPsippWW |
G_HPsippWHWH
| G_HPsippWHW | G_Psi00ZH | G_Psi00AH | G_Psi00ZHAH
| G_Psi0pWA | G_Psi0pWHA | G_Psi0pWZ | G_Psi0pWHZ |
G_Psi0pWAH
| G_Psi0pWHAH | G_Psi0pWZH | G_Psi0pWHZH | G_Psi0ppWW |
G_Psi0ppWHWH
| G_Psi0ppWHW | I_G_Psi0pWA | I_G_Psi0pWHA | I_G_Psi0pWZ |
I_G_Psi0pWHZ
| I_G_Psi0pWAH | I_G_Psi0pWHAH | I_G_Psi0pWZH | I_G_Psi0pWHZH
| I_G_Psi0ppWW | I_G_Psi0ppWHWH | I_G_Psi0ppWHW
| G_PsippZZ | G_PsippZHZH | G_PsippAZ | G_PsippAAH |
G_PsippZAH
| G_PsippWA | G_PsippWHA | G_PsippWZ | G_PsippWHZ |
G_PsippWAH
| G_PsippWHAH | G_PsippWZH | G_PsippWHZH
| G_PsiccZZ | G_PsiccAZ | G_PsiccAAH | G_PsiccZZH |
G_PsiccAZH
| G_PsiccZAH
| G_Htt | G_Hbb | G_Hcc | G_Htautau | G_H3 | G_H4
| G_Hthth | G_Htht | G_Ethth | G_Etht | G_Ett
| G_HHtt | G_HHthth | G_HHtht
| G_Psi0tt | G_Psi0bb | G_Psi0cc | G_Psi0tautau
| G_Psi1tt | G_Psi1bb | G_Psi1cc | G_Psi1tautau

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      | *G_Psipq3* | *G_Psipq2* | *G_Psipl3* | *G_Psi0tth* | *G_Psi1tth*
      | *G_Psipbth* | *G_Ebb*
      | *G_HGaGa* | *G_HGaZ* | *G_EGaGa* | *G_EGaZ* | *G_EGlGl*
      | *G_HWHW* | *G_HWHWH* | *G_HAHAH* | *G_HZHZ* | *G_HZHAH* |
G_HAHZ
      | *Mass* of *flavor* | *Width* of *flavor*

Two integer counters for the QCD and EW order of the couplings.

type *orders* = *int* × *int*

let *orders* = function
  | _ → (0, 0)

let *input_parameters* =
  [ ]

let *derived_parameters* =
  [ ]

let *g_over_2_costh* =
  Quot (*Neg* (*Atom G_weak*), *Prod* [*Const* 2; *Atom Costhw*])

let *nc_coupling c t3 q* =
  (*Real_Array c*,
   [*Prod* [*g_over_2_costh*; *Diff* (*t3*, *Prod* [*Const* 2; *q*; *Atom Sin2thw*])];
    Prod [*g_over_2_costh*; *t3*]])

let *half* = *Quot* (*Const* 1, *Const* 2)

let *derived_parameter_arrays* =
  [*nc_coupling G_NC_neutrino half* (*Const* 0);
   nc_coupling G_NC_lepton (*Neg half*) (*Const* (−1));
   nc_coupling G_NC_up half (*Quot* (*Const* 2, *Const* 3));
   nc_coupling G_NC_down (*Neg half*) (*Quot* (*Const* (−1), *Const* 3));
   nc_coupling G_NC_h_neutrino half (*Const* 0);
   nc_coupling G_NC_h_lepton (*Neg half*) (*Const* (−1));
   nc_coupling G_NC_h_up half (*Quot* (*Const* 2, *Const* 3));
   nc_coupling G_NC_h_down (*Neg half*) (*Quot* (*Const* (−1), *Const* 3))]

let *parameters* () =
  { *input* = *input_parameters*;
   derived = *derived_parameters*;
   derived_arrays = *derived_parameter_arrays* }

module *F* = *Modeltools.Fusions* (struct
  type *f* = *flavor*
  type *c* = *constant*
  let *compare* = *compare*
  let *conjugate* = *conjugate*
end)

let *electromagnetic_currents n* =
   [((*L* (−*n*), *Ga*, *L n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_lepton*);
    ((*U* (−*n*), *Ga*, *U n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
    ((*D* (−*n*), *Ga*, *D n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

let *color_currents* *n* =
    [((*U* (−*n*), *Gl*, *U* *n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*);
      ((*D* (−*n*), *Gl*, *D* *n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*)]

let *neutral_currents* *n* =
    [((*L* (−*n*), *Z*, *L* *n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_lepton*);
      ((*N* (−*n*), *Z*, *N* *n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_neutrino*);
      ((*U* (−*n*), *Z*, *U* *n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_up*);
      ((*D* (−*n*), *Z*, *D* *n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_down*)]

The sign of this coupling is just the one of the T3, being -(1/2) for leptons and down quarks, and +(1/2) for neutrinos and up quarks.

let *neutral_heavy_currents* *n* =
    ([((*L* (−*n*), *ZH*, *L* *n*), *FBF* ((−1), *Psibar*, *VL*, *Psi*), *G_NC_heavy*);
       ((*N* (−*n*), *ZH*, *N* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_NC_heavy*);
       ((*U* (−*n*), *ZH*, *U* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_NC_heavy*);
       ((*D* (−*n*), *ZH*, *D* *n*), *FBF* ((−1), *Psibar*, *VL*, *Psi*), *G_NC_heavy*)]
     @
       (if *Flags.u1_gauged* then
       [((*L* (−*n*), *AH*, *L* *n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_h_lepton*);
         ((*N* (−*n*), *AH*, *N* *n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_h_neutrino*);
         ((*D* (−*n*), *AH*, *D* *n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_h_down*)]
       else
         []))

let *heavy_top_currents* =
    ([((*Toppb*, *Ga*, *Topp*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
       ((*Toppb*, *Z*, *Topp*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_Z_up*);
       ((*Toppb*, *Z*, *U* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZTHT*);
       ((*U* (−3), *Z*, *Topp*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZTHT*);
       ((*Toppb*, *ZH*, *U* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZHTHT*);
       ((*U* (−3), *ZH*, *Topp*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_ZHTHT*);
       ((*U* (−3), *Wp*, *D* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CCtop*);
       ((*D* (−3), *Wm*, *U* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CCtop*);
       ((*Toppb*, *WHp*, *D* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC_WH*);
       ((*D* (−3), *WHm*, *Topp*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC_WH*);
       ((*Toppb*, *Wp*, *D* 3), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC_W*);
       ((*D* (−3), *Wm*, *Topp*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC_W*)]
     @
       (if *Flags.u1_gauged* then
       [((*U* (−3), *AH*, *U* 3), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_AHTT*);
         ((*Toppb*, *AH*, *Topp*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_AHTHTH*);
         ((*Toppb*, *AH*, *U* 3), *FBF* (1, *Psibar*, *VR*, *Psi*), *G_AHTHT*);
         ((*U* (−3), *AH*, *Topp*), *FBF* (1, *Psibar*, *VR*, *Psi*), *G_AHTHT*)]
       else
         []))

$$\mathcal{L}_{\text{CC}} = -\frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i (T^+ W\!\!\!/^+ + T^- W\!\!\!/^-)(1 - \gamma_5)\psi_i \qquad (13.48)$$

let *charged_currents* *n* =

[((L (−n), Wm, N n), FBF (1, Psibar, VL, Psi), G_CC);
  ((N (−n), Wp, L n), FBF (1, Psibar, VL, Psi), G_CC);
  ((L (−n), WHm, N n), FBF (1, Psibar, VL, Psi), G_CC_heavy);
  ((N (−n), WHp, L n), FBF (1, Psibar, VL, Psi), G_CC_heavy);
  ((D (−n), WHm, U n), FBF (1, Psibar, VL, Psi), G_CC_heavy);
  ((U (−n), WHp, D n), FBF (1, Psibar, VL, Psi), G_CC_heavy)]

let *quark_currents* n =
  ([((D (−n), Wm, U n), FBF (1, Psibar, VL, Psi), G_CC);
     ((U (−n), Wp, D n), FBF (1, Psibar, VL, Psi), G_CC)]
   @
      (if *Flags.u1_gauged* then
   [((U (−n), AH, U n), FBF (1, Psibar, VA, Psi), G_NC_h_up)]
        else
          []))

We specialize the third generation since there is an additional shift coming from the admixture of the heavy top quark. The universal shift, coming from the mixing in the non-Abelian gauge boson sector is unobservable. (Redefinition of coupling constants by measured ones.

let *yukawa* =
  [((U (−3), H, U 3), FBF (1, Psibar, S, Psi), G_Htt);
    ((D (−3), H, D 3), FBF (1, Psibar, S, Psi), G_Hbb);
    ((U (−2), H, U 2), FBF (1, Psibar, S, Psi), G_Hcc);
    ((L (−3), H, L 3), FBF (1, Psibar, S, Psi), G_Htautau)]

let *yukawa_add′* =
  [((Toppb, H, Topp), FBF (1, Psibar, S, Psi), G_Hthth);
    ((Toppb, H, U 3), FBF (1, Psibar, SLR, Psi), G_Htht);
    ((U (−3), H, Topp), FBF (1, Psibar, SLR, Psi), G_Htht);
    ((U (−3), Psi0, U 3), FBF (1, Psibar, S, Psi), G_Psi0tt);
    ((D (−3), Psi0, D 3), FBF (1, Psibar, S, Psi), G_Psi0bb);
    ((U (−2), Psi0, U 2), FBF (1, Psibar, S, Psi), G_Psi0cc);
    ((L (−3), Psi0, L 3), FBF (1, Psibar, S, Psi), G_Psi0tautau);
    ((U (−3), Psi1, U 3), FBF (1, Psibar, P, Psi), G_Psi1tt);
    ((D (−3), Psi1, D 3), FBF (1, Psibar, P, Psi), G_Psi1bb);
    ((U (−2), Psi1, U 2), FBF (1, Psibar, P, Psi), G_Psi1cc);
    ((L (−3), Psi1, L 3), FBF (1, Psibar, P, Psi), G_Psi1tautau);
    ((U (−3), Psip, D 3), FBF (1, Psibar, SLR, Psi), G_Psipq3);
    ((U (−2), Psip, D 2), FBF (1, Psibar, SLR, Psi), G_Psipq2);
    ((N (−3), Psip, L 3), FBF (1, Psibar, SR, Psi), G_Psipl3);
    ((D (−3), Psim, U 3), FBF (1, Psibar, SLR, Psi), G_Psipq3);
    ((D (−2), Psim, U 2), FBF (1, Psibar, SLR, Psi), G_Psipq2);
    ((L (−3), Psim, N 3), FBF (1, Psibar, SL, Psi), G_Psipl3);
    ((Toppb, Psi0, U 3), FBF (1, Psibar, SL, Psi), G_Psi0tth);
    ((U (−3), Psi0, Topp), FBF (1, Psibar, SR, Psi), G_Psi0tth);
    ((Toppb, Psi1, U 3), FBF (1, Psibar, SL, Psi), G_Psi1tth);
    ((U (−3), Psi1, Topp), FBF (1, Psibar, SR, Psi), G_Psi1tth);
    ((Toppb, Psip, D 3), FBF (1, Psibar, SL, Psi), G_Psipbth);
    ((D (−3), Psim, Topp), FBF (1, Psibar, SR, Psi), G_Psipbth)]

let *yukawa_add* =
  if *Flags.u1_gauged* then
    yukawa_add′
  else
    yukawa_add′ @
  [((*U* (−3), *Eta*, *U* 3), *FBF* (1, *Psibar*, *P*, *Psi*), *G_Ett*);
    ((*Toppb*, *Eta*, *U* 3), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_Etht*);
    ((*D* (−3), *Eta*, *D* 3), *FBF* (1, *Psibar*, *P*, *Psi*), *G_Ebb*);
    ((*U* (−3), *Eta*, *Topp*), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_Etht*)]

$$\mathcal{L}_{\mathrm{TGC}} = -e\partial_\mu A_\nu W_+^\mu W_-^\nu + \ldots - e\cot\theta_w \partial_\mu Z_\nu W_+^\mu W_-^\nu + \ldots \qquad (13.49)$$

Check.

let *standard_triple_gauge* =
    [((*Ga*, *Wm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_Q_W*);
      ((*Z*, *Wm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_G_ZWW*);
      ((*Gl*, *Gl*, *Gl*), *Gauge_Gauge_Gauge* 1, *I_Gs*)]

let *heavy_triple_gauge* =
    ([((*Ga*, *WHm*, *WHp*), *Gauge_Gauge_Gauge* 1, *I_Q_W*);
      ((*Z*, *WHm*, *WHp*), *Gauge_Gauge_Gauge* 1, *I_G_ZWW*);
      ((*ZH*, *Wm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_G_ZHWW*);
      ((*Z*, *WHm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_G_ZWHW*);
      ((*Z*, *Wm*, *WHp*), *Gauge_Gauge_Gauge* (−1), *I_G_ZWHW*);
      ((*ZH*, *WHm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_G_WWW*);
      ((*ZH*, *Wm*, *WHp*), *Gauge_Gauge_Gauge* (−1), *I_G_WWW*);
      ((*ZH*, *WHm*, *WHp*), *Gauge_Gauge_Gauge* (−1), *I_G_ZHWHWH*)]
      @
        (if *Flags.u1_gauged* then
    [((*AH*, *Wm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_G_AHWW*);
      ((*AH*, *WHm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_G_AHWHW*);
      ((*AH*, *Wm*, *WHp*), *Gauge_Gauge_Gauge* (−1), *I_G_AHWHW*);
      ((*AH*, *WHm*, *WHp*), *Gauge_Gauge_Gauge* 1, *I_G_AHWHWH*)]
        else
          []))

let *triple_gauge* =
    standard_triple_gauge @ *heavy_triple_gauge*

let *gauge4* = *Vector4* [(2, *C_13_42*); (−1, *C_12_34*); (−1, *C_14_23*)]
let *minus_gauge4* = *Vector4* [(−2, *C_13_42*); (1, *C_12_34*); (1, *C_14_23*)]
let *standard_quartic_gauge* =
    [(*Wm*, *Wp*, *Wm*, *Wp*), *gauge4*, *G_WWWW*;
      (*Wm*, *Z*, *Wp*, *Z*), *minus_gauge4*, *G_ZZWW*;
      (*Wm*, *Z*, *Wp*, *Ga*), *minus_gauge4*, *G_AZWW*;
      (*Wm*, *Ga*, *Wp*, *Ga*), *minus_gauge4*, *G_AAWW*;
      (*Gl*, *Gl*, *Gl*, *Gl*), *gauge4*, *G2*]

let *heavy_quartic_gauge* =
  [(*WHm*, *Wp*, *WHm*, *Wp*), *gauge4*, *G_WWWW*;
    (*Wm*, *WHp*, *Wm*, *WHp*), *gauge4*, *G_WWWW*;

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  (*WHm, WHp, WHm, WHp*), *gauge4*, *G_WH4*;
  (*Wm, Wp, WHm, WHp*), *gauge4*, *G_WHWHWW*;
  (*Wm, Wp, Wm, WHp*), *gauge4*, *G_WHWWW*;
  (*Wm, Wp, WHm, Wp*), *gauge4*, *G_WHWWW*;
  (*WHm, WHp, Wm, WHp*), *gauge4*, *G_WH3W*;
  (*WHm, WHp, WHm, Wp*), *gauge4*, *G_WH3W*;
  (*WHm, Z, WHp, Z*), *minus_gauge4*, *G_ZZWW*;
  (*WHm, Z, WHp, Ga*), *minus_gauge4*, *G_AZWW*;
  (*WHm, Ga, WHp, ZH*), *minus_gauge4*, *G_AAWW*;
  (*WHm, Z, WHp, ZH*), *minus_gauge4*, *G_ZZWW*;
  (*Wm, ZH, Wp, ZH*), *minus_gauge4*, *G_WWWW*;
  (*Wm, Ga, Wp, ZH*), *minus_gauge4*, *G_WWAZH*;
  (*Wm, Z, Wp, ZH*), *minus_gauge4*, *G_WWZZH*;
  (*WHm, Ga, WHp, ZH*), *minus_gauge4*, *G_WHWHAZH*;
  (*WHm, Z, WHp, ZH*), *minus_gauge4*, *G_WHWHZZH*;
  (*WHm, ZH, WHp, ZH*), *minus_gauge4*, *G_WH4*;
  (*WHm, Z, Wp, Z*), *minus_gauge4*, *G_WHWZZ*;
  (*Wm, Z, WHp, Z*), *minus_gauge4*, *G_WHWZZ*;
  (*WHm, Ga, Wp, Z*), *minus_gauge4*, *G_WHWAZ*;
  (*Wm, Ga, WHp, Z*), *minus_gauge4*, *G_WHWAZ*;
  (*WHm, ZH, Wp, ZH*), *minus_gauge4*, *G_WHWZHZH*;
  (*Wm, ZH, WHp, ZH*), *minus_gauge4*, *G_WHWZHZH*;
  (*WHm, Ga, Wp, ZH*), *minus_gauge4*, *G_WHWAZH*;
  (*Wm, Ga, WHp, ZH*), *minus_gauge4*, *G_WHWAZH*;
  (*WHm, Z, Wp, ZH*), *minus_gauge4*, *G_WHWZZH*;
  (*Wm, Z, WHp, ZH*), *minus_gauge4*, *G_WHWZZH*]
  @
   (if *Flags.u1_gauged* then
 [(*Wm, Ga, Wp, AH*), *minus_gauge4*, *G_WWAAH*;
  (*Wm, Z, Wp, AH*), *minus_gauge4*, *G_WWZAH*;
  (*WHm, Ga, WHp, AH*), *minus_gauge4*, *G_WHWHAAH*;
  (*WHm, Z, WHp, AH*), *minus_gauge4*, *G_WHWHZAH*;
  (*Wm, ZH, Wp, AH*), *minus_gauge4*, *G_WWZHAH*;
  (*WHm, ZH, WHp, AH*), *minus_gauge4*, *G_WHWHZHAH*;
  (*WHm, Ga, Wp, AH*), *minus_gauge4*, *G_WHWAAH*;
  (*Wm, Ga, WHp, AH*), *minus_gauge4*, *G_WHWAAH*;
  (*WHm, Z, Wp, AH*), *minus_gauge4*, *G_WHWZAH*;
  (*Wm, Z, WHp, AH*), *minus_gauge4*, *G_WHWZAH*;
  (*WHm, ZH, Wp, AH*), *minus_gauge4*, *G_WHWZHAH*;
  (*Wm, ZH, WHp, AH*), *minus_gauge4*, *G_WHWZHAH*]
   else
    [])

let *quartic_gauge* =
 standard_quartic_gauge @ *heavy_quartic_gauge*

let *standard_gauge_higgs′* =
 [((*H, Wp, Wm*), *Scalar_Vector_Vector* 1, *G_HWW*);
  ((*H, Z, Z*), *Scalar_Vector_Vector* 1, *G_HZZ*)]

let *heavy_gauge_higgs* =
 [((*H, Wp, WHm*), *Scalar_Vector_Vector* 1, *G_HWHW*);

```
                ((H, WHp, Wm), Scalar_Vector_Vector 1, G_HWHW);
                ((H, WHp, WHm), Scalar_Vector_Vector 1, G_HWHWH);
                ((H, ZH, ZH), Scalar_Vector_Vector 1, G_HWHWH);
                ((H, ZH, Z), Scalar_Vector_Vector 1, G_HZHZ);
                ((H, Wp, Wm), Scalar_Vector_Vector 1, G_HZHAH)]
                @
                  (if Flags.u1_gauged then
                [((H, AH, AH), Scalar_Vector_Vector 1, G_HAHAH);
                 ((H, Z, AH), Scalar_Vector_Vector 1, G_HAHZ)]
                    else
                      [])

let triplet_gauge_higgs =
   [ ((Psi0, Wp, Wm), Scalar_Vector_Vector 1, G_PsiWW);
     ((Psi0, WHp, WHm), Scalar_Vector_Vector (−1), G_PsiWW);
     ((Psi0, WHp, Wm), Scalar_Vector_Vector 1, G_PsiWHW);
     ((Psi0, WHm, Wp), Scalar_Vector_Vector 1, G_PsiWHW);
     ((Psi0, Z, Z), Scalar_Vector_Vector 1, G_PsiZZ);
     ((Psi0, ZH, ZH), Scalar_Vector_Vector 1, G_PsiZHZH);
     ((Psi0, ZH, Z), Scalar_Vector_Vector 1, G_PsiZHZ);
     ((Psim, Wp, Z), Scalar_Vector_Vector 1, G_PsiZW);
     ((Psip, Wm, Z), Scalar_Vector_Vector 1, G_PsiZW);
     ((Psim, WHp, Z), Scalar_Vector_Vector 1, G_PsiZWH);
     ((Psip, WHm, Z), Scalar_Vector_Vector 1, G_PsiZWH);
     ((Psim, Wp, ZH), Scalar_Vector_Vector 1, G_PsiZHW);
     ((Psip, Wm, ZH), Scalar_Vector_Vector 1, G_PsiZHW);
     ((Psim, WHp, ZH), Scalar_Vector_Vector 1, G_PsiZHWH);
     ((Psip, WHm, ZH), Scalar_Vector_Vector 1, G_PsiZHWH);
     ((Psimm, Wp, Wp), Scalar_Vector_Vector 1, G_PsippWW);
     ((Psipp, Wm, Wm), Scalar_Vector_Vector 1, G_PsippWW);
     ((Psimm, WHp, Wp), Scalar_Vector_Vector 1, G_PsippWHW);
     ((Psipp, WHm, Wm), Scalar_Vector_Vector 1, G_PsippWHW);
     ((Psimm, WHp, WHp), Scalar_Vector_Vector 1, G_PsippWHWH);
     ((Psipp, WHm, WHm), Scalar_Vector_Vector 1, G_PsippWHWH)]
     @
       (if Flags.u1_gauged then
     [((Psi0, AH, Z), Scalar_Vector_Vector 1, G_PsiZAH);
      ((Psi0, AH, ZH), Scalar_Vector_Vector 1, G_PsiZHAH);
      ((Psi0, AH, AH), Scalar_Vector_Vector 1, G_PsiAHAH);
      ((Psim, Wp, AH), Scalar_Vector_Vector 1, G_PsiAHW);
      ((Psip, Wm, AH), Scalar_Vector_Vector 1, G_PsiAHW);
      ((Psim, WHp, AH), Scalar_Vector_Vector 1, G_PsiAHWH);
      ((Psip, WHm, AH), Scalar_Vector_Vector 1, G_PsiAHWH)]
        else
          [])

let triplet_gauge2_higgs =
   [ ((Wp, H, Psim), Vector_Scalar_Scalar 1, G_PsiHW);
     ((Wm, H, Psip), Vector_Scalar_Scalar 1, G_PsiHW);
     ((WHp, H, Psim), Vector_Scalar_Scalar 1, G_PsiHWH);
     ((WHm, H, Psip), Vector_Scalar_Scalar 1, G_PsiHWH);
```

```
((Wp, Psi0, Psim), Vector_Scalar_Scalar 1, G_Psi0W);
((Wm, Psi0, Psip), Vector_Scalar_Scalar 1, G_Psi0W);
((WHp, Psi0, Psim), Vector_Scalar_Scalar 1, G_Psi0WH);
((WHm, Psi0, Psip), Vector_Scalar_Scalar 1, G_Psi0WH);
((Wp, Psi1, Psim), Vector_Scalar_Scalar 1, G_Psi1W);
((Wm, Psi1, Psip), Vector_Scalar_Scalar (−1), G_Psi1W);
((WHp, Psi1, Psim), Vector_Scalar_Scalar 1, G_Psi1WH);
((WHm, Psi1, Psip), Vector_Scalar_Scalar (−1), G_Psi1WH);
((Wp, Psip, Psimm), Vector_Scalar_Scalar 1, G_PsiPPW);
((Wm, Psim, Psipp), Vector_Scalar_Scalar 1, G_PsiPPW);
((WHp, Psip, Psimm), Vector_Scalar_Scalar 1, G_PsiPPWH);
((WHm, Psim, Psipp), Vector_Scalar_Scalar 1, G_PsiPPWH);
((Ga, Psip, Psim), Vector_Scalar_Scalar 1, Q_lepton);
((Ga, Psipp, Psimm), Vector_Scalar_Scalar 2, Q_lepton);
((Z, H, Psi1), Vector_Scalar_Scalar 1, G_Psi1HZ);
((ZH, H, Psi1), Vector_Scalar_Scalar 1, G_Psi1HZH);
((Z, Psi0, Psi1), Vector_Scalar_Scalar 1, G_Psi01Z);
((ZH, Psi0, Psi1), Vector_Scalar_Scalar 1, G_Psi01ZH);
((Z, Psip, Psim), Vector_Scalar_Scalar 1, G_ZPsip);
((Z, Psipp, Psimm), Vector_Scalar_Scalar 2, G_ZPsipp);
((ZH, Psipp, Psimm), Vector_Scalar_Scalar 2, G_ZHPsipp)]
@
  (if Flags.u1_gauged then
[((AH, H, Psi1), Vector_Scalar_Scalar 1, G_Psi1HAH);
 ((AH, Psi0, Psi1), Vector_Scalar_Scalar 1, G_Psi01AH);
 ((AH, Psip, Psim), Vector_Scalar_Scalar 1, G_AHPsip);
 ((AH, Psipp, Psimm), Vector_Scalar_Scalar 2, G_AHPsip)]
    else [])
```

let *standard_gauge_higgs* =
  standard_gauge_higgs' @ *heavy_gauge_higgs* @ *triplet_gauge_higgs* @
  triplet_gauge2_higgs

let *standard_gauge_higgs4* =
  [(H, H, Wp, Wm), Scalar2_Vector2 1, G_HHWW;
    (H, H, Z, Z), Scalar2_Vector2 1, G_HHZZ]

let *littlest_gauge_higgs4* =
  [(H, H, WHp, WHm), Scalar2_Vector2 (−1), G_HHWW;
    (H, H, ZH, ZH), Scalar2_Vector2 (−1), G_HHWW;
    (H, H, Wp, WHm), Scalar2_Vector2 1, G_HHWHW;
    (H, H, WHp, Wm), Scalar2_Vector2 1, G_HHWHW;
    (H, H, ZH, Z), Scalar2_Vector2 (−1), G_HHZHZ;
    (H, Psi0, Wp, Wm), Scalar2_Vector2 1, G_HPsi0WW;
    (H, Psi0, WHp, WHm), Scalar2_Vector2 (−1), G_HPsi0WW;
    (H, Psi0, WHp, Wm), Scalar2_Vector2 1, G_HPsi0WHW;
    (H, Psi0, Wp, WHm), Scalar2_Vector2 1, G_HPsi0WHW;
    (H, Psi0, Z, Z), Scalar2_Vector2 1, G_HPsi0ZZ;
    (H, Psi0, ZH, ZH), Scalar2_Vector2 1, G_HPsi0ZHZH;
    (H, Psi0, ZH, Z), Scalar2_Vector2 1, G_HPsi0ZHZ;
    (H, Psim, Wp, Ga), Scalar2_Vector2 1, G_HPsipWA;
    (H, Psip, Wm, Ga), Scalar2_Vector2 1, G_HPsipWA;
```

$(H,\ Psim,\ WHp,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHA;$
$(H,\ Psip,\ WHm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHA;$
$(H,\ Psim,\ Wp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_HPsipWZ;$
$(H,\ Psip,\ Wm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_HPsipWZ;$
$(H,\ Psim,\ WHp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHZ;$
$(H,\ Psip,\ WHm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHZ;$
$(H,\ Psim,\ Wp,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWZH;$
$(H,\ Psip,\ Wm,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWZH;$
$(H,\ Psim,\ WHp,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHZH;$
$(H,\ Psip,\ WHm,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_HPsipWHZH;$
$(H,\ Psimm,\ Wp,\ Wp),\ Scalar2\_Vector2\ 1,\ G\_HPsippWW;$
$(H,\ Psipp,\ Wm,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_HPsippWW;$
$(H,\ Psimm,\ WHp,\ WHp),\ Scalar2\_Vector2\ 1,\ G\_HPsippWHWH;$
$(H,\ Psipp,\ WHm,\ WHm),\ Scalar2\_Vector2\ 1,\ G\_HPsippWHWH;$
$(H,\ Psimm,\ WHp,\ Wp),\ Scalar2\_Vector2\ 1,\ G\_HPsippWHW;$
$(H,\ Psipp,\ WHm,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_HPsippWHW;$
$(Psi0,\ Psi0,\ Wp,\ Wm),\ Scalar2\_Vector2\ 2,\ G\_HHWW;$
$(Psi0,\ Psi0,\ WHp,\ WHm),\ Scalar2\_Vector2\ (-2),\ G\_HHWW;$
$(Psi0,\ Psi0,\ Z,\ Z),\ Scalar2\_Vector2\ 4,\ G\_HHZZ;$
$(Psi0,\ Psi0,\ ZH,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi00ZH;$
$(Psi0,\ Psi0,\ WHp,\ Wm),\ Scalar2\_Vector2\ 2,\ G\_HHWHW;$
$(Psi0,\ Psi0,\ Wp,\ WHm),\ Scalar2\_Vector2\ 2,\ G\_HHWHW;$
$(Psi0,\ Psi0,\ Z,\ ZH),\ Scalar2\_Vector2\ 4,\ G\_HHZHZ;$
$(Psi0,\ Psim,\ Wp,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWA;$
$(Psi0,\ Psip,\ Wm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWA;$
$(Psi0,\ Psim,\ WHp,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHA;$
$(Psi0,\ Psip,\ WHm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHA;$
$(Psi0,\ Psim,\ Wp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWZ;$
$(Psi0,\ Psip,\ Wm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWZ;$
$(Psi0,\ Psim,\ WHp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHZ;$
$(Psi0,\ Psip,\ WHm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHZ;$
$(Psi0,\ Psim,\ Wp,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWZH;$
$(Psi0,\ Psip,\ Wm,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWZH;$
$(Psi0,\ Psim,\ WHp,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHZH;$
$(Psi0,\ Psip,\ WHm,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi0pWHZH;$
$(Psi0,\ Psimm,\ Wp,\ Wp),\ Scalar2\_Vector2\ 1,\ G\_Psi0ppWW;$
$(Psi0,\ Psipp,\ Wm,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_Psi0ppWW;$
$(Psi0,\ Psimm,\ WHp,\ WHp),\ Scalar2\_Vector2\ 1,\ G\_Psi0ppWHWH;$
$(Psi0,\ Psipp,\ WHm,\ WHm),\ Scalar2\_Vector2\ 1,\ G\_Psi0ppWHWH;$
$(Psi0,\ Psimm,\ WHp,\ Wp),\ Scalar2\_Vector2\ 1,\ G\_Psi0ppWHW;$
$(Psi0,\ Psipp,\ WHm,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_Psi0ppWHW;$
$(Psi1,\ Psi1,\ Wp,\ Wm),\ Scalar2\_Vector2\ 2,\ G\_HHWW;$
$(Psi1,\ Psi1,\ WHp,\ WHm),\ Scalar2\_Vector2\ (-2),\ G\_HHWW;$
$(Psi1,\ Psi1,\ Z,\ Z),\ Scalar2\_Vector2\ 4,\ G\_HHZZ;$
$(Psi1,\ Psi1,\ ZH,\ ZH),\ Scalar2\_Vector2\ 1,\ G\_Psi00ZH;$
$(Psi1,\ Psi1,\ WHp,\ Wm),\ Scalar2\_Vector2\ 2,\ G\_HHWHW;$
$(Psi1,\ Psi1,\ Wp,\ WHm),\ Scalar2\_Vector2\ 2,\ G\_HHWHW;$
$(Psi1,\ Psi1,\ Z,\ ZH),\ Scalar2\_Vector2\ 4,\ G\_HHZHZ;$
$(Psi1,\ Psim,\ Wp,\ Ga),\ Scalar2\_Vector2\ 1,\ I\_G\_Psi0pWA;$
$(Psi1,\ Psip,\ Wm,\ Ga),\ Scalar2\_Vector2\ (-1),\ I\_G\_Psi0pWA;$

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$(Psi1, Psim, WHp, Ga), Scalar2\_Vector2\ 1, I\_G\_Psi0pWHA;$
$(Psi1, Psip, WHm, Ga), Scalar2\_Vector2\ (-1), I\_G\_Psi0pWHA;$
$(Psi1, Psim, Wp, Z), Scalar2\_Vector2\ 1, I\_G\_Psi0pWZ;$
$(Psi1, Psip, Wm, Z), Scalar2\_Vector2\ (-1), I\_G\_Psi0pWZ;$
$(Psi1, Psim, WHp, Z), Scalar2\_Vector2\ 1, I\_G\_Psi0pWHZ;$
$(Psi1, Psip, WHm, Z), Scalar2\_Vector2\ (-1), I\_G\_Psi0pWHZ;$
$(Psi1, Psim, Wp, ZH), Scalar2\_Vector2\ 1, I\_G\_Psi0pWZH;$
$(Psi1, Psip, Wm, ZH), Scalar2\_Vector2\ (-1), I\_G\_Psi0pWZH;$
$(Psi1, Psim, WHp, ZH), Scalar2\_Vector2\ 1, I\_G\_Psi0pWHZH;$
$(Psi1, Psip, WHm, ZH), Scalar2\_Vector2\ (-1), I\_G\_Psi0pWHZH;$
$(Psi1, Psimm, Wp, Wp), Scalar2\_Vector2\ 1, I\_G\_Psi0ppWW;$
$(Psi1, Psipp, Wm, Wm), Scalar2\_Vector2\ (-1), I\_G\_Psi0ppWW;$
$(Psi1, Psimm, WHp, WHp), Scalar2\_Vector2\ 1, I\_G\_Psi0ppWHWH;$
$(Psi1, Psipp, WHm, WHm), Scalar2\_Vector2\ (-1), I\_G\_Psi0ppWHWH;$
$(Psi1, Psimm, WHp, Wp), Scalar2\_Vector2\ 1, I\_G\_Psi0ppWHW;$
$(Psi1, Psipp, WHm, Wm), Scalar2\_Vector2\ (-1), I\_G\_Psi0ppWHW;$
$(Psip, Psim, Wp, Wm), Scalar2\_Vector2\ 4, G\_HHWW;$
$(Psip, Psim, WHp, WHm), Scalar2\_Vector2\ 1, G\_Psi00ZH;$
$(Psip, Psim, WHp, Wm), Scalar2\_Vector2\ 4, G\_HHWHW;$
$(Psip, Psim, Wp, WHm), Scalar2\_Vector2\ 4, G\_HHWHW;$
$(Psip, Psim, Z, Z), Scalar2\_Vector2\ 1, G\_PsippZZ;$
$(Psip, Psim, Ga, Ga), Scalar2\_Vector2\ 2, G\_AAWW;$
$(Psip, Psim, ZH, ZH), Scalar2\_Vector2\ 1, G\_PsippZHZH;$
$(Psip, Psim, Ga, Z), Scalar2\_Vector2\ 4, G\_PsippAZ;$
$(Psip, Psimm, Wp, Ga), Scalar2\_Vector2\ 1, G\_PsippWA;$
$(Psim, Psipp, Wm, Ga), Scalar2\_Vector2\ 1, G\_PsippWA;$
$(Psip, Psimm, WHp, Ga), Scalar2\_Vector2\ 1, G\_PsippWHA;$
$(Psim, Psipp, WHm, Ga), Scalar2\_Vector2\ 1, G\_PsippWHA;$
$(Psip, Psimm, Wp, Z), Scalar2\_Vector2\ 1, G\_PsippWZ;$
$(Psim, Psipp, Wm, Z), Scalar2\_Vector2\ 1, G\_PsippWZ;$
$(Psip, Psimm, WHp, Z), Scalar2\_Vector2\ 1, G\_PsippWHZ;$
$(Psim, Psipp, WHm, Z), Scalar2\_Vector2\ 1, G\_PsippWHZ;$
$(Psip, Psimm, Wp, ZH), Scalar2\_Vector2\ 1, G\_PsippWZH;$
$(Psim, Psipp, Wm, ZH), Scalar2\_Vector2\ 1, G\_PsippWZH;$
$(Psip, Psimm, WHp, ZH), Scalar2\_Vector2\ 1, G\_PsippWHZH;$
$(Psim, Psipp, WHm, ZH), Scalar2\_Vector2\ 1, G\_PsippWHZH;$
$(Psipp, Psimm, Wp, Wm), Scalar2\_Vector2\ 2, G\_HHWW;$
$(Psipp, Psimm, WHp, WHm), Scalar2\_Vector2\ (-2), G\_HHWW;$
$(Psipp, Psimm, WHp, Wm), Scalar2\_Vector2\ 2, G\_HHWHW;$
$(Psipp, Psimm, Wp, WHm), Scalar2\_Vector2\ 2, G\_HHWHW;$
$(Psipp, Psimm, Z, Z), Scalar2\_Vector2\ 1, G\_PsiccZZ;$
$(Psipp, Psimm, Ga, Ga), Scalar2\_Vector2\ 8, G\_AAWW;$
$(Psipp, Psimm, ZH, ZH), Scalar2\_Vector2\ 1, G\_Psi00ZH;$
$(Psipp, Psimm, Ga, Z), Scalar2\_Vector2\ 1, G\_PsiccAZ;$
$(Psipp, Psimm, Z, ZH), Scalar2\_Vector2\ 4, G\_PsiccZZH;$
$(Psipp, Psimm, Ga, ZH), Scalar2\_Vector2\ 4, G\_PsiccAZH]$
@
  (if *Flags.u1_gauged* then
$[(H, H, AH, AH), Scalar2\_Vector2\ 1, G\_HHAA;$
$(H, H, AH, Z), Scalar2\_Vector2\ (-1), G\_HHAHZ;$

$(H,\ H,\ ZH,\ AH)$, *Scalar2_Vector2* (-1), *G_HHZHAH*;
$(H,\ Psi0,\ AH,\ AH)$, *Scalar2_Vector2* 1, *G_HPsi0AHAH*;
$(H,\ Psi0,\ Z,\ AH)$, *Scalar2_Vector2* 1, *G_HPsi0ZAH*;
$(H,\ Psi0,\ ZH,\ AH)$, *Scalar2_Vector2* 1, *G_HPsi0ZHAH*;
$(H,\ Psim,\ Wp,\ AH)$, *Scalar2_Vector2* 1, *G_HPsipWAH*;
$(H,\ Psip,\ Wm,\ AH)$, *Scalar2_Vector2* 1, *G_HPsipWAH*;
$(H,\ Psim,\ WHp,\ AH)$, *Scalar2_Vector2* 1, *G_HPsipWHAH*;
$(H,\ Psip,\ WHm,\ AH)$, *Scalar2_Vector2* 1, *G_HPsipWHAH*;
$(Psi0,\ Psi0,\ AH,\ AH)$, *Scalar2_Vector2* 1, *G_Psi00AH*;
$(Psi0,\ Psi0,\ Z,\ AH)$, *Scalar2_Vector2* 4, *G_HHAHZ*;
$(Psi0,\ Psi0,\ AH,\ ZH)$, *Scalar2_Vector2* 1, *G_Psi00ZHAH*;
$(Psi0,\ Psim,\ Wp,\ AH)$, *Scalar2_Vector2* 1, *G_Psi0pWAH*;
$(Psi0,\ Psip,\ Wm,\ AH)$, *Scalar2_Vector2* 1, *G_Psi0pWAH*;
$(Psi0,\ Psim,\ WHp,\ AH)$, *Scalar2_Vector2* 1, *G_Psi0pWHAH*;
$(Psi0,\ Psip,\ WHm,\ AH)$, *Scalar2_Vector2* 1, *G_Psi0pWHAH*;
$(Psi1,\ Psi1,\ AH,\ AH)$, *Scalar2_Vector2* 1, *G_Psi00AH*;
$(Psi1,\ Psi1,\ Z,\ AH)$, *Scalar2_Vector2* 4, *G_HHAHZ*;
$(Psi1,\ Psi1,\ AH,\ ZH)$, *Scalar2_Vector2* 1, *G_Psi00ZHAH*;
$(Psi1,\ Psim,\ Wp,\ AH)$, *Scalar2_Vector2* 1, *I_G_Psi0pWAH*;
$(Psi1,\ Psip,\ Wm,\ AH)$, *Scalar2_Vector2* (-1), *I_G_Psi0pWAH*;
$(Psi1,\ Psim,\ WHp,\ AH)$, *Scalar2_Vector2* 1, *I_G_Psi0pWHAH*;
$(Psi1,\ Psip,\ WHm,\ AH)$, *Scalar2_Vector2* (-1), *I_G_Psi0pWHAH*;
$(Psip,\ Psim,\ AH,\ AH)$, *Scalar2_Vector2* 1, *G_Psi00AH*;
$(Psip,\ Psim,\ Ga,\ AH)$, *Scalar2_Vector2* 4, *G_PsippAAH*;
$(Psip,\ Psim,\ Z,\ AH)$, *Scalar2_Vector2* 4, *G_PsippZAH*;
$(Psip,\ Psimm,\ Wp,\ AH)$, *Scalar2_Vector2* 1, *G_PsippWAH*;
$(Psim,\ Psipp,\ Wm,\ AH)$, *Scalar2_Vector2* 1, *G_PsippWAH*;
$(Psip,\ Psimm,\ WHp,\ AH)$, *Scalar2_Vector2* 1, *G_PsippWHAH*;
$(Psim,\ Psipp,\ WHm,\ AH)$, *Scalar2_Vector2* 1, *G_PsippWHAH*;
$(Psipp,\ Psimm,\ AH,\ AH)$, *Scalar2_Vector2* 1, *G_Psi00AH*;
$(Psipp,\ Psimm,\ AH,\ ZH)$, *Scalar2_Vector2* (-1), *G_Psi00ZHAH*;
$(Psipp,\ Psimm,\ Ga,\ AH)$, *Scalar2_Vector2* 4, *G_PsiccAAH*;
$(Psipp,\ Psimm,\ Z,\ AH)$, *Scalar2_Vector2* 4, *G_PsiccZAH*]
     else [])

  let *standard_higgs* =
    [$(H,\ H,\ H)$, *Scalar_Scalar_Scalar* 1, *G_H3*]

  let *anomaly_higgs* =
    [$(Eta,\ Gl,\ Gl)$, *Dim5_Scalar_Gauge2_Skew* 1, *G_EGlGl*;
     $(Eta,\ Ga,\ Ga)$, *Dim5_Scalar_Gauge2_Skew* 1, *G_EGaGa*;
     $(Eta,\ Ga,\ Z)$, *Dim5_Scalar_Gauge2_Skew* 1, *G_EGaZ*]
$(*$ @ $(H,\ Ga,\ Ga)$, *Dim5_Scalar_Gauge2* 1, *G_HGaGa*; $(H,\ Ga,\ Z)$, *Dim5_Scalar_Gauge2* 1, *G_HGaZ*
$*)$

  let *standard_higgs4* =
    [$(H,\ H,\ H,\ H)$, *Scalar4* 1, *G_H4*]

  let *gauge_higgs* =
    standard_gauge_higgs

  let *gauge_higgs4* =
    standard_gauge_higgs4

```
let higgs  =
    standard_higgs

let higgs4  =
    standard_higgs4

let top_quartic  =
  [ ((U (−3), H, H, U 3), GBBG (1, Psibar, S2, Psi), G_HHtt);
((Toppb, H, H, Topp), GBBG (1, Psibar, S2, Psi), G_HHthth);
((U (−3), H, H, Topp), GBBG (1, Psibar, S2LR, Psi), G_HHtht);
((Toppb, H, H, U 3), GBBG (1, Psibar, S2LR, Psi), G_HHtht)]

let goldstone_vertices  =
  [ ((Phi0, Wm, Wp), Scalar_Vector_Vector 1, I_G_ZWW);
    ((Phip, Ga, Wm), Scalar_Vector_Vector 1, I_Q_W);
    ((Phip, Z, Wm), Scalar_Vector_Vector 1, I_G_ZWW);
    ((Phim, Wp, Ga), Scalar_Vector_Vector 1, I_Q_W);
    ((Phim, Wp, Z), Scalar_Vector_Vector 1, I_G_ZWW) ]

let vertices3  =
  (ThoList.flatmap electromagnetic_currents [1; 2; 3] @
   ThoList.flatmap neutral_currents [1; 2; 3] @
   ThoList.flatmap color_currents [1; 2; 3] @
   ThoList.flatmap neutral_heavy_currents [1; 2; 3] @
   ThoList.flatmap charged_currents [1; 2; 3] @
   ThoList.flatmap quark_currents [1; 2] @
   heavy_top_currents @
   (if Flags.u1_gauged then [ ]
        else anomaly_higgs) @
   yukawa @ yukawa_add @ triple_gauge @
   gauge_higgs @ higgs @ goldstone_vertices)

let vertices4  =
  quartic_gauge @ gauge_higgs4 @ higgs4 @ top_quartic

let vertices ()  =  (vertices3, vertices4, [])
```

For efficiency, make sure that *F.of_vertices vertices* is evaluated only once.

```
let table  =  F.of_vertices (vertices ())
let fuse2  =  F.fuse2 table
let fuse3  =  F.fuse3 table
let fuse  =  F.fuse table
let max_degree ()  =  4

let flavor_of_string  = function
  | "e-" → L 1 | "e+" → L (−1)
  | "mu-" → L 2 | "mu+" → L (−2)
  | "tau-" → L 3 | "tau+" → L (−3)
  | "nue" → N 1 | "nuebar" → N (−1)
  | "numu" → N 2 | "numubar" → N (−2)
  | "nutau" → N 3 | "nutaubar" → N (−3)
  | "u" → U 1 | "ubar" → U (−1)
  | "c" → U 2 | "cbar" → U (−2)
  | "t" → U 3 | "tbar" → U (−3)
```

```
  | "d" → D 1 | "dbar" → D (−1)
  | "s" → D 2 | "sbar" → D (−2)
  | "b" → D 3 | "bbar" → D (−3)
  | "tp" → Topp | "tpbar" → Toppb
  | "g" → Gl
  | "A" → Ga | "Z" | "Z0" → Z
  | "AH" | "AH0" | "Ah" | "Ah0" → AH
  | "ZH" | "ZH0" | "Zh" | "Zh0" → ZH
  | "W+" → Wp | "W-" → Wm
  | "WH+" → WHp | "WH-" → WHm
  | "H" | "h" → H | "eta" | "Eta" → Eta
  | "Psi" | "Psi0" | "psi" | "psi0" → Psi0
  | "Psi1" | "psi1" → Psi1
  | "Psi+" | "psi+" | "Psip" | "psip" → Psip
  | "Psi-" | "psi-" | "Psim" | "psim" → Psim
  | "Psi++" | "psi++" | "Psipp" | "psipp" → Psipp
  | "Psi--" | "psi--" | "Psimm" | "psimm" → Psimm
  | _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_of_string"

let flavor_to_string = function
  | L 1 → "e-" | L (−1) → "e+"
  | L 2 → "mu-" | L (−2) → "mu+"
  | L 3 → "tau-" | L (−3) → "tau+"
  | L _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
  | N 1 → "nue" | N (−1) → "nuebar"
  | N 2 → "numu" | N (−2) → "numubar"
  | N 3 → "nutau" | N (−3) → "nutaubar"
  | N _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
  | Lodd 1 → "l1odd-" | Lodd (−1) → "l1odd+"
  | Lodd 2 → "l2odd-" | Lodd (−2) → "l2odd+"
  | Lodd 3 → "l3odd-" | Lodd (−3) → "l3odd+"
  | Lodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
  | Nodd 1 → "n1odd" | Nodd (−1) → "n1oddbar"
  | Nodd 2 → "n2odd" | Nodd (−2) → "n2oddbar"
  | Nodd 3 → "n3odd" | Nodd (−3) → "n3oddbar"
  | Nodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
  | U 1 → "u" | U (−1) → "ubar"
  | U 2 → "c" | U (−2) → "cbar"
  | U 3 → "t" | U (−3) → "tbar"
  | U _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
  | D 1 → "d" | D (−1) → "dbar"
  | D 2 → "s" | D (−2) → "sbar"
  | D 3 → "b" | D (−3) → "bbar"
  | D _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
  | Uodd 1 → "uodd" | Uodd (−1) → "uoddbar"
  | Uodd 2 → "codd" | Uodd (−2) → "coddbar"
  | Uodd 3 → "t1odd" | Uodd (−3) → "t1oddbar"
  | Uodd 4 → "t2odd" | Uodd (−4) → "t2oddbar"
  | Uodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
  | Dodd 1 → "dodd" | Dodd (−1) → "doddbar"
```

```
    | Dodd 2 → "sodd" | Dodd (−2) → "soddbar"
    | Dodd 3 → "bodd" | Dodd (−3) → "boddbar"
    | Dodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_string"
    | Topp → "tp" | Toppb → "tpbar"
    | Gl → "g"
    | Ga → "A" | Z → "Z"
    | Wp → "W+" | Wm → "W-"
    | ZH → "ZH" | AH → "AH" | WHp → "WHp" | WHm → "WHm"
    | Phip → "phi+" | Phim → "phi-" | Phi0 → "phi0"
    | H → "H" | Eta → "Eta"
    | Psi0 → "Psi0" | Psi1 → "Psi1" | Psip → "Psi+"
    | Psim → "Psi-" | Psipp → "Psi++" | Psimm → "Psi--"
let flavor_to_TeX = function
    | L 1 → "e^-" | L (−1) → "e^+"
    | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
    | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
    | L _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
    | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
    | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
    | N _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | Lodd 1 → "L_1^-" | Lodd (−1) → "L_1^+"
    | Lodd 2 → "L_2^-" | Lodd (−2) → "L_2^+"
    | Lodd 3 → "L_3^-" | Lodd (−3) → "L_3^+"
    | Lodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | Nodd 1 → "N_1" | Nodd (−1) → "\\bar{N}_1"
    | Nodd 2 → "N_2" | Nodd (−2) → "\\bar{N}_2"
    | Nodd 3 → "N_3" | Nodd (−3) → "\\bar{N}_3"
    | Nodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | U 1 → "u" | U (−1) → "\\bar{u}"
    | U 2 → "c" | U (−2) → "\\bar{c}"
    | U 3 → "t" | U (−3) → "\\bar{t}"
    | U _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | D 1 → "d" | D (−1) → "\\bar{d}"
    | D 2 → "s" | D (−2) → "\\bar{s}"
    | D 3 → "b" | D (−3) → "\\bar{b}"
    | D _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | Uodd 1 → "U" | Uodd (−1) → "\\bar{U}"
    | Uodd 2 → "C" | Uodd (−2) → "\\bar{C}"
    | Uodd 3 → "T_1" | Uodd (−3) → "\\bar{T}_1"
    | Uodd 4 → "T_2" | Uodd (−4) → "\\bar{T}_2"
    | Uodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | Dodd 1 → "D" | Dodd (−1) → "\\bar{D}"
    | Dodd 2 → "S" | Dodd (−2) → "\\bar{S}"
    | Dodd 3 → "B" | Dodd (−3) → "\\bar{B}"
    | Dodd _ → invalid_arg "Modellib_BSM.Littlest_Tpar.flavor_to_TeX"
    | Topp → "T^\\prime" | Toppb → "\\bar{T}^\\prime"
    | Gl → "g"
    | Ga → "\\gamma" | Z → "Z"
```

```
        | Wp → "W^+" | Wm → "W^-"
        | ZH → "Z_H" | AH → "\\gamma_H" | WHp → "W_H^+" |
  WHm → "W_H^-"
        | Phip → "\\Phi^+" | Phim → "\\Phi^-" | Phi0 → "\\Phi^0"
        | H → "H" | Eta → "\\eta"
        | Psi0 → "\\Psi_S" | Psi1 → "\\Psi_P" | Psip → "\\Psi^+"
        | Psim → "\\Psi^-" | Psipp → "\\Psi^{++}" | Psimm →
  "\\Psi^{--}"
```

```
    let flavor_symbol = function
        | L n when n > 0 → "l" ^ string_of_int n
        | L n → "l" ^ string_of_int (abs n) ^ "b"
        | Lodd n when n > 0 → "lodd" ^ string_of_int n
        | Lodd n → "lodd" ^ string_of_int (abs n) ^ "b"
        | N n when n > 0 → "n" ^ string_of_int n
        | N n → "n" ^ string_of_int (abs n) ^ "b"
        | Nodd n when n > 0 → "nodd" ^ string_of_int n
        | Nodd n → "nodd" ^ string_of_int (abs n) ^ "b"
        | U n when n > 0 → "u" ^ string_of_int n
        | U n → "u" ^ string_of_int (abs n) ^ "b"
        | D n when n > 0 → "d" ^ string_of_int n
        | D n → "d" ^ string_of_int (abs n) ^ "b"
        | Uodd n when n > 0 → "uodd" ^ string_of_int n
        | Uodd n → "uodd" ^ string_of_int (abs n) ^ "b"
        | Dodd n when n > 0 → "dodd" ^ string_of_int n
        | Dodd n → "dodd" ^ string_of_int (abs n) ^ "b"
        | Topp → "tp" | Toppb → "tpb"
        | Gl → "gl"
        | Ga → "a" | Z → "z"
        | Wp → "wp" | Wm → "wm"
        | ZH → "zh" | AH → "ah" | WHp → "whp" | WHm → "whm"
        | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
        | H → "h" | Eta → "eta"
        | Psi0 → "psi0" | Psi1 → "psi1" | Psip → "psip"
        | Psim → "psim" | Psipp → "psipp" | Psimm → "psimm"
```

There are PDG numbers for Z', Z'', W', 32-34, respectively. We just introduce a number 38 for Y0 as a Z'''. As well, there is the number 8 for a t'. But we cheat a little bit and take the number 35 which is reserved for a heavy scalar Higgs for the Eta scalar. For the heavy Higgs states we take 35 and 36 for the neutral ones, 37 for the charged and 38 for the doubly-charged. The pseudoscalar gets the 39. For the odd fermions we add 40 to the values for the SM particles.

```
    let pdg = function
        | L n when n > 0 → 9 + 2 × n
        | L n → − 9 + 2 × n
        | N n when n > 0 → 10 + 2 × n
        | N n → − 10 + 2 × n
        | U n when n > 0 → 2 × n
        | U n → 2 × n
        | D n when n > 0 → − 1 + 2 × n
        | D n → 1 + 2 × n
```

```
          |  Lodd n when n > 0  →  49 + 2 × n
          |  Lodd n  →  − 49 + 2 × n
          |  Nodd n when n > 0  →  50 + 2 × n
          |  Nodd n  →  − 50 + 2 × n
          |  Uodd n when n > 0  →  40 + 2 × n
          |  Uodd n  →  − 40 + 2 × n
          |  Dodd n when n > 0  →  39 + 2 × n
          |  Dodd n  →  − 39 + 2 × n
          |  Topp  →  8  |  Toppb  →  (−8)
          |  Gl  →  21
          |  Ga  →  22  |  Z  →  23
          |  Wp  →  24  |  Wm  →  (−24)
          |  AH  →  32  |  ZH  →  33  |  WHp  →  34  |  WHm  →  (−34)
          |  Phip  |  Phim  →  27  |  Phi0  →  26
          |  Psi0  →  35  |  Psi1  →  36  |  Psip  →  37  |  Psim  →  (−37)
          |  Psipp  →  38  |  Psimm  →  (−38)
          |  H  →  25  |  Eta  →  39
```

let *mass_symbol f* =
  `"mass("` ^ *string_of_int* (*abs* (*pdg f*)) ^ `")"`

let *width_symbol f* =
  `"width("` ^ *string_of_int* (*abs* (*pdg f*)) ^ `")"`

let *constant_symbol* = function
```
    |  Unit  →  "unit"  |  Pi  →  "PI"  |  VHeavy  →  "vheavy"
    |  Alpha_QED  →  "alpha"  |  E  →  "e"  |  G_weak  →  "g"  |  Vev  →
"vev"
    |  Sin2thw  →  "sin2thw"  |  Sinthw  →  "sinthw"  |  Costhw  →
"costhw"
    |  Sinpsi  →  "sinpsi"  |  Cospsi  →  "cospsi"
    |  Atpsi  →  "atpsi"  |  Sccs  →  "sccs"
    |  Supp  →  "vF"  |  Supp2  →  "v2F2"
    |  Q_lepton  →  "qlep"  |  Q_up  →  "qup"  |  Q_down  →  "qdwn"
    |  Q_Z_up  →  "qzup"
    |  G_ZHTHT  →  "gzhtht"  |  G_ZTHT  →  "gzhtht"
    |  G_AHTHTH  →  "gahthth"  |  G_AHTHT  →  "gahtht"  |  G_AHTT  →
"gahtt"
    |  G_NC_lepton  →  "gnclep"  |  G_NC_neutrino  →  "gncneu"
    |  G_NC_up  →  "gncup"  |  G_NC_down  →  "gncdwn"
    |  G_CC  →  "gcc"  |  G_CCtop  →  "gcctop"  |  G_CC_heavy  →
"gcch"
    |  G_CC_WH  →  "gccwh"  |  G_CC_W  →  "gccw"
    |  G_NC_h_lepton  →  "gnchlep"  |  G_NC_h_neutrino  →  "gnchneu"
    |  G_NC_h_up  →  "gnchup"  |  G_NC_h_down  →  "gnchdwn"
    |  G_NC_heavy  →  "gnch"
    |  I_Q_W  →  "iqw"  |  I_G_ZWW  →  "igzww"  |  I_G_WWW  →
"igwww"
    |  I_G_AHWW  →  "igahww"  |  I_G_ZHWW  →  "igzhww"  |  I_G_ZWHW  →
"igzwhw"
    |  I_G_AHWHWH  →  "igahwhwh"  |  I_G_ZHWHWH  →  "igzhwhwh"
    |  I_G_AHWHW  →  "igahwhw"
```

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$\quad | \; I\_Q\_H \; \rightarrow \;$ "iqh"
$\quad | \; Gs \; \rightarrow \;$ "gs" $| \; I\_Gs \; \rightarrow \;$ "igs" $| \; G2 \; \rightarrow \;$ "gs**2"
$\quad | \; G\_WWWW \; \rightarrow \;$ "gw4" $| \; G\_ZZWW \; \rightarrow \;$ "gzzww"
$\quad | \; G\_AZWW \; \rightarrow \;$ "gazww" $| \; G\_AAWW \; \rightarrow \;$ "gaaww"
$\quad | \; G\_WH4 \; \rightarrow \;$ "gwh4" $| \; G\_WHWHWW \; \rightarrow \;$ "gwhwhww" $| \; G\_WHWWW \; \rightarrow$
"gwhwww"
$\quad | \; G\_WH3W \; \rightarrow \;$ "gwh3w"
$\quad | \; G\_WWWAAH \; \rightarrow \;$ "gwwaah" $| \; G\_WWAZH \; \rightarrow \;$ "gwwazh" $| \; G\_WWZZH \; \rightarrow$
"gwwzzh"
$\quad | \; G\_WWZAH \; \rightarrow \;$ "gwwzah" $| \; G\_WHWHAAH \; \rightarrow \;$ "gwhwhaah"
$\quad | \; G\_WHWHAZH \; \rightarrow \;$ "gwhwhazh" $| \; G\_WHWHZZH \; \rightarrow \;$ "gwhwhzzh"
$\quad | \; G\_WHWHZAH \; \rightarrow \;$ "gwhwhzah"
$\quad | \; G\_WWZHAH \; \rightarrow \;$ "gwwzhah" $| \; G\_WHWHZHAH \; \rightarrow \;$ "gwhwhzhah"
$\quad | \; G\_WHWZZ \; \rightarrow \;$ "gwhwzz" $| \; G\_WHWAZ \; \rightarrow \;$ "gwhwaz"
$\quad | \; G\_WHWAAH \; \rightarrow \;$ "gwhwaah" $| \; G\_WHWZAH \; \rightarrow \;$ "gwhwzah"
$\quad | \; G\_WHWZHZH \; \rightarrow \;$ "gwhwzhzh" $| \; G\_WHWZHAH \; \rightarrow \;$ "gwhwzhah"
$\quad | \; G\_WHWAZH \; \rightarrow \;$ "gwhwazh" $| \; G\_WHWZZH \; \rightarrow \;$ "gwhwzzh"
$\quad | \; G\_HWW \; \rightarrow \;$ "ghww" $| \; G\_HZZ \; \rightarrow \;$ "ghzz"
$\quad | \; G\_HHWW \; \rightarrow \;$ "ghhww" $| \; G\_HHZZ \; \rightarrow \;$ "ghhzz"
$\quad | \; G\_HWHW \; \rightarrow \;$ "ghwhw" $| \; G\_HWHWH \; \rightarrow \;$ "ghwhwh" $| \; G\_HAHAH \; \rightarrow$
"ghahah"
$\quad | \; G\_HZHZ \; \rightarrow \;$ "ghzhz" $| \; G\_HZHAH \; \rightarrow \;$ "ghzhah"
$\quad | \; G\_HAHZ \; \rightarrow \;$ "ghahz"
$\quad | \; G\_Htt \; \rightarrow \;$ "ghtt" $| \; G\_Hbb \; \rightarrow \;$ "ghbb"
$\quad | \; G\_Htautau \; \rightarrow \;$ "ghtautau" $| \; G\_Hcc \; \rightarrow \;$ "ghcc"
$\quad | \; G\_Hthth \; \rightarrow \;$ "ghthth" $| \; G\_Htht \; \rightarrow \;$ "ghtht"
$\quad | \; G\_HHtt \; \rightarrow \;$ "ghhtt" $| \; G\_HHthth \; \rightarrow \;$ "ghhthth" $| \; G\_HHtht \; \rightarrow$
"ghhtht"
$\quad | \; G\_Psi0tt \; \rightarrow \;$ "gpsi0tt" $| \; G\_Psi0bb \; \rightarrow \;$ "gpsi0bb"
$\quad | \; G\_Psi0cc \; \rightarrow \;$ "gpsi0cc" $| \; G\_Psi0tautau \; \rightarrow \;$ "gpsi0tautau"
$\quad | \; G\_Psi1tt \; \rightarrow \;$ "gpsi1tt" $| \; G\_Psi1bb \; \rightarrow \;$ "gpsi1bb"
$\quad | \; G\_Psi1cc \; \rightarrow \;$ "gpsi1cc" $| \; G\_Psi1tautau \; \rightarrow \;$ "gpsi1tautau"
$\quad | \; G\_Psipq3 \; \rightarrow \;$ "gpsipq3" $| \; G\_Psipq2 \; \rightarrow \;$ "gpsipq2" $| \; G\_Psipl3 \; \rightarrow$
"gpsil3"
$\quad | \; G\_Psi0tth \; \rightarrow \;$ "gpsi0tth" $| \; G\_Psi1tth \; \rightarrow \;$ "gpsi1tth"
$\quad | \; G\_Psipbth \; \rightarrow \;$ "gpsipbth"
$\quad | \; G\_Ethth \; \rightarrow \;$ "gethth" $| \; G\_Etht \; \rightarrow \;$ "getht"
$\quad | \; G\_Ett \; \rightarrow \;$ "gett" $| \; G\_Ebb \; \rightarrow \;$ "gebb"
$\quad | \; G\_HGaGa \; \rightarrow \;$ "ghgaga" $| \; G\_HGaZ \; \rightarrow \;$ "ghgaz"
$\quad | \; G\_EGaGa \; \rightarrow \;$ "geaa" $| \; G\_EGaZ \; \rightarrow \;$ "geaz" $| \; G\_EGlGl \; \rightarrow \;$ "gegg"
$\quad | \; G\_H3 \; \rightarrow \;$ "gh3" $| \; G\_H4 \; \rightarrow \;$ "gh4"
$\quad | \; G\_PsiWW \; \rightarrow \;$ "gpsiww" $| \; G\_PsiWHW \; \rightarrow \;$ "gpsiwhw"
$\quad | \; G\_PsiZZ \; \rightarrow \;$ "gpsizz" $| \; G\_PsiZHZH \; \rightarrow \;$ "gpsizhzh"
$\quad | \; G\_PsiZHZ \; \rightarrow \;$ "gpsizhz" $| \; G\_PsiZAH \; \rightarrow \;$ "gpsizah"
$\quad | \; G\_PsiZHAH \; \rightarrow \;$ "gpsizhah" $| \; G\_PsiAHAH \; \rightarrow \;$ "gpsiahah"
$\quad | \; G\_PsiZW \; \rightarrow \;$ "gpsizw" $| \; G\_PsiZWH \; \rightarrow \;$ "gpsizwh" $| \; G\_PsiAHW \; \rightarrow$
"gpsiahw"
$\quad | \; G\_PsiAHWH \; \rightarrow \;$ "gpsiahwh" $| \; G\_PsiZHW \; \rightarrow \;$ "gpsizhw"
$\quad | \; G\_PsiZHWH \; \rightarrow \;$ "gpsizhwh"
$\quad | \; G\_PsippWW \; \rightarrow \;$ "gpsippww" $| \; G\_PsippWHW \; \rightarrow \;$ "gpsippwhw"

```
      | G_PsippWHWH → "gpsippwhwh"
      | G_PsiHW → "gpsihw" | G_PsiHWH → "gpsihwh"
      | G_Psi0W → "gpsi0w" | G_Psi0WH → "gpsi0wh"
      | G_Psi1W → "gpsi1w" | G_Psi1WH → "gpsi1wh"
      | G_PsiPPW → "gpsippw" | G_PsiPPWH → "gpsippwh"
      | G_Psi1HAH → "gpsihah" | G_Psi01AH → "gpsi0ah"
      | G_AHPsip → "gahpsip" | G_Psi1HZ → "gpsi1hz"
      | G_Psi1HZH → "gpsi1hzh" | G_Psi01Z → "gpsi01z"
      | G_Psi01ZH → "gpsi01zh" | G_ZPsip → "gzpsip"
      | G_ZPsipp → "gzpsipp" | G_ZHPsipp → "gzhpsipp"
      | G_HHAA → "ghhaa" | G_HHWHW → "ghhwhw" | G_HHZHZ →
"ghhzhz"
      | G_HHAHZ → "ghhahz" | G_HHZHAH → "ghhzhah"
      | G_HPsi0WW → "ghpsi0ww" | G_HPsi0WHW → "ghpsi0whw"
      | G_HPsi0ZZ → "ghpsi0zz" | G_HPsi0ZHZH → "ghpsi0zhzh"
      | G_HPsi0ZHZ → "ghpsi0zhz" | G_HPsi0AHAH → "ghpsi0ahah"
      | G_HPsi0ZAH → "ghpsi0zah" | G_HPsi0ZHAH → "ghpsi0zhah"
      | G_HPsipWA → "ghpsipwa" | G_HPsipWHA → "ghpsipwha"
      | G_HPsipWZ → "ghpsipwz" | G_HPsipWHZ → "ghpsiwhz"
      | G_HPsipWAH → "ghpsipwah" | G_HPsipWHAH → "ghpsipwhah"
      | G_HPsipWZH → "ghpsipwzh" | G_HPsipWHZH → "ghpsipwhzh"
      | G_HPsippWW → "ghpsippww" | G_HPsippWHWH → "ghpsippwhwh"
      | G_HPsippWHW → "ghpsippwhw" | G_Psi00ZH → "gpsi00zh"
      | G_Psi00AH → "gpsi00ah" | G_Psi00ZHAH → "gpsi00zhah"
      | G_Psi0pWA → "gpsi0pwa" | G_Psi0pWHA → "gpsi0pwha"
      | G_Psi0pWZ → "gpsi0pwz" | G_Psi0pWHZ → "gpsi0pwhz"
      | G_Psi0pWAH → "gpsi0pwah" | G_Psi0pWHAH → "gpsi0pwhah"
      | G_Psi0pWZH → "gpsi0pwzh" | G_Psi0pWHZH → "gpsi0pwhzh"
      | G_Psi0ppWW → "gpsi0ppww" | G_Psi0ppWHWH → "gpsi0ppwhwh"
      | G_Psi0ppWHW → "gpsi0ppwhw"
      | I_G_Psi0pWA → "i_gpsi0pwa" | I_G_Psi0pWHA → "i_gpsi0pwha"
      | I_G_Psi0pWZ → "i_gpsi0pwz" | I_G_Psi0pWHZ → "i_gpsi0pwhz"
      | I_G_Psi0pWAH → "i_gpsi0pwah" | I_G_Psi0pWHAH → "i_gpsi0pwhah"
      | I_G_Psi0pWZH → "i_gpsi0pwzh" | I_G_Psi0pWHZH → "i_gpsi0pwhzh"
      | I_G_Psi0ppWW → "i_gpsi0ppww" | I_G_Psi0ppWHWH →
"i_gpsi0ppwhwh"
      | I_G_Psi0ppWHW → "i_gpsi0ppwhw"
      | G_PsippZZ → "gpsippzz" | G_PsippZHZH → "gpsippzhzh"
      | G_PsippAZ → "gpsippaz" | G_PsippAAH → "gpsippaah"
      | G_PsippZAH → "gpsippzah"
      | G_PsippWA → "gpsippwa" | G_PsippWHA → "gpsippwha"
      | G_PsippWZ → "gpsippwz" | G_PsippWHZ → "gpsippwhz"
      | G_PsippWAH → "gpsippwah" | G_PsippWHAH → "gpsippwhah"
      | G_PsippWZH → "gpsippwzh" | G_PsippWHZH → "gpsippwhzh"
      | G_PsiccZZ → "gpsicczz" | G_PsiccAZ → "gpsiccaz"
      | G_PsiccAAH → "gpsiccaah" | G_PsiccZZH → "gpsiccczzh"
      | G_PsiccAZH → "gpsiccazh" | G_PsiccZAH → "gpsicczah"
      | Mass f → "mass" ^ flavor_symbol f
      | Width f → "width" ^ flavor_symbol f
  end
```

```
module Simplest (Flags : BSM_flags) =
  struct
    let rcs = rcs_file

    open Coupling

    let default_width = ref Timelike
    let use_fudged_width = ref false

    let options = Options.create
      [ "constant_width", Arg.Unit (fun () → default_width := Constant),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", Arg.Set use_fudged_width,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", Arg.String (fun f → default_width := Custom f),
        "use␣custom␣width";
        "cancel_widths", Arg.Unit (fun () → default_width := Vanishing),
        "use␣vanishing␣width" ]
```

We do not introduce the Goldstones for the heavy vectors here. The heavy quarks are simply numerated by their generation, the assignments whether they are up- or down-type will be defined by the model.

```
    type flavor = L of int | N of int | U of int | D of int | QH of int
        | NH of int | Wp | Wm | Ga | Z | Xp | Xm | X0 | Y0 | ZH
        | Phip | Phim | Phi0 | H | Eta | Gl

    type gauge = unit

    let gauge_symbol () =
      failwith "Modellib_BSM.Simplest.gauge_symbol:␣internal␣error"

    let family n = [ L n; N n; U n; D n; QH n; NH n ]
```

Note that we add all heavy quarks, *U*, *D*, *C*, *S*, in order to have both embeddings included.

```
    let external_flavors () =
      [ "1st␣Generation␣(incl.␣heavy)", ThoList.flatmap family [1; − 1];
        "2nd␣Generation␣(incl.␣heavy)", ThoList.flatmap family [2; − 2];
        "3rd␣Generation␣(incl.␣heavy)", ThoList.flatmap family [3; − 3];
        "Gauge␣Bosons", [Ga; Z; Wp; Wm; Gl; Xp; Xm; X0; Y0; ZH];
        "Higgs", [H; Eta];
        "Goldstone␣Bosons", [Phip; Phim; Phi0] ]

    let flavors () = ThoList.flatmap snd (external_flavors ())

    let spinor n =
      if n ≥ 0 then
        Spinor
      else
        ConjSpinor

    let lorentz = function
      | L n → spinor n | N n → spinor n
      | U n → spinor n | D n → spinor n
      | QH n → spinor n | NH n → spinor n
```

```
    |  Ga  |  Gl  →  Vector
    |  Wp  |  Wm  |  Z  |  Xp  |  Xm  |  X0  |  Y0  |  ZH  →  Massive_Vector
    |  _  →  Scalar

let color  =  function
    |  U n  →  Color.SUN (if n  >  0 then 3 else -3)
    |  D n  →  Color.SUN (if n  >  0 then 3 else -3)
    |  QH n  →  Color.SUN (if n  >  0 then 3 else -3)
    |  Gl  →  Color.AdjSUN 3
    |  _  →  Color.Singlet

let prop_spinor n  =
    if n  ≥  0 then
        Prop_Spinor
    else
        Prop_ConjSpinor

let propagator  =  function
    |  L n  →  prop_spinor n  |  N n  →  prop_spinor n
    |  U n  →  prop_spinor n  |  D n  →  prop_spinor n
    |  QH n  →  prop_spinor n  |  NH n  →  prop_spinor n
    |  Ga  |  Gl  →  Prop_Feynman
    |  Wp  |  Wm  |  Z  |  Xp  |  Xm  |  X0  |  Y0  |  ZH  →  Prop_Unitarity
    |  Phip  |  Phim  |  Phi0  →  Only_Insertion
    |  H  |  Eta  →  Prop_Scalar
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

```
let width f  =
    if !use_fudged_width then
        match f with
        |  Wp  |  Wm  |  U 3  |  U (−3)  |  QH _  |  NH _  →  Fudged
        |  _  →  !default_width
    else
        !default_width

let goldstone  =  function
    |  Wp  →  Some (Phip, Coupling.Const 1)
    |  Wm  →  Some (Phim, Coupling.Const 1)
    |  Z  →  Some (Phi0, Coupling.Const 1)
    |  _  →  None

let conjugate  =  function
    |  L n  →  L (−n)  |  N n  →  N (−n)
    |  U n  →  U (−n)  |  D n  →  D (−n)
    |  QH n  →  QH (−n)  |  NH n  →  NH (−n)
    |  Ga  →  Ga  |  Gl  →  Gl  |  Z  →  Z
    |  Wp  →  Wm  |  Wm  →  Wp
    |  Xp  →  Xm  |  Xm  →  Xp  |  X0  →  X0  |  Y0  →  Y0  |  ZH  →  ZH
    |  Phip  →  Phim  |  Phim  →  Phip  |  Phi0  →  Phi0
    |  H  →  H  |  Eta  →  Eta

let fermion  =  function
```

```
    | L n  →  if n  >  0 then 1 else -1
    | N n  →  if n  >  0 then 1 else -1
    | U n  →  if n  >  0 then 1 else -1
    | D n  →  if n  >  0 then 1 else -1
    | QH n  →  if n  >  0 then 1 else -1
    | NH n  →  if n  >  0 then 1 else -1
    | Ga  |  Gl  |  Z  |  Wp  |  Wm  |  Xp  |  Xm  |  X0  |  Y0  |  ZH  →  0
    | _  →  0

module Ch  =  Charges.QQ
let ( // )  =  Algebra.Small_Rational.make

let charge  =  function
    | L n  →  if n  >  0 then -1//1 else 1//1
    | N n  |  NH n  →  0//1
    | U n  →  if n  >  0 then 2//3 else -2//3
    | QH 3  →  2//3  |  QH (−3)  →  − 2//3
    | QH (1 | 2)  →
        if Flags.anom_ferm_ass then
            2//3
        else
            -1//3
    | QH ((−1) | (−2))  →
        if Flags.anom_ferm_ass then
            -2//3
        else
            1//3
    | QH n  →  invalid_arg ("Simplest.charge:␣QH␣" ^ string_of_int n)
    | D n  →  if n  >  0 then -1//3 else 1//3
    | Gl  |  Ga  |  Z  |  ZH  |  X0  |  Y0  →  0//1
    | Wp  |  Xp  →  1//1
    | Wm  |  Xm  →  − 1//1
    | H  |  Phi0  |  Eta  →  0//1
    | Phip  →  1//1
    | Phim  →  − 1//1

let lepton  =  function
    | L n  |  N n  |  NH n
        →  if n  >  0 then 1//1 else -1//1
    | U _  |  D _  |  _  →  0//1

let baryon  =  function
    | L _  |  N _  →  0//1
    | U n  |  D n  |  QH n
        →  if n  >  0 then 1//1 else -1//1
    | _  →  0//1

let charges f  =
  [ charge f; lepton f; baryon f]

type constant  =
    | Unit  |  Pi  |  Alpha_QED  |  Sin2thw
    | Sinthw  |  Costhw  |  E  |  G_weak  |  Vev  |  VHeavy
```

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> | *Supp* | *Supp2*
> | *Sinpsi* | *Cospsi* | *Atpsi* | *Sccs* (∗ Mixing angles of SU(2) ∗)
> | *Q_lepton* | *Q_up* | *Q_down* | *Q_Z_up* | *G_CC* | *I_G_CC*
> | *G_NC_neutrino* | *G_NC_lepton* | *G_NC_up* | *G_NC_down*
> | *G_NC_X* | *G_NC_X_t* | *G_NC_Y* | *G_NC_Y_t* | *G_NC_H*
> | *G_NC_h_neutrino* | *G_NC_h_lepton* | *G_NC_h_up* | *G_NC_h_down*
> | *G_NC_h_top* | *G_NC_h_bot* | *G_NCH_N* | *G_NCH_U* |
> *G_NCH_D* | *G_NCHt*
> | *G_zhthth*
> | *I_Q_W* | *I_G_ZWW* | *I_G_WWW*
> | *I_G_Z1* | *I_G_Z2* | *I_G_Z3* | *I_G_Z4* | *I_G_Z5* | *I_G_Z6*
> | *I_Q_H* | *Gs* | *I_Gs* | *G2*
> | *G_WWWW* | *G_ZZWW* | *G_AZWW* | *G_AAWW*
> | *I_Q_ZH*
> | *G_HWW* | *G_HHWW* | *G_HZZ* | *G_HHZZ* | *G_HHZZH*
> | *G_heavy_HVV* | *G_heavy_HWW* | *G_heavy_HZZ* | *G_HHthth*
> | *G_Htt* | *G_Hbb* | *G_Hcc* | *G_Htautau* | *G_H3* | *G_H4*
> | *G_Hthth* | *G_Htht* | *G_Ethth* | *G_Etht* | *G_Ett* | *G_Hqhq*
> | *G_Ebb* | *G_ZEH* | *G_ZHEH* | *G_Hgg*
> | *G_HGaGa* | *G_HGaZ* | *G_EGaGa* | *G_EGaZ* | *G_EGlGl*
> | *Mass* of *flavor* | *Width* of *flavor*

Two integer counters for the QCD and EW order of the couplings.

```
type orders = int × int

let orders = function
  | _ → (0, 0)
```

⚠ The current abstract syntax for parameter dependencies is admittedly tedious. Later, there will be a parser for a convenient concrete syntax as a part of a concrete syntax for models. But as these examples show, it should include simple functions.

```
let input_parameters =
  []

let derived_parameters =
  []

let g_over_2_costh =
  Quot (Neg (Atom G_weak), Prod [Const 2; Atom Costhw])

let nc_coupling c t3 q =
  (Real_Array c,
    [Prod [g_over_2_costh; Diff (t3, Prod [Const 2; q; Atom Sin2thw])];
      Prod [g_over_2_costh; t3]])

let half = Quot (Const 1, Const 2)

let derived_parameter_arrays =
  [ nc_coupling G_NC_neutrino half (Const 0);
    nc_coupling G_NC_lepton (Neg half) (Const (−1));
    nc_coupling G_NC_up half (Quot (Const 2, Const 3));
```

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```
        nc_coupling G_NC_down (Neg half) (Quot (Const (−1), Const 3));
        nc_coupling G_NC_h_neutrino half (Const 0);
        nc_coupling G_NC_h_lepton (Neg half) (Const (−1));
        nc_coupling G_NC_h_up half (Quot (Const 2, Const 3));
        nc_coupling G_NC_h_down (Neg half) (Quot (Const (−1), Const 3)) ]

let parameters () =
  { input = input_parameters;
    derived = derived_parameters;
    derived_arrays = derived_parameter_arrays }

module F = Modeltools.Fusions (struct
  type f = flavor
  type c = constant
  let compare = compare
  let conjugate = conjugate
end)

let electromagnetic_currents n =
    [ ((L (−n), Ga, L n), FBF (1, Psibar, V, Psi), Q_lepton);
      ((U (−n), Ga, U n), FBF (1, Psibar, V, Psi), Q_up);
      ((D (−n), Ga, D n), FBF (1, Psibar, V, Psi), Q_down) ]

let color_currents n =
  [ ((D (−n), Gl, D n), FBF ((−1), Psibar, V, Psi), Gs);
    ((U (−n), Gl, U n), FBF ((−1), Psibar, V, Psi), Gs);
    ((QH (−n), Gl, QH n), FBF ((−1), Psibar, V, Psi), Gs)]

let neutral_currents n =
    [ ((L (−n), Z, L n), FBF (1, Psibar, VA, Psi), G_NC_lepton);
      ((N (−n), Z, N n), FBF (1, Psibar, VA, Psi), G_NC_neutrino);
      ((U (−n), Z, U n), FBF (1, Psibar, VA, Psi), G_NC_up);
      ((D (−n), Z, D n), FBF (1, Psibar, VA, Psi), G_NC_down) ]

let xy_currents =
  ThoList.flatmap
    (fun n → [ ((N (−n), X0, N n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
                ((L (−n), Xm, N n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
                ((N (−n), Xp, L n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
                ((N (−n), Y0, N n), FBF ((−1), Psibar, VL, Psi), G_NC_Y);
                ((NH (−n), X0, N n), FBF ((−1), Psibar, VL, Psi), G_CC);
                ((N (−n), X0, NH n), FBF ((−1), Psibar, VL, Psi), G_CC);
                ((NH (−n), Y0, N n), FBF ((−1), Psibar, VL, Psi), I_G_CC);
                ((N (−n), Y0, NH n), FBF ((−1), Psibar, VL, Psi), I_G_CC);
                ((L (−n), Xm, NH n), FBF ((−1), Psibar, VL, Psi), G_CC);
                ((NH (−n), Xp, L n), FBF ((−1), Psibar, VL, Psi), G_CC)])
    [1; 2; 3]
  @
    [ ((U (−3), X0, U 3), FBF (1, Psibar, VL, Psi), G_NC_X_t);
      ((U (−3), Y0, U 3), FBF (1, Psibar, VL, Psi), G_NC_Y_t);
      ((U (−3), X0, QH 3), FBF (1, Psibar, VL, Psi), G_CC);
      ((QH (−3), X0, U 3), FBF (1, Psibar, VL, Psi), G_CC);
      ((U (−3), Y0, QH 3), FBF (1, Psibar, VL, Psi), I_G_CC);
```

```
            ((QH (−3), Y0, U 3), FBF (1, Psibar, VL, Psi), I_G_CC);
            ((D (−3), Xm, U 3), FBF (1, Psibar, VL, Psi), G_NC_X_t);
            ((U (−3), Xp, D 3), FBF (1, Psibar, VL, Psi), G_NC_X_t);
            ((D (−3), Xm, QH 3), FBF (1, Psibar, VL, Psi), G_CC);
            ((QH (−3), Xp, D 3), FBF (1, Psibar, VL, Psi), G_CC);
            ((QH (−3), Wp, D 3), FBF (1, Psibar, VL, Psi), G_NC_X_t);
            ((D (−3), Wm, QH 3), FBF (1, Psibar, VL, Psi), G_NC_X_t);
            ((QH (−3), Z, U 3), FBF (1, Psibar, VL, Psi), G_NCHt);
            ((U (−3), Z, QH 3), FBF (1, Psibar, VL, Psi), G_NCHt)]
  @
     ThoList.flatmap
       (fun n →
          if Flags.anom_ferm_ass then
            [ ((U (−n), X0, U n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
              ((U (−n), Y0, U n), FBF ((−1), Psibar, VL, Psi), G_NC_Y);
              ((D (−n), Xm, U n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
              ((U (−n), Xp, D n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
              ((QH (−n), X0, U n), FBF ((−1), Psibar, VL, Psi), G_CC);
              ((U (−n), X0, QH n), FBF ((−1), Psibar, VL, Psi), G_CC);
              ((QH (−n), Y0, U n), FBF ((−1), Psibar, VL, Psi), I_G_CC);
              ((U (−n), Y0, QH n), FBF ((−1), Psibar, VL, Psi), I_G_CC);
              ((D (−n), Xm, QH n), FBF ((−1), Psibar, VL, Psi), G_CC);
              ((QH (−n), Xp, D n), FBF ((−1), Psibar, VL, Psi), G_CC);
              ((QH (−n), Wp, D n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
              ((D (−n), Wm, QH n), FBF ((−1), Psibar, VL, Psi), G_NC_X);
              ((QH (−n), Z, U n), FBF (1, Psibar, VL, Psi), G_NC_H);
              ((U (−n), Z, QH n), FBF (1, Psibar, VL, Psi), G_NC_H)]
          else
            [ ((D (−n), X0, D n), FBF (1, Psibar, VL, Psi), G_NC_X);
              ((D (−n), Y0, D n), FBF (1, Psibar, VL, Psi), G_NC_Y);
              ((D (−n), Xm, U n), FBF (1, Psibar, VL, Psi), G_NC_X);
              ((U (−n), Xp, D n), FBF (1, Psibar, VL, Psi), G_NC_X);
              ((QH (−n), X0, D n), FBF ((−1), Psibar, VL, Psi), G_CC);
              ((D (−n), X0, QH n), FBF ((−1), Psibar, VL, Psi), G_CC);
              ((QH (−n), Y0, D n), FBF ((−1), Psibar, VL, Psi), I_G_CC);
              ((D (−n), Y0, QH n), FBF ((−1), Psibar, VL, Psi), I_G_CC);
              ((QH (−n), Xm, U n), FBF (1, Psibar, VL, Psi), G_CC);
              ((U (−n), Xp, QH n), FBF (1, Psibar, VL, Psi), G_CC);
              ((QH (−n), Wm, U n), FBF (1, Psibar, VL, Psi), G_NC_X);
              ((U (−n), Wp, QH n), FBF (1, Psibar, VL, Psi), G_NC_X);
              ((QH (−n), Z, D n), FBF (1, Psibar, VL, Psi), G_NC_H);
              ((D (−n), Z, QH n), FBF (1, Psibar, VL, Psi), G_NC_H)])
       [1; 2]
```

The sign of this coupling is just the one of the T3, being -(1/2) for leptons and down quarks, and +(1/2) for neutrinos and up quarks.

```
    let neutral_heavy_currents n =
      [ ((L (−n), ZH, L n), FBF (1, Psibar, VLR, Psi), G_NC_h_lepton);
        ((N (−n), ZH, N n), FBF ((−1), Psibar, VLR, Psi), G_NC_h_neutrino);
        ((U (−n), ZH, U n), FBF ((−1), Psibar, VLR, Psi), (if n = 3 then
```

$$G\_NC\_h\_top \text{ else } G\_NC\_h\_up));$$
$$((D\ (-n),\ ZH,\ D\ n),\ FBF\ (1,\ Psibar,\ VLR,\ Psi),\ (\text{if } n\ =\ 3 \text{ then}$$
$$G\_NC\_h\_bot \text{ else } G\_NC\_h\_down));$$
$$((NH\ (-n),\ ZH,\ NH\ n),\ FBF\ (1,\ Psibar,\ VLR,\ Psi),\ G\_NCH\_N);$$
$$((QH\ (-n),\ ZH,\ QH\ n),\ FBF\ (1,\ Psibar,\ VLR,\ Psi),\ (\text{if } n\ =\ 3 \text{ then}$$
$$G\_NCH\_U \text{ else if } Flags.anom\_ferm\_ass \text{ then } G\_NCH\_U \text{ else } G\_NCH\_D))]$$

let *heavy_currents* $n\ =$
  $[\ (((QH\ (-n),\ Ga,\ QH\ n),\ FBF\ (1,\ Psibar,\ V,\ Psi),\ (\text{if } n = 3 \text{ then } Q\_up \text{ else}$
        if $Flags.anom\_ferm\_ass$ then Q_up else $Q\_down))]$

let *charged_currents* $n\ =$
  $[\ ((L\ (-n),\ Wm,\ N\ n),\ FBF\ (1,\ Psibar,\ VL,\ Psi),\ G\_CC);$
    $((N\ (-n),\ Wp,\ L\ n),\ FBF\ (1,\ Psibar,\ VL,\ Psi),\ G\_CC);$
    $((D\ (-n),\ Wm,\ U\ n),\ FBF\ (1,\ Psibar,\ VL,\ Psi),\ G\_CC);$
    $((U\ (-n),\ Wp,\ D\ n),\ FBF\ (1,\ Psibar,\ VL,\ Psi),\ G\_CC)\ ]$

let *yukawa* $=$
  $[\ ((U\ (-3),\ H,\ U\ 3),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Htt);$
    $((D\ (-3),\ H,\ D\ 3),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Hbb);$
    $((U\ (-2),\ H,\ U\ 2),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Hcc);$
    $((L\ (-3),\ H,\ L\ 3),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Htautau)\ ]$

let *yukawa_add* $=$
  $[\ ((QH\ (-3),\ H,\ U\ 3),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Htht);$
    $((U\ (-3),\ H,\ QH\ 3),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Htht);$
    $((QH\ (-3),\ Eta,\ U\ 3),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Etht);$
    $((U\ (-3),\ Eta,\ QH\ 3),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Etht);$
    $((D\ (-3),\ Eta,\ D\ 3),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Ebb);$
    $((U\ (-3),\ Eta,\ U\ 3),\ FBF\ (1,\ Psibar,\ P,\ Psi),\ G\_Ett)]$
    @
      ThoList.flatmap
        (fun $n\ \rightarrow$
        if *Flags.anom_ferm_ass* then
  $[\ ((QH\ (-n),\ H,\ U\ n),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Hqhq);$
    $((U\ (-n),\ H,\ QH\ n),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Hqhq)]$
        else
  $[\ ((QH\ (-n),\ H,\ D\ n),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_Hqhq);$
    $((D\ (-n),\ H,\ QH\ n),\ FBF\ (1,\ Psibar,\ SR,\ Psi),\ G\_Hqhq)])$
        $[1;2]$

let *standard_triple_gauge* $=$
  $[\ ((Ga,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_W);$
    $((Z,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWW);$
    $((Gl,\ Gl,\ Gl),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Gs)]$

let *heavy_triple_gauge* $=$
  $[\ ((Ga,\ Xm,\ Xp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_W);$
    $((Z,\ Xm,\ Xp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_ZH);$
    $((Z,\ X0,\ Y0),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_Z1);$
    $((ZH,\ X0,\ Y0),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_Z2);$
    $((Y0,\ Wm,\ Xp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_Z3);$
    $((Y0,\ Wp,\ Xm),\ Gauge\_Gauge\_Gauge\ (-1),\ I\_G\_Z3);$

```
    ((X0, Wm, Xp), Gauge_Gauge_Gauge 1, I_G_Z4);
    ((X0, Wp, Xm), Gauge_Gauge_Gauge 1, I_G_Z4);
    ((ZH, Xm, Xp), Gauge_Gauge_Gauge 1, I_G_Z5);
    ((ZH, Wm, Wp), Gauge_Gauge_Gauge 1, I_G_Z6)]
```

let *triple_gauge* =
  standard_triple_gauge @ *heavy_triple_gauge*

let *gauge4* = *Vector4* [(2, C_13_42); (−1, C_12_34); (−1, C_14_23)]
let *minus_gauge4* = *Vector4* [(−2, C_13_42); (1, C_12_34); (1, C_14_23)]
let *standard_quartic_gauge* =
  [(Wm, Wp, Wm, Wp), *gauge4*, G_WWWW;
    (Wm, Z, Wp, Z), *minus_gauge4*, G_ZZWW;
    (Wm, Z, Wp, Ga), *minus_gauge4*, G_AZWW;
    (Wm, Ga, Wp, Ga), *minus_gauge4*, G_AAWW;
    (Gl, Gl, Gl, Gl), *gauge4*, G2]

let *heavy_quartic_gauge* =
  []

let *quartic_gauge* =
  standard_quartic_gauge @ *heavy_quartic_gauge*

let *standard_gauge_higgs′* =
  [((H, Wp, Wm), Scalar_Vector_Vector 1, G_HWW);
    ((H, Z, Z), Scalar_Vector_Vector 1, G_HZZ)]

let *heavy_gauge_higgs* =
  [((H, Wp, Xm), Scalar_Vector_Vector 1, G_heavy_HWW);
    ((H, Wm, Xp), Scalar_Vector_Vector 1, G_heavy_HWW);
    ((H, Z, X0), Scalar_Vector_Vector 1, G_heavy_HVV);
    ((H, ZH, X0), Scalar_Vector_Vector 1, G_heavy_HVV)]

let *standard_gauge_higgs* =
  standard_gauge_higgs′ @ *heavy_gauge_higgs*

let *standard_gauge_higgs4* =
  [(H, H, Wp, Wm), Scalar2_Vector2 1, G_HHWW;
    (H, H, Z, Z), Scalar2_Vector2 1, G_HHZZ]

let *heavy_gauge_higgs4* =
  [(H, H, Z, ZH), Scalar2_Vector2 1, G_HHZZH;
    (H, H, Xp, Xm), Scalar2_Vector2 (−1), G_HHWW;
    (H, H, ZH, ZH), Scalar2_Vector2 (−1), G_HHZZ]

let *standard_higgs* =
  [(H, H, H), Scalar_Scalar_Scalar 1, G_H3]

let *anomaly_higgs* =
  [(Eta, Gl, Gl), Dim5_Scalar_Gauge2_Skew 1, G_EGlGl;
    (Eta, Ga, Ga), Dim5_Scalar_Gauge2_Skew 1, G_EGaGa;
    (Eta, Ga, Z), Dim5_Scalar_Gauge2_Skew 1, G_EGaZ]
(* @ (H, Ga, Ga), Dim5_Scalar_Gauge2 1, G_HGaGa; (H, Ga, Z), Dim5_Scalar_Gauge2 1, G_HGaZ
*)

let *standard_higgs4* =
```
```

```
    [ (H, H, H, H), Scalar4 1, G_H4 ]

let gauge_higgs =
    standard_gauge_higgs

let gauge_higgs4 =
    standard_gauge_higgs4 @ heavy_gauge_higgs4

let higgs =
    standard_higgs

let eta_higgs_gauge =
  [ (Z, Eta, H), Vector_Scalar_Scalar 1, G_ZEH;
    (ZH, Eta, H), Vector_Scalar_Scalar 1, G_ZHEH;
    (X0, Eta, H), Vector_Scalar_Scalar 1, G_CC ]

let top_quartic =
  [ ((QH (−3), H, H, QH 3), GBBG (1, Psibar, S2, Psi), G_HHthth)]

let higgs4 =
    standard_higgs4

let goldstone_vertices =
  [ ((Phi0, Wm, Wp), Scalar_Vector_Vector 1, I_G_ZWW);
    ((Phip, Ga, Wm), Scalar_Vector_Vector 1, I_Q_W);
    ((Phip, Z, Wm), Scalar_Vector_Vector 1, I_G_ZWW);
    ((Phim, Wp, Ga), Scalar_Vector_Vector 1, I_Q_W);
    ((Phim, Wp, Z), Scalar_Vector_Vector 1, I_G_ZWW) ]

let vertices3 =
  (ThoList.flatmap electromagnetic_currents [1; 2; 3] @
   ThoList.flatmap color_currents [1; 2; 3] @
   ThoList.flatmap neutral_currents [1; 2; 3] @
   ThoList.flatmap neutral_heavy_currents [1; 2; 3] @
   ThoList.flatmap heavy_currents [1; 2; 3] @
   ThoList.flatmap charged_currents [1; 2; 3] @
   xy_currents @ anomaly_higgs @
   eta_higgs_gauge @
   yukawa @ yukawa_add @
   triple_gauge @
   gauge_higgs @ higgs @ goldstone_vertices)

let vertices4 =
  quartic_gauge @ gauge_higgs4 @ higgs4

let vertices () = (vertices3, vertices4, [])
```

For efficiency, make sure that *F.of_vertices vertices* is evaluated only once.

```
let table = F.of_vertices (vertices ())
let fuse2 = F.fuse2 table
let fuse3 = F.fuse3 table
let fuse = F.fuse table
let max_degree () = 4

let flavor_of_string = function
  | "e-" → L 1 | "e+" → L (−1)
```

```
    | "mu-" → L 2 | "mu+" → L (−2)
    | "tau-" → L 3 | "tau+" → L (−3)
    | "nue" → N 1 | "nuebar" → N (−1)
    | "numu" → N 2 | "numubar" → N (−2)
    | "nutau" → N 3 | "nutaubar" → N (−3)
    | "nh1" → NH 1 | "nh1bar" → NH (−1)
    | "nh2" → NH 2 | "nh2bar" → NH (−2)
    | "nh3" → NH 3 | "nh3bar" → NH (−3)
    | "u" → U 1 | "ubar" → U (−1)
    | "c" → U 2 | "cbar" → U (−2)
    | "t" → U 3 | "tbar" → U (−3)
    | "d" → D 1 | "dbar" → D (−1)
    | "s" → D 2 | "sbar" → D (−2)
    | "b" → D 3 | "bbar" → D (−3)
    | "uh" → if Flags.anom_ferm_ass then QH 1 else invalid_arg
        "Modellib_BSM.Simplest.flavor_of_string"
    | "dh" → if Flags.anom_ferm_ass then invalid_arg
          "Modellib_BSM.Simplest.flavor_of_string" else QH 1
    | "uhbar" → if Flags.anom_ferm_ass then QH (−1) else invalid_arg
        "Modellib_BSM.Simplest.flavor_of_string"
    | "dhbar" → if Flags.anom_ferm_ass then invalid_arg
          "Modellib_BSM.Simplest.flavor_of_string" else QH (−1)
    | "ch" → if Flags.anom_ferm_ass then QH 2 else invalid_arg
        "Modellib_BSM.Simplest.flavor_of_string"
    | "sh" → if Flags.anom_ferm_ass then invalid_arg
          "Modellib_BSM.Simplest.flavor_of_string" else QH 2
    | "chbar" → if Flags.anom_ferm_ass then QH (−2) else invalid_arg
        "Modellib_BSM.Simplest.flavor_of_string"
    | "shbar" → if Flags.anom_ferm_ass then invalid_arg
          "Modellib_BSM.Simplest.flavor_of_string" else QH (−2)
    | "th" → QH 3 | "thbar" → QH (−3)
    | "eta" | "Eta" → Eta
    | "A" → Ga | "Z" | "Z0" → Z | "g" | "gl" → Gl
    | "ZH" | "ZH0" | "Zh" | "Zh0" → ZH
    | "W+" → Wp | "W-" → Wm
    | "X+" → Xp | "X-" → Xm
    | "X0" → X0 | "Y0" → Y0
    | "H" → H
    | _ → invalid_arg "Modellib_BSM.Simplest.flavor_of_string"

let flavor_to_string = function
  | L 1 → "e-" | L (−1) → "e+"
  | L 2 → "mu-" | L (−2) → "mu+"
  | L 3 → "tau-" | L (−3) → "tau+"
  | L _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_string:␣invalid␣lepton"
  | N 1 → "nue" | N (−1) → "nuebar"
  | N 2 → "numu" | N (−2) → "numubar"
  | N 3 → "nutau" | N (−3) → "nutaubar"
  | N _ → invalid_arg
```

```
        "Modellib_BSM.Simplest.flavor_to_string:␣invalid␣neutrino"
  | U 1 → "u" | U (−1) → "ubar"
  | U 2 → "c" | U (−2) → "cbar"
  | U 3 → "t" | U (−3) → "tbar"
  | U _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_string:␣invalid␣up␣type␣quark"
  | D 1 → "d" | D (−1) → "dbar"
  | D 2 → "s" | D (−2) → "sbar"
  | D 3 → "b" | D (−3) → "bbar"
  | D _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_string:␣invalid␣down␣type␣quark"
  | QH 1 → if Flags.anom_ferm_ass then "uh" else "dh"
  | QH 2 → if Flags.anom_ferm_ass then "ch" else "sh"
  | QH 3 → "th"
  | QH (−1) → if Flags.anom_ferm_ass then "uhbar" else "dhbar"
  | QH (−2) → if Flags.anom_ferm_ass then "chbar" else "shbar"
  | QH (−3) → "thbar"
  | QH _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_string:␣invalid␣heavy␣quark"
  | NH n when n > 0 → "nh" ˆ string_of_int n
  | NH n → "nh" ˆ string_of_int (abs n) ˆ "bar"
  | Ga → "A" | Z → "Z" | Gl → "gl"
  | Wp → "W+" | Wm → "W-"
  | Xp → "X+" | Xm → "X-" | X0 → "X0" | Y0 → "Y0" | ZH →
"ZH"
  | Phip → "phi+" | Phim → "phi-" | Phi0 → "phi0"
  | H → "H" | Eta → "Eta"

let flavor_to_TeX = function
  | L 1 → "e^-" | L (−1) → "\\e^+"
  | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
  | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
  | L _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_TeX:␣invalid␣lepton"
  | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
  | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
  | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
  | N _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_TeX:␣invalid␣neutrino"
  | U 1 → "u" | U (−1) → "\\bar{u}"
  | U 2 → "c" | U (−2) → "\\bar{c}"
  | U 3 → "t" | U (−3) → "\\bar{t}"
  | U _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_TeX:␣invalid␣up␣type␣quark"
  | D 1 → "d" | D (−1) → "\\bar{d}"
  | D 2 → "s" | D (−2) → "\\bar{s}"
  | D 3 → "b" | D (−3) → "\\bar{b}"
  | D _ → invalid_arg
        "Modellib_BSM.Simplest.flavor_to_TeX:␣invalid␣down␣type␣quark"
  | QH 1 → if Flags.anom_ferm_ass then "U" else "D"
```

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```
      |  QH 2  →  if Flags.anom_ferm_ass then "C" else "S"
      |  QH 3  →  "T"
      |  QH (−1)  →  if Flags.anom_ferm_ass then "\\bar{U}" else "\\bar{D}"
      |  QH (−2)  →  if Flags.anom_ferm_ass then "\\bar{C}" else "\\bar{S}"
      |  QH (−3)  →  "thbar"
      |  QH _  →  invalid_arg
            "Modellib_BSM.Simplest.flavor_to_TeX:␣invalid␣heavy␣quark"
      |  NH n when n > 0  →  "N_" ^ string_of_int n
      |  NH n  →  "\\bar{N}_" ^ string_of_int (abs n)
      |  Ga  →  "\\gamma" |  Z  →  "Z" |  Gl  →  "g"
      |  Wp  →  "W^+" |  Wm  →  "W^-"
      |  Xp  →  "X^+" |  Xm  →  "X^-" |  X0  →  "X^0" |  Y0  →  "Y^0" |
ZH  →  "Z_H"
      |  Phip  →  "\\phi^+" |  Phim  →  "\\phi^-" |  Phi0  →  "\\phi^0"
      |  H  →  "H" |  Eta  →  "\\eta"

    let flavor_symbol  =  function
      |  L n when n > 0  →  "l" ^ string_of_int n
      |  L n  →  "l" ^ string_of_int (abs n) ^ "b"
      |  N n when n > 0  →  "n" ^ string_of_int n
      |  N n  →  "n" ^ string_of_int (abs n) ^ "b"
      |  U n when n > 0  →  "u" ^ string_of_int n
      |  U n  →  "u" ^ string_of_int (abs n) ^ "b"
      |  D n when n > 0  →  "d" ^ string_of_int n
      |  D n  →  "d" ^ string_of_int (abs n) ^ "b"
      |  NH n when n > 0  →  "nh" ^ string_of_int n
      |  NH n  →  "nh" ^ string_of_int (abs n) ^ "b"
      |  QH n when n > 0  →  "qh" ^ string_of_int n
      |  QH n  →  "qh" ^ string_of_int (abs n) ^ "b"
      |  Ga  →  "a" |  Z  →  "z" |  Gl  →  "gl"
      |  Wp  →  "wp" |  Wm  →  "wm"
      |  Xp  →  "xp" |  Xm  →  "xm" |  X0  →  "x0" |  Y0  →  "y0" |  ZH  →
"zh"
      |  Phip  →  "pp" |  Phim  →  "pm" |  Phi0  →  "p0"
      |  H  →  "h" |  Eta  →  "eta"
```

There are PDG numbers for Z', Z", W', 32-34, respectively. We just introduce a number 38 for Y0 as a Z"'. As well, there is the number 8 for a t'. But we cheat a little bit and take the number 35 which is reserved for a heavy scalar Higgs for the Eta scalar.

We abuse notation for the heavy quarks and take the PDG code for their SUSY partners!!! (What about an update of the PDG numbering scheme?) Thereby we take only those for up-type (s)quarks. The heavy neutrinos get the numbers of the sneutrinos.

```
    let pdg  =  function
      |  L n when n > 0  →  9 + 2 × n
      |  L n  →  − 9 + 2 × n
      |  N n when n > 0  →  10 + 2 × n
      |  N n  →  − 10 + 2 × n
      |  U n when n > 0  →  2 × n
      |  U n  →  2 × n
```

```
| D n when n > 0 →  − 1 + 2 × n
| D n →  1 + 2 × n
| NH n when n > 0 →  1000010 + 2 × n
| NH n →  − 1000010 + 2 × n
| QH 3 →  1000006
| QH (−3) →  − 1000006
| QH n when n > 0 →  if Flags.anom_ferm_ass then
      1000000 + 2×n else 999999 + 2×n
| QH n →  if Flags.anom_ferm_ass then
      - 1000000 + 2×n else - 999999 + 2×n
| Gl →  21
| Ga →  22 | Z →  23
| Wp →  24 | Wm →  (−24)
| Xp →  34 | Xm →  (−34) | ZH →  32 | X0 →  33 | Y0 →  38
| Phip | Phim →  27 | Phi0 →  26
| H →  25 | Eta →  36
```

As in the case of SUSY we introduce an internal dummy pdf code in order to have manageable arrays. Heavy neutrinos get numbers 41,43,45, while the heavy quarks have the numbers 40,42,44. I take them all as up type here.

```
let pdg_mw  =  function
  | L n when n > 0 →  9 + 2 × n
  | L n →  − 9 + 2 × n
  | N n when n > 0 →  10 + 2 × n
  | N n →  − 10 + 2 × n
  | U n when n > 0 →  2 × n
  | U n →  2 × n
  | D n when n > 0 →  − 1 + 2 × n
  | D n →  1 + 2 × n
  | NH n when n > 0 →  39 + 2 × n
  | NH n →  − 39 + 2 × n
  | QH n when n > 0 →  38 + 2 × n
  | QH n →  − 38 + 2 × n
  | Gl →  21
  | Ga →  22 | Z →  23
  | Wp →  24 | Wm →  (−24)
  | Xp →  34 | Xm →  (−34) | ZH →  32 | X0 →  33 | Y0 →  38
  | Phip | Phim →  27 | Phi0 →  26
  | H →  25 | Eta →  36

let mass_symbol f  =
  "mass(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let width_symbol f  =
  "width(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let constant_symbol  =  function
  | Unit →  "unit" | Pi →  "PI" | VHeavy →  "vheavy"
  | Alpha_QED →  "alpha" | E →  "e" | G_weak →  "g" | Vev →
"vev"
  | Sin2thw →  "sin2thw" | Sinthw →  "sinthw" | Costhw →
"costhw"
```

```
      | Sinpsi → "sinpsi" | Cospsi → "cospsi"
      | Atpsi → "atpsi" | Sccs → "sccs"
      | Supp → "vF" | Supp2 → "v2F2"
      | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
      | Q_Z_up → "qzup"
      | G_zhthth → "gzhthth"
      | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
      | G_NC_up → "gncup" | G_NC_down → "gncdwn"
      | G_NC_X → "gncx" | G_NC_X_t → "gncxt"
      | G_NC_Y → "gncy" | G_NC_Y_t → "gncyt" | G_NC_H →
"gnch"
      | G_CC → "gcc" | I_G_CC → "i_gcc"
      | G_NC_h_lepton → "gnchlep" | G_NC_h_neutrino → "gnchneu"
      | G_NC_h_up → "gnchup" | G_NC_h_down → "gnchdwn"
      | G_NC_h_top → "gnchtop" | G_NC_h_bot → "gnchbot"
      | G_NCH_N → "gnchn" | G_NCH_U → "gnchu" | G_NCH_D →
"gnchd"
      | G_NCHt → "gncht"
      | I_Q_W → "iqw" | I_G_ZWW → "igzww" | I_G_WWW →
"igwww"
      | I_Q_H → "iqh" | I_Q_ZH → "iqzh"
      | I_G_Z1 → "igz1" | I_G_Z2 → "igz2" | I_G_Z3 → "igz3"
      | I_G_Z4 → "igz4" | I_G_Z5 → "igz5" | I_G_Z6 → "igz6"
      | G_HHthth → "ghhthth"
      | G_WWWW → "gw4" | G_ZZWW → "gzzww"
      | G_AZWW → "gazww" | G_AAWW → "gaaww"
      | G_HWW → "ghww" | G_HZZ → "ghzz"
      | G_heavy_HVV → "ghyhvv"
      | G_heavy_HWW → "ghyhww"
      | G_heavy_HZZ → "ghyhzz"
      | G_HHWW → "ghhww" | G_HHZZ → "ghhzz"
      | G_HHZZH → "ghhzzh"
      | G_Hgg → "ghgg"
      | G_Htt → "ghtt" | G_Hbb → "ghbb"
      | G_Htautau → "ghtautau" | G_Hcc → "ghcc"
      | G_Hthth → "ghthth" | G_Htht → "ghtht"
      | G_Hqhq → "ghqhq"
      | G_Ethth → "gethth" | G_Etht → "getht"
      | G_Ett → "gett" | G_Ebb → "gebb"
      | G_HGaGa → "ghgaga" | G_HGaZ → "ghgaz"
      | G_EGaGa → "geaa" | G_EGaZ → "geaz" | G_EGlGl → "gegg"
      | G_ZEH → "gzeh" | G_ZHEH → "gzheh"
      | G_H3 → "gh3" | G_H4 → "gh4"
      | Mass f → "mass" ^ flavor_symbol f
      | Width f → "width" ^ flavor_symbol f
      | Gs → "gs" | I_Gs → "igs" | G2 → "gs**2"
  end

module Xdim (Flags : BSM_flags) =
  struct
```

```
let rcs = RCS.rename rcs_file "Modellib_BSM.Xdim"
    [ "SM␣with␣extradimensional␣resonances"]
```

open *Coupling*

```
let default_width = ref Timelike
let use_fudged_width = ref false
```

```
let options = Options.create
  [ "constant_width", Arg.Unit (fun () → default_width := Constant),
    "use␣constant␣width␣(also␣in␣t-channel)";
    "fudged_width", Arg.Set use_fudged_width,
    "use␣fudge␣factor␣for␣charge␣particle␣width";
    "custom_width", Arg.String (fun f → default_width := Custom f),
    "use␣custom␣width";
    "cancel_widths", Arg.Unit (fun () → default_width := Vanishing),
    "use␣vanishing␣width"]
```

```
type matter_field = L of int | N of int | U of int | D of int
type gauge_boson = Ga | Wp | Wm | Z | Gl
type other = Phip | Phim | Phi0 | H | Grav
type flavor = M of matter_field | G of gauge_boson | O of other
```

```
let matter_field f = M f
let gauge_boson f = G f
let other f = O f
```

```
type field =
  | Matter of matter_field
  | Gauge of gauge_boson
  | Other of other
```

```
let field = function
  | M f → Matter f
  | G f → Gauge f
  | O f → Other f
```

type *gauge* = *unit*

```
let gauge_symbol () =
  failwith "Modellib_BSM.Xdim.gauge_symbol:␣internal␣error"
```

let *family n* = *List.map matter_field* [*L n*; *N n*; *U n*; *D n*]

```
let external_flavors () =
  [ "1st␣Generation", ThoList.flatmap family [1; − 1];
    "2nd␣Generation", ThoList.flatmap family [2; − 2];
    "3rd␣Generation", ThoList.flatmap family [3; − 3];
    "Gauge␣Bosons", List.map gauge_boson [Ga; Z; Wp; Wm; Gl];
    "Higgs", List.map other [H];
    "Graviton", List.map other [Grav];
    "Goldstone␣Bosons", List.map other [Phip; Phim; Phi0] ]
```

let *flavors* () = *ThoList.flatmap snd* (*external_flavors* ())

```
let spinor n =
  if n ≥ 0 then
```

```
        Spinor
    else
        ConjSpinor

let lorentz = function
  | M f →
        begin match f with
        | L n → spinor n | N n → spinor n
        | U n → spinor n | D n → spinor n
        end
  | G f →
        begin match f with
        | Ga | Gl → Vector
        | Wp | Wm | Z → Massive_Vector
        end
  | O f →
        begin match f with
        | Grav → Tensor_2
        | _ → Scalar
        end

let color = function
  | M (U n) → Color.SUN (if n > 0 then 3 else -3)
  | M (D n) → Color.SUN (if n > 0 then 3 else -3)
  | G Gl → Color.AdjSUN 3
  | _ → Color.Singlet

let prop_spinor n =
    if n ≥ 0 then
        Prop_Spinor
    else
        Prop_ConjSpinor

let propagator = function
  | M f →
        begin match f with
        | L n → prop_spinor n | N n → prop_spinor n
        | U n → prop_spinor n | D n → prop_spinor n
        end
  | G f →
        begin match f with
        | Ga | Gl → Prop_Feynman
        | Wp | Wm | Z → Prop_Unitarity
        end
  | O f →
        begin match f with
        | Phip | Phim | Phi0 → Only_Insertion
        | H → Prop_Scalar
        | Grav → Prop_Tensor_2
        end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

```
let width f  =
  if !use_fudged_width then
    match f with
    | G Wp | G Wm | M (U 3) | M (U (−3)) | O Grav → Fudged
    | _ → !default_width
  else
    !default_width

let goldstone = function
  | G f →
      begin match f with
      | Wp → Some (O Phip, Coupling.Const 1)
      | Wm → Some (O Phim, Coupling.Const 1)
      | Z → Some (O Phi0, Coupling.Const 1)
      | _ → None
      end
  | _ → None

let conjugate = function
  | M f →
      M (begin match f with
      | L n → L (−n) | N n → N (−n)
      | U n → U (−n) | D n → D (−n)
      end)
  | G f →
      G (begin match f with
      | Gl → Gl | Ga → Ga | Z → Z
      | Wp → Wm | Wm → Wp
      end)
  | O f →
      O (begin match f with
      | Phip → Phim | Phim → Phip | Phi0 → Phi0
      | H → H | Grav → Grav
      end)

let fermion = function
  | M f →
      begin match f with
      | L n → if n > 0 then 1 else -1
      | N n → if n > 0 then 1 else -1
      | U n → if n > 0 then 1 else -1
      | D n → if n > 0 then 1 else -1
      end
  | G f →
      begin match f with
      | Gl | Ga | Z | Wp | Wm → 0
      end
  | O _ → 0

module Ch = Charges.QQ
let ( // ) = Algebra.Small_Rational.make
```

```
let generation' = function
  | 1 → [ 1//1; 0//1; 0//1]
  | 2 → [ 0//1; 1//1; 0//1]
  | 3 → [ 0//1; 0//1; 1//1]
  | − 1 → [−1//1; 0//1; 0//1]
  | − 2 → [ 0//1; − 1//1; 0//1]
  | − 3 → [ 0//1; 0//1; − 1//1]
  | n → invalid_arg ("Xdim.generation':␣" ^ string_of_int n)

let generation f =
  match f with
  | M (L n | N n | U n | D n) → generation' n
  | G _ | O _ → [0//1; 0//1; 0//1]

let charge = function
  | M f →
      begin match f with
      | L n → if n > 0 then -1//1 else 1//1
      | N n → 0//1
      | U n → if n > 0 then 2//3 else -2//3
      | D n → if n > 0 then -1//3 else 1//3
      end
  | G f →
      begin match f with
      | Gl | Ga | Z → 0//1
      | Wp → 1//1
      | Wm → − 1//1
      end
  | O f →
      begin match f with
      | H | Phi0 | Grav → 0//1
      | Phip → 1//1
      | Phim → − 1//1
      end

let lepton = function
  | M f →
      begin match f with
      | L n | N n → if n > 0 then 1//1 else -1//1
      | U _ | D _ → 0//1
      end
  | G _ | O _ → 0//1

let baryon = function
  | M f →
      begin match f with
      | L _ | N _ → 0//1
      | U n | D n → if n > 0 then 1//1 else -1//1
      end
  | G _ | O _ → 0//1

let charges f =
  [ charge f; lepton f; baryon f] @ generation f
```

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```
type constant =
  | Unit | Pi | Alpha_QED | Sin2thw
  | Sinthw | Costhw | E | G_weak | Vev
  | Q_lepton | Q_up | Q_down | G_CC | G_CCQ of int×int
  | G_NC_neutrino | G_NC_lepton | G_NC_up | G_NC_down
  | Gs | I_Gs | G2
  | I_Q_W | I_G_ZWW
  | G_WWWW | G_ZZWW | G_AZWW | G_AAWW
  | G_HWW | G_HHWW | G_HZZ | G_HHZZ
  | G_Htt | G_Hbb | G_Hcc | G_Htautau | G_H3 | G_H4
  | G_HGaZ | G_HGaGa | G_Hgg | G_Grav
  | Mass of flavor | Width of flavor
```

Two integer counters for the QCD and EW order of the couplings.

```
type orders = int × int

let orders = function
  | _ → (0,0)

let input_parameters =
  []

let derived_parameters =
  []

let g_over_2_costh =
  Quot (Neg (Atom G_weak), Prod [Const 2; Atom Costhw])

let nc_coupling c t3 q =
  (Real_Array c,
   [Prod [g_over_2_costh; Diff (t3, Prod [Const 2; q; Atom Sin2thw])];
    Prod [g_over_2_costh; t3]])

let half = Quot (Const 1, Const 2)

let derived_parameter_arrays =
  [ nc_coupling G_NC_neutrino half (Const 0);
    nc_coupling G_NC_lepton (Neg half) (Const (−1));
    nc_coupling G_NC_up half (Quot (Const 2, Const 3));
    nc_coupling G_NC_down (Neg half) (Quot (Const (−1), Const 3)) ]

let parameters () =
  { input = input_parameters;
    derived = derived_parameters;
    derived_arrays = derived_parameter_arrays }

module F = Modeltools.Fusions (struct
  type f = flavor
  type c = constant
  let compare = compare
  let conjugate = conjugate
end)

let mgm ((m1, g, m2), fbf, c) = ((M m1, G g, M m2), fbf, c)
let mom ((m1, o, m2), fbf, c) = ((M m1, O o, M m2), fbf, c)
```

let *electromagnetic_currents n* =
  List.map mgm
    [((L (−n), Ga, L n), FBF (1, Psibar, V, Psi), Q_lepton);
      ((U (−n), Ga, U n), FBF (1, Psibar, V, Psi), Q_up);
      ((D (−n), Ga, D n), FBF (1, Psibar, V, Psi), Q_down)]

let *neutral_currents n* =
  List.map mgm
    [((L (−n), Z, L n), FBF (1, Psibar, VA, Psi), G_NC_lepton);
      ((N (−n), Z, N n), FBF (1, Psibar, VA, Psi), G_NC_neutrino);
      ((U (−n), Z, U n), FBF (1, Psibar, VA, Psi), G_NC_up);
      ((D (−n), Z, D n), FBF (1, Psibar, VA, Psi), G_NC_down)]

let *color_currents n* =
  List.map mgm
    [((U (−n), Gl, U n), FBF ((−1), Psibar, V, Psi), Gs);
      ((D (−n), Gl, D n), FBF ((−1), Psibar, V, Psi), Gs)]

let *charged_currents n* =
  List.map mgm
    [((L (−n), Wm, N n), FBF (1, Psibar, VL, Psi), G_CC);
      ((N (−n), Wp, L n), FBF (1, Psibar, VL, Psi), G_CC);
      ((D (−n), Wm, U n), FBF (1, Psibar, VL, Psi), G_CC);
      ((U (−n), Wp, D n), FBF (1, Psibar, VL, Psi), G_CC)]

let *gravity_currents n* =
  List.map mom
    [((L (−n), Grav, L n), Graviton_Spinor_Spinor 1, G_Grav);
      ((N (−n), Grav, N n), Graviton_Spinor_Spinor 1, G_Grav);
      ((U (−n), Grav, U n), Graviton_Spinor_Spinor 1, G_Grav);
      ((D (−n), Grav, D n), Graviton_Spinor_Spinor 1, G_Grav)]

let *yukawa* =
  List.map mom
    [((U (−3), H, U 3), FBF (1, Psibar, S, Psi), G_Htt);
      ((D (−3), H, D 3), FBF (1, Psibar, S, Psi), G_Hbb);
      ((U (−2), H, U 2), FBF (1, Psibar, S, Psi), G_Hcc);
      ((L (−3), H, L 3), FBF (1, Psibar, S, Psi), G_Htautau)]

let *tgc* ((g1, g2, g3), t, c) = ((G g1, G g2, G g3), t, c)

let *standard_triple_gauge* =
  List.map tgc
    [((Ga, Wm, Wp), Gauge_Gauge_Gauge 1, I_Q_W);
      ((Z, Wm, Wp), Gauge_Gauge_Gauge 1, I_G_ZWW);
      ((Gl, Gl, Gl), Gauge_Gauge_Gauge 1, I_Gs)]

let *triple_gauge* =
  standard_triple_gauge

let *qgc* ((g1, g2, g3, g4), t, c) = ((G g1, G g2, G g3, G g4), t, c)

let *gauge4* = Vector4 [(2, C_13_42); (−1, C_12_34); (−1, C_14_23)]
let *minus_gauge4* = Vector4 [(−2, C_13_42); (1, C_12_34); (1, C_14_23)]
let *standard_quartic_gauge* =

```
    List.map qgc
      [ (Wm, Wp, Wm, Wp), gauge4, G_WWWW;
        (Wm, Z, Wp, Z), minus_gauge4, G_ZZWW;
        (Wm, Z, Wp, Ga), minus_gauge4, G_AZWW;
        (Wm, Ga, Wp, Ga), minus_gauge4, G_AAWW;
        (Gl, Gl, Gl, Gl), gauge4, G2]

let quartic_gauge =
  standard_quartic_gauge

let gravity_gauge =
  [ (O Grav, G Z, G Z), Graviton_Vector_Vector 1, G_Grav;
    (O Grav, G Wp, G Wm), Graviton_Vector_Vector 1, G_Grav;
    (O Grav, G Ga, G Ga), Graviton_Vector_Vector 1, G_Grav;
    (O Grav, G Gl, G Gl), Graviton_Vector_Vector 1, G_Grav ]

let standard_gauge_higgs =
  [ ((O H, G Wp, G Wm), Scalar_Vector_Vector 1, G_HWW);
    ((O H, G Z, G Z), Scalar_Vector_Vector 1, G_HZZ) ]

let standard_gauge_higgs4 =
  [ (O H, O H, G Wp, G Wm), Scalar2_Vector2 1, G_HHWW;
    (O H, O H, G Z, G Z), Scalar2_Vector2 1, G_HHZZ ]

let standard_higgs =
  [ (O H, O H, O H), Scalar_Scalar_Scalar 1, G_H3 ]

let standard_higgs4 =
  [ (O H, O H, O H, O H), Scalar4 1, G_H4 ]

let gravity_higgs =
  [ (O Grav, O H, O H), Graviton_Scalar_Scalar 1, G_Grav]

let anomalous_gauge_higgs =
  []

let anomalous_gauge_higgs4 =
  []

let anomalous_higgs =
  []

let anomaly_higgs =
  [ (O H, G Ga, G Ga), Dim5_Scalar_Gauge2 1, G_HGaGa;
    (O H, G Ga, G Z), Dim5_Scalar_Gauge2 1, G_HGaZ;
    (O H, G Gl, G Gl), Dim5_Scalar_Gauge2 1, G_Hgg ]

let anomalous_higgs4 =
  []

let gauge_higgs =
    standard_gauge_higgs

let gauge_higgs4 =
    standard_gauge_higgs4

let higgs =
    standard_higgs @ gravity_higgs
```

let *higgs4* =
    standard_higgs4

let *goldstone_vertices* =
  [$((O\ Phi0,\ G\ Wm,\ G\ Wp),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)$;
    $((O\ Phip,\ G\ Ga,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W)$;
    $((O\ Phip,\ G\ Z,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)$;
    $((O\ Phim,\ G\ Wp,\ G\ Ga),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W)$;
    $((O\ Phim,\ G\ Wp,\ G\ Z),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)$]

let *vertices3* =
  (*ThoList.flatmap electromagnetic_currents* $[1; 2; 3]$ @
   ThoList.flatmap neutral_currents $[1; 2; 3]$ @
   ThoList.flatmap color_currents $[1; 2; 3]$ @
   ThoList.flatmap charged_currents $[1; 2; 3]$ @
   ThoList.flatmap gravity_currents $[1; 2; 3]$ @
   yukawa @ *triple_gauge* @ *gravity_gauge* @
   gauge_higgs @ *higgs* @ *anomaly_higgs*
   @ *goldstone_vertices*)

let *vertices4* =
  quartic_gauge @ *gauge_higgs4* @ *higgs4*

let *vertices* () = (*vertices3*, *vertices4*, [])

For efficiency, make sure that *F.of_vertices vertices* is evaluated only once.

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *flavor_of_string* = function
  | "e-" → $M\ (L\ 1)$ | "e+" → $M\ (L\ (-1))$
  | "mu-" → $M\ (L\ 2)$ | "mu+" → $M\ (L\ (-2))$
  | "tau-" → $M\ (L\ 3)$ | "tau+" → $M\ (L\ (-3))$
  | "nue" → $M\ (N\ 1)$ | "nuebar" → $M\ (N\ (-1))$
  | "numu" → $M\ (N\ 2)$ | "numubar" → $M\ (N\ (-2))$
  | "nutau" → $M\ (N\ 3)$ | "nutaubar" → $M\ (N\ (-3))$
  | "u" → $M\ (U\ 1)$ | "ubar" → $M\ (U\ (-1))$
  | "c" → $M\ (U\ 2)$ | "cbar" → $M\ (U\ (-2))$
  | "t" → $M\ (U\ 3)$ | "tbar" → $M\ (U\ (-3))$
  | "d" → $M\ (D\ 1)$ | "dbar" → $M\ (D\ (-1))$
  | "s" → $M\ (D\ 2)$ | "sbar" → $M\ (D\ (-2))$
  | "b" → $M\ (D\ 3)$ | "bbar" → $M\ (D\ (-3))$
  | "g" | "gl" → $G\ Gl$
  | "A" → $G\ Ga$ | "Z" | "Z0" → $G\ Z$
  | "W+" → $G\ Wp$ | "W-" → $G\ Wm$
  | "H" → $O\ H$
  | "GG" → $O\ Grav$
  | _ → *invalid_arg* "Modellib_BSM.Xdim.flavor_of_string"

let *flavor_to_string* = function

```
  | M f →
      begin match f with
      | L 1 → "e-" | L (−1) → "e+"
      | L 2 → "mu-" | L (−2) → "mu+"
      | L 3 → "tau-" | L (−3) → "tau+"
      | L _ → invalid_arg
            "Modellib_BSM.Xdim.flavor_to_string:␣invalid␣lepton"
      | N 1 → "nue" | N (−1) → "nuebar"
      | N 2 → "numu" | N (−2) → "numubar"
      | N 3 → "nutau" | N (−3) → "nutaubar"
      | N _ → invalid_arg
            "Modellib_BSM.Xdim.flavor_to_string:␣invalid␣neutrino"
      | U 1 → "u" | U (−1) → "ubar"
      | U 2 → "c" | U (−2) → "cbar"
      | U 3 → "t" | U (−3) → "tbar"
      | U _ → invalid_arg
            "Modellib_BSM.Xdim.flavor_to_string:␣invalid␣up␣type␣quark"
      | D 1 → "d" | D (−1) → "dbar"
      | D 2 → "s" | D (−2) → "sbar"
      | D 3 → "b" | D (−3) → "bbar"
      | D _ → invalid_arg
            "Modellib_BSM.Xdim.flavor_to_string:␣invalid␣down␣type␣quark"
      end
  | G f →
      begin match f with
      | Gl → "g"
      | Ga → "A" | Z → "Z"
      | Wp → "W+" | Wm → "W-"
      end
  | O f →
      begin match f with
      | Phip → "phi+" | Phim → "phi-" | Phi0 → "phi0"
      | H → "H" | Grav → "GG"
      end
let flavor_to_TeX = function
  | M f →
      begin match f with
      | L 1 → "e^-" | L (−1) → "e^+"
      | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
      | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
      | L _ → invalid_arg
            "Modellib_BSM.Xdim.flavor_to_TeX:␣invalid␣lepton"
      | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
      | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
      | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
      | N _ → invalid_arg
            "Modellib_BSM.Xdim.flavor_to_TeX:␣invalid␣neutrino"
      | U 1 → "u" | U (−1) → "\\bar{u}"
      | U 2 → "c" | U (−2) → "\\bar{c}"
```

```
        | U 3 → "t" | U (−3) → "\\bar{t}"
        | U _ → invalid_arg
               "Modellib_BSM.Xdim.flavor_to_TeX:␣invalid␣up␣type␣quark"
        | D 1 → "d" | D (−1) → "\\bar{d}"
        | D 2 → "s" | D (−2) → "\\bar{s}"
        | D 3 → "b" | D (−3) → "\\bar{b}"
        | D _ → invalid_arg
               "Modellib_BSM.Xdim.flavor_to_TeX:␣invalid␣down␣type␣quark"
        end
    | G f →
        begin match f with
        | Gl → "g"
        | Ga → "\\gamma" | Z → "Z"
        | Wp → "W^+" | Wm → "W^-"
        end
    | O f →
        begin match f with
        | Phip  →  "\\phi^+" |  Phim  →  "\\phi^-" |  Phi0  →
"\\phi^0"
        | H → "H" | Grav → "G"
        end

let flavor_symbol = function
    | M f →
        begin match f with
        | L n when n > 0 → "l" ^ string_of_int n
        | L n → "l" ^ string_of_int (abs n) ^ "b"
        | N n when n > 0 → "n" ^ string_of_int n
        | N n → "n" ^ string_of_int (abs n) ^ "b"
        | U n when n > 0 → "u" ^ string_of_int n
        | U n → "u" ^ string_of_int (abs n) ^ "b"
        | D n when n > 0 → "d" ^ string_of_int n
        | D n → "d" ^ string_of_int (abs n) ^ "b"
        end
    | G f →
        begin match f with
        | Gl → "gl"
        | Ga → "a" | Z → "z"
        | Wp → "wp" | Wm → "wm"
        end
    | O f →
        begin match f with
        | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
        | H → "h" | Grav → "gv"
        end

let pdg = function
    | M f →
        begin match f with
        | L n when n > 0 → 9 + 2 × n
        | L n → − 9 + 2 × n
```

```
                | N n when n > 0 → 10 + 2 × n
                | N n → − 10 + 2 × n
                | U n when n > 0 → 2 × n
                | U n → 2 × n
                | D n when n > 0 → − 1 + 2 × n
                | D n → 1 + 2 × n
                end
        | G f →
            begin match f with
            | Gl → 21
            | Ga → 22 | Z → 23
            | Wp → 24 | Wm → (−24)
            end
        | O f →
            begin match f with
            | Phip | Phim → 27 | Phi0 → 26
            | H → 25 | Grav → 39
            end

    let mass_symbol f =
        "mass(" ˆ string_of_int (abs (pdg f)) ˆ ")"

    let width_symbol f =
        "width(" ˆ string_of_int (abs (pdg f)) ˆ ")"

    let constant_symbol = function
        | Unit → "unit" | Pi → "PI"
        | Alpha_QED → "alpha" | E → "e" | G_weak → "g" | Vev →
"vev"
        | Sin2thw → "sin2thw" | Sinthw → "sinthw" | Costhw →
"costhw"
        | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
        | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
        | G_NC_up → "gncup" | G_NC_down → "gncdwn"
        | Gs → "gs" | I_Gs → "igs" | G2 → "gs**2"
        | G_CC → "gcc"
        | G_CCQ (n1, n2) → "gccq" ˆ string_of_int n1 ˆ string_of_int n2
        | I_Q_W → "iqw" | I_G_ZWW → "igzww"
        | G_WWWW → "gw4" | G_ZZWW → "gzzww"
        | G_AZWW → "gazww" | G_AAWW → "gaaww"
        | G_HWW → "ghww" | G_HZZ → "ghzz"
        | G_HHWW → "ghhww" | G_HHZZ → "ghhzz"
        | G_Htt → "ghtt" | G_Hbb → "ghbb"
        | G_Htautau → "ghtautau" | G_Hcc → "ghcc"
        | G_HGaZ → "ghgaz" | G_HGaGa → "ghgaga" | G_Hgg →
"ghgg"
        | G_H3 → "gh3" | G_H4 → "gh4" | G_Grav → "ggrav"
        | Mass f → "mass" ˆ flavor_symbol f
        | Width f → "width" ˆ flavor_symbol f

  end

module UED (Flags : BSM_flags) =
```

```
struct
    let rcs = RCS.rename rcs_file "Modellib_BSM.UED"
        [ "Universal␣Extra␣Dimensions"]

    open Coupling

    let default_width = ref Timelike
    let use_fudged_width = ref false

    let options = Options.create
      [ "constant_width", Arg.Unit (fun () → default_width := Constant),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", Arg.Set use_fudged_width,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", Arg.String (fun f → default_width := Custom f),
        "use␣custom␣width";
        "cancel_widths", Arg.Unit (fun () → default_width := Vanishing),
        "use␣vanishing␣width"]

    type matter_field = L of int | N of int | U of int | D of int
            | L_K1_L of int | L_K1_R of int | N_K1 of int
            | L_K2_L of int | L_K2_R of int | N_K2 of int
            | U_K1_L of int | U_K2_L of int | D_K1_L of int | D_K2_L of int
            | U_K1_R of int | U_K2_R of int | D_K1_R of int | D_K2_R of int
    type gauge_boson = Ga | Wp | Wm | Z | Gl | Gl_K1 | Gl_K2
            | B1 | B2 | Z1 | Z2 | Wp1 | Wm1 | Wp2 | Wm2
    type other = Phip | Phim | Phi0 | H | H1up | H1um
            | H1dp | H1dm | H2up | H2um | H2dp | H2dm
            | Grav
    type flavor = M of matter_field | G of gauge_boson | O of other

    let matter_field f = M f
    let gauge_boson f = G f
    let other f = O f

    type field =
      | Matter of matter_field
      | Gauge of gauge_boson
      | Other of other

    let field = function
      | M f → Matter f
      | G f → Gauge f
      | O f → Other f

    type gauge = unit

    let gauge_symbol () =
        failwith "Modellib_BSM.UED.gauge_symbol:␣internal␣error"

    let family n = List.map matter_field [ L n; N n; U n; D n; L_K1_L n;
        L_K1_R n; L_K2_L n; L_K2_R n; N_K1 n; N_K2 n; U_K1_L n; U_K2_L n;
        D_K1_L n; D_K2_L n; U_K1_R n; U_K2_R n; D_K1_R n; D_K2_R n]
```

We don't introduce a special index for the higher excitations but make them parts of the particles' names.

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let *external_flavors* () =
  ["1st␣Generation", *ThoList.flatmap family* [1; − 1];
    "2nd␣Generation", *ThoList.flatmap family* [2; − 2];
    "3rd␣Generation", *ThoList.flatmap family* [3; − 3];
    "Gauge␣Bosons", *List.map gauge_boson* [*Ga*; *Z*; *Wp*; *Wm*; *Gl*;
        Gl_K1; *Gl_K2*; *B1*; *B2*; *Z1*; *Z2*; *Wp1*; *Wm1*; *Wp2*; *Wm2*];
    "Higgs", *List.map other* [*H*; *H1up*; *H1um*; *H1dp*; *H1dm*;
        H2up; *H2um*; *H2dp*; *H2dm*];
    "Graviton", *List.map other* [*Grav*];
    "Goldstone␣Bosons", *List.map other* [*Phip*; *Phim*; *Phi0*]]

let *flavors* () = *ThoList.flatmap snd* (*external_flavors* ())

let *spinor n* =
  if *n* ≥ 0 then
    Spinor
  else
    ConjSpinor

let *lorentz* = function
  | *M f* →
    begin match *f* with
    | *L n* → *spinor n* | *N n* → *spinor n*
    | *U n* → *spinor n* | *D n* → *spinor n*
    | *L_K1_L n* → *spinor n* | *L_K1_R n* → *spinor n*
    | *L_K2_L n* → *spinor n* | *L_K2_R n* → *spinor n*
    | *N_K1 n* → *spinor n* | *N_K2 n* → *spinor n*
    | *U_K1_L n* → *spinor n* | *U_K1_R n* → *spinor n*
    | *U_K2_L n* → *spinor n* | *U_K2_R n* → *spinor n*
    | *D_K1_L n* → *spinor n* | *D_K1_R n* → *spinor n*
    | *D_K2_L n* → *spinor n* | *D_K2_R n* → *spinor n*
    end
  | *G f* →
    begin match *f* with
    | *Ga* | *Gl* → *Vector*
    | *Wp* | *Wm* | *Z* | *Gl_K1* | *Gl_K2* | *B1* | *B2*
    | *Z1* | *Z2* | *Wp1* | *Wm1* | *Wp2* | *Wm2* → *Massive_Vector*
    end
  | *O f* →
    begin match *f* with
    | *Grav* → *Tensor_2*
    | _ → *Scalar*
    end

let *color* = function
  | *M* (*U n*) → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *M* (*D n*) → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *M* (*U_K1_L n*) → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *M* (*D_K1_L n*) → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *M* (*U_K1_R n*) → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *M* (*D_K1_R n*) → *Color.SUN* (if *n* > 0 then 3 else -3)
  | *M* (*U_K2_L n*) → *Color.SUN* (if *n* > 0 then 3 else -3)

```
        | M (D_K2_L n)  →  Color.SUN (if n > 0 then 3 else -3)
        | M (U_K2_R n)  →  Color.SUN (if n > 0 then 3 else -3)
        | M (D_K2_R n)  →  Color.SUN (if n > 0 then 3 else -3)
        | G Gl  |  G Gl_K1  |  G Gl_K2  →  Color.AdjSUN 3
        | _  →  Color.Singlet
    let prop_spinor n  =
      if n ≥ 0 then
        Prop_Spinor
      else
        Prop_ConjSpinor

    let propagator  = function
      | M f  →
          begin match f with
          | L n  →  prop_spinor n  |  N n  →  prop_spinor n
          | U n  →  prop_spinor n  |  D n  →  prop_spinor n
          | L_K1_L n  →  prop_spinor n  |  L_K1_R n  →  prop_spinor n
          | L_K2_L n  →  prop_spinor n  |  L_K2_R n  →  prop_spinor n
          | N_K1 n  →  prop_spinor n  |  N_K2 n  →  prop_spinor n
          | U_K1_L n  →  prop_spinor n  |  U_K1_R n  →  prop_spinor n
          | U_K2_L n  →  prop_spinor n  |  U_K2_R n  →  prop_spinor n
          | D_K1_L n  →  prop_spinor n  |  D_K1_R n  →  prop_spinor n
          | D_K2_L n  →  prop_spinor n  |  D_K2_R n  →  prop_spinor n
          end
      | G f  →
          begin match f with
          | Ga  |  Gl  →  Prop_Feynman
          | Wp  |  Wm  |  Z  |  Gl_K1  |  Gl_K2  |  B1  |  B2
          | Z1  |  Z2  |  Wp1  |  Wm1  |  Wp2  |  Wm2  →  Prop_Unitarity
          end
      | O f  →
          begin match f with
          | Phip  |  Phim  |  Phi0  →  Only_Insertion
          | H  |  H1up  |  H1um  |  H1dp  |  H1dm  |  H2up
          | H2um  |  H2dp  |  H2dm  →  Prop_Scalar
          | Grav  →  Prop_Tensor_2
          end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^\pm$ and top.

```
    let width f  =
      if !use_fudged_width then
        match f with
        | G Wp  |  G Wm  |  M (U 3)  |  M (U (−3))  |  O Grav  →  Fudged
        | _  →  !default_width
      else
        !default_width

    let goldstone  = function
      | G f  →
          begin match f with
```

```
        | Wp → Some (O Phip, Coupling.Const 1)
        | Wm → Some (O Phim, Coupling.Const 1)
        | Z → Some (O Phi0, Coupling.Const 1)
        | _ → None
        end
   | _ → None

let conjugate = function
  | M f →
      M (begin match f with
      | L n → L (−n) | N n → N (−n)
      | U n → U (−n) | D n → D (−n)
      | L_K1_L n → L_K1_L (−n) | L_K1_R n → L_K1_R (−n)
      | L_K2_L n → L_K2_L (−n) | L_K2_R n → L_K2_R (−n)
      | N_K1 n → N_K1 (−n) | N_K2 n → N_K2 (−n)
      | U_K1_L n → U_K1_L (−n) | U_K1_R n → U_K1_R (−n)
      | U_K2_L n → U_K2_L (−n) | U_K2_R n → U_K2_R (−n)
      | D_K1_L n → D_K1_L (−n) | D_K1_R n → D_K1_R (−n)
      | D_K2_L n → D_K2_L (−n) | D_K2_R n → D_K2_R (−n)
      end)
   | G f →
      G (begin match f with
      | Gl → Gl | Ga → Ga | Z → Z
      | Wp → Wm | Wm → Wp
      | Gl_K1 → Gl_K1 | Gl_K2 → Gl_K2 | B1 → B1 | B2 →
B2
      | Z1 → Z1 | Z2 → Z2 | Wp1 → Wm1 | Wm1 → Wp1
      | Wp2 → Wm2 | Wm2 → Wp2
      end)
   | O f →
      O (begin match f with
      | Phip → Phim | Phim → Phip | Phi0 → Phi0
      | H → H | H1up → H1um | H1um → H1up
      | H1dp → H1dm | H1dm → H1dp
      | H2up → H2um | H2um → H2up
      | H2dp → H2dm | H2dm → H2dp
      | Grav → Grav
      end)

let fermion = function
  | M f →
      begin match f with
      | L n → if n > 0 then 1 else -1
      | N n → if n > 0 then 1 else -1
      | U n → if n > 0 then 1 else -1
      | D n → if n > 0 then 1 else -1
      | L_K1_L n → if n > 0 then 1 else -1
      | L_K2_L n → if n > 0 then 1 else -1
      | L_K1_R n → if n > 0 then 1 else -1
      | L_K2_R n → if n > 0 then 1 else -1
      | U_K1_L n → if n > 0 then 1 else -1
```

```
          | U_K2_L n  →  if n > 0 then 1 else -1
          | U_K1_R n  →  if n > 0 then 1 else -1
          | U_K2_R n  →  if n > 0 then 1 else -1
          | D_K1_L n  →  if n > 0 then 1 else -1
          | D_K2_L n  →  if n > 0 then 1 else -1
          | D_K1_R n  →  if n > 0 then 1 else -1
          | D_K2_R n  →  if n > 0 then 1 else -1
          | N_K1 n  →  if n > 0 then 1 else -1
          | N_K2 n  →  if n > 0 then 1 else -1
        end
  | G f  →
      begin match f with
      | Gl  |  Ga  |  Z  |  Wp  |  Wm  |  Gl_K1  |  Gl_K2
      | B1  |  B2  |  Z1  |  Z2  |  Wp1  |  Wm1  |  Wp2
      | Wm2  →  0
      end
  | O _  →  0

module Ch  =  Charges.QQ

let ( // )  =  Algebra.Small_Rational.make

let generation' = function
  | 1  →  [ 1//1; 0//1; 0//1]
  | 2  →  [ 0//1; 1//1; 0//1]
  | 3  →  [ 0//1; 0//1; 1//1]
  | − 1  →  [−1//1; 0//1; 0//1]
  | − 2  →  [ 0//1; − 1//1; 0//1]
  | − 3  →  [ 0//1; 0//1; − 1//1]
  | n  →  invalid_arg ("SM.generation':␣" ^ string_of_int n)

let generation f  =
  match f with
  | M (L n  |  N n  |  U n  |  D n  |  L_K1_L n  |  L_K2_L n
      | L_K1_R n  |  L_K2_R n  |  N_K1 n  |  N_K2 n  |  U_K1_L n
      | U_K2_L n  |  U_K1_R n  |  U_K2_R n  |  D_K1_L n  |
D_K2_L n
      | D_K1_R n  |  D_K2_R n )  →  generation' n
  | G _  |  O _  →  [0//1; 0//1; 0//1]

let charge = function
  | M f  →
      begin match f with
      | L n  |  L_K1_L n  |  L_K2_L n  |  L_K1_R n
      | L_K2_R n  →  if n > 0 then -1//1 else 1//1
      | N n  |  N_K1 n  |  N_K2 n  →  0//1
      | U n  |  U_K1_L n  |  U_K2_L n  |  U_K1_R n
      | U_K2_R n  →  if n > 0 then 2//3 else -2//3
      | D n  |  D_K1_L n  |  D_K2_L n  |  D_K1_R n
      | D_K2_R n  →  if n > 0 then -1//3 else 1//3
      end
  | G f  →
      begin match f with
```

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```
            |  Gl  |  Gl_K1  |  Gl_K2  |  Ga  |  Z
            |  B1  |  B2  |  Z1  |  Z2  →  0//1
            |  Wp  |  Wp1  |  Wp2  →  1//1
            |  Wm  |  Wm1  |  Wm2  →  − 1//1
            end
      |  O f  →
            begin match f with
            |  H  |  Phi0  |  Grav  →  0//1
            |  H1up  |  H1dp  |  H2up  |  H2dp  |  Phip  →  1//1
            |  H1um  |  H1dm  |  H2um  |  H2dm  |  Phim  →  − 1//1
            end

    let lepton  =  function
      |  M f  →
            begin match f with
            |  L n  |  N n  |  L_K1_L n  |  L_K1_R n  |  L_K2_L n
            |  L_K2_R n  |  N_K1 n  |  N_K2 n  →  if n > 0 then 1//1 else -
1//1
            |  U _  |  D _  |  _  →  0//1
            end
      |  G _  |  O _  →  0//1

    let baryon  =  function
      |  M f  →
            begin match f with
            |  U n  |  D n  |  U_K1_L n  |  U_K1_R n  |  U_K2_L n
            |  U_K2_R n  |  D_K1_L n  |  D_K1_R n  |  D_K2_L n
            |  D_K2_R n  →  if n > 0 then 1//1 else -1//1
            |  L _  |  N _  |  _  →  0//1
            end
      |  G _  |  O _  →  0//1

    let charges f  =
      [ charge f; lepton f; baryon f] @ generation f

    type constant  =
      |  Unit  |  Pi  |  Alpha_QED  |  Sin2thw
      |  Sinthw  |  Costhw  |  E  |  G_weak  |  Vev
      |  Q_lepton  |  Q_up  |  Q_down  |  G_CC  |  G_CCQ of int×int
      |  G_NC_neutrino  |  G_NC_lepton  |  G_NC_up  |  G_NC_down
      |  I_Q_W  |  I_G_ZWW  |  I_Q_W_K  |  I_G_ZWW_K1  |
I_G_ZWW_K2
      |  I_G_ZWW_K3
      |  G_WWWW  |  G_ZZWW  |  G_AZWW  |  G_AAWW
      |  G_HWW  |  G_HHWW  |  G_HZZ  |  G_HHZZ
      |  G_Htt  |  G_Hbb  |  G_Hcc  |  G_Htautau  |  G_H3  |  G_H4
      |  G_HGaZ  |  G_HGaGa  |  G_Hgg
      |  Gs  |  I_Gs  |  I_GsRt2  |  G2  |  G22  |  G_Grav
      |  Mass of flavor  |  Width of flavor
```

Two integer counters for the QCD and EW order of the couplings.

```
    type orders  =  int × int
```

let *orders* = function
  | _ → (0, 0)

let *input_parameters* =
  [ ]

let *derived_parameters* =
  [ ]

let *g_over_2_costh* =
  Quot (*Neg* (*Atom G_weak*), *Prod* [*Const* 2; *Atom Costhw*])

let *nc_coupling c t3 q* =
  (*Real_Array c*,
   [*Prod* [*g_over_2_costh*; *Diff* (*t3*, *Prod* [*Const* 2; *q*; *Atom Sin2thw*])];
     Prod [*g_over_2_costh*; *t3*]])

let *half* = *Quot* (*Const* 1, *Const* 2)

let *derived_parameter_arrays* =
  [*nc_coupling G_NC_neutrino half* (*Const* 0);
      nc_coupling G_NC_lepton (*Neg half*) (*Const* (−1));
      nc_coupling G_NC_up half (*Quot* (*Const* 2, *Const* 3));
      nc_coupling G_NC_down (*Neg half*) (*Quot* (*Const* (−1), *Const* 3))]

let *parameters* () =
  { *input* = *input_parameters*;
    derived = *derived_parameters*;
    derived_arrays = *derived_parameter_arrays* }

module *F* = *Modeltools.Fusions* (struct
  type *f* = *flavor*
  type *c* = *constant*
  let *compare* = *compare*
  let *conjugate* = *conjugate*
end)

let *mgm* ((*m1*, *g*, *m2*), *fbf*, *c*) = ((*M m1*, *G g*, *M m2*), *fbf*, *c*)
let *mom* ((*m1*, *o*, *m2*), *fbf*, *c*) = ((*M m1*, *O o*, *M m2*), *fbf*, *c*)

let *electromagnetic_currents n* =
  List.map mgm
    [((*L* (−*n*), *Ga*, *L n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_lepton*);
       ((*U* (−*n*), *Ga*, *U n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
       ((*D* (−*n*), *Ga*, *D n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

let *neutral_currents n* =
  List.map mgm
    [((*L* (−*n*), *Z*, *L n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_lepton*);
       ((*N* (−*n*), *Z*, *N n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_neutrino*);
       ((*U* (−*n*), *Z*, *U n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_up*);
       ((*D* (−*n*), *Z*, *D n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_down*)]

let *charged_currents n* =
  List.map mgm
    [((*L* (−*n*), *Wm*, *N n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);

```
        ((N (−n), Wp, L n), FBF (1, Psibar, VL, Psi), G_CC);
        ((D (−n), Wm, U n), FBF (1, Psibar, VL, Psi), G_CC);
        ((U (−n), Wp, D n), FBF (1, Psibar, VL, Psi), G_CC) ]

  let color_currents n =
      List.map mgm
        [ ((U (−n), Gl, U n), FBF ((−1), Psibar, V, Psi), Gs);
          ((D (−n), Gl, D n), FBF ((−1), Psibar, V, Psi), Gs) ]

  let gravity_currents n =
    List.map mom
        [ ((L (−n), Grav, L n), Graviton_Spinor_Spinor 1, G_Grav);
          ((N (−n), Grav, N n), Graviton_Spinor_Spinor 1, G_Grav);
          ((U (−n), Grav, U n), Graviton_Spinor_Spinor 1, G_Grav);
          ((D (−n), Grav, D n), Graviton_Spinor_Spinor 1, G_Grav) ]

  let yukawa =
    List.map mom
        [ ((U (−3), H, U 3), FBF (1, Psibar, S, Psi), G_Htt);
          ((D (−3), H, D 3), FBF (1, Psibar, S, Psi), G_Hbb);
          ((U (−2), H, U 2), FBF (1, Psibar, S, Psi), G_Hcc);
          ((L (−3), H, L 3), FBF (1, Psibar, S, Psi), G_Htautau) ]

  let tgc ((g1, g2, g3), t, c) = ((G g1, G g2, G g3), t, c)
```

Gluons should be included in just that way.

```
  let standard_triple_gauge =
    List.map tgc
        [ ((Ga, Wm, Wp), Gauge_Gauge_Gauge 1, I_Q_W);
          ((Ga, Wm1, Wp1), Gauge_Gauge_Gauge 1, I_Q_W_K);
          ((Z, Wm, Wp), Gauge_Gauge_Gauge 1, I_G_ZWW);
          ((Z, Wm1, Wp1), Gauge_Gauge_Gauge 1, I_G_ZWW_K1);
          ((Z1, Wm, Wp1), Gauge_Gauge_Gauge 1, I_G_ZWW_K2);
          ((Z1, Wm1, Wp), Gauge_Gauge_Gauge 1, I_G_ZWW_K2);
          ((Z2, Wm1, Wp2), Gauge_Gauge_Gauge 1, I_G_ZWW_K3);
          ((Gl, Gl, Gl), Gauge_Gauge_Gauge 1, I_Gs);
          ((Gl, Gl_K2, Gl_K2), Gauge_Gauge_Gauge (−1), I_Gs);
          ((Gl, Gl_K1, Gl_K1), Gauge_Gauge_Gauge 1, I_Gs);
          ((Gl_K2, Gl_K1, Gl_K1), Gauge_Gauge_Gauge 1, I_GsRt2)]

  let triple_gauge =
      standard_triple_gauge

  let qgc ((g1, g2, g3, g4), t, c) = ((G g1, G g2, G g3, G g4), t, c)

  let gauge4 = Vector4 [(2, C_13_42); (−1, C_12_34); (−1, C_14_23)]
  let minus_gauge4 = Vector4 [(−2, C_13_42); (1, C_12_34); (1, C_14_23)]
  let standard_quartic_gauge =
    List.map qgc
        [ (Wm, Wp, Wm, Wp), gauge4, G_WWWW;
          (Wm, Z, Wp, Z), minus_gauge4, G_ZZWW;
          (Wm, Z, Wp, Ga), minus_gauge4, G_AZWW;
          (Wm, Ga, Wp, Ga), minus_gauge4, G_AAWW;
          ((Gl, Gl, Gl, Gl), gauge4, G2);
```

```
            ((Gl,  Gl,  Gl_K1, Gl_K1), gauge4, G2);
            ((Gl,  Gl,  Gl_K2, Gl_K2), gauge4, G2);
            ((Gl_K1, Gl_K1, Gl_K2, Gl_K2), gauge4, G2);
            ((Gl_K2, Gl_K2, Gl_K2, Gl_K2), gauge4, G22)]
```

let *quartic_gauge* =
  standard_quartic_gauge

let *gravity_gauge* =
  [(*O Grav*, *G Z*, *G Z*), *Graviton_Vector_Vector* 1, *G_Grav*;
    (*O Grav*, *G Wp*, *G Wm*), *Graviton_Vector_Vector* 1, *G_Grav*;
    (*O Grav*, *G Ga*, *G Ga*), *Graviton_Vector_Vector* 1, *G_Grav*;
    (*O Grav*, *G Gl*, *G Gl*), *Graviton_Vector_Vector* 1, *G_Grav*]

let *standard_gauge_higgs* =
  [((*O H*, *G Wp*, *G Wm*), *Scalar_Vector_Vector* 1, *G_HWW*);
    ((*O H*, *G Z*, *G Z*), *Scalar_Vector_Vector* 1, *G_HZZ*)]

let *standard_gauge_higgs4* =
  [(*O H*, *O H*, *G Wp*, *G Wm*), *Scalar2_Vector2* 1, *G_HHWW*;
    (*O H*, *O H*, *G Z*, *G Z*), *Scalar2_Vector2* 1, *G_HHZZ*]

let *standard_higgs* =
  [(*O H*, *O H*, *O H*), *Scalar_Scalar_Scalar* 1, *G_H3*]

let *standard_higgs4* =
  [(*O H*, *O H*, *O H*, *O H*), *Scalar4* 1, *G_H4*]

let *gravity_higgs* =
  [(*O Grav*, *O H*, *O H*), *Graviton_Scalar_Scalar* 1, *G_Grav*]

let *anomalous_gauge_higgs* =
  []

let *anomalous_gauge_higgs4* =
  []

let *anomalous_higgs* =
  []

let *anomaly_higgs* =
  [(*O H*, *G Ga*, *G Ga*), *Dim5_Scalar_Gauge2* 1, *G_HGaGa*;
    (*O H*, *G Ga*, *G Z*), *Dim5_Scalar_Gauge2* 1, *G_HGaZ*;
    (*O H*, *G Gl*, *G Gl*), *Dim5_Scalar_Gauge2* 1, *G_Hgg*]

let *anomalous_higgs4* =
  []

let *gauge_higgs* =
  standard_gauge_higgs

let *gauge_higgs4* =
  standard_gauge_higgs4

let *higgs* =
  standard_higgs @ *gravity_higgs*

let *higgs4* =

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$standard\_higgs4$

let $goldstone\_vertices$ =
  [$((O\ Phi0,\ G\ Wm,\ G\ Wp),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)$;
    $((O\ Phip,\ G\ Ga,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W)$;
    $((O\ Phip,\ G\ Z,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)$;
    $((O\ Phim,\ G\ Wp,\ G\ Ga),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W)$;
    $((O\ Phim,\ G\ Wp,\ G\ Z),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)$]

let $vertices3$ =
  ($ThoList.flatmap\ electromagnetic\_currents\ [1;2;3]$ @
   $ThoList.flatmap\ neutral\_currents\ [1;2;3]$ @
   $ThoList.flatmap\ charged\_currents\ [1;2;3]$ @
   $ThoList.flatmap\ color\_currents\ [1;2;3]$ @
   $ThoList.flatmap\ gravity\_currents\ [1;2;3]$ @
   $yukawa$ @ $triple\_gauge$ @ $gravity\_gauge$ @
   $gauge\_higgs$ @ $higgs$ @ $anomaly\_higgs$
   @ $goldstone\_vertices$)

let $vertices4$ =
  $quartic\_gauge$ @ $gauge\_higgs4$ @ $higgs4$

let $vertices$ () = ($vertices3$, $vertices4$, [])

For efficiency, make sure that $F.of\_vertices\ vertices$ is evaluated only once.

let $table$ = $F.of\_vertices\ (vertices\ ())$
let $fuse2$ = $F.fuse2\ table$
let $fuse3$ = $F.fuse3\ table$
let $fuse$ = $F.fuse\ table$
let max_degree () = 4

let $flavor\_of\_string$ = function
  | "e-" $\rightarrow$ $M\ (L\ 1)$ | "e+" $\rightarrow$ $M\ (L\ (-1))$
  | "mu-" $\rightarrow$ $M\ (L\ 2)$ | "mu+" $\rightarrow$ $M\ (L\ (-2))$
  | "tau-" $\rightarrow$ $M\ (L\ 3)$ | "tau+" $\rightarrow$ $M\ (L\ (-3))$
  | "nue" $\rightarrow$ $M\ (N\ 1)$ | "nuebar" $\rightarrow$ $M\ (N\ (-1))$
  | "numu" $\rightarrow$ $M\ (N\ 2)$ | "numubar" $\rightarrow$ $M\ (N\ (-2))$
  | "nutau" $\rightarrow$ $M\ (N\ 3)$ | "nutaubar" $\rightarrow$ $M\ (N\ (-3))$
  | "u" $\rightarrow$ $M\ (U\ 1)$ | "ubar" $\rightarrow$ $M\ (U\ (-1))$
  | "c" $\rightarrow$ $M\ (U\ 2)$ | "cbar" $\rightarrow$ $M\ (U\ (-2))$
  | "t" $\rightarrow$ $M\ (U\ 3)$ | "tbar" $\rightarrow$ $M\ (U\ (-3))$
  | "d" $\rightarrow$ $M\ (D\ 1)$ | "dbar" $\rightarrow$ $M\ (D\ (-1))$
  | "s" $\rightarrow$ $M\ (D\ 2)$ | "sbar" $\rightarrow$ $M\ (D\ (-2))$
  | "b" $\rightarrow$ $M\ (D\ 3)$ | "bbar" $\rightarrow$ $M\ (D\ (-3))$
  | "uk1l" $\rightarrow$ $M\ (U\_K1\_L\ 1)$ | "uk1lbar" $\rightarrow$ $M\ (U\_K1\_L\ (-1))$
  | "ck1l" $\rightarrow$ $M\ (U\_K1\_L\ 2)$ | "ck1lbar" $\rightarrow$ $M\ (U\_K1\_L\ (-2))$
  | "tk1l" $\rightarrow$ $M\ (U\_K1\_L\ 3)$ | "tk1lbar" $\rightarrow$ $M\ (U\_K1\_L\ (-3))$
  | "dk1l" $\rightarrow$ $M\ (D\_K1\_L\ 1)$ | "dk1lbar" $\rightarrow$ $M\ (D\_K1\_L\ (-1))$
  | "sk1l" $\rightarrow$ $M\ (D\_K1\_L\ 2)$ | "sk1lbar" $\rightarrow$ $M\ (D\_K1\_L\ (-2))$
  | "bk1l" $\rightarrow$ $M\ (D\_K1\_L\ 3)$ | "bk1lbar" $\rightarrow$ $M\ (D\_K1\_L\ (-3))$
  | "uk1r" $\rightarrow$ $M\ (U\_K1\_R\ 1)$ | "uk1rbar" $\rightarrow$ $M\ (U\_K1\_R\ (-1))$
  | "ck1r" $\rightarrow$ $M\ (U\_K1\_R\ 2)$ | "ck1rbar" $\rightarrow$ $M\ (U\_K1\_R\ (-2))$
  | "tk1r" $\rightarrow$ $M\ (U\_K1\_R\ 3)$ | "tk1rbar" $\rightarrow$ $M\ (U\_K1\_R\ (-3))$

```
    | "dk1r" → M (D_K1_R 1) | "dk1rbar" → M (D_K1_R (−1))
    | "sk1r" → M (D_K1_R 2) | "sk1rbar" → M (D_K1_R (−2))
    | "bk1r" → M (D_K1_R 3) | "bk1rbar" → M (D_K1_R (−3))
    | "uk2l" → M (U_K2_L 1) | "uk2lbar" → M (U_K2_L (−1))
    | "ck2l" → M (U_K2_L 2) | "ck2lbar" → M (U_K2_L (−2))
    | "tk2l" → M (U_K2_L 3) | "tk2lbar" → M (U_K2_L (−3))
    | "dk2l" → M (D_K2_L 1) | "dk2lbar" → M (D_K2_L (−1))
    | "sk2l" → M (D_K2_L 2) | "sk2lbar" → M (D_K2_L (−2))
    | "bk2l" → M (D_K2_L 3) | "bk2lbar" → M (D_K2_L (−3))
    | "uk2r" → M (U_K2_R 1) | "uk2rbar" → M (U_K2_R (−1))
    | "ck2r" → M (U_K2_R 2) | "ck2rbar" → M (U_K2_R (−2))
    | "tk2r" → M (U_K2_R 3) | "tk2rbar" → M (U_K2_R (−3))
    | "dk2r" → M (D_K2_R 1) | "dk2rbar" → M (D_K2_R (−1))
    | "sk2r" → M (D_K2_R 2) | "sk2rbar" → M (D_K2_R (−2))
    | "bk2r" → M (D_K2_R 3) | "bk2rbar" → M (D_K2_R (−3))
    | "g" | "gl" → G Gl
    | "g_k1" | "gl_k1" → G Gl_K1
    | "g_k2" | "gl_k2" → G Gl_K2
    | "b1" → G B1 | "b2" → G B2 | "z1" → G Z1 | "z2" → G Z2
    | "W1+" → G Wp1 | "W1-" → G Wm1
    | "W2+" → G Wp2 | "W2-" → G Wm2
    | "A" → G Ga | "Z" | "Z0" → G Z
    | "W+" → G Wp | "W-" → G Wm
    | "H" → O H | "H1u+" → O H1up | "H1u-" → O H1um
    | "H1d+" → O H1dp | "H1d-" → O H1dm
    | "H2u+" → O H2up | "H2u-" → O H2um
    | "H2d+" → O H2dp | "H2d-" → O H2dm
    | "GG" → O Grav
    | "ek1l-" → M (L_K1_L 1) | "ek1l+" → M (L_K1_L (−1))
    | "muk1l-" → M (L_K1_L 2) | "mu1l+" → M (L_K1_L (−2))
    | "tauk1l-" → M (L_K1_L 3) | "tauk1l+" → M (L_K1_L (−3))
    | "ek1r-" → M (L_K1_R 1) | "ek1r+" → M (L_K1_R (−1))
    | "muk1r-" → M (L_K1_R 2) | "mu1r+" → M (L_K1_R (−2))
    | "tau1r-" → M (L_K1_R 3) | "tauk1r+" → M (L_K1_R (−3))
    | "ek2l-" → M (L_K2_L 1) | "ek2l+" → M (L_K2_L (−1))
    | "muk2l-" → M (L_K2_L 2) | "mu2l+" → M (L_K2_L (−2))
    | "tauk2l-" → M (L_K2_L 3) | "tauk2l+" → M (L_K2_L (−3))
    | "ek2r-" → M (L_K2_R 1) | "ek2r+" → M (L_K2_R (−1))
    | "muk2r-" → M (L_K2_R 2) | "mu2r+" → M (L_K2_R (−2))
    | "tau2r-" → M (L_K2_R 3) | "tauk2r+" → M (L_K2_R (−3))
    | "nuek1" → M (N_K1 1) | "nuek1bar" → M (N_K1 (−1))
    | "numuk1" → M (N_K1 2) | "numuk1bar" → M (N_K1 (−2))
    | "nutauk1" → M (N_K1 3) | "nutauk1bar" → M (N_K1 (−3))
    | "nuek2" → M (N_K2 1) | "nuek2bar" → M (N_K2 (−1))
    | "numuk2" → M (N_K2 2) | "numuk2bar" → M (N_K2 (−2))
    | "nutauk2" → M (N_K2 3) | "nutauk2bar" → M (N_K2 (−3))
    | _ → invalid_arg "Modellib_BSM.UED.flavor_of_string"

let flavor_to_string = function
  | M f →
```

begin match f with
| $L\ 1\ \rightarrow$ "e-" | $L\ (-1)\ \rightarrow$ "e+"
| $L\ 2\ \rightarrow$ "mu-" | $L\ (-2)\ \rightarrow$ "mu+"
| $L\ 3\ \rightarrow$ "tau-" | $L\ (-3)\ \rightarrow$ "tau+"
| $L\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣lepton"
| $N\ 1\ \rightarrow$ "nue" | $N\ (-1)\ \rightarrow$ "nuebar"
| $N\ 2\ \rightarrow$ "numu" | $N\ (-2)\ \rightarrow$ "numubar"
| $N\ 3\ \rightarrow$ "nutau" | $N\ (-3)\ \rightarrow$ "nutaubar"
| $N\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣neutrino"
| $U\ 1\ \rightarrow$ "u" | $U\ (-1)\ \rightarrow$ "ubar"
| $U\ 2\ \rightarrow$ "c" | $U\ (-2)\ \rightarrow$ "cbar"
| $U\ 3\ \rightarrow$ "t" | $U\ (-3)\ \rightarrow$ "tbar"
| $U\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣up␣type␣quark"
| $D\ 1\ \rightarrow$ "d" | $D\ (-1)\ \rightarrow$ "dbar"
| $D\ 2\ \rightarrow$ "s" | $D\ (-2)\ \rightarrow$ "sbar"
| $D\ 3\ \rightarrow$ "b" | $D\ (-3)\ \rightarrow$ "bbar"
| $D\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣down␣type␣quark"
| $U\_K1\_L\ 1\ \rightarrow$ "uk1l" | $U\_K1\_L\ (-1)\ \rightarrow$ "uk1lbar"
| $U\_K1\_L\ 2\ \rightarrow$ "ck1l" | $U\_K1\_L\ (-2)\ \rightarrow$ "ck1lbar"
| $U\_K1\_L\ 3\ \rightarrow$ "tk1l" | $U\_K1\_L\ (-3)\ \rightarrow$ "tk1lbar"
| $U\_K1\_L\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣up␣type␣quark"
| $D\_K1\_L\ 1\ \rightarrow$ "dk1l" | $D\_K1\_L\ (-1)\ \rightarrow$ "dk1lbar"
| $D\_K1\_L\ 2\ \rightarrow$ "sk1l" | $D\_K1\_L\ (-2)\ \rightarrow$ "sk1lbar"
| $D\_K1\_L\ 3\ \rightarrow$ "bk1l" | $D\_K1\_L\ (-3)\ \rightarrow$ "bk1lbar"
| $D\_K1\_L\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣down␣type␣quark"
| $U\_K1\_R\ 1\ \rightarrow$ "uk1r" | $U\_K1\_R\ (-1)\ \rightarrow$ "uk1rbar"
| $U\_K1\_R\ 2\ \rightarrow$ "ck1r" | $U\_K1\_R\ (-2)\ \rightarrow$ "ck1rbar"
| $U\_K1\_R\ 3\ \rightarrow$ "tk1r" | $U\_K1\_R\ (-3)\ \rightarrow$ "tk1rbar"
| $U\_K1\_R\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣up␣type␣quark"
| $D\_K1\_R\ 1\ \rightarrow$ "dk1r" | $D\_K1\_R\ (-1)\ \rightarrow$ "dk1rbar"
| $D\_K1\_R\ 2\ \rightarrow$ "sk1r" | $D\_K1\_R\ (-2)\ \rightarrow$ "sk1rbar"
| $D\_K1\_R\ 3\ \rightarrow$ "bk1r" | $D\_K1\_R\ (-3)\ \rightarrow$ "bk1rbar"
| $D\_K1\_R\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣down␣type␣quark"
| $U\_K2\_L\ 1\ \rightarrow$ "uk2l" | $U\_K2\_L\ (-1)\ \rightarrow$ "uk2lbar"
| $U\_K2\_L\ 2\ \rightarrow$ "ck2l" | $U\_K2\_L\ (-2)\ \rightarrow$ "ck2lbar"
| $U\_K2\_L\ 3\ \rightarrow$ "tk2l" | $U\_K2\_L\ (-3)\ \rightarrow$ "tk2lbar"
| $U\_K2\_L\ \_\ \rightarrow$ *invalid_arg*
      "Modellib_BSM.UED.flavor_to_string:␣invalid␣up␣type␣quark"
| $D\_K2\_L\ 1\ \rightarrow$ "dk2l" | $D\_K2\_L\ (-1)\ \rightarrow$ "dk2lbar"
| $D\_K2\_L\ 2\ \rightarrow$ "sk2l" | $D\_K2\_L\ (-2)\ \rightarrow$ "sk2lbar"
| $D\_K2\_L\ 3\ \rightarrow$ "bk2l" | $D\_K2\_L\ (-3)\ \rightarrow$ "bk2lbar"
| $D\_K2\_L\ \_\ \rightarrow$ *invalid_arg*

```
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣down␣type␣quark"
    | U_K2_R 1  →  "uk2r" |  U_K2_R (−1)  →  "uk2rbar"
    | U_K2_R 2  →  "ck2r" |  U_K2_R (−2)  →  "ck2rbar"
    | U_K2_R 3  →  "tk2r" |  U_K2_R (−3)  →  "tk2rbar"
    | U_K2_R _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣up␣type␣quark"
    | D_K2_R 1  →  "dk2r" |  D_K2_R (−1)  →  "dk2rbar"
    | D_K2_R 2  →  "sk2r" |  D_K2_R (−2)  →  "sk2rbar"
    | D_K2_R 3  →  "bk2r" |  D_K2_R (−3)  →  "bk2rbar"
    | D_K2_R _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣down␣type␣quark"
    | L_K1_L 1  →  "ek1l-" |  L_K1_L (−1)  →  "ek1l+"
    | L_K1_L 2  →  "muk1l-" |  L_K1_L (−2)  →  "muk1l+"
    | L_K1_L 3  →  "tauk1l-" |  L_K1_L (−3)  →  "tauk1l+"
    | L_K1_L _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣lepton"
    | L_K1_R 1  →  "ek1r-" |  L_K1_R (−1)  →  "ek1r+"
    | L_K1_R 2  →  "muk1r-" |  L_K1_R (−2)  →  "muk1r+"
    | L_K1_R 3  →  "tauk1r-" |  L_K1_R (−3)  →  "tauk1r+"
    | L_K1_R _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣lepton"
    | L_K2_L 1  →  "ek2l-" |  L_K2_L (−1)  →  "ek2l+"
    | L_K2_L 2  →  "muk2l-" |  L_K2_L (−2)  →  "muk2l+"
    | L_K2_L 3  →  "tauk2l-" |  L_K2_L (−3)  →  "tauk2l+"
    | L_K2_L _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣lepton"
    | L_K2_R 1  →  "ek2r-" |  L_K2_R (−1)  →  "ek2r+"
    | L_K2_R 2  →  "muk2r-" |  L_K2_R (−2)  →  "muk2r+"
    | L_K2_R 3  →  "tauk2r-" |  L_K2_R (−3)  →  "tauk2r+"
    | L_K2_R _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣lepton"
    | N_K1 1  →  "nuek1" |  N_K1 (−1)  →  "nuek1bar"
    | N_K1 2  →  "numuk1" |  N_K1 (−2)  →  "numuk1bar"
    | N_K1 3  →  "nutauk1" |  N_K1 (−3)  →  "nutauk1bar"
    | N_K1 _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣neutrino"
    | N_K2 1  →  "nuek2" |  N_K2 (−1)  →  "nuek2bar"
    | N_K2 2  →  "numuk2" |  N_K2 (−2)  →  "numuk2bar"
    | N_K2 3  →  "nutauk2" |  N_K2 (−3)  →  "nutauk2bar"
    | N_K2 _  →  invalid_arg
            "Modellib_BSM.UED.flavor_to_string:␣invalid␣neutrino"
    end
| G f  →
    begin match f with
    | Gl  →  "g"
    | Ga  →  "A" |  Z  →  "Z"
    | Wp  →  "W+" |  Wm  →  "W-"
    | Gl_K1  →  "gk1" |  Gl_K2  →  "gk2"
    | B1  →  "b1" |  B2  →  "b2" |  Z1  →  "z1" |  Z2  →  "z2"
    | Wp1  →  "W1+" |  Wm1  →  "W1-"
```

```
              |  Wp2  →  "W2+" |  Wm2  →  "W2-"
              end
      |  O f  →
          begin match f with
          |  Phip  →  "phi+" |  Phim  →  "phi-" |  Phi0  →  "phi0"
          |  H  →  "H" |  H1up  →  "H1u+" |  H1um  →  "H1u-"
          |  H1dp  →  "H1d+" |  H1dm  →  "H1d-"
          |  H2up  →  "H2u+" |  H2um  →  "H2u-"
          |  H2dp  →  "H2d+" |  H2dm  →  "H2d-"
          |  Grav  →  "GG"
          end

let flavor_to_TeX  =  function
  |  M f  →
      begin match f with
      |  L 1  →  "e^-" |  L (−1)  →  "e^+"
      |  L 2  →  "\\mu^-" |  L (−2)  →  "\\mu^+"
      |  L 3  →  "\\tau^-" |  L (−3)  →  "\\tau^+"
      |  L _  →  invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣lepton"
      |  N 1  →  "\\nu_e" |  N (−1)  →  "\\bar{\\nu}_e"
      |  N 2  →  "\\nu_\\mu" |  N (−2)  →  "\\bar{\\nu}_\\mu"
      |  N 3  →  "\\nu_\\tau" |  N (−3)  →  "\\bar{\\nu}_\\tau"
      |  N _  →  invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣neutrino"
      |  U 1  →  "u" |  U (−1)  →  "\\bar{u}"
      |  U 2  →  "c" |  U (−2)  →  "\\bar{c}"
      |  U 3  →  "t" |  U (−3)  →  "\\bar{t}"
      |  U _  →  invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣up␣type␣quark"
      |  D 1  →  "d" |  D (−1)  →  "dbar"
      |  D 2  →  "s" |  D (−2)  →  "sbar"
      |  D 3  →  "b" |  D (−3)  →  "bbar"
      |  D _  →  invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣down␣type␣quark"
      |  U_K1_L 1 →  "u^\\prime_L" |  U_K1_L (−1)  →  "\\bar{u}^\\prime_L"
      |  U_K1_L 2 →  "c^\\prime_L" |  U_K1_L (−2)  →  "\\bar{c}^\\prime_L"
      |  U_K1_L 3 →  "t^\\prime_L" |  U_K1_L (−3)  →  "\\bar{t}^\\prime_L"
      |  U_K1_L _  →  invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣up␣type␣quark"
      |  D_K1_L 1 →  "d^\\prime_L" |  D_K1_L (−1)  →  "\\bar{d}^\\prime_L"
      |  D_K1_L 2 →  "s^\\prime_L" |  D_K1_L (−2)  →  "\\bar{s}^\\prime_L"
      |  D_K1_L 3 →  "b^\\prime_L" |  D_K1_L (−3)  →  "\\bar{b}^\\prime_L"
      |  D_K1_L _  →  invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣down␣type␣quark"
      |  U_K1_R 1 →  "u^\\prime_R" |  U_K1_R (−1)  →  "\\bar{u}^\\prime_R"
      |  U_K1_R 2 →  "c^\\prime_R" |  U_K1_R (−2)  →  "\\bar{c}^\\prime_R"
      |  U_K1_R 3 →  "t^\\prime_R" |  U_K1_R (−3)  →  "\\bar{t}^\\prime_R"
      |  U_K1_R _  →  invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣up␣type␣quark"
```

```
         | D_K1_R 1 → "d^\\prime_R" | D_K1_R (−1) → "\\bar{d}^\\prime_R"
         | D_K1_R 2 → "s^\\prime_R" | D_K1_R (−2) → "\\bar{s}^\\prime_R"
         | D_K1_R 3 → "b^\\prime_R" | D_K1_R (−3) → "\\bar{b}^\\prime_R"
         | D_K1_R _ → invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣down␣type␣quark"
         | U_K2_L 1 → "u^{\\prime\\prime}_L" | U_K2_L (−1) →
"\\bar{u}^{\\prime\\prime}_L"
         | U_K2_L 2 → "c^{\\prime\\prime}_L" | U_K2_L (−2) →
"\\bar{c}^{\\prime\\prime}_L"
         | U_K2_L 3 → "t^{\\prime\\prime}_L" | U_K2_L (−3) →
"\\bar{t}^{\\prime\\prime}_L"
         | U_K2_L _ → invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣up␣type␣quark"
         | D_K2_L 1 → "d^{\\prime\\prime}_L" | D_K2_L (−1) →
"\\bar{d}^{\\prime\\prime}_L"
         | D_K2_L 2 → "s^{\\prime\\prime}_L" | D_K2_L (−2) →
"\\bar{s}^{\\prime\\prime}_L"
         | D_K2_L 3 → "b^{\\prime\\prime}_L" | D_K2_L (−3) →
"\\bar{b}^{\\prime\\prime}_L"
         | D_K2_L _ → invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣down␣type␣quark"
         | U_K2_R 1 → "u^{\\prime\\prime}_R" | U_K2_R (−1) →
"\\bar{u}^{\\prime\\prime}_R"
         | U_K2_R 2 → "c^{\\prime\\prime}_R" | U_K2_R (−2) →
"\\bar{c}^{\\prime\\prime}_R"
         | U_K2_R 3 → "t^{\\prime\\prime}_R" | U_K2_R (−3) →
"\\bar{t}^{\\prime\\prime}_R"
         | U_K2_R _ → invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣up␣type␣quark"
         | D_K2_R 1 → "d^\\prime_R" | D_K2_R (−1) → "\\bar{d}^{\\prime\\prime}_R"
         | D_K2_R 2 → "s^\\prime_R" | D_K2_R (−2) → "\\bar{s}^{\\prime\\prime}_R"
         | D_K2_R 3 → "b^\\prime_R" | D_K2_R (−3) → "\\bar{b}^{\\prime\\prime}_R"
         | D_K2_R _ → invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣down␣type␣quark"
         | L_K1_L 1 → "e_L^{\\prime,,-}" | L_K1_L (−1) → "\\bar{e}_L^{\\prime,,+}"
         | L_K1_L 2 → "\\mu_L^{\\prime,,-}" | L_K1_L (−2) →
"\\bar{\\mu}_L^{{\\prime,,+}"
         | L_K1_L 3 → "\\tau_L^{\\prime,,-}" | L_K1_L (−3) →
"\\bar{\\tau}_L^{\\prime,,+}"
         | L_K1_L _ → invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣lepton"
         | L_K1_R 1 → "e_R^{\\prime,,-}" | L_K1_R (−1) →
"\\bar{e}_R^{\\prime,,+}"
         | L_K1_R 2 → "\\mu_R{\\prime,,-}" | L_K1_R (−2) →
"\\bar{\\mu}_R^{\\prime,,+}"
         | L_K1_R 3 → "\\tau_R{\\prime,,-}" | L_K1_R (−3) →
"\\bar{\\tau}_R{\\prime,,+}"
         | L_K1_R _ → invalid_arg
             "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣lepton"
         | L_K2_L 1 → "e^{\\prime\\prime,,-}_L" | L_K2_L (−1) →
```

```
"\\bar{e}_L^{\\prime\\prime,,+}"
        | L_K2_L 2 → "\\mu_L^{\\prime\\prime,,-}" | L_K2_L (−2) →
"\\bar{\\mu}_L^{\\prime\\prime,,+}"
        | L_K2_L 3 → "\\tau_L^{\\prime\\prime,,-}" | L_K2_L (−3) →
"\\bar{\\tau}_L^{\\prime\\prime,,+}"
        | L_K2_L _ → invalid_arg
            "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣lepton"
        | L_K2_R 1 → "e_R^{\\prime\\prime,,-}" | L_K2_R (−1) →
"\\bar{e}_R^{\\prime\\prime,,+}"
        | L_K2_R 2 → "\\mu_R^{\\prime\\prime,,-}" | L_K2_R (−2) →
"\\bar{\\mu}_R^{\\prime\\prime,,+}"
        | L_K2_R 3 → "\\tau_R{\\prime\\prime,,-}" | L_K2_R (−3) →
"\\bar{\\tau}_R^{\\prime\\prime,,+}"
        | L_K2_R _ → invalid_arg
            "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣lepton"
        | N_K1 1 → "\\nu_e^\\prime" | N_K1 (−1) → "\\bar{\\nu}_e^\\prime"
        | N_K1 2 → "\\nu_\\mu^\\prime" | N_K1 (−2) → "\\bar{\\nu}_\\mu^\\prime"
        | N_K1 3 → "\\nu_\\tau^\\prime" | N_K1 (−3) → "\\bar{\\nu}_\\tau^\\prime"
        | N_K1 _ → invalid_arg
            "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣neutrino"
        | N_K2 1 → "\\nu_e^{\\prime\\prime}" | N_K2 (−1) →
"\\bar{\\nu}_e^{\\prime\\prime}"
        | N_K2 2 → "\\nu_\\mu^{\\prime\\prime}" | N_K2 (−2) →
"\\bar{\\nu}_\\mu^{\\prime\\prime}"
        | N_K2 3 → "\\nu_\\tau^{\\prime\\prime}" | N_K2 (−3) →
"\\bar{\\nu}_\\tau^{\\prime\\prime}"
        | N_K2 _ → invalid_arg
            "Modellib_BSM.UED.flavor_to_TeX:␣invalid␣neutrino"
      end
  | G f →
      begin match f with
      | Gl → "g"
      | Ga → "\\gamma" | Z → "Z"
      | Wp → "W^+" | Wm → "W^-"
      | Gl_K1 → "g^\\prime" | Gl_K2 → "g^{\\prime\\prime}"
      | B1 → "B^\\prime" | B2 → "B^{\\prime\\prime}"
      | Z1 → "Z^\\prime" | Z2 → "Z^{\\prime\\prime}"
      | Wp1 → "W^{\\prime,,+}" | Wm1 → "W^{\\prime,,-}"
      | Wp2 → "W^{\\prime\\prime,,+}" | Wm2 → "W^{\\prime\\prime,,-}"
      end
  | O f →
      begin match f with
      | Phip → "\\phi^+" | Phim → "\\phi^-" | Phi0 →
"\\phi^0"
      | H → "H" | H1up → "H1u+" | H1um → "H1u-"
      | H1dp → "H1d+" | H1dm → "H1d-"
      | H2up → "H2u+" | H2um → "H2u-"
      | H2dp → "H2d+" | H2dm → "H2d-"
      | Grav → "G^\\prime"
      end
```

```
let flavor_symbol = function
  | M f →
      begin match f with
      | L n when n > 0 → "l" ^ string_of_int n
      | L n → "l" ^ string_of_int (abs n) ^ "b"
      | N n when n > 0 → "n" ^ string_of_int n
      | N n → "n" ^ string_of_int (abs n) ^ "b"
      | U n when n > 0 → "u" ^ string_of_int n
      | U n → "u" ^ string_of_int (abs n) ^ "b"
      | D n when n > 0 → "d" ^ string_of_int n
      | D n → "d" ^ string_of_int (abs n) ^ "b"
      | L_K1_L n when n > 0 → "lk1l" ^ string_of_int n
      | L_K1_L n → "lk1l" ^ string_of_int (abs n) ^ "b"
      | L_K1_R n when n > 0 → "lk1r" ^ string_of_int n
      | L_K1_R n → "lk1r" ^ string_of_int (abs n) ^ "b"
      | L_K2_L n when n > 0 → "lk2l" ^ string_of_int n
      | L_K2_L n → "lk2l" ^ string_of_int (abs n) ^ "b"
      | L_K2_R n when n > 0 → "lk2r" ^ string_of_int n
      | L_K2_R n → "lk2r" ^ string_of_int (abs n) ^ "b"
      | U_K1_L n when n > 0 → "uk1l" ^ string_of_int n
      | U_K1_L n → "uk1l" ^ string_of_int (abs n) ^ "b"
      | U_K1_R n when n > 0 → "uk1r" ^ string_of_int n
      | U_K1_R n → "uk1r" ^ string_of_int (abs n) ^ "b"
      | U_K2_L n when n > 0 → "uk2l" ^ string_of_int n
      | U_K2_L n → "uk2l" ^ string_of_int (abs n) ^ "b"
      | U_K2_R n when n > 0 → "uk2r" ^ string_of_int n
      | U_K2_R n → "uk2r" ^ string_of_int (abs n) ^ "b"
      | D_K1_L n when n > 0 → "dk1l" ^ string_of_int n
      | D_K1_L n → "dk1l" ^ string_of_int (abs n) ^ "b"
      | D_K1_R n when n > 0 → "dk1r" ^ string_of_int n
      | D_K1_R n → "dk1r" ^ string_of_int (abs n) ^ "b"
      | D_K2_L n when n > 0 → "dk2l" ^ string_of_int n
      | D_K2_L n → "dk2l" ^ string_of_int (abs n) ^ "b"
      | D_K2_R n when n > 0 → "dk2r" ^ string_of_int n
      | D_K2_R n → "dk2r" ^ string_of_int (abs n) ^ "b"
      | N_K1 n when n > 0 → "nk1" ^ string_of_int n
      | N_K1 n → "nk1" ^ string_of_int (abs n) ^ "b"
      | N_K2 n when n > 0 → "nk2" ^ string_of_int n
      | N_K2 n → "nk2" ^ string_of_int (abs n) ^ "b"
      end
  | G f →
      begin match f with
      | Gl → "gl"
      | Ga → "a" | Z → "z"
      | Wp → "wp" | Wm → "wm"
      | Gl_K1 → "gk1" | Gl_K2 → "gk2"
      | B1 → "b1" | B2 → "b2" | Z1 → "z1" | Z2 → "z2"
      | Wp1 → "wp1" | Wm1 → "wm1"
      | Wp2 → "wp2" | Wm2 → "wm2"
      end
```

```
    | O f →
        begin match f with
        | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
        | H → "h" | H1up → "h1up" | H1um → "h1um"
        | H1dp → "h1dp" | H1dm → "h1dm"
        | H2up → "h2up" | H2um → "h2um"
        | H2dp → "h2dp" | H2dm → "h2dm"
        | Grav → "gv"
        end

let pdg = function
  | M f →
        begin match f with
        | L n when n > 0 → 9 + 2 × n
        | L n → − 9 + 2 × n
        | N n when n > 0 → 10 + 2 × n
        | N n → − 10 + 2 × n
        | U n when n > 0 → 2 × n
        | U n → 2 × n
        | D n when n > 0 → − 1 + 2 × n
        | D n → 1 + 2 × n
        | U_K1_L n when n > 0 → 4000000 + 2 × n
        | U_K1_L n → − 4000000 + 2 × n
        | D_K1_L n when n > 0 → 3999999 + 2 × n
        | D_K1_L n → − 3999999 + 2 × n
        | U_K1_R n when n > 0 → 5000000 + 2 × n
        | U_K1_R n → − 5000000 + 2 × n
        | D_K1_R n when n > 0 → 4999999 + 2 × n
        | D_K1_R n → − 4999999 + 2 × n
        | U_K2_L n when n > 0 → 6000000 + 2 × n
        | U_K2_L n → − 6000000 + 2 × n
        | D_K2_L n when n > 0 → 5999999 + 2 × n
        | D_K2_L n → − 5999999 + 2 × n
        | U_K2_R n when n > 7000000 → 2 × n
        | U_K2_R n → − 7000000 + 2 × n
        | D_K2_R n when n > 0 → 6999999 + 2 × n
        | D_K2_R n → − 6999999 + 2 × n
        | L_K1_L n when n > 0 → 4000009 + 2 × n
        | L_K1_L n → − 4000009 + 2 × n
        | L_K1_R n when n > 0 → 5000009 + 2 × n
        | L_K1_R n → − 5000009 + 2 × n
        | L_K2_L n when n > 0 → 6000009 + 2 × n
        | L_K2_L n → − 6000009 + 2 × n
        | L_K2_R n when n > 0 → 7000009 + 2 × n
        | L_K2_R n → − 7000009 + 2 × n
        | N_K1 n when n > 0 → 4000010 + 2 × n
        | N_K1 n → − 4000010 + 2 × n
        | N_K2 n when n > 0 → 6000010 + 2 × n
        | N_K2 n → − 6000010 + 2 × n
        end
```

```
        | G f →
            begin match f with
            | Gl → 21
            | Ga → 22 | Z → 23
            | Wp → 24 | Wm → (−24)
            | Gl_K1 → 4000021 | Gl_K2 → 6000021
            | B1 → 4000022 | B2 → 6000022
            | Z1 → 4000023 | Z2 → 6000024
            | Wp1 → 4000024 | Wm1 → (−4000024)
            | Wp2 → 6000024 | Wm2 → (−6000024)
            end
        | O f →
            begin match f with
            | Phip | Phim → 27 | Phi0 → 26
            | H → 25 | H1up → 4000036 | H1um → (−4000036)
            | H1dp → 4000037 | H1dm → (−4000037)
            | H2up → 6000036 | H2um → (−6000036)
            | H2dp → 6000037 | H2dm → (−6000037)
            | Grav → 39
            end

    let mass_symbol f =
        "mass(" ^ string_of_int (abs (pdg f)) ^ ")"

    let width_symbol f =
        "width(" ^ string_of_int (abs (pdg f)) ^ ")"

    let constant_symbol = function
        | Unit → "unit" | Pi → "PI"
        | Alpha_QED → "alpha" | E → "e" | G_weak → "g" | Vev →
"vev"
        | Sin2thw → "sin2thw" | Sinthw → "sinthw" | Costhw →
"costhw"
        | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
        | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
        | G_NC_up → "gncup" | G_NC_down → "gncdwn"
        | G_CC → "gcc"
        | G_CCQ (n1,n2) → "gccq" ^ string_of_int n1 ^ string_of_int n2
        | I_Q_W → "iqw" | I_G_ZWW → "igzww"
        | I_Q_W_K → "iqwk" | I_G_ZWW_K1 → "igzwwk1"
        | I_G_ZWW_K2 → "igzwwk2" | I_G_ZWW_K3 → "igzwwk3"
        | G_WWWW → "gw4" | G_ZZWW → "gzzww"
        | G_AZWW → "gazww" | G_AAWW → "gaaww"
        | G_HWW → "ghww" | G_HZZ → "ghzz"
        | G_HHWW → "ghhww" | G_HHZZ → "ghhzz"
        | G_Htt → "ghtt" | G_Hbb → "ghbb"
        | G_Htautau → "ghtautau" | G_Hcc → "ghcc"
        | G_HGaZ → "ghgaz" | G_HGaGa → "ghgaga" | G_Hgg →
"ghgg"
        | G_H3 → "gh3" | G_H4 → "gh4"
        | G2 → "gs**2" | Gs → "gs" | I_Gs → "igs" | I_GsRt2 →
"igs/sqrt(2.0_default)"
```

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```
        | G22 → "gs**2/2.0_default"
        | G_Grav → "ggrav"
        | Mass f → "mass" ˆ flavor_symbol f
        | Width f → "width" ˆ flavor_symbol f
```

end

module *GravTest* (*Flags* : *BSM_flags*) =
  struct
    let *rcs* = *RCS.rename rcs_file* "Modellib_BSM.GravTest"
        ["Testing␣of␣Gravitinos"]

    open *Coupling*

    let *default_width* = *ref Timelike*
    let *use_fudged_width* = *ref* false

    let *options* = *Options.create*
      ["constant_width", *Arg.Unit* (fun () → *default_width* := *Constant*),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", *Arg.Set use_fudged_width*,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", *Arg.String* (fun *f* → *default_width* := *Custom f*),
        "use␣custom␣width";
        "cancel_widths", *Arg.Unit* (fun () → *default_width* := *Vanishing*),
        "use␣vanishing␣width"]

    type *matter_field* = *L* of *int* | *N* of *int* | *U* of *int* | *D* of *int* | *SL* of *int*
    type *gauge_boson* = *Ga* | *Wp* | *Wm* | *Z* | *Gl* | *Phino*
    type *other* = *Phip* | *Phim* | *Phi0* | *H* | *Grino*
    type *flavor* = *M* of *matter_field* | *G* of *gauge_boson* | *O* of *other*

    let *matter_field f* = *M f*
    let *gauge_boson f* = *G f*
    let *other f* = *O f*

    type *field* =
      | *Matter* of *matter_field*
      | *Gauge* of *gauge_boson*
      | *Other* of *other*

    let *field* = function
      | *M f* → *Matter f*
      | *G f* → *Gauge f*
      | *O f* → *Other f*

    type *gauge* = *unit*

    let *gauge_symbol* () =
      failwith "Modellib_BSM.SM.gauge_symbol:␣internal␣error"

    let *family n* = *List.map matter_field* [*L n; SL n; N n; U n; D n*]

    let *external_flavors* () =
      ["1st␣Generation", *ThoList.flatmap family* [1; − 1];
        "2nd␣Generation", *ThoList.flatmap family* [2; − 2];
        "3rd␣Generation", *ThoList.flatmap family* [3; − 3];
```

```
    "Gauge␣Bosons", List.map gauge_boson [Ga; Z; Wp; Wm; Gl; Phino];
    "Higgs", List.map other [H];
    "Gravitino", List.map other [Grino];
    "Goldstone␣Bosons", List.map other [Phip; Phim; Phi0] ]

let flavors () = ThoList.flatmap snd (external_flavors ())

let spinor n =
  if n ≥ 0 then
    Spinor
  else
    ConjSpinor

let lorentz = function
  | M f →
      begin match f with
      | L n → spinor n | N n → spinor n
      | U n → spinor n | D n → spinor n
      | SL _ → Scalar
      end
  | G f →
      begin match f with
      | Ga | Gl → Vector
      | Wp | Wm | Z → Massive_Vector
      | Phino → Majorana
      end
  | O f →
      begin match f with
      | Grino → Vectorspinor
      | _ → Scalar
      end

let color = function
  | M (U n) → Color.SUN (if n > 0 then 3 else -3)
  | M (D n) → Color.SUN (if n > 0 then 3 else -3)
  | G Gl → Color.AdjSUN 3
  | _ → Color.Singlet

let prop_spinor n =
  if n ≥ 0 then
    Prop_Spinor
  else
    Prop_ConjSpinor

let propagator = function
  | M f →
      begin match f with
      | L n → prop_spinor n | N n → prop_spinor n
      | U n → prop_spinor n | D n → prop_spinor n
      | SL n → Prop_Scalar
      end
  | G f →
      begin match f with
```

```
        |  Ga  |  Gl  →  Prop_Feynman
        |  Wp  |  Wm  |  Z  →  Prop_Unitarity
        |  Phino  →  Prop_Majorana
        end
    |  O f  →
        begin match f with
        |  Phip  |  Phim  |  Phi0  →  Only_Insertion
        |  H  →  Prop_Scalar
        |  Grino  →  Prop_Vectorspinor
        end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^\pm$ and top.

```
let width f  =
    if !use_fudged_width then
        match f with
        |  G Wp  |  G Wm  |  M (U 3)  |  M (U (−3))  |  O Grino  →  Fudged
        |  _  →  !default_width
    else
        !default_width

let goldstone  = function
    |  G f  →
        begin match f with
        |  Wp  →  Some (O Phip, Coupling.Const 1)
        |  Wm  →  Some (O Phim, Coupling.Const 1)
        |  Z  →  Some (O Phi0, Coupling.Const 1)
        |  _  →  None
        end
    |  _  →  None

let conjugate  = function
    |  M f  →
        M (begin match f with
        |  L n  →  L (−n)  |  N n  →  N (−n)
        |  U n  →  U (−n)  |  D n  →  D (−n)
        |  SL n  →  SL (−n)
        end)
    |  G f  →
        G (begin match f with
        |  Gl  →  Gl  |  Ga  →  Ga  |  Z  →  Z
        |  Wp  →  Wm  |  Wm  →  Wp  |  Phino  →  Phino
        end)
    |  O f  →
        O (begin match f with
        |  Phip  →  Phim  |  Phim  →  Phip  |  Phi0  →  Phi0
        |  H  →  H  |  Grino  →  Grino
        end)

let fermion  = function
    |  M f  →
        begin match f with
```

```
      |  L n  →  if n > 0 then 1 else -1
      |  N n  →  if n > 0 then 1 else -1
      |  U n  →  if n > 0 then 1 else -1
      |  D n  →  if n > 0 then 1 else -1
      |  SL _  →  0
      end
  |  G f  →
      begin match f with
      |  Gl  |  Ga  |  Z  |  Wp  |  Wm  →  0
      |  Phino  →  2
      end
  |  O f  →
      begin match f with
      |  Grino  →  2
      |  _  →  0
      end
```

module *Ch* = *Charges.QQ*

let (//) = *Algebra.Small_Rational.make*

```
let generation′ = function
  |  1  →  [ 1//1; 0//1; 0//1]
  |  2  →  [ 0//1; 1//1; 0//1]
  |  3  →  [ 0//1; 0//1; 1//1]
  | − 1  →  [−1//1; 0//1; 0//1]
  | − 2  →  [ 0//1; − 1//1; 0//1]
  | − 3  →  [ 0//1; 0//1; − 1//1]
  |  n  →  invalid_arg ("SM3.generation':␣" ^ string_of_int n)
```

```
let generation f =
  match f with
  |  M (L n  |  N n  |  U n  |  D n  |  SL n)  →  generation′ n
  |  G _  |  O _  →  [0//1; 0//1; 0//1]
```

```
let charge = function
  |  M f  →
      begin match f with
      |  L n  →  if n > 0 then -1//1 else 1//1
      |  SL n  →  if n > 0 then -1//1 else 1//1
      |  N n  →  0//1
      |  U n  →  if n > 0 then 2//3 else -2//3
      |  D n  →  if n > 0 then -1//3 else 1//3
      end
  |  G f  →
      begin match f with
      |  Gl  |  Ga  |  Z  |  Phino  →  0//1
      |  Wp  →  1//1
      |  Wm  →  − 1//1
      end
  |  O f  →
      begin match f with
      |  H  |  Phi0  |  Grino  →  0//1
```

```
        | Phip → 1//1
        | Phim → − 1//1
        end

let lepton = function
  | M f →
      begin match f with
      | L n | N n | SL n → if n > 0 then 1//1 else -1//1
      | U _ | D _ → 0//1
      end
  | G _ | O _ → 0//1

let baryon = function
  | M f →
      begin match f with
      | L _ | N _ | SL _ → 0//1
      | U n | D n → if n > 0 then 1//1 else -1//1
      end
  | G _ | O _ → 0//1

let charges f =
  [ charge f; lepton f; baryon f] @ generation f

type constant =
  | Unit | Pi | Alpha_QED | Sin2thw
  | Sinthw | Costhw | E | G_weak | Vev
  | Q_lepton | Q_up | Q_down | G_CC | G_CCQ of int×int
  | G_NC_neutrino | G_NC_lepton | G_NC_up | G_NC_down
  | I_Q_W | I_G_ZWW
  | G_WWWW | G_ZZWW | G_AZWW | G_AAWW
  | G_HWW | G_HHWW | G_HZZ | G_HHZZ
  | G_Htt | G_Hbb | G_Hcc | G_Htautau | G_H3 | G_H4
  | G_HGaZ | G_HGaGa | G_Hgg
  | G_strong | G_Grav
  | Mass of flavor | Width of flavor
```

Two integer counters for the QCD and EW order of the couplings.

```
type orders = int × int

let orders = function
  | _ → (0, 0)

let input_parameters =
  []

let derived_parameters =
  []

let g_over_2_costh =
  Quot (Neg (Atom G_weak), Prod [Const 2; Atom Costhw])

let nc_coupling c t3 q =
  (Real_Array c,
   [Prod [g_over_2_costh; Diff (t3, Prod [Const 2; q; Atom Sin2thw])];
     Prod [g_over_2_costh; t3]])
```

let *half* = *Quot* (*Const* 1, *Const* 2)

let *derived_parameter_arrays* =
  [*nc_coupling* *G_NC_neutrino* *half* (*Const* 0);
    nc_coupling *G_NC_lepton* (*Neg half*) (*Const* (−1));
    nc_coupling *G_NC_up* *half* (*Quot* (*Const* 2, *Const* 3));
    nc_coupling *G_NC_down* (*Neg half*) (*Quot* (*Const* (−1), *Const* 3))]

let *parameters* () =
  { *input* = *input_parameters*;
    derived = *derived_parameters*;
    derived_arrays = *derived_parameter_arrays* }

module *F* = *Modeltools.Fusions* (struct
  type *f* = *flavor*
  type *c* = *constant*
  let *compare* = *compare*
  let *conjugate* = *conjugate*
end)

let *mgm* ((*m1*, *g*, *m2*), *fbf*, *c*) = ((*M m1*, *G g*, *M m2*), *fbf*, *c*)
let *mom* ((*m1*, *o*, *m2*), *fbf*, *c*) = ((*M m1*, *O o*, *M m2*), *fbf*, *c*)

let *electromagnetic_currents* *n* =
  List.map *mgm*
    [((*L* (−*n*), *Ga*, *L n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_lepton*);
      ((*U* (−*n*), *Ga*, *U n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
      ((*D* (−*n*), *Ga*, *D n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

let *neutral_currents* *n* =
  List.map *mgm*
    [((*L* (−*n*), *Z*, *L n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_lepton*);
      ((*N* (−*n*), *Z*, *N n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_neutrino*);
      ((*U* (−*n*), *Z*, *U n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_up*);
      ((*D* (−*n*), *Z*, *D n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_down*)]

let *charged_currents* *n* =
  List.map *mgm*
    [((*L* (−*n*), *Wm*, *N n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
      ((*N* (−*n*), *Wp*, *L n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
      ((*D* (−*n*), *Wm*, *U n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
      ((*U* (−*n*), *Wp*, *D n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*)]

let *yukawa* =
  List.map *mom*
    [((*U* (−3), *H*, *U* 3), *FBF* (1, *Psibar*, *S*, *Psi*), *G_Htt*);
      ((*D* (−3), *H*, *D* 3), *FBF* (1, *Psibar*, *S*, *Psi*), *G_Hbb*);
      ((*U* (−2), *H*, *U* 2), *FBF* (1, *Psibar*, *S*, *Psi*), *G_Hcc*);
      ((*L* (−3), *H*, *L* 3), *FBF* (1, *Psibar*, *S*, *Psi*), *G_Htautau*)]

let *tgc* ((*g1*, *g2*, *g3*), *t*, *c*) = ((*G g1*, *G g2*, *G g3*), *t*, *c*)

let *standard_triple_gauge* =
  List.map *tgc*
    [((*Ga*, *Wm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_Q_W*);

$\qquad$ $((Z,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWW)\ ]$

let *triple_gauge* =
$\qquad$ *standard_triple_gauge*

let *qgc* $((g1,\ g2,\ g3,\ g4),\ t,\ c)\ =\ ((G\ g1,\ G\ g2,\ G\ g3,\ G\ g4),\ t,\ c)$

let *gauge4* $=\ Vector4\ [(2,\ C\_13\_42);\ (-1,\ C\_12\_34);\ (-1,\ C\_14\_23)]$
let *minus_gauge4* $=\ Vector4\ [(-2,\ C\_13\_42);\ (1,\ C\_12\_34);\ (1,\ C\_14\_23)]$
let *standard_quartic_gauge* =
$\quad$ *List.map qgc*
$\qquad$ $[\ (Wm,\ Wp,\ Wm,\ Wp),\ gauge4,\ G\_WWWW;$
$\qquad\quad (Wm,\ Z,\ Wp,\ Z),\ minus\_gauge4,\ G\_ZZWW;$
$\qquad\quad (Wm,\ Z,\ Wp,\ Ga),\ minus\_gauge4,\ G\_AZWW;$
$\qquad\quad (Wm,\ Ga,\ Wp,\ Ga),\ minus\_gauge4,\ G\_AAWW\ ]$

let *quartic_gauge* =
$\quad$ *standard_quartic_gauge*

let *standard_gauge_higgs* =
$\quad$ $[\ ((O\ H,\ G\ Wp,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ G\_HWW);$
$\quad\ ((O\ H,\ G\ Z,\ G\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_HZZ)\ ]$

let *standard_gauge_higgs4* =
$\quad$ $[\ (O\ H,\ O\ H,\ G\ Wp,\ G\ Wm),\ Scalar2\_Vector2\ 1,\ G\_HHWW;$
$\quad\ (O\ H,\ O\ H,\ G\ Z,\ G\ Z),\ Scalar2\_Vector2\ 1,\ G\_HHZZ\ ]$

let *standard_higgs* =
$\quad$ $[\ (O\ H,\ O\ H,\ O\ H),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ ]$

let *standard_higgs4* =
$\quad$ $[\ (O\ H,\ O\ H,\ O\ H,\ O\ H),\ Scalar4\ 1,\ G\_H4\ ]$

let *anomalous_gauge_higgs* =
$\quad$ $[\,]$

let *anomalous_gauge_higgs4* =
$\quad$ $[\,]$

let *anomalous_higgs* =
$\quad$ $[\,]$

let *anomaly_higgs* =
$\quad$ $[\ (O\ H,\ G\ Ga,\ G\ Ga),\ Dim5\_Scalar\_Gauge2\ 1,\ G\_HGaGa;$
$\quad\ (O\ H,\ G\ Ga,\ G\ Z),\ Dim5\_Scalar\_Gauge2\ 1,\ G\_HGaZ;$
$\quad\ (O\ H,\ G\ Gl,\ G\ Gl),\ Dim5\_Scalar\_Gauge2\ 1,\ G\_Hgg\ ]$

let *gravitino_coup n* =
$\quad$ $[\ (O\ Grino,\ M\ (SL\ (-n)),\ M\ (L\ n)),\ GBG\ (1,\ Gravbar,\ POT,\ Psi),\ G\_Grav;$
$\quad\ (M\ (L\ (-n)),\ M\ (SL\ n),\ O\ Grino),\ GBG\ (1,\ Psibar,\ POT,\ Grav),\ G\_Grav]$

let *gravitino_gauge* =
$\quad$ $[\ (O\ Grino,\ G\ Ga,\ G\ Phino),\ GBG\ (1,\ Gravbar,\ V,\ Chi),\ G\_Grav\ ]$

let *anomalous_higgs4* =
$\quad$ $[\,]$

let *gauge_higgs* =

$standard\_gauge\_higgs$

let $gauge\_higgs4$ =
$standard\_gauge\_higgs4$

let $higgs$ =
$standard\_higgs$

let $higgs4$ =
$standard\_higgs4$

let $goldstone\_vertices$ =
$[((O\ Phi0,\ G\ Wm,\ G\ Wp),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW);$
$((O\ Phip,\ G\ Ga,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W);$
$((O\ Phip,\ G\ Z,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW);$
$((O\ Phim,\ G\ Wp,\ G\ Ga),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W);$
$((O\ Phim,\ G\ Wp,\ G\ Z),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)]$

let $vertices3$ =
$(ThoList.flatmap\ electromagnetic\_currents\ [1; 2; 3]\ @$
$ThoList.flatmap\ neutral\_currents\ [1; 2; 3]\ @$
$ThoList.flatmap\ charged\_currents\ [1; 2; 3]\ @$
$ThoList.flatmap\ gravitino\_coup\ [1; 2; 3]\ @$
$gravitino\_gauge\ @$
$yukawa\ @\ triple\_gauge\ @$
$gauge\_higgs\ @\ higgs\ @\ anomaly\_higgs$
$@\ goldstone\_vertices)$

let $vertices4$ =
$quartic\_gauge\ @\ gauge\_higgs4\ @\ higgs4$

let $vertices$ () = $(vertices3,\ vertices4,\ [])$

For efficiency, make sure that $F.of\_vertices\ vertices$ is evaluated only once.

let $table$ = $F.of\_vertices\ (vertices\ ())$
let $fuse2$ = $F.fuse2\ table$
let $fuse3$ = $F.fuse3\ table$
let $fuse$ = $F.fuse\ table$
let max_degree () = 4

let $flavor\_of\_string$ = function
| "e-" $\to$ $M\ (L\ 1)$ | "e+" $\to$ $M\ (L\ (-1))$
| "mu-" $\to$ $M\ (L\ 2)$ | "mu+" $\to$ $M\ (L\ (-2))$
| "tau-" $\to$ $M\ (L\ 3)$ | "tau+" $\to$ $M\ (L\ (-3))$
| "se-" $\to$ $M\ (SL\ 1)$ | "se+" $\to$ $M\ (SL\ (-1))$
| "smu-" $\to$ $M\ (SL\ 2)$ | "smu+" $\to$ $M\ (SL\ (-2))$
| "stau-" $\to$ $M\ (SL\ 3)$ | "stau+" $\to$ $M\ (SL\ (-3))$
| "nue" $\to$ $M\ (N\ 1)$ | "nuebar" $\to$ $M\ (N\ (-1))$
| "numu" $\to$ $M\ (N\ 2)$ | "numubar" $\to$ $M\ (N\ (-2))$
| "nutau" $\to$ $M\ (N\ 3)$ | "nutaubar" $\to$ $M\ (N\ (-3))$
| "u" $\to$ $M\ (U\ 1)$ | "ubar" $\to$ $M\ (U\ (-1))$
| "c" $\to$ $M\ (U\ 2)$ | "cbar" $\to$ $M\ (U\ (-2))$
| "t" $\to$ $M\ (U\ 3)$ | "tbar" $\to$ $M\ (U\ (-3))$
| "d" $\to$ $M\ (D\ 1)$ | "dbar" $\to$ $M\ (D\ (-1))$

```
        | "s" →  M (D 2) | "sbar" →  M (D (−2))
        | "b" →  M (D 3) | "bbar" →  M (D (−3))
        | "g" | "gl" →  G Gl
        | "A" →  G Ga | "Z" | "Z0" →  G Z
        | "W+" →  G Wp | "W-" →  G Wm
        | "H" →  O H
        | "GG" →  O Grino
        | "phino" | "Phino" →  G Phino
        | _ →  invalid_arg "Modellib_BSM.GravTest.flavor_of_string"

let flavor_to_string  = function
  | M f  →
      begin match f with
      | L 1 →  "e-" | L (−1) →  "e+"
      | L 2 →  "mu-" | L (−2) →  "mu+"
      | L 3 →  "tau-" | L (−3) →  "tau+"
      | L _ →  invalid_arg
              "Modellib_BSM.GravTest.flavor_to_string:␣invalid␣lepton"
      | SL 1 →  "se-" | SL (−1) →  "se+"
      | SL 2 →  "smu-" | SL (−2) →  "smu+"
      | SL 3 →  "stau-" | SL (−3) →  "stau+"
      | SL _ →  invalid_arg
              "Modellib_BSM.GravTest.flavor_to_string:␣invalid␣slepton"
      | N 1 →  "nue" | N (−1) →  "nuebar"
      | N 2 →  "numu" | N (−2) →  "numubar"
      | N 3 →  "nutau" | N (−3) →  "nutaubar"
      | N _ →  invalid_arg
              "Modellib_BSM.GravTest.flavor_to_string:␣invalid␣neutrino"
      | U 1 →  "u" | U (−1) →  "ubar"
      | U 2 →  "c" | U (−2) →  "cbar"
      | U 3 →  "t" | U (−3) →  "tbar"
      | U _ →  invalid_arg
              "Modellib_BSM.SM.flavor_to_string:␣invalid␣up␣type␣quark"
      | D 1 →  "d" | D (−1) →  "dbar"
      | D 2 →  "s" | D (−2) →  "sbar"
      | D 3 →  "b" | D (−3) →  "bbar"
      | D _ →  invalid_arg
              "Modellib_BSM.GravTest.flavor_to_string:␣invalid␣down␣type␣quark"
      end
  | G f  →
      begin match f with
      | Gl →  "g"
      | Ga →  "A" | Z →  "Z"
      | Wp →  "W+" | Wm →  "W-"
      | Phino →  "phino"
      end
  | O f  →
      begin match f with
      | Phip →  "phi+" | Phim →  "phi-" | Phi0 →  "phi0"
      | H →  "H" | Grino →  "GG"
```

```
                end
      let flavor_to_TeX  =  function
        |  M f  →
              begin match f with
              |  L 1  →  "e^-" |  L (−1)  →  "e^+"
              |  L 2  →  "\\mu^-" |  L (−2)  →  "\\mu^+"
              |  L 3  →  "\\tau^-" |  L (−3)  →  "\\tau^+"
              |  L _  →  invalid_arg
                     "Modellib_BSM.GravTest.flavor_to_TeX:␣invalid␣lepton"
              |  SL 1  →  "\\tilde{e}^-" |  SL (−1)  →  "\\tilde{e}^+"
              |  SL 2  →  "\\tilde{\\mu}^-" |  SL (−2)  →  "\\tilde{\\mu}^+"
              |  SL 3  →  "\\tilde{\\tau}^-" |  SL (−3)  →  "\\tilde{\\tau}^+"
              |  SL _  →  invalid_arg
                     "Modellib_BSM.GravTest.flavor_to_TeX:␣invalid␣slepton"
              |  N 1  →  "\\nu_e" |  N (−1)  →  "\\bar{\\nu}_e"
              |  N 2  →  "\\nu_\\mu" |  N (−2)  →  "\\bar{\\nu}_\\mu"
              |  N 3  →  "\\nu_\\tau" |  N (−3)  →  "\\bar{\\nu}_\\tau"
              |  N _  →  invalid_arg
                     "Modellib_BSM.GravTest.flavor_to_TeX:␣invalid␣neutrino"
              |  U 1  →  "u" |  U (−1)  →  "\\bar{u}"
              |  U 2  →  "c" |  U (−2)  →  "\\bar{c}"
              |  U 3  →  "t" |  U (−3)  →  "\\bar{t}"
              |  U _  →  invalid_arg
                     "Modellib_BSM.SM.flavor_to_TeX:␣invalid␣up␣type␣quark"
              |  D 1  →  "d" |  D (−1)  →  "\\bar{d}"
              |  D 2  →  "s" |  D (−2)  →  "\\bar{s}"
              |  D 3  →  "b" |  D (−3)  →  "\\bar{b}"
              |  D _  →  invalid_arg
                     "Modellib_BSM.GravTest.flavor_to_TeX:␣invalid␣down␣type␣quark"
              end
        |  G f  →
              begin match f with
              |  Gl  →  "g"
              |  Ga  →  "\\gamma" |  Z  →  "Z"
              |  Wp  →  "W^+" |  Wm  →  "W^-"
              |  Phino  →  "\\tilde{\\phi}"
              end
        |  O f  →
              begin match f with
              |  Phip  →  "\\phi^+" |  Phim  →  "\\phi^-" |  Phi0  →
"\\phi^0"
              |  H  →  "H" |  Grino  →  "\\tilde{G}"
              end

      let flavor_symbol  =  function
        |  M f  →
              begin match f with
              |  L n when n > 0  →  "l" ^ string_of_int n
              |  L n  →  "l" ^ string_of_int (abs n) ^ "b"
              |  SL n when n > 0  →  "sl" ^ string_of_int n
```

```
            | SL n → "sl" ˆ string_of_int (abs n) ˆ "b"
            | N n when n > 0 → "n" ˆ string_of_int n
            | N n → "n" ˆ string_of_int (abs n) ˆ "b"
            | U n when n > 0 → "u" ˆ string_of_int n
            | U n → "u" ˆ string_of_int (abs n) ˆ "b"
            | D n when n > 0 → "d" ˆ string_of_int n
            | D n → "d" ˆ string_of_int (abs n) ˆ "b"
            end
      | G f →
            begin match f with
            | Gl → "gl"
            | Ga → "a" | Z → "z"
            | Wp → "wp" | Wm → "wm"
            | Phino → "phino"
            end
      | O f →
            begin match f with
            | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
            | H → "h" | Grino → "gv"
            end

let pdg = function
  | M f →
            begin match f with
            | L n when n > 0 → 9 + 2 × n
            | L n → − 9 + 2 × n
            | SL n when n > 0 → 39 + 2 × n
            | SL n → − 39 + 2 × n
            | N n when n > 0 → 10 + 2 × n
            | N n → − 10 + 2 × n
            | U n when n > 0 → 2 × n
            | U n → 2 × n
            | D n when n > 0 → − 1 + 2 × n
            | D n → 1 + 2 × n
            end
      | G f →
            begin match f with
            | Gl → 21
            | Ga → 22 | Z → 23
            | Wp → 24 | Wm → (−24)
            | Phino → 46
            end
      | O f →
            begin match f with
            | Phip | Phim → 27 | Phi0 → 26
            | H → 25 | Grino → 39
            end

let mass_symbol f =
  "mass(" ˆ string_of_int (abs (pdg f)) ˆ ")"
```

  let *width_symbol f* =
   "width(" ^ *string_of_int* (*abs* (*pdg f*)) ^ ")"

  let *constant_symbol* = function
   | *Unit* → "unit" | *Pi* → "PI"
   | *Alpha_QED* → "alpha" | *E* → "e" | *G_weak* → "g" | *Vev* →
"vev"
   | *Sin2thw* → "sin2thw" | *Sinthw* → "sinthw" | *Costhw* →
"costhw"
   | *Q_lepton* → "qlep" | *Q_up* → "qup" | *Q_down* → "qdwn"
   | *G_NC_lepton* → "gnclep" | *G_NC_neutrino* → "gncneu"
   | *G_NC_up* → "gncup" | *G_NC_down* → "gncdwn"
   | *G_CC* → "gcc"
   | *G_CCQ* (*n1, n2*) → "gccq" ^ *string_of_int n1* ^ *string_of_int n2*
   | *I_Q_W* → "iqw" | *I_G_ZWW* → "igzww"
   | *G_WWWW* → "gw4" | *G_ZZWW* → "gzzww"
   | *G_AZWW* → "gazww" | *G_AAWW* → "gaaww"
   | *G_HWW* → "ghww" | *G_HZZ* → "ghzz"
   | *G_HHWW* → "ghhww" | *G_HHZZ* → "ghhzz"
   | *G_Htt* → "ghtt" | *G_Hbb* → "ghbb"
   | *G_Htautau* → "ghtautau" | *G_Hcc* → "ghcc"
   | *G_HGaZ* → "ghgaz" | *G_HGaGa* → "ghgaga" | *G_Hgg* →
"ghgg"
   | *G_H3* → "gh3" | *G_H4* → "gh4"
   | *G_strong* → "gs" | *G_Grav* → "ggrav"
   | *Mass f* → "mass" ^ *flavor_symbol f*
   | *Width f* → "width" ^ *flavor_symbol f*

 end
module *Template* (*Flags* : *BSM_flags*) =
 struct
  let *rcs* = *RCS.rename rcs_file* "Modellib_BSM.Template"
   ["Template␣for␣user-defined␣BSM␣model"]

  open *Coupling*

  let *default_width* = *ref Timelike*
  let *use_fudged_width* = *ref* false

  let *options* = *Options.create*
   ["constant_width", *Arg.Unit* (fun () → *default_width* := *Constant*),
    "use␣constant␣width␣(also␣in␣t-channel)";
    "fudged_width", *Arg.Set use_fudged_width*,
    "use␣fudge␣factor␣for␣charge␣particle␣width";
    "custom_width", *Arg.String* (fun *f* → *default_width* := *Custom f*),
    "use␣custom␣width";
    "cancel_widths", *Arg.Unit* (fun () → *default_width* := *Vanishing*),
    "use␣vanishing␣width"]

  type *matter_field* = *L* of *int* | *N* of *int* | *U* of *int* | *D* of *int*
  type *gauge_boson* = *Ga* | *Wp* | *Wm* | *Z* | *Gl*
  type *other* = *Phip* | *Phim* | *Phi0* | *H*
  type *flavor* = *M* of *matter_field* | *G* of *gauge_boson* | *O* of *other*

```
let matter_field f  =  M f
let gauge_boson f  =  G f
let other f  =  O f

type field  =
   |  Matter of matter_field
   |  Gauge of gauge_boson
   |  Other of other

let field  = function
   |  M f  →  Matter f
   |  G f  →  Gauge f
   |  O f  →  Other f

type gauge  =  unit

let gauge_symbol ()  =
   failwith "Modellib_BSM.Template.gauge_symbol:␣internal␣error"

let family n  =  List.map matter_field [ L n;  N n;  U n;  D n ]

let external_flavors ()  =
   [ "1st␣Generation", ThoList.flatmap family [1;  − 1];
      "2nd␣Generation", ThoList.flatmap family [2;  − 2];
      "3rd␣Generation", ThoList.flatmap family [3;  − 3];
      "Gauge␣Bosons", List.map gauge_boson [Ga;  Z;  Wp;  Wm;  Gl];
      "Higgs", List.map other [H];
      "Goldstone␣Bosons", List.map other [Phip;  Phim;  Phi0] ]

let flavors ()  =  ThoList.flatmap snd (external_flavors ())

let spinor n  =
   if n  ≥  0 then
      Spinor
   else
      ConjSpinor

let lorentz  = function
   |  M f  →
         begin match f with
         |  L n  →  spinor n  |  N n  →  spinor n
         |  U n  →  spinor n  |  D n  →  spinor n
         end
   |  G f  →
         begin match f with
         |  Ga  |  Gl  →  Vector
         |  Wp  |  Wm  |  Z  →  Massive_Vector
         end
   |  O f  →  Scalar

let color  = function
   |  M (U n)  →  Color.SUN (if n  >  0 then 3 else -3)
   |  M (D n)  →  Color.SUN (if n  >  0 then 3 else -3)
   |  G Gl  →  Color.AdjSUN 3
   |  _  →  Color.Singlet
```

```
let prop_spinor n  =
  if n  ≥  0 then
    Prop_Spinor
  else
    Prop_ConjSpinor

let propagator  = function
  | M f  →
      begin match f with
      |  L n  →  prop_spinor n  |  N n  →  prop_spinor n
      |  U n  →  prop_spinor n  |  D n  →  prop_spinor n
      end
  | G f  →
      begin match f with
      |  Ga  |  Gl  →  Prop_Feynman
      |  Wp  |  Wm  |  Z  →  Prop_Unitarity
      end
  | O f  →
      begin match f with
      |  Phip  |  Phim  |  Phi0  →  Only_Insertion
      |  H  →  Prop_Scalar
      end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^\pm$ and top.

```
let width f  =
  if !use_fudged_width then
    match f with
    |  G Wp  |  G Wm  |  M (U 3)  |  M (U (−3))  →  Fudged
    |  _  →  !default_width
  else
    !default_width

let goldstone  = function
  | G f  →
      begin match f with
      |  Wp  →  Some (O Phip, Coupling.Const 1)
      |  Wm  →  Some (O Phim,  Coupling.Const 1)
      |  Z  →  Some (O Phi0, Coupling.Const 1)
      |  _  →  None
      end
  | _  →  None

let conjugate  = function
  | M f  →
      M (begin match f with
      |  L n  →  L (−n)  |  N n  →  N (−n)
      |  U n  →  U (−n)  |  D n  →  D (−n)
      end)
  | G f  →
      G (begin match f with
      |  Gl  →  Gl  |  Ga  →  Ga  |  Z  →  Z
```

```
          | Wp → Wm | Wm → Wp
          end)
    | O f →
        O (begin match f with
        | Phip → Phim | Phim → Phip | Phi0 → Phi0
        | H → H
        end)

let fermion = function
  | M f →
      begin match f with
      | L n → if n > 0 then 1 else -1
      | N n → if n > 0 then 1 else -1
      | U n → if n > 0 then 1 else -1
      | D n → if n > 0 then 1 else -1
      end
  | G f →
      begin match f with
      | Gl | Ga | Z | Wp | Wm → 0
      end
  | O _ → 0

module Ch = Charges.QQ

let ( // ) = Algebra.Small_Rational.make

let generation' = function
  | 1 → [ 1//1; 0//1; 0//1]
  | 2 → [ 0//1; 1//1; 0//1]
  | 3 → [ 0//1; 0//1; 1//1]
  | − 1 → [−1//1; 0//1; 0//1]
  | − 2 → [ 0//1; − 1//1; 0//1]
  | − 3 → [ 0//1; 0//1; − 1//1]
  | n → invalid_arg ("Template.generation':␣" ^ string_of_int n)

let generation f =
  match f with
  | M (L n | N n | U n | D n) → generation' n
  | G _ | O _ → [0//1; 0//1; 0//1]

let charge = function
  | M f →
      begin match f with
      | L n → if n > 0 then -1//1 else 1//1
      | N n → 0//1
      | U n → if n > 0 then 2//3 else -2//3
      | D n → if n > 0 then -1//3 else 1//3
      end
  | G f →
      begin match f with
      | Gl | Ga | Z → 0//1
      | Wp → 1//1
      | Wm → − 1//1
```

```
        end
      | O f →
          begin match f with
          | H | Phi0 → 0//1
          | Phip → 1//1
          | Phim → − 1//1
          end

    let lepton = function
      | M f →
          begin match f with
          | L n | N n → if n > 0 then 1//1 else -1//1
          | U _ | D _ → 0//1
          end
      | G _ | O _ → 0//1

    let baryon = function
      | M f →
          begin match f with
          | L _ | N _ → 0//1
          | U n | D n → if n > 0 then 1//1 else -1//1
          end
      | G _ | O _ → 0//1

    let charges f =
      [ charge f; lepton f; baryon f] @ generation f

    type constant =
      | Unit | Pi | Alpha_QED | Sin2thw
      | Sinthw | Costhw | E | G_weak | Vev
      | Q_lepton | Q_up | Q_down | G_CC
      | G_NC_neutrino | G_NC_lepton | G_NC_up | G_NC_down
      | I_Q_W | I_G_ZWW
      | G_WWWW | G_ZZWW | G_AZWW | G_AAWW
      | G_HWW | G_HHWW | G_HZZ | G_HHZZ
      | G_Htt | G_Hbb | G_Hcc | G_Hmm | G_Htautau | G_H3 |
G_H4
      | G_HGaZ | G_HGaGa | G_Hgg
      | Gs | I_Gs | G2
      | Mass of flavor | Width of flavor
```

Two integer counters for the QCD and EW order of the couplings.

```
    type orders = int × int

    let orders = function
      | _ → (0, 0)

    let input_parameters = []

    let derived_parameters = []

    let g_over_2_costh =
      Quot (Neg (Atom G_weak), Prod [Const 2; Atom Costhw])
```

```
let nc_coupling c t3 q =
  (Real_Array c,
   [Prod [g_over_2_costh; Diff (t3, Prod [Const 2; q; Atom Sin2thw])];
    Prod [g_over_2_costh; t3]])

let half = Quot (Const 1, Const 2)

let derived_parameter_arrays =
  [ nc_coupling G_NC_neutrino half (Const 0);
    nc_coupling G_NC_lepton (Neg half) (Const (−1));
    nc_coupling G_NC_up half (Quot (Const 2, Const 3));
    nc_coupling G_NC_down (Neg half) (Quot (Const (−1), Const 3)) ]

let parameters () =
  { input = input_parameters;
    derived = derived_parameters;
    derived_arrays = derived_parameter_arrays }

module F = Modeltools.Fusions (struct
  type f = flavor
  type c = constant
  let compare = compare
  let conjugate = conjugate
end)

let mgm ((m1, g, m2), fbf, c) = ((M m1, G g, M m2), fbf, c)

let electromagnetic_currents n =
  List.map mgm
    [ ((L (−n), Ga, L n), FBF (1, Psibar, V, Psi), Q_lepton);
      ((U (−n), Ga, U n), FBF (1, Psibar, V, Psi), Q_up);
      ((D (−n), Ga, D n), FBF (1, Psibar, V, Psi), Q_down) ]

let color_currents n =
  List.map mgm
    [ ((U (−n), Gl, U n), FBF ((−1), Psibar, V, Psi), Gs);
      ((D (−n), Gl, D n), FBF ((−1), Psibar, V, Psi), Gs) ]

let neutral_currents n =
  List.map mgm
    [ ((L (−n), Z, L n), FBF (1, Psibar, VA, Psi), G_NC_lepton);
      ((N (−n), Z, N n), FBF (1, Psibar, VA, Psi), G_NC_neutrino);
      ((U (−n), Z, U n), FBF (1, Psibar, VA, Psi), G_NC_up);
      ((D (−n), Z, D n), FBF (1, Psibar, VA, Psi), G_NC_down) ]

let charged_currents n =
  List.map mgm
    [ ((L (−n), Wm, N n), FBF (1, Psibar, VL, Psi), G_CC);
      ((N (−n), Wp, L n), FBF (1, Psibar, VL, Psi), G_CC);
      ((D (−n), Wm, U n), FBF (1, Psibar, VL, Psi), G_CC);
      ((U (−n), Wp, D n), FBF (1, Psibar, VL, Psi), G_CC) ]

let yukawa =
  [ ((M (U (−3)), O H, M (U 3)), FBF (1, Psibar, S, Psi), G_Htt);
    ((M (D (−3)), O H, M (D 3)), FBF (1, Psibar, S, Psi), G_Hbb);
```

$((M\ (U\ (-2)),\ O\ H,\ M\ (U\ 2)),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Hcc);$
$((M\ (L\ (-2)),\ O\ H,\ M\ (L\ 2)),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Hmm);$
$((M\ (L\ (-3)),\ O\ H,\ M\ (L\ 3)),\ FBF\ (1,\ Psibar,\ S,\ Psi),\ G\_Htautau)\ ]$

let *tgc* $((g1,\ g2,\ g3),\ t,\ c)\ =\ ((G\ g1,\ G\ g2,\ G\ g3),\ t,\ c)$

let *triple_gauge* $=$
  List.map tgc
    $[\ ((Ga,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_W);$
      $((Z,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWW);$
      $((Gl,\ Gl,\ Gl),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Gs)\ ]$

let *qgc* $((g1,\ g2,\ g3,\ g4),\ t,\ c)\ =\ ((G\ g1,\ G\ g2,\ G\ g3,\ G\ g4),\ t,\ c)$

let *gauge4* $=\ Vector4\ [(2,\ C\_13\_42);\ (-1,\ C\_12\_34);\ (-1,\ C\_14\_23)]$
let *minus_gauge4* $=\ Vector4\ [(-2,\ C\_13\_42);\ (1,\ C\_12\_34);\ (1,\ C\_14\_23)]$
let *quartic_gauge* $=$
  List.map qgc
    $[\ (Wm,\ Wp,\ Wm,\ Wp),\ gauge4,\ G\_WWWW;$
      $(Wm,\ Z,\ Wp,\ Z),\ minus\_gauge4,\ G\_ZZWW;$
      $(Wm,\ Z,\ Wp,\ Ga),\ minus\_gauge4,\ G\_AZWW;$
      $(Wm,\ Ga,\ Wp,\ Ga),\ minus\_gauge4,\ G\_AAWW;$
      $(Gl,\ Gl,\ Gl,\ Gl),\ gauge4,\ G2]$

let *gauge_higgs* $=$
  $[\ ((O\ H,\ G\ Wp,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ G\_HWW);$
    $((O\ H,\ G\ Z,\ G\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_HZZ)\ ]$

let *gauge_higgs4* $=$
  $[\ (O\ H,\ O\ H,\ G\ Wp,\ G\ Wm),\ Scalar2\_Vector2\ 1,\ G\_HHWW;$
    $(O\ H,\ O\ H,\ G\ Z,\ G\ Z),\ Scalar2\_Vector2\ 1,\ G\_HHZZ\ ]$

let *higgs* $=$
  $[\ (O\ H,\ O\ H,\ O\ H),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ ]$

let *higgs4* $=$
  $[\ (O\ H,\ O\ H,\ O\ H,\ O\ H),\ Scalar4\ 1,\ G\_H4\ ]$

let *anomaly_higgs* $=$
  $[\ ]$
(* $(O\ H,\ G\ Ga,\ G\ Ga),\ Dim5\_Scalar\_Gauge2\ 1,\ G\_HGaGa;\ (O\ H,\ G\ Ga,\ G\ Z),\ Dim5\_Scalar\_Gauge2\ 1,$
*)

let *goldstone_vertices* $=$
  $[\ ((O\ Phi0,\ G\ Wm,\ G\ Wp),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW);$
    $((O\ Phip,\ G\ Ga,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W);$
    $((O\ Phip,\ G\ Z,\ G\ Wm),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW);$
    $((O\ Phim,\ G\ Wp,\ G\ Ga),\ Scalar\_Vector\_Vector\ 1,\ I\_Q\_W);$
    $((O\ Phim,\ G\ Wp,\ G\ Z),\ Scalar\_Vector\_Vector\ 1,\ I\_G\_ZWW)\ ]$

let *vertices3* $=$
  (*ThoList.flatmap electromagnetic_currents* $[1; 2; 3]$ @
    ThoList.flatmap color_currents $[1; 2; 3]$ @
    ThoList.flatmap neutral_currents $[1; 2; 3]$ @
    ThoList.flatmap charged_currents $[1; 2; 3]$ @
    yukawa @ *triple_gauge* @ *gauge_higgs* @ *higgs* @

$anomaly\_higgs$ @ $goldstone\_vertices$)

let $vertices4$ =
  $quartic\_gauge$ @ $gauge\_higgs4$ @ $higgs4$

let $vertices$ () = ($vertices3$, $vertices4$, [])

let $table$ = $F.of\_vertices$ ($vertices$ ())
let $fuse2$ = $F.fuse2$ $table$
let $fuse3$ = $F.fuse3$ $table$
let $fuse$ = $F.fuse$ $table$
let max_degree () = 4

let $flavor\_of\_string$ = function
  | "e-" → M (L 1) | "e+" → M (L (−1))
  | "mu-" → M (L 2) | "mu+" → M (L (−2))
  | "tau-" → M (L 3) | "tau+" → M (L (−3))
  | "nue" → M (N 1) | "nuebar" → M (N (−1))
  | "numu" → M (N 2) | "numubar" → M (N (−2))
  | "nutau" → M (N 3) | "nutaubar" → M (N (−3))
  | "u" → M (U 1) | "ubar" → M (U (−1))
  | "c" → M (U 2) | "cbar" → M (U (−2))
  | "t" → M (U 3) | "tbar" → M (U (−3))
  | "d" → M (D 1) | "dbar" → M (D (−1))
  | "s" → M (D 2) | "sbar" → M (D (−2))
  | "b" → M (D 3) | "bbar" → M (D (−3))
  | "g" | "gl" → G Gl
  | "A" → G Ga | "Z" | "Z0" → G Z
  | "W+" → G Wp | "W-" → G Wm
  | "H" → O H
  | _ → $invalid\_arg$ "Modellib_BSM.Template.flavor_of_string"

let $flavor\_to\_string$ = function
  | M f →
      begin match f with
      | L 1 → "e-" | L (−1) → "e+"
      | L 2 → "mu-" | L (−2) → "mu+"
      | L 3 → "tau-" | L (−3) → "tau+"
      | L _ → $invalid\_arg$
          "Modellib_BSM.Template.flavor_to_string:␣invalid␣lepton"
      | N 1 → "nue" | N (−1) → "nuebar"
      | N 2 → "numu" | N (−2) → "numubar"
      | N 3 → "nutau" | N (−3) → "nutaubar"
      | N _ → $invalid\_arg$
          "Modellib_BSM.Template.flavor_to_string:␣invalid␣neutrino"
      | U 1 → "u" | U (−1) → "ubar"
      | U 2 → "c" | U (−2) → "cbar"
      | U 3 → "t" | U (−3) → "tbar"
      | U _ → $invalid\_arg$
          "Modellib_BSM.Template.flavor_to_string:␣invalid␣up␣type␣quark"
      | D 1 → "d" | D (−1) → "dbar"
      | D 2 → "s" | D (−2) → "sbar"
      | D 3 → "b" | D (−3) → "bbar"

```
            |  D _  →  invalid_arg
                    "Modellib_BSM.Template.flavor_to_string:␣invalid␣down␣type␣quark"
            end
      |  G f  →
            begin match f with
            |  Gl  →  "g"
            |  Ga  →  "A" |  Z  →  "Z"
            |  Wp  →  "W+" |  Wm  →  "W-"
            end
      |  O f  →
            begin match f with
            |  Phip  →  "phi+" |  Phim  →  "phi-" |  Phi0  →  "phi0"
            |  H  →  "H"
            end
    let flavor_to_TeX  =  function
      |  M f  →
            begin match f with
            |  L 1  →  "e^-" |  L (−1)  →  "e^+"
            |  L 2  →  "\\mu^-" |  L (−2)  →  "\\mu^+"
            |  L 3  →  "\\tau^-" |  L (−3)  →  "\\tau^+"
            |  L _  →  invalid_arg
                    "Modellib_BSM.Template.flavor_to_TeX:␣invalid␣lepton"
            |  N 1  →  "\\nu_e" |  N (−1)  →  "\\bar{\\nu}_e"
            |  N 2  →  "\\nu_\\mu" |  N (−2)  →  "\\bar{\\nu}_\\mu"
            |  N 3  →  "\\nu_\\tau" |  N (−3)  →  "\\bar{\\nu}_\\tau"
            |  N _  →  invalid_arg
                    "Modellib_BSM.Template.flavor_to_TeX:␣invalid␣neutrino"
            |  U 1  →  "u" |  U (−1)  →  "\\bar{u}"
            |  U 2  →  "c" |  U (−2)  →  "\\bar{c}"
            |  U 3  →  "t" |  U (−3)  →  "\\bar{t}"
            |  U _  →  invalid_arg
                    "Modellib_BSM.Template.flavor_to_TeX:␣invalid␣up␣type␣quark"
            |  D 1  →  "d" |  D (−1)  →  "\\bar{d}"
            |  D 2  →  "s" |  D (−2)  →  "\\bar{s}"
            |  D 3  →  "b" |  D (−3)  →  "\\bar{b}"
            |  D _  →  invalid_arg
                    "Modellib_BSM.Template.flavor_to_TeX:␣invalid␣down␣type␣quark"
            end
      |  G f  →
            begin match f with
            |  Gl  →  "g"
            |  Ga  →  "\\gamma" |  Z  →  "Z"
            |  Wp  →  "W^+" |  Wm  →  "W^-"
            end
      |  O f  →
            begin match f with
            |  Phip  →  "\\phi^+" |  Phim  →  "\\phi^-" |  Phi0  →
"\\phi^0"
            |  H  →  "H"
```

```
        end
let flavor_symbol = function
  | M f →
      begin match f with
      | L n when n > 0 → "l" ^ string_of_int n
      | L n → "l" ^ string_of_int (abs n) ^ "b"
      | N n when n > 0 → "n" ^ string_of_int n
      | N n → "n" ^ string_of_int (abs n) ^ "b"
      | U n when n > 0 → "u" ^ string_of_int n
      | U n → "u" ^ string_of_int (abs n) ^ "b"
      | D n when n > 0 → "d" ^ string_of_int n
      | D n → "d" ^ string_of_int (abs n) ^ "b"
      end
  | G f →
      begin match f with
      | Gl → "gl"
      | Ga → "a" | Z → "z"
      | Wp → "wp" | Wm → "wm"
      end
  | O f →
      begin match f with
      | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
      | H → "h"
      end
let pdg = function
  | M f →
      begin match f with
      | L n when n > 0 → 9 + 2 × n
      | L n → − 9 + 2 × n
      | N n when n > 0 → 10 + 2 × n
      | N n → − 10 + 2 × n
      | U n when n > 0 → 2 × n
      | U n → 2 × n
      | D n when n > 0 → − 1 + 2 × n
      | D n → 1 + 2 × n
      end
  | G f →
      begin match f with
      | Gl → 21
      | Ga → 22 | Z → 23
      | Wp → 24 | Wm → (−24)
      end
  | O f →
      begin match f with
      | Phip | Phim → 27 | Phi0 → 26
      | H → 25
      end
let mass_symbol f =
  "mass(" ^ string_of_int (abs (pdg f)) ^ ")"
```

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```
    let width_symbol f =
      "width(" ^ string_of_int (abs (pdg f)) ^ ")"

    let constant_symbol = function
      | Unit → "unit" | Pi → "PI"
      | Alpha_QED → "alpha" | E → "e" | G_weak → "g" | Vev →
"vev"
      | Sin2thw → "sin2thw" | Sinthw → "sinthw" | Costhw →
"costhw"
      | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
      | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
      | G_NC_up → "gncup" | G_NC_down → "gncdwn"
      | G_CC → "gcc"
      | I_Q_W → "iqw" | I_G_ZWW → "igzww"
      | G_WWWW → "gw4" | G_ZZWW → "gzzww"
      | G_AZWW → "gazww" | G_AAWW → "gaaww"
      | G_HWW → "ghww" | G_HZZ → "ghzz"
      | G_HHWW → "ghhww" | G_HHZZ → "ghhzz"
      | G_Htt → "ghtt" | G_Hbb → "ghbb"
      | G_Htautau → "ghtautau" | G_Hcc → "ghcc" | G_Hmm →
"ghmm"
      | G_HGaZ → "ghgaz" | G_HGaGa → "ghgaga" | G_Hgg →
"ghgg"
      | G_H3 → "gh3" | G_H4 → "gh4"
      | Gs → "gs" | I_Gs → "igs" | G2 → "gs**2"
      | Mass f → "mass" ^ flavor_symbol f
      | Width f → "width" ^ flavor_symbol f

  end
module HSExt (Flags : BSM_flags) =
  struct
    let rcs = RCS.rename rcs_file "Modellib_BSM.HSExt"
        [ "Higgs␣Singlet␣Extension␣of␣the␣SM"]

    open Coupling

    let default_width = ref Timelike
    let use_fudged_width = ref false

    let options = Options.create
      [ "constant_width", Arg.Unit (fun () → default_width := Constant),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", Arg.Set use_fudged_width,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", Arg.String (fun f → default_width := Custom f),
        "use␣custom␣width";
        "cancel_widths", Arg.Unit (fun () → default_width := Vanishing),
        "use␣vanishing␣width"]

    type matter_field = L of int | N of int | U of int | D of int
    type gauge_boson = Ga | Wp | Wm | Z | Gl
    type other = Phip | Phim | Phi0 | H | S
    type flavor = M of matter_field | G of gauge_boson | O of other
```

```
let matter_field f = M f
let gauge_boson f = G f
let other f = O f

type field =
  | Matter of matter_field
  | Gauge of gauge_boson
  | Other of other

let field = function
  | M f → Matter f
  | G f → Gauge f
  | O f → Other f

type gauge = unit

let gauge_symbol () =
  failwith "Modellib_BSM.HSExt.gauge_symbol:␣internal␣error"

let family n = List.map matter_field [ L n; N n; U n; D n ]

let external_flavors () =
  [ "1st␣Generation", ThoList.flatmap family [1; − 1];
    "2nd␣Generation", ThoList.flatmap family [2; − 2];
    "3rd␣Generation", ThoList.flatmap family [3; − 3];
    "Gauge␣Bosons", List.map gauge_boson [Ga; Z; Wp; Wm; Gl];
    "Higgs", List.map other [H; S];
    "Goldstone␣Bosons", List.map other [Phip; Phim; Phi0] ]

let flavors () = ThoList.flatmap snd (external_flavors ())

let spinor n =
  if n ≥ 0 then
    Spinor
  else
    ConjSpinor

let lorentz = function
  | M f →
      begin match f with
      | L n → spinor n | N n → spinor n
      | U n → spinor n | D n → spinor n
      end
  | G f →
      begin match f with
      | Ga | Gl → Vector
      | Wp | Wm | Z → Massive_Vector
      end
  | O f → Scalar

let color = function
  | M (U n) → Color.SUN (if n > 0 then 3 else -3)
  | M (D n) → Color.SUN (if n > 0 then 3 else -3)
  | G Gl → Color.AdjSUN 3
  | _ → Color.Singlet
```

```
let prop_spinor n =
  if n ≥ 0 then
    Prop_Spinor
  else
    Prop_ConjSpinor

let propagator = function
  | M f →
      begin match f with
      | L n → prop_spinor n | N n → prop_spinor n
      | U n → prop_spinor n | D n → prop_spinor n
      end
  | G f →
      begin match f with
      | Ga | Gl → Prop_Feynman
      | Wp | Wm | Z → Prop_Unitarity
      end
  | O f →
      begin match f with
      | Phip | Phim | Phi0 → Only_Insertion
      | H | S → Prop_Scalar
      end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

```
let width f =
  if !use_fudged_width then
    match f with
    | G Wp | G Wm | M (U 3) | M (U (−3)) → Fudged
    | _ → !default_width
  else
    !default_width

let goldstone = function
  | G f →
      begin match f with
      | Wp → Some (O Phip, Coupling.Const 1)
      | Wm → Some (O Phim, Coupling.Const 1)
      | Z → Some (O Phi0, Coupling.Const 1)
      | _ → None
      end
  | _ → None

let conjugate = function
  | M f →
      M (begin match f with
      | L n → L (−n) | N n → N (−n)
      | U n → U (−n) | D n → D (−n)
      end)
  | G f →
      G (begin match f with
      | Gl → Gl | Ga → Ga | Z → Z
```

```
          |  Wp  →  Wm  |  Wm  →  Wp
          end)
    |  O f  →
          O (begin match f with
          |  Phip  →  Phim  |  Phim  →  Phip  |  Phi0  →  Phi0
          |  H  →  H  |  S  →  S
          end)

let fermion  =  function
  |  M f  →
        begin match f with
        |  L n  →  if n  >  0 then 1 else -1
        |  N n  →  if n  >  0 then 1 else -1
        |  U n  →  if n  >  0 then 1 else -1
        |  D n  →  if n  >  0 then 1 else -1
        end
  |  G f  →
        begin match f with
        |  Gl  |  Ga  |  Z  |  Wp  |  Wm  →  0
        end
  |  O _  →  0

module Ch  =  Charges.QQ

let ( // )  =  Algebra.Small_Rational.make

let generation′  =  function
  |  1  →  [ 1//1; 0//1; 0//1]
  |  2  →  [ 0//1; 1//1; 0//1]
  |  3  →  [ 0//1; 0//1; 1//1]
  |  − 1  →  [−1//1; 0//1; 0//1]
  |  − 2  →  [ 0//1; − 1//1; 0//1]
  |  − 3  →  [ 0//1; 0//1; − 1//1]
  |  n  →  invalid_arg ("HSExt.generation′:␣" ^ string_of_int n)

let generation f  =
  match f with
  |  M (L n  |  N n  |  U n  |  D n)  →  generation′ n
  |  G _  |  O _  →  [0//1; 0//1; 0//1]

let charge  =  function
  |  M f  →
        begin match f with
        |  L n  →  if n  >  0 then -1//1 else 1//1
        |  N n  →  0//1
        |  U n  →  if n  >  0 then 2//3 else -2//3
        |  D n  →  if n  >  0 then -1//3 else 1//3
        end
  |  G f  →
        begin match f with
        |  Gl  |  Ga  |  Z  →  0//1
        |  Wp  →  1//1
        |  Wm  →  − 1//1
```

```
            end
        | O f →
            begin match f with
            | H | Phi0 | S → 0//1
            | Phip → 1//1
            | Phim → - 1//1
            end

    let lepton = function
        | M f →
            begin match f with
            | L n | N n → if n > 0 then 1//1 else -1//1
            | U _ | D _ → 0//1
            end
        | G _ | O _ → 0//1

    let baryon = function
        | M f →
            begin match f with
            | L _ | N _ → 0//1
            | U n | D n → if n > 0 then 1//1 else -1//1
            end
        | G _ | O _ → 0//1

    let charges f =
        [ charge f; lepton f; baryon f] @ generation f

    type constant =
        | Unit | Pi | Alpha_QED | Sin2thw
        | Sinthw | Costhw | E | G_weak | Vev
        | Q_lepton | Q_up | Q_down | G_CC
        | G_NC_neutrino | G_NC_lepton | G_NC_up | G_NC_down
        | I_Q_W | I_G_ZWW
        | G_WWWW | G_ZZWW | G_AZWW | G_AAWW
        | G_HWW | G_HHWW | G_HZZ | G_HHZZ
        | G_SWW | G_SSWW | G_SZZ | G_SSZZ | G_HSWW |
G_HSZZ
        | G_Htt | G_Hbb | G_Hcc | G_Hmm | G_Htautau | G_H3 |
G_H4_1
        | G_H4_2 | G_H4_3 | G_H4_4 | G_H4_5
        | G_Stt | G_Sbb | G_Scc | G_Smm | G_Stautau | G_HSS |
G_HHS
        | G_HGaZ | G_HGaGa | G_Hgg | G_SGaZ | G_SGaGa | G_Sgg
        | Gs | I_Gs | G2
        | Mass of flavor | Width of flavor
```

Two integer counters for the QCD and EW order of the couplings.

```
    type orders = int × int

    let orders = function
        | _ → (0,0)

    let input_parameters = []
```

let *derived_parameters* = [ ]

let *g_over_2_costh* =
  Quot (*Neg* (*Atom G_weak*), *Prod* [*Const* 2; *Atom Costhw*])

let *nc_coupling c t3 q* =
  (*Real_Array c*,
   [*Prod* [*g_over_2_costh*; *Diff* (*t3*, *Prod* [*Const* 2; *q*; *Atom Sin2thw*])];
    Prod [*g_over_2_costh*; *t3*]])

let *half* = *Quot* (*Const* 1, *Const* 2)

let *derived_parameter_arrays* =
  [*nc_coupling G_NC_neutrino half* (*Const* 0);
    nc_coupling G_NC_lepton (*Neg half*) (*Const* (−1));
    nc_coupling G_NC_up half (*Quot* (*Const* 2, *Const* 3));
    nc_coupling G_NC_down (*Neg half*) (*Quot* (*Const* (−1), *Const* 3))]

let *parameters* ( ) =
  { *input* = *input_parameters*;
    derived = *derived_parameters*;
    derived_arrays = *derived_parameter_arrays* }

module *F* = *Modeltools.Fusions* (struct
  type *f* = *flavor*
  type *c* = *constant*
  let *compare* = *compare*
  let *conjugate* = *conjugate*
end)

let *mgm* ((*m1*, *g*, *m2*), *fbf*, *c*) = ((*M m1*, *G g*, *M m2*), *fbf*, *c*)

let *electromagnetic_currents n* =
  List.map mgm
    [((*L* (−*n*), *Ga*, *L n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_lepton*);
      ((*U* (−*n*), *Ga*, *U n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
      ((*D* (−*n*), *Ga*, *D n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

let *color_currents n* =
  List.map mgm
    [((*U* (−*n*), *Gl*, *U n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*);
      ((*D* (−*n*), *Gl*, *D n*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*)]

let *neutral_currents n* =
  List.map mgm
    [((*L* (−*n*), *Z*, *L n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_lepton*);
      ((*N* (−*n*), *Z*, *N n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_neutrino*);
      ((*U* (−*n*), *Z*, *U n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_up*);
      ((*D* (−*n*), *Z*, *D n*), *FBF* (1, *Psibar*, *VA*, *Psi*), *G_NC_down*)]

let *charged_currents n* =
  List.map mgm
    [((*L* (−*n*), *Wm*, *N n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
      ((*N* (−*n*), *Wp*, *L n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
      ((*D* (−*n*), *Wm*, *U n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*);
      ((*U* (−*n*), *Wp*, *D n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_CC*)]

let *yukawa* =
  [((M (U (-3)), O H, M (U 3)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Htt*);
    ((M (D (-3)), O H, M (D 3)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Hbb*);
    ((M (U (-2)), O H, M (U 2)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Hcc*);
    ((M (L (-2)), O H, M (L 2)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Hmm*);
    ((M (L (-3)), O H, M (L 3)),
            FBF (1, *Psibar*, *Coupling.S*, *Psi*), *G_Htautau*);
    ((M (U (-3)), O S, M (U 3)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Stt*);
    ((M (D (-3)), O S, M (D 3)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Sbb*);
    ((M (U (-2)), O S, M (U 2)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Scc*);
    ((M (L (-2)), O S, M (L 2)), *FBF* (1, *Psibar*, *Coupling.S*, *Psi*), *G_Smm*);
    ((M (L (-3)), O S, M (L 3)),
            FBF (1, *Psibar*, *Coupling.S*, *Psi*), *G_Stautau*)]

let *tgc* (($g1$, $g2$, $g3$), t, c) = ((G $g1$, G $g2$, G $g3$), t, c)

let *triple_gauge* =
  List.map *tgc*
    [((Ga, Wm, Wp), *Gauge_Gauge_Gauge* 1, *I_Q_W*);
      ((Z, Wm, Wp), *Gauge_Gauge_Gauge* 1, *I_G_ZWW*);
      ((Gl, Gl, Gl), *Gauge_Gauge_Gauge* 1, *I_Gs*)]

let *qgc* (($g1$, $g2$, $g3$, $g4$), t, c) = ((G $g1$, G $g2$, G $g3$, G $g4$), t, c)

let *gauge4* = *Vector4* [(2, *C_13_42*); (-1, *C_12_34*); (-1, *C_14_23*)]
let *minus_gauge4* = *Vector4* [(-2, *C_13_42*); (1, *C_12_34*); (1, *C_14_23*)]
let *quartic_gauge* =
  List.map *qgc*
    [(Wm, Wp, Wm, Wp), *gauge4*, *G_WWWW*;
      (Wm, Z, Wp, Z), *minus_gauge4*, *G_ZZWW*;
      (Wm, Z, Wp, Ga), *minus_gauge4*, *G_AZWW*;
      (Wm, Ga, Wp, Ga), *minus_gauge4*, *G_AAWW*;
      (Gl, Gl, Gl, Gl), *gauge4*, *G2*]

let *gauge_higgs* =
  [((O H, G Wp, G Wm), *Scalar_Vector_Vector* 1, *G_HWW*);
    ((O H, G Z, G Z), *Scalar_Vector_Vector* 1, *G_HZZ*);
    ((O S, G Wp, G Wm), *Scalar_Vector_Vector* 1, *G_SWW*);
    ((O S, G Z, G Z), *Scalar_Vector_Vector* 1, *G_SZZ*)]

let *gauge_higgs4* =
  [(O H, O H, G Wp, G Wm), *Scalar2_Vector2* 1, *G_HHWW*;
    (O H, O H, G Z, G Z), *Scalar2_Vector2* 1, *G_HHZZ*;
    (O H, O S, G Wp, G Wm), *Scalar2_Vector2* 1, *G_HSWW*;
    (O H, O S, G Z, G Z), *Scalar2_Vector2* 1, *G_HSZZ*;
    (O S, O S, G Wp, G Wm), *Scalar2_Vector2* 1, *G_SSWW*;
    (O S, O S, G Z, G Z), *Scalar2_Vector2* 1, *G_SSZZ*]

let *higgs* =
  [(O H, O H, O H), *Scalar_Scalar_Scalar* 1, *G_H3*;
    (O S, O H, O H), *Scalar_Scalar_Scalar* 1, *G_HHS*;
    (O S, O S, O H), *Scalar_Scalar_Scalar* 1, *G_HSS*]

let *higgs4* =

```
    [ (O H, O H, O H, O H), Scalar4 1, G_H4_1;
      (O H, O H, O H, O S), Scalar4 1, G_H4_2;
      (O H, O H, O S, O S), Scalar4 1, G_H4_3;
      (O H, O S, O S, O S), Scalar4 1, G_H4_4;
      (O S, O S, O S, O S), Scalar4 1, G_H4_5 ]
let anomaly_higgs =
  [ (O H, G Ga, G Ga), Dim5_Scalar_Gauge2 1, G_HGaGa;
    (O H, G Ga, G Z), Dim5_Scalar_Gauge2 1, G_HGaZ;
    (O H, G Gl, G Gl), Dim5_Scalar_Gauge2 1, G_Hgg;
    (O S, G Ga, G Ga), Dim5_Scalar_Gauge2 1, G_SGaGa;
    (O S, G Ga, G Z), Dim5_Scalar_Gauge2 1, G_SGaZ;
    (O S, G Gl, G Gl), Dim5_Scalar_Gauge2 1, G_Sgg ]
let goldstone_vertices =
  [ ((O Phi0, G Wm, G Wp), Scalar_Vector_Vector 1, I_G_ZWW);
    ((O Phip, G Ga, G Wm), Scalar_Vector_Vector 1, I_Q_W);
    ((O Phip, G Z, G Wm), Scalar_Vector_Vector 1, I_G_ZWW);
    ((O Phim, G Wp, G Ga), Scalar_Vector_Vector 1, I_Q_W);
    ((O Phim, G Wp, G Z), Scalar_Vector_Vector 1, I_G_ZWW) ]
let vertices3 =
  (ThoList.flatmap electromagnetic_currents [1; 2; 3] @
   ThoList.flatmap color_currents [1; 2; 3] @
   ThoList.flatmap neutral_currents [1; 2; 3] @
   ThoList.flatmap charged_currents [1; 2; 3] @
   yukawa @ triple_gauge @ gauge_higgs @ higgs @
   anomaly_higgs @ goldstone_vertices)
let vertices4 =
  quartic_gauge @ gauge_higgs4 @ higgs4
let vertices () = (vertices3, vertices4, [])
let table = F.of_vertices (vertices ())
let fuse2 = F.fuse2 table
let fuse3 = F.fuse3 table
let fuse = F.fuse table
let max_degree () = 4
let flavor_of_string = function
  | "e-" → M (L 1) | "e+" → M (L (−1))
  | "mu-" → M (L 2) | "mu+" → M (L (−2))
  | "tau-" → M (L 3) | "tau+" → M (L (−3))
  | "nue" → M (N 1) | "nuebar" → M (N (−1))
  | "numu" → M (N 2) | "numubar" → M (N (−2))
  | "nutau" → M (N 3) | "nutaubar" → M (N (−3))
  | "u" → M (U 1) | "ubar" → M (U (−1))
  | "c" → M (U 2) | "cbar" → M (U (−2))
  | "t" → M (U 3) | "tbar" → M (U (−3))
  | "d" → M (D 1) | "dbar" → M (D (−1))
  | "s" → M (D 2) | "sbar" → M (D (−2))
  | "b" → M (D 3) | "bbar" → M (D (−3))
  | "g" | "gl" → G Gl
```

```
    | "A" → G Ga | "Z" | "Z0" → G Z
    | "W+" → G Wp | "W-" → G Wm
    | "H" → O H
    | "S" → O S
    | _ → invalid_arg "Modellib_BSM.HSExt.flavor_of_string"
let flavor_to_string = function
  | M f →
      begin match f with
      | L 1 → "e-" | L (−1) → "e+"
      | L 2 → "mu-" | L (−2) → "mu+"
      | L 3 → "tau-" | L (−3) → "tau+"
      | L _ → invalid_arg
          "Modellib_BSM.HSExt.flavor_to_string:␣invalid␣lepton"
      | N 1 → "nue" | N (−1) → "nuebar"
      | N 2 → "numu" | N (−2) → "numubar"
      | N 3 → "nutau" | N (−3) → "nutaubar"
      | N _ → invalid_arg
          "Modellib_BSM.HSExt.flavor_to_string:␣invalid␣neutrino"
      | U 1 → "u" | U (−1) → "ubar"
      | U 2 → "c" | U (−2) → "cbar"
      | U 3 → "t" | U (−3) → "tbar"
      | U _ → invalid_arg
          "Modellib_BSM.HSExt.flavor_to_string:␣invalid␣up␣type␣quark"
      | D 1 → "d" | D (−1) → "dbar"
      | D 2 → "s" | D (−2) → "sbar"
      | D 3 → "b" | D (−3) → "bbar"
      | D _ → invalid_arg
          "Modellib_BSM.HSExt.flavor_to_string:␣invalid␣down␣type␣quark"
      end
  | G f →
      begin match f with
      | Gl → "g"
      | Ga → "A" | Z → "Z"
      | Wp → "W+" | Wm → "W-"
      end
  | O f →
      begin match f with
      | Phip → "phi+" | Phim → "phi-" | Phi0 → "phi0"
      | H → "H" | S → "S"
      end
let flavor_to_TeX = function
  | M f →
      begin match f with
      | L 1 → "e^-" | L (−1) → "e^+"
      | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
      | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
      | L _ → invalid_arg
          "Modellib_BSM.HSExt.flavor_to_TeX:␣invalid␣lepton"
      | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
```

```
          | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
          | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
          | N _ → invalid_arg
                "Modellib_BSM.HSExt.flavor_to_TeX:␣invalid␣neutrino"
          | U 1 → "u" | U (−1) → "\\bar{u}"
          | U 2 → "c" | U (−2) → "\\bar{c}"
          | U 3 → "t" | U (−3) → "\\bar{t}"
          | U _ → invalid_arg
                "Modellib_BSM.HSExt.flavor_to_TeX:␣invalid␣up␣type␣quark"
          | D 1 → "d" | D (−1) → "\\bar{d}"
          | D 2 → "s" | D (−2) → "\\bar{s}"
          | D 3 → "b" | D (−3) → "\\bar{b}"
          | D _ → invalid_arg
                "Modellib_BSM.HSExt.flavor_to_TeX:␣invalid␣down␣type␣quark"
          end
      | G f →
          begin match f with
          | Gl → "g"
          | Ga → "\\gamma" | Z → "Z"
          | Wp → "W^+" | Wm → "W^-"
          end
      | O f →
          begin match f with
          | Phip → "\\phi^+" | Phim → "\\phi^-" | Phi0 →
"\\phi^0"
          | H → "H" | S → "S"
          end

    let flavor_symbol = function
      | M f →
          begin match f with
          | L n when n > 0 → "l" ^ string_of_int n
          | L n → "l" ^ string_of_int (abs n) ^ "b"
          | N n when n > 0 → "n" ^ string_of_int n
          | N n → "n" ^ string_of_int (abs n) ^ "b"
          | U n when n > 0 → "u" ^ string_of_int n
          | U n → "u" ^ string_of_int (abs n) ^ "b"
          | D n when n > 0 → "d" ^ string_of_int n
          | D n → "d" ^ string_of_int (abs n) ^ "b"
          end
      | G f →
          begin match f with
          | Gl → "gl"
          | Ga → "a" | Z → "z"
          | Wp → "wp" | Wm → "wm"
          end
      | O f →
          begin match f with
          | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
          | H → "h" | S → "s"
```

```
                end

      let pdg = function
        | M f →
            begin match f with
            | L n when n > 0 → 9 + 2 × n
            | L n → − 9 + 2 × n
            | N n when n > 0 → 10 + 2 × n
            | N n → − 10 + 2 × n
            | U n when n > 0 → 2 × n
            | U n → 2 × n
            | D n when n > 0 → − 1 + 2 × n
            | D n → 1 + 2 × n
            end
        | G f →
            begin match f with
            | Gl → 21
            | Ga → 22 | Z → 23
            | Wp → 24 | Wm → (−24)
            end
        | O f →
            begin match f with
            | Phip | Phim → 27 | Phi0 → 26
            | H → 25 | S → 35
            end

      let mass_symbol f =
        "mass(" ^ string_of_int (abs (pdg f)) ^ ")"

      let width_symbol f =
        "width(" ^ string_of_int (abs (pdg f)) ^ ")"

      let constant_symbol = function
        | Unit → "unit" | Pi → "PI"
        | Alpha_QED → "alpha" | E → "e" | G_weak → "g" | Vev →
"vev"
        | Sin2thw → "sin2thw" | Sinthw → "sinthw" | Costhw →
"costhw"
        | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
        | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
        | G_NC_up → "gncup" | G_NC_down → "gncdwn"
        | G_CC → "gcc"
        | I_Q_W → "iqw" | I_G_ZWW → "igzww"
        | G_WWWW → "gw4" | G_ZZWW → "gzzww"
        | G_AZWW → "gazww" | G_AAWW → "gaaww"
        | G_HWW → "ghww" | G_HZZ → "ghzz"
        | G_HHWW → "ghhww" | G_HHZZ → "ghhzz"
        | G_SWW → "gsww" | G_SZZ → "gszz"
        | G_SSWW → "gssww" | G_SSZZ → "gsszz"
        | G_HSWW → "ghsww" | G_HSZZ → "ghszz"
        | G_Htt → "ghtt" | G_Hbb → "ghbb"
```

```
      |  G_Htautau  →  "ghtautau" |  G_Hcc  →  "ghcc" |  G_Hmm  →
"ghmm"
      | G_Stt → "gstt" | G_Sbb → "gsbb"
      |  G_Stautau  →  "gstautau" |  G_Scc  →  "gscc" |  G_Smm  →
"gsmm"
      |  G_HGaZ  →  "ghgaz" |  G_HGaGa  →  "ghgaga" |  G_Hgg  →
"ghgg"
      | G_SGaZ → "gsgaz" | G_SGaGa → "gsgaga" | G_Sgg → "gsgg"
      | G_H3 → "gh3" | G_H4_1 → "gh4_1" | G_H4_2 → "gh4_2"
      |  G_H4_3  →  "gh4_3" |  G_H4_4  →  "gh4_4" |  G_H4_5  →
"gh4_5"
      | G_HHS → "ghhs" | G_HSS → "ghss"
      | Gs → "gs" | I_Gs → "igs" | G2 → "gs**2"
      | Mass f  →  "mass" ^ flavor_symbol f
      | Width f  →  "width" ^ flavor_symbol f

  end
```

13.6.2   Three-Site Higgsless Model

```
module type Threeshl_options  =
  sig
      val include_ckm :  bool
      val include_hf :  bool
      val diet :  bool
  end

module Threeshl_no_ckm :  Threeshl_options  =
  struct
      let include_ckm  =  false
      let include_hf  =  true
      let diet  =  false
  end

module Threeshl_ckm :  Threeshl_options  =
  struct
      let include_ckm  =  true
      let include_hf  =  true
      let diet  =  false
  end

module Threeshl_no_ckm_no_hf :  Threeshl_options  =
  struct
      let include_ckm  =  false
      let include_hf  =  false
      let diet  =  false
  end

module Threeshl_ckm_no_hf :  Threeshl_options  =
  struct
      let include_ckm  =  true
      let include_hf  =  false
```

```
        let diet = false
    end
```

module *Threeshl_diet_no_hf* : *Threeshl_options* =
    struct
        let *include_ckm* = false
        let *include_hf* = false
        let *diet* = true
    end

module *Threeshl_diet* : *Threeshl_options* =
    struct
        let *include_ckm* = false
        let *include_hf* = true
        let *diet* = true
    end

We use one generic implementation of the model and implement different features via option modules given to a functor

module *Threeshl* (*Module_options* : *Threeshl_options*) =
    struct

        open *Coupling*

        let *modname* = `"Modellib_BSM.Threeshl"`

        let *rcs* =
        let *renderbool* = function true → `"true"` | false → `"false"`
        in *RCS.rename rcs_file* `"Modellib_BSM.Threeshl"`
            [`"Three-Site␣Higgsless␣Model,␣"` ^
                `"flavor␣mixing:␣"` ^ (*renderbool Module_options.include_ckm*) ^
                `",␣heavy␣fermions:␣"` ^ (*renderbool Module_options.include_hf*) ^
                `",␣reduced␣set␣of␣couplings:␣"` ^ (*renderbool Module_options.diet*)
            ]

Shamelessly stolen from Modellib.SM3, but with no support for fudged width yet

        let *default_width* = *ref Timelike*

If this flag is set true, all gauge bosons are assumed to be massless and are assigned feynman gauge propagators. This in conjunction with the unbroken three site model is intended for checking gauge invariance via the ward identites.

        let *all_feynman* = *ref* false

        let *options* = *Options.create* [
            `"constant_width"`, *Arg.Unit* (fun _ → *default_width* := *Constant*),
                `"use␣constant␣width␣(also␣in␣t-channel)"`;
            `"custom_width"`, *Arg.String* (fun *x* → *default_width* := *Custom x*),
                `"use␣custom␣width"`;
            `"cancel_widths"`, *Arg.Unit* (fun _ → *default_width* := *Vanishing*),
                `"use␣vanishing␣width"`;
            `"all_feynman"`, *Arg.Unit* (fun _ → *all_feynman* := true),
                `"assign␣feynman␣gauge␣propagators␣to␣all␣gauge␣bosons\n"`

```
        ^ "\t(for␣checking␣the␣ward␣identities);␣use␣only␣if␣you␣*really*␣know\n"
        ^ "\twhat␣you␣are␣doing"]
```

The quantum numbers that are carried by the particles. `csign` is *not* the charge carried by the particle, but differentiates between particles (`Pos`) and antiparticles (`Neg`)

> type *kkmode* = *Light* | *Heavy*
> type *generation* = *Gen0* | *Gen1* | *Gen2*
> type *csign* = *Pos* | *Neg*
> type *isospin* = *Iso_up* | *Iso_down*

Necessary to represent the indices of the couplings defined in FORTRAN

> type *kk2* = *Light2* | *Heavy2* | *Light_Heavy*

Map the different types to the constants used in the FORTRAN module

> let *fspec_of_kkmode* = function *Light* → `"l_mode"` | *Heavy* → `"h_mode"`
> let *fspec_of_kk2* = function
>   *Light2* → `"l_mode"` | *Heavy2* → `"h_mode"` | *Light_Heavy* → `"lh_mode"`
> let *fspec_of_gen* = function *Gen0* → `"gen_0"` | *Gen1* → `"gen_1"` | *Gen2* → `"gen_2"`
> let *fspec_of_iso* = function *Iso_up* → `"iso-up"` | *Iso_down* → `"iso_down"`

Covert the "charge sign" into a numeric sign (used e.g. in the determination of the MCID codes)

> let *int_of_csign* = function *Pos* → 1 | *Neg* → −1

Convert the generation into an integer (dito)

> let *int_of_gen* = function *Gen0* → 1 | *Gen1* → 2 | *Gen2* → 3

The type `flavor` is implemented as a variant. Fermions are implemented as a variant differentating between leptons and quarks (seemed the most natural way as this is also the way in which the FORTRAN code is structured). Bosons are implemented as a variant the differentiates between W, Z and A. All other quantum numbers that are required for identifying the particles are carried by the variant constructors.

> type *fermion* =
> | *Lepton* of (*kkmode* × *csign* × *generation* × *isospin*)
> | *Quark* of (*kkmode* × *csign* × *generation* × *isospin*)
>
> type *boson* =
> | *W* of (*kkmode* × *csign*)
> | *Z* of *kkmode*
> | *A*
> | *G*
>
> type *flavor* = *Fermion* of *fermion* | *Boson* of *boson*

Helpers to construct particles from quantum numbers

let *lepton kk cs gen iso* = *Lepton* (*kk, cs, gen, iso*)
let *quark kk cs gen iso* = *Quark* (*kk, cs, gen, iso*)
let *w kk cs* = *W* (*kk, cs*)
let *z kk* = *Z kk*
let *flavor_of_f x* = *Fermion x*
let *flavor_of_b x* = *Boson x*

Map a list of functions to the list (partially) applied to a value

let *revmap funs v* = *List.map* (fun *x* → *x v*) *funs*

The same for a list of values; the result is flattened

let *revmap2 funs vals* = *ThoList.flatmap* (*revmap funs*) *vals*

Functions to loop the constructors over quantum numbers for list creation purposes

let *loop_kk flist* = *revmap2 flist* [*Light; Heavy*]
let *loop_cs flist* = *revmap2 flist* [*Pos; Neg*]
let *loop_gen flist* = *revmap2 flist* [*Gen0; Gen1; Gen2*]
let *loop_iso flist* = *revmap2 flist* [*Iso_up; Iso_down*]
let *loop_kk2 flist* = *revmap2 flist* [*Light2; Heavy2; Light_Heavy*]

Conditional looping over kk modes depending on whether to include heavy fermions

let *cloop_kk flist* = match *Module_options.include_hf* with
   | true → *loop_kk flist*
   | false → *revmap flist Light*
let *cloop_kk2 flist* = match *Module_options.include_hf* with
   | true → *loop_kk2 flist*
   | false → *revmap flist Light2*

Having defined the necessary helpers, the magic of currying makes building lists of particles as easy as nesting the loop functions in the correct order...

let *all_leptons* = *loop_iso* (*loop_gen* (*loop_cs* (*cloop_kk* [*lepton*])))
let *all_quarks* = *loop_iso*(*loop_gen* (*loop_cs* (*cloop_kk* [*quark*])))
let *all_bosons* = (*loop_cs* (*loop_kk* [*w*])) @ [*Z Light; Z Heavy; A; G*]

Converts a flavor spec to the BCD identifier defined in the FORTRAN module. Splitting the function into two parts `prefix` and `rump` removes a lot of redundancy.

let *bcdi_of_flavor* =
let *prefix* = function
   | *Fermion* (*Lepton* (*Heavy, _, _, _*)) | *Fermion* (*Quark* (*Heavy, _, _, _*))
   | *Boson* (*W* (*Heavy, _*)) | *Boson* (*Z Heavy*) → "h"
   | _ → ""
in let *rump* = function
   | *Fermion* (*Lepton spec*) → (match *spec* with
      | (_, _, *Gen0, Iso_up*) → "nue"
      | (_, _, *Gen0, Iso_down*) → "e"
      | (_, _, *Gen1, Iso_up*) → "numu"
      | (_, _, *Gen1, Iso_down*) → "mu"

```
                    |  (_, _, Gen2, Iso_up)  →  "nutau"
                    |  (_, _, Gen2, Iso_down)  →  "tau")
              |  Fermion (Quark spec)  →  (match spec with
                    |  (_, _, Gen0, Iso_up)  →  "u"
                    |  (_, _, Gen0, Iso_down)  →  "d"
                    |  (_, _, Gen1, Iso_up)  →  "c"
                    |  (_, _, Gen1, Iso_down)  →  "s"
                    |  (_, _, Gen2, Iso_up)  →  "t"
                    |  (_, _, Gen2, Iso_down)  →  "b")
              |  Boson (W _)  →  "w"  |  Boson (Z _)  →  "z"
              |  Boson A  →  invalid_arg (modname ˆ ".bcd_of_flavor:␣no␣bcd␣for␣photon!")
              |  Boson G  →  invalid_arg (modname ˆ ".bcd_of_flavor:␣no␣bcd␣for␣gluon!")
       in function x  →  (prefix x) ˆ (rump x) ˆ "_bcd"
```

The function defined in the model signature which returns the colour representation of a particle

```
       let color  =
       let quarkrep  = function
              |  (_, Pos, _, _)  →  Color.SUN 3
              |  (_, Neg, _, _)  →  Color.SUN (−3)
       in function
              |  Fermion (Quark x)  →  quarkrep x
              |  Boson G  →  Color.AdjSUN 3
              |  _  →  Color.Singlet
```

Function for calculating the MCID code of a particle. Convenctions have been choosen such that the heavy modes are identified by the same numbers as the light ones, prefixed with 99. This is supposedly in accord with the conventions for adding new particles to the list of MCID codes. This function is required by the signature.

```
       let pdg  =
       let iso_delta  = function Iso_down  →  0  |  Iso_up  →  1
       in let gen_delta  = function Gen0  →  0  |  Gen1  →  2  |  Gen2  →  4
       in let kk_delta  = function Light  →  0  |  Heavy  →  9900
       in function
              |  Fermion ( Lepton (kk, cs, gen, iso))  →
                    (int_of_csign cs) × (11 + (gen_delta gen) + (iso_delta iso) + (kk_delta kk))
              |  Fermion ( Quark (kk, cs, gen, iso))  →
                    (int_of_csign cs) × (1 + (gen_delta gen) + (iso_delta iso) + (kk_delta kk))
              |  Boson (W (kk, cs))  →  (int_of_csign cs) × (24 + (kk_delta kk))
              |  Boson (Z kk)  →  23 + (kk_delta kk)
              |  Boson A  →  22
              |  Boson G  →  21
```

Returns the lorentz representation of a particle; required by the signature.

```
       let lorentz  =
       let spinor  = function
              |  (_, Pos, _, _)  →  Spinor
              |  (_, Neg, _, _)  →  ConjSpinor
       in function
```

```
    | Fermion (Lepton x) | Fermion (Quark x) → spinor x
    | Boson (W _) | Boson (Z _) → Massive_Vector
    | Boson A → Vector
    | Boson G → Vector
```

O'Mega supports models that allow different gauges; however, we only implement unitary gauge and therefore stub this (SM3 does the same thing). The `gauge` type as well as `gauge_symbol` are required by the signature.

```
type gauge = unit

let gauge_symbol () =
    failwith (modname ^ ".gauge_symbol:␣internal␣error")
```

Returns the propagator for a given particle type. Required by signature.

```
let propagator =
let spinorprop = function
    | (_, Pos, _, _) → Prop_Spinor
    | (_, Neg, _, _) → Prop_ConjSpinor
in function
    | Fermion (Lepton x) | Fermion (Quark x) → spinorprop x
    | Boson (W _) | Boson (Z _) →
        (match !all_feynman with false → Prop_Unitarity | true →
Prop_Feynman)
    | Boson A → Prop_Feynman
    | Boson G → Prop_Feynman
```

Return the width of a particle, required by signature.
TODO: Refine such that stable particles always are treade via vanishing width, as this might speed up the generated code a bit.

```
let width _ = !default_width
```

Returns the conjugate particle; required by signature.

```
let conjugate =
let conj_csign = function
    | Pos → Neg
    | Neg → Pos
in function
    | Fermion (Lepton (kk, cs, gen, iso)) → Fermion (Lepton (kk, conj_csign cs, gen, iso))
    | Fermion (Quark (kk, cs, gen, iso)) → Fermion (Quark (kk, conj_csign cs, gen, iso))
    | Boson (W (kk, cs)) → Boson (W (kk, conj_csign cs))
    | x → x
```

Tells the diagram generator whether a particle is a fermion, a conjugate fermion or a boson. Required by signature

```
let fermion = function
    | Fermion (Lepton (_, cs, _, _)) | Fermion (Quark (_, cs, _, _)) →
int_of_csign cs
    | Boson _ → 0
```

Charges are: charge, lepton number, baryon number, generation. Required by signature

```
module Ch = Charges.QQ
let ( // ) = Algebra.Small_Rational.make

let qn_charge = function
    | Boson b → (match b with
        | W (_, c) → (int_of_csign (c)) // 1
        | _ → 0//1)
    | Fermion f → (match f with
        | Lepton (_, c, _, Iso_up) → 0//1
        | Lepton (_, c, _, Iso_down) → (−1 × int_of_csign (c)) // 1
        | Quark (_, c, _, Iso_up) → (2 × int_of_csign (c)) // 3
        | Quark (_, c, _, Iso_down) → (−1 × int_of_csign (c)) // 3)

let qn_lepton = function
    | Fermion (Lepton (_, c, _, _)) → int_of_csign (c) // 1
    | _ → 0//1

let qn_baryon = function
    | Fermion (Quark (_, c, _, _)) → int_of_csign (c) // 1
    | _ → 0//1
```

Generation is conditional: if we enable the nontrivial CKM matrix, all particles carry generation 0; 0; 0

```
let qn_generation x =
    let qn cs gen =
        let c = int_of_csign (cs) in
        match gen with
            | Gen0 → [c//1; 0//1; 0//1]
            | Gen1 → [0//1; c//1; 0//1]
            | Gen2 → [0//1; 0//1; c//1]
    in
    if Module_options.include_ckm then
        [0//1; 0//1; 0//1]
    else
        match x with
            | Fermion (Lepton (_, c, g, _)) → qn c g
            | Fermion (Quark (_, c, g, _)) → qn c g
            | _ → [0//1; 0//1; 0//1]

let charges x =
    [qn_charge x; qn_lepton x; qn_baryon x] @ (qn_generation x)
```

A variant to represent the different coupling constants, choosen to mimic the FORTRAN part. Required by signature.

```
type constant =
    | G_a_lep | G_a_quark of isospin
    | G_aww | G_aaww
    | G_w_lep of (kkmode × kkmode × generation × kkmode ×
generation)
    | G_w_quark of (kkmode × kkmode × generation × kkmode ×
generation)
    | G_z_lep of (kkmode × kk2 × generation × isospin)
```

| G_z_$quark$ of ($kkmode$ × $kk2$ × $generation$ × $isospin$)
| G_wwz of ($kk2$ × $kkmode$)
| G_$wwzz$ of ($kk2$ × $kk2$)
| G_$wwza$ of ($kk2$ × $kkmode$)
| G_$wwww$ of int
| G_s
| IG_s
| G_$s2$

Two integer counters for the QCD and EW order of the couplings.

type $orders$ = int × int

let $orders$ = function
| _ → $(0, 0)$

Functions for the construction of constants from indices

let g_a_$quark$ x = G_a_$quark$ x
let g_w_lep $kk1$ $kk2$ $gen1$ $kk3$ $gen2$ = G_w_lep ($kk1$, $kk2$, $gen1$, $kk3$, $gen2$)
let g_w_$quark$ $kk1$ $kk2$ $gen1$ $kk3$ $gen2$ = G_w_$quark$ ($kk1$, $kk2$, $gen1$, $kk3$, $gen2$)
let g_z_lep $kk1$ $kk2$ gen iso = G_z_lep ($kk1$, $kk2$, gen, iso)
let g_z_$quark$ $kk1$ $kk2$ gen iso = G_z_$quark$ ($kk1$, $kk2$, gen, iso)
let g_wwz $kk1$ $kk2$ = G_wwz ($kk1$, $kk2$)
let g_$wwzz$ $kk1$ $kk2$ = G_$wwzz$ ($kk1$, $kk2$)
let g_$wwza$ $kk1$ $kk2$ = G_$wwza$ ($kk1$, $kk2$)
let g_$wwww$ nhw = if (nhw ≥ 0) ∧ (nhw ≤ 4) then G_$wwww$ nhw
     else $failwith$ ($modname$ ˆ ".g_wwww:␣invalid␣integer,␣very␣bad")

Build a list of the different constants

let $clist$ = [G_a_lep; G_aww; G_$aaww$] @ ($loop$_iso [g_a_$quark$]) @
($loop$_gen ($cloop$_kk ($loop$_gen ($cloop$_kk ($loop$_kk [g_w_lep]))))) @
($loop$_gen ($cloop$_kk ($loop$_gen ($cloop$_kk ($loop$_kk [g_w_$quark$]))))) @
($loop$_iso ($loop$_gen ($cloop$_$kk2$ ($loop$_kk [g_z_lep])))) @
($loop$_iso ($loop$_gen ($cloop$_$kk2$ ($loop$_kk [g_z_$quark$])))) @
($loop$_kk ($loop$_$kk2$ [g_wwz])) @ ($loop$_$kk2$ ($loop$_$kk2$ [g_$wwzz$])) @
($loop$_kk ($loop$_$kk2$ [g_$wwza$])) @ ($List.map$ g_$wwww$ [0; 1; 2; 3; 4])

Maximum number of lines meeting at a vertex, required by signature.

let max_$degree$ () = 4

Transform a pair of kk identifiers into a kk2 identifier

let get_$kk2$ = function ($Light$, $Light$) → $Light2$ | ($Heavy$, $Heavy$) → $Heavy2$
| ($Light$, $Heavy$) | ($Heavy$, $Light$) → $Light$_$Heavy$

Flip isospin

let $conj$_iso = function Iso_up → Iso_$down$ | Iso_$down$ → Iso_up

Below, lists of couplings are generated which ultimately are joined into a list of all couplings in the model. The generated lists can be viewed using the dump.ml script in the O'Mega toplevel directory.
The individual couplings are defined as 5-tupels resp. 6-tupels consisting in this

order of the particles meeting at the vertex, the coupling type (see `couplings.ml`) and the coupling constant.
List of *llA* type vertices

> let *vertices_all* =
> let *vgen kk gen* =
> ((*Fermion* (*Lepton* (*kk*, *Neg*, *gen*, *Iso_down*)), *Boson A*, *Fermion* (*Lepton* (*kk*, *Pos*, *gen*, *Iso_down*))), *FBF*(1, *Psibar*, *V*, *Psi*), *G_a_lep*)
> in *loop_gen* (*cloop_kk* [*vgen*])

List of *qqA* type vertices

> let *vertices_aqq* =
> let *vgen kk gen iso* =
> ((*Fermion* (*Quark* (*kk*, *Neg*, *gen*, *iso*)), *Boson A*, *Fermion* (*Quark* (*kk*, *Pos*, *gen*, *iso*))), *FBF*(1, *Psibar*, *V*, *Psi*), *G_a_quark iso*)
> in *loop_iso* (*loop_gen* (*cloop_kk* [*vgen*]))

List of $\nu l W$ type vertices

> let *vertices_wll* =
> let *vgen kkw kk_f kk_fbar iso_f gen* =
> ((*Fermion* (*Lepton* (*kk_fbar*, *Neg*, *gen*, *conj_iso iso_f*)),
> *Boson* (*W* (*kkw*, (match *iso_f* with *Iso_up* → *Neg* | _ → *Pos*))),
> *Fermion* (*Lepton* (*kk_f*, *Pos*, *gen*, *iso_f*))),
> *FBF* (1, *Psibar*, *VA2*, *Psi*),
> *G_w_lep* (*kkw*, (match *iso_f* with *Iso_up* → *kk_f* | _ → *kk_fbar*), *gen*,
> (match *iso_f* with *Iso_up* → *kk_fbar* | _ → *kk_f*), *gen*))
> in *loop_gen* (*loop_iso* (*cloop_kk* (*cloop_kk* (*loop_kk* [*vgen*]))))

The same list, but without couplings between the W' and light fermions

> let *vertices_wll_diet* =
> let *filter* = function
> | ((*Fermion* (*Lepton* (*Light*, _, _, _)), *Boson* (*W* (*Heavy*, _)),
> *Fermion* (*Lepton* (*Light*, _, _, _))), _, _) → false
> | _ → true
> in *List.filter filter vertices_wll*

List of *udW* type vertices, flavor-diagonal

> let *vertices_wqq_no_ckm* =
> let *vgen kkw kk_f kk_fbar iso_f gen* =
> ((*Fermion* (*Quark* (*kk_fbar*, *Neg*, *gen*, *conj_iso iso_f*)),
> *Boson* (*W* (*kkw*, (match *iso_f* with *Iso_up* → *Neg* | _ → *Pos*))),
> *Fermion* (*Quark* (*kk_f*, *Pos*, *gen*, *iso_f*))),
> *FBF* (1, *Psibar*, *VA2*, *Psi*),
> *G_w_quark* (*kkw*, (match *iso_f* with *Iso_up* → *kk_f* | _ → *kk_fbar*), *gen*,
> (match *iso_f* with *Iso_up* → *kk_fbar* | _ → *kk_f*), *gen*))
> in *loop_gen* (*loop_iso* (*cloop_kk* (*cloop_kk* (*loop_kk* [*vgen*]))))

The same list, but without couplings between the W' and the first two generations of quarks

> let *vertices_wqq_no_ckm_diet* =
> let *filter* = function
> | ((Fermion (*Quark* (*Light*, _, *gen*, _)), *Boson* (*W* (*Heavy*, _)),
>   *Fermion* (*Quark* (*Light*, _, _, _))), _, _) →
>   (match *gen* with *Gen2* → true | _ → false)
> | _ → true
> in *List.filter filter vertices_wqq_no_ckm*

List of *udW* type vertices, including non flavor-diagonal couplings

> let *vertices_wqq* =
> let *vgen kkw kk_f gen_f kk_fbar gen_fbar iso_f* =
>   ((Fermion (*Quark* (*kk_fbar*, *Neg*, *gen_fbar*, *conj_iso iso_f*)),
>   *Boson* (*W* (*kkw*, (match *iso_f* with *Iso_up* → *Neg* | _ →
> *Pos*))),
>   *Fermion* (*Quark* (*kk_f*, *Pos*, *gen_f*, *iso_f*)),
>   *FBF* (1, *Psibar*, *VA2*, *Psi*),
>   *G_w_quark* (match *iso_f* with
> | *Iso_up* → (*kkw*, *kk_f*, *gen_f*, *kk_fbar*, *gen_fbar*)
> | *Iso_down* → (*kkw*, *kk_fbar*, *gen_fbar*, *kk_f*, *gen_f*)))
> in *loop_iso* (*loop_gen* (*cloop_kk* (*loop_gen* (*cloop_kk* (*loop_kk* [*vgen*])))))

List of *llZ* / $\nu\nu Z$ type vertices

> let *vertices_zll* =
> let *vgen kkz kk_f kk_fbar gen iso* =
>   ((Fermion (*Lepton* (*kk_fbar*, *Neg*, *gen*, *iso*)), *Boson* (*Z kkz*),
>   *Fermion* (*Lepton* (*kk_f*, *Pos*, *gen*, *iso*))),
>   *FBF* (1, *Psibar*, *VA2*, *Psi*),
>   *G_z_lep* (*kkz*, *get_kk2* (*kk_f*, *kk_fbar*), *gen*, *iso*))
> in *loop_iso* (*loop_gen* (*cloop_kk* (*cloop_kk* (*loop_kk* [*vgen*]))))

List of *qqZ* type vertices

> let *vertices_zqq* =
> let *vgen kkz kk_f kk_fbar gen iso* =
>   ((Fermion (*Quark* (*kk_fbar*, *Neg*, *gen*, *iso*)), *Boson* (*Z kkz*),
>   *Fermion* (*Quark* (*kk_f*, *Pos*, *gen*, *iso*))),
>   *FBF* (1, *Psibar*, *VA2*, *Psi*),
>   *G_z_quark* (*kkz*, *get_kk2* (*kk_f*, *kk_fbar*), *gen*, *iso*))
> in *loop_iso* (*loop_gen* (*cloop_kk* (*cloop_kk* (*loop_kk* [*vgen*]))))

$gq\bar{q}$

> let *vertices_gqq* =
> let *vgen kk gen iso* =
>   ((Fermion (*Quark* (*kk*, *Neg*, *gen*, *iso*)), *Boson* *G*, *Fermion* (*Quark* (*kk*, *Pos*, *gen*, *iso*))),
>   *FBF* (1, *Psibar*, *V*, *Psi*), *G_s*)
> in *loop_iso* (*loop_gen* (*cloop_kk* [*vgen*]))

AWW

> let *vertices_aww* =

let *vgen kk* =
  ((*Boson A*, *Boson* (*W* (*kk*, *Pos*)), *Boson* (*W* (*kk*, *Neg*))), *Gauge_Gauge_Gauge* 1, *G_aww*)
in *loop_kk* [*vgen*]

ZWW

let *vertices_zww* =
let *vgen kkz kkwp kkwm* =
  ((*Boson* (*Z kkz*), *Boson* (*W* (*kkwp*, *Pos*)), *Boson* (*W* (*kkwm*, *Neg*))), *Gauge_Gauge_Gauge* 1,
    G_wwz (*get_kk2* (*kkwp*, *kkwm*), *kkz*))
in *loop_kk* (*loop_kk* (*loop_kk* [*vgen*]))

ggg

let *vertices_ggg* = [(*Boson G*, *Boson G*, *Boson G*), *Gauge_Gauge_Gauge* (−1), *IG_s*]

Stolen from Modellib.SM; the signs seem to be OK. See `couplings.ml` for more docs.

let *gauge4* = *Vector4* [(2, *C_13_42*); (−1, *C_12_34*); (−1, *C_14_23*)]
let *minus_gauge4* = *Vector4* [(−2, *C_13_42*); (1, *C_12_34*); (1, *C_14_23*)]

AAWW

let *vertices_aaww* =
let *vgen kk* =
  ((*Boson A*, *Boson* (*W* (*kk*, *Pos*)), *Boson A*, *Boson* (*W* (*kk*, *Neg*))), *minus_gauge4*, *G_aaww*)
in *loop_kk* [*vgen*]

WWZZ

let *vertices_wwzz* =
let *vgen kkwp kkwm kk2z* =
  ((*Boson* (*Z* (match *kk2z* with *Heavy2* → *Heavy* | *Light2* |
Light_Heavy → *Light*)),
        Boson (*W* (*kkwp*, *Pos*)),
        Boson (*Z* (match *kk2z* with *Heavy2* | *Light_Heavy* → *Heavy* |
Light2 → *Light*)),
        Boson (*W* (*kkwm*, *Neg*))), *minus_gauge4*, *G_wwzz* (*get_kk2* (*kkwp*, *kkwm*), *kk2z*))
in *loop_kk2* (*loop_kk* (*loop_kk* [*vgen*]))

WWZA

let *vertices_wwza* =
let *vgen kkwp kkwm kkz* =
  ((*Boson A*, *Boson* (*W* (*kkwp*, *Pos*)), *Boson* (*Z kkz*), *Boson* (*W* (*kkwm*, *Neg*))),
        minus_gauge4, *G_wwza* (*get_kk2* (*kkwp*, *kkwm*), *kkz*))
in *loop_kk* (*loop_kk* (*loop_kk* [*vgen*]))

WWWW

let *vertices_wwww* =
let *count* = function *Light2* → 0 | *Light_Heavy* → 1 | *Heavy2* → 2
in let *vgen kk2wp kk2wm* =
  ((*Boson* (*W* ((match *kk2wp* with *Heavy2* → *Heavy* | *Light2* |
Light_Heavy → *Light*), *Pos*)),
        Boson (*W* ((match *kk2wm* with *Heavy2* → *Heavy* | *Light2* |
Light_Heavy → *Light*), *Neg*)),

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            Boson (*W* ((match *kk2wp* with *Heavy2* | *Light_Heavy* → *Heavy* |
Light2 → *Light*), *Pos*)),
                Boson (*W* ((match *kk2wm* with *Heavy2* | *Light_Heavy* →
Heavy | *Light2* → *Light*), *Neg*))),
                gauge4, *G_wwww* ((*count kk2wp*) + (*count kk2wm*)))
        in *loop_kk2* (*loop_kk2* [*vgen*])

gggg

        let *vertices_gggg* = [(*Boson G*, *Boson G*, *Boson G*, *Boson G*), *gauge4*, *G_s2*]

The list of couplings is transformed into the fusion lists required by the generator
by the Model.Fusions functor.
This is copy& paste from the other models; check again with Thorsten if it is
correct

        module *F* = *Modeltools.Fusions* (struct
            type *f* = *flavor*
            type *c* = *constant*
            let *compare* = *compare*
            let *conjugate* = *conjugate*
        end)

Not sure yet whether F.fusex also creates the conjugate vertices; by looking at
the implementation of the other models, I assume it doesn't. Still, better ask
Thorsten to be sure!!!
Update: Still didn't get to ask, but since the results are consistent, I suspect
my assertion is correct.
The stuff below is required by the signature.

        let *vertices* () = (*vertices_all* @ *vertices_aqq* @
            (match *Module_options.diet* with
                | false → *vertices_wll*
                | true → *vertices_wll_diet*) @
            (match (*Module_options.include_ckm*, *Module_options.diet*) with
                | (true, false) → *vertices_wqq*
                | (false, false) → *vertices_wqq_no_ckm*
                | (false, true) → *vertices_wqq_no_ckm_diet*
                | (true, true) → *raise* (*Failure*
                    ("Modules4.Threeshl.vertices:␣CKM␣matrix␣together␣with␣option␣diet␣is␣not" ^
                    "␣implemented␣yet!"))) @
            vertices_zll @ *vertices_zqq* @ *vertices_aww* @ *vertices_zww* @ *vertices_gqq* @ *vertices_ggg*,
            vertices_aaww @ *vertices_wwzz* @ *vertices_wwza* @ *vertices_wwww* @ *vertices_gggg*
            , [])
        let *table* = *F.of_vertices* (*vertices* ())
        let *fuse2* = *F.fuse2 table*
        let *fuse3* = *F.fuse3 table*
        let *fuse* = *F.fuse table*

A function that returns a list of a flavours known to the model, required by the
signature.

        let *flavors* () = (*List.map flavor_of_f* (*all_leptons* @ *all_quarks*)) @
            (*List.map flavor_of_b all_bosons*)

dito, external flavours, also required.

> let *external_flavors* () = [
>   "light␣leptons", *List.map flavor_of_f* (*loop_iso* (*loop_gen*(*loop_cs* [*lepton Light*])));
>   "light␣quarks", *List.map flavor_of_f* (*loop_iso* (*loop_gen*(*loop_cs* [*quark Light*])));
>   "light␣gauge␣bosons", *List.map flavor_of_b* [*W* (*Light*, *Pos*); *W* (*Light*, *Neg*); *Z Light*; *A*];
>   "heavy␣gauge␣bosons", *List.map flavor_of_b* [*W* (*Heavy*, *Pos*); *W* (*Heavy*, *Neg*); *Z Heavy*]] @
>   (match *Module_options.include_hf* with
> | true → [
>   "heavy␣leptons", *List.map flavor_of_f* (*loop_iso* (*loop_gen*(*loop_cs* [*lepton Heavy*])));
>   "heavy␣quarks", *List.map flavor_of_f* (*loop_iso* (*loop_gen*(*loop_cs* [*quark Heavy*])))]
> | false → []) @ ["gluons", [*Boson G*]]

Which of the particles are goldstones? → none. Required by the signature.

> let *goldstone x* = *None*

This is wrong but handy for debugging the constant identifier generation via -params. Usually, this function would return a record consisting of the parameters as well as expression for the dependent quantities that can be used to generate FORTRAN code for calculating them. However, we have a seperate module for the threeshl, so we can abuse this for debugging. Required by signature.

> let *parameters* () = {*input* = *List.map* (fun *x* → (*x*, 0.)) *clist*;
>   *derived* = []; *derived_arrays* = []}

Convert a flavour into a ID string with which it will be referred by the user interface of the compiled generator. Required by signature

> let *flavor_to_string* =
> let *prefix* = function
> | *Fermion* (*Lepton* (*Heavy*, _, _, _)) | *Fermion* (*Quark* (*Heavy*, _, _, _))
> | *Boson* (*W* (*Heavy*, _)) | *Boson* (*Z Heavy*) → "H"
> | _ → ""
> in let *postfix* = function
> | *Fermion* (*Lepton* (_, *cs*, _, *Iso_down*)) → (match *cs* with *Pos* →
> "-" | *Neg* → "+")
> | *Fermion* (*Quark* (_, *Neg*, _, _)) | *Fermion* (*Lepton* (_, *Neg*, _, *Iso_up*)) →
> "bar"
> | *Boson* (*W* (_, *cs*)) → (match *cs* with *Pos* → "+" | *Neg* → "-")
> | _ → ""
> in let *rump* = function
> | *Fermion* (*Lepton desc*) → (match *desc* with
> | (_, _, *Gen0*, *Iso_up*) → "nue"
> | (_, _, *Gen0*, *Iso_down*) → "e"
> | (_, _, *Gen1*, *Iso_up*) → "numu"
> | (_, _, *Gen1*, *Iso_down*) → "mu"
> | (_, _, *Gen2*, *Iso_up*) → "nutau"
> | (_, _, *Gen2*, *Iso_down*) → "tau")
> | *Fermion* (*Quark desc*) → (match *desc* with
> | (_, _, *Gen0*, *Iso_up*) → "u"
> | (_, _, *Gen0*, *Iso_down*) → "d"
> | (_, _, *Gen1*, *Iso_up*) → "c"

```
              | (_, _, Gen1, Iso_down)  →  "s"
              | (_, _, Gen2, Iso_up)  →  "t"
              | (_, _, Gen2, Iso_down)  →  "b")
       | Boson (W _)  →  "W" |  Boson (Z _)  →  "Z" |  Boson A  →  "A" |
 Boson G  →  "gl"
       in function  x  →  (prefix x) ^ (rump x) ^ (postfix x)
```

Conversion of the ID string into a particle flavor. Instead of going through all cases again, we generate a "dictionary" of flavor / ID pairs which we use to identify the correct flavor. Required by signature.

```
       let flavor_of_string x  =
       let dict  =  List.map (fun x  →  (x, flavor_to_string x)) (flavors ())
       in let get_ident  =  function (x, _)  →  x
       in try
              get_ident (List.find (fun (_, y)  →  (x = y)) dict)
           with
              Not_found  →  (match x with
                  | "g" →  Boson G
                  | _  →  invalid_arg (modname ^ ".flavor_of_string")
              )
```

Converts a flavor into a symbol used as identification in the generated FOR-TRAN code (has to comply to the conventions of valid FORTRAN identifiers therefore). We stick to the same convenctions as SM3, prefixing heavy modes with a H. Required by signature.

```
       let flavor_symbol  =
       let prefix  =  function
           | Fermion (Lepton (Heavy, _, _, _)) |  Fermion (Quark (Heavy, _, _, _))
           | Boson (W (Heavy, _)) |  Boson (Z Heavy)  →  "H"
           | _  →  ""
       in let postfix  =  function
           | Fermion (Lepton (_, Neg, _, _)) |  Fermion (Quark (_, Neg, _, _))  →
"b"
           | _  →  ""
       in let rump  =  function
           | Fermion spec  →  (match spec with
               | Lepton (_, _, gen, Iso_up)  →  "n" ^ (string_of_int (int_of_gen gen))
               | Lepton (_, _, gen, Iso_down)  →  "l" ^ (string_of_int (int_of_gen gen))
               | Quark (_, _, gen, Iso_up)  →  "u" ^ (string_of_int (int_of_gen gen))
               | Quark (_, _, gen, Iso_down)  →  "d" ^ (string_of_int (int_of_gen gen)))
           | Boson spec  →  (match spec with
               | W (_, Pos)  →  "wp" |  W (_, Neg)  →  "wm"
               | Z _  →  "z" |  A  →  "a" |  G  →  "gl" )
       in function
              x  →  (prefix x) ^ (rump x) ^ (postfix x)
```

Generate TeX for a flavor

```
       let flavor_to_TeX  =
       let bar x y  =  match x with Neg  →  "\\overline{" ^ y ^ "}" |  Pos  →
y
```

in let *pm x y* = match *x* with *Neg* → `"{"` ˆ *y* ˆ `"}^+"` | *Pos* →
`"{"` ˆ *y* ˆ `"}^-"`
      in let *prime x y* = match *x* with *Light* → *y* | *Heavy* → `"{"` ˆ *y* ˆ `"}^\\prime"`
      in function
          | *Fermion* (*Lepton desc*) → (match *desc* with
             | (*kk*, *cs*, *gen*, *Iso_up*) → *prime kk* (*bar cs* (match *gen* with
               | *Gen0* → `"\\nu_e"`
               | *Gen1* → `"\\nu_\\mu"`
               | *Gen2* → `"\\nu_\\tau"`))
             | (*kk*, *cs*, *gen*, *Iso_down*) → *prime kk* (*pm cs* (match *gen* with
               | *Gen0* → `"e"` | *Gen1* → `"\\mu"` | *Gen2* → `"\\tau"`)))
          | *Fermion* (*Quark* (*kk*, *cs*, *gen*, *iso*)) → *prime kk* (*bar cs* (match (*gen*, *iso*) with
             | (*Gen0*, *Iso_up*) → `"u"`
             | (*Gen0*, *Iso_down*) → `"d"`
             | (*Gen1*, *Iso_up*) → `"c"`
             | (*Gen1*, *Iso_down*) → `"s"`
             | (*Gen2*, *Iso_up*) → `"t"`
             | (*Gen2*, *Iso_down*) → `"b"`))
          | *Boson spec* → (match *spec* with
             | *W* (*kk*, *cs*) → *prime kk* (*pm* (match *cs* with *Pos* → *Neg* |
Neg → *Pos*) `"W"`)
             | *Z kk* → *prime kk* `"Z"`
             | *A* → `"A"` | *G* → `"g"`)

Returns the string referring to the particle mass in the generated FORTRAN
code. Required by signature.

      let *mass_symbol* = function
        | *Boson A* | *Boson G* → `"0._default"`
        | *x* → `"mass_array("` ˆ (*bcdi_of_flavor x*) ˆ `")"`

Dito, for width. Required by signature.

      let *width_symbol* = function
        | *Boson A* | *Boson G* → `"0._default"`
        | *x* → `"width_array("` ˆ (*bcdi_of_flavor x*) ˆ `")"`

Determines the string referring to a coupling constant in the generated FOR-
TRAN code. Required by signature.

      let *constant_symbol* =
      let *c* = `", "`
      in let *g_w_ferm* = function
        (*kk1*, *kk2*, *gen1*, *kk3*, *gen2*) →
          `":, "` ˆ (*fspec_of_kkmode kk1*) ˆ *c* ˆ (*fspec_of_kkmode kk2*) ˆ *c* ˆ (*fspec_of_gen gen1*) ˆ *c* ˆ
          (*fspec_of_kkmode kk3*) ˆ *c* ˆ (*fspec_of_gen gen2*)
      in let *g_z_ferm* = function
        (*kk1*, *kk2*, *gen*, *iso*) →
          `":, "` ˆ (*fspec_of_kkmode kk1*) ˆ *c* ˆ (*fspec_of_kk2 kk2*) ˆ *c* ˆ (*fspec_of_gen gen*) ˆ *c* ˆ
          (*fspec_of_iso iso*)
      in function
        | *G_a_lep* → `"g_a_lep"`
        | *G_s* → `"g_s_norm"`

```
        | IG_s → "ig_s_norm"
        | G_s2 → "g_s_norm2"
        | G_a_quark iso → "g_a_quark(" ^ (fspec_of_iso iso) ^ ")"
        | G_aww → "ig_aww"
        | G_aaww → "g_aaww"
        | G_w_lep spec → "g_w_lep_va(" ^ (g_w_ferm spec) ^ ")"
        | G_w_quark spec → "g_w_quark_va(" ^ (g_w_ferm spec) ^ ")"
        | G_z_lep spec → "g_z_lep_va(" ^ (g_z_ferm spec) ^ ")"
        | G_z_quark spec → "g_z_quark_va(" ^ (g_z_ferm spec) ^ ")"
        | G_wwz (kk1, kk2) → "ig_wwz(" ^ (fspec_of_kk2 kk1) ^ c ^
            (fspec_of_kkmode kk2) ^ ")"
        | G_wwzz (kk1, kk2) → "g_wwzz(" ^ (fspec_of_kk2 kk1) ^ c ^
            (fspec_of_kk2 kk2) ^ ")"
        | G_wwza (kk1, kk2) → "g_wwza(" ^(fspec_of_kk2 kk1) ^ c ^
            (fspec_of_kkmode kk2) ^ ")"
        | G_wwww nhw → if (0 ≤ nhw) ∧ (nhw ≤ 4) then
            "g_wwww(" ^ (string_of_int nhw) ^ ")"
            else failwith "Modules4.Threeshl.constant_symbol:␣invalid␣int␣for␣G_wwww;␣very␣bad'
    end
```

13.6.3 2HDM with and without non-trivial flavor structure

```
module type THDM_flags =
  sig
    val ckm_present : bool
  end

module THDM : THDM_flags =
  struct
    let ckm_present = false
  end

module THDM_CKM : THDM_flags =
  struct
    let ckm_present = true
  end

module TwoHiggsDoublet (Flags : THDM_flags) =
  struct
    let rcs = rcs_file

    open Coupling

    let default_width = ref Timelike
    let use_fudged_width = ref false

    let options = Options.create
      [ "constant_width", Arg.Unit (fun () → default_width := Constant),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", Arg.Set use_fudged_width,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", Arg.String (fun f → default_width := Custom f),
```

```
        "use␣custom␣width";
        "cancel_widths", Arg.Unit (fun () → default_width := Vanishing),
        "use␣vanishing␣width" ]
```

type *matter_field* = *L* of *int* | *N* of *int* | *U* of *int* | *D* of *int*
type *gauge_boson* = *Ga* | *Wp* | *Wm* | *Z* | *Gl*
type *other* = *Phip* | *Phim* | *Phi0* | *Hh* | *HA* | *HH* | *Hp* | *Hm*
type *flavor* = *M* of *matter_field* | *G* of *gauge_boson* | *O* of *other*

let *matter_field f* = *M f*
let *gauge_boson f* = *G f*
let *other f* = *O f*

type *field* =
   | *Matter* of *matter_field*
   | *Gauge* of *gauge_boson*
   | *Other* of *other*

let *field* = function
   | *M f* → *Matter f*
   | *G f* → *Gauge f*
   | *O f* → *Other f*

type *gauge* = *unit*

let *gauge_symbol* () =
   failwith "Modellib_BSM.TwoHiggsDoublet.gauge_symbol:␣internal␣error"

let *family n* = *List.map matter_field* [*L n*; *N n*; *U n*; *D n*]

let *external_flavors* () =
   ["1st␣Generation", *ThoList.flatmap family* [1; −1];
     "2nd␣Generation", *ThoList.flatmap family* [2; −2];
     "3rd␣Generation", *ThoList.flatmap family* [3; −3];
     "Gauge␣Bosons", *List.map gauge_boson* [*Ga*; *Z*; *Wp*; *Wm*; *Gl*];
     "Higgs", *List.map other* [*Hh*; *HH*; *HA*; *Hp*; *Hm*];
     "Goldstone␣Bosons", *List.map other* [*Phip*; *Phim*; *Phi0*]]

let *flavors* () = *ThoList.flatmap snd* (*external_flavors* ())

let *spinor n* =
   if *n* ≥ 0 then
     Spinor
   else
     ConjSpinor

let *lorentz* = function
   | *M f* →
       begin match *f* with
       | *L n* → *spinor n* | *N n* → *spinor n*
       | *U n* → *spinor n* | *D n* → *spinor n*
       end
   | *G f* →
       begin match *f* with
       | *Ga* | *Gl* → *Vector*
       | *Wp* | *Wm* | *Z* → *Massive_Vector*
```

```
          end
    | O f → Scalar

let color = function
    | M (U n) → Color.SUN (if n > 0 then 3 else -3)
    | M (D n) → Color.SUN (if n > 0 then 3 else -3)
    | G Gl → Color.AdjSUN 3
    | _ → Color.Singlet

let prop_spinor n =
    if n ≥ 0 then
        Prop_Spinor
    else
        Prop_ConjSpinor

let propagator = function
    | M f →
        begin match f with
        | L n → prop_spinor n | N n → prop_spinor n
        | U n → prop_spinor n | D n → prop_spinor n
        end
    | G f →
        begin match f with
        | Ga | Gl → Prop_Feynman
        | Wp | Wm | Z → Prop_Unitarity
        end
    | O f →
        begin match f with
        | Phip | Phim | Phi0 → Only_Insertion
        | Hh | HH | HA | Hp | Hm → Prop_Scalar
        end
```

Optionally, ask for the fudge factor treatment for the widths of charged particles.
Currently, this only applies to $W^{\pm}$ and top.

```
let width f =
    if !use_fudged_width then
        match f with
        | G Wp | G Wm | M (U 3) | M (U (−3)) → Fudged
        | _ → !default_width
    else
        !default_width

let goldstone = function
    | G f →
        begin match f with
        | Wp → Some (O Phip, Coupling.Const 1)
        | Wm → Some (O Phim, Coupling.Const 1)
        | Z → Some (O Phi0, Coupling.Const 1)
        | _ → None
        end
    | _ → None
```

```
let conjugate = function
  | M f →
      M (begin match f with
      | L n → L (−n) | N n → N (−n)
      | U n → U (−n) | D n → D (−n)
      end)
  | G f →
      G (begin match f with
      | Gl → Gl | Ga → Ga | Z → Z
      | Wp → Wm | Wm → Wp
      end)
  | O f →
      O (begin match f with
      | Phip → Phim | Phim → Phip | Phi0 → Phi0
      | Hh → Hh | HH → HH | HA → HA
      | Hp → Hm | Hm → Hp
      end)

let fermion = function
  | M f →
      begin match f with
      | L n → if n > 0 then 1 else -1
      | N n → if n > 0 then 1 else -1
      | U n → if n > 0 then 1 else -1
      | D n → if n > 0 then 1 else -1
      end
  | G f →
      begin match f with
      | Gl | Ga | Z | Wp | Wm → 0
      end
  | O _ → 0
```

Electrical charge, lepton number, baryon number. We could avoid the rationals altogether by multiplying the first and last by 3 . . .

```
module Ch = Charges.QQ
let ( // ) = Algebra.Small_Rational.make

let generation' = function
  | 1 → [ 1//1; 0//1; 0//1]
  | 2 → [ 0//1; 1//1; 0//1]
  | 3 → [ 0//1; 0//1; 1//1]
  | − 1 → [−1//1; 0//1; 0//1]
  | − 2 → [ 0//1; − 1//1; 0//1]
  | − 3 → [ 0//1; 0//1; − 1//1]
  | n → invalid_arg ("Modellib_BSM.TwoHiggsDoublet.generation':␣" ^ string_of_int n)

let generation f =
  if Flags.ckm_present then
    []
  else
    match f with
      | M (L n | N n | U n | D n) → generation' n
```

```
                | G _ | O _ → [ 0//1; 0//1; 0//1]
let charge = function
  | M f →
      begin match f with
      | L n → if n > 0 then -1//1 else 1//1
      | N n → 0//1
      | U n → if n > 0 then 2//3 else -2//3
      | D n → if n > 0 then -1//3 else 1//3
      end
  | G f →
      begin match f with
      | Gl | Ga | Z → 0//1
      | Wp → 1//1
      | Wm → − 1//1
      end
  | O f →
      begin match f with
      | Hh | HH | HA | Phi0 → 0//1
      | Hp | Phip → 1//1
      | Hm | Phim → − 1//1
      end

let lepton = function
  | M f →
      begin match f with
      | L n | N n → if n > 0 then 1//1 else -1//1
      | U _ | D _ → 0//1
      end
  | G _ | O _ → 0//1

let baryon = function
  | M f →
      begin match f with
      | L _ | N _ → 0//1
      | U n | D n → if n > 0 then 1//1 else -1//1
      end
  | G _ | O _ → 0//1

let charges f =
    [ charge f; lepton f; baryon f] @ generation f

type constant =
  | Unit | Pi | Alpha_QED | Sin2thw
  | Sinthw | Costhw | E | G_weak | Vev
  | Q_lepton | Q_up | Q_down | G_CC
  | G_NC_neutrino | G_NC_lepton | G_NC_up | G_NC_down
  | I_Q_W | I_G_ZWW | I_G_WWW
  | G_WWWW | G_ZZWW | G_AZWW | G_AAWW
  | G_htt | G_hbb | G_hcc | G_htautau | G_hmumu
  | G_Htt | G_Hbb | G_Hcc | G_Htautau | G_Hmumu
  | I_G_Att | I_G_Abb | I_G_Acc | I_G_Atautau | I_G_Amumu
  | G_Htb | G_Hcs | G_Htaunu | G_Hmunu
```

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| *Gs* | *I_Gs* | *G2*
| *G_AHpHm* | *G_ZHpHm* | *G_Zh1h2* | *G_Zh1h3* | *G_Zh2h3*
| *G_WpHmh1* | *G_WpHmh2* | *G_WpHmh3* | *G_WmHph1* |
G_WmHph2
| *G_WmHph3* | *G_h1ZZ* | *G_h2ZZ* | *G_h3ZZ* | *G_h1WpWm* |
G_h2WpWm
| *G_h3WpWm* | *G_hhWpWm* | *G_hhZZ* | *G_HpHmAA* |
G_HpHmZZ | *G_HpHmAZ*
| *G_HpHmWpWm* | *G_h1HpAWm* | *G_h2HpAWm* | *G_h3HpAWm* |
G_h1HpZWm
| *G_h2HpZWm* | *G_h3HpZWm* | *G_h1HpAWmC* | *G_h2HpAWmC* |
G_h3HpAWmC
| *G_h1HpZWmC* | *G_h2HpZWmC* | *G_h3HpZWmC* | *G_h1HpHm* |
G_h2HpHm | *G_h3HpHm*
| *G_h111* | *G_h112* | *G_h113* | *G_h221* | *G_h222* | *G_h223* |
G_h331 | *G_h332*
| *G_h333* | *G_h123* | *G_HpHmHpHm* | *G_HpHm11* | *G_HpHm12* |
G_HpHm13
| *G_HpHm22* | *G_HpHm23* | *G_HpHm33* | *G_h1111* | *G_h1112* |
G_h1113
| *G_h1122* | *G_h1123* | *G_h1133* | *G_h1222* | *G_h1223* |
G_h1233 | *G_h1333*
| *G_h2222* | *G_h2223* | *G_h2233* | *G_h2333* | *G_h3333*
| *G_h1uu* | *G_h2uu* | *G_h3uu* | *G_h1uc* | *G_h2uc* | *G_h3uc* |
G_h1ut
| *G_h2ut* | *G_h3ut* | *G_h1cu* | *G_h2cu* | *G_h3cu* | *G_h1cc* |
G_h2cc
| *G_h3cc* | *G_h1ct* | *G_h2ct* | *G_h3ct* | *G_h1tu* | *G_h2tu* |
G_h3tu
| *G_h1tc* | *G_h2tc* | *G_h3tc* | *G_h1tt* | *G_h2tt* | *G_h3tt*
| *G_h1dd* | *G_h2dd* | *G_h3dd* | *G_h1ds* | *G_h2ds* | *G_h3ds* |
G_h1db
| *G_h2db* | *G_h3db* | *G_h1sd* | *G_h2sd* | *G_h3sd* | *G_h1ss* |
G_h2ss
| *G_h3ss* | *G_h1sb* | *G_h2sb* | *G_h3sb* | *G_h1bd* | *G_h2bd* |
G_h3bd
| *G_h1bs* | *G_h2bs* | *G_h3bs* | *G_h1bb* | *G_h2bb* | *G_h3bb*
| *G_hud* | *G_hus* | *G_hub* | *G_hcd* | *G_hcs* | *G_hcb* | *G_htd* |
G_hts | *G_htb*
| *G_hdu* | *G_hdc* | *G_hdt* | *G_hsu* | *G_hsc* | *G_hst* | *G_hbu* |
G_hbc | *G_hbt*
| *G_he1n1* | *G_he1n2* | *G_he1n3* | *G_he2n1* | *G_he2n2* |
G_he2n3 | *G_he3n1*
| *G_he3n2* | *G_he3n3* | *G_hn1e1* | *G_hn1e2* | *G_hn1e3* |
G_hn2e1 | *G_hn2e2*
| *G_hn2e3* | *G_hn3e1* | *G_hn3e2* | *G_hn3e3*
| *G_h1e1e1* | *G_h2e1e1* | *G_h3e1e1* | *G_h1e1e2* | *G_h2e1e2* |
G_h3e1e2
| *G_h1e1e3* | *G_h2e1e3* | *G_h3e1e3* | *G_h1e2e1* | *G_h2e2e1* |
G_h3e2e1

    | *G_h1e2e2* | *G_h2e2e2* | *G_h3e2e2* | *G_h1e2e3* | *G_h2e2e3* |
G_h3e2e3
    | *G_h1e3e1* | *G_h2e3e1* | *G_h3e3e1* | *G_h1e3e2* | *G_h2e3e2* |
G_h3e3e2
    | *G_h1e3e3* | *G_h2e3e3* | *G_h3e3e3*
    | *Mass* of *flavor* | *Width* of *flavor*

Two integer counters for the QCD and EW order of the couplings.

type *orders* = *int* × *int*

let *orders* = function
  | _ → (0, 0)

let *g_over_2_costh* =
  Quot (*Neg* (*Atom G_weak*), *Prod* [*Const* 2; *Atom Costhw*])

let *nc_coupling c t3 q* =
  (*Real_Array c*,
   [*Prod* [*g_over_2_costh*; *Diff* (*t3*, *Prod* [*Const* 2; *q*; *Atom Sin2thw*])];
    Prod [*g_over_2_costh*; *t3*]])

let *half* = *Quot* (*Const* 1, *Const* 2)

let *array_list* = [*G_h1uu*; *G_h2uu*; *G_h3uu*; *G_h1uc*; *G_h2uc*; *G_h3uc*; *G_h1ut*;
   G_h2ut; *G_h3ut*; *G_h1cu*; *G_h2cu*; *G_h3cu*; *G_h1cc*; *G_h2cc*;
   G_h3cc; *G_h1ct*; *G_h2ct*; *G_h3ct*; *G_h1tu*; *G_h2tu*; *G_h3tu*;
   G_h1tc; *G_h2tc*; *G_h3tc*; *G_h1tt*; *G_h2tt*; *G_h3tt*;
   G_h1dd; *G_h2dd*; *G_h3dd*; *G_h1ds*; *G_h2ds*; *G_h3ds*; *G_h1db*;
   G_h2db; *G_h3db*; *G_h1sd*; *G_h2sd*; *G_h3sd*; *G_h1ss*; *G_h2ss*;
   G_h3ss; *G_h1sb*; *G_h2sb*; *G_h3sb*; *G_h1bd*; *G_h2bd*; *G_h3bd*;
   G_h1bs; *G_h2bs*; *G_h3bs*; *G_h1bb*; *G_h2bb*; *G_h3bb*;
   G_hud; *G_hus*; *G_hub*; *G_hcd*; *G_hcs*; *G_hcb*; *G_htd*; *G_hts*; *G_htb*;
   G_hdu; *G_hdc*; *G_hdt*; *G_hsu*; *G_hsc*; *G_hst*; *G_hbu*; *G_hbc*; *G_hbt*;
   G_he1n1; *G_he1n2*; *G_he1n3*; *G_he2n1*; *G_he2n2*; *G_he2n3*; *G_he3n1*;
   G_he3n2; *G_he3n3*; *G_hn1e1*; *G_hn1e2*; *G_hn1e3*; *G_hn2e1*; *G_hn2e2*;
   G_hn2e3; *G_hn3e1*; *G_hn3e2*; *G_hn3e3*;
   G_h1e1e1; *G_h2e1e1*; *G_h3e1e1*; *G_h1e1e2*; *G_h2e1e2*; *G_h3e1e2*;
   G_h1e1e3; *G_h2e1e3*; *G_h3e1e3*; *G_h1e2e1*; *G_h2e2e1*; *G_h3e2e1*;
   G_h1e2e2; *G_h2e2e2*; *G_h3e2e2*; *G_h1e2e3*; *G_h2e2e3*; *G_h3e2e3*;
   G_h1e3e1; *G_h2e3e1*; *G_h3e3e1*; *G_h1e3e2*; *G_h2e3e2*; *G_h3e3e2*;
   G_h1e3e3; *G_h2e3e3*; *G_h3e3e3*]

let *add_complex_array_tag c* = (*Complex_Array c*, [*Const* 0; *Const* 0])

let *derived_parameter_arrays* =
  [*nc_coupling G_NC_neutrino half* (*Const* 0);
   nc_coupling G_NC_lepton (*Neg half*) (*Const* (−1));
   nc_coupling G_NC_up half (*Quot* (*Const* 2, *Const* 3));
   nc_coupling G_NC_down (*Neg half*) (*Quot* (*Const* (−1), *Const* 3));
   ] @ (*List.map add_complex_array_tag array_list*)

let *parameters* () =
  { *input* = []; *derived* = []; *derived_arrays* = *derived_parameter_arrays* }

```
module F = Modeltools.Fusions (struct
  type f = flavor
  type c = constant
  let compare = compare
  let conjugate = conjugate
end)
```

$$\mathcal{L}_{\mathrm{EM}} = -e \sum_i q_i \bar{\psi}_i \slashed{A} \psi_i \tag{13.50}$$

```
let mgm ((m1, g, m2), fbf, c) = ((M m1, G g, M m2), fbf, c)
let electromagnetic_currents n =
  List.map mgm
    [ ((L (−n), Ga, L n), FBF (1, Psibar, V, Psi), Q_lepton);
      ((U (−n), Ga, U n), FBF (1, Psibar, V, Psi), Q_up);
      ((D (−n), Ga, D n), FBF (1, Psibar, V, Psi), Q_down) ]

let color_currents n =
  List.map mgm
    [ ((U (−n), Gl, U n), FBF (1, Psibar, V, Psi), Gs);
      ((D (−n), Gl, D n), FBF (1, Psibar, V, Psi), Gs) ]
```

$$\mathcal{L}_{\mathrm{NC}} = -\frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i \slashed{Z}(g_V^i - g_A^i \gamma_5)\psi_i \tag{13.51}$$

```
let neutral_currents n =
  List.map mgm
    [ ((L (−n), Z, L n), FBF (1, Psibar, VA, Psi), G_NC_lepton);
      ((N (−n), Z, N n), FBF (1, Psibar, VA, Psi), G_NC_neutrino);
      ((U (−n), Z, U n), FBF (1, Psibar, VA, Psi), G_NC_up);
      ((D (−n), Z, D n), FBF (1, Psibar, VA, Psi), G_NC_down) ]
```

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i (T^+ \slashed{W}^+ + T^- \slashed{W}^-)(1-\gamma_5)\psi_i \tag{13.52}$$

```
let charged_currents n =
  List.map mgm
    [ ((L (−n), Wm, N n), FBF (1, Psibar, VL, Psi), G_CC);
      ((N (−n), Wp, L n), FBF (1, Psibar, VL, Psi), G_CC);
      ((D (−n), Wm, U n), FBF (1, Psibar, VL, Psi), G_CC);
      ((U (−n), Wp, D n), FBF (1, Psibar, VL, Psi), G_CC) ]

let yukawa =
  [ ((M (U (−1)), O Hh, M (U 1)), FBF (1, Psibar, SP, Psi), G_h1uu);
    ((M (U (−1)), O HH, M (U 1)), FBF (1, Psibar, SP, Psi), G_h2uu);
    ((M (U (−1)), O HA, M (U 1)), FBF (1, Psibar, SP, Psi), G_h3uu);
    ((M (U (−2)), O Hh, M (U 2)), FBF (1, Psibar, SP, Psi), G_h1cc);
    ((M (U (−2)), O HH, M (U 2)), FBF (1, Psibar, SP, Psi), G_h2cc);
    ((M (U (−2)), O HA, M (U 2)), FBF (1, Psibar, SP, Psi), G_h3cc);
    ((M (U (−3)), O Hh, M (U 3)), FBF (1, Psibar, SP, Psi), G_h1tt);
```

$((M\ (U\ (-3)), O\ HH, M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2tt);$
$((M\ (U\ (-3)), O\ HA, M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3tt);$

$((M\ (D\ (-1)), O\ Hh, M\ (D\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1dd);$
$((M\ (D\ (-1)), O\ HH, M\ (D\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2dd);$
$((M\ (D\ (-1)), O\ HA, M\ (D\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3dd);$
$((M\ (D\ (-2)), O\ Hh, M\ (D\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1ss);$
$((M\ (D\ (-2)), O\ HH, M\ (D\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2ss);$
$((M\ (D\ (-2)), O\ HA, M\ (D\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3ss);$
$((M\ (D\ (-3)), O\ Hh, M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1bb);$
$((M\ (D\ (-3)), O\ HH, M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2bb);$
$((M\ (D\ (-3)), O\ HA, M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3bb);$

$((M\ (U\ (-1)), O\ Hp, M\ (D\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_hud);$
$((M\ (U\ (-2)), O\ Hp, M\ (D\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_hcs);$
$((M\ (U\ (-3)), O\ Hp, M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_htb);$

$((M\ (D\ (-1)), O\ Hm, M\ (U\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_hdu);$
$((M\ (D\ (-2)), O\ Hm, M\ (U\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_hsc);$
$((M\ (D\ (-3)), O\ Hm, M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_hbt);$

$((M\ (L\ (-1)), O\ Hh, M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e1e1);$
$((M\ (L\ (-1)), O\ HH, M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e1e1);$
$((M\ (L\ (-1)), O\ HA, M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e1e1);$
$((M\ (L\ (-2)), O\ Hh, M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e2e2);$
$((M\ (L\ (-2)), O\ HH, M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e2e2);$
$((M\ (L\ (-2)), O\ HA, M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e2e2);$
$((M\ (L\ (-3)), O\ Hh, M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e3e3);$
$((M\ (L\ (-3)), O\ HH, M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e3e3);$
$((M\ (L\ (-3)), O\ HA, M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e3e3)$

] @

if *Flags.ckm_present* then

$[((M\ (U\ (-1)), O\ Hh,\ M\ (U\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1uc);$
$((M\ (U\ (-1)), O\ Hh,\ M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1ut);$
$((M\ (U\ (-2)), O\ Hh,\ M\ (U\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1cu);$
$((M\ (U\ (-2)), O\ Hh,\ M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1ct);$
$((M\ (U\ (-1)), O\ HH,\ M\ (U\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2uc);$
$((M\ (U\ (-1)), O\ HH,\ M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2ut);$
$((M\ (U\ (-1)), O\ HA,\ M\ (U\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3uc);$
$((M\ (U\ (-1)), O\ HA,\ M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3ut);$
$((M\ (U\ (-2)), O\ HH,\ M\ (U\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2cu);$
$((M\ (U\ (-2)), O\ HH,\ M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2ct);$
$((M\ (U\ (-2)), O\ HA,\ M\ (U\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3cu);$
$((M\ (U\ (-2)), O\ HA,\ M\ (U\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3ct);$
$((M\ (U\ (-3)), O\ Hh,\ M\ (U\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1tu);$
$((M\ (U\ (-3)), O\ Hh,\ M\ (U\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1tc);$
$((M\ (U\ (-3)), O\ HH,\ M\ (U\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2tu);$
$((M\ (U\ (-3)), O\ HH,\ M\ (U\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2tc);$
$((M\ (U\ (-3)), O\ HA,\ M\ (U\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3tu);$
$((M\ (U\ (-3)), O\ HA,\ M\ (U\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3tc);$

$((M\ (D\ (-1)), O\ Hh,\ M\ (D\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1ds);$

$((M\ (D\ (-1)),\ O\ Hh,\ M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1db);$
$((M\ (D\ (-1)),\ O\ HH,\ M\ (D\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2ds);$
$((M\ (D\ (-1)),\ O\ HH,\ M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2db);$
$((M\ (D\ (-1)),\ O\ HA,\ M\ (D\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3ds);$
$((M\ (D\ (-1)),\ O\ HA,\ M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3db);$
$((M\ (D\ (-2)),\ O\ Hh,\ M\ (D\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1sd);$
$((M\ (D\ (-2)),\ O\ Hh,\ M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1sb);$
$((M\ (D\ (-2)),\ O\ HH,\ M\ (D\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2sd);$
$((M\ (D\ (-2)),\ O\ HH,\ M\ (D\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2sb);$
$((M\ (D\ (-2)),\ O\ HA,\ M\ (D\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3sd);$

$((M\ (L\ (-1)),\ O\ Hh,\ M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e1e2);$
$((M\ (L\ (-1)),\ O\ Hh,\ M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e1e3);$
$((M\ (L\ (-1)),\ O\ HH,\ M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e1e2);$
$((M\ (L\ (-1)),\ O\ HH,\ M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e1e3);$
$((M\ (L\ (-1)),\ O\ HA,\ M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e1e2);$
$((M\ (L\ (-1)),\ O\ HA,\ M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e1e3);$
$((M\ (L\ (-2)),\ O\ Hh,\ M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e2e1);$
$((M\ (L\ (-2)),\ O\ Hh,\ M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e2e3);$
$((M\ (L\ (-2)),\ O\ HH,\ M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e2e1);$
$((M\ (L\ (-2)),\ O\ HH,\ M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e2e3);$
$((M\ (L\ (-2)),\ O\ HA,\ M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e2e1);$
$((M\ (L\ (-2)),\ O\ HA,\ M\ (L\ 3)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e2e3);$
$((M\ (L\ (-3)),\ O\ Hh,\ M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e3e1);$
$((M\ (L\ (-3)),\ O\ Hh,\ M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h1e3e2);$
$((M\ (L\ (-3)),\ O\ HH,\ M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e3e1);$
$((M\ (L\ (-3)),\ O\ HH,\ M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h2e3e2);$
$((M\ (L\ (-3)),\ O\ HA,\ M\ (L\ 1)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e3e1);$
$((M\ (L\ (-3)),\ O\ HA,\ M\ (L\ 2)),\ FBF\ (1, Psibar, SP, Psi),\ G\_h3e3e2)\ ]$
else
  [ ]

$$\mathcal{L}_{\mathrm{TGC}} = -e\partial_\mu A_\nu W_+^\mu W_-^\nu + \ldots - e\cot\theta_w \partial_\mu Z_\nu W_+^\mu W_-^\nu + \ldots \tag{13.53}$$

let *tgc* $((g1,\ g2,\ g3),\ t,\ c)\ =\ ((G\ g1,\ G\ g2,\ G\ g3),\ t,\ c)$

let *triple_gauge* =
  List.map tgc
    [$((Ga,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_W);$
     $((Z,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWW);$
     $((Gl,\ Gl,\ Gl),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Gs)\ ]$

$$\mathcal{L}_{\mathrm{QGC}} = -g^2 W_{+,\mu} W_{-,\nu} W_+^\mu W_-^\nu + \ldots \tag{13.54}$$

Actually, quartic gauge couplings are a little bit more straightforward using auxiliary fields. Here we have to impose the antisymmetry manually:

$$(W_1^{+,\mu} W_2^{-,\nu} - W_1^{+,\nu} W_2^{-,\mu})(W_{3,\mu}^+ W_{4,\nu}^- - W_{3,\nu}^+ W_{4,\mu}^-)$$
$$= 2(W_1^+ W_3^+)(W_2^- W_4^-) - 2(W_1^+ W_4^-)(W_2^- W_3^+) \tag{13.55a}$$

also (V can be A or Z)

$$(W_1^{+,\mu}V_2^\nu - W_1^{+,\nu}V_2^\mu)(W_{3,\mu}^- V_{4,\nu} - W_{3,\nu}^- V_{4,\mu})$$
$$= 2(W_1^+ W_3^-)(V_2 V_4) - 2(W_1^+ V_4)(V_2 W_3^-) \quad (13.55\text{b})$$

$W^{+,\mu}W^{-,\nu}W_\mu^+ W_\nu^- \quad (13.56\text{a})$

```
let qgc ((g1, g2, g3, g4), t, c) = ((G g1, G g2, G g3, G g4), t, c)

let gauge4 = Vector4 [(2, C_13_42); (−1, C_12_34); (−1, C_14_23)]
let minus_gauge4 = Vector4 [(−2, C_13_42); (1, C_12_34); (1, C_14_23)]
let quartic_gauge =
  List.map qgc
    [ (Wm, Wp, Wm, Wp), gauge4, G_WWWW;
      (Wm, Z, Wp, Z), minus_gauge4, G_ZZWW;
      (Wm, Z, Wp, Ga), minus_gauge4, G_AZWW;
      (Wm, Ga, Wp, Ga), minus_gauge4, G_AAWW;
      (Gl, Gl, Gl, Gl), gauge4, G2]

let gauge_higgs =
  [ (G Ga, O Hp, O Hm), Vector_Scalar_Scalar 1, G_AHpHm;
    (G Z, O Hp, O Hm), Vector_Scalar_Scalar 1, G_ZHpHm;
    (G Z, O Hh, O HH), Vector_Scalar_Scalar 1, G_Zh1h2;
    (G Z, O Hh, O HA), Vector_Scalar_Scalar 1, G_Zh1h3;
    (G Z, O HH, O HA), Vector_Scalar_Scalar 1, G_Zh2h3;
    (G Wp, O Hm, O Hh), Vector_Scalar_Scalar 1, G_WpHmh1;
    (G Wp, O Hm, O HH), Vector_Scalar_Scalar 1, G_WpHmh2;
    (G Wp, O Hm, O HA), Vector_Scalar_Scalar 1, G_WpHmh3;
    (G Wm, O Hp, O Hh), Vector_Scalar_Scalar 1, G_WmHph1;
    (G Wm, O Hp, O HH), Vector_Scalar_Scalar 1, G_WmHph2;
    (G Wm, O Hp, O HA), Vector_Scalar_Scalar 1, G_WmHph3;
    (O Hh, G Z, G Z), Scalar_Vector_Vector 1, G_h1ZZ;
    (O HH, G Z, G Z), Scalar_Vector_Vector 1, G_h2ZZ;
    (O HA, G Z, G Z), Scalar_Vector_Vector 1, G_h3ZZ;
    (O Hh, G Wp, G Wm), Scalar_Vector_Vector 1, G_h1WpWm;
    (O HH, G Wp, G Wm), Scalar_Vector_Vector 1, G_h2WpWm;
    (O HA, G Wp, G Wm), Scalar_Vector_Vector 1, G_h3WpWm ]

let gauge_higgs4 =
  [ (O Hh, O Hh, G Wp, G Wm), Scalar2_Vector2 1, G_hhWpWm;
    (O HH, O HH, G Wp, G Wm), Scalar2_Vector2 1, G_hhWpWm;
    (O HA, O HA, G Wp, G Wm), Scalar2_Vector2 1, G_hhWpWm;
    (O Hh, O Hh, G Z, G Z), Scalar2_Vector2 1, G_hhZZ;
    (O HH, O HH, G Z, G Z), Scalar2_Vector2 1, G_hhZZ;
    (O HA, O HA, G Z, G Z), Scalar2_Vector2 1, G_hhZZ;
    (O Hp, O Hm, G Ga, G Ga), Scalar2_Vector2 1, G_HpHmAA;
    (O Hp, O Hm, G Z, G Z), Scalar2_Vector2 1, G_HpHmZZ;
    (O Hp, O Hm, G Ga, G Z), Scalar2_Vector2 1, G_HpHmAZ;
    (O Hp, O Hm, G Wp, G Wm), Scalar2_Vector2 1, G_HpHmWpWm;
```

    (*O Hh*, *O Hp*, *G Ga*, *G Wm*), *Scalar2_Vector2* 1, *G_h1HpAWm*;
    (*O HH*, *O Hp*, *G Ga*, *G Wm*), *Scalar2_Vector2* 1, *G_h2HpAWm*;
    (*O HA*, *O Hp*, *G Ga*, *G Wm*), *Scalar2_Vector2* 1, *G_h3HpAWm*;
    (*O Hh*, *O Hp*, *G Z*, *G Wm*), *Scalar2_Vector2* 1, *G_h1HpZWm*;
    (*O HH*, *O Hp*, *G Z*, *G Wm*), *Scalar2_Vector2* 1, *G_h2HpZWm*;
    (*O HA*, *O Hp*, *G Z*, *G Wm*), *Scalar2_Vector2* 1, *G_h3HpZWm*;
    (*O Hh*, *O Hm*, *G Ga*, *G Wp*), *Scalar2_Vector2* 1, *G_h1HpAWmC*;
    (*O HH*, *O Hm*, *G Ga*, *G Wp*), *Scalar2_Vector2* 1, *G_h2HpAWmC*;
    (*O HA*, *O Hm*, *G Ga*, *G Wp*), *Scalar2_Vector2* 1, *G_h3HpAWmC*;
    (*O Hh*, *O Hm*, *G Z*, *G Wp*), *Scalar2_Vector2* 1, *G_h1HpZWmC*;
    (*O HH*, *O Hm*, *G Z*, *G Wp*), *Scalar2_Vector2* 1, *G_h2HpZWmC*;
    (*O HA*, *O Hm*, *G Z*, *G Wp*), *Scalar2_Vector2* 1, *G_h3HpZWmC*]

let *higgs* =
  [(*O Hh*, *O Hp*, *O Hm*), *Scalar_Scalar_Scalar* 1, *G_h1HpHm*;
   (*O HH*, *O Hp*, *O Hm*), *Scalar_Scalar_Scalar* 1, *G_h2HpHm*;
   (*O HA*, *O Hp*, *O Hm*), *Scalar_Scalar_Scalar* 1, *G_h3HpHm*;
   (*O Hh*, *O Hh*, *O Hh*), *Scalar_Scalar_Scalar* 1, *G_h111*;
   (*O Hh*, *O Hh*, *O HH*), *Scalar_Scalar_Scalar* 1, *G_h112*;
   (*O Hh*, *O Hh*, *O HA*), *Scalar_Scalar_Scalar* 1, *G_h113*;
   (*O HH*, *O HH*, *O Hh*), *Scalar_Scalar_Scalar* 1, *G_h221*;
   (*O HH*, *O HH*, *O HH*), *Scalar_Scalar_Scalar* 1, *G_h222*;
   (*O HH*, *O HH*, *O HA*), *Scalar_Scalar_Scalar* 1, *G_h223*;
   (*O HA*, *O HA*, *O Hh*), *Scalar_Scalar_Scalar* 1, *G_h331*;
   (*O HA*, *O HA*, *O HH*), *Scalar_Scalar_Scalar* 1, *G_h332*;
   (*O HA*, *O HA*, *O HA*), *Scalar_Scalar_Scalar* 1, *G_h333*;
   (*O Hh*, *O HH*, *O HA*), *Scalar_Scalar_Scalar* 1, *G_h123*]

let *higgs4* =
  [(*O Hp*, *O Hm*, *O Hp*, *O Hm*), *Scalar4* 1, *G_HpHmHpHm*;
   (*O Hp*, *O Hm*, *O Hh*, *O Hh*), *Scalar4* 1, *G_HpHm11*;
   (*O Hp*, *O Hm*, *O Hh*, *O HH*), *Scalar4* 1, *G_HpHm12*;
   (*O Hp*, *O Hm*, *O Hh*, *O HA*), *Scalar4* 1, *G_HpHm13*;
   (*O Hp*, *O Hm*, *O HH*, *O HH*), *Scalar4* 1, *G_HpHm22*;
   (*O Hp*, *O Hm*, *O HH*, *O HA*), *Scalar4* 1, *G_HpHm23*;
   (*O Hp*, *O Hm*, *O HA*, *O HA*), *Scalar4* 1, *G_HpHm33*;
   (*O Hh*, *O Hh*, *O Hh*, *O Hh*), *Scalar4* 1, *G_h1111*;
   (*O Hh*, *O Hh*, *O Hh*, *O HH*), *Scalar4* 1, *G_h1112*;
   (*O Hh*, *O Hh*, *O Hh*, *O HA*), *Scalar4* 1, *G_h1113*;
   (*O Hh*, *O Hh*, *O HH*, *O HH*), *Scalar4* 1, *G_h1122*;
   (*O Hh*, *O Hh*, *O HH*, *O HA*), *Scalar4* 1, *G_h1123*;
   (*O Hh*, *O Hh*, *O HA*, *O HA*), *Scalar4* 1, *G_h1133*;
   (*O Hh*, *O HH*, *O HH*, *O HH*), *Scalar4* 1, *G_h1222*;
   (*O Hh*, *O HH*, *O HH*, *O HA*), *Scalar4* 1, *G_h1223*;
   (*O Hh*, *O HH*, *O HA*, *O HA*), *Scalar4* 1, *G_h1233*;
   (*O Hh*, *O HA*, *O HA*, *O HA*), *Scalar4* 1, *G_h1333*;
   (*O HH*, *O HH*, *O HH*, *O HH*), *Scalar4* 1, *G_h2222*;
   (*O HH*, *O HH*, *O HH*, *O HA*), *Scalar4* 1, *G_h2223*;
   (*O HH*, *O HH*, *O HA*, *O HA*), *Scalar4* 1, *G_h2233*;
   (*O HH*, *O HA*, *O HA*, *O HA*), *Scalar4* 1, *G_h2333*;
   (*O HA*, *O HA*, *O HA*, *O HA*), *Scalar4* 1, *G_h3333*]

let *goldstone_vertices* =
  [((*O Phi0*, *G Wm*, *G Wp*), *Scalar_Vector_Vector* 1, *I_G_ZWW*);
    ((*O Phip*, *G Ga*, *G Wm*), *Scalar_Vector_Vector* 1, *I_Q_W*);
    ((*O Phip*, *G Z*, *G Wm*), *Scalar_Vector_Vector* 1, *I_G_ZWW*);
    ((*O Phim*, *G Wp*, *G Ga*), *Scalar_Vector_Vector* 1, *I_Q_W*);
    ((*O Phim*, *G Wp*, *G Z*), *Scalar_Vector_Vector* 1, *I_G_ZWW*)]

let *vertices3* =
  (*ThoList.flatmap electromagnetic_currents* [1; 2; 3] @
   ThoList.flatmap color_currents [1; 2; 3] @
   ThoList.flatmap neutral_currents [1; 2; 3] @
   ThoList.flatmap charged_currents [1; 2; 3] @
   yukawa @ *triple_gauge* @
   gauge_higgs @ *higgs* @ *goldstone_vertices*)

let *vertices4* =
  quartic_gauge @ *gauge_higgs4* @ *higgs4*

let *vertices* () = (*vertices3*, *vertices4*, [])

For efficiency, make sure that *F.of_vertices vertices* is evaluated only once.

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *flavor_of_string* = function
  | "e-" → *M* (*L* 1) | "e+" → *M* (*L* (−1))
  | "mu-" → *M* (*L* 2) | "mu+" → *M* (*L* (−2))
  | "tau-" → *M* (*L* 3) | "tau+" → *M* (*L* (−3))
  | "nue" → *M* (*N* 1) | "nuebar" → *M* (*N* (−1))
  | "numu" → *M* (*N* 2) | "numubar" → *M* (*N* (−2))
  | "nutau" → *M* (*N* 3) | "nutaubar" → *M* (*N* (−3))
  | "u" → *M* (*U* 1) | "ubar" → *M* (*U* (−1))
  | "c" → *M* (*U* 2) | "cbar" → *M* (*U* (−2))
  | "t" → *M* (*U* 3) | "tbar" → *M* (*U* (−3))
  | "d" → *M* (*D* 1) | "dbar" → *M* (*D* (−1))
  | "s" → *M* (*D* 2) | "sbar" → *M* (*D* (−2))
  | "b" → *M* (*D* 3) | "bbar" → *M* (*D* (−3))
  | "g" → *G Gl*
  | "A" → *G Ga* | "Z" | "Z0" → *G Z*
  | "W+" → *G Wp* | "W-" → *G Wm*
  | "h0" → *O Hh*
  | "H0" → *O HH*
  | "A0" → *O HA*
  | _ → *invalid_arg* "Modellib_BSM.TwoHiggsDoublet.flavor_of_string"

let *flavor_to_string* = function
  | *M f* →
    begin match *f* with
    | *L* 1 → "e-" | *L* (−1) → "e+"
    | *L* 2 → "mu-" | *L* (−2) → "mu+"

    | *L* 3 → `"tau-"` | *L* (−3) → `"tau+"`
    | *L* _ → *invalid_arg*
        `"Modellib_BSM.TwoHiggsDoublet.flavor_to_string:␣invalid␣lepton"`
    | *N* 1 → `"nue"` | *N* (−1) → `"nuebar"`
    | *N* 2 → `"numu"` | *N* (−2) → `"numubar"`
    | *N* 3 → `"nutau"` | *N* (−3) → `"nutaubar"`
    | *N* _ → *invalid_arg*
        `"Modellib_BSM.TwoHiggsDoublet.flavor_to_string:␣invalid␣neutrino"`
    | *U* 1 → `"u"` | *U* (−1) → `"ubar"`
    | *U* 2 → `"c"` | *U* (−2) → `"cbar"`
    | *U* 3 → `"t"` | *U* (−3) → `"tbar"`
    | *U* _ → *invalid_arg*
        `"Modellib_BSM.TwoHiggsDoublet.flavor_to_string:␣invalid␣up␣type␣quark"`
    | *D* 1 → `"d"` | *D* (−1) → `"dbar"`
    | *D* 2 → `"s"` | *D* (−2) → `"sbar"`
    | *D* 3 → `"b"` | *D* (−3) → `"bbar"`
    | *D* _ → *invalid_arg*
        `"Modellib_BSM.TwoHiggsDoublet.flavor_to_string:␣invalid␣down␣type␣quark"`
   end
| *G f* →
   begin match *f* with
   | *Gl* → `"g"`
   | *Ga* → `"A"` | *Z* → `"Z"`
   | *Wp* → `"W+"` | *Wm* → `"W-"`
   end
| *O f* →
   begin match *f* with
   | *Phip* → `"phi+"` | *Phim* → `"phi-"` | *Phi0* → `"phi0"`
   | *Hh* → `"h0"` | *HH* → `"H0"` | *HA* → `"A0"`
   | *Hp* → `"H+"` | *Hm* → `"H-"`
   end

let *flavor_to_TeX* = function
| *M f* →
   begin match *f* with
   | *L* 1 → `"e^-"` | *L* (−1) → `"e^+"`
   | *L* 2 → `"\\mu^-"` | *L* (−2) → `"\\mu^+"`
   | *L* 3 → `"\\tau^-"` | *L* (−3) → `"\\tau^+"`
   | *L* _ → *invalid_arg*
       `"Modellib_BSM.TwoHiggsDoublet.flavor_to_TeX:␣invalid␣lepton"`
   | *N* 1 → `"\\nu_e"` | *N* (−1) → `"\\bar{\\nu}_e"`
   | *N* 2 → `"\\nu_\\mu"` | *N* (−2) → `"\\bar{\\nu}_\\mu"`
   | *N* 3 → `"\\nu_\\tau"` | *N* (−3) → `"\\bar{\\nu}_\\tau"`
   | *N* _ → *invalid_arg*
       `"Modellib_BSM.TwoHiggsDoublet.flavor_to_TeX:␣invalid␣neutrino"`
   | *U* 1 → `"u"` | *U* (−1) → `"\\bar{u}"`
   | *U* 2 → `"c"` | *U* (−2) → `"\\bar{c}"`
   | *U* 3 → `"t"` | *U* (−3) → `"\\bar{t}"`
   | *U* _ → *invalid_arg*
       `"Modellib_BSM.TwoHiggsDoublet.flavor_to_TeX:␣invalid␣up␣type␣quark"`

```
                | D 1 → "d" | D (−1) → "\\bar{d}"
                | D 2 → "s" | D (−2) → "\\bar{s}"
                | D 3 → "b" | D (−3) → "\\bar{b}"
                | D _ → invalid_arg
                        "Modellib_BSM.TwoHiggsDoublet.flavor_to_TeX:␣invalid␣down␣type␣quark"
            end
        | G f →
            begin match f with
            | Gl → "g"
            | Ga → "\\gamma" | Z → "Z"
            | Wp → "W^+" | Wm → "W^-"
            end
        | O f →
            begin match f with
            | Phip → "\\phi^+" | Phim → "\\phi^-" | Phi0 →
"\\phi^0"
            | Hh → "h^0" | HH → "H^0" | HA → "A^0"
            | Hp → "H^+" | Hm → "H^-"
            end

    let flavor_symbol = function
        | M f →
            begin match f with
            | L n when n > 0 → "l" ^ string_of_int n
            | L n → "l" ^ string_of_int (abs n) ^ "b"
            | N n when n > 0 → "n" ^ string_of_int n
            | N n → "n" ^ string_of_int (abs n) ^ "b"
            | U n when n > 0 → "u" ^ string_of_int n
            | U n → "u" ^ string_of_int (abs n) ^ "b"
            | D n when n > 0 → "d" ^ string_of_int n
            | D n → "d" ^ string_of_int (abs n) ^ "b"
            end
        | G f →
            begin match f with
            | Gl → "gl"
            | Ga → "a" | Z → "z"
            | Wp → "wp" | Wm → "wm"
            end
        | O f →
            begin match f with
            | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
            | Hh → "h" | HH → "h0" | HA → "a0"
            | Hp → "hp" | Hm → "hm"
            end

    let pdg = function
        | M f →
            begin match f with
            | L n when n > 0 → 9 + 2 × n
            | L n → − 9 + 2 × n
            | N n when n > 0 → 10 + 2 × n
```

```
          | N n  →  − 10 + 2 × n
          | U n when n > 0 → 2 × n
          | U n  →  2 × n
          | D n when n > 0  →  − 1 + 2 × n
          | D n  →  1 + 2 × n
          end
      | G f  →
          begin match f with
          | Gl  →  21
          | Ga  →  22  | Z  →  23
          | Wp  →  24  | Wm  →  (−24)
          end
      | O f  →
          begin match f with
          | Phip  →  27  | Phim  →  − 27  | Phi0  →  26
          | Hh  →  25
          | HH  →  35
          | HA  →  36
          | Hp  →  37
          | Hm  →  − 37
          end

    let mass_symbol f  =
      "mass(" ^ string_of_int (abs (pdg f)) ^ ")"

    let width_symbol f  =
      "width(" ^ string_of_int (abs (pdg f)) ^ ")"

    let constant_symbol  =  function
      | Unit  →  "unit"  | Pi  →  "PI"
      | Alpha_QED  →  "alpha"  | E  →  "e"  | G_weak  →  "g"  | Vev  →
"vev"
      | Sin2thw  →  "sin2thw"  | Sinthw  →  "sinthw"  | Costhw  →
"costhw"
      | Q_lepton  →  "qlep"  | Q_up  →  "qup"  | Q_down  →  "qdwn"
      | G_NC_lepton  →  "gnclep"  | G_NC_neutrino  →  "gncneu"
      | G_NC_up  →  "gncup"  | G_NC_down  →  "gncdwn"
      | G_CC  →  "gcc"
      | I_Q_W  →  "iqw"  | I_G_ZWW  →  "igzww"  | I_G_WWW  →
"igwww"
      | G_WWWW  →  "gw4"  | G_ZZWW  →  "gzzww"
      | G_AZWW  →  "gazww"  | G_AAWW  →  "gaaww"
      | G_htt  →  "ghtt"  | G_hbb  →  "ghbb"  | G_hcc  →  "ghcc"
      | G_Htt  →  "gh0tt"  | G_Hbb  →  "gh0bb"  | G_Hcc  →  "gh0cc"
      | I_G_Att  →  "iga0tt"  | I_G_Abb  →  "iga0bb"  | I_G_Acc  →
"iga0cc"
      | G_htautau  →  "ghtautau"  | G_hmumu  →  "ghmumu"
      | G_Htautau  →  "gh0tautau"  | G_Hmumu  →  "gh0mumu"
      | I_G_Atautau  →  "iga0tautau"  | I_G_Amumu  →  "iga0mumu"
      | G_Htb  →  "ghptb"  | G_Hcs  →  "ghpcs"
      | G_Htaunu  →  "ghptaunu"  | G_Hmunu  →  "ghpmunu"
      | Gs  →  "gs"  | I_Gs  →  "igs"  | G2  →  "gs**2"
```

| G_AHpHm → "gAHpHm" | G_ZHpHm → "gZHpHm"
| G_Zh1h2 → "gZh1h2" | G_Zh1h3 → "gZh1h3"
| G_Zh2h3 → "gZh2h3" | G_WpHmh1 → "gWpHmh1"
| G_WpHmh2 → "gWpHmh2" | G_WpHmh3 → "gWpHmh3"
| G_WmHph1 → "gWmHph1" | G_WmHph2 → "gWmHph2"
| G_WmHph3 → "gWmHph3" | G_h1ZZ → "gh1ZZ"
| G_h2ZZ → "gh2ZZ" | G_h3ZZ → "gh3ZZ"
| G_h1WpWm → "gh1WpWm" | G_h2WpWm → "gh2WpWm"
| G_h3WpWm → "gh3WpWm" | G_hhWpWm → "ghhWpWm"
| G_hhZZ → "ghhZZ" | G_HpHmAA → "gHpHmAA"
| G_HpHmZZ → "gHpHmZZ" | G_HpHmAZ → "gHpHmAZ"
| $G\_HpHmWpWm$ → "gHpHmWpWm" | $G\_h1HpAWm$ → "gh1HpAWm"
| $G\_h2HpAWm$ → "gh2HpAWm" | $G\_h3HpAWm$ → "gh3HpAWm"
| $G\_h1HpZWm$ → "gh1HpZWm" | $G\_h2HpZWm$ → "gh2HpZWm"
| $G\_h3HpZWm$ → "gh3HpZWm" | $G\_h1HpAWmC$ → "gh1HpAWmC"
| $G\_h2HpAWmC$ → "gh2HpAWmC" | $G\_h3HpAWmC$ → "gh3HpAWmC"
| $G\_h1HpZWmC$ → "gh1HpZWmC" | $G\_h2HpZWmC$ → "gh2HpZWmC"
| $G\_h3HpZWmC$ → "gh3HpZWmC" | G_h1HpHm → "gh1HpHm"
| G_h2HpHm → "gh2HpHm" | G_h3HpHm → "gh3HpHm"
| G_h111 → "gh111" | G_h112 → "gh112" | G_h113 → "gh113"
| G_h221 → "gh221" | G_h222 → "gh222" | G_h223 → "gh223"
| G_h331 → "gh331" | G_h332 → "gh332"
| G_h333 → "gh333" | G_h123 → "gh123"
| $G\_HpHmHpHm$ → "gHpHmHpHm" | G_HpHm11 → "gHpHm11"
| G_HpHm12 → "gHpHm12" | G_HpHm13 → "gHpHm13"
| G_HpHm22 → "gHpHm22" | G_HpHm23 → "gHpHm23"
| G_HpHm33 → "gHpHm33" | G_h1111 → "gh1111"
| G_h1112 → "gh1112" | G_h1113 → "gh1113"
| G_h1122 → "gh1122" | G_h1123 → "gh1123"
| G_h1133 → "gh1133" | G_h1222 → "gh1222"
| G_h1223 → "gh1223" | G_h1233 → "gh1233"
| G_h1333 → "gh1333" | G_h2222 → "gh2222"
| G_h2223 → "gh2223" | G_h2233 → "gh2233"
| G_h2333 → "gh2333" | G_h3333 → "gh3333"
| G_h1uu → "gh1uu" | G_h2uu → "gh2uu"
| G_h3uu → "gh3uu" | G_h1uc → "gh1uc"
| G_h2uc → "gh2uc" | G_h3uc → "gh3uc"
| G_h1ut → "gh1ut" | G_h2ut → "gh2ut"
| G_h3ut → "gh3ut" | G_h1cu → "gh1cu"
| G_h2cu → "gh2cu" | G_h3cu → "gh3cu"
| G_h1cc → "gh1cc" | G_h2cc → "gh2cc"
| G_h3cc → "gh3cc" | G_h1ct → "gh1ct"
| G_h2ct → "gh2ct" | G_h3ct → "gh3ct"
| G_h1tu → "gh1tu" | G_h2tu → "gh2tu"
| G_h3tu → "gh3tu" | G_h1tc → "gh1tc"
| G_h2tc → "gh2tc" | G_h3tc → "gh3tc"
| G_h1tt → "gh1tt" | G_h2tt → "gh2tt"
| G_h3tt → "gh3tt"
| G_h1dd → "gh1dd" | G_h2dd → "gh2dd"
| G_h3dd → "gh3dd" | G_h1ds → "gh1ds"

```
        | G_h2ds → "gh2ds" | G_h3ds → "gh3ds"
        | G_h1db → "gh1db" | G_h2db → "gh2db"
        | G_h3db → "gh3db" | G_h1sd → "gh1sd"
        | G_h2sd → "gh2sd" | G_h3sd → "gh3sd"
        | G_h1ss → "gh1ss" | G_h2ss → "gh2ss"
        | G_h3ss → "gh3ss" | G_h1sb → "gh1sb"
        | G_h2sb → "gh2sb" | G_h3sb → "gh3sb"
        | G_h1bd → "gh1bd" | G_h2bd → "gh2bd"
        | G_h3bd → "gh3bd" | G_h1bs → "gh1bs"
        | G_h2bs → "gh2bs" | G_h3bs → "gh3bs"
        | G_h1bb → "gh1bb" | G_h2bb → "gh2bb"
        | G_h3bb → "gh3bb"
        | G_hud → "ghud" | G_hus → "ghus"
        | G_hub → "ghub" | G_hcd → "ghcd"
        | G_hcs → "ghcs" | G_hcb → "ghcb"
        | G_htd → "ghtd" | G_hts → "ghts"
        | G_htb → "ghtb"
        | G_hdu → "ghdu" | G_hdc → "ghdc"
        | G_hdt → "ghdt" | G_hsu → "ghsu"
        | G_hsc → "ghsc" | G_hst → "ghst"
        | G_hbu → "ghbu" | G_hbc → "ghbc"
        | G_hbt → "ghbt"
        | G_he1n1 → "ghe1n1" | G_he1n2 → "ghe1n2"
        | G_he1n3 → "ghe1n3" | G_he2n1 → "ghe2n1"
        | G_he2n2 → "ghe2n2" | G_he2n3 → "ghe2n3"
        | G_he3n1 → "ghe3n1" | G_he3n2 → "ghe3n2"
        | G_he3n3 → "ghe3n3" | G_hn1e1 → "ghn1e1"
        | G_hn1e2 → "ghn1e2" | G_hn1e3 → "ghn1e3"
        | G_hn2e1 → "ghn2e1" | G_hn2e2 → "ghn2e2"
        | G_hn2e3 → "ghn2e3" | G_hn3e1 → "ghn3e1"
        | G_hn3e2 → "ghn3e2" | G_hn3e3 → "ghn3e3"
        | G_h1e1e1 → "gh1e1e1" | G_h2e1e1 → "gh2e1e1"
        | G_h3e1e1 → "gh3e1e1" | G_h1e1e2 → "gh1e1e2"
        | G_h2e1e2 → "gh2e1e2" | G_h3e1e2 → "gh3e1e2"
        | G_h1e1e3 → "gh1e1e3" | G_h2e1e3 → "gh2e1e3"
        | G_h3e1e3 → "gh3e1e3" | G_h1e2e1 → "gh1e2e1"
        | G_h2e2e1 → "gh2e2e1" | G_h3e2e1 → "gh3e2e1"
        | G_h1e2e2 → "gh1e2e2" | G_h2e2e2 → "gh2e2e2"
        | G_h3e2e2 → "gh3e2e2" | G_h1e2e3 → "gh1e2e3"
        | G_h2e2e3 → "gh2e2e3" | G_h3e2e3 → "gh3e2e3"
        | G_h1e3e1 → "gh1e3e1" | G_h2e3e1 → "gh2e3e1"
        | G_h3e3e1 → "gh3e3e1" | G_h1e3e2 → "gh1e3e2"
        | G_h2e3e2 → "gh2e3e2" | G_h3e3e2 → "gh3e3e2"
        | G_h1e3e3 → "gh1e3e3" | G_h2e3e3 → "gh2e3e3"
        | G_h3e3e3 → "gh3e3e3"
        | Mass f → "mass" ^ flavor_symbol f
        | Width f → "width" ^ flavor_symbol f

    end

module type SSC_flags =
```

```
sig
  val higgs_triangle : bool (* Hγγ, Hgγ and Hgg couplings *)
  val higgs_hmm : bool
  val triple_anom : bool
  val quartic_anom : bool
  val higgs_anom : bool
  val k_matrix : bool
  val ckm_present : bool
  val top_anom : bool
  val top_anom_4f : bool
  val cf_arbitrary : bool
  val higgs_matrix : bool
end

module SSC_kmatrix : SSC_flags =
  struct
    let higgs_triangle = false
    let higgs_hmm = false
    let triple_anom = false
    let quartic_anom = true
    let higgs_anom = false
    let k_matrix = true
    let ckm_present = false
    let top_anom = false
    let top_anom_4f = false
    let cf_arbitrary = false
    let higgs_matrix = false
  end

module SSC_kmatrix_2 : SSC_flags =
  struct
    let higgs_triangle = false
    let higgs_hmm = false
    let triple_anom = false
    let quartic_anom = true
    let higgs_anom = false
    let k_matrix = true
    let ckm_present = false
    let top_anom = false
    let top_anom_4f = false
    let cf_arbitrary = true
    let higgs_matrix = true
  end
```

13.6.4 Complete Minimal Standard Model including additional Resonances

```
module SSC (Flags : SSC_flags) =
  struct
    let rcs = RCS.rename rcs_file "Modellib_BSM.SSC"
```

["minimal␣electroweak␣standard␣model␣in␣unitarity␣gauge␣with␣additional␣Vectorboson

open *Coupling*

let *default_width* = *ref Timelike*
let *use_fudged_width* = *ref* false

let *options* = *Options.create*
  ["constant_width", *Arg.Unit* (fun () → *default_width* := *Constant*),
    "use␣constant␣width␣(also␣in␣t-channel)";
    "fudged_width", *Arg.Set use_fudged_width*,
    "use␣fudge␣factor␣for␣charge␣particle␣width";
    "custom_width", *Arg.String* (fun *f* → *default_width* := *Custom f*),
    "use␣custom␣width";
    "cancel_widths", *Arg.Unit* (fun () → *default_width* := *Vanishing*),
    "use␣vanishing␣width"]

type *f_aux_top* = *TTGG* | *TBWA* | *TBWZ* | *TTWW* | *BBWW* |
let *quartic_gauge* =
  standard_quartic_gauge @ *anomalous_quartic_gauge* @ *k_matrix_quartic_gauge*

let *standard_gauge_higgs* =
  [((*O H*, *G Wp*, *G Wm*), *Scalar_Vector_Vector* 1, *G_HWW*);
    ((*O H*, *G Z*, *G Z*), *Scalar_Vector_Vector* 1, *G_HZZ*)]

let *standard_gauge_higgs4* =
  [(*O H*, *O H*, *G Wp*, *G Wm*), *Scalar2_Vector2* 1, *G_HHWW*;
    (*O H*, *O H*, *G Z*, *G Z*), *Scalar2_Vector2* 1, *G_HHZZ*]

let *dim8_gauge_higgs4* =
  [(*O H*, *O H*, *G Wp*, *G Wm*), *Dim8_Scalar2_Vector2_1* 1, *FS0_HHWW*;
    (*O H*, *O H*, *G Z*, *G Z*), *Dim8_Scalar2_Vector2_1* 1, *FS0_HHZZ*;
    (*O H*, *O H*, *G Wp*, *G Wm*), *Dim8_Scalar2_Vector2_2* 1, *FS1_HHWW*;
    (*O H*, *O H*, *G Z*, *G Z*), *Dim8_Scalar2_Vector2_2* 1, *FS1_HHZZ*]

let *standard_higgs* =
  [(*O H*, *O H*, *O H*), *Scalar_Scalar_Scalar* 1, *G_H3*]

let *standard_higgs4* =
  [(*O H*, *O H*, *O H*, *O H*), *Scalar4* 1, *G_H4*]

let *fs_higgs4* =
  [(*O H*, *O H*, *O H*, *O H*), *Dim8_Scalar4* 1, *FS_H4*]

WK's couplings (apparently, he still intends to divide by $\Lambda_{\text{EWSB}}^2 = 16\pi^2 v_{\text{F}}^2$):

$$\mathcal{L}_4^\tau = \left[(\partial_\mu H)(\partial^\mu H) + \frac{g^2 v_{\text{F}}^2}{4} V_\mu V^\mu \right]^2 \tag{13.57a}$$

$$\mathcal{L}_5^\tau = \left[(\partial_\mu H)(\partial_\nu H) + \frac{g^2 v_{\text{F}}^2}{4} V_\mu V_\nu \right]^2 \tag{13.57b}$$

with

$$V_\mu V_\nu = \frac{1}{2} \left(W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^- \right) + \frac{1}{2\cos^2\theta_w} Z_\mu Z_\nu \tag{13.58}$$

(note the symmetrization!), i.e.

$$\mathcal{L}_4 = \alpha_4 \frac{g^4 v_{\mathrm{F}}^4}{16}(V_\mu V_\nu)^2 \tag{13.59a}$$

$$\mathcal{L}_5 = \alpha_5 \frac{g^4 v_{\mathrm{F}}^4}{16}(V_\mu V^\mu)^2 \tag{13.59b}$$

Breaking thinks up

$$\mathcal{L}_4^{\tau,H^4} = [(\partial_\mu H)(\partial^\mu H)]^2 \tag{13.60a}$$

$$\mathcal{L}_5^{\tau,H^4} = [(\partial_\mu H)(\partial^\mu H)]^2 \tag{13.60b}$$

and

$$\mathcal{L}_4^{\tau,H^2V^2} = \frac{g^2 v_{\mathrm{F}}^2}{2}(\partial_\mu H)(\partial^\mu H)V_\mu V^\mu \tag{13.61a}$$

$$\mathcal{L}_5^{\tau,H^2V^2} = \frac{g^2 v_{\mathrm{F}}^2}{2}(\partial_\mu H)(\partial_\nu H)V_\mu V_\nu \tag{13.61b}$$

i.e.

$$\mathcal{L}_4^{\tau,H^2V^2} = \frac{g^2 v_{\mathrm{F}}^2}{2}\left[(\partial_\mu H)(\partial^\mu H)W_\nu^+ W^{-,\nu} + \frac{1}{2\cos^2\theta_w}(\partial_\mu H)(\partial^\mu H)Z_\nu Z^\nu\right] \tag{13.62a}$$

$$\mathcal{L}_5^{\tau,H^2V^2} = \frac{g^2 v_{\mathrm{F}}^2}{2}\left[(W^{+,\mu}\partial_\mu H)(W^{-,\nu}\partial_\nu H) + \frac{1}{2\cos^2\theta_w}(Z^\mu\partial_\mu H)(Z^\nu\partial_\nu H)\right] \tag{13.62b}$$

$$\tau_8^4 \mathcal{L}_4^{\tau,H^2V^2} + \tau_8^5 \mathcal{L}_5^{\tau,H^2V^2} =$$
$$-\frac{g^2 v_{\mathrm{F}}^2}{2}\left[2\tau_8^4 \frac{1}{2}(\mathrm{i}\partial_\mu H)(\mathrm{i}\partial^\mu H)W_\nu^+ W^{-,\nu} + \tau_8^5(W^{+,\mu}\mathrm{i}\partial_\mu H)(W^{-,\nu}\mathrm{i}\partial_\nu H)\right.$$
$$\left.+ \frac{2\tau_8^4}{\cos^2\theta_w}\frac{1}{4}(\mathrm{i}\partial_\mu H)(\mathrm{i}\partial^\mu H)Z_\nu Z^\nu + \frac{\tau_8^5}{\cos^2\theta_w}\frac{1}{2}(Z^\mu\mathrm{i}\partial_\mu H)(Z^\nu\mathrm{i}\partial_\nu H)\right] \tag{13.63}$$

where the two powers of i make the sign conveniently negative, i.e.

$$\alpha_{(\partial H)^2 W^2}^2 = \tau_8^4 g^2 v_{\mathrm{F}}^2 \tag{13.64a}$$

$$\alpha_{(\partial H W)^2}^2 = \frac{\tau_8^5 g^2 v_{\mathrm{F}}^2}{2} \tag{13.64b}$$

$$\alpha_{(\partial H)^2 Z^2}^2 = \frac{\tau_8^4 g^2 v_{\mathrm{F}}^2}{\cos^2\theta_w} \tag{13.64c}$$

$$\alpha_{(\partial H Z)^2}^2 = \frac{\tau_8^5 g^2 v_{\mathrm{F}}^2}{2\cos^2\theta_w} \tag{13.64d}$$

```
let anomalous_gauge_higgs =
  [ (O H, G Ga, G Ga), Dim5_Scalar_Gauge2 1, G_HGaGa_anom;
    (O H, G Ga, G Z), Dim5_Scalar_Gauge2 1, G_HGaZ_anom;
```

      (*O H*, *G Z*, *G Z*), *Dim5_Scalar_Gauge2* 1, *G_HZZ_anom*;
      (*O H*, *G Wp*, *G Wm*), *Dim5_Scalar_Gauge2* 1, *G_HWW_anom*;
      (*O H*, *G Ga*, *G Z*), *Dim5_Scalar_Vector_Vector_U* 1, *G_HGaZ_u*;
      (*O H*, *G Z*, *G Z*), *Dim5_Scalar_Vector_Vector_U* 1, *G_HZZ_u*;
      (*O H*, *G Wp*, *G Wm*), *Dim5_Scalar_Vector_Vector_U* 1, *G_HWW_u*;
      (*O H*, *G Wm*, *G Wp*), *Dim5_Scalar_Vector_Vector_U* 1, *G_HWW_u*
  ]

let *anomalous_gauge_higgs4* =
  [ ]

let *anomalous_higgs* =
  [ ]

let *higgs_triangle_vertices* =
  if *Flags.higgs_triangle* then
    [(*O H*, *G Ga*, *G Ga*), *Dim5_Scalar_Gauge2* 1, *G_HGaGa*;
     (*O H*, *G Ga*, *G Z*), *Dim5_Scalar_Gauge2* 1, *G_HGaZ*;
     (*O H*, *G Gl*, *G Gl*), *Dim5_Scalar_Gauge2* 1, *G_Hgg*]
  else
    [ ]

let *anomalous_higgs4* =
  [ ]

let *gauge_higgs* =
  if *Flags.higgs_anom* then
    standard_gauge_higgs @ *anomalous_gauge_higgs*
  else
    standard_gauge_higgs

let *gauge_higgs4* =
  (if *Flags.higgs_anom* then
     standard_gauge_higgs4 @ *anomalous_gauge_higgs4*
    else
     standard_gauge_higgs4) @
  (if *Flags.higgs_matrix* then
     (*dim8_gauge_higgs4* @ *k_matrix_2scalar_2gauge*)
    else
     [ ])

let *higgs* =
  if *Flags.higgs_anom* then
    standard_higgs @ *anomalous_higgs*
  else
    standard_higgs

let *higgs4* =
  (if *Flags.higgs_anom* then
     standard_higgs4 @ *anomalous_higgs4*
    else
     standard_higgs4) @
  (if *Flags.higgs_matrix* then
     (*fs_higgs4* @ *k_matrix_4scalar*)

```
    else
      [] )
let goldstone_vertices =
  [ ((O Phi0, G Wm, G Wp), Scalar_Vector_Vector 1, I_G_ZWW);
    ((O Phip, G Ga, G Wm), Scalar_Vector_Vector 1, I_Q_W);
    ((O Phip, G Z, G Wm), Scalar_Vector_Vector 1, I_G_ZWW);
    ((O Phim, G Wp, G Ga), Scalar_Vector_Vector 1, I_Q_W);
    ((O Phim, G Wp, G Z), Scalar_Vector_Vector 1, I_G_ZWW) ]
```

New Resonances

⚠ There is an extra minus in the Lagrangian to have the same sign as HWW or HZZ vertex. Effectivly this doesn't matter for SSC, because $(-1)^2 = 1$. This is only for completeness.

$$\mathbf{V}_\mu = -\mathrm{i}g\mathbf{W}_\mu + \mathrm{i}g'\mathbf{B}_\mu \tag{13.65a}$$

$$\mathbf{W}_\mu = W_\mu^a \frac{\tau^a}{2} \tag{13.65b}$$

$$\mathbf{B}_\mu = W_\mu^a \frac{\tau^3}{2} \tag{13.65c}$$

$$\tau^{++} = \tau^+ \otimes \tau^+ \tag{13.65d}$$

$$\tau^+ = \frac{1}{2}\left(\tau^+ \otimes \tau^3 + \tau^3 + \tau^+\right) \tag{13.65e}$$

$$\tau^0 = \frac{1}{\sqrt{6}}\left(\tau^3 \otimes \tau^3 - \tau^+ \otimes \tau^- - \tau^- + \tau^+\right) \tag{13.65f}$$

$$\tau^- = \frac{1}{2}\left(\tau^- \otimes \tau^3 + \tau^3 + \tau^-\right) \tag{13.65g}$$

$$\tau^{--} = \tau^- \otimes \tau^- \tag{13.65h}$$

Scalar Isoscalar Old representation

$$\mathcal{L}_\sigma = -\frac{g_\sigma v}{2}\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\sigma \tag{13.66}$$

⚠ Transversal couplings like rsigma3t and rf3t are to be calculated in the new higgs matrix representation.

```
let rsigma3 =
  [ ((O Rsigma, G Wp, G Wm), Scalar_Vector_Vector 1, G_SWW);
    ((O Rsigma, G Z, G Z), Scalar_Vector_Vector 1, G_SZZ) ]

let rsigma3h =
  [ ((O Rsigma, O H, O H), Dim5_Scalar_Scalar2 1, G_SHH) ]

let rsigma3t =
  [ ((O Rsigma, G Wp, G Wm), Scalar_Vector_Vector_t 1, G_SWW_T);
    ((O Rsigma, G Z, G Z), Scalar_Vector_Vector_t 1, G_SZZ_T) ]

let rsigma4 =
  [ (O Rsigma, O Rsigma, G Wp, G Wm), Scalar2_Vector2 1, G_SSWW;
    (O Rsigma, O Rsigma, G Z, G Z), Scalar2_Vector2 1, G_SSZZ ]
```

Scalar Isotensor

$$\mathcal{L}_\phi = \frac{g_\phi v}{4} \mathrm{Tr} \left[\left(\mathbf{V}_\mu \otimes \mathbf{V}^\mu - \frac{\tau^{aa}}{6} \mathrm{Tr} \left[\mathbf{V}_\mu \mathbf{V}^\mu \right] \right) \phi \right] \tag{13.67a}$$

$$\phi = \sqrt{2} \left(\phi^{++} \tau^{++} + \phi^+ \tau^+ + \phi^0 \tau^0 + \phi^- \tau^- + \phi^{--} \tau^{--} \right) \tag{13.67b}$$

let *rphi3* =
 [((*O Rphin*, *G Wp*, *G Wm*), *Scalar_Vector_Vector* 1, *G_PNWW*);
  ((*O Rphin*, *G Z*, *G Z*), *Scalar_Vector_Vector* 1, *G_PNZZ*) ;
  ((*O Rphisn*, *G Wp*, *G Wm*), *Scalar_Vector_Vector* 1, *G_PSNWW*);
  ((*O Rphisn*, *G Z*, *G Z*), *Scalar_Vector_Vector* 1, *G_PSNZZ*) ;
  ((*O Rphip*, *G Z*, *G Wm*), *Scalar_Vector_Vector* 1, *G_PWZ*) ;
  ((*O Rphipp*, *G Wm*, *G Wm*), *Scalar_Vector_Vector* 1, *G_PWW*) ;
  ((*O Rphim*, *G Wp*, *G Z*), *Scalar_Vector_Vector* 1, *G_PWZ*) ;
  ((*O Rphimm*, *G Wp*, *G Wp*), *Scalar_Vector_Vector* 1, *G_PWW*)]

let *rphi3h* =
 [((*O Rphisn*, *O H*, *O H*), *Dim5_Scalar_Scalar2* 1, *G_PSNHH*)]

Tensor IsoScalar

let *rf3* =
 [((*O Rf*, *G Wp*, *G Wm*), *Tensor_2_Vector_Vector_1* 1, *G_FWW*);
  ((*O Rf*, *G Z*, *G Z*), *Tensor_2_Vector_Vector_1* 1, *G_FZZ*)]

let *rf3cf* =
 [((*O Rf*, *G Wp*, *G Wm*), *Tensor_2_Vector_Vector* 1, *G_FWW*);
  ((*O Rf*, *G Z*, *G Z*), *Tensor_2_Vector_Vector* 1, *G_FZZ*);
  ((*O Rf*, *G Wp*, *G Wm*), *Tensor_2_Vector_Vector_cf* 1, *G_FWW_CF*);
  ((*O Rf*, *G Z*, *G Z*), *Tensor_2_Vector_Vector_cf* 1, *G_FZZ_CF*)]

let *rf3h* =
 [((*O Rf*, *O H*, *O H*), *Tensor_2_Scalar_Scalar* 1, *G_FHH*);
  ((*O Rf*, *O H*, *O H*), *Tensor_2_Scalar_Scalar_cf* 1, *G_FHH_CF*)]

let *rf3t* =
 [((*O Rf*, *G Wp*, *G Wm*), *Tensor_2_Vector_Vector_t* 1, *G_FWW_T*);
  ((*O Rf*, *G Z*, *G Z*), *Tensor_2_Vector_Vector_t* 1, *G_FZZ_T*)]

Tensor Isotensor

$$\mathcal{L}_t \tag{13.68a}$$

let *rt3* =
 [((*O Rtn*, *G Wp*, *G Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TNWW*);
  ((*O Rtn*, *G Z*, *G Z*), *Tensor_2_Vector_Vector_1* 1, *G_TNZZ*) ;
  ((*O Rtsn*, *G Wp*, *G Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TSNWW*);
  ((*O Rtsn*, *G Z*, *G Z*), *Tensor_2_Vector_Vector_1* 1, *G_TSNZZ*) ;
  ((*O Rtp*, *G Z*, *G Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TWZ*) ;
  ((*O Rtpp*, *G Wm*, *G Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TWW*) ;
  ((*O Rtm*, *G Wp*, *G Z*), *Tensor_2_Vector_Vector_1* 1, *G_TWZ*) ;
  ((*O Rtmm*, *G Wp*, *G Wp*), *Tensor_2_Vector_Vector_1* 1, *G_TWW*)]

let *rt3cf* =

[((O Rtn, G Wp, G Wm), Tensor_2_Vector_Vector 1, G_TNWW);
((O Rtn, G Z, G Z), Tensor_2_Vector_Vector 1, G_TNZZ) ;
((O Rtsn, G Wp, G Wm), Tensor_2_Vector_Vector 1, G_TSNWW);
((O Rtsn, G Z, G Z), Tensor_2_Vector_Vector 1, G_TSNZZ) ;
((O Rtp, G Z, G Wm), Tensor_2_Vector_Vector 1, G_TWZ) ;
((O Rtpp, G Wm, G Wm), Tensor_2_Vector_Vector 1, G_TWW) ;
((O Rtm, G Wp, G Z), Tensor_2_Vector_Vector 1, G_TWZ) ;
((O Rtmm, G Wp, G Wp), Tensor_2_Vector_Vector 1, G_TWW);
((O Rtn, G Wp, G Wm), Tensor_2_Vector_Vector_cf 1, G_TNWW_CF);
((O Rtn, G Z, G Z), Tensor_2_Vector_Vector_cf 1, G_TNZZ_CF) ;
((O Rtsn, G Wp, G Wm), Tensor_2_Vector_Vector_cf 1, G_TSNWW_CF);
((O Rtsn, G Z, G Z), Tensor_2_Vector_Vector_cf 1, G_TSNZZ_CF) ;
((O Rtp, G Z, G Wm), Tensor_2_Vector_Vector_cf 1, G_TWZ_CF) ;
((O Rtpp, G Wm, G Wm), Tensor_2_Vector_Vector_cf 1, G_TWW_CF) ;
((O Rtm, G Wp, G Z), Tensor_2_Vector_Vector_cf 1, G_TWZ_CF) ;
((O Rtmm, G Wp, G Wp), Tensor_2_Vector_Vector_cf 1, G_TWW_CF)]

Anomalous trilinear interactions $f_i f_j V$ and ttH:

$$\Delta\mathcal{L}_{tt\gamma} = -e\frac{\upsilon}{\Lambda^2}\bar{t}i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)tA_\mu \tag{13.69}$$

let *anomalous_ttA* =
if *Flags.top_anom* then
[((M (U (−3)), G Ga, M (U 3)), FBF (1, Psibar, TVAM, Psi), G_TVA_ttA)]
else
[ ]

$$\Delta\mathcal{L}_{bb\gamma} = -e\frac{\upsilon}{\Lambda^2}\bar{b}i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)bA_\mu \tag{13.70}$$

let *anomalous_bbA* =
if *Flags.top_anom* then
[((M (D (−3)), G Ga, M (D 3)), FBF (1, Psibar, TVAM, Psi), G_TVA_bbA)]
else
[ ]

$$\Delta\mathcal{L}_{ttg} = -g_s\frac{\upsilon}{\Lambda^2}\bar{t}\lambda^a i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)tG_\mu^a \tag{13.71}$$

let *anomalous_ttG* =
if *Flags.top_anom* then
[((M (U (−3)), G Gl, M (U 3)), FBF (1, Psibar, TVAM, Psi), G_TVA_ttG)]
else
[ ]

$$\Delta\mathcal{L}_{ttZ} = -\frac{g}{2c_W}\frac{\upsilon^2}{\Lambda^2}\left[\bar{t}\not{Z}(X_L(k^2)P_L + X_R(k^2)P_R)t + \bar{t}\frac{i\sigma^{\mu\nu}k_\nu}{m_Z}(d_V(k^2) + id_A(k^2)\gamma_5)tZ_\mu\right] \tag{13.72}$$

let *anomalous_ttZ* =

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if *Flags.top_anom* then
  [((*M* (*U* (−3)), *G Z*, *M* (*U* 3)), *FBF* (1, *Psibar*, *VLRM*, *Psi*), *G_VLR_ttZ*);
   ((*M* (*U* (−3)), *G Z*, *M* (*U* 3)), *FBF* (1, *Psibar*, *TVAM*, *Psi*), *G_TVA_ttZ*)]
else
  [ ]

$$\Delta\mathcal{L}_{bbZ} = -\frac{g}{2c_W}\frac{\upsilon^2}{\Lambda^2}\bar{b}\frac{i\sigma^{\mu\nu}k_\nu}{m_Z}(d_V(k^2) + id_A(k^2)\gamma_5)bZ_\mu \qquad (13.73)$$

let *anomalous_bbZ* =
  if *Flags.top_anom* then
  [((*M* (*D* (−3)), *G Z*, *M* (*D* 3)), *FBF* (1, *Psibar*, *TVAM*, *Psi*), *G_TVA_bbZ*)]
  else
  [ ]

$$\Delta\mathcal{L}_{tbW} = -\frac{g}{\sqrt{2}}\frac{\upsilon^2}{\Lambda^2}\left[\bar{b}W^-(V_L(k^2)P_L + V_R(k^2)P_R)t + \bar{b}\frac{i\sigma^{\mu\nu}k_\nu}{m_W}(g_L(k^2)P_L + g_R(k^2)P_R)tW^-_\mu\right] + \text{H.c.}$$
$$(13.74)$$

let *anomalous_tbW* =
  if *Flags.top_anom* then
  [((*M* (*D* (−3)), *G Wm*, *M* (*U* 3)), *FBF* (1, *Psibar*, *VLRM*, *Psi*), *G_VLR_btW*);
   ((*M* (*U* (−3)), *G Wp*, *M* (*D* 3)), *FBF* (1, *Psibar*, *VLRM*, *Psi*), *G_VLR_tbW*);
   ((*M* (*D* (−3)), *G Wm*, *M* (*U* 3)), *FBF* (1, *Psibar*, *TLRM*, *Psi*), *G_TLR_btW*);
   ((*M* (*U* (−3)), *G Wp*, *M* (*D* 3)), *FBF* (1, *Psibar*, *TRLM*, *Psi*), *G_TRL_tbW*)]
  else
  [ ]

$$\Delta\mathcal{L}_{ttH} = -\frac{1}{\sqrt{2}}\bar{t}(Y_V(k^2) + iY_A(k^2)\gamma_5)tH \qquad (13.75)$$

let *anomalous_ttH* =
  if *Flags.top_anom* then
  [((*M* (*U* (−3)), *O H*, *M* (*U* 3)), *FBF* (1, *Psibar*, *SPM*, *Psi*), *G_SP_ttH*)]
  else
  [ ]

quartic fermion-gauge interactions $f_i f_j V_1 V_2$ emerging from gauge-invariant effective operators:

$$\Delta\mathcal{L}_{ttgg} = -\frac{g_s^2}{2}f_{abc}\frac{\upsilon}{\Lambda^2}\bar{t}\lambda^a\sigma^{\mu\nu}(d_V(k^2) + id_A(k^2)\gamma_5)tG^b_\mu G^c_\nu \qquad (13.76)$$

let *anomalous_ttGG* =
  if *Flags.top_anom* then
  [((*M* (*U* (−3)), *O* (*Aux_top* (2, 1, 0,true, *TTGG*)), *M* (*U* 3)), *FBF* (1, *Psibar*, *TVA*, *Psi*), *G_TVA*
   ((*O* (*Aux_top* (2, 1, 0,false, *TTGG*)), *G Gl*, *G Gl*), *Aux_Gauge_Gauge* 1, *I_Gs*)]
  else
  [ ]

$$\Delta \mathcal{L}_{tbWA} = -i \sin \theta_w \frac{g^2}{2\sqrt{2}} \frac{v^2}{\Lambda^2} \left[\bar{b} \frac{\sigma^{\mu\nu}}{m_W} (g_L(k^2) P_L + g_R(k^2) P_R) t A_\mu W_\nu^- \right] + \text{H.c.}$$

(13.77)

let *anomalous_tbWA* =
   if *Flags.top_anom* then
      [((M (D (-3)), O (Aux_top ($2, 0, -1$,true,$TBWA$)), M (U 3)), FBF (1, *Psibar*, *TLR*, *Psi*), G_TL
         ((O (Aux_top ($2, 0, 1$,false,$TBWA$)), G Ga, G Wm), Aux_Gauge_Gauge 1, I_G_weak);
         ((M (U (-3)), O (Aux_top ($2, 0, 1$,true,$TBWA$)), M (D 3)), FBF (1, *Psibar*, *TRL*, *Psi*), G_TRL
         ((O (Aux_top ($2, 0, -1$,false,$TBWA$)), G Wp, G Ga), Aux_Gauge_Gauge 1, I_G_weak)]
   else
      [ ]

$$\Delta \mathcal{L}_{tbWZ} = -i \cos \theta_w \frac{g^2}{2\sqrt{2}} \frac{v^2}{\Lambda^2} \left[\bar{b} \frac{\sigma^{\mu\nu}}{m_W} (g_L(k^2) P_L + g_R(k^2) P_R) t Z_\mu W_\nu^- \right] + \text{H.c.}$$

(13.78)

let *anomalous_tbWZ* =
   if *Flags.top_anom* then
      [((M (D (-3)), O (Aux_top ($2, 0, -1$,true,$TBWZ$)), M (U 3)), FBF (1, *Psibar*, *TLR*, *Psi*), G_TL
         ((O (Aux_top ($2, 0, 1$,false,$TBWZ$)), G Z, G Wm), Aux_Gauge_Gauge 1, I_G_weak);
         ((M (U (-3)), O (Aux_top ($2, 0, 1$,true,$TBWZ$)), M (D 3)), FBF (1, *Psibar*, *TRL*, *Psi*), G_TRL
         ((O (Aux_top ($2, 0, -1$,false,$TBWZ$)), G Wp, G Z), Aux_Gauge_Gauge 1, I_G_weak)]
   else
      [ ]

$$\Delta \mathcal{L}_{ttWW} = -i \frac{g^2}{2} \frac{v^2}{\Lambda^2} \bar{t} \frac{\sigma^{\mu\nu}}{m_W} (d_V(k^2) + i d_A(k^2) \gamma_5) t W_\mu^- W_\nu^+$$

(13.79)

let *anomalous_ttWW* =
   if *Flags.top_anom* then
      [((M (U (-3)), O (Aux_top ($2, 0, 0$,true,$TTWW$)), M (U 3)), FBF (1, *Psibar*, *TVA*, *Psi*), G_TVA
         ((O (Aux_top ($2, 0, 0$,false,$TTWW$)), G Wm, G Wp), Aux_Gauge_Gauge 1, I_G_weak)]
   else
      [ ]

$$\Delta \mathcal{L}_{bbWW} = -i \frac{g^2}{2} \frac{v^2}{\Lambda^2} \bar{b} \frac{\sigma^{\mu\nu}}{m_W} (d_V(k^2) + i d_A(k^2) \gamma_5) b W_\mu^- W_\nu^+$$

(13.80)

let *anomalous_bbWW* =
   if *Flags.top_anom* then
      [((M (D (-3)), O (Aux_top ($2, 0, 0$,true,$BBWW$)), M (D 3)), FBF (1, *Psibar*, *TVA*, *Psi*), G_TVA
         ((O (Aux_top ($2, 0, 0$,false,$BBWW$)), G Wm, G Wp), Aux_Gauge_Gauge 1, I_G_weak)]
   else
      [ ]

4-fermion contact terms emerging from operator rewriting:

let *anomalous_top_qGuG_tt* =

[(((*M* (*U* (−3)), *O* (*Aux_top* (1, 1, 0,true,*QGUG*)), *M* (*U* 3)), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_q*

let *anomalous_top_qGuG_ff* *n* =
  List.map mom
    [(((*U* (−*n*)), *Aux_top* (1, 1, 0,false,*QGUG*), *U* *n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Unit*);
      ((*D* (−*n*)), *Aux_top* (1, 1, 0,false,*QGUG*), *D* *n*), *FBF* (1, *Psibar*, *V*, *Psi*), *Unit*)]

let *anomalous_top_qGuG* =
  if *Flags.top_anom_4f* then
    anomalous_top_qGuG_tt @ *ThoList.flatmap anomalous_top_qGuG_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_qBuB_tt* =
  [(((*M* (*U* (−3)), *O* (*Aux_top* (1, 0, 0,true,*QBUB*)), *M* (*U* 3)), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_q*

let *anomalous_top_qBuB_ff* *n* =
  List.map mom
    [(((*U* (−*n*)), *Aux_top* (1, 0, 0,false,*QBUB*), *U* *n*), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_u*);
      ((*D* (−*n*)), *Aux_top* (1, 0, 0,false,*QBUB*), *D* *n*), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_d*);
      ((*L* (−*n*)), *Aux_top* (1, 0, 0,false,*QBUB*), *L* *n*), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_e*);
      ((*N* (−*n*)), *Aux_top* (1, 0, 0,false,*QBUB*), *N* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qBuB_n*)]

let *anomalous_top_qBuB* =
  if *Flags.top_anom_4f* then
    anomalous_top_qBuB_tt @ *ThoList.flatmap anomalous_top_qBuB_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_qW_tq* =
  [(((*M* (*U* (−3)), *O* (*Aux_top* (1, 0, 0,true,*QW*)), *M* (*U* 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*);
    ((*M* (*D* (−3)), *O* (*Aux_top* (1, 0, −1,true,*QW*)), *M* (*U* 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*)
    ((*M* (*U* (−3)), *O* (*Aux_top* (1, 0, 1,true,*QW*)), *M* (*D* 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*)]

let *anomalous_top_qW_ff* *n* =
  List.map mom
    [(((*U* (−*n*)), *Aux_top* (1, 0, 0,false,*QW*), *U* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_u*);
      ((*D* (−*n*)), *Aux_top* (1, 0, 0,false,*QW*), *D* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_d*);
      ((*N* (−*n*)), *Aux_top* (1, 0, 0,false,*QW*), *N* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_u*);
      ((*L* (−*n*)), *Aux_top* (1, 0, 0,false,*QW*), *L* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_d*);
      ((*D* (−*n*)), *Aux_top* (1, 0, −1,false,*QW*), *U* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
      ((*U* (−*n*)), *Aux_top* (1, 0, 1,false,*QW*), *D* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
      ((*L* (−*n*)), *Aux_top* (1, 0, −1,false,*QW*), *N* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
      ((*N* (−*n*)), *Aux_top* (1, 0, 1,false,*QW*), *L* *n*), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*)]

let *anomalous_top_qW* =
  if *Flags.top_anom_4f* then
    anomalous_top_qW_tq @ *ThoList.flatmap anomalous_top_qW_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_DuDd* =
  if *Flags.top_anom_4f* then
    [(((*M* (*U* (−3)), *O* (*Aux_top* (0, 0, 0,true,*DR*)), *M* (*U* 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *Half*);
      ((*M* (*U* (−3)), *O* (*Aux_top* (0, 0, 0,false,*DR*)), *M* (*U* 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *G_SL_DttR*

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$((M (D (-3)), O (Aux\_top (0, 0, 0, \text{false}, DR)), M (D 3)), FBF (1, Psibar, SR, Psi), G\_SR\_DttR$

$((M (U (-3)), O (Aux\_top (0, 0, 0, \text{true}, DL)), M (U 3)), FBF (1, Psibar, SL, Psi), Half);$

$((M (D (-3)), O (Aux\_top (0, 0, 0, \text{false}, DL)), M (D 3)), FBF (1, Psibar, SL, Psi), G\_SL\_DttL)$

$((M (D (-3)), O (Aux\_top (0, 0, -1, \text{true}, DR)), M (U 3)), FBF (1, Psibar, SR, Psi), Half);$

$((M (U (-3)), O (Aux\_top (0, 0, 1, \text{false}, DR)), M (D 3)), FBF (1, Psibar, SLR, Psi), G\_SLR\_D$

$((M (D (-3)), O (Aux\_top (0, 0, -1, \text{true}, DL)), M (U 3)), FBF (1, Psibar, SL, Psi), Half);$

$((M (U (-3)), O (Aux\_top (0, 0, 1, \text{false}, DL)), M (D 3)), FBF (1, Psibar, SL, Psi), G\_SL\_DbtL)$

    else

      [ ]

let *vertices3* =

  (*ThoList.flatmap electromagnetic_currents* [1; 2; 3] @

   ThoList.flatmap color_currents [1; 2; 3] @

   ThoList.flatmap neutral_currents [1; 2; 3] @

  (if *Flags.ckm_present* then

    charged_currents_ckm

  else

    charged_currents_triv) @

  yukawa @ *triple_gauge* @

  gauge_higgs @ *higgs* @ *higgs_triangle_vertices*

  @ *goldstone_vertices* @

  rsigma3 @ *rsigma3t* @ *rphi3* @

  (if *Flags.cf_arbitrary* then

      (*rf3cf* @ *rt3cf*)

    else

      (*rf3* @ *rt3*)) @

  rf3t @

  (if *Flags.higgs_matrix* then

      (*rsigma3h* @ *rphi3h* @ *rf3h*)

    else

      [ ]) @

  anomalous_ttA @ *anomalous_bbA* @

  anomalous_ttZ @ *anomalous_bbZ* @

  anomalous_tbW @ *anomalous_tbWA* @ *anomalous_tbWZ* @

  anomalous_ttWW @ *anomalous_bbWW* @

  anomalous_ttG @ *anomalous_ttGG* @

  anomalous_ttH @

  anomalous_top_qGuG @ *anomalous_top_qBuB* @

  anomalous_top_qW @ *anomalous_top_DuDd*)

let *vertices4* =

  quartic_gauge @ *gauge_higgs4* @ *higgs4*

let *vertices* () = (*vertices3*, *vertices4*, [ ])

For efficiency, make sure that *F.of_vertices vertices* is evaluated only once.

let *table* = *F.of_vertices* (*vertices* ())

let *fuse2* = *F.fuse2 table*

let *fuse3* = *F.fuse3 table*

let *fuse* = *F.fuse table*

let *max_degree* () = 4

let *flavor_of_string* = function
  | "e-" → $M\ (L\ 1)$ | "e+" → $M\ (L\ (-1))$
  | "mu-" → $M\ (L\ 2)$ | "mu+" → $M\ (L\ (-2))$
  | "tau-" → $M\ (L\ 3)$ | "tau+" → $M\ (L\ (-3))$
  | "nue" → $M\ (N\ 1)$ | "nuebar" → $M\ (N\ (-1))$
  | "numu" → $M\ (N\ 2)$ | "numubar" → $M\ (N\ (-2))$
  | "nutau" → $M\ (N\ 3)$ | "nutaubar" → $M\ (N\ (-3))$
  | "u" → $M\ (U\ 1)$ | "ubar" → $M\ (U\ (-1))$
  | "c" → $M\ (U\ 2)$ | "cbar" → $M\ (U\ (-2))$
  | "t" → $M\ (U\ 3)$ | "tbar" → $M\ (U\ (-3))$
  | "d" → $M\ (D\ 1)$ | "dbar" → $M\ (D\ (-1))$
  | "s" → $M\ (D\ 2)$ | "sbar" → $M\ (D\ (-2))$
  | "b" → $M\ (D\ 3)$ | "bbar" → $M\ (D\ (-3))$
  | "g" | "gl" → $G\ Gl$
  | "A" → $G\ Ga$ | "Z" | "Z0" → $G\ Z$
  | "W+" → $G\ Wp$ | "W-" → $G\ Wm$
  | "H" → $O\ H$ | "Rsigma" → $O\ Rsigma$
  | "Rphi0" → $O\ Rphin$
  | "Rphis0" → $O\ Rphisn$
  | "Rphi+" → $O\ Rphip$ | "Rphi-" → $O\ Rphim$
  | "Rphi++" → $O\ Rphip$ | "Rphi--" → $O\ Rphimm$
  | "Rf" → $O\ Rf$
  | "Rt0" → $O\ Rtn$
  | "Rts0" → $O\ Rtsn$
  | "Rt+" → $O\ Rtp$ | "Rt-" → $O\ Rtm$
  | "Rt++" → $O\ Rtp$ | "Rt--" → $O\ Rtmm$
  | "Aux_t_ttGG0" → $O\ (Aux\_top\ (2,1,\ 0,\text{true},TTGG))$ | "Aux_ttGG0" → $O\ (Aux\_top\ (2,1,\ 0,\text{false},TTGG))$
  | "Aux_t_tbWA+" → $O\ (Aux\_top\ (2,0,\ 1,\text{true},TBWA))$ | "Aux_tbWA+" → $O\ (Aux\_top\ (2,0,\ 1,\text{false},TBWA))$
  | "Aux_t_tbWA-" → $O\ (Aux\_top\ (2,0,-1,\text{true},TBWA))$ | "Aux_tbWA-" → $O\ (Aux\_top\ (2,0,-1,\text{false},TBWA))$
  | "Aux_t_tbWZ+" → $O\ (Aux\_top\ (2,0,\ 1,\text{true},TBWZ))$ | "Aux_tbWZ+" → $O\ (Aux\_top\ (2,0,\ 1,\text{false},TBWZ))$
  | "Aux_t_tbWZ-" → $O\ (Aux\_top\ (2,0,-1,\text{true},TBWZ))$ | "Aux_tbWZ-" → $O\ (Aux\_top\ (2,0,-1,\text{false},TBWZ))$
  | "Aux_t_ttWW0" → $O\ (Aux\_top\ (2,0,\ 0,\text{true},TTWW))$ | "Aux_ttWW0" → $O\ (Aux\_top\ (2,0,\ 0,\text{false},TTWW))$
  | "Aux_t_bbWW0" → $O\ (Aux\_top\ (2,0,\ 0,\text{true},BBWW))$ | "Aux_bbWW0" → $O\ (Aux\_top\ (2,0,\ 0,\text{false},BBWW))$
  | "Aux_t_qGuG0" → $O\ (Aux\_top\ (1,1,\ 0,\text{true},QGUG))$ | "Aux_qGuG0" → $O\ (Aux\_top\ (1,1,\ 0,\text{false},QGUG))$
  | "Aux_t_qBuB0" → $O\ (Aux\_top\ (1,0,\ 0,\text{true},QBUB))$ | "Aux_qBuB0" → $O\ (Aux\_top\ (1,0,\ 0,\text{false},QBUB))$
  | "Aux_t_qW0" → $O\ (Aux\_top\ (1,0,\ 0,\text{true},QW))$ | "Aux_qW0" → $O\ (Aux\_top\ (1,0,\ 0,\text{false},QW))$
  | "Aux_t_qW+" → $O\ (Aux\_top\ (1,0,\ 1,\text{true},QW))$ | "Aux_qW+" → $O\ (Aux\_top\ (1,0,\ 1,\text{false},QW))$
  | "Aux_t_qW-" → $O\ (Aux\_top\ (1,0,-1,\text{true},QW))$ | "Aux_qW-" → $O\ (Aux\_top\ (1,0,-1,\text{false},QW))$

```
      |  "Aux_t_dL0"  →   O (Aux_top (0,0, 0,true,DL))  |  "Aux_dL0"  →
O (Aux_top (0,0, 0,false,DL))
      |  "Aux_t_dL+"  →   O (Aux_top (0,0, 1,true,DL))  |  "Aux_dL+"  →
O (Aux_top (0,0, 1,false,DL))
      |  "Aux_t_dL-"  →   O (Aux_top (0,0,−1,true,DL))  |  "Aux_dL-"  →
O (Aux_top (0,0,−1,false,DL))
      |  "Aux_t_dR0"  →   O (Aux_top (0,0, 0,true,DR))  |  "Aux_dR0"  →
O (Aux_top (0,0, 0,false,DR))
      |  "Aux_t_dR+"  →   O (Aux_top (0,0, 1,true,DR))  |  "Aux_dR+"  →
O (Aux_top (0,0, 1,false,DR))
      |  "Aux_t_dR-"  →   O (Aux_top (0,0,−1,true,DR))  |  "Aux_dR-"  →
O (Aux_top (0,0,−1,false,DR))
      |  _  →  invalid_arg "Modellib_BSM.SSC.flavor_of_string"

   let flavor_to_string  =  function
    |  M f  →
        begin match f with
        |  L 1  →  "e-" |  L (−1)  →  "e+"
        |  L 2  →  "mu-" |  L (−2)  →  "mu+"
        |  L 3  →  "tau-" |  L (−3)  →  "tau+"
        |  L _  →  invalid_arg
              "Modellib_BSM.SSC.flavor_to_string:␣invalid␣lepton"
        |  N 1  →  "nue" |  N (−1)  →  "nuebar"
        |  N 2  →  "numu" |  N (−2)  →  "numubar"
        |  N 3  →  "nutau" |  N (−3)  →  "nutaubar"
        |  N _  →  invalid_arg
              "Modellib_BSM.SSC.flavor_to_string:␣invalid␣neutrino"
        |  U 1  →  "u" |  U (−1)  →  "ubar"
        |  U 2  →  "c" |  U (−2)  →  "cbar"
        |  U 3  →  "t" |  U (−3)  →  "tbar"
        |  U _  →  invalid_arg
              "Modellib_BSM.SSC.flavor_to_string:␣invalid␣up␣type␣quark"
        |  D 1  →  "d" |  D (−1)  →  "dbar"
        |  D 2  →  "s" |  D (−2)  →  "sbar"
        |  D 3  →  "b" |  D (−3)  →  "bbar"
        |  D _  →  invalid_arg
              "Modellib_BSM.SSC.flavor_to_string:␣invalid␣down␣type␣quark"
        end
    |  G f  →
        begin match f with
        |  Gl  →  "gl"
        |  Ga  →  "A" |  Z  →  "Z"
        |  Wp  →  "W+" |  Wm  →  "W-"
        end
    |  O f  →
        begin match f with
        |  Phip  →  "phi+" |  Phim  →  "phi-" |  Phi0  →  "phi0"
        |  H  →  "H" |  Rsigma  →  "Rsigma"
        |  Rphin  →  "Rphin" |  Rphisn  →  "Rphisn"
        |  Rphip  →  "Rphi+" |  Rphim  →  "Rphi-"
```

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```
            | Rphipp → "Rphi++" | Rphimm → "Rphi--"
            | Rf → "Rf"
            | Rtn → "Rtn" | Rtsn → "Rtsn" | Rtp → "Rt+" | Rtm →
"Rt-"
            | Rtpp → "Rt++" | Rtmm → "Rt--"
            | Aux_top (_, _, ch, n, v) → "Aux_" ^ (if n then "t_" else "") ^ (
                begin match v with
                | TTGG → "ttGG" | TBWA → "tbWA" | TBWZ →
"tbWZ"
                | TTWW → "ttWW" | BBWW → "bbWW"
                | QGUG → "qGuG" | QBUB → "qBuB"
                | QW → "qW" | DL → "dL" | DR → "dR"
                end ) ^ ( if ch > 0 then "+" else if ch < 0 then "-" else "0" )
            end
    let flavor_to_TeX = function
      | M f →
          begin match f with
          | L 1 → "e^-" | L (−1) → "e^+"
          | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
          | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
          | L _ → invalid_arg
                "Modellib_BSM.SSC.flavor_to_TeX:␣invalid␣lepton"
          | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
          | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
          | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
          | N _ → invalid_arg
                "Modellib_BSM.SSC.flavor_to_TeX:␣invalid␣neutrino"
          | U 1 → "u" | U (−1) → "\\bar{u}"
          | U 2 → "c" | U (−2) → "\\bar{c}"
          | U 3 → "t" | U (−3) → "\\bar{t}"
          | U _ → invalid_arg
                "Modellib_BSM.SSC.flavor_to_TeX:␣invalid␣up␣type␣quark"
          | D 1 → "d" | D (−1) → "\\bar{d}"
          | D 2 → "s" | D (−2) → "\\bar{s}"
          | D 3 → "b" | D (−3) → "\\bar{b}"
          | D _ → invalid_arg
                "Modellib_BSM.SSC.flavor_to_TeX:␣invalid␣down␣type␣quark"
          end
      | G f →
          begin match f with
          | Gl → "g"
          | Ga → "\\gamma" | Z → "Z"
          | Wp → "W^+" | Wm → "W^-"
          end
      | O f →
          begin match f with
          | Phip → "\\phi^+" | Phim → "\\phi^-" | Phi0 →
"\\phi^0"
          | H → "H" | Rsigma → "\\sigma"
```

```
        | Rphip → "\\phi^+" | Rphim → "\\phi^-" | Rphin →
"\\phi^0"
        | Rphisn → "\\phi_s^0"
        | Rphipp → "\\phi^{++}" | Rphimm → "\\phi^{--}"
        | Rf → "f"
        | Rtp → "t^+" | Rtm → "t^-" | Rtn → "t^0" | Rtsn →
"t_s^0"
        | Rtpp → "t^{++}" | Rtmm → "t^{--}"
        | Aux_top (_, _, ch, n, v) → "\\textnormal{Aux_" ^ (if n then "t_" else "") ^ (
            begin match v with
            | TTGG → "ttGG" | TBWA → "tbWA" | TBWZ →
"tbWZ"
            | TTWW → "ttWW" | BBWW → "bbWW"
            | QGUG → "qGuG" | QBUB → "qBuB"
            | QW → "qW" | DL → "dL" | DR → "dR"
            end ) ^ ( if ch > 0 then "^+" else if ch < 0 then "^-" else "^0" ) ^ "}"
        end

    let flavor_symbol = function
      | M f →
        begin match f with
        | L n when n > 0 → "l" ^ string_of_int n
        | L n → "l" ^ string_of_int (abs n) ^ "b"
        | N n when n > 0 → "n" ^ string_of_int n
        | N n → "n" ^ string_of_int (abs n) ^ "b"
        | U n when n > 0 → "u" ^ string_of_int n
        | U n → "u" ^ string_of_int (abs n) ^ "b"
        | D n when n > 0 → "d" ^ string_of_int n
        | D n → "d" ^ string_of_int (abs n) ^ "b"
        end
      | G f →
        begin match f with
        | Gl → "gl"
        | Ga → "a" | Z → "z"
        | Wp → "wp" | Wm → "wm"
        end
      | O f →
        begin match f with
        | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
        | H → "h" | Rsigma → "rsi"
        | Rphip → "rpp" | Rphim → "rpm" | Rphin → "rpn"
        | Rphisn → "rpsn"
        | Rphipp → "rppp" | Rphimm → "rpmm"
        | Rf → "rf"
        | Rtp → "rtp" | Rtm → "rtm" | Rtn → "rtn"
        | Rtsn → "rtsn"
        | Rtpp → "rtpp" | Rtmm → "rtmm"
        | Aux_top (_, _, ch, n, v) → "aux_" ^ (if n then "t_" else "") ^ (
            begin match v with
            | TTGG → "ttgg" | TBWA → "tbwa" | TBWZ →
```

```
"tbwz"
                      |  TTWW  →  "ttww" |  BBWW  →  "bbww"
                      |  QGUG  →  "qgug" |  QBUB  →  "qbub"
                      |  QW  →  "qw" |  DL  →  "dl" |  DR  →  "dr"
                      end ) ^ "_" ^ ( if ch > 0 then "p" else if ch < 0 then "m" else "0" )
              end
```

Introducing new Resonances from 45, there are no PDG values

```
      let pdg  =  function
        |  M f  →
              begin match f with
              |  L n when n > 0 → 9 + 2 × n
              |  L n →  − 9 + 2 × n
              |  N n when n > 0 → 10 + 2 × n
              |  N n →  − 10 + 2 × n
              |  U n when n > 0 → 2 × n
              |  U n → 2 × n
              |  D n when n > 0 →  − 1 + 2 × n
              |  D n → 1 + 2 × n
              end
        |  G f  →
              begin match f with
              |  Gl → 21
              |  Ga → 22 |  Z → 23
              |  Wp → 24 |  Wm → (−24)
              end
        |  O f  →
              begin match f with
              |  Phip |  Phim → 27 |  Phi0 → 26
              |  H → 25 |  Rsigma → 45
              |  Rphin → 46 |  Rphip |  Rphim → 47
              |  Rphipp |  Rphimm → 48
              |  Rphisn → 49
              |  Rf → 52
              |  Rtn → 53 |  Rtp |  Rtm → 54
              |  Rtpp |  Rtmm → 55
              |  Rtsn → 59
              |  Aux_top (_, _, _, _, _) → 81
              end

      let mass_symbol f  =
        "mass(" ^ string_of_int (abs (pdg f)) ^ ")"

      let width_symbol f  =
        "width(" ^ string_of_int (abs (pdg f)) ^ ")"

      let constant_symbol  =  function
        |  Unit → "unit" |  Half → "half" |  Pi → "PI"
        |  Alpha_QED → "alpha" |  E → "e" |  G_weak → "g" |  Vev →
"vev"
        |  I_G_weak → "ig"
```

```
        |   Sin2thw   →   "sin2thw" |   Sinthw   →   "sinthw" |   Costhw   →
"costhw"
        | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
        | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
        | G_NC_up → "gncup" | G_NC_down → "gncdwn"
        | G_TVA_ttA → "gtva_tta" | G_TVA_bbA → "gtva_bba"
        |  G_VLR_ttZ  →  "gvlr_ttz" |  G_TVA_ttZ  →  "gtva_ttz" |
G_TVA_bbZ → "gtva_bbz"
        | G_VLR_btW → "gvlr_btw" | G_VLR_tbW → "gvlr_tbw"
        | G_TLR_btW → "gtlr_btw" | G_TRL_tbW → "gtrl_tbw"
        | G_TLR_btWA → "gtlr_btwa" | G_TRL_tbWA → "gtrl_tbwa"
        | G_TLR_btWZ → "gtlr_btwz" | G_TRL_tbWZ → "gtrl_tbwz"
        | G_TVA_ttWW → "gtva_ttww" | G_TVA_bbWW → "gtva_bbww"
        | G_TVA_ttG → "gtva_ttg" | G_TVA_ttGG → "gtva_ttgg"
        | G_SP_ttH → "gsp_tth"
        | G_VLR_qGuG → "gvlr_qgug"
        | G_VLR_qBuB → "gvlr_qbub"
        | G_VLR_qBuB_u → "gvlr_qbub_u" | G_VLR_qBuB_d → "gvlr_qbub_d"
        | G_VLR_qBuB_e → "gvlr_qbub_e" | G_VL_qBuB_n → "gvl_qbub_n"
        | G_VL_qW → "gvl_qw"
        | G_VL_qW_u → "gvl_qw_u" | G_VL_qW_d → "gvl_qw_d"
        |  G_SL_DttR  →  "gsl_dttr" |  G_SR_DttR  →  "gsr_dttr" |
G_SL_DttL → "gsl_dttl"
        | G_SLR_DbtR → "gslr_dbtr" | G_SL_DbtL → "gsl_dbtl"
        | G_CC → "gcc"
        | G_CCQ (n1, n2) → "gccq" ^ string_of_int n1 ^ string_of_int n2
        | I_Q_W → "iqw" | I_G_ZWW → "igzww"
        | G_WWWW → "gw4" | G_ZZWW → "gzzww"
        | G_AZWW → "gazww" | G_AAWW → "gaaww"
        | I_G1_AWW → "ig1a" | I_G1_ZWW → "ig1z"
        | I_G1_plus_kappa_plus_G4_AWW → "ig1pkpg4a"
        | I_G1_plus_kappa_plus_G4_ZWW → "ig1pkpg4z"
        | I_G1_plus_kappa_minus_G4_AWW → "ig1pkmg4a"
        | I_G1_plus_kappa_minus_G4_ZWW → "ig1pkmg4z"
        | I_G1_minus_kappa_plus_G4_AWW → "ig1mkpg4a"
        | I_G1_minus_kappa_plus_G4_ZWW → "ig1mkpg4z"
        | I_G1_minus_kappa_minus_G4_AWW → "ig1mkmg4a"
        | I_G1_minus_kappa_minus_G4_ZWW → "ig1mkmg4z"
        | I_lambda_AWW → "ila"
        | I_lambda_ZWW → "ilz"
        | G5_AWW → "rg5a"
        | G5_ZWW → "rg5z"
        | I_kappa5_AWW → "ik5a"
        | I_kappa5_ZWW → "ik5z"
        | I_lambda5_AWW → "il5a" | I_lambda5_ZWW → "il5z"
        | Alpha_WWWW0 → "alww0" | Alpha_WWWW2 → "alww2"
        | Alpha_ZZWW0 → "alzw0" | Alpha_ZZWW1 → "alzw1"
        | Alpha_ZZZZ → "alzz"
        | FS0_HHWW → "fs0hhww" | FS0_HHZZ → "fs0hhzz"
        | FS1_HHWW → "fs1hhww" | FS1_HHZZ → "fs1hhzz"
```

| FS_H4 → "fsh4"
| $D\_Alpha\_ZZWW0\_S$ → "dalzz0_s(gkm,mkm,"
| $D\_Alpha\_ZZWW0\_T$ → "dalzz0_t(gkm,mkm,"
| $D\_Alpha\_ZZWW1\_S$ → "dalzz1_s(gkm,mkm,"
| $D\_Alpha\_ZZWW1\_T$ → "dalzz1_t(gkm,mkm,"
| $D\_Alpha\_ZZWW1\_U$ → "dalzz1_u(gkm,mkm,"
| $D\_Alpha\_WWWW0\_S$ → "dalww0_s(gkm,mkm,"
| $D\_Alpha\_WWWW0\_T$ → "dalww0_t(gkm,mkm,"
| $D\_Alpha\_WWWW0\_U$ → "dalww0_u(gkm,mkm,"
| $D\_Alpha\_WWWW2\_S$ → "dalww2_s(gkm,mkm,"
| $D\_Alpha\_WWWW2\_T$ → "dalww2_t(gkm,mkm,"
| $D\_Alpha\_ZZZZ\_S$ → "dalz4_s(gkm,mkm,"
| $D\_Alpha\_ZZZZ\_T$ → "dalz4_t(gkm,mkm,"
| $D\_Alpha\_HHHH\_S$ → "dalh4_s(gkm,mkm,"
| $D\_Alpha\_HHHH\_T$ → "dalh4_t(gkm,mkm,"
| $D\_Alpha\_HHWW0\_S$ → "dalhw0_s(gkm,mkm,"
| $D\_Alpha\_HHWW0\_T$ → "dalhw0_t(gkm,mkm,"
| $D\_Alpha\_HHZZ0\_S$ → "dalhz0_s(gkm,mkm,"
| $D\_Alpha\_HHZZ0\_T$ → "dalhz0_t(gkm,mkm,"
| $D\_Alpha\_HHWW1\_S$ → "dalhw1_s(gkm,mkm,"
| $D\_Alpha\_HHWW1\_T$ → "dalhw1_t(gkm,mkm,"
| $D\_Alpha\_HHWW1\_U$ → "dalhw1_u(gkm,mkm,"
| $D\_Alpha\_HHZZ1\_S$ → "dalhz1_s(gkm,mkm,"
| $D\_Alpha\_HHZZ1\_T$ → "dalhz1_t(gkm,mkm,"
| $D\_Alpha\_HHZZ1\_U$ → "dalhz1_u(gkm,mkm,"
| G_HWW → "ghww" | G_HZZ → "ghzz"
| G_HHWW → "ghhww" | G_HHZZ → "ghhzz"
| G_SWW → "gsww" | G_SZZ → "gszz"
| G_SHH → "gshh"
| G_SWW_T → "gswwt" | G_SZZ_T → "gszzt"
| G_PNWW → "gpnww" | G_PNZZ → "gpnzz"
| G_PSNWW → "gpsnww" | G_PSNZZ → "gpsnzz"
| G_PSNHH → "gpsnhh"
| G_PWZ → "gpwz" | G_PWW → "gpww"
| G_FWW → "gfww" | G_FZZ → "gfzz"
| G_FWW_CF → "gfwwcf" | G_FZZ_CF → "gfzzcf"
| G_FHH → "gfhh" | G_FHH_CF → "gfhhcf"
| G_FWW_T → "gfwwt" | G_FZZ_T → "gfzzt"
| G_TNWW → "gtnww" | G_TNZZ → "gtnzz"
| G_TNWW_CF → "gtnwwcf" | G_TNZZ_CF → "gtnzzcf"
| G_TSNWW → "gtsnww" | G_TSNZZ → "gtsnzz"
| G_TSNWW_CF → "gtsnwwcf" | G_TSNZZ_CF → "gtsnzzcf"
| G_TWZ → "gtwz" | G_TWW → "gtww"
| G_TWZ_CF → "gtwzcf" | G_TWW_CF → "gtwwcf"
| G_SSWW → "gssww" | G_SSZZ → "gsszz"
| G_Htt → "ghtt" | G_Hbb → "ghbb"
| $G\_Htautau$ → "ghtautau" | G_Hcc → "ghcc" | G_Hmm → "ghmm"
| G_HGaZ → "ghgaz" | G_HGaGa → "ghgaga" | G_Hgg → "ghgg"

    | G_HGaGa_anom → "ghgaga_ac" | G_HGaZ_anom → "ghgaz_ac"
    | G_HZZ_anom → "ghzz_ac" | G_HWW_anom → "ghww_ac"
    | G_HGaZ_u → "ghgaz_u" | G_HZZ_u → "ghzz_u"
    | G_HWW_u → "ghww_u"
    | G_H3 → "gh3" | G_H4 → "gh4"
    | Gs → "gs" | I_Gs → "igs" | $G2$ → "gs**2"
    | $Mass\ f$ → "mass" ^ *flavor_symbol f*
    | $Width\ f$ → "width" ^ *flavor_symbol f*
    | $K\_Matrix\_Coeff\ i$ → "kc" ^ *string_of_int i*
    | $K\_Matrix\_Pole\ i$ → "kp" ^ *string_of_int i*

  end

13.6.5 Complete Minimal Standard Model including additional Resonances (alternate Tensor)

module *SSC_AltT* (*Flags* : *SSC_flags*) =
  struct
    let *rcs* = *RCS.rename rcs_file* "Modellib_BSM.SSC_AltT"
      ["minimal␣electroweak␣standard␣model␣in␣unitarity␣gauge␣with␣additional␣Vectorboson

    open *Coupling*

    let *default_width* = *ref Timelike*
    let *use_fudged_width* = *ref* false

    let *options* = *Options.create*
      ["constant_width", *Arg.Unit* (fun () → *default_width* := *Constant*),
       "use␣constant␣width␣(also␣in␣t-channel)";
       "fudged_width", *Arg.Set use_fudged_width*,
       "use␣fudge␣factor␣for␣charge␣particle␣width";
       "custom_width", *Arg.String* (fun *f* → *default_width* := *Custom f*),
       "use␣custom␣width";
       "cancel_widths", *Arg.Unit* (fun () → *default_width* := *Vanishing*),
       "use␣vanishing␣width"]

    type f_aux_top = *TTGG* | *TBWA* | *TBWZ* | *TTWW* | *BBWW* |
    let *quartic_gauge* =
      standard_quartic_gauge @ *anomalous_quartic_gauge* @ *k_matrix_quartic_gauge*

    let *standard_gauge_higgs* =
      [((*O H*, *G Wp*, *G Wm*), *Scalar_Vector_Vector* 1, *G_HWW*);
       ((*O H*, *G Z*, *G Z*), *Scalar_Vector_Vector* 1, *G_HZZ*)]

    let *standard_gauge_higgs4* =
      [(*O H*, *O H*, *G Wp*, *G Wm*), *Scalar2_Vector2* 1, *G_HHWW*;
       (*O H*, *O H*, *G Z*, *G Z*), *Scalar2_Vector2* 1, *G_HHZZ*]

    let *dim8_gauge_higgs4* =
      [(*O H*, *O H*, *G Wp*, *G Wm*), *Dim8_Scalar2_Vector2_1* 1, *FS0_HHWW*;
       (*O H*, *O H*, *G Z*, *G Z*), *Dim8_Scalar2_Vector2_1* 1, *FS0_HHZZ*;
       (*O H*, *O H*, *G Wp*, *G Wm*), *Dim8_Scalar2_Vector2_2* 1, *FS1_HHWW*;
       (*O H*, *O H*, *G Z*, *G Z*), *Dim8_Scalar2_Vector2_2* 1, *FS1_HHZZ*]

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let *standard_higgs* =
  [(O H, O H, O H), *Scalar_Scalar_Scalar* 1, *G_H3*]

let *standard_higgs4* =
  [(O H, O H, O H, O H), *Scalar4* 1, *G_H4*]

let *fs_higgs4* =
  [(O H, O H, O H, O H), *Dim8_Scalar4* 1, *FS_H4*]

WK's couplings (apparently, he still intends to divide by $\Lambda_{\text{EWSB}}^2 = 16\pi^2 v_{\text{F}}^2$):

$$\mathcal{L}_4^\tau = \left[(\partial_\mu H)(\partial^\mu H) + \frac{g^2 v_{\text{F}}^2}{4} V_\mu V^\mu \right]^2 \tag{13.81a}$$

$$\mathcal{L}_5^\tau = \left[(\partial_\mu H)(\partial_\nu H) + \frac{g^2 v_{\text{F}}^2}{4} V_\mu V_\nu \right]^2 \tag{13.81b}$$

with

$$V_\mu V_\nu = \frac{1}{2} \left(W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^- \right) + \frac{1}{2\cos^2\theta_w} Z_\mu Z_\nu \tag{13.82}$$

(note the symmetrization!), i. e.

$$\mathcal{L}_4 = \alpha_4 \frac{g^4 v_{\text{F}}^4}{16} (V_\mu V_\nu)^2 \tag{13.83a}$$

$$\mathcal{L}_5 = \alpha_5 \frac{g^4 v_{\text{F}}^4}{16} (V_\mu V^\mu)^2 \tag{13.83b}$$

Breaking thinks up

$$\mathcal{L}_4^{\tau,H^4} = [(\partial_\mu H)(\partial^\mu H)]^2 \tag{13.84a}$$

$$\mathcal{L}_5^{\tau,H^4} = [(\partial_\mu H)(\partial^\mu H)]^2 \tag{13.84b}$$

and

$$\mathcal{L}_4^{\tau,H^2V^2} = \frac{g^2 v_{\text{F}}^2}{2} (\partial_\mu H)(\partial^\mu H) V_\mu V^\mu \tag{13.85a}$$

$$\mathcal{L}_5^{\tau,H^2V^2} = \frac{g^2 v_{\text{F}}^2}{2} (\partial_\mu H)(\partial_\nu H) V_\mu V_\nu \tag{13.85b}$$

i. e.

$$\mathcal{L}_4^{\tau,H^2V^2} = \frac{g^2 v_{\text{F}}^2}{2} \left[(\partial_\mu H)(\partial^\mu H) W_\nu^+ W^{-,\nu} + \frac{1}{2\cos^2\theta_w} (\partial_\mu H)(\partial^\mu H) Z_\nu Z^\nu \right] \tag{13.86a}$$

$$\mathcal{L}_5^{\tau,H^2V^2} = \frac{g^2 v_{\text{F}}^2}{2} \left[(W^{+,\mu}\partial_\mu H)(W^{-,\nu}\partial_\nu H) + \frac{1}{2\cos^2\theta_w} (Z^\mu \partial_\mu H)(Z^\nu \partial_\nu H) \right] \tag{13.86b}$$

$$\tau_8^4 \mathcal{L}_4^{\tau,H^2V^2} + \tau_8^5 \mathcal{L}_5^{\tau,H^2V^2} =$$

$$- \frac{g^2 v_{\text{F}}^2}{2} \left[2\tau_8^4 \frac{1}{2} (\mathrm{i}\partial_\mu H)(\mathrm{i}\partial^\mu H) W_\nu^+ W^{-,\nu} + \tau_8^5 (W^{+,\mu}\mathrm{i}\partial_\mu H)(W^{-,\nu}\mathrm{i}\partial_\nu H) \right.$$

$$+ \frac{2\tau_8^4}{\cos^2\theta_w}\frac{1}{4}(\mathrm{i}\partial_\mu H)(\mathrm{i}\partial^\mu H)Z_\nu Z^\nu + \frac{\tau_8^5}{\cos^2\theta_w}\frac{1}{2}(Z^\mu\mathrm{i}\partial_\mu H)(Z^\nu\mathrm{i}\partial_\nu H)\Bigg] \quad (13.87)$$

where the two powers of i make the sign conveniently negative, i. e.

$$\alpha^2_{(\partial H)^2 W^2} = \tau_8^4 g^2 v_{\mathrm{F}}^2 \qquad (13.88\mathrm{a})$$

$$\alpha^2_{(\partial HW)^2} = \frac{\tau_8^5 g^2 v_{\mathrm{F}}^2}{2} \qquad (13.88\mathrm{b})$$

$$\alpha^2_{(\partial H)^2 Z^2} = \frac{\tau_8^4 g^2 v_{\mathrm{F}}^2}{\cos^2\theta_w} \qquad (13.88\mathrm{c})$$

$$\alpha^2_{(\partial HZ)^2} = \frac{\tau_8^5 g^2 v_{\mathrm{F}}^2}{2\cos^2\theta_w} \qquad (13.88\mathrm{d})$$

```
let anomalous_gauge_higgs =
  [ (O H, G Ga, G Ga), Dim5_Scalar_Gauge2 1, G_HGaGa_anom;
    (O H, G Ga, G Z), Dim5_Scalar_Gauge2 1, G_HGaZ_anom;
    (O H, G Z, G Z), Dim5_Scalar_Gauge2 1, G_HZZ_anom;
    (O H, G Wp, G Wm), Dim5_Scalar_Gauge2 1, G_HWW_anom;
    (O H, G Ga, G Z), Dim5_Scalar_Vector_Vector_U 1, G_HGaZ_u;
    (O H, G Z, G Z), Dim5_Scalar_Vector_Vector_U 1, G_HZZ_u;
    (O H, G Wp, G Wm), Dim5_Scalar_Vector_Vector_U 1, G_HWW_u;
    (O H, G Wm, G Wp), Dim5_Scalar_Vector_Vector_U 1, G_HWW_u
  ]

let anomalous_gauge_higgs4 =
  []

let anomalous_higgs =
  []

let higgs_triangle_vertices =
  if Flags.higgs_triangle then
    [ (O H, G Ga, G Ga), Dim5_Scalar_Gauge2 1, G_HGaGa;
      (O H, G Ga, G Z), Dim5_Scalar_Gauge2 1, G_HGaZ;
      (O H, G Gl, G Gl), Dim5_Scalar_Gauge2 1, G_Hgg ]
  else
    []

let anomalous_higgs4 =
  []

let gauge_higgs =
  if Flags.higgs_anom then
    standard_gauge_higgs @ anomalous_gauge_higgs
  else
    standard_gauge_higgs

let gauge_higgs4 =
  ( if Flags.higgs_anom then
      standard_gauge_higgs4 @ anomalous_gauge_higgs4
    else
      standard_gauge_higgs4 ) @
```

(if *Flags.higgs_matrix* then
    (*dim8_gauge_higgs4* @ *k_matrix_2scalar_2gauge*)
  else
    [])

let *higgs* =
  if *Flags.higgs_anom* then
    standard_higgs @ *anomalous_higgs*
  else
    standard_higgs

let *higgs4* =
  (if *Flags.higgs_anom* then
    standard_higgs4 @ *anomalous_higgs4*
    else
    standard_higgs4) @
  (if *Flags.higgs_matrix* then
    (*fs_higgs4* @ *k_matrix_4scalar*)
    else
    [])

let *goldstone_vertices* =
[((*O Phi0*, *G Wm*, *G Wp*), *Scalar_Vector_Vector* 1, *I_G_ZWW*);
  ((*O Phip*, *G Ga*, *G Wm*), *Scalar_Vector_Vector* 1, *I_Q_W*);
  ((*O Phip*, *G Z*, *G Wm*), *Scalar_Vector_Vector* 1, *I_G_ZWW*);
  ((*O Phim*, *G Wp*, *G Ga*), *Scalar_Vector_Vector* 1, *I_Q_W*);
  ((*O Phim*, *G Wp*, *G Z*), *Scalar_Vector_Vector* 1, *I_G_ZWW*)]

New Resonances

There is an extra minus in the Lagrangian to have the same sign as HWW or HZZ vertex. Effectivly this doesn't matter for SSC, because $(-1)^2 = 1$. This is only for completeness.

$$\mathbf{V}_\mu = -\mathrm{i}g\mathbf{W}_\mu + \mathrm{i}g'\mathbf{B}_\mu \tag{13.89a}$$

$$\mathbf{W}_\mu = W^a_\mu \frac{\tau^a}{2} \tag{13.89b}$$

$$\mathbf{B}_\mu = W^a_\mu \frac{\tau^3}{2} \tag{13.89c}$$

$$\tau^{++} = \tau^+ \otimes \tau^+ \tag{13.89d}$$

$$\tau^+ = \frac{1}{2}\left(\tau^+ \otimes \tau^3 + \tau^3 + \tau^+\right) \tag{13.89e}$$

$$\tau^0 = \frac{1}{\sqrt{6}}\left(\tau^3 \otimes \tau^3 - \tau^+ \otimes \tau^- - \tau^- + \tau^+\right) \tag{13.89f}$$

$$\tau^- = \frac{1}{2}\left(\tau^- \otimes \tau^3 + \tau^3 + \tau^-\right) \tag{13.89g}$$

$$\tau^{--} = \tau^- \otimes \tau^- \tag{13.89h}$$

Scalar Isoscalar Old representation

$$\mathcal{L}_\sigma = -\frac{g_\sigma v}{2}\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\sigma \tag{13.90}$$

⚡ Transversal couplings like rsigma3t and rf3t are to be calculated in the new higgs matrix representation.

let *rsigma3* =
  [((O *Rsigma*, G *Wp*, G *Wm*), *Scalar_Vector_Vector* 1, *G_SWW*);
    ((O *Rsigma*, G *Z*, G *Z*), *Scalar_Vector_Vector* 1, *G_SZZ*)]

let *rsigma3h* =
  [((O *Rsigma*, O *H*, O *H*), *Dim5_Scalar_Scalar2* 1, *G_SHH*)]

let *rsigma3t* =
  [((O *Rsigma*, G *Wp*, G *Wm*), *Scalar_Vector_Vector_t* 1, *G_SWW_T*);
    ((O *Rsigma*, G *Z*, G *Z*), *Scalar_Vector_Vector_t* 1, *G_SZZ_T*)]

let *rsigma4* =
  [(O *Rsigma*, O *Rsigma*, G *Wp*, G *Wm*), *Scalar2_Vector2* 1, *G_SSWW*;
    (O *Rsigma*, O *Rsigma*, G *Z*, G *Z*), *Scalar2_Vector2* 1, *G_SSZZ*]

Scalar Isotensor

$$\mathcal{L}_\phi = \frac{g_\phi v}{4} \mathrm{Tr}\left[\left(\mathbf{V}_\mu \otimes \mathbf{V}^\mu - \frac{\tau^{aa}}{6} \mathrm{Tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu \right] \right) \phi \right] \tag{13.91a}$$

$$\phi = \sqrt{2}\left(\phi^{++}\tau^{++} + \phi^{+}\tau^{+} + \phi^{0}\tau^{0} + \phi^{-}\tau^{-} + \phi^{--}\tau^{--} \right) \tag{13.91b}$$

let *rphi3* =
  [((O *Rphin*, G *Wp*, G *Wm*), *Scalar_Vector_Vector* 1, *G_PNWW*);
    ((O *Rphin*, G *Z*, G *Z*), *Scalar_Vector_Vector* 1, *G_PNZZ*) ;
    ((O *Rphisn*, G *Wp*, G *Wm*), *Scalar_Vector_Vector* 1, *G_PSNWW*);
    ((O *Rphisn*, G *Z*, G *Z*), *Scalar_Vector_Vector* 1, *G_PSNZZ*) ;
    ((O *Rphip*, G *Z*, G *Wm*), *Scalar_Vector_Vector* 1, *G_PWZ*) ;
    ((O *Rphipp*, G *Wm*, G *Wm*), *Scalar_Vector_Vector* 1, *G_PWW*) ;
    ((O *Rphim*, G *Wp*, G *Z*), *Scalar_Vector_Vector* 1, *G_PWZ*) ;
    ((O *Rphimm*, G *Wp*, G *Wp*), *Scalar_Vector_Vector* 1, *G_PWW*)]

let *rphi3h* =
  [((O *Rphisn*, O *H*, O *H*), *Dim5_Scalar_Scalar2* 1, *G_PSNHH*)]

Tensor IsoScalar

let *rf3* =
  [((O *Rf*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector_1* 1, *G_FWW*);
    ((O *Rf*, G *Z*, G *Z*), *Tensor_2_Vector_Vector_1* 1, *G_FZZ*)]

let *rf3cf* =
  [((O *Rf*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector* 1, *G_FWW*);
    ((O *Rf*, G *Z*, G *Z*), *Tensor_2_Vector_Vector* 1, *G_FZZ*);
    ((O *Rf*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector_cf* 1, *G_FWW_CF*);
    ((O *Rf*, G *Z*, G *Z*), *Tensor_2_Vector_Vector_cf* 1, *G_FZZ_CF*)]

let *rff3cf* =
  [((O *Rff*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector* 1, *G_FFWW*);
    ((O *Rff*, G *Z*, G *Z*), *Tensor_2_Vector_Vector* 1, *G_FFZZ*);
    ((O *Rff*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector_cf* 1, *G_FFWW_CF*);
    ((O *Rff*, G *Z*, G *Z*), *Tensor_2_Vector_Vector_cf* 1, *G_FFZZ_CF*)]

let *rfv3cf* =
  [((O *Rfv*, G *Wp*, G *Wm*), *TensorVector_Vector_Vector* 1, *G_FVWW*);
    ((O *Rfv*, G *Z*, G *Z*), *TensorVector_Vector_Vector* 1, *G_FVZZ*);
    ((O *Rfv*, G *Wp*, G *Wm*), *TensorVector_Vector_Vector_cf* 1, *G_FVWW_CF*);
    ((O *Rfv*, G *Z*, G *Z*), *TensorVector_Vector_Vector_cf* 1, *G_FVZZ_CF*)]

let *rfddphi3cf* =
  [((O *Rfphi*, G *Wp*, G *Wm*), *TensorScalar_Vector_Vector* 1, *G_FDDSWW*);
    ((O *Rfphi*, G *Z*, G *Z*), *TensorScalar_Vector_Vector* 1, *G_FDDSZZ*);
    ((O *Rfphi*, G *Wp*, G *Wm*), *TensorScalar_Vector_Vector_cf* 1, *G_FDDSWW_CF*);
    ((O *Rfphi*, G *Z*, G *Z*), *TensorScalar_Vector_Vector_cf* 1, *G_FDDSZZ_CF*)]

let *rfphi3cf* =
  [((O *Rfphi*, G *Wp*, G *Wm*), *Scalar_Vector_Vector* 1, *G_FSWW*);
    ((O *Rfphi*, G *Z*, G *Z*), *Scalar_Vector_Vector* 1, *G_FSZZ*)]

let *rf3h* =
  [((O *Rf*, O *H*, O *H*), *Tensor_2_Scalar_Scalar* 1, *G_FHH*);
    ((O *Rf*, O *H*, O *H*), *Tensor_2_Scalar_Scalar_cf* 1, *G_FHH_CF*)]

let *rff3h* =
  [((O *Rff*, O *H*, O *H*), *Tensor_2_Scalar_Scalar* 1, *G_FFHH*);
    ((O *Rff*, O *H*, O *H*), *Tensor_2_Scalar_Scalar_cf* 1, *G_FFHH_CF*);
    ((O *Rfv*, O *H*, O *H*), *TensorVector_Scalar_Scalar* 1, *G_FVHH*);
    ((O *Rfv*, O *H*, O *H*), *TensorVector_Scalar_Scalar_cf* 1, *G_FVHH_CF*);
    ((O *Rfphi*, O *H*, O *H*), *TensorScalar_Scalar_Scalar* 1, *G_FDDSHH*);
    ((O *Rfphi*, O *H*, O *H*), *TensorScalar_Scalar_Scalar_cf* 1, *G_FDDSHH_CF*);
    ((O *Rfphi*, O *H*, O *H*), *Dim5_Scalar_Scalar2* 1, *G_FSHH*)]

let *rf3t* =
  [((O *Rf*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector_t* 1, *G_FWW_T*);
    ((O *Rf*, G *Z*, G *Z*), *Tensor_2_Vector_Vector_t* 1, *G_FZZ_T*)]

Tensor Isotensor

$$\mathcal{L}_t \tag{13.92a}$$

let *rt3* =
  [((O *Rtn*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TNWW*);
    ((O *Rtn*, G *Z*, G *Z*), *Tensor_2_Vector_Vector_1* 1, *G_TNZZ*) ;
    ((O *Rtsn*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TSNWW*);
    ((O *Rtsn*, G *Z*, G *Z*), *Tensor_2_Vector_Vector_1* 1, *G_TSNZZ*) ;
    ((O *Rtp*, G *Z*, G *Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TWZ*) ;
    ((O *Rtpp*, G *Wm*, G *Wm*), *Tensor_2_Vector_Vector_1* 1, *G_TWW*);
    ((O *Rtm*, G *Wp*, G *Z*), *Tensor_2_Vector_Vector_1* 1, *G_TWZ*) ;
    ((O *Rtmm*, G *Wp*, G *Wp*), *Tensor_2_Vector_Vector_1* 1, *G_TWW*)]

let *rt3cf* =
  [((O *Rtn*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector* 1, *G_TNWW*);
    ((O *Rtn*, G *Z*, G *Z*), *Tensor_2_Vector_Vector* 1, *G_TNZZ*) ;
    ((O *Rtsn*, G *Wp*, G *Wm*), *Tensor_2_Vector_Vector* 1, *G_TSNWW*);
    ((O *Rtsn*, G *Z*, G *Z*), *Tensor_2_Vector_Vector* 1, *G_TSNZZ*) ;
    ((O *Rtp*, G *Z*, G *Wm*), *Tensor_2_Vector_Vector* 1, *G_TWZ*) ;

$((O\ Rtpp,\ G\ Wm,\ G\ Wm),\ Tensor\_2\_Vector\_Vector\ 1,\ G\_TWW)$;
$((O\ Rtm,\ G\ Wp,\ G\ Z),\ Tensor\_2\_Vector\_Vector\ 1,\ G\_TWZ)$;
$((O\ Rtmm,\ G\ Wp,\ G\ Wp),\ Tensor\_2\_Vector\_Vector\ 1,\ G\_TWW)$;
$((O\ Rtn,\ G\ Wp,\ G\ Wm),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TNWW\_CF)$;
$((O\ Rtn,\ G\ Z,\ G\ Z),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TNZZ\_CF)$;
$((O\ Rtsn,\ G\ Wp,\ G\ Wm),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TSNWW\_CF)$;
$((O\ Rtsn,\ G\ Z,\ G\ Z),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TSNZZ\_CF)$;
$((O\ Rtp,\ G\ Z,\ G\ Wm),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TWZ\_CF)$;
$((O\ Rtpp,\ G\ Wm,\ G\ Wm),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TWW\_CF)$;
$((O\ Rtm,\ G\ Wp,\ G\ Z),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TWZ\_CF)$;
$((O\ Rtmm,\ G\ Wp,\ G\ Wp),\ Tensor\_2\_Vector\_Vector\_cf\ 1,\ G\_TWW\_CF)$]

Anomalous trilinear interactions $f_i f_j V$ and ttH:

$$\Delta\mathcal{L}_{tt\gamma} = -e\frac{\upsilon}{\Lambda^2}\bar{t}i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)tA_\mu \qquad (13.93)$$

let *anomalous_ttA* =
  if *Flags.top_anom* then
    [((M (U (-3)), G Ga, M (U 3)), FBF (1, $Psibar$, $TVAM$, Psi), G_TVA_ttA)]
  else
    []

$$\Delta\mathcal{L}_{bb\gamma} = -e\frac{\upsilon}{\Lambda^2}\bar{b}i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)bA_\mu \qquad (13.94)$$

let *anomalous_bbA* =
  if *Flags.top_anom* then
    [((M (D (-3)), G Ga, M (D 3)), FBF (1, $Psibar$, $TVAM$, Psi), G_TVA_bbA)]
  else
    []

$$\Delta\mathcal{L}_{ttg} = -g_s\frac{\upsilon}{\Lambda^2}\bar{t}\lambda^a i\sigma^{\mu\nu}k_\nu(d_V(k^2) + id_A(k^2)\gamma_5)tG^a_\mu \qquad (13.95)$$

let *anomalous_ttG* =
  if *Flags.top_anom* then
    [((M (U (-3)), G Gl, M (U 3)), FBF (1, $Psibar$, $TVAM$, Psi), G_TVA_ttG)]
  else
    []

$$\Delta\mathcal{L}_{ttZ} = -\frac{g}{2c_W}\frac{\upsilon^2}{\Lambda^2}\left[\bar{t}\not{Z}(X_L(k^2)P_L + X_R(k^2)P_R)t + \bar{t}\frac{i\sigma^{\mu\nu}k_\nu}{m_Z}(d_V(k^2) + id_A(k^2)\gamma_5)tZ_\mu\right]$$
$$(13.96)$$

let *anomalous_ttZ* =
  if *Flags.top_anom* then
    [((M (U (-3)), G Z, M (U 3)), FBF (1, $Psibar$, $VLRM$, Psi), G_VLR_ttZ);
      ((M (U (-3)), G Z, M (U 3)), FBF (1, $Psibar$, $TVAM$, Psi), G_TVA_ttZ)]
  else
    []

$$\Delta\mathcal{L}_{bbZ} = -\frac{g}{2c_W}\frac{v^2}{\Lambda^2}\bar{b}\frac{i\sigma^{\mu\nu}k_\nu}{m_Z}(d_V(k^2) + id_A(k^2)\gamma_5)bZ_\mu \qquad (13.97)$$

let *anomalous_bbZ* =
  if *Flags.top_anom* then
    [((M (D (−3)), G Z, M (D 3)), FBF (1, Psibar, TVAM, Psi), G_TVA_bbZ)]
  else
    [ ]

$$\Delta\mathcal{L}_{tbW} = -\frac{g}{\sqrt{2}}\frac{v^2}{\Lambda^2}\left[\bar{b}W^-(V_L(k^2)P_L + V_R(k^2)P_R)t + \bar{b}\frac{i\sigma^{\mu\nu}k_\nu}{m_W}(g_L(k^2)P_L + g_R(k^2)P_R)tW_\mu^-\right] + \text{H.c.}$$
$$(13.98)$$

let *anomalous_tbW* =
  if *Flags.top_anom* then
    [((M (D (−3)), G Wm, M (U 3)), FBF (1, Psibar, VLRM, Psi), G_VLR_btW);
      ((M (U (−3)), G Wp, M (D 3)), FBF (1, Psibar, VLRM, Psi), G_VLR_tbW);
      ((M (D (−3)), G Wm, M (U 3)), FBF (1, Psibar, TLRM, Psi), G_TLR_btW);
      ((M (U (−3)), G Wp, M (D 3)), FBF (1, Psibar, TRLM, Psi), G_TRL_tbW)]
  else
    [ ]

$$\Delta\mathcal{L}_{ttH} = -\frac{1}{\sqrt{2}}\bar{t}(Y_V(k^2) + iY_A(k^2)\gamma_5)tH \qquad (13.99)$$

let *anomalous_ttH* =
  if *Flags.top_anom* then
    [((M (U (−3)), O H, M (U 3)), FBF (1, Psibar, SPM, Psi), G_SP_ttH)]
  else
    [ ]

quartic fermion-gauge interactions $f_i f_j V_1 V_2$ emerging from gauge-invariant effective operators:

$$\Delta\mathcal{L}_{ttgg} = -\frac{g_s^2}{2}f_{abc}\frac{v}{\Lambda^2}\bar{t}\lambda^a\sigma^{\mu\nu}(d_V(k^2) + id_A(k^2)\gamma_5)tG_\mu^b G_\nu^c \qquad (13.100)$$

let *anomalous_ttGG* =
  if *Flags.top_anom* then
    [((M (U (−3)), O (Aux_top (2, 1, 0,true, TTGG)), M (U 3)), FBF (1, Psibar, TVA, Psi), G_TVA_
      ((O (Aux_top (2, 1, 0,false, TTGG)), G Gl, G Gl), Aux_Gauge_Gauge 1, I_Gs)]
  else
    [ ]

$$\Delta\mathcal{L}_{tbWA} = -i\sin\theta_w\frac{g^2}{2\sqrt{2}}\frac{v^2}{\Lambda^2}\left[\bar{b}\frac{\sigma^{\mu\nu}}{m_W}(g_L(k^2)P_L + g_R(k^2)P_R)tA_\mu W_\nu^-\right] + \text{H.c.}$$
$$(13.101)$$

let *anomalous_tbWA* =

if *Flags.top_anom* then
[((M (D (−3)), O (*Aux_top* (2, 0, −1,true, *TBWA*)), M (U 3)), *FBF* (1, *Psibar*, *TLR*, *Psi*), *G_TL*
((O (*Aux_top* (2, 0, 1,false, *TBWA*)), G *Ga*, G *Wm*), *Aux_Gauge_Gauge* 1, *I_G_weak*);
((M (U (−3)), O (*Aux_top* (2, 0, 1,true, *TBWA*)), M (D 3)), *FBF* (1, *Psibar*, *TRL*, *Psi*), *G_TRL*
((O (*Aux_top* (2, 0, −1,false, *TBWA*)), G *Wp*, G *Ga*), *Aux_Gauge_Gauge* 1, *I_G_weak*)]
else
[ ]

$$\Delta\mathcal{L}_{tbWZ} = -i\cos\theta_w \frac{g^2}{2\sqrt{2}} \frac{v^2}{\Lambda^2} \left[\bar{b} \frac{\sigma^{\mu\nu}}{m_W}(g_L(k^2)P_L + g_R(k^2)P_R)tZ_\mu W_\nu^- \right] + \text{H.c.}$$
(13.102)

let *anomalous_tbWZ* =
if *Flags.top_anom* then
[((M (D (−3)), O (*Aux_top* (2, 0, −1,true, *TBWZ*)), M (U 3)), *FBF* (1, *Psibar*, *TLR*, *Psi*), *G_TL*
((O (*Aux_top* (2, 0, 1,false, *TBWZ*)), G *Z*, G *Wm*), *Aux_Gauge_Gauge* 1, *I_G_weak*);
((M (U (−3)), O (*Aux_top* (2, 0, 1,true, *TBWZ*)), M (D 3)), *FBF* (1, *Psibar*, *TRL*, *Psi*), *G_TR*
((O (*Aux_top* (2, 0, −1,false, *TBWZ*)), G *Wp*, G *Z*), *Aux_Gauge_Gauge* 1, *I_G_weak*)]
else
[ ]

$$\Delta\mathcal{L}_{ttWW} = -i\frac{g^2}{2} \frac{v^2}{\Lambda^2} \bar{t} \frac{\sigma^{\mu\nu}}{m_W}(d_V(k^2) + id_A(k^2)\gamma_5)tW_\mu^- W_\nu^+$$
(13.103)

let *anomalous_ttWW* =
if *Flags.top_anom* then
[((M (U (−3)), O (*Aux_top* (2, 0, 0,true, *TTWW*)), M (U 3)), *FBF* (1, *Psibar*, *TVA*, *Psi*), *G_TVA*
((O (*Aux_top* (2, 0, 0,false, *TTWW*)), G *Wm*, G *Wp*), *Aux_Gauge_Gauge* 1, *I_G_weak*)]
else
[ ]

$$\Delta\mathcal{L}_{bbWW} = -i\frac{g^2}{2} \frac{v^2}{\Lambda^2} \bar{b} \frac{\sigma^{\mu\nu}}{m_W}(d_V(k^2) + id_A(k^2)\gamma_5)bW_\mu^- W_\nu^+$$
(13.104)

let *anomalous_bbWW* =
if *Flags.top_anom* then
[((M (D (−3)), O (*Aux_top* (2, 0, 0,true, *BBWW*)), M (D 3)), *FBF* (1, *Psibar*, *TVA*, *Psi*), *G_TVA*
((O (*Aux_top* (2, 0, 0,false, *BBWW*)), G *Wm*, G *Wp*), *Aux_Gauge_Gauge* 1, *I_G_weak*)]
else
[ ]

4-fermion contact terms emerging from operator rewriting:

let *anomalous_top_qGuG_tt* =
[((M (U (−3)), O (*Aux_top* (1, 1, 0,true, *QGUG*)), M (U 3)), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_q*

let *anomalous_top_qGuG_ff* n =
List.map mom
[((U (−n), *Aux_top* (1, 1, 0,false, *QGUG*), U n), *FBF* (1, *Psibar*, *V*, *Psi*), *Unit*);
((D (−n), *Aux_top* (1, 1, 0,false, *QGUG*), D n), *FBF* (1, *Psibar*, *V*, *Psi*), *Unit*)]

let *anomalous_top_qGuG* =
  if *Flags.top_anom_4f* then
    anomalous_top_qGuG_tt @ *ThoList.flatmap anomalous_top_qGuG_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_qBuB_tt* =
  [((M (U (−3)), O (*Aux_top* (1, 0, 0,true,*QBUB*)), M (U 3)), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_q*

let *anomalous_top_qBuB_ff* n =
  List.map mom
    [((U (−n), *Aux_top* (1, 0, 0,false,*QBUB*), U n), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_u*);
      ((D (−n), *Aux_top* (1, 0, 0,false,*QBUB*), D n), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_d*);
      ((L (−n), *Aux_top* (1, 0, 0,false,*QBUB*), L n), *FBF* (1, *Psibar*, *VLR*, *Psi*), *G_VLR_qBuB_e*);
      ((N (−n), *Aux_top* (1, 0, 0,false,*QBUB*), N n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qBuB_n*)]

let *anomalous_top_qBuB* =
  if *Flags.top_anom_4f* then
    anomalous_top_qBuB_tt @ *ThoList.flatmap anomalous_top_qBuB_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_qW_tq* =
  [((M (U (−3)), O (*Aux_top* (1, 0, 0,true,*QW*)), M (U 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*);
    ((M (D (−3)), O (*Aux_top* (1, 0, −1,true,*QW*)), M (U 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*);
    ((M (U (−3)), O (*Aux_top* (1, 0, 1,true,*QW*)), M (D 3)), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW*)]

let *anomalous_top_qW_ff* n =
  List.map mom
    [((U (−n), *Aux_top* (1, 0, 0,false,*QW*), U n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_u*);
      ((D (−n), *Aux_top* (1, 0, 0,false,*QW*), D n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_d*);
      ((N (−n), *Aux_top* (1, 0, 0,false,*QW*), N n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_u*);
      ((L (−n), *Aux_top* (1, 0, 0,false,*QW*), L n), *FBF* (1, *Psibar*, *VL*, *Psi*), *G_VL_qW_d*);
      ((D (−n), *Aux_top* (1, 0, −1,false,*QW*), U n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
      ((U (−n), *Aux_top* (1, 0, 1,false,*QW*), D n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
      ((L (−n), *Aux_top* (1, 0, −1,false,*QW*), N n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*);
      ((N (−n), *Aux_top* (1, 0, 1,false,*QW*), L n), *FBF* (1, *Psibar*, *VL*, *Psi*), *Half*)]

let *anomalous_top_qW* =
  if *Flags.top_anom_4f* then
    anomalous_top_qW_tq @ *ThoList.flatmap anomalous_top_qW_ff* [1; 2; 3]
  else
    [ ]

let *anomalous_top_DuDd* =
  if *Flags.top_anom_4f* then
    [((M (U (−3)), O (*Aux_top* (0, 0, 0,true,*DR*)), M (U 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *Half*);
      ((M (U (−3)), O (*Aux_top* (0, 0, 0,false,*DR*)), M (U 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *G_SL_DttR*
      ((M (D (−3)), O (*Aux_top* (0, 0, 0,false,*DR*)), M (D 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *G_SR_DttR*
      ((M (U (−3)), O (*Aux_top* (0, 0, 0,true,*DL*)), M (U 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *Half*);
      ((M (D (−3)), O (*Aux_top* (0, 0, 0,false,*DL*)), M (D 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *G_SL_DttL*
      ((M (D (−3)), O (*Aux_top* (0, 0, −1,true,*DR*)), M (U 3)), *FBF* (1, *Psibar*, *SR*, *Psi*), *Half*);
      ((M (U (−3)), O (*Aux_top* (0, 0, 1,false,*DR*)), M (D 3)), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_SLR_D*
      ((M (D (−3)), O (*Aux_top* (0, 0, −1,true,*DL*)), M (U 3)), *FBF* (1, *Psibar*, *SL*, *Psi*), *Half*);

$((M\ (U\ (-3)),\ O\ (Aux\_top\ (0,0,1,\mathsf{false},DL)),\ M\ (D\ 3)),\ FBF\ (1,\ Psibar,\ SL,\ Psi),\ G\_SL\_DbtL)$
else
  $[\,]$

let *vertices3* =
  (*ThoList.flatmap electromagnetic_currents* $[1;2;3]$ @
   ThoList.flatmap color_currents $[1;2;3]$ @
   ThoList.flatmap neutral_currents $[1;2;3]$ @
   (if *Flags.ckm_present* then
      charged_currents_ckm
    else
      charged_currents_triv) @
   yukawa @ *triple_gauge* @
   gauge_higgs @ *higgs* @ *higgs_triangle_vertices*
   @ *goldstone_vertices* @
   rsigma3 @ *rsigma3t* @ *rphi3* @
   (if *Flags.cf_arbitrary* then
        (*rt3cf* @ *rff3cf* @
          rfv3cf @ *rfphi3cf* @ *rfddphi3cf*)
      else
        (*rf3* @ *rt3*)) @
   rf3t @
   (if *Flags.higgs_matrix* then
        (*rsigma3h* @ *rff3h*)
      else
        $[\,]$) @
   anomalous_ttA @ *anomalous_bbA* @
   anomalous_ttZ @ *anomalous_bbZ* @
   anomalous_tbW @ *anomalous_tbWA* @ *anomalous_tbWZ* @
   anomalous_ttWW @ *anomalous_bbWW* @
   anomalous_ttG @ *anomalous_ttGG* @
   anomalous_ttH @
   anomalous_top_qGuG @ *anomalous_top_qBuB* @
   anomalous_top_qW @ *anomalous_top_DuDd*)

let *vertices4* =
  quartic_gauge @ *gauge_higgs4* @ *higgs4*

let *vertices* () = (*vertices3*, *vertices4*, $[\,]$)

For efficiency, make sure that $F.of\_vertices\ vertices$ is evaluated only once.

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *flavor_of_string* = function
  | "e-" → $M\ (L\ 1)$ | "e+" → $M\ (L\ (-1))$
  | "mu-" → $M\ (L\ 2)$ | "mu+" → $M\ (L\ (-2))$
  | "tau-" → $M\ (L\ 3)$ | "tau+" → $M\ (L\ (-3))$
  | "nue" → $M\ (N\ 1)$ | "nuebar" → $M\ (N\ (-1))$

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| "numu" → M (N 2) | "numubar" → M (N (−2))
| "nutau" → M (N 3) | "nutaubar" → M (N (−3))
| "u" → M (U 1) | "ubar" → M (U (−1))
| "c" → M (U 2) | "cbar" → M (U (−2))
| "t" → M (U 3) | "tbar" → M (U (−3))
| "d" → M (D 1) | "dbar" → M (D (−1))
| "s" → M (D 2) | "sbar" → M (D (−2))
| "b" → M (D 3) | "bbar" → M (D (−3))
| "g" | "gl" → G Gl
| "A" → G Ga | "Z" | "Z0" → G Z
| "W+" → G Wp | "W-" → G Wm
| "H" → O H | "Rsigma" → O $Rsigma$
| "Rphi0" → O $Rphin$
| "Rphis0" → O $Rphisn$
| "Rphi+" → O $Rphip$ | "Rphi-" → O $Rphim$
| "Rphi++" → O $Rphip$ | "Rphi--" → O $Rphimm$
| "Rf" → O Rf
| "Rff" → O Rff
| "Rfv" → O Rfv
| "Rfphi" → O $Rfphi$
| "Rt0" → O Rtn
| "Rts0" → O $Rtsn$
| "Rt+" → O Rtp | "Rt-" → O Rtm
| "Rt++" → O Rtp | "Rt--" → O $Rtmm$
| "Aux_t_ttGG0" → O (Aux_top (2, 1, 0,true,$TTGG$)) | "Aux_ttGG0" → O (Aux_top (2, 1, 0,false,$TTGG$))
| "Aux_t_tbWA+" → O (Aux_top (2, 0, 1,true,$TBWA$)) | "Aux_tbWA+" → O (Aux_top (2, 0, 1,false,$TBWA$))
| "Aux_t_tbWA-" → O (Aux_top (2, 0, −1,true,$TBWA$)) | "Aux_tbWA-" → O (Aux_top (2, 0, −1,false,$TBWA$))
| "Aux_t_tbWZ+" → O (Aux_top (2, 0, 1,true,$TBWZ$)) | "Aux_tbWZ+" → O (Aux_top (2, 0, 1,false,$TBWZ$))
| "Aux_t_tbWZ-" → O (Aux_top (2, 0, −1,true,$TBWZ$)) | "Aux_tbWZ-" → O (Aux_top (2, 0, −1,false,$TBWZ$))
| "Aux_t_ttWW0" → O (Aux_top (2, 0, 0,true,$TTWW$)) | "Aux_ttWW0" → O (Aux_top (2, 0, 0,false,$TTWW$))
| "Aux_t_bbWW0" → O (Aux_top (2, 0, 0,true,$BBWW$)) | "Aux_bbWW0" → O (Aux_top (2, 0, 0,false,$BBWW$))
| "Aux_t_qGuG0" → O (Aux_top (1, 1, 0,true,$QGUG$)) | "Aux_qGuG0" → O (Aux_top (1, 1, 0,false,$QGUG$))
| "Aux_t_qBuB0" → O (Aux_top (1, 0, 0,true,$QBUB$)) | "Aux_qBuB0" → O (Aux_top (1, 0, 0,false,$QBUB$))
| "Aux_t_qW0" → O (Aux_top (1, 0, 0,true,QW)) | "Aux_qW0" → O (Aux_top (1, 0, 0,false,QW))
| "Aux_t_qW+" → O (Aux_top (1, 0, 1,true,QW)) | "Aux_qW+" → O (Aux_top (1, 0, 1,false,QW))
| "Aux_t_qW-" → O (Aux_top (1, 0, −1,true,QW)) | "Aux_qW-" → O (Aux_top (1, 0, −1,false,QW))
| "Aux_t_dL0" → O (Aux_top (0, 0, 0,true,DL)) | "Aux_dL0" → O (Aux_top (0, 0, 0,false,DL))

```
        |  "Aux_t_dL+" →   O (Aux_top (0, 0, 1,true,DL))  |  "Aux_dL+" →
O (Aux_top (0, 0, 1,false,DL))
        |  "Aux_t_dL-" →   O (Aux_top (0, 0, −1,true,DL))  |  "Aux_dL-" →
O (Aux_top (0, 0, −1,false,DL))
        |  "Aux_t_dR0" →   O (Aux_top (0, 0, 0,true,DR))  |  "Aux_dR0" →
O (Aux_top (0, 0, 0,false,DR))
        |  "Aux_t_dR+" →   O (Aux_top (0, 0, 1,true,DR))  |  "Aux_dR+" →
O (Aux_top (0, 0, 1,false,DR))
        |  "Aux_t_dR-" →   O (Aux_top (0, 0, −1,true,DR))  |  "Aux_dR-" →
O (Aux_top (0, 0, −1,false,DR))
        |  _ →  invalid_arg "Modellib_BSM.SSC_AltT.flavor_of_string"
```

```
    let flavor_to_string = function
      | M f →
          begin match f with
          | L 1 → "e-" | L (−1) → "e+"
          | L 2 → "mu-" | L (−2) → "mu+"
          | L 3 → "tau-" | L (−3) → "tau+"
          | L _ → invalid_arg
              "Modellib_BSM.SSC_AltT.flavor_to_string:␣invalid␣lepton"
          | N 1 → "nue" | N (−1) → "nuebar"
          | N 2 → "numu" | N (−2) → "numubar"
          | N 3 → "nutau" | N (−3) → "nutaubar"
          | N _ → invalid_arg
              "Modellib_BSM.SSC_AltT.flavor_to_string:␣invalid␣neutrino"
          | U 1 → "u" | U (−1) → "ubar"
          | U 2 → "c" | U (−2) → "cbar"
          | U 3 → "t" | U (−3) → "tbar"
          | U _ → invalid_arg
              "Modellib_BSM.SSC_AltT.flavor_to_string:␣invalid␣up␣type␣quark"
          | D 1 → "d" | D (−1) → "dbar"
          | D 2 → "s" | D (−2) → "sbar"
          | D 3 → "b" | D (−3) → "bbar"
          | D _ → invalid_arg
              "Modellib_BSM.SSC_AltT.flavor_to_string:␣invalid␣down␣type␣quark"
          end
      | G f →
          begin match f with
          | Gl → "gl"
          | Ga → "A" | Z → "Z"
          | Wp → "W+" | Wm → "W-"
          end
      | O f →
          begin match f with
          | Phip → "phi+" | Phim → "phi-" | Phi0 → "phi0"
          | H → "H" | Rsigma → "Rsigma"
          | Rphin → "Rphin" | Rphip → "Rphi+" | Rphim → "Rphi-"
          | Rphipp → "Rphi++" | Rphimm → "Rphi--"
          | Rphisn → "Rphisn"
          | Rf → "Rf"
```

```
                | Rff  →  "Rff" |  Rfv  →  "Rfv" |  Rfphi  →  "Rfphi"
                | Rtn  →  "Rtn" |  Rtsn  →  "Rtsn" |  Rtp  →  "Rt+" |  Rtm  →
"Rt-"
                | Rtpp  →  "Rt++" |  Rtmm  →  "Rt--"
                | Aux_top (_, _, ch, n, v)  →  "Aux_" ^ (if n then "t_" else "") ^ (
                    begin match v with
                    | TTGG  →  "ttGG" |  TBWA  →  "tbWA" |  TBWZ  →
"tbWZ"
                    | TTWW  →  "ttWW" |  BBWW  →  "bbWW"
                    | QGUG  →  "qGuG" |  QBUB  →  "qBuB"
                    | QW  →  "qW" |  DL  →  "dL" |  DR  →  "dR"
                    end ) ^ ( if ch > 0 then "+" else if ch < 0 then "-" else "0" )
                end

    let flavor_to_TeX  =  function
      | M f  →
          begin match f with
          | L 1  →  "e^-" |  L (−1)  →  "e^+"
          | L 2  →  "\\mu^-" |  L (−2)  →  "\\mu^+"
          | L 3  →  "\\tau^-" |  L (−3)  →  "\\tau^+"
          | L _  →  invalid_arg
                "Modellib_BSM.SSC_AltT.flavor_to_TeX:␣invalid␣lepton"
          | N 1  →  "\\nu_e" |  N (−1)  →  "\\bar{\\nu}_e"
          | N 2  →  "\\nu_\\mu" |  N (−2)  →  "\\bar{\\nu}_\\mu"
          | N 3  →  "\\nu_\\tau" |  N (−3)  →  "\\bar{\\nu}_\\tau"
          | N _  →  invalid_arg
                "Modellib_BSM.SSC_AltT.flavor_to_TeX:␣invalid␣neutrino"
          | U 1  →  "u" |  U (−1)  →  "\\bar{u}"
          | U 2  →  "c" |  U (−2)  →  "\\bar{c}"
          | U 3  →  "t" |  U (−3)  →  "\\bar{t}"
          | U _  →  invalid_arg
                "Modellib_BSM.SSC_AltT.flavor_to_TeX:␣invalid␣up␣type␣quark"
          | D 1  →  "d" |  D (−1)  →  "\\bar{d}"
          | D 2  →  "s" |  D (−2)  →  "\\bar{s}"
          | D 3  →  "b" |  D (−3)  →  "\\bar{b}"
          | D _  →  invalid_arg
                "Modellib_BSM.SSC_AltT.flavor_to_TeX:␣invalid␣down␣type␣quark"
          end
      | G f  →
          begin match f with
          | Gl  →  "g"
          | Ga  →  "\\gamma" |  Z  →  "Z"
          | Wp  →  "W^+" |  Wm  →  "W^-"
          end
      | O f  →
          begin match f with
          | Phip  →  "\\phi^+" |  Phim  →  "\\phi^-" |  Phi0  →
"\\phi^0"
          | H  →  "H" |  Rsigma  →  "\\sigma"
          | Rphip  →  "\\phi^+" |  Rphim  →  "\\phi^-" |  Rphin  →
```

```
"\\phi^0"
          |  Rphisn  →  "\\phi_s^0"
          |  Rphipp  →  "\\phi^{++}" |  Rphimm  →  "\\phi^{--}"
          |  Rf  →  "f"
          |  Rff  →  "f^f" |  Rfv  →  "f^v" |  Rfphi  →  "f^s"
          |  Rtp  →  "t^+" |  Rtm  →  "t^-" |  Rtn  →  "t^0" |  Rtsn  →
"t_s^0"
          |  Rtpp  →  "t^{++}" |  Rtmm  →  "t^{--}"
          |  Aux_top (_, _, ch, n, v)  →  "\\textnormal{Aux_" ^ (if n then "t_" else "") ^ (
              begin match v with
              |  TTGG  →  "ttGG" |  TBWA  →  "tbWA" |  TBWZ  →
"tbWZ"
              |  TTWW  →  "ttWW" |  BBWW  →  "bbWW"
              |  QGUG  →  "qGuG" |  QBUB  →  "qBuB"
              |  QW  →  "qW" |  DL  →  "dL" |  DR  →  "dR"
              end ) ^ ( if ch > 0 then "^+" else if ch < 0 then "^-" else "^0" ) ^ "}"
          end
```

```
    let flavor_symbol = function
      |  M f  →
          begin match f with
          |  L n when n > 0  →  "l" ^ string_of_int n
          |  L n  →  "l" ^ string_of_int (abs n) ^ "b"
          |  N n when n > 0  →  "n" ^ string_of_int n
          |  N n  →  "n" ^ string_of_int (abs n) ^ "b"
          |  U n when n > 0  →  "u" ^ string_of_int n
          |  U n  →  "u" ^ string_of_int (abs n) ^ "b"
          |  D n when n > 0  →  "d" ^ string_of_int n
          |  D n  →  "d" ^ string_of_int (abs n) ^ "b"
          end
      |  G f  →
          begin match f with
          |  Gl  →  "gl"
          |  Ga  →  "a" |  Z  →  "z"
          |  Wp  →  "wp" |  Wm  →  "wm"
          end
      |  O f  →
          begin match f with
          |  Phip  →  "pp" |  Phim  →  "pm" |  Phi0  →  "p0"
          |  H  →  "h" |  Rsigma  →  "rsi"
          |  Rphip  →  "rpp" |  Rphim  →  "rpm" |  Rphin  →  "rpn"
          |  Rphisn  →  "rpsn"
          |  Rphipp  →  "rppp" |  Rphimm  →  "rpmm"
          |  Rf  →  "rf"
          |  Rff  →  "rff" |  Rfv  →  "rfv" |  Rfphi  →  "rfphi"
          |  Rtp  →  "rtp" |  Rtm  →  "rtm" |  Rtn  →  "rtn"
          |  Rtsn  →  "rtsn"
          |  Rtpp  →  "rtpp" |  Rtmm  →  "rtmm"
          |  Aux_top (_, _, ch, n, v)  →  "aux_" ^ (if n then "t_" else "") ^ (
              begin match v with
```

```
                    |  TTGG  →  "ttgg" |  TBWA  →  "tbwa" |  TBWZ  →
"tbwz"
                    |  TTWW  →  "ttww" |  BBWW  →  "bbww"
                    |  QGUG  →  "qgug" |  QBUB  →  "qbub"
                    |  QW  →  "qw" |  DL  →  "dl" |  DR  →  "dr"
                    end ) ˆ "_" ˆ ( if ch > 0 then "p" else if ch < 0 then "m" else "0" )
              end
```

Introducing new Resonances from 45, there are no PDG values

```
      let pdg  =  function
        |  M f  →
              begin match f with
              |  L n when n > 0 → 9 + 2 × n
              |  L n  →  − 9 + 2 × n
              |  N n when n > 0 → 10 + 2 × n
              |  N n  →  − 10 + 2 × n
              |  U n when n > 0 → 2 × n
              |  U n  →  2 × n
              |  D n when n > 0  →  − 1 + 2 × n
              |  D n  →  1 + 2 × n
              end
        |  G f  →
              begin match f with
              |  Gl  →  21
              |  Ga  →  22  |  Z  →  23
              |  Wp  →  24  |  Wm  →  (−24)
              end
        |  O f  →
              begin match f with
              |  Phip  |  Phim  →  27  |  Phi0  →  26
              |  H  →  25  |  Rsigma  →  45
              |  Rphin  →  46  |  Rphip  |  Rphim  →  47
              |  Rphipp  |  Rphimm  →  48
              |  Rphisn  →  49
              |  Rf  →  52
              |  Rtn  →  53  |  Rtp  |  Rtm  →  54
              |  Rtpp  |  Rtmm  →  55
              |  Rff  →  56  |  Rfv  →  57  |  Rfphi  →  58
              |  Rtsn  →  59
              |  Aux_top (_, _, _, _, _)  →  81
              end

    let mass_symbol f  =
      "mass(" ˆ string_of_int (abs (pdg f)) ˆ ")"

    let width_symbol f  =
      "width(" ˆ string_of_int (abs (pdg f)) ˆ ")"

    let constant_symbol  =  function
      |  Unit  →  "unit" |  Half  →  "half" |  Pi  →  "PI"
      |  Alpha_QED  →  "alpha" |  E  →  "e" |  G_weak  →  "g" |  Vev  →
"vev"
```

```
    | I_G_weak → "ig"
    | Sin2thw → "sin2thw" | Sinthw → "sinthw" | Costhw →
"costhw"
    | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
    | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
    | G_NC_up → "gncup" | G_NC_down → "gncdwn"
    | G_TVA_ttA → "gtva_tta" | G_TVA_bbA → "gtva_bba"
    | G_VLR_ttZ → "gvlr_ttz" | G_TVA_ttZ → "gtva_ttz" |
G_TVA_bbZ → "gtva_bbz"
    | G_VLR_btW → "gvlr_btw" | G_VLR_tbW → "gvlr_tbw"
    | G_TLR_btW → "gtlr_btw" | G_TRL_tbW → "gtrl_tbw"
    | G_TLR_btWA → "gtlr_btwa" | G_TRL_tbWA → "gtrl_tbwa"
    | G_TLR_btWZ → "gtlr_btwz" | G_TRL_tbWZ → "gtrl_tbwz"
    | G_TVA_ttWW → "gtva_ttww" | G_TVA_bbWW → "gtva_bbww"
    | G_TVA_ttG → "gtva_ttg" | G_TVA_ttGG → "gtva_ttgg"
    | G_SP_ttH → "gsp_tth"
    | G_VLR_qGuG → "gvlr_qgug"
    | G_VLR_qBuB → "gvlr_qbub"
    | G_VLR_qBuB_u → "gvlr_qbub_u" | G_VLR_qBuB_d → "gvlr_qbub_d"
    | G_VLR_qBuB_e → "gvlr_qbub_e" | G_VL_qBuB_n → "gvl_qbub_n"
    | G_VL_qW → "gvl_qw"
    | G_VL_qW_u → "gvl_qw_u" | G_VL_qW_d → "gvl_qw_d"
    | G_SL_DttR → "gsl_dttr" | G_SR_DttR → "gsr_dttr" |
G_SL_DttL → "gsl_dttl"
    | G_SLR_DbtR → "gslr_dbtr" | G_SL_DbtL → "gsl_dbtl"
    | G_CC → "gcc"
    | G_CCQ (n1, n2) → "gccq" ^ string_of_int n1 ^ string_of_int n2
    | I_Q_W → "iqw" | I_G_ZWW → "igzww"
    | G_WWWW → "gw4" | G_ZZWW → "gzzww"
    | G_AZWW → "gazww" | G_AAWW → "gaaww"
    | I_G1_AWW → "ig1a" | I_G1_ZWW → "ig1z"
    | I_G1_plus_kappa_plus_G4_AWW → "ig1pkpg4a"
    | I_G1_plus_kappa_plus_G4_ZWW → "ig1pkpg4z"
    | I_G1_plus_kappa_minus_G4_AWW → "ig1pkmg4a"
    | I_G1_plus_kappa_minus_G4_ZWW → "ig1pkmg4z"
    | I_G1_minus_kappa_plus_G4_AWW → "ig1mkpg4a"
    | I_G1_minus_kappa_plus_G4_ZWW → "ig1mkpg4z"
    | I_G1_minus_kappa_minus_G4_AWW → "ig1mkmg4a"
    | I_G1_minus_kappa_minus_G4_ZWW → "ig1mkmg4z"
    | I_lambda_AWW → "ila"
    | I_lambda_ZWW → "ilz"
    | G5_AWW → "rg5a"
    | G5_ZWW → "rg5z"
    | I_kappa5_AWW → "ik5a"
    | I_kappa5_ZWW → "ik5z"
    | I_lambda5_AWW → "il5a" | I_lambda5_ZWW → "il5z"
    | Alpha_WWWW0 → "alww0" | Alpha_WWWW2 → "alww2"
    | Alpha_ZZWW0 → "alzw0" | Alpha_ZZWW1 → "alzw1"
    | Alpha_ZZZZ → "alzz"
    | FS0_HHWW → "fs0hhww" | FS0_HHZZ → "fs0hhzz"
```

| $FS1\_HHWW \rightarrow$ "fs1hhww" | $FS1\_HHZZ \rightarrow$ "fs1hhzz"
| $FS\_H4 \rightarrow$ "fsh4"
| $D\_Alpha\_ZZWW0\_S \rightarrow$ "dalzz0_s(gkm,mkm,"
| $D\_Alpha\_ZZWW0\_T \rightarrow$ "dalzz0_t(gkm,mkm,"
| $D\_Alpha\_ZZWW1\_S \rightarrow$ "dalzz1_s(gkm,mkm,"
| $D\_Alpha\_ZZWW1\_T \rightarrow$ "dalzz1_t(gkm,mkm,"
| $D\_Alpha\_ZZWW1\_U \rightarrow$ "dalzz1_u(gkm,mkm,"
| $D\_Alpha\_WWWW0\_S \rightarrow$ "dalww0_s(gkm,mkm,"
| $D\_Alpha\_WWWW0\_T \rightarrow$ "dalww0_t(gkm,mkm,"
| $D\_Alpha\_WWWW0\_U \rightarrow$ "dalww0_u(gkm,mkm,"
| $D\_Alpha\_WWWW2\_S \rightarrow$ "dalww2_s(gkm,mkm,"
| $D\_Alpha\_WWWW2\_T \rightarrow$ "dalww2_t(gkm,mkm,"
| $D\_Alpha\_ZZZZ\_S \rightarrow$ "dalz4_s(gkm,mkm,"
| $D\_Alpha\_ZZZZ\_T \rightarrow$ "dalz4_t(gkm,mkm,"
| $D\_Alpha\_HHHH\_S \rightarrow$ "dalh4_s(gkm,mkm,"
| $D\_Alpha\_HHHH\_T \rightarrow$ "dalh4_t(gkm,mkm,"
| $D\_Alpha\_HHWW0\_S \rightarrow$ "dalhw0_s(gkm,mkm,"
| $D\_Alpha\_HHWW0\_T \rightarrow$ "dalhw0_t(gkm,mkm,"
| $D\_Alpha\_HHZZ0\_S \rightarrow$ "dalhz0_s(gkm,mkm,"
| $D\_Alpha\_HHZZ0\_T \rightarrow$ "dalhz0_t(gkm,mkm,"
| $D\_Alpha\_HHWW1\_S \rightarrow$ "dalhw1_s(gkm,mkm,"
| $D\_Alpha\_HHWW1\_T \rightarrow$ "dalhw1_t(gkm,mkm,"
| $D\_Alpha\_HHWW1\_U \rightarrow$ "dalhw1_u(gkm,mkm,"
| $D\_Alpha\_HHZZ1\_S \rightarrow$ "dalhz1_s(gkm,mkm,"
| $D\_Alpha\_HHZZ1\_T \rightarrow$ "dalhz1_t(gkm,mkm,"
| $D\_Alpha\_HHZZ1\_U \rightarrow$ "dalhz1_u(gkm,mkm,"
| $G\_HWW \rightarrow$ "ghww" | $G\_HZZ \rightarrow$ "ghzz"
| $G\_HHWW \rightarrow$ "ghhww" | $G\_HHZZ \rightarrow$ "ghhzz"
| $G\_SWW \rightarrow$ "gsww" | $G\_SZZ \rightarrow$ "gszz"
| $G\_SHH \rightarrow$ "gshh"
| $G\_SWW\_T \rightarrow$ "gswwt" | $G\_SZZ\_T \rightarrow$ "gszzt"
| $G\_PNWW \rightarrow$ "gpnww" | $G\_PNZZ \rightarrow$ "gpnzz"
| $G\_PSNWW \rightarrow$ "gpsnww" | $G\_PSNZZ \rightarrow$ "gpsnzz"
| $G\_PSNHH \rightarrow$ "gpsnhh"
| $G\_PWZ \rightarrow$ "gpwz" | $G\_PWW \rightarrow$ "gpww"
| $G\_FWW \rightarrow$ "gfww" | $G\_FZZ \rightarrow$ "gfzz"
| $G\_FWW\_CF \rightarrow$ "gfwwcf" | $G\_FZZ\_CF \rightarrow$ "gfzzcf"
| $G\_FHH \rightarrow$ "gfhh" | $G\_FHH\_CF \rightarrow$ "gfhhcf"
| $G\_FWW\_T \rightarrow$ "gfwwt" | $G\_FZZ\_T \rightarrow$ "gfzzt"
| $G\_FFWW \rightarrow$ "gffww" | $G\_FFZZ \rightarrow$ "gffzz"
| $G\_FFWW\_CF \rightarrow$ "gffwwcf" | $G\_FFZZ\_CF \rightarrow$ "gffzzcf"
| $G\_FFHH \rightarrow$ "gffhh" | $G\_FFHH\_CF \rightarrow$ "gffhhcf"
| $G\_FVWW \rightarrow$ "gfvww" | $G\_FVZZ \rightarrow$ "gfvzz"
| $G\_FVWW\_CF \rightarrow$ "gfvwwcf" | $G\_FVZZ\_CF \rightarrow$ "gfvzzcf"
| $G\_FVHH \rightarrow$ "gfvhh" | $G\_FVHH\_CF \rightarrow$ "gfvhhcf"
| $G\_FDDSWW \rightarrow$ "gfddsww" | $G\_FDDSZZ \rightarrow$ "gfddszz"
| $G\_FDDSWW\_CF \rightarrow$ "gfddswwcf" | $G\_FDDSZZ\_CF \rightarrow$ "gfddszzcf"
| $G\_FDDSHH \rightarrow$ "gfddshh" | $G\_FDDSHH\_CF \rightarrow$ "gfddshhcf"
| $G\_FSWW \rightarrow$ "gfsww" | $G\_FSZZ \rightarrow$ "gfszz"
| $G\_FSHH \rightarrow$ "gfshh"

```
      |  G_TNWW  →  "gtnww" |  G_TNZZ  →  "gtnzz"
      |  G_TNWW_CF  →  "gtnwwcf" |  G_TNZZ_CF  →  "gtnzzcf"
      |  G_TSNWW  →  "gtsnww" |  G_TSNZZ  →  "gtsnzz"
      |  G_TSNWW_CF  →  "gtsnwwcf" |  G_TSNZZ_CF  →  "gtsnzzcf"
      |  G_TWZ  →  "gtwz" |  G_TWW  →  "gtww"
      |  G_TWZ_CF  →  "gtwzcf" |  G_TWW_CF  →  "gtwwcf"
      |  G_SSWW  →  "gssww" |  G_SSZZ  →  "gsszz"
      |  G_Htt  →  "ghtt" |  G_Hbb  →  "ghbb"
      |  G_Htautau  →  "ghtautau" |  G_Hcc  →  "ghcc" |  G_Hmm  →
"ghmm"
      |  G_HGaZ  →  "ghgaz" |  G_HGaGa  →  "ghgaga" |  G_Hgg  →
"ghgg"
      |  G_HGaGa_anom  →  "ghgaga_ac" |  G_HGaZ_anom  →  "ghgaz_ac"
      |  G_HZZ_anom  →  "ghzz_ac" |  G_HWW_anom  →  "ghww_ac"
      |  G_HGaZ_u  →  "ghgaz_u" |  G_HZZ_u  →  "ghzz_u"
      |  G_HWW_u  →  "ghww_u"
      |  G_H3  →  "gh3" |  G_H4  →  "gh4"
      |  Gs  →  "gs" |  I_Gs  →  "igs" |  G2  →  "gs**2"
      |  Mass f  →  "mass" ^ flavor_symbol f
      |  Width f  →  "width" ^ flavor_symbol f
      |  K_Matrix_Coeff i  →  "kc" ^ string_of_int i
      |  K_Matrix_Pole i  →  "kp" ^ string_of_int i
```

end

13.7   Interface of *Modellib_MSSM*

13.7.1   More Hardcoded Models

```
module type MSSM_flags  =
  sig
     val include_goldstone  :  bool
     val include_four  :  bool
     val ckm_present  :  bool
     val gravitino  :  bool
     val higgs_triangle  :  bool
  end

module MSSM_no_goldstone  :  MSSM_flags
module MSSM_goldstone  :  MSSM_flags
module MSSM_no_4  :  MSSM_flags
module MSSM_no_4_ckm  :  MSSM_flags
module MSSM_Grav  :  MSSM_flags
module MSSM_Hgg  :  MSSM_flags
module MSSM  : functor (F : MSSM_flags) → Model.T with module Ch = Charges.QQ
```

13.8   Implementation of *Modellib_MSSM*

$Id : modellib_M SSM.ml 6465 2015 − 01 − 10 15 : 22 : 31Z jr_r euter$

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```
let rcs_file = RCS.parse "Modellib_MSSM" ["MSSM"]
    { RCS.revision = "$Revision:␣6465␣$";
      RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
      RCS.author = "$Author:␣jr_reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr
```

13.8.1 Minimal Supersymmetric Standard Model

```
module type MSSM_flags =
  sig
    val include_goldstone : bool
    val include_four : bool
    val ckm_present : bool
    val gravitino : bool
    val higgs_triangle : bool
  end

module MSSM_no_goldstone : MSSM_flags =
  struct
    let include_goldstone = false
    let include_four = true
    let ckm_present = false
    let gravitino = false
    let higgs_triangle = false
  end

module MSSM_goldstone : MSSM_flags =
  struct
    let include_goldstone = true
    let include_four = true
    let ckm_present = false
    let gravitino = false
    let higgs_triangle = false
  end

module MSSM_no_4 : MSSM_flags =
  struct
    let include_goldstone = false
    let include_four = false
    let ckm_present = false
    let gravitino = false
    let higgs_triangle = false
  end

module MSSM_no_4_ckm : MSSM_flags =
  struct
    let include_goldstone = false
    let include_four = false
    let ckm_present = true
    let gravitino = false
```

```
    let higgs_triangle  =  false
  end
module MSSM_Grav  :  MSSM_flags  =
  struct
    let include_goldstone  =  false
    let include_four  =  false
    let ckm_present  =  false
    let gravitino  =  true
    let higgs_triangle  =  false
  end
module MSSM_Hgg  :  MSSM_flags  =
  struct
    let include_goldstone  =  false
    let include_four  =  false
    let ckm_present  =  false
    let gravitino  =  false
    let higgs_triangle  =  true
  end
module MSSM (Flags  :  MSSM_flags)  =
  struct
    let rcs  =  RCS.rename rcs_file "Modellib_MSSM.MSSM"
        [ "MSSM" ]

    open Coupling

    let default_width  =  ref Timelike
    let use_fudged_width  =  ref false

    let options  =  Options.create
      [ "constant_width", Arg.Unit (fun ()  →  default_width :=  Constant),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", Arg.Set use_fudged_width,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", Arg.String (fun f  →  default_width :=  Custom f),
        "use␣custom␣width";
        "cancel_widths", Arg.Unit (fun ()  →  default_width :=  Vanishing),
        "use␣vanishing␣width"]

    type gen  =
      | G of int |  GG of gen × gen

    let rec string_of_gen  =  function
      | G n when n  >  0  →  string_of_int n
      | G n  →  string_of_int (abs n) ^ "c"
      | GG (g1, g2)  →  string_of_gen g1 ^ "_" ^ string_of_gen g2
```
With this we distinguish the flavour.
```
    type sff  =
      | SL | SN | SU | SD

    let string_of_sff  =  function
      | SL  →  "sl" | SN  →  "sn" | SU  →  "su" | SD  →  "sd"
```

With this we distinguish the mass eigenstates. At the moment we have to cheat a little bit for the sneutrinos. Because we are dealing with massless neutrinos there is only one sort of sneutrino.

```
type sfm =
  | M1 | M2

let string_of_sfm = function
  | M1 → "1" | M2 → "2"
```

We also introduce special types for the charginos and neutralinos.

```
type char =
  | C1 | C2 | C1c | C2c

type neu =
  | N1 | N2 | N3 | N4

let int_of_char = function
  | C1 → 1 | C2 → 2 | C1c → −1 | C2c → −2

let string_of_char = function
  | C1 → "1" | C2 → "2" | C1c → "-1" | C2c → "-2"

let conj_char = function
  | C1 → C1c | C2 → C2c | C1c → C1 | C2c → C2

let string_of_neu = function
  | N1 → "1" | N2 → "2" | N3 → "3" | N4 → "4"
```

Also we need types to distinguish the Higgs bosons. We follow the conventions of Kuroda, which means

$$H_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_1 + H^0 \cos\alpha - h^0 \sin\alpha + \mathrm{i}A^0 \sin\beta - \mathrm{i}\phi^0 \cos\beta\right) \\ H^- \sin\beta - \phi^- \cos\beta \end{pmatrix}, \qquad (13.105)$$

$$H_2 = \begin{pmatrix} H^+ \cos\beta + \phi^+ \sin\beta \\ \frac{1}{\sqrt{2}}\left(v_2 + H^0 \sin\alpha + h^0 \cos\alpha + \mathrm{i}A^0 \cos\beta + \mathrm{i}\phi^0 \sin\beta\right) \end{pmatrix} \qquad (13.106)$$

This is a different sign convention compared to, e.g., Weinberg's volume iii. We will refer to it as *GS+*.

```
type higgs =
  | H1 (* the light scalar Higgs *)
  | H2 (* the heavy scalar Higgs *)
  | H3 (* the pseudoscalar Higgs *)
  | H4 (* the charged Higgs *)
  | H5 (* the neutral Goldstone boson *)
  | H6 (* the charged Goldstone boson *)
  | DH of higgs × higgs

let rec string_of_higgs = function
  | H1 → "h1" | H2 → "h2" | H3 → "h3" | H4 → "h4"
  | H5 → "p1" | H6 → "p2"
  | DH (h1, h2) → string_of_higgs h1 ^ string_of_higgs h2
```

type *flavor* =
  | *L* of *int* | *N* of *int*
  | *U* of *int* | *D* of *int*
  | *Sup* of *sfm* × *int* | *Sdown* of *sfm* × *int*
  | *Ga* | *Wp* | *Wm* | *Z* | *Gl*
  | *Slepton* of *sfm* × *int* | *Sneutrino* of *int*
  | *Neutralino* of *neu* | *Chargino* of *char*
  | *Gluino* | *Grino*
  | *Phip* | *Phim* | *Phi0* | *H_Heavy* | *H_Light* | *Hp* | *Hm* | *A*

type *gauge* = *unit*

let *gauge_symbol* () =
  failwith "Modellib_MSSM.MSSM.gauge_symbol:␣internal␣error"

At this point we will forget graviton and -tino.

let *lep_family* g = [*L* g; *N* g; *Slepton* $(M1, g)$;
                 Slepton $(M2, g)$; *Sneutrino* g]

let *family* g =
  [*L* g; *N* g; *Slepton* $(M1, g)$; *Slepton* $(M2, g)$; *Sneutrino* g;
    U g; *D* g; *Sup* $(M1, g)$; *Sup* $(M2, g)$; *Sdown* $(M1, g)$;
    Sdown $(M2, g)$]

let *external_flavors″* =
  ["1st␣Generation", *ThoList.flatmap family* $[1; -1]$;
    "2nd␣Generation", *ThoList.flatmap family* $[2; -2]$;
    "3rd␣Generation", *ThoList.flatmap family* $[3; -3]$;
    "Gauge␣Bosons", $[Ga; Z; Wp; Wm; Gl]$;
    "Charginos", $[Chargino\ C1; Chargino\ C2; Chargino\ C1c; Chargino\ C2c]$;
    "Neutralinos", $[Neutralino\ N1; Neutralino\ N2; Neutralino\ N3;$
                 $Neutralino\ N4]$;
    "Higgs␣Bosons", $[H\_Heavy; H\_Light; Hp; Hm; A]$;
    "Gluinos", $[Gluino]$]

let *external_flavors′* =
  if *Flags.gravitino* then *external_flavors″* @ ["Gravitino", $[Grino]$]
  else
    external_flavors″

let *external_flavors* () =
  if *Flags.include_goldstone* then *external_flavors′* @ ["Goldstone␣Bosons",
                                  $[Phip; Phim; Phi0]]$
  else
    external_flavors′

let *flavors* () = *ThoList.flatmap snd* (*external_flavors* ())

let *spinor* n =
  if $n ≥ 0$ then
    Spinor
  else if
    $n ≤ 0$ then
    ConjSpinor
  else
    invalid_arg "Modellib_MSSM.MSSM.spinor:␣internal␣error"

```
let lorentz = function
  | L g  →  spinor g  |  N g  →  spinor g
  | U g  →  spinor g  |  D g  →  spinor g
  | Chargino c  →  spinor (int_of_char c)
  | Ga  →  Vector
  | Gl  →  Vector
  | Wp  |  Wm  |  Z  →  Massive_Vector
  | H_Heavy  |  H_Light  |  Hp  |  Hm  |  A  →  Scalar
  | Phip  |  Phim  |  Phi0  →  Scalar
  | Sup _  |  Sdown _  |  Slepton _  |  Sneutrino _  →  Scalar
  | Neutralino _  →  Majorana
  | Gluino  →  Majorana
  | Grino  →  Vectorspinor

let color = function
  | U g  →  Color.SUN (if g > 0 then 3 else -3)
  | Sup (m, g)  →  Color.SUN (if g > 0 then 3 else -3)
  | D g  →  Color.SUN (if g > 0 then 3 else -3)
  | Sdown (m, g)  →  Color.SUN (if g > 0 then 3 else -3)
  | Gl  |  Gluino  →  Color.AdjSUN 3
  | _  →  Color.Singlet

let prop_spinor n =
  if n ≥ 0 then
    Prop_Spinor
  else if
    n ≤ 0 then
    Prop_ConjSpinor
  else
    invalid_arg "Modellib_MSSM.MSSM.prop_spinor:␣internal␣error"

let propagator = function
  | L g  →  prop_spinor g  |  N g  →  prop_spinor g
  | U g  →  prop_spinor g  |  D g  →  prop_spinor g
  | Chargino c  →  prop_spinor (int_of_char c)
  | Ga  |  Gl  →  Prop_Feynman
  | Wp  |  Wm  |  Z  →  Prop_Unitarity
  | H_Heavy  |  H_Light  |  Hp  |  Hm  |  A  →  Prop_Scalar
  | Phip  |  Phim  |  Phi0  →  if Flags.include_goldstone then Prop_Scalar
                                        else Only_Insertion
  | Slepton _  |  Sneutrino _  |  Sup _  |  Sdown _  →  Prop_Scalar
  | Gluino  →  Prop_Majorana  |  Neutralino _  →  Prop_Majorana
  | Grino  →  Only_Insertion
```

Note, that we define the gravitino only as an insertion since when using propagators we are effectively going to a higher order in the gravitational coupling. This would enforce us to also include higher-dimensional vertices with two gravitinos for a consistent power counting in $1/M_{\text{Planck}}$.

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

```
let width f =
  if !use_fudged_width then
```

```
    match f with
    |  Wp  |  Wm  |  U 3  |  U (−3)  →  Fudged
    |  _  →  !default_width
  else
    !default_width
```

For the Goldstone bosons we adopt the conventions of the Kuroda paper.

$$H_1 \equiv \left(\begin{array}{c} \left(v_1 + H^0 \cos\alpha - h^0 \sin\alpha + \mathrm{i}A^0 \sin\beta - \mathrm{i}\cos\beta\phi^0\right)/\sqrt{2} \\ H^- \sin\beta - \phi^- \cos\beta \end{array} \right) \quad (13.107\mathrm{a})$$

$$H_2 \equiv \left(\begin{array}{c} H^+ \cos\beta + \phi^+ \sin\beta \\ \left(v_2 + H^0 \sin\alpha + h^0 \cos\alpha + \mathrm{i}A^0 \cos\beta + \mathrm{i}\phi^0 \sin\beta\right)/\sqrt{2} \end{array} \right) \quad (13.107\mathrm{b})$$

```
let goldstone  =  function
  |  Wp  →  Some (Phip,  Coupling.Const 1)
  |  Wm  →  Some (Phim,  Coupling.Const 1)
  |  Z  →  Some (Phi0,  Coupling.Const 1)
  |  _  →  None

let conjugate  =  function
  |  L g  →  L (−g)  |  N g  →  N (−g)
  |  U g  →  U (−g)  |  D g  →  D (−g)
  |  Sup (m, g)  →  Sup (m, −g)
  |  Sdown (m, g)  →  Sdown (m, −g)
  |  Slepton (m, g)  →  Slepton (m, −g)
  |  Sneutrino g  →  Sneutrino (−g)
  |  Gl  →  Gl (* — Gl0 -¿ Gl0 *)
  |  Ga  →  Ga  |  Z  →  Z  |  Wp  →  Wm  |  Wm  →  Wp
  |  H_Heavy  →  H_Heavy  |  H_Light  →  H_Light  |  A  →  A
  |  Hp  →  Hm  |  Hm  →  Hp
  |  Phip  →  Phim  |  Phim  →  Phip  |  Phi0  →  Phi0
  |  Gluino  →  Gluino
  |  Grino  →  Grino
  |  Neutralino n  →  Neutralino n  |  Chargino c  →  Chargino (conj_char c)

let fermion  =  function
  |  L g  →  if g > 0 then 1 else -1
  |  N g  →  if g > 0 then 1 else -1
  |  U g  →  if g > 0 then 1 else -1
  |  D g  →  if g > 0 then 1 else -1
  |  Gl  |  Ga  |  Z  |  Wp  |  Wm  →  0 (* — Gl0 -¿ 0 *)
  |  H_Heavy  |  H_Light  |  Hp  |  Hm  |  A  →  0
  |  Phip  |  Phim  |  Phi0  →  0
  |  Neutralino _  →  2
  |  Chargino c  →  if (int_of_char c) > 0 then 1 else -1
  |  Sup _  →  0  |  Sdown _  →  0
  |  Slepton _  →  0  |  Sneutrino _  →  0
  |  Gluino  |  Grino  →  2
```

Because the O'Caml compiler only allows 248 constructors we must divide the constants into subgroups of constants, e.g. for the Higgs couplings. In the

MSSM there are a lot of angles among the parameters, the Weinberg-angle, the angle describing the Higgs vacuum structure, the mixing angle of the real parts of the Higgs dubletts, the mixing angles of the sfermions. Therefore we are going to define the trigonometric functions of those angles not as constants but as functors of the angels. Sums and differences of angles are only used as arguments for the α and β angles, so it makes no sense to define special functions for differences and sums of angles.

type *angle* =
    | *Thw* | *Al* | *Be* | *Th_SF* of *sff* × *int* | *Delta* | *CKM_12* | *CKM_13* | *CKM_23*

let *string_of_angle* = function
    | *Thw* → "thw" | *Al* → "al" | *Be* → "be" | *Delta* → "d"
    | *CKM_12* → "ckm12" | *CKM_13* → "ckm13" | *CKM_23* → "ckm23"
    | *Th_SF* (*f*, *g*) → "th" ^ *string_of_sff* *f* ^ *string_of_int* *g*

We introduce a Boolean type vc as a pseudonym for Vertex Conjugator to distinguish between vertices containing complex mixing matrices like the CKM–matrix or the sfermion or neutralino/chargino–mixing matrices, which have to become complex conjugated. The true–option stands for the conjugated vertex, the false–option for the unconjugated vertex.

type *vc* = *bool*

type *constant* =
    | *Unit* | *Pi* | *Alpha_QED* | *Sin2thw*
    | *Sin* of *angle* | *Cos* of *angle* | *E* | *G* | *Vev* | *Tanb* | *Tana*
    | *Cos2be* | *Cos2al* | *Sin2be* | *Sin2al* | *Sin4al* | *Sin4be* | *Cos4be*
    | *Cosapb* | *Cosamb* | *Sinapb* | *Sinamb* | *Cos2am2b* | *Sin2am2b*
    | *Eidelta*
    | *Mu* | *AU* of *int* | *AD* of *int* | *AL* of *int*
    | *V_CKM* of *int*×*int* | *M_SF* of *sff* × *int*×*sfm* × *sfm*
    | *M_V* of *char*×*char* (∗ left chargino mixing matrix ∗)
    | *M_U* of *char*×*char* (∗ right chargino mixing matrix ∗)
    | *M_N* of *neu* × *neu* (∗ neutralino mixing matrix ∗)
    | *V_0* of *neu* × *neu* | *A_0* of *neu* × *neu* | *V_P* of *char*×*char* | *A_P* of *char*×*char*
    | *L_CN* of *char*×*neu* | *R_CN* of *char*×*neu* | *L_NC* of *neu* × *char* | *R_NC* of *neu* × *char*
    | *S_NNH1* of *neu* × *neu* | *P_NNH1* of *neu* × *neu*
    | *S_NNH2* of *neu* × *neu* | *P_NNH2* of *neu* × *neu*
    | *S_NNA* of *neu* × *neu* | *P_NNA* of *neu* × *neu*
    | *S_NNG* of *neu* × *neu* | *P_NNG* of *neu* × *neu*
    | *L_CNG* of *char*×*neu* | *R_CNG* of *char*×*neu*
    | *L_NCH* of *neu* × *char* | *R_NCH* of *neu* × *char*
    | *Q_lepton* | *Q_up* | *Q_down* | *Q_charg*
    | *G_Z* | *G_CC* | *G_CCQ* of *vc* × *int*×*int*
    | *G_NC_neutrino* | *G_NC_lepton* | *G_NC_up* | *G_NC_down*
    | *I_Q_W* | *I_G_ZWW* | *G_WWWW* | *G_ZZWW* | *G_PZWW* | *G_PPWW*
    | *G_strong* | *G_SS* | *I_G_S* | *G_S_Sqrt*

| Gs
| M of *flavor* | W of *flavor*
| G_NZN of $neu \times neu$ | G_CZC of $char \times char$
| G_YUK of $int \times int$
| G_YUK_1 of $int \times int$ | G_YUK_2 of $int \times int$ | G_YUK_3 of $int \times int$
| G_YUK_4 of $int \times int$ | G_NHC of $neu \times char$ | G_CHN of $char \times neu$
| G_YUK_C of $vc \times int \times char \times sff \times sfm$
| G_YUK_Q of $vc \times int \times int \times char \times sff \times sfm$
| G_YUK_N of $vc \times int \times neu \times sff \times sfm$
| G_YUK_G of $vc \times int \times sff \times sfm$
| G_NGC of $neu \times char$ | G_CGN of $char \times neu$
| SUM_1
| G_NWC of $neu \times char$ | G_CWN of $char \times neu$
| G_CH1C of $char \times char$ | G_CH2C of $char \times char$ | G_CAC of $char \times char$
| G_CGC of $char \times char$
| G_SWS of $vc \times int \times int \times sfm \times sfm$
| G_SLSNW of $vc \times int \times sfm$
| G_ZSF of $sff \times int \times sfm \times sfm$
| G_CICIH1 of $neu \times neu$ | G_CICIH2 of $neu \times neu$ | G_CICIA of $neu \times$
neu
| G_CICIG of $neu \times neu$
| G_GH of int | G_GHGo of int
| G_GLGLH | G_GLGLHH | G_GLGLA | G_PPH | G_PPHH |
G_PPA
| G_WWSFSF of $sff \times int \times sfm \times sfm$
| G_WPSLSN of $vc \times int \times sfm$
| G_H3 of int | G_H4 of int
| G_HGo3 of int | G_HGo4 of int | G_GG4 of int
| G_H1SFSF of $sff \times int \times sfm \times sfm$ | G_H2SFSF of $sff \times int \times sfm \times sfm$
| G_ASFSF of $sff \times int \times sfm \times sfm$
| G_HSNSL of $vc \times int \times sfm$
| G_GoSFSF of $sff \times int \times sfm \times sfm$
| G_GoSNSL of $vc \times int \times sfm$
| G_HSUSD of $vc \times sfm \times sfm \times int \times int$ | G_GSUSD of $vc \times sfm \times$
$sfm \times int \times int$
| G_WPSUSD of $vc \times sfm \times sfm \times int \times int$
| G_WZSUSD of $vc \times sfm \times sfm \times int \times int$
| G_WZSLSN of $vc \times int \times sfm$ | $G\_GlGlSQSQ$
| G_PPSFSF of sff
| G_ZZSFSF of $sff \times int \times sfm \times sfm$ | G_ZPSFSF of $sff \times int \times sfm \times sfm$
| $G\_GlZSFSF$ of $sff \times int \times sfm \times sfm$ | $G\_GlPSQSQ$
| $G\_GlWSUSD$ of $vc \times sfm \times sfm \times int \times int$
| G_GH4 of int | G_GHGo4 of int
| $G\_H1H2SFSF$ of $sff \times sfm \times sfm \times int$
| $G\_H1H1SFSF$ of $sff \times sfm \times sfm \times int$
| $G\_H2H2SFSF$ of $sff \times sfm \times sfm \times int$
| G_HHSFSF of $sff \times sfm \times sfm \times int$
| G_AASFSF of $sff \times sfm \times sfm \times int$
| $G\_HH1SLSN$ of $vc \times sfm \times int$ | $G\_HH2SLSN$ of $vc \times sfm \times int$
| G_HASLSN of $vc \times sfm \times int$

| | $G\_HH1SUSD$ of $vc \times sfm \times sfm \times int \times int$
| | $G\_HH2SUSD$ of $vc \times sfm \times sfm \times int \times int$
| | G_HASUSD of $vc \times sfm \times sfm \times int \times int$
| | $G\_AG0SFSF$ of $sff \times sfm \times sfm \times int$
| | G_HGSFSF of $sff \times sfm \times sfm \times int$
| | G_GGSFSF of $sff \times sfm \times sfm \times int$
| | $G\_G0G0SFSF$ of $sff \times sfm \times sfm \times int$
| | G_HGSNSL of $vc \times sfm \times int$ | $G\_H1GSNSL$ of $vc \times sfm \times int$
| | $G\_H2GSNSL$ of $vc \times sfm \times int$ | G_AGSNSL of $vc \times sfm \times int$
| | G_GGSNSL of $vc \times sfm \times int$
| | G_HGSUSD of $vc \times sfm \times sfm \times int \times int$
| | $G\_H1GSUSD$ of $vc \times sfm \times sfm \times int \times int$
| | $G\_H2GSUSD$ of $vc \times sfm \times sfm \times int \times int$
| | G_AGSUSD of $vc \times sfm \times sfm \times int \times int$
| | G_GGSUSD of $vc \times sfm \times sfm \times int \times int$
| | G_SN4 of $int \times int$
| | G_SN2SL2_1 of $sfm \times sfm \times int \times int$ | G_SN2SL2_2 of $sfm \times sfm \times$
$int \times int$
| | G_SF4 of $sff \times sff \times sfm \times sfm \times sfm \times sfm \times int \times int$
| | G_SF4_3 of $sff \times sff \times sfm \times sfm \times sfm \times sfm \times int \times int \times int$
| | G_SF4_4 of $sff \times sff \times sfm \times sfm \times sfm \times sfm \times int \times int \times int \times int$
| | G_SL4 of $sfm \times sfm \times sfm \times sfm \times int$
| | G_SL4_2 of $sfm \times sfm \times sfm \times sfm \times int \times int$
| | G_SN2SQ2 of $sff \times sfm \times sfm \times int \times int$
| | G_SL2SQ2 of $sff \times sfm \times sfm \times sfm \times sfm \times int \times int$
| | $G\_SUSDSNSL$ of $vc \times sfm \times sfm \times sfm \times int \times int \times int$
| | G_SU4 of $sfm \times sfm \times sfm \times sfm \times int$
| | G_SU4_2 of $sfm \times sfm \times sfm \times sfm \times int \times int$
| | G_SD4 of $sfm \times sfm \times sfm \times sfm \times int$
| | G_SD4_2 of $sfm \times sfm \times sfm \times sfm \times int \times int$
| | G_SU2SD2 of $sfm \times sfm \times sfm \times sfm \times int \times int \times int \times int$
| | G_HSF31 of $higgs \times int \times sfm \times sfm \times sff \times sff$
| | G_HSF32 of $higgs \times int \times int \times sfm \times sfm \times sff \times sff$
| | G_HSF41 of $higgs \times int \times sfm \times sfm \times sff \times sff$
| | G_HSF42 of $higgs \times int \times int \times sfm \times sfm \times sff \times sff$
| | G_Grav | G_Gr_Ch of $char$ | $G\_Gr\_Z\_Neu$ of neu
| | $G\_Gr\_A\_Neu$ of neu | G_Gr4_Neu of neu
| | $G\_Gr4\_A\_Ch$ of $char$ | $G\_Gr4\_Z\_Ch$ of $char$
| | G_Grav_N | G_Grav_U of $int \times sfm$ | G_Grav_D of $int \times sfm$
| | G_Grav_L of $int \times sfm$ | G_Grav_Uc of $int \times sfm$ | G_Grav_Dc of $int \times sfm$
| | G_Grav_Lc of $int \times sfm$ | G_GravGl
| | $G\_Gr\_H\_Ch$ of $char$ | $G\_Gr\_H1\_Neu$ of neu
| | $G\_Gr\_H2\_Neu$ of neu | $G\_Gr\_H3\_Neu$ of neu
| | G_Gr4A_Sl of $int \times sfm$ | G_Gr4A_Slc of $int \times sfm$
| | G_Gr4A_Su of $int \times sfm$ | G_Gr4A_Suc of $int \times sfm$
| | G_Gr4A_Sd of $int \times sfm$ | G_Gr4A_Sdc of $int \times sfm$
| | G_Gr4Z_Sn | G_Gr4Z_Snc
| | G_Gr4Z_Sl of $int \times sfm$ | G_Gr4Z_Slc of $int \times sfm$
| | G_Gr4Z_Su of $int \times sfm$ | G_Gr4Z_Suc of $int \times sfm$
| | G_Gr4Z_Sd of $int \times sfm$ | G_Gr4Z_Sdc of $int \times sfm$

```
      |  G_Gr4W_Sl of int×sfm  |  G_Gr4W_Slc of int×sfm
      |  G_Gr4W_Su of int×sfm  |  G_Gr4W_Suc of int×sfm
      |  G_Gr4W_Sd of int×sfm  |  G_Gr4W_Sdc of int×sfm
      |  G_Gr4W_Sn  |  G_Gr4W_Snc
      |  G_Gr4Gl_Su of int×sfm  |  G_Gr4Gl_Suc of int×sfm
      |  G_Gr4Gl_Sd of int×sfm  |  G_Gr4Gl_Sdc of int×sfm
      |  G_Gr4_Z_H1 of neu  |  G_Gr4_Z_H2 of neu  |  G_Gr4_Z_H3 of neu
      |  G_Gr4_W_H of neu  |  G_Gr4_W_Hc of neu  |  G_Gr4_H_A of char
      |  G_Gr4_H_Z of char
```

Two integer counters for the QCD and EW order of the couplings.

```
type orders  =  int × int

let orders  =  function
  |  _  →  (0,0)

let ferm_of_sff  =  function
  |  SL, g  →  (L g)  |  SN, g  →  (N g)
  |  SU, g  →  (U g)  |  SD, g  →  (D g)
```

$$\alpha_{\mathrm{QED}} = \frac{1}{137.0359895} \tag{13.108a}$$

$$\sin^2 \theta_w = 0.23124 \tag{13.108b}$$

Here we must perhaps allow for complex input parameters. So split them into their modulus and their phase. At first, we leave them real; the generalization to complex parameters is obvious.

```
module Ch  =  Charges.QQ

let ( // )  =  Algebra.Small_Rational.make

let generation'  =  function
  |  1  →  [ 1//1; 0//1; 0//1]
  |  2  →  [ 0//1; 1//1; 0//1]
  |  3  →  [ 0//1; 0//1; 1//1]
  |  − 1  →  [−1//1; 0//1; 0//1]
  |  − 2  →  [ 0//1; − 1//1; 0//1]
  |  − 3  →  [ 0//1; 0//1; − 1//1]
  |  n  →  invalid_arg ("MSSM.generation':␣" ˆ string_of_int n)

let generation f  =
  if Flags.ckm_present then
    []
  else
    match f with
    |  L n  |  N n  |  U n  |  D n  |  Sup (_,n)
    |  Sdown (_,n)  |  Slepton (_,n)
    |  Sneutrino n  →  generation' n
    |  _  →  [0//1; 0//1; 0//1]

let charge  =  function
  |  L n  →  if n > 0 then -1//1 else 1//1
```

```
    |  Slepton (_, n)  →  if n > 0 then -1//1 else 1//1
    |  N n  →  0//1
    |  Sneutrino n  →  0//1
    |  U n  →  if n > 0 then 2//3 else -2//3
    |  Sup (_, n)  →  if n > 0 then 2//3 else -2//3
    |  D n  →  if n > 0 then -1//3 else 1//3
    |  Sdown (_, n)  →  if n > 0 then -1//3 else 1//3
    |  Gl  |  Ga  |  Z  |  Neutralino _  |  Gluino  →  0//1
    |  Wp  →  1//1
    |  Wm  →  − 1//1
    |  H_Heavy  |  H_Light  |  Phi0  →  0//1
    |  Hp  |  Phip  →  1//1
    |  Hm  |  Phim  →  − 1//1
    |  Chargino (C1  |  C2)  →  1//1
    |  Chargino (C1c  |  C2c)  →  − 1//1
    |  _  →  0//1

let lepton  =  function
    |  L n  |  N n  →  if n > 0 then 1//1 else -1//1
    |  Slepton (_, n)
    |  Sneutrino n  →  if n > 0 then 1//1 else -1//1
    |  _  →  0//1

let baryon  =  function
    |  U n  |  D n  →  if n > 0 then 1//1 else -1//1
    |  Sup (_, n)  |  Sdown (_, n)  →  if n > 0 then 1//1 else -1//1
    |  _  →  0//1

let charges f  =
    [ charge f;  lepton f;  baryon f] @ generation f

let parameters ()  =
    { input  =  [];
      derived  =  [];
      derived_arrays  =  [] }

module F  =  Modeltools.Fusions (struct
    type f  =  flavor
    type c  =  constant
    let compare  =  compare
    let conjugate  =  conjugate
end)
```

For the couplings there are generally two possibilities concerning the sign of the covariant derivative.

$$\text{CD}^{\pm} = \partial_\mu \pm \mathrm{i}gT^a A^a_\mu \tag{13.109}$$

The particle data group defines the signs consistently to be positive. Since the convention for that signs also influence the phase definitions of the gaugino/higgsino fields via the off-diagonal entries in their mass matrices it would be the best to adopt that convention.
** REVISED: Compatible with CD+. **

```
    let electromagnetic_currents_3 g  =
```

```
[(((U (−g), Ga, U g), FBF (1, Psibar, V, Psi), Q_up);
 ((D (−g), Ga, D g), FBF (1, Psibar, V, Psi), Q_down);
 ((L (−g), Ga, L g), FBF (1, Psibar, V, Psi), Q_lepton) ]
```

** REVISED: Compatible with CD+. **

```
let electromagnetic_sfermion_currents g m  =
   [ ((Ga, Slepton (m, −g), Slepton (m, g)), Vector_Scalar_Scalar 1, Q_lepton);
     ((Ga, Sup (m, −g), Sup (m, g)), Vector_Scalar_Scalar 1, Q_up);
     ((Ga, Sdown (m, −g), Sdown (m, g)), Vector_Scalar_Scalar 1, Q_down) ]
```

** REVISED: Compatible with CD+. **

```
let electromagnetic_currents_2 c  =
   let cc  =  conj_char c in
   [ ((Chargino cc, Ga, Chargino c), FBF (1, Psibar, V, Psi), Q_charg) ]
```

** REVISED: Compatible with CD+. **

```
let neutral_currents g  =
   [ ((L (−g), Z, L g), FBF (1, Psibar, VA, Psi), G_NC_lepton);
     ((N (−g), Z, N g), FBF (1, Psibar, VA, Psi), G_NC_neutrino);
     ((U (−g), Z, U g), FBF (1, Psibar, VA, Psi), G_NC_up);
     ((D (−g), Z, D g), FBF (1, Psibar, VA, Psi), G_NC_down) ]
```

$$\mathcal{L}_{\mathrm{CC}} = \mp \frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i \gamma^\mu (1 - \gamma_5)(T^+ W_\mu^+ + T^- W_\mu^-)\psi_i, \qquad (13.110)$$

where the sign corresponds to CD$_\pm$, respectively.
** REVISED: Compatible with CD+. **
Remark: The definition with the other sign compared to the SM files comes from the fact that $g_{cc} = 1/(2\sqrt{2})$ is used overwhelmingly often in the SUSY Feynman rules, so that JR decided to use a different definiton for g_cc in SM and MSSM.

```
let charged_currents g  =
   [ ((L (−g), Wm, N g), FBF ((−1), Psibar, VL, Psi), G_CC);
     ((N (−g), Wp, L g), FBF ((−1), Psibar, VL, Psi), G_CC) ]
```

The quark with the inverted generation (the antiparticle) is the outgoing one, the other the incoming. The vertex attached to the outgoing up-quark contains the CKM matrix element *not* complex conjugated, while the vertex with the outgoing down-quark has the conjugated CKM matrix element.
** REVISED: Compatible with CD+. **

```
let charged_quark_currents g h  =
   [ ((D (−g), Wm, U h), FBF ((−1), Psibar, VL, Psi), G_CCQ (true,g, h));
     ((U (−g), Wp, D h), FBF ((−1), Psibar, VL, Psi), G_CCQ (false,h, g))]
```

** REVISED: Compatible with CD+. **

```
let charged_chargino_currents n c  =
   let cc  =  conj_char c in
   [ ((Chargino cc, Wp, Neutralino n),
               FBF (1, Psibar, VLR, Chi), G_CWN (c, n));
     ((Neutralino n, Wm, Chargino c),
```

$$FBF\ (1,\ Chibar,\ VLR,\ Psi),\ G\_NWC\ (n, c))\ ]$$

** REVISED: Compatible with CD+. **

    let *charged_slepton_currents g m* =
      [((*Wm, Slepton* (*m*, −*g*), *Sneutrino g*), *Vector_Scalar_Scalar* (−1), *G_SLSNW*
          (true,*g*, *m*));
        ((*Wp, Slepton* (*m*, *g*), *Sneutrino* (−*g*)), *Vector_Scalar_Scalar* 1, *G_SLSNW*
          (false,*g*, *m*))]

** REVISED: Compatible with CD+. **

    let *charged_squark_currents′ g h m1 m2* =
      [((*Wm, Sup* (*m1*, *g*), *Sdown* (*m2*, −*h*)), *Vector_Scalar_Scalar* (−1), *G_SWS*
          (true,*g*, *h*, *m1*, *m2*));
        ((*Wp, Sup* (*m1*, −*g*), *Sdown* (*m2*, *h*)), *Vector_Scalar_Scalar* 1, *G_SWS*
          (false,*g*, *h*, *m1*, *m2*))]
    let *charged_squark_currents g h* = *List.flatten* (*Product.list2*
          (*charged_squark_currents′ g h*) [*M1*; *M2*] [*M1*; *M2*])

** REVISED: Compatible with CD+. **

    let *neutral_sfermion_currents′ g m1 m2* =
      [((*Z, Slepton* (*m1*, −*g*), *Slepton* (*m2*, *g*)), *Vector_Scalar_Scalar* (−1), *G_ZSF*
          (*SL*, *g*, *m1*, *m2*));
        ((*Z, Sup* (*m1*, −*g*), *Sup* (*m2*, *g*)), *Vector_Scalar_Scalar* (−1), *G_ZSF*
          (*SU*, *g*, *m1*, *m2*));
        ((*Z, Sdown* (*m1*, −*g*), *Sdown* (*m2*, *g*)), *Vector_Scalar_Scalar* (−1), *G_ZSF*
          (*SD*, *g*, *m1*, *m2*))]
    let *neutral_sfermion_currents g* =
      List.flatten (*Product.list2* (*neutral_sfermion_currents′*
            g) [*M1*; *M2*] [*M1*; *M2*]) @
      [((*Z, Sneutrino* (−*g*), *Sneutrino g*), *Vector_Scalar_Scalar* (−1), *G_ZSF*
          (*SN*, *g*, *M1*, *M1*))]

The reality of the coupling of the Z-boson to two identical neutralinos makes
the vector part of the coupling vanish. So we distinguish them not by the name
but by the structure of the couplings.
** REVISED: Compatible with CD+. **

    let *neutral_Z_1* (*n*, *m*) =
      [((*Neutralino n, Z, Neutralino m*), *FBF* (1, *Chibar, VA, Chi*),
          (*G_NZN* (*n*, *m*)))]
(*** REVISED: Compatible with CD+. ***)
    let *neutral_Z_2 n* =
      [((*Neutralino n, Z, Neutralino n*), *FBF* (1, *Chibar, Coupling.A, Chi*),
          (*G_NZN* (*n*, *n*)))]

** REVISED: Compatible with CD+. **

    let *charged_Z c1 c2* =
      let *cc1* = *conj_char c1* in
      ((*Chargino cc1, Z, Chargino c2*), *FBF* ((−1), *Psibar, VA, Psi*),
          G_CZC (*c1*, *c2*))

** REVISED: Compatible with CD+. **

let *yukawa_v* =
  [((*Gluino*, *Gl*, *Gluino*), *FBF* (1, *Chibar*, *V*, *Chi*), *Gs*)]

** REVISED: Independent of the sign of CD. **

let *yukawa_higgs* g =
  [((*N* ($-g$), *Hp*, *L* g), *FBF* (1, *Psibar*, *Coupling.SR*, *Psi*), *G_YUK* (6, g));
    ((*L* ($-g$), *Hm*, *N* g), *FBF* (1, *Psibar*, *Coupling.SL*, *Psi*), *G_YUK* (6, g));
    ((*L* ($-g$), *H_Heavy*, *L* g), *FBF* (1, *Psibar*, *S*, *Psi*), *G_YUK* (7, g));
    ((*L* ($-g$), *H_Light*, *L* g), *FBF* (1, *Psibar*, *S*, *Psi*), *G_YUK* (8, g));
    ((*L* ($-g$), *A*, *L* g), *FBF* (1, *Psibar*, *P*, *Psi*), *G_YUK* (9, g));
    ((*U* ($-g$), *H_Heavy*, *U* g), *FBF* (1, *Psibar*, *S*, *Psi*), *G_YUK* (10, g));
    ((*U* ($-g$), *H_Light*, *U* g), *FBF* (1, *Psibar*, *S*, *Psi*), *G_YUK* (11, g));
    ((*U* ($-g$), *A*, *U* g), *FBF* (1, *Psibar*, *P*, *Psi*), *G_YUK* (12, g));
    ((*D* ($-g$), *H_Heavy*, *D* g), *FBF* (1, *Psibar*, *S*, *Psi*), *G_YUK* (13, g));
    ((*D* ($-g$), *H_Light*, *D* g), *FBF* (1, *Psibar*, *S*, *Psi*), *G_YUK* (14, g));
    ((*D* ($-g$), *A*, *D* g), *FBF* (1, *Psibar*, *P*, *Psi*), *G_YUK* (15, g))]

** REVISED: Compatible with CD+ and GS+. **

let *yukawa_goldstone* g =
  [((*N* ($-g$), *Phip*, *L* g), *FBF* (1, *Psibar*, *Coupling.SR*, *Psi*), *G_YUK* (19, g));
    ((*L* ($-g$), *Phim*, *N* g), *FBF* (1, *Psibar*, *Coupling.SL*, *Psi*), *G_YUK* (19, g));
    ((*L* ($-g$), *Phi0*, *L* g), *FBF* (1, *Psibar*, *P*, *Psi*), *G_YUK* (16, g));
    ((*U* ($-g$), *Phi0*, *U* g), *FBF* (1, *Psibar*, *P*, *Psi*), *G_YUK* (17, g));
    ((*D* ($-g$), *Phi0*, *D* g), *FBF* (1, *Psibar*, *P*, *Psi*), *G_YUK* (18, g))]

** REVISED: Independent of the sign of CD. **

let *yukawa_higgs_quark* (g, h) =
  [((*U* ($-g$), *Hp*, *D* h), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_YUK_1* (g, h));
    ((*D* ($-h$), *Hm*, *U* g), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_YUK_2* (g, h))]

** REVISED: Compatible with CD+ and GS+. **

let *yukawa_goldstone_quark* g h =
  [((*U* ($-g$), *Phip*, *D* h), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_YUK_3* (g, h));
    ((*D* ($-h$), *Phim*, *U* g), *FBF* (1, *Psibar*, *SLR*, *Psi*), *G_YUK_4* (g, h))]

** REVISED: Compatible with CD+.

let *yukawa_higgs_2′* (*c1, c2*) =
  let *cc1* = *conj_char c1* in
  [((*Chargino cc1*, *H_Heavy*, *Chargino c2*), *FBF* (1, *Psibar*, *SLR*, *Psi*),
        G_CH2C (*c1, c2*));
    ((*Chargino cc1*, *H_Light*, *Chargino c2*), *FBF* (1, *Psibar*, *SLR*, *Psi*),
        G_CH1C (*c1, c2*));
    ((*Chargino cc1*, *A*, *Chargino c2*), *FBF* (1, *Psibar*, *SLR*, *Psi*),
        G_CAC (*c1, c2*))]
let *yukawa_higgs_2″* c =
  let *cc* = *conj_char c* in
  [((*Chargino cc*, *H_Heavy*, *Chargino c*), *FBF* (1, *Psibar*, *S*, *Psi*),
        G_CH2C (*c, c*));
    ((*Chargino cc*, *H_Light*, *Chargino c*), *FBF* (1, *Psibar*, *S*, *Psi*),
        G_CH1C (*c, c*));

```
            ((Chargino cc, A, Chargino c), FBF (1, Psibar, P, Psi),
                    G_CAC (c, c)) ]
    let yukawa_higgs_2 =
      ThoList.flatmap yukawa_higgs_2′ [(C1, C2); (C2, C1)] @
      ThoList.flatmap yukawa_higgs_2″ [C1; C2]
```

** REVISED: Compatible with CD+ and GS+. **

```
    let yukawa_goldstone_2′ (c1, c2) =
      let cc1 = conj_char c1 in
      [ ((Chargino cc1, Phi0, Chargino c2), FBF (1, Psibar, SLR, Psi),
              G_CGC (c1, c2)) ]
    let yukawa_goldstone_2″ c =
      let cc = conj_char c in
      [ ((Chargino cc, Phi0, Chargino c), FBF (1, Psibar, P, Psi),
              G_CGC (c, c)) ]
    let yukawa_goldstone_2 =
      ThoList.flatmap yukawa_goldstone_2′ [(C1, C2); (C2, C1)] @
      ThoList.flatmap yukawa_goldstone_2″ [C1; C2]
```

** REVISED: Compatible with CD+. **

```
    let higgs_charg_neutr n c =
      let cc = conj_char c in
      [ ((Neutralino n, Hm, Chargino c), FBF (−1, Chibar, SLR, Psi),
                  G_NHC (n, c));
        ((Chargino cc, Hp, Neutralino n), FBF (−1, Psibar, SLR, Chi),
                  G_CHN (c, n)) ]
```

** REVISED: Compatible with CD+ and GS+. **

```
    let goldstone_charg_neutr n c =
      let cc = conj_char c in
      [ ((Neutralino n, Phim, Chargino c), FBF (1, Chibar, SLR, Psi),
              G_NGC (n, c));
        ((Chargino cc, Phip, Neutralino n), FBF (1, Psibar, SLR, Chi),
                G_CGN (c, n)) ]
```

** REVISED: Compatible with CD+. **

```
  let higgs_neutr′ (n, m) =
    [ ((Neutralino n, H_Heavy, Neutralino m), FBF (1, Chibar, SP, Chi),
              G_CICIH2 (n, m));
      ((Neutralino n, H_Light, Neutralino m), FBF (1, Chibar, SP, Chi),
              G_CICIH1 (n, m));
      ((Neutralino n, A, Neutralino m), FBF (1, Chibar, SP, Chi),
              G_CICIA (n, m)) ]
  let higgs_neutr″ n =
    [ ((Neutralino n, H_Heavy, Neutralino n), FBF (1, Chibar, S, Chi),
              G_CICIH2 (n, n));
      ((Neutralino n, H_Light, Neutralino n), FBF (1, Chibar, S, Chi),
              G_CICIH1 (n, n));
      ((Neutralino n, A, Neutralino n), FBF (1, Chibar, P, Chi),
              G_CICIA (n, n)) ]
```

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let *higgs_neutr* =
  ThoList.flatmap higgs_neutr' [($N1, N2$); ($N1, N3$); ($N1, N4$);
                      ($N2, N3$); ($N2, N4$); ($N3, N4$)] @
  ThoList.flatmap higgs_neutr'' [$N1; N2; N3; N4$]

** REVISED: Compatible with CD+ and GS+. **

let *goldstone_neutr'* (n, m) =
  [(($Neutralino\ n$, $Phi0$, $Neutralino\ m$), FBF (1, $Chibar$, SP, Chi),
          $G\_CICIG\ (n, m)$)]
let *goldstone_neutr''* n =
  [(($Neutralino\ n$, $Phi0$, $Neutralino\ n$), FBF (1, $Chibar$, P, Chi),
          $G\_CICIG\ (n, n)$)]
let *goldstone_neutr* =
  ThoList.flatmap goldstone_neutr' [($N1, N2$); ($N1, N3$); ($N1, N4$);
                      ($N2, N3$); ($N2, N4$); ($N3, N4$)] @
  ThoList.flatmap goldstone_neutr'' [$N1; N2; N3; N4$]

** REVISED: Compatible with CD+. **

let *yukawa_n_1* $n\ g$ =
  [(($Neutralino\ n$, $Slepton\ (M1, -g)$, $L\ g$), FBF (1, $Chibar$, $Coupling.SL$,
          Psi), G_YUK_N (true,$g, n, SL, M1$));
    (($Neutralino\ n$, $Slepton\ (M2, -g)$, $L\ g$), FBF (1, $Chibar$, SR, Psi),
          G_YUK_N (true,$g, n, SL, M2$));
    (($L\ (-g)$, $Slepton\ (M1, g)$, $Neutralino\ n$), FBF (1, $Psibar$, SR, Chi),
          G_YUK_N (false,$g, n, SL, M1$));
    (($L\ (-g)$, $Slepton\ (M2, g)$, $Neutralino\ n$), FBF (1, $Psibar$, $Coupling.SL$,
          Chi), G_YUK_N (false,$g, n, SL, M2$));
    (($Neutralino\ n$, $Sup\ (M1, -g)$, $U\ g$), FBF (1, $Chibar$, $Coupling.SL$,
          Psi), G_YUK_N (true,$g, n, SU, M1$));
    (($Neutralino\ n$, $Sup\ (M2, -g)$, $U\ g$), FBF (1, $Chibar$, SR, Psi),
          G_YUK_N (true,$g, n, SU, M2$));
    (($U\ (-g)$, $Sup\ (M1, g)$, $Neutralino\ n$), FBF (1, $Psibar$, SR, Chi),
          G_YUK_N (false,$g, n, SU, M1$));
    (($U\ (-g)$, $Sup\ (M2, g)$, $Neutralino\ n$), FBF (1, $Psibar$, $Coupling.SL$,
          Chi), G_YUK_N (false,$g, n, SU, M2$));
    (($Neutralino\ n$, $Sdown\ (M1, -g)$, $D\ g$), FBF (1, $Chibar$, $Coupling.SL$,
          Psi), G_YUK_N (true,$g, n, SD, M1$));
    (($Neutralino\ n$, $Sdown\ (M2, -g)$, $D\ g$), FBF (1, $Chibar$, SR, Psi),
          G_YUK_N (true,$g, n, SD, M2$));
    (($D\ (-g)$, $Sdown\ (M1, g)$, $Neutralino\ n$), FBF (1, $Psibar$, SR, Chi),
          G_YUK_N (false,$g, n, SD, M1$));
    (($D\ (-g)$, $Sdown\ (M2, g)$, $Neutralino\ n$), FBF (1, $Psibar$, $Coupling.SL$,
          Chi), G_YUK_N (false,$g, n, SD, M2$))]
let *yukawa_n_2* $n\ m$ =
  [(($Neutralino\ n$, $Slepton\ (m, -3)$, $L\ 3$), FBF (1, $Chibar$, SLR, Psi),
          G_YUK_N (true,3,n, SL, m));
    (($L\ (-3)$, $Slepton\ (m, 3)$, $Neutralino\ n$), FBF (1, $Psibar$, SLR, Chi),
          G_YUK_N (false,3,n, SL, m));
    (($Neutralino\ n$, $Sup\ (m, -3)$, $U\ 3$), FBF (1, $Chibar$, SLR, Psi),
          G_YUK_N (true,3,n, SU, m));

$\quad\quad\quad((U\ (-3),\ Sup\ (m,3),\ Neutralino\ n),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
$\quad\quad\quad\quad G_{-}YUK_{-}N$ (false,3,n,SU,m));
$\quad\quad\quad((Neutralino\ n,\ Sdown\ (m,-3),\ D\ 3),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
$\quad\quad\quad\quad G_{-}YUK_{-}N$ (true,3,SD,m));
$\quad\quad\quad((D\ (-3),\ Sdown\ (m,3),\ Neutralino\ n),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
$\quad\quad\quad\quad G_{-}YUK_{-}N$ (false,3,n,SD,m))]
$\quad$ let *yukawa_n_3 n g* =
$\quad\quad\quad [((Neutralino\ n,\ Sneutrino\ (-g),\ N\ g),\ FBF\ (1,\ Chibar,\ Coupling.SL,$
$\quad\quad\quad\quad Psi),\ G_{-}YUK_{-}N$ (true,$g,n,SN,M1$));
$\quad\quad\quad ((N\ (-g),\ Sneutrino\ g,\ Neutralino\ n),\ FBF\ (1,\ Psibar,\ SR,\ Chi),$
$\quad\quad\quad\quad G_{-}YUK_{-}N$ (false,$g,n,SN,M1$))]
$\quad$ let *yukawa_n_4 g* =
$\quad\quad\quad [((U\ (-g),\ Sup\ (M1,g),\ Gluino),\ FBF\ ((-1),\ Psibar,\ SR,\ Chi),\ G_{-}S_{-}Sqrt);$
$\quad\quad\quad ((D\ (-g),\ Sdown\ (M1,g),\ Gluino),\ FBF\ ((-1),\ Psibar,\ SR,\ Chi),\ G_{-}S_{-}Sqrt);$
$\quad\quad\quad ((Gluino,\ Sup\ (M1,-g),\ U\ g),\ FBF\ ((-1),\ Chibar,\ Coupling.SL,\ Psi),\ G_{-}S_{-}Sqrt);$
$\quad\quad\quad ((Gluino,\ Sdown\ (M1,-g),\ D\ g),\ FBF\ ((-1),\ Chibar,\ Coupling.SL,\ Psi),\ G_{-}S_{-}Sqrt);$
$\quad\quad\quad ((U\ (-g),\ Sup\ (M2,g),\ Gluino),\ FBF\ (1,\ Psibar,\ Coupling.SL,\ Chi),\ G_{-}S_{-}Sqrt);$
$\quad\quad\quad ((D\ (-g),\ Sdown\ (M2,g),\ Gluino),\ FBF\ (1,\ Psibar,\ Coupling.SL,\ Chi),\ G_{-}S_{-}Sqrt);$
$\quad\quad\quad ((Gluino,\ Sup\ (M2,-g),\ U\ g),\ FBF\ (1,\ Chibar,\ SR,\ Psi),\ G_{-}S_{-}Sqrt);$
$\quad\quad\quad ((Gluino,\ Sdown\ (M2,-g),\ D\ g),\ FBF\ (1,\ Chibar,\ SR,\ Psi),\ G_{-}S_{-}Sqrt)]$
$\quad$ let *yukawa_n_5 m* =
$\quad\quad\quad [((U\ (-3),\ Sup\ (m,3),\ Gluino),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
$\quad\quad\quad\quad G_{-}YUK_{-}G$ (false,3,SU,m));
$\quad\quad\quad ((D\ (-3),\ Sdown\ (m,3),\ Gluino),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
$\quad\quad\quad\quad G_{-}YUK_{-}G$ (false,3,SD,m));
$\quad\quad\quad ((Gluino,\ Sup\ (m,-3),\ U\ 3),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
$\quad\quad\quad\quad G_{-}YUK_{-}G$ (true,3,SU,m));
$\quad\quad\quad ((Gluino,\ Sdown\ (m,-3),\ D\ 3),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
$\quad\quad\quad\quad G_{-}YUK_{-}G$ (true,3,SD,m))]
$\quad$ let *yukawa_n* =
$\quad\quad$ *List.flatten* (*Product.list2 yukawa_n_1* [*N1*; *N2*; *N3*; *N4*] [1; 2]) @
$\quad\quad$ *List.flatten* (*Product.list2 yukawa_n_2* [*N1*; *N2*; *N3*; *N4*] [*M1*; *M2*]) @
$\quad\quad$ *List.flatten* (*Product.list2 yukawa_n_3* [*N1*; *N2*; *N3*; *N4*] [1; 2; 3]) @
$\quad\quad$ *ThoList.flatmap yukawa_n_4* [1; 2] @
$\quad\quad$ *ThoList.flatmap yukawa_n_5* [*M1*; *M2*]

** REVISED: Compatible with CD+. **

$\quad$ let *yukawa_c_1 c g* =
$\quad\quad\quad$ let *cc* = *conj_char c* in
$\quad\quad\quad [((L\ (-g),\ Sneutrino\ g,\ Chargino\ cc),\ BBB\ (1,\ Psibar,\ Coupling.SR,$
$\quad\quad\quad\quad Psibar),\ G_{-}YUK_{-}C$ (true,$g,c,SN,M1$));
$\quad\quad\quad ((Chargino\ c,\ Sneutrino\ (-g),\ L\ g),\ PBP\ (1,\ Psi,\ Coupling.SL,\ Psi),$
$\quad\quad\quad\quad G_{-}YUK_{-}C$ (false,$g,c,SN,M1$))]
$\quad$ let *yukawa_c_2 c* =
$\quad\quad\quad$ let *cc* = *conj_char c* in
$\quad\quad\quad [((L\ (-3),\ Sneutrino\ 3,\ Chargino\ cc),\ BBB\ (1,\ Psibar,\ SLR,$
$\quad\quad\quad\quad Psibar),\ G_{-}YUK_{-}C$ (true,3,$c,SN,M1$));
$\quad\quad\quad ((Chargino\ c,\ Sneutrino\ (-3),\ L\ 3),\ PBP\ (1,\ Psi,\ SLR,\ Psi),$
$\quad\quad\quad\quad G_{-}YUK_{-}C$ (false,3,$c,SN,M1$))]
$\quad$ let *yukawa_c_3 c m g* =

```
        let cc = conj_char c in
        [ ((N (−g), Slepton (m, g), Chargino c), FBF (1, Psibar, Coupling.SR,
              Psi), G_YUK_C (true,g, c, SL, m));
            ((Chargino cc, Slepton (m, −g), N g), FBF (1, Psibar, Coupling.SL,
              Psi), G_YUK_C (false,g, c, SL, m)) ]
    let yukawa_c c =
      ThoList.flatmap (yukawa_c_1 c) [1; 2] @
      yukawa_c_2 c @
      List.flatten (Product.list2 (yukawa_c_3 c) [M1] [1; 2]) @
      List.flatten (Product.list2 (yukawa_c_3 c) [M1; M2] [3])
```

** REVISED: Compatible with CD+. **

```
    let yukawa_cq′ c (g, h) m =
        let cc = conj_char c in
        [ ((Chargino c, Sup (m, −g), D h), PBP (1, Psi, SLR, Psi),
            G_YUK_Q (false,g, h, c, SU, m));
            ((D (−h), Sup (m, g), Chargino cc), BBB (1, Psibar, SLR, Psibar),
            G_YUK_Q (true,g, h, c, SU, m));
            ((Chargino cc, Sdown (m, −h), U g), FBF (1, Psibar, SLR, Psi),
            G_YUK_Q (true,g, h, c, SD, m));
            ((U (−g), Sdown (m, h), Chargino c), FBF (1, Psibar, SLR, Psi),
            G_YUK_Q (false,g, h, c, SD, m)) ]
    let yukawa_cq″ c (g, h) =
        let cc = conj_char c in
        [ ((Chargino c, Sup (M1, −g), D h), PBP (1, Psi, Coupling.SL, Psi),
              G_YUK_Q (false,g, h, c, SU, M1));
            ((D (−h), Sup (M1, g), Chargino cc),
              BBB (1, Psibar, Coupling.SR, Psibar), G_YUK_Q (true,g, h, c, SU, M1));
            ((Chargino cc, Sdown (M1, −h), U g),
              FBF (1, Psibar, Coupling.SL, Psi), G_YUK_Q (true,g, h, c, SD, M1));
            ((U (−g), Sdown (M1, h), Chargino c),
              FBF (1, Psibar, Coupling.SR, Psi), G_YUK_Q (false,g, h, c, SD, M1)) ]
    let yukawa_cq c =
      if Flags.ckm_present then
        List.flatten (Product.list2 (yukawa_cq′ c) [(1, 3); (2, 3); (3, 3);
                                          (3, 2); (3, 1)] [M1; M2]) @
        ThoList.flatmap (yukawa_cq″ c) [(1, 1); (1, 2); (2, 1); (2, 2)]
      else
        ThoList.flatmap (yukawa_cq′ c (3, 3)) [M1; M2] @
        ThoList.flatmap (yukawa_cq″ c) [(1, 1); (2, 2)]
```

** REVISED: Compatible with CD+. Remark: Singlet and octet gluon exchange. The coupling is divided by sqrt(2) to account for the correct normalization of the Lie algebra generators. **

```
    let col_currents g =
      [ ((D (−g), Gl, D g), FBF ((−1), Psibar, V, Psi), Gs);
          ((U (−g), Gl, U g), FBF ((−1), Psibar, V, Psi), Gs)]
```

** REVISED: Compatible with CD+. Remark: Singlet and octet gluon exchange. The coupling is divided by sqrt(2) to account for the correct normalization of the Lie algebra generators. **

let *col_sfermion_currents g m* =
    [((*Gl, Sup* (*m*, −*g*), *Sup* (*m*, *g*)), *Vector_Scalar_Scalar* (−1), *Gs*);
        ((*Gl, Sdown* (*m*, −*g*), *Sdown* (*m*, *g*)), *Vector_Scalar_Scalar* (−1), *Gs*)]

The gravitino coupling is generically $1/(4M_{Pl.})$
** Triple vertices containing graivitinos. **

let *triple_gravitino′ g* =
    [((*Grino, Sneutrino* (−*g*), *N g*), *GBG* (1, *Gravbar, Coupling.SL, Psi*), *G_Grav_N*);
        ((*N* (−*g*), *Sneutrino g, Grino*), *GBG* (1, *Psibar, Coupling.SL, Grav*), *G_Grav_N*)]

let *triple_gravitino″ g m* =
    [((*Grino, Slepton* (*m*, −*g*), *L g*), *GBG* (1, *Gravbar, SLR, Psi*), *G_Grav_L* (*g, m*));
        ((*L* (−*g*), *Slepton* (*m*, *g*), *Grino*), *GBG* (1, *Psibar, SLR, Grav*), *G_Grav_Lc* (*g, m*));
        ((*Grino, Sup* (*m*, −*g*), *U g*), *GBG* (1, *Gravbar, SLR, Psi*), *G_Grav_U* (*g, m*));
        ((*U* (−*g*), *Sup* (*m*, *g*), *Grino*), *GBG* (1, *Psibar, SLR, Grav*), *G_Grav_Uc* (*g, m*));
        ((*Grino, Sdown* (*m*, −*g*), *D g*), *GBG* (1, *Gravbar, SLR, Psi*), *G_Grav_D* (*g, m*));
        ((*D* (−*g*), *Sdown* (*m*, *g*), *Grino*), *GBG* (1, *Psibar, SLR, Grav*), *G_Grav_Dc* (*g, m*))]

let *higgs_ch_gravitino c* =
    let *cc* = *conj_char c* in
    [((*Grino, Hm, Chargino c*), *GBG* (1, *Gravbar, SLR, Psi*), *G_Gr_H_Ch c*);
        ((*Chargino cc, Hp, Grino*), *GBG* (1, *Psibar, SLR, Grav*), *G_Gr_H_Ch cc*)]

let *higgs_neu_gravitino n* =
    [((*Grino, H_Light, Neutralino n*), *GBG* (1, *Gravbar, SLR, Chi*), *G_Gr_H1_Neu n*);
        ((*Grino, H_Heavy, Neutralino n*), *GBG* (1, *Gravbar, SLR, Chi*), *G_Gr_H2_Neu n*);
        ((*Grino, A, Neutralino n*), *GBG* (1, *Gravbar, SLR, Chi*), *G_Gr_H3_Neu n*)]

let *gravitino_gaugino_3* =
    [((*Grino, Gl, Gluino*), *GBG* (1, *Gravbar, V, Chi*), *G_Grav*);
        ((*Gluino, Gl, Grino*), *GBG* (1, *Chibar, V, Grav*), *G_Grav*);
        ((*Chargino C1c, Wp, Grino*), *GBG* (1, *Psibar, VLR, Grav*), *G_Gr_Ch C1*);
        ((*Chargino C2c, Wp, Grino*), *GBG* (1, *Psibar, VLR, Grav*), *G_Gr_Ch C2*);
        ((*Grino, Wm, Chargino C1*), *GBG* (1, *Gravbar, VLR, Psi*), *G_Gr_Ch C1c*);
        ((*Grino, Wm, Chargino C2*), *GBG* (1, *Gravbar, VLR, Psi*), *G_Gr_Ch C2c*);
        ((*Grino, Z, Neutralino N1*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_Z_Neu N1*);
        ((*Grino, Z, Neutralino N2*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_Z_Neu N2*);
        ((*Grino, Z, Neutralino N3*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_Z_Neu N3*);
        ((*Grino, Z, Neutralino N4*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_Z_Neu N4*);
        ((*Grino, Ga, Neutralino N1*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_A_Neu N1*);
        ((*Grino, Ga, Neutralino N2*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_A_Neu N2*);
        ((*Grino, Ga, Neutralino N3*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_A_Neu N3*);
        ((*Grino, Ga, Neutralino N4*), *GBG* (1, *Gravbar, VLR, Chi*), *G_Gr_A_Neu N4*)]

let *triple_gravitino* =
    ThoList.flatmap triple_gravitino′ [1; 2; 3] @
    List.flatten (*Product.list2 triple_gravitino″* [1; 2; 3] [*M1*; *M2*]) @
    ThoList.flatmap higgs_ch_gravitino [*C1*; *C2*] @
    ThoList.flatmap higgs_neu_gravitino [*N1*; *N2*; *N3*; *N4*] @
    gravitino_gaugino_3

** REVISED: Compatible with CD+. **

let *triple_gauge* =

  [((Ga, Wm, Wp), $Gauge\_Gauge\_Gauge$ 1, I_Q_W);
    ((Z, Wm, Wp), $Gauge\_Gauge\_Gauge$ 1, I_G_ZWW);
    ((Gl, Gl, Gl), $Gauge\_Gauge\_Gauge$ 1, I_G_S)]

** REVISED: Independent of the sign of CD. **

  let $gauge4$ = $Vector4$ [(2, C_13_42); (−1, C_12_34); (−1, C_14_23)]
  let $gluon4$ = $Vector4$ [(−1, C_13_42); (−1, C_12_34); (−1, C_14_23)]
  let $minus\_gauge4$ = $Vector4$ [(−2, C_13_42); (1, C_12_34); (1, C_14_23)]
  let $quartic\_gauge$ =
    [(Wm, Wp, Wm, Wp), $gauge4$, G_WWWW;
      (Wm, Z, Wp, Z), $minus\_gauge4$, G_ZZWW;
      (Wm, Z, Wp, Ga), $minus\_gauge4$, G_PZWW;
      (Wm, Ga, Wp, Ga), $minus\_gauge4$, G_PPWW;
      (Gl, Gl, Gl, Gl), $gauge4$, G_SS]

The *Scalar_Vector_Vector* couplings do not depend on the choice of the sign of
the covariant derivative since they are quadratic in the gauge couplings.
** REVISED: Compatible with CD+. **
** Revision: 2005-03-10: first two vertices corrected. **

  let $gauge\_higgs$ =
    [((Wm, Hp, A), $Vector\_Scalar\_Scalar$ 1, G_GH 1);
      ((Wp, Hm, A), $Vector\_Scalar\_Scalar$ 1, G_GH 1);
      ((Z, H_Heavy, A), $Vector\_Scalar\_Scalar$ 1, G_GH 3);
      ((Z, H_Light, A), $Vector\_Scalar\_Scalar$ 1, G_GH 2);
      ((H_Heavy, Wp, Wm), $Scalar\_Vector\_Vector$ 1, G_GH 5);
      ((H_Light, Wp, Wm), $Scalar\_Vector\_Vector$ 1, G_GH 4);
      ((Wm, Hp, H_Heavy), $Vector\_Scalar\_Scalar$ 1, G_GH 7);
      ((Wp, Hm, H_Heavy), $Vector\_Scalar\_Scalar$ (−1), G_GH 7);
      ((Wm, Hp, H_Light), $Vector\_Scalar\_Scalar$ 1, G_GH 6);
      ((Wp, Hm, H_Light), $Vector\_Scalar\_Scalar$ (−1), G_GH 6);
      ((H_Heavy, Z, Z), $Scalar\_Vector\_Vector$ 1, G_GH 9);
      ((H_Light, Z, Z), $Scalar\_Vector\_Vector$ 1, G_GH 8);
      ((Z, Hp, Hm), $Vector\_Scalar\_Scalar$ 1, G_GH 10);
      ((Ga, Hp, Hm), $Vector\_Scalar\_Scalar$ 1, G_GH 11)] @
    (if $Flags.higgs\_triangle$ then
     [((H_Light, Gl, Gl), $Dim5\_Scalar\_Gauge2$ 1, G_GLGLH);
      ((H_Heavy, Gl, Gl), $Dim5\_Scalar\_Gauge2$ 1, G_GLGLHH);
      ((A, Gl, Gl), $Dim5\_Scalar\_Gauge2\_Skew$ 1, G_GLGLA);
      ((H_Light, Ga, Ga), $Dim5\_Scalar\_Gauge2$ 1, G_PPH);
      ((H_Heavy, Ga, Ga), $Dim5\_Scalar\_Gauge2$ 1, G_PPHH);
      ((A, Ga, Ga), $Dim5\_Scalar\_Gauge2$ 1, G_PPA)]
     else
       [])

** REVISED: Compatible with CD+ and GS+. **

  let $gauge\_higgs\_gold$ =
    [((Wp, $Phi0$, $Phim$), $Vector\_Scalar\_Scalar$ 1, G_GH 1);
      ((Wm, $Phi0$, $Phip$), $Vector\_Scalar\_Scalar$ 1, G_GH 1);
      ((Z, H_Heavy, $Phi0$), $Vector\_Scalar\_Scalar$ 1, G_GH 2);
      ((Z, H_Light, $Phi0$), $Vector\_Scalar\_Scalar$ (−1), G_GH 3);

$((Wp,\ H\_Heavy,\ Phim),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\ 6);$
$((Wm,\ H\_Heavy,\ Phip),\ Vector\_Scalar\_Scalar\ (-1),\ G\_GH\ 6);$
$((Wp,\ H\_Light,\ Phim),\ Vector\_Scalar\_Scalar\ (-1),\ G\_GH\ 7);$
$((Wm,\ H\_Light,\ Phip),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\ 7);$
$((Phim,\ Wp,\ Ga),\ Scalar\_Vector\_Vector\ 1,\ G\_GHGo\ 1);$
$((Phip,\ Wm,\ Ga),\ Scalar\_Vector\_Vector\ 1,\ G\_GHGo\ 1);$
$((Phim,\ Wp,\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_GHGo\ 2);$
$((Phip,\ Wm,\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_GHGo\ 2);$
$((Z,\ Phip,\ Phim),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\ 10);$
$((Ga,\ Phip,\ Phim),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\ 11)\ ]$

let *gauge_higgs4* =
  [$((A,\ A,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 1);$
  $((H\_Heavy,\ H\_Heavy,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 3);$
  $((H\_Light,\ H\_Light,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 2);$
  $((Hp,\ Hm,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 4);$
  $((Hp,\ Hm,\ Ga,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 5);$
  $((Hp,\ Hm,\ Ga,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 6);$
  $((Hp,\ H\_Heavy,\ Wm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 8);$
  $((Hm,\ H\_Heavy,\ Wp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 8);$
  $((Hp,\ H\_Light,\ Wm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 7);$
  $((Hm,\ H\_Light,\ Wp,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 7);$
  $((Hp,\ H\_Heavy,\ Wm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 10);$
  $((Hm,\ H\_Heavy,\ Wp,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 10);$
  $((Hp,\ H\_Light,\ Wm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 9);$
  $((Hm,\ H\_Light,\ Wp,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 9);$
  $((A,\ A,\ Wp,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 11);$
  $((H\_Heavy,\ H\_Heavy,\ Wp,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 13);$
  $((H\_Light,\ H\_Light,\ Wp,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 12);$
  $((Hp,\ Hm,\ Wp,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 14);$
  $((Hp,\ A,\ Wm,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 15);$
  $((Hm,\ A,\ Wp,\ Z),\ Scalar2\_Vector2\ (-1),\ G\_GH4\ 15);$
  $((Hp,\ A,\ Wm,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\ 16);$
  $((Hm,\ A,\ Wp,\ Ga),\ Scalar2\_Vector2\ (-1),\ G\_GH4\ 16)\ ]$

let *gauge_higgs_gold4* =
  [$((Z,\ Z,\ Phi0,\ Phi0),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 1);$
  $((Z,\ Z,\ Phip,\ Phim),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 2);$
  $((Ga,\ Ga,\ Phip,\ Phim),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 3);$
  $((Z,\ Ga,\ Phip,\ Phim),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 4);$
  $((Wp,\ Wm,\ Phip,\ Phim),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 5);$
  $((Wp,\ Wm,\ Phi0,\ Phi0),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 5);$
  $((Wp,\ Z,\ Phim,\ Phi0),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 6);$
  $((Wm,\ Z,\ Phip,\ Phi0),\ Scalar2\_Vector2\ (-1),\ G\_GHGo4\ 6);$
  $((Wp,\ Ga,\ Phim,\ Phi0),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 7);$
  $((Wm,\ Ga,\ Phip,\ Phi0),\ Scalar2\_Vector2\ (-1),\ G\_GHGo4\ 7);$
  $((Wp,\ Z,\ Phim,\ H\_Heavy),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 9);$
  $((Wm,\ Z,\ Phip,\ H\_Heavy),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 9);$
  $((Wp,\ Ga,\ Phim,\ H\_Heavy),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 11);$
  $((Wm,\ Ga,\ Phip,\ H\_Heavy),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 11);$
  $((Wp,\ Z,\ Phim,\ H\_Light),\ Scalar2\_Vector2\ 1,\ G\_GHGo4\ 8);$

$((Wm, Z, Phip, H\_Light), Scalar2\_Vector2\ 1, G\_GHGo4\ 8);$
$((Wp, Ga, Phim, H\_Light), Scalar2\_Vector2\ 1, G\_GHGo4\ 10);$
$((Wm, Ga, Phip, H\_Light), Scalar2\_Vector2\ 1, G\_GHGo4\ 10)\ ]$

let *gauge_sfermion4′* g $m1$ $m2$ =
  $[\ ((Wp, Wm, Slepton\ (m1, g), Slepton\ (m2, -g)), Scalar2\_Vector2\ 1,$
      $G\_WWSFSF\ (SL, g, m1, m2));$
  $((Z, Ga, Slepton\ (m1, g), Slepton\ (m2, -g)), Scalar2\_Vector2\ 1,$
    $G\_ZPSFSF\ (SL, g, m1, m2));$
  $((Z, Z, Slepton\ (m1, g), Slepton\ (m2, -g)), Scalar2\_Vector2\ 1, G\_ZZSFSF$
    $(SL, g, m1, m2));$
  $((Wp, Wm, Sup\ (m1, g), Sup\ (m2, -g)), Scalar2\_Vector2\ 1, G\_WWSFSF$
    $(SU, g, m1, m2));$
  $((Wp, Wm, Sdown\ (m1, g), Sdown\ (m2, -g)), Scalar2\_Vector2\ 1, G\_WWSFSF$
    $(SD, g, m1, m2));$
  $((Z, Z, Sup\ (m1, g), Sup\ (m2, -g)), Scalar2\_Vector2\ 1, G\_ZZSFSF$
    $(SU, g, m1, m2));$
  $((Z, Z, Sdown\ (m1, g), Sdown\ (m2, -g)), Scalar2\_Vector2\ 1, G\_ZZSFSF$
    $(SD, g, m1, m2));$
  $((Z, Ga, Sup\ (m1, g), Sup\ (m2, -g)), Scalar2\_Vector2\ 1, G\_ZPSFSF$
    $(SU, g, m1, m2));$
  $((Z, Ga, Sdown\ (m1, g), Sdown\ (m2, -g)), Scalar2\_Vector2\ 1, G\_ZPSFSF$
    $(SD, g, m1, m2))\ ]$

let *gauge_sfermion4″* g m =
  $[\ ((Wp, Ga, Slepton\ (m, g), Sneutrino\ (-g)), Scalar2\_Vector2\ 1, G\_WPSLSN$
      $(\mathsf{false}, g, m));$
  $((Wm, Ga, Slepton\ (m, -g), Sneutrino\ g), Scalar2\_Vector2\ 1,$
    $G\_WPSLSN\ (\mathsf{true}, g, m));$
  $((Wp, Z, Slepton\ (m, g), Sneutrino\ (-g)), Scalar2\_Vector2\ 1, G\_WZSLSN$
    $(\mathsf{false}, g, m));$
  $((Wm, Z, Slepton\ (m, -g), Sneutrino\ g), Scalar2\_Vector2\ 1,$
    $G\_WZSLSN\ (\mathsf{true}, g, m));$
  $((Ga, Ga, Slepton\ (m, g), Slepton\ (m, -g)), Scalar2\_Vector2\ 1, G\_PPSFSF\ SL);$
  $((Ga, Ga, Sup\ (m, g), Sup\ (m, -g)), Scalar2\_Vector2\ 1, G\_PPSFSF\ SU);$
  $((Ga, Ga, Sdown\ (m, g), Sdown\ (m, -g)), Scalar2\_Vector2\ 1, G\_PPSFSF\ SD)]$

let *gauge_sfermion4* g =
  $List.flatten\ (Product.list2\ (gauge\_sfermion4'\ g)\ [M1; M2]\ [M1; M2])\ @$
  $ThoList.flatmap\ (gauge\_sfermion4''\ g)\ [M1; M2]\ @$
  $[\ ((Wp, Wm, Sneutrino\ g, Sneutrino\ (-g)), Scalar2\_Vector2\ 1, G\_WWSFSF$
      $(SN, g, M1, M1));$
  $((Z, Z, Sneutrino\ g, Sneutrino\ (-g)), Scalar2\_Vector2\ 1, G\_ZZSFSF$
    $(SN, g, M1, M1))\ ]$

let *gauge_squark4″* g h $m1$ $m2$ =
  $[\ ((Wp, Ga, Sup\ (m1, -g), Sdown\ (m2, h)), Scalar2\_Vector2\ 1, G\_WPSUSD$
      $(\mathsf{false}, m1, m2, g, h));$
  $((Wm, Ga, Sup\ (m1, g), Sdown\ (m2, -h)), Scalar2\_Vector2\ 1, G\_WPSUSD$
    $(\mathsf{true}, m1, m2, g, h));$
  $((Wp, Z, Sup\ (m1, -g), Sdown\ (m2, h)), Scalar2\_Vector2\ 1, G\_WZSUSD$
    $(\mathsf{false}, m1, m2, g, h));$
  $((Wm, Z, Sup\ (m1, g), Sdown\ (m2, -h)), Scalar2\_Vector2\ 1, G\_WZSUSD$

$(\mathsf{true}, m1, m2, g, h))]$
let *gauge_squark4′* $g\ h\ =\ List.flatten\ (Product.list2\ (gauge\_squark4''\ g\ h)$
$[M1; M2]\ [M1; M2])$
let *gauge_squark4* $=$
  if *Flags.ckm_present* then
    List.flatten $(Product.list2\ gauge\_squark4'\ [1; 2; 3]\ [1; 2; 3])$
  else
    ThoList.flatmap (fun $g\ \rightarrow\ gauge\_squark4'\ g\ g$) $[1; 2; 3]$

let *gluon_w_squark″* $g\ h\ m1\ m2\ =$
$[\ ((Gl,\ Wp,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),$
    $Scalar2\_Vector2\ 1,\ G\_GlWSUSD\ (\mathsf{false}, m1, m2, g, h));$
  $((Gl,\ Wm,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),$
    $Scalar2\_Vector2\ 1,\ G\_GlWSUSD\ (\mathsf{true}, m1, m2, g, h))\ ]$
let *gluon_w_squark′* $g\ h\ =$
  List.flatten $(Product.list2\ (gluon\_w\_squark''\ g\ h)\ [M1; M2]\ [M1; M2])$
let *gluon_w_squark* $=$
  if *Flags.ckm_present* then
    List.flatten $(Product.list2\ gluon\_w\_squark'\ [1; 2; 3]\ [1; 2; 3])$
  else
    ThoList.flatmap (fun $g\ \rightarrow\ gluon\_w\_squark'\ g\ g$) $[1; 2; 3]$

let *gluon_gauge_squark′* $g\ m1\ m2\ =$
$[\ ((Gl,\ Z,\ Sup\ (m1, g),\ Sup\ (m2, -g)),\ Scalar2\_Vector2\ 2,\ G\_GlZSFSF\ (SU, g, m1, m2));$
  $((Gl,\ Z,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar2\_Vector2\ 2,\ G\_GlZSFSF\ (SD, g, m1, m2))\ ]$
let *gluon_gauge_squark″* $g\ m\ =$
$[\ ((Gl,\ Ga,\ Sup\ (m, g),\ Sup\ (m, -g)),\ Scalar2\_Vector2\ 2,\ G\_GlPSQSQ);$
  $((Gl,\ Ga,\ Sdown\ (m, g),\ Sdown\ (m, -g)),\ Scalar2\_Vector2\ (-1),\ G\_GlPSQSQ)\ ]$
let *gluon_gauge_squark* $g\ =$
  List.flatten $(Product.list2\ (gluon\_gauge\_squark'\ g)\ [M1; M2]\ [M1; M2])$ @
  ThoList.flatmap $(gluon\_gauge\_squark''\ g)\ [M1; M2]$

let *gluon2_squark2* $g\ m\ =$
$[\ ((Gl,\ Gl,\ Sup\ (m, g),\ Sup\ (m, -g)),\ Scalar2\_Vector2\ 1,\ G\_GlGlSQSQ);$
  $((Gl,\ Gl,\ Sdown\ (m, g),\ Sdown\ (m, -g)),\ Scalar2\_Vector2\ 1,\ G\_GlGlSQSQ)]$

** REVISED: Independent of the sign of CD. **

let *higgs* $=$
$[\ ((Hp,\ Hm,\ H\_Heavy),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 1);$
  $((Hp,\ Hm,\ H\_Light),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 2);$
  $((H\_Heavy,\ H\_Heavy,\ H\_Light),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 3);$
  $((H\_Heavy,\ H\_Heavy,\ H\_Heavy),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 4);$
  $((H\_Light,\ H\_Light,\ H\_Light),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 5);$
  $((H\_Heavy,\ H\_Light,\ H\_Light),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 6);$
  $((H\_Heavy,\ A,\ A),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 7);$
  $((H\_Light,\ A,\ A),\ Scalar\_Scalar\_Scalar\ 1,\ G\_H3\ 8)\ ]$

** REVISED: Compatible with GS+, independent of the sign of CD. **

let *higgs_gold* $=$
$[\ ((H\_Heavy,\ A,\ Phi0),\ Scalar\_Scalar\_Scalar\ 1,\ G\_HGo3\ 1);$
  $((H\_Light,\ A,\ Phi0),\ Scalar\_Scalar\_Scalar\ 1,\ G\_HGo3\ 2);$
  $((H\_Heavy,\ Hp,\ Phim),\ Scalar\_Scalar\_Scalar\ 1,\ G\_HGo3\ 3);$

$((H\_Heavy, \ Hm, \ Phip), \ Scalar\_Scalar\_Scalar \ 1, \ G\_HGo3 \ 3);$
$((H\_Light, \ Hp, \ Phim), \ Scalar\_Scalar\_Scalar \ 1, \ G\_HGo3 \ 4);$
$((H\_Light, \ Hm, \ Phip), \ Scalar\_Scalar\_Scalar \ 1, \ G\_HGo3 \ 4);$
$((A, \ Hp, \ Phim), \ Scalar\_Scalar\_Scalar \ (-1), \ G\_HGo3 \ 5);$
$((A, \ Hm, \ Phip), \ Scalar\_Scalar\_Scalar \ 1, \ G\_HGo3 \ 5);$
$((H\_Heavy, \ Phi0, \ Phi0), \ Scalar\_Scalar\_Scalar \ (-1), \ G\_H3 \ 7);$
$((H\_Heavy, \ Phip, \ Phim), \ Scalar\_Scalar\_Scalar \ (-1), \ G\_H3 \ 7);$
$((H\_Light, \ Phi0, \ Phi0), \ Scalar\_Scalar\_Scalar \ (-1), \ G\_H3 \ 8);$
$((H\_Light, \ Phip, \ Phim), \ Scalar\_Scalar\_Scalar \ (-1), \ G\_H3 \ 8) \ ]$

Here follow purely scalar quartic vertices which are only available for the no-Whizard colored version.
** REVISED: Independent of the sign of CD. **

let *higgs4* =
$[\ ((Hp, \ Hm, \ Hp, \ Hm), \ Scalar4 \ 1, \ G\_H4 \ 1);$
$((Hp, \ Hm, \ H\_Heavy, \ H\_Heavy), \ Scalar4 \ 1, \ G\_H4 \ 2);$
$((Hp, \ Hm, \ H\_Light, \ H\_Light), \ Scalar4 \ 1, \ G\_H4 \ 3);$
$((Hp, \ Hm, \ H\_Heavy, \ H\_Light), \ Scalar4 \ 1, \ G\_H4 \ 4);$
$((Hp, \ Hm, \ A, \ A), \ Scalar4 \ 1, \ G\_H4 \ 5);$
$((H\_Heavy, \ H\_Heavy, \ H\_Heavy, \ H\_Heavy), \ Scalar4 \ 1, \ G\_H4 \ 6);$
$((H\_Light, \ H\_Light, \ H\_Light, \ H\_Light), \ Scalar4 \ 1, \ G\_H4 \ 6);$
$((H\_Heavy, \ H\_Heavy, \ H\_Light, \ H\_Light), \ Scalar4 \ 1, \ G\_H4 \ 7);$
$((H\_Heavy, \ H\_Light, \ H\_Light, \ H\_Light), \ Scalar4 \ 1, \ G\_H4 \ 8);$
$((H\_Heavy, \ H\_Heavy, \ H\_Heavy, \ H\_Light), \ Scalar4 \ (-1), \ G\_H4 \ 8);$
$((H\_Heavy, \ H\_Heavy, \ A, \ A), \ Scalar4 \ 1, \ G\_H4 \ 9);$
$((H\_Light, \ H\_Light, \ A, \ A), \ Scalar4 \ (-1), \ G\_H4 \ 9);$
$((H\_Heavy, \ H\_Light, \ A, \ A), \ Scalar4 \ 1, \ G\_H4 \ 10);$
$((A, \ A, \ A, \ A), \ Scalar4 \ 1, \ G\_H4 \ 11) \ ]$

** REVISED: Compatible with GS+, independent of the sign of CD. **

let *higgs_gold4* =
$[\ ((H\_Heavy, \ H\_Heavy, \ A, \ Phi0), \ Scalar4 \ 1, \ G\_HGo4 \ 1);$
$((H\_Heavy, \ H\_Light, \ A, \ Phi0), \ Scalar4 \ 1, \ G\_HGo4 \ 2);$
$((H\_Light, \ H\_Light, \ A, \ Phi0), \ Scalar4 \ (-1), \ G\_HGo4 \ 1);$
$((A, \ A, \ A, \ Phi0), \ Scalar4 \ 3, \ G\_HGo4 \ 3);$
$((Hp, \ Hm, \ A, \ Phi0), \ Scalar4 \ 1, \ G\_HGo4 \ 3);$
$((H\_Heavy, \ H\_Heavy, \ Hp, \ Phim), \ Scalar4 \ 1, \ G\_HGo4 \ 4);$
$((H\_Heavy, \ H\_Heavy, \ Hm, \ Phip), \ Scalar4 \ 1, \ G\_HGo4 \ 4);$
$((H\_Heavy, \ H\_Light, \ Hp, \ Phim), \ Scalar4 \ 1, \ G\_HGo4 \ 5);$
$((H\_Heavy, \ H\_Light, \ Hm, \ Phip), \ Scalar4 \ 1, \ G\_HGo4 \ 5);$
$((H\_Light, \ H\_Light, \ Hp, \ Phim), \ Scalar4 \ (-1), \ G\_HGo4 \ 4);$
$((H\_Light, \ H\_Light, \ Hm, \ Phip), \ Scalar4 \ (-1), \ G\_HGo4 \ 4);$
$((A, \ A, \ Hp, \ Phim), \ Scalar4 \ 1, \ G\_HGo4 \ 6);$
$((A, \ A, \ Hm, \ Phip), \ Scalar4 \ 1, \ G\_HGo4 \ 6);$
$((H\_Heavy, \ A, \ Hp, \ Phim), \ Scalar4 \ 1, \ G\_HGo4 \ 7);$
$((H\_Heavy, \ A, \ Hm, \ Phip), \ Scalar4 \ (-1), \ G\_HGo4 \ 7);$
$((H\_Light, \ A, \ Hp, \ Phim), \ Scalar4 \ 1, \ G\_HGo4 \ 8);$
$((H\_Light, \ A, \ Hm, \ Phip), \ Scalar4 \ (-1), \ G\_HGo4 \ 8);$
$((Hp, \ Hm, \ Hp, \ Phim), \ Scalar4 \ 2, \ G\_HGo4 \ 6);$
$((Hp, \ Hm, \ Hm, \ Phip), \ Scalar4 \ 2, \ G\_HGo4 \ 6);$

$((H\_Heavy,\ H\_Heavy,\ Phi0,\ Phi0),\ Scalar4\ (-1),\ G\_H4\ 9);$
$((H\_Heavy,\ H\_Light,\ Phi0,\ Phi0),\ Scalar4\ (-1),\ G\_H4\ 10);$
$((H\_Light,\ H\_Light,\ Phi0,\ Phi0),\ Scalar4\ 1,\ G\_H4\ 9);$
$((A,\ A,\ Phi0,\ Phi0),\ Scalar4\ 1,\ G\_HGo4\ 9);$
$((Hp,\ Hm,\ Phi0,\ Phi0),\ Scalar4\ 1,\ G\_HGo4\ 10);$
$((H\_Heavy,\ Hp,\ Phim,\ Phi0),\ Scalar4\ 1,\ G\_HGo4\ 8);$
$((H\_Heavy,\ Hm,\ Phip,\ Phi0),\ Scalar4\ (-1),\ G\_HGo4\ 8);$
$((H\_Light,\ Hp,\ Phim,\ Phi0),\ Scalar4\ (-1),\ G\_HGo4\ 7);$
$((H\_Light,\ Hm,\ Phip,\ Phi0),\ Scalar4\ 1,\ G\_HGo4\ 7);$
$((A,\ Hp,\ Phim,\ Phi0),\ Scalar4\ 1,\ G\_HGo4\ 11);$
$((A,\ Hm,\ Phip,\ Phi0),\ Scalar4\ 1,\ G\_HGo4\ 11);$
$((H\_Heavy,\ H\_Heavy,\ Phip,\ Phim),\ Scalar4\ 1,\ G\_HGo4\ 12);$
$((H\_Heavy,\ H\_Light,\ Phip,\ Phim),\ Scalar4\ 1,\ G\_HGo4\ 13);$
$((H\_Light,\ H\_Light,\ Phip,\ Phim),\ Scalar4\ 1,\ G\_HGo4\ 14);$
$((A,\ A,\ Phip,\ Phim),\ Scalar4\ 1,\ G\_HGo4\ 15);$
$((Hp,\ Hm,\ Phip,\ Phim),\ Scalar4\ 1,\ G\_HGo4\ 16);$
$((Hp,\ Hp,\ Phim,\ Phim),\ Scalar4\ 1,\ G\_HGo4\ 17);$
$((Hm,\ Hm,\ Phip,\ Phip),\ Scalar4\ 1,\ G\_HGo4\ 17);$
$((Hp,\ Phim,\ Phi0,\ Phi0),\ Scalar4\ (-1),\ G\_HGo4\ 6);$
$((Hm,\ Phip,\ Phi0,\ Phi0),\ Scalar4\ (-1),\ G\_HGo4\ 6);$
$((A,\ Phi0,\ Phi0,\ Phi0),\ Scalar4\ (-3),\ G\_HGo4\ 6);$
$((A,\ Phi0,\ Phip,\ Phim),\ Scalar4\ (-1),\ G\_HGo4\ 6);$
$((Hp,\ Phim,\ Phip,\ Phim),\ Scalar4\ (-2),\ G\_HGo4\ 6);$
$((Hm,\ Phip,\ Phip,\ Phim),\ Scalar4\ (-2),\ G\_HGo4\ 6)\ ]$

** REVISED: Independent of the sign of CD and GS. **

let *goldstone4* =
$[\ ((Phi0,\ Phi0,\ Phi0,\ Phi0),\ Scalar4\ 1,\ G\_GG4\ 1);$
$((Phip,\ Phim,\ Phi0,\ Phi0),\ Scalar4\ 1,\ G\_GG4\ 2);$
$((Phip,\ Phim,\ Phip,\ Phim),\ Scalar4\ 1,\ G\_GG4\ 3)\ ]$

The vertices of the type Higgs - Sfermion - Sfermion are independent of the choice of the CD sign since they are quadratic in the gauge coupling.
** REVISED: Independent of the sign of CD. **

let *higgs_sneutrino′* g =
$[\ ((H\_Heavy,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_H2SFSF\ (SN,\ g,\ M1,\ M1));$
$((H\_Light,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_H1SFSF\ (SN,\ g,\ M1,\ M1));$
$((Hp,\ Sneutrino\ (-g),\ Slepton\ (M1,\ g)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_HSNSL\ (\mathsf{false},g,\ M1));$
$((Hm,\ Sneutrino\ g,\ Slepton\ (M1,-g)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_HSNSL\ (\mathsf{true},g,\ M1))\ ]$
let *higgs_sneutrino″* =
$[\ ((Hp,\ Sneutrino\ (-3),\ Slepton\ (M2,3)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_HSNSL\ (\mathsf{false},3,M2));$
$((Hm,\ Sneutrino\ 3,\ Slepton\ (M2,-3)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_HSNSL\ (\mathsf{false},3,M2))\ ]$
let *higgs_sneutrino* =
$ThoList.flatmap\ higgs\_sneutrino′\ [1;2;3]\ @\ higgs\_sneutrino″$

Under the assumption that there is no mixing between the left- and right-handed sfermions for the first two generations there is only a coupling of the form Higgs - sfermion1 - sfermion2 for the third generation. All the others are suppressed by m_f/M_W.
** REVISED: Independent of the sign of CD. **

> let *higgs_sfermion′ g m1 m2* =
>   [((*H_Heavy*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_H2SFSF* (*SL*, *g*, *m1*, *m2*));
>   ((*H_Light*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_H1SFSF* (*SL*, *g*, *m1*, *m2*));
>   ((*H_Heavy*, *Sup* (*m1*, *g*), *Sup* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_H2SFSF* (*SU*, *g*, *m1*, *m2*));
>   ((*H_Heavy*, *Sdown* (*m1*, *g*), *Sdown* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_H2SFSF* (*SD*, *g*, *m1*, *m2*));
>   ((*H_Light*, *Sup* (*m1*, *g*), *Sup* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_H1SFSF* (*SU*, *g*, *m1*, *m2*));
>   ((*H_Light*, *Sdown* (*m1*, *g*), *Sdown* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_H1SFSF* (*SD*, *g*, *m1*, *m2*))]
> let *higgs_sfermion″ m1 m2* =
>   [((*A*, *Slepton* (*m1*, 3), *Slepton* (*m2*, −3)), *Scalar_Scalar_Scalar* 1,
>   *G_ASFSF* (*SL*, 3, *m1*, *m2*));
>   ((*A*, *Sup* (*m1*, 3), *Sup* (*m2*, −3)), *Scalar_Scalar_Scalar* 1,
>   *G_ASFSF* (*SU*, 3, *m1*, *m2*));
>   ((*A*, *Sdown* (*m1*, 3), *Sdown* (*m2*, −3)), *Scalar_Scalar_Scalar* 1,
>   *G_ASFSF* (*SD*, 3, *m1*, *m2*))]
> let *higgs_sfermion* = *List.flatten* (*Product.list2* (*higgs_sfermion′* 3)
>   [*M1*; *M2*] [*M1*; *M2*]) @
> (*higgs_sfermion′* 1 *M1* *M1*) @ (*higgs_sfermion′* 1 *M2* *M2*) @
> (*higgs_sfermion′* 2 *M1* *M1*) @ (*higgs_sfermion′* 2 *M2* *M2*) @
> *List.flatten* (*Product.list2* *higgs_sfermion″* [*M1*; *M2*] [*M1*; *M2*])

** REVISED: Independent of the sign of CD, compatible with GS+. **

> let *goldstone_sfermion′ g m1 m2* =
>   [((*Phi0*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_GoSFSF* (*SL*, *g*, *m1*, *m2*));
>   ((*Phi0*, *Sup* (*m1*, *g*), *Sup* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_GoSFSF* (*SU*, *g*, *m1*, *m2*));
>   ((*Phi0*, *Sdown* (*m1*, *g*), *Sdown* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_GoSFSF* (*SD*, *g*, *m1*, *m2*))]
> let *goldstone_sfermion″ g* =
>   [((*Phip*, *Sneutrino* (−*g*), *Slepton* (*M1*, *g*)), *Scalar_Scalar_Scalar* 1,
>   *G_GoSNSL* (false,*g*, *M1*));
>   ((*Phim*, *Sneutrino* *g*, *Slepton* (*M1*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_GoSNSL* (true,*g*, *M1*))]
> let *goldstone_sfermion‴ g* =
>   [((*Phip*, *Sneutrino* (−*g*), *Slepton* (*M2*, *g*)), *Scalar_Scalar_Scalar* 1,
>   *G_GoSNSL* (false,*g*, *M2*));
>   ((*Phim*, *Sneutrino* *g*, *Slepton* (*M2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_GoSNSL* (true,*g*, *M2*))]
> let *goldstone_sfermion* =

$List.flatten\ (Product.list2\ (goldstone\_sfermion'\ 3)\ [M1; M2]\ [M1; M2])\ @$
$ThoList.flatmap\ goldstone\_sfermion''\ [1; 2; 3]\ @$
$goldstone\_sfermion'''\ 3$

** REVISED: Independent of the sign of CD. **

let *higgs_squark'* $g\ h\ m1\ m2\ =$
$[\ ((Hp,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_HSUSD\ (\mathsf{false}, m1, m2, g, h));$
$\quad ((Hm,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_HSUSD\ (\mathsf{true}, m1, m2, g, h))\ ]$
let *higgs_squark_a* $g\ h\ =\ higgs\_squark'\ g\ h\ M1\ M1$
let *higgs_squark_b* $(g, h)\ =\ List.flatten\ (Product.list2\ (higgs\_squark'\ g\ h)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [M1; M2]\ [M1; M2])$

let *higgs_squark* $=$
 if *Flags.ckm_present* then
  $List.flatten\ (Product.list2\ higgs\_squark\_a\ [1; 2]\ [1; 2])\ @$
  $ThoList.flatmap\ higgs\_squark\_b\ [(1, 3); (2, 3); (3, 3); (3, 1); (3, 2)]$
 else
  $higgs\_squark\_a\ 1\ 1\ @\ higgs\_squark\_a\ 2\ 2\ @\ higgs\_squark\_b\ (3, 3)$

** REVISED: Independent of the sign of CD, compatible with GS+. **

let *goldstone_squark'* $g\ h\ m1\ m2\ =$
$[\ ((Phip,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_GSUSD\ (\mathsf{false}, m1, m2, g, h));$
$\quad ((Phim,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar\_Scalar\_Scalar\ 1,$
$\qquad G\_GSUSD\ (\mathsf{true}, m1, m2, g, h))\ ]$
let *goldstone_squark_a* $g\ h\ =\ goldstone\_squark'\ g\ h\ M1\ M1$
let *goldstone_squark_b* $(g, h)\ =\ List.flatten\ (Product.list2$
$\qquad\qquad (goldstone\_squark'\ g\ h)\ [M1; M2]\ [M1; M2])$
let *goldstone_squark* $=$
  $List.flatten\ (Product.list2\ goldstone\_squark\_a\ [1; 2]\ [1; 2])\ @$
  $ThoList.flatmap\ goldstone\_squark\_b\ [(1, 3); (2, 3); (3, 3); (3, 1); (3, 2)]$

BAUSTELLE: For the quartic scalar coupligs we does not allow *whiz_col*.

let *higgs_sneutrino4'* $g\ m\ =$
$[\ ((Hp,\ H\_Heavy,\ Slepton\ (m, g),\ Sneutrino\ (-g)),\ Scalar4\ 1,$
$\qquad G\_HH2SLSN\ (\mathsf{false}, m, g));$
$\quad ((Hm,\ H\_Heavy,\ Slepton\ (m, -g),\ Sneutrino\ g),\ Scalar4\ 1,$
$\qquad G\_HH2SLSN\ (\mathsf{true}, m, g));$
$\quad ((Hp,\ H\_Light,\ Slepton\ (m, g),\ Sneutrino\ (-g)),\ Scalar4\ 1,$
$\qquad G\_HH1SLSN\ (\mathsf{false}, m, g));$
$\quad ((Hm,\ H\_Light,\ Slepton\ (m, -g),\ Sneutrino\ g),\ Scalar4\ 1,$
$\qquad G\_HH1SLSN\ (\mathsf{true}, m, g));$
$\quad ((Hp,\ A,\ Slepton\ (m, g),\ Sneutrino\ (-g)),\ Scalar4\ 1,$
$\qquad G\_HASLSN\ (\mathsf{false}, m, g));$
$\quad ((Hm,\ A,\ Slepton\ (m, -g),\ Sneutrino\ g),\ Scalar4\ 1,$
$\qquad G\_HASLSN\ (\mathsf{true}, m, g))\ ]$
let *higgs_sneutrino4* $g\ =$
 $ThoList.flatmap\ (higgs\_sneutrino4'\ g)\ [M1; M2]\ @$
 $[\ ((H\_Heavy,\ H\_Heavy,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar4\ 1,$

$$G\_H2H2SFSF\ (SN, M1, M1, g));$$
$$((H\_Heavy,\ H\_Light,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar4\ 1,$$
$$G\_H1H2SFSF\ (SN, M1, M1, g));$$
$$((H\_Light,\ H\_Light,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar4\ 1,$$
$$G\_H1H1SFSF\ (SN, M1, M1, g));$$
$$((Hp,\ Hm,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar4\ 1,\ G\_HHSFSF\ (SN, M1, M1, g))\ ]$$

let *higgs_sfermion4′ g m1 m2* =
  $[\ ((H\_Heavy,\ H\_Heavy,\ Slepton\ (m1, g),\ Slepton\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H2H2SFSF\ (SL, m1, m2, g));$
    $((H\_Heavy,\ H\_Light,\ Slepton\ (m1, g),\ Slepton\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H1H2SFSF\ (SL, m1, m2, g));$
    $((H\_Light,\ H\_Light,\ Slepton\ (m1, g),\ Slepton\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H1H1SFSF\ (SL, m1, m2, g));$
    $((A,\ A,\ Slepton\ (m1, g),\ Slepton\ (m2, -g)),\ Scalar4\ 1,$
        $G\_AASFSF\ (SL, m1, m2, g));$
    $((Hp,\ Hm,\ Slepton\ (m1, g),\ Slepton\ (m2, -g)),\ Scalar4\ 1,$
        $G\_HHSFSF\ (SL, m1, m2, g));$
    $((H\_Heavy,\ H\_Heavy,\ Sup\ (m1, g),\ Sup\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H2H2SFSF\ (SU, m1, m2, g));$
    $((H\_Heavy,\ H\_Heavy,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H2H2SFSF\ (SD, m1, m2, g));$
    $((H\_Light,\ H\_Light,\ Sup\ (m1, g),\ Sup\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H1H1SFSF\ (SU, m1, m2, g));$
    $((H\_Light,\ H\_Light,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H1H1SFSF\ (SD, m1, m2, g));$
    $((H\_Light,\ H\_Heavy,\ Sup\ (m1, g),\ Sup\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H1H2SFSF\ (SU, m1, m2, g));$
    $((H\_Light,\ H\_Heavy,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar4\ 1,$
        $G\_H1H2SFSF\ (SD, m1, m2, g));$
    $((Hp,\ Hm,\ Sup\ (m1, g),\ Sup\ (m2, -g)),\ Scalar4\ 1,\ G\_HHSFSF\ (SU, m1, m2, g));$
    $((Hp,\ Hm,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar4\ 1,\ G\_HHSFSF\ (SD, m1, m2, g));$
    $((A,\ A,\ Sup\ (m1, g),\ Sup\ (m2, -g)),\ Scalar4\ 1,\ G\_AASFSF\ (SU, m1, m2, g));$
    $((A,\ A,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar4\ 1,\ G\_AASFSF\ (SD, m1, m2, g))\ ]$
let *higgs_sfermion4 g* = *List.flatten* (*Product.list2* (*higgs_sfermion4′ g*)
$$[M1; M2]\ [M1; M2])$$

let *higgs_squark4′ g h m1 m2* =
  $[\ ((Hp,\ H\_Light,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
        $G\_HH1SUSD\ (\mathsf{false}, m1, m2, g, h));$
    $((Hm,\ H\_Light,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
        $G\_HH1SUSD\ (\mathsf{true}, m1, m2, g, h));$
    $((Hp,\ H\_Heavy,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
        $G\_HH2SUSD\ (\mathsf{false}, m1, m2, g, h));$
    $((Hm,\ H\_Heavy,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
        $G\_HH2SUSD\ (\mathsf{true}, m1, m2, g, h));$
    $((Hp,\ A,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
        $G\_HASUSD\ (\mathsf{false}, m1, m2, g, h));$
    $((Hm,\ A,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
        $G\_HASUSD\ (\mathsf{true}, m1, m2, g, h))\ ]$
let *higgs_squark4 g h* = *List.flatten* (*Product.list2* (*higgs_squark4′ g h*)

$$[M1; M2] \; [M1; M2])$$

let *higgs_gold_sneutrino′* g m =
  [$((Hp, \; Phi0, \; Sneutrino \; (-g), \; Slepton \; (m, g)), \; Scalar4 \; 1, \; G\_HGSNSL$ (false,m, g));
    $((Hm, \; Phi0, \; Sneutrino \; g, \; Slepton \; (m, -g)), \; Scalar4 \; 1, \; G\_HGSNSL$ (true,m, g));
    $((H\_Heavy, \; Phip, \; Sneutrino \; (-g), \; Slepton \; (m, g)), \; Scalar4 \; 1,$
        $G\_H2GSNSL$ (false,m, g));
    $((H\_Heavy, \; Phim, \; Sneutrino \; g, \; Slepton \; (m, -g)), \; Scalar4 \; 1,$
        $G\_H2GSNSL$ (true,m, g));
    $((H\_Light, \; Phip, \; Sneutrino \; (-g), \; Slepton \; (m, g)), \; Scalar4 \; 1,$
        $G\_H1GSNSL$ (false,m, g));
    $((H\_Light, \; Phim, \; Sneutrino \; g, \; Slepton \; (m, -g)), \; Scalar4 \; 1,$
        $G\_H1GSNSL$ (true,m, g));
    $((A, \; Phip, \; Sneutrino \; (-g), \; Slepton \; (m, g)), \; Scalar4 \; 1, \; G\_AGSNSL$ (false,m, g));
    $((A, \; Phim, \; Sneutrino \; g, \; Slepton \; (m, -g)), \; Scalar4 \; 1, \; G\_AGSNSL$ (true,m, g));
    $((Phi0, \; Phip, \; Sneutrino \; (-g), \; Slepton \; (m, g)), \; Scalar4 \; 1, \; G\_GGSNSL$ (false,m, g));
    $((Phi0, \; Phim, \; Sneutrino \; g, \; Slepton \; (m, -g)), \; Scalar4 \; 1, \; G\_GGSNSL$ (true,m, g))]
let *higgs_gold_sneutrino* g =
  ThoList.flatmap (*higgs_gold_sneutrino′* g) $[M1; M2]$ @
  [$((A, \; Phi0, \; Sneutrino \; g, \; Sneutrino \; (-g)), \; Scalar4 \; 1,$
        $G\_AG0SFSF \; (SN, M1, M1, g))$;
    $((Hp, \; Phim, \; Sneutrino \; g, \; Sneutrino \; (-g)), \; Scalar4 \; 1,$
        $G\_HGSFSF \; (SN, M1, M1, g))$;
    $((Hm, \; Phip, \; Sneutrino \; g, \; Sneutrino \; (-g)), \; Scalar4 \; 1,$
        $G\_HGSFSF \; (SN, M1, M1, g))$;
    $((Phip, \; Phim, \; Sneutrino \; g, \; Sneutrino \; (-g)), \; Scalar4 \; 1,$
        $G\_GGSFSF \; (SN, M1, M1, g))$;
    $((Phi0, \; Phi0, \; Sneutrino \; g, \; Sneutrino \; (-g)), \; Scalar4 \; 1,$
        $G\_G0G0SFSF \; (SN, M1, M1, g))$]

let *higgs_gold_sfermion′* g $m1$ $m2$ =
  [$((A, \; Phi0, \; Slepton \; (m1, g), \; Slepton \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_AG0SFSF \; (SL, m1, m2, g))$;
    $((Hp, \; Phim, \; Slepton \; (m1, g), \; Slepton \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_HGSFSF \; (SL, m1, m2, g))$;
    $((Hm, \; Phip, \; Slepton \; (m1, g), \; Slepton \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_HGSFSF \; (SL, m1, m2, g))$;
    $((Phip, \; Phim, \; Slepton \; (m1, g), \; Slepton \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_GGSFSF \; (SL, m1, m2, g))$;
    $((Phi0, \; Phi0, \; Slepton \; (m1, g), \; Slepton \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_G0G0SFSF \; (SL, m1, m2, g))$;
    $((A, \; Phi0, \; Sup \; (m1, g), \; Sup \; (m2, -g)), \; Scalar4 \; 1, \; G\_AG0SFSF \; (SU, m1, m2, g))$;
    $((A, \; Phi0, \; Sdown \; (m1, g), \; Sdown \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_AG0SFSF \; (SD, m1, m2, g))$;
    $((Hp, \; Phim, \; Sup \; (m1, g), \; Sup \; (m2, -g)), \; Scalar4 \; 1, \; G\_HGSFSF \; (SU, m1, m2, g))$;
    $((Hm, \; Phip, \; Sup \; (m1, g), \; Sup \; (m2, -g)), \; Scalar4 \; 1, \; G\_HGSFSF \; (SU, m1, m2, g))$;
    $((Hp, \; Phim, \; Sdown \; (m1, g), \; Sdown \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_HGSFSF \; (SD, m1, m2, g))$;
    $((Hm, \; Phip, \; Sdown \; (m1, g), \; Sdown \; (m2, -g)), \; Scalar4 \; 1,$
        $G\_HGSFSF \; (SD, m1, m2, g))$;
    $((Phip, \; Phim, \; Sup \; (m1, g), \; Sup \; (m2, -g)), \; Scalar4 \; 1,$

$G\_GGSFSF\ (SU, m1, m2, g));$
$((Phip,\ Phim,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar4\ 1,$
$\quad G\_GGSFSF\ (SD, m1, m2, g));$
$((Phi0,\ Phi0,\ Sup\ (m1, g),\ Sup\ (m2, -g)),\ Scalar4\ 1,$
$\quad G\_G0G0SFSF\ (SU, m1, m2, g));$
$((Phi0,\ Phi0,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),\ Scalar4\ 1,$
$\quad G\_G0G0SFSF\ (SD, m1, m2, g))\ ]$
let *higgs_gold_sfermion* $g\ =\ List.flatten\ (Product.list2$
$\quad (higgs\_gold\_sfermion'\ g)\ [M1; M2]\ [M1; M2])$

let *higgs_gold_squark'* $g\ h\ m1\ m2\ =$
$[\ ((Hp,\ Phi0,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
$\quad G\_HGSUSD\ (\mathsf{false}, m1, m2, g, h));$
$((Hm,\ Phi0,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
$\quad G\_HGSUSD\ (\mathsf{true}, m1, m2, g, h));$
$((H\_Heavy,\ Phip,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
$\quad G\_H2GSUSD\ (\mathsf{false}, m1, m2, g, h));$
$((H\_Heavy,\ Phim,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
$\quad G\_H2GSUSD\ (\mathsf{true}, m1, m2, g, h));$
$((H\_Light,\ Phip,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
$\quad G\_H1GSUSD\ (\mathsf{false}, m1, m2, g, h));$
$((H\_Light,\ Phim,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
$\quad G\_H1GSUSD\ (\mathsf{true}, m1, m2, g, h));$
$((A,\ Phip,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
$\quad G\_AGSUSD\ (\mathsf{false}, m1, m2, g, h));$
$((A,\ Phim,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
$\quad G\_AGSUSD\ (\mathsf{true}, m1, m2, g, h));$
$((Phi0,\ Phip,\ Sup\ (m1, -g),\ Sdown\ (m2, h)),\ Scalar4\ 1,$
$\quad G\_GGSUSD\ (\mathsf{false}, m1, m2, g, h));$
$((Phi0,\ Phim,\ Sup\ (m1, g),\ Sdown\ (m2, -h)),\ Scalar4\ 1,$
$\quad G\_GGSUSD\ (\mathsf{true}, m1, m2, g, h))\ ]$
let *higgs_gold_squark* $g\ h\ =\ List.flatten\ (Product.list2\ (higgs\_gold\_squark'$
$\quad g\ h)\ [M1; M2]\ [M1; M2])$

let *sneutrino4'* $(g, h)\ =$
$[\ ((Sneutrino\ g,\ Sneutrino\ h,\ Sneutrino\ (-g),\ Sneutrino\ (-h)),\ Scalar4\ 1,$
$\quad G\_SN4\ (g, h))]$
let *sneutrino4* $=\ ThoList.flatmap\ sneutrino4'$
$\quad [(1, 1); (1, 2); (1, 3); (2, 2); (2, 3); (3, 3)]$

let *sneu2_slep2_1'* $g\ h\ m1\ m2\ =$
$\quad ((Sneutrino\ (-g),\ Sneutrino\ g,\ Slepton\ (m1, -h),\ Slepton\ (m2, h)),\ Scalar4\ 1,$
$\quad\quad G\_SN2SL2\_1\ (m1, m2, g, h))$
let *sneu2_slep2_2'* $(g, h)\ m1\ m2\ =$
$\quad ((Sneutrino\ g,\ Sneutrino\ (-h),\ Slepton\ (m1, -g),\ Slepton\ (m2, h)),\ Scalar4\ 1,$
$\quad\quad G\_SN2SL2\_2\ (m1, m2, g, h))$
let *sneu2_slep2_1* $g\ h\ =\ Product.list2\ (sneu2\_slep2\_1'\ g\ h)\ [M1; M2]\ [M1; M2]$
let *sneu2_slep2_2* $(g, h)\ =\ Product.list2\ (sneu2\_slep2\_2'\ (g, h))\ [M1; M2]\ [M1; M2]$

The 4-slepton-vertices have the following structure: The sleptons come up in pairs of a positive and a negative slepton of the same generation; there is no vertex with e.g. two negative selectrons and two positive smuons, that of course

would be a contradiction to the conservation of the separate slepton numbers of each generation which is not implemented in the MSSM. Because there is no CKM-mixing for the sleptons (in case of massless neutrinos) we maximally have two different generations of sleptons in a 4-slepton-vertex.

let *slepton4_1gen'* g $(m1, m2, m3, m4)$ =
  [$((Slepton\,(m1, -g),\ Slepton\,(m2, g),\ Slepton\,(m3, -g),\ Slepton\,(m4, g)),$
      $Scalar4$ 1, $G\_SL4\,(m1, m2, m3, m4, g))$]
let *slepton4_1gen* g = *ThoList.flatmap* (*slepton4_1gen'* g) $[(M1, M1, M1, M1);$
    $(M1, M1, M1, M2);\ (M1, M1, M2, M1);\ (M1, M1, M2, M2);\ (M1, M2, M1, M2);\ (M1, M2, M2, M$
    $(M1, M2, M2, M2);\ (M2, M1, M2, M2);\ (M2, M2, M2, M2)$]
let *slepton4_2gen'* (g, h) $(m1, m2)$ $(m3, m4)$ =
  $((Slepton\,(m1, -g),\ Slepton\,(m2, g),\ Slepton\,(m3, -h),\ Slepton\,(m4, h)),$
      $Scalar4$ 1, $G\_SL4\_2\,(m1, m2, m3, m4, g, h))$
let *slepton4_2gen* (g, h) =
  Product.list2 (*slepton4_2gen'* (g, h)) $[(M1, M1); (M1, M2); (M2, M1); (M2, M2)]$
  $[(M1, M1); (M1, M2); (M2, M1); (M2, M2)]$

let *sneu2_squark2'* g h $m1$ $m2$ =
  [$((Sneutrino\,(-g),\ Sneutrino\ g,\ Sup\,(m1, -h),\ Sup\,(m2, h)),\ Scalar4$ 1,
      $G\_SN2SQ2\,(SU, m1, m2, g, h));$
    $((Sneutrino\,(-g),\ Sneutrino\ g,\ Sdown\,(m1, -h),\ Sdown\,(m2, h)),\ Scalar4$ 1,
      $G\_SN2SQ2\,(SD, m1, m2, g, h))$]
let *sneu2_squark2* g h = *List.flatten* (*Product.list2* (*sneu2_squark2'* g h)
                      $[M1; M2]\ [M1; M2])$

let *slepton2_squark2''* g h $m1$ $m2$ $m3$ $m4$ =
  [$((Slepton\,(m1, -g),\ Slepton\,(m2, g),\ Sup\,(m3, -h),\ Sup\,(m4, h)),\ Scalar4$ 1,
      $G\_SL2SQ2\,(SU, m1, m2, m3, m4, g, h));$
    $((Slepton\,(m1, -g),\ Slepton\,(m2, g),\ Sdown\,(m3, -h),\ Sdown\,(m4, h)),$
      $Scalar4$ 1, $G\_SL2SQ2\,(SD, m1, m2, m3, m4, g, h))$]
let *slepton2_squark2'* g h $m1$ $m2$ =
  List.flatten (*Product.list2* (*slepton2_squark2''* g h $m1$ $m2$) $[M1; M2]\ [M1; M2])$
let *slepton2_squark2* g h =
  List.flatten (*Product.list2* (*slepton2_squark2'* g h) $[M1; M2]\ [M1; M2])$

let *slep_sneu_squark2''* $g1$ $g2$ $g3$ $m1$ $m2$ $m3$ =
  [$((Sup\,(m1, -g1),\ Sdown\,(m2, g2),\ Slepton\,(m3, -g3),\ Sneutrino\ g3),$
      $Scalar4$ 1, $G\_SUSDSNSL\,(\mathsf{false}, m1, m2, m3, g1, g2, g3));$
    $((Sup\,(m1, g1),\ Sdown\,(m2, -g2),\ Slepton\,(m3, g3),\ Sneutrino\,(-g3)),$
      $Scalar4$ 1, $G\_SUSDSNSL\,(\mathsf{true}, m1, m2, m3, g1, g2, g3))$]
let *slep_sneu_squark2'* $g1$ $g2$ $g3$ $m1$ =
  List.flatten (*Product.list2* (*slep_sneu_squark2''* $g1$ $g2$ $g3$ $m1$)
               $[M1; M2]\ [M1; M2])$
let *slep_sneu_squark2* $g1$ $g2$ =
  List.flatten (*Product.list2* (*slep_sneu_squark2'* $g1$ $g2$) $[1; 2; 3]\ [M1; M2])$

There are three kinds of 4-squark-vertices: Four up-Squarks, four down-squarks or two up- and two down-squarks.

let *sup4_1gen'* g $(m1, m2, m3, m4)$ =
  [$((Sup\,(m1, -g),\ Sup\,(m2, g),\ Sup\,(m3, -g),\ Sup\,(m4, g)),\ Scalar4$ 1,
      $G\_SU4\,(m1, m2, m3, m4, g))$]

let *sup4_1gen g* = *ThoList.flatmap* (*sup4_1gen′ g*) [(*M1, M1, M1, M1*);
    (*M1, M1, M1, M2*); (*M1, M1, M2, M1*); (*M1, M1, M2, M2*); (*M1, M2, M1, M2*); (*M1, M2, M2, M*
    (*M1, M2, M2, M2*); (*M2, M1, M2, M2*); (*M2, M2, M2, M2*)]
let *sup4_2gen′* (*g, h*) (*m1, m2*) (*m3, m4*) =
  ((*Sup* (*m1, −g*), *Sup* (*m2, g*), *Sup* (*m3, −h*), *Sup* (*m4, h*)), *Scalar4* 1,
     G_SU4_2 (*m1, m2, m3, m4, g, h*))
let *sup4_2gen* (*g, h*) =
  Product.list2 (*sup4_2gen′* (*g, h*)) [(*M1, M1*); (*M1, M2*); (*M2, M1*); (*M2, M2*)]
    [(*M1, M1*); (*M1, M2*); (*M2, M1*); (*M2, M2*)]

let *sdown4_1gen′ g* (*m1, m2, m3, m4*) =
  [((*Sdown* (*m1, −g*), *Sdown* (*m2, g*), *Sdown* (*m3, −g*), *Sdown* (*m4, g*)), *Scalar4* 1,
     G_SD4 (*m1, m2, m3, m4, g*))]
let *sdown4_1gen g* = *ThoList.flatmap* (*sdown4_1gen′ g*) [(*M1, M1, M1, M1*);
    (*M1, M1, M1, M2*); (*M1, M1, M2, M1*); (*M1, M1, M2, M2*); (*M1, M2, M1, M2*); (*M1, M2, M2, M*
    (*M1, M2, M2, M2*); (*M2, M1, M2, M2*); (*M2, M2, M2, M2*)]
let *sdown4_2gen′* (*g, h*) (*m1, m2*) (*m3, m4*) =
  ((*Sdown* (*m1, −g*), *Sdown* (*m2, g*), *Sdown* (*m3, −h*), *Sdown* (*m4, h*)), *Scalar4* 1,
     G_SD4_2 (*m1, m2, m3, m4, g, h*))
let *sdown4_2gen* (*g, h*) =
  Product.list2 (*sdown4_2gen′* (*g, h*)) [(*M1, M1*); (*M1, M2*); (*M2, M1*); (*M2, M2*)]
    [(*M1, M1*); (*M1, M2*); (*M2, M1*); (*M2, M2*)]

let *sup2_sdown2_3 g1 g2 g3 g4 m1 m2 m3 m4* =
  ((*Sup* (*m1, −g1*), *Sup* (*m2, g2*), *Sdown* (*m3, −g3*), *Sdown*
    (*m4, g4*)), *Scalar4* 1, *G_SU2SD2* (*m1, m2, m3, m4, g1, g2, g3, g4*))
let *sup2_sdown2_2 g1 g2 g3 g4 m1 m2* =
  Product.list2 (*sup2_sdown2_3 g1 g2 g3 g4 m1 m2*) [*M1; M2*] [*M1; M2*]
let *sup2_sdown2_1 g1 g2 g3 g4* =
  List.flatten (*Product.list2* (*sup2_sdown2_2 g1 g2 g3 g4*) [*M1; M2*] [*M1; M2*])
let *sup2_sdown2 g1 g2* =
  List.flatten (*Product.list2* (*sup2_sdown2_1 g1 g2*) [1; 2; 3] [1; 2; 3])

let *quartic_grav_gauge g m* =
  [(((*Grino, Slepton* (*m, −g*), *Ga, L g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4A_Sl* (*g, m*));
    ((*L* (*−g*), *Slepton* (*m, g*), *Ga, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4A_Slc* (*g, m*));
    ((*Grino, Sup* (*m, −g*), *Ga, U g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4A_Su* (*g, m*));
    ((*U* (*−g*), *Sup* (*m, g*), *Ga, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4A_Suc* (*g, m*));
    ((*Grino, Sdown* (*m, −g*), *Ga, D g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4A_Sd* (*g, m*));
    ((*D* (*−g*), *Sdown* (*m, g*), *Ga, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4A_Sdc* (*g, m*));
    ((*Grino, Slepton* (*m, −g*), *Z, L g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4Z_Sl* (*g, m*));
    ((*L* (*−g*), *Slepton* (*m, g*), *Z, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4Z_Slc* (*g, m*));
    ((*Grino, Sup* (*m, −g*), *Z, U g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4Z_Su* (*g, m*));
    ((*U* (*−g*), *Sup* (*m, g*), *Z, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4Z_Suc* (*g, m*));
    ((*Grino, Sdown* (*m, −g*), *Z, D g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4Z_Sd* (*g, m*));
    ((*D* (*−g*), *Sdown* (*m, g*), *Z, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4Z_Sdc* (*g, m*));
    ((*Grino, Sup* (*m, −g*), *Gl, U g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4Gl_Su* (*g, m*));
    ((*U* (*−g*), *Sup* (*m, g*), *Gl, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4Gl_Suc* (*g, m*));
    ((*Grino, Sdown* (*m, −g*), *Gl, D g*), *GBBG* (1, *Gravbar, SLRV, Psi*), *G_Gr4Gl_Sd* (*g, m*));
    ((*D* (*−g*), *Sdown* (*m, g*), *Gl, Grino*), *GBBG* (1, *Psibar, SLRV, Grav*), *G_Gr4Gl_Sdc* (*g, m*));
    ((*Grino, Slepton* (*m, −g*), *Wm, N g*), *GBBG* (1, *Gravbar, SLV, Psi*), *G_Gr4W_Sl* (*g, m*));
    ((*N* (*−g*), *Slepton* (*m, g*), *Wp, Grino*), *GBBG* (1, *Psibar, SLV, Grav*), *G_Gr4Z_Slc* (*g, m*));

```
      ((Grino, Sup (m, −g), Wp, D g), GBBG (1, Gravbar, SLV, Psi), G_Gr4W_Su (g, m));
      ((D (−g), Sup (m, g), Wm, Grino), GBBG (1, Psibar, SLV, Grav), G_Gr4W_Suc (g, m));
      ((Grino, Sdown (m, −g), Wm, U g), GBBG (1, Gravbar, SLV, Psi), G_Gr4W_Sd (g, m));
      ((U (−g), Sdown (m, g), Wp, Grino), GBBG (1, Psibar, SLV, Grav), G_Gr4W_Sdc (g, m)) ]
```

let *quartic_grav_sneutrino g* =

```
  [ ((Grino, Sneutrino (−g), Z, N g), GBBG (1, Gravbar, SLV, Psi), G_Gr4Z_Sn);
    ((N (−g), Sneutrino g, Z, Grino), GBBG (1, Psibar, SLV, Grav), G_Gr4Z_Snc);
    ((Grino, Sneutrino (−g), Wp, L g), GBBG (1, Gravbar, SLV, Psi), G_Gr4W_Sn);
    ((L (−g), Sneutrino g, Wm, Grino), GBBG (1, Psibar, SLV, Grav), G_Gr4W_Snc) ]
```

let *quartic_grav_neu n* =

```
  [ ((Grino, Wp, Wm, Neutralino n), GBBG (1, Gravbar, V2LR, Chi), G_Gr4_Neu n);
    ((Grino, H_Light, Z, Neutralino n), GBBG (1, Gravbar, SLRV, Chi), G_Gr4_Z_H1 n);
    ((Grino, H_Heavy, Z, Neutralino n), GBBG (1, Gravbar, SLRV, Chi), G_Gr4_Z_H2 n);
    ((Grino, A, Z, Neutralino n), GBBG (1, Gravbar, SLRV, Chi), G_Gr4_Z_H3 n);
    ((Grino, Hm, Wp, Neutralino n), GBBG (1, Gravbar, SLRV, Chi), G_Gr4_W_H n);
    ((Grino, Hp, Wm, Neutralino n), GBBG (1, Gravbar, SLRV, Chi), G_Gr4_W_Hc n) ]
```

let *quartic_grav_char c* =

  let *cc* = *conj_char c* in

```
  [ ((Grino, Wm, Ga, Chargino c), GBBG (1, Gravbar, V2LR, Psi), G_Gr4_A_Ch c);
    ((Grino, Wm, Z, Chargino c), GBBG (1, Gravbar, V2LR, Psi), G_Gr4_Z_Ch c);
    ((Chargino cc, Wp, Ga, Grino), GBBG ((−1), Psibar, V2LR, Grav), G_Gr4_A_Ch cc);
    ((Chargino cc, Wp, Z, Grino), GBBG ((−1), Psibar, V2LR, Grav), G_Gr4_Z_Ch cc);
    ((Grino, Hm, Ga, Chargino c), GBBG (1, Gravbar, SLRV, Psi), G_Gr4_H_A c);
    ((Chargino cc, Hp, Ga, Grino), GBBG (1, Psibar, SLRV, Grav), G_Gr4_H_A cc);
    ((Grino, Hm, Z, Chargino c), GBBG (1, Gravbar, SLRV, Psi), G_Gr4_H_Z c);
    ((Chargino cc, Hp, Z, Grino), GBBG (1, Psibar, SLRV, Grav), G_Gr4_H_Z cc)]
```

let *quartic_gravitino* =

```
  [ ((Grino, Gl, Gl, Gluino), GBBG (1, Gravbar, V2, Chi), G_GravGl)] @
```
  ThoList.flatmap quartic_grav_neu [*N1*; *N2*; *N3*; *N4*] @
  ThoList.flatmap quartic_grav_char [*C1*; *C2*] @
  List.flatten (*Product.list2 quartic_grav_gauge* [1; 2; 3] [*M1*; *M2*]) @
  ThoList.flatmap quartic_grav_sneutrino [1; 2; 3]

let *vertices3″* =

  if *Flags.ckm_present* then

    (*ThoList.flatmap electromagnetic_currents_3* [1; 2; 3] @
    ThoList.flatmap electromagnetic_currents_2 [*C1*; *C2*] @
    List.flatten (*Product.list2*
          electromagnetic_sfermion_currents [1; 2; 3] [*M1*; *M2*]) @
    ThoList.flatmap neutral_currents [1; 2; 3] @
    ThoList.flatmap neutral_sfermion_currents [1; 2; 3] @
    ThoList.flatmap charged_currents [1; 2; 3] @
    List.flatten (*Product.list2 charged_slepton_currents* [1; 2; 3]
               [*M1*; *M2*]) @
    List.flatten (*Product.list2 charged_quark_currents* [1; 2; 3]
               [1; 2; 3]) @
    List.flatten (*Product.list2 charged_squark_currents* [1; 2; 3]
               [1; 2; 3]) @
    ThoList.flatmap yukawa_higgs_quark [(1, 3); (2, 3); (3, 3); (3, 1); (3, 2)] @

          yukawa_higgs 3 @ *yukawa_n* @
          ThoList.flatmap yukawa_c [*C1*; *C2*] @
          ThoList.flatmap yukawa_cq [*C1*; *C2*] @
          List.flatten (*Product.list2 charged_chargino_currents* [*N1*; *N2*; *N3*; *N4*]
                    [*C1*; *C2*]) @ *triple_gauge* @
          ThoList.flatmap neutral_Z_1 [(*N1, N2*); (*N1, N3*); (*N1, N4*); (*N2, N3*); (*N2, N4*);
                       (*N3, N4*)] @
          ThoList.flatmap neutral_Z_2 [*N1*; *N2*; *N3*; *N4*] @
          Product.list2 charged_Z [*C1*; *C2*] [*C1*; *C2*] @
          gauge_higgs @ *higgs* @ *yukawa_higgs_2* @
          List.flatten (*Product.list2 higgs_charg_neutr* [*N1*; *N2*; *N3*; *N4*] [*C1*; *C2*]) @
          higgs_neutr @ *higgs_sneutrino* @ *higgs_sfermion* @
          higgs_squark @ *yukawa_v* @
          ThoList.flatmap col_currents [1; 2; 3] @
          List.flatten (*Product.list2 col_sfermion_currents* [1; 2; 3] [*M1*; *M2*]))
      else
       (*ThoList.flatmap electromagnetic_currents_3* [1; 2; 3] @
       ThoList.flatmap electromagnetic_currents_2 [*C1*; *C2*] @
       List.flatten (*Product.list2*
             electromagnetic_sfermion_currents [1; 2; 3] [*M1*; *M2*]) @
       ThoList.flatmap neutral_currents [1; 2; 3] @
       ThoList.flatmap neutral_sfermion_currents [1; 2; 3] @
       ThoList.flatmap charged_currents [1; 2; 3] @
       List.flatten (*Product.list2 charged_slepton_currents* [1; 2; 3]
                   [*M1*; *M2*]) @
       charged_quark_currents 1 1 @
       charged_quark_currents 2 2 @
       charged_quark_currents 3 3 @
       charged_squark_currents 1 1 @
       charged_squark_currents 2 2 @
       charged_squark_currents 3 3 @
       ThoList.flatmap yukawa_higgs_quark [(3, 3)] @
       yukawa_higgs 3 @ *yukawa_n* @
       ThoList.flatmap yukawa_c [*C1*; *C2*] @
       ThoList.flatmap yukawa_cq [*C1*; *C2*] @
       List.flatten (*Product.list2 charged_chargino_currents* [*N1*; *N2*; *N3*; *N4*]
                  [*C1*; *C2*]) @ *triple_gauge* @
       ThoList.flatmap neutral_Z_1 [(*N1, N2*); (*N1, N3*); (*N1, N4*); (*N2, N3*); (*N2, N4*);
                     (*N3, N4*)] @
       ThoList.flatmap neutral_Z_2 [*N1*; *N2*; *N3*; *N4*] @
       Product.list2 charged_Z [*C1*; *C2*] [*C1*; *C2*] @
       gauge_higgs @ *higgs* @ *yukawa_higgs_2* @
       List.flatten (*Product.list2 higgs_charg_neutr* [*N1*; *N2*; *N3*; *N4*] [*C1*; *C2*]) @
       higgs_neutr @ *higgs_sneutrino* @ *higgs_sfermion* @
       higgs_squark @ *yukawa_v* @
       ThoList.flatmap col_currents [1; 2; 3] @
       List.flatten (*Product.list2 col_sfermion_currents* [1; 2; 3] [*M1*; *M2*]))

  let *vertices3′* =
    if *Flags.gravitino* **then** (*vertices3″* @ *triple_gravitino*)

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```
          else vertices3″
  let vertices3 =
    if Flags.include_goldstone then
      (vertices3′ @ yukawa_goldstone 3 @
       gauge_higgs_gold @ higgs_gold @ yukawa_goldstone_2 @
       (if Flags.ckm_present then
          List.flatten (Product.list2 yukawa_goldstone_quark [1; 2; 3]
                          [1; 2; 3]) @
          List.flatten (Product.list2 goldstone_charg_neutr [N1; N2; N3; N4]
                          [C1; C2])
        else
          yukawa_goldstone_quark 1 1 @
          yukawa_goldstone_quark 2 2 @
          yukawa_goldstone_quark 3 3) @
        goldstone_neutr @ goldstone_sfermion @ goldstone_squark)
    else vertices3′

 let vertices4‴ =
   (quartic_gauge @ higgs4 @ gauge_higgs4 @
    ThoList.flatmap gauge_sfermion4 [1; 2; 3] @
    gauge_squark4 @ gluon_w_squark @
    List.flatten (Product.list2 gluon2_squark2 [1; 2; 3] [M1; M2]) @
    ThoList.flatmap gluon_gauge_squark [1; 2; 3])
 let vertices4″ =
   if Flags.gravitino then (vertices4‴ @ quartic_gravitino)
     else vertices4‴
let vertices4′ =
 if Flags.include_four then
     (vertices4″ @
      ThoList.flatmap higgs_sfermion4 [1; 2; 3] @
      ThoList.flatmap higgs_sneutrino4 [1; 2; 3] @
      List.flatten (Product.list2 higgs_squark4 [1; 2; 3] [1; 2; 3]) @
      sneutrino4 @
      List.flatten (Product.list2 sneu2_slep2_1 [1; 2; 3] [1; 2; 3]) @
      ThoList.flatmap sneu2_slep2_2 [(1, 2); (1, 3); (2, 3); (2, 1); (3, 1); (3, 2)] @
      ThoList.flatmap slepton4_1gen [1; 2; 3] @
      ThoList.flatmap slepton4_2gen [(1, 2); (1, 3); (2, 3)] @
      List.flatten (Product.list2 sneu2_squark2 [1; 2; 3] [1; 2; 3]) @
      List.flatten (Product.list2 slepton2_squark2 [1; 2; 3] [1; 2; 3]) @
      List.flatten (Product.list2 slep_sneu_squark2 [1; 2; 3] [1; 2; 3]) @
      ThoList.flatmap sup4_1gen [1; 2; 3] @
      ThoList.flatmap sup4_2gen [(1, 2); (1, 3); (2, 3)] @
      ThoList.flatmap sdown4_1gen [1; 2; 3] @
      ThoList.flatmap sdown4_2gen [(1, 2); (1, 3); (2, 3)] @
      List.flatten (Product.list2 sup2_sdown2 [1; 2; 3] [1; 2; 3]))
     else
   vertices4″
let vertices4 =
  if Flags.include_goldstone then
    (vertices4′ @ higgs_gold4 @ gauge_higgs_gold4 @ goldstone4 @
```

      ThoList.flatmap higgs_gold_sneutrino [1; 2; 3] @
      ThoList.flatmap higgs_gold_sfermion [1; 2; 3] @
      List.flatten (*Product.list2 higgs_gold_squark* [1; 2; 3] [1; 2; 3]))
  else
    vertices4′

let *vertices* () = (*vertices3*, *vertices4*, [])

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *flavor_of_string s* =
  match *s* with
  | "e-" → *L* 1 | "e+" → *L* (−1)
  | "mu-" → *L* 2 | "mu+" → *L* (−2)
  | "tau-" → *L* 3 | "tau+" → *L* (−3)
  | "nue" → *N* 1 | "nuebar" → *N* (−1)
  | "numu" → *N* 2 | "numubar" → *N* (−2)
  | "nutau" → *N* 3 | "nutaubar" → *N* (−3)
  | "se1-" → *Slepton* (*M1*, 1) | "se1+" → *Slepton* (*M1*, −1)
  | "smu1-" → *Slepton* (*M1*, 2) | "smu1+" → *Slepton* (*M1*, −2)
  | "stau1-" → *Slepton* (*M1*, 3) | "stau1+" → *Slepton* (*M1*, −3)
  | "se2-" → *Slepton* (*M2*, 1) | "se2+" → *Slepton* (*M2*, −1)
  | "smu2-" → *Slepton* (*M2*, 2) | "smu2+" → *Slepton* (*M2*, −2)
  | "stau2-" → *Slepton* (*M2*, 3) | "stau2+" → *Slepton* (*M2*, −3)
  | "snue" → *Sneutrino* 1 | "snue*" → *Sneutrino* (−1)
  | "snumu" → *Sneutrino* 2 | "snumu*" → *Sneutrino* (−2)
  | "snutau" → *Sneutrino* 3 | "snutau*" → *Sneutrino* (−3)
  | "u" → *U* 1 | "ubar" → *U* (−1)
  | "c" → *U* 2 | "cbar" → *U* (−2)
  | "t" → *U* 3 | "tbar" → *U* (−3)
  | "d" → *D* 1 | "dbar" → *D* (−1)
  | "s" → *D* 2 | "sbar" → *D* (−2)
  | "b" → *D* 3 | "bbar" → *D* (−3)
  | "A" → *Ga* | "Z" | "Z0" → *Z*
  | "W+" → *Wp* | "W-" → *Wm*
  | "gl" | "g" → *Gl*
  | "H" → *H_Heavy* | "h" → *H_Light* | "A0" → *A*
  | "H+" → *Hp* | "H-" → *Hm*
  | "phi0" → *Phi0* | "phi+" → *Phip* | "phim" → *Phim*
  | "su1" → *Sup* (*M1*, 1) | "su1c" → *Sup* (*M1*, −1)
  | "sc1" → *Sup* (*M1*, 2) | "sc1c" → *Sup* (*M1*, −2)
  | "st1" → *Sup* (*M1*, 3) | "st1c" → *Sup* (*M1*, −3)
  | "su2" → *Sup* (*M2*, 1) | "su2c" → *Sup* (*M2*, −1)
  | "sc2" → *Sup* (*M2*, 2) | "sc2c" → *Sup* (*M2*, −2)
  | "st2" → *Sup* (*M2*, 3) | "st2c" → *Sup* (*M2*, −3)
  | "sgl" | "sg" → *Gluino*
  | "sd1" → *Sdown* (*M1*, 1) | "sd1c" → *Sdown* (*M1*, −1)
  | "ss1" → *Sdown* (*M1*, 2) | "ss1c" → *Sdown* (*M1*, −2)

```
      | "sb1" → Sdown (M1, 3) | "sb1c" → Sdown (M1, −3)
      | "sd2" → Sdown (M2, 1) | "sd2c" → Sdown (M2, −1)
      | "ss2" → Sdown (M2, 2) | "ss2c" → Sdown (M2, −2)
      | "sb2" → Sdown (M2, 3) | "sb2c" → Sdown (M2, −3)
      | "neu1" → Neutralino N1 | "neu2" → Neutralino N2
      | "neu3" → Neutralino N3 | "neu4" → Neutralino N4
      | "ch1+" → Chargino C1 | "ch2+" → Chargino C2
      | "ch1-" → Chargino C1c | "ch2-" → Chargino C2c
      | "GR" → Grino
      | _ → invalid_arg "Modellib_MSSM.MSSM.flavor_of_string"

let flavor_to_string = function
      | L 1 → "e-" | L (−1) → "e+"
      | L 2 → "mu-" | L (−2) → "mu+"
      | L 3 → "tau-" | L (−3) → "tau+"
      | N 1 → "nue" | N (−1) → "nuebar"
      | N 2 → "numu" | N (−2) → "numubar"
      | N 3 → "nutau" | N (−3) → "nutaubar"
      | U 1 → "u" | U (−1) → "ubar"
      | U 2 → "c" | U (−2) → "cbar"
      | U 3 → "t" | U (−3) → "tbar"
      | D 1 → "d" | D (−1) → "dbar"
      | D 2 → "s" | D (−2) → "sbar"
      | D 3 → "b" | D (−3) → "bbar"
      | L _ → invalid_arg
            "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣lepton"
      | N _ → invalid_arg
            "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣neutrino"
      | U _ → invalid_arg
            "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣up␣type␣quark"
      | D _ → invalid_arg
            "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣down␣type␣quark"
      | Gl → "gl" | Gluino → "sgl"
      | Ga → "A" | Z → "Z" | Wp → "W+" | Wm → "W-"
      | Phip → "phi+" | Phim → "phi-" | Phi0 → "phi0"
      | H_Heavy → "H" | H_Light → "h" | A → "A0"
      | Hp → "H+" | Hm → "H-"
      | Slepton (M1, 1) → "se1-" | Slepton (M1, −1) → "se1+"
      | Slepton (M1, 2) → "smu1-" | Slepton (M1, −2) → "smu1+"
      | Slepton (M1, 3) → "stau1-" | Slepton (M1, −3) → "stau1+"
      | Slepton (M2, 1) → "se2-" | Slepton (M2, −1) → "se2+"
      | Slepton (M2, 2) → "smu2-" | Slepton (M2, −2) → "smu2+"
      | Slepton (M2, 3) → "stau2-" | Slepton (M2, −3) → "stau2+"
      | Sneutrino 1 → "snue" | Sneutrino (−1) → "snue*"
      | Sneutrino 2 → "snumu" | Sneutrino (−2) → "snumu*"
      | Sneutrino 3 → "snutau" | Sneutrino (−3) → "snutau*"
      | Sup (M1, 1) → "su1" | Sup (M1, −1) → "su1c"
      | Sup (M1, 2) → "sc1" | Sup (M1, −2) → "sc1c"
      | Sup (M1, 3) → "st1" | Sup (M1, −3) → "st1c"
      | Sup (M2, 1) → "su2" | Sup (M2, −1) → "su2c"
```

```
    | Sup (M2, 2) → "sc2" | Sup (M2, −2) → "sc2c"
    | Sup (M2, 3) → "st2" | Sup (M2, −3) → "st2c"
    | Sdown (M1, 1) → "sd1" | Sdown (M1, −1) → "sd1c"
    | Sdown (M1, 2) → "ss1" | Sdown (M1, −2) → "ss1c"
    | Sdown (M1, 3) → "sb1" | Sdown (M1, −3) → "sb1c"
    | Sdown (M2, 1) → "sd2" | Sdown (M2, −1) → "sd2c"
    | Sdown (M2, 2) → "ss2" | Sdown (M2, −2) → "ss2c"
    | Sdown (M2, 3) → "sb2" | Sdown (M2, −3) → "sb2c"
    | Neutralino N1 → "neu1"
    | Neutralino N2 → "neu2"
    | Neutralino N3 → "neu3"
    | Neutralino N4 → "neu4"
    | Slepton _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣slepton"
    | Sneutrino _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣sneutrino"
    | Sup _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣up␣type␣squark"
    | Sdown _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_string:␣invalid␣down␣type␣squark"
    | Chargino C1 → "ch1+" | Chargino C1c → "ch1-"
    | Chargino C2 → "ch2+" | Chargino C2c → "ch2-"
    | Grino → "GR"

let flavor_symbol = function
    | L g when g > 0 → "l" ^ string_of_int g
    | L g → "l" ^ string_of_int (abs g) ^ "b"
    | N g when g > 0 → "n" ^ string_of_int g
    | N g → "n" ^ string_of_int (abs g) ^ "b"
    | U g when g > 0 → "u" ^ string_of_int g
    | U g → "u" ^ string_of_int (abs g) ^ "b"
    | D g when g > 0 → "d" ^ string_of_int g
    | D g → "d" ^ string_of_int (abs g) ^ "b"
    | Gl → "gl" | Ga → "a" | Z → "z"
    | Wp → "wp" | Wm → "wm"
    | Slepton (M1, g) when g > 0 → "sl1" ^ string_of_int g
    | Slepton (M1, g) → "sl1c" ^ string_of_int (abs g)
    | Slepton (M2, g) when g > 0 → "sl2" ^ string_of_int g
    | Slepton (M2, g) → "sl2c" ^ string_of_int (abs g)
    | Sneutrino g when g > 0 → "sn" ^ string_of_int g
    | Sneutrino g → "snc" ^ string_of_int (abs g)
    | Sup (M1, g) when g > 0 → "su1" ^ string_of_int g
    | Sup (M1, g) → "su1c" ^ string_of_int (abs g)
    | Sup (M2, g) when g > 0 → "su2" ^ string_of_int g
    | Sup (M2, g) → "su2c" ^ string_of_int (abs g)
    | Sdown (M1, g) when g > 0 → "sd1" ^ string_of_int g
    | Sdown (M1, g) → "sd1c" ^ string_of_int (abs g)
    | Sdown (M2, g) when g > 0 → "sd2" ^ string_of_int g
    | Sdown (M2, g) → "sd2c" ^ string_of_int (abs g)
    | Neutralino n → "neu" ^ (string_of_neu n)
```

```
    | Chargino c when (int_of_char c) > 0 → "cp" ^ string_of_char c
    | Chargino c → "cm" ^ string_of_int (abs (int_of_char c))
    | Gluino → "sgl" | Phip → "pp" | Phim → "pm" | Phi0 → "p0"
    | H_Heavy → "h0h" | H_Light → "h0l" | A → "a0"
    | Hp → "hp" | Hm → "hm" | Grino → "gv"

let flavor_to_TeX = function
    | L 1 → "e^-" | L (−1) → "e^+"
    | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
    | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
    | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
    | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
    | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
    | U 1 → "u" | U (−1) → "\\bar{u}"
    | U 2 → "c" | U (−2) → "\\bar{c}"
    | U 3 → "t" | U (−3) → "\\bar{t}"
    | D 1 → "d" | D (−1) → "\\bar{d}"
    | D 2 → "s" | D (−2) → "\\bar{s}"
    | D 3 → "b" | D (−3) → "\\bar{b}"
    | L _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣lepton"
    | N _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣neutrino"
    | U _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣up␣type␣quark"
    | D _ → invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣down␣type␣quark"
    | Gl → "g" | Gluino → "\\widetilde{g}"
    | Ga → "\\gamma" | Z → "Z" | Wp → "W^+" | Wm → "W^-"
    | Phip → "\\phi^+" | Phim → "\\phi^-" | Phi0 → "\\phi^0"
    | H_Heavy → "H^0" | H_Light → "h^0" | A → "A^0"
    | Hp → "H^+" | Hm → "H^-"
    | Slepton (M1,1) → "\\widetilde{e}_1^-"
    | Slepton (M1,−1) → "\\widetilde{e}_1^+"
    | Slepton (M1,2) → "\\widetilde{\\mu}_1^-"
    | Slepton (M1,−2) → "\\widetilde{\\mu}_1^+"
    | Slepton (M1,3) → "\\widetilde{\\tau}_1^-"
    | Slepton (M1,−3) → "\\widetilde{\\tau}_1^+"
    | Slepton (M2,1) → "\\widetilde{e}_2^-"
    | Slepton (M2,−1) → "\\widetilde{e}_2^+"
    | Slepton (M2,2) → "\\widetilde{\\mu}_2^-"
    | Slepton (M2,−2) → "\\widetilde{\\mu}_2^+"
    | Slepton (M2,3) → "\\widetilde{\\tau}_2^-"
    | Slepton (M2,−3) → "\\widetilde{\\tau}_2^+"
    | Sneutrino 1 → "\\widetilde{\\nu}_e"
    | Sneutrino (−1) → "\\widetilde{\\nu}_e^*"
    | Sneutrino 2 → "\\widetilde{\\nu}_\\mu"
    | Sneutrino (−2) → "\\widetilde{\\nu}_\\mu^*"
    | Sneutrino 3 → "\\widetilde{\\nu}_\\tau"
    | Sneutrino (−3) → "\\widetilde{\\nu}_\\tau^*"
```

```
    | Sup (M1, 1)  →  "\\widetilde{u}_1"
    | Sup (M1, −1)  →  "\\widetilde{u}_1^*"
    | Sup (M1, 2)  →  "\\widetilde{c}_1"
    | Sup (M1, −2)  →  "\\widetilde{c}_1^*"
    | Sup (M1, 3)  →  "\\widetilde{t}_1"
    | Sup (M1, −3)  →  "\\widetilde{t}_1^*"
    | Sup (M2, 1)  →  "\\widetilde{u}_2"
    | Sup (M2, −1)  →  "\\widetilde{u}_2^*"
    | Sup (M2, 2)  →  "\\widetilde{c}_2"
    | Sup (M2, −2)  →  "\\widetilde{c}_2^*"
    | Sup (M2, 3)  →  "\\widetilde{t}_2"
    | Sup (M2, −3)  →  "\\widetilde{t}_2^*"
    | Sdown (M1, 1)  →  "\\widetilde{d}_1"
    | Sdown (M1, −1)  →  "\\widetilde{d}_1^*"
    | Sdown (M1, 2)  →  "\\widetilde{s}_1"
    | Sdown (M1, −2)  →  "\\widetilde{s}_1^*"
    | Sdown (M1, 3)  →  "\\widetilde{b}_1"
    | Sdown (M1, −3)  →  "\\widetilde{b}_1^*"
    | Sdown (M2, 1)  →  "\\widetilde{d}_2"
    | Sdown (M2, −1)  →  "\\widetilde{d}_2^*"
    | Sdown (M2, 2)  →  "\\widetilde{s}_2"
    | Sdown (M2, −2)  →  "\\widetilde{s}_2^*"
    | Sdown (M2, 3)  →  "\\widetilde{b}_2"
    | Sdown (M2, −3)  →  "\\widetilde{b}_2^*"
    | Neutralino N1  →  "\\widetilde{\\chi}^0_1"
    | Neutralino N2  →  "\\widetilde{\\chi}^0_2"
    | Neutralino N3  →  "\\widetilde{\\chi}^0_3"
    | Neutralino N4  →  "\\widetilde{\\chi}^0_4"
    | Slepton _  →  invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣slepton"
    | Sneutrino _  →  invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣sneutrino"
    | Sup _  →  invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣up␣type␣squark"
    | Sdown _  →  invalid_arg
          "Modellib_MSSM.MSSM.flavor_to_TeX:␣invalid␣down␣type␣squark"
    | Chargino C1  →  "\\widetilde{\\chi}_1^+"
    | Chargino C1c  →  "\\widetilde{\\chi}_1^-"
    | Chargino C2  →  "\\widetilde{\\chi}_2^+"
    | Chargino C2c  →  "\\widetilde{\\chi}_2^-"
    | Grino  →  "\\widetilde{G}"

let pdg  =  function
    | L g when g > 0  →  9 + 2 × g
    | L g  →  − 9 + 2 × g
    | N g when g > 0  →  10 + 2 × g
    | N g  →  − 10 + 2 × g
    | U g when g > 0  →  2 × g
    | U g  →  2 × g
    | D g when g > 0  →  − 1 + 2 × g
```

| $D\ g\ \to\ 1\ +\ 2\times g$
| $Gl\ \to\ 21$ | $Ga\ \to\ 22$ | $Z\ \to\ 23$
| $Wp\ \to\ 24$ | $Wm\ \to\ (-24)$
| $H\_Light\ \to\ 25$ | $H\_Heavy\ \to\ 35$ | $A\ \to\ 36$
| $Hp\ \to\ 37$ | $Hm\ \to\ (-37)$
| $Phip$ | $Phim\ \to\ 27$ | $Phi0\ \to\ 26$
| $Slepton\ (M1, g)$ when $g\ >\ 0\ \to\ 1000009\ +\ 2\times g$
| $Slepton\ (M1, g)\ \to\ -\ 1000009\ +\ 2\times g$
| $Slepton\ (M2, g)$ when $g\ >\ 0\ \to\ 2000009\ +\ 2\times g$
| $Slepton\ (M2, g)\ \to\ -\ 2000009\ +\ 2\times g$
| $Sneutrino\ g$ when $g\ >\ 0\ \to\ 1000010\ +\ 2\times g$
| $Sneutrino\ g\ \to\ -\ 1000010\ +\ 2\times g$
| $Sup\ (M1, g)$ when $g\ >\ 0\ \to\ 1000000\ +\ 2\times g$
| $Sup\ (M1, g)\ \to\ -\ 1000000\ +\ 2\times g$
| $Sup\ (M2, g)$ when $g\ >\ 0\ \to\ 2000000\ +\ 2\times g$
| $Sup\ (M2, g)\ \to\ -\ 2000000\ +\ 2\times g$
| $Sdown\ (M1, g)$ when $g\ >\ 0\ \to\ 999999\ +\ 2\times g$
| $Sdown\ (M1, g)\ \to\ -\ 999999\ +\ 2\times g$
| $Sdown\ (M2, g)$ when $g\ >\ 0\ \to\ 1999999\ +\ 2\times g$
| $Sdown\ (M2, g)\ \to\ -\ 1999999\ +\ 2\times g$
| $Gluino\ \to\ 1000021$
| $Grino\ \to\ 1000039$
| $Chargino\ C1\ \to\ 1000024$ | $Chargino\ C1c\ \to\ (-1000024)$
| $Chargino\ C2\ \to\ 1000037$ | $Chargino\ C2c\ \to\ (-1000037)$
| $Neutralino\ N1\ \to\ 1000022$ | $Neutralino\ N2\ \to\ 1000023$
| $Neutralino\ N3\ \to\ 1000025$ | $Neutralino\ N4\ \to\ 1000035$

We must take care of the pdg numbers for the two different kinds of sfermions in the MSSM. The particle data group in its Monte Carlo particle numbering scheme takes only into account mixtures of the third generation squarks and the stau. For the other sfermions we will use the number of the lefthanded field for the lighter mixed state and the one for the righthanded for the heavier. Below are the official pdg numbers from the Particle Data Group. In order not to produce arrays with some million entries in the Fortran code for the masses and the widths we introduce our private pdg numbering scheme which only extends not too far beyond 42. Our private scheme then has the following pdf numbers (for the sparticles the subscripts L/R and $1/2$ are taken synonymously):

d	down-quark	1
u	up-quark	2
s	strange-quark	3
c	charm-quark	4
b	bottom-quark	5
t	top-quark	6
e^-	electron	11
ν_e	electron-neutrino	12
μ^-	muon	13
ν_μ	muon-neutrino	14
τ^-	tau	15
ν_τ	tau-neutrino	16
g	gluon	(9) 21
γ	photon	22
Z^0	Z-boson	23
W^+	W-boson	24
h^0	light Higgs boson	25
H^0	heavy Higgs boson	35
A^0	pseudoscalar Higgs	36
H^+	charged Higgs	37
$\widetilde{\psi}_\mu$	gravitino	39
$\widetilde{d}_L$	down-squark 1	41
$\widetilde{u}_L$	up-squark 1	42
$\widetilde{s}_L$	strange-squark 1	43
$\widetilde{c}_L$	charm-squark 1	44
$\widetilde{b}_L$	bottom-squark 1	45
$\widetilde{t}_L$	top-squark 1	46
$\widetilde{d}_R$	down-squark 2	47
$\widetilde{u}_R$	up-squark 2	48
$\widetilde{s}_R$	strange-squark 2	49
$\widetilde{c}_R$	charm-squark 2	50
$\widetilde{b}_R$	bottom-squark 2	51
$\widetilde{t}_R$	top-squark 2	52
$\widetilde{e}_L$	selectron 1	53
$\widetilde{\nu}_{e,L}$	electron-sneutrino	54
$\widetilde{\mu}_L$	smuon 1	55
$\widetilde{\nu}_{\mu,L}$	muon-sneutrino	56
$\widetilde{\tau}_L$	stau 1	57
$\widetilde{\nu}_{\tau,L}$	tau-sneutrino	58
$\widetilde{e}_R$	selectron 2	59
$\widetilde{\mu}_R$	smuon 2	61
$\widetilde{\tau}_R$	stau 2 519	63
$\widetilde{g}$	gluino	64
$\widetilde{\chi}_1^0$	neutralino 1	65
$\widetilde{\chi}_2^0$	neutralino 2	66
$\widetilde{\chi}_3^0$	neutralino 3	67
$\widetilde{\chi}_4^0$	neutralino 4	68
$\widetilde{\chi}^+$	chargino 1	69

```
let pdg_mw = function
  | L g when g > 0 → 9 + 2 × g
  | L g → − 9 + 2 × g
  | N g when g > 0 → 10 + 2 × g
  | N g → − 10 + 2 × g
  | U g when g > 0 → 2 × g
  | U g → 2 × g
  | D g when g > 0 → − 1 + 2 × g
  | D g → 1 + 2 × g
  | Gl → 21 | Ga → 22 | Z → 23
  | Wp → 24 | Wm → (−24)
  | H_Light → 25 | H_Heavy → 35 | A → 36
  | Hp → 37 | Hm → (−37)
  | Phip | Phim → 27 | Phi0 → 26
  | Sup (M1, g) when g > 0 → 40 + 2 × g
  | Sup (M1, g) → − 40 + 2 × g
  | Sup (M2, g) when g > 0 → 46 + 2 × g
  | Sup (M2, g) → − 46 + 2 × g
  | Sdown (M1, g) when g > 0 → 39 + 2 × g
  | Sdown (M1, g) → − 39 + 2 × g
  | Sdown (M2, g) when g > 0 → 45 + 2 × g
  | Sdown (M2, g) → − 45 + 2 × g
  | Slepton (M1, g) when g > 0 → 51 + 2 × g
  | Slepton (M1, g) → − 51 + 2 × g
  | Slepton (M2, g) when g > 0 → 57 + 2 × g
  | Slepton (M2, g) → − 57 + 2 × g
  | Sneutrino g when g > 0 → 52 + 2 × g
  | Sneutrino g → − 52 + 2 × g
  | Grino → 39
  | Gluino → 64
  | Chargino C1 → 69 | Chargino C1c → (−69)
  | Chargino C2 → 70 | Chargino C2c → (−70)
  | Neutralino N1 → 65 | Neutralino N2 → 66
  | Neutralino N3 → 67 | Neutralino N4 → 68

let mass_symbol f =
  "mass(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let width_symbol f =
  "width(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let conj_symbol = function
  | false, str → str
  | true, str → str ^ "_c"

let constant_symbol = function
  | Unit → "unit" | Pi → "PI"
  | Alpha_QED → "alpha" | E → "e" | G → "g" | Vev → "vev"
  | Sin2thw → "sin2thw" | Eidelta → "eidelta" | Mu → "mu" |
G_Z → "gz"
  | Sin a → "sin" ^ string_of_angle a | Cos a → "cos" ^ string_of_angle a
```

       | *Sin2am2b* → "sin2am2b" | *Cos2am2b* → "cos2am2b" | *Sinamb* →
"sinamb"
       | *Sinapb* → "sinapb" | *Cosamb* → "cosamb" | *Cosapb* → "cosapb"
       | *Cos4be* → "cos4be" | *Sin4be* → "sin4be" | *Sin4al* → "sin4al"
       | *Sin2al* → "sin2al" | *Cos2al* → "cos2al" | *Sin2be* → "sin2be"
       | *Cos2be* → "cos2be" | *Tana* → "tana" | *Tanb* → "tanb"
       | *Q_lepton* → "qlep" | *Q_up* → "qup" | *Q_down* → "qdwn"
       | *Q_charg* → "qchar"
       | *V_CKM* (*g1*, *g2*) → "vckm_" ^ *string_of_int g1* ^ *string_of_int g2*
       | *M_SF* (*f*, *g*, *m1*, *m2*) → "mix_" ^ *string_of_sff f* ^ *string_of_int g*
          ^ *string_of_sfm m1* ^ *string_of_sfm m2*
       | *AL g* → "al_" ^ *string_of_int g*
       | *AD g* → "ad_" ^ *string_of_int g*
       | *AU g* → "au_" ^ *string_of_int g*
       | *A_0* (*n1*, *n2*) → "a0_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *A_P* (*c1*, *c2*) → "ap_" ^ *string_of_char c1* ^ *string_of_char c2*
       | *V_0* (*n1*, *n2*) → "v0_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *V_P* (*c1*, *c2*) → "vp_" ^ *string_of_char c1* ^ *string_of_char c2*
       | *M_N* (*n1*, *n2*) → "mn_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *M_U* (*c1*, *c2*) → "mu_" ^ *string_of_char c1* ^ *string_of_char c2*
       | *M_V* (*c1*, *c2*) → "mv_" ^ *string_of_char c1* ^ *string_of_char c2*
       | *L_NC* (*n*, *c*) → "lnc_" ^ *string_of_neu n* ^ *string_of_char c*
       | *R_NC* (*n*, *c*) → "rnc_" ^ *string_of_neu n* ^ *string_of_char c*
       | *L_CN* (*c*, *n*) → "lcn_" ^ *string_of_char c* ^ *string_of_neu n*
       | *R_CN* (*c*, *n*) → "rcn_" ^ *string_of_char c* ^ *string_of_neu n*
       | *L_NCH* (*n*, *c*) → "lnch_" ^ *string_of_neu n* ^ *string_of_char c*
       | *R_NCH* (*n*, *c*) → "rnch_" ^ *string_of_neu n* ^ *string_of_char c*
       | *L_CNG* (*c*, *n*) → "lcng_" ^ *string_of_char c* ^ *string_of_neu n*
       | *R_CNG* (*c*, *n*) → "rcng_" ^ *string_of_char c* ^ *string_of_neu n*
       | *S_NNA* (*n1*, *n2*) → "snna_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *P_NNA* (*n1*, *n2*) → "pnna_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *S_NNG* (*n1*, *n2*) → "snng_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *P_NNG* (*n1*, *n2*) → "pnng_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *S_NNH1* (*n1*, *n2*) → "snnh1_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *P_NNH1* (*n1*, *n2*) → "pnnh1_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *S_NNH2* (*n1*, *n2*) → "snnh2_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *P_NNH2* (*n1*, *n2*) → "pnnh2_" ^ *string_of_neu n1* ^ *string_of_neu n2*
       | *G_NC_lepton* → "gnclep" | *G_NC_neutrino* → "gncneu"
       | *G_NC_up* → "gncup" | *G_NC_down* → "gncdwn"
       | *G_CC* → "gcc"
       | *G_CCQ* (*vc*, *g1*, *g2*) → *conj_symbol* (*vc*, "gccq_" ^ *string_of_int g1* ^ "_"
          ^ *string_of_int g2*)
       | *I_Q_W* → "iqw" | *I_G_ZWW* → "igzww"
       | *G_WWWW* → "gw4" | *G_ZZWW* → "gzzww"
       | *G_PZWW* → "gpzww" | *G_PPWW* → "gppww"
       | *G_GH* 1 → "ghaw"
       | *G_GH* 2 → "gh1az" | *G_GH* 3 → "gh2az"
       | *G_GH* 4 → "gh1ww" | *G_GH* 5 → "gh2ww"
       | *G_GH* 6 → "ghh1w" | *G_GH* 7 → "ghh2w"
       | *G_GH* 8 → "gh1zz" | *G_GH* 9 → "gh2zz"

```
    | G_GH 10 → "ghhz" | G_GH 11 → "ghhp"
    | G_GH _ → failwith "this␣G_GH␣coupling␣is␣not␣available"
    | G_GLGLH → "gglglh" | G_GLGLHH → "gglglhh"
    | G_GLGLA → "gglgla" | G_PPH → "gpph"
    | G_PPHH → "gpphh" | G_PPA → "gppa"
    | G_GHGo n → "g_hgh(" ^ string_of_int n ^ ")"
    | G_GH4 1 → "gaazz" | G_GH4 2 → "gh1h1zz" | G_GH4 3 →
"gh2h2zz"
    | G_GH4 4 → "ghphmzz" | G_GH4 5 → "ghphmpp" | G_GH4 6 →
"ghphmpz"
    | G_GH4 7 → "ghh1wz" | G_GH4 8 → "ghh2wz"
    | G_GH4 9 → "ghh1wp" | G_GH4 10 → "ghh2wp"
    | G_GH4 11 → "gaaww" | G_GH4 12 → "gh1h1ww" | G_GH4 13 →
"gh2h2ww"
    | G_GH4 14 → "ghhww" | G_GH4 15 → "ghawz" | G_GH4 16 →
"ghawp"
    | G_GH4 _ → failwith "this␣G_GH4␣coupling␣is␣not␣available"
    | G_CICIH1 (n1, n2) → "gcicih1_" ^ string_of_neu n1 ^ "_"
        ^ string_of_neu n2
    | G_CICIH2 (n1, n2) → "gcicih2_" ^ string_of_neu n1 ^ "_"
        ^ string_of_neu n2
    | G_CICIA (n1, n2) → "gcicia_" ^ string_of_neu n1 ^ "_"
        ^ string_of_neu n2
    | G_CICIG (n1, n2) → "gcicig_" ^ string_of_neu n1 ^ "_"
        ^ string_of_neu n2
    | G_H3 n → "gh3_" ^ string_of_int n
    | G_H4 n → "gh4_" ^ string_of_int n
    | G_HGo3 n → "ghg3_" ^ string_of_int n
    | G_HGo4 n → "ghg4_" ^ string_of_int n
    | G_GG4 n → "ggg4_" ^ string_of_int n
    | G_strong → "gs" | G_SS → "gs**2"
    | Gs → "gs"
    | I_G_S → "igs"
    | G_S_Sqrt → "gssq"
    | G_NWC (n, c) → "gnwc_" ^ string_of_neu n ^ "_" ^ string_of_char c
    | G_CWN (c, n) → "gcwn_" ^ string_of_char c ^ "_" ^ string_of_neu n
    | G_CH1C (c1, c2) → "gch1c_" ^ string_of_char c1 ^ "_" ^ string_of_char c2
    | G_CH2C (c1, c2) → "gch2c_" ^ string_of_char c1 ^ "_" ^ string_of_char c2
    | G_CAC (c1, c2) → "gcac_" ^ string_of_char c1 ^ "_" ^ string_of_char c2
    | G_CGC (c1, c2) → "gcgc_" ^ string_of_char c1 ^ "_" ^ string_of_char c2
    | G_YUK (i, g) → "g_yuk" ^ string_of_int i ^ "_" ^ string_of_int g
    | G_NZN (n1, n2) → "gnzn_" ^ string_of_neu n1 ^ "_" ^ string_of_neu n2
    | G_CZC (c1, c2) → "gczc_" ^ string_of_char c1 ^ "_" ^ string_of_char
        c2
    | G_YUK_1 (n, m) → "g_yuk1_" ^ string_of_int n ^ "_" ^ string_of_int m
    | G_YUK_2 (n, m) → "g_yuk2_" ^ string_of_int n ^ "_" ^ string_of_int m
    | G_YUK_3 (n, m) → "g_yuk3_" ^ string_of_int n ^ "_" ^ string_of_int m
    | G_YUK_4 (n, m) → "g_yuk4_" ^ string_of_int n ^ "_" ^ string_of_int m
    | G_YUK_C (vc, g, c, sf, m) → conj_symbol (vc, "g_yuk_ch" ^ string_of_char c
        ^ "_" ^ string_of_sff sf ^ string_of_sfm m ^ "_" ^ string_of_int g )
```

| $G\_YUK\_N\,(vc, g, n, sf, m) \rightarrow conj\_symbol\,(vc,$ `"g_yuk_n"` $\hat{}\ string\_of\_neu\ n$
$\quad \hat{}\ $ `"_"` $\hat{}\ string_of_sff\ sf \ \hat{}\ string_of_sfm\ m\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g\,)$

| $G\_YUK\_G\,(vc, g, sf, m) \rightarrow conj\_symbol\,(vc,$ `"g_yuk_g"` $\hat{}\ string\_of\_sff\ sf$
$\quad \hat{}\ string_of_sfm\ m\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g)$

| $G\_YUK\_Q\,(vc, g1, g2, c, sf, m) \rightarrow conj\_symbol\,(vc,$ `"g_yuk_ch"` $\hat{}\ string\_of\_char\ c$
$\quad \hat{}\ $ `"_"` $\hat{}\ string_of_sff\ sf\ \hat{}\ string_of_sfm\ m\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g1$
$\quad \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g2)$

| $G\_NHC\,(n, c) \rightarrow$ `"g_nhc_"` $\hat{}\ string_of_neu\ n\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_char\ c$

| $G\_CHN\,(c, n) \rightarrow$ `"g_chn_"` $\hat{}\ string_of_neu\ n\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_char\ c$

| $G\_NGC\,(n, c) \rightarrow$ `"g_ngc_"` $\hat{}\ string\_of\_neu\ n\ \hat{}\ string\_of\_char\ c$

| $G\_CGN\,(c, n) \rightarrow$ `"g_cgn_"` $\hat{}\ string\_of\_char\ c\ \hat{}\ string\_of\_neu\ n$

| $SUM\_1 \rightarrow$ `"sum1"`

| $G\_SLSNW\,(vc, g, m) \rightarrow conj\_symbol\,(vc,$ `"gsl"` $\hat{}\ string_of_sfm\ m\ \hat{}\ $ `"_"`
$\quad \hat{}\ string_of_int\ g\ \hat{}\ $ `"snw")`

| $G\_ZSF\,(f, g, m1, m2) \rightarrow$ `"g"` $\hat{}\ string_of_sff\ f\ \hat{}\ string_of_sfm\ m1\ \hat{}\ $ `"z"`
$\quad \hat{}\ string_of_sff\ f\ \hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g$

| $G\_WWSFSF\,(f, g, m1, m2) \rightarrow$ `"gww"` $\hat{}\ string\_of\_sff\ f\ \hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ string_of_sff\ f\ \hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g$

| $G\_WPSLSN\,(vc, g, m) \rightarrow conj\_symbol\,(vc,$ `"gpwsl"` $\hat{}\ string\_of\_sfm\ m$
$\quad \hat{}\ $ `"sn_"` $\hat{}\ string\_of\_int\ g)$

| $G\_WZSLSN\,(vc, g, m) \rightarrow conj\_symbol\,(vc,$ `"gwzsl"` $\hat{}\ string\_of\_sfm\ m$
$\quad \hat{}\ $ `"sn_"` $\hat{}\ string\_of\_int\ g)$

| $G\_H1SFSF\,(f, g, m1, m2) \rightarrow$ `"gh1"` $\hat{}\ string\_of\_sff\ f\ \hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ string_of_sff\ f\ \hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g$

| $G\_H2SFSF\,(f, g, m1, m2) \rightarrow$ `"gh2"` $\hat{}\ string\_of\_sff\ f\ \hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ string_of_sff\ f\ \hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g$

| $G\_ASFSF\,(f, g, m1, m2) \rightarrow$ `"ga"` $\hat{}\ string\_of\_sff\ f\ \hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ string_of_sff\ f\ \hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g$

| $G\_HSNSL\,(vc, g, m) \rightarrow conj\_symbol\,(vc,$ `"ghsnsl"` $\hat{}\ string_of_sfm\ m\ \hat{}\ $ `"_"`
$\quad \hat{}\ string\_of\_int\ g)$

| $G\_GoSFSF\,(f, g, m1, m2) \rightarrow$ `"ggo"` $\hat{}\ string\_of\_sff\ f\ \hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ string_of_sff\ f\ \hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g$

| $G\_GoSNSL\,(vc, g, m) \rightarrow conj\_symbol\,(vc,$ `"ggosnsl"` $\hat{}\ string_of_sfm\ m\ \hat{}\ $ `"_"`
$\quad \hat{}\ string\_of\_int\ g)$

| $G\_HSUSD\,(vc, m1, m2, g1, g2) \rightarrow conj\_symbol\,(vc,$ `"ghsu"` $\hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ $ `"sd"` $\hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string_of_int\ g1\ \hat{}\ $ `"_"`
$\quad \hat{}\ string\_of\_int\ g2)$

| $G\_GSUSD\,(vc, m1, m2, g1, g2) \rightarrow conj\_symbol\,(vc,$ `"ggsu"` $\hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ $ `"sd"` $\hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string_of_int\ g1\ \hat{}\ $ `"_"`
$\quad \hat{}\ string\_of\_int\ g2)$

| $G\_WPSUSD\,(vc, m1, m2, n, m) \rightarrow conj\_symbol\,(vc,$ `"gpwpsu"` $\hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ $ `"sd"` $\hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string_of_int\ n\ \hat{}\ $ `"_"`
$\quad \hat{}\ string\_of\_int\ m)$

| $G\_WZSUSD\,(vc, m1, m2, n, m) \rightarrow conj\_symbol\,(vc,$ `"gzwpsu"` $\hat{}\ string\_of\_sfm\ m1$
$\quad \hat{}\ $ `"sd"` $\hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string_of_int\ n\ \hat{}\ $ `"_"`
$\quad \hat{}\ string\_of\_int\ m)$

| $G\_SWS\,(vc, g1, g2, m1, m2) \rightarrow conj\_symbol\,(vc,$ `"gs"` $\hat{}\ string_of_sfm\ m1\ \hat{}\ $ `"ws"`
$\quad \hat{}\ string_of_sfm\ m2\ \hat{}\ $ `"_"` $\hat{}\ string_of_int\ g1\ \hat{}\ $ `"_"` $\hat{}\ string\_of\_int\ g2)$

| $G\_GlGlSQSQ \rightarrow$ `"gglglsqsq"`

| $G\_PPSFSF\ f \rightarrow$ `"gpp"` $\hat{}\ string\_of\_sff\ f\ \hat{}\ string\_of\_sff\ f$

| G_ZZSFSF $(f, g, m1, m2)$ $\to$ `"gzz"` ˆ *string_of_sff f* ˆ *string_of_sfm m1*
        ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int g*
| G_ZPSFSF $(f, g, m1, m2)$ $\to$ `"gzp"` ˆ *string_of_sff f* ˆ *string_of_sfm m1*
        ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int g*
| $G\_GlPSQSQ$ $\to$ `"gglpsqsq"`
| $G\_GlZSFSF$ $(f, g, m1, m2)$ $\to$ `"ggl"` ˆ *string_of_sff f* ˆ *string_of_sfm m1*
        ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int g*
| $G\_GlWSUSD$ $(vc, m1, m2, g1, g2)$ $\to$ *conj_symbol* $(vc,$ `"gglwsu"`
        ˆ *string_of_sfm m1* ˆ `"sd"` ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int g1*
        ˆ `"_"` ˆ *string_of_int g2*$)$
| G_GHGo4 1 $\to$ `"gzzg0g0"` | G_GHGo4 2 $\to$ `"gzzgpgm"`
| G_GHGo4 3 $\to$ `"gppgpgm"` | G_GHGo4 4 $\to$ `"gzpgpgm"`
| G_GHGo4 5 $\to$ `"gwwgpgm"` | G_GHGo4 6 $\to$ `"gwwg0g0"`
| G_GHGo4 7 $\to$ `"gwzg0g"` | G_GHGo4 8 $\to$ `"gwzg0g"`
| G_GHGo4 9 $\to$ `"gwzh1g"` | G_GHGo4 10 $\to$ `"gwzh2g"`
| G_GHGo4 11 $\to$ `"gwph1g"` | G_GHGo4 12 $\to$ `"gwph2g"`
| G_GHGo4 _ $\to$ *failwith* `"Coupling␣G_GHGo4␣is␣not␣available"`
| G_HSF31 $(h, g, m1, m2, f1, f2)$ $\to$ `"g_"` ˆ *string_of_higgs h* ˆ
        string_of_int g ˆ *string_of_sfm m1* ˆ *string_of_sfm m2* ˆ
        string_of_sff f1 ˆ *string_of_sff f2*
| G_HSF32 $(h, g1, g2, m1, m2, f1, f2)$ $\to$ `"g_"` ˆ *string_of_higgs h* ˆ
        string_of_int g1 ˆ `"_"` ˆ *string_of_int g2* ˆ *string_of_sfm m1* ˆ
        string_of_sfm m2 ˆ *string_of_sff f1* ˆ *string_of_sff f2*
| G_HSF41 $(h, g, m1, m2, f1, f2)$ $\to$ `"g_"` ˆ *string_of_higgs h* ˆ
        string_of_int g ˆ *string_of_sfm m1* ˆ *string_of_sfm m2* ˆ
        string_of_sff f1 ˆ *string_of_sff f2*
| G_HSF42 $(h, g1, g2, m1, m2, f1, f2)$ $\to$ `"g_"` ˆ *string_of_higgs h* ˆ
        string_of_int g1 ˆ `"_"` ˆ *string_of_int g2* ˆ *string_of_sfm m1* ˆ
        string_of_sfm m2 ˆ *string_of_sff f1* ˆ *string_of_sff f2*
| $G\_H1H1SFSF$ $(f, m1, m2, n)$ $\to$ `"gh1h1"` ˆ *string_of_sff f* ˆ *string_of_sfm*
        m1 ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int n*
| $G\_H1H2SFSF$ $(f, m1, m2, n)$ $\to$ `"gh1h2"` ˆ *string_of_sff f* ˆ *string_of_sfm*
        m1 ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int n*
| $G\_H2H2SFSF$ $(f, m1, m2, n)$ $\to$ `"gh2h2"` ˆ *string_of_sff f* ˆ *string_of_sfm*
        m1 ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int n*
| G_HHSFSF $(f, m1, m2, n)$ $\to$ `"ghh"` ˆ *string_of_sff f* ˆ *string_of_sfm m1*
        ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int n*
| G_AASFSF $(f, m1, m2, n)$ $\to$ `"gaa"` ˆ *string_of_sff f* ˆ *string_of_sfm m1*
        ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int n*
| $G\_HH1SUSD$ $(vc, m1, m2, g1, g2)$ $\to$ *conj_symbol* $(vc,$ `"ghh1su"`
        ˆ *string_of_sfm m1* ˆ `"sd"` ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int g1*
        ˆ `"_"` ˆ *string_of_int g2*$)$
| $G\_HH2SUSD$ $(vc, m1, m2, g1, g2)$ $\to$ *conj_symbol* $(vc,$ `"ghh2su"`
        ˆ *string_of_sfm m1* ˆ `"sd"` ˆ *string_of_sfm m2* ˆ `"_"` ˆ *string_of_int g1*
        ˆ `"_"` ˆ *string_of_int g2*$)$
| G_HASUSD $(vc, m1, m2, g1, g2)$ $\to$ *conj_symbol* $(vc,$ `"ghasu"`
        ˆ *string_of_sfm m1* ˆ `"sd"` ˆ *string_of_sfm m2* ˆ `"_"`
        ˆ *string_of_int g1* ˆ `"_"` ˆ *string_of_int g2* ˆ `"_c"`$)$
| $G\_HH1SLSN$ (vc, m, g) $\to$ *conj_symbol* $(vc,$ `"ghh1sl"` ˆ *string_of_sfm m*
                            ˆ `"sn_"` ˆ *string_of_int g*$)$

| $G\_HH2SLSN\,(vc, m, g) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"ghh2sl"}\,\hat{}\;\; string\_of\_sfm\;m$
$\hat{}\;\;\texttt{"sn_"}\,\hat{}\;\; string\_of\_int\;g)$
| $G\_HASLSN\,(vc, m, g) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"ghasl"}\,\hat{}\;\; string\_of\_sfm\;m$
$\hat{}\;\;\texttt{"sn_"}\,\hat{}\;\; string\_of\_int\;g)$
| $G\_AG0SFSF\,(f, m1, m2, n) \;\rightarrow\; \texttt{"gag0"}\,\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;n$
| $G\_HGSFSF\,(f, m1, m2, n) \;\rightarrow\; \texttt{"ghg"}\,\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m1\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;n$
| $G\_GGSFSF\,(f, m1, m2, n) \;\rightarrow\; \texttt{"ggg"}\,\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;n$
| $G\_G0G0SFSF\,(f, m1, m2, n) \;\rightarrow\; \texttt{"gg0g0"}\,\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\; string\_of\_sff\;f\;\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;n$
| $G\_HGSNSL\,(vc, m, n) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"ghgsnsl"}\,\hat{}\;\; string\_of\_sfm\;m\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;n)$
| $G\_H1GSNSL\,(vc, m, n) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gh1gsnsl"}\,\hat{}\;\; string\_of\_sfm\;m\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;n)$
| $G\_H2GSNSL\,(vc, m, n) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gh2gsnsl"}\,\hat{}\;\; string\_of\_sfm\;m\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;n)$
| $G\_AGSNSL\,(vc, m, n) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gagsnsl"}\,\hat{}\;\; string\_of\_sfm\;m\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;n)$
| $G\_GGSNSL\,(vc, m, n) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gggsnsl"}\,\hat{}\;\; string\_of\_sfm\;m\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;n)$
| $G\_HGSUSD\,(vc, m1, m2, g1, g2) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gghpsu"}\,\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\;\texttt{"sd"}\,\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;g2)$
| $G\_H1GSUSD\,(vc, m1, m2, g1, g2) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gh1gpsu"}\,\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\;\texttt{"sd"}\,\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;g2)$
| $G\_H2GSUSD\,(vc, m1, m2, g1, g2) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gh2gpsu"}\,\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\;\texttt{"sd"}\,\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;g2)$
| $G\_AGSUSD\,(vc, m1, m2, g1, g2) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gagpsu"}\,\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\;\texttt{"sd"}\,\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;g2)$
| $G\_GGSUSD\,(vc, m1, m2, g1, g2) \;\rightarrow\; conj\_symbol\,(vc,\, \texttt{"gggpsu"}\,\hat{}\;\; string\_of\_sfm\;m1$
$\hat{}\;\;\texttt{"sd"}\,\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_"}$
$\hat{}\;\; string\_of\_int\;g2)$
| $G\_SN4\,(g1, g2) \;\rightarrow\; \texttt{"gsn4_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g2$
| $G\_SN2SL2\_1\,(m1, m2, g1, g2) \;\rightarrow\; \texttt{"gsl_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_sl_"}$
$\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_sl"}\,\hat{}\;\; string\_of\_sfm\;m1\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g2$
$\hat{}\;\;\texttt{"_sl"}\,\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g2$
| $G\_SN2SL2\_2\,(m1, m2, g1, g2) \;\rightarrow\; \texttt{"gsl_"}\,\hat{}\;\; string\_of\_int\;g1\;\hat{}\;\;\texttt{"_sl_"}$
$\hat{}\;\; string\_of\_int\;g2\;\hat{}\;\;\texttt{"_sl"}\,\hat{}\;\; string\_of\_sfm\;m1\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g1$
$\hat{}\;\;\texttt{"_sl"}\,\hat{}\;\; string\_of\_sfm\;m2\;\hat{}\;\;\texttt{"_"}\,\hat{}\;\; string\_of\_int\;g2\;\hat{}\;\;\texttt{"_mix"}$
| $G\_SF4\,(f1, f2, m1, m2, m3, m4, g1, g2) \;\rightarrow\; \texttt{"gsf"}\,\hat{}\;\; string\_of\_sff\;f1\;\hat{}$
$string\_of\_sff\;f2\;\hat{}\;\; string\_of\_sfm\;m1\;\hat{}\;\; string\_of\_sfm\;m2\;\hat{}$
$string\_of\_sfm\;m3\;\hat{}\;\; string\_of\_sfm\;m4\;\hat{}\;\; string\_of\_int\;g1\;\hat{}$
$string\_of\_int\;g2$
| $G\_SF4\_3\,(f1, f2, m1, m2, m3, m4, g1, g2, g3) \;\rightarrow\; \texttt{"gsf"}\,\hat{}\;\; string\_of\_sff\;f1\;\hat{}$
$string\_of\_sff\;f2\;\hat{}\;\; string\_of\_sfm\;m1\;\hat{}\;\; string\_of\_sfm\;m2\;\hat{}$

```
                string_of_sfm m3 ^ string_of_sfm m4 ^ string_of_int g1 ^
                string_of_int g2 ^ "_" ^ string_of_int g3
      | G_SF4_4 (f1, f2, m1, m2, m3, m4, g1, g2, g3, g4) → "gsf" ^ string_of_sff f1 ^
                string_of_sff f2 ^ string_of_sfm m1 ^ string_of_sfm m2 ^
                string_of_sfm m3 ^ string_of_sfm m4 ^ string_of_int g1 ^ "_" ^
                string_of_int g2 ^ string_of_int g3 ^ "_" ^ string_of_int g4
      | G_SL4 (m1, m2, m3, m4, g) → "gsl" ^ string_of_sfm m1 ^ "_"
                ^ "sl" ^ string_of_sfm m2 ^ "_" ^ "sl" ^ string_of_sfm m3 ^ "_"
                ^ "sl" ^ string_of_sfm m4 ^ "_" ^ string_of_int g
      | G_SL4_2 (m1, m2, m3, m4, g1, g2) → "gsl" ^ string_of_sfm m1 ^ "_"
                ^ "sl" ^ string_of_sfm m2 ^ "_" ^ "sl" ^ string_of_sfm m3 ^ "_"
                ^ "sl" ^ string_of_sfm m4 ^ "_" ^ string_of_int g1 ^ "_" ^
                string_of_int g2
      | G_SN2SQ2 (f, m1, m2, g1, g2) → "gsn_" ^ string_of_int g1 ^ "_sn_"
                ^ string_of_int g1 ^ "_" ^ string_of_sff f ^ string_of_sfm m1 ^ "_"
                ^ string_of_int g2 ^ "_" ^ string_of_sff f ^ string_of_sfm m2 ^ "_"
                ^ string_of_int g2
      | G_SL2SQ2 (f, m1, m2, m3, m4, g1, g2) → "gsl" ^ string_of_sfm m1 ^ "_"
                ^ string_of_int g1 ^ "_sl" ^ string_of_sfm m2 ^ "_" ^ string_of_int g1
                ^ "_" ^ string_of_sff f ^ string_of_sfm m3 ^ "_" ^ string_of_int g2
                ^ "_" ^ string_of_sff f ^ string_of_sfm m4 ^ "_" ^ string_of_int g2
      | G_SUSDSNSL (vc, m1, m2, m3, g1, g2, g3) → conj_symbol (vc, "gsl"
                ^ string_of_sfm m3 ^ "_" ^ string_of_int g3 ^ "_sn_" ^ string_of_int g3
                ^ "_su" ^ string_of_sfm m1 ^ "_" ^ string_of_int g1 ^ "_sd"
                ^ string_of_sfm m2 ^ "_" ^ string_of_int g2)
      | G_SU4 (m1, m2, m3, m4, g) → "gsu" ^ string_of_sfm m1 ^ "_"
                ^ "_su" ^ string_of_sfm m2 ^ "_" ^ "_su" ^ string_of_sfm m3 ^ "_" ^
                "_su" ^ string_of_sfm m4 ^ "_" ^ string_of_int g
      | G_SU4_2 (m1, m2, m3, m4, g1, g2) → "gsu" ^ string_of_sfm m1 ^ "_"
                ^ "_su" ^ string_of_sfm m2 ^ "_" ^ "_su" ^ string_of_sfm m3 ^ "_" ^
                "_su" ^ string_of_sfm m4 ^ "_" ^ string_of_int g1 ^ "_" ^
                string_of_int g2
      | G_SD4 (m1, m2, m3, m4, g) → "gsd" ^ string_of_sfm m1 ^ "_"
                ^ "_sd" ^ string_of_sfm m2 ^ "_" ^ "_sd" ^ string_of_sfm m3 ^ "_"
                ^ "_sd" ^ string_of_sfm m4 ^ "_" ^ string_of_int g
      | G_SD4_2 (m1, m2, m3, m4, g1, g2) → "gsd" ^ string_of_sfm m1 ^ "_"
                ^ "_sd" ^ string_of_sfm m2 ^ "_" ^ "_sd" ^ string_of_sfm m3 ^ "_"
                ^ "_sd" ^ string_of_sfm m4 ^ "_" ^ string_of_int g1 ^ "_" ^
                string_of_int g2
      | G_SU2SD2 (m1, m2, m3, m4, g1, g2, g3, g4) → "gsu" ^ string_of_sfm m1
                ^ "_" ^ string_of_int g1 ^ "_su" ^ string_of_sfm m2 ^ "_"
                ^ string_of_int g2 ^ "_sd" ^ string_of_sfm m3 ^ "_" ^ string_of_int g3
                ^ "_sd" ^ string_of_sfm m4 ^ "_" ^ string_of_int g4
      | M f → "mass" ^ flavor_symbol f
      | W f → "width" ^ flavor_symbol f
      | G_Grav → "ggrav" | G_Gr_Ch C1 → "ggrch1" | G_Gr_Ch C2 →
                "ggrch2"
      | G_Gr_Ch C1c → "ggrch1c" | G_Gr_Ch C2c → "ggrch2c"
      | G_Gr_Z_Neu n → "ggrzneu" ^ string_of_neu n
      | G_Gr_A_Neu n → "ggraneu" ^ string_of_neu n
```

```
    | G_Gr4_Neu n  →  "ggr4neu" ^ string_of_neu n
    | G_Gr4_A_Ch C1 → "ggr4ach1" | G_Gr4_A_Ch C2 → "ggr4ach2"
    |  G_Gr4_A_Ch  C1c  →  "ggr4ach1c" |  G_Gr4_A_Ch  C2c  →
"ggr4ach2c"
    | G_Gr4_Z_Ch C1 → "ggr4zch1" | G_Gr4_Z_Ch C2 → "ggr4zch2"
    |  G_Gr4_Z_Ch  C1c  →  "ggr4zch1c" |  G_Gr4_Z_Ch  C2c  →
"ggr4zch2c"
    | G_Grav_N  →  "ggravn"
    | G_GravGl  →  "gs␣*␣ggrav"
    | G_Grav_L (g, m) → "ggravl" ^ string_of_int g ^ string_of_sfm m
    | G_Grav_Lc (g, m) → "ggravl" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Grav_U (g, m) → "ggravu" ^ string_of_int g ^ string_of_sfm m
    | G_Grav_Uc (g, m) → "ggravu" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Grav_D (g, m) → "ggravd" ^ string_of_int g ^ string_of_sfm m
    | G_Grav_Dc (g, m) → "ggravd" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr_H_Ch C1 → "ggrhch1" | G_Gr_H_Ch C2  →  "ggrhch2"
    | G_Gr_H_Ch C1c → "ggrhch1c" | G_Gr_H_Ch C2c  →  "ggrhch2c"
    | G_Gr_H1_Neu n  →  "ggrh1neu" ^ string_of_neu n
    | G_Gr_H2_Neu n  →  "ggrh2neu" ^ string_of_neu n
    | G_Gr_H3_Neu n  →  "ggrh3neu" ^ string_of_neu n
    | G_Gr4A_Sl (g, m) → "ggr4asl" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4A_Slc (g, m) → "ggr4asl" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4A_Su (g, m) → "ggr4asu" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4A_Suc (g, m) → "ggr4asu" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4A_Sd (g, m) → "ggr4asd" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4A_Sdc (g, m) → "ggr4asd" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4Z_Sn → "ggr4zsn" | G_Gr4Z_Snc → "ggr4zsnc"
    | G_Gr4Z_Sl (g, m) → "ggr4zsl" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4Z_Slc (g, m) → "ggr4zsl" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4Z_Su (g, m) → "ggr4zsu" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4Z_Suc (g, m) → "ggr4zsu" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4Z_Sd (g, m) → "ggr4zsd" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4Z_Sdc (g, m) → "ggr4zsd" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4W_Sl (g, m) → "ggr4wsl" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4W_Slc (g, m) → "ggr4wsl" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4W_Su (g, m) → "ggr4wsu" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4W_Suc (g, m) → "ggr4wsu" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4W_Sd (g, m) → "ggr4wsd" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4W_Sdc (g, m) → "ggr4wsd" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4Gl_Su (g, m) → "ggr4glsu" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4Gl_Suc (g, m) → "ggr4glsu" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4Gl_Sd (g, m) → "ggr4glsd" ^ string_of_int g ^ string_of_sfm m
    | G_Gr4Gl_Sdc (g, m) → "ggr4glsd" ^ string_of_int g ^ string_of_sfm m ^ "c"
    | G_Gr4_Z_H1 n  →  "ggr4zh1_" ^ string_of_neu n
    | G_Gr4_Z_H2 n  →  "ggr4zh2_" ^ string_of_neu n
    | G_Gr4_Z_H3 n  →  "ggr4zh3_" ^ string_of_neu n
    | G_Gr4_W_H n  →  "ggr4wh_" ^ string_of_neu n
    | G_Gr4_W_Hc n  →  "ggr4whc_" ^ string_of_neu n
    | G_Gr4_H_A C1 → "ggr4ha1" | G_Gr4_H_A C2  →  "ggr4ha2"
    | G_Gr4_H_A C1c → "ggr4ha1c" | G_Gr4_H_A C2c  →  "ggr4ha2c"
```

```
    | G_Gr4_H_Z C1  →  "ggr4hz1" | G_Gr4_H_Z C2  →  "ggr4hz2"
    | G_Gr4_H_Z C1c →  "ggr4hz1c"| G_Gr4_H_Z C2c →  "ggr4hz2c"
    | G_Gr4W_Sn    →  "ggr4wsn"
    | G_Gr4W_Snc   →  "ggr4wsnc"
```

end

13.9   Interface of Modellib_NMSSM

13.9.1   Extended Supersymmetric Models

We do not introduce the possibility here of using four point couplings or not. We simply add the relevant and leave the rest out. No possibility for Goldstone bosons is given. But we allow for CKM mixing.

module type *NMSSM_flags* =
  sig
    val *ckm_present* : *bool*
    val *higgs_triangle* : *bool* (* $H\gamma\gamma$, $Hg\gamma$ and $Hggcouplings$*)
  end

module *NMSSM* : *NMSSM_flags*
module *NMSSM_CKM* : *NMSSM_flags*
module *NMSSM_Hgg* : *NMSSM_flags*
module *NMSSM_func* : functor (*F* : *NMSSM_flags*) → *Model.T* with module *Ch* = *Charges.QQ*

13.10   Implementation of Modellib_NMSSM

let *rcs_file* = *RCS.parse* "Modellib_NMSSM" ["NMSSM"]
  { *RCS.revision* = "\$Revision:␣6465␣\$";
   RCS.date = "\$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣\$";
   RCS.author = "\$Author:␣jr_reuter␣\$";
   RCS.source
    = "\$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr

13.10.1   Next-to-Minimal Supersymmetric Standard Model

This is based on the NMSSM implementation by Felix Braam. Note that for the Higgs sector vertices the conventions of the Franke/Fraas paper have been used.

module type *NMSSM_flags* =
  sig
    val *ckm_present* : *bool*
    val *higgs_triangle* : *bool*
  end

module *NMSSM* : *NMSSM_flags* =
  struct
    let *ckm_present* = false

```
      let higgs_triangle  =  false
   end

module NMSSM_CKM  :  NMSSM_flags  =
   struct
      let ckm_present  =  true
      let higgs_triangle  =  false
   end

module NMSSM_Hgg  :  NMSSM_flags  =
   struct
      let ckm_present  =  false
      let higgs_triangle  =  true
   end

module NMSSM_func (Flags  :  NMSSM_flags)  =
   struct
      let rcs  =  RCS.rename rcs_file "Modellib_NMSSM.NMSSM"
          [ "NMSSM" ]

      open Coupling

      let default_width  =  ref Timelike
      let use_fudged_width  =  ref false

      let options  =  Options.create
         [ "constant_width", Arg.Unit (fun ()  →  default_width :=  Constant),
           "use␣constant␣width␣(also␣in␣t-channel)";
           "fudged_width", Arg.Set use_fudged_width,
           "use␣fudge␣factor␣for␣charge␣particle␣width";
           "custom_width", Arg.String (fun f  →  default_width :=  Custom f),
           "use␣custom␣width";
           "cancel_widths", Arg.Unit (fun ()  →  default_width :=  Vanishing),
           "use␣vanishing␣width"]
```

Yields a list of tuples consistig of the off-diag combinations of the elements in
"set".

```
      let choose2 set  =
         List.map (function [x; y]  →  (x, y)  |  _  →  failwith "choose2")
            (Combinatorics.choose 2 set)
```

pairs appends the diagonal combinations to *choose2*.

```
      let rec diag  =  function
         |  []  →  []
         |  x1 ::  rest  →  (x1, x1) ::  diag rest

      let pairs l  =  choose2 l @ diag l

      let rec cloop set i j k  =
         if i  >  ((List.length set) − 1) then []
         else if j  >  i then cloop set (succ i) (j − i − 1) (j − i − 1)
         else if k  >  j then cloop set i (succ j) (k − j − 1)
         else (List.nth set i, List.nth set j, List.nth set k) ::  cloop set i j (succ k)
```

```
let triples set  =  cloop set 0 0 0
```

```
let rec two_and_one' l1 z n  =
    if  n  <  0 then [ ]
    else
    ((fst (List.nth (pairs l1) n)), (snd (List.nth (pairs l1) n)), z) :: two_and_one' l1 z (pred n)
```

```
let two_and_one l1 l2  =
    let f z  =  two_and_one' l1 z ((List.length (pairs l1)) − 1)
    in
    List.flatten ( List.map f l2 )
```

```
type gen  =
    |  G of int |  GG of gen × gen
```

```
let rec string_of_gen  =  function
    |  G n when n  >  0  →  string_of_int n
    |  G n  →  string_of_int (abs n) ˆ "c"
    |  GG (g1, g2)  →  string_of_gen g1 ˆ "_" ˆ string_of_gen g2
```

With this we distinguish the flavour.

```
type sff  =
    |  SL  |  SN  |  SU  |  SD
```

```
let string_of_sff  =  function
    |  SL  →  "sl" |  SN  →  "sn" |  SU  →  "su" |  SD  →  "sd"
```

With this we distinguish the mass eigenstates. At the moment we have to cheat
a little bit for the sneutrinos. Because we are dealing with massless neutrinos
there is only one sort of sneutrino.

```
type sfm  =
    |  M1  |  M2
```

```
let string_of_sfm  =  function
    |  M1  →  "1" |  M2  →  "2"
```

We also introduce special types for the charginos and neutralinos.

```
type char =
    |  C1  |  C2  |  C1c  |  C2c
```

```
type neu  =
    |  N1  |  N2  |  N3  |  N4  |  N5
```

```
let int_of_char  =  function
    |  C1  →  1  |  C2  →  2  |  C1c  →  − 1  |  C2c  →  − 2
```

```
let string_of_char  =  function
    |  C1  →  "1" |  C2  →  "2" |  C1c  →  "-1" |  C2c  →  "-2"
```

```
let conj_char  =  function
    |  C1  →  C1c  |  C2  →  C2c  |  C1c  →  C1  |  C2c  →  C2
```

```
let string_of_neu  =  function
    |  N1  →  "1" |  N2  →  "2" |  N3  →  "3" |  N4  →  "4" |  N5  →  "5"
```

For the Higgs bosons, we follow the conventions of Franke/Fraas.

type *shiggs* =
  | *S1* | *S2* | *S3*

type *phiggs* =
  | *P1* | *P2*

let *string_of_shiggs* = function
  | *S1* → "1" | *S2* → "2" | *S3* → "3"

let *string_of_phiggs* = function
  | *P1* → "1" | *P2* → "2"

type *flavor* =
  | *L* of *int* | *N* of *int*
  | *U* of *int* | *D* of *int*
  | *Sup* of *sfm* × *int* | *Sdown* of *sfm* × *int*
  | *Ga* | *Wp* | *Wm* | *Z* | *Gl*
  | *Slepton* of *sfm* × *int* | *Sneutrino* of *int*
  | *Neutralino* of *neu* | *Chargino* of *char*
  | *Gluino*
  | *SHiggs* of *shiggs* | *Hp* | *Hm* | *PHiggs* of *phiggs*

let *string_of_fermion_type* = function
  | *L* _ → "l" | *U* _ → "u" | *D* _ → "d" | *N* _ → "n"
  | _ → *failwith* "Modellib_NMSSM.NMSSM.string_of_fermion_type:␣invalid␣fermion␣type"

let *string_of_fermion_gen* = function
  | *L g* | *U g* | *D g* | *N g* → *string_of_int* (*abs* (*g*))
  | _ → *failwith* "Modellib_NMSSM.NMSSM.string_of_fermion_gen:␣invalid␣fermion␣type"

type *gauge* = *unit*

let *gauge_symbol* () =
  failwith "Modellib_NMSSM.NMSSM.gauge_symbol:␣internal␣error"

At this point we will forget graviton and -ino.

let *family g* = [*L g*; *N g*; *Slepton* (*M1, g*);
              Slepton (*M2, g*); *Sneutrino g*;
              U g; *D g*; *Sup* (*M1, g*); *Sup* (*M2, g*);
              Sdown (*M1, g*); *Sdown* (*M2, g*)]

let *external_flavors* () =
    ["1st␣Generation␣matter", *ThoList.flatmap family* [1; − 1];
      "2nd␣Generation␣matter", *ThoList.flatmap family* [2; − 2];
      "3rd␣Generation␣matter", *ThoList.flatmap family* [3; − 3];
      "Gauge␣Bosons", [*Ga*; *Z*; *Wp*; *Wm*; *Gl*];
      "Charginos", [*Chargino C1*; *Chargino C2*; *Chargino C1c*; *Chargino C2c*];
      "Neutralinos", [*Neutralino N1*; *Neutralino N2*; *Neutralino N3*;
                           Neutralino N4; *Neutralino N5*];
      "Higgs␣Bosons", [*SHiggs S1*; *SHiggs S2*; *SHiggs S3*; *Hp*; *Hm*; *PHiggs P1*; *PHiggs P2*];
      "Gluino", [*Gluino*]]

let *flavors* () = *ThoList.flatmap snd* (*external_flavors* ())

let *spinor n m* =
  if *n* ≥ 0 ∧ *m* ≥ 0 then

   Spinor
  else if
   $n \leq 0 \wedge m \leq 0$ then
   ConjSpinor
  else
   invalid_arg "Modellib_NMSSM.NMSSM.spinor:␣internal␣error"

let *lorentz* = function
 | *L g* → *spinor g* 0 | *N g* → *spinor g* 0
 | *U g* → *spinor g* 0 | *D g* → *spinor g* 0
 | *Chargino c* → *spinor* (*int_of_char c*) 0
 | *Ga* | *Gl* → *Vector*
 | *Wp* | *Wm* | *Z* → *Massive_Vector*
 | *SHiggs* _ | *PHiggs* _ | *Hp* | *Hm*
 | *Sup* _ | *Sdown* _ | *Slepton* _ | *Sneutrino* _ → *Scalar*
 | *Neutralino* _ | *Gluino* → *Majorana*

let *color* = function
 | *U g* → *Color.SUN* (if $g > 0$ then 3 else -3)
 | *Sup* (m, g) → *Color.SUN* (if $g > 0$ then 3 else -3)
 | *D g* → *Color.SUN* (if $g > 0$ then 3 else -3)
 | *Sdown* (m, g) → *Color.SUN* (if $g > 0$ then 3 else -3)
 | *Gl* | *Gluino* → *Color.AdjSUN* 3
 | _ → *Color.Singlet*

let *prop_spinor n m* =
 if $n \geq 0 \wedge m \geq 0$ then
  Prop_Spinor
 else if
  $n \leq 0 \wedge m \leq 0$ then
  Prop_ConjSpinor
 else
  invalid_arg "Modellib_NMSSM.NMSSM.prop_spinor:␣internal␣error"

let *propagator* = function
 | *L g* → *prop_spinor g* 0 | *N g* → *prop_spinor g* 0
 | *U g* → *prop_spinor g* 0 | *D g* → *prop_spinor g* 0
 | *Chargino c* → *prop_spinor* (*int_of_char c*) 0
 | *Ga* | *Gl* → *Prop_Feynman*
 | *Wp* | *Wm* | *Z* → *Prop_Unitarity*
 | *SHiggs* _ | *PHiggs* _ → *Prop_Scalar*
 | *Hp* | *Hm* → *Prop_Scalar*
 | *Sup* _ | *Sdown* _ | *Slepton* _ | *Sneutrino* _ → *Prop_Scalar*
 | *Gluino* → *Prop_Majorana*
 | *Neutralino* _ → *Prop_Majorana*

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

let *width f* =
 if !*use_fudged_width* then
  match *f* with
  | *Wp* | *Wm* | *U* 3 | *U* (-3) → *Fudged*

```
      | _  →  !default_width
    else
      !default_width

  let goldstone _ =  None

  let conjugate  = function
    | L g  →  L (−g) | N g  →  N (−g)
    | U g  →  U (−g) | D g  →  D (−g)
    | Sup (m, g)  →  Sup (m, −g)
    | Sdown (m, g)  →  Sdown (m, −g)
    | Slepton (m, g)  →  Slepton (m, −g)
    | Sneutrino g  →  Sneutrino (−g)
    | Gl  →  Gl | Ga  →  Ga | Z  →  Z
    | Wp  →  Wm | Wm  →  Wp
    | SHiggs s  →  SHiggs s
    | PHiggs p  →  PHiggs p
    | Hp  →  Hm | Hm  →  Hp
    | Gluino  →  Gluino
    | Neutralino n  →  Neutralino n | Chargino c  →  Chargino (conj_char c)

let fermion  = function
  | L g  →  if g > 0 then 1 else -1
  | N g  →  if g > 0 then 1 else -1
  | U g  →  if g > 0 then 1 else -1
  | D g  →  if g > 0 then 1 else -1
  | Gl | Ga | Z | Wp | Wm  →  0
  | SHiggs _ | Hp | Hm | PHiggs _  →  0
  | Neutralino _  →  2
  | Chargino c  →  if (int_of_char c) > 0 then 1 else -1
  | Sup _  →  0 | Sdown _  →  0
  | Slepton _  →  0 | Sneutrino _  →  0
  | Gluino  →  2

module Ch  =  Charges.QQ

let ( // )  =  Algebra.Small_Rational.make

let generation′  = function
  | 1  →  [ 1//1;  0//1;  0//1]
  | 2  →  [ 0//1;  1//1;  0//1]
  | 3  →  [ 0//1;  0//1;  1//1]
  | −1  →  [−1//1;  0//1;  0//1]
  | −2  →  [ 0//1;  −1//1;  0//1]
  | −3  →  [ 0//1;  0//1;  −1//1]
  | n  →  invalid_arg ("NMSSM.generation′:␣" ^ string_of_int n)

let generation f  =
  if Flags.ckm_present then
    []
  else
    match f with
    | L n | N n | U n | D n | Sup (_, n)
    | Sdown (_, n) | Slepton (_, n)
```

```
     | Sneutrino n → generation' n
     | _ → [0//1; 0//1; 0//1]
 let charge = function
     | L n → if n > 0 then -1//1 else 1//1
     | Slepton (_, n) → if n > 0 then -1//1 else 1//1
     | N n → 0//1
     | Sneutrino n → 0//1
     | U n → if n > 0 then 2//3 else -2//3
     | Sup (_, n) → if n > 0 then 2//3 else -2//3
     | D n → if n > 0 then -1//3 else 1//3
     | Sdown (_, n) → if n > 0 then -1//3 else 1//3
     | Gl | Ga | Z | Neutralino _ | Gluino → 0//1
     | Wp → 1//1
     | Wm → − 1//1
     | SHiggs _ | PHiggs _ → 0//1
     | Hp → 1//1
     | Hm → − 1//1
     | Chargino (C1 | C2) → 1//1
     | Chargino (C1c | C2c) → − 1//1

 let lepton = function
     | L n | N n → if n > 0 then 1//1 else -1//1
     | Slepton (_, n)
     | Sneutrino n → if n > 0 then 1//1 else -1//1
     | _ → 0//1

 let baryon = function
     | U n | D n → if n > 0 then 1//1 else -1//1
     | Sup (_, n) | Sdown (_, n) → if n > 0 then 1//1 else -1//1
     | _ → 0//1

 let charges f =
     [ charge f; lepton f; baryon f] @ generation f
```

We introduce a Boolean type vc as a pseudonym for Vertex Conjugator to distinguish between vertices containing complex mixing matrices like the CKM–matrix or the sfermion or neutralino/chargino–mixing matrices, which have to become complex conjugated. The true–option stands for the conjugated vertex, the false–option for the unconjugated vertex.

```
 type vc = bool

 type constant =
     | E | G
     | Mu (*lambda*¡s¿*) | Lambda
     | Q_lepton | Q_up | Q_down | Q_charg
     | G_Z | G_CC | G_CCQ of vc × int×int
     | G_NC_neutrino | G_NC_lepton | G_NC_up | G_NC_down
     | I_Q_W | I_G_ZWW | G_WWWW | G_ZZWW | G_PZWW |
 G_PPWW
     | G_SS | I_G_S | Gs
     | G_NZN of neu × neu | G_CZC of char×char
     | G_YUK_FFS of flavor × flavor × shiggs
```

    | *G_YUK_FFP* of *flavor* × *flavor* × *phiggs*
    | *G_YUK_LCN* of *int*
    | *G_YUK_UCD* of *int*×*int* | *G_YUK_DCU* of *int*×*int*
    | *G_NHC* of *vc* × *neu* × *char*
    | *G_YUK_C* of *vc* × *flavor* × *char*×*sff* × *sfm*
    | *G_YUK_Q* of *vc* × *int*×*flavor* × *char*×*sff* × *sfm*
    | *G_YUK_N* of *vc* × *flavor* × *neu* × *sff* × *sfm*
    | *G_YUK_G* of *vc* × *flavor* × *sff* × *sfm*
    | *G_NWC* of *neu* × *char* | *G_CWN* of *char*×*neu*
    | *G_CSC* of *char*×*char*×*shiggs*
    | *G_CPC* of *char*×*char*×*phiggs*
    | *G_WSQ* of *vc* × *int*×*int*×*sfm* × *sfm*
    | *G_SLSNW* of *vc* × *int*×*sfm*
    | *G_ZSF* of *sff* × *int*×*sfm* × *sfm*
    | *G_CICIS* of *neu* × *neu* × *shiggs*
    | *G_CICIP* of *neu* × *neu* × *phiggs*
    | *G_GH_WPC* of *phiggs* | *G_GH_WSC* of *shiggs*
    | *G_GH_ZSP* of *shiggs* × *phiggs* | *G_GH_WWS* of *shiggs*
    | *G_GH_ZZS* of *shiggs* | *G_GH_ZCC*
    | *G_GH_GaCC*
    | *G_GH4_ZZPP* of *phiggs* × *phiggs*
    | *G_GH4_ZZSS* of *shiggs* × *shiggs*
    | *G_GH4_ZZCC* | *G_GH4_GaGaCC*
    | *G_GH4_ZGaCC* | *G_GH4_WWCC*
    | *G_GH4_WWPP* of *phiggs* × *phiggs*
    | *G_GH4_WWSS* of *shiggs* × *shiggs*
    | *G_GH4_ZWSC* of *shiggs*
    | *G_GH4_GaWSC* of *shiggs*
    | *G_GH4_ZWPC* of *phiggs*
    | *G_GH4_GaWPC* of *phiggs*
    | *G_WWSFSF* of *sff* × *int*×*sfm* × *sfm*
    | *G_WPSLSN* of *vc* × *int*×*sfm*
    | *G_H3_SCC* of *shiggs*
    | *G_H3_SSS* of *shiggs* × *shiggs* × *shiggs*
    | *G_H3_SPP* of *shiggs* × *phiggs* × *phiggs*
    | *G_SFSFS* of *shiggs* × *sff* × *int*×*sfm* × *sfm*
    | *G_SFSFP* of *phiggs* × *sff* × *int*×*sfm* × *sfm*
    | *G_HSNSL* of *vc* × *int*×*sfm*
    | *G_HSUSD* of *vc* × *sfm* × *sfm* × *int*×*int*
    | *G_WPSUSD* of *vc* × *sfm* × *sfm* × *int*×*int*
    | *G_WZSUSD* of *vc* × *sfm* × *sfm* × *int*×*int*
    | *G_WZSLSN* of *vc* × *int*×*sfm* | *G_GlGlSQSQ*
    | *G_PPSFSF* of *sff*
    | *G_ZZSFSF* of *sff*×*int*×*sfm*×*sfm* | *G_ZPSFSF* of *sff*×*int*×*sfm*×*sfm*
    | *G_GlZSFSF* of *sff* × *int*×*sfm* × *sfm* | *G_GlPSQSQ*
    | *G_GlWSUSD* of *vc* × *sfm* × *sfm* × *int*×*int*
    | *G_GLUGLUA0* of *phiggs* | *G_GLUGLUH0* of *shiggs*

Two integer counters for the QCD and EW order of the couplings.

    type *orders* = *int* × *int*

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```
let orders  =  function
  | _  →  (0, 0)
```

$$\alpha_{\mathrm{QED}} = \frac{1}{137.0359895} \tag{13.111a}$$

$$\sin^2 \theta_w = 0.23124 \tag{13.111b}$$

Here we must perhaps allow for complex input parameters. So split them into their modulus and their phase. At first, we leave them real; the generalization to complex parameters is obvious.

```
let parameters ()  =
  { input  =  [];
    derived  =  [];
    derived_arrays  =  [] }

module F  =  Modeltools.Fusions (struct
  type f  =  flavor
  type c  =  constant
  let compare  =  compare
  let conjugate  =  conjugate
end)
```

For the couplings there are generally two possibilities concerning the sign of the covariant derivative.

$$\mathrm{CD}^{\pm} = \partial_\mu \pm \mathrm{i} g T^a A_\mu^a \tag{13.112}$$

The particle data group defines the signs consistently to be positive. Since the convention for that signs also influence the phase definitions of the gaugino/higgsino fields via the off-diagonal entries in their mass matrices it would be the best to adopt that convention.
** REVISED: Compatible with CD+. FB **

```
let electromagnetic_currents_3 g  =
    [ ((L (−g), Ga, L g), FBF (1, Psibar, V, Psi), Q_lepton);
      ((U (−g), Ga, U g), FBF (1, Psibar, V, Psi), Q_up);
      ((D (−g), Ga, D g), FBF (1, Psibar, V, Psi), Q_down)]
```

** REVISED: Compatible with CD+. FB**

```
let electromagnetic_sfermion_currents g m  =
    [ ((Ga, Slepton (m, −g), Slepton (m, g)), Vector_Scalar_Scalar 1, Q_lepton);
      ((Ga, Sup (m, −g), Sup (m, g)), Vector_Scalar_Scalar 1, Q_up);
      ((Ga, Sdown (m, −g), Sdown (m, g)), Vector_Scalar_Scalar 1, Q_down)]
```

** REVISED: Compatible with CD+. FB**

```
let electromagnetic_currents_2 c  =
  let cc  =  conj_char c in
    [ ((Chargino cc, Ga, Chargino c), FBF (1, Psibar, V, Psi), Q_charg) ]
```

** REVISED: Compatible with CD+. FB**

```
let neutral_currents g  =
    [ ((L (−g), Z, L g), FBF (1, Psibar, VA, Psi), G_NC_lepton);
```

$((N\ (-g),\ Z,\ N\ g),\ FBF\ (1,\ Psibar,\ VA,\ Psi),\ G\_NC\_neutrino);$
$((U\ (-g),\ Z,\ U\ g),\ FBF\ (1,\ Psibar,\ VA,\ Psi),\ G\_NC\_up);$
$((D\ (-g),\ Z,\ D\ g),\ FBF\ (1,\ Psibar,\ VA,\ Psi),\ G\_NC\_down)]$

$$\mathcal{L}_{\mathrm{CC}} = \mp\frac{g}{2\sqrt{2}}\sum_i \bar{\psi}_i\gamma^\mu(1-\gamma_5)(T^+W_\mu^+ + T^-W_\mu^-)\psi_i, \qquad (13.113)$$

where the sign corresponds to $CD_\pm$, respectively.
** REVISED: Compatible with CD+. **
Remark: The definition with the other sign compared to the SM files comes from the fact that $g_{cc} = 1/(2\sqrt{2})$ is used overwhelmingly often in the SUSY Feynman rules, so that JR decided to use a different definiton for g_cc in SM and MSSM.
* FB *

let *charged_currents g* =
[$((L\ (-g),\ Wm,\ N\ g),\ FBF\ ((-1),\ Psibar,\ VL,\ Psi),\ G\_CC);$
$((N\ (-g),\ Wp,\ L\ g),\ FBF\ ((-1),\ Psibar,\ VL,\ Psi),\ G\_CC)\ ]$

The quark with the inverted generation (the antiparticle) is the outgoing one, the other the incoming. The vertex attached to the outgoing up-quark contains the CKM matrix element *not* complex conjugated, while the vertex with the outgoing down-quark has the conjugated CKM matrix element.
** REVISED: Compatible with CD+. FB **

let *charged_quark_currents g h* =
[$((D\ (-g),\ Wm,\ U\ h),\ FBF\ ((-1),\ Psibar,\ VL,\ Psi),\ G\_CCQ$ (true,g,h));
$((U\ (-g),\ Wp,\ D\ h),\ FBF\ ((-1),\ Psibar,\ VL,\ Psi),\ G\_CCQ$ (false,h,g))]

** REVISED: Compatible with CD+.FB **

let *charged_chargino_currents n c* =
let *cc* = *conj_char c* in
[$((Chargino\ cc,\ Wp,\ Neutralino\ n),$
$FBF\ (1,\ Psibar,\ VLR,\ Chi),\ G\_CWN\ (c,n));$
$((Neutralino\ n,\ Wm,\ Chargino\ c),$
$FBF\ (1,\ Chibar,\ VLR,\ Psi),\ G\_NWC\ (n,c))\ ]$

** REVISED: Compatible with CD+. FB**

let *charged_slepton_currents g m* =
[$((Wm,\ Slepton\ (m,-g),\ Sneutrino\ g),\ Vector\_Scalar\_Scalar\ (-1),\ G\_SLSNW$
(true,g,m));
$((Wp,\ Slepton\ (m,g),\ Sneutrino\ (-g)),\ Vector\_Scalar\_Scalar\ 1,\ G\_SLSNW$
(false,g,m))]

** REVISED: Compatible with CD+. FB**

let *charged_squark_currents' g h m1 m2* =
[$((Wm,\ Sup\ (m1,g),\ Sdown\ (m2,-h)),\ Vector\_Scalar\_Scalar\ (-1),\ G\_WSQ$
(true,$g,h,m1,m2$));
$((Wp,\ Sup\ (m1,-g),\ Sdown\ (m2,h)),\ Vector\_Scalar\_Scalar\ 1,\ G\_WSQ$
(false,$g,h,m1,m2$))]
let *charged_squark_currents g h* =
List.flatten (Product.list2 (charged_squark_currents' g h) $[M1;M2]\ [M1;M2]$)

** REVISED: Compatible with CD+. FB **

    let *neutral_sfermion_currents′ g m1 m2 =*
      [((*Z, Slepton* (*m1, −g*), *Slepton* (*m2, g*)), *Vector_Scalar_Scalar* (−1),
         G_ZSF (*SL, g, m1, m2*));
       ((*Z, Sup* (*m1, −g*), *Sup* (*m2, g*)), *Vector_Scalar_Scalar* (−1),
         G_ZSF(*SU, g, m1, m2*));
       ((*Z, Sdown* (*m1, −g*), *Sdown* (*m2, g*)), *Vector_Scalar_Scalar* (−1),
         G_ZSF (*SD, g, m1, m2*))]
    let *neutral_sfermion_currents g =*
      List.flatten (*Product.list2* (*neutral_sfermion_currents′*
              g) [*M1; M2*] [*M1; M2*]) @
      [((*Z, Sneutrino* (−*g*), *Sneutrino g*), *Vector_Scalar_Scalar* (−1),
        G_ZSF (*SN, g, M1, M1*))]

** REVISED: Compatible with CD+. FB**

    let *neutral_Z* (*n, m*) =
      [((*Neutralino n, Z, Neutralino m*), *FBF* (1, *Chibar, VLR, Chi*),
        (*G_NZN* (*n, m*)))]

** REVISED: Compatible with CD+. FB**

    let *charged_Z c1 c2 =*
      let *cc1 = conj_char c1* in
      ((*Chargino cc1, Z, Chargino c2*), *FBF* ((−1), *Psibar, VA , Psi*),
        G_CZC (*c1, c2*))

** REVISED: Compatible with CD+. Remark: This is pure octet. FB**

    let *yukawa_v =*
      [(*Gluino, Gl, Gluino*), *FBF* (1, *Chibar, V, Chi*), *Gs*]

** REVISED: Independent of the sign of CD. **
** REVISED: Felix Braam: Compact version using new COMBOS + FF-
Couplings

    let *yukawa_higgs_FFS f s =*
      [((*conjugate f, SHiggs s, f*), *FBF* (1, *Psibar, S, Psi*),
        G_YUK_FFS (*conjugate f, f, s*))]
    let *yukawa_higgs_FFP f p =*
      [((*conjugate f, PHiggs p, f*), *FBF* (1, *Psibar, P, Psi*),
        G_YUK_FFP (*conjugate f ,f , p*))]
    let *yukawa_higgs_NLC g =*
      [((*N* (−*g*), *Hp, L g*), *FBF* (1, *Psibar, Coupling.SR, Psi*), *G_YUK_LCN g*);
       ((*L* (−*g*), *Hm, N g*), *FBF* (1, *Psibar, Coupling.SL, Psi*), *G_YUK_LCN g*)]

    let *yukawa_higgs g =*
      yukawa_higgs_NLC g @
      List.flatten (*Product.list2 yukawa_higgs_FFS* [*L g; U g; D g*] [*S1; S2; S3*]) @
      List.flatten (*Product.list2 yukawa_higgs_FFP* [*L g; U g; D g*] [*P1; P2*])

** REVISED: Independent of the sign of CD. FB**

    let *yukawa_higgs_quark* (*g, h*) =
      [((*U* (−*g*), *Hp, D h*), *FBF* (1, *Psibar, SLR, Psi*), *G_YUK_UCD* (*g, h*));

$$((D\ (-h),\ Hm,\ U\ g),\ FBF\ (1,\ Psibar,\ SLR,\ Psi),\ G\_YUK\_DCU\ (g,\ h))\,]$$

** REVISED: Compatible with CD+. **
** REVISED: Felix Braam: Compact version using new COMBOS

    let *yukawa_shiggs_2 c1 c2 s* =
      let *cc1* = *conj_char c1* in
      ((*Chargino cc1*, *SHiggs s*, *Chargino c2*), *FBF* (1, *Psibar*, *SLR*, *Psi*),
         G_CSC (*c1*, *c2*, *s*))

    let *yukawa_phiggs_2 c1 c2 p* =
      let *cc1* = *conj_char c1* in
      ((*Chargino cc1*, *PHiggs p*, *Chargino c2*), *FBF* (1, *Psibar*, *SLR*, *Psi*),
         G_CPC (*c1*, *c2*, *p*))

    let *yukawa_higgs_2* =
      Product.list3 yukawa_shiggs_2 [*C1*; *C2*] [*C1*; *C2*] [*S1*; *S2*; *S3*] @
      Product.list3 yukawa_phiggs_2 [*C1*; *C2*] [*C1*; *C2*] [*P1*; *P2*]

** REVISED: Compatible with CD+.FB **

    let *higgs_charg_neutr n c* =
      let *cc* = *conj_char c* in
    [((*Neutralino n*, *Hm*, *Chargino c*), *FBF* (−1, *Chibar*, *SLR*, *Psi*),
            G_NHC (false,*n*, *c*));
     ((*Chargino cc*, *Hp*, *Neutralino n*), *FBF* (−1, *Psibar*, *SLR*, *Chi*),
            G_NHC (true,*n*, *c*))]

** REVISED: Compatible with CD+. **
** REVISED: Felix Braam: Compact version using new COMBOS

    let *shiggs_neutr* (*n*, *m*, *s*) =
      ((*Neutralino n*, *SHiggs s*, *Neutralino m*), *FBF* (1, *Chibar*, *SLR*, *Chi*),
      G_CICIS (*n*, *m*, *s*))
    let *phiggs_neutr* (*n*, *m*, *p*) =
      ((*Neutralino n*, *PHiggs p*, *Neutralino m*), *FBF* (1, *Chibar*, *SLR*, *Chi*),
      G_CICIP (*n*, *m*, *p*))

    let *higgs_neutr* =
      List.map shiggs_neutr (*two_and_one* [*N1*; *N2*; *N3*; *N4*; *N5*] [*S1*; *S2*; *S3*]) @
      List.map phiggs_neutr (*two_and_one* [*N1*; *N2*; *N3*; *N4*; *N5*] [*P1*; *P2*])

** REVISED: Compatible with CD+. FB**

     let *yukawa_n_2 n m g* =
      [((*Neutralino n*, *Slepton* (*m*, −*g*), *L g*), *FBF* (1, *Chibar*, *SLR*, *Psi*),
         G_YUK_N (true,*L g*, *n*, *SL*, *m*));
       ((*L* (−*g*), *Slepton* (*m*, *g*), *Neutralino n*), *FBF* (1, *Psibar*, *SLR*, *Chi*),
         G_YUK_N (false,*L g*, *n*, *SL*, *m*));
       ((*Neutralino n*, *Sup* (*m*, −*g*), *U g*), *FBF* (1, *Chibar*, *SLR*, *Psi*),
         G_YUK_N (true,*U g*, *n*, *SU*, *m*));
       ((*U* (−*g*), *Sup* (*m*, *g*), *Neutralino n*), *FBF* (1, *Psibar*, *SLR*, *Chi*),
         G_YUK_N (false,*U g*, *n*, *SU*, *m*));
       ((*Neutralino n*, *Sdown* (*m*, −*g*), *D g*), *FBF* (1, *Chibar*, *SLR*, *Psi*),
         G_YUK_N (true,*D g*, *n*, *SD*, *m*));
       ((*D* (−*g*), *Sdown* (*m*, *g*), *Neutralino n*), *FBF* (1, *Psibar*, *SLR*, *Chi*),

$$G\_YUK\_N \ (\mathsf{false}, D \ g, n, SD, m)) \ ]$$

let *yukawa_n_3 n g* =
$$[\ ((Neutralino \ n, \ Sneutrino \ (-g), \ N \ g), \ FBF \ (1, \ Chibar, \ SLR, \ Psi),$$
$$G\_YUK\_N \ (\mathsf{true}, N \ g, n, SN, M1));$$
$$((N \ (-g), \ Sneutrino \ g, \ Neutralino \ n), \ FBF \ (1, \ Psibar, \ SLR, \ Chi),$$
$$G\_YUK\_N \ (\mathsf{false}, N \ g, \ n, SN, M1)) \ ]$$

let *yukawa_n_5 g m* =
$$[\ ((U \ (-g), \ Sup \ (m, g), \ Gluino), \ FBF \ (1, \ Psibar, \ SLR, \ Chi),$$
$$G\_YUK\_G \ (\mathsf{false}, U \ g, SU, m));$$
$$((D \ (-g), \ Sdown \ (m, g), \ Gluino), \ FBF \ (1, \ Psibar, \ SLR, \ Chi),$$
$$G\_YUK\_G \ (\mathsf{false}, D \ g, SD, m));$$
$$((Gluino, \ Sup \ (m, -g), \ U \ g), \ FBF \ (1, \ Chibar, \ SLR, \ Psi),$$
$$G\_YUK\_G \ (\mathsf{true}, U \ g, SU, m));$$
$$((Gluino, \ Sdown \ (m, -g), \ D \ g), \ FBF \ (1, \ Chibar, \ SLR, \ Psi),$$
$$G\_YUK\_G \ (\mathsf{true}, D \ g, SD, m))]$$

let *yukawa_n* =
List.flatten (*Product.list3 yukawa_n_2* [*N1*; *N2*; *N3*; *N4*; *N5*] [*M1*; *M2*] [1; 2; 3]) @
List.flatten (*Product.list2 yukawa_n_3* [*N1*; *N2*; *N3*; *N4*; *N5*] [1; 2; 3]) @
List.flatten (*Product.list2 yukawa_n_5* [1; 2; 3] [*M1*; *M2*])

** REVISED: Compatible with CD+.FB **

let *yukawa_c_2 c g* =
    let *cc* = *conj_char c* in
$$[\ ((L \ (-g), \ Sneutrino \ g, \ Chargino \ cc), \ BBB \ (1, \ Psibar, \ SLR,$$
$$Psibar), \ G\_YUK\_C \ (\mathsf{true}, L \ g, c, SN, M1));$$
$$((Chargino \ c, \ Sneutrino \ (-g), \ L \ g), \ PBP \ (1, \ Psi, \ SLR, \ Psi),$$
$$G\_YUK\_C \ (\mathsf{false}, L \ g, c, SN, M1)) \ ]$$

let *yukawa_c_3 c m g* =
    let *cc* = *conj_char c* in
$$[\ ((N \ (-g), \ Slepton \ (m, g), \ Chargino \ c), \ FBF \ (1, \ Psibar, \ SLR,$$
$$Psi), \ G\_YUK\_C \ (\mathsf{true}, N \ g, c, SL, m));$$
$$((Chargino \ cc, \ Slepton \ (m, -g), \ N \ g), \ FBF \ (1, \ Psibar, \ SLR,$$
$$Psi), \ G\_YUK\_C \ (\mathsf{false}, N \ g, c, SL, m)) \ ]$$

let *yukawa_c c* =
ThoList.flatmap (*yukawa_c_2 c*) [1; 2; 3] @
List.flatten (*Product.list2* (*yukawa_c_3 c*) [*M1*; *M2*] [1; 2; 3])

** REVISED: Compatible with CD+. FB**

let *yukawa_cq′ c* (*g, h*) *m* =
    let *cc* = *conj_char c* in
$$[\ ((Chargino \ c, \ Sup \ (m, -g), \ D \ h), \ PBP \ (1, \ Psi, \ SLR, \ Psi),$$
$$G\_YUK\_Q \ (\mathsf{false}, g, D \ h, c, SU, m));$$
$$((D \ (-h), \ Sup \ (m, g), \ Chargino \ cc), \ BBB \ (1, \ Psibar, \ SLR, \ Psibar),$$
$$G\_YUK\_Q \ (\mathsf{true}, g, D \ h, c, SU, m));$$
$$((Chargino \ cc, \ Sdown \ (m, -g), \ U \ h), \ FBF \ (1, \ Psibar, \ SLR, \ Psi),$$
$$G\_YUK\_Q \ (\mathsf{true}, g, U \ h, c, SD, m));$$
$$((U \ (-h), \ Sdown \ (m, g), \ Chargino \ c), \ FBF \ (1, \ Psibar, \ SLR, \ Psi),$$
$$G\_YUK\_Q \ (\mathsf{false}, g, U \ h, c, SD, m)) \ ]$$

let *yukawa_cq c* =
if *Flags.ckm_present* then

$List.flatten\ (Product.list2\ (yukawa\_cq'\ c)\ [(1,1);(1,2);(2,1);(2,2);(1,3);(2,3);(3,3);(3,2);(3,1)]\ [M$
else
$List.flatten\ (Product.list2\ (yukawa\_cq'\ c)\ [(1,1);(2,2);(3,3)]\ [M1;M2])$

** REVISED: Compatible with CD+. Remark: Singlet and octet gluon exchange. The coupling is divided by sqrt(2) to account for the correct normalization of the Lie algebra generators. **FB

   let *col_currents* g =
    [$((D\ (-g),\ Gl,\ D\ g),\ FBF\ ((-1),\ Psibar,\ V,\ Psi),\ Gs);$
     $((U\ (-g),\ Gl,\ U\ g),\ FBF\ ((-1),\ Psibar,\ V,\ Psi),\ Gs)]$

** REVISED: Compatible with CD+. Remark: Singlet and octet gluon exchange. The coupling is divided by sqrt(2) to account for the correct normalization of the Lie algebra generators. **FB

   let *chg* = function
   | $M1\ \rightarrow\ M2\ |\ M2\ \rightarrow\ M1$

   let *col_sfermion_currents* $g\ m$ =
    [$((Gl,\ Sup\ (m,-g),\ Sup\ (m,g)),\ Vector\_Scalar\_Scalar\ (-1),\ Gs);$
     $((Gl,\ Sdown\ (m,-g),\ Sdown\ (m,g)),\ Vector\_Scalar\_Scalar\ (-1),\ Gs)]$

** REVISED: Compatible with CD+. **FB

   let *triple_gauge* =
    [$((Ga,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_Q\_W);$
     $((Z,\ Wm,\ Wp),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_ZWW);$
     $((Gl,\ Gl,\ Gl),\ Gauge\_Gauge\_Gauge\ 1,\ I\_G\_S)]$

** REVISED: Independent of the sign of CD. **FB

   let *gauge4* = $Vector4\ [(2,\ C\_13\_42);\ (-1,\ C\_12\_34);\ (-1,\ C\_14\_23)]$
   let *minus_gauge4* = $Vector4\ [(-2,\ C\_13\_42);\ (1,\ C\_12\_34);\ (1,\ C\_14\_23)]$
   let *quartic_gauge* =
    [$(Wm,\ Wp,\ Wm,\ Wp),\ gauge4,\ G\_WWWW;$
     $(Wm,\ Z,\ Wp,\ Z),\ minus\_gauge4,\ G\_ZZWW;$
     $(Wm,\ Z,\ Wp,\ Ga),\ minus\_gauge4,\ G\_PZWW;$
     $(Wm,\ Ga,\ Wp,\ Ga),\ minus\_gauge4,\ G\_PPWW;$
     $(Gl,\ Gl,\ Gl,\ Gl),\ gauge4,\ G\_SS]$

The *Scalar_Vector_Vector* couplings do not depend on the choice of the sign of the covariant derivative since they are quadratic in the gauge couplings.
* Effective Higgs-Gluon-Gluon coupling. *

   let *gauge_higgs_GlGlS* s =
    $((SHiggs\ s,\ Gl,\ Gl),\ Dim5\_Scalar\_Gauge2\ 1,\ G\_GLUGLUH0\ s)$
   let *gauge_higgs_GlGlP* p =
    $((PHiggs\ p,\ Gl,\ Gl),\ Dim5\_Scalar\_Gauge2\_Skew\ 1,\ G\_GLUGLUA0\ p)$

** REVISED: Compatible with CD+. FB**
** Revision: 2005-03-10: first two vertices corrected. **
** REVISED: Compact version using new COMBOS
** REVISED: Couplings adjusted to FF-convention

   let *gauge_higgs_WPC* p =
    [$((Wm,\ Hp,\ PHiggs\ p),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_WPC\ p);$

$((Wp,\ Hm,\ PHiggs\ p),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_WPC\ p)]$
let *gauge_higgs_WSC s* =
$[((Wm,\ Hp,\ SHiggs\ s),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_WSC\ s);$
$((Wp,\ Hm,\ SHiggs\ s),\ Vector\_Scalar\_Scalar\ (-1),\ G\_GH\_WSC\ s)]$
let *gauge_higgs_ZSP s p* =
$[((Z,\ SHiggs\ s,\ PHiggs\ p),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_ZSP\ (s,p))]$
let *gauge_higgs_WWS s* =
$((SHiggs\ s,\ Wp,\ Wm),\ Scalar\_Vector\_Vector\ 1,\ G\_GH\_WWS\ s)$
let *gauge_higgs_ZZS s* =
$((SHiggs\ s,\ Z,\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_GH\_ZZS\ s)$
let *gauge_higgs_ZCC* =
$((Z,\ Hp,\ Hm),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_ZCC\ )$
let *gauge_higgs_GaCC* =
$((Ga,\ Hp,\ Hm),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_GaCC\ )$

let *gauge_higgs* =
ThoList.flatmap gauge_higgs_WPC [*P1*; *P2*] @
ThoList.flatmap gauge_higgs_WSC [*S1*; *S2*; *S3*] @
List.flatten (*Product.list2 gauge_higgs_ZSP* [*S1*; *S2*; *S3*] [*P1*; *P2*]) @
List.map gauge_higgs_WWS [*S1*; *S2*; *S3*] @
List.map gauge_higgs_ZZS [*S1*; *S2*; *S3*] @
[*gauge_higgs_ZCC*] @ [*gauge_higgs_GaCC*] @
(if *Flags.higgs_triangle* then
List.map gauge_higgs_GlGlS [*S1*; *S2*; *S3*] @
List.map gauge_higgs_GlGlP [*P1*; *P2*]
else
[])
** REVISED: Compact version using new COMBOS
** REVISED: Couplings adjusted to FF-convention

let *gauge_higgs4_ZZPP* $(p1, p2)$ =
$((PHiggs\ p1,\ PHiggs\ p2,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\_ZZPP\ (p1, p2))$

let *gauge_higgs4_ZZSS* $(s1, s2)$ =
$((SHiggs\ s1,\ SHiggs\ s2\ ,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\_ZZSS\ (s1, s2))$

let *gauge_higgs4_ZZCC* =
$((Hp,\ Hm,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\_ZZCC)$

let *gauge_higgs4_GaGaCC* =
$((Hp,\ Hm,\ Ga,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\_GaGaCC)$

let *gauge_higgs4_ZGaCC* =
$((Hp,\ Hm,\ Ga,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\_ZGaCC\ )$

let *gauge_higgs4_WWCC* =
$((Hp,\ Hm,\ Wp,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_GH4\_WWCC\ )$

let *gauge_higgs4_WWPP* $(p1, p2)$ =
$((PHiggs\ p1,\ PHiggs\ p2,\ Wp,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_GH4\_WWPP\ (p1, p2))$

let *gauge_higgs4_WWSS* $(s1, s2)$ =
$((SHiggs\ s1,\ SHiggs\ s2,\ Wp,\ Wm),\ Scalar2\_Vector2\ 1,\ G\_GH4\_WWSS\ (s1, s2))$

let *gauge_higgs4_ZWSC s* =
  [((Hp, SHiggs s, Wm, Z), Scalar2_Vector2 1, G_GH4_ZWSC s);
    ((Hm, SHiggs s, Wp, Z), Scalar2_Vector2 1, G_GH4_ZWSC s)]

let *gauge_higgs4_GaWSC s* =
  [((Hp, SHiggs s, Wm, Ga), Scalar2_Vector2 1, G_GH4_GaWSC s);
    ((Hm, SHiggs s, Wp, Ga), Scalar2_Vector2 1, G_GH4_GaWSC s)]

let *gauge_higgs4_ZWPC p* =
  [((Hp, PHiggs p, Wm, Z), Scalar2_Vector2 1, G_GH4_ZWPC p);
    ((Hm, PHiggs p, Wp, Z), Scalar2_Vector2 (−1), G_GH4_ZWPC p)]

let *gauge_higgs4_GaWPC p* =
  [((Hp, PHiggs p, Wm, Ga), Scalar2_Vector2 1, G_GH4_GaWPC p);
    ((Hm, PHiggs p, Wp, Ga), Scalar2_Vector2 (−1), G_GH4_GaWPC p)]

let *gauge_higgs4* =
  List.map gauge_higgs4_ZZPP (pairs [P1; P2]) @
  List.map gauge_higgs4_ZZSS (pairs [S1; S2; S3]) @
  [gauge_higgs4_ZZCC] @ [gauge_higgs4_GaGaCC] @
  [gauge_higgs4_ZGaCC] @ [gauge_higgs4_WWCC] @
  List.map gauge_higgs4_WWPP (pairs [P1; P2]) @
  List.map gauge_higgs4_WWSS (pairs [S1; S2; S3]) @
  ThoList.flatmap gauge_higgs4_ZWSC [S1; S2; S3] @
  ThoList.flatmap gauge_higgs4_GaWSC [S1; S2; S3] @
  ThoList.flatmap gauge_higgs4_ZWPC [P1; P2] @
  ThoList.flatmap gauge_higgs4_GaWPC [P1; P2]

FB*

let *gauge_sfermion4' g m1 m2* =
  [((Wp, Wm, Slepton (m1, g), Slepton (m2, −g)), Scalar2_Vector2 1,
      G_WWSFSF (SL, g, m1, m2));
    ((Z, Ga, Slepton (m1, g), Slepton (m2, −g)), Scalar2_Vector2 1,
      G_ZPSFSF (SL, g, m1, m2));
    ((Z, Z, Slepton (m1, g), Slepton (m2, −g)), Scalar2_Vector2 1,
      G_ZZSFSF(SL, g, m1, m2));
    ((Wp, Wm, Sup (m1, g), Sup (m2, −g)), Scalar2_Vector2 1, G_WWSFSF
      (SU, g, m1, m2));
    ((Wp, Wm, Sdown (m1, g), Sdown (m2, −g)), Scalar2_Vector2 1,
      G_WWSFSF(SD, g, m1, m2));
    ((Z, Z, Sup (m1, g), Sup (m2, −g)), Scalar2_Vector2 1, G_ZZSFSF
      (SU, g, m1, m2));
    ((Z, Z, Sdown (m1, g), Sdown (m2, −g)), Scalar2_Vector2 1, G_ZZSFSF
      (SD, g, m1, m2));
    ((Z, Ga, Sup (m1, g), Sup (m2, −g)), Scalar2_Vector2 1, G_ZPSFSF
      (SU, g, m1, m2));
    ((Z, Ga, Sdown (m1, g), Sdown (m2, −g)), Scalar2_Vector2 1, G_ZPSFSF
      (SD, g, m1, m2))]

let *gauge_sfermion4'' g m* =
  [((Wp, Ga, Slepton (m, g), Sneutrino (−g)), Scalar2_Vector2 1,
      G_WPSLSN (false,g, m));
    ((Wm, Ga, Slepton (m, −g), Sneutrino g), Scalar2_Vector2 1,

            G_WPSLSN (true,g,m));
        ((Wp, Z, *Slepton* (m, g), *Sneutrino* $(-g)$), *Scalar2_Vector2* 1,
            G_WZSLSN(false,g,m));
        ((Wm, Z, *Slepton* $(m, -g)$, *Sneutrino* g), *Scalar2_Vector2* 1,
            G_WZSLSN (true,g,m));
        ((Ga, Ga, *Slepton* (m, g), *Slepton* $(m, -g)$), *Scalar2_Vector2* 1,
            G_PPSFSF SL);
        ((Ga, Ga, *Sup* (m, g), *Sup* $(m, -g)$), *Scalar2_Vector2* 1, *G_PPSFSF SU*);
        ((Ga, Ga, *Sdown* (m, g), *Sdown* $(m, -g)$), *Scalar2_Vector2* 1, *G_PPSFSF SD*)]

     let *gauge_sfermion4* g =
      List.flatten (*Product.list2* (*gauge_sfermion4′* g) [*M1*; *M2*] [*M1*; *M2*]) @
      ThoList.flatmap (*gauge_sfermion4″* g) [*M1*; *M2*] @
      [((Wp, Wm, *Sneutrino* g, *Sneutrino* $(-g)$), *Scalar2_Vector2* 1, *G_WWSFSF*
         (SN, g, *M1*, *M1*));
       ((Z, Z, *Sneutrino* g, *Sneutrino* $(-g)$), *Scalar2_Vector2* 1, *G_ZZSFSF*
         (SN, g, *M1*, *M1*))]

** Added by Felix Braam. **

     let *gauge_squark4″* g h $m1$ $m2$ =
      [((Wp, Ga, *Sup* $(m1, -g)$, *Sdown* $(m2, h)$), *Scalar2_Vector2* 1, *G_WPSUSD*
         (false,$m1$, $m2$, g, h));
       ((Wm, Ga, *Sup* $(m1, g)$, *Sdown* $(m2, -h)$), *Scalar2_Vector2* 1, *G_WPSUSD*
         (true,$m1$, $m2$, g, h));
       ((Wp, Z, *Sup* $(m1, -g)$, *Sdown* $(m2, h)$), *Scalar2_Vector2* 1, *G_WZSUSD*
         (false,$m1$, $m2$, g, h));
       ((Wm, Z, *Sup* $(m1, g)$, *Sdown* $(m2, -h)$), *Scalar2_Vector2* 1, *G_WZSUSD*
         (true,$m1$, $m2$, g, h))]
     let *gauge_squark4′* g h = *List.flatten* (*Product.list2* (*gauge_squark4″* g h)
                               [*M1*; *M2*] [*M1*; *M2*])
     let *gauge_squark4* =
      if *Flags.ckm_present* then
       List.flatten (*Product.list2* *gauge_squark4′* [1; 2; 3] [1; 2; 3])
      else
       ThoList.flatmap (fun g → *gauge_squark4′* g g) [1; 2; 3]

*********************************FB*******************

     let *gluon_w_squark″* g h $m1$ $m2$ =
      [((Gl, Wp, *Sup* $(m1, -g)$, *Sdown* $(m2, h)$),
         Scalar2_Vector2 1, *G_GlWSUSD* (false,$m1$, $m2$, g, h));
       ((Gl, Wm, *Sup* $(m1, g)$, *Sdown* $(m2, -h)$),
         Scalar2_Vector2 1, *G_GlWSUSD* (true,$m1$, $m2$, g, h))]
     let *gluon_w_squark′* g h =
      List.flatten (*Product.list2* (*gluon_w_squark″* g h) [*M1*; *M2*] [*M1*; *M2*])
     let *gluon_w_squark* =
      if *Flags.ckm_present* then
       List.flatten (*Product.list2* *gluon_w_squark′* [1; 2; 3] [1; 2; 3])
      else
       ThoList.flatmap (fun g → *gluon_w_squark′* g g) [1; 2; 3]

********************************FB******************

let *gluon_gauge_squark′ g m1 m2* =
[((Gl, Z, Sup ($m1, g$), Sup ($m2, -g$)),
  $Scalar2\_Vector2$ 2, $G\_GlZSFSF$ ($SU, g, m1, m2$));
  ((Gl, Z, $Sdown$ ($m1, g$), $Sdown$ ($m2, -g$)),
  $Scalar2\_Vector2$ 2, $G\_GlZSFSF$ ($SD, g, m1, m2$))]
let *gluon_gauge_squark″ g m* =
[((Gl, Ga, Sup (m, g), Sup ($m, -g$)), $Scalar2\_Vector2$ 2, $G\_GlPSQSQ$);
  ((Gl, Ga, $Sdown$ (m, g), $Sdown$ ($m, -g$)), $Scalar2\_Vector2$ (-1), $G\_GlPSQSQ$)]

let *gluon_gauge_squark g* =
List.flatten (*Product.list2* (*gluon_gauge_squark′ g*) [*M1*; *M2*] [*M1*; *M2*]) @
ThoList.flatmap (*gluon_gauge_squark″ g*) [*M1*; *M2*]
(∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗FB∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗∗)

let *gluon2_squark2′ g m* =
[((Gl, Gl, Sup (m, g), Sup ($m, -g$)), $Scalar2\_Vector2$ 2, $G\_GlGlSQSQ$);
  ((Gl, Gl, $Sdown$ (m, g), $Sdown$ ($m, -g$)), $Scalar2\_Vector2$ 2, $G\_GlGlSQSQ$)]
let *gluon2_squark2 g* =
ThoList.flatmap (*gluon2_squark2′ g*) [*M1*; *M2*]

** REVISED: Independent of the sign of CD. *FB*
** REVISED: Compact version using new COMBOS
** REVISED: Couplings adjusted to FF-convention

let *higgs_SCC s* =
((Hp, Hm, $SHiggs$ s), $Scalar\_Scalar\_Scalar$ 1, G_H3_SCC s)
let *higgs_SSS* ($s1, s2, s3$) =
(($SHiggs$ $s1$, $SHiggs$ $s2$, $SHiggs$ $s3$), $Scalar\_Scalar\_Scalar$ 1,
G_H3_SSS ($s1, s2, s3$))
let *higgs_SPP* ($p1, p2, s$) =
(($SHiggs$ s, $PHiggs$ $p1$, $PHiggs$ $p2$), $Scalar\_Scalar\_Scalar$ 1,
G_H3_SPP ($s, p1, p2$))

let *higgs* =
List.map higgs_SCC [$S1$; $S2$; $S3$]@
List.map higgs_SSS (*triples* [$S1$; $S2$; $S3$])@
List.map higgs_SPP (*two_and_one* [$P1$; $P2$] [$S1$; $S2$; $S3$])

let *higgs4* = [ ]
(∗ The vertices of the type Higgs - Sfermion - Sfermion are independent of the
choice of the CD sign since they are quadratic in the gauge coupling. ∗)

** REVISED: Independent of the sign of CD. **

let *higgs_sneutrino′ s g* =
(($SHiggs$ s, $Sneutrino$ g, $Sneutrino$ ($-g$)), $Scalar\_Scalar\_Scalar$ 1,
G_SFSFS ($s, SN, g, M1, M1$))
let *higgs_sneutrino″ g m* =
[((Hp, $Sneutrino$ ($-g$), $Slepton$ (m, g)), $Scalar\_Scalar\_Scalar$ 1,
G_HSNSL (false,g, m));
((Hm, $Sneutrino$ g, $Slepton$ ($m, -g$)), $Scalar\_Scalar\_Scalar$ 1,
G_HSNSL (true,g, m))]
let *higgs_sneutrino* =
Product.list2 higgs_sneutrino′ [$S1$; $S2$; $S3$] [1; 2; 3] @

$$List.flatten \ (\ Product.list2 \ higgs\_sneutrino'' \ [1; 2; 3] \ [M1; M2] \ )$$

Under the assumption that there is no mixing between the left- and right-handed sfermions for the first two generations there is only a coupling of the form Higgs - sfermion1 - sfermion2 for the third generation. All the others are suppressed by m_f/M_W.

** REVISED: Independent of the sign of CD. **

```
let higgs_sfermion_S s g m1 m2 =
  [ ((SHiggs s, Slepton (m1, g), Slepton (m2, −g)), Scalar_Scalar_Scalar 1,
        G_SFSFS (s, SL, g, m1, m2));
    ((SHiggs s, Sup (m1, g), Sup (m2, −g)), Scalar_Scalar_Scalar 1,
        G_SFSFS (s, SU, g, m1, m2));
    ((SHiggs s, Sdown (m1, g), Sdown (m2, −g)), Scalar_Scalar_Scalar 1,
        G_SFSFS (s, SD, g, m1, m2))]
```

```
let higgs_sfermion' g m1 m2 =
  (higgs_sfermion_S S1 g m1 m2) @ (higgs_sfermion_S S2 g m1 m2) @ (higgs_sfermion_S S3 g m1
```

```
let higgs_sfermion_P p g m1 m2 =
  [ ((PHiggs p, Slepton (m1, g), Slepton (m2, −g)), Scalar_Scalar_Scalar 1,
        G_SFSFP (p, SL, g, m1, m2));
    ((PHiggs p, Sup (m1, g), Sup (m2, −g)), Scalar_Scalar_Scalar 1,
        G_SFSFP (p, SU, g, m1, m2));
    ((PHiggs p, Sdown (m1, g), Sdown (m2, −g)), Scalar_Scalar_Scalar 1,
        G_SFSFP (p, SD, g, m1, m2)) ]
```

```
let higgs_sfermion'' g m1 m2 =
  (higgs_sfermion_P P1 g m1 m2) @ (higgs_sfermion_P P2 g m1 m2)
let higgs_sfermion = List.flatten (Product.list3 higgs_sfermion' [1; 2; 3] [M1; M2] [M1; M2]) @
  List.flatten (Product.list3 higgs_sfermion'' [1; 2; 3] [M1; M2] [M1; M2])
```

** REVISED: Independent of the sign of CD. **

```
let higgs_squark' g h m1 m2 =
  [ ((Hp, Sup (m1, −g), Sdown (m2, h)), Scalar_Scalar_Scalar 1,
        G_HSUSD (false, m1, m2, g, h));
    ((Hm, Sup (m1, g), Sdown (m2, −h)), Scalar_Scalar_Scalar 1,
        G_HSUSD (true, m1, m2, g, h)) ]
let higgs_squark_a g h = higgs_squark' g h M1 M1
let higgs_squark_b (g, h) = List.flatten (Product.list2 (higgs_squark' g h)
                                          [M1; M2] [M1; M2])
```

```
let higgs_squark =
  if Flags.ckm_present then
    List.flatten (Product.list2 higgs_squark_a [1; 2] [1; 2]) @
    ThoList.flatmap higgs_squark_b [(1, 3); (2, 3); (3, 3); (3, 1); (3, 2)]
  else
    higgs_squark_a 1 1 @ higgs_squark_a 2 2 @ higgs_squark_b (3, 3)
```

```
let vertices3 =
  (ThoList.flatmap electromagnetic_currents_3 [1; 2; 3] @
   ThoList.flatmap electromagnetic_currents_2 [C1; C2] @
   List.flatten (Product.list2 electromagnetic_sfermion_currents [1; 2; 3]
                   [M1; M2]) @
```

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      ThoList.flatmap neutral_currents $[1; 2; 3]$ @
      ThoList.flatmap neutral_sfermion_currents $[1; 2; 3]$ @
      ThoList.flatmap charged_currents $[1; 2; 3]$ @
      List.flatten (*Product.list2 charged_slepton_currents* $[1; 2; 3]$
                       $[M1; M2]$) @
     (if *Flags.ckm_present* then
      List.flatten (*Product.list2 charged_quark_currents* $[1; 2; 3]$
                     $[1; 2; 3]$) @
      List.flatten (*Product.list2 charged_squark_currents* $[1; 2; 3]$
                     $[1; 2; 3]$) @
      ThoList.flatmap yukawa_higgs_quark $[(1, 3); (2, 3); (3, 3); (3, 1); (3, 2)]$
     else
      charged_quark_currents 1 1 @
      charged_quark_currents 2 2 @
      charged_quark_currents 3 3 @
      charged_squark_currents 1 1 @
      charged_squark_currents 2 2 @
      charged_squark_currents 3 3 @
      ThoList.flatmap yukawa_higgs_quark $[(3, 3)]$) @
    yukawa_higgs 3 @ *yukawa_n* @
    ThoList.flatmap yukawa_c $[C1; C2]$ @
    ThoList.flatmap yukawa_cq $[C1; C2]$ @
    List.flatten (*Product.list2 charged_chargino_currents* $[N1; N2; N3; N4; N5]$
                  $[C1; C2]$) @ *triple_gauge* @
    ThoList.flatmap neutral_Z (*pairs* $[N1; N2; N3; N4; N5]$) @
    Product.list2 charged_Z $[C1; C2]$ $[C1; C2]$ @
    gauge_higgs @ *higgs* @ *yukawa_higgs_2* @
    List.flatten (*Product.list2 higgs_charg_neutr* $[N1; N2; N3; N4; N5]$ $[C1; C2]$) @
    higgs_neutr @ *higgs_sneutrino* @ *higgs_sfermion* @
    higgs_squark @ *yukawa_v* @
    ThoList.flatmap col_currents $[1; 2; 3]$ @
    List.flatten (*Product.list2 col_sfermion_currents* $[1; 2; 3]$ $[M1; M2]$))

  let *vertices4* =
    (*quartic_gauge* @ *higgs4* @ *gauge_higgs4* @
    ThoList.flatmap gauge_sfermion4 $[1; 2; 3]$ @
    gauge_squark4 @ *gluon_w_squark* @
    ThoList.flatmap gluon2_squark2 $[1; 2; 3]$ @
    ThoList.flatmap gluon_gauge_squark $[1; 2; 3]$)

  let *vertices* () = (*vertices3*, *vertices4*, $[\,]$)

  let *table* = *F.of_vertices* (*vertices* ())
  let *fuse2* = *F.fuse2 table*
  let *fuse3* = *F.fuse3 table*
  let *fuse* = *F.fuse table*
  let *max_degree* () = 4

SLHA2-Nomenclature for neutral Higgses

  let *flavor_of_string s* =
    match *s* with
      | "e-" $\to$ L 1 | "e+" $\to$ L (-1)

```
          | "mu-" →  L 2  |  "mu+" →  L (−2)
          | "tau-" →  L 3  |  "tau+" →  L (−3)
          | "nue" →  N 1  |  "nuebar" →  N (−1)
          | "numu" →  N 2  |  "numubar" →  N (−2)
          | "nutau" →  N 3  |  "nutaubar" →  N (−3)
          | "se1-" →  Slepton (M1, 1)  |  "se1+" →  Slepton (M1, −1)
          | "smu1-" →  Slepton (M1, 2)  |  "smu1+" →  Slepton (M1, −2)
          | "stau1-" →  Slepton (M1, 3)  |  "stau1+" →  Slepton (M1, −3)
          | "se2-" →  Slepton (M2, 1)  |  "se2+" →  Slepton (M2, −1)
          | "smu2-" →  Slepton (M2, 2)  |  "smu2+" →  Slepton (M2, −2)
          | "stau2-" →  Slepton (M2, 3)  |  "stau2+" →  Slepton (M2, −3)
          | "snue" →  Sneutrino 1  |  "snue*" →  Sneutrino (−1)
          | "snumu" →  Sneutrino 2  |  "snumu*" →  Sneutrino (−2)
          | "snutau" →  Sneutrino 3  |  "snutau*" →  Sneutrino (−3)
          | "u" →  U 1  |  "ubar" →  U (−1)
          | "c" →  U 2  |  "cbar" →  U (−2)
          | "t" →  U 3  |  "tbar" →  U (−3)
          | "d" →  D 1  |  "dbar" →  D (−1)
          | "s" →  D 2  |  "sbar" →  D (−2)
          | "b" →  D 3  |  "bbar" →  D (−3)
          | "A" →  Ga  |  "Z"  |  "Z0" →  Z
          | "W+" →  Wp  |  "W-" →  Wm
          | "gl"  |  "g" →  Gl
          | "h01" →  SHiggs S1  |  "h02" →  SHiggs S2  |  "h03" →
SHiggs S3
          | "A01" →  PHiggs P1  |  "A02" →  PHiggs P2
          | "H+" →  Hp  |  "H-" →  Hm
          | "su1" →  Sup (M1, 1)  |  "su1c" →  Sup (M1, −1)
          | "sc1" →  Sup (M1, 2)  |  "sc1c" →  Sup (M1, −2)
          | "st1" →  Sup (M1, 3)  |  "st1c" →  Sup (M1, −3)
          | "su2" →  Sup (M2, 1)  |  "su2c" →  Sup (M2, −1)
          | "sc2" →  Sup (M2, 2)  |  "sc2c" →  Sup (M2, −2)
          | "st2" →  Sup (M2, 3)  |  "st2c" →  Sup (M2, −3)
          | "sgl"  |  "sg" →  Gluino
          | "sd1" →  Sdown (M1, 1)  |  "sd1c" →  Sdown (M1, −1)
          | "ss1" →  Sdown (M1, 2)  |  "ss1c" →  Sdown (M1, −2)
          | "sb1" →  Sdown (M1, 3)  |  "sb1c" →  Sdown (M1, −3)
          | "sd2" →  Sdown (M2, 1)  |  "sd2c" →  Sdown (M2, −1)
          | "ss2" →  Sdown (M2, 2)  |  "ss2c" →  Sdown (M2, −2)
          | "sb2" →  Sdown (M2, 3)  |  "sb2c" →  Sdown (M2, −3)
          | "neu1" →  Neutralino N1  |  "neu2" →  Neutralino N2
          | "neu3" →  Neutralino N3  |  "neu4" →  Neutralino N4
          | "neu5" →  Neutralino N5
          | "ch1+" →  Chargino C1  |  "ch2+" →  Chargino C2
          | "ch1-" →  Chargino C1c  |  "ch2-" →  Chargino C2c
          | s →  invalid_arg ("Fatal error:_%s_Modellib_NMSSM.NMSSM.flavor_of_string:" ˆ s)

  let flavor_to_string = function
    | L 1 →  "e-"  |  L (−1) →  "e+"
    | L 2 →  "mu-"  |  L (−2) →  "mu+"
```

```
| L 3 → "tau-" | L (−3) → "tau+"
| N 1 → "nue" | N (−1) → "nuebar"
| N 2 → "numu" | N (−2) → "numubar"
| N 3 → "nutau" | N (−3) → "nutaubar"
| U 1 → "u" | U (−1) → "ubar"
| U 2 → "c" | U (−2) → "cbar"
| U 3 → "t" | U (−3) → "tbar"
| U _ → invalid_arg
      "Modellib_NMSSM.NMSSM.flavor_to_string:␣invalid␣up␣type␣quark"
| D 1 → "d" | D (−1) → "dbar"
| D 2 → "s" | D (−2) → "sbar"
| D 3 → "b" | D (−3) → "bbar"
| D _ → invalid_arg
      "Modellib_NMSSM.NMSSM.flavor_to_string:␣invalid␣down␣type␣quark"
| Gl → "gl" | Gluino → "sgl"
| Ga → "A" | Z → "Z"
| Wp → "W+" | Wm → "W-"
| SHiggs S1 → "h01" | SHiggs S2 → "h02" | SHiggs S3 → "h03"
| PHiggs P1 → "A01" | PHiggs P2 → "A02"
| Hp → "H+" | Hm → "H-"
| Slepton (M1, 1) → "se1-" | Slepton (M1, −1) → "se1+"
| Slepton (M1, 2) → "smu1-" | Slepton (M1, −2) → "smu1+"
| Slepton (M1, 3) → "stau1-" | Slepton (M1, −3) → "stau1+"
| Slepton (M2, 1) → "se2-" | Slepton (M2, −1) → "se2+"
| Slepton (M2, 2) → "smu2-" | Slepton (M2, −2) → "smu2+"
| Slepton (M2, 3) → "stau2-" | Slepton (M2, −3) → "stau2+"
| Sneutrino 1 → "snue" | Sneutrino (−1) → "snue*"
| Sneutrino 2 → "snumu" | Sneutrino (−2) → "snumu*"
| Sneutrino 3 → "snutau" | Sneutrino (−3) → "snutau*"
| Sup (M1, 1) → "su1" | Sup (M1, −1) → "su1c"
| Sup (M1, 2) → "sc1" | Sup (M1, −2) → "sc1c"
| Sup (M1, 3) → "st1" | Sup (M1, −3) → "st1c"
| Sup (M2, 1) → "su2" | Sup (M2, −1) → "su2c"
| Sup (M2, 2) → "sc2" | Sup (M2, −2) → "sc2c"
| Sup (M2, 3) → "st2" | Sup (M2, −3) → "st2c"
| Sdown (M1, 1) → "sd1" | Sdown (M1, −1) → "sd1c"
| Sdown (M1, 2) → "ss1" | Sdown (M1, −2) → "ss1c"
| Sdown (M1, 3) → "sb1" | Sdown (M1, −3) → "sb1c"
| Sdown (M2, 1) → "sd2" | Sdown (M2, −1) → "sd2c"
| Sdown (M2, 2) → "ss2" | Sdown (M2, −2) → "ss2c"
| Sdown (M2, 3) → "sb2" | Sdown (M2, −3) → "sb2c"
| Neutralino N1 → "neu1"
| Neutralino N2 → "neu2"
| Neutralino N3 → "neu3"
| Neutralino N4 → "neu4"
| Neutralino N5 → "neu5"
| Chargino C1 → "ch1+" | Chargino C1c → "ch1-"
| Chargino C2 → "ch2+" | Chargino C2c → "ch2-"
| _ → invalid_arg "Modellib_NMSSM.NMSSM.flavor_to_string"
```

```
let flavor_to_TeX = function
  | L 1 → "e^-" | L (−1) → "e^+"
  | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
  | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
  | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
  | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
  | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
  | U 1 → "u" | U (−1) → "\\bar{u}"
  | U 2 → "c" | U (−2) → "\\bar{c}"
  | U 3 → "t" | U (−3) → "\\bar{t}"
  | D 1 → "d" | D (−1) → "\\bar{d}"
  | D 2 → "s" | D (−2) → "\\bar{s}"
  | D 3 → "b" | D (−3) → "\\bar{b}"
  | L _ → invalid_arg
      "Modellib_NMSSM.NMSSM.flavor_to_TeX:_invalid_lepton"
  | N _ → invalid_arg
      "Modellib_NMSSM.NMSSM.flavor_to_TeX:_invalid_neutrino"
  | U _ → invalid_arg
      "Modellib_NMSSM.NMSSM.flavor_to_TeX:_invalid_up_type_quark"
  | D _ → invalid_arg
      "Modellib_NMSSM.NMSSM.flavor_to_TeX:_invalid_down_type_quark"
  | Gl → "g" | Gluino → "\\widetilde{g}"
  | Ga → "\\gamma" | Z → "Z" | Wp → "W^+" | Wm → "W^-"
  | SHiggs S1 → "S_1" | SHiggs S2 → "S_2" | SHiggs S3 → "S_3"
  | PHiggs P1 → "P_1" | PHiggs P2 → "P_2"
  | Hp → "H^+" | Hm → "H^-"
  | Slepton (M1,1) → "\\widetilde{e}_1^-"
  | Slepton (M1,−1) → "\\widetilde{e}_1^+"
  | Slepton (M1,2) → "\\widetilde{\\mu}_1^-"
  | Slepton (M1,−2) → "\\widetilde{\\mu}_1^+"
  | Slepton (M1,3) → "\\widetilde{\\tau}_1^-"
  | Slepton (M1,−3) → "\\widetilde{\\tau}_1^+"
  | Slepton (M2,1) → "\\widetilde{e}_2^-"
  | Slepton (M2,−1) → "\\widetilde{e}_2^+"
  | Slepton (M2,2) → "\\widetilde{\\mu}_2^-"
  | Slepton (M2,−2) → "\\widetilde{\\mu}_2^+"
  | Slepton (M2,3) → "\\widetilde{\\tau}_2^-"
  | Slepton (M2,−3) → "\\widetilde{\\tau}_2^+"
  | Sneutrino 1 → "\\widetilde{\\nu}_e"
  | Sneutrino (−1) → "\\widetilde{\\nu}_e^*"
  | Sneutrino 2 → "\\widetilde{\\nu}_\\mu"
  | Sneutrino (−2) → "\\widetilde{\\nu}_\\mu^*"
  | Sneutrino 3 → "\\widetilde{\\nu}_\\tau"
  | Sneutrino (−3) → "\\widetilde{\\nu}_\\tau^*"
  | Sup (M1,1) → "\\widetilde{u}_1"
  | Sup (M1,−1) → "\\widetilde{u}_1^*"
  | Sup (M1,2) → "\\widetilde{c}_1"
  | Sup (M1,−2) → "\\widetilde{c}_1^*"
  | Sup (M1,3) → "\\widetilde{t}_1"
  | Sup (M1,−3) → "\\widetilde{t}_1^*"
```

```
    | Sup (M2, 1) → "\\widetilde{u}_2"
    | Sup (M2, −1) → "\\widetilde{u}_2^*"
    | Sup (M2, 2) → "\\widetilde{c}_2"
    | Sup (M2, −2) → "\\widetilde{c}_2^*"
    | Sup (M2, 3) → "\\widetilde{t}_2"
    | Sup (M2, −3) → "\\widetilde{t}_2^*"
    | Sdown (M1, 1) → "\\widetilde{d}_1"
    | Sdown (M1, −1) → "\\widetilde{d}_1^*"
    | Sdown (M1, 2) → "\\widetilde{s}_1"
    | Sdown (M1, −2) → "\\widetilde{s}_1^*"
    | Sdown (M1, 3) → "\\widetilde{b}_1"
    | Sdown (M1, −3) → "\\widetilde{b}_1^*"
    | Sdown (M2, 1) → "\\widetilde{d}_2"
    | Sdown (M2, −1) → "\\widetilde{d}_2^*"
    | Sdown (M2, 2) → "\\widetilde{s}_2"
    | Sdown (M2, −2) → "\\widetilde{s}_2^*"
    | Sdown (M2, 3) → "\\widetilde{b}_2"
    | Sdown (M2, −3) → "\\widetilde{b}_2^*"
    | Neutralino N1 → "\\widetilde{\\chi}^0_1"
    | Neutralino N2 → "\\widetilde{\\chi}^0_2"
    | Neutralino N3 → "\\widetilde{\\chi}^0_3"
    | Neutralino N4 → "\\widetilde{\\chi}^0_4"
    | Neutralino N5 → "\\widetilde{\\chi}^0_5"
    | Slepton _ → invalid_arg
          "Modellib_NMSSM.NMSSM.flavor_to_TeX:␣invalid␣slepton"
    | Sneutrino _ → invalid_arg
          "Modellib_NMSSM.NMSSM.flavor_to_TeX:␣invalid␣sneutrino"
    | Sup _ → invalid_arg
          "Modellib_NMSSM.NMSSM.flavor_to_TeX:␣invalid␣up␣type␣squark"
    | Sdown _ → invalid_arg
          "Modellib_NMSSM.NMSSM.flavor_to_TeX:␣invalid␣down␣type␣squark"
    | Chargino C1 → "\\widetilde{\\chi}_1^+"
    | Chargino C1c → "\\widetilde{\\chi}_1^-"
    | Chargino C2 → "\\widetilde{\\chi}_2^+"
    | Chargino C2c → "\\widetilde{\\chi}_2^-"

let flavor_symbol = function
  | L g when g > 0 → "l" ^ string_of_int g
  | L g → "l" ^ string_of_int (abs g) ^ "b"
  | N g when g > 0 → "n" ^ string_of_int g
  | N g → "n" ^ string_of_int (abs g) ^ "b"
  | U g when g > 0 → "u" ^ string_of_int g
  | U g → "u" ^ string_of_int (abs g) ^ "b"
  | D g when g > 0 → "d" ^ string_of_int g
  | D g → "d" ^ string_of_int (abs g) ^ "b"
  | Gl → "gl"
  | Ga → "a" | Z → "z"
  | Wp → "wp" | Wm → "wm"
  | Slepton (M1, g) when g > 0 → "sl1" ^ string_of_int g
  | Slepton (M1, g) → "sl1c" ^ string_of_int (abs g)
```

```
  | Slepton (M2, g) when g > 0 → "sl2" ^ string_of_int g
  | Slepton (M2, g) → "sl2c" ^ string_of_int (abs g)
  | Sneutrino g when g > 0 → "sn" ^ string_of_int g
  | Sneutrino g → "snc" ^ string_of_int (abs g)
  | Sup (M1, g) when g > 0 → "su1" ^ string_of_int g
  | Sup (M1, g) → "su1c" ^ string_of_int (abs g)
  | Sup (M2, g) when g > 0 → "su2" ^ string_of_int g
  | Sup (M2, g) → "su2c" ^ string_of_int (abs g)
  | Sdown (M1, g) when g > 0 → "sd1" ^ string_of_int g
  | Sdown (M1, g) → "sd1c" ^ string_of_int (abs g)
  | Sdown (M2, g) when g > 0 → "sd2" ^ string_of_int g
  | Sdown (M2, g) → "sd2c" ^ string_of_int (abs g)
  | Neutralino n → "neu" ^ (string_of_neu n)
  | Chargino c when (int_of_char c) > 0 → "cp" ^ string_of_char c
  | Chargino c → "cm" ^ string_of_int (abs (int_of_char c))
  | Gluino → "sgl"
  | SHiggs s → "h0" ^ (string_of_shiggs s)
  | PHiggs p → "A0" ^ (string_of_phiggs p)
  | Hp → "hp" | Hm → "hm"
let pdg = function
  | L g when g > 0 → 9 + 2 × g
  | L g → − 9 + 2 × g
  | N g when g > 0 → 10 + 2 × g
  | N g → − 10 + 2 × g
  | U g when g > 0 → 2 × g
  | U g → 2 × g
  | D g when g > 0 → − 1 + 2 × g
  | D g → 1 + 2 × g
  | Gl → 21
  | Ga → 22 | Z → 23
  | Wp → 24 | Wm → (−24)
  | SHiggs S1 → 25 | SHiggs S2 → 35 | SHiggs S3 → 45
  | PHiggs P1 → 36 | PHiggs P2 → 46
  | Hp → 37 | Hm → (−37)
  | Slepton (M1, g) when g > 0 → 1000009 + 2 × g
  | Slepton (M1, g) → − 1000009 + 2 × g
  | Slepton (M2, g) when g > 0 → 2000009 + 2 × g
  | Slepton (M2, g) → − 2000009 + 2 × g
  | Sneutrino g when g > 0 → 1000010 + 2 × g
  | Sneutrino g → − 1000010 + 2 × g
  | Sup (M1, g) when g > 0 → 1000000 + 2 × g
  | Sup (M1, g) → − 1000000 + 2 × g
  | Sup (M2, g) when g > 0 → 2000000 + 2 × g
  | Sup (M2, g) → − 2000000 + 2 × g
  | Sdown (M1, g) when g > 0 → 999999 + 2 × g
  | Sdown (M1, g) → − 999999 + 2 × g
  | Sdown (M2, g) when g > 0 → 1999999 + 2 × g
  | Sdown (M2, g) → − 1999999 + 2 × g
  | Gluino → 1000021
```

| Chargino C1 $\rightarrow$ 1000024 | Chargino C1c $\rightarrow$ (−1000024)
| Chargino C2 $\rightarrow$ 1000037 | Chargino C2c $\rightarrow$ (−1000037)
| Neutralino N1 $\rightarrow$ 1000022 | Neutralino N2 $\rightarrow$ 1000023
| Neutralino N3 $\rightarrow$ 1000025 | Neutralino N4 $\rightarrow$ 1000035
| Neutralino N5 $\rightarrow$ 1000045

We must take care of the pdg numbers for the two different kinds of sfermions in the MSSM. The particle data group in its Monte Carlo particle numbering scheme takes only into account mixtures of the third generation squarks and the stau. For the other sfermions we will use the number of the lefthanded field for the lighter mixed state and the one for the righthanded for the heavier. Below are the official pdg numbers from the Particle Data Group. In order not to produce arrays with some million entries in the Fortran code for the masses and the widths we introduce our private pdg numbering scheme which only extends not too far beyond 42. Our private scheme then has the following pdf numbers (for the sparticles the subscripts L/R and $1/2$ are taken synonymously):

d	down-quark	1
u	up-quark	2
s	strange-quark	3
c	charm-quark	4
b	bottom-quark	5
t	top-quark	6
e^-	electron	11
ν_e	electron-neutrino	12
μ^-	muon	13
ν_μ	muon-neutrino	14
τ^-	tau	15
ν_τ	tau-neutrino	16
g	gluon	(9) 21
γ	photon	22
Z^0	Z-boson	23
W^+	W-boson	24
h^0	light Higgs boson	25
H^0	heavy Higgs boson	35
A^0	pseudoscalar Higgs	36
H^+	charged Higgs	37
$\tilde{d}_L$	down-squark 1	41
$\tilde{u}_L$	up-squark 1	42
$\tilde{s}_L$	strange-squark 1	43
$\tilde{c}_L$	charm-squark 1	44
$\tilde{b}_L$	bottom-squark 1	45
$\tilde{t}_L$	top-squark 1	46
$\tilde{d}_R$	down-squark 2	47
$\tilde{u}_R$	up-squark 2	48
$\tilde{s}_R$	strange-squark 2	49
$\tilde{c}_R$	charm-squark 2	50
$\tilde{b}_R$	bottom-squark 2	51
$\tilde{t}_R$	top-squark 2	52
$\tilde{e}_L$	selectron 1	53
$\tilde{\nu}_{e,L}$	electron-sneutrino	54
$\tilde{\mu}_L$	smuon 1	55
$\tilde{\nu}_{\mu,L}$	muon-sneutrino	56
$\tilde{\tau}_L$	stau 1	57
$\tilde{\nu}_{\tau,L}$	tau-sneutrino	58
$\tilde{e}_R$	selectron 2	59
$\tilde{\mu}_R$	smuon 2	61
$\tilde{\tau}_R$	stau 2	63
$\tilde{g}$	gluino	64
$\tilde{\chi}_1^0$	neutralino 1	65
$\tilde{\chi}_2^0$	neutralino 2	66
$\tilde{\chi}_3^0$	neutralino 3	67
$\tilde{\chi}_4^0$	neutralino 4	68
$\tilde{\chi}_5^0$	neutralino 5	69
$\tilde{\chi}^+$	chargino 1	70

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let pdg_mw = function
  | L g when g > 0 → 9 + 2 × g
  | L g → − 9 + 2 × g
  | N g when g > 0 → 10 + 2 × g
  | N g → − 10 + 2 × g
  | U g when g > 0 → 2 × g
  | U g → 2 × g
  | D g when g > 0 → − 1 + 2 × g
  | D g → 1 + 2 × g
  | Gl → 21
  | Ga → 22 | Z → 23
  | Wp → 24 | Wm → (−24)
  | SHiggs S1 → 25 | SHiggs S2 → 35 | PHiggs P1 → 36
  | Hp → 37 | Hm → (−37)
  | Sup (M1, g) when g > 0 → 40 + 2 × g
  | Sup (M1, g) → − 40 + 2 × g
  | Sup (M2, g) when g > 0 → 46 + 2 × g
  | Sup (M2, g) → − 46 + 2 × g
  | Sdown (M1, g) when g > 0 → 39 + 2 × g
  | Sdown (M1, g) → − 39 + 2 × g
  | Sdown (M2, g) when g > 0 → 45 + 2 × g
  | Sdown (M2, g) → − 45 + 2 × g
  | Slepton (M1, g) when g > 0 → 51 + 2 × g
  | Slepton (M1, g) → − 51 + 2 × g
  | Slepton (M2, g) when g > 0 → 57 + 2 × g
  | Slepton (M2, g) → − 57 + 2 × g
  | Sneutrino g when g > 0 → 52 + 2 × g
  | Sneutrino g → − 52 + 2 × g
  | Gluino → 64
  | Chargino C1 → 70 | Chargino C1c → (−70)
  | Chargino C2 → 71 | Chargino C2c → (−71)
  | Neutralino N1 → 65 | Neutralino N2 → 66
  | Neutralino N3 → 67 | Neutralino N4 → 68
  | Neutralino N5 → 69
  | PHiggs P2 → 72 | SHiggs S3 → 73

let mass_symbol f =
  "mass(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let width_symbol f =
  "width(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let conj_symbol = function
  | false, str → str
  | true, str → str ^ "_c"

let constant_symbol = function
  | E → "e" | G → "g"
  | Mu → "mu" | Lambda → "lambda" | G_Z → "gz"
  | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
  | Q_charg → "qchar"
  | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
```

```
| G_NC_up → "gncup" | G_NC_down → "gncdwn"
| G_CC → "gcc"
| G_CCQ (vc, g1, g2) → conj_symbol (vc, "g_ccq" ) ^ "(" ^
    string_of_int g1 ^ "," ^ string_of_int g2 ^ ")"
| I_Q_W → "iqw" | I_G_ZWW → "igzww"
| G_WWWW → "gw4" | G_ZZWW → "gzzww"
| G_PZWW → "gpzww" | G_PPWW → "gppww"
| G_GH4_ZZPP (p1, p2) → "g_ZZA0A0(" ^ string_of_phiggs p1 ^ ","
    ^ string_of_phiggs p2 ^ ")"
| G_GH4_ZZSS (s1, s2) → "g_ZZh0h0(" ^ string_of_shiggs s1 ^ ","
    ^ string_of_shiggs s2 ^ ")"
| G_GH4_ZZCC → "g_zzhphm"
| G_GH4_GaGaCC → "g_AAhphm"
| G_GH4_ZGaCC → "g_zAhphm"
| G_GH4_WWCC → "g_wwhphm"
| G_GH4_WWPP (p1, p2) → "g_WWA0A0(" ^ string_of_phiggs p1 ^ "," ^
    string_of_phiggs p2 ^ ")"
| G_GH4_WWSS (s1, s2) → "g_WWh0h0(" ^ string_of_shiggs s1 ^ "," ^
    string_of_shiggs s2 ^ ")"
| G_GH4_ZWSC s → "g_ZWhph0(" ^ string_of_shiggs s ^")"
| G_GH4_GaWSC s → "g_AWhph0(" ^ string_of_shiggs s ^")"
| G_GH4_ZWPC p → "g_ZWhpA0(" ^ string_of_phiggs p ^")"
| G_GH4_GaWPC p → "g_AWhpA0(" ^ string_of_phiggs p ^")"
| G_CICIS (n1, n2, s) → "g_neuneuh0(" ^ string_of_neu n1 ^ "," ^
    string_of_neu n2 ^ "," ^ string_of_shiggs s ^ ")"
| G_CICIP (n1, n2, p) → "g_neuneuA0(" ^ string_of_neu n1 ^ "," ^
    string_of_neu n2 ^ "," ^ string_of_phiggs p ^ ")"
| G_H3_SCC s → "g_h0hphm(" ^ string_of_shiggs s ^ ")"
| G_H3_SPP (s, p1, p2) → "g_h0A0A0(" ^ string_of_shiggs s ^ "," ^
    string_of_phiggs p1 ^ "," ^ string_of_phiggs p2 ^ ")"
| G_H3_SSS (s1, s2, s3) → "g_h0h0h0(" ^ string_of_shiggs s1 ^ "," ^
    string_of_shiggs s2 ^ "," ^ string_of_shiggs s3 ^ ")"
| G_CSC (c1, c2, s) → "g_chchh0(" ^ string_of_char c1 ^ "," ^
    string_of_char c2 ^ "," ^ string_of_shiggs s ^")"
| G_CPC (c1, c2, p) → "g_chchA0(" ^ string_of_char c1 ^ "," ^
    string_of_char c2 ^ "," ^ string_of_phiggs p ^")"
| G_YUK_FFS (f1, f2, s) → "g_yuk_h0_" ^ string_of_fermion_type f1 ^
    string_of_fermion_type f2 ^ "(" ^ string_of_shiggs s ^ "," ^
    string_of_fermion_gen f1 ^ ")"
| G_YUK_FFP (f1, f2, p) → "g_yuk_A0_" ^ string_of_fermion_type f1 ^
    string_of_fermion_type f2 ^ "(" ^ string_of_phiggs p ^ "," ^
    string_of_fermion_gen f1 ^ ")"
| G_YUK_LCN g → "g_yuk_hp_ln(" ^ string_of_int g ^ ")"
| G_NWC (n, c) → "g_nwc(" ^ string_of_char c ^ "," ^ string_of_neu n
    ^ ")"
| G_CWN (c, n) → "g_cwn(" ^ string_of_char c ^ "," ^ string_of_neu n
    ^ ")"
| G_SLSNW (vc, g, m) → conj_symbol (vc, "g_wslsn") ^ "(" ^
    string_of_int g ^ "," ^ string_of_sfm m ^ ")"
| G_NZN (n1, n2) → "g_zneuneu(" ^ string_of_neu n1 ^ ","
```

```
              ^ string_of_neu n2 ^ ")"
  | G_CZC (c1, c2) → "g_zchch(" ^ string_of_char c1 ^ "," ^
      string_of_char c2 ^ ")"
  | Gs → "gs"
  | G_YUK_UCD (n, m) → "g_yuk_hp_ud(" ^ string_of_int n ^ "," ^
      string_of_int m ^ ")"
  | G_YUK_DCU (n, m) → "g_yuk_hm_du(" ^ string_of_int n ^ "," ^
      string_of_int m ^ ")"
  | G_YUK_N (vc, f, n, sf, m) → conj_symbol (vc, "g_yuk_neu_" ^
      string_of_fermion_type f ^ string_of_sff sf) ^ "(" ^
      string_of_fermion_gen f ^ "," ^ string_of_neu n ^ "," ^
      string_of_sfm m ^ ")"
  | G_YUK_G (vc, f, sf, m) → conj_symbol (vc, "g_yuk_gluino_" ^
      string_of_fermion_type f ^ string_of_sff sf) ^ "(" ^
      string_of_fermion_gen f ^ "," ^ string_of_sfm m ^ ")"
  | G_YUK_C (vc, f, c, sf, m) → conj_symbol (vc, "g_yuk_char_" ^
      string_of_fermion_type f ^ string_of_sff sf) ^ "(" ^
      string_of_fermion_gen f ^ "," ^ string_of_char c ^ "," ^
      string_of_sfm m ^ ")"
  | G_YUK_Q (vc, g1, f, c, sf, m) → conj_symbol (vc, "g_yuk_char_" ^
      string_of_fermion_type f ^ string_of_sff sf) ^ "(" ^ string_of_int
      g1 ^ "," ^ string_of_fermion_gen f ^ "," ^ string_of_char c ^ ","
      ^ string_of_sfm m ^ ")"
  | G_WPSUSD (vc, m1, m2, g1, g2) → conj_symbol (vc, "g_wA_susd") ^ "(" ^
      string_of_int g1 ^ "," ^ string_of_int g2 ^ "," ^ string_of_sfm m1 ^
      "," ^ string_of_sfm m2 ^ ")"
  | G_WZSUSD (vc, m1, m2, g1, g2) → conj_symbol (vc, "g_wz_susd") ^ "(" ^
      string_of_int g1 ^ "," ^ string_of_int g2 ^ "," ^ string_of_sfm m1 ^
      "," ^ string_of_sfm m2 ^ ")"
  | G_GH_ZSP (s, p) → "g_zh0a0(" ^ string_of_shiggs s ^ "," ^
      string_of_phiggs p ^ ")"
  | G_GH_WSC s → "g_Whph0(" ^ string_of_shiggs s ^ ")"
  | G_GH_WPC p → "g_WhpA0(" ^ string_of_phiggs p ^ ")"
  | G_GH_ZZS s → "g_ZZh0(" ^ string_of_shiggs s ^ ")"
  | G_GH_WWS s → "g_WWh0(" ^ string_of_shiggs s ^ ")"
  | G_GLUGLUH0 s → "g_glugluh0(" ^ string_of_shiggs s ^ ")"
  | G_GLUGLUA0 p → "g_gluglua0(" ^ string_of_phiggs p ^ ")"
  | G_GH_ZCC → "g_Zhmhp"
  | G_GH_GaCC → "g_Ahmhp"
  | G_ZSF (f, g, m1, m2) → "g_z" ^ string_of_sff f ^ string_of_sff f ^ "(" ^
      string_of_int g ^ "," ^ string_of_sfm m1 ^ "," ^ string_of_sfm m2
      ^ ")"
  | G_HSNSL (vc, g, m) → conj_symbol (vc, "g_hp_sl" ^ string_of_sfm m ^
      "sn1") ^ "(" ^ string_of_int g ^ ")"
  | G_GlGlSQSQ → "g_gg_sqsq"
  | G_PPSFSF f → "g_AA_" ^ string_of_sff f ^ string_of_sff f
  | G_ZZSFSF (f, g, m1, m2) → "g_zz_" ^ string_of_sff f ^ string_of_sff f ^
      "(" ^ string_of_int g ^"," ^ string_of_sfm m1
      ^ "," ^ string_of_sfm m2 ^ ")"
  | G_ZPSFSF (f, g, m1, m2) → "g_zA_" ^ string_of_sff f ^ string_of_sff f ^
```

  "(" ˆ *string_of_int g* ˆ"","ˆ *string_of_sfm m1*
  ˆ "," ˆ *string_of_sfm m2* ˆ ")"
 | *G_GlPSQSQ* → "g_gA_sqsq"
 | *G_GlZSFSF* (*f, g, m1, m2*) → "g_gz_" ˆ *string_of_sff f* ˆ *string_of_sff f* ˆ
  "(" ˆ *string_of_int g* ˆ "," ˆ *string_of_sfm m1* ˆ "," ˆ *string_of_sfm*
  m2 ˆ ")"
 | *G_GlWSUSD* (*vc, m1, m2, g1, g2*) → *conj_symbol* (*vc*, "g-gw_susd") ˆ "(" ˆ
  string_of_int g1 ˆ "," ˆ *string_of_int g2* ˆ "," ˆ *string_of_sfm m1* ˆ
  "," ˆ *string_of_sfm m2* ˆ ")"
 | *G_SS* → "gs**2"
 | *I_G_S* → "igs"
 | *G_NHC* (*vc, n, c*) → *conj_symbol*(*vc*,"g_neuhmchar") ˆ "(" ˆ
  string_of_neu n ˆ "," ˆ *string_of_char c* ˆ")"
 | *G_WWSFSF* (*f, g, m1, m2*) → "g_ww_" ˆ *string_of_sff f*
  ˆ *string_of_sff f* ˆ"(" ˆ *string_of_int g* ˆ "," ˆ *string_of_sfm m1* ˆ
  "," ˆ *string_of_sfm m2* ˆ ")"
 | *G_WPSLSN* (*vc, g, m*) → *conj_symbol* (*vc*, "g_wA_slsn") ˆ "(" ˆ
  string_of_int g ˆ "," ˆ *string_of_sfm m* ˆ ")"
 | *G_WZSLSN* (*vc, g, m*) → *conj_symbol* (*vc*, "g_wz_slsn") ˆ"("ˆ *string_of_int*
  g ˆ "," ˆ *string_of_sfm m* ˆ ")"
 | *G_SFSFS* (*s, f, g, m1, m2*) → "g_h0_"ˆ *string_of_sff f* ˆ *string_of_sfm m1*
  ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ "(" ˆ *string_of_shiggs s* ˆ ","
  ˆ *string_of_int g* ˆ ")"
 | *G_SFSFP* (*p, f, g, m1, m2*) → "g_A0_"ˆ *string_of_sff f* ˆ *string_of_sfm m1*
  ˆ *string_of_sff f* ˆ *string_of_sfm m2* ˆ "(" ˆ *string_of_phiggs p* ˆ ","
  ˆ *string_of_int g* ˆ ")"
 | *G_HSUSD* (*vc, m1, m2, g1, g2*) → *conj_symbol* (*vc*, "g_hp_su" ˆ *string_of_sfm*
  m1 ˆ "sd" ˆ *string_of_sfm m2*)ˆ "(" ˆ *string_of_int g1* ˆ ","
  ˆ *string_of_int g2* ˆ")"
 | *G_WSQ* (*vc, g1, g2, m1, m2*) → *conj_symbol* (*vc*, "g_wsusd") ˆ "("
  ˆ *string_of_int g1* ˆ "," ˆ *string_of_int g2* ˆ "," ˆ *string_of_sfm m1*
  ˆ "," ˆ *string_of_sfm m2* ˆ ")"

 end

13.11 Interface of Modellib_PSSSM

13.11.1 Extended Supersymmetric Models

We do not introduce the possibility here of using four point couplings or not.
We simply add the relevant and leave the rest out. No possibility for Goldstone
bosons is given. But we allow for CKM mixing.

module type *extMSSM_flags* =
 sig
  val *ckm_present* : *bool*
 end

module *PSSSM* : *extMSSM_flags*
module *PSSSM_QCD* : *extMSSM_flags*

module *ExtMSSM* : functor (*F* : *extMSSM_flags*) → *Model.T* with module *Ch* = *Charges.QQ*

13.12 Implementation of Modellib_PSSSM

let *rcs_file* = *RCS.parse* "Modellib_PSSSM" ["Extended␣SUSY␣models"]
    { *RCS.revision* = "$Revision:␣6465␣$";
      RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
      RCS.author = "$Author:␣jr_reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr

13.12.1 Extended Supersymmetric Standard Model(s)

This is based on the NMSSM implementation by Felix Braam, and extended to the exotica – leptoquarks, leptoquarkinos, additional Higgses etc. – by Daniel Wiesler. Note that for the Higgs sector vertices the conventions of the Franke/Fraas paper have been used.

module type *extMSSM_flags* =
  sig
    val *ckm_present* : *bool*
  end

module *PSSSM* : *extMSSM_flags* =
  struct
    let *ckm_present* = false
  end

module *PSSSM_QCD* : *extMSSM_flags* =
  struct
    let *ckm_present* = false
  end

module *ExtMSSM* (*Flags* : *extMSSM_flags*) =
  struct
    let *rcs* = *RCS.rename rcs_file* "Modellib_PSSSM.NMSSM"
      ["Extended␣SUSY␣models"]

    open *Coupling*

    let *default_width* = *ref Timelike*
    let *use_fudged_width* = *ref* false

    let *options* = *Options.create*
      ["constant_width", *Arg.Unit* (fun () → *default_width* := *Constant*),
        "use␣constant␣width␣(also␣in␣t-channel)";
        "fudged_width", *Arg.Set use_fudged_width*,
        "use␣fudge␣factor␣for␣charge␣particle␣width";
        "custom_width", *Arg.String* (fun *f* → *default_width* := *Custom f*),
        "use␣custom␣width";
        "cancel_widths", *Arg.Unit* (fun () → *default_width* := *Vanishing*),

```
        "use␣vanishing␣width"]
```

additional combinatorics

yields a list of tuples consistig of the off-diag combinations of the elements in
"set"

```
    let choose2 set  =
      List.map (function [x; y]  →  (x, y)  |  _  →  failwith "choose2")
        (Combinatorics.choose 2 set)
```

pairs

pairs appends the diagonal combinations to *choose2*

```
    let rec diag  =  function
      | []  →  []
      | x1  ::  rest  →  (x1,  x1)  ::  diag rest

    let pairs l  =  choose2 l @ diag l
```

triples

rank 3 generalization of *pairs*

```
   let rec cloop set i j k  =
      if i  >  ((List.length set) − 1) then []
      else if j  >  i then cloop set (succ i) (j − i − 1) (j − i − 1)
      else if k  >  j then cloop set i (succ j) (k − j − 1)
      else (List.nth set i, List.nth set j, List.nth set k) ::  cloop set i j (succ k)

    let triples set  =  cloop set 0 0 0
(∗ two_and_one ∗)
    let rec two_and_one′ l1  z  n  =
        if n  <  0 then []
        else
        ((fst (List.nth (pairs l1) n)), (snd (List.nth (pairs l1) n)), z) ::  two_and_one′ l1 z (pred n)

    let two_and_one l1 l2  =
        let f z  =  two_and_one′ l1 z ((List.length (pairs l1)) − 1)
        in
        List.flatten ( List.map f l2 )

    type gen  =
      |  G of int |  GG of gen × gen

    let rec string_of_gen  =  function
      |  G n when n  >  0  →  string_of_int n
      |  G n  →  string_of_int (abs n) ˆ "c"
      |  GG (g1, g2)  →  string_of_gen g1 ˆ "_" ˆ string_of_gen g2
```

With this we distinguish the flavour.

```
    type sff  =
      |  SL  |  SN  |  SU  |  SD

    let string_of_sff  =  function
      |  SL  →  "sl" |  SN  →  "sn" |  SU  →  "su" |  SD  →  "sd"
```

With this we distinguish the mass eigenstates. At the moment we have to cheat a little bit for the sneutrinos. Because we are dealing with massless neutrinos there is only one sort of sneutrino.

```
type sfm =
  | M1 | M2

let string_of_sfm = function
  | M1 → "1" | M2 → "2"
```

We also introduce special types for the charginos and neutralinos.

```
type char =
  | C1 | C2 | C1c | C2c | C3 | C3c | C4 | C4c

type neu =
  | N1 | N2 | N3 | N4 | N5 | N6 | N7 | N8 | N9 | N10 | N11

let int_of_char = function
  | C1 → 1 | C2 → 2 | C1c → −1 | C2c → −2
  | C3 → 3 | C4 → 4 | C3c → −3 | C4c → −4

let string_of_char c = string_of_int (int_of_char c)

let conj_char = function
  | C1 → C1c | C2 → C2c | C1c → C1 | C2c → C2
  | C3 → C3c | C4 → C4c | C3c → C3 | C4c → C4

let string_of_neu = function
  | N1 → "1" | N2 → "2" | N3 → "3" | N4 → "4" | N5 →
"5" | N6 → "6"
  | N7 → "7" | N8 → "8" | N9 → "9" | N10 → "10" | N11 → "11"
```

For NMSSM-like the Higgs bosons, we follow the conventions of Franke/Fraas. Daniel Wiesler: extended to E6 models.

```
type shiggs =
  | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9

type phiggs =
  | P1 | P2 | P3 | P4 | P5 | P6 | P7
```

HCx is always the H^+, $HCxc$ the H^-.

```
type chiggs =
  | HC1 | HC2 | HC3 | HC4 | HC5 | HC1c | HC2c | HC3c |
HC4c | HC5c

let conj_chiggs = function
  | HC1 → HC1c | HC2 → HC2c | HC1c → HC1 | HC2c → HC2
  | HC3 → HC3c | HC4 → HC4c | HC3c → HC3 | HC4c → HC4
  | HC5 → HC5c | HC5c → HC5

let string_of_shiggs = function
  | S1 → "1" | S2 → "2" | S3 → "3" | S4 → "4" | S5 → "5"
  | S6 → "6" | S7 → "7" | S8 → "8" | S9 → "9"

let string_of_phiggs = function
  | P1 → "1" | P2 → "2" | P3 → "3" | P4 → "4" | P5 → "5"
```

```
    | P6 → "6" | P7 → "7"
```

let *nlist* = [*N1*; *N2*; *N3*; *N4*; *N5*; *N6*; *N7*; *N8*; *N9*; *N10*; *N11*]
let *slist* = [*S1*; *S2*; *S3*; *S4*; *S5*; *S6*; *S7*; *S8*; *S9*]
let *plist* = [*P1*; *P2*; *P3*; *P4*; *P5*; *P6*; *P7*]
let *clist* = [*HC1*; *HC2*; *HC3*; *HC4*; *HC5*; *HC1c*; *HC2c*; *HC3c*; *HC4c*; *HC5c*]
let *charlist* = [*C1*; *C2*; *C3*; *C4*; *C1c*; *C2c*; *C3c*; *C4c*]

type *flavor* =
    | *L* of *int* | *N* of *int*
    | *U* of *int* | *D* of *int*
    | *Sup* of *sfm* × *int* | *Sdown* of *sfm* × *int*
    | *Ga* | *Wp* | *Wm* | *Z* | *Gl*
    | *Slepton* of *sfm* × *int* | *Sneutrino* of *int*
    | *Neutralino* of *neu* | *Chargino* of *char*
    | *Gluino* | *SHiggs* of *shiggs* | *PHiggs* of *phiggs*
    | *CHiggs* of *chiggs*
    | *LQ* of *sfm* × *int*
    | *LQino* of *int*

let *string_of_fermion_type* = function
    | *L* _ → "l" | *U* _ → "u" | *D* _ → "d" | *N* _ → "n"
    | _ → *failwith*
        "Modellib_PSSSM.ExtMSSM.string_of_fermion_type:␣invalid␣fermion␣type"

let *string_of_fermion_gen* = function
    | *L g* | *U g* | *D g* | *N g* → *string_of_int* (*abs* (*g*))
    | _ → *failwith*
        "Modellib_PSSSM.ExtMSSM.string_of_fermion_gen:␣invalid␣fermion␣type"

type *gauge* = *unit*

let *gauge_symbol* () =
    failwith "Modellib_PSSSM.ExtMSSM.gauge_symbol:␣internal␣error"

At this point we will forget graviton and -ino.

let *family g* = [*L g*; *N g*; *Slepton* (*M1, g*);
                Slepton (*M2, g*); *Sneutrino g*;
                U g; *D g*; *Sup* (*M1, g*); *Sup* (*M2, g*);
                Sdown (*M1, g*); *Sdown* (*M2, g*);
                LQ (*M1, g*); *LQ* (*M2, g*); *LQino g*]

let *external_flavors* () =
    ["1st␣Generation␣matter", *ThoList.flatmap family* [1; −1];
      "2nd␣Generation␣matter", *ThoList.flatmap family* [2; −2];
      "3rd␣Generation␣matter", *ThoList.flatmap family* [3; −3];
      "Gauge␣Bosons", [*Ga*; *Z*; *Wp*; *Wm*; *Gl*];
      "Charginos", *List.map* (fun *a* → *Chargino a*) *charlist*;
      "Neutralinos", *List.map* (fun *a* → *Neutralino a*) *nlist*;
      "Higgs␣Bosons", *List.map* (fun *a* → *SHiggs a*) *slist* @
                    List.map (fun *a* → *PHiggs a*) *plist* @
                    List.map (fun *a* → *CHiggs a*) *clist*;
      "Gluino", [*Gluino*]]
```

let *flavors* () = *ThoList.flatmap snd* (*external_flavors* ())

let *spinor* n m =
  if n ≥ 0 ∧ m ≥ 0 then
    *Spinor*
  else if
    n ≤ 0 ∧ m ≤ 0 then
    *ConjSpinor*
  else
    *invalid_arg* "Modellib_PSSSM.ExtMSSM.spinor:␣internal␣error"

let *lorentz* = function
  | *L g* → *spinor g* 0 | *N g* → *spinor g* 0
  | *U g* → *spinor g* 0 | *D g* → *spinor g* 0
  | *LQino g* → *spinor g* 0
  | *Chargino c* → *spinor* (*int_of_char c*) 0
  | *Ga* | *Gl* → *Vector*
  | *Wp* | *Wm* | *Z* → *Massive_Vector*
  | *SHiggs* _ | *PHiggs* _ | *CHiggs* _
  | *Sup* _ | *Sdown* _ | *Slepton* _ | *Sneutrino* _ | *LQ* _ → *Scalar*
  | *Neutralino* _ | *Gluino* → *Majorana*

let *color* = function
  | *U g* → *Color.SUN* (if g > 0 then 3 else -3)
  | *Sup* (m, g) → *Color.SUN* (if g > 0 then 3 else -3)
  | *D g* → *Color.SUN* (if g > 0 then 3 else -3)
  | *Sdown* (m, g) → *Color.SUN* (if g > 0 then 3 else -3)
  | *LQ* (m, g) → *Color.SUN* (if g > 0 then 3 else -3)
  | *LQino g* → *Color.SUN* (if g > 0 then 3 else -3)
  | *Gl* | *Gluino* → *Color.AdjSUN* 3
  | _ → *Color.Singlet*

let *prop_spinor* n m =
  if n ≥ 0 ∧ m ≥ 0 then
    *Prop_Spinor*
  else if
    n ≤ 0 ∧ m ≤ 0 then
    *Prop_ConjSpinor*
  else
    *invalid_arg* "Modellib_PSSSM.ExtMSSM.prop_spinor:␣internal␣error"

let *propagator* = function
  | *L g* → *prop_spinor g* 0 | *N g* → *prop_spinor g* 0
  | *U g* → *prop_spinor g* 0 | *D g* → *prop_spinor g* 0
  | *LQino g* → *prop_spinor g* 0
  | *Chargino c* → *prop_spinor* (*int_of_char c*) 0
  | *Ga* | *Gl* → *Prop_Feynman*
  | *Wp* | *Wm* | *Z* → *Prop_Unitarity*
  | *SHiggs* _ | *PHiggs* _ | *CHiggs* _ → *Prop_Scalar*
  | *Sup* _ | *Sdown* _ | *Slepton* _ | *Sneutrino* _ → *Prop_Scalar*
  | *LQ* _ → *Prop_Scalar*
  | *Gluino* → *Prop_Majorana*
  | *Neutralino* _ → *Prop_Majorana*

Optionally, ask for the fudge factor treatment for the widths of charged particles. Currently, this only applies to $W^{\pm}$ and top.

```
let width f  =
  if !use_fudged_width then
    match f with
    |  Wp  |  Wm  |  U 3  |  U (−3)  →  Fudged
    |  _  →  !default_width
  else
    !default_width
```

```
let goldstone _  =  None
```

```
let conjugate  = function
  |  L g  →  L (−g)  |  N g  →  N (−g)
  |  U g  →  U (−g)  |  D g  →  D (−g)
  |  Sup (m, g)  →  Sup (m, −g)
  |  Sdown (m, g)  →  Sdown (m, −g)
  |  Slepton (m, g)  →  Slepton (m, −g)
  |  Sneutrino g  →  Sneutrino (−g)
  |  Gl  →  Gl  |  Ga  →  Ga  |  Z  →  Z
  |  Wp  →  Wm  |  Wm  →  Wp
  |  SHiggs s  →  SHiggs s
  |  PHiggs p  →  PHiggs p
  |  CHiggs c  →  CHiggs (conj_chiggs c)
  |  Gluino  →  Gluino
  |  Neutralino n  →  Neutralino n  |  Chargino c  →  Chargino (conj_char c)
  |  LQino g  →  LQino (−g)
  |  LQ (m, g)  →  LQ (m, −g)
```

```
let fermion  = function
  |  L g  →  if g  >  0 then 1 else -1
  |  N g  →  if g  >  0 then 1 else -1
  |  U g  →  if g  >  0 then 1 else -1
  |  D g  →  if g  >  0 then 1 else -1
  |  Gl  |  Ga  |  Z  |  Wp  |  Wm  →  0
  |  SHiggs _  |  PHiggs _  |  CHiggs _  →  0
  |  Neutralino _  →  2
  |  Chargino c  →  if (int_of_char c)  >  0 then 1 else -1
  |  Sup _  →  0  |  Sdown _  →  0
  |  Slepton _  →  0  |  Sneutrino _  →  0
  |  Gluino  →  2
  |  LQ _  →  0
  |  LQino g  →  if g  >  0 then 1 else -1
```

This model does NOT have a conserved generation quantum number.

```
module Ch  =  Charges.QQ
```

```
let ( // )  =  Algebra.Small_Rational.make
```

```
let charge  = function
  |  L n  →  if n  >  0 then -1//1 else 1//1
  |  Slepton (_, n)  →  if n  >  0 then -1//1 else 1//1
```

```
    |  N n  →  0//1
    |  Sneutrino n  →  0//1
    |  U n  →  if n > 0 then 2//3 else -2//3
    |  Sup (_, n)  →  if n > 0 then 2//3 else -2//3
    |  D n  |  LQ (_, n)  |  LQino n  →  if n > 0 then -1//3 else 1//3
    |  Sdown (_, n)  →  if n > 0 then -1//3 else 1//3
    |  Gl  |  Ga  |  Z  |  Neutralino _  |  Gluino  →  0//1
    |  Wp  →  1//1
    |  Wm  →  − 1//1
    |  SHiggs _  |  PHiggs _  →  0//1
    |  CHiggs (HC1 | HC2 | HC3 | HC4 | HC5)  →  1//1
    |  CHiggs (HC1c | HC2c | HC3c | HC4c | HC5c)  →  − 1//1
    |  Chargino (C1 | C2 | C3 | C4)  →  1//1
    |  Chargino (C1c | C2c | C3c | C4c)  →  − 1//1

let lepton  =  function
    |  L n  |  N n  →  if n > 0 then 1//1 else -1//1
    |  Slepton (_, n)
    |  Sneutrino n  →  if n > 0 then 1//1 else -1//1
    |  LQ (_, n)  |  LQino n  →  if n > 0 then 1//1 else -1//1
    |  _  →  0//1

let baryon  =  function
    |  U n  |  D n  →  if n > 0 then 1//1 else -1//1
    |  Sup (_, n)  |  Sdown (_, n)  →  if n > 0 then 1//1 else -1//1
    |  LQ (_, n)  |  LQino n  →  if n > 0 then 1//1 else -1//1
    |  _  →  0//1

let charges f  =
    [ charge f; lepton f; baryon f]
```

We introduce a Boolean type vc as a pseudonym for Vertex Conjugator to distinguish between vertices containing complex mixing matrices like the CKM–matrix or the sfermion or neutralino/chargino–mixing matrices, which have to become complex conjugated. The true–option stands for the conjugated vertex, the false–option for the unconjugated vertex.

```
type vc  =  bool

type constant  =
    |  E  |  G
    |  Q_lepton  |  Q_up  |  Q_down  |  Q_charg
    |  G_Z  |  G_CC  |  G_CCQ of vc × int×int
    |  G_NC_neutrino  |  G_NC_lepton  |  G_NC_up  |  G_NC_down
    |  I_Q_W  |  I_G_ZWW  |  G_WWWW  |  G_ZZWW  |  G_PZWW  |
G_PPWW
    |  G_strong  |  G_SS  |  I_G_S
    |  Gs
    |  G_NZN of neu × neu  |  G_CZC of char×char
    |  G_YUK_FFS of flavor × flavor × shiggs
    |  G_YUK_FFP of flavor × flavor × phiggs
    |  G_YUK_LCN of int
    |  G_YUK_UCD of int×int  |  G_YUK_DCU of int×int
```

| G_NHC of $vc \times neu \times char$
| G_YUK_C of $vc \times flavor \times char \times sff \times sfm$
| G_YUK_Q of $vc \times int \times flavor \times char \times sff \times sfm$
| G_YUK_N of $vc \times flavor \times neu \times sff \times sfm$
| G_YUK_G of $vc \times flavor \times sff \times sfm$
| G_NWC of $neu \times char$ | G_CWN of $char \times neu$
| G_CSC of $char \times char \times shiggs$
| G_CPC of $char \times char \times phiggs$
| G_WSQ of $vc \times int \times int \times sfm \times sfm$
| G_SLSNW of $vc \times int \times sfm$
| G_ZSF of $sff \times int \times sfm \times sfm$
| G_CICIS of $neu \times neu \times shiggs$
| G_CICIP of $neu \times neu \times phiggs$
| G_GH_WPC of $phiggs$ | G_GH_WSC of $shiggs$
| G_GH_ZSP of $shiggs \times phiggs$ | G_GH_WWS of $shiggs$
| G_GH_ZZS of $shiggs$ | G_GH_ZCC
| G_GH_GaCC
| G_GH4_ZZPP of $phiggs \times phiggs$
| G_GH4_ZZSS of $shiggs \times shiggs$
| G_GH4_ZZCC | G_GH4_GaGaCC
| G_GH4_ZGaCC | G_GH4_WWCC
| G_GH4_WWPP of $phiggs \times phiggs$
| G_GH4_WWSS of $shiggs \times shiggs$
| G_GH4_ZWSC of $shiggs$
| G_GH4_GaWSC of $shiggs$
| G_GH4_ZWPC of $phiggs$
| G_GH4_GaWPC of $phiggs$
| G_WWSFSF of $sff \times int \times sfm \times sfm$
| G_WPSLSN of $vc \times int \times sfm$
| G_H3_SCC of $shiggs$
| G_H3_SSS of $shiggs \times shiggs \times shiggs$
| G_H3_SPP of $shiggs \times phiggs \times phiggs$
| G_SFSFS of $shiggs \times sff \times int \times sfm \times sfm$
| G_SFSFP of $phiggs \times sff \times int \times sfm \times sfm$
| G_HSNSL of $vc \times int \times sfm$
| G_HSUSD of $vc \times sfm \times sfm \times int \times int$
| G_WPSUSD of $vc \times sfm \times sfm \times int \times int$
| G_WZSUSD of $vc \times sfm \times sfm \times int \times int$
| G_WZSLSN of $vc \times int \times sfm$ | $G\_GlGlSQSQ$
| G_PPSFSF of sff
| G_ZZSFSF of $sff \times int \times sfm \times sfm$ | G_ZPSFSF of $sff \times int \times sfm \times sfm$
| $G\_GlZSFSF$ of $sff \times int \times sfm \times sfm$ | $G\_GlPSQSQ$
| $G\_GlWSUSD$ of $vc \times sfm \times sfm \times int \times int$
| $G\_YUK\_LQ\_S$ of $int \times shiggs \times int$
| $G\_YUK\_LQ\_P$ of $int \times phiggs \times int$
| G_LQ_NEU of $sfm \times int \times int \times neu$
| $G\_LQ\_EC\_UC$ of $vc \times sfm \times int \times int \times int$
| G_LQ_GG of $sfm \times int \times int$
| G_LQ_SSU of $sfm \times sfm \times sfm \times int \times int \times int$
| G_LQ_SSD of $sfm \times sfm \times int \times int \times int$

     | *G_LQ_S* of *sfm* × *sfm* × *int*×*shiggs* × *int*
     | *G_LQ_P* of *sfm* × *sfm* × *int*×*phiggs* × *int*
     | *G_ZLQ* of *int*×*sfm* × *sfm*
     | *G_ZZLQLQ* | *G_ZPLQLQ* | *G_PPLQLQ* | *G_ZGlLQLQ* |
G_PGlLQLQ | *G_NLQC* | *G_GlGlLQLQ*

Two integer counters for the QCD and EW order of the couplings.

    type *orders* = *int* × *int*

    let *orders* = function
     | _ → (0, 0)

$$\alpha_{\text{QED}} = \frac{1}{137.0359895} \tag{13.114a}$$

$$\sin^2 \theta_w = 0.23124 \tag{13.114b}$$

Here we must perhaps allow for complex input parameters. So split them into their modulus and their phase. At first, we leave them real; the generalization to complex parameters is obvious.

    let *parameters* () =
     { *input* = [];
      derived = [];
      derived_arrays = [] }

    module *F* = *Modeltools.Fusions* (struct
     type *f* = *flavor*
     type *c* = *constant*
     let *compare* = *compare*
     let *conjugate* = *conjugate*
    end)

For the couplings there are generally two possibilities concerning the sign of the covariant derivative.

$$\text{CD}^{\pm} = \partial_\mu \pm \mathrm{i} g T^a A_\mu^a \tag{13.115}$$

The particle data group defines the signs consistently to be positive. Since the convention for that signs also influence the phase definitions of the gaugino/higgsino fields via the off-diagonal entries in their mass matrices it would be the best to adopt that convention.
** REVISED: Compatible with CD+. FB **

    let *electromagnetic_currents_3 g* =
     [((*L* (−*g*), *Ga*, *L g*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_lepton*);
      ((*U* (−*g*), *Ga*, *U g*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_up*);
      ((*D* (−*g*), *Ga*, *D g*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

** REVISED: Compatible with CD+. FB**

    let *electromagnetic_sfermion_currents g m* =
     [((*Ga*, *Slepton* (*m*, −*g*), *Slepton* (*m*, *g*)), *Vector_Scalar_Scalar* 1, *Q_lepton*);
      ((*Ga*, *Sup* (*m*, −*g*), *Sup* (*m*, *g*)), *Vector_Scalar_Scalar* 1, *Q_up*);
      ((*Ga*, *Sdown* (*m*, −*g*), *Sdown* (*m*, *g*)), *Vector_Scalar_Scalar* 1, *Q_down*)]

** REVISED: Compatible with CD+. FB**

> let *electromagnetic_currents_2 c* =
> let *cc* = *conj_char c* in
> [((*Chargino cc, Ga, Chargino c*), *FBF* (1, *Psibar, V, Psi*), *Q_charg*)]

** REVISED: Compatible with CD+. FB**

> let *neutral_currents g* =
> [((*L* (−*g*), *Z, L g*), *FBF* (1, *Psibar, VA, Psi*), *G_NC_lepton*);
>   ((*N* (−*g*), *Z, N g*), *FBF* (1, *Psibar, VA, Psi*), *G_NC_neutrino*);
>   ((*U* (−*g*), *Z, U g*), *FBF* (1, *Psibar, VA, Psi*), *G_NC_up*);
>   ((*D* (−*g*), *Z, D g*), *FBF* (1, *Psibar, VA, Psi*), *G_NC_down*)]

$$\mathcal{L}_{\mathrm{CC}} = \mp \frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i \gamma^\mu (1 - \gamma_5)(T^+ W_\mu^+ + T^- W_\mu^-)\psi_i, \qquad (13.116)$$

where the sign corresponds to CD$_\pm$, respectively.
** REVISED: Compatible with CD+. **
Remark: The definition with the other sign compared to the SM files comes from the fact that $g_{cc} = 1/(2\sqrt{2})$ is used overwhelmingly often in the SUSY Feynman rules, so that JR decided to use a different definiton for *g_cc* in SM and MSSM.
* FB *

> let *charged_currents g* =
> [((*L* (−*g*), *Wm, N g*), *FBF* ((−1), *Psibar, VL, Psi*), *G_CC*);
>   ((*N* (−*g*), *Wp, L g*), *FBF* ((−1), *Psibar, VL, Psi*), *G_CC*)]

The quark with the inverted generation (the antiparticle) is the outgoing one, the other the incoming. The vertex attached to the outgoing up-quark contains the CKM matrix element *not* complex conjugated, while the vertex with the outgoing down-quark has the conjugated CKM matrix element.
** REVISED: Compatible with CD+. FB **

> let *charged_quark_currents g h* =
> [((*D* (−*g*), *Wm, U h*), *FBF* ((−1), *Psibar, VL, Psi*), *G_CCQ* (true,*g,h*));
>   ((*U* (−*g*), *Wp, D h*), *FBF* ((−1), *Psibar, VL, Psi*), *G_CCQ* (false,*h, g*))]

** REVISED: Compatible with CD+.FB **

> let *charged_chargino_currents n c* =
> let *cc* = *conj_char c* in
> [((*Chargino cc, Wp, Neutralino n*),
>   *FBF* (1, *Psibar, VLR, Chi*), *G_CWN* (*c, n*));
>   ((*Neutralino n, Wm, Chargino c*),
>   *FBF* (1, *Chibar, VLR, Psi*), *G_NWC* (*n, c*))]

** REVISED: Compatible with CD+. FB**

> let *charged_slepton_currents g m* =
> [((*Wm, Slepton* (*m*, −*g*), *Sneutrino g*), *Vector_Scalar_Scalar* (−1), *G_SLSNW*
>   (true,*g, m*));
>   ((*Wp, Slepton* (*m, g*), *Sneutrino* (−*g*)), *Vector_Scalar_Scalar* 1, *G_SLSNW*
>   (false,*g, m*))]

** REVISED: Compatible with CD+. FB**

> let *charged_squark_currents′ g h m1 m2* =
> [((*Wm*, *Sup* (*m1*, *g*), *Sdown* (*m2*, −*h*)), *Vector_Scalar_Scalar* (−1), *G_WSQ*
>   (true, *g*, *h*, *m1*, *m2*));
>   ((*Wp*, *Sup* (*m1*, −*g*), *Sdown* (*m2*, *h*)), *Vector_Scalar_Scalar* 1, *G_WSQ*
>   (false, *g*, *h*, *m1*, *m2*))]
> let *charged_squark_currents g h* =
> *List.flatten* (*Product.list2* (*charged_squark_currents′ g h*) [*M1*; *M2*] [*M1*; *M2*])

** REVISED: Compatible with CD+. FB **

> let *neutral_sfermion_currents′ g m1 m2* =
> [((*Z*, *Slepton* (*m1*, −*g*), *Slepton* (*m2*, *g*)), *Vector_Scalar_Scalar* (−1),
>   *G_ZSF* (*SL*, *g*, *m1*, *m2*));
>   ((*Z*, *Sup* (*m1*, −*g*), *Sup* (*m2*, *g*)), *Vector_Scalar_Scalar* (−1),
>   *G_ZSF* (*SU*, *g*, *m1*, *m2*));
>   ((*Z*, *Sdown* (*m1*, −*g*), *Sdown* (*m2*, *g*)), *Vector_Scalar_Scalar* (−1),
>   *G_ZSF* (*SD*, *g*, *m1*, *m2*))]
> let *neutral_sfermion_currents g* =
> *List.flatten* (*Product.list2* (*neutral_sfermion_currents′*
>   *g*) [*M1*; *M2*] [*M1*; *M2*]) @
> [((*Z*, *Sneutrino* (−*g*), *Sneutrino g*), *Vector_Scalar_Scalar* (−1),
>   *G_ZSF* (*SN*, *g*, *M1*, *M1*))]

The reality of the coupling of the Z-boson to two identical neutralinos makes
the vector part of the coupling vanish. So we distinguish them not by the name
but by the structure of the couplings.
** REVISED: Compatible with CD+. FB**

> let *neutral_Z* (*n*, *m*) =
> [((*Neutralino n*, *Z*, *Neutralino m*), *FBF* (1, *Chibar*, *VA*, *Chi*),
>   (*G_NZN* (*n*, *m*)))]

** REVISED: Compatible with CD+. FB**

> let *charged_Z c1 c2* =
> let *cc1* = *conj_char c1* in
> ((*Chargino cc1*, *Z*, *Chargino c2*), *FBF* ((−1), *Psibar*, *VA* , *Psi*),
>   *G_CZC* (*c1*, *c2*))

** REVISED: Compatible with CD+. Remark: This is pure octet. FB**

> let *yukawa_v* =
> [(*Gluino*, *Gl*, *Gluino*), *FBF* (1, *Chibar*, *V*, *Chi*), *Gs*]

** REVISED: Independent of the sign of CD. **
** REVISED: Felix Braam: Compact version using new COMBOS + FF-
Couplings

> let *yukawa_higgs_FFS f s* =
>   [((*conjugate f*, *SHiggs s*, *f*), *FBF* (1, *Psibar*, *S*, *Psi*),
>   *G_YUK_FFS* (*conjugate f*, *f*, *s*))]
> let *yukawa_higgs_FFP f p* =
>   [((*conjugate f*, *PHiggs p*, *f*), *FBF* (1, *Psibar*, *P*, *Psi*),
>   *G_YUK_FFP* (*conjugate f* , *f* , *p*))]

JR: Only the first charged Higgs.

```
let yukawa_higgs_NLC g =
  [ ((N (−g), CHiggs HC1, L g), FBF (1, Psibar, Coupling.SR, Psi),
        G_YUK_LCN g);
    ((L (−g), CHiggs HC1c, N g), FBF (1, Psibar, Coupling.SL, Psi),
        G_YUK_LCN g)]
```

```
let yukawa_higgs g =
  yukawa_higgs_NLC g @
  List.flatten ( Product.list2 yukawa_higgs_FFS [L g; U g; D g] [S1; S2; S3]) @
  List.flatten ( Product.list2 yukawa_higgs_FFP [L g; U g; D g] [P1; P2])
```

JR: Only the first charged Higgs.
** REVISED: Independent of the sign of CD. FB**

```
let yukawa_higgs_quark (g, h) =
  [ ((U (−g), CHiggs HC1, D h), FBF (1, Psibar, SLR, Psi),
        G_YUK_UCD (g, h));
    ((D (−h), CHiggs HC1c, U g), FBF (1, Psibar, SLR, Psi),
        G_YUK_DCU (g, h)) ]
```

** REVISED: Compatible with CD+.FB
** REVISED: Compact version using new COMBOS

```
let yukawa_shiggs_2 c1 c2 s =
  let cc1 = conj_char c1 in
    ((Chargino cc1, SHiggs s, Chargino c2), FBF (1, Psibar, SLR, Psi),
        G_CSC (c1, c2, s))
```

```
let yukawa_phiggs_2 c1 c2 p =
  let cc1 = conj_char c1 in
    ((Chargino cc1, PHiggs p, Chargino c2), FBF (1, Psibar, SLR, Psi),
        G_CPC (c1, c2, p))
```

```
let yukawa_higgs_2 =
  Product.list3 yukawa_shiggs_2 [C1; C2] [C1; C2] [S1; S2; S3] @
  Product.list3 yukawa_phiggs_2 [C1; C2] [C1; C2] [P1; P2]
```

JR: Only the first charged Higgs.
** REVISED: Compatible with CD+.FB **

```
let higgs_charg_neutr n c =
  let cc = conj_char c in
  [ ((Neutralino n, CHiggs HC1c, Chargino c), FBF (−1, Chibar, SLR, Psi),
            G_NHC (false,n, c));
    ((Chargino cc, CHiggs HC1, Neutralino n), FBF (−1, Psibar, SLR, Chi),
            G_NHC (true,n, c)) ]
```

** REVISED: Compatible with CD+. FB**
** REVISED: Compact version using new COMBOS

```
let shiggs_neutr (n, m, s) =
  ((Neutralino n, SHiggs s, Neutralino m), FBF (1, Chibar, SP, Chi),
        G_CICIS (n, m, s))
let phiggs_neutr (n, m, p) =
```

$((Neutralino\ n,\ PHiggs\ p,\ Neutralino\ m),\ FBF\ (1,\ Chibar,\ SP,\ Chi),$
$\qquad G\_CICIP\ (n, m, p))$

let *higgs_neutr* =
  List.map shiggs_neutr (*two_and_one* [*N1*; *N2*; *N3*; *N4*; *N5*] [*S1*; *S2*; *S3*]) @
  List.map phiggs_neutr (*two_and_one* [*N1*; *N2*; *N3*; *N4*; *N5*] [*P1*; *P2*])

** REVISED: Compatible with CD+. FB**

  let *yukawa_n_2 n m g* =
   [$((Neutralino\ n,\ Slepton\ (m, -g),\ L\ g),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
     G_YUK_N (true,$L\ g, n, SL, m$));
    $((L\ (-g),\ Slepton\ (m, g),\ Neutralino\ n),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
     G_YUK_N (false,$L\ g, n, SL, m$));
    $((Neutralino\ n,\ Sup\ (m, -g),\ U\ g),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
     G_YUK_N (true,$U\ g, n, SU, m$));
    $((U\ (-g),\ Sup\ (m, g),\ Neutralino\ n),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
     G_YUK_N (false,$U\ g, n, SU, m$));
    $((Neutralino\ n,\ Sdown\ (m, -g),\ D\ g),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
     G_YUK_N (true,$D\ g, n, SD, m$));
    $((D\ (-g),\ Sdown\ (m, g),\ Neutralino\ n),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
     G_YUK_N (false,$D\ g, n, SD, m$))]
  let *yukawa_n_3 n g* =
   [$((Neutralino\ n,\ Sneutrino\ (-g),\ N\ g),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
     G_YUK_N (true,$N\ g, n, SN, M1$));
    $((N\ (-g),\ Sneutrino\ g,\ Neutralino\ n),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
     G_YUK_N (false,$N\ g,\ n, SN, M1$))]

  let *yukawa_n_5 g m* =
   [$((U\ (-g),\ Sup\ (m, g),\ Gluino),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
     G_YUK_G (false,$U\ g, SU, m$));
    $((D\ (-g),\ Sdown\ (m, g),\ Gluino),\ FBF\ (1,\ Psibar,\ SLR,\ Chi),$
     G_YUK_G (false,$D\ g, SD, m$));
    $((Gluino,\ Sup\ (m, -g),\ U\ g),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
     G_YUK_G (true,$U\ g, SU, m$));
    $((Gluino,\ Sdown\ (m, -g),\ D\ g),\ FBF\ (1,\ Chibar,\ SLR,\ Psi),$
     G_YUK_G (true,$D\ g, SD, m$))]
  let *yukawa_n* =
  List.flatten (*Product.list3 yukawa_n_2* [*N1*; *N2*; *N3*; *N4*; *N5*] [*M1*; *M2*] [1; 2; 3]) @
  List.flatten (*Product.list2 yukawa_n_3* [*N1*; *N2*; *N3*; *N4*; *N5*] [1; 2; 3]) @
  List.flatten (*Product.list2 yukawa_n_5* [1; 2; 3] [*M1*; *M2*])

** REVISED: Compatible with CD+.FB **

let *yukawa_c_2 c g* =
  let *cc* = *conj_char c* in
  [$((L\ (-g),\ Sneutrino\ g,\ Chargino\ cc),\ BBB\ (1,\ Psibar,\ SLR,$
    $Psibar),\ G\_YUK\_C$ (true,$L\ g, c, SN, M1$));
   $((Chargino\ c,\ Sneutrino\ (-g),\ L\ g),\ PBP\ (1,\ Psi,\ SLR,\ Psi),$
    G_YUK_C (false,$L\ g, c, SN, M1$))]
let *yukawa_c_3 c m g* =
  let *cc* = *conj_char c* in
  [$((N\ (-g),\ Slepton\ (m, g),\ Chargino\ c),\ FBF\ (1,\ Psibar,\ SLR,$

```
              Psi),  G_YUK_C (true,N g, c, SL, m));
          ((Chargino cc, Slepton (m, −g), N g), FBF (1, Psibar, SLR,
              Psi),  G_YUK_C (false,N g, c, SL, m)) ]
    let yukawa_c c =
      ThoList.flatmap (yukawa_c_2 c) [1; 2; 3] @
      List.flatten (Product.list2 (yukawa_c_3 c) [M1; M2] [1; 2; 3])
```

** REVISED: Compatible with CD+. FB**

```
    let yukawa_cq' c (g, h) m  =
        let cc = conj_char c in
          [ ((Chargino c, Sup (m, −g), D h), PBP (1, Psi, SLR, Psi),
              G_YUK_Q (false,g, D h, c, SU, m));
            ((D (−h), Sup (m, g), Chargino cc), BBB (1, Psibar, SLR, Psibar),
              G_YUK_Q (true,g, D h, c, SU, m));
            ((Chargino cc, Sdown (m, −g), U h), FBF (1, Psibar, SLR, Psi),
              G_YUK_Q (true,g, U h, c, SD, m));
            ((U (−h), Sdown (m, g), Chargino c), FBF (1, Psibar, SLR, Psi),
              G_YUK_Q (false,g, U h, c, SD, m)) ]
    let yukawa_cq c  =
     if Flags.ckm_present then
       List.flatten (Product.list2 (yukawa_cq' c) [(1, 1); (1, 2); (2, 1); (2, 2); (1, 3); (2, 3); (3, 3); (3, 2); (3, 1)] [M
     else
        List.flatten (Product.list2 (yukawa_cq' c) [(1, 1); (2, 2); (3, 3)] [M1; M2])
```

** REVISED: Compatible with CD+. Remark: Singlet and octet gluon exchange. The coupling is divided by sqrt(2) to account for the correct normalization of the Lie algebra generators. **FB

```
    let col_currents g  =
     [ ((D (−g),  Gl,  D g),  FBF ((−1),  Psibar,  V,  Psi),  Gs);
       ((U (−g),  Gl,  U g),  FBF ((−1),  Psibar,  V,  Psi),  Gs)]
```

** REVISED: Compatible with CD+. Remark: Singlet and octet gluon exchange. The coupling is divided by sqrt(2) to account for the correct normalization of the Lie algebra generators. **FB
* LQ-coupl. **DW*

```
    let chg  = function
     | M1 → M2 | M2 → M1
```

* LQ - Yuk's *

```
    let yuk_lqino_se_uc1' g1 g2 g3 m  =
      let cm = chg m in
        [ ((U (−g3), Slepton (m, −g2), LQino g1), FBF (1, Psibar, SLR, Psi),
             G_LQ_EC_UC (true,cm, g1, g2, g3)) ]

    let yuk_lqino_se_uc1 g1 g2 g3  =
       ThoList.flatmap (yuk_lqino_se_uc1' g1 g2 g3) [M1; M2]

    let yuk_lqino_se_uc2' g1 g2 g3 m  =
      let cm = chg m in
        [ ((LQino (−g1), Slepton (m, g2), U g3), FBF (1, Psibar, SLR, Psi),
             G_LQ_EC_UC (false,cm, g1, g2, g3)) ]
```

let *yuk_lqino_se_uc2 g1 g2 g3* =
    ThoList.flatmap (*yuk_lqino_se_uc2′ g1 g2 g3*) [*M1*; *M2*]

let *yuk_lqino_sn_dc1 g1 g2 g3* =
  [((D ($-g3$), $Sneutrino$ ($-g2$), $LQino$ $g1$), *FBF* (-1, *Psibar*, *SLR*, *Psi*),
      $G\_LQ\_EC\_UC$ (true,$M2$,$g1$,$g2$,$g3$))]

let *yuk_lqino_sn_dc2 g1 g2 g3* =
  [(($LQino$ ($-g1$), $Sneutrino$ $g2$, D $g3$), *FBF* (-1, *Psibar*, *SLR*, *Psi*),
      $G\_LQ\_EC\_UC$ (false,$M2$,$g1$,$g2$,$g3$))]

let *yuk_lqino_ec_su1′ g1 g2 g3 m* =
  let *cm* = *chg m* in
    [(($LQino$ ($-g1$), Sup (m,$g3$), L $g2$), *FBF* (1, *Psibar*, *SLR*, *Psi*),
        $G\_LQ\_EC\_UC$ (true,cm,$g1$,$g2$,$g3$))]

let *yuk_lqino_ec_su1 g1 g2 g3* =
  ThoList.flatmap (*yuk_lqino_ec_su1′ g1 g2 g3*) [*M1*; *M2*]

let *yuk_lqino_ec_su2′ g1 g2 g3 m* =
  let *cm* = *chg m* in
    [((L ($-g2$), Sup (m,$-g3$), $LQino$ ($g1$)), *FBF* (1, *Psibar*, *SLR*, *Psi*),
        $G\_LQ\_EC\_UC$ (false,cm,$g1$,$g2$,$g3$))]

let *yuk_lqino_ec_su2 g1 g2 g3* =
  ThoList.flatmap (*yuk_lqino_ec_su2′ g1 g2 g3*) [*M1*; *M2*]

let *yuk_lqino_nc_sd1 g1 g2 g3* =
  [(($LQino$ ($-g1$), $Sdown$ ($M1$,$g3$), N $g2$), *FBF* (-1, *Psibar*, *SLR*, *Psi*),
      $G\_LQ\_EC\_UC$ (true,$M2$,$g1$,$g2$,$g3$))]

let *yuk_lqino_nc_sd2 g1 g2 g3* =
  [((N ($-g2$), $Sdown$ ($M1$,$-g3$), $LQino$ ($g1$)), *FBF* (-1, *Psibar*, *SLR*, *Psi*),
      $G\_LQ\_EC\_UC$ (false,$M2$,$g1$,$g2$,$g3$))]

let *yuk_lq_ec_uc′ g1 g2 g3 m* =
  [((L ($-g2$), LQ (m,$g1$), U ($-g3$)), *BBB* (1, *Psibar*, *SLR*, *Psibar*),
      $G\_LQ\_EC\_UC$ (false,m,$g1$,$g2$,$g3$))]

let *yuk_lq_ec_uc g1 g2 g3* =
  ThoList.flatmap (*yuk_lq_ec_uc′ g1 g2 g3*) [*M1*; *M2*]

let *yuk_lq_ec_uc2′ g1 g2 g3 m* =
  [((L ($g2$), LQ (m,$-g1$), U ($g3$)), *PBP* (1, *Psi*, *SLR*, *Psi*),
      $G\_LQ\_EC\_UC$ (true,m,$g1$,$g2$,$g3$))]

let *yuk_lq_ec_uc2 g1 g2 g3* =
  ThoList.flatmap (*yuk_lq_ec_uc2′ g1 g2 g3*) [*M1*; *M2*]

let *yuk_lq_nc_dc g1 g2 g3* =
  [((N ($-g2$), LQ ($M2$,$g1$), D ($-g3$)), *BBB* (-1, *Psibar*, *SLR*, *Psibar*),
      $G\_LQ\_EC\_UC$ (false,$M2$,$g1$,$g2$,$g3$))]

let *yuk_lq_nc_dc2 g1 g2 g3* =
  [((N ($g2$), LQ ($M2$,$-g1$), D ($g3$)), *PBP* (-1, *Psi*, *SLR*, *Psi*),
      $G\_LQ\_EC\_UC$ (true,$M2$,$g1$,$g2$,$g3$))]

** Daniel Wiesler: LQ - F-Term w/ vev **

let *lq_se_su'* *g1* *g2* *g3* *m1* *m2* *m3* =
  [((LQ ($m1, g1$), $Slepton$ ($m2, -g2$), Sup ($m3, -g3$)), $Scalar\_Scalar\_Scalar$ 1,
      G_LQ_SSU ($m1, m2, m3, g1, g2, g3$))]

let *lq_se_su* *g1* *g2* *g3* =
  $List.flatten$ ($Product.list3$ (*lq_se_su'* *g1* *g2* *g3*) [$M1; M2$] [$M1; M2$] [$M1; M2$])

let *lq_snu_sd'* *g1* *g2* *g3* *m1* *m2* =
  [((LQ ($m1, g1$), $Sdown$ ($m2, -g2$), $Sneutrino$ ($-g3$)), $Scalar\_Scalar\_Scalar$ 1,
      G_LQ_SSD ($m1, m2, g1, g2, g3$))]

let *lq_snu_sd* *g1* *g2* *g3* =
  $List.flatten$ ($Product.list2$ (*lq_snu_sd'* *g1* *g2* *g3*) [$M1; M2$] [$M1; M2$])

** Daniel Wiesler: LQ - Higgs **

let *lq_shiggs'* *g1* *s* *g2* *m1* *m2* =
  [((LQ ($m1, g1$), $SHiggs$ s, LQ ($m2, -g2$)), $Scalar\_Scalar\_Scalar$ 1, G_LQ_S ($m1, m2, g1, s, g2$))]

let *lq_shiggs* *g1* *s* *g2* =
  $List.flatten$ ($Product.list2$ (*lq_shiggs'* *g1* *s* *g2*) [$M1; M2$] [$M1; M2$])

let *lq_phiggs'* *g1* *p* *g2* *m1* *m2* =
  [((LQ ($m1, g1$), $PHiggs$ p, LQ ($m2, -g2$)), $Scalar\_Scalar\_Scalar$ 1, G_LQ_P ($m1, m2, g1, p, g2$))]

let *lq_phiggs* *g1* *p* *g2* =
  $List.flatten$ ($Product.list2$ (*lq_phiggs'* *g1* *p* *g2*) [$M1; M2$] [$M1; M2$])

let *yuk_lqino_shiggs* *g1* *s* *g2* =
  [(($LQino$ ($-g1$), $SHiggs$ s, $LQino$ $g2$), FBF (1, $Psibar$, SLR, Psi),
      $G\_YUK\_LQ\_S$ ($g1, s, g2$))]

let *yuk_lqino_phiggs* *g1* *p* *g2* =
  [(($LQino$ ($-g1$), $PHiggs$ p, $LQino$ $g2$), FBF (1, $Psibar$, SLR, Psi),
      $G\_YUK\_LQ\_P$ ($g1, p, g2$))]

** Daniel Wiesler: LQ - Neutralinos. **

let *lqino_lq_neu'* *n* *g1* *g2* *m* =
  [(($Neutralino$ n, LQ ($m, -g1$), $LQino$ $g2$), FBF (1, $Chibar$, SLR, Psi),
      G_LQ_NEU ($m, g1, g2, n$))]

let *lqino_lq_neu* *n* *g1* *g2* =
  $ThoList.flatmap$ (*lqino_lq_neu'* *n* *g1* *g2*) [$M1; M2$]

let *lqino_lq_neu2'* *n* *g1* *g2* *m* =
  [(($LQino$ ($-g2$), LQ ($m, g1$), $Neutralino$ n), FBF (1, $Psibar$, SLR, Chi),
      G_LQ_NEU ($m, g1, g2, n$))]

let *lqino_lq_neu2* *n* *g1* *g2* =
  $ThoList.flatmap$ (*lqino_lq_neu2'* *n* *g1* *g2*) [$M1; M2$]

** Daniel Wiesler: LQ-LQino-Gluino **

let *lqino_lq_gg'* *g1* *g2* *m* =
  [(($Gluino$, LQ ($m, -g1$), $LQino$ $g2$), FBF (1, $Chibar$, SLR, Psi),
      G_LQ_GG ($m, g1, g2$))]

let *lqino_lq_gg* *g1* *g2* =

        ThoList.flatmap (*lqino_lq_gg′ g1 g2*) [*M1*; *M2*]

** Daniel Wiesler: LQ - Gauge **

  let *col_lqino_currents g* =
    [((*LQino* (−*g*), *Gl*, *LQino g*), *FBF* ((−1), *Psibar*, *V*, *Psi*), *Gs*)]

  let *neutr_lqino_current g* =
    [((*LQino* (−*g*), *Z*, *LQino g*), *FBF* (1, *Psibar*, *V*, *Psi*), *G_NLQC*)]

  let *col_lq_currents m g* =
    [((*Gl*, *LQ* (*m*, −*g*), *LQ* (*m*, *g*)), *Vector_Scalar_Scalar* (−1), *Gs*)]

  let *lq_neutr_Z g m1 m2* =
    [((*Z*, *LQ* (*m1*, −*g*), *LQ* (*m2*, *g*)), *Vector_Scalar_Scalar* (−1), *G_ZLQ* (*g*, *m1*, *m2*))]

  let *em_lq_currents g m* =
    [((*Ga*, *LQ* (*m*, −*g*), *LQ* (*m*, *g*)), *Vector_Scalar_Scalar* 1, *Q_down*)]

  let *em_lqino_currents g* =
    [((*LQino* (−*g*), *Ga*, *LQino g*), *FBF* (1, *Psibar*, *V*, *Psi*), *Q_down*)]

  let *gluon2_lq2′ g m* =
    [((*LQ* (*m*, *g*), *LQ* (*m*, −*g*), *Gl*, *Gl*), *Scalar2_Vector2* 2, *G_GlGlLQLQ*)]
  let *gluon2_lq2 g* =
    ThoList.flatmap (*gluon2_lq2′ g*) [*M1*; *M2*]

  let *lq_gauge4′ g m* =
    [((*Z*, *Z*, *LQ* (*m*, *g*), *LQ* (*m*, −*g*)), *Scalar2_Vector2* 1, *G_ZZLQLQ*);
     ((*Z*, *Ga*, *LQ* (*m*, *g*), *LQ* (*m*, −*g*)), *Scalar2_Vector2* 1, *G_ZPLQLQ*);
     ((*Ga*, *Ga*, *LQ* (*m*, *g*), *LQ* (*m*, −*g*)), *Scalar2_Vector2* 1, *G_PPLQLQ*)]

  let *lq_gauge4 g* =
    ThoList.flatmap (*lq_gauge4′ g*) [*M1*; *M2*]

  let *lq_gg_gauge2′ g m* =
    [((*Z*, *Gl*, *LQ* (*m*, *g*), *LQ* (*m*, −*g*)), *Scalar2_Vector2* 1, *G_ZGlLQLQ*);
     ((*Ga*, *Gl*, *LQ* (*m*, *g*), *LQ* (*m*, −*g*)), *Scalar2_Vector2* 1, *G_PGlLQLQ*)]

  let *lq_gg_gauge2 g* =
    ThoList.flatmap (*lq_gg_gauge2′ g*) [*M1*; *M2*]

  let *col_sfermion_currents g m* =
    [((*Gl*, *Sup* (*m*, −*g*), *Sup* (*m*, *g*)), *Vector_Scalar_Scalar* (−1), *Gs*);
     ((*Gl*, *Sdown* (*m*, −*g*), *Sdown* (*m*, *g*)), *Vector_Scalar_Scalar* (−1), *Gs*)]

** REVISED: Compatible with CD+. **FB

  let *triple_gauge* =
    [((*Ga*, *Wm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_Q_W*);
     ((*Z*, *Wm*, *Wp*), *Gauge_Gauge_Gauge* 1, *I_G_ZWW*);
     ((*Gl*, *Gl*, *Gl*), *Gauge_Gauge_Gauge* 1, *I_G_S*)]

** REVISED: Independent of the sign of CD. **FB

  let *gauge4* = *Vector4* [(2, *C_13_42*); (−1, *C_12_34*); (−1, *C_14_23*)]
  let *minus_gauge4* = *Vector4* [(−2, *C_13_42*); (1, *C_12_34*); (1, *C_14_23*)]
  let *quartic_gauge* =
    [(*Wm*, *Wp*, *Wm*, *Wp*), *gauge4*, *G_WWWW*;

$\qquad$ $(Wm,\ Z,\ Wp,\ Z),\ minus\_gauge4,\ G\_ZZWW;$
$(Wm,\ Z,\ Wp,\ Ga),\ minus\_gauge4,\ G\_PZWW;$
$(Wm,\ Ga,\ Wp,\ Ga),\ minus\_gauge4,\ G\_PPWW;$
$(Gl,\ Gl,\ Gl,\ Gl),\ gauge4,\ G\_SS]$

The *Scalar_Vector_Vector* couplings do not depend on the choice of the sign of the covariant derivative since they are quadratic in the gauge couplings.
JR: Only the first charged Higgs.
** REVISED: Compatible with CD+. **
** Revision: 2005-03-10: first two vertices corrected. **
** REVISED: Felix Braam: Compact version using new COMBOS
** REVISED: Felix Braam: Couplings adjusted to FF-convention

let *gauge_higgs_WPC* $p =$
$[\ ((Wm,\ CHiggs\ HC1,\ PHiggs\ p),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_WPC\ p);$
$\quad ((Wp,\ CHiggs\ HC1c,\ PHiggs\ p),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_WPC\ p)]$
let *gauge_higgs_WSC* $s =$
$[((Wm,\ CHiggs\ HC1,\ SHiggs\ s),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_WSC\ s);$
$\quad ((Wp,\ CHiggs\ HC1c,\ SHiggs\ s),\ Vector\_Scalar\_Scalar\ (-1),\ G\_GH\_WSC\ s)]$
let *gauge_higgs_ZSP* $s\ p\ =$
$[((Z,\ SHiggs\ s,\ PHiggs\ p),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_ZSP\ (s,p))]$
let *gauge_higgs_WWS* $s =$
$((SHiggs\ s,\ Wp,\ Wm),\ Scalar\_Vector\_Vector\ 1,\ G\_GH\_WWS\ s)$
let *gauge_higgs_ZZS* $s =$
$((SHiggs\ s,\ Z,\ Z),\ Scalar\_Vector\_Vector\ 1,\ G\_GH\_ZZS\ s)$
let *gauge_higgs_ZCC* $\ =$
$((Z,\ CHiggs\ HC1,\ CHiggs\ HC1c),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_ZCC\ )$
let *gauge_higgs_GaCC* $\ =$
$((Ga,\ CHiggs\ HC1,\ CHiggs\ HC1c),\ Vector\_Scalar\_Scalar\ 1,\ G\_GH\_GaCC\ )$

let *gauge_higgs* $\ =$
ThoList.flatmap gauge_higgs_WPC $[P1; P2]$ @
ThoList.flatmap gauge_higgs_WSC $[S1; S2; S3]$ @
List.flatten (*Product.list2 gauge_higgs_ZSP* $[S1; S2; S3]$ $[P1; P2]$) @
List.map gauge_higgs_WWS $[S1; S2; S3]$ @
List.map gauge_higgs_ZZS $[S1; S2; S3]$ @
$[gauge\_higgs\_ZCC]$ @ $[gauge\_higgs\_GaCC]$

** REVISED: Compact version using new COMBOS
** REVISED: Couplings adjusted to FF-convention

let *gauge_higgs4_ZZPP* $(p1, p2)\ =$
$((PHiggs\ p1,\ PHiggs\ p2,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\_ZZPP\ (p1, p2))$

let *gauge_higgs4_ZZSS* $(s1, s2)\ =$
$((SHiggs\ s1,\ SHiggs\ s2,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\_ZZSS\ (s1, s2))$

JR: Only the first charged Higgs.

let *gauge_higgs4_ZZCC* $\ =$
$((CHiggs\ HC1,\ CHiggs\ HC1c,\ Z,\ Z),\ Scalar2\_Vector2\ 1,\ G\_GH4\_ZZCC)$

let *gauge_higgs4_GaGaCC* $\ =$
$((CHiggs\ HC1,\ CHiggs\ HC1c,\ Ga,\ Ga),\ Scalar2\_Vector2\ 1,\ G\_GH4\_GaGaCC)$

let *gauge_higgs4_ZGaCC* =
  ((*CHiggs HC1*, *CHiggs HC1c*, *Ga*, *Z*), *Scalar2_Vector2* 1, *G_GH4_ZGaCC*)

let *gauge_higgs4_WWCC* =
  ((*CHiggs HC1*, *CHiggs HC1c*, *Wp*, *Wm*), *Scalar2_Vector2* 1, *G_GH4_WWCC*)

let *gauge_higgs4_WWPP* (*p1*, *p2*) =
  ((*PHiggs p1*, *PHiggs p2*, *Wp*, *Wm*), *Scalar2_Vector2* 1, *G_GH4_WWPP* (*p1*, *p2*))

let *gauge_higgs4_WWSS* (*s1*, *s2*) =
  ((*SHiggs s1*, *SHiggs s2*, *Wp*, *Wm*), *Scalar2_Vector2* 1, *G_GH4_WWSS* (*s1*, *s2*))

JR: Only the first charged Higgs.

let *gauge_higgs4_ZWSC* *s* =
  [((*CHiggs HC1*, *SHiggs s*, *Wm*, *Z*), *Scalar2_Vector2* 1, *G_GH4_ZWSC s*);
    ((*CHiggs HC1c*, *SHiggs s*, *Wp*, *Z*), *Scalar2_Vector2* 1, *G_GH4_ZWSC s*)]

let *gauge_higgs4_GaWSC* *s* =
  [((*CHiggs HC1*, *SHiggs s*, *Wm*, *Ga*), *Scalar2_Vector2* 1, *G_GH4_GaWSC s*);
    ((*CHiggs HC1c*, *SHiggs s*, *Wp*, *Ga*), *Scalar2_Vector2* 1, *G_GH4_GaWSC s*)]

let *gauge_higgs4_ZWPC* *p* =
  [((*CHiggs HC1*, *PHiggs p*, *Wm*, *Z*), *Scalar2_Vector2* 1, *G_GH4_ZWPC p*);
    ((*CHiggs HC1c*, *PHiggs p*, *Wp*, *Z*), *Scalar2_Vector2* (−1), *G_GH4_ZWPC p*)]

let *gauge_higgs4_GaWPC* *p* =
  [((*CHiggs HC1*, *PHiggs p*, *Wm*, *Ga*), *Scalar2_Vector2* 1, *G_GH4_GaWPC p*);
    ((*CHiggs HC1c*, *PHiggs p*, *Wp*, *Ga*), *Scalar2_Vector2* (−1),
       G_GH4_GaWPC p)]

let *gauge_higgs4* =
  List.map gauge_higgs4_ZZPP (*pairs* [*P1*; *P2*]) @
  List.map gauge_higgs4_ZZSS (*pairs* [*S1*; *S2*; *S3*]) @
  [*gauge_higgs4_ZZCC*] @ [*gauge_higgs4_GaGaCC*] @
  [*gauge_higgs4_ZGaCC*] @ [*gauge_higgs4_WWCC*] @
  List.map gauge_higgs4_WWPP (*pairs* [*P1*; *P2*]) @
  List.map gauge_higgs4_WWSS (*pairs* [*S1*; *S2*; *S3*]) @
  ThoList.flatmap gauge_higgs4_ZWSC [*S1*; *S2*; *S3*] @
  ThoList.flatmap gauge_higgs4_GaWSC [*S1*; *S2*; *S3*] @
  ThoList.flatmap gauge_higgs4_ZWPC [*P1*; *P2*] @
  ThoList.flatmap gauge_higgs4_GaWPC [*P1*; *P2*]

** Added by Felix Braam. **

let *gauge_sfermion4′* *g* *m1* *m2* =
  [((*Wp*, *Wm*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar2_Vector2* 1,
       G_WWSFSF (*SL*, *g*, *m1*, *m2*));
    ((*Z*, *Ga*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar2_Vector2* 1,
       G_ZPSFSF (*SL*, *g*, *m1*, *m2*));
    ((*Z*, *Z*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar2_Vector2* 1,
       G_ZZSFSF (*SL*, *g*, *m1*, *m2*));
    ((*Wp*, *Wm*, *Sup* (*m1*, *g*), *Sup* (*m2*, −*g*)), *Scalar2_Vector2* 1, *G_WWSFSF*
       (*SU*, *g*, *m1*, *m2*));
    ((*Wp*, *Wm*, *Sdown* (*m1*, *g*), *Sdown* (*m2*, −*g*)), *Scalar2_Vector2* 1, *G_WWSFSF*
       (*SD*, *g*, *m1*, *m2*));

          $((Z,\ Z,\ Sup\ (m1,g),\ Sup\ (m2,-g)),\ Scalar2\_Vector2\ 1,\ G\_ZZSFSF$
             $(SU,g,m1,m2));$
          $((Z,\ Z,\ Sdown\ (m1,g),\ Sdown\ (m2,-g)),\ Scalar2\_Vector2\ 1,\ G\_ZZSFSF$
             $(SD,g,m1,m2));$
          $((Z,\ Ga,\ Sup\ (m1,g),\ Sup\ (m2,-g)),\ Scalar2\_Vector2\ 1,\ G\_ZPSFSF$
             $(SU,g,m1,m2));$
          $((Z,\ Ga,\ Sdown\ (m1,g),\ Sdown\ (m2,-g)),\ Scalar2\_Vector2\ 1,\ G\_ZPSFSF$
             $(SD,g,m1,m2))\ ]$

let *gauge_sfermion4″ g m* =
  $[\ ((Wp,\ Ga,\ Slepton\ (m,g),\ Sneutrino\ (-g)),\ Scalar2\_Vector2\ 1,$
     $G\_WPSLSN\ (\mathsf{false},g,m));$
    $((Wm,\ Ga,\ Slepton\ (m,-g),\ Sneutrino\ g),\ Scalar2\_Vector2\ 1,$
     $G\_WPSLSN\ (\mathsf{true},g,m));$
    $((Wp,\ Z,\ Slepton\ (m,g),\ Sneutrino\ (-g)),\ Scalar2\_Vector2\ 1,$
     $G\_WZSLSN\ (\mathsf{false},g,m));$
    $((Wm,\ Z,\ Slepton\ (m,-g),\ Sneutrino\ g),\ Scalar2\_Vector2\ 1,$
     $G\_WZSLSN\ (\mathsf{true},g,m));$
    $((Ga,\ Ga,\ Slepton\ (m,g),\ Slepton\ (m,-g)),\ Scalar2\_Vector2\ 1,\ G\_PPSFSF\ SL);$
    $((Ga,\ Ga,\ Sup\ (m,g),\ Sup\ (m,-g)),\ Scalar2\_Vector2\ 1,\ G\_PPSFSF\ SU);$
    $((Ga,\ Ga,\ Sdown\ (m,g),\ Sdown\ (m,-g)),\ Scalar2\_Vector2\ 1,\ G\_PPSFSF\ SD)]$

let *gauge_sfermion4 g* =
  $List.flatten\ (Product.list2\ (gauge\_sfermion4'\ g)\ [M1;M2]\ [M1;M2])\ @$
  $ThoList.flatmap\ (gauge\_sfermion4''\ g)\ [M1;M2]\ @$
  $[\ ((Wp,\ Wm,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar2\_Vector2\ 1,\ G\_WWSFSF$
     $(SN,g,M1,M1));$
    $((Z,\ Z,\ Sneutrino\ g,\ Sneutrino\ (-g)),\ Scalar2\_Vector2\ 1,\ G\_ZZSFSF$
     $(SN,g,M1,M1))\ ]$

** Modified by Felix Braam. **

let *gauge_squark4″ g h m1 m2* =
  $[\ ((Wp,\ Ga,\ Sup\ (m1,-g),\ Sdown\ (m2,h)),\ Scalar2\_Vector2\ 1,\ G\_WPSUSD$
     $(\mathsf{false},m1,m2,g,h));$
    $((Wm,\ Ga,\ Sup\ (m1,g),\ Sdown\ (m2,-h)),\ Scalar2\_Vector2\ 1,\ G\_WPSUSD$
     $(\mathsf{true},m1,m2,g,h));$
    $((Wp,\ Z,\ Sup\ (m1,-g),\ Sdown\ (m2,h)),\ Scalar2\_Vector2\ 1,\ G\_WZSUSD$
     $(\mathsf{false},m1,m2,g,h));$
    $((Wm,\ Z,\ Sup\ (m1,g),\ Sdown\ (m2,-h)),\ Scalar2\_Vector2\ 1,\ G\_WZSUSD$
     $(\mathsf{true},m1,m2,g,h))\ ]$

let *gauge_squark4′ g h* = $List.flatten\ (Product.list2\ (gauge\_squark4''\ g\ h)$
                                       $[M1;M2]\ [M1;M2])$

let *gauge_squark4* =
  if *Flags.ckm_present* then
    $List.flatten\ (Product.list2\ gauge\_squark4'\ [1;2;3]\ [1;2;3])$
  else
    $ThoList.flatmap\ (\mathsf{fun}\ g\ \rightarrow\ gauge\_squark4'\ g\ g)\ [1;2;3]$

let *gluon_w_squark″ g h m1 m2* =
  $[\ ((Gl,\ Wp,\ Sup\ (m1,-g),\ Sdown\ (m2,h)),$
     $Scalar2\_Vector2\ 1,\ G\_GlWSUSD\ (\mathsf{false},m1,m2,g,h));$
    $((Gl,\ Wm,\ Sup\ (m1,g),\ Sdown\ (m2,-h)),$

       $Scalar2\_Vector2$ 1, $G\_GlWSUSD$ (true,$m1$, $m2$, g, h))]
   let $gluon\_w\_squark'$ g h =
    $List.flatten$ ($Product.list2$ ($gluon\_w\_squark''$ g h) $[M1; M2]$ $[M1; M2]$)
   let $gluon\_w\_squark$ =
    if $Flags.ckm\_present$ then
     $List.flatten$ ($Product.list2$ $gluon\_w\_squark'$ $[1; 2; 3]$ $[1; 2; 3]$)
    else
     $ThoList.flatmap$ (fun g $\rightarrow$ $gluon\_w\_squark'$ g g) $[1; 2; 3]$

** Modified by Felix Braam. **

   let $gluon\_gauge\_squark'$ g $m1$ $m2$ =
   $[$ $((Gl,\ Z,\ Sup\ (m1, g),\ Sup\ (m2, -g)),$
     $Scalar2\_Vector2$ 2, $G\_GlZSFSF$ $(SU, g, m1, m2)$);
    $((Gl,\ Z,\ Sdown\ (m1, g),\ Sdown\ (m2, -g)),$
     $Scalar2\_Vector2$ 2, $G\_GlZSFSF$ $(SD, g, m1, m2)$) $]$
   let $gluon\_gauge\_squark''$ g m =
   $[$ $((Gl,\ Ga,\ Sup\ (m, g),\ Sup\ (m, -g)),\ Scalar2\_Vector2$ 2, $G\_GlPSQSQ$);
    $((Gl,\ Ga,\ Sdown\ (m, g),\ Sdown\ (m, -g)),\ Scalar2\_Vector2$ (-1), $G\_GlPSQSQ$) $]$

** Modified by Felix Braam. **

   let $gluon\_gauge\_squark$ g =
    $List.flatten$ ($Product.list2$ ($gluon\_gauge\_squark'$ g) $[M1; M2]$ $[M1; M2]$) @
    $ThoList.flatmap$ ($gluon\_gauge\_squark''$ g) $[M1; M2]$

   let $gluon2\_squark2'$ g m =
   $[$ $((Gl,\ Gl,\ Sup\ (m, g),\ Sup\ (m, -g)),\ Scalar2\_Vector2$ 2, $G\_GlGlSQSQ$);
    $((Gl,\ Gl,\ Sdown\ (m, g),\ Sdown\ (m, -g)),\ Scalar2\_Vector2$ 2, $G\_GlGlSQSQ$) $]$
   let $gluon2\_squark2$ g =
    $ThoList.flatmap$ ($gluon2\_squark2'$ g) $[M1; M2]$

JR: Only the first charged Higgs.
** REVISED: Independent of the sign of CD. **
** REVISED: Felix Braam: Compact version using new COMBOS **
** REVISED: Felix Braam: Couplings adjusted to FF-convention

   let $higgs\_SCC$ s =
    $((CHiggs\ HC1,\ CHiggs\ HC1c,\ SHiggs\ s),\ Scalar\_Scalar\_Scalar$ 1,
     G_H3_SCC s)
   let $higgs\_SSS$ $(s1, s2, s3)$ =
    $((SHiggs\ s1,\ SHiggs\ s2,\ SHiggs\ s3),\ Scalar\_Scalar\_Scalar$ 1,
     G_H3_SSS $(s1, s2, s3)$)
   let $higgs\_SPP$ $(p1, p2, s)$ =
    $((SHiggs\ s,\ PHiggs\ p1,\ PHiggs\ p2),\ Scalar\_Scalar\_Scalar$ 1,
     G_H3_SPP $(s, p1, p2)$)

   let $higgs$ =
    $List.map$ $higgs\_SCC$ $[S1; S2; S3]$@
    $List.map$ $higgs\_SSS$ ($triples$ $[S1; S2; S3]$)@
    $List.map$ $higgs\_SPP$ (two_and_one $[P1; P2]$ $[S1; S2; S3]$)

   let $higgs4$ = $[]$
(∗ The vertices of the type Higgs - Sfermion - Sfermion are independent of the
choice of the CD sign since they are quadratic in the gauge coupling. ∗)

JR: Only the first charged Higgs.
** REVISED: Independent of the sign of CD. **

> let *higgs_sneutrino′ s g* =
>   ((*SHiggs s*, *Sneutrino g*, *Sneutrino* (−*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_SFSFS* (*s*, *SN*, *g*, *M1*, *M1*))
> let *higgs_sneutrino″ g m* =
>   [((*CHiggs HC1*, *Sneutrino* (−*g*), *Slepton* (*m*, *g*)),
>   *Scalar_Scalar_Scalar* 1, *G_HSNSL* (false,*g*, *m*));
>   ((*CHiggs HC1c*, *Sneutrino g*, *Slepton* (*m*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_HSNSL* (true,*g*, *m*))]
> let *higgs_sneutrino* =
>   *Product.list2 higgs_sneutrino′* [*S1*; *S2*; *S3*] [1; 2; 3] @
>   *List.flatten* (*Product.list2 higgs_sneutrino″* [1; 2; 3] [*M1*; *M2*])

Under the assumption that there is no mixing between the left- and right-handed
sfermions for the first two generations there is only a coupling of the form Higgs
- sfermion1 - sfermion2 for the third generation. All the others are suppressed
by m_f/M_W.
** REVISED: Independent of the sign of CD. **

> let *higgs_sfermion_S s g m1 m2* =
>   [((*SHiggs s*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_SFSFS* (*s*, *SL*, *g*, *m1*, *m2*));
>   ((*SHiggs s*, *Sup* (*m1*, *g*), *Sup* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_SFSFS* (*s*, *SU*, *g*, *m1*, *m2*));
>   ((*SHiggs s*, *Sdown* (*m1*, *g*), *Sdown* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_SFSFS* (*s*, *SD*, *g*, *m1*, *m2*))]
>
> let *higgs_sfermion′ g m1 m2* =
>   (*higgs_sfermion_S S1 g m1 m2*) @ (*higgs_sfermion_S S2 g m1 m2*) @ (*higgs_sfermion_S S3 g m1*
>
> let *higgs_sfermion_P p g m1 m2* =
>   [((*PHiggs p*, *Slepton* (*m1*, *g*), *Slepton* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_SFSFP* (*p*, *SL*, *g*, *m1*, *m2*));
>   ((*PHiggs p*, *Sup* (*m1*, *g*), *Sup* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_SFSFP* (*p*, *SU*, *g*, *m1*, *m2*));
>   ((*PHiggs p*, *Sdown* (*m1*, *g*), *Sdown* (*m2*, −*g*)), *Scalar_Scalar_Scalar* 1,
>   *G_SFSFP* (*p*, *SD*, *g*, *m1*, *m2*))]
>
> let *higgs_sfermion″ g m1 m2* =
>   (*higgs_sfermion_P P1 g m1 m2*) @ (*higgs_sfermion_P P2 g m1 m2*)
> let *higgs_sfermion* = *List.flatten* (*Product.list3 higgs_sfermion′* [1; 2; 3] [*M1*; *M2*] [*M1*; *M2*]) @
>   *List.flatten* (*Product.list3 higgs_sfermion″* [1; 2; 3] [*M1*; *M2*] [*M1*; *M2*])

JR: Only the first charged Higgs.
** REVISED: Independent of the sign of CD. **

> let *higgs_squark′ g h m1 m2* =
>   [((*CHiggs HC1*, *Sup* (*m1*, −*g*), *Sdown* (*m2*, *h*)), *Scalar_Scalar_Scalar* 1,
>   *G_HSUSD* (false,*m1*, *m2*, *g*, *h*));
>   ((*CHiggs HC1c*, *Sup* (*m1*, *g*), *Sdown* (*m2*, −*h*)), *Scalar_Scalar_Scalar* 1,
>   *G_HSUSD* (true,*m1*, *m2*, *g*, *h*))]
> let *higgs_squark_a g h* = *higgs_squark′ g h M1 M1*

let *higgs_squark_b* (g, h) = *List.flatten* (*Product.list2* (*higgs_squark′ g h*)
$$[M1; M2]\ [M1; M2])$$

let *higgs_squark* =
  if *Flags.ckm_present* then
    List.flatten (*Product.list2 higgs_squark_a* $[1; 2]\ [1; 2]$) @
    ThoList.flatmap higgs_squark_b $[(1, 3); (2, 3); (3, 3); (3, 1); (3, 2)]$
  else
    higgs_squark_a 1 1 @ *higgs_squark_a* 2 2 @ *higgs_squark_b* $(3, 3)$

let *vertices3* =
    (*ThoList.flatmap electromagnetic_currents_3* $[1; 2; 3]$ @
     ThoList.flatmap electromagnetic_currents_2 $[C1; C2]$ @
     List.flatten (*Product.list2 electromagnetic_sfermion_currents* $[1; 2; 3]$
$$[M1; M2])\ @$$
     ThoList.flatmap neutral_currents $[1; 2; 3]$ @
     ThoList.flatmap neutral_sfermion_currents $[1; 2; 3]$ @
     ThoList.flatmap charged_currents $[1; 2; 3]$ @
     List.flatten (*Product.list2 charged_slepton_currents* $[1; 2; 3]$
$$[M1; M2])\ @$$
     (if *Flags.ckm_present* then
      List.flatten (*Product.list2 charged_quark_currents* $[1; 2; 3]$
$$[1; 2; 3])\ @$$
      List.flatten (*Product.list2 charged_squark_currents* $[1; 2; 3]$
$$[1; 2; 3])\ @$$
      ThoList.flatmap yukawa_higgs_quark $[(1, 3); (2, 3); (3, 3); (3, 1); (3, 2)]$
     else
      charged_quark_currents 1 1 @
      charged_quark_currents 2 2 @
      charged_quark_currents 3 3 @
      charged_squark_currents 1 1 @
      charged_squark_currents 2 2 @
      charged_squark_currents 3 3 @
      ThoList.flatmap yukawa_higgs_quark $[(3, 3)])\ @$
     yukawa_higgs 3 @ *yukawa_n* @
     ThoList.flatmap yukawa_c $[C1; C2]$ @
     ThoList.flatmap yukawa_cq $[C1; C2]$ @
     List.flatten (*Product.list2 charged_chargino_currents* $[N1; N2; N3; N4; N5]$
$$[C1; C2])\ @\ triple\_gauge\ @$$
     ThoList.flatmap neutral_Z (*pairs* $[N1; N2; N3; N4; N5]$) @
     Product.list2 charged_Z $[C1; C2]\ [C1; C2]$ @
     gauge_higgs @ *higgs* @ *yukawa_higgs_2* @
     List.flatten (*Product.list2 higgs_charg_neutr* $[N1; N2; N3; N4; N5]\ [C1; C2]$) @
     higgs_neutr @ *higgs_sneutrino* @ *higgs_sfermion* @
     higgs_squark @ *yukawa_v* @
     ThoList.flatmap col_currents $[1; 2; 3]$ @
     List.flatten (*Product.list2 col_sfermion_currents* $[1; 2; 3]\ [M1; M2]$)) @
     List.flatten (*Product.list2 col_lq_currents* $[M1; M2]\ [1; 2; 3]$) @
     ThoList.flatmap col_lqino_currents $[1; 2; 3]$ @
     ThoList.flatmap em_lqino_currents $[1; 2; 3]$ @
     ThoList.flatmap neutr_lqino_current $[1; 2; 3]$ @

$List.flatten\ (Product.list3\ yuk\_lqino\_se\_uc1\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_se\_uc2\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_ec\_su1\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_ec\_su2\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_sn\_dc1\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_sn\_dc2\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_nc\_sd1\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_nc\_sd2\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lq\_ec\_uc\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lq\_ec\_uc2\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lq\_nc\_dc\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lq\_nc\_dc2\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ lq\_neutr\_Z\ [1;2;3]\ [M1;M2]\ [M1;M2])\ @$
$List.flatten\ (Product.list2\ em\_lq\_currents\ [1;2;3]\ [M1;M2])\ @$
$List.flatten\ (Product.list3\ lq\_shiggs\ [1;2;3]\ [S1;S2;S3;S4;S5;S6;S7;S8;S9]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ lq\_phiggs\ [1;2;3]\ [P1;P2;P3;P4;P5;P6;P7]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_shiggs\ [1;2;3]\ [S1;S2;S3;S4;S5;S6;S7;S8;S9]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ yuk\_lqino\_phiggs\ [1;2;3]\ [P1;P2;P3;P4;P5;P6;P7]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ lqino\_lq\_neu\ nlist\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ lqino\_lq\_neu2\ nlist\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ lq\_se\_su\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list3\ lq\_snu\_sd\ [1;2;3]\ [1;2;3]\ [1;2;3])\ @$
$List.flatten\ (Product.list2\ lqino\_lq\_gg\ [1;2;3]\ [1;2;3])$

let $vertices4$ =
 $(quartic\_gauge\ @\ higgs4\ @\ gauge\_higgs4\ @$
 $ThoList.flatmap\ gauge\_sfermion4\ [1;2;3]\ @$
 $gauge\_squark4\ @\ gluon\_w\_squark\ @$
 $ThoList.flatmap\ gluon2\_squark2\ [1;2;3]\ @$
 $ThoList.flatmap\ gluon\_gauge\_squark\ [1;2;3]\ @$
 $ThoList.flatmap\ gluon2\_lq2\ [1;2;3]\ @$
 $ThoList.flatmap\ lq\_gauge4\ [1;2;3]\ @$
 $ThoList.flatmap\ lq\_gg\_gauge2\ [1;2;3])$

let $vertices\ ()$ = $(vertices3,\ vertices4,\ [])$

let $table$ = $F.of\_vertices\ (vertices\ ())$
let $fuse2$ = $F.fuse2\ table$
let $fuse3$ = $F.fuse3\ table$
let $fuse$ = $F.fuse\ table$
let $max\_degree\ ()$ = 4

SLHA2-Nomenclature for neutral Higgses

let $flavor\_of\_string\ s$ =
 match s with
  | "e-" $\to$ $L\ 1$ | "e+" $\to$ $L\ (-1)$
  | "mu-" $\to$ $L\ 2$ | "mu+" $\to$ $L\ (-2)$
  | "tau-" $\to$ $L\ 3$ | "tau+" $\to$ $L\ (-3)$
  | "nue" $\to$ $N\ 1$ | "nuebar" $\to$ $N\ (-1)$
  | "numu" $\to$ $N\ 2$ | "numubar" $\to$ $N\ (-2)$
  | "nutau" $\to$ $N\ 3$ | "nutaubar" $\to$ $N\ (-3)$
  | "se1-" $\to$ $Slepton\ (M1,1)$ | "se1+" $\to$ $Slepton\ (M1,-1)$

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      | "smu1-" → Slepton (M1, 2) | "smu1+" → Slepton (M1, −2)
      | "stau1-" → Slepton (M1, 3) | "stau1+" → Slepton (M1, −3)
      | "se2-" → Slepton (M2, 1) | "se2+" → Slepton (M2, −1)
      | "smu2-" → Slepton (M2, 2) | "smu2+" → Slepton (M2, −2)
      | "stau2-" → Slepton (M2, 3) | "stau2+" → Slepton (M2, −3)
      | "snue" → Sneutrino 1 | "snue*" → Sneutrino (−1)
      | "snumu" → Sneutrino 2 | "snumu*" → Sneutrino (−2)
      | "snutau" → Sneutrino 3 | "snutau*" → Sneutrino (−3)
      | "u" → U 1 | "ubar" → U (−1)
      | "c" → U 2 | "cbar" → U (−2)
      | "t" → U 3 | "tbar" → U (−3)
      | "d" → D 1 | "dbar" → D (−1)
      | "s" → D 2 | "sbar" → D (−2)
      | "b" → D 3 | "bbar" → D (−3)
      | "A" → Ga | "Z" | "Z0" → Z
      | "W+" → Wp | "W-" → Wm
      | "gl" | "g" → Gl
      | "h01" → SHiggs S1 | "h02" → SHiggs S2 | "h03" →
SHiggs S3
      | "A01" → PHiggs P1 | "A02" → PHiggs P2
      | "h04" → SHiggs S4 | "h05" → SHiggs S5 | "h06" →
SHiggs S6
      | "A03" → PHiggs P3 | "A04" → PHiggs P4
      | "h07" → SHiggs S7 | "h08" → SHiggs S8 | "h09" →
SHiggs S9
      | "A05" → PHiggs P5 | "A06" → PHiggs P6 | "A07" →
PHiggs P7
(∗ JR: Only the first charged Higgs. ∗)
      | "H+" → CHiggs HC1 | "H-" → CHiggs HC1c
      | "su1" → Sup (M1, 1) | "su1c" → Sup (M1, −1)
      | "sc1" → Sup (M1, 2) | "sc1c" → Sup (M1, −2)
      | "st1" → Sup (M1, 3) | "st1c" → Sup (M1, −3)
      | "su2" → Sup (M2, 1) | "su2c" → Sup (M2, −1)
      | "sc2" → Sup (M2, 2) | "sc2c" → Sup (M2, −2)
      | "st2" → Sup (M2, 3) | "st2c" → Sup (M2, −3)
      | "sgl" | "sg" → Gluino
      | "sd1" → Sdown (M1, 1) | "sd1c" → Sdown (M1, −1)
      | "ss1" → Sdown (M1, 2) | "ss1c" → Sdown (M1, −2)
      | "sb1" → Sdown (M1, 3) | "sb1c" → Sdown (M1, −3)
      | "sd2" → Sdown (M2, 1) | "sd2c" → Sdown (M2, −1)
      | "ss2" → Sdown (M2, 2) | "ss2c" → Sdown (M2, −2)
      | "sb2" → Sdown (M2, 3) | "sb2c" → Sdown (M2, −3)
      | "neu1" → Neutralino N1 | "neu2" → Neutralino N2
      | "neu3" → Neutralino N3 | "neu4" → Neutralino N4
      | "neu5" → Neutralino N5 | "neu6" → Neutralino N6
      | "neu7" → Neutralino N7 | "neu8" → Neutralino N8
      | "neu9" → Neutralino N9 | "neu10" → Neutralino N10
      | "neu11" → Neutralino N11
      | "ch1+" → Chargino C1 | "ch2+" → Chargino C2
      | "ch1-" → Chargino C1c | "ch2-" → Chargino C2c
```

    | "ch3+" $\rightarrow$ *Chargino C3* | "ch4+" $\rightarrow$ *Chargino C4*
    | "ch3-" $\rightarrow$ *Chargino C3c* | "ch4-" $\rightarrow$ *Chargino C4c*
    | "lq11" $\rightarrow$ *LQ* (*M1*, 1) | "lq11c" $\rightarrow$ *LQ* (*M1*, -1)
    | "lq12" $\rightarrow$ *LQ* (*M2*, 1) | "lq12c" $\rightarrow$ *LQ* (*M2*, -1)
    | "lq21" $\rightarrow$ *LQ* (*M1*, 2) | "lq21c" $\rightarrow$ *LQ* (*M1*, -2)
    | "lq22" $\rightarrow$ *LQ* (*M2*, 2) | "lq22c" $\rightarrow$ *LQ* (*M2*, -2)
    | "lq31" $\rightarrow$ *LQ* (*M1*, 3) | "lq31c" $\rightarrow$ *LQ* (*M1*, -3)
    | "lq32" $\rightarrow$ *LQ* (*M2*, 3) | "lq32c" $\rightarrow$ *LQ* (*M2*, -3)
    | "lqino1" $\rightarrow$ *LQino* 1 | "lqino1b" $\rightarrow$ *LQino* (-1)
    | "lqino2" $\rightarrow$ *LQino* 2 | "lqino2b" $\rightarrow$ *LQino* (-2)
    | "lqino3" $\rightarrow$ *LQino* 3 | "lqino3b" $\rightarrow$ *LQino* (-3)
    | s $\rightarrow$ *invalid_arg* ("HUBABUBA:␣%s␣Modellib_PSSSM.ExtMSSM.flavor_of_string:" ˆ s)

  let *flavor_to_string* $=$ function
   | *L* 1 $\rightarrow$ "e-" | *L* (-1) $\rightarrow$ "e+"
   | *L* 2 $\rightarrow$ "mu-" | *L* (-2) $\rightarrow$ "mu+"
   | *L* 3 $\rightarrow$ "tau-" | *L* (-3) $\rightarrow$ "tau+"
   | *N* 1 $\rightarrow$ "nue" | *N* (-1) $\rightarrow$ "nuebar"
   | *N* 2 $\rightarrow$ "numu" | *N* (-2) $\rightarrow$ "numubar"
   | *N* 3 $\rightarrow$ "nutau" | *N* (-3) $\rightarrow$ "nutaubar"
   | *U* 1 $\rightarrow$ "u" | *U* (-1) $\rightarrow$ "ubar"
   | *U* 2 $\rightarrow$ "c" | *U* (-2) $\rightarrow$ "cbar"
   | *U* 3 $\rightarrow$ "t" | *U* (-3) $\rightarrow$ "tbar"
   | *U* _ $\rightarrow$ *invalid_arg*
     "Modellib_PSSSM.ExtMSSM.flavor_to_string:␣invalid␣up␣type␣quark"
   | *D* 1 $\rightarrow$ "d" | *D* (-1) $\rightarrow$ "dbar"
   | *D* 2 $\rightarrow$ "s" | *D* (-2) $\rightarrow$ "sbar"
   | *D* 3 $\rightarrow$ "b" | *D* (-3) $\rightarrow$ "bbar"
   | *D* _ $\rightarrow$ *invalid_arg*
     "Modellib_PSSSM.ExtMSSM.flavor_to_string:␣invalid␣down␣type␣quark"
   | *Gl* $\rightarrow$ "gl" | *Gluino* $\rightarrow$ "sgl"
   | *Ga* $\rightarrow$ "A" | *Z* $\rightarrow$ "Z"
   | *Wp* $\rightarrow$ "W+" | *Wm* $\rightarrow$ "W-"
   | *SHiggs S1* $\rightarrow$ "h01" | *SHiggs S2* $\rightarrow$ "h02" | *SHiggs S3* $\rightarrow$ "h03"
   | *PHiggs P1* $\rightarrow$ "A01" | *PHiggs P2* $\rightarrow$ "A02"
   | *SHiggs S4* $\rightarrow$ "h04" | *SHiggs S5* $\rightarrow$ "h05" | *SHiggs S6* $\rightarrow$ "h06"
   | *PHiggs P3* $\rightarrow$ "A03" | *PHiggs P4* $\rightarrow$ "A04"
   | *SHiggs S7* $\rightarrow$ "h07" | *SHiggs S8* $\rightarrow$ "h08" | *SHiggs S9* $\rightarrow$ "h09"
   | *PHiggs P5* $\rightarrow$ "A05" | *PHiggs P6* $\rightarrow$ "A06" | *PHiggs P7* $\rightarrow$ "A07"
($*$ JR: Only the first charged Higgs. $*$)
   | *CHiggs HC1* $\rightarrow$ "H+" | *CHiggs HC1c* $\rightarrow$ "H-"
   | *CHiggs HC2* $\rightarrow$ "HX_1+" | *CHiggs HC2c* $\rightarrow$ "HX_1-"
   | *CHiggs HC3* $\rightarrow$ "HX_2+" | *CHiggs HC3c* $\rightarrow$ "HX_2-"
   | *CHiggs HC4* $\rightarrow$ "HX_3+" | *CHiggs HC4c* $\rightarrow$ "HX_3-"
   | *CHiggs HC5* $\rightarrow$ "HX_4+" | *CHiggs HC5c* $\rightarrow$ "HX_4-"
   | *Slepton* (*M1*, 1) $\rightarrow$ "se1-" | *Slepton* (*M1*, -1) $\rightarrow$ "se1+"
   | *Slepton* (*M1*, 2) $\rightarrow$ "smu1-" | *Slepton* (*M1*, -2) $\rightarrow$ "smu1+"
   | *Slepton* (*M1*, 3) $\rightarrow$ "stau1-" | *Slepton* (*M1*, -3) $\rightarrow$ "stau1+"
   | *Slepton* (*M2*, 1) $\rightarrow$ "se2-" | *Slepton* (*M2*, -1) $\rightarrow$ "se2+"
   | *Slepton* (*M2*, 2) $\rightarrow$ "smu2-" | *Slepton* (*M2*, -2) $\rightarrow$ "smu2+"

```
  | Slepton (M2, 3) → "stau2-" | Slepton (M2, −3) → "stau2+"
  | Sneutrino 1 → "snue" | Sneutrino (−1) → "snue*"
  | Sneutrino 2 → "snumu" | Sneutrino (−2) → "snumu*"
  | Sneutrino 3 → "snutau" | Sneutrino (−3) → "snutau*"
  | Sup (M1, 1) → "su1" | Sup (M1, −1) → "su1c"
  | Sup (M1, 2) → "sc1" | Sup (M1, −2) → "sc1c"
  | Sup (M1, 3) → "st1" | Sup (M1, −3) → "st1c"
  | Sup (M2, 1) → "su2" | Sup (M2, −1) → "su2c"
  | Sup (M2, 2) → "sc2" | Sup (M2, −2) → "sc2c"
  | Sup (M2, 3) → "st2" | Sup (M2, −3) → "st2c"
  | Sdown (M1, 1) → "sd1" | Sdown (M1, −1) → "sd1c"
  | Sdown (M1, 2) → "ss1" | Sdown (M1, −2) → "ss1c"
  | Sdown (M1, 3) → "sb1" | Sdown (M1, −3) → "sb1c"
  | Sdown (M2, 1) → "sd2" | Sdown (M2, −1) → "sd2c"
  | Sdown (M2, 2) → "ss2" | Sdown (M2, −2) → "ss2c"
  | Sdown (M2, 3) → "sb2" | Sdown (M2, −3) → "sb2c"
  | Neutralino n → "neu" ^ string_of_neu n
  | Chargino C1 → "ch1+" | Chargino C1c → "ch1-"
  | Chargino C2 → "ch2+" | Chargino C2c → "ch2-"
  | Chargino C3 → "ch3+" | Chargino C3c → "ch3-"
  | Chargino C4 → "ch4+" | Chargino C4c → "ch4-"
  | LQ (M1, 1) → "lq11" | LQ (M1, −1) → "lq11c"
  | LQ (M2, 1) → "lq12" | LQ (M2, −1) → "lq12c"
  | LQ (M1, 2) → "lq21" | LQ (M1, −2) → "lq21c"
  | LQ (M2, 2) → "lq22" | LQ (M2, −2) → "lq22c"
  | LQ (M1, 3) → "lq31" | LQ (M1, −3) → "lq31c"
  | LQ (M2, 3) → "lq32" | LQ (M2, −3) → "lq32c"
  | LQino 1 → "lqino1" | LQino (−1) → "lqino1b"
  | LQino 2 → "lqino2" | LQino (−2) → "lqino2b"
  | LQino 3 → "lqino3" | LQino (−3) → "lqino3b"
  | _ → invalid_arg "Modellib_PSSSM.ExtMSSM.flavor_to_string"

let flavor_to_TeX = function
  | L 1 → "e^-" | L (−1) → "e^+"
  | L 2 → "\\mu^-" | L (−2) → "\\mu^+"
  | L 3 → "\\tau^-" | L (−3) → "\\tau^+"
  | N 1 → "\\nu_e" | N (−1) → "\\bar{\\nu}_e"
  | N 2 → "\\nu_\\mu" | N (−2) → "\\bar{\\nu}_\\mu"
  | N 3 → "\\nu_\\tau" | N (−3) → "\\bar{\\nu}_\\tau"
  | U 1 → "u" | U (−1) → "\\bar{u}"
  | U 2 → "c" | U (−2) → "\\bar{c}"
  | U 3 → "t" | U (−3) → "\\bar{t}"
  | D 1 → "d" | D (−1) → "\\bar{d}"
  | D 2 → "s" | D (−2) → "\\bar{s}"
  | D 3 → "b" | D (−3) → "\\bar{b}"
  | L _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣lepton"
  | N _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣neutrino"
  | U _ → invalid_arg
```

```
      "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣up␣type␣quark"
```
| D _ $\rightarrow$ *invalid_arg*
```
      "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣down␣type␣quark"
```
| Gl $\rightarrow$ "g" | *Gluino* $\rightarrow$ "\\widetilde{g}"
| Ga $\rightarrow$ "\\gamma" | Z $\rightarrow$ "Z" | Wp $\rightarrow$ "W^+" | Wm $\rightarrow$ "W^-"
| *SHiggs S1* $\rightarrow$ "S_1" | *SHiggs S2* $\rightarrow$ "S_2" | *SHiggs S3* $\rightarrow$ "S_3"
| *SHiggs S4* $\rightarrow$ "S_4" | *SHiggs S5* $\rightarrow$ "S_5" | *SHiggs S6* $\rightarrow$ "S_6"
| *SHiggs S7* $\rightarrow$ "S_7" | *SHiggs S8* $\rightarrow$ "S_8" | *SHiggs S9* $\rightarrow$ "S_9"
| *PHiggs P1* $\rightarrow$ "P_1" | *PHiggs P2* $\rightarrow$ "P_2" | *PHiggs P3* $\rightarrow$ "P_3"
| *PHiggs P4* $\rightarrow$ "P_4" | *PHiggs P5* $\rightarrow$ "P_5" | *PHiggs P6* $\rightarrow$ "P_6"
| *PHiggs P7* $\rightarrow$ "P_7"
| *CHiggs HC1* $\rightarrow$ "H^+" | *CHiggs HC1c* $\rightarrow$ "H^-"
| *CHiggs HC2* $\rightarrow$ "X_{H,1}^+" | *CHiggs HC2c* $\rightarrow$ "X_{H,1}^-"
| *CHiggs HC3* $\rightarrow$ "X_{H,2}^+" | *CHiggs HC3c* $\rightarrow$ "X_{H,2}^-"
| *CHiggs HC4* $\rightarrow$ "X_{H,3}^+" | *CHiggs HC4c* $\rightarrow$ "X_{H,3}^-"
| *CHiggs HC5* $\rightarrow$ "X_{H,4}^+" | *CHiggs HC5c* $\rightarrow$ "X_{H,4}^-"
| *Slepton* $(M1,1)$ $\rightarrow$ "\\widetilde{e}_1^-"
| *Slepton* $(M1,-1)$ $\rightarrow$ "\\widetilde{e}_1^+"
| *Slepton* $(M1,2)$ $\rightarrow$ "\\widetilde{\\mu}_1^-"
| *Slepton* $(M1,-2)$ $\rightarrow$ "\\widetilde{\\mu}_1^+"
| *Slepton* $(M1,3)$ $\rightarrow$ "\\widetilde{\\tau}_1^-"
| *Slepton* $(M1,-3)$ $\rightarrow$ "\\widetilde{\\tau}_1^+"
| *Slepton* $(M2,1)$ $\rightarrow$ "\\widetilde{e}_2^-"
| *Slepton* $(M2,-1)$ $\rightarrow$ "\\widetilde{e}_2^+"
| *Slepton* $(M2,2)$ $\rightarrow$ "\\widetilde{\\mu}_2^-"
| *Slepton* $(M2,-2)$ $\rightarrow$ "\\widetilde{\\mu}_2^+"
| *Slepton* $(M2,3)$ $\rightarrow$ "\\widetilde{\\tau}_2^-"
| *Slepton* $(M2,-3)$ $\rightarrow$ "\\widetilde{\\tau}_2^+"
| *Sneutrino* 1 $\rightarrow$ "\\widetilde{\\nu}_e"
| *Sneutrino* (-1) $\rightarrow$ "\\widetilde{\\nu}_e^*"
| *Sneutrino* 2 $\rightarrow$ "\\widetilde{\\nu}_\\mu"
| *Sneutrino* (-2) $\rightarrow$ "\\widetilde{\\nu}_\\mu^*"
| *Sneutrino* 3 $\rightarrow$ "\\widetilde{\\nu}_\\tau"
| *Sneutrino* (-3) $\rightarrow$ "\\widetilde{\\nu}_\\tau^*"
| *Sup* $(M1,1)$ $\rightarrow$ "\\widetilde{u}_1"
| *Sup* $(M1,-1)$ $\rightarrow$ "\\widetilde{u}_1^*"
| *Sup* $(M1,2)$ $\rightarrow$ "\\widetilde{c}_1"
| *Sup* $(M1,-2)$ $\rightarrow$ "\\widetilde{c}_1^*"
| *Sup* $(M1,3)$ $\rightarrow$ "\\widetilde{t}_1"
| *Sup* $(M1,-3)$ $\rightarrow$ "\\widetilde{t}_1^*"
| *Sup* $(M2,1)$ $\rightarrow$ "\\widetilde{u}_2"
| *Sup* $(M2,-1)$ $\rightarrow$ "\\widetilde{u}_2^*"
| *Sup* $(M2,2)$ $\rightarrow$ "\\widetilde{c}_2"
| *Sup* $(M2,-2)$ $\rightarrow$ "\\widetilde{c}_2^*"
| *Sup* $(M2,3)$ $\rightarrow$ "\\widetilde{t}_2"
| *Sup* $(M2,-3)$ $\rightarrow$ "\\widetilde{t}_2^*"
| *Sdown* $(M1,1)$ $\rightarrow$ "\\widetilde{d}_1"
| *Sdown* $(M1,-1)$ $\rightarrow$ "\\widetilde{d}_1^*"
| *Sdown* $(M1,2)$ $\rightarrow$ "\\widetilde{s}_1"
| *Sdown* $(M1,-2)$ $\rightarrow$ "\\widetilde{s}_1^*"
```

```
  | Sdown (M1, 3) → "\\widetilde{b}_1"
  | Sdown (M1, -3) → "\\widetilde{b}_1^*"
  | Sdown (M2, 1) → "\\widetilde{d}_2"
  | Sdown (M2, -1) → "\\widetilde{d}_2^*"
  | Sdown (M2, 2) → "\\widetilde{s}_2"
  | Sdown (M2, -2) → "\\widetilde{s}_2^*"
  | Sdown (M2, 3) → "\\widetilde{b}_2"
  | Sdown (M2, -3) → "\\widetilde{b}_2^*"
  | Neutralino N1 → "\\widetilde{\\chi}^0_1"
  | Neutralino N2 → "\\widetilde{\\chi}^0_2"
  | Neutralino N3 → "\\widetilde{\\chi}^0_3"
  | Neutralino N4 → "\\widetilde{\\chi}^0_4"
  | Neutralino N5 → "\\widetilde{\\chi}^0_5"
  | Neutralino N6 → "\\widetilde{\\chi}^0_6"
  | Neutralino N7 → "\\widetilde{\\chi}^0_7"
  | Neutralino N8 → "\\widetilde{\\chi}^0_8"
  | Neutralino N9 → "\\widetilde{\\chi}^0_9"
  | Neutralino N10 → "\\widetilde{\\chi}^0_{10}"
  | Neutralino N11 → "\\widetilde{\\chi}^0_{11}"
  | Slepton _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣slepton"
  | Sneutrino _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣sneutrino"
  | Sup _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣up␣type␣squark"
  | Sdown _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣down␣type␣squark"
  | Chargino C1 → "\\widetilde{\\chi}_1^+"
  | Chargino C1c → "\\widetilde{\\chi}_1^-"
  | Chargino C2 → "\\widetilde{\\chi}_2^+"
  | Chargino C2c → "\\widetilde{\\chi}_2^-"
  | Chargino C3 → "\\widetilde{\\chi}_3^+"
  | Chargino C3c → "\\widetilde{\\chi}_3^-"
  | Chargino C4 → "\\widetilde{\\chi}_4^+"
  | Chargino C4c → "\\widetilde{\\chi}_4^-"
  | LQ (M1, 1) → "D_{1,,1}" | LQ (M1, -1) → "D_{1,,1}^*"
  | LQ (M2, 1) → "D_{1,,2}" | LQ (M2, -1) → "D_{1,,2}^*"
  | LQ (M1, 2) → "D_{2,,1}" | LQ (M1, -2) → "D_{2,,1}^*"
  | LQ (M2, 2) → "D_{2,,2}" | LQ (M2, -2) → "D_{2,,2}^*"
  | LQ (M1, 3) → "D_{3,,1}" | LQ (M1, -3) → "D_{3,,1}^*"
  | LQ (M2, 3) → "D_{3,,2}" | LQ (M2, -3) → "D_{3,,2}^*"
  | LQino 1 → "\\widetilde{D}_1" | LQino (-1) → "\\bar\\widetilde{D}_1"
  | LQino 2 → "\\widetilde{D}_2" | LQino (-2) → "\\bar\\widetilde{D}_2"
  | LQino 3 → "\\widetilde{D}_3" | LQino (-3) → "\\bar\\widetilde{D}_3"
  | LQ _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣leptoquark␣type"
  | LQino _ → invalid_arg
        "Modellib_PSSSM.ExtMSSM.flavor_to_TeX:␣invalid␣leptoquarkino␣type"

let flavor_symbol = function
```

```
  | L g when g  >  0  →  "l" ^ string_of_int g
  | L g  →  "l" ^ string_of_int (abs g) ^ "b"
  | N g when g  >  0  →  "n" ^ string_of_int g
  | N g  →  "n" ^ string_of_int (abs g) ^ "b"
  | U g when g  >  0  →  "u" ^ string_of_int g
  | U g  →  "u" ^ string_of_int (abs g) ^ "b"
  | D g when g  >  0  →  "d" ^ string_of_int g
  | D g  →  "d" ^ string_of_int (abs g) ^ "b"
  | Gl  →  "gl"
  | Ga  →  "a" | Z  →  "z"
  | Wp  →  "wp" |  Wm  →  "wm"
  | Slepton (M1, g) when g  >  0  →  "sl1" ^ string_of_int g
  | Slepton (M1, g)  →  "sl1c" ^ string_of_int (abs g)
  | Slepton (M2, g) when g  >  0  →  "sl2" ^ string_of_int g
  | Slepton (M2, g)  →  "sl2c" ^ string_of_int (abs g)
  | Sneutrino g when g  >  0  →  "sn" ^ string_of_int g
  | Sneutrino g  →  "snc" ^ string_of_int (abs g)
  | Sup (M1, g) when g  >  0  →  "su1" ^ string_of_int g
  | Sup (M1, g)  →  "su1c" ^ string_of_int (abs g)
  | Sup (M2, g) when g  >  0  →  "su2" ^ string_of_int g
  | Sup (M2, g)  →  "su2c" ^ string_of_int (abs g)
  | Sdown (M1, g) when g  >  0  →  "sd1" ^ string_of_int g
  | Sdown (M1, g)  →  "sd1c" ^ string_of_int (abs g)
  | Sdown (M2, g) when g  >  0  →  "sd2" ^ string_of_int g
  | Sdown (M2, g)  →  "sd2c" ^ string_of_int (abs g)
  | Neutralino n  →  "neu" ^ (string_of_neu n)
  | Chargino c when (int_of_char c)  >  0  →  "cp" ^ string_of_char c
  | Chargino c  →  "cm" ^ string_of_int (abs (int_of_char c))
  | Gluino  →  "sgl"
  | SHiggs s  →  "h0" ^ (string_of_shiggs s)
  | PHiggs p  →  "A0" ^ (string_of_phiggs p)
  | CHiggs HC1  →  "hp" | CHiggs HC1c  →  "hm"
  | CHiggs _  →  invalid_arg "charged␣Higgs␣not␣yet␣implemented"
  | LQ (M1, g) when g  >  0  →  "lq" ^ string_of_int g ^ "1"
  | LQ (M1, g)  →  "lq" ^ string_of_int (abs g) ^ "1c"
  | LQ (M2, g) when g  >  0  →  "lq" ^ string_of_int g ^ "2"
  | LQ (M2, g)  →  "lq" ^ string_of_int (abs g) ^ "2c"
  | LQino g when g  >  0  →  "lqino" ^ string_of_int g
  | LQino g  →  "lqino" ^ string_of_int (abs g) ^ "b"

let pdg  =  function
  | L g when g  >  0  →  9  +  2 × g
  | L g  →  − 9  +  2 × g
  | N g when g  >  0  →  10  +  2 × g
  | N g  →  − 10  +  2 × g
  | U g when g  >  0  →  2 × g
  | U g  →  2 × g
  | D g when g  >  0  →  − 1  +  2 × g
  | D g  →  1  +  2 × g
  | Gl  →  21
```

```
        |  Ga  →  22  |  Z  →  23
        |  Wp  →  24  |  Wm  →  (−24)
        |  SHiggs S1  →  25  |  SHiggs S2  →  35  |  PHiggs P1  →  36
(∗ JR: Only the first charged Higgs. ∗)
        |  CHiggs HC1  →  37  |  CHiggs HC1c  →  (−37)
        |  CHiggs _  →  invalid_arg "charged␣Higgs␣not␣yet␣implemented"
        |  Slepton (M1, g) when g > 0  →  1000009 + 2 × g
        |  Slepton (M1, g)  →  − 1000009 + 2 × g
        |  Slepton (M2, g) when g > 0  →  2000009 + 2 × g
        |  Slepton (M2, g)  →  − 2000009 + 2 × g
        |  Sneutrino g when g > 0  →  1000010 + 2 × g
        |  Sneutrino g  →  − 1000010 + 2 × g
        |  Sup (M1, g) when g > 0  →  1000000 + 2 × g
        |  Sup (M1, g)  →  − 1000000 + 2 × g
        |  Sup (M2, g) when g > 0  →  2000000 + 2 × g
        |  Sup (M2, g)  →  − 2000000 + 2 × g
        |  Sdown (M1, g) when g > 0  →  999999 + 2 × g
        |  Sdown (M1, g)  →  − 999999 + 2 × g
        |  Sdown (M2, g) when g > 0  →  1999999 + 2 × g
        |  Sdown (M2, g)  →  − 1999999 + 2 × g
        |  Gluino  →  1000021
(∗ JR: only the first two charginos. ∗)
        |  Chargino C1  →  1000024  |  Chargino C1c  →  (−1000024)
        |  Chargino C2  →  1000037  |  Chargino C2c  →  (−1000037)
        |  Chargino C3  →  1000039  |  Chargino C3c  →  (−1000039)
        |  Chargino C4  →  1000041  |  Chargino C4c  →  (−1000041)
        |  Neutralino N1  →  1000022  |  Neutralino N2  →  1000023
        |  Neutralino N3  →  1000025  |  Neutralino N4  →  1000035
(∗ According to SLHA2 (not anymore ?!?)∗)
        |  Neutralino N5  →  1000045  |  Neutralino N6  →  1000046
        |  Neutralino N7  →  1000047  |  Neutralino N8  →  1000048
        |  Neutralino N9  →  1000049  |  Neutralino N10  →  1000050
        |  Neutralino N11  →  1000051
        |  PHiggs P2  →  46  |  PHiggs P3  →  47  |  PHiggs P4  →  48
        |  PHiggs P5  →  49  |  PHiggs P6  →  50  |  PHiggs P7  →  51
        |  SHiggs S3  →  45  |  SHiggs S4  →  52  |  SHiggs S5  →  53
        |  SHiggs S6  →  54  |  SHiggs S7  →  55  |  SHiggs S8  →  56
        |  SHiggs S9  →  57
        |  LQ (M1, g) when g > 0  →  1000059 + g
        |  LQ (M1, g)  →  − 1000059 + g
        |  LQ (M2, g) when g > 0  →  2000059 + g
        |  LQ (M2, g)  →  − 2000059 + g
        |  LQino g when g > 0  →  59 + g
        |  LQino g  →  − 59 + g
```

We must take care of the pdg numbers for the two different kinds of sfermions in the MSSM. The particle data group in its Monte Carlo particle numbering scheme takes only into account mixtures of the third generation squarks and the stau. For the other sfermions we will use the number of the lefthanded field for the lighter mixed state and the one for the righthanded for the heavier. Below

are the official pdg numbers from the Particle Data Group. In order not to produce arrays with some million entries in the Fortran code for the masses and the widths we introduce our private pdg numbering scheme which only extends not too far beyond 42. Our private scheme then has the following pdf numbers (for the sparticles the subscripts L/R and $1/2$ are taken synonymously):

d	down-quark	1
u	up-quark	2
s	strange-quark	3
c	charm-quark	4
b	bottom-quark	5
t	top-quark	6
e^-	electron	11
ν_e	electron-neutrino	12
μ^-	muon	13
ν_μ	muon-neutrino	14
τ^-	tau	15
ν_τ	tau-neutrino	16
g	gluon	(9) 21
γ	photon	22
Z^0	Z-boson	23
W^+	W-boson	24
h^0	light Higgs boson	25
H^0	heavy Higgs boson	35
A^0	pseudoscalar Higgs	36
H^+	charged Higgs	37
$\tilde{d}_L$	down-squark 1	41
$\tilde{u}_L$	up-squark 1	42
$\tilde{s}_L$	strange-squark 1	43
$\tilde{c}_L$	charm-squark 1	44
$\tilde{b}_L$	bottom-squark 1	45
$\tilde{t}_L$	top-squark 1	46
$\tilde{d}_R$	down-squark 2	47
$\tilde{u}_R$	up-squark 2	48
$\tilde{s}_R$	strange-squark 2	49
$\tilde{c}_R$	charm-squark 2	50
$\tilde{b}_R$	bottom-squark 2	51
$\tilde{t}_R$	top-squark 2	52
$\tilde{e}_L$	selectron 1	53
$\tilde{\nu}_{e,L}$	electron-sneutrino	54
$\tilde{\mu}_L$	smuon 1	55
$\tilde{\nu}_{\mu,L}$	muon-sneutrino	56
$\tilde{\tau}_L$	stau 1	57
$\tilde{\nu}_{\tau,L}$	tau-sneutrino	58
$\tilde{e}_R$	selectron 2	59
$\tilde{\mu}_R$	smuon 2	61
$\tilde{\tau}_R$	stau 2	63
$\tilde{g}$	gluino $\quad$ 591	64
$\tilde{\chi}_1^0$	neutralino 1	65
$\tilde{\chi}_2^0$	neutralino 2	66
$\tilde{\chi}_3^0$	neutralino 3	67
$\tilde{\chi}_4^0$	neutralino 4	68
$\tilde{\chi}_4^0$	neutralino 5	69
$\tilde{\chi}_4^+$	chargino 1	70

```
let pdg_mw = function
  | L g when g > 0 → 9 + 2 × g
  | L g → − 9 + 2 × g
  | N g when g > 0 → 10 + 2 × g
  | N g → − 10 + 2 × g
  | U g when g > 0 → 2 × g
  | U g → 2 × g
  | D g when g > 0 → − 1 + 2 × g
  | D g → 1 + 2 × g
  | Gl → 21
  | Ga → 22 | Z → 23
  | Wp → 24 | Wm → (−24)
  | SHiggs S1 → 25 | SHiggs S2 → 35 | PHiggs P1 → 36
```
(∗ JR: Only the first charged Higgs. ∗)
```
  | CHiggs HC1 → 37 | CHiggs HC1c → (−37)
  | CHiggs _ → invalid_arg "charged␣Higgs␣not␣yet␣implemented"
  | Sup (M1, g) when g > 0 → 40 + 2 × g
  | Sup (M1, g) → − 40 + 2 × g
  | Sup (M2, g) when g > 0 → 46 + 2 × g
  | Sup (M2, g) → − 46 + 2 × g
  | Sdown (M1, g) when g > 0 → 39 + 2 × g
  | Sdown (M1, g) → − 39 + 2 × g
  | Sdown (M2, g) when g > 0 → 45 + 2 × g
  | Sdown (M2, g) → − 45 + 2 × g
  | Slepton (M1, g) when g > 0 → 51 + 2 × g
  | Slepton (M1, g) → − 51 + 2 × g
  | Slepton (M2, g) when g > 0 → 57 + 2 × g
  | Slepton (M2, g) → − 57 + 2 × g
  | Sneutrino g when g > 0 → 52 + 2 × g
  | Sneutrino g → − 52 + 2 × g
  | Gluino → 64
```
(∗ JR: Only the first two charginos. ∗)
```
  | Chargino C1 → 70 | Chargino C1c → (−70)
  | Chargino C2 → 71 | Chargino C2c → (−71)
  | Chargino C3 → 106 | Chargino C3c → (−106)
  | Chargino C4 → 107 | Chargino C4c → (−107)
  | Neutralino N1 → 65 | Neutralino N2 → 66
  | Neutralino N3 → 67 | Neutralino N4 → 68
  | Neutralino N5 → 69 | Neutralino N6 → 100
  | Neutralino N7 → 101 | Neutralino N8 → 102
  | Neutralino N9 → 103 | Neutralino N10 → 104
  | Neutralino N11 → 105
  | PHiggs P2 → 72 | PHiggs P3 → 89 | PHiggs P4 → 90
  | PHiggs P5 → 91 | PHiggs P6 → 92 | PHiggs P7 → 93
  | SHiggs S3 → 73 | SHiggs S4 → 94 | SHiggs S5 → 95
  | SHiggs S6 → 96 | SHiggs S7 → 97 | SHiggs S8 → 98
  | SHiggs S9 → 99
  | LQ (M1, g) when g > 0 → 78 + 2 × g
  | LQ (M1, g) → − 78 + 2 × g
  | LQ (M2, g) when g > 0 → 79 + 2 × g
```

```
    | LQ (M2, g) → − 79 + 2 × g
    | LQino g when g > 0 → 85 + g
    | LQino g → − 85 + g

let mass_symbol f =
  "mass(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let width_symbol f =
  "width(" ^ string_of_int (abs (pdg_mw f)) ^ ")"

let conj_symbol = function
  | false, str → str
  | true, str → str ^ "_c"

let constant_symbol = function
  | E → "e" | G → "g" | G_Z → "gz"
  | Q_lepton → "qlep" | Q_up → "qup" | Q_down → "qdwn"
  | Q_charg → "qchar"
  | G_NC_lepton → "gnclep" | G_NC_neutrino → "gncneu"
  | G_NC_up → "gncup" | G_NC_down → "gncdwn"
  | G_CC → "gcc"
  | G_CCQ (vc, g1, g2) → conj_symbol (vc, "g_ccq" ) ^ "("
      ^ string_of_int g1 ^ "," ^ string_of_int g2 ^ ")"
  | I_Q_W → "iqw" | I_G_ZWW → "igzww"
  | G_WWWW → "gw4" | G_ZZWW → "gzzww"
  | G_PZWW → "gpzww" | G_PPWW → "gppww"
  | G_GH4_ZZPP (p1, p2) → "g_ZZA0A0(" ^ string_of_phiggs p1 ^ "," ^
      string_of_phiggs p2 ^ ")"
  | G_GH4_ZZSS (s1, s2) → "g_ZZh0h0(" ^ string_of_shiggs s1 ^ "," ^
      string_of_shiggs s2 ^ ")"
  | G_GH4_ZZCC → "g_zzhphm"
  | G_GH4_GaGaCC → "g_AAhphm"
  | G_GH4_ZGaCC → "g_zAhphm"
  | G_GH4_WWCC → "g_wwhphm"
  | G_GH4_WWPP (p1, p2) → "g_WWA0A0(" ^ string_of_phiggs p1 ^ "," ^
      string_of_phiggs p2 ^ ")"
  | G_GH4_WWSS (s1, s2) → "g_WWh0h0(" ^ string_of_shiggs s1 ^ "," ^
      string_of_shiggs s2 ^ ")"
  | G_GH4_ZWSC s → "g_ZWhph0(" ^ string_of_shiggs s ^")"
  | G_GH4_GaWSC s → "g_AWhph0(" ^ string_of_shiggs s ^")"
  | G_GH4_ZWPC p → "g_ZWhpA0(" ^ string_of_phiggs p ^")"
  | G_GH4_GaWPC p → "g_AWhpA0(" ^ string_of_phiggs p ^")"
  | G_CICIS (n1, n2, s) → "g_neuneuh0(" ^ string_of_neu n1 ^ "," ^
      string_of_neu n2 ^ "," ^ string_of_shiggs s ^ ")"
  | G_CICIP (n1, n2, p) → "g_neuneuA0(" ^ string_of_neu n1 ^ "," ^
      string_of_neu n2 ^ "," ^ string_of_phiggs p ^ ")"
  | G_H3_SCC s → "g_h0hphm(" ^ string_of_shiggs s ^ ")"
  | G_H3_SPP (s, p1, p2) → "g_h0A0A0(" ^ string_of_shiggs s ^ "," ^
      string_of_phiggs p1 ^ "," ^ string_of_phiggs p2 ^ ")"
  | G_H3_SSS (s1, s2, s3) → "g_h0h0h0(" ^ string_of_shiggs s1 ^ "," ^
      string_of_shiggs s2 ^ "," ^ string_of_shiggs s3 ^ ")"
  | G_CSC (c1, c2, s) → "g_chchh0(" ^ string_of_char c1 ^ "," ^
```

$string\_of\_char\ c2$ ˆ $","$ ˆ $string\_of\_shiggs\ s$ ˆ $")"$
$|\ G_CPC\ (c1, c2, p)\ \rightarrow\ $ `"g_chchA0("` ˆ $string\_of\_char\ c1$ ˆ $","$ ˆ
$\quad string\_of\_char\ c2$ ˆ $","$ ˆ $string\_of\_phiggs\ p$ ˆ $")"$
$|\ G_YUK_FFS\ (f1, f2, s)\ \rightarrow\ $ `"g_yuk_h0_"` ˆ $string\_of\_fermion\_type\ f1$ ˆ
$\quad string\_of\_fermion\_type\ f2$ ˆ $"("$ ˆ $string\_of\_shiggs\ s$ ˆ $","$ ˆ
$\quad string\_of\_fermion\_gen\ f1$ ˆ $")"$
$|\ G_YUK_FFP\ (f1, f2, p)\ \rightarrow\ $ `"g_yuk_A0_"` ˆ $string\_of\_fermion\_type\ f1$ ˆ
$\quad string\_of\_fermion\_type\ f2$ ˆ $"("$ ˆ $string\_of\_phiggs\ p$ ˆ $","$ ˆ
$\quad string\_of\_fermion\_gen\ f1$ ˆ $")"$
$|\ G_YUK_LCN\ g\ \rightarrow\ $ `"g_yuk_hp_ln("` ˆ $string\_of\_int\ g$ ˆ $")"$
$|\ G_NWC\ (n, c)\ \rightarrow\ $ `"g_nwc("` ˆ $string\_of\_char\ c$ ˆ $","$ ˆ $string\_of\_neu\ n$ ˆ $")"$
$|\ G_CWN\ (c, n)\ \rightarrow\ $ `"g_cwn("` ˆ $string\_of\_char\ c$ ˆ $","$ ˆ $string\_of\_neu\ n$ ˆ $")"$
$|\ G_SLSNW\ (vc, g, m)\ \rightarrow\ conj_symbol\ (vc,\ $ `"g_wslsn"`$)$ ˆ $"("$ ˆ $string\_of\_int\ g$
$\quad$ ˆ $","$ ˆ $string\_of\_sfm\ m$ ˆ $")"$
$|\ G_NZN\ (n1, n2)\ \rightarrow\ $ `"g_zneuneu("` ˆ $string\_of\_neu\ n1$ ˆ $","$
$\quad$ ˆ $string\_of\_neu\ n2$ ˆ $")"$
$|\ G_CZC\ (c1, c2)\ \rightarrow\ $ `"g_zchch("` ˆ $string\_of\_char\ c1$ ˆ $","$
$\quad$ ˆ $string\_of\_char\ c2$ ˆ $")"$
$|\ Gs\ \rightarrow\ $ `"gs"`
$|\ G_YUK_UCD\ (n, m)\ \rightarrow\ $ `"g_yuk_hp_ud("` ˆ $string\_of\_int\ n$ ˆ $","$ ˆ
$\quad string\_of\_int\ m$ ˆ $")"$
$|\ G_YUK_DCU\ (n, m)\ \rightarrow\ $ `"g_yuk_hm_du("` ˆ $string\_of\_int\ n$ ˆ $","$ ˆ
$\quad string\_of\_int\ m$ ˆ $")"$
$|\ G_YUK_N\ (vc, f, n, sf, m)\ \rightarrow\ conj_symbol\ (vc,\ $ `"g_yuk_neu_"` ˆ
$\quad string\_of\_fermion\_type\ f$ ˆ $string\_of\_sff\ sf)$ ˆ $"("$ ˆ
$\quad string\_of\_fermion\_gen\ f$ ˆ $","$ ˆ $string\_of\_neu\ n$ ˆ $","$ ˆ
$\quad string\_of\_sfm\ m$ ˆ $")"$
$|\ G_YUK_G\ (vc, f, sf, m)\ \rightarrow\ conj_symbol\ (vc,\ $ `"g_yuk_gluino_"` ˆ
$\quad string\_of\_fermion\_type\ f$ ˆ $string\_of\_sff\ sf)$ ˆ $"("$ ˆ
$\quad string\_of\_fermion\_gen\ f$ ˆ $","$ ˆ $string\_of\_sfm\ m$ ˆ $")"$
$|\ G_YUK_C\ (vc, f, c, sf, m)\ \rightarrow\ conj_symbol\ (vc,\ $ `"g_yuk_char_"` ˆ
$\quad string\_of\_fermion\_type\ f$ ˆ $string\_of\_sff\ sf)$ ˆ $"("$ ˆ
$\quad string\_of\_fermion\_gen\ f$ ˆ $","$ ˆ $string\_of\_char\ c$ ˆ $","$ ˆ
$\quad string\_of\_sfm\ m$ ˆ $")"$
$|\ G_YUK_Q\ (vc, g1, f, c, sf, m)\ \rightarrow\ conj_symbol\ (vc,\ $ `"g_yuk_char_"` ˆ
$\quad string\_of\_fermion\_type\ f$ ˆ $string\_of\_sff\ sf)$ ˆ $"("$ ˆ $string\_of\_int\ g1$ ˆ
$\quad$ $","$ ˆ $string\_of\_fermion\_gen\ f$ ˆ $","$ ˆ $string\_of\_char\ c$ ˆ $","$ ˆ
$\quad string\_of\_sfm\ m$ ˆ $")"$
$|\ G_WPSUSD\ (vc, m1, m2, g1, g2)\ \rightarrow\ conj_symbol\ (vc,\ $ `"g_wA_susd"`$)$ ˆ $"("$ ˆ
$\quad string\_of\_int\ g1$ ˆ $","$ ˆ $string\_of\_int\ g2$ ˆ $","$ ˆ $string\_of\_sfm\ m1$ ˆ
$\quad$ $","$ ˆ $string\_of\_sfm\ m2$ ˆ $")"$
$|\ G_WZSUSD\ (vc, m1, m2, g1, g2)\ \rightarrow\ conj_symbol\ (vc,\ $ `"g_wz_susd"`$)$ ˆ $"("$ ˆ
$\quad string\_of\_int\ g1$ ˆ $","$ ˆ $string\_of\_int\ g2$ ˆ $","$ ˆ $string\_of\_sfm\ m1$
$\quad$ ˆ $","$ ˆ $string\_of\_sfm\ m2$ ˆ $")"$
$(* \ 3\text{vertex: Higgs-Gauge a la Franke-Fraas} \ *)$

Nomenclature consistent with $flavor\_of\_string$

$\quad |\ G_GH_ZSP\ (s, p)\ \rightarrow\ $ `"g_zh0a0("` ˆ $string\_of\_shiggs\ s$ ˆ $","$ ˆ
$\quad string\_of\_phiggs\ p$ ˆ $")"$
$\quad |\ G_GH_WSC\ s\ \rightarrow\ $ `"g_Whph0("` ˆ $string\_of\_shiggs\ s$ ˆ $")"$

```
  | G_GH_WPC p  →  "g_WhpA0(" ^ string_of_phiggs p^ ")"
  | G_GH_ZZS s  →  "g_ZZh0(" ^ string_of_shiggs s ^ ")"
  | G_GH_WWS s  →  "g_WWh0(" ^ string_of_shiggs s ^ ")"
  | G_GH_ZCC  →  "g_Zhmhp"
  | G_GH_GaCC  →  "g_Ahmhp"
  | G_ZSF (f, g, m1, m2)  →  "g_z" ^ string_of_sff f ^ string_of_sff f ^ "(" ^
      string_of_int g ^ "," ^ string_of_sfm m1 ^ "," ^ string_of_sfm m2 ^ ")"
  | G_HSNSL (vc, g, m)  →  conj_symbol (vc, "g_hp_sl" ^ string_of_sfm m ^ "sn1")
      ^ "(" ^ string_of_int g ^ ")"
  | G_GlGlSQSQ  →  "g_gg_sqsq"
  | G_PPSFSF f  →  "g_AA_" ^ string_of_sff f ^ string_of_sff f
  | G_ZZSFSF (f, g, m1, m2)  →  "g_zz_" ^ string_of_sff f ^string_of_sff f ^ "(" ^
      string_of_int g ^ "," ^ string_of_sfm m1 ^ "," ^ string_of_sfm m2 ^ ")"
  | G_ZPSFSF (f, g, m1, m2)  →  "g_zA_" ^ string_of_sff f ^string_of_sff f ^ "("
      ^ string_of_int g ^"," ^ string_of_sfm m1 ^ "," ^ string_of_sfm m2 ^ ")"
  | G_GlPSQSQ  →  "g_gA_sqsq"
  | G_GlZSFSF (f, g, m1, m2)  →  "g_gz_" ^ string_of_sff f ^ string_of_sff f ^
      "(" ^ string_of_int g ^ "," ^ string_of_sfm m1 ^ "," ^ string_of_sfm
      m2 ^ ")"
  | G_GlWSUSD (vc, m1, m2, g1, g2)  →  conj_symbol (vc, "g_gw_susd") ^ "(" ^
      string_of_int g1 ^ "," ^string_of_int g2 ^ "," ^ string_of_sfm m1 ^ ","
      ^ string_of_sfm m2 ^ ")"
  | G_strong  →  "gs" |  G_SS  →  "gs**2"
  | I_G_S  →  "igs"
  | G_NHC (vc, n, c)  →  conj_symbol(vc,"g_neuhmchar") ^ "(" ^
      string_of_neu n ^ "," ^ string_of_char c ^ ")"
  | G_WWSFSF (f, g, m1, m2)  →  "g_ww_" ^ string_of_sff f ^ string_of_sff f ^
      "(" ^ string_of_int g ^ "," ^ string_of_sfm m1 ^ "," ^ string_of_sfm m2
      ^ ")"
  | G_WPSLSN (vc, g, m)  →  conj_symbol (vc, "g_wA_slsn") ^"("^ string_of_int g
      ^ "," ^ string_of_sfm m ^ ")"
  | G_WZSLSN (vc, g, m)  →  conj_symbol (vc, "g_wz_slsn") ^ "(" ^ string_of_int
      g ^ "," ^ string_of_sfm m ^ ")"
  | G_SFSFS (s, f, g, m1, m2)  →  "g_h0_"^ string_of_sff f ^ string_of_sfm m1
      ^ string_of_sff f ^ string_of_sfm m2 ^ "(" ^ string_of_shiggs s ^ "," ^
      string_of_int g ^ ")"
  | G_SFSFP (p, f, g, m1, m2)  →  "g_A0_"^ string_of_sff f ^ string_of_sfm m1
      ^ string_of_sff f ^ string_of_sfm m2 ^ "(" ^ string_of_phiggs p ^ "," ^
      string_of_int g ^ ")"
  | G_HSUSD (vc, m1, m2, g1, g2)  →  conj_symbol (vc, "g_hp_su" ^ string_of_sfm m1
      ^ "sd" ^ string_of_sfm m2 ) ^ "(" ^ string_of_int g1 ^ ","
      ^ string_of_int g2 ^ ")"
  | G_WSQ (vc, g1, g2, m1, m2)  →  conj_symbol (vc, "g_wsusd") ^ "("
      ^ string_of_int g1 ^ "," ^ string_of_int g2 ^ "," ^ string_of_sfm m1 ^
      "," ^ string_of_sfm m2 ^ ")"
  | G_YUK_LQ_S (g1, s, g3)  →  "g_yuk_lq_s(" ^ string_of_int g1 ^ "," ^
      string_of_shiggs s ^"," ^ string_of_int g3 ^")"
  | G_YUK_LQ_P (g1, p, g3)  →  "g_yuk_lq_p(" ^ string_of_int g1 ^ "," ^
      string_of_phiggs p ^ "," ^ string_of_int g3 ^ ")"
  | G_LQ_NEU (m, g1, g2, n)  →  "g_lq_neu(" ^ string_of_sfm m ^ "," ^
```

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```
            string_of_int g1 ^ "," ^ string_of_int g2 ^ "," ^ string_of_neu n ^ ")"
    | G_LQ_GG (m, g1, g2) → "g_lq-gg(" ^ string_of_sfm m ^ "," ^
            string_of_int g1 ^ "," ^ string_of_int g2 ^ ")"
    | G_LQ_EC_UC (vc, m, g1, g2, g3) → conj_symbol(vc,"g_lq_ec_uc") ^ "(" ^
            string_of_sfm m ^ "," ^ string_of_int g1 ^ "," ^ string_of_int g2 ^ ","
            ^ string_of_int g3 ^ ")"
    | G_LQ_SSU (m1, m2, m3, g1, g2, g3) → "g_lq-sst(" ^ string_of_sfm m1 ^ "," ^
            string_of_sfm m2 ^ "," ^ string_of_sfm m3 ^ "," ^ string_of_int g1 ^ "," ^
            string_of_int g2 ^ "," ^ string_of_int g3 ^ ")"
    | G_LQ_SSD (m1, m2, g1, g2, g3) → "g_lq-ssta(" ^ string_of_sfm m1 ^ "," ^
            string_of_sfm m2 ^ "," ^ string_of_int g1 ^ "," ^ string_of_int g2 ^
            "," ^ string_of_int g3 ^ ")"
    | G_LQ_S (m1, m2, g1, s, g2) → "g_lq-s(" ^ string_of_sfm m1 ^ "," ^ string_of_sfm m2 ^ ","
            ^ string_of_int g1 ^ "," ^ string_of_shiggs s ^ "," ^ string_of_int g2 ^ ")"
    | G_LQ_P (m1, m2, g1, p, g2) → "g_lq-s(" ^ string_of_sfm m1 ^ "," ^ string_of_sfm m2 ^ ","
            ^ string_of_int g1 ^ "," ^ string_of_phiggs p ^ "," ^ string_of_int g2 ^ ")"
    | G_ZLQ (g, m1, m2) → "g_zlqlq(" ^ string_of_int g ^ "," ^
            string_of_sfm m1 ^ "," ^ string_of_sfm m2 ^ ")"
    | G_ZZLQLQ → "g_zz_lqlq"
    | G_ZPLQLQ → "g_zA_lqlq"
    | G_PPLQLQ → "g_AA_lqlq"
    | G_ZGlLQLQ → "g_zg_lqlq"
    | G_PGlLQLQ → "g_Ag_lqlq"
    | G_GlGlLQLQ → "g_gg_lqlq"
    | G_NLQC → "g_nlqc"

end
```

—14—
Comphep Models

14.1 Interface of Comphep_syntax

```
type raw  =
   |  I  |  Integer of int |  Symbol of string
   |  Application of string × raw
   |  Dotproduct of raw  ×  raw
   |  Product of (raw  ×  int) list
   |  Sum of (raw  ×  int) list

val symbol  :  string →  raw
val integer  :  int →  raw
val imag  :  raw

val apply  :  string →  raw  →  raw
val dot  :  raw  →  raw  →  raw
val multiply  :  raw  →  raw  →  raw
val divide  :  raw  →  raw  →  raw
val power  :  raw  →  int →  raw
val add  :  raw  →  raw  →  raw
val subtract  :  raw  →  raw  →  raw
val neg  :  raw  →  raw
```

14.2 Implementation of Comphep_syntax

```
type raw  =
   |  I  |  Integer of int |  Symbol of string
   |  Application of string × raw
   |  Dotproduct of raw  ×  raw
   |  Product of (raw  ×  int) list
   |  Sum of (raw  ×  int) list

let symbol name  =  Symbol name
let integer n  =  Integer n
let imag  =  I

let apply f x  =  Application (f,  x)
let dot x y  =  Dotproduct (x,  y)
```

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```
let negate  =  List.map (fun (x, c) → (x, − c))
let scale n  =  List.map (fun (x, c) → (x, n × c))

let add1 (x, c) y  =
  if c = 0 then
    y
  else
    try
      let c′ = List.assoc x y + c in
      if c′ = 0 then
        List.remove_assoc x y
      else
        (x, c′) :: (List.remove_assoc x y)
    with
    | Not_found → (x, c) :: y

let addn  =  List.fold_right add1

let multiply x y  =
  match x, y with
  | Product x′, Product y′ → Product (addn x′ y′)
  | Integer n, Product y′ → Product (scale n y′)
  | Product x′, Integer n → Product (scale n x′)
  | _, Product y′ → Product (add1 (x, 1) y′)
  | Product x′, _ → Product (add1 (y, 1) x′)
  | _ when x = y → Product ([(x, 2)])
  | _ → Product ([(x, 1); (y, 1)])

let divide x y  =
  match y with
  | Product y′ → multiply x (Product (negate y′))
  | _ when x = y → Product ([])
  | _ → Product ([(x, 1); (y, − 1)])

let power x n  =
  match x with
  | Product x′ → Product (scale n x′)
  | x → Product ([(x, n)])

let add x y  =
  match x, y with
  | Sum x′, Sum y′ → Sum (addn x′ y′)
  | _, Sum y′ → Sum (add1 (x, 1) y′)
  | Sum x′, _ → Sum (add1 (y, 1) x′)
  | _ when x = y → Sum ([(x, 2)])
  | _ → Sum ([(x, 1); (y, 1)])

let subtract x y  =
  match y with
  | Sum y′ → add x (Sum (negate y′))
  | _ when x = y → Sum ([])
  | _ → Sum ([(x, 1); (y, − 1)])

let neg  =  function
```

```
    | Sum x  →  Sum (negate x)
    | x  →  Sum ([(x, − 1)])

type vector  =
    | Momentum of int
    | Index of int
    | Index′ of int

let vector_keyword  = function
    | "p1" →  Some (Momentum 1)
    | "p2" →  Some (Momentum 2)
    | "p3" →  Some (Momentum 3)
    | "p4" →  Some (Momentum 4)
    | "m1" →  Some (Index 1)
    | "m2" →  Some (Index 2)
    | "m3" →  Some (Index 3)
    | "m4" →  Some (Index 4)
    | "M1" →  Some (Index′ 1)
    | "M2" →  Some (Index′ 2)
    | "M3" →  Some (Index′ 3)
    | "M4" →  Some (Index′ 4)
    | _ →  None
```

14.3   Lexer

```
{
open Comphep_parser
}

let digit  =  ['0'−'9']
let upper  =  ['A'−'Z']
let lower  =  ['a'−'z']
let alpha  =  upper | lower
let alphanum  =  alpha | digit

let symbol  =  alpha alphanum⋆
let integer  =  digit⁺

rule token  = parse
      [' ' '\t'] { token lexbuf } (∗ skip blanks ∗)
    | "(" { LPAREN }
    | ")" { RPAREN }
    | "i" { I }
    | "." { DOT }
    | "**" { POWER }
    | "*" { MULT }
    | "/" { DIV }
    | "+" { PLUS }
    | "−" { MINUS }
    | symbol { SYMBOL (Lexing.lexeme lexbuf) }
    | integer { INT (int_of_string (Lexing.lexeme lexbuf)) }
```

```
| _ { failwith ("lexer␣fails␣@" ^ Lexing.lexeme lexbuf) }
| eof { END }
```

14.4   Parser

Header

module S = $Comphep\_syntax$

Token declarations

%token < $string$ > $SYMBOL$
%token < int > INT
%token I
%token $LPAREN$ $RPAREN$
%token DOT $MULT$ DIV $POWER$ $PLUS$ $MINUS$
%token END

%left $PLUS$ $MINUS$
%left $MULT$ DIV
%nonassoc $UNARY$
%nonassoc $POWER$
%nonassoc DOT

%start $expr$
%type < $Comphep\_syntax.raw$ > $expr$

Grammar rules

$expr$ ::=
     e END { \$1 }

e ::=
     $SYMBOL$ { $S.symbol$ \$1 }
  | INT { $S.integer$ \$1 }
  | I { $S.imag$ }
  | $SYMBOL$ $LPAREN$ e $RPAREN$ { $S.apply$ \$1 \$3 }
  | $LPAREN$ e $RPAREN$ { \$2 }
  | e DOT e { $S.dot$ \$1 \$3 }
  | e $MULT$ e { $S.multiply$ \$1 \$3 }
  | e DIV e { $S.divide$ \$1 \$3 }
  | e $PLUS$ e { $S.add$ \$1 \$3 }
  | e $MINUS$ e { $S.subtract$ \$1 \$3 }

```
| PLUS e %prec UNARY {  $2 }
| MINUS e %prec UNARY {  S.neg $2 }
| e POWER INT {  S.power $1 $3 }
```

14.5   Interface of Comphep

Wolfgang's idea: read Comphep's model files:

module *Model* : *Model.T*

14.6   Implementation of Comphep

let *rcs_file* = *RCS.parse* "Comphep" ["Plagiarizing␣CompHEP␣models␣..."]
    { *RCS.revision* = "$Revision:␣6465␣$";
       RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
       RCS.author = "$Author:␣jr_reuter␣$";
       RCS.source
          = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s

A friendlier *String.sub* that returns an empty string instead of raising an exception. Instead of the length, the second argument denotes the last position.

let *substring buffer i1 i2* =
  let *imax* = *String.length buffer* $-$ 1 in
  let *i1* = *max i1* 0
  and *i2* = *min i2 imax* in
  let *len* = *i2* $-$ *i1* $+$ 1 in
  if *len* $>$ 0 then
    String.sub buffer i1 len
  else
    ""

let *first_non_white buffer* =
  let *len* = *String.length buffer* in
  let rec *skip_white i* =
    if *i* $\geq$ *len* then
      i
    else if *buffer.[i]* $\neq$ ' ' $\wedge$ *buffer.[i]* $\neq$ '\t' then
      i
    else
      skip_white (*succ i*) in
  skip_white 0

let *last_non_white buffer* =
  let *len* = *String.length buffer* in
  let rec *skip_white i* =
    if *i* $<$ 0 then
      i
    else if *buffer.[i]* $\neq$ ' ' $\wedge$ *buffer.[i]* $\neq$ '\t' then
      i

```ocaml
      else
          skip_white (pred i) in
      skip_white (pred len)

let gobble_white buffer =
   substring buffer (first_non_white buffer) (last_non_white buffer)

let gobble_arrows buffer =
   let imax = String.length buffer − 1 in
   if imax ≥ 0 then
      gobble_white
        (substring buffer
            (if buffer.[0] = '>' then 1 else 0)
            (if buffer.[imax] = '<' then pred imax else imax))
   else
      ""

let fold_lines ic f init =
   let rec fold_lines' acc =
      let continue =
         try
            let acc' = f (input_line ic) acc in
            fun () → fold_lines' acc'
         with
         | End_of_file → fun () → acc in
      continue () in
   fold_lines' init

let column_tabs line =
   let len = String.length line in
   let rec tabs' acc i =
      if i ≥ len then
         List.rev acc
      else if line.[i] = '|' then
         tabs' (i :: acc) (succ i)
      else
         tabs' acc (succ i)
   in
   tabs' [] 0

let columns tabs line =
   let imax = String.length line − 1 in
   let rec columns' acc i = function
      | [] → List.rev_map gobble_white (substring line i imax :: acc)
      | tab :: rest →
          if tab < i then
             invalid_arg "columns:␣clash"
          else if (match rest with [] → false | _ → true)
                 ∧ line.[tab] ≠ '|' then
             invalid_arg "columns:␣expecting␣'|'"
          else
             columns' (substring line i (pred tab) :: acc) (succ tab) rest
   in
```

$columns'\ [\ ]\ 0\ tabs$

let $input\_table\ name\ =$
  let $ic\ =\ open\_in\ name$ in
  let $model\ =\ input\_line\ ic$ in
  let $table\ =\ input\_line\ ic$ in
  let $line\ =\ input\_line\ ic$ in
  let $tabs\ =\ column\_tabs\ line$ in
  let $titles\ =\ columns\ tabs\ line$ in
  let $rows\ =\ fold\_lines\ ic$ (fun $line\ acc\ \rightarrow$
    if $String.length\ line\ >\ 0\ \wedge\ line.[0]\ =\ $ '=' then
      acc
    else
      $columns\ tabs\ line\ ::\ acc)\ [\ ]$ in
  $close\_in\ ic;$
  ($gobble\_white\ model,\ gobble\_white\ table,\ List.map\ gobble\_arrows\ titles,\ rows$)

let $substitute\_char\ (cold,\ cnew)\ s\ =$
  for $i\ =\ 0$ to $String.length\ s\ -\ 1$ do
    if $s.[i]\ =\ cold$ then
      $s.[i]\ \leftarrow\ cnew$
  done;
  s

let $sanitize\_symbol\ s\ =$
  $List.fold\_right\ substitute\_char\ [('+',\ 'p');\ ('-',\ 'm')]\ (String.copy\ s)$

⚠ Fodder for a future _Coupling_ module . . .

let rec $fermion\_of\_lorentz\ =$ function
  | $Coupling.Spinor\ \rightarrow\ 1$
  | $Coupling.ConjSpinor\ \rightarrow\ -1$
  | $Coupling.Majorana\ \rightarrow\ 1$
  | $Coupling.Maj\_Ghost\ \rightarrow\ 1$
  | $Coupling.Vectorspinor\ \rightarrow\ 1$
  | $Coupling.Vector\ |\ Coupling.Massive\_Vector\ \rightarrow\ 0$
  | $Coupling.Scalar\ |\ Coupling.Tensor\_1\ |\ Coupling.Tensor\_2\ \rightarrow\ 0$
  | $Coupling.BRS\ f\ \rightarrow\ fermion\_of\_lorentz\ f$

let rec $conjugate\_lorentz\ =$ function
  | $Coupling.Spinor\ \rightarrow\ Coupling.ConjSpinor$
  | $Coupling.ConjSpinor\ \rightarrow\ Coupling.Spinor$
  | $Coupling.BRS\ f\ \rightarrow\ Coupling.BRS\ (conjugate\_lorentz\ f)$
  | $f\ \rightarrow\ f$

⚠ Currently, this operates on the sanitized symbol names.

let $pdg\_heuristic\ name\ =$
  match $name$ with
  | "e1" $\rightarrow$ 11 | "E1" $\rightarrow$ -11 | "n1" $\rightarrow$ 12 | "N1" $\rightarrow$ -12
  | "e2" $\rightarrow$ 13 | "E2" $\rightarrow$ -13 | "n2" $\rightarrow$ 14 | "N2" $\rightarrow$ -14

```
      |  "e3" →  15  |  "E3" →  − 15  |  "n3" →  16  |  "N3" →  − 16
      |  "u"  →  2   |  "U"  →  − 2   |  "d"  →  1   |  "D"  →  − 1
      |  "c"  →  4   |  "C"  →  − 4   |  "s"  →  3   |  "S"  →  − 3
      |  "t"  →  6   |  "T"  →  − 6   |  "b"  →  5   |  "B"  →  − 5
      |  "G"  →  21  |  "A"  →  22  |  "Z"  →  23
      |  "Wp" →  24  |  "Wm" →  − 24  |  "H"  →  25
      |  _ →  invalid_arg ("pdg_heuristic failed: " ^ name)
```

module *Model* =
  struct

    type *flavor* = *int*
    type *constant* = *string*
    type *gauge* = *unit*

    module *M* = *Modeltools.Mutable*
        (struct type *f* = *flavor* type *g* = *gauge* type *c* = *constant* end)

    let *flavors* = *M.flavors*
    let *external_flavors* = *M.external_flavors*
    let *lorentz* = *M.lorentz*
    let *color* = *M.color*
    let *propagator* = *M.propagator*
    let *width* = *M.width*
    let *goldstone* = *M.goldstone*
    let *conjugate* = *M.conjugate*
    let *fermion* = *M.fermion*
    let *vertices* = *M.vertices*
    let *fuse2* = *M.fuse2*
    let *fuse3* = *M.fuse3*
    let *fuse* = *M.fuse*
    let *max_degree* = *M.max_degree*
    let *parameters* = *M.parameters*
    let *flavor_of_string* = *M.flavor_of_string*
    let *flavor_to_string* = *M.flavor_to_string*
    let *flavor_to_TeX* = *M.flavor_to_TeX*
    let *flavor_symbol* = *M.flavor_symbol*
    let *gauge_symbol* = *M.gauge_symbol*
    let *pdg* = *M.pdg*
    let *mass_symbol* = *M.mass_symbol*
    let *width_symbol* = *M.width_symbol*
    let *constant_symbol* = *M.constant_symbol*
    module *Ch* = *M.Ch*
    let *charges* = *M.charges*

    let *rcs* = *rcs_file*

    type *symbol* =
      | *Selfconjugate* of *string*
      | *Conjugates* of *string* × *string*

    type *particle* =
        { *p_name* : *string*;
          p_symbol : *symbol*;
```

```
          p_spin  :  Coupling.lorentz;
          p_mass  :  Comphep_syntax.raw;
          p_width  :  Comphep_syntax.raw;
          p_color  :  Color.t;
          p_aux  :  string option }
```

let *count_flavors particles* =
  List.fold_left (fun *n p* → *n* +
    match *p.p_symbol* with
    | *Selfconjugate* _ → 1
    | *Conjugates* _ → 2) 0 *particles*

type *particle_flavor* =
  { *f_name* : *string*;
    f_conjugate : *int*;
    f_symbol : *string*;
    f_pdg : *int*;
    f_spin : *Coupling.lorentz*;
    f_propagator : *gauge Coupling.propagator*;
    f_fermion : *int*;
    f_mass : *string*;
    f_width : *string*;
    f_color : *Color.t*;
    f_aux : *string option* }

let *real_variable* = function
  | *Comphep_syntax.Integer* 0 → `"zero"`
  | *Comphep_syntax.Symbol s* → *s*
  | _ → *invalid_arg* `"real_variable"`

let *dummy_flavor* =
  { *f_name* = `""`;
    f_conjugate = −1;
    f_symbol = `""`;
    f_pdg = 0;
    f_spin = *Coupling.Scalar*;
    f_propagator = *Coupling.Prop_Scalar*;
    f_fermion = 0;
    f_mass = *real_variable* (*Comphep_syntax.integer* 0);
    f_width = *real_variable* (*Comphep_syntax.integer* 0);
    f_color = *Color.Singlet*;
    f_aux = *None* }

let *propagator_of_lorentz* = function
  | *Coupling.Scalar* → *Coupling.Prop_Scalar*
  | *Coupling.Spinor* → *Coupling.Prop_Spinor*
  | *Coupling.ConjSpinor* → *Coupling.Prop_ConjSpinor*
  | *Coupling.Majorana* → *Coupling.Prop_Majorana*
  | *Coupling.Maj_Ghost* → *invalid_arg*
      `"propagator_of_lorentz:␣SUSY␣ghosts␣do␣not␣propagate"`
  | *Coupling.Vector* → *Coupling.Prop_Feynman*
  | *Coupling.Massive_Vector* → *Coupling.Prop_Unitarity*
  | *Coupling.Vectorspinor* →

      invalid_arg `"propagator_of_lorentz:␣Vectorspinor"`
  | *Coupling.Tensor_1* → *invalid_arg* `"propagator_of_lorentz:␣Tensor_1"`
  | *Coupling.Tensor_2* → *invalid_arg* `"propagator_of_lorentz:␣Tensor_2"`
  | *Coupling.BRS* _ → *invalid_arg* `"propagator_of_lorentz:␣no␣BRST"`

let *flavor_of_particle symbol conjg particle* =
  let *spin* = *particle.p_spin* in
  { *f_name* = *particle.p_name*;
   f_conjugate = *conjg*;
   f_symbol = *symbol*;
   f_pdg = *pdg_heuristic symbol*;
   f_spin = *spin*;
   f_propagator = *propagator_of_lorentz spin*;
   f_fermion = *fermion_of_lorentz spin*;
   f_mass = *real_variable particle.p_mass*;
   f_width = *real_variable particle.p_width*;
   f_color = *particle.p_color*;
   f_aux = *particle.p_aux* }

let *flavor_of_antiparticle symbol conjg particle* =
  let *spin* = *conjugate_lorentz particle.p_spin* in
  { *f_name* = `"anti-"` ˆ *particle.p_name*;
   f_conjugate = *conjg*;
   f_symbol = *symbol*;
   f_pdg = *pdg_heuristic symbol*;
   f_spin = *spin*;
   f_propagator = *propagator_of_lorentz spin*;
   f_fermion = *fermion_of_lorentz spin*;
   f_mass = *real_variable particle.p_mass*;
   f_width = *real_variable particle.p_width*;
   f_color = *Color.conjugate particle.p_color*;
   f_aux = *particle.p_aux* }

let *parse_expr text* =
  try
   Comphep_parser.expr Comphep_lexer.token (*Lexing.from_string text*)
  with
  | *Parsing.Parse_error* → *invalid_arg* (`"parse␣error:␣"` ˆ *text*)

let *parse_function_row* = function
  | *name* :: *fct* :: *comment* :: _ → (*name, parse_expr fct, comment*)
  | _ → *invalid_arg* `"parse_function_row"`

let *parse_lagragian_row* = function
  | *p1* :: *p2* :: *p3* :: *p4* :: *c* :: *t* :: _ →
    ((*p1, p2, p3, p4*), *parse_expr c, parse_expr t*)
  | _ → *invalid_arg* `"parse_lagragian_row"`

let *parse_symbol s1 s2* =
  if *s1* = *s2* then
   Selfconjugate (*sanitize_symbol s1*)
  else
   Conjugates (*sanitize_symbol s1, sanitize_symbol s2*)

let *parse_spin spin* =
  match *int_of_string spin* with
  | 0 → *Coupling.Scalar*
  | 1 → *Coupling.Spinor*
  | 2 → *Coupling.Vector*
  | _ → *invalid_arg* ("`parse_spin:␣spin␣=␣`" ̂ *spin*)

let *parse_color color* =
  match *int_of_string color* with
  | 1 → *Color.Singlet*
  | 3 → *Color.SUN* 3
  | 8 → *Color.AdjSUN* 3
  | _ → *invalid_arg* ("`parse_color:␣color␣=␣`" ̂ *color*)

let *parse_particle_row* = function
  | *name* :: *symbol* :: *symbol_cc* :: *spin* :: *mass* :: *width* :: *color* ::
    aux :: _ →
      { *p_name* = *name*;
        p_symbol = *parse_symbol symbol symbol_cc*;
        p_spin = *parse_spin spin*;
        p_mass = *parse_expr mass*;
        p_width = *parse_expr width*;
        p_color = *parse_color color*;
        p_aux = match *aux* with "" → *None* | _ → *Some aux* }
  | _ → *invalid_arg* "`parse_particle_row`"

let *parse_variable_row* = function
  | *name* :: *value* :: *comment* :: _ →
    (*name*, *float_of_string value*, *comment*)
  | _ → *invalid_arg* "`parse_variable_row`"

let *parse_table parse_row name* =
  let *model*, *table*, *titles*, *rows* = *input_table name* in
  (*model*, *table*, *titles*, *List.rev_map parse_row rows*)

let *input_functions* = *parse_table parse_function_row*
let *input_lagrangian* = *parse_table parse_lagragian_row*
let *input_particles* = *parse_table parse_particle_row*
let *input_variables* = *parse_table parse_variable_row*

let *input_model dir idx* =
  let *idx* = *string_of_int idx* in
  (*input_particles* (*dir* ̂ "`/prtcls`" ̂ *idx* ̂ "`.mdl`"),
   input_variables (*dir* ̂ "`/vars`" ̂ *idx* ̂ "`.mdl`"),
   input_functions (*dir* ̂ "`/func`" ̂ *idx* ̂ "`.mdl`"),
   input_lagrangian (*dir* ̂ "`/lgrng`" ̂ *idx* ̂ "`.mdl`"))

let *flavors_of_particles particles* =
  let *flavors* = *Array.make* (*count_flavors particles*) *dummy_flavor* in
  ignore (*List.fold_left* (fun *n p* →
    match *p.p_symbol* with
    | *Selfconjugate f* →
      flavors.(*n*) ← *flavor_of_particle f n p*;
      n + 1

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```
        | Conjugates (f1, f2) →
            flavors.(n) ← flavor_of_particle f1 (n + 1) p;
            flavors.(n + 1) ← flavor_of_antiparticle f2 n p;
            n + 2) 0 particles);
    flavors
module F = Modeltools.Fusions (struct
    type f = flavor
    type c = constant
    let compare = compare
    let conjugate = conjugate
end)

let translate_tensor3 _ = Coupling.Scalar_Scalar_Scalar 1
let translate_tensor4 _ = Coupling.Scalar4 1
let translate_constant _ = ""

let init flavors variables functions vertices =
    let fmax = Array.length flavors − 1 in
    let flist = ThoList.range 0 fmax in
    let clamp_flavor msg f =
        if f ≥ 0 ∨ f ≤ fmax then
            f
        else
            invalid_arg (msg ^ ":␣invalid␣flavor:␣" ^ string_of_int f) in
    let flavor_hash = Hashtbl.create 37 in
    let flavor_of_string s =
        try
            Hashtbl.find flavor_hash s
        with
        | Not_found → invalid_arg ("flavor_of_string:␣" ^ s) in
    for f = 0 to fmax do
        Hashtbl.add flavor_hash flavors.(f).f_symbol f
    done;
    let vertices3, vertices4 =
        List.fold_left (fun (v3, v4) ((p1, p2, p3, p4), c, t) →
            if p4 = "" then
                (((flavor_of_string p1, flavor_of_string p2, flavor_of_string p3),
                    translate_tensor3 t, translate_constant c) :: v3, v4)
            else
                (v3, ((flavor_of_string p1, flavor_of_string p2,
                        flavor_of_string p3, flavor_of_string p4),
                        translate_tensor4 t, translate_constant c) :: v4))
        ([], []) vertices in
    let max_degree = match vertices4 with [] → 3 | _ → 4 in
    let all_vertices () = (vertices3, vertices4, []) in
    let table = F.of_vertices (all_vertices ()) in
    let input_parameters =
        (real_variable (Comphep_syntax.integer 0), 0.0) ::
        (List.map (fun (n, v, _) → (n, v)) variables) in
    let derived_parameters =
        List.map (fun (n, f, _) → (Coupling.Real n, Coupling.Const 0))
```

   functions in
  M.setup
   ˜*color* : (fun $f \rightarrow$ *flavors.*(*clamp_flavor* "color" f).f_color)
   ˜*pdg* : (fun $f \rightarrow$ *flavors.*(*clamp_flavor* "pdg" f).f_pdg)
   ˜*lorentz* : (fun $f \rightarrow$ *flavors.*(*clamp_flavor* "spin" f).f_spin)
   ˜*propagator* : (fun $f \rightarrow$
    flavors.(*clamp_flavor* "propagator" f).$f\_propagator$)
   ˜*width* : (fun $f \rightarrow$ *Coupling.Constant*)
   ˜*goldstone* : (fun $f \rightarrow$ *None*)
   ˜*conjugate* : (fun $f \rightarrow$ *flavors.*(*clamp_flavor* "conjugate" f).$f\_conjugate$)
   ˜*fermion* : (fun $f \rightarrow$ *flavors.*(*clamp_flavor* "fermion" f).$f\_fermion$)
   ˜*max_degree*
   ˜*vertices* : *all_vertices*
   ˜*fuse* : (*F.fuse2 table, F.fuse3 table, F.fuse table*)
   ˜*flavors* : ([("All␣Flavors", *flist*)])
   ˜*parameters* : (fun () $\rightarrow$
    { *Coupling.input* = *input_parameters*;
     Coupling.derived = *derived_parameters*;
     Coupling.derived_arrays = [] })
   ˜*flavor_of_string*
   ˜*flavor_to_string* : (fun $f \rightarrow$
    flavors.(*clamp_flavor* "flavor_to_string" f).f_name)
   ˜*flavor_to_TeX* : (fun $f \rightarrow$
    flavors.(*clamp_flavor* "flavor_to_TeX" f).f_name)
   ˜*flavor_symbol* : (fun $f \rightarrow$
    flavors.(*clamp_flavor* "flavor_symbol" f).f_symbol)
   ˜*gauge_symbol* : (fun () $\rightarrow$ "")
   ˜*mass_symbol* : (fun $f \rightarrow$
    flavors.(*clamp_flavor* "mass_symbol" f).f_mass)
   ˜*width_symbol* : (fun $f \rightarrow$
    flavors.(*clamp_flavor* "width_symbol" f).f_width)
   ˜*constant_symbol* : (fun $c \rightarrow$ *failwith* "constant_symbol")

let *particles_file* = *ref* "prtcls1.mdl"
let *variables_file* = *ref* "vars1.mdl"
let *functions_file* = *ref* "func1.mdl"
let *lagrangian_file* = *ref* "lgrng1.mdl"

let *load* () =
 let (_, _, _, p), v, f, l =
  (*input_particles* !*particles_file*, *input_variables* !*variables_file*,
   input_functions !*functions_file*, *input_lagrangian* !*lagrangian_file*) in
 init (*flavors_of_particles* p) [] [] []

let *options* = *Options.create*
  [("p", *Arg.String* (fun *name* $\rightarrow$ *particles_file* := *name*),
   "CompHEP␣particles␣file␣(default:␣" ˆ !*particles_file* ˆ ")");
   ("v", *Arg.String* (fun *name* $\rightarrow$ *variables_file* := *name*),
   "CompHEP␣variables␣file␣(default:␣" ˆ !*variables_file* ˆ ")");
   ("f", *Arg.String* (fun *name* $\rightarrow$ *functions_file* := *name*),
   "CompHEP␣functions␣file␣(default:␣" ˆ !*functions_file* ˆ ")");
   ("l", *Arg.String* (fun *name* $\rightarrow$ *lagrangian_file* := *name*),

```
        "CompHEP␣lagrangian␣file␣(default:␣" ^ !lagrangian_file ^ ")");
      ("exec", Arg.Unit load,
        "load␣the␣model␣files␣(required␣_before_␣any␣particle)");
      ("help", Arg.Unit (fun ()  →
          print_endline
            ("[" ^ String.concat "|"
                      (List.map M.flavor_to_string (M.flavors ())) ^ "]")),
        "print␣information␣on␣the␣model")]

end
```

—15—
HARDCODED TARGETS

15.1 Interface of Targets

module *Dummy* : *Target.Maker*

15.1.1 Supported Targets

module *Fortran* : *Target.Maker*
module *Fortran_Majorana* : *Target.Maker*
module *VM* : *Target.Maker*

15.1.2 Potential Targets

module *Fortran77* : *Target.Maker*
module *C* : *Target.Maker*
module *Cpp* : *Target.Maker*
module *Java* : *Target.Maker*
module *Ocaml* : *Target.Maker*
module *LaTeX* : *Target.Maker*

15.2 Implementation of Targets

let *rcs_file* = *RCS.parse* "Targets" ["Code␣Generation"]
    { *RCS.revision* = "$Revision:␣7372␣$";
      RCS.date = "$Date:␣2015-11-18␣19:07:59␣+0100␣(Wed,␣18␣Nov␣2015)␣$";
      RCS.author = "$Author:␣jr_reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s

module *Dummy* (*F* : *Fusion.Maker*) (*P* : *Momentum.T*) (*M* : *Model.T*) =
  struct
    let *rcs_list* = []
    type *amplitudes* = *Fusion.Multi*(*F*)(*P*)(*M*).*amplitudes*
    type *diagnostic* = *All* | *Arguments* | *Momenta* | *Gauge*
    let *options* = *Options.empty*

```
    let amplitudes_to_channel _ _ _ = failwith "Targets.Dummy"
    let parameters_to_channel _ = failwith "Targets.Dummy"
  end
```

15.2.1   O'Mega Virtual Machine with `Fortran 90/95`

Preliminaries

```
module VM (Fusion_Maker : Fusion.Maker) (P : Momentum.T) (M : Model.T) =
  struct
    let rcs_list =
      [RCS.rename rcs_file "Targets.VM"
                  ["Virtual␣Machine␣able␣to␣compute␣amplitudes␣from␣byte␣code"]]

    open Coupling
    open Format

    module CM = Colorize.It(M)
    module F = Fusion_Maker(P)(M)
    module CF = Fusion.Multi(Fusion_Maker)(P)(M)
    module CFlow = Color.Flow
    type amplitudes = CF.amplitudes
```

Options.

```
    type diagnostic = All | Arguments | Momenta | Gauge

    let wrapper_module = ref "ovm_wrapper"
    let parameter_module_external = ref "some_external_module_with_model_info"
    let bytecode_file = ref "bytecode.hbc"
    let md5sum = ref None
    let openmp = ref false
    let kind = ref "default"
    let whizard = ref false

    let options = Options.create
      [ "wrapper_module", Arg.String (fun s → wrapper_module := s),
          "name␣of␣wrapper␣module";
        "bytecode_file", Arg.String (fun s → bytecode_file := s),
          "bytecode␣file␣to␣be␣used␣in␣wrapper";
        "parameter_module_external", Arg.String (fun s →
                                        parameter_module_external := s),
          "external␣parameter␣module␣to␣be␣used␣in␣wrapper";
        "md5sum", Arg.String (fun s → md5sum := Some s),
          "transfer␣MD5␣checksum␣in␣wrapper";
        "whizard", Arg.Set whizard, "include␣WHIZARD␣interface␣in␣wrapper";
        "openmp", Arg.Set openmp,
          "activate␣parallel␣computation␣of␣amplitude␣with␣OpenMP"]
```

This is part of OCaml 4.01.

```
    let (|>) fn x = x fn
```

let (@@) *fn x* = *fn x*

Integers encode the opcodes (operation codes).

let *ovm_ADD_MOMENTA* = 1
let *ovm_CALC_BRAKET* = 2

let *ovm_LOAD_SCALAR* = 10
let *ovm_LOAD_SPINOR_INC* = 11
let *ovm_LOAD_SPINOR_OUT* = 12
let *ovm_LOAD_CONJSPINOR_INC* = 13
let *ovm_LOAD_CONJSPINOR_OUT* = 14
let *ovm_LOAD_MAJORANA_INC* = 15
let *ovm_LOAD_MAJORANA_OUT* = 16
let *ovm_LOAD_VECTOR_INC* = 17
let *ovm_LOAD_VECTOR_OUT* = 18
let *ovm_LOAD_VECTORSPINOR_INC* = 19
let *ovm_LOAD_VECTORSPINOR_OUT* = 20
let *ovm_LOAD_TENSOR2_INC* = 21
let *ovm_LOAD_TENSOR2_OUT* = 22
let *ovm_LOAD_BRS_SCALAR* = 30
let *ovm_LOAD_BRS_SPINOR_INC* = 31
let *ovm_LOAD_BRS_SPINOR_OUT* = 32
let *ovm_LOAD_BRS_CONJSPINOR_INC* = 33
let *ovm_LOAD_BRS_CONJSPINOR_OUT* = 34
let *ovm_LOAD_BRS_VECTOR_INC* = 37
let *ovm_LOAD_BRS_VECTOR_OUT* = 38
let *ovm_LOAD_MAJORANA_GHOST_INC* = 23
let *ovm_LOAD_MAJORANA_GHOST_OUT* = 24
let *ovm_LOAD_BRS_MAJORANA_INC* = 35
let *ovm_LOAD_BRS_MAJORANA_OUT* = 36

let *ovm_PROPAGATE_SCALAR* = 51
let *ovm_PROPAGATE_COL_SCALAR* = 52
let *ovm_PROPAGATE_GHOST* = 53
let *ovm_PROPAGATE_SPINOR* = 54
let *ovm_PROPAGATE_CONJSPINOR* = 55
let *ovm_PROPAGATE_MAJORANA* = 56
let *ovm_PROPAGATE_COL_MAJORANA* = 57
let *ovm_PROPAGATE_UNITARITY* = 58
let *ovm_PROPAGATE_COL_UNITARITY* = 59
let *ovm_PROPAGATE_FEYNMAN* = 60
let *ovm_PROPAGATE_COL_FEYNMAN* = 61
let *ovm_PROPAGATE_VECTORSPINOR* = 62
let *ovm_PROPAGATE_TENSOR2* = 63

ovm_PROPAGATE_NONE has to be split up to different types to work in conjunction with color MC . . .

let *ovm_PROPAGATE_NONE* = 64

let *ovm_FUSE_V_FF* = −1

let $ovm\_FUSE\_F\_VF \ = \ -2$
let $ovm\_FUSE\_F\_FV \ = \ -3$
let $ovm\_FUSE\_VA\_FF \ = \ -4$
let $ovm\_FUSE\_F\_VAF \ = \ -5$
let $ovm\_FUSE\_F\_FVA \ = \ -6$
let $ovm\_FUSE\_VA2\_FF \ = \ -7$
let $ovm\_FUSE\_F\_VA2F \ = \ -8$
let $ovm\_FUSE\_F\_FVA2 \ = \ -9$
let $ovm\_FUSE\_A\_FF \ = \ -10$
let $ovm\_FUSE\_F\_AF \ = \ -11$
let $ovm\_FUSE\_F\_FA \ = \ -12$
let $ovm\_FUSE\_VL\_FF \ = \ -13$
let $ovm\_FUSE\_F\_VLF \ = \ -14$
let $ovm\_FUSE\_F\_FVL \ = \ -15$
let $ovm\_FUSE\_VR\_FF \ = \ -16$
let $ovm\_FUSE\_F\_VRF \ = \ -17$
let $ovm\_FUSE\_F\_FVR \ = \ -18$
let $ovm\_FUSE\_VLR\_FF \ = \ -19$
let $ovm\_FUSE\_F\_VLRF \ = \ -20$
let $ovm\_FUSE\_F\_FVLR \ = \ -21$
let $ovm\_FUSE\_SP\_FF \ = \ -22$
let $ovm\_FUSE\_F\_SPF \ = \ -23$
let $ovm\_FUSE\_F\_FSP \ = \ -24$
let $ovm\_FUSE\_S\_FF \ = \ -25$
let $ovm\_FUSE\_F\_SF \ = \ -26$
let $ovm\_FUSE\_F\_FS \ = \ -27$
let $ovm\_FUSE\_P\_FF \ = \ -28$
let $ovm\_FUSE\_F\_PF \ = \ -29$
let $ovm\_FUSE\_F\_FP \ = \ -30$
let $ovm\_FUSE\_SL\_FF \ = \ -31$
let $ovm\_FUSE\_F\_SLF \ = \ -32$
let $ovm\_FUSE\_F\_FSL \ = \ -33$
let $ovm\_FUSE\_SR\_FF \ = \ -34$
let $ovm\_FUSE\_F\_SRF \ = \ -35$
let $ovm\_FUSE\_F\_FSR \ = \ -36$
let $ovm\_FUSE\_SLR\_FF \ = \ -37$
let $ovm\_FUSE\_F\_SLRF \ = \ -38$
let $ovm\_FUSE\_F\_FSLR \ = \ -39$

let $ovm\_FUSE\_G\_GG \ = \ -40$
let $ovm\_FUSE\_V\_SS \ = \ -41$
let $ovm\_FUSE\_S\_VV \ = \ -42$
let $ovm\_FUSE\_S\_VS \ = \ -43$
let $ovm\_FUSE\_V\_SV \ = \ -44$
let $ovm\_FUSE\_S\_SS \ = \ -45$
let $ovm\_FUSE\_S\_SVV \ = \ -46$
let $ovm\_FUSE\_V\_SSV \ = \ -47$
let $ovm\_FUSE\_S\_SSS \ = \ -48$
let $ovm\_FUSE\_V\_VVV \ = \ -49$

let $ovm\_FUSE\_S\_G2 \ = \ -50$

```
let ovm_FUSE_G_SG   =  − 51
let ovm_FUSE_G_GS   =  − 52
let ovm_FUSE_S_G2_SKEW   =  − 53
let ovm_FUSE_G_SG_SKEW   =  − 54
let ovm_FUSE_G_GS_SKEW   =  − 55

let inst_length  =  8
```

Some helper functions.

```
let printi ˜lhs : l ˜rhs1 : r1 ?coupl : (cp  =  0) ?coeff : (co  =  0)
            ?rhs2 : (r2  =  0) ?rhs3 : (r3  =  0) ?rhs4 : (r4  =  0) code  =
  printf "@\n%d␣%d␣%d␣%d␣%d␣%d␣%d" code cp co l r1 r2 r3 r4

let nl ()  =  printf "@\n"

let print_int_lst lst  =  nl (); lst | >  List.iter (printf "%d␣␣␣")

let print_str_lst lst  =  nl (); lst | >  List.iter (printf "%s␣")

let break ()  =  printi ˜lhs : 0 ˜rhs1 : 0 0
```

Copied from below. Needed for header.

⚠ Could be fused with *lorentz_ordering*.

```
type declarations  =
  { scalars  :  F.wf list;
    spinors  :  F.wf list;
    conjspinors  :  F.wf list;
    realspinors  :  F.wf list;
    ghostspinors  :  F.wf list;
    vectorspinors  :  F.wf list;
    vectors  :  F.wf list;
    ward_vectors  :  F.wf list;
    massive_vectors  :  F.wf list;
    tensors_1  :  F.wf list;
    tensors_2  :  F.wf list;
    brs_scalars  :  F.wf list;
    brs_spinors  :  F.wf list;
    brs_conjspinors  :  F.wf list;
    brs_realspinors  :  F.wf list;
    brs_vectorspinors  :  F.wf list;
    brs_vectors  :  F.wf list;
    brs_massive_vectors  :  F.wf list }

let rec classify_wfs' acc  = function
  | []  →  acc
  | wf  ::  rest  →
      classify_wfs'
        (match CM.lorentz (F.flavor wf) with
        | Scalar  →  {acc with scalars  =  wf  ::  acc.scalars}
        | Spinor  →  {acc with spinors  =  wf  ::  acc.spinors}
        | ConjSpinor  →  {acc with conjspinors  =  wf  ::  acc.conjspinors}
        | Majorana  →  {acc with realspinors  =  wf  ::  acc.realspinors}
```

$\mid$ *Maj_Ghost* $\rightarrow$ $\{acc$ with *ghostspinors* $=$ *wf* $::$ *acc.ghostspinors*$\}$
$\mid$ *Vectorspinor* $\rightarrow$
  $\{acc$ with *vectorspinors* $=$ *wf* $::$ *acc.vectorspinors*$\}$
$\mid$ *Vector* $\rightarrow$ $\{acc$ with *vectors* $=$ *wf* $::$ *acc.vectors*$\}$
$\mid$ *Massive_Vector* $\rightarrow$
  $\{acc$ with *massive_vectors* $=$ *wf* $::$ *acc.massive_vectors*$\}$
$\mid$ *Tensor_1* $\rightarrow$ $\{acc$ with *tensors_1* $=$ *wf* $::$ *acc.tensors_1*$\}$
$\mid$ *Tensor_2* $\rightarrow$ $\{acc$ with *tensors_2* $=$ *wf* $::$ *acc.tensors_2*$\}$
$\mid$ *BRS Scalar* $\rightarrow$ $\{acc$ with *brs_scalars* $=$ *wf* $::$ *acc.brs_scalars*$\}$
$\mid$ *BRS Spinor* $\rightarrow$ $\{acc$ with *brs_spinors* $=$ *wf* $::$ *acc.brs_spinors*$\}$
$\mid$ *BRS ConjSpinor* $\rightarrow$ $\{acc$ with *brs_conjspinors* $=$
      wf $::$ *acc.brs_conjspinors*$\}$
$\mid$ *BRS Majorana* $\rightarrow$ $\{acc$ with *brs_realspinors* $=$
     wf $::$ *acc.brs_realspinors*$\}$
$\mid$ *BRS Vectorspinor* $\rightarrow$ $\{acc$ with *brs_vectorspinors* $=$
      wf $::$ *acc.brs_vectorspinors*$\}$
$\mid$ *BRS Vector* $\rightarrow$ $\{acc$ with *brs_vectors* $=$ *wf* $::$ *acc.brs_vectors*$\}$
$\mid$ *BRS Massive_Vector* $\rightarrow$ $\{acc$ with *brs_massive_vectors* $=$
      wf $::$ *acc.brs_massive_vectors*$\}$
$\mid$ *BRS _* $\rightarrow$ *invalid_arg* `"Targets.classify_wfs':␣not␣needed␣here")`
rest

let *classify_wfs wfs* $=$ *classify_wfs'*
 $\{$ *scalars* $=$ $[];$
  spinors $=$ $[];$
  conjspinors $=$ $[];$
  realspinors $=$ $[];$
  ghostspinors $=$ $[];$
  vectorspinors $=$ $[];$
  vectors $=$ $[];$
  ward_vectors $=$ $[];$
  massive_vectors $=$ $[];$
  tensors_1 $=$ $[];$
  tensors_2 $=$ $[];$
  brs_scalars $=$ $[];$
  brs_spinors $=$ $[];$
  brs_conjspinors $=$ $[];$
  brs_realspinors $=$ $[];$
  brs_vectorspinors $=$ $[];$
  brs_vectors $=$ $[];$
  brs_massive_vectors $=$ $[]$ $\}$ *wfs*

Sets and maps

The OVM identifies all objects via integers. Therefore, we need maps which assign the abstract object a unique ID.

I want *int list*s with less elements to come first. Used in conjunction with the int list representation of momenta, this will set the outer particles at first position and allows the OVM to set them without further instructions.

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Using the Momentum module might give better performance than integer lists?

```
let rec int_lst_compare (e1 : int list) (e2 : int list) =
  match e1, e2 with
  | [], [] → 0
  | _, [] → +1
  | [], _ → −1
  | [_; _], [_] → +1
  | [_], [_; _] → −1
  | hd1 :: tl1, hd2 :: tl2 →
      let c = compare hd1 hd2 in
      if (c ≢ 0 ∧ List.length tl1 = List.length tl2) then
        c
      else
        int_lst_compare tl1 tl2
```

We need a canonical ordering for the different types of wfs. Copied, and slightly modified to order *wf*s, from `fusion.ml`.

```
let lorentz_ordering wf =
  match CM.lorentz (F.flavor wf) with
  | Scalar → 0
  | Spinor → 1
  | ConjSpinor → 2
  | Majorana → 3
  | Vector → 4
  | Massive_Vector → 5
  | Tensor_2 → 6
  | Tensor_1 → 7
  | Vectorspinor → 8
  | BRS Scalar → 9
  | BRS Spinor → 10
  | BRS ConjSpinor → 11
  | BRS Majorana → 12
  | BRS Vector → 13
  | BRS Massive_Vector → 14
  | BRS Tensor_2 → 15
  | BRS Tensor_1 → 16
  | BRS Vectorspinor → 17
  | Maj_Ghost → invalid_arg "lorentz_ordering:␣not␣implemented"
  | BRS _ → invalid_arg "lorentz_ordering:␣not␣needed"

let wf_compare (wf1, mult1) (wf2, mult2) =
  let c1 = compare (lorentz_ordering wf1) (lorentz_ordering wf2) in
  if c1 ≠ 0 then
    c1
  else
    let c2 = compare wf1 wf2 in
    if c2 ≠ 0 then
      c2
```

```
          else
              compare mult1 mult2

    let amp_compare amp1 amp2  =
      let cflow a  =  CM.flow (F.incoming a) (F.outgoing a) in
      let c1  =  compare (cflow amp1) (cflow amp2) in
      if c1  ≠  0 then
        c1
      else
        let process_sans_color a  =
            (List.map CM.flavor_sans_color (F.incoming a),
             List.map CM.flavor_sans_color (F.outgoing a)) in
        compare (process_sans_color amp1) (process_sans_color amp2)

    let level_compare (f1, amp1) (f2, amp2)  =
      let p1  =  F.momentum_list (F.lhs f1)
      and p2  =  F.momentum_list (F.lhs f2) in
      let c1  =  int_lst_compare p1 p2 in
      if c1  ≠  0 then
        c1
      else
        let c2  =  compare f1 f2 in
        if c2  ≠  0 then
          c2
        else
          amp_compare amp1 amp2

    module ISet  =  Set.Make (struct type t  =  int list
                                let compare  =  int_lst_compare end)

    module WFSet  =  Set.Make (struct type t  =  CF.wf  ×  int
                                 let compare  =  wf_compare end)

    module CSet  =  Set.Make (struct type t  =  CM.constant
                                let compare  =  compare end)

    module FSet  =  Set.Make (struct type t  =  F.fusion  ×  F.amplitude
                                let compare  =  level_compare end)
```

It might be preferable to use a *PMap* which maps mom to int, instead of this way. More standard functions like *mem* could be used. Also, *get_ID* would be faster, $\mathcal{O}(\log N)$ instead of $\mathcal{O}(N)$, and simpler. For 8 gluons: N=127 momenta. Minor performance issue.

```
    module IMap  =  Map.Make (struct type t  =  int let compare  =  compare end)
```

For *wf*s it is crucial for the performance to use a different type of *Map*s.

```
    module WFMap  =  Map.Make (struct type t  =  CF.wf  ×  int
                                 let compare  =  wf_compare end)

    type lookups  =  { pmap : int list IMap.t;
                       wfmap : int WFMap.t;
                       cmap : CM.constant IMap.t  ×  CM.constant IMap.t;
```

```
                      amap  :  F.amplitude IMap.t;
                      n_wfs  :  int list;
                      amplitudes  :  CF.amplitudes;
                      dict  :  F.amplitude  →  F.wf  →  int }
```

let *largest_key imap* =
  if (*IMap.is_empty imap*) then
    failwith `"largest_key:␣Map␣is␣empty!"`
  else
    fst (*IMap.max_binding imap*)

OCaml's *compare* from pervasives cannot compare functional types, e.g. for type *amplitude*, if no specific equality function is given ("equal: functional value"). Therefore, we allow to specify the ordering.

let *get_ID′ comp map elt* : *int* =
  let *smallmap* = *IMap.filter* (fun _ *x* → (*comp x elt*) = 0) *map* in
  if *IMap.is_empty smallmap* then
    raise Not_found
  else
    fst (*IMap.min_binding smallmap*)

⚠ Trying to curry *map* here leads to type errors of the polymorphic function *get_ID*?

let *get_ID map* = match *map* with
  | *map* → *get_ID′ compare map*

let *get_const_ID map x* = match *map* with
  | (*map1*, *map2*) → try *get_ID′ compare map1 x* with
                         _ → try *get_ID′ compare map2 x* with
                         _ → *failwith* `"Impossible"`

Creating an integer map of a list with an optional argument that indicates where the map should start counting.

let *map_of_list ?start* : (*st* = 1) *lst* =
  let *g* (*ind*, *map*) *wf* = (*succ ind*, *IMap.add ind wf map*) in
  lst | > *List.fold_left g* (*st*, *IMap.empty*) | > *snd*

let *wf_map_of_list ?start* : (*st* = 1) *lst* =
  let *g* (*ind*, *map*) *wf* = (*succ ind*, *WFMap.add wf ind map*) in
  lst | > *List.fold_left g* (*st*, *WFMap.empty*) | > *snd*

Header

⚠ It would be nice to safe the creation date as comment. However, the Unix module doesn't seem to be loaded on default.

let *version_string x* = *RCS.name x* ˆ `"-rev"` ˆ *RCS.revision x*

let *print_description cmdline* =
  rcs_list @ [*M.rcs*] | > *List.map version_string*

$\longrightarrow$ *List.iter* (*printf* `"%s\n"`);

printf `"@\nBytecode␣file␣generated␣automatically␣by␣O'Mega␣for␣OVM"`;
printf `"@\nDo␣not␣delete␣any␣lines.␣You␣called␣O'Mega␣with"`;
printf `"@\n␣␣%s"` *cmdline*;
       printf `"@\n"`

let *num_classified_wfs wfs* =
  let *wfs′* = *classify_wfs wfs* in
  List.map List.length
    [*wfs′.scalars* @ *wfs′.brs_scalars*;
     wfs′.spinors @ *wfs′.brs_spinors*;
     wfs′.conjspinors @ *wfs′.brs_conjspinors*;
     wfs′.realspinors @ *wfs′.brs_realspinors* @ *wfs′.ghostspinors*;
     wfs′.vectors @ *wfs′.massive_vectors* @ *wfs′.brs_vectors*
      @ *wfs′.brs_massive_vectors* @ *wfs′.ward_vectors*;
     wfs′.tensors_2;
     wfs′.tensors_1;
     wfs′.vectorspinors]

let *description_classified_wfs* =
  [`"N_scalars"`;
   `"N_spinors"`;
   `"N_conjspinors"`;
   `"N_bispinors"`;
   `"N_vectors"`;
   `"N_tensors_2"`;
   `"N_tensors_1"`;
   `"N_vectorspinors"`]

let *num_particles_in amp* =
  match *CF.flavors amp* with
  | [] $\to$ 0
  | (*fin*, _) :: _ $\to$ *List.length fin*

let *num_particles_out amp* =
  match *CF.flavors amp* with
  | [] $\to$ 0
  | (_, *fout*) :: _ $\to$ *List.length fout*

let *num_particles amp* =
  match *CF.flavors amp* with
  | [] $\to$ 0
  | (*fin*, *fout*) :: _ $\to$ *List.length fin* + *List.length fout*

let *num_color_indices_default* = 2 (∗ Standard model and non-color-exotica ∗)

let *num_color_indices amp* =
  try *CFlow.rank* (*List.hd* (*CF.color_flows amp*)) with
  _ $\to$ *num_color_indices_default*

let *num_color_factors amp* =
  let *table* = *CF.color_factors amp* in
  let *n_cflow* = *Array.length table*

```
          and n_cfactors  =  ref 0 in
          for c1  =  0 to pred n_cflow do
            for c2  =  0 to pred n_cflow do
              if c1  ≤  c2 then begin
                match table.(c1).(c2) with
                | [] → ()
                | _ → incr n_cfactors
              end
            done
          done;
          !n_cfactors
```

let *num_helicities amp* = *amp* | > *CF.helicities* | > *List.length*

let *num_flavors amp* = *amp* | > *CF.flavors* | > *List.length*

let *num_ks amp* = *amp* | > *CF.processes* | > *List.length*

let *num_color_flows amp* = *amp* | > *CF.color_flows* | > *List.length*

Use *fst* since *WFSet.t = F.wf × int*.

let *num_wfs wfset* = *wfset* | > *WFSet.elements* | > *List.map fst*
                                  —> *num_classified_wfs*

largest_key gives the number of momenta if applied to *pmap*.

let *num_lst lookups wfset* =
  [*largest_key lookups.pmap*;
    num_particles lookups.amplitudes;
    num_particles_in lookups.amplitudes;
    num_particles_out lookups.amplitudes;
    num_ks lookups.amplitudes;
    num_helicities lookups.amplitudes;
    num_color_flows lookups.amplitudes;
    num_color_indices lookups.amplitudes;
    num_flavors lookups.amplitudes;
    num_color_factors lookups.amplitudes] @ *num_wfs wfset*

let *description_lst* =
  ["N_momenta";
    "N_particles";
    "N_prt_in";
    "N_prt_out";
    "N_amplitudes";
    "N_helicities";
    "N_col_flows";
    "N_col_indices";
    "N_flavors";
    "N_col_factors"] @ *description_classified_wfs*

let *print_header′ numbers* =
  let *chopped_num_lst = ThoList.chopn inst_length numbers*
  and *chopped_desc_lst = ThoList.chopn inst_length description_lst*
  and *printer a b = print_str_lst a*; *print_int_lst b* in

> *List.iter2 printer chopped_desc_lst chopped_num_lst*

let *print_header lookups wfset* = *print_header′ (num_lst lookups wfset)*

let *print_zero_header* () =
  let rec *zero_list′ j* =
    if *j* < 1 then [ ]
    else 0 :: *zero_list′ (j* − 1) in
  let *zero_list i* = *zero_list′ (i* + 1) in
  description_lst | > *List.length* | > *zero_list* | > *print_header′*

<div align="center">

Tables

</div>

let *print_spin_table′ tuples* =
  match *tuples* with
  | [ ] → ()
  | _ → *tuples* | > *List.iter* (fun (*tuple1, tuple2*) →
    tuple1 @ *tuple2* | > *List.map* (*Printf.sprintf* "%d␣")
        —> *String.concat* "" —> *printf* "@\n%s")

let *print_spin_table amplitudes* =
  printf "@\nSpin␣states␣table";
  print_spin_table′ @@ *CF.helicities amplitudes*

let *print_flavor_table tuples* =
  match *tuples* with
  | [ ] → ()
  | _ → *List.iter* (fun *tuple* → *tuple*
      —> *List.map* (fun *f* → *Printf.sprintf* "%d␣" @@ *M.pdg f*)
      —> *String.concat* "" —> *printf* "@\n%s"
) *tuples*

let *print_flavor_tables amplitudes* =
  printf "@\nFlavor␣states␣table";
  print_flavor_table @@ *List.map* (fun (*fin, fout*) → *fin* @ *fout*)
      @@ *CF.flavors amplitudes*

let *print_color_flows_table′ tuple* =
  match *CFlow.to_lists tuple* with
  | [ ] → ()
  | *cfs* → *printf* "@\n%s" @@ *String.concat* "" @@ *List.map*
    (fun *cf* → *cf* | > *List.map* (*Printf.sprintf* "%d␣")
      —> *String.concat* ""
) *cfs*

let *print_color_flows_table tuples* =
  match *tuples* with
  | [ ] → ()
  | _ → *List.iter print_color_flows_table′ tuples*

let *print_ghost_flags_table tuples* =
  match *tuples* with
  | [ ] → ()

```
| _ →
  List.iter (fun tuple →
  match CFlow.ghost_flags tuple with
      | [] → ()
      | gfs → printf "@\n"; List.iter (fun gf → printf "%s␣"
        (if gf then "1" else "0") ) gfs
  ) tuples
```

let *format_power*
  { *CFlow.num* = *num*; *CFlow.den* = *den*; *CFlow.power* = *pwr* } =
  match *num*, *den*, *pwr* with
  | _, 0, _ → *invalid_arg* `"targets.format_power:␣zero␣denominator"`
  | *n*, *d*, *p* → [*n*; *d*; *p*]

let *format_powers* = function
  | [] → [0]
  | *powers* → *List.flatten* (*List.map format_power powers*)

Straightforward iteration gives a great speedup compared to the fancier approach which only collects nonzero colorfactors.

let *print_color_factor_table table* =
  let *n_cflow* = *Array.length table* in
  if *n_cflow* > 0 then begin
    for *c1* = 0 to *pred n_cflow* do
      for *c2* = 0 to *pred n_cflow* do
        if *c1* ≤ *c2* then begin
          match *table.(c1).(c2)* with
          | [] → ()
          | *cf* → *printf* `"@\n"`; *List.iter* (*printf* `"%9d"`)
            ([*succ c1*; *succ c2*] @ (*format_powers cf*));
        end
      done
    done
  end

let *option_to_binary* = function
  | *Some* _ → `"1"`
  | *None* → `"0"`

let *print_flavor_color_table n_flv n_cflow table* =
  if *n_flv* > 0 then begin
    for *c* = 0 to *pred n_cflow* do
      printf `"@\n"`;
      for *f* = 0 to *pred n_flv* do
        printf `"%s␣"` (*option_to_binary table.(f).(c)*)
      done;
    done;
  end

let *print_color_tables amplitudes* =
  let *cflows* = *CF.color_flows amplitudes*
  and *cfactors* = *CF.color_factors amplitudes* in
  printf `"@\nColor␣flows␣table:␣[␣(i,␣j)␣(k,␣l)␣->␣(m,␣n)␣...]"`;

print_color_flows_table cflows;
printf "@\nColor␣ghost␣flags␣table:";
print_ghost_flags_table cflows;
printf "@\nColor␣factors␣table:␣[␣i,␣j:␣num␣den␣power],␣%s"
  "i,␣j␣are␣indexed␣color␣flows";
print_color_factor_table cfactors;
printf "@\nFlavor␣color␣combination␣is␣allowed:";
print_flavor_color_table (*num_flavors amplitudes*) (*List.length*
  (*CF.color_flows amplitudes*)) (*CF.process_table amplitudes*)

<div align="center">

Momenta

</div>

Add the momenta of a WFSet to a Iset. For now, we are throwing away the information to which amplitude the momentum belongs. This could be optimized for random color flow computations.

let *momenta_set wfset* =
  let *get_mom wf* = *wf* | > *fst* | > *F.momentum_list* in
  let *momenta* = *List.map get_mom* (*WFSet.elements wfset*) in
  momenta | > *List.fold_left* (fun *set x* → *set* | > *ISet.add x*) *ISet.empty*

let *chop_in_3 lst* =
  let *ceil_div i j* = if (*i* mod *j* = 0) then *i/j* else *i/j* + 1 in
  ThoList.chopn (*ceil_div* (*List.length lst*) 3) *lst*

Assign momenta via instruction code. External momenta [_] are already set by the OVM. To avoid unnecessary look-ups of IDs we seperate two cases. If we have more, we split up in two or three parts.

let *add_mom p pmap* =
  let *print_mom lhs rhs1 rhs2 rhs3* = if (*rhs1* ≢ 0) then
    printi ˜lhs : *lhs ˜rhs1* : *rhs1 ˜rhs2* : *rhs2 ˜rhs3* : *rhs3 ovm_ADD_MOMENTA* in
  let *get_p_ID* = *get_ID pmap* in
  match *p* with
  | [] | [_] → *print_mom* 0 0 0 0
  | [*rhs1*; *rhs2*] → *print_mom* (*get_p_ID* [*rhs1*; *rhs2*]) *rhs1 rhs2* 0
  | [*rhs1*; *rhs2*; *rhs3*] → *print_mom* (*get_p_ID* [*rhs1*; *rhs2*; *rhs3*]) *rhs1 rhs2 rhs3*
  | *more* →
    let *ids* = *List.map get_p_ID* (*chop_in_3 more*) in
    if (*List.length ids* = 3) then
      print_mom (*get_p_ID more*) (*List.nth ids* 0) (*List.nth ids* 1)
        (*List.nth ids* 2)
    else
      print_mom (*get_p_ID more*) (*List.nth ids* 0) (*List.nth ids* 1) 0

Hand through the current level and print level seperators if necessary.

let *add_all_mom lookups pset* =
  let *add_all′ level p* =
    let *level′* = *List.length p* in
    if (*level′* > *level* ∧ *level′* > 3) then *break* ();
    add_mom p lookups.pmap; *level′*
  in

> $ignore\ (pset\ |>\ ISet.elements\ |>\ List.fold\_left\ add\_all'\ 1)$

Expand a set of momenta to contain all needed momenta for the computation in the OVM. For this, we create a list of sets which contains the chopped momenta and unify them afterwards. If the set has become larger, we expand again.

> let rec $expand\_pset\ p\ =$
>   let $momlst\ =\ ISet.elements\ p$ in
>   let $pset\_of\ lst\ =\ List.fold\_left$ (fun $s\ x\ \rightarrow\ ISet.add\ x\ s)\ ISet.empty$
> lst in
>   let $sets\ =\ List.map$ (fun $x\ \rightarrow\ pset\_of\ (chop\_in\_3\ x)$) $momlst$ in
>   let $bigset\ =\ List.fold\_left\ ISet.union\ ISet.empty\ sets$ in
>   let $biggerset\ =\ ISet.union\ bigset\ p$ in
>   if ($List.length\ momlst\ <\ List.length\ (ISet.elements\ biggerset)$) then
> $expand\_pset\ biggerset$
>   else
> $biggerset$
>
> let $mom\_ID\ pmap\ wf\ =\ get\_ID\ pmap\ (F.momentum\_list\ wf)$

Wavefunctions and externals

$mult\_wf$ is needed because the wf with same combination of flavor and momentum can have different dependencies and content.

> let $mult\_wf\ dict\ amplitude\ wf\ =$
>   try
> $wf,\ dict\ amplitude\ wf$
>   with
> | $Not\_found\ \rightarrow\ wf,\ 0$

Build the union of all wfs of all amplitudes and a map of the amplitudes.

> let $wfset\_amps\ amplitudes\ =$
>   let $amap\ =\ amplitudes\ |>\ CF.processes\ |>\ List.sort\ amp\_compare$
> $—>\ map\_of\_list$
>   and $dict\ =\ CF.dictionary\ amplitudes$ in
>   let $wfset\_amp\ amp\ =$
>   let $f\ =\ mult\_wf\ dict\ amp$ in
>   let $lst\ =\ List.map\ f\ ((F.externals\ amp)\ @\ (F.variables\ amp))$ in
> $lst\ |>\ List.fold\_left$ (fun $s\ x\ \rightarrow\ WFSet.add\ x\ s)\ WFSet.empty$ in
>   let $list\_of\_sets\ =\ amplitudes\ |>\ CF.processes\ |>\ List.map\ wfset\_amp$ in
> $List.fold\_left\ WFSet.union\ WFSet.empty\ list\_of\_sets,\ amap$

To obtain the Fortran index, we substract the number of precedent wave functions.

> let $lorentz\_ordering\_reduced\ wf\ =$
>   match $CM.lorentz\ (F.flavor\ wf)$ with
> | $Scalar$ | $BRS\ Scalar\ \rightarrow\ 0$
> | $Spinor$ | $BRS\ Spinor\ \rightarrow\ 1$
> | $ConjSpinor$ | $BRS\ ConjSpinor\ \rightarrow\ 2$
> | $Majorana$ | $BRS\ Majorana\ \rightarrow\ 3$
> | $Vector$ | $BRS\ Vector$ | $Massive\_Vector$ | $BRS\ Massive\_Vector\ \rightarrow\ 4$

```
        |  Tensor_2  |  BRS Tensor_2  →  5
        |  Tensor_1  |  BRS Tensor_1  →  6
        |  Vectorspinor  |  BRS Vectorspinor  →  7
        |  Maj_Ghost  →  invalid_arg "lorentz_ordering:␣not␣implemented"
        |  BRS _  →  invalid_arg "lorentz_ordering:␣not␣needed"

let wf_index wfmap num_lst (wf, i)  =
    let wf_ID  =  WFMap.find (wf, i) wfmap
    and sum lst  =  List.fold_left (fun x y  →  x + y) 0 lst in
        wf_ID − sum (ThoList.hdn (lorentz_ordering_reduced wf) num_lst)

let print_ext lookups amp_ID inc (wf, i)  =
    let mom  =  (F.momentum_list wf) in
    let outer_index  =  if List.length mom = 1 then List.hd mom else
        failwith "targets.print_ext:␣called␣with␣non-external␣particle"
    and f  =  F.flavor wf in
    let pdg  =  CM.pdg f
    and wf_code  =
        match CM.lorentz f with
        |  Scalar  →  ovm_LOAD_SCALAR
        |  BRS Scalar  →  ovm_LOAD_BRS_SCALAR
        |  Spinor  →
            if inc then ovm_LOAD_SPINOR_INC
            else ovm_LOAD_SPINOR_OUT
        |  BRS Spinor  →
            if inc then ovm_LOAD_BRS_SPINOR_INC
            else ovm_LOAD_BRS_SPINOR_OUT
        |  ConjSpinor  →
            if inc then ovm_LOAD_CONJSPINOR_INC
            else ovm_LOAD_CONJSPINOR_OUT
        |  BRS ConjSpinor  →
            if inc then ovm_LOAD_BRS_CONJSPINOR_INC
            else ovm_LOAD_BRS_CONJSPINOR_OUT
        |  Vector  |  Massive_Vector  →
            if inc then ovm_LOAD_VECTOR_INC
            else ovm_LOAD_VECTOR_OUT
        |  BRS Vector  |  BRS Massive_Vector  →
            if inc then ovm_LOAD_BRS_VECTOR_INC
            else ovm_LOAD_BRS_VECTOR_OUT
        |  Tensor_2  →
            if inc then ovm_LOAD_TENSOR2_INC
            else ovm_LOAD_TENSOR2_OUT
        |  Vectorspinor  |  BRS Vectorspinor  →
            if inc then ovm_LOAD_VECTORSPINOR_INC
            else ovm_LOAD_VECTORSPINOR_OUT
        |  Majorana  →
            if inc then ovm_LOAD_MAJORANA_INC
            else ovm_LOAD_MAJORANA_OUT
        |  BRS Majorana  →
            if inc then ovm_LOAD_BRS_MAJORANA_INC
            else ovm_LOAD_BRS_MAJORANA_OUT
```

```
        |  Maj_Ghost  →
              if inc then ovm_LOAD_MAJORANA_GHOST_INC
              else ovm_LOAD_MAJORANA_GHOST_OUT
        |  Tensor_1  →
              invalid_arg "targets.print_ext:␣Tensor_1␣only␣internal"
        |  BRS _  →
              failwith "targets.print_ext:␣Not␣implemented"
    and wf_ind  =  wf_index lookups.wfmap lookups.n_wfs (wf, i)
    in
        printi wf_code ˜lhs : wf_ind ˜coupl : (abs(pdg)) ˜rhs1 : outer_index ˜rhs4 :
amp_ID
```

```
    let print_ext_amp lookups amplitude  =
      let incoming  =  (List.map (fun _  →  true) (F.incoming amplitude) @
                              List.map (fun _  →  false) (F.outgoing amplitude))
      and amp_ID  =  get_ID' amp_compare lookups.amap amplitude in
      let wf_tpl wf  =  mult_wf lookups.dict amplitude wf in
      let print_ext_wf inc wf  =  wf | >  wf_tpl | >  print_ext lookups amp_ID inc in
        List.iter2 print_ext_wf incoming (F.externals amplitude)
```

```
    let print_externals lookups seen_wfs amplitude  =
      let externals  =
        List.combine
          (F.externals amplitude)
          (List.map (fun _  →  true) (F.incoming amplitude) @
            List.map (fun _  →  false) (F.outgoing amplitude)) in
      List.fold_left (fun seen (wf, incoming)  →
        let amp_ID  =  get_ID' amp_compare lookups.amap amplitude in
        let wf_tpl  =  mult_wf lookups.dict amplitude wf in
        if ¬ (WFSet.mem wf_tpl seen) then begin
          wf_tpl | >  print_ext lookups amp_ID incoming
        end;
        WFSet.add wf_tpl seen) seen_wfs externals
```

print_externals and *print_ext_amp* do in principle the same thing but *print_externals* filters out dublicate external wave functions. Even with *print_externals* the same (numerically) external wave function will be loaded if it belongs to a different color flow, just as in the native Fortran code. For color MC, *print_ext_amp* has to be used (redundant instructions but only one flow is computed) and the filtering of duplicate fusions has to be disabled.

```
    let print_ext_amps lookups  =
      let print_external_amp s x  =  print_externals lookups s x in
      ignore (
        List.fold_left print_external_amp WFSet.empty
          (CF.processes lookups.amplitudes)
        )
```

```
(*
```

Currents

```
*)
```

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Parallelization issues: All fusions have to be completed before the propagation takes place. Preferably each fusion and propagation is done by one thread. Solution: All fusions are subinstructions, i.e. if they are read by the main loop they are skipped. If a propagation occurs, all fusions have to be computed first. The additional control bit is the sign of the first int of an instruction.

```
let print_fermion_current code_a code_b code_c coeff lhs c wf1 wf2 fusion =
  let printc code r1 r2 = printi code ˜lhs : lhs ˜coupl : c ˜coeff : coeff
    ˜rhs1 : r1 ˜rhs2 : r2 in
  match fusion with
  | F13 → printc code_a wf1 wf2
  | F31 → printc code_a wf2 wf1
  | F23 → printc code_b wf1 wf2
  | F32 → printc code_b wf2 wf1
  | F12 → printc code_c wf1 wf2
  | F21 → printc code_c wf2 wf1

let ferm_print_current = function
  | coeff, Psibar, V, Psi → print_fermion_current
    ovm_FUSE_V_FF ovm_FUSE_F_VF ovm_FUSE_F_FV coeff
  | coeff, Psibar, VA, Psi → print_fermion_current
    ovm_FUSE_VA_FF ovm_FUSE_F_VAF ovm_FUSE_F_FVA coeff
  | coeff, Psibar, VA2, Psi → print_fermion_current
    ovm_FUSE_VA2_FF ovm_FUSE_F_VA2F ovm_FUSE_F_FVA2 coeff
  | coeff, Psibar, A, Psi → print_fermion_current
    ovm_FUSE_A_FF ovm_FUSE_F_AF ovm_FUSE_F_FA coeff
  | coeff, Psibar, VL, Psi → print_fermion_current
    ovm_FUSE_VL_FF ovm_FUSE_F_VLF ovm_FUSE_F_FVL coeff
  | coeff, Psibar, VR, Psi → print_fermion_current
    ovm_FUSE_VR_FF ovm_FUSE_F_VRF ovm_FUSE_F_FVR coeff
  | coeff, Psibar, VLR, Psi → print_fermion_current
    ovm_FUSE_VLR_FF ovm_FUSE_F_VLRF ovm_FUSE_F_FVLR coeff
  | coeff, Psibar, SP, Psi → print_fermion_current
    ovm_FUSE_SP_FF ovm_FUSE_F_SPF ovm_FUSE_F_FSP coeff
  | coeff, Psibar, S, Psi → print_fermion_current
    ovm_FUSE_S_FF ovm_FUSE_F_SF ovm_FUSE_F_FS coeff
  | coeff, Psibar, P, Psi → print_fermion_current
    ovm_FUSE_P_FF ovm_FUSE_F_PF ovm_FUSE_F_FP coeff
  | coeff, Psibar, SL, Psi → print_fermion_current
    ovm_FUSE_SL_FF ovm_FUSE_F_SLF ovm_FUSE_F_FSL coeff
  | coeff, Psibar, SR, Psi → print_fermion_current
    ovm_FUSE_SR_FF ovm_FUSE_F_SRF ovm_FUSE_F_FSR coeff
  | coeff, Psibar, SLR, Psi → print_fermion_current
    ovm_FUSE_SLR_FF ovm_FUSE_F_SLRF ovm_FUSE_F_FSLR coeff
  | _, Psibar, _, Psi → invalid_arg
    "Targets.Fortran.VM:␣no␣superpotential␣here"
  | _, Chibar, _, _ | _, _, _, Chi → invalid_arg
    "Targets.Fortran.VM:␣Majorana␣spinors␣not␣handled"
  | _, Gravbar, _, _ | _, _, _, Grav → invalid_arg
    "Targets.Fortran.VM:␣Gravitinos␣not␣handled"
```

let *children2 rhs* =
  match *F.children rhs* with
  | [*wf1*; *wf2*] → (*wf1*, *wf2*)
  | _ → *failwith* "Targets.children2:␣can't␣happen"

let *children3 rhs* =
  match *F.children rhs* with
  | [*wf1*; *wf2*; *wf3*] → (*wf1*, *wf2*, *wf3*)
  | _ → *invalid_arg* "Targets.children3:␣can't␣happen"

let *print_vector4 c lhs wf1 wf2 wf3 fusion* (*coeff*, *contraction*) =
  let *printc r1 r2 r3* = *printi ovm_FUSE_V_VVV* ˜*lhs* : *lhs* ˜*coupl* : *c*
    ˜*coeff* : *coeff* ˜*rhs1* : *r1* ˜*rhs2* : *r2* ˜*rhs3* : *r3* in
  match *contraction*, *fusion* with
  | *C_12_34*, (*F341* | *F431* | *F342* | *F432* | *F123* | *F213* | *F124* | *F214*)
  | *C_13_42*, (*F241* | *F421* | *F243* | *F423* | *F132* | *F312* | *F134* | *F314*)
  | *C_14_23*, (*F231* | *F321* | *F234* | *F324* | *F142* | *F412* | *F143* |
F413) →
        printc wf1 wf2 wf3
  | *C_12_34*, (*F134* | *F143* | *F234* | *F243* | *F312* | *F321* | *F412* | *F421*)
  | *C_13_42*, (*F124* | *F142* | *F324* | *F342* | *F213* | *F231* | *F413* | *F431*)
  | *C_14_23*, (*F123* | *F132* | *F423* | *F432* | *F214* | *F241* | *F314* |
F341) →
        printc wf2 wf3 wf1
  | *C_12_34*, (*F314* | *F413* | *F324* | *F423* | *F132* | *F231* | *F142* | *F241*)
  | *C_13_42*, (*F214* | *F412* | *F234* | *F432* | *F123* | *F321* | *F143* | *F341*)
  | *C_14_23*, (*F213* | *F312* | *F243* | *F342* | *F124* | *F421* | *F134* |
F431) →
        printc wf1 wf3 wf2

let *print_current lookups lhs amplitude rhs* =
  let *f* = *mult_wf lookups.dict amplitude* in
  match *F.coupling rhs* with
  | *V3* (*vertex*, *fusion*, *constant*) →
      let *ch1*, *ch2* = *children2 rhs* in
      let *wf1* = *wf_index lookups.wfmap lookups.n_wfs* (*f ch1*)
      and *wf2* = *wf_index lookups.wfmap lookups.n_wfs* (*f ch2*)
      and *p1* = *mom_ID lookups.pmap ch1*
      and *p2* = *mom_ID lookups.pmap ch2*
      and *const_ID* = *get_const_ID lookups.cmap constant* in
      let *c* = if (*F.sign rhs*) < 0 then - *const_ID* else *const_ID* in
      begin match *vertex* with
      | *FBF* (*coeff*, *fb*, *b*, *f*) →
          begin match *coeff*, *fb*, *b*, *f* with
          | _, *Psibar*, *VLRM*, *Psi* | _, *Psibar*, *SPM*, *Psi*
          | _, *Psibar*, *TVA*, *Psi* | _, *Psibar*, *TVAM*, *Psi*
          | _, *Psibar*, *TLR*, *Psi* | _, *Psibar*, *TLRM*, *Psi*
          | _, *Psibar*, *TRL*, *Psi* | _, *Psibar*, *TRLM*, *Psi* → *failwith*
    "print_current:␣V3:␣Momentum␣dependent␣fermion␣couplings␣not␣implemented"
          | _, _, _, _ →
              ferm_print_current (*coeff*, *fb*, *b*, *f*) *lhs c wf1 wf2 fusion*
          end

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```
| PBP (_, _, _, _) →
    failwith "print_current:␣V3:␣PBP␣not␣implemented"
| BBB (_, _, _, _) →
    failwith "print_current:␣V3:␣BBB␣not␣implemented"
| GBG (_, _, _, _) →
    failwith "print_current:␣V3:␣GBG␣not␣implemented"

| Gauge_Gauge_Gauge coeff →
    let printc r1 r2 r3 r4 = printi ovm_FUSE_G_GG
        ~lhs : lhs ~coupl : c ~coeff : coeff ~rhs1 : r1 ~rhs2 : r2 ~rhs3 :
r3
        ~rhs4 : r4 in
    begin match fusion with
    | (F23 | F31 | F12) → printc wf1 p1 wf2 p2
    | (F32 | F13 | F21) → printc wf2 p2 wf1 p1
    end

| Aux_Gauge_Gauge _ →
    failwith "print_current:␣V3:␣not␣implemented"

| Scalar_Vector_Vector coeff →
    let printc code r1 r2 = printi code
        ~lhs : lhs ~coupl : c ~coeff : coeff ~rhs1 : r1 ~rhs2 : r2 in
    begin match fusion with
    | (F23 | F32) → printc ovm_FUSE_S_VV wf1 wf2
    | (F12 | F13) → printc ovm_FUSE_V_SV wf1 wf2
    | (F21 | F31) → printc ovm_FUSE_V_SV wf2 wf1
    end

| Scalar_Scalar_Scalar coeff →
    printi ovm_FUSE_S_SS ~lhs : lhs ~coupl : c ~coeff : coeff ~rhs1 :
wf1 ~rhs2 : wf2

| Vector_Scalar_Scalar coeff →
    let printc code ?flip : (f = 1) r1 r2 r3 r4 = printi code
        ~lhs : lhs ~coupl : (c × f) ~coeff : coeff ~rhs1 : r1 ~rhs2 :
r2 ~rhs3 : r3
        ~rhs4 : r4 in
    begin match fusion with
    | F23 → printc ovm_FUSE_V_SS wf1 p1 wf2 p2
    | F32 → printc ovm_FUSE_V_SS wf2 p2 wf1 p1
    | F12 → printc ovm_FUSE_S_VS wf1 p1 wf2 p2
    | F21 → printc ovm_FUSE_S_VS wf2 p2 wf1 p1
    | F13 → printc ovm_FUSE_S_VS wf1 p1 wf2 p2 ~flip : (−1)
    | F31 → printc ovm_FUSE_S_VS wf2 p2 wf1 p1 ~flip : (−1)
    end

| Aux_Vector_Vector _ →
    failwith "print_current:␣V3:␣not␣implemented"

| Aux_Scalar_Scalar _ →
    failwith "print_current:␣V3:␣not␣implemented"

| Aux_Scalar_Vector _ →
```

        failwith `"print_current:␣V3:␣not␣implemented"`

    | *Graviton_Scalar_Scalar* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Graviton_Vector_Vector* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Graviton_Spinor_Spinor* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Dim4_Vector_Vector_Vector_T* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Dim4_Vector_Vector_Vector_L* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Dim6_Gauge_Gauge_Gauge* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Dim4_Vector_Vector_Vector_T5* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Dim4_Vector_Vector_Vector_L5* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Dim6_Gauge_Gauge_Gauge_5* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Aux_DScalar_DScalar* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Aux_Vector_DScalar* _ →
       failwith `"print_current:␣V3:␣not␣implemented"`

    | *Dim5_Scalar_Gauge2 coeff* →
       let *printc code r1 r2 r3 r4* = *printi code*
         ~*lhs : lhs* ~*coupl : c* ~*coeff : coeff* ~*rhs1 : r1* ~*rhs2 : r2* ~*rhs3 :*
r3

         ~*rhs4 : r4* in
       begin match *fusion* with
       | *(F23 | F32)* → *printc ovm_FUSE_S_G2 wf1 p1 wf2 p2*
       | *(F12 | F13)* → *printc ovm_FUSE_G_SG wf1 p1 wf2 p2*
       | *(F21 | F31)* → *printc ovm_FUSE_G_GS wf2 p2 wf1 p1*
       end

    | *Dim5_Scalar_Gauge2_Skew coeff* →
       let *printc code ?flip : (f = 1) r1 r2 r3 r4* = *printi code*
         ~*lhs : lhs* ~*coupl : (c × f)* ~*coeff : coeff* ~*rhs1 : r1* ~*rhs2 :*
r2 ~*rhs3 : r3*

         ~*rhs4 : r4* in
       begin match *fusion* with
       | *(F23 | F32)* → *printc ovm_FUSE_S_G2_SKEW wf1 p1 wf2 p2*
       | *(F12 | F13)* → *printc ovm_FUSE_G_SG_SKEW wf1 p1 wf2 p2*
       | *(F21 | F31)* → *printc ovm_FUSE_G_GS_SKEW wf2 p1 wf1 p2* ~*flip :*
(−1)

       end

| *Dim5 _ Scalar _ Vector _ Vector _ T* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Dim5 _ Scalar _ Vector _ Vector _ U* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Dim5 _ Scalar _ Scalar2* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Dim6 _ Vector _ Vector _ Vector _ T* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Tensor _ 2 _ Vector _ Vector* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Tensor _ 2 _ Scalar _ Scalar* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Dim5 _ Tensor _ 2 _ Vector _ Vector _ 1* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Dim5 _ Tensor _ 2 _ Vector _ Vector _ 2* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Dim7 _ Tensor _ 2 _ Vector _ Vector _ T* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Dim5 _ Scalar _ Vector _ Vector _ TU* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Scalar _ Vector _ Vector _ t* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Tensor _ 2 _ Vector _ Vector _ cf* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Tensor _ 2 _ Scalar _ Scalar _ cf* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Tensor _ 2 _ Vector _ Vector _ 1* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *Tensor _ 2 _ Vector _ Vector _ t* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *TensorVector _ Vector _ Vector* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *TensorVector _ Vector _ Vector _ cf* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *TensorVector _ Scalar _ Scalar* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *TensorVector _ Scalar _ Scalar _ cf* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

| *TensorScalar _ Vector _ Vector* _ →
 failwith `"print␣current:␣V3:␣not␣implemented"`

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      | *TensorScalar_Vector_Vector_cf* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *TensorScalar_Scalar_Scalar* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *TensorScalar_Scalar_Scalar_cf* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_Scalar_Vector_Vector_D* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_Scalar_Vector_Vector_DP* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_HAZ_D* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_HAZ_DP* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_HHH* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_Gauge_Gauge_Gauge_i* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Gauge_Gauge_Gauge_i* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_GGG* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_AWW_DP* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_AWW_DW* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_WWZ_DPWDW* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_WWZ_DW* _ →
        failwith "print_current:␣V3:␣not␣implemented"

      | *Dim6_WWZ_D* _ →
        failwith "print_current:␣V3:␣not␣implemented"

    end

Flip the sign in c to account for the i^2 relative to diagrams with only cubic couplings.

     | *V4* (*vertex*, *fusion*, *constant*) →
       let *ch1*, *ch2*, *ch3* = *children3 rhs* in
       let *wf1* = *wf_index lookups.wfmap lookups.n_wfs* (*f ch1*)
       and *wf2* = *wf_index lookups.wfmap lookups.n_wfs* (*f ch2*)
       and *wf3* = *wf_index lookups.wfmap lookups.n_wfs* (*f ch3*)
          and *const_ID* = *get_const_ID lookups.cmap constant* in

```
let c =
    if (F.sign rhs) < 0 then const_ID else - const_ID in
begin match vertex with
| Scalar4 coeff →
    printi ovm_FUSE_S_SSS ~lhs : lhs ~coupl : c ~coeff : coeff ~rhs1 :
```
wf1
```
        ~rhs2 : wf2 ~rhs3 : wf3
| Scalar2_Vector2 coeff →
    let printc code r1 r2 r3 = printi code
        ~lhs : lhs ~coupl : c ~coeff : coeff ~rhs1 : r1 ~rhs2 : r2 ~rhs3 :
```
r3 in
```
        begin match fusion with
        | F134 | F143 | F234 | F243 →
            printc ovm_FUSE_S_SVV wf1 wf2 wf3
        | F314 | F413 | F324 | F423 →
            printc ovm_FUSE_S_SVV wf2 wf1 wf3
        | F341 | F431 | F342 | F432 →
            printc ovm_FUSE_S_SVV wf3 wf1 wf2
        | F312 | F321 | F412 | F421 →
            printc ovm_FUSE_V_SSV wf2 wf3 wf1
        | F231 | F132 | F241 | F142 →
            printc ovm_FUSE_V_SSV wf1 wf3 wf2
        | F123 | F213 | F124 | F214 →
            printc ovm_FUSE_V_SSV wf1 wf2 wf3
        end

| Vector4 contractions →
    List.iter (print_vector4 c lhs wf1 wf2 wf3 fusion) contractions

| Vector4_K_Matrix_tho _
| Vector4_K_Matrix_jr _
| DScalar2_Vector2_K_Matrix_ms _
| DScalar4_K_Matrix_ms _ →
    failwith "print_current:␣V4:␣K_Matrix␣not␣implemented"
| Dim8_Scalar2_Vector2_1 _
| Dim8_Scalar2_Vector2_2 _
| Dim8_Scalar4 _ →
    failwith "print_current:␣V4:␣not␣implemented"
| GBBG _ →
    failwith "print_current:␣V4:␣GBBG␣not␣implemented"
| DScalar4 _
| DScalar2_Vector2 _ →
    failwith "print_current:␣V4:␣DScalars␣not␣implemented"
| Dim6_H4_P2 _ →
    failwith "print_current:␣V3:␣not␣implemented"
| Dim6_AHWW_DPB _ →
    failwith "print_current:␣V3:␣not␣implemented"
| Dim6_AHWW_DPW _ →
    failwith "print_current:␣V3:␣not␣implemented"
| Dim6_AHWW_DW _ →
    failwith "print_current:␣V3:␣not␣implemented"
```

$\quad\quad$ | *Dim6_Vector4_DW* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_Vector4_W* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_Scalar2_Vector2_D* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_Scalar2_Vector2_DP* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_HWWZ_DW* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_HWWZ_DPB* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_HWWZ_DDPW* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_HWWZ_DPW* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_AHHZ_D* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_AHHZ_DP* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_AHHZ_PB* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_Scalar2_Vector2_PB* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ | *Dim6_HHZZ_T* _ $\rightarrow$
$\quad\quad\quad$ *failwith* `"print_current:␣V3:␣not␣implemented"`
$\quad\quad$ end

$\quad$ | *Vn* (_, _, _) $\rightarrow$ *invalid_arg* `"Targets.print_current:␣n-ary␣fusion."`

Fusions

let *print_fusion lookups lhs_momID fusion amplitude* $=$
$\quad$ if *F.on_shell amplitude* (*F.lhs fusion*) then
$\quad\quad$ *failwith* `"print_fusion:␣on_shell␣projectors␣not␣implemented!"`;
$\quad$ if *F.is_gauss amplitude* (*F.lhs fusion*) then
$\quad\quad$ *failwith* `"print_fusion:␣gauss␣amplitudes␣not␣implemented!"`;
$\quad$ let *lhs_wf* $=$ *mult_wf lookups.dict amplitude* (*F.lhs fusion*) in
$\quad$ let *lhs_wfID* $=$ *wf_index lookups.wfmap lookups.n_wfs lhs_wf* in
$\quad$ let *f* $=$ *F.flavor* (*F.lhs fusion*) in
$\quad$ let *pdg* $=$ *CM.pdg f* in
$\quad$ let *w* $=$
$\quad\quad$ begin match *CM.width f* with
$\quad\quad$ | *Vanishing* | *Fudged* $\rightarrow$ 0
$\quad\quad$ | *Constant* $\rightarrow$ 1
$\quad\quad$ | *Timelike* $\rightarrow$ 2
$\quad\quad$ | *Running* $\rightarrow$ *failwith* `"Targets.VM:␣running␣width␣not␣available"`
$\quad\quad$ | *Custom* _ $\rightarrow$ *failwith* `"Targets.VM:␣custom␣width␣not␣available"`
$\quad\quad$ end
$\quad$ in

let *propagate code* = *printi code* ˜*lhs* : *lhs_wfID* ˜*rhs1* : *lhs_momID*
  ˜*coupl* : (*abs*(*pdg*)) ˜*coeff* : *w* ˜*rhs4* : (*get_ID′ amp_compare lookups.amap amplitude*)
in
begin match *CM.propagator f* with
| *Prop_Scalar* →
    propagate ovm_PROPAGATE_SCALAR
| *Prop_Col_Scalar* →
    propagate ovm_PROPAGATE_COL_SCALAR
| *Prop_Ghost* →
    propagate ovm_PROPAGATE_GHOST
| *Prop_Spinor* →
    propagate ovm_PROPAGATE_SPINOR
| *Prop_ConjSpinor* →
    propagate ovm_PROPAGATE_CONJSPINOR
| *Prop_Majorana* →
    propagate ovm_PROPAGATE_MAJORANA
| *Prop_Col_Majorana* →
    propagate ovm_PROPAGATE_COL_MAJORANA
| *Prop_Unitarity* →
    propagate ovm_PROPAGATE_UNITARITY
| *Prop_Col_Unitarity* →
    propagate ovm_PROPAGATE_COL_UNITARITY
| *Prop_Feynman* →
    propagate ovm_PROPAGATE_FEYNMAN
| *Prop_Col_Feynman* →
    propagate ovm_PROPAGATE_COL_FEYNMAN
| *Prop_Vectorspinor* →
    propagate ovm_PROPAGATE_VECTORSPINOR
| *Prop_Tensor_2* →
    propagate ovm_PROPAGATE_TENSOR2
| *Aux_Col_Scalar* | *Aux_Col_Vector* | *Aux_Col_Tensor_1* →
    failwith `"print_fusion:␣Aux_Col_*␣not␣implemented!"`
| *Aux_Vector* | *Aux_Tensor_1* | *Aux_Scalar* | *Aux_Spinor* |
Aux_ConjSpinor
| *Aux_Majorana* | *Only_Insertion* →
    propagate ovm_PROPAGATE_NONE
| *Prop_Gauge* _ →
    failwith `"print_fusion:␣Prop_Gauge␣not␣implemented!"`
| *Prop_Tensor_pure* →
    failwith `"print_fusion:␣Prop_Tensor_pure␣not␣implemented!"`
| *Prop_Vector_pure* →
    failwith `"print_fusion:␣Prop_Vector_pure␣not␣implemented!"`
| *Prop_Rxi* _ →
    failwith `"print_fusion:␣Prop_Rxi␣not␣implemented!"`
end;

Since the OVM knows that we want to propagate a wf, we can send the necessary fusions now.

List.iter (*print_current lookups lhs_wfID amplitude*) (*F.rhs fusion*)

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```
let print_all_fusions lookups  =
   let fusions  =  CF.fusions lookups.amplitudes in
   let fset  =  List.fold_left (fun s x  →  FSet.add x s) FSet.empty fusions in
   ignore (List.fold_left (fun level (f,  amplitude)  →
      let wf  =  F.lhs f in
      let lhs_momID  =  mom_ID lookups.pmap wf in
      let level'  =  List.length (F.momentum_list wf) in
      if (level'  >  level  ∧  level'  >  2) then break ();
      print_fusion lookups lhs_momID f amplitude;
      level')
   1 (FSet.elements fset) )
```

<div align="center">

Brakets

</div>

```
let print_braket lookups amplitude braket  =
   let bra  =  F.bra braket
   and ket  =  F.ket braket in
   let braID  =  wf_index lookups.wfmap lookups.n_wfs
      (mult_wf lookups.dict amplitude bra) in
   List.iter (print_current lookups braID amplitude) ket
```

$$iT = i^{\#\text{vertices}} i^{\#\text{propagators}} \cdots = i^{n-2} i^{n-3} \cdots = -i(-1)^n \cdots \tag{15.1}$$

All brakets for one cflow amplitude should be calculated by one thread to avoid
multiple access on the same memory (amplitude).

```
let print_brakets lookups (amplitude,  i)  =
   let n  =  List.length (F.externals amplitude) in
   let sign  =  if n mod 2 = 0 then -1 else 1
   and sym  =  F.symmetry amplitude in
   printi ovm_CALC_BRAKET ˜lhs : i ˜rhs1 : sym ˜coupl : sign;
   amplitude | >  F.brakets | >  List.iter (print_braket lookups amplitude)
```

Fortran arrays/OCaml lists start on 1/0. The amplitude list is sorted by
amp_compare according to their color flows. In this way the amp array is
sorted in the same way as *table_color_factors*.

```
let print_all_brakets lookups  =
   let g i elt  =  print_brakets lookups (elt,  i + 1) in
   lookups.amplitudes | >  CF.processes | >  List.sort amp_compare
                      —>  ThoList.iteri g 0
```

<div align="center">

Couplings

</div>

For now we only care to catch the arrays *gncneu*, *gnclep*, *gncup* and *gncdown*
of the SM. This will need an overhaul when it is clear how we store the type
information of coupling constants.

```
let strip_array_tag  =  function
   |  Real_Array x  →  x
   |  Complex_Array x  →  x
```

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</div>

let *array_constants_list* =
  let *params* = *M.parameters*()
  and *strip_to_constant* (*lhs*, _) = *strip_array_tag lhs* in
    List.map strip_to_constant params.derived_arrays

let *is_array x* = *List.mem x array_constants_list*

let *constants_map* =
  let *first* = fun (*x*, _, _) → *x* in
  let *second* = fun (_, *y*, _) → *y* in
  let *third* = fun (_, _, *z*) → *z* in
  let *v3* = *List.map third* (*first* (*M.vertices* ()))
  and *v4* = *List.map third* (*second* (*M.vertices* ())) in
  let *set* = *List.fold_left* (fun *s x* → *CSet.add x s*) *CSet.empty* (*v3* @ *v4*) in
  let (*arrays*, *singles*) = *CSet.partition is_array set* in
    (*singles* | > *CSet.elements* | > *map_of_list*,
     arrays | > *CSet.elements* | > *map_of_list*)

Output calls

let *amplitudes_to_channel* (*cmdline* : *string*) (*oc* : *out_channel*)
  (*diagnostics* : (*diagnostic* × *bool*) *list*) (*amplitudes* : *CF.amplitudes*) =

  set_formatter_out_channel oc;
  if (*num_particles amplitudes* = 0) then begin
    print_description cmdline;
    print_zero_header (); *nl* ()
  end else begin
    let (*wfset*, *amap*) = *wfset_amps amplitudes* in
    let *pset* = *expand_pset* (*momenta_set wfset*)
    and *n_wfs* = *num_wfs wfset* in
    let *wfmap* = *wf_map_of_list* (*WFSet.elements wfset*)
    and *pmap* = *map_of_list* (*ISet.elements pset*)
    and *cmap* = *constants_map* in

    let *lookups* = {*pmap* = *pmap*; *wfmap* = *wfmap*; *cmap* = *cmap*; *amap* = *amap*;
     n_wfs = *n_wfs*; *amplitudes* = *amplitudes*;
     dict = *CF.dictionary amplitudes*} in

    print_description cmdline;
    print_header lookups wfset;
    print_spin_table amplitudes;
    print_flavor_tables amplitudes;
    print_color_tables amplitudes;
    printf `"@\n%s"` (`"OVM␣instructions␣for␣momenta␣addition,"` ^
                `"␣fusions␣and␣brakets␣start␣here:␣"`);
    break ();
    add_all_mom lookups pset;
    print_ext_amps lookups;
    break ();
    print_all_fusions lookups;
    break ();

```
        print_all_brakets lookups;
        break (); nl ();
        print_flush ()
     end

let parameters_to_fortran oc _ =
          set_formatter_out_channel oc;
     let arrays_to_set = ¬ (IMap.is_empty (snd constants_map)) in
     let set_coupl ty dim cmap = IMap.iter (fun key elt →
        printf "␣␣␣␣%s(%s%d)␣=␣%s" ty dim key (M.constant_symbol elt);
        nl () ) cmap in
     let declarations () =
        printf "␣␣complex(%s),␣dimension(%d)␣::␣ovm_coupl_cmplx"
           !kind (constants_map |> fst |> largest_key); nl ();
        if arrays_to_set then
           printf "␣␣complex(%s),␣dimension(2,␣%d)␣::␣ovm_coupl_cmplx2"
              !kind (constants_map |> snd |> largest_key); nl () in
     let print_line str = printf "%s" str; nl() in
     let print_md5sum = function
         | Some s →
           print_line "␣␣function␣md5sum␣()";
           print_line "␣␣␣␣character(len=32)␣::␣md5sum";
           print_line ("␣␣␣␣bytecode_file␣=␣'" ^ !bytecode_file ^ "'");
           print_line "␣␣␣␣call␣initialize_vm␣(vm,␣bytecode_file)";
           print_line "␣␣␣␣!␣DON'T␣EVEN␣THINK␣of␣modifying␣the␣following␣line!";
           print_line ("␣␣␣␣md5sum␣=␣'" ^ s ^ "'");
           print_line "␣␣end␣function␣md5sum";
         | None → ()
     in
     let print_inquiry_function_openmp () = begin
       print_line "␣␣pure␣function␣openmp_supported␣()␣result␣(status)";
       print_line "␣␣␣␣logical␣::␣status";
       print_line ("␣␣␣␣status␣=␣" ^ (if !openmp then ".true." else ".false."));
       print_line "␣␣end␣function␣openmp_supported";
       nl ()
     end in
     let print_interface whizard =
     if whizard then begin
       print_line "␣␣subroutine␣init␣(par)";
       print_line "␣␣␣␣real(kind=default),␣dimension(*),␣intent(in)␣::␣par";
       print_line ("␣␣␣␣␣bytecode_file␣=␣'" ^ !bytecode_file ^ "'");
       print_line "␣␣␣␣call␣import_from_whizard␣(par)";
       print_line "␣␣␣␣call␣initialize_vm␣(vm,␣bytecode_file)";
       print_line "␣␣end␣subroutine␣init";
       nl ();
       print_line "␣␣subroutine␣final␣()";
       print_line "␣␣␣␣call␣vm%final␣()";
       print_line "␣␣end␣subroutine␣final";
       nl ();
       print_line "␣␣subroutine␣update_alpha_s␣(alpha_s)";
```

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```
      print_line ("␣␣␣␣real(kind=" ^ !kind ^ "),␣intent(in)␣::␣alpha_s");
      print_line "␣␣␣␣call␣model_update_alpha_s␣(alpha_s)";
      print_line "␣␣end␣subroutine␣update_alpha_s";
      nl ()
  end
  else begin
      print_line "␣␣subroutine␣init␣()";
      print_line ("␣␣␣␣␣bytecode_file␣=␣'" ^ !bytecode_file ^ "'");
      print_line "␣␣␣␣␣call␣init_parameters␣()";
      print_line "␣␣␣␣␣call␣initialize_vm␣(vm,␣bytecode_file)";
      print_line "␣␣end␣subroutine"
  end in
let print_lookup_functions () = begin
  print_line "␣␣pure␣function␣number_particles_in␣()␣result␣(n)";
  print_line "␣␣␣␣integer␣::␣n";
  print_line "␣␣␣␣n␣=␣vm%number_particles_in␣()";
  print_line "␣␣end␣function␣number_particles_in";
  nl();
  print_line "␣␣pure␣function␣number_particles_out␣()␣result␣(n)";
  print_line "␣␣␣␣integer␣::␣n";
  print_line "␣␣␣␣n␣=␣vm%number_particles_out␣()";
  print_line "␣␣end␣function␣number_particles_out";
  nl();
  print_line "␣␣pure␣function␣number_spin_states␣()␣result␣(n)";
  print_line "␣␣␣␣integer␣::␣n";
  print_line "␣␣␣␣n␣=␣vm%number_spin_states␣()";
  print_line "␣␣end␣function␣number_spin_states";
  nl();
  print_line "␣␣pure␣subroutine␣spin_states␣(a)";
  print_line "␣␣␣␣integer,␣dimension(:,:),␣intent(out)␣::␣a";
  print_line "␣␣␣␣call␣vm%spin_states␣(a)";
  print_line "␣␣end␣subroutine␣spin_states";
  nl();
  print_line "␣␣pure␣function␣number_flavor_states␣()␣result␣(n)";
  print_line "␣␣␣␣integer␣::␣n";
  print_line "␣␣␣␣n␣=␣vm%number_flavor_states␣()";
  print_line "␣␣end␣function␣number_flavor_states";
  nl();
  print_line "␣␣pure␣subroutine␣flavor_states␣(a)";
  print_line "␣␣␣␣integer,␣dimension(:,:),␣intent(out)␣::␣a";
  print_line "␣␣␣␣call␣vm%flavor_states␣(a)";
  print_line "␣␣end␣subroutine␣flavor_states";
  nl();
  print_line "␣␣pure␣function␣number_color_indices␣()␣result␣(n)";
  print_line "␣␣␣␣integer␣::␣n";
  print_line "␣␣␣␣n␣=␣vm%number_color_indices␣()";
  print_line "␣␣end␣function␣number_color_indices";
  nl();
  print_line "␣␣pure␣function␣number_color_flows␣()␣result␣(n)";
  print_line "␣␣␣␣integer␣::␣n";
```

print_line "␣␣␣␣n␣=␣vm%number_color_flows␣()";
print_line "␣␣end␣function␣number_color_flows";
nl();
print_line "␣␣pure␣subroutine␣color_flows␣(a,␣g)";
print_line "␣␣␣␣integer,␣dimension(:,:,:),␣intent(out)␣::␣a";
print_line "␣␣␣␣logical,␣dimension(:,:),␣intent(out)␣::␣g";
print_line "␣␣␣␣call␣vm%color_flows␣(a,␣g)";
print_line "␣␣end␣subroutine␣color_flows";
nl();
print_line "␣␣pure␣function␣number_color_factors␣()␣result␣(n)";
print_line "␣␣␣␣integer␣::␣n";
print_line "␣␣␣␣n␣=␣vm%number_color_factors␣()";
print_line "␣␣end␣function␣number_color_factors";
nl();
print_line "␣␣pure␣subroutine␣color_factors␣(cf)";
print_line "␣␣␣␣use␣omega_color";
print_line "␣␣␣␣type(omega_color_factor),␣dimension(:),␣intent(out)␣::␣cf";
print_line "␣␣␣␣call␣vm%color_factors␣(cf)";
print_line "␣␣end␣subroutine␣color_factors";
nl();
print_line "␣␣!pure␣unless␣OpenMP";
print_line "␣␣!pure␣function␣color_sum␣(flv,␣hel)␣result␣(amp2)";
print_line "␣␣function␣color_sum␣(flv,␣hel)␣result␣(amp2)";
print_line "␣␣␣␣use␣kinds";
print_line "␣␣␣␣integer,␣intent(in)␣::␣flv,␣hel";
print_line "␣␣␣␣real(kind=default)␣::␣amp2";
print_line "␣␣␣␣amp2␣=␣vm%color_sum␣(flv,␣hel)";
print_line "␣␣end␣function␣color_sum";
nl();
print_line "␣␣subroutine␣new_event␣(p)";
print_line "␣␣␣␣use␣kinds";
print_line "␣␣␣␣real(kind=default),␣dimension(0:3,*),␣intent(in)␣::␣p";
print_line "␣␣␣␣call␣vm%new_event␣(p)";
print_line "␣␣end␣subroutine␣new_event";
nl();
print_line "␣␣subroutine␣reset_helicity_selection␣(threshold,␣cutoff)";
print_line "␣␣␣␣use␣kinds";
print_line "␣␣␣␣real(kind=default),␣intent(in)␣::␣threshold";
print_line "␣␣␣␣integer,␣intent(in)␣::␣cutoff";
print_line "␣␣␣␣call␣vm%reset_helicity_selection␣(threshold,␣cutoff)";
print_line "␣␣end␣subroutine␣reset_helicity_selection";
nl();
print_line "␣␣pure␣function␣is_allowed␣(flv,␣hel,␣col)␣result␣(yorn)";
print_line "␣␣␣␣logical␣::␣yorn";
print_line "␣␣␣␣integer,␣intent(in)␣::␣flv,␣hel,␣col";
print_line "␣␣␣␣yorn␣=␣vm%is_allowed␣(flv,␣hel,␣col)";
print_line "␣␣end␣function␣is_allowed";
nl();
print_line "␣␣pure␣function␣get_amplitude␣(flv,␣hel,␣col)␣result␣(amp_result)";
print_line "␣␣␣␣use␣kinds";

```
    print_line "␣␣␣␣complex(kind=default)␣::␣amp_result";
    print_line "␣␣␣␣integer,␣intent(in)␣::␣flv,␣hel,␣col";
    print_line "␣␣␣␣amp_result␣=␣vm%get_amplitude(flv,␣hel,␣col)";
    print_line "␣␣end␣function␣get_amplitude";
    nl ();
end in
let rcs_str s = String.sub s 0 (String.length s − 1) in
let rcs_tags = rcs_list @ [M.rcs] |> List.map version_string
                              —> List.map rcs_str in

print_line ("module␣" ^ !wrapper_module);
print_line ("␣␣use␣" ^ !parameter_module_external);
print_line "␣␣use␣iso_varying_string,␣string_t␣=>␣varying_string";
print_line "␣␣use␣kinds";
print_line "␣␣use␣omegavm95";
print_line "␣␣implicit␣none";
print_line "␣␣private";
print_line "␣␣type(vm_t)␣::␣vm";
print_line "␣␣type(string_t)␣::␣bytecode_file";
print_line ("␣␣public␣::␣number_particles_in,␣number_particles_out," ^
    "␣number_spin_states,␣&");
print_line ("␣␣␣␣spin_states,␣number_flavor_states,␣flavor_states," ^
    "␣number_color_indices,␣&");
print_line ("␣␣␣␣number_color_flows,␣color_flows," ^
    "␣number_color_factors,␣color_factors,␣&");
print_line ("␣␣␣␣color_sum,␣new_event,␣reset_helicity_selection," ^
    "␣is_allowed,␣get_amplitude,␣&");
print_line ("␣␣␣␣init,␣" ^
    (match !md5sum with Some _ → "md5sum,␣"
                        | None → "") ^ "openmp_supported");
if !whizard then
    print_line ("␣␣public␣::␣final,␣update_alpha_s")
else
    print_line ("␣␣public␣::␣initialize_vm");
declarations ();
print_line "contains";

print_line "␣␣subroutine␣setup_couplings␣()";
set_coupl "ovm_coupl_cmplx" "" (fst constants_map);
if arrays_to_set then
    set_coupl "ovm_coupl_cmplx2" ":," (snd constants_map);
print_line "␣␣end␣subroutine␣setup_couplings";
print_line "␣␣subroutine␣initialize_vm␣(vm,␣bytecode_file)";
print_line "␣␣␣␣class(vm_t),␣intent(out)␣::␣vm";
print_line "␣␣␣␣type(string_t),␣intent(in)␣::␣bytecode_file";
print_line "␣␣␣␣type(string_t)␣::␣version";
print_line "␣␣␣␣type(string_t)␣::␣model";
print_line ("␣␣␣␣version␣=␣'" ^ List.nth rcs_tags 0 ^ "'");
print_line ("␣␣␣␣model␣=␣'" ^ List.nth rcs_tags 1 ^ "'");
print_line "␣␣␣␣call␣setup_couplings␣()";
print_line "␣␣␣␣call␣vm%init␣(bytecode_file,␣version,␣model,␣verbose=.False.,␣&";
```

print_line "␣␣␣␣␣␣coupl_cmplx=ovm_coupl_cmplx,␣&";
if *arrays_to_set* then
    print_line "␣␣␣␣␣␣coupl_cmplx2=ovm_coupl_cmplx2,␣&";
print_line ("␣␣␣␣␣␣mass=mass,␣width=width,␣openmp=" ˆ (if !*openmp* then
    ".true." else ".false.") ˆ ")");
print_line "␣␣end␣subroutine␣initialize_vm";
nl();
print_md5sum !*md5sum*;
print_inquiry_function_openmp ();
print_interface !*whizard*;
print_lookup_functions ();

print_line ("end␣module␣" ˆ !*wrapper_module*);
print_line "!␣O'Mega␣revision␣control␣information:";
ThoList.flatmap RCS.summary (*M.rcs* :: *rcs_list*) | >
    List.iter (fun *s* → *printf* "!␣␣␣␣%s" *s*; *nl* ())

let *parameters_to_channel oc* =
    parameters_to_fortran oc (*CM.parameters* ())

end

15.2.2   *Fortran 90/95*

Dirac Fermions

We factor out the code for fermions so that we can use the simpler implementation for Dirac fermions if the model contains no Majorana fermions.

module type *Fermions* =
  sig
    open *Coupling*
    val *psi_type* : *string*
    val *psibar_type* : *string*
    val *chi_type* : *string*
    val *grav_type* : *string*
    val *psi_incoming* : *string*
    val *brs_psi_incoming* : *string*
    val *psibar_incoming* : *string*
    val *brs_psibar_incoming* : *string*
    val *chi_incoming* : *string*
    val *brs_chi_incoming* : *string*
    val *grav_incoming* : *string*
    val *psi_outgoing* : *string*
    val *brs_psi_outgoing* : *string*
    val *psibar_outgoing* : *string*
    val *brs_psibar_outgoing* : *string*
    val *chi_outgoing* : *string*
    val *brs_chi_outgoing* : *string*
    val *grav_outgoing* : *string*
    val *psi_propagator* : *string*

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```
    val psibar_propagator : string
    val chi_propagator : string
    val grav_propagator : string
    val psi_projector : string
    val psibar_projector : string
    val chi_projector : string
    val grav_projector : string
    val psi_gauss : string
    val psibar_gauss : string
    val chi_gauss : string
    val grav_gauss : string
    val print_current : int × fermionbar × boson × fermion →
      string → string → string → fuse2 → unit
    val print_current_mom : int × fermionbar × boson × fermion →
      string → string → string → string → string → string
      → fuse2 → unit
    val print_current_p : int × fermion × boson × fermion →
      string → string → string → fuse2 → unit
    val print_current_b : int × fermionbar × boson × fermionbar →
      string → string → string → fuse2 → unit
    val print_current_g : int × fermionbar × boson × fermion →
      string → string → string → string → string → string
      → fuse2 → unit
    val print_current_g4 : int × fermionbar × boson2 × fermion →
      string → string → string → string → fuse3 → unit
    val reverse_braket : lorentz → bool
    val use_module : string
    val require_library : string list
    val rcs : RCS.t
  end

module Fortran_Fermions : Fermions =
  struct
    let rcs = RCS.rename rcs_file "Targets.Fortran_Fermions()"
        [ "generates␣Fortran95␣code␣for␣Dirac␣fermions";
          "using␣revision␣2000␣10␣A␣of␣module␣omega95" ]

    open Coupling
    open Format

    let psi_type = "spinor"
    let psibar_type = "conjspinor"
    let chi_type = "???"
    let grav_type = "???"

    let psi_incoming = "u"
    let brs_psi_incoming = "brs_u"
    let psibar_incoming = "vbar"
    let brs_psibar_incoming = "brs_vbar"
    let chi_incoming = "???"
    let brs_chi_incoming = "???"
    let grav_incoming = "???"
```

```
let psi_outgoing = "v"
let brs_psi_outgoing = "brs_v"
let psibar_outgoing = "ubar"
let brs_psibar_outgoing = "brs_ubar"
let chi_outgoing = "???"
let brs_chi_outgoing = "???"
let grav_outgoing = "???"

let psi_propagator = "pr_psi"
let psibar_propagator = "pr_psibar"
let chi_propagator = "???"
let grav_propagator = "???"

let psi_projector = "pj_psi"
let psibar_projector = "pj_psibar"
let chi_projector = "???"
let grav_projector = "???"

let psi_gauss = "pg_psi"
let psibar_gauss = "pg_psibar"
let chi_gauss = "???"
let grav_gauss = "???"

let format_coupling coeff c =
  match coeff with
  | 1 → c
  | −1 → "(-" ^ c ^")"
  | coeff → string_of_int coeff ^ "*" ^ c

let format_coupling_2 coeff c =
  match coeff with
  | 1 → c
  | −1 → "-" ^ c
  | coeff → string_of_int coeff ^ "*" ^ c
```

JR's coupling constant HACK, necessitated by tho's bad design descition.

```
let fastener s i ?p ?q () =
  try
    let offset = (String.index s '(') in
    if ((String.get s (String.length s − 1)) ≢ ')') then
      failwith "fastener:␣wrong␣usage␣of␣parentheses"
    else
      let func_name = (String.sub s 0 offset) and
          tail =
        (String.sub s (succ offset) (String.length s − offset − 2)) in
      if (String.contains func_name ')') ∨
         (String.contains tail '(') ∨
         (String.contains tail ')') then
        failwith "fastener:␣wrong␣usage␣of␣parentheses"
      else
        func_name ^ "(" ^ string_of_int i ^ "," ^ tail ^ ")"
```

```
      with
      | Not_found →
          if (String.contains s ')') then
              failwith "fastener:␣wrong␣usage␣of␣parentheses"
          else
              match p with
              | None → s ^ "(" ^ string_of_int i ^ ")"
              | Some p →
                match q with
                | None → s ^ "(" ^ p ^ "*" ^ p ^ "," ^ string_of_int i ^ ")"
                | Some q → s ^ "(" ^ p ^ "," ^ q ^ "," ^ string_of_int i ^ ")"
  let print_fermion_current coeff f c wf1 wf2 fusion =
    let c = format_coupling coeff c in
    match fusion with
    | F13 → printf "%s_ff(%s,%s,%s)" f c wf1 wf2
    | F31 → printf "%s_ff(%s,%s,%s)" f c wf2 wf1
    | F23 → printf "f_%sf(%s,%s,%s)" f c wf1 wf2
    | F32 → printf "f_%sf(%s,%s,%s)" f c wf2 wf1
    | F12 → printf "f_f%s(%s,%s,%s)" f c wf1 wf2
    | F21 → printf "f_f%s(%s,%s,%s)" f c wf2 wf1
```

Using a two element array for the combined vector-axial and scalar-pseudo couplings helps to support HELAS as well. Since we will probably never support general boson couplings with HELAS, it might be retired in favor of two separate variables. For this *Model.constant_symbol* has to be generalized.

NB: passing the array instead of two separate constants would be a *bad* idea, because the support for Majorana spinors below will have to flip signs!

```
  let print_fermion_current2 coeff f c wf1 wf2 fusion =
    let c = format_coupling_2 coeff c in
    let c1 = fastener c 1 ()
    and c2 = fastener c 2 () in
    match fusion with
    | F13 → printf "%s_ff(%s,%s,%s,%s)" f c1 c2 wf1 wf2
    | F31 → printf "%s_ff(%s,%s,%s,%s)" f c1 c2 wf2 wf1
    | F23 → printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf1 wf2
    | F32 → printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf2 wf1
    | F12 → printf "f_f%s(%s,%s,%s,%s)" f c1 c2 wf1 wf2
    | F21 → printf "f_f%s(%s,%s,%s,%s)" f c1 c2 wf2 wf1

  let print_fermion_current_mom_v1 coeff f c wf1 wf2 p1 p2 p12 fusion =
    let c = format_coupling coeff c in
    let c1 = fastener c 1 and
        c2 = fastener c 2 in
    match fusion with
    | F13 → printf "%s_ff(%s,%s,%s,%s)" f (c1 ~p : p12 ()) (c2 ~p :
p12 ()) wf1 wf2
```

| _F31_ → _printf_ `"%s_ff(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p12_ ()) (_c2_ ˜_p_ : _p12_ ()) _wf2_ _wf1_
| _F23_ → _printf_ `"f_%sf(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p1_ ()) (_c2_ ˜_p_ : _p1_ ()) _wf1_ _wf2_
| _F32_ → _printf_ `"f_%sf(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p2_ ()) (_c2_ ˜_p_ : _p2_ ()) _wf2_ _wf1_
| _F12_ → _printf_ `"f_f%s(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p2_ ()) (_c2_ ˜_p_ : _p2_ ()) _wf1_ _wf2_
| _F21_ → _printf_ `"f_f%s(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p1_ ()) (_c2_ ˜_p_ : _p1_ ()) _wf2_ _wf1_

let _print_fermion_current_mom_v2 coeff f c wf1 wf2 p1 p2 p12 fusion_ =
let _c_ = _format_coupling coeff c_ in
let _c1_ = _fastener c_ 1 and
_c2_ = _fastener c_ 2 in
match _fusion_ with
| _F13_ → _printf_ `"%s_ff(%s,%s,@,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p12_ ()) (_c2_ ˜_p_ : _p12_ ()) _wf1_ _wf2_ _p12_
| _F31_ → _printf_ `"%s_ff(%s,%s,@,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p12_ ()) (_c2_ ˜_p_ : _p12_ ()) _wf2_ _wf1_ _p12_
| _F23_ → _printf_ `"f_%sf(%s,%s,@,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p1_ ()) (_c2_ ˜_p_ : _p1_ ()) _wf1_ _wf2_ _p1_
| _F32_ → _printf_ `"f_%sf(%s,%s,@,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p2_ ()) (_c2_ ˜_p_ : _p2_ ()) _wf2_ _wf1_ _p2_
| _F12_ → _printf_ `"f_f%s(%s,%s,@,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p2_ ()) (_c2_ ˜_p_ : _p2_ ()) _wf1_ _wf2_ _p2_
| _F21_ → _printf_ `"f_f%s(%s,%s,@,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p1_ ()) (_c2_ ˜_p_ : _p1_ ()) _wf2_ _wf1_ _p1_

let _print_fermion_current_mom_ff coeff f c wf1 wf2 p1 p2 p12 fusion_ =
let _c_ = _format_coupling coeff c_ in
let _c1_ = _fastener c_ 1 and
_c2_ = _fastener c_ 2 in
match _fusion_ with
| _F13_ → _printf_ `"%s_ff(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p1_ ˜_q_ : _p2_ ()) (_c2_ ˜_p_ : _p1_ ˜_q_ : _p2_ ()) _wf1_ _wf2_
| _F31_ → _printf_ `"%s_ff(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p1_ ˜_q_ : _p2_ ()) (_c2_ ˜_p_ : _p1_ ˜_q_ : _p2_ ()) _wf2_ _wf1_
| _F23_ → _printf_ `"f_%sf(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p12_ ˜_q_ : _p2_ ()) (_c2_ ˜_p_ : _p12_ ˜_q_ : _p2_ ()) _wf1_ _wf2_
| _F32_ → _printf_ `"f_%sf(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p12_ ˜_q_ : _p1_ ()) (_c2_ ˜_p_ : _p12_ ˜_q_ : _p1_ ()) _wf2_ _wf1_
| _F12_ → _printf_ `"f_f%s(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p12_ ˜_q_ : _p1_ ()) (_c2_ ˜_p_ : _p12_ ˜_q_ : _p1_ ()) _wf1_ _wf2_
| _F21_ → _printf_ `"f_f%s(%s,%s,%s,%s)"` _f_ (_c1_ ˜_p_ : _p12_ ˜_q_ : _p2_ ()) (_c2_ ˜_p_ : _p12_ ˜_q_ : _p2_ ()) _wf2_ _wf1_

let _print_current_ = function
| _coeff_, _Psibar_, _VA_, _Psi_ → _print_fermion_current2 coeff_ `"va"`
| _coeff_, _Psibar_, _VA2_, _Psi_ → _print_fermion_current coeff_ `"va2"`
| _coeff_, _Psibar_, _VA3_, _Psi_ → _print_fermion_current coeff_ `"va3"`
| _coeff_, _Psibar_, _V_, _Psi_ → _print_fermion_current coeff_ `"v"`

```
    | coeff, Psibar, A, Psi → print_fermion_current coeff "a"
    | coeff, Psibar, VL, Psi → print_fermion_current coeff "vl"
    | coeff, Psibar, VR, Psi → print_fermion_current coeff "vr"
    | coeff, Psibar, VLR, Psi → print_fermion_current2 coeff "vlr"
    | coeff, Psibar, SP, Psi → print_fermion_current2 coeff "sp"
    | coeff, Psibar, S, Psi → print_fermion_current coeff "s"
    | coeff, Psibar, P, Psi → print_fermion_current coeff "p"
    | coeff, Psibar, SL, Psi → print_fermion_current coeff "sl"
    | coeff, Psibar, SR, Psi → print_fermion_current coeff "sr"
    | coeff, Psibar, SLR, Psi → print_fermion_current2 coeff "slr"
    | _, Psibar, _, Psi → invalid_arg
          "Targets.Fortran_Fermions:␣no␣superpotential␣here"
    | _, Chibar, _, _ | _, _, _, Chi → invalid_arg
          "Targets.Fortran_Fermions:␣Majorana␣spinors␣not␣handled"
    | _, Gravbar, _, _ | _, _, _, Grav → invalid_arg
          "Targets.Fortran_Fermions:␣Gravitinos␣not␣handled"

let print_current_mom = function
    | coeff, Psibar, VLRM, Psi → print_fermion_current_mom_v1 coeff "vlr"
    | coeff, Psibar, VAM, Psi → print_fermion_current_mom_ff coeff "va"
    | coeff, Psibar, VA3M, Psi → print_fermion_current_mom_ff coeff "va3"
    | coeff, Psibar, SPM, Psi → print_fermion_current_mom_v1 coeff "sp"
    | coeff, Psibar, TVA, Psi → print_fermion_current_mom_v1 coeff "tva"
    | coeff, Psibar, TVAM, Psi → print_fermion_current_mom_v2 coeff "tvam"
    | coeff, Psibar, TLR, Psi → print_fermion_current_mom_v1 coeff "tlr"
    | coeff, Psibar, TLRM, Psi → print_fermion_current_mom_v2 coeff "tlrm"
    | coeff, Psibar, TRL, Psi → print_fermion_current_mom_v1 coeff "trl"
    | coeff, Psibar, TRLM, Psi → print_fermion_current_mom_v2 coeff "trlm"
    | _, Psibar, _, Psi → invalid_arg
          "Targets.Fortran_Fermions:␣only␣sigma␣tensor␣coupling␣here"
    | _, Chibar, _, _ | _, _, _, Chi → invalid_arg
          "Targets.Fortran_Fermions:␣Majorana␣spinors␣not␣handled"
    | _, Gravbar, _, _ | _, _, _, Grav → invalid_arg
          "Targets.Fortran_Fermions:␣Gravitinos␣not␣handled"

let print_current_p = function
    | _, _, _, _ → invalid_arg
          "Targets.Fortran_Fermions:␣No␣clashing␣arrows␣here"

let print_current_b = function
    | _, _, _, _ → invalid_arg
          "Targets.Fortran_Fermions:␣No␣clashing␣arrows␣here"

let print_current_g = function
    | _, _, _, _ → invalid_arg
          "Targets.Fortran_Fermions:␣No␣gravitinos␣here"

let print_current_g4 = function
    | _, _, _, _ → invalid_arg
          "Targets.Fortran_Fermions:␣No␣gravitinos␣here"

let reverse_braket = function
    | Spinor → true
```

```
      |  _  →  false
```

```
    let use_module  =  "omega95"
    let require_library  =
      ["omega_spinors_2010_01_A"; "omega_spinor_cpls_2010_01_A"]
  end
```

<div align="center">

Main Functor

</div>

```
module Make_Fortran (Fermions  :  Fermions)
    (Fusion_Maker  :  Fusion.Maker) (P  :  Momentum.T) (M  :  Model.T)  =
  struct
    let rcs_list  =
      [ RCS.rename rcs_file "Targets.Make_Fortran()"
          [ "Interface␣for␣Whizard␣2.X";
            "NB:␣non-gauge␣vector␣couplings␣are␣not␣available␣yet" ];
        Fermions.rcs ]
```

```
    let require_library  =
      Fermions.require_library @
      [ "omega_vectors_2010_01_A"; "omega_polarizations_2010_01_A";
        "omega_couplings_2010_01_A"; "omega_color_2010_01_A";
        "omega_utils_2010_01_A" ]
```

```
    module CM  =  Colorize.It(M)
    module F  =  Fusion_Maker(P)(M)
```

```
    module CF  =  Fusion.Multi(Fusion_Maker)(P)(M)
    type amplitudes  =  CF.amplitudes
```

```
    open Coupling
    open Format
```

```
    type output_mode  =
      | Single_Function
      | Single_Module of int
      | Single_File of int
      | Multi_File of int
```

```
    let line_length  =  ref 80
    let continuation_lines  =  ref (−1) (∗ 255 ∗)
    let kind  =  ref "default"
    let fortran95  =  ref true
    let module_name  =  ref "omega_amplitude"
    let output_mode  =  ref (Single_Module 10)
    let use_modules  =  ref []
    let whizard  =  ref false
    let parameter_module  =  ref ""
    let md5sum  =  ref None
    let no_write  =  ref false
    let km_write  =  ref false
    let km_pure  =  ref false
    let km_2_write  =  ref false
```

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```
let km_2_pure = ref false
let openmp = ref false
let pure_unless_openmp = false

let options = Options.create
  [ "90", Arg.Clear fortran95,
    "don't␣use␣Fortran95␣features␣that␣are␣not␣in␣Fortran90";
    "kind", Arg.String (fun s → kind := s),
    "real␣and␣complex␣kind␣(default:␣" ^ !kind ^ ")";
    "width", Arg.Int (fun w → line_length := w), "maximum␣line␣length";
    "continuation", Arg.Int (fun l → continuation_lines := l),
    "maximum␣#␣of␣continuation␣lines";
    "module", Arg.String (fun s → module_name := s), "module␣name";
    "single_function", Arg.Unit (fun () → output_mode := Single_Function),
    "compute␣the␣matrix␣element(s)␣in␣a␣monolithic␣function";
    "split_function", Arg.Int (fun n → output_mode := Single_Module n),
    "split␣the␣matrix␣element(s)␣into␣small␣functions␣[default,␣size␣=␣10]";
    "split_module", Arg.Int (fun n → output_mode := Single_File n),
    "split␣the␣matrix␣element(s)␣into␣small␣modules";
    "split_file", Arg.Int (fun n → output_mode := Multi_File n),
    "split␣the␣matrix␣element(s)␣into␣small␣files";
    "use", Arg.String (fun s → use_modules := s :: !use_modules),
    "use␣module";
    "parameter_module", Arg.String (fun s → parameter_module := s),
    "parameter_module";
    "md5sum", Arg.String (fun s → md5sum := Some s),
    "transfer␣MD5␣checksum";
    "whizard", Arg.Set whizard, "include␣WHIZARD␣interface";
    "no_write", Arg.Set no_write, "no␣'write'␣statements";
    "kmatrix_write", Arg.Set km_2_write, "write␣K␣matrix␣functions";
    "kmatrix_2_write", Arg.Set km_write, "write␣K␣matrix␣2␣functions";
    "kmatrix_write_pure", Arg.Set km_pure, "write␣K␣matrix␣pure␣functions";
    "kmatrix_2_write_pure", Arg.Set km_2_pure, "write␣Kmatrix2pure␣functions";
    "openmp", Arg.Set openmp, "activate␣OpenMP␣support␣in␣generated␣code"]
```

Fortran style line continuation:
Default function to output spaces (copied from `format.ml`).

```
let blank_line = String.make 80 ' '
let rec display_blanks oc n =
  if n > 0 then
    if n ≤ 80 then
      output oc blank_line 0 n
    else begin
      output oc blank_line 0 80;
      display_blanks oc (n − 80)
    end
```

Default function to output new lines (copied from `format.ml`).

```
let display_newline oc () =
  output oc "\n" 0 1
```

current_continuation_line

- ≤ 0: not continuing: print a straight newline,

- > 0: continuing: append "␣&" until we run up to !*continuation_lines*.
  NB: !*continuation_lines* < 0 means *unlimited* continuation lines.

  let *current_continuation_line* $=$ *ref* 1
  exception *Continuation_Lines* of *int*

  let *fortran_newline oc* () $=$
    if !*current_continuation_line* $>$ 0 then begin
      if !*continuation_lines* $\geq$ 0 $\wedge$!*current_continuation_line* $>$!*continuation_lines* then
        raise (*Continuation_Lines* !*current_continuation_line*)
      else begin
        output oc "␣&" 0 2;
        incr current_continuation_line
      end
    end;
    display_newline oc ()

  let *nl* () $=$
    current_continuation_line $:=$ 0;
    print_newline ();
    current_continuation_line $:=$ 1

Make a formatter with default functions to output spaces and new lines.

  let *setup_fortran_formatter width oc* $=$
    set_all_formatter_output_functions
      ~*out* : (*output oc*)
      ~*flush* : (fun () $\rightarrow$ *flush oc*)
      ~*newline* : (*fortran_newline oc*)
      ~*spaces* : (*display_blanks oc*);
    set_margin (*width* $-$ 2)

  let *print_list* $=$ function
    | [] $\rightarrow$ ()
    | *a* :: *rest* $\rightarrow$
        print_string a;
        List.iter (fun *s* $\rightarrow$ *printf* ",@␣%s" *s*) *rest*

Variables and Declarations

"NC" is already used up in the module "constants":

  let *nc_parameter* $=$ "N_"
  let *omega_color_factor_abbrev* $=$ "OCF"
  let *openmp_tld_type* $=$ "thread_local_data"
  let *openmp_tld* $=$ "tld"

  let *flavors_symbol* ?(*decl* $=$ false) *flavors* $=$
    (if !*openmp* $\wedge$ $\neg$ *decl* then *openmp_tld* ^ "%" else "") ^
    "oks_" ^ *String.concat* "" (*List.map CM.flavor_symbol flavors*)

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let *p2s p* =
  if *p* ≥ 0 ∧ *p* ≤ 9 then
    string_of_int p
  else if *p* ≤ 36 then
    String.make 1 (*Char.chr* (*Char.code* 'A' + *p* − 10))
  else
    "_"

let *format_momentum p* =
  "p" ^ *String.concat* "" (*List.map p2s p*)

let *format_p wf* =
  String.concat "" (*List.map p2s* (*F.momentum_list wf*))

let *ext_momentum wf* =
  match *F.momentum_list wf* with
  | [*n*] → *n*
  | _ → *invalid_arg* "Targets.Fortran.ext_momentum"

module *PSet* = *Set.Make* (struct type *t* = *int list* let *compare* = *compare* end)
module *WFSet* = *Set.Make* (struct type *t* = *F.wf* let *compare* = *compare* end)

let *add_tag wf name* =
  match *F.wf_tag wf* with
  | *None* → *name*
  | *Some tag* → *name* ^ "_" ^ *tag*

let *variable* ?(*decl* = false) *wf* =
  (if !*openmp* ∧ ¬ *decl* then *openmp_tld* ^ "%" else "")
  ^ *add_tag wf* ("owf_" ^ *CM.flavor_symbol* (*F.flavor wf*) ^ "_" ^ *format_p wf*)

let *momentum wf* = "p" ^ *format_p wf*
let *spin wf* = "s(" ^ *string_of_int* (*ext_momentum wf*) ^ ")"

let *format_multiple_variable* ?(*decl* = false) *wf i* =
  variable ~*decl* : *decl wf* ^ "_X" ^ *string_of_int i*

let *multiple_variable* ?(*decl* = false) *amplitude dictionary wf* =
  try
    format_multiple_variable ~*decl* : *decl wf* (*dictionary amplitude wf*)
  with
  | *Not_found* → *variable wf*

let *multiple_variables* ?(*decl* = false) *multiplicity wf* =
  try
    List.map
      (*format_multiple_variable* ~*decl* : *decl wf*)
      (*ThoList.range* 1 (*multiplicity wf*))
  with
  | *Not_found* → [*variable* ~*decl* : *decl wf*]

let *declaration_chunk_size* = 64

let *declare_list_chunk multiplicity t* = function
  | [] → ()
  | *wfs* →

```
            printf "␣␣␣␣@[<2>%s␣::␣" t;
            print_list (ThoList.flatmap (multiple_variables ~decl :true multiplicity) wfs); nl ()
let declare_list multiplicity t  =  function
   | []  →  ()
   | wfs  →
       List.iter
         (declare_list_chunk multiplicity t)
         (ThoList.chopn declaration_chunk_size wfs)

type declarations  =
     { scalars  :  F.wf list;
       spinors  :  F.wf list;
       conjspinors  :  F.wf list;
       realspinors  :  F.wf list;
       ghostspinors  :  F.wf list;
       vectorspinors  :  F.wf list;
       vectors  :  F.wf list;
       ward_vectors  :  F.wf list;
       massive_vectors  :  F.wf list;
       tensors_1  :  F.wf list;
       tensors_2  :  F.wf list;
       brs_scalars  :  F.wf list;
       brs_spinors  :  F.wf list;
       brs_conjspinors  :  F.wf list;
       brs_realspinors  :  F.wf list;
       brs_vectorspinors  :  F.wf list;
       brs_vectors  :  F.wf list;
       brs_massive_vectors  :  F.wf list }

let rec classify_wfs' acc  =  function
   | []  →  acc
   | wf  ::  rest  →
       classify_wfs'
          (match CM.lorentz (F.flavor wf) with
           | Scalar  →  {acc with scalars  =  wf  ::  acc.scalars}
           | Spinor  →  {acc with spinors  =  wf  ::  acc.spinors}
           | ConjSpinor  →  {acc with conjspinors  =  wf  ::  acc.conjspinors}
           | Majorana  →  {acc with realspinors  =  wf  ::  acc.realspinors}
           | Maj_Ghost  →  {acc with ghostspinors  =  wf  ::  acc.ghostspinors}
           | Vectorspinor  →
               {acc with vectorspinors  =  wf  ::  acc.vectorspinors}
           | Vector  →  {acc with vectors  =  wf  ::  acc.vectors}
           | Massive_Vector  →
               {acc with massive_vectors  =  wf  ::  acc.massive_vectors}
           | Tensor_1  →  {acc with tensors_1  =  wf  ::  acc.tensors_1}
           | Tensor_2  →  {acc with tensors_2  =  wf  ::  acc.tensors_2}
           | BRS Scalar  →  {acc with brs_scalars  =  wf  ::  acc.brs_scalars}
           | BRS Spinor  →  {acc with brs_spinors  =  wf  ::  acc.brs_spinors}
           | BRS ConjSpinor  →  {acc with brs_conjspinors  =
                                       wf  ::  acc.brs_conjspinors}
           | BRS Majorana  →  {acc with brs_realspinors  =
```

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$$wf \; :: \; acc.brs\_realspinors\}$$
$$| \; BRS \; Vectorspinor \; \rightarrow \; \{acc \; \text{with} \; brs\_vectorspinors \; =$$
$$wf \; :: \; acc.brs\_vectorspinors\}$$
$$| \; BRS \; Vector \; \rightarrow \; \{acc \; \text{with} \; brs\_vectors \; = \; wf \; :: \; acc.brs\_vectors\}$$
$$| \; BRS \; Massive\_Vector \; \rightarrow \; \{acc \; \text{with} \; brs\_massive\_vectors \; =$$
$$wf \; :: \; acc.brs\_massive\_vectors\}$$
$$| \; BRS \; \_ \; \rightarrow \; invalid\_arg \; \texttt{"Targets.wfs\_classify':\textvisiblespace not\textvisiblespace needed\textvisiblespace here"})$$
$$rest$$

let *classify_wfs wfs* = *classify_wfs′*
  { *scalars* = []; *spinors* = []; *conjspinors* = []; *realspinors* = [];
    ghostspinors = []; *vectorspinors* = []; *vectors* = [];
    ward_vectors = [];
    massive_vectors = []; *tensors_1* = []; *tensors_2* = [];
    brs_scalars = [] ; *brs_spinors* = []; *brs_conjspinors* = [];
    brs_realspinors = []; *brs_vectorspinors* = [];
    brs_vectors = []; *brs_massive_vectors* = []}
  wfs

<div align="center">*Parameters*</div>

type α *parameters* =
  { *real_singles* : α *list*;
    real_arrays : ($\alpha \; \times \; int$) *list*;
    complex_singles : α *list*;
    complex_arrays : ($\alpha \; \times \; int$) *list* }

let rec *classify_singles acc* = function
  | [] $\rightarrow$ *acc*
  | *Real p* :: *rest* $\rightarrow$ *classify_singles*
        { *acc* with *real_singles* = *p* :: *acc.real_singles* } *rest*
  | *Complex p* :: *rest* $\rightarrow$ *classify_singles*
        { *acc* with *complex_singles* = *p* :: *acc.complex_singles* } *rest*

let rec *classify_arrays acc* = function
  | [] $\rightarrow$ *acc*
  | (*Real_Array p, rhs*) :: *rest* $\rightarrow$ *classify_arrays*
        { *acc* with *real_arrays* =
          (*p, List.length rhs*) :: *acc.real_arrays* } *rest*
  | (*Complex_Array p, rhs*) :: *rest* $\rightarrow$ *classify_arrays*
        { *acc* with *complex_arrays* =
          (*p, List.length rhs*) :: *acc.complex_arrays* } *rest*

let *classify_parameters params* =
  classify_arrays
    (*classify_singles*
        { *real_singles* = [];
          real_arrays = [];
          complex_singles = [];
          complex_arrays = [] }
        (*List.map fst params.derived*)) *params.derived_arrays*

⚡ Unify this with the other code using *ThoList.chopn*.

```
let rec schisma n l =
  if List.length l ≤ n then
    [l]
  else
    let a, b = ThoList.splitn n l in
    [a] @ (schisma n b)

let rec schisma_num i n l =
  if List.length l ≤ n then
    [(i, l)]
  else
    let a, b = ThoList.splitn n l in
    [(i, a)] @ (schisma_num (i + 1) n b)

let declare_parameters' t = function
  | [] → ()
  | plist →
      printf "␣␣@[<2>%s(kind=%s),␣public,␣save␣::␣" t !kind;
      print_list (List.map CM.constant_symbol plist); nl ()

let declare_parameters t plist =
  List.iter (declare_parameters' t) plist

let declare_parameter_array t (p, n) =
  printf "␣␣@[<2>%s(kind=%s),␣dimension(%d),␣public,␣save␣::␣%s"
    t !kind n (CM.constant_symbol p); nl ()

let default_parameter (x, v) =
  printf "@␣%s␣=␣%g_%s" (CM.constant_symbol x) v !kind

let declare_default_parameters t = function
  | [] → ()
  | p :: plist →
      printf "␣␣@[<2>%s(kind=%s),␣public,␣save␣::" t !kind;
      default_parameter p;
      List.iter (fun p' → printf ","; default_parameter p') plist;
      nl ()

let format_constant = function
  | I → sprintf "cmplx␣(0.0_%s,␣1.0_%s)" !kind !kind
  | Const c when c < 0 → sprintf "(%d.0_%s)" c !kind
  | Const c → sprintf "%d.0_%s" c !kind
  | _ → invalid_arg "format_constant"

let rec eval_parameter' = function
  | I → printf "cmplx␣(0.0_%s,␣1.0_%s)" !kind !kind
  | Const c when c < 0 → printf "(%d.0_%s)" c !kind
  | Const c → printf "%d.0_%s" c !kind
  | Atom x → printf "%s" (CM.constant_symbol x)
  | Sum [] → printf "0.0_%s" !kind
  | Sum [x] → eval_parameter' x
  | Sum (x :: xs) →
```

```
              printf "@,("; eval_parameter' x;
              List.iter (fun x → printf "@, + "; eval_parameter' x) xs;
              printf ")"
        | Diff (x, y) →
              printf "@,("; eval_parameter' x;
              printf " - "; eval_parameter' y; printf ")"
        | Neg x → printf "@,( - "; eval_parameter' x; printf ")"
        | Prod [] → printf "1.0_%s" !kind
        | Prod [x] → eval_parameter' x
        | Prod (x :: xs) →
              printf "@,("; eval_parameter' x;
              List.iter (fun x → printf " * "; eval_parameter' x) xs;
              printf ")"
        | Quot (x, y) →
              printf "@,("; eval_parameter' x;
              printf " / "; eval_parameter' y; printf ")"
        | Rec x →
              printf "@, (1.0_%s / " !kind; eval_parameter' x; printf ")"
        | Pow (x, n) →
              printf "@,("; eval_parameter' x; printf "**%d" n; printf ")"
        | Sqrt x → printf "@,sqrt ("; eval_parameter' x; printf ")"
        | Sin x → printf "@,sin ("; eval_parameter' x; printf ")"
        | Cos x → printf "@,cos ("; eval_parameter' x; printf ")"
        | Tan x → printf "@,tan ("; eval_parameter' x; printf ")"
        | Cot x → printf "@,cot ("; eval_parameter' x; printf ")"
        | Atan2 (y, x) → printf "@,atan2 ("; eval_parameter' y;
              printf ",@ "; eval_parameter' x; printf ")"
        | Conj x → printf "@,conjg ("; eval_parameter' x; printf ")"

let strip_single_tag = function
    | Real x → x
    | Complex x → x

let strip_array_tag = function
    | Real_Array x → x
    | Complex_Array x → x

let eval_parameter (lhs, rhs) =
    let x = CM.constant_symbol (strip_single_tag lhs) in
    printf "    @[<2>%s = " x; eval_parameter' rhs; nl ()

let eval_para_list n l =
    printf " subroutine setup_parameters%s ()" (string_of_int n); nl ();
    List.iter eval_parameter l;
    printf " end subroutine setup_parameters%s" (string_of_int n); nl ()

let eval_parameter_pair (lhs, rhs) =
    let x = CM.constant_symbol (strip_array_tag lhs) in
    let _ = List.fold_left (fun i rhs' →
        printf "    @[<2>%s(%d) = " x i; eval_parameter' rhs'; nl ();
        succ i) 1 rhs in
    ()
```

```
let eval_para_pair_list n l =
  printf " subroutine setup_parameters%s ()" (string_of_int n); nl ();
  List.iter eval_parameter_pair l;
  printf " end subroutine setup_parameters%s" (string_of_int n); nl ()

let print_echo fmt p =
  let s = CM.constant_symbol p in
  printf "     write (unit = *, fmt = fmt_%s) \"%s\", %s"
    fmt s s; nl ()

let print_echo_array fmt (p, n) =
  let s = CM.constant_symbol p in
  for i = 1 to n do
    printf "     write (unit = *, fmt = fmt_%s_array) " fmt ;
    printf "\"%s\", %d, %s(%d)" s i s i; nl ()
  done

let parameters_to_fortran oc params =
  setup_fortran_formatter !line_length oc;
  let declarations = classify_parameters params in
  printf "module %s" !parameter_module; nl ();
  printf "  use kinds"; nl ();
  printf "  use constants"; nl ();
  printf "  implicit none"; nl ();
  printf "  private"; nl ();
  printf "  @[<2>public :: setup_parameters";
  if !no_write then begin
    printf "! No print_parameters"; nl ();
  end else begin
    printf "@,, print_parameters"; nl ();
  end;
  declare_default_parameters "real" params.input;
  declare_parameters "real" (schisma 69 declarations.real_singles);
  List.iter (declare_parameter_array "real") declarations.real_arrays;
  declare_parameters "complex" (schisma 69 declarations.complex_singles);
  List.iter (declare_parameter_array "complex") declarations.complex_arrays;
  printf "contains"; nl ();
  printf "     ! derived parameters:"; nl ();
  let shredded = schisma_num 1 120 params.derived in
  let shredded_arrays = schisma_num 1 120 params.derived_arrays in
    let num_sub = List.length shredded in
    let num_sub_arrays = List.length shredded_arrays in
  printf "      !length: %s" (string_of_int (List.length params.derived));
    nl ();
  printf "      !Num_Sub: %s" (string_of_int num_sub); nl ();
  List.iter (fun (i, l) -> eval_para_list i l) shredded;
  List.iter (fun (i, l) -> eval_para_pair_list (num_sub + i) l)
    shredded_arrays;
  printf "  subroutine setup_parameters ()"; nl ();
  let sum_sub = num_sub + num_sub_arrays in
  for i = 1 to sum_sub do
    printf "     call setup_parameters%s" (string_of_int i); nl ();
```

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```
    done;
    printf "␣␣end␣subroutine␣setup_parameters"; nl ();
    if !no_write then begin
      printf "!␣No␣print_parameters"; nl ();
    end else begin
      printf "␣␣subroutine␣print_parameters␣()"; nl ();
      printf "␣␣␣␣@[<2>character(len=*),␣parameter␣::";
      printf "@␣fmt_real␣=␣\"(A12,4X,'␣=␣',E25.18)\",";
      printf "@␣fmt_complex␣=␣\"(A12,4X,'␣=␣',E25.18,'␣+␣i*',E25.18)\",";
      printf "@␣fmt_real_array␣=␣\"(A12,'(',I2.2,')','␣=␣',E25.18)\",";
      printf "@␣fmt_complex_array␣=␣";
      printf "\"(A12,'(',I2.2,')','␣=␣',E25.18,'␣+␣i*',E25.18)\""; nl ();
      printf "␣␣␣␣@[<2>write␣(unit␣=␣*,␣fmt␣=␣\"(A)\")␣@,";
      printf "\"default␣values␣for␣the␣input␣parameters:\""; nl ();
      List.iter (fun (p, _) → print_echo "real" p) params.input;
      printf "␣␣␣␣@[<2>write␣(unit␣=␣*,␣fmt␣=␣\"(A)\")␣@,";
      printf "\"derived␣parameters:\""; nl ();
      List.iter (print_echo "real") declarations.real_singles;
      List.iter (print_echo "complex") declarations.complex_singles;
      List.iter (print_echo_array "real") declarations.real_arrays;
      List.iter (print_echo_array "complex") declarations.complex_arrays;
      printf "␣␣end␣subroutine␣print_parameters"; nl ();
    end;
    printf "end␣module␣%s" !parameter_module; nl ();
    printf "!␣O'Mega␣revision␣control␣information:"; nl ();
    List.iter (fun s → printf "!␣␣␣␣%s" s; nl ())
      (ThoList.flatmap RCS.summary (M.rcs :: rcs_list));
    printf "!!!␣program␣test_parameters"; nl ();
    printf "!!!␣␣␣use␣%s" !parameter_module; nl ();
    printf "!!!␣␣␣call␣setup_parameters␣()"; nl ();
    printf "!!!␣␣␣call␣print_parameters␣()"; nl ();
    printf "!!!␣end␣program␣test_parameters"; nl ()
```

Run-Time Diagnostics

```
type diagnostic = All | Arguments | Momenta | Gauge

type diagnostic_mode = Off | Warn | Panic

let warn mode =
  match !mode with
  | Off → false
  | Warn → true
  | Panic → true

let panic mode =
  match !mode with
  | Off → false
  | Warn → false
  | Panic → true
```

```
let suffix mode =
  if panic mode then
    "panic"
  else
    "warn"

let diagnose_arguments = ref Off
let diagnose_momenta = ref Off
let diagnose_gauge = ref Off

let rec parse_diagnostic = function
  | All, panic →
      parse_diagnostic (Arguments, panic);
      parse_diagnostic (Momenta, panic);
      parse_diagnostic (Gauge, panic)
  | Arguments, panic →
      diagnose_arguments := if panic then Panic else Warn
  | Momenta, panic →
      diagnose_momenta := if panic then Panic else Warn
  | Gauge, panic →
      diagnose_gauge := if panic then Panic else Warn
```

If diagnostics are required, we have to switch off Fortran95 features like pure functions.

```
let parse_diagnostics = function
  | [] → ()
  | diagnostics →
      fortran95 := false;
      List.iter parse_diagnostic diagnostics
```

Amplitude

```
let declare_momenta_chunk = function
  | [] → ()
  | momenta →
      printf "␣␣␣␣@[<2>type(momentum)␣::␣";
      print_list (List.map format_momentum momenta); nl ()

let declare_momenta = function
  | [] → ()
  | momenta →
      List.iter
        declare_momenta_chunk
        (ThoList.chopn declaration_chunk_size momenta)

let declare_wavefunctions multiplicity wfs =
  let wfs' = classify_wfs wfs in
  declare_list multiplicity ("complex(kind=" ^ !kind ^ ")")
    (wfs'.scalars @ wfs'.brs_scalars);
  declare_list multiplicity ("type(" ^ Fermions.psi_type ^ ")")
    (wfs'.spinors @ wfs'.brs_spinors);
```

    declare_list multiplicity ("type(" ^ *Fermions.psibar_type* ^ ")")
      (*wfs'.conjspinors* @ *wfs'.brs_conjspinors*);
    declare_list multiplicity ("type(" ^ *Fermions.chi_type* ^ ")")
      (*wfs'.realspinors* @ *wfs'.brs_realspinors* @ *wfs'.ghostspinors*);
    declare_list multiplicity ("type(" ^ *Fermions.grav_type* ^ ")") *wfs'.vectorspinors*;
    declare_list multiplicity "type(vector)" (*wfs'.vectors* @ *wfs'.massive_vectors* @
      wfs'.brs_vectors @ *wfs'.brs_massive_vectors* @ *wfs'.ward_vectors*);
    declare_list multiplicity "type(tensor2odd)" *wfs'.tensors_1*;
    declare_list multiplicity "type(tensor)" *wfs'.tensors_2*

let *flavors a* = *F.incoming a* @ *F.outgoing a*

let *declare_brakets_chunk* = function
  | [] → ()
  | *amplitudes* →
      printf "␣␣␣␣@[<2>complex(kind=%s)␣::␣" !*kind*;
      print_list (*List.map* (fun *a* → *flavors_symbol* ~*decl* :true (*flavors a*)) *amplitudes*); *nl* ()

let *declare_brakets* = function
  | [] → ()
  | *amplitudes* →
    List.iter
      declare_brakets_chunk
      (*ThoList.chopn declaration_chunk_size amplitudes*)

let *print_variable_declarations amplitudes* =
  let *multiplicity* = *CF.multiplicity amplitudes*
  and *processes* = *CF.processes amplitudes* in
  declare_momenta
    (*PSet.elements*
      (*List.fold_left*
        (fun *set a* →
          PSet.union set (*List.fold_right*
                     (fun *wf* → *PSet.add* (*F.momentum_list wf*))
                     (*F.externals a*) *PSet.empty*))
        PSet.empty processes));
  declare_momenta
    (*PSet.elements*
      (*List.fold_left*
        (fun *set a* →
          PSet.union set (*List.fold_right*
                     (fun *wf* → *PSet.add* (*F.momentum_list wf*))
                     (*F.variables a*) *PSet.empty*))
        PSet.empty processes));
  if !*openmp* then begin
    printf "␣␣type␣%s@[<2>" *openmp_tld_type*;
    nl ();
  end ;
  declare_wavefunctions multiplicity
    (*WFSet.elements*
      (*List.fold_left*
        (fun *set a* →

```
              WFSet.union set (List.fold_right WFSet.add (F.externals a) WFSet.empty))
            WFSet.empty processes));
      declare_wavefunctions multiplicity
        (WFSet.elements
           (List.fold_left
              (fun set a →
                 WFSet.union set (List.fold_right WFSet.add (F.variables a) WFSet.empty))
              WFSet.empty processes));
      declare_brakets processes;
      if !openmp then begin
          printf "@]␣␣end␣type␣%s\n" openmp_tld_type;
          printf "␣␣type(%s)␣::␣%s" openmp_tld_type openmp_tld;
          nl ();
      end
```

print_current is the most important function that has to match the functions in
omega95 (see appendix X). It offers plentiful opportunities for making mistakes,
in particular those related to signs. We start with a few auxiliary functions:

```
let children2 rhs =
  match F.children rhs with
  | [wf1; wf2] → (wf1, wf2)
  | _ → failwith "Targets.children2:␣can't␣happen"

let children3 rhs =
  match F.children rhs with
  | [wf1; wf2; wf3] → (wf1, wf2, wf3)
  | _ → invalid_arg "Targets.children3:␣can't␣happen"
```

Note that it is (marginally) faster to multiply the two scalar products with the
coupling constant than the four vector components.

⬙ This could be part of omegalib as well . . .

```
let format_coeff = function
  | 1 → ""
  | − 1 → "-"
  | coeff → "(" ^ string_of_int coeff ^ ")*"

let format_coupling coeff c =
  match coeff with
  | 1 → c
  | − 1 → "(-" ^ c ^")"
  | coeff → string_of_int coeff ^ "*" ^ c
```

⬙ The following is error prone and should be generated automagically.

```
let print_vector4 c wf1 wf2 wf3 fusion (coeff, contraction) =
  match contraction, fusion with
  | C_12_34, (F341 | F431 | F342 | F432 | F123 | F213 | F124 | F214)
  | C_13_42, (F241 | F421 | F243 | F423 | F132 | F312 | F134 | F314)
  | C_14_23, (F231 | F321 | F234 | F324 | F142 | F412 | F143 |
F413) →
```

       printf "((%s%s)*(%s*%s))*%s" (*format_coeff coeff*) *c wf1 wf2 wf3*
    | *C_12_34,* (*F134* | *F143* | *F234* | *F243* | *F312* | *F321* | *F412* | *F421*)
    | *C_13_42,* (*F124* | *F142* | *F324* | *F342* | *F213* | *F231* | *F413* | *F431*)
    | *C_14_23,* (*F123* | *F132* | *F423* | *F432* | *F214* | *F241* | *F314* |
F341) →
        printf "((%s%s)*(%s*%s))*%s" (*format_coeff coeff*) *c wf2 wf3 wf1*
    | *C_12_34,* (*F314* | *F413* | *F324* | *F423* | *F132* | *F231* | *F142* | *F241*)
    | *C_13_42,* (*F214* | *F412* | *F234* | *F432* | *F123* | *F321* | *F143* | *F341*)
    | *C_14_23,* (*F213* | *F312* | *F243* | *F342* | *F124* | *F421* | *F134* |
F431) →
        printf "((%s%s)*(%s*%s))*%s" (*format_coeff coeff*) *c wf1 wf3 wf2*

  let *print_add_vector4 c wf1 wf2 wf3 fusion* (*coeff, contraction*) =
    printf "@␣+␣";
    print_vector4 c wf1 wf2 wf3 fusion (*coeff, contraction*)

  let *print_vector4_km c pa pb wf1 wf2 wf3 fusion* (*coeff, contraction*) =
    match *contraction, fusion* with
    | *C_12_34,* (*F341* | *F431* | *F342* | *F432* | *F123* | *F213* | *F124* | *F214*)
    | *C_13_42,* (*F241* | *F421* | *F243* | *F423* | *F132* | *F312* | *F134* | *F314*)
    | *C_14_23,* (*F231* | *F321* | *F234* | *F324* | *F142* | *F412* | *F143* |
F413) →
      printf "((%s%s%s+%s))*(%s*%s))*%s"
       (*format_coeff coeff*) *c pa pb wf1 wf2 wf3*
    | *C_12_34,* (*F134* | *F143* | *F234* | *F243* | *F312* | *F321* | *F412* | *F421*)
    | *C_13_42,* (*F124* | *F142* | *F324* | *F342* | *F213* | *F231* | *F413* | *F431*)
    | *C_14_23,* (*F123* | *F132* | *F423* | *F432* | *F214* | *F241* | *F314* |
F341) →
      printf "((%s%s%s+%s))*(%s*%s))*%s"
       (*format_coeff coeff*) *c pa pb wf2 wf3 wf1*
    | *C_12_34,* (*F314* | *F413* | *F324* | *F423* | *F132* | *F231* | *F142* | *F241*)
    | *C_13_42,* (*F214* | *F412* | *F234* | *F432* | *F123* | *F321* | *F143* | *F341*)
    | *C_14_23,* (*F213* | *F312* | *F243* | *F342* | *F124* | *F421* | *F134* |
F431) →
      printf "((%s%s%s+%s))*(%s*%s))*%s"
       (*format_coeff coeff*) *c pa pb wf1 wf3 wf2*

  let *print_add_vector4_km c pa pb wf1 wf2 wf3 fusion* (*coeff, contraction*) =
    printf "@␣+␣";
    print_vector4_km c pa pb wf1 wf2 wf3 fusion (*coeff, contraction*)

  let *print_dscalar4 c wf1 wf2 wf3 p1 p2 p3 p123*
     fusion (*coeff, contraction*) =
    match *contraction, fusion* with
    | *C_12_34,* (*F341* | *F431* | *F342* | *F432* | *F123* | *F213* | *F124* | *F214*)
    | *C_13_42,* (*F241* | *F421* | *F243* | *F423* | *F132* | *F312* | *F134* | *F314*)
    | *C_14_23,* (*F231* | *F321* | *F234* | *F324* | *F142* | *F412* | *F143* |
F413) →
      printf "((%s%s)*(%s*%s)*(%s*%s)*%s*%s*%s)"
       (*format_coeff coeff*) *c p1 p2 p3 p123 wf1 wf2 wf3*
    | *C_12_34,* (*F134* | *F143* | *F234* | *F243* | *F312* | *F321* | *F412* | *F421*)
    | *C_13_42,* (*F124* | *F142* | *F324* | *F342* | *F213* | *F231* | *F413* | *F431*)

```
        |   C_14_23, (F123 | F132 | F423 | F432 | F214 | F241 | F314 |
F341) →
              printf "((%s%s)*(%s*%s)*(%s*%s)*%s*%s*%s)"
                (format_coeff coeff) c p2 p3 p1 p123 wf1 wf2 wf3
        |   C_12_34, (F314 | F413 | F324 | F423 | F132 | F231 | F142 | F241)
        |   C_13_42, (F214 | F412 | F234 | F432 | F123 | F321 | F143 | F341)
        |   C_14_23, (F213 | F312 | F243 | F342 | F124 | F421 | F134 |
F431) →
              printf "((%s%s)*(%s*%s)*(%s*%s)*%s*%s*%s)"
                (format_coeff coeff) c p1 p3 p2 p123 wf1 wf2 wf3

    let print_add_dscalar4 c wf1 wf2 wf3 p1 p2 p3 p123
        fusion (coeff, contraction) =
      printf "@␣+␣";
      print_dscalar4 c wf1 wf2 wf3 p1 p2 p3 p123 fusion (coeff, contraction)

    let print_dscalar2_vector2 c wf1 wf2 wf3 p1 p2 p3 p123 fusion (coeff, contraction) =
      match contraction, fusion with
      |  C_12_34, (F123 | F213 | F124 | F214) →
          printf "(%s%s)*(%s*%s)*(%s*%s)*%s"
            (format_coeff coeff) c p1 p2 wf1 wf2 wf3
      |  C_12_34, (F134 | F143 | F234 | F243) →
          printf "(%s%s)*(%s*%s)*(%s*%s)*%s"
            (format_coeff coeff) c p1 p123 wf2 wf3 wf1
      |  C_12_34, (F132 | F231 | F142 | F241) →
          printf "(%s%s)*(%s*%s)*(%s*%s)*%s"
            (format_coeff coeff) c p1 p3 wf1 wf3 wf2
      |  C_12_34, (F312 | F321 | F412 | F421) →
          printf "(%s%s)*(%s*%s)*(%s*%s)*%s"
            (format_coeff coeff) c p2 p3 wf2 wf3 wf1
      |  C_12_34, (F314 | F413 | F324 | F423) →
          printf "(%s%s)*(%s*%s)*(%s*%s)*%s"
            (format_coeff coeff) c p2 p123 wf1 wf3 wf2
      |  C_12_34, (F341 | F431 | F342 | F432) →
          printf "(%s%s)*(%s*%s)*(%s*%s)*%s"
            (format_coeff coeff) c p3 p123 wf1 wf2 wf3
      |  C_13_42, (F123 | F214)
      |  C_14_23, (F124 | F213) →
          printf "((%s%s)*(%s*%s*%s)*%s*%s)"
            (format_coeff coeff) c wf1 p1 wf3 wf2 p2
      |  C_13_42, (F124 | F213)
      |  C_14_23, (F123 | F214) →
          printf "((%s%s)*(%s*%s*%s)*%s*%s)"
            (format_coeff coeff) c wf2 p2 wf3 wf1 p1
      |  C_13_42, (F132 | F241)
      |  C_14_23, (F142 | F231) →
          printf "((%s%s)*(%s*%s*%s)*%s*%s)"
            (format_coeff coeff) c wf1 p1 wf2 wf3 p3
      |  C_13_42, (F142 | F231)
      |  C_14_23, (F132 | F241) →
          printf "((%s%s)*(%s*%s*%s)*%s*%s)"
```

```
        (format_coeff coeff) c wf3 p3 wf2 wf1 p1
    | C_13_42, (F312 | F421)
    | C_14_23, (F412 | F321) →
        printf "((%s%s)*(%s*%s*%s)*%s*%s)"
          (format_coeff coeff) c wf2 p2 wf1 wf3 p3
    | C_13_42, (F321 | F412)
    | C_14_23, (F421 | F312) →
        printf "((%s%s)*(%s*%s*%s)*%s*%s)"
          (format_coeff coeff) c wf3 p3 wf1 wf2 p2
    | C_13_42, (F134 | F243)
    | C_14_23, (F143 | F234) →
        printf "((%s%s)*(%s*%s)*(%s*%s*%s))"
          (format_coeff coeff) c wf3 p123 wf1 p1 wf2
    | C_13_42, (F143 | F234)
    | C_14_23, (F134 | F243) →
        printf "((%s%s)*(%s*%s)*(%s*%s*%s))"
          (format_coeff coeff) c wf2 p123 wf1 p1 wf3
    | C_13_42, (F314 | F423)
    | C_14_23, (F413 | F324) →
        printf "((%s%s)*(%s*%s)*(%s*%s*%s))"
          (format_coeff coeff) c wf3 p123 wf2 p2 wf1
    | C_13_42, (F324 | F413)
    | C_14_23, (F423 | F314) →
        printf "((%s%s)*(%s*%s)*(%s*%s*%s))"
          (format_coeff coeff) c wf1 p123 wf2 p2 wf3
    | C_13_42, (F341 | F432)
    | C_14_23, (F431 | F342) →
        printf "((%s%s)*(%s*%s)*(%s*%s*%s))"
          (format_coeff coeff) c wf2 p123 wf3 p3 wf1
    | C_13_42, (F342 | F431)
    | C_14_23, (F432 | F341) →
        printf "((%s%s)*(%s*%s)*(%s*%s*%s))"
          (format_coeff coeff) c wf1 p123 wf3 p3 wf2

let print_add_dscalar2_vector2 c wf1 wf2 wf3 p1 p2 p3 p123
    fusion (coeff, contraction) =
  printf "@ + ";
  print_dscalar2_vector2 c wf1 wf2 wf3 p1 p2 p3 p123
    fusion (coeff, contraction)

let print_dscalar2_vector2_km c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion (coeff, contraction) =
  match contraction, fusion with
  | C_12_34, (F123 | F213 | F124 | F214) →
      printf "(%s%s%s+%s))*(%s*%s)*(%s*%s)*%s"
        (format_coeff coeff) c pa pb p1 p2 wf1 wf2 wf3
  | C_12_34, (F134 | F143 | F234 | F243) →
      printf "(%s%s%s+%s))*(%s*%s)*(%s*%s)*%s"
        (format_coeff coeff) c pa pb p1 p123 wf2 wf3 wf1
  | C_12_34, (F132 | F231 | F142 | F241) →
      printf "(%s%s%s+%s))*(%s*%s)*(%s*%s)*%s"
        (format_coeff coeff) c pa pb p1 p3 wf1 wf3 wf2
```

```
| C_12_34, (F312 | F321 | F412 | F421) →
    printf "(%s%s%s+%s))*(%s*%s)*(%s*%s)*%s"
      (format_coeff coeff) c pa pb p2 p3 wf2 wf3 wf1
| C_12_34, (F314 | F413 | F324 | F423) →
    printf "(%s%s%s+%s))*(%s*%s)*(%s*%s)*%s"
      (format_coeff coeff) c pa pb p2 p123 wf1 wf3 wf2
| C_12_34, (F341 | F431 | F342 | F432) →
    printf "(%s%s%s+%s))*(%s*%s)*(%s*%s)*%s"
      (format_coeff coeff) c pa pb p3 p123 wf1 wf2 wf3
| C_13_42, (F123 | F214)
| C_14_23, (F124 | F213) →
    printf "((%s%s%s+%s))*(%s*%s*%s)*%s*%s)"
      (format_coeff coeff) c pa pb wf1 p1 wf3 wf2 p2
| C_13_42, (F124 | F213)
| C_14_23, (F123 | F214) →
    printf "((%s%s%s+%s))*(%s*%s*%s)*%s*%s)"
      (format_coeff coeff) c pa pb wf2 p2 wf3 wf1 p1
| C_13_42, (F132 | F241)
| C_14_23, (F142 | F231) →
    printf "((%s%s%s+%s))*(%s*%s*%s)*%s*%s)"
      (format_coeff coeff) c pa pb wf1 p1 wf2 wf3 p3
| C_13_42, (F142 | F231)
| C_14_23, (F132 | F241) →
    printf "((%s%s%s+%s))*(%s*%s*%s)*%s*%s)"
      (format_coeff coeff) c pa pb wf3 p3 wf2 wf1 p1
| C_13_42, (F312 | F421)
| C_14_23, (F412 | F321) →
    printf "((%s%s%s+%s))*(%s*%s*%s)*%s*%s)"
      (format_coeff coeff) c pa pb wf2 p2 wf1 wf3 p3
| C_13_42, (F321 | F412)
| C_14_23, (F421 | F312) →
    printf "((%s%s%s+%s))*(%s*%s*%s)*%s*%s)"
      (format_coeff coeff) c pa pb wf3 p3 wf1 wf2 p2
| C_13_42, (F134 | F243)
| C_14_23, (F143 | F234) →
    printf "((%s%s%s+%s))*(%s*%s)*(%s*%s*%s))"
      (format_coeff coeff) c pa pb wf3 p123 wf1 p1 wf2
| C_13_42, (F143 | F234)
| C_14_23, (F134 | F243) →
    printf "((%s%s%s+%s))*(%s*%s)*(%s*%s*%s))"
      (format_coeff coeff) c pa pb wf2 p123 wf1 p1 wf3
| C_13_42, (F314 | F423)
| C_14_23, (F413 | F324) →
    printf "((%s%s%s+%s))*(%s*%s)*(%s*%s*%s))"
      (format_coeff coeff) c pa pb wf3 p123 wf2 p2 wf1
| C_13_42, (F324 | F413)
| C_14_23, (F423 | F314) →
    printf "((%s%s%s+%s))*(%s*%s)*(%s*%s*%s))"
      (format_coeff coeff) c pa pb wf1 p123 wf2 p2 wf3
| C_13_42, (F341 | F432)
```

```
        |  C_14_23, (F431 | F342)  →
            printf "((%s%s%s+%s))*(%s*%s)*(%s*%s*%s))"
               (format_coeff coeff) c pa pb wf2 p123 wf3 p3 wf1
        |  C_13_42, (F342 | F431)
        |  C_14_23, (F432 | F341)  →
            printf "((%s%s%s+%s))*(%s*%s)*(%s*%s*%s))"
               (format_coeff coeff) c pa pb wf1 p123 wf3 p3 wf2
```

let *print_add_dscalar2_vector2_km c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion* (*coeff*, *contraction*) =
printf "@␣+␣";
print_dscalar2_vector2_km c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion (*coeff*, *contraction*)

let *print_dscalar4_km c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion* (*coeff*, *contraction*) =
match *contraction*, *fusion* with

```
        |  C_12_34, (F341 | F431 | F342 | F432 | F123 | F213 | F124 | F214)
        |  C_13_42, (F241 | F421 | F243 | F423 | F132 | F312 | F134 | F314)
        |  C_14_23, (F231 | F321 | F234 | F324 | F142 | F412 | F143 |
F413)  →
            printf "((%s%s%s+%s))*(%s*%s)*(%s*%s)*%s*%s*%s)"
               (format_coeff coeff) c pa pb p1 p2 p3 p123 wf1 wf2 wf3
        |  C_12_34, (F134 | F143 | F234 | F243 | F312 | F321 | F412 | F421)
        |  C_13_42, (F124 | F142 | F324 | F342 | F213 | F231 | F413 | F431)
        |  C_14_23, (F123 | F132 | F423 | F432 | F214 | F241 | F314 |
F341)  →
            printf "((%s%s%s+%s))*(%s*%s)*(%s*%s)*%s*%s*%s)"
               (format_coeff coeff) c pa pb p2 p3 p1 p123 wf1 wf2 wf3
        |  C_12_34, (F314 | F413 | F324 | F423 | F132 | F231 | F142 | F241)
        |  C_13_42, (F214 | F412 | F234 | F432 | F123 | F321 | F143 | F341)
        |  C_14_23, (F213 | F312 | F243 | F342 | F124 | F421 | F134 |
F431)  →
            printf "((%s%s%s+%s))*(%s*%s)*(%s*%s)*%s*%s*%s)"
               (format_coeff coeff) c pa pb p1 p3 p2 p123 wf1 wf2 wf3
```

let *print_add_dscalar4_km c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion* (*coeff*, *contraction*) =
printf "@␣+␣";
print_dscalar4_km c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion (*coeff*, *contraction*)

let *print_current amplitude dictionary rhs* =
match *F.coupling rhs* with
| *V3* (*vertex*, *fusion*, *constant*) →
    let *ch1*, *ch2* = *children2 rhs* in
    let *wf1* = *multiple_variable amplitude dictionary ch1*
    and *wf2* = *multiple_variable amplitude dictionary ch2*
    and *p1* = *momentum ch1*
    and *p2* = *momentum ch2*
    and *m1* = *CM.mass_symbol* (*F.flavor ch1*)
    and *m2* = *CM.mass_symbol* (*F.flavor ch2*) in
    let *c* = *CM.constant_symbol constant* in
    printf "@,␣%s␣" (if (*F.sign rhs*) < 0 then "-" else "+");
    begin match *vertex* with

Fermionic currents $\bar{\psi} A\!\!\!/ \psi$ and $\bar{\psi} \phi \psi$ are handled by the *Fermions* module, since
they depend on the choice of Feynman rules: Dirac or Majorana.

```
| FBF (coeff, fb, b, f) →
    begin match coeff, fb, b, f with
    | _, Psibar, VLRM, Psi  | _, Psibar, SPM, Psi
    | _, Psibar, VAM, Psi  | _, Psibar, VA3M, Psi
    | _, Psibar, TVA, Psi  | _, Psibar, TVAM, Psi
    | _, Psibar, TLR, Psi  | _, Psibar, TLRM, Psi
    | _, Psibar, TRL, Psi  | _, Psibar, TRLM, Psi →
        let p12 = Printf.sprintf "(-%s-%s)" p1 p2 in
        Fermions.print_current_mom (coeff, fb, b, f) c wf1 wf2 p1 p2
            p12 fusion
    | _, _, _, _ →
        Fermions.print_current (coeff, fb, b, f) c wf1 wf2 fusion
    end
| PBP (coeff, f1, b, f2) →
    Fermions.print_current_p (coeff, f1, b, f2) c wf1 wf2 fusion
| BBB (coeff, fb1, b, fb2) →
    Fermions.print_current_b (coeff, fb1, b, fb2) c wf1 wf2 fusion
| GBG (coeff, fb, b, f) → let p12 =
    Printf.sprintf "(-%s-%s)" p1 p2 in
    Fermions.print_current_g (coeff, fb, b, f) c wf1 wf2 p1 p2
        p12 fusion
```

Table 9.13 is a bit misleading, since if includes totally antisymmetric structure constants. The space-time part alone is also totally antisymmetric:

```
| Gauge_Gauge_Gauge coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | (F23 | F31 | F12) →
        printf "g_gg(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | (F32 | F13 | F21) →
        printf "g_gg(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    end
```

In *Aux_Gauge_Gauge*, we can not rely on antisymmetry alone, because of the different Lorentz representations of the auxialiary and the gauge field. Instead we have to provide the sign in

$$(V_2 \wedge V_3) \cdot T_1 = \begin{cases} V_2 \cdot (T_1 \cdot V_3) = -V_2 \cdot (V_3 \cdot T_1) \\ V_3 \cdot (V_2 \cdot T_1) = -V_3 \cdot (T_1 \cdot V_2) \end{cases} \tag{15.2}$$

ourselves. Alternatively, one could provide `g_xg` mirroring `g_gx`.

```
| Aux_Gauge_Gauge coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | F23 → printf "x_gg(%s,%s,%s)" c wf1 wf2
    | F32 → printf "x_gg(%s,%s,%s)" c wf2 wf1
    | F12 → printf "g_gx(%s,%s,%s)" c wf2 wf1
    | F21 → printf "g_gx(%s,%s,%s)" c wf1 wf2
    | F13 → printf "(-1)*g_gx(%s,%s,%s)" c wf2 wf1
    | F31 → printf "(-1)*g_gx(%s,%s,%s)" c wf1 wf2
```

           end

These cases are symmetric and we just have to juxtapose the correct fields and
provide parentheses to minimize the number of multiplications.

       | *Scalar_Vector_Vector coeff* →
         let *c* = *format_coupling coeff c* in
         begin match *fusion* with
         | (*F23* | *F32*) → *printf* `"%s*(%s*%s)"` *c wf1 wf2*
         | (*F12* | *F13*) → *printf* `"(%s*%s)*%s"` *c wf1 wf2*
         | (*F21* | *F31*) → *printf* `"(%s*%s)*%s"` *c wf2 wf1*
         end

       | *Aux_Vector_Vector coeff* →
         let *c* = *format_coupling coeff c* in
         begin match *fusion* with
         | (*F23* | *F32*) → *printf* `"%s*(%s*%s)"` *c wf1 wf2*
         | (*F12* | *F13*) → *printf* `"(%s*%s)*%s"` *c wf1 wf2*
         | (*F21* | *F31*) → *printf* `"(%s*%s)*%s"` *c wf2 wf1*
         end

Even simpler:

       | *Scalar_Scalar_Scalar coeff* →
         printf `"(%s*%s*%s)"` (*format_coupling coeff c*) *wf1 wf2*

       | *Aux_Scalar_Scalar coeff* →
         printf `"(%s*%s*%s)"` (*format_coupling coeff c*) *wf1 wf2*

       | *Aux_Scalar_Vector coeff* →
         let *c* = *format_coupling coeff c* in
         begin match *fusion* with
         | (*F13* | *F31*) → *printf* `"%s*(%s*%s)"` *c wf1 wf2*
         | (*F23* | *F21*) → *printf* `"(%s*%s)*%s"` *c wf1 wf2*
         | (*F32* | *F12*) → *printf* `"(%s*%s)*%s"` *c wf2 wf1*
         end

       | *Vector_Scalar_Scalar coeff* →
         let *c* = *format_coupling coeff c* in
         begin match *fusion* with
         | *F23* → *printf* `"v_ss(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
         | *F32* → *printf* `"v_ss(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
         | *F12* → *printf* `"s_vs(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
         | *F21* → *printf* `"s_vs(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
         | *F13* → *printf* `"(-1)*s_vs(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
         | *F31* → *printf* `"(-1)*s_vs(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
         end

       | *Graviton_Scalar_Scalar coeff* →
         let *c* = *format_coupling coeff c* in
         begin match *fusion* with
         | *F12* → *printf* `"s_gravs(%s,%s,-(%s+%s),%s,%s,%s)"` *c m2 p1 p2 p2 wf1 wf2*
         | *F21* → *printf* `"s_gravs(%s,%s,-(%s+%s),%s,%s,%s)"` *c m1 p1 p2 p1 wf2 wf1*
         | *F13* → *printf* `"s_gravs(%s,%s,%s,-(%s+%s),%s,%s)"` *c m2 p2 p1 p2 wf1 wf2*
         | *F31* → *printf* `"s_gravs(%s,%s,%s,-(%s+%s),%s,%s)"` *c m1 p1 p1 p2 wf2 wf1*

      | *F23* → *printf* `"grav_ss(%s,%s,%s,%s,%s,%s)"` *c m1 p1 p2 wf1 wf2*
      | *F32* → *printf* `"grav_ss(%s,%s,%s,%s,%s,%s)"` *c m1 p2 p1 wf2 wf1*
     end

In producing a vector in the fusion we always contract the rightmost index with the vector wavefunction from *rhs*. So the first momentum is always the one of the vector boson produced in the fusion, while the second one is that from the *rhs*. This makes the cases *F12* and *F13* as well as *F21* and *F31* equal. In principle, we could have already done this for the *Graviton_Scalar_Scalar* case.

    | *Graviton_Vector_Vector coeff* →
      let *c* = *format_coupling coeff c* in
      begin match *fusion* with
      | (*F12* | *F13*) → *printf* `"v_gravv(%s,%s,-(%s+%s),%s,%s,%s)"` *c m2 p1 p2 p2 wf1 wf2*
      | (*F21* | *F31*) → *printf* `"v_gravv(%s,%s,-(%s+%s),%s,%s,%s)"` *c m1 p1 p2 p1 wf2 wf1*
      | *F23* → *printf* `"grav_vv(%s,%s,%s,%s,%s,%s)"` *c m1 p1 p2 wf1 wf2*
      | *F32* → *printf* `"grav_vv(%s,%s,%s,%s,%s,%s)"` *c m1 p2 p1 wf2 wf1*
     end

    | *Graviton_Spinor_Spinor coeff* →
      let *c* = *format_coupling coeff c* in
      begin match *fusion* with
      | *F23* → *printf* `"f_gravf(%s,%s,-(%s+%s),(-%s),%s,%s)"` *c m2 p1 p2 p2 wf1 wf2*
      | *F32* → *printf* `"f_gravf(%s,%s,-(%s+%s),(-%s),%s,%s)"` *c m1 p1 p2 p1 wf2 wf1*
      | *F12* → *printf* `"f_fgrav(%s,%s,%s,%s+%s,%s,%s)"` *c m1 p1 p1 p2 wf1 wf2*
      | *F21* → *printf* `"f_fgrav(%s,%s,%s,%s+%s,%s,%s)"` *c m2 p2 p1 p2 wf2 wf1*
      | *F13* → *printf* `"grav_ff(%s,%s,%s,(-%s),%s,%s)"` *c m1 p1 p2 wf1 wf2*
      | *F31* → *printf* `"grav_ff(%s,%s,%s,(-%s),%s,%s)"` *c m1 p2 p1 wf2 wf1*
     end

    | *Dim4_Vector_Vector_Vector_T coeff* →
      let *c* = *format_coupling coeff c* in
      begin match *fusion* with
      | *F23* → *printf* `"tkv_vv(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
      | *F32* → *printf* `"tkv_vv(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
      | *F12* → *printf* `"tv_kvv(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
      | *F21* → *printf* `"tv_kvv(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
      | *F13* → *printf* `"(-1)*tv_kvv(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
      | *F31* → *printf* `"(-1)*tv_kvv(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
     end

    | *Dim4_Vector_Vector_Vector_L coeff* →
      let *c* = *format_coupling coeff c* in
      begin match *fusion* with
      | *F23* → *printf* `"lkv_vv(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
      | *F32* → *printf* `"lkv_vv(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
      | *F12* | *F13* → *printf* `"lv_kvv(%s,%s,%s,%s)"` *c wf1 p1 wf2*
      | *F21* | *F31* → *printf* `"lv_kvv(%s,%s,%s,%s)"` *c wf2 p2 wf1*
     end

    | *Dim6_Gauge_Gauge_Gauge coeff* →
      let *c* = *format_coupling coeff c* in
      begin match *fusion* with

        | *F23* | *F31* | *F12* →
          printf `"kg_kgkg(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
        | *F32* | *F13* | *F21* →
          printf `"kg_kgkg(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
      end

| *Dim4_Vector_Vector_Vector_T5 coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | *F23* → *printf* `"t5kv_vv(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *F32* → *printf* `"t5kv_vv(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    | *F12* | *F13* → *printf* `"t5v_kvv(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *F21* | *F31* → *printf* `"t5v_kvv(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    end

| *Dim4_Vector_Vector_Vector_L5 coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | *F23* → *printf* `"l5kv_vv(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *F32* → *printf* `"l5kv_vv(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    | *F12* → *printf* `"l5v_kvv(%s,%s,%s,%s)"` *c wf1 p1 wf2*
    | *F21* → *printf* `"l5v_kvv(%s,%s,%s,%s)"` *c wf2 p2 wf1*
    | *F13* → *printf* `"(-1)*l5v_kvv(%s,%s,%s,%s)"` *c wf1 p1 wf2*
    | *F31* → *printf* `"(-1)*l5v_kvv(%s,%s,%s,%s)"` *c wf2 p2 wf1*
    end

| *Dim6_Gauge_Gauge_Gauge_5 coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | *F23* → *printf* `"kg5_kgkg(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *F32* → *printf* `"kg5_kgkg(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    | *F12* → *printf* `"kg_kg5kg(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *F21* → *printf* `"kg_kg5kg(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    | *F13* → *printf* `"(-1)*kg_kg5kg(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *F31* → *printf* `"(-1)*kg_kg5kg(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    end

| *Aux_DScalar_DScalar coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) →
      printf `"%s*(%s*%s)*(%s*%s)"` *c p1 p2 wf1 wf2*
    | (*F12* | *F13*) →
      printf `"%s*(-((%s+%s)*%s))*(%s*%s)"` *c p1 p2 p2 wf1 wf2*
    | (*F21* | *F31*) →
      printf `"%s*(-((%s+%s)*%s))*(%s*%s)"` *c p1 p2 p1 wf1 wf2*
    end

| *Aux_Vector_DScalar coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | *F23* → *printf* `"%s*(%s*%s)*%s"` *c wf1 p2 wf2*
    | *F32* → *printf* `"%s*(%s*%s)*%s"` *c wf2 p1 wf1*

   | *F12* → *printf* "%s*(-((%s+%s)*%s))*%s" *c p1 p2 wf2 wf1*
   | *F21* → *printf* "%s*(-((%s+%s)*%s))*%s" *c p1 p2 wf1 wf2*
   | (*F13* | *F31*) → *printf* "(-(%s+%s))*(%s*%s*%s)" *p1 p2 c wf1 wf2*
   end

 | *Dim5_Scalar_Gauge2 coeff* →
   let *c* = *format_coupling coeff c* in
   begin match *fusion* with
   | (*F23* | *F32*) → *printf* "(%s)*((%s*%s)*(%s*%s)␣-␣(%s*%s)*(%s*%s))"
     c p1 wf2 p2 wf1 p1 p2 wf2 wf1
   | (*F12* | *F13*) → *printf* "(%s)*%s*((-((%s+%s)*%s))*%s␣-␣((-(%s+%s)*%s))*%s)"
     c wf1 p1 p2 wf2 p2 p1 p2 p2 wf2
   | (*F21* | *F31*) → *printf* "(%s)*%s*((-((%s+%s)*%s))*%s␣-␣((-(%s+%s)*%s))*%s)"
     c wf2 p2 p1 wf1 p1 p1 p2 p1 wf1
   end

 | *Dim5_Scalar_Gauge2_Skew coeff* →
   let *c* = *format_coupling coeff c* in
   begin match *fusion* with
   | (*F23* | *F32*) → *printf* "(-␣phi_vv␣(%s,␣%s,␣%s,␣%s,␣%s))" *c p1 p2 wf1 wf2*
   | (*F12* | *F13*) → *printf* "(-␣v_phiv␣(%s,␣%s,␣%s,␣%s,␣%s))" *c wf1 p1 p2 wf2*
   | (*F21* | *F31*) → *printf* "v_phiv␣(%s,␣%s,␣%s,␣%s,␣%s)" *c wf2 p1 p2 wf1*
   end

 | *Dim5_Scalar_Vector_Vector_T coeff* →
   let *c* = *format_coupling coeff c* in
   begin match *fusion* with
   | (*F23* | *F32*) → *printf* "(%s)*(%s*%s)*(%s*%s)" *c p1 wf2 p2 wf1*
   | (*F12* | *F13*) → *printf* "(%s)*%s*(-((%s+%s)*%s))*%s" *c wf1 p1 p2 wf2 p2*
   | (*F21* | *F31*) → *printf* "(%s)*%s*(-((%s+%s)*%s))*%s" *c wf2 p2 p1 wf1 p1*
   end

 | *Dim5_Scalar_Vector_Vector_U coeff* →
   let *c* = *format_coupling coeff c* in
   begin match *fusion* with
   | (*F23* | *F32*) → *printf* "phi_u_vv␣(%s,␣%s,␣%s,␣%s,␣%s)" *c p1 p2 wf1 wf2*
   | (*F12* | *F13*) → *printf* "v_u_phiv␣(%s,␣%s,␣%s,␣%s,␣%s)" *c wf1 p1 p2 wf2*
   | (*F21* | *F31*) → *printf* "v_u_phiv␣(%s,␣%s,␣%s,␣%s,␣%s)" *c wf2 p2 p1 wf1*
   end

 | *Dim5_Scalar_Vector_Vector_TU coeff* →
   let *c* = *format_coupling coeff c* in
   begin match *fusion* with
   | *F23* → *printf* "(%s)*((%s*%s)*(-(%s+%s)*%s)␣-␣(-(%s+%s)*%s)*(%s*%s))"
     c p1 wf2 p1 p2 wf1 p1 p2 p1 wf1 wf2
   | *F32* → *printf* "(%s)*((%s*%s)*(-(%s+%s)*%s)␣-␣(-(%s+%s)*%s)*(%s*%s))"
     c p2 wf1 p1 p2 wf2 p1 p2 p2 wf1 wf2
   | *F12* → *printf* "(%s)*%s*((%s*%s)*%s␣-␣(%s*%s)*%s)"
     c wf1 p1 wf2 p2 p1 p2 wf2
   | *F21* → *printf* "(%s)*%s*((%s*%s)*%s␣-␣(%s*%s)*%s)"
     c wf2 p2 wf1 p1 p1 p2 wf1
   | *F13* → *printf* "(%s)*%s*((-(%s+%s)*%s)*%s␣-␣(-(%s+%s)*%s)*%s)"
     c wf1 p1 p2 wf2 p1 p1 p2 p1 wf2

    | *F31* → *printf* `"(%s)*%s*((-(%s+%s)*%s)*%s␣-␣(-(%s+%s)*%s)*%s)"`
      c wf2 p1 p2 wf1 p2 p1 p2 p2 wf1
    end

  | *Dim5_Scalar_Scalar2 coeff* →
    let *c = format_coupling coeff c* in
    begin match *fusion* with
    | *(F23 | F32)* → *printf* `"phi_dim5s2(%s,␣%s␣,%s,␣%s,␣%s)"`
      c wf1 p1 wf2 p2
    | *(F12 | F13)* → let *p12 = Printf.sprintf* `"(-%s-%s)"` *p1 p2* in
      printf `"phi_dim5s2(%s,%s,%s,%s,%s)"` *c wf1 p12 wf2 p2*
    | *(F21 | F31)* → let *p12 = Printf.sprintf* `"(-%s-%s)"` *p1 p2* in
      printf `"phi_dim5s2(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p12*
    end

  | *Scalar_Vector_Vector_t coeff* →
    let *c = format_coupling coeff c* in
    begin match *fusion* with
    | *(F23 | F32)* → *printf* `"s_vv_t(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *(F12 | F13)* → *printf* `"v_sv_t(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *(F21 | F31)* → *printf* `"v_sv_t(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    end

  | *Dim6_Vector_Vector_Vector_T coeff* →
    let *c = format_coupling coeff c* in
    begin match *fusion* with
    | *F23* → *printf* `"(%s)*(%s*%s)*(%s*%s)*(%s-%s)"` *c p2 wf1 p1 wf2 p1 p2*
    | *F32* → *printf* `"(%s)*(%s*%s)*(%s*%s)*(%s-%s)"` *c p1 wf2 p2 wf1 p2 p1*
    | *(F12 | F13)* → *printf* `"(%s)*((%s+2*%s)*%s)*(-((%s+%s)*%s))*%s"`
      c p1 p2 wf1 p1 p2 wf2 p2
    | *(F21 | F31)* → *printf* `"(%s)*((-((%s+%s)*%s))*(%s+2*%s)*%s)*%s"`
      c p2 p1 wf1 p2 p1 wf2 p1
    end

  | *Tensor_2_Vector_Vector coeff* →
    let *c = format_coupling coeff c* in
    begin match *fusion* with
    | *(F23 | F32)* → *printf* `"t2_vv(%s,%s,%s)"` *c wf1 wf2*
    | *(F12 | F13)* → *printf* `"v_t2v(%s,%s,%s)"` *c wf1 wf2*
    | *(F21 | F31)* → *printf* `"v_t2v(%s,%s,%s)"` *c wf2 wf1*
    end

  | *Tensor_2_Scalar_Scalar coeff* →
    let *c = format_coupling coeff c* in
    begin match *fusion* with
    | *(F23 | F32)* → *printf* `"t2_phi2(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *(F12 | F13)* → *printf* `"phi_t2phi(%s,%s,%s,%s,%s)"` *c wf1 p1 wf2 p2*
    | *(F21 | F31)* → *printf* `"phi_t2phi(%s,%s,%s,%s,%s)"` *c wf2 p2 wf1 p1*
    end

  | *Tensor_2_Vector_Vector_1 coeff* →
    let *c = format_coupling coeff c* in
    begin match *fusion* with

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            | (F23 | F32) →  printf "t2_vv_1(%s,%s,%s)" c wf1 wf2
            | (F12 | F13) →  printf "v_t2v_1(%s,%s,%s)" c wf1 wf2
            | (F21 | F31) →  printf "v_t2v_1(%s,%s,%s)" c wf2 wf1
            end

    | Tensor_2_Vector_Vector_cf coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | (F23 | F32) →  printf "t2_vv_cf(%s,%s,%s)" c wf1 wf2
        | (F12 | F13) →  printf "v_t2v_cf(%s,%s,%s)" c wf1 wf2
        | (F21 | F31) →  printf "v_t2v_cf(%s,%s,%s)" c wf2 wf1
        end

    | Tensor_2_Scalar_Scalar_cf coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | (F23 | F32) →  printf "t2_phi2_cf(%s,%s,%s,%s, %s)" c wf1 p1 wf2 p2
        | (F12 | F13) →  printf "phi_t2phi_cf(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | (F21 | F31) →  printf "phi_t2phi_cf(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | Dim5_Tensor_2_Vector_Vector_1 coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | (F23 | F32) →  printf "t2_vv_d5_1(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | (F12 | F13) →  printf "v_t2v_d5_1(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | (F21 | F31) →  printf "v_t2v_d5_1(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | Tensor_2_Vector_Vector_t coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | (F23 | F32) →  printf "t2_vv_t(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | (F12 | F13) →  printf "v_t2v_t(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | (F21 | F31) →  printf "v_t2v_t(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | Dim5_Tensor_2_Vector_Vector_2 coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | F23 →  printf "t2_vv_d5_2(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F32 →  printf "t2_vv_d5_2(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        | (F12 | F13) →  printf "v_t2v_d5_2(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | (F21 | F31) →  printf "v_t2v_d5_2(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | TensorVector_Vector_Vector coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | (F23 | F32) →  printf "dv_vv(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | (F12 | F13) →  printf "v_dvv(%s,%s,%s,%s)" c wf1 p1 wf2
        | (F21 | F31) →  printf "v_dvv(%s,%s,%s,%s)" c wf2 p2 wf1
        end
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| *TensorVector_Vector_Vector_cf coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) → *printf* "dv_vv_cf(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F12* | *F13*) → *printf* "v_dvv_cf(%s,%s,%s,%s)" *c wf1 p1 wf2*
    | (*F21* | *F31*) → *printf* "v_dvv_cf(%s,%s,%s,%s)" *c wf2 p2 wf1*
    end

| *TensorVector_Scalar_Scalar coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) → *printf* "dv_phi2(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F12* | *F13*) → *printf* "phi_dvphi(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F21* | *F31*) → *printf* "phi_dvphi(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
    end

| *TensorVector_Scalar_Scalar_cf coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) → *printf* "dv_phi2_cf(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F12* | *F13*) → *printf* "phi_dvphi_cf(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F21* | *F31*) → *printf* "phi_dvphi_cf(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
    end

| *TensorScalar_Vector_Vector coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) → *printf* "tphi_vv(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F12* | *F13*) → *printf* "v_tphiv(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F21* | *F31*) → *printf* "v_tphiv(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
    end

| *TensorScalar_Vector_Vector_cf coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) → *printf* "tphi_vv_cf(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F12* | *F13*) → *printf* "v_tphiv_cf(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F21* | *F31*) → *printf* "v_tphiv_cf(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
    end

| *TensorScalar_Scalar_Scalar coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) → *printf* "tphi_ss(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F12* | *F13*) → *printf* "s_tphis(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F21* | *F31*) → *printf* "s_tphis(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
    end

| *TensorScalar_Scalar_Scalar_cf coeff* →
    let *c* = *format_coupling coeff c* in
    begin match *fusion* with
    | (*F23* | *F32*) → *printf* "tphi_ss_cf(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
    | (*F12* | *F13*) → *printf* "s_tphis_cf(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*

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    | (F21 | F31) → printf "s_tphis_cf(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    end

| Dim7_Tensor_2_Vector_Vector_T coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | F23 → printf "t2_vv_d7(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | F32 → printf "t2_vv_d7(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    | (F12 | F13) → printf "v_t2v_d7(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | (F21 | F31) → printf "v_t2v_d7(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    end

| Dim6_Scalar_Vector_Vector_D coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | (F23 | F32) → printf "s_vv_6D(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | (F12 | F13) → printf "v_sv_6D(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | (F21 | F31) → printf "v_sv_6D(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    end

| Dim6_Scalar_Vector_Vector_DP coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | (F23 | F32) → printf "s_vv_6DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | (F12 | F13) → printf "v_sv_6DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | (F21 | F31) → printf "v_sv_6DP(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    end

| Dim6_HAZ_D coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | F23 → printf "h_az_D(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | F32 → printf "h_az_D(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    | F13 → printf "a_hz_D(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | F31 → printf "a_hz_D(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    | F12 → printf "z_ah_D(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    | F21 → printf "z_ah_D(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    end
| Dim6_HAZ_DP coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | F23 → printf "h_az_DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | F32 → printf "h_az_DP(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    | F13 → printf "a_hz_DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    | F31 → printf "a_hz_DP(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    | F12 → printf "z_ah_DP(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
    | F21 → printf "z_ah_DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
    end
| Gauge_Gauge_Gauge_i coeff →
    let c = format_coupling coeff c in
    begin match fusion with
    | F23 → printf "g_gg_23(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
```

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            | F32 → printf "g_gg_23(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
            | F13 → printf "g_gg_13(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
            | F31 → printf "g_gg_13(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
            | F12 → printf "(-1)␣*␣g_gg_13(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
            | F21 → printf "(-1)␣*␣g_gg_13(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
            end

    | Dim6_GGG coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | F23 → printf "g_gg_6(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F32 → printf "g_gg_6(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        | F12 → printf "g_gg_6(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F21 → printf "g_gg_6(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        | F13 → printf "(-1)␣*␣g_gg_6(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F31 → printf "(-1)␣*␣g_gg_6(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | Dim6_AWW_DP coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | F23 → printf "a_ww_DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F32 → printf "a_ww_DP(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        | F13 → printf "w_aw_DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F31 → printf "w_aw_DP(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        | F12 → printf "(-1)␣*␣w_aw_DP(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F21 → printf "(-1)␣*␣w_aw_DP(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | Dim6_AWW_DW coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | F23 → printf "a_ww_DW(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F32 → printf "a_ww_DW(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        | F13 → printf "(-1)␣*␣a_ww_DW(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F31 → printf "(-1)␣*␣a_ww_DW(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        | F12 → printf "a_ww_DW(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F21 → printf "a_ww_DW(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | Dim6_Gauge_Gauge_Gauge_i coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | F23 | F31 | F12 →
            printf "kg_kgkg_i(%s,%s,%s,%s,%s)" c wf1 p1 wf2 p2
        | F32 | F13 | F21 →
            printf "kg_kgkg_i(%s,%s,%s,%s,%s)" c wf2 p2 wf1 p1
        end

    | Dim6_HHH coeff →
        let c = format_coupling coeff c in
        begin match fusion with
        | (F23 | F32 | F12 | F21 | F13 | F31) →
```

         printf "h_hh_6(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
        end
     | *Dim6_WWZ_DPWDW coeff* →
       let *c* = *format_coupling coeff c* in
       begin match *fusion* with
       | *F23* → *printf* "w_wz_DPW(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F32* → *printf* "w_wz_DPW(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       | *F13* → *printf* "(-1)␣*␣w_wz_DPW(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F31* → *printf* "(-1)␣*␣w_wz_DPW(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       | *F12* → *printf* "z_ww_DPW(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F21* → *printf* "z_ww_DPW(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       end
     | *Dim6_WWZ_DW coeff* →
       let *c* = *format_coupling coeff c* in
       begin match *fusion* with
       | *F23* → *printf* "w_wz_DW(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F32* → *printf* "w_wz_DW(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       | *F13* → *printf* "(-1)␣*␣w_wz_DW(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F31* → *printf* "(-1)␣*␣w_wz_DW(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       | *F12* → *printf* "z_ww_DW(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F21* → *printf* "z_ww_DW(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       end
     | *Dim6_WWZ_D coeff* →
       let *c* = *format_coupling coeff c* in
       begin match *fusion* with
       | *F23* → *printf* "w_wz_D(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F32* → *printf* "w_wz_D(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       | *F13* → *printf* "(-1)␣*␣w_wz_D(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F31* → *printf* "(-1)␣*␣w_wz_D(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       | *F12* → *printf* "z_ww_D(%s,%s,%s,%s,%s)" *c wf1 p1 wf2 p2*
       | *F21* → *printf* "z_ww_D(%s,%s,%s,%s,%s)" *c wf2 p2 wf1 p1*
       end

    end

Flip the sign to account for the i^2 relative to diagrams with only cubic couplings.

   | *V4 (vertex, fusion, constant)* →
     let *c* = *CM.constant_symbol constant*
     and *ch1, ch2, ch3* = *children3 rhs* in
     let *wf1* = *multiple_variable amplitude dictionary ch1*
     and *wf2* = *multiple_variable amplitude dictionary ch2*
     and *wf3* = *multiple_variable amplitude dictionary ch3*
     and *p1* = *momentum ch1*
     and *p2* = *momentum ch2*
     and *p3* = *momentum ch3* in
     printf "@,␣%s␣" (if (*F.sign rhs*) < 0 then "+" else "-");
     begin match *vertex* with
     | *Scalar4 coeff* →
       printf "(%s*%s*%s*%s)" (*format_coupling coeff c*) *wf1 wf2 wf3*
     | *Scalar2_Vector2 coeff* →

```
            let c = format_coupling coeff c in
            begin match fusion with
            | F134 | F143 | F234 | F243 →
                printf "%s*%s*(%s*%s)" c wf1 wf2 wf3
            | F314 | F413 | F324 | F423 →
                printf "%s*%s*(%s*%s)" c wf2 wf1 wf3
            | F341 | F431 | F342 | F432 →
                printf "%s*%s*(%s*%s)" c wf3 wf1 wf2
            | F312 | F321 | F412 | F421 →
                printf "(%s*%s*%s)*%s" c wf2 wf3 wf1
            | F231 | F132 | F241 | F142 →
                printf "(%s*%s*%s)*%s" c wf1 wf3 wf2
            | F123 | F213 | F124 | F214 →
                printf "(%s*%s*%s)*%s" c wf1 wf2 wf3
            end
        | Vector4 contractions →
            begin match contractions with
            | [] → invalid_arg "Targets.print_current:␣Vector4␣[]"
            | head :: tail →
                printf "(";
                print_vector4 c wf1 wf2 wf3 fusion head;
                List.iter (print_add_vector4 c wf1 wf2 wf3 fusion) tail;
                printf ")"
            end
        | Vector4_K_Matrix_tho (_, poles) →
            let pa, pb =
                begin match fusion with
                | (F341 | F431 | F342 | F432 | F123 | F213 | F124 |
F214) → (p1, p2)
                | (F134 | F143 | F234 | F243 | F312 | F321 | F412 |
F421) → (p2, p3)
                | (F314 | F413 | F324 | F423 | F132 | F231 | F142 |
F241) → (p1, p3)
                end in
            printf "(%s*(%s*%s)*(%s*%s)*(%s*%s)@,*("
            c p1 wf1 p2 wf2 p3 wf3;
            List.iter (fun (coeff, pole) →
                printf "+%s/((%s+%s)*(%s+%s)-%s)"
                    (CM.constant_symbol coeff) pa pb pa pb
                    (CM.constant_symbol pole))
                poles;
            printf ")*(-%s-%s-%s))" p1 p2 p3
        | Vector4_K_Matrix_jr (disc, contractions) →
            let pa, pb =
                begin match disc, fusion with
                | 3, (F143 | F413 | F142 | F412 | F321 | F231 | F324 |
F234) → (p1, p2)
                | 3, (F314 | F341 | F214 | F241 | F132 | F123 | F432 |
F423) → (p2, p3)
                | 3, (F134 | F431 | F124 | F421 | F312 | F213 | F342 |
```

```
F243) → (p1, p3)
                          | _, (F341 | F431 | F342 | F432 | F123 | F213 | F124 |
F214) → (p1, p2)
                          | _, (F134 | F143 | F234 | F243 | F312 | F321 | F412 |
F421) → (p2, p3)
                          | _, (F314 | F413 | F324 | F423 | F132 | F231 | F142 |
F241) → (p1, p3)
                    end in
                begin match contractions with
                | [] → invalid_arg "Targets.print_current:␣Vector4_K_Matrix_jr␣[]"
                | head :: tail →
                    printf "(";
                    print_vector4_km c pa pb wf1 wf2 wf3 fusion head;
                    List.iter (print_add_vector4_km c pa pb wf1 wf2 wf3 fusion)
                      tail;
                    printf ")"
                end
            | DScalar2_Vector2_K_Matrix_ms (disc, contractions) →
                let p123 = Printf.sprintf "(-%s-%s-%s)" p1 p2 p3 in
                let pa, pb =
                    begin match disc, fusion with
                    | 3, (F143 | F413 | F142 | F412 | F321 | F231 | F324 |
F234) → (p1, p2)
                          | 3, (F314 | F341 | F214 | F241 | F132 | F123 | F432 |
F423) → (p2, p3)
                          | 3, (F134 | F431 | F124 | F421 | F312 | F213 | F342 |
F243) → (p1, p3)
                          | 4, (F143 | F413 | F142 | F412 | F321 | F231 | F324 |
F234) → (p1, p2)
                          | 4, (F314 | F341 | F214 | F241 | F132 | F123 | F432 |
F423) → (p2, p3)
                          | 4, (F134 | F431 | F124 | F421 | F312 | F213 | F342 |
F243) → (p1, p3)
                          | 5, (F143 | F413 | F142 | F412 | F321 | F231 | F324 |
F234) → (p1, p2)
                          | 5, (F314 | F341 | F214 | F241 | F132 | F123 | F432 |
F423) → (p2, p3)
                          | 5, (F134 | F431 | F124 | F421 | F312 | F213 | F342 |
F243) → (p1, p3)
                          | 6, (F134 | F132 | F314 | F312 | F241 | F243 | F421 |
F423) → (p1, p2)
                          | 6, (F213 | F413 | F231 | F431 | F124 | F324 | F142 |
F342) → (p2, p3)
                          | 6, (F143 | F123 | F341 | F321 | F412 | F214 | F432 |
F234) → (p1, p3)
                          | 7, (F134 | F132 | F314 | F312 | F241 | F243 | F421 |
F423) → (p1, p2)
                          | 7, (F213 | F413 | F231 | F431 | F124 | F324 | F142 |
F342) → (p2, p3)
                          | 7, (F143 | F123 | F341 | F321 | F412 | F214 | F432 |
```

F234) → (*p1*, *p3*)
                    | 8, (*F134* | *F132* | *F314* | *F312* | *F241* | *F243* | *F421* |
F423) → (*p1*, *p2*)
                    | 8, (*F213* | *F413* | *F231* | *F431* | *F124* | *F324* | *F142* |
F342) → (*p2*, *p3*)
                    | 8, (*F143* | *F123* | *F341* | *F321* | *F412* | *F214* | *F432* |
F234) → (*p1*, *p3*)
                    | _, (*F341* | *F431* | *F342* | *F432* | *F123* | *F213* | *F124* |
F214) → (*p1*, *p2*)
                    | _, (*F134* | *F143* | *F234* | *F243* | *F312* | *F321* | *F412* |
F421) → (*p2*, *p3*)
                    | _, (*F314* | *F413* | *F324* | *F423* | *F132* | *F231* | *F142* |
F241) → (*p1*, *p3*)
                end in
            begin match *contractions* with
            | [] → *invalid _ arg* "Targets.print_current:␣DScalar2_Vector4_K_Matrix_ms␣[]"
            | *head* :: *tail* →
                printf "(";
                print _ dscalar2 _ vector2 _ km
                    c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion head;
                List.iter (*print _ add _ dscalar2 _ vector2 _ km*
                                c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion)
                    tail;
                printf ")"
            end
        | *DScalar4 _ K _ Matrix _ ms* (*disc*, *contractions*) →
            let *p123* = *Printf.sprintf* "(-%s-%s-%s)" *p1 p2 p3* in
            let *pa*, *pb* =
                begin match *disc*, *fusion* with
                | 3, (*F143* | *F413* | *F142* | *F412* | *F321* | *F231* | *F324* |
F234) → (*p1*, *p2*)
                    | 3, (*F314* | *F341* | *F214* | *F241* | *F132* | *F123* | *F432* |
F423) → (*p2*, *p3*)
                    | 3, (*F134* | *F431* | *F124* | *F421* | *F312* | *F213* | *F342* |
F243) → (*p1*, *p3*)
                    | _, (*F341* | *F431* | *F342* | *F432* | *F123* | *F213* | *F124* |
F214) → (*p1*, *p2*)
                    | _, (*F134* | *F143* | *F234* | *F243* | *F312* | *F321* | *F412* |
F421) → (*p2*, *p3*)
                    | _, (*F314* | *F413* | *F324* | *F423* | *F132* | *F231* | *F142* |
F241) → (*p1*, *p3*)
                end in
            begin match *contractions* with
            | [] → *invalid _ arg* "Targets.print_current:␣DScalar4_K_Matrix_ms␣[]"
            | *head* :: *tail* →
                printf "(";
                print _ dscalar4 _ km
                    c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion head;
                List.iter (*print _ add _ dscalar4 _ km*
                                c pa pb wf1 wf2 wf3 p1 p2 p3 p123 fusion)

```
                    tail;
                    printf ")"
              end
| Dim8_Scalar2_Vector2_1 coeff →
      let c = format_coupling coeff c in
            begin match fusion with
            | F134 | F143 | F234 | F243 →
                  printf "phi_phi2v_1(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
            | F314 | F413 | F324 | F423 →
                  printf "phi_phi2v_1(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf1 p1 wf3 p3
            | F341 | F431 | F342 | F432 →
                  printf "phi_phi2v_1(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
            | F312 | F321 | F412 | F421 →
                  printf "v_phi2v_1(%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1
            | F231 | F132 | F241 | F142 →
                  printf "v_phi2v_1(%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf3 p3 wf2
            | F123 | F213 | F124 | F214 →
                  printf "v_phi2v_1(%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3
            end
| Dim8_Scalar2_Vector2_2 coeff →
      let c = format_coupling coeff c in
            begin match fusion with
            | F134 | F143 | F234 | F243 →
                  printf "phi_phi2v_2(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
            | F314 | F413 | F324 | F423 →
                  printf "phi_phi2v_2(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf1 p1 wf3 p3
            | F341 | F431 | F342 | F432 →
                  printf "phi_phi2v_2(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
            | F312 | F321 | F412 | F421 →
                  printf "v_phi2v_2(%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1
            | F231 | F132 | F241 | F142 →
                  printf "v_phi2v_2(%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf3 p3 wf2
            | F123 | F213 | F124 | F214 →
                  printf "v_phi2v_2(%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3
            end
| Dim8_Scalar4 coeff →
      let c = format_coupling coeff c in
            begin match fusion with
```

$$| \ F134 \ | \ F143 \ | \ F234 \ | \ F243 \ | \ F314 \ | \ F413 \ |$$
$F324 \ | \ F423$
$$| \ F341 \ | \ F431 \ | \ F342 \ | \ F432 \ | \ F312 \ | \ F321 \ |$$
$F412 \ | \ F421$
$$| \ F231 \ | \ F132 \ | \ F241 \ | \ F142 \ | \ F123 \ | \ F213 \ |$$
$F124 \ | \ F214 \ \rightarrow$

        printf `"s_dim8s3␣(%s,%s,%s,%s,%s,%s,%s)"`
          c wf1 p1 wf2 p2 wf3 p3
      end
  | *GBBG* (*coeff*, *fb*, *b*, *f*) $\rightarrow$
    Fermions.print_current_g4 (*coeff*, *fb*, *b*, *f*) *c wf1 wf2 wf3*
      fusion

  | *Dim6_H4_P2 coeff* $\rightarrow$
    let *c* = *format_coupling coeff c* in
      begin match *fusion* with
        | *F134* | *F143* | *F234* | *F243* | *F314* | *F413* |
$F324 \ | \ F423$
$$| \ F341 \ | \ F431 \ | \ F342 \ | \ F432 \ | \ F312 \ | \ F321 \ |$$
$F412 \ | \ F421$
$$| \ F231 \ | \ F132 \ | \ F241 \ | \ F142 \ | \ F123 \ | \ F213 \ |$$
$F124 \ | \ F214 \ \rightarrow$

        printf `"hhhh_p2␣(%s,%s,%s,%s,%s,%s,%s)"`
          c wf1 p1 wf2 p2 wf3 p3
      end
  | *Dim6_AHWW_DPB coeff* $\rightarrow$
    let *c* = *format_coupling coeff c* in
      begin match *fusion* with
    | *F234* $\rightarrow$
      printf `"a_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"`
        c wf1 p1 wf2 p2 wf3 p3
    | *F243* $\rightarrow$
      printf `"a_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"`
        c wf1 p1 wf3 p3 wf2 p2
    | *F342* $\rightarrow$
      printf `"a_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"`
        c wf3 p3 wf1 p1 wf2 p2
    | *F324* $\rightarrow$
      printf `"a_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"`
        c wf2 p2 wf1 p1 wf3 p3
    | *F423* $\rightarrow$
      printf `"a_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"`
        c wf2 p2 wf3 p3 wf1 p1
    | *F432* $\rightarrow$
      printf `"a_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"`
        c wf3 p3 wf2 p2 wf1 p1
    | *F134* $\rightarrow$
      printf `"h_aww_DPB(%s,%s,%s,%s,%s,%s,%s)"`
        c wf1 p1 wf2 p2 wf3 p3
    | *F143* $\rightarrow$

```
        printf "h_aww_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf1 p1 wf3 p3 wf2 p2
    | F341 →
        printf "h_aww_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf3 p3 wf1 p1 wf2 p2
    | F314 →
        printf "h_aww_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf2 p2 wf1 p1 wf3 p3
    | F413 →
        printf "h_aww_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf2 p2 wf3 p3 wf1 p1
    | F431 →
        printf "h_aww_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf3 p3 wf2 p2 wf1 p1
    | F124 →
        printf "w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf1 p1 wf2 p2 wf3 p3
    | F142 →
        printf "w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf1 p1 wf3 p3 wf2 p2
    | F241 →
        printf "w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf3 p3 wf1 p1 wf2 p2
    | F214 →
        printf "w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf2 p2 wf1 p1 wf3 p3
    | F412 →
        printf "w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf2 p2 wf3 p3 wf1 p1
    | F421 →
        printf "w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf3 p3 wf2 p2 wf1 p1
    | F123 →
        printf "(-1)*w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf1 p1 wf2 p2 wf3 p3
    | F132 →
        printf "(-1)*w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf1 p1 wf3 p3 wf2 p2
    | F231 →
        printf "(-1)*w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf3 p3 wf1 p1 wf2 p2
    | F213 →
        printf "(-1)*w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf2 p2 wf1 p1 wf3 p3
    | F312 →
        printf "(-1)*w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf2 p2 wf3 p3 wf1 p1
    | F321 →
        printf "(-1)*w_ahw_DPB(%s,%s,%s,%s,%s,%s,%s)"
            c wf3 p3 wf2 p2 wf1 p1
```

```
              end
| Dim6_AHWW_DPW coeff →
     let c = format_coupling coeff c in
         begin match fusion with
         | F234 →
             printf "a_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf2 p2 wf3 p3
         | F243 →
             printf "a_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf3 p3 wf2 p2
         | F342 →
             printf "a_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf1 p1 wf2 p2
         | F324 →
             printf "a_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf1 p1 wf3 p3
         | F423 →
             printf "a_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf3 p3 wf1 p1
         | F432 →
             printf "a_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf2 p2 wf1 p1
         | F134 →
             printf "h_aww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf2 p2 wf3 p3
         | F143 →
             printf "h_aww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf3 p3 wf2 p2
         | F341 →
             printf "h_aww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf1 p1 wf2 p2
         | F314 →
             printf "h_aww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf1 p1 wf3 p3
         | F413 →
             printf "h_aww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf3 p3 wf1 p1
         | F431 →
             printf "h_aww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf2 p2 wf1 p1
         | F124 →
             printf "w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf2 p2 wf3 p3
         | F142 →
             printf "w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf3 p3 wf2 p2
         | F241 →
             printf "w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf1 p1 wf2 p2
         | F214 →
```

        printf `"w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf2 p2 wf1 p1 wf3 p3
     | *F412* →
        printf `"w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf2 p2 wf3 p3 wf1 p1
     | *F421* →
        printf `"w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf3 p3 wf2 p2 wf1 p1
     | *F123* →
        printf `"(-1)*w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf1 p1 wf2 p2 wf3 p3
     | *F132* →
        printf `"(-1)*w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf1 p1 wf3 p3 wf2 p2
     | *F231* →
        printf `"(-1)*w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf3 p3 wf1 p1 wf2 p2
     | *F213* →
        printf `"(-1)*w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf2 p2 wf1 p1 wf3 p3
     | *F312* →
        printf `"(-1)*w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf2 p2 wf3 p3 wf1 p1
     | *F321* →
        printf `"(-1)*w_ahw_DPW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf3 p3 wf2 p2 wf1 p1
    end
| *Dim6_AHWW_DW coeff* →
    let *c* = *format_coupling coeff c* in
      begin match *fusion* with
     | *F234* →
        printf `"a_hww_DW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf1 p1 wf2 p2 wf3 p3
     | *F243* →
        printf `"a_hww_DW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf1 p1 wf3 p3 wf2 p2
     | *F342* →
        printf `"a_hww_DW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf3 p3 wf1 p1 wf2 p2
     | *F324* →
        printf `"a_hww_DW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf2 p2 wf1 p1 wf3 p3
     | *F423* →
        printf `"a_hww_DW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf2 p2 wf3 p3 wf1 p1
     | *F432* →
        printf `"a_hww_DW(%s,%s,%s,%s,%s,%s,%s)"`
           c wf3 p3 wf2 p2 wf1 p1
     | *F134* →
        printf `"h_aww_DW(%s,%s,%s,%s,%s,%s,%s)"`

```
              c wf1 p1 wf2 p2 wf3 p3
| F143 →
    printf "h_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
| F341 →
    printf "h_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
| F314 →
    printf "h_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
| F413 →
    printf "h_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
| F431 →
    printf "h_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf2 p2 wf1 p1
| F124 →
    printf "w3_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf2 p2 wf3 p3
| F142 →
    printf "w3_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
| F241 →
    printf "w3_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
| F214 →
    printf "w3_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
| F412 →
    printf "w3_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
| F421 →
    printf "w3_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf2 p2 wf1 p1
| F123 →
    printf "w4_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf2 p2 wf3 p3
| F132 →
    printf "w4_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
| F231 →
    printf "w4_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
| F213 →
    printf "w4_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
| F312 →
    printf "w4_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
| F321 →
```

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```
                printf "w4_ahw_DW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
            end
    | Dim6_Scalar2_Vector2_D coeff →
      let c = format_coupling coeff c in
            begin match fusion with
            | F234 | F134 →
                printf "h_hww_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
            | F243 | F143 →
                printf "h_hww_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf3 p3 wf2 p2
            | F342 | F341 →
                printf "h_hww_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf1 p1 wf2 p2
            | F324 | F314 →
                printf "h_hww_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf1 p1 wf3 p3
            | F423 | F413 →
                printf "h_hww_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf3 p3 wf1 p1
            | F432 | F431 →
                printf "h_hww_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
            | F124 | F123 →
                printf "w_hhw_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
            | F142 | F132 →
                printf "w_hhw_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf3 p3 wf2 p2
            | F241 | F231 →
                printf "w_hhw_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf1 p1 wf2 p2
            | F214 | F213 →
                printf "w_hhw_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf1 p1 wf3 p3
            | F412 | F312 →
                printf "w_hhw_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf3 p3 wf1 p1
            | F421 | F321 →
                printf "w_hhw_D(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
            end
    | Dim6_Scalar2_Vector2_DP coeff →
      let c = format_coupling coeff c in
            begin match fusion with
            | F234 | F134 →
                printf "h_hww_DP(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
            | F342 | F341 →
```

```
          printf "h_hww_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf1 p1 wf2 p2
       | F423 | F413 →
          printf "h_hww_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf3 p3 wf1 p1
       | F243 | F143 →
          printf "h_hww_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf3 p3 wf2 p2
       | F324 | F314 →
          printf "h_hww_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf1 p1 wf3 p3
       | F432 | F431 →
          printf "h_hww_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf2 p2 wf1 p1
       | F123 | F124 →
          printf "w_hhw_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf2 p2 wf3 p3
       | F231 | F241 →
          printf "w_hhw_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf1 p1 wf2 p2
       | F312 | F412 →
          printf "w_hhw_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf3 p3 wf1 p1
       | F132 | F142 →
          printf "w_hhw_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf1 p1 wf3 p3 wf2 p2
       | F213 | F214 →
          printf "w_hhw_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf2 p2 wf1 p1 wf3 p3
       | F321 | F421 →
          printf "w_hhw_DP(%s,%s,%s,%s,%s,%s,%s)"
                 c wf3 p3 wf2 p2 wf1 p1
       end
  | Dim6_Scalar2_Vector2_PB coeff →
     let c = format_coupling coeff c in
        begin match fusion with
        | F234 | F134 →
           printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf1 p1 wf2 p2 wf3 p3
        | F342 | F341 →
           printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf3 p3 wf1 p1 wf2 p2
        | F423 | F413 →
           printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf2 p2 wf3 p3 wf1 p1
        | F243 | F143 →
           printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf1 p1 wf3 p3 wf2 p2
        | F324 | F314 →
           printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
```

> > > > *c wf2 p2 wf1 p1 wf3 p3*
> > > | *F432* | *F431* →
> > > > *printf* `"h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf3 p3 wf2 p2 wf1 p1*
> > > | *F123* | *F124* →
> > > > *printf* `"v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf1 p1 wf2 p2 wf3 p3*
> > > | *F231* | *F241* →
> > > > *printf* `"v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf3 p3 wf1 p1 wf2 p2*
> > > | *F312* | *F412* →
> > > > *printf* `"v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf2 p2 wf3 p3 wf1 p1*
> > > | *F132* | *F142* →
> > > > *printf* `"v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf1 p1 wf3 p3 wf2 p2*
> > > | *F213* | *F214* →
> > > > *printf* `"v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf2 p2 wf1 p1 wf3 p3*
> > > | *F321* | *F421* →
> > > > *printf* `"v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf3 p3 wf2 p2 wf1 p1*
> > > end

> | *Dim6_HHZZ_T coeff* →
> > let *c* = *format_coupling coeff c* in
> > > begin match *fusion* with
> > > | *F234* | *F134* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf1 wf2 wf3*
> > > | *F342* | *F341* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf3 wf1 wf2*
> > > | *F423* | *F413* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf2 wf3 wf1*
> > > | *F243* | *F143* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf1 wf3 wf2*
> > > | *F324* | *F314* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf2 wf1 wf3*
> > > | *F432* | *F431* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf3 wf2 wf1*
> > > | *F123* | *F124* | *F231* | *F241* | *F312* | *F412* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf1 wf2 wf3*
> > > | *F132* | *F142* | *F213* | *F214* | *F321* | *F421* →
> > > > *printf* `"(%s)*(%s)*(%s)*(%s)"` *c wf1 wf2 wf3*
> > > end
> | *Dim6_Vector4_DW coeff* →
> > let *c* = *format_coupling coeff c* in
> > > begin match *fusion* with
> > > | *F234* | *F134* →
> > > > *printf* `"a_aww_DW(%s,%s,%s,%s,%s,%s,%s)"`
> > > > > *c wf1 p1 wf2 p2 wf3 p3*

```
            | F342 | F341 →
                printf "a_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf1 p1 wf2 p2
            | F423 | F413 →
                printf "a_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf3 p3 wf1 p1
            | F243 | F143 →
                printf "a_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf3 p3 wf2 p2
            | F324 | F314 →
                printf "a_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf1 p1 wf3 p3
            | F432 | F431 →
                printf "a_aww_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf2 p2 wf1 p1
            | F124 | F123 →
                printf "w_aaw_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf2 p2 wf3 p3
            | F241 | F231 →
                printf "w_aaw_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf1 p1 wf2 p2
            | F412 | F312 →
                printf "w_aaw_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf3 p3 wf1 p1
            | F142 | F132 →
                printf "w_aaw_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf3 p3 wf2 p2
            | F214 | F213 →
                printf "w_aaw_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf1 p1 wf3 p3
            | F421 | F321 →
                printf "w_aaw_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf2 p2 wf1 p1
            end
    | Dim6_Vector4_W coeff →
        let c = format_coupling coeff c in
            begin match fusion with
            | F234 | F134 →
                printf "a_aww_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf2 p2 wf3 p3
            | F342 | F341 →
                printf "a_aww_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf1 p1 wf2 p2
            | F423 | F413 →
                printf "a_aww_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf3 p3 wf1 p1
            | F243 | F143 →
                printf "a_aww_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf3 p3 wf2 p2
            | F324 | F314 →
```

```
              printf "a_aww_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf1 p1 wf3 p3
          | F432 | F431 →
              printf "a_aww_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf2 p2 wf1 p1
          | F123 | F124 →
              printf "w_aaw_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf2 p2 wf3 p3
          | F231 | F241 →
              printf "w_aaw_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf1 p1 wf2 p2
          | F312 | F412 →
              printf "w_aaw_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf3 p3 wf1 p1
          | F132 | F142 →
              printf "w_aaw_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf3 p3 wf2 p2
          | F213 | F214 →
              printf "w_aaw_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf1 p1 wf3 p3
          | F321 | F421 →
              printf "w_aaw_W(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf2 p2 wf1 p1
          end
    | Dim6_HWWZ_DW coeff →
      let c = format_coupling coeff c in
          begin match fusion with
          | F234 →
              printf "h_wwz_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf2 p2 wf3 p3
          | F243 →
              printf "h_wwz_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf3 p3 wf2 p2
          | F342 →
              printf "h_wwz_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf1 p1 wf2 p2
          | F324 →
              printf "h_wwz_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf1 p1 wf3 p3
          | F423 →
              printf "h_wwz_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf3 p3 wf1 p1
          | F432 →
              printf "h_wwz_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf2 p2 wf1 p1
          | F124 →
              printf "(-1)*w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf2 p2 wf3 p3
          | F142 →
              printf "(-1)*w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
```

```
              c wf1 p1 wf3 p3 wf2 p2
  | F241 →
      printf "(-1)*w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
  | F214 →
      printf "(-1)*w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
  | F412 →
      printf "(-1)*w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
  | F421 →
      printf "(-1)*w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf2 p2 wf1 p1
  | F134 →
      printf "w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf2 p2 wf3 p3
  | F143 →
      printf "w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
  | F341 →
      printf "w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
  | F314 →
      printf "w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
  | F413 →
      printf "w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
  | F431 →
      printf "w_hwz_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf2 p2 wf1 p1
  | F123 →
      printf "z_hww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf2 p2 wf3 p3
  | F132 →
      printf "z_hww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
  | F231 →
      printf "z_hww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
  | F213 →
      printf "z_hww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
  | F312 →
      printf "z_hww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
  | F321 →
      printf "z_hww_DW(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf2 p2 wf1 p1
  end
```

```
| Dim6_HWWZ_DPB coeff  →
  let c  =  format_coupling coeff c in
      begin match fusion with
      | F234  →
          printf "h_wwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf2 p2 wf3 p3
      | F243  →
          printf "h_wwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
      | F342  →
          printf "h_wwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
      | F324  →
          printf "h_wwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
      | F423  →
          printf "h_wwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
      | F432  →
          printf "h_wwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf2 p2 wf1 p1
      | F124  →
          printf "(-1)*w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf2 p2 wf3 p3
      | F142  →
          printf "(-1)*w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
      | F241  →
          printf "(-1)*w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
      | F214  →
          printf "(-1)*w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf1 p1 wf3 p3
      | F412  →
          printf "(-1)*w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf2 p2 wf3 p3 wf1 p1
      | F421  →
          printf "(-1)*w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf2 p2 wf1 p1
      | F134  →
          printf "w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf2 p2 wf3 p3
      | F143  →
          printf "w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf1 p1 wf3 p3 wf2 p2
      | F341  →
          printf "w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
              c wf3 p3 wf1 p1 wf2 p2
      | F314  →
          printf "w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
```

```
                        c wf2 p2 wf1 p1 wf3 p3
            | F413 →
                printf "w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf3 p3 wf1 p1
            | F431 →
                printf "w_hwz_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
            | F123 →
                printf "z_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
            | F132 →
                printf "z_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf3 p3 wf2 p2
            | F231 →
                printf "z_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf1 p1 wf2 p2
            | F213 →
                printf "z_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf1 p1 wf3 p3
            | F312 →
                printf "z_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf3 p3 wf1 p1
            | F321 →
                printf "z_hww_DPB(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
          end
      | Dim6_HWWZ_DDPW coeff →
        let c = format_coupling coeff c in
            begin match fusion with
            | F234 →
                printf "h_wwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
            | F243 →
                printf "h_wwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf3 p3 wf2 p2
            | F342 →
                printf "h_wwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf1 p1 wf2 p2
            | F324 →
                printf "h_wwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf1 p1 wf3 p3
            | F423 →
                printf "h_wwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf2 p2 wf3 p3 wf1 p1
            | F432 →
                printf "h_wwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf3 p3 wf2 p2 wf1 p1
            | F124 →
                printf "(-1)*w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                        c wf1 p1 wf2 p2 wf3 p3
```

```
        | F142  →
            printf "(-1)*w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf3 p3 wf2 p2
        | F241  →
            printf "(-1)*w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf1 p1 wf2 p2
        | F214  →
            printf "(-1)*w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf1 p1 wf3 p3
        | F412  →
            printf "(-1)*w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf3 p3 wf1 p1
        | F421  →
            printf "(-1)*w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf2 p2 wf1 p1
        | F134  →
            printf "w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf2 p2 wf3 p3
        | F143  →
            printf "w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf3 p3 wf2 p2
        | F341  →
            printf "w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf1 p1 wf2 p2
        | F314  →
            printf "w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf1 p1 wf3 p3
        | F413  →
            printf "w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf3 p3 wf1 p1
        | F431  →
            printf "w_hwz_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf2 p2 wf1 p1
        | F123  →
            printf "z_hww_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf2 p2 wf3 p3
        | F132  →
            printf "z_hww_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf3 p3 wf2 p2
        | F231  →
            printf "z_hww_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf1 p1 wf2 p2
        | F213  →
            printf "z_hww_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf1 p1 wf3 p3
        | F312  →
            printf "z_hww_DDPW(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf3 p3 wf1 p1
        | F321  →
            printf "z_hww_DDPW(%s,%s,%s,%s,%s,%s,%s)"
```

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```
                         c wf3 p3 wf2 p2 wf1 p1
              end
    | Dim6_HWWZ_DPW coeff →
      let c = format_coupling coeff c in
            begin match fusion with
            | F234 →
                printf "h_wwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf1 p1 wf2 p2 wf3 p3
            | F243 →
                printf "h_wwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf1 p1 wf3 p3 wf2 p2
            | F342 →
                printf "h_wwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf3 p3 wf1 p1 wf2 p2
            | F324 →
                printf "h_wwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf2 p2 wf1 p1 wf3 p3
            | F423 →
                printf "h_wwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf2 p2 wf3 p3 wf1 p1
            | F432 →
                printf "h_wwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf3 p3 wf2 p2 wf1 p1
            | F124 →
                printf "(-1)*w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf1 p1 wf2 p2 wf3 p3
            | F142 →
                printf "(-1)*w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf1 p1 wf3 p3 wf2 p2
            | F241 →
                printf "(-1)*w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf3 p3 wf1 p1 wf2 p2
            | F214 →
                printf "(-1)*w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf2 p2 wf1 p1 wf3 p3
            | F412 →
                printf "(-1)*w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf2 p2 wf3 p3 wf1 p1
            | F421 →
                printf "(-1)*w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf3 p3 wf2 p2 wf1 p1
            | F134 →
                printf "w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf1 p1 wf2 p2 wf3 p3
            | F143 →
                printf "w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf1 p1 wf3 p3 wf2 p2
            | F341 →
                printf "w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                       c wf3 p3 wf1 p1 wf2 p2
```

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```
                  | F314 →
                      printf "w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf1 p1 wf3 p3
                  | F413 →
                      printf "w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf3 p3 wf1 p1
                  | F431 →
                      printf "w_hwz_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf3 p3 wf2 p2 wf1 p1
                  | F123 →
                      printf "z_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf1 p1 wf2 p2 wf3 p3
                  | F132 →
                      printf "z_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf1 p1 wf3 p3 wf2 p2
                  | F231 →
                      printf "z_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf3 p3 wf1 p1 wf2 p2
                  | F213 →
                      printf "z_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf1 p1 wf3 p3
                  | F312 →
                      printf "z_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf3 p3 wf1 p1
                  | F321 →
                      printf "z_hww_DPW(%s,%s,%s,%s,%s,%s,%s)"
                          c wf3 p3 wf2 p2 wf1 p1
                  end
     | Dim6_AHHZ_D coeff →
       let c = format_coupling coeff c in
            begin match fusion with
            | F234 →
                printf "a_hhz_D(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf2 p2 wf3 p3
            | F243 →
                printf "a_hhz_D(%s,%s,%s,%s,%s,%s,%s)"
                    c wf1 p1 wf3 p3 wf2 p2
            | F342 →
                printf "a_hhz_D(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf1 p1 wf2 p2
            | F324 →
                printf "a_hhz_D(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf1 p1 wf3 p3
            | F423 →
                printf "a_hhz_D(%s,%s,%s,%s,%s,%s,%s)"
                    c wf2 p2 wf3 p3 wf1 p1
            | F432 →
                printf "a_hhz_D(%s,%s,%s,%s,%s,%s,%s)"
                    c wf3 p3 wf2 p2 wf1 p1
            | F124 →
```

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```
        printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
            c wf1 p1 wf2 p2 wf3 p3
| F142 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf1 p1 wf3 p3 wf2 p2
| F241 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf3 p3 wf1 p1 wf2 p2
| F214 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf2 p2 wf1 p1 wf3 p3
| F412 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf2 p2 wf3 p3 wf1 p1
| F421 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf3 p3 wf2 p2 wf1 p1
| F134 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf1 p1 wf2 p2 wf3 p3
| F143 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf1 p1 wf3 p3 wf2 p2
| F341 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf3 p3 wf1 p1 wf2 p2
| F314 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf2 p2 wf1 p1 wf3 p3
| F413 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf2 p2 wf3 p3 wf1 p1
| F431 →
    printf "h_ahz_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf3 p3 wf2 p2 wf1 p1
| F123 →
    printf "z_ahh_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf1 p1 wf2 p2 wf3 p3
| F132 →
    printf "z_ahh_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf1 p1 wf3 p3 wf2 p2
| F231 →
    printf "z_ahh_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf3 p3 wf1 p1 wf2 p2
| F213 →
    printf "z_ahh_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf2 p2 wf1 p1 wf3 p3
| F312 →
    printf "z_ahh_D(%s,%s,%s,%s,%s,%s,%s)"
        c wf2 p2 wf3 p3 wf1 p1
```

```
          | F321 →
              printf "z_ahh_D(%s,%s,%s,%s,%s,%s,%s)"
                  c wf3 p3 wf2 p2 wf1 p1
          end
| Dim6_AHHZ_DP coeff →
    let c = format_coupling coeff c in
        begin match fusion with
        | F234 →
            printf "a_hhz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf2 p2 wf3 p3
        | F243 →
            printf "a_hhz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf3 p3 wf2 p2
        | F342 →
            printf "a_hhz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf1 p1 wf2 p2
        | F324 →
            printf "a_hhz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf1 p1 wf3 p3
        | F423 →
            printf "a_hhz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf3 p3 wf1 p1
        | F432 →
            printf "a_hhz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf2 p2 wf1 p1
        | F124 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf2 p2 wf3 p3
        | F142 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf3 p3 wf2 p2
        | F241 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf1 p1 wf2 p2
        | F214 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf1 p1 wf3 p3
        | F412 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf2 p2 wf3 p3 wf1 p1
        | F421 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf3 p3 wf2 p2 wf1 p1
        | F134 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf2 p2 wf3 p3
        | F143 →
            printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                c wf1 p1 wf3 p3 wf2 p2
        | F341 →
```

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```
              printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf3 p3 wf1 p1 wf2 p2
          | F314 →
              printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf2 p2 wf1 p1 wf3 p3
          | F413 →
              printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf2 p2 wf3 p3 wf1 p1
          | F431 →
              printf "h_ahz_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf3 p3 wf2 p2 wf1 p1
          | F123 →
              printf "z_ahh_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf1 p1 wf2 p2 wf3 p3
          | F132 →
              printf "z_ahh_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf1 p1 wf3 p3 wf2 p2
          | F231 →
              printf "z_ahh_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf3 p3 wf1 p1 wf2 p2
          | F213 →
              printf "z_ahh_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf2 p2 wf1 p1 wf3 p3
          | F312 →
              printf "z_ahh_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf2 p2 wf3 p3 wf1 p1
          | F321 →
              printf "z_ahh_DP(%s,%s,%s,%s,%s,%s,%s)"
                  c wf3 p3 wf2 p2 wf1 p1
          end
  | Dim6_AHHZ_PB coeff →
      let c = format_coupling coeff c in
          begin match fusion with
          | F234 →
              printf "a_hhz_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf1 p1 wf2 p2 wf3 p3
          | F243 →
              printf "a_hhz_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf1 p1 wf3 p3 wf2 p2
          | F342 →
              printf "a_hhz_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf3 p3 wf1 p1 wf2 p2
          | F324 →
              printf "a_hhz_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf2 p2 wf1 p1 wf3 p3
          | F423 →
              printf "a_hhz_PB(%s,%s,%s,%s,%s,%s,%s)"
                  c wf2 p2 wf3 p3 wf1 p1
          | F432 →
              printf "a_hhz_PB(%s,%s,%s,%s,%s,%s,%s)"
```

```
                      c wf3 p3 wf2 p2 wf1 p1
        | F124 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf1 p1 wf2 p2 wf3 p3
        | F142 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf1 p1 wf3 p3 wf2 p2
        | F241 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf3 p3 wf1 p1 wf2 p2
        | F214 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf2 p2 wf1 p1 wf3 p3
        | F412 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf2 p2 wf3 p3 wf1 p1
        | F421 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf3 p3 wf2 p2 wf1 p1
        | F134 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf1 p1 wf2 p2 wf3 p3
        | F143 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf1 p1 wf3 p3 wf2 p2
        | F341 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf3 p3 wf1 p1 wf2 p2
        | F314 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf2 p2 wf1 p1 wf3 p3
        | F413 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf2 p2 wf3 p3 wf1 p1
        | F431 →
              printf "h_ahz_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf3 p3 wf2 p2 wf1 p1
        | F123 →
              printf "z_ahh_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf1 p1 wf2 p2 wf3 p3
        | F132 →
              printf "z_ahh_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf1 p1 wf3 p3 wf2 p2
        | F231 →
              printf "z_ahh_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf3 p3 wf1 p1 wf2 p2
        | F213 →
              printf "z_ahh_PB(%s,%s,%s,%s,%s,%s,%s)"
                      c wf2 p2 wf1 p1 wf3 p3
        | F312 →
```

```
                        printf "z_ahh_PB(%s,%s,%s,%s,%s,%s,%s)"
                              c wf2 p2 wf3 p3 wf1 p1
                  | F321 →
                        printf "z_ahh_PB(%s,%s,%s,%s,%s,%s,%s)"
                              c wf3 p3 wf2 p2 wf1 p1
            end
    | Dim6_Scalar2_Vector2_PB coeff →
        let c = format_coupling coeff c in
              begin match fusion with
              | F234 | F134 →
                    printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf1 p1 wf2 p2 wf3 p3
              | F342 | F341 →
                    printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf3 p3 wf1 p1 wf2 p2
              | F423 | F413 →
                    printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf3 p3 wf1 p1
              | F243 | F143 →
                    printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf1 p1 wf3 p3 wf2 p2
              | F324 | F314 →
                    printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf1 p1 wf3 p3
              | F432 | F431 →
                    printf "h_hvv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf3 p3 wf2 p2 wf1 p1
              | F123 | F124 →
                    printf "v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf1 p1 wf2 p2 wf3 p3
              | F231 | F241 →
                    printf "v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf3 p3 wf1 p1 wf2 p2
              | F312 | F412 →
                    printf "v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf3 p3 wf1 p1
              | F132 | F142 →
                    printf "v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf1 p1 wf3 p3 wf2 p2
              | F213 | F214 →
                    printf "v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf2 p2 wf1 p1 wf3 p3
              | F321 | F421 →
                    printf "v_hhv_PB(%s,%s,%s,%s,%s,%s,%s)"
                          c wf3 p3 wf2 p2 wf1 p1
            end
```

In principle, $p4$ could be obtained from the left hand side ...

```
    | DScalar4 contractions →
```

```
                  let p123  =  Printf.sprintf "(-%s-%s-%s)" p1 p2 p3 in
                  begin match contractions with
                  | [] → invalid_arg "Targets.print_current:␣DScalar4␣[]"
                  | head :: tail →
                      printf "(";
                      print_dscalar4 c wf1 wf2 wf3 p1 p2 p3 p123 fusion head;
                      List.iter (print_add_dscalar4
                                      c wf1 wf2 wf3 p1 p2 p3 p123 fusion) tail;
                      printf ")"
                  end
            |  DScalar2_Vector2 contractions →
                  let p123  =  Printf.sprintf "(-%s-%s-%s)" p1 p2 p3 in
                  begin match contractions with
                  | [] → invalid_arg "Targets.print_current:␣DScalar4␣[]"
                  | head :: tail →
                      printf "(";
                      print_dscalar2_vector2
                        c wf1 wf2 wf3 p1 p2 p3 p123 fusion head;
                      List.iter (print_add_dscalar2_vector2
                                      c wf1 wf2 wf3 p1 p2 p3 p123 fusion) tail;
                      printf ")"
                  end
            end
      |  Vn (_, _, _) →
            invalid_arg "Targets.print_current:␣n-ary␣fusion"

let print_propagator f p m gamma =
  let minus_third  =  "(-1.0_" ^ !kind ^ "/3.0_" ^ !kind ^ ")" in
  let w =
    begin match CM.width f with
    | Vanishing | Fudged → "0.0_" ^ !kind
    | Constant → gamma
    | Timelike → "wd_tl(" ^ p ^ "," ^ gamma ^ ")"
    | Running →
        failwith "Targets.Fortran:␣running␣width␣not␣yet␣available"
    | Custom f → f ^ "(" ^ p ^ "," ^ gamma ^ ")"
    end in
  match CM.propagator f with
  | Prop_Scalar →
      printf "pr_phi(%s,%s,%s," p m w
  | Prop_Col_Scalar →
      printf "%s␣*␣pr_phi(%s,%s,%s," minus_third p m w
  | Prop_Ghost → printf "(0,1)␣*␣pr_phi(%s,␣%s,␣%s," p m w
  | Prop_Spinor →
      printf "%s(%s,%s,%s," Fermions.psi_propagator p m w
  | Prop_ConjSpinor →
      printf "%s(%s,%s,%s," Fermions.psibar_propagator p m w
  | Prop_Majorana →
      printf "%s(%s,%s,%s," Fermions.chi_propagator p m w
  | Prop_Col_Majorana →
```

```
        printf "%s␣*␣%s(%s,%s,%s," minus_third Fermions.chi_propagator p m w
    | Prop_Unitarity →
        printf "pr_unitarity(%s,%s,%s," p m w
    | Prop_Col_Unitarity →
        printf "%s␣*␣pr_unitarity(%s,%s,%s," minus_third p m w
    | Prop_Feynman →
        printf "pr_feynman(%s," p
    | Prop_Col_Feynman →
        printf "%s␣*␣pr_feynman(%s," minus_third p
    | Prop_Gauge xi →
        printf "pr_gauge(%s,%s," p (CM.gauge_symbol xi)
    | Prop_Rxi xi →
        printf "pr_rxi(%s,%s,%s,%s," p m w (CM.gauge_symbol xi)
    | Prop_Tensor_2 →
        printf "pr_tensor(%s,%s,%s," p m w
    | Prop_Tensor_pure →
        printf "pr_tensor_pure(%s,%s,%s," p m w
    | Prop_Vector_pure →
        printf "pr_vector_pure(%s,%s,%s," p m w
    | Prop_Vectorspinor →
        printf "pr_grav(%s,%s,%s," p m w
    | Aux_Scalar | Aux_Spinor | Aux_ConjSpinor | Aux_Majorana
    | Aux_Vector | Aux_Tensor_1 → printf "("
    | Aux_Col_Scalar | Aux_Col_Vector | Aux_Col_Tensor_1 →
printf "%s␣*␣(" minus_third
    | Only_Insertion → printf "("

let print_projector f p m gamma =
    let minus_third = "(-1.0_" ^ !kind ^ "/3.0_" ^ !kind ^ ")" in
    match CM.propagator f with
    | Prop_Scalar →
        printf "pj_phi(%s,%s," m gamma
    | Prop_Col_Scalar →
        printf "%s␣*␣pj_phi(%s,%s," minus_third m gamma
    | Prop_Ghost →
        printf "(0,1)␣*␣pj_phi(%s,%s," m gamma
    | Prop_Spinor →
        printf "%s(%s,%s,%s," Fermions.psi_projector p m gamma
    | Prop_ConjSpinor →
        printf "%s(%s,%s,%s," Fermions.psibar_projector p m gamma
    | Prop_Majorana →
        printf "%s(%s,%s,%s," Fermions.chi_projector p m gamma
    | Prop_Col_Majorana →
        printf "%s␣*␣%s(%s,%s,%s," minus_third Fermions.chi_projector p m gamma
    | Prop_Unitarity →
        printf "pj_unitarity(%s,%s,%s," p m gamma
    | Prop_Col_Unitarity →
        printf "%s␣*␣pj_unitarity(%s,%s,%s," minus_third p m gamma
    | Prop_Feynman | Prop_Col_Feynman →
        invalid_arg "no␣on-shell␣Feynman␣propagator!"
```

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```
        | Prop_Gauge _ →
            invalid_arg "no on-shell massless gauge propagator!"
        | Prop_Rxi _ →
            invalid_arg "no on-shell Rxi propagator!"
        | Prop_Vectorspinor →
            printf "pj_grav(%s,%s,%s," p m gamma
        | Prop_Tensor_2 →
            printf "pj_tensor(%s,%s,%s," p m gamma
        | Prop_Tensor_pure →
            invalid_arg "no on-shell pure Tensor propagator!"
        | Prop_Vector_pure →
            invalid_arg "no on-shell pure Vector propagator!"
        | Aux_Scalar | Aux_Spinor | Aux_ConjSpinor | Aux_Majorana
        | Aux_Vector | Aux_Tensor_1 → printf "("
        | Aux_Col_Scalar | Aux_Col_Vector | Aux_Col_Tensor_1 →
printf "%s * (" minus_third
        | Only_Insertion → printf "("

    let print_gauss f p m gamma =
        let minus_third = "(-1.0_" ^ !kind ^ "/3.0_" ^ !kind ^ ")" in
        match CM.propagator f with
        | Prop_Scalar →
            printf "pg_phi(%s,%s,%s," p m gamma
        | Prop_Ghost →
            printf "(0,1) * pg_phi(%s,%s,%s," p m gamma
        | Prop_Spinor →
            printf "%s(%s,%s,%s," Fermions.psi_projector p m gamma
        | Prop_ConjSpinor →
            printf "%s(%s,%s,%s," Fermions.psibar_projector p m gamma
        | Prop_Majorana →
            printf "%s(%s,%s,%s," Fermions.chi_projector p m gamma
        | Prop_Col_Majorana →
            printf "%s * %s(%s,%s,%s," minus_third Fermions.chi_projector p m gamma
        | Prop_Unitarity →
            printf "pg_unitarity(%s,%s,%s," p m gamma
        | Prop_Feynman | Prop_Col_Feynman →
            invalid_arg "no on-shell Feynman propagator!"
        | Prop_Gauge _ →
            invalid_arg "no on-shell massless gauge propagator!"
        | Prop_Rxi _ →
            invalid_arg "no on-shell Rxi propagator!"
        | Prop_Tensor_2 →
            printf "pg_tensor(%s,%s,%s," p m gamma
        | Prop_Tensor_pure →
            invalid_arg "no pure tensor propagator!"
        | Prop_Vector_pure →
            invalid_arg "no pure vector propagator!"
        | Aux_Scalar | Aux_Spinor | Aux_ConjSpinor | Aux_Majorana
        | Aux_Vector | Aux_Tensor_1 → printf "("
        | Only_Insertion → printf "("
```

   | _ → *invalid_arg* `"targets:print_gauss:␣not␣available"`

let *print_fusion_diagnostics amplitude dictionary fusion* =
 if *warn diagnose_gauge* then begin
  let *lhs* = *F.lhs fusion* in
  let *f* = *F.flavor lhs*
  and *v* = *variable lhs*
  and *p* = *momentum lhs* in
  let *mass* = *CM.mass_symbol f* in
  match *CM.propagator f* with
  | *Prop_Gauge _* | *Prop_Feynman*
  | *Prop_Rxi _* | *Prop_Unitarity* →
   printf `"␣␣␣␣␣␣@[<2>%s␣="` *v*;
   List.iter (*print_current amplitude dictionary*) (*F.rhs fusion*); *nl* ();
   begin match *CM.goldstone f* with
   | *None* →
    printf `"␣␣␣␣␣␣call␣omega_ward_%s(\"%s\",%s,%s,%s)"`
     (*suffix diagnose_gauge*) *v mass p v*; *nl* ()
   | *Some* (*g, phase*) →
    let *gv* = *add_tag lhs* (*CM.flavor_symbol g* ^ `"_"` ^ *format_p lhs*) in
    printf `"␣␣␣␣␣␣call␣omega_slavnov_%s"`
     (*suffix diagnose_gauge*);
    printf `"(@[\"%s\",%s,%s,%s,@,%s*%s)"`
     v mass p v (*format_constant phase*) *gv*; *nl* ()
   end
  | _ → ()
 end

let *print_fusion amplitude dictionary fusion* =
 let *lhs* = *F.lhs fusion* in
 let *f* = *F.flavor lhs* in
 printf `"␣␣␣␣␣␣@[<2>%s␣=@,␣"` (*multiple_variable amplitude dictionary lhs*);
 if *F.on_shell amplitude lhs* then
  print_projector f (*momentum lhs*)
   (*CM.mass_symbol f*) (*CM.width_symbol f*)
 else
  if *F.is_gauss amplitude lhs* then
   print_gauss f (*momentum lhs*)
    (*CM.mass_symbol f*) (*CM.width_symbol f*)
  else
   print_propagator f (*momentum lhs*)
    (*CM.mass_symbol f*) (*CM.width_symbol f*);
 List.iter (*print_current amplitude dictionary*) (*F.rhs fusion*);
 printf `")"`; *nl* ()

let *print_momenta seen_momenta amplitude* =
 List.fold_left (fun *seen f* →
  let *wf* = *F.lhs f* in
  let *p* = *F.momentum_list wf* in
  if ¬ (*PSet.mem p seen*) then begin
   let *rhs1* = *List.hd* (*F.rhs f*) in
   printf `"␣␣␣␣%s␣=␣%s"` (*momentum wf*)

```
        (String.concat "␣+␣"
            (List.map momentum (F.children rhs1))); nl ()
    end;
    PSet.add p seen)
    seen_momenta (F.fusions amplitude)

let print_fusions dictionary fusions =
  List.iter
    (fun (f, amplitude) →
      print_fusion_diagnostics amplitude dictionary f;
      print_fusion amplitude dictionary f)
    fusions

let print_braket amplitude dictionary name braket =
  let bra = F.bra braket
  and ket = F.ket braket in
  printf "␣␣␣␣␣␣␣@[<2>%s␣=␣%s@,␣+␣" name name;
  begin match Fermions.reverse_braket (CM.lorentz (F.flavor bra)) with
  | false →
      printf "%s*(@," (multiple_variable amplitude dictionary bra);
      List.iter (print_current amplitude dictionary) ket;
      printf ")"
  | true →
      printf "(@,";
      List.iter (print_current amplitude dictionary) ket;
      printf ")*%s" (multiple_variable amplitude dictionary bra)
  end; nl ()
```

$$iT = i^{\#\mathrm{vertices}} i^{\#\mathrm{propagators}} \cdots = i^{n-2} i^{n-3} \cdots = -i(-1)^n \cdots \tag{15.3}$$

tho : we write some brakets twice using different names. Is it useful to cache them?

```
let print_brakets dictionary amplitude =
  let name = flavors_symbol (flavors amplitude) in
  printf "␣␣␣␣␣␣%s␣=␣0" name; nl ();
  List.iter (print_braket amplitude dictionary name) (F.brakets amplitude);
  let n = List.length (F.externals amplitude) in
  if n mod 2 = 0 then begin
    printf "␣␣␣␣␣␣␣@[<2>%s␣=@,␣-␣%s␣!␣%d␣vertices,␣%d␣propagators"
      name name (n − 2) (n − 3); nl ()
  end else begin
    printf "␣␣␣␣␣␣␣!␣%s␣=␣%s␣!␣%d␣vertices,␣%d␣propagators"
      name name (n − 2) (n − 3); nl ()
  end;
  let s = F.symmetry amplitude in
  if s > 1 then
    printf "␣␣␣␣␣␣␣@[<2>%s␣=@,␣%s@,␣/␣sqrt(%d.0_%s)␣!␣symmetry␣factor" name name s !kind
  else
    printf "␣␣␣␣␣␣␣!␣unit␣symmetry␣factor";
```

nl ()

let *print_incoming wf* =
  let *p* = *momentum wf*
  and *s* = *spin wf*
  and *f* = *F.flavor wf* in
  let *m* = *CM.mass_symbol f* in
  match *CM.lorentz f* with
  | *Scalar* → *printf* "1"
  | *BRS Scalar* → *printf* "(0,-1)␣*␣(%s␣*␣%s␣-␣%s**2)" *p p m*
  | *Spinor* →
    printf "%s␣(%s,␣-␣%s,␣%s)" *Fermions.psi_incoming m p s*
  | *BRS Spinor* →
    printf "%s␣(%s,␣-␣%s,␣%s)" *Fermions.brs_psi_incoming m p s*
  | *ConjSpinor* →
    printf "%s␣(%s,␣-␣%s,␣%s)" *Fermions.psibar_incoming m p s*
  | *BRS ConjSpinor* →
    printf "%s␣(%s,␣-␣%s,␣%s)" *Fermions.brs_psibar_incoming m p s*
  | *Majorana* →
    printf "%s␣(%s,␣-␣%s,␣%s)" *Fermions.chi_incoming m p s*
  | *Maj_Ghost* → *printf* "ghost␣(%s,␣-␣%s,␣%s)" *m p s*
  | *BRS Majorana* →
    printf "%s␣(%s,␣-␣%s,␣%s)" *Fermions.brs_chi_incoming m p s*
  | *Vector* | *Massive_Vector* →
    printf "eps␣(%s,␣-␣%s,␣%s)" *m p s*
  | *BRS Vector* | *BRS Massive_Vector* → *printf*
    "(0,1)␣*␣(%s␣*␣%s␣-␣%s**2)␣*␣eps␣(%s,␣-%s,␣%s)" *p p m m p s*
  | *Vectorspinor* | *BRS Vectorspinor* →
    printf "%s␣(%s,␣-␣%s,␣%s)" *Fermions.grav_incoming m p s*
  | *Tensor_1* → *invalid_arg* "Tensor_1␣only␣internal"
  | *Tensor_2* → *printf* "eps2␣(%s,␣-␣%s,␣%s)" *m p s*
  | _ → *invalid_arg* "no␣such␣BRST␣transformations"

let *print_outgoing wf* =
  let *p* = *momentum wf*
  and *s* = *spin wf*
  and *f* = *F.flavor wf* in
  let *m* = *CM.mass_symbol f* in
  match *CM.lorentz f* with
  | *Scalar* → *printf* "1"
  | *BRS Scalar* → *printf* "(0,-1)␣*␣(%s␣*␣%s␣-␣%s**2)" *p p m*
  | *Spinor* →
    printf "%s␣(%s,␣%s,␣%s)" *Fermions.psi_outgoing m p s*
  | *BRS Spinor* →
    printf "%s␣(%s,␣%s,␣%s)" *Fermions.brs_psi_outgoing m p s*
  | *ConjSpinor* →
    printf "%s␣(%s,␣%s,␣%s)" *Fermions.psibar_outgoing m p s*
  | *BRS ConjSpinor* →
    printf "%s␣(%s,␣%s,␣%s)" *Fermions.brs_psibar_outgoing m p s*
  | *Majorana* →
    printf "%s␣(%s,␣%s,␣%s)" *Fermions.chi_outgoing m p s*

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```
| BRS Majorana →
    printf "%s␣(%s,␣%s,␣%s)" Fermions.brs_chi_outgoing m p s
| Maj_Ghost → printf "ghost␣(%s,␣%s,␣%s)" m p s
| Vector | Massive_Vector →
    printf "conjg␣(eps␣(%s,␣%s,␣%s))" m p s
| BRS Vector | BRS Massive_Vector → printf
    "(0,1)␣*␣(%s*%s-%s**2)␣*␣(conjg␣(eps(%s,␣%s,␣%s)))" p p m m p s
| Vectorspinor | BRS Vectorspinor →
    printf "%s␣(%s,␣%s,␣%s)" Fermions.grav_incoming m p s
| Tensor_1 → invalid_arg "Tensor_1␣only␣internal"
| Tensor_2 → printf "conjg␣(eps2␣(%s,␣%s,␣%s))" m p s
| BRS _ → invalid_arg "no␣such␣BRST␣transformations"
```

let *print_external_momenta amplitude* =
  let *externals* =
    List.combine
      (*F.externals amplitude*)
      (*List.map* (fun _ → true) (*F.incoming amplitude*) @
       List.map (fun _ → false) (*F.outgoing amplitude*)) in
  List.iter (fun (*wf*, *incoming*) →
    if *incoming* then
      printf "␣␣␣␣%s␣=␣-␣k(:,%d)␣!␣incoming"
        (*momentum wf*) (*ext_momentum wf*)
    else
      printf "␣␣␣␣%s␣=␣␣␣k(:,%d)␣!␣outgoing"
        (*momentum wf*) (*ext_momentum wf*); *nl* ()) *externals*

let *print_externals seen_wfs amplitude* =
  let *externals* =
    List.combine
      (*F.externals amplitude*)
      (*List.map* (fun _ → true) (*F.incoming amplitude*) @
       List.map (fun _ → false) (*F.outgoing amplitude*)) in
  List.fold_left (fun *seen* (*wf*, *incoming*) →
    if ¬ (*WFSet.mem wf seen*) then begin
      printf "␣␣␣␣␣␣␣@[<2>%s␣=@,␣" (*variable wf*);
      (if *incoming* then *print_incoming* else *print_outgoing*) *wf*; *nl* ()
    end;
    WFSet.add wf seen) *seen_wfs externals*

let *flavors_sans_color_to_string flavors* =
  String.concat "␣" (*List.map M.flavor_to_string flavors*)

let *process_sans_color_to_string* (*fin*, *fout*) =
  flavors_sans_color_to_string fin ^ "␣->␣" ^
  flavors_sans_color_to_string fout

let *print_fudge_factor amplitude* =
  let *name* = *flavors_symbol* (*flavors amplitude*) in
  List.iter (fun *wf* →
    let *p* = *momentum wf*
    and *f* = *F.flavor wf* in
    match *CM.width f* with

```
    |  Fudged  →
        let m  =  CM.mass_symbol f
        and w  =  CM.width_symbol f in
        printf "␣␣␣␣␣␣␣if␣(%s␣>␣0.0_%s)␣then" w !kind; nl ();
        printf "␣␣␣␣␣␣␣␣␣@[<2>%s␣=␣%s@␣*␣(%s*%s␣-␣%s**2)"
            name name p p m;
        printf "@␣/␣cmplx␣(%s*%s␣-␣%s**2,␣%s*%s,␣kind=%s)"
            p p m m w !kind; nl ();
        printf "␣␣␣␣␣␣␣end␣if"; nl ()
    |  _  →  ()) (F.s_channel amplitude)

let num_helicities amplitudes  =
    List.length (CF.helicities amplitudes)
```

Spin, Flavor & Color Tables

The following abomination is required to keep the number of continuation lines
as low as possible. FORTRAN77-style `DATA` statements are actually a bit nicer
here, but they are nor available for *constant* arrays.

We used to have a more elegant design with a sentinel 0 added to each
initializer, but some revisions of the Compaq/Digital Compiler have a bug
that causes it to reject this variant.

The actual table writing code using `reshape` should be factored, since it's
the same algorithm every time.

```
let print_integer_parameter name value  =
    printf "␣␣@[<2>integer,␣parameter␣::␣%s␣=␣%d" name value; nl ()

let print_real_parameter name value  =
    printf "␣␣@[<2>real(kind=%s),␣parameter␣::␣%s␣=␣%d"
        !kind name value; nl ()

let print_logical_parameter name value  =
    printf "␣␣@[<2>logical,␣parameter␣::␣%s␣=␣.%s."
        name (if value then "true" else "false"); nl ()

let num_particles_in amplitudes  =
    match CF.flavors amplitudes with
    | []  →  0
    | (fin, _) :: _  →  List.length fin

let num_particles_out amplitudes  =
    match CF.flavors amplitudes with
    | []  →  0
    | (_, fout) :: _  →  List.length fout

let num_particles amplitudes  =
    match CF.flavors amplitudes with
    | []  →  0
    | (fin, fout) :: _  →  List.length fin + List.length fout
```

module *CFlow* = *Color.Flow*

let *num_color_flows amplitudes* =
  List.length (*CF.color_flows amplitudes*)

let *num_color_indices_default* = 2 (∗ Standard model ∗)

let *num_color_indices amplitudes* =
  try *CFlow.rank* (*List.hd* (*CF.color_flows amplitudes*)) with _ → *num_color_indices_default*

let *color_to_string c* =
  "(" ^ (*String.concat* "," (*List.map* (*Printf.sprintf* "%3d") *c*)) ^ ")"

let *cflow_to_string cflow* =
  String.concat "␣" (*List.map color_to_string* (*CFlow.in_to_lists cflow*)) ^ "␣->␣" ^
  String.concat "␣" (*List.map color_to_string* (*CFlow.out_to_lists cflow*))

let *protected* = ",␣protected" (∗ Fortran 2003! ∗)

let *print_spin_table name tuples* =
  printf "␣␣@[<2>integer,␣dimension(n_prt,n_hel),␣save%s␣::␣table_spin_%s"
    protected name; *nl* ();
  match *tuples* with
  | [] → ()
  | _ →
      ignore (*List.fold_left* (fun *i* (*tuple1, tuple2*) →
        printf "␣␣@[<2>data␣table_spin_%s(:,%4d)␣/␣%s␣/" *name i*
          (*String.concat* ",␣" (*List.map* (*Printf.sprintf* "%2d") (*tuple1* @ *tuple2*)));
        nl (); *succ i*) 1 *tuples*)

let *print_spin_tables amplitudes* =
  (∗ *print_spin_table_old* "s" "states_old" (*CF.helicities amplitudes*);
∗)
  print_spin_table "states" (*CF.helicities amplitudes*);
  nl ()

let *print_flavor_table name tuples* =
  printf "␣␣@[<2>integer,␣dimension(n_prt,n_flv),␣save%s␣::␣table_flavor_%s"
    protected name; *nl* ();
  match *tuples* with
  | [] → ()
  | _ →
      ignore (*List.fold_left* (fun *i tuple* →
        printf "␣␣@[<2>data␣table_flavor_%s(:,%4d)␣/␣%s␣/␣!␣%s" *name i*
          (*String.concat* ",␣"
             (*List.map* (fun *f* → *Printf.sprintf* "%3d" (*M.pdg f*)) *tuple*))
          (*String.concat* "␣" (*List.map M.flavor_to_string tuple*));
        nl (); *succ i*) 1 *tuples*)

let *print_flavor_tables amplitudes* =
  (∗ let *n* = *num_particles amplitudes* in ∗)
  (∗ *print_flavor_table_old n* "f" "states_old" (*List.map* (fun (*fin, fout*) →
fin @ *fout*) (*CF.flavors amplitudes*)); ∗)
  print_flavor_table "states"
    (*List.map* (fun (*fin, fout*) → *fin* @ *fout*) (*CF.flavors amplitudes*));

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nl ()

let *num_flavors amplitudes* =
  List.length (*CF.flavors amplitudes*)

let *print_color_flows_table tuples* =
  printf
    "␣␣@[<2>integer,␣dimension(n_cindex,n_prt,n_cflow),␣save%s␣::␣table_color_flows"
    protected; *nl* ();
  match *tuples* with
  | [] → ()
  | _ :: _ as *tuples* →
      ignore (*List.fold_left* (fun *i tuple* →
        begin match *CFlow.to_lists tuple* with
        | [] → ()
        | *cf1* :: *cfn* →
            printf "␣␣@[<2>data␣table_color_flows(:,:,%4d)␣/" *i*;
            printf "@␣%s" (*String.concat* "," (*List.map string_of_int cf1*));
            List.iter (function *cf* →
              printf ",@␣␣%s" (*String.concat* "," (*List.map string_of_int cf*))) *cfn*;
            printf "@␣/"; *nl* ()
        end;
        succ i) 1 *tuples*)

let *print_ghost_flags_table tuples* =
  printf
    "␣␣@[<2>logical,␣dimension(n_prt,n_cflow),␣save%s␣::␣table_ghost_flags"
    protected; *nl* ();
  match *tuples* with
  | [] → ()
  | _ →
      ignore (*List.fold_left* (fun *i tuple* →
        begin match *CFlow.ghost_flags tuple* with
        | [] → ()
        | *gf1* :: *gfn* →
            printf "␣␣@[<2>data␣table_ghost_flags(:,%4d)␣/" *i*;
            printf "@␣%s" (if *gf1* then "T" else "F");
            List.iter (function *gf* → *printf* ",@␣␣%s" (if *gf* then "T" else "F")) *gfn*;
            printf "␣/";
            nl ()
        end;
        succ i) 1 *tuples*)

let *format_power_of x*
    { *Color.Flow.num* = *num*; *Color.Flow.den* = *den*; *Color.Flow.power* = *pwr* } =
  match *num*, *den*, *pwr* with
  | _, 0, _ → *invalid_arg* "format_power_of:␣zero␣denominator"
  | 0, _, _ → "+zero"
  | 1, 1, 0 | −1, −1, 0 → "+one"
  | −1, 1, 0 | 1, −1, 0 → "-one"
  | 1, 1, 1 | −1, −1, 1 → "+" ^ *x*
  | −1, 1, 1 | 1, −1, 1 → "-" ^ *x*

```
| 1, 1, −1 | −1, −1, −1 → "+1/" ^ x
| −1, 1, −1 | 1, −1, −1 → "-1/" ^ x
| 1, 1, p | −1, −1, p →
    "+" ^ (if p > 0 then "" else "1/") ^ x ^ "**" ^ string_of_int (abs p)
| −1, 1, p | 1, −1, p →
    "-" ^ (if p > 0 then "" else "1/") ^ x ^ "**" ^ string_of_int (abs p)
| n, 1, 0 →
    (if n < 0 then "-" else "+") ^ string_of_int (abs n) ^ ".0_" ^ !kind
| n, d, 0 →
    (if n × d < 0 then "-" else "+") ^
    string_of_int (abs n) ^ ".0_" ^ !kind ^ "/" ^
    string_of_int (abs d)
| n, 1, 1 →
    (if n < 0 then "-" else "+") ^ string_of_int (abs n) ^ "*" ^ x
| n, 1, −1 →
    (if n < 0 then "-" else "+") ^ string_of_int (abs n) ^ "/" ^ x
| n, d, 1 →
    (if n × d < 0 then "-" else "+") ^
    string_of_int (abs n) ^ ".0_" ^ !kind ^ "/" ^
    string_of_int (abs d) ^ "*" ^ x
| n, d, −1 →
    (if n × d < 0 then "-" else "+") ^
    string_of_int (abs n) ^ ".0_" ^ !kind ^ "/" ^
    string_of_int (abs d) ^ "/" ^ x
| n, 1, p →
    (if n < 0 then "-" else "+") ^ string_of_int (abs n) ^
    (if p > 0 then "*" else "/") ^ x ^ "**" ^ string_of_int (abs p)
| n, d, p →
    (if n × d < 0 then "-" else "+") ^
    string_of_int (abs n) ^ ".0_" ^ !kind ^ "/" ^
    string_of_int (abs d) ^
    (if p > 0 then "*" else "/") ^ x ^ "**" ^ string_of_int (abs p)

let format_powers_of x = function
    | [] → "zero"
    | powers → String.concat "" (List.map (format_power_of x) powers)
```

We can optimize the following slightly by reusing common color factor *parameter*s.

```
let print_color_factor_table table =
    let n_cflow = Array.length table in
    let n_cfactors = ref 0 in
    for c1 = 0 to pred n_cflow do
        for c2 = 0 to pred n_cflow do
            match table.(c1).(c2) with
            | [] → ()
            | _ → incr n_cfactors
        done
    done;
```

```
    print_integer_parameter "n_cfactors" !n_cfactors;
    printf "␣␣@[<2>type(%s),␣dimension(n_cfactors),␣save%s␣::"
      omega_color_factor_abbrev protected;
    printf "@␣table_color_factors"; nl ();
    let i = ref 1 in
    if n_cflow > 0 then begin
      for c1 = 0 to pred n_cflow do
        for c2 = 0 to pred n_cflow do
          match table.(c1).(c2) with
          | [] → ()
          | cf →
              printf "␣␣@[<2>real(kind=%s),␣parameter,␣private␣::␣color_factor_%06d␣=␣%s"
                !kind !i (format_powers_of nc_parameter cf);
              nl ();
              printf "␣␣@[<2>data␣table_color_factors(%6d)␣/␣%s(%d,%d,color_factor_%06d)␣␣
                !i omega_color_factor_abbrev (succ c1) (succ c2) !i;
              incr i;
              nl ();
        done
      done
    end

let print_color_tables amplitudes =
  let cflows = CF.color_flows amplitudes
  and cfactors = CF.color_factors amplitudes in
  (* print_color_flows_table_old "c" cflows; nl (); *)
  print_color_flows_table cflows; nl ();
  (* print_ghost_flags_table_old "g" cflows; nl (); *)
  print_ghost_flags_table cflows; nl ();
  (* print_color_factor_table_old cfactors; nl (); *)
  print_color_factor_table cfactors; nl ()

let option_to_logical = function
  | Some _ → "T"
  | None → "F"

let print_flavor_color_table n_flv n_cflow table =
  printf
    "␣␣@[<2>logical,␣dimension(n_flv,␣n_cflow),␣save%s␣::␣@␣flv_col_is_allowed"
    protected; nl ();
  if n_flv > 0 then begin
    for c = 0 to pred n_cflow do
      printf
        "␣␣@[<2>data␣flv_col_is_allowed(:,%4d)␣/" (succ c);
      printf "@␣%s" (option_to_logical table.(0).(c));
      for f = 1 to pred n_flv do
        printf ",@␣%s" (option_to_logical table.(f).(c))
      done;
      printf "@␣/"; nl ()
    done;
  end
```

let *print_amplitude_table a* =
  (∗ *print_flavor_color_table_old* "a" (*num_flavors a*) (*List.length* (*CF.color_flows a*)) (*CF.process_ta*
∗)
  print_flavor_color_table
    (*num_flavors a*) (*List.length* (*CF.color_flows a*)) (*CF.process_table a*);
  nl ();
  printf
    "␣␣@[<2>complex(kind=%s),␣dimension(n_flv,␣n_cflow,␣n_hel),␣save␣::␣amp" !*kind*;
  nl ();
  nl ()

let *print_helicity_selection_table* () =
  printf "␣␣␣@[<2>logical,␣dimension(n_hel),␣save␣::␣";
  printf "hel_is_allowed␣=␣T"; *nl* ();
  printf "␣␣␣@[<2>real(kind=%s),␣dimension(n_hel),␣save␣::␣" !*kind*;
  printf "hel_max_abs␣=␣0"; *nl* ();
  printf "␣␣␣@[<2>real(kind=%s),␣save␣::␣" !*kind*;
  printf "hel_sum_abs␣=␣0,␣";
  printf "hel_threshold␣=␣1E10"; *nl* ();
  printf "␣␣␣@[<2>integer,␣save␣::␣";
  printf "hel_count␣=␣0,␣";
  printf "hel_cutoff␣=␣100"; *nl* ();
  printf "␣␣␣@[<2>integer␣::␣";
  printf "i"; *nl* ();
  printf "␣␣␣@[<2>integer,␣save,␣dimension(n_hel)␣::␣";
  printf "hel_map␣=␣(/(i,␣i␣=␣1,␣n_hel)/)"; *nl* ();
  printf "␣␣␣@[<2>integer,␣save␣::␣hel_finite␣=␣n_hel"; *nl* ();
  nl ()

Optional MD5 sum function

let *print_md5sum_functions* = function
  | *Some s* →
      printf "␣␣@[<5>"; if !*fortran95* then *printf* "pure␣";
      printf "function␣md5sum␣()"; *nl* ();
      printf "␣␣␣␣character(len=32)␣::␣md5sum"; *nl* ();
      printf "␣␣␣␣␣!␣DON'T␣EVEN␣THINK␣of␣modifying␣the␣following␣line!"; *nl* ();
      printf "␣␣␣␣md5sum␣=␣\"%s\"" *s*; *nl* ();
      printf "␣␣end␣function␣md5sum"; *nl* ();
      nl ()
  | *None* → ()

Maintenance & Inquiry Functions

let *print_maintenance_functions* () =
  if !*whizard* then begin
    printf "␣␣subroutine␣init␣(par)"; *nl* ();
    printf "␣␣␣␣real(kind=%s),␣dimension(*),␣intent(in)␣::␣par" !*kind*; *nl* ();
    printf "␣␣␣␣call␣import_from_whizard␣(par)"; *nl* ();

```
      printf "␣␣end␣subroutine␣init"; nl ();
      nl ();
      printf "␣␣subroutine␣final␣()"; nl ();
      printf "␣␣end␣subroutine␣final"; nl ();
      nl ();
      printf "␣␣subroutine␣update_alpha_s␣(alpha_s)"; nl ();
      printf "␣␣␣␣␣real(kind=%s),␣intent(in)␣::␣alpha_s" !kind; nl ();
      printf "␣␣␣␣␣call␣model_update_alpha_s␣(alpha_s)"; nl ();
      printf "␣␣end␣subroutine␣update_alpha_s"; nl ();
      nl ()
  end

let print_inquiry_function_openmp () = begin
  printf "␣␣pure␣function␣openmp_supported␣()␣result␣(status)"; nl ();
  printf "␣␣␣␣␣logical␣::␣status"; nl ();
  printf "␣␣␣␣␣status␣=␣%s" (if !openmp then ".true." else ".false."); nl ();
  printf "␣␣end␣function␣openmp_supported"; nl ();
  nl ()
end

let print_numeric_inquiry_functions (f, v) =
  printf "␣␣@[<5>"; if !fortran95 then printf "pure␣";
  printf "function␣%s␣()␣result␣(n)" f; nl ();
  printf "␣␣␣␣␣integer␣::␣n"; nl ();
  printf "␣␣␣␣␣n␣=␣%s" v; nl ();
  printf "␣␣end␣function␣%s" f; nl ();
  nl ()

let print_inquiry_functions name =
  printf "␣␣@[<5>"; if !fortran95 then printf "pure␣";
  printf "function␣number_%s␣()␣result␣(n)" name; nl ();
  printf "␣␣␣␣␣integer␣::␣n"; nl ();
  printf "␣␣␣␣␣n␣=␣size␣(table_%s,␣dim=2)" name; nl ();
  printf "␣␣end␣function␣number_%s" name; nl ();
  nl ();
  printf "␣␣@[<5>"; if !fortran95 then printf "pure␣";
  printf "subroutine␣%s␣(a)" name; nl ();
  printf "␣␣␣␣␣integer,␣dimension(:,:),␣intent(out)␣::␣a"; nl ();
  printf "␣␣␣␣␣a␣=␣table_%s" name; nl ();
  printf "␣␣end␣subroutine␣%s" name; nl ();
  nl ()

let print_color_flows () =
  printf "␣␣@[<5>"; if !fortran95 then printf "pure␣";
  printf "function␣number_color_indices␣()␣result␣(n)"; nl ();
  printf "␣␣␣␣␣integer␣::␣n"; nl ();
  printf "␣␣␣␣␣n␣=␣size␣(table_color_flows,␣dim=1)"; nl ();
  printf "␣␣end␣function␣number_color_indices"; nl ();
  nl ();
  printf "␣␣@[<5>"; if !fortran95 then printf "pure␣";
  printf "function␣number_color_flows␣()␣result␣(n)"; nl ();
  printf "␣␣␣␣␣integer␣::␣n"; nl ();
```

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  printf "␣␣␣␣n␣=␣size␣(table_color_flows,␣dim=3)"; *nl* ();
  printf "␣␣end␣function␣number_color_flows"; *nl* ();
  nl ();
  printf "␣␣@[<5>"; if !*fortran95* then *printf* "pure␣";
  printf "subroutine␣color_flows␣(a,␣g)"; *nl* ();
  printf "␣␣␣␣␣integer,␣dimension(:,:,:),␣intent(out)␣::␣a"; *nl* ();
  printf "␣␣␣␣␣logical,␣dimension(:,:),␣intent(out)␣::␣g"; *nl* ();
  printf "␣␣␣␣␣a␣=␣table_color_flows"; *nl* ();
  printf "␣␣␣␣␣g␣=␣table_ghost_flags"; *nl* ();
  printf "␣␣end␣subroutine␣color_flows"; *nl* ();
  nl ()

let *print_color_factors* () =
  printf "␣␣@[<5>"; if !*fortran95* then *printf* "pure␣";
  printf "function␣number_color_factors␣()␣result␣(n)"; *nl* ();
  printf "␣␣␣␣integer␣::␣n"; *nl* ();
  printf "␣␣␣␣␣n␣=␣size␣(table_color_factors)"; *nl* ();
  printf "␣␣end␣function␣number_color_factors"; *nl* ();
  nl ();
  printf "␣␣@[<5>"; if !*fortran95* then *printf* "pure␣";
  printf "subroutine␣color_factors␣(cf)"; *nl* ();
  printf "␣␣␣␣␣type(%s),␣dimension(:),␣intent(out)::␣cf"
   omega_color_factor_abbrev; *nl* ();
  printf "␣␣␣␣␣cf␣=␣table_color_factors"; *nl* ();
  printf "␣␣end␣subroutine␣color_factors"; *nl* ();
  nl ();
  printf "␣␣@[<5>"; if !*fortran95* ∧ *pure_unless_openmp* then *printf* "pure␣";
  printf "function␣color_sum␣(flv,␣hel)␣result␣(amp2)"; *nl* ();
  printf "␣␣␣␣integer,␣intent(in)␣::␣flv,␣hel"; *nl* ();
  printf "␣␣␣␣␣real(kind=%s)␣::␣amp2" !*kind*; *nl* ();
  printf "␣␣␣␣amp2␣=␣real␣(omega_color_sum(flv,␣hel,␣amp,␣table_color_factors))"; *nl* ();
  printf "␣␣end␣function␣color_sum"; *nl* ();
  nl ()

let *print_dispatch_functions* () =
  printf "␣␣@[<5>";
  printf "subroutine␣new_event␣(p)"; *nl* ();
  printf "␣␣␣␣real(kind=%s),␣dimension(0:3,*),␣intent(in)␣::␣p" !*kind*; *nl* ();
  printf "␣␣␣␣logical␣::␣mask_dirty"; *nl* ();
  printf "␣␣␣␣integer␣::␣hel"; *nl* ();
  printf "␣␣␣␣call␣calculate_amplitudes␣(amp,␣p,␣hel_is_allowed)"; *nl* ();
  printf "␣␣␣␣if␣((hel_threshold␣.gt.␣0)␣.and.␣(hel_count␣.le.␣hel_cutoff))␣then"; *nl* ();
  printf "␣␣␣␣␣␣call␣@[<3>omega_update_helicity_selection@␣(hel_count,␣@␣amp,␣@␣";
  printf "hel_max_abs,␣@␣hel_sum_abs,␣@␣hel_is_allowed,␣@␣hel_threshold,␣@␣hel_cutoff,␣@␣mask";
  printf "␣␣␣␣␣␣if␣(mask_dirty)␣then"; *nl* ();
  printf "␣␣␣␣␣␣␣␣hel_finite␣=␣0"; *nl* ();
  printf "␣␣␣␣␣␣␣␣do␣hel␣=␣1,␣n_hel"; *nl* ();
  printf "␣␣␣␣␣␣␣␣␣␣if␣(hel_is_allowed(hel))␣then"; *nl* ();
  printf "␣␣␣␣␣␣␣␣␣␣␣␣hel_finite␣=␣hel_finite␣+␣1"; *nl* ();
  printf "␣␣␣␣␣␣␣␣␣␣␣␣hel_map(hel_finite)␣=␣hel"; *nl* ();
  printf "␣␣␣␣␣␣␣␣␣␣end␣if"; *nl* ();

printf "␣␣␣␣␣␣␣␣end␣do"; _nl_ ();
printf "␣␣␣␣␣␣␣end␣if"; _nl_ ();
printf "␣␣␣␣␣end␣if"; _nl_ ();
printf "␣␣end␣subroutine␣new_event"; _nl_ ();
_nl_ ();
printf "␣␣@[<5>";
printf "subroutine␣reset_helicity_selection␣(threshold,␣cutoff)"; _nl_ ();
printf "␣␣␣␣␣real(kind=%s),␣intent(in)␣::␣threshold" !_kind_; _nl_ ();
printf "␣␣␣␣␣integer,␣intent(in)␣::␣cutoff"; _nl_ ();
printf "␣␣␣␣␣integer␣::␣i"; _nl_ ();
printf "␣␣␣␣␣hel_is_allowed␣=␣T"; _nl_ ();
printf "␣␣␣␣␣hel_max_abs␣=␣0"; _nl_ ();
printf "␣␣␣␣␣hel_sum_abs␣=␣0"; _nl_ ();
printf "␣␣␣␣␣hel_count␣=␣0"; _nl_ ();
printf "␣␣␣␣␣hel_threshold␣=␣threshold"; _nl_ ();
printf "␣␣␣␣␣hel_cutoff␣=␣cutoff"; _nl_ ();
printf "␣␣␣␣␣hel_map␣=␣(/(i,␣i␣=␣1,␣n_hel)/)"; _nl_ ();
printf "␣␣␣␣␣hel_finite␣=␣n_hel"; _nl_ ();
printf "␣␣end␣subroutine␣reset_helicity_selection"; _nl_ ();
_nl_ ();
printf "␣␣@[<5>"; if !_fortran95_ then _printf_ "pure␣";
printf "function␣is_allowed␣(flv,␣hel,␣col)␣result␣(yorn)"; _nl_ ();
printf "␣␣␣␣␣logical␣::␣yorn"; _nl_ ();
printf "␣␣␣␣␣integer,␣intent(in)␣::␣flv,␣hel,␣col"; _nl_ ();
printf "␣␣␣␣␣yorn␣=␣hel_is_allowed(hel)␣.and.␣";
printf "flv_col_is_allowed(flv,col)"; _nl_ ();
printf "␣␣end␣function␣is_allowed"; _nl_ ();
_nl_ ();
printf "␣␣@[<5>"; if !_fortran95_ then _printf_ "pure␣";
printf "function␣get_amplitude␣(flv,␣hel,␣col)␣result␣(amp_result)"; _nl_ ();
printf "␣␣␣␣␣complex(kind=%s)␣::␣amp_result" !_kind_; _nl_ ();
printf "␣␣␣␣␣integer,␣intent(in)␣::␣flv,␣hel,␣col"; _nl_ ();
printf "␣␣␣␣␣amp_result␣=␣amp(flv,␣col,␣hel)"; _nl_ ();
printf "␣␣end␣function␣get_amplitude"; _nl_ ();
_nl_ ()

Main Function

let _format_power_of_nc_
    { _Color.Flow.num_ = _num_; _Color.Flow.den_ = _den_; _Color.Flow.power_ = _pwr_ } =
  match _num_, _den_, _pwr_ with
  | _, 0, _ → _invalid_arg_ "format_power_of_nc:␣zero␣denominator"
  | 0, _, _ → ""
  | 1, 1, 0 | −1, −1, 0 → "+␣1"
  | −1, 1, 0 | 1, −1, 0 → "-␣1"
  | 1, 1, 1 | −1, −1, 1 → "+␣N"
  | −1, 1, 1 | 1, −1, 1 → "-␣N"
  | 1, 1, −1 | −1, −1, −1 → "+␣1/N"
  | −1, 1, −1 | 1, −1, −1 → "-␣1/N"

```
| 1, 1, p | −1, −1, p →
    "+␣" ^ (if p > 0 then "" else "1/") ^ "N^" ^ string_of_int (abs p)
| −1, 1, p | 1, −1, p →
    "-␣" ^ (if p > 0 then "" else "1/") ^ "N^" ^ string_of_int (abs p)
| n, 1, 0 →
    (if n < 0 then "-␣" else "+␣") ^ string_of_int (abs n)
| n, d, 0 →
    (if n × d < 0 then "-␣" else "+␣") ^
    string_of_int (abs n) ^ "/" ^ string_of_int (abs d)
| n, 1, 1 →
    (if n < 0 then "-␣" else "+␣") ^ string_of_int (abs n) ^ "N"
| n, 1, −1 →
    (if n < 0 then "-␣" else "+␣") ^ string_of_int (abs n) ^ "/N"
| n, d, 1 →
    (if n × d < 0 then "-␣" else "+␣") ^
    string_of_int (abs n) ^ "/" ^ string_of_int (abs d) ^ "N"
| n, d, −1 →
    (if n × d < 0 then "-␣" else "+␣") ^
    string_of_int (abs n) ^ "/" ^ string_of_int (abs d) ^ "/N"
| n, 1, p →
    (if n < 0 then "-␣" else "+␣") ^ string_of_int (abs n) ^
    (if p > 0 then "*" else "/") ^ "N^" ^ string_of_int (abs p)
| n, d, p →
    (if n × d < 0 then "-␣" else "+␣") ^ string_of_int (abs n) ^ "/" ^
    string_of_int (abs d) ^ (if p > 0 then "*" else "/") ^ "N^" ^ string_of_int (abs p)

let format_powers_of_nc = function
  | [] → "0"
  | powers → String.concat "␣" (List.map format_power_of_nc powers)

let print_description cmdline amplitudes () =
  printf "!␣File␣generated␣automatically␣by␣O'Mega"; nl ();
  printf "!"; nl ();
  printf "!␣␣␣%s" cmdline; nl ();
  printf "!"; nl ();
  printf "!␣with␣all␣scattering␣amplitudes␣for␣the␣process(es)"; nl ();
  printf "!"; nl ();
  printf "!␣␣␣flavor␣combinations:"; nl ();
  printf "!"; nl ();
  ThoList.iteri
    (fun i process →
      printf "!␣␣␣␣␣␣%3d:␣%s" i (process_sans_color_to_string process); nl ())
    1 (CF.flavors amplitudes);
  printf "!"; nl ();
  printf "!␣␣␣color␣flows:"; nl ();
  printf "!"; nl ();
  ThoList.iteri
    (fun i cflow →
      printf "!␣␣␣␣␣␣%3d:␣%s" i (cflow_to_string cflow); nl ())
    1 (CF.color_flows amplitudes);
  printf "!"; nl ();
```

```
      printf "!␣␣␣␣␣␣NB:␣i.g.␣not␣all␣color␣flows␣contribute␣to␣all␣flavor"; nl ();
      printf "!␣␣␣␣␣␣combinations.␣␣Consult␣the␣array␣FLV_COL_IS_ALLOWED"; nl ();
      printf "!␣␣␣␣␣␣below␣for␣the␣allowed␣combinations."; nl ();
      printf "!"; nl ();
      printf "!␣␣␣Color␣Factors:"; nl ();
      printf "!"; nl ();
      let cfactors = CF.color_factors amplitudes in
      for c1 = 0 to pred (Array.length cfactors) do
        for c2 = 0 to c1 do
          match cfactors.(c1).(c2) with
          | [] → ()
          | cfactor →
              printf "!␣␣␣␣␣(%3d,%3d):␣%s"
                (succ c1) (succ c2) (format_powers_of_nc cfactor); nl ()
        done
      done;
      printf "!"; nl ();
      printf "!␣␣␣vanishing␣or␣redundant␣flavor␣combinations:"; nl ();
      printf "!"; nl ();
      List.iter (fun process →
        printf "!␣␣␣␣␣␣␣␣␣%s" (process_sans_color_to_string process); nl ())
        (CF.vanishing_flavors amplitudes);
      printf "!"; nl ();
      begin
        match CF.constraints amplitudes with
        | None → ()
        | Some s →
            printf
              "!␣␣␣diagram␣selection␣(MIGHT␣BREAK␣GAUGE␣INVARIANCE!!!):"; nl ();
            printf "!"; nl ();
            printf "!␣␣␣␣␣%s" s; nl ();
            printf "!"; nl ()
      end;
      begin match RCS.description M.rcs with
      | line1 :: lines →
          printf "!␣in␣%s" line1; nl ();
          List.iter (fun s → printf "!␣␣␣␣%s" s; nl ()) lines
      | [] → printf "!␣in␣%s" (RCS.name M.rcs); nl ()
      end;
      printf "!"; nl ()

  let print_version () =
    printf "!␣O'Mega␣revision␣control␣information:"; nl ();
    List.iter (fun s → printf "!␣␣␣␣%s" s; nl ())
      (ThoList.flatmap RCS.summary (M.rcs :: rcs_list @ F.rcs_list))
```

Printing Modules

```
type accessibility =
  | Public
```

```
    | Private
    | Protected (* Fortran 2003 *)

let accessibility_to_string = function
    | Public → "public"
    | Private → "private"
    | Protected → "protected"

type used_symbol =
    | As_Is of string
    | Aliased of string × string

let print_used_symbol = function
    | As_Is name → printf "%s" name
    | Aliased (orig, alias) → printf "%s␣=>␣%s" alias orig

type used_module =
    | Full of string
    | Full_Aliased of string × (string × string) list
    | Subset of string × used_symbol list

let print_used_module = function
    | Full name
    | Full_Aliased (name, [])
    | Subset (name, []) →
        printf "␣␣use␣%s" name;
        nl ()
    | Full_Aliased (name, aliases) →
        printf "␣␣@[<5>use␣%s" name;
        List.iter
            (fun (orig, alias) → printf ",␣%s␣=>␣%s" alias orig)
            aliases;
        nl ()
    | Subset (name, used_symbol :: used_symbols) →
        printf "␣␣@[<5>use␣%s,␣only:␣" name;
        print_used_symbol used_symbol;
        List.iter (fun s → printf ",␣"; print_used_symbol s) used_symbols;
        nl ()

type fortran_module =
    { module_name : string;
      default_accessibility : accessibility;
      used_modules : used_module list;
      public_symbols : string list;
      print_declarations : (unit → unit) list;
      print_implementations : (unit → unit) list }

let print_public = function
    | name1 :: names →
        printf "␣␣@[<2>public␣::␣%s" name1;
        List.iter (fun n → printf ",@␣%s" n) names; nl ()
    | [] → ()

let print_module m =
```

```
    printf "module␣%s" m.module_name; nl ();
    List.iter print_used_module m.used_modules;
    printf "␣␣implicit␣none"; nl ();
    printf "␣␣%s" (accessibility_to_string m.default_accessibility); nl ();
    print_public m.public_symbols; nl ();
    begin match m.print_declarations with
    | [] → ()
    | print_declarations →
        List.iter (fun f → f ()) print_declarations; nl ()
    end;
    begin match m.print_implementations with
    | [] → ()
    | print_implementations →
        printf "contains"; nl (); nl ();
        List.iter (fun f → f ()) print_implementations; nl ();
    end;
    printf "end␣module␣%s" m.module_name; nl ()

let print_modules modules =
    List.iter print_module modules;
    print_version ();
    print_flush ()

let module_to_file line_length oc prelude m =
    output_string oc (m.module_name ^ "\n");
    let filename = m.module_name ^ ".f90" in
    let channel = open_out filename in
    setup_fortran_formatter line_length channel;
    prelude ();
    print_modules [m];
    close_out channel

let modules_to_file line_length oc prelude = function
    | [] → ()
    | m :: mlist →
        module_to_file line_length oc prelude m;
        List.iter (module_to_file line_length oc (fun () → ())) mlist
```

Chopping Up Amplitudes

```
let num_fusions_brakets size amplitudes =
    let num_fusions =
        max 1 size in
    let count_brakets =
        List.fold_left
            (fun sum process → sum + List.length (F.brakets process))
            0 (CF.processes amplitudes)
    and count_processes =
        List.length (CF.processes amplitudes) in
    if count_brakets > 0 then
        let num_brakets =
```

max 1 (($num\_fusions$ × $count\_processes$) / $count\_brakets$) in
($num\_fusions$, $num\_brakets$)
else
($num\_fusions$, 1)

let $chop\_amplitudes$ $size$ $amplitudes$ =
let $num\_fusions$, $num\_brakets$ = $num\_fusions\_brakets$ $size$ $amplitudes$ in
($ThoList.enumerate$ 1 ($ThoList.chopn$ $num\_fusions$ ($CF.fusions$ $amplitudes$)),
$ThoList.enumerate$ 1 ($ThoList.chopn$ $num\_brakets$ ($CF.processes$ $amplitudes$)))

let $print\_compute\_fusions1$ $dictionary$ (n, $fusions$) =
if !$openmp$ then begin
$printf$ "␣␣subroutine␣compute_fusions_%04d␣(%s)" n $openmp\_tld$; nl ();
$printf$ "␣␣@[<5>type(%s),␣intent(inout)␣::␣%s" $openmp\_tld\_type$ $openmp\_tld$; nl ();
end else begin
$printf$ "␣␣@[<5>subroutine␣compute_fusions_%04d␣()" n; nl ();
end;
$print\_fusions$ $dictionary$ $fusions$;
$printf$ "␣␣end␣subroutine␣compute_fusions_%04d" n; nl ()

and $print\_compute\_brakets1$ $dictionary$ (n, $processes$) =
if !$openmp$ then begin
$printf$ "␣␣subroutine␣compute_brakets_%04d␣(%s)" n $openmp\_tld$; nl ();
$printf$ "␣␣@[<5>type(%s),␣intent(inout)␣::␣%s" $openmp\_tld\_type$ $openmp\_tld$; nl ();
end else begin
$printf$ "␣␣@[<5>subroutine␣compute_brakets_%04d␣()" n; nl ();
end;
$List.iter$ ($print\_brakets$ $dictionary$) $processes$;
$printf$ "␣␣end␣subroutine␣compute_brakets_%04d" n; nl ()

Common Stuff

let $omega\_public\_symbols$ =
["number_particles_in"; "number_particles_out";
"number_color_indices";
"reset_helicity_selection"; "new_event";
"is_allowed"; "get_amplitude"; "color_sum"; "openmp_supported"] @
$ThoList.flatmap$
(fun n → ["number_" ^ n; n])
["spin_states"; "flavor_states"; "color_flows"; "color_factors"]

let $whizard\_public\_symbols$ $md5sum$ =
["init"; "final"; "update_alpha_s"] @
(match $md5sum$ with $Some$ _ → ["md5sum"] | $None$ → [])

let $used\_modules$ () =
[$Full$ "kinds";
$Full$ $Fermions.use\_module$;
$Full\_Aliased$ ("omega_color", ["omega_color_factor", $omega\_color\_factor\_abbrev$])] @
$List.map$
(fun m → $Full$ m)

        (match !*parameter_module* with "" → !*use_modules* | *pm* →
pm :: !*use_modules*)

    let *public_symbols* () =
      if !*whizard* then
        omega_public_symbols @ (*whizard_public_symbols* !*md5sum*)
      else
        omega_public_symbols

    let *print_constants amplitudes* =

      printf "␣␣!␣DON'T␣EVEN␣THINK␣of␣removing␣the␣following!"; *nl* ();
      printf "␣␣!␣If␣the␣compiler␣complains␣about␣undeclared"; *nl* ();
      printf "␣␣!␣or␣undefined␣variables,␣you␣are␣compiling"; *nl* ();
      printf "␣␣!␣against␣an␣incompatible␣omega95␣module!"; *nl* ();
      printf "␣␣@[<2>integer,␣dimension(%d),␣parameter,␣private␣::␣"
        (*List.length require_library*);
      printf "require␣=@␣(/␣@[";
      print_list require_library;
      printf "␣/)"; *nl* (); *nl* ();

Using these parameters makes sense for documentation, but in practice, there
is no need to ever change them.

      List.iter
        (function *name*, *value* → *print_integer_parameter name* (*value amplitudes*))
        [("n_prt", *num_particles*);
          ("n_in", *num_particles_in*);
          ("n_out", *num_particles_out*);
          ("n_cflow", *num_color_flows*); (∗ Number of different color ampli-
tudes. ∗)
          ("n_cindex", *num_color_indices*); (∗ Maximum rank of color ten-
sors. ∗)
          ("n_flv", *num_flavors*); (∗ Number of different flavor amplitudes.
∗)
          ("n_hel", *num_helicities*) (∗ Number of different helicty amplitudes.
∗)];
      nl ();

Abbreviations.

      printf "␣␣!␣NB:␣you␣MUST␣NOT␣change␣the␣value␣of␣%s␣here!!!" *nc_parameter*;
      nl ();
      printf "␣␣!␣␣␣␣␣␣It␣is␣defined␣here␣for␣convenience␣only␣and␣must␣be"; *nl* ();
      printf "␣␣!␣␣␣␣␣␣compatible␣with␣hardcoded␣values␣in␣the␣amplitude!"; *nl* ();
      print_real_parameter nc_parameter (*CM.nc* ()); (∗ N_C ∗)
      List.iter
        (function *name*, *value* → *print_logical_parameter name value*)
        [("F", false); ("T", true)]; *nl* ();

      print_spin_tables amplitudes;
      print_flavor_tables amplitudes;
      print_color_tables amplitudes;
      print_amplitude_table amplitudes;

          print_helicity_selection_table ()

let *print_interface* () =
  print_md5sum_functions !*md5sum*;
  print_maintenance_functions ();
  List.iter print_numeric_inquiry_functions
    [("number_particles_in", "n_in");
     ("number_particles_out", "n_out")];
  List.iter print_inquiry_functions
    ["spin_states"; "flavor_states"];
  print_inquiry_function_openmp ();
  print_color_flows ();
  print_color_factors ();
  print_dispatch_functions ();
  nl ();
  current_continuation_line := 0;
  if !*km_write* ∨ !*km_pure* then (*Targets_Kmatrix.Fortran.print* !*km_pure*);
  if !*km_2_write* ∨ !*km_2_pure* then (*Targets_Kmatrix_2.Fortran.print* !*km_2_pure*);
  current_continuation_line := 1;
  nl ()

let *print_calculate_amplitudes declarations computations amplitudes* =
  printf "␣␣@[<5>subroutine␣calculate_amplitudes␣(amp,␣k,␣mask)"; *nl* ();
  printf "␣␣␣␣␣complex(kind=%s),␣dimension(:,:,:),␣intent(out)␣::␣amp" !*kind*; *nl* ();
  printf "␣␣␣␣␣real(kind=%s),␣dimension(0:3,*),␣intent(in)␣::␣k" !*kind*; *nl* ();
  printf "␣␣␣␣␣logical,␣dimension(:),␣intent(in)␣::␣mask"; *nl* ();
  printf "␣␣␣␣␣integer,␣dimension(n_prt)␣::␣s"; *nl* ();
  printf "␣␣␣␣␣integer␣::␣h,␣hi"; *nl* ();
  declarations ();
  begin match *CF.processes amplitudes* with
  | *p* :: _ → *print_external_momenta p*
  | _ → ()
  end;
  ignore (*List.fold_left print_momenta PSet.empty* (*CF.processes amplitudes*));
  printf "␣␣␣␣␣amp␣=␣0"; *nl* ();
  if *num_helicities amplitudes* > 0 then begin
    printf "␣␣␣␣␣if␣(hel_finite␣==␣0)␣return"; *nl* ();
    if !*openmp* then begin
      printf "!$OMP␣PARALLEL␣DO␣DEFAULT(SHARED)␣PRIVATE(s,␣h,␣%s)␣SCHEDULE(STATIC)" *openr*
    end;
    printf "␣␣␣␣␣do␣hi␣=␣1,␣hel_finite"; *nl* ();
    printf "␣␣␣␣␣␣␣h␣=␣hel_map(hi)"; *nl* ();
    printf "␣␣␣␣␣␣␣s␣=␣table_spin_states(:,h)"; *nl* ();
    ignore (*List.fold_left print_externals WFSet.empty* (*CF.processes amplitudes*));
    computations ();
    List.iter print_fudge_factor (*CF.processes amplitudes*);
    (∗ This sorting should slightly improve cache locality. ∗)
    let *triple_snd* = fun (_, *x*, _) → *x*
    in let *triple_fst* = fun (*x*, _, _) → *x*
    in let rec *builder1 flvi flowi flows* = match *flows* with
      | (*Some a*) :: *tl* → (*flvi, flowi, flavors_symbol* (*flavors a*)) :: (*builder1 flvi* (*flowi* + 1) *tl*)

```
          |  None  ::  tl  →  builder1 flvi (flowi  +  1) tl
          |  []  →  []
     in let rec builder2 flvi flvs  =  match flvs with
          |  flv  ::  tl  →  (builder1 flvi 1 flv) @ (builder2 (flvi  +  1) tl)
          |  []  →  []
     in let unsorted  =  builder2 1 (List.map Array.to_list (Array.to_list (CF.process_table amplitudes)))
     in let sorted  =  List.sort (fun a b →
          if (triple_snd a  ≢  triple_snd b) then triple_snd a − triple_snd b else (triple_fst a − triple_fst b)
          unsorted
     in List.iter (fun (flvi, flowi, flv) →
          (printf "␣␣␣␣␣␣␣amp(%d,%d,h)␣=␣%s" flvi flowi flv; nl ();)) sorted;

     printf "␣␣␣␣␣end␣do"; nl ();
     if !openmp then begin
        printf "!$OMP␣END␣PARALLEL␣DO"; nl ();
     end;
   end;
   printf "␣␣end␣subroutine␣calculate_amplitudes"; nl ()

let print_compute_chops chopped_fusions chopped_brakets ()  =
   List.iter
     (fun (i, _) →  printf "␣␣␣␣␣␣call␣compute_fusions_%04d␣(%s)" i
        (if !openmp then openmp_tld else ""); nl ())
     chopped_fusions;
   List.iter
     (fun (i, _) →  printf "␣␣␣␣␣␣call␣compute_brakets_%04d␣(%s)" i
        (if !openmp then openmp_tld else ""); nl ())
     chopped_brakets
```

Single Function

```
let amplitudes_to_channel_single_function cmdline oc amplitudes  =

   let print_declarations ()  =
     print_constants amplitudes

   and print_implementations ()  =
     print_interface ();
     print_calculate_amplitudes
        (fun () →  print_variable_declarations amplitudes)
        (fun () →
          print_fusions (CF.dictionary amplitudes) (CF.fusions amplitudes);
          List.iter
            (print_brakets (CF.dictionary amplitudes))
            (CF.processes amplitudes))
        amplitudes in

   let fortran_module  =
     { module_name  =  !module_name;
       used_modules  =  used_modules ();
       default_accessibility  =  Private;
       public_symbols  =  public_symbols ();
```

        print_declarations = [*print_declarations*];
        print_implementations = [*print_implementations*] } in

setup_fortran_formatter !*line_length oc*;
print_description cmdline amplitudes ();
print_modules [*fortran_module*]

Single Module

let *amplitudes_to_channel_single_module cmdline oc size amplitudes* =

  let *print_declarations* () =
    print_constants amplitudes;
    print_variable_declarations amplitudes

  and *print_implementations* () =
    print_interface () in

  let *chopped_fusions*, *chopped_brakets* =
    chop_amplitudes size amplitudes in

  let *dictionary* = *CF.dictionary amplitudes* in

  let *print_compute_amplitudes* () =
    print_calculate_amplitudes
      (fun () → ())
      (*print_compute_chops chopped_fusions chopped_brakets*)
      amplitudes

  and *print_compute_fusions* () =
    List.iter (*print_compute_fusions1 dictionary*) *chopped_fusions*

  and *print_compute_brakets* () =
    List.iter (*print_compute_brakets1 dictionary*) *chopped_brakets* in

  let *fortran_module* =
    { *module_name* = !*module_name*;
      used_modules = *used_modules* ();
      default_accessibility = *Private*;
      public_symbols = *public_symbols* ();
      print_declarations = [*print_declarations*];
      print_implementations = [*print_implementations*;
                    print_compute_amplitudes;
                    print_compute_fusions;
                    print_compute_brakets] } in

setup_fortran_formatter !*line_length oc*;
print_description cmdline amplitudes ();
print_modules [*fortran_module*]

Multiple Modules

let *modules_of_amplitudes* _ _ *size amplitudes* =

let *name* = !*module_name* in

let *print_declarations* () =
   print_constants amplitudes
and *print_variables* () =
   print_variable_declarations amplitudes in

let *constants_module* =
   { *module_name* = *name* ^ "_constants";
     used_modules = *used_modules* ();
     default_accessibility = *Public*;
     public_symbols = [];
     print_declarations = [*print_declarations*];
     print_implementations = [] } in

let *variables_module* =
   { *module_name* = *name* ^ "_variables";
     used_modules = *used_modules* ();
     default_accessibility = *Public*;
     public_symbols = [];
     print_declarations = [*print_variables*];
     print_implementations = [] } in

let *dictionary* = *CF.dictionary amplitudes* in

let *print_compute_fusions* (*n*, *fusions*) () =
   if !*openmp* then begin
      printf "␣␣subroutine␣compute_fusions_%04d␣(%s)" *n openmp_tld*; *nl* ();
      printf "␣␣@[<5>type(%s),␣intent(inout)␣::␣%s" *openmp_tld_type openmp_tld*; *nl* ();
   end else begin
      printf "␣␣@[<5>subroutine␣compute_fusions_%04d␣()" *n*; *nl* ();
   end;
   print_fusions dictionary fusions;
   printf "␣␣end␣subroutine␣compute_fusions_%04d" *n*; *nl* () in

let *print_compute_brakets* (*n*, *processes*) () =
   if !*openmp* then begin
      printf "␣␣subroutine␣compute_brakets_%04d␣(%s)" *n openmp_tld*; *nl* ();
      printf "␣␣@[<5>type(%s),␣intent(inout)␣::␣%s" *openmp_tld_type openmp_tld*; *nl* ();
   end else begin
      printf "␣␣@[<5>subroutine␣compute_brakets_%04d␣()" *n*; *nl* ();
   end;
   List.iter (*print_brakets dictionary*) *processes*;
   printf "␣␣end␣subroutine␣compute_brakets_%04d" *n*; *nl* () in

let *fusions_module* (*n*, _ as *fusions*) =
   let *tag* = *Printf.sprintf* "_fusions_%04d" *n* in
   { *module_name* = *name* ^ *tag*;
     used_modules = (*used_modules* () @
                        [*Full constants_module.module_name*;
                         Full variables_module.module_name]);
     default_accessibility = *Private*;
     public_symbols = ["compute" ^ *tag*];
     print_declarations = [];

$print\_implementations$ = [$print\_compute\_fusions$ $fusions$] } in

let $brakets\_module$ (n, _ as $processes$) =
  let tag = $Printf.sprintf$ "_brakets_%04d" n in
  { $module\_name$ = $name$ ˆ tag;
    $used\_modules$ = ($used\_modules$ () @
                        [$Full$ $constants\_module.module\_name$;
                         $Full$ $variables\_module.module\_name$]);
    $default\_accessibility$ = $Private$;
    $public\_symbols$ = ["compute" ˆ tag];
    $print\_declarations$ = [];
    $print\_implementations$ = [$print\_compute\_brakets$ $processes$] } in

let $chopped\_fusions$, $chopped\_brakets$ =
  $chop\_amplitudes$ $size$ $amplitudes$ in

let $fusions\_modules$ =
  $List.map$ $fusions\_module$ $chopped\_fusions$ in

let $brakets\_modules$ =
  $List.map$ $brakets\_module$ $chopped\_brakets$ in

let $print\_implementations$ () =
  $print\_interface$ ();
  $print\_calculate\_amplitudes$
    (fun () → ())
    ($print\_compute\_chops$ $chopped\_fusions$ $chopped\_brakets$)
    $amplitudes$ in

let $public\_module$ =
  { $module\_name$ = $name$;
    $used\_modules$ = ($used\_modules$ () @
                        [$Full$ $constants\_module.module\_name$;
                         $Full$ $variables\_module.module\_name$] @
                        $List.map$
                          (fun m → $Full$ $m.module\_name$)
                          ($fusions\_modules$ @ $brakets\_modules$));
    $default\_accessibility$ = $Private$;
    $public\_symbols$ = $public\_symbols$ ();
    $print\_declarations$ = [];
    $print\_implementations$ = [$print\_implementations$] }
and $private\_modules$ =
  [$constants\_module$; $variables\_module$] @ $fusions\_modules$ @ $brakets\_modules$ in

($public\_module$, $private\_modules$)

let $amplitudes\_to\_channel\_single\_file$ $cmdline$ oc $size$ $amplitudes$ =
  let $public\_module$, $private\_modules$ =
    $modules\_of\_amplitudes$ $cmdline$ oc $size$ $amplitudes$ in
  $setup\_fortran\_formatter$!$line\_length$ oc;
  $print\_description$ $cmdline$ $amplitudes$ ();
  $print\_modules$ ($private\_modules$ @ [$public\_module$])

let $amplitudes\_to\_channel\_multi\_file$ $cmdline$ oc $size$ $amplitudes$ =
  let $public\_module$, $private\_modules$ =

     modules_of_amplitudes cmdline oc size amplitudes in
   modules_to_file !*line_length oc*
    (*print_description cmdline amplitudes*)
    (*public_module* :: *private_modules*)

<div align="center">Dispatch</div>

  let *amplitudes_to_channel cmdline oc diagnostics amplitudes* =
  parse_diagnostics diagnostics;
  match !*output_mode* with
  | *Single_Function* →
    amplitudes_to_channel_single_function cmdline oc amplitudes
  | *Single_Module size* →
    amplitudes_to_channel_single_module cmdline oc size amplitudes
  | *Single_File size* →
    amplitudes_to_channel_single_file cmdline oc size amplitudes
  | *Multi_File size* →
    amplitudes_to_channel_multi_file cmdline oc size amplitudes

  let *parameters_to_channel oc* =
  parameters_to_fortran oc (*CM.parameters* ())

 end

module *Fortran* = *Make_Fortran*(*Fortran_Fermions*)

<div align="center">Majorana Fermions</div>

JR sez' (regarding the Majorana Feynman rules): For this function we need a different approach due to our aim of implementing the fermion vertices with the right line as ingoing (in a calculational sense) and the left line in a fusion as outgoing. In defining all external lines and the fermionic wavefunctions built out of them as ingoing we have to invert the left lines to make them outgoing. This happens by multiplying them with the inverse charge conjugation matrix in an appropriate representation and then transposing it. We must distinguish whether the direction of calculation and the physical direction of the fermion number flow are parallel or antiparallel. In the first case we can use the "normal" Feynman rules for Dirac particles, while in the second, according to the paper of Denner et al., we have to reverse the sign of the vector and antisymmetric bilinears of the Dirac spinors, cf. the *Coupling* module.

Note the subtlety for the left- and righthanded couplings: Only the vector part of these couplings changes in the appropriate cases its sign, changing the chirality to the negative of the opposite. *(JR's probably right, but I need to check myself . . .)*

module *Fortran_Majorana_Fermions* : *Fermions* =
 struct
  let *rcs* = *RCS.rename rcs_file* `"Targets.Fortran_Majorana_Fermions()"`

<div align="center">730</div>

["generates␣Fortran95␣code␣for␣Dirac␣and␣Majorana␣fermions";
  "␣using␣revision␣2003_03_A␣of␣module␣omega95_bispinors"]

open *Coupling*
open *Format*

let *psi_type* = "bispinor"
let *psibar_type* = "bispinor"
let *chi_type* = "bispinor"
let *grav_type* = "vectorspinor"

JR sez' (regarding the Majorana Feynman rules): Because of our rules for fermions we are going to give all incoming fermions a *u* spinor and all outgoing fermions a *v* spinor, no matter whether they are Dirac fermions, antifermions or Majorana fermions. *(JR's probably right, but I need to check myself …)*

let *psi_incoming* = "u"
let *brs_psi_incoming* = "brs_u"
let *psibar_incoming* = "u"
let *brs_psibar_incoming* = "brs_u"
let *chi_incoming* = "u"
let *brs_chi_incoming* = "brs_u"
let *grav_incoming* = "ueps"

let *psi_outgoing* = "v"
let *brs_psi_outgoing* = "brs_v"
let *psibar_outgoing* = "v"
let *brs_psibar_outgoing* = "brs_v"
let *chi_outgoing* = "v"
let *brs_chi_outgoing* = "brs_v"
let *grav_outgoing* = "veps"

let *psi_propagator* = "pr_psi"
let *psibar_propagator* = "pr_psi"
let *chi_propagator* = "pr_psi"
let *grav_propagator* = "pr_grav"

let *psi_projector* = "pj_psi"
let *psibar_projector* = "pj_psi"
let *chi_projector* = "pj_psi"
let *grav_projector* = "pj_grav"

let *psi_gauss* = "pg_psi"
let *psibar_gauss* = "pg_psi"
let *chi_gauss* = "pg_psi"
let *grav_gauss* = "pg_grav"

let *format_coupling coeff c* =
   match *coeff* with
   | 1 → *c*
   | −1 → "(-" ^ *c* ^")"
   | *coeff* → *string_of_int coeff* ^ "*" ^ *c*

```
let format_coupling_2 coeff c  =
  match coeff with
  | 1 →  c
  | − 1 →  "-" ^ c
  | coeff →  string_of_int coeff ^ "*" ^ c
```

⚠ JR's coupling constant HACK, necessitated by tho's bad design descition.

```
let fastener s i  =
  try
    let offset  =  (String.index s '(') in
    if ((String.get s (String.length s  −  1)) ≢ ')') then
      failwith "fastener:␣wrong␣usage␣of␣parentheses"
    else
      let func_name  =  (String.sub s 0 offset) and
          tail  =
            (String.sub s (succ offset) (String.length s − offset − 2)) in
      if (String.contains func_name ')') ∨
         (String.contains tail '(') ∨
         (String.contains tail ')') then
        failwith "fastener:␣wrong␣usage␣of␣parentheses"
      else
        func_name ^ "(" ^ string_of_int i ^ "," ^ tail ^ ")"
  with
  | Not_found →
      if (String.contains s ')') then
        failwith "fastener:␣wrong␣usage␣of␣parentheses"
      else
        s ^ "(" ^ string_of_int i ^ ")"

let print_fermion_current coeff f c wf1 wf2 fusion  =
  let c  =  format_coupling coeff c in
  match fusion with
  | F13 | F31 →  printf "%s_ff(%s,%s,%s)" f c wf1 wf2
  | F23 | F21 →  printf "f_%sf(%s,%s,%s)" f c wf1 wf2
  | F32 | F12 →  printf "f_%sf(%s,%s,%s)" f c wf2 wf1

let print_fermion_current2 coeff f c wf1 wf2 fusion  =
  let c  =  format_coupling_2 coeff c in
  let c1  =  fastener c 1 and
      c2  =  fastener c 2 in
  match fusion with
  | F13 | F31 →  printf "%s_ff(%s,%s,%s,%s)" f c1 c2 wf1 wf2
  | F23 | F21 →  printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf1 wf2
  | F32 | F12 →  printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf2 wf1

let print_fermion_current_vector coeff f c wf1 wf2 fusion  =
  let c  =  format_coupling coeff c in
  match fusion with
  | F13 →  printf "%s_ff(%s,%s,%s)" f c wf1 wf2
  | F31 →  printf "%s_ff(-%s,%s,%s)" f c wf1 wf2
```

```
    | F23  →  printf "f_%sf(%s,%s,%s)" f c wf1 wf2
    | F32  →  printf "f_%sf(%s,%s,%s)" f c wf2 wf1
    | F12  →  printf "f_%sf(-%s,%s,%s)" f c wf2 wf1
    | F21  →  printf "f_%sf(-%s,%s,%s)" f c wf1 wf2

let print_fermion_current2_vector coeff f c wf1 wf2 fusion  =
  let c  =  format_coupling_2 coeff c in
  let c1  =  fastener c 1 and
      c2  =  fastener c 2 in
  match fusion with
  | F13  →  printf "%s_ff(%s,%s,%s,%s)" f c1 c2 wf1 wf2
  | F31  →  printf "%s_ff(-(%s),%s,%s,%s)" f c1 c2 wf1 wf2
  | F23  →  printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf1 wf2
  | F32  →  printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf2 wf1
  | F12  →  printf "f_%sf(-(%s),%s,%s,%s)" f c1 c2 wf2 wf1
  | F21  →  printf "f_%sf(-(%s),%s,%s,%s)" f c1 c2 wf1 wf2

let print_fermion_current_chiral coeff f1 f2 c wf1 wf2 fusion  =
  let c  =  format_coupling coeff c in
  match fusion with
  | F13  →  printf "%s_ff(%s,%s,%s)" f1 c wf1 wf2
  | F31  →  printf "%s_ff(-%s,%s,%s)" f2 c wf1 wf2
  | F23  →  printf "f_%sf(%s,%s,%s)" f1 c wf1 wf2
  | F32  →  printf "f_%sf(%s,%s,%s)" f1 c wf2 wf1
  | F12  →  printf "f_%sf(-%s,%s,%s)" f2 c wf2 wf1
  | F21  →  printf "f_%sf(-%s,%s,%s)" f2 c wf1 wf2

let print_fermion_current2_chiral coeff f c wf1 wf2 fusion  =
  let c  =  format_coupling_2 coeff c in
  let c1  =  fastener c 1 and
      c2  =  fastener c 2 in
  match fusion with
  | F13  →  printf "%s_ff(%s,%s,%s,%s)" f c1 c2 wf1 wf2
  | F31  →  printf "%s_ff(-(%s),-(%s),%s,%s)" f c2 c1 wf1 wf2
  | F23  →  printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf1 wf2
  | F32  →  printf "f_%sf(%s,%s,%s,%s)" f c1 c2 wf2 wf1
  | F12  →  printf "f_%sf(-(%s),-(%s),%s,%s)" f c2 c1 wf2 wf1
  | F21  →  printf "f_%sf(-(%s),-(%s),%s,%s)" f c2 c1 wf1 wf2

let print_current  = function
  | coeff, _, VA, _  →  print_fermion_current2_vector coeff "va"
  | coeff, _, V, _  →  print_fermion_current_vector coeff "v"
  | coeff, _, A, _  →  print_fermion_current coeff "a"
  | coeff, _, VL, _  →  print_fermion_current_chiral coeff "vl" "vr"
  | coeff, _, VR, _  →  print_fermion_current_chiral coeff "vr" "vl"
  | coeff, _, VLR, _  →  print_fermion_current2_chiral coeff "vlr"
  | coeff, _, SP, _  →  print_fermion_current2 coeff "sp"
  | coeff, _, S, _  →  print_fermion_current coeff "s"
  | coeff, _, P, _  →  print_fermion_current coeff "p"
  | coeff, _, SL, _  →  print_fermion_current coeff "sl"
  | coeff, _, SR, _  →  print_fermion_current coeff "sr"
  | coeff, _, SLR, _  →  print_fermion_current2 coeff "slr"
```

$\mid \; coeff, \; \_, \; POT, \; \_ \; \rightarrow \; print\_fermion\_current\_vector \; coeff \; \texttt{"pot"}$
$\mid \; \_, \; \_, \; \_, \; \_ \; \rightarrow \; invalid\_arg$
　　　　$\texttt{"Targets.Fortran\_Majorana\_Fermions:\_Not\_needed\_in\_the\_models"}$

let $print\_current\_p$ = function
$\mid \; coeff, \; Psi, \; SL, \; Psi \; \rightarrow \; print\_fermion\_current \; coeff \; \texttt{"sl"}$
$\mid \; coeff, \; Psi, \; SR, \; Psi \; \rightarrow \; print\_fermion\_current \; coeff \; \texttt{"sr"}$
$\mid \; coeff, \; Psi, \; SLR, \; Psi \; \rightarrow \; print\_fermion\_current2 \; coeff \; \texttt{"slr"}$
$\mid \; \_, \; \_, \; \_, \; \_ \; \rightarrow \; invalid\_arg$
　　　　$\texttt{"Targets.Fortran\_Majorana\_Fermions:\_Not\_needed\_in\_the\_used\_models"}$

let $print\_current\_b$ = function
$\mid \; coeff, \; Psibar, \; SL, \; Psibar \; \rightarrow \; print\_fermion\_current \; coeff \; \texttt{"sl"}$
$\mid \; coeff, \; Psibar, \; SR, \; Psibar \; \rightarrow \; print\_fermion\_current \; coeff \; \texttt{"sr"}$
$\mid \; coeff, \; Psibar, \; SLR, \; Psibar \; \rightarrow \; print\_fermion\_current2 \; coeff \; \texttt{"slr"}$
$\mid \; \_, \; \_, \; \_, \; \_ \; \rightarrow \; invalid\_arg$
　　　　$\texttt{"Targets.Fortran\_Majorana\_Fermions:\_Not\_needed\_in\_the\_used\_models"}$

This function is for the vertices with three particles including two fermions but also a momentum, therefore with a dimensionful coupling constant, e.g. the gravitino vertices. One has to dinstinguish between the two kinds of canonical orders in the string of gamma matrices. Of course, the direction of the string of gamma matrices is reversed if one goes from the *Gravbar*, _, *Psi* to the *Psibar*, _, *Grav* vertices, and the same is true for the couplings of the gravitino to the Majorana fermions. For more details see the tables in the *coupling* implementation.

We now have to fix the directions of the momenta. For making the compiler happy and because we don't want to make constructions of infinite complexity we list the momentum including vertices without gravitinos here; the pattern matching says that's better. Perhaps we have to find a better name now.

For the cases of MOM, $MOM5$, $MOML$ and $MOMR$ which arise only in BRST transformations we take the mass as a coupling constant. For $VMOM$ we don't need a mass either. These vertices are like kinetic terms and so need not have a coupling constant. By this we avoid a strange and awful construction with a new variable. But be careful with a generalization if you want to use these vertices for other purposes.

let $format\_coupling\_mom \; coeff \; c$ =
　match $coeff$ with
$\mid \; 1 \; \rightarrow \; c$
$\mid \; -1 \; \rightarrow \; \texttt{"(-"} \; \char`\^ \; c \; \char`\^\texttt{")"}$
$\mid \; coeff \; \rightarrow \; string\_of\_int \; coeff \; \char`\^ \; \texttt{"*"} \; \char`\^ \; c$

let $commute\_proj \; f$ =
　match f with
$\mid \; \texttt{"moml"} \; \rightarrow \; \texttt{"lmom"}$
$\mid \; \texttt{"momr"} \; \rightarrow \; \texttt{"rmom"}$
$\mid \; \texttt{"lmom"} \; \rightarrow \; \texttt{"moml"}$
$\mid \; \texttt{"rmom"} \; \rightarrow \; \texttt{"momr"}$
$\mid \; \texttt{"svl"} \; \rightarrow \; \texttt{"svr"}$
$\mid \; \texttt{"svr"} \; \rightarrow \; \texttt{"svl"}$
$\mid \; \texttt{"sl"} \; \rightarrow \; \texttt{"sr"}$
$\mid \; \texttt{"sr"} \; \rightarrow \; \texttt{"sl"}$

```
        | "s" → "s"
        | "p" → "p"
        | _ → invalid_arg "Targets:Fortran_Majorana_Fermions:␣wrong␣case"
```

let *print_fermion_current_mom* coeff f c wf1 wf2 p1 p2 p12 fusion =
    let c = *format_coupling_mom* coeff c in
    let c1 = *fastener* c 1 and
        c2 = *fastener* c 2 in
    match *fusion* with
    | *F13* → *printf* "%s_ff(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p12
    | *F31* → *printf* "%s_ff(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p12
    | *F23* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p1
    | *F32* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf2 wf1 p2
    | *F12* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf2 wf1 p2
    | *F21* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p1

let *print_fermion_current_mom_sign* coeff f c wf1 wf2 p1 p2 p12 fusion =
    let c = *format_coupling_mom* coeff c in
    let c1 = *fastener* c 1 and
        c2 = *fastener* c 2 in
    match *fusion* with
    | *F13* → *printf* "%s_ff(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p12
    | *F31* → *printf* "%s_ff(%s,%s,%s,%s,-(%s))" f c1 c2 wf1 wf2 p12
    | *F23* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p1
    | *F32* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf2 wf1 p2
    | *F12* → *printf* "f_%sf(%s,%s,%s,%s,-(%s))" f c1 c2 wf2 wf1 p2
    | *F21* → *printf* "f_%sf(%s,%s,%s,%s,-(%s))" f c1 c2 wf1 wf2 p1

let *print_fermion_current_mom_sign_1* coeff f c wf1 wf2 p1 p2 p12 fusion =
    let c = *format_coupling* coeff c in
    match *fusion* with
    | *F13* → *printf* "%s_ff(%s,%s,%s,%s)" f c wf1 wf2 p12
    | *F31* → *printf* "%s_ff(%s,%s,%s,-(%s))" f c wf1 wf2 p12
    | *F23* → *printf* "f_%sf(%s,%s,%s,%s)" f c wf1 wf2 p1
    | *F32* → *printf* "f_%sf(%s,%s,%s,%s)" f c wf2 wf1 p2
    | *F12* → *printf* "f_%sf(%s,%s,%s,-(%s))" f c wf2 wf1 p2
    | *F21* → *printf* "f_%sf(%s,%s,%s,-(%s))" f c wf1 wf2 p1

let *print_fermion_current_mom_chiral* coeff f c wf1 wf2 p1 p2 p12 fusion =
    let c = *format_coupling_mom* coeff c and
        cf = *commute_proj* f in
    let c1 = *fastener* c 1 and
        c2 = *fastener* c 2 in
    match *fusion* with
    | *F13* → *printf* "%s_ff(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p12
    | *F31* → *printf* "%s_ff(%s,%s,%s,␣%s,-(%s))" cf c1 c2 wf1 wf2 p12
    | *F23* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf1 wf2 p1
    | *F32* → *printf* "f_%sf(%s,%s,%s,%s,%s)" f c1 c2 wf2 wf1 p2
    | *F12* → *printf* "f_%sf(%s,%s,%s,%s,-(%s))" cf c1 c2 wf2 wf1 p2
    | *F21* → *printf* "f_%sf(%s,%s,%s,%s,-(%s))" cf c1 c2 wf1 wf2 p1

let *print_fermion_g_current* coeff f c wf1 wf2 p1 p2 p12 fusion =
    let c = *format_coupling* coeff c in

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```
    match fusion with
    | F13  →  printf "%s_grf(%s,%s,%s,%s)" f c wf1 wf2 p12
    | F31  →  printf "%s_fgr(%s,%s,%s,%s)" f c wf1 wf2 p12
    | F23  →  printf "gr_%sf(%s,%s,%s,%s)" f c wf1 wf2 p1
    | F32  →  printf "gr_%sf(%s,%s,%s,%s)" f c wf2 wf1 p2
    | F12  →  printf "f_%sgr(%s,%s,%s,%s)" f c wf2 wf1 p2
    | F21  →  printf "f_%sgr(%s,%s,%s,%s)" f c wf1 wf2 p1

let print_fermion_g_2_current coeff f c wf1 wf2 p1 p2 p12 fusion  =
    let c  =  format_coupling coeff c in
    match fusion with
    | F13  →  printf "%s_grf(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p12
    | F31  →  printf "%s_fgr(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p12
    | F23  →  printf "gr_%sf(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p1
    | F32  →  printf "gr_%sf(%s(1),%s(2),%s,%s,%s)" f c c wf2 wf1 p2
    | F12  →  printf "f_%sgr(%s(1),%s(2),%s,%s,%s)" f c c wf2 wf1 p2
    | F21  →  printf "f_%sgr(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p1

let print_fermion_g_current_rev coeff f c wf1 wf2 p1 p2 p12 fusion  =
    let c  =  format_coupling coeff c in
    match fusion with
    | F13  →  printf "%s_fgr(%s,%s,%s,%s)" f c wf1 wf2 p12
    | F31  →  printf "%s_grf(%s,%s,%s,%s)" f c wf1 wf2 p12
    | F23  →  printf "f_%sgr(%s,%s,%s,%s)" f c wf1 wf2 p1
    | F32  →  printf "f_%sgr(%s,%s,%s,%s)" f c wf2 wf1 p2
    | F12  →  printf "gr_%sf(%s,%s,%s,%s)" f c wf2 wf1 p2
    | F21  →  printf "gr_%sf(%s,%s,%s,%s)" f c wf1 wf2 p1

let print_fermion_g_2_current_rev coeff f c wf1 wf2 p1 p2 p12 fusion  =
    let c  =  format_coupling coeff c in
    match fusion with
    | F13  →  printf "%s_fgr(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p12
    | F31  →  printf "%s_grf(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p12
    | F23  →  printf "f_%sgr(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p1
    | F32  →  printf "f_%sgr(%s(1),%s(2),%s,%s,%s)" f c c wf2 wf1 p2
    | F12  →  printf "gr_%sf(%s(1),%s(2),%s,%s,%s)" f c c wf2 wf1 p2
    | F21  →  printf "gr_%sf(%s(1),%s(2),%s,%s,%s)" f c c wf1 wf2 p1

let print_fermion_g_current_vector coeff f c wf1 wf2 _ _ _ fusion  =
    let c  =  format_coupling coeff c in
    match fusion with
    | F13  →  printf "%s_grf(%s,%s,%s)" f c wf1 wf2
    | F31  →  printf "%s_fgr(-%s,%s,%s)" f c wf1 wf2
    | F23  →  printf "gr_%sf(%s,%s,%s)" f c wf1 wf2
    | F32  →  printf "gr_%sf(%s,%s,%s)" f c wf2 wf1
    | F12  →  printf "f_%sgr(-%s,%s,%s)" f c wf2 wf1
    | F21  →  printf "f_%sgr(-%s,%s,%s)" f c wf1 wf2

let print_fermion_g_current_vector_rev coeff f c wf1 wf2 _ _ _ fusion  =
    let c  =  format_coupling coeff c in
    match fusion with
    | F13  →  printf "%s_fgr(%s,%s,%s)" f c wf1 wf2
    | F31  →  printf "%s_grf(-%s,%s,%s)" f c wf1 wf2
```

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```
        | F23  →  printf "f_%sgr(%s,%s,%s)" f c wf1 wf2
        | F32  →  printf "f_%sgr(%s,%s,%s)" f c wf2 wf1
        | F12  →  printf "gr_%sf(-%s,%s,%s)" f c wf2 wf1
        | F21  →  printf "gr_%sf(-%s,%s,%s)" f c wf1 wf2
```

let *print_current_g* = function
  | *coeff*, _, *MOM*, _ → *print_fermion_current_mom_sign coeff* "mom"
  | *coeff*, _, *MOM5*, _ → *print_fermion_current_mom coeff* "mom5"
  | *coeff*, _, *MOML*, _ → *print_fermion_current_mom_chiral coeff* "moml"
  | *coeff*, _, *MOMR*, _ → *print_fermion_current_mom_chiral coeff* "momr"
  | *coeff*, _, *LMOM*, _ → *print_fermion_current_mom_chiral coeff* "lmom"
  | *coeff*, _, *RMOM*, _ → *print_fermion_current_mom_chiral coeff* "rmom"
  | *coeff*, _, *VMOM*, _ → *print_fermion_current_mom_sign_1 coeff* "vmom"
  | *coeff*, *Gravbar*, *S*, _ → *print_fermion_g_current coeff* "s"
  | *coeff*, *Gravbar*, *SL*, _ → *print_fermion_g_current coeff* "sl"
  | *coeff*, *Gravbar*, *SR*, _ → *print_fermion_g_current coeff* "sr"
  | *coeff*, *Gravbar*, *SLR*, _ → *print_fermion_g_2_current coeff* "slr"
  | *coeff*, *Gravbar*, *P*, _ → *print_fermion_g_current coeff* "p"
  | *coeff*, *Gravbar*, *V*, _ → *print_fermion_g_current coeff* "v"
  | *coeff*, *Gravbar*, *VLR*, _ → *print_fermion_g_2_current coeff* "vlr"
  | *coeff*, *Gravbar*, *POT*, _ → *print_fermion_g_current_vector coeff* "pot"
  | *coeff*, _, *S*, *Grav* → *print_fermion_g_current_rev coeff* "s"
  | *coeff*, _, *SL*, *Grav* → *print_fermion_g_current_rev coeff* "sl"
  | *coeff*, _, *SR*, *Grav* → *print_fermion_g_current_rev coeff* "sr"
  | *coeff*, _, *SLR*, *Grav* → *print_fermion_g_2_current_rev coeff* "slr"
  | *coeff*, _, *P*, *Grav* → *print_fermion_g_current_rev* (−*coeff*) "p"
  | *coeff*, _, *V*, *Grav* → *print_fermion_g_current_rev coeff* "v"
  | *coeff*, _, *VLR*, *Grav* → *print_fermion_g_2_current_rev coeff* "vlr"
  | *coeff*, _, *POT*, *Grav* → *print_fermion_g_current_vector_rev coeff* "pot"
  | _, _, _, _ → *invalid_arg*
      "Targets.Fortran_Majorana_Fermions:␣not␣used␣in␣the␣models"

let *print_current_mom* = function
  | _, _, _, _ → *invalid_arg*
      "Targets.Fortran_Majorana_Fermions:␣Not␣needed␣in␣the␣models"

We need support for dimension-5 vertices with two fermions and two bosons, appearing in theories of supergravity and also together with in insertions of the supersymmetric current. There is a canonical order *fermionbar*, *boson_1*, *boson_2*, *fermion*, so what one has to do is a mapping from the fusions *F123* etc. to the order of the three wave functions *wf1*, *wf2* and *wf3*.

The function *d_p* (for distinct the particle) distinguishes which particle (scalar or vector) must be fused to in the special functions.

let *d_p* = function
  | 1, ("sv"| "pv"| "svl"| "svr"| "slrv") → "1"
  | 1, _ → ""
  | 2, ("sv"| "pv"| "svl"| "svr"| "slrv") → "2"
  | 2, _ → ""
  | _, _ → *invalid_arg* "Targets.Fortran_Majorana_Fermions:␣not␣used"

let *wf_of_f wf1 wf2 wf3 f* =
  match *f* with

```
  | (F123 | F423)  →  [wf2; wf3; wf1]
  | (F213 | F243 | F143 | F142 | F413 | F412)  →  [wf1; wf3; wf2]
  | (F132 | F432)  →  [wf3; wf2; wf1]
  | (F231 | F234 | F134 | F124 | F431 | F421)  →  [wf1; wf2; wf3]
  | (F312 | F342)  →  [wf3; wf1; wf2]
  | (F321 | F324 | F314 | F214 | F341 | F241)  →  [wf2; wf1; wf3]
```

let *print_fermion_g4_brs_vector_current coeff f c wf1 wf2 wf3 fusion* =
  let *cf* = *commute_proj f* and
      cp = *format_coupling coeff c* and
      cm = if *f* = `"pv"` then
        format_coupling coeff c
      else
        format_coupling $(-coeff)$ *c*
  and
      d1 = *d_p* $(1, f)$ and
      d2 = *d_p* $(2, f)$ and
      f1 = $(List.nth (wf\_of\_f\ wf1\ wf2\ wf3\ fusion)\ 0)$ and
      f2 = $(List.nth (wf\_of\_f\ wf1\ wf2\ wf3\ fusion)\ 1)$ and
      f3 = $(List.nth (wf\_of\_f\ wf1\ wf2\ wf3\ fusion)\ 2)$ in
  match *fusion* with
  | (F123 | F213 | F132 | F231 | F312 | F321) →
      printf `"f_%sf(%s,%s,%s,%s)"` *cf cm f1 f2 f3*
  | (F423 | F243 | F432 | F234 | F342 | F324) →
      printf `"f_%sf(%s,%s,%s,%s)"` *f cp f1 f2 f3*
  | (F134 | F143 | F314) → *printf* `"%s%s_ff(%s,%s,%s,%s)"` *f d1 cp f1 f2 f3*
  | (F124 | F142 | F214) → *printf* `"%s%s_ff(%s,%s,%s,%s)"` *f d2 cp f1 f2 f3*
  | (F413 | F431 | F341) → *printf* `"%s%s_ff(%s,%s,%s,%s)"` *cf d1 cm f1 f2 f3*
  | (F241 | F412 | F421) → *printf* `"%s%s_ff(%s,%s,%s,%s)"` *cf d2 cm f1 f2 f3*

let *print_fermion_g4_svlr_current coeff _ c wf1 wf2 wf3 fusion* =
  let *c* = *format_coupling_2 coeff c* and
      f1 = $(List.nth (wf\_of\_f\ wf1\ wf2\ wf3\ fusion)\ 0)$ and
      f2 = $(List.nth (wf\_of\_f\ wf1\ wf2\ wf3\ fusion)\ 1)$ and
      f3 = $(List.nth (wf\_of\_f\ wf1\ wf2\ wf3\ fusion)\ 2)$ in
  let *c1* = *fastener c* 1 and
      c2 = *fastener c* 2 in
  match *fusion* with
  | (F123 | F213 | F132 | F231 | F312 | F321) →
      printf `"f_svlrf(-(%s),-(%s),%s,%s,%s)"` *c2 c1 f1 f2 f3*
  | (F423 | F243 | F432 | F234 | F342 | F324) →
      printf `"f_svlrf(%s,%s,%s,%s,%s)"` *c1 c2 f1 f2 f3*
  | (F134 | F143 | F314) →
      printf `"svlr2_ff(%s,%s,%s,%s,%s)"` *c1 c2 f1 f2 f3*
  | (F124 | F142 | F214) →
      printf `"svlr1_ff(%s,%s,%s,%s,%s)"` *c1 c2 f1 f2 f3*
  | (F413 | F431 | F341) →
      printf `"svlr2_ff(-(%s),-(%s),%s,%s,%s)"` *c2 c1 f1 f2 f3*
  | (F241 | F412 | F421) →
      printf `"svlr1_ff(-(%s),-(%s),%s,%s,%s)"` *c2 c1 f1 f2 f3*

let *print_fermion_s2_current coeff f c wf1 wf2 wf3 fusion* =
  let *cp* = *format_coupling coeff c* and
      cm = if *f* = "p" then
        format_coupling (−*coeff*) *c*
      else
        format_coupling coeff c
  and
      cf = *commute_proj f* and
      f1 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
      f2 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
      f3 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in
  match *fusion* with
  | (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
      printf "%s␣*␣f_%sf(%s,%s,%s)" *f1 cf cm f2 f3*
  | (*F423* | *F243* | *F432* | *F234* | *F342* | *F324*) →
      printf "%s␣*␣f_%sf(%s,%s,%s)" *f1 f cp f2 f3*
  | (*F134* | *F143* | *F314*) →
      printf "%s␣*␣%s_ff(%s,%s,%s)" *f2 f cp f1 f3*
  | (*F124* | *F142* | *F214*) →
      printf "%s␣*␣%s_ff(%s,%s,%s)" *f2 f cp f1 f3*
  | (*F413* | *F431* | *F341*) →
      printf "%s␣*␣%s_ff(%s,%s,%s)" *f2 cf cm f1 f3*
  | (*F241* | *F412* | *F421*) →
      printf "%s␣*␣%s_ff(%s,%s,%s)" *f2 cf cm f1 f3*

let *print_fermion_s2p_current coeff f c wf1 wf2 wf3 fusion* =
  let *c* = *format_coupling_2 coeff c* and
      f1 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
      f2 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
      f3 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in
  let *c1* = *fastener c* 1 and
      c2 = *fastener c* 2 in
  match *fusion* with
  | (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
      printf "%s␣*␣f_%sf(%s,-(%s),%s,%s)" *f1 f c1 c2 f2 f3*
  | (*F423* | *F243* | *F432* | *F234* | *F342* | *F324*) →
      printf "%s␣*␣f_%sf(%s,%s,%s,%s)" *f1 f c1 c2 f2 f3*
  | (*F134* | *F143* | *F314*) →
      printf "%s␣*␣%s_ff(%s,%s,%s,%s)" *f2 f c1 c2 f1 f3*
  | (*F124* | *F142* | *F214*) →
      printf "%s␣*␣%s_ff(%s,%s,%s,%s)" *f2 f c1 c2 f1 f3*
  | (*F413* | *F431* | *F341*) →
      printf "%s␣*␣%s_ff(%s,-(%s),%s,%s)" *f2 f c1 c2 f1 f3*
  | (*F241* | *F412* | *F421*) →
      printf "%s␣*␣%s_ff(%s,-(%s),%s,%s)" *f2 f c1 c2 f1 f3*

let *print_fermion_s2lr_current coeff f c wf1 wf2 wf3 fusion* =
  let *c* = *format_coupling_2 coeff c* and
      f1 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
      f2 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
      f3 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in

let *c1* = *fastener c* 1 and
    c2 = *fastener c* 2 in
match *fusion* with
| (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
    printf "%s␣*␣f_%sf(%s,%s,%s,%s)" *f1 f c2 c1 f2 f3*
| (*F423* | *F243* | *F432* | *F234* | *F342* | *F324*) →
    printf "%s␣*␣f_%sf(%s,%s,%s,%s)" *f1 f c1 c2 f2 f3*
| (*F134* | *F143* | *F314*) →
    printf "%s␣*␣%s_ff(%s,%s,%s,%s)" *f2 f c1 c2 f1 f3*
| (*F124* | *F142* | *F214*) →
    printf "%s␣*␣%s_ff(%s,%s,%s,%s)" *f2 f c1 c2 f1 f3*
| (*F413* | *F431* | *F341*) →
    printf "%s␣*␣%s_ff(%s,%s,%s,%s)" *f2 f c2 c1 f1 f3*
| (*F241* | *F412* | *F421*) →
    printf "%s␣*␣%s_ff(%s,%s,%s,%s)" *f2 f c2 c1 f1 f3*

let *print_fermion_g4_current coeff f c wf1 wf2 wf3 fusion* =
  let *c* = *format_coupling coeff c* and
      f1 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
      f2 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
      f3 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in
  match *fusion* with
  | (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
      printf "f_%sgr(-%s,%s,%s,%s)" *f c f1 f2 f3*
  | (*F423* | *F243* | *F432* | *F234* | *F342* | *F324*) →
      printf "gr_%sf(%s,%s,%s,%s)" *f c f1 f2 f3*
  | (*F134* | *F143* | *F314* | *F124* | *F142* | *F214*) →
      printf "%s_grf(%s,%s,%s,%s)" *f c f1 f2 f3*
  | (*F413* | *F431* | *F341* | *F241* | *F412* | *F421*) →
      printf "%s_fgr(-%s,%s,%s,%s)" *f c f1 f2 f3*

let *print_fermion_2_g4_current coeff f c wf1 wf2 wf3 fusion* =
  let *f1* = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
      f2 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
      f3 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in
  let *c* = *format_coupling_2 coeff c* in
  let *c1* = *fastener c* 1 and
      c2 = *fastener c* 2 in
  match *fusion* with
  | (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
      printf "f_%sgr(-(%s),-(%s),%s,%s,%s)" *f c2 c1 f1 f2 f3*
  | (*F423* | *F243* | *F432* | *F234* | *F342* | *F324*) →
      printf "gr_%sf(%s,%s,%s,%s,%s)" *f c1 c2 f1 f2 f3*
  | (*F134* | *F143* | *F314* | *F124* | *F142* | *F214*) →
      printf "%s_grf(%s,%s,%s,%s,%s)" *f c1 c2 f1 f2 f3*
  | (*F413* | *F431* | *F341* | *F241* | *F412* | *F421*) →
      printf "%s_fgr(-(%s),-(%s),%s,%s,%s)" *f c2 c1 f1 f2 f3*

let *print_fermion_g4_current_rev coeff f c wf1 wf2 wf3 fusion* =
  let *c* = *format_coupling coeff c* and
      f1 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
      f2 = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and

```
      f3  =  (List.nth (wf_of_f wf1 wf2 wf3 fusion) 2) in
    match fusion with
    | (F123 | F213 | F132 | F231 | F312 | F321) →
        printf "f_%sgr(%s,%s,%s,%s)" f c f1 f2 f3
    | (F423 | F243 | F432 | F234 | F342 | F324) →
        printf "gr_%sf(-%s,%s,%s,%s)" f c f1 f2 f3
    | (F134 | F143 | F314 | F124 | F142 | F214) →
        printf "%s_grf(-%s,%s,%s,%s)" f c f1 f2 f3
    | (F413 | F431 | F341 | F241 | F412 | F421) →
        printf "%s_fgr(%s,%s,%s,%s)" f c f1 f2 f3
```

Here we have to distinguish which of the two bosons is produced in the fusion of three particles which include both fermions.

```
    let print_fermion_g4_vector_current coeff f c wf1 wf2 wf3 fusion =
      let c  =  format_coupling coeff c and
          d1  =  d_p (1,f) and
          d2  =  d_p (2,f) and
          f1  =  (List.nth (wf_of_f wf1 wf2 wf3 fusion) 0) and
          f2  =  (List.nth (wf_of_f wf1 wf2 wf3 fusion) 1) and
          f3  =  (List.nth (wf_of_f wf1 wf2 wf3 fusion) 2) in
      match fusion with
      | (F123 | F213 | F132 | F231 | F312 | F321) →
          printf "f_%sgr(%s,%s,%s,%s)" f c f1 f2 f3
      | (F423 | F243 | F432 | F234 | F342 | F324) →
          printf "gr_%sf(%s,%s,%s,%s)" f c f1 f2 f3
      | (F134 | F143 | F314) → printf "%s%s_grf(%s,%s,%s,%s)" f d1 c f1 f2 f3
      | (F124 | F142 | F214) → printf "%s%s_grf(%s,%s,%s,%s)" f d2 c f1 f2 f3
      | (F413 | F431 | F341) → printf "%s%s_fgr(%s,%s,%s,%s)" f d1 c f1 f2 f3
      | (F241 | F412 | F421) → printf "%s%s_fgr(%s,%s,%s,%s)" f d2 c f1 f2 f3

    let print_fermion_2_g4_vector_current coeff f c wf1 wf2 wf3 fusion =
      let d1  =  d_p (1,f) and
          d2  =  d_p (2,f) and
          f1  =  (List.nth (wf_of_f wf1 wf2 wf3 fusion) 0) and
          f2  =  (List.nth (wf_of_f wf1 wf2 wf3 fusion) 1) and
          f3  =  (List.nth (wf_of_f wf1 wf2 wf3 fusion) 2) in
      let c  =  format_coupling_2 coeff c in
      let c1  =  fastener c 1 and
          c2  =  fastener c 2 in
      match fusion with
      | (F123 | F213 | F132 | F231 | F312 | F321) →
          printf "f_%sgr(%s,%s,%s,%s,%s)" f c1 c2 f1 f2 f3
      | (F423 | F243 | F432 | F234 | F342 | F324) →
          printf "gr_%sf(%s,%s,%s,%s,%s)" f c1 c2 f1 f2 f3
      | (F134 | F143 | F314) → printf "%s%s_grf(%s,%s,%s,%s,%s)" f d1 c1 c2 f1 f2 f3
      | (F124 | F142 | F214) → printf "%s%s_grf(%s,%s,%s,%s,%s)" f d2 c1 c2 f1 f2 f3
      | (F413 | F431 | F341) → printf "%s%s_fgr(%s,%s,%s,%s,%s)" f d1 c1 c2 f1 f2 f3
      | (F241 | F412 | F421) → printf "%s%s_fgr(%s,%s,%s,%s,%s)" f d2 c1 c2 f1 f2 f3

    let print_fermion_g4_vector_current_rev coeff f c wf1 wf2 wf3 fusion =
      let c  =  format_coupling coeff c and
```

      $d1$ = d_p (1, f) and
      $d2$ = d_p (2, f) and
      $f1$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
      $f2$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
      $f3$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in
   match *fusion* with
   | (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
     printf `"gr_%sf(%s,%s,%s,%s)"` *f c f1 f2 f3*
   | (*F423* | *F243* | *F432* | *F234* | *F342* | *F324*) →
     printf `"f_%sgr(%s,%s,%s,%s)"` *f c f1 f2 f3*
   | (*F134* | *F143* | *F314*) → *printf* `"%s%s_fgr(%s,%s,%s,%s)"` *f d1 c f1 f2 f3*
   | (*F124* | *F142* | *F214*) → *printf* `"%s%s_fgr(%s,%s,%s,%s)"` *f d2 c f1 f2 f3*
   | (*F413* | *F431* | *F341*) → *printf* `"%s%s_grf(%s,%s,%s,%s)"` *f d1 c f1 f2 f3*
   | (*F241* | *F412* | *F421*) → *printf* `"%s%s_grf(%s,%s,%s,%s)"` *f d2 c f1 f2 f3*

let *print_fermion_2_g4_current_rev coeff f c wf1 wf2 wf3 fusion* =
  let c = *format_coupling_2 coeff c* in
  let $c1$ = *fastener c* 1 and
     $c2$ = *fastener c* 2 and
     $d1$ = d_p (1, f) and
     $d2$ = d_p (2, f) in
  let $f1$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
     $f2$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
     $f3$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in
  match *fusion* with
  | (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
    printf `"gr_%sf(%s,%s,%s,%s,%s)"` *f c1 c2 f1 f2 f3*
  | (*F423* | *F243* | *F432* | *F234* | *F342* | *F324*) →
    printf `"f_%sgr(-(%s),-(%s),%s,%s,%s)"` *f c1 c2 f1 f2 f3*
  | (*F134* | *F143* | *F314*) →
    printf `"%s%s_fgr(-(%s),-(%s),%s,%s,%s)"` *f d1 c1 c2 f1 f2 f3*
  | (*F124* | *F142* | *F214*) →
    printf `"%s%s_fgr(-(%s),-(%s),%s,%s,%s)"` *f d2 c1 c2 f1 f2 f3*
  | (*F413* | *F431* | *F341*) →
    printf `"%s%s_grf(%s,%s,%s,%s,%s)"` *f d1 c1 c2 f1 f2 f3*
  | (*F241* | *F412* | *F421*) →
    printf `"%s%s_grf(%s,%s,%s,%s,%s)"` *f d2 c1 c2 f1 f2 f3*

let *print_fermion_2_g4_vector_current_rev coeff f c wf1 wf2 wf3 fusion* =
  (∗ Here we put in the extra minus sign from the coeff. ∗)
  let c = *format_coupling coeff c* in
  let $c1$ = *fastener c* 1 and
     $c2$ = *fastener c* 2 in
  let $d1$ = d_p (1, f) and
     $d2$ = d_p (2, f) and
     $f1$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 0) and
     $f2$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 1) and
     $f3$ = (*List.nth* (*wf_of_f wf1 wf2 wf3 fusion*) 2) in
  match *fusion* with
  | (*F123* | *F213* | *F132* | *F231* | *F312* | *F321*) →
    printf `"gr_%sf(%s,%s,%s,%s,%s)"` *f c1 c2 f1 f2 f3*

```
        | (F423 | F243 | F432 | F234 | F342 | F324) →
            printf "f-%sgr(%s,%s,%s,%s,%s)" f c1 c2 f1 f2 f3
        | (F134 | F143 | F314) → printf "%s%s-fgr(%s,%s,%s,%s,%s)" f d1 c1 c2 f1 f2 f3
        | (F124 | F142 | F214) → printf "%s%s-fgr(%s,%s,%s,%s,%s)" f d2 c1 c2 f1 f2 f3
        | (F413 | F431 | F341) → printf "%s%s-grf(%s,%s,%s,%s,%s)" f d1 c1 c2 f1 f2 f3
        | (F241 | F412 | F421) → printf "%s%s-grf(%s,%s,%s,%s,%s)" f d2 c1 c2 f1 f2 f3

    let print_current_g4 = function
      | coeff, Gravbar, S2, _ → print_fermion_g4_current coeff "s2"
      | coeff, Gravbar, SV, _ → print_fermion_g4_vector_current coeff "sv"
      | coeff, Gravbar, SLV, _ → print_fermion_g4_vector_current coeff "slv"
      | coeff, Gravbar, SRV, _ → print_fermion_g4_vector_current coeff "srv"
      | coeff, Gravbar, SLRV, _ → print_fermion_2_g4_vector_current coeff "slrv"
      | coeff, Gravbar, PV, _ → print_fermion_g4_vector_current coeff "pv"
      | coeff, Gravbar, V2, _ → print_fermion_g4_current coeff "v2"
      | coeff, Gravbar, V2LR, _ → print_fermion_2_g4_current coeff "v2lr"
      | _, Gravbar, _, _ → invalid_arg "print_current_g4:␣not␣implemented"
      | coeff, _, S2, Grav → print_fermion_g4_current_rev coeff "s2"
      | coeff, _, SV, Grav → print_fermion_g4_vector_current_rev (−coeff) "sv"
      | coeff, _, SLV, Grav → print_fermion_g4_vector_current_rev (−coeff) "slv"
      | coeff, _, SRV, Grav → print_fermion_g4_vector_current_rev (−coeff) "srv"
      | coeff, _, SLRV, Grav → print_fermion_2_g4_vector_current_rev coeff "slrv"
      | coeff, _, PV, Grav → print_fermion_g4_vector_current_rev coeff "pv"
      | coeff, _, V2, Grav → print_fermion_g4_vector_current_rev coeff "v2"
      | coeff, _, V2LR, Grav → print_fermion_2_g4_current_rev coeff "v2lr"
      | _, _, _, Grav → invalid_arg "print_current_g4:␣not␣implemented"
      | coeff, _, S2, _ → print_fermion_s2_current coeff "s"
      | coeff, _, P2, _ → print_fermion_s2_current coeff "p"
      | coeff, _, S2P, _ → print_fermion_s2p_current coeff "sp"
      | coeff, _, S2L, _ → print_fermion_s2_current coeff "sl"
      | coeff, _, S2R, _ → print_fermion_s2_current coeff "sr"
      | coeff, _, S2LR, _ → print_fermion_s2lr_current coeff "slr"
      | coeff, _, V2, _ → print_fermion_g4_brs_vector_current coeff "v2"
      | coeff, _, SV, _ → print_fermion_g4_brs_vector_current coeff "sv"
      | coeff, _, PV, _ → print_fermion_g4_brs_vector_current coeff "pv"
      | coeff, _, SLV, _ → print_fermion_g4_brs_vector_current coeff "svl"
      | coeff, _, SRV, _ → print_fermion_g4_brs_vector_current coeff "svr"
      | coeff, _, SLRV, _ → print_fermion_g4_svlr_current coeff "svlr"
      | _, _, V2LR, _ → invalid_arg "Targets.print_current:␣not␣available"

    let reverse_braket _ = false

    let use_module = "omega95_bispinors"
    let require_library =
      ["omega_bispinors_2010_01_A"; "omega_bispinor_cpls_2010_01_A"]
  end

module Fortran_Majorana = Make_Fortran(Fortran_Majorana_Fermions)
```

FORTRAN 77

```
module Fortran77 = Dummy
```

15.2.3 *C*

module *C* = *Dummy*

C++

module *Cpp* = *Dummy*

Java

module *Java* = *Dummy*

15.2.4   *O'Caml*

module *Ocaml* = *Dummy*

15.2.5   LaTeX

module *LaTeX* = *Dummy*

15.3   Interface of *Targets_Kmatrix*

module *Fortran* : sig val *print* : *bool* → *unit* end

15.4   Implementation of *Targets_Kmatrix*

let *rcs_file* = *RCS.parse* "Targets_Kmatrix" ["K-Matrix␣Support␣routines"]
    { *RCS.revision* = "$Revision:␣7367␣$";
      RCS.date = "$Date:␣2015-11-13␣18:28:39␣+0100␣(Fri,␣13␣Nov␣2015)␣$";
      RCS.author = "$Author:␣jr_reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr

module *Fortran* =
  struct

    open *Format*

    let *nl* = *print_newline*

Special functions for the K matrix approach. This might be generalized to other
functions that have to have access to the parameters and coupling constants.
At the moment, this is hardcoded.

    let *print pure_functions* =
      let *pure* =
        if *pure_functions* then

```
      "pure␣"
    else
      "" in
printf "␣␣!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!"; nl ();
printf "␣␣!!!␣Special␣K␣matrix␣functions"; nl ();
printf "␣␣!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!"; nl ();
nl();
printf "␣␣%sfunction␣width_res␣(z,res,w_wkm,m,g)␣result␣(w)" pure; nl ();
printf "␣␣␣␣␣␣real(kind=default),␣intent(in)␣::␣z,␣w_wkm,␣m,␣g"; nl ();
printf "␣␣␣␣␣␣integer,␣intent(in)␣::␣res"; nl ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣w"; nl ();
printf "␣␣␣␣␣␣if␣(z.eq.0␣.AND.␣w_wkm.eq.0␣)␣then"; nl ();
printf "␣␣␣␣␣␣␣␣w␣=␣0"; nl ();
printf "␣␣␣␣␣␣else"; nl ();
printf "␣␣␣␣␣␣␣␣if␣(w_wkm.eq.0)␣then"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣select␣case␣(res)"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣case␣(1)␣!!!␣Scalar␣isosinglet"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣w␣=␣3.*g**2/32./Pi␣*␣m**3/vev**2"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣case␣(2)␣!!!␣Scalar␣isoquintet"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣w␣=␣g**2/64./Pi␣*␣m**3/vev**2"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣case␣(3)␣!!!␣Vector␣isotriplet"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣w␣=␣g**2/48./Pi␣*␣m"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣case␣(4)␣!!!␣Tensor␣isosinglet"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣w␣=␣g**2/320./Pi␣*␣m**3/vev**2"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣case␣(5)␣!!!␣Tensor␣isoquintet"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣w␣=␣g**2/1920./Pi␣*␣m**3/vev**2"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣case␣default"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣w␣=␣0"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣end␣select"; nl ();
printf "␣␣␣␣␣␣␣␣else"; nl ();
printf "␣␣␣␣␣␣␣␣␣␣w␣=␣w_wkm"; nl ();
printf "␣␣␣␣␣␣␣␣end␣if"; nl ();
printf "␣␣␣␣␣␣end␣if"; nl ();
printf "␣␣end␣function␣width_res"; nl ();
nl ();
printf "␣␣%sfunction␣s0stu␣(s,␣m)␣result␣(s0)" pure; nl ();
printf "␣␣␣␣␣␣real(kind=default),␣intent(in)␣::␣s,␣m"; nl ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s0"; nl ();
printf "␣␣␣␣␣␣if␣(m.ge.1.0e08)␣then"; nl ();
printf "␣␣␣␣␣␣␣␣s0␣=␣0"; nl ();
printf "␣␣␣␣␣␣else"; nl ();
printf "␣␣␣␣␣␣␣␣s0␣=␣m**2␣-␣s/2␣+␣m**4/s␣*␣log(m**2/(s+m**2))"; nl ();
printf "␣␣␣␣␣␣end␣if"; nl ();
printf "␣␣end␣function␣s0stu"; nl();
nl ();
printf "␣␣%sfunction␣s1stu␣(s,␣m)␣result␣(s1)" pure; nl ();
printf "␣␣␣␣␣␣real(kind=default),␣intent(in)␣::␣s,␣m"; nl ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s1"; nl ();
printf "␣␣␣␣␣␣if␣(m.ge.1.0e08)␣then"; nl ();
printf "␣␣␣␣␣␣␣␣s1␣=␣0"; nl ();
```

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```
printf "       else"; nl ();
printf "          s1 = 2*m**4/s + s/6 + m**4/s**2*(2*m**2+s) &"; nl();
printf "              * log(m**2/(s+m**2))"; nl ();
printf "       end if"; nl ();
printf "  end function s1stu"; nl();
nl ();
printf "  %sfunction s2stu (s, m) result (s2)" pure; nl ();
printf "       real(kind=default), intent(in) :: s, m"; nl ();
printf "       real(kind=default) :: s2"; nl ();
printf "       if (m.ge.1.0e08) then"; nl ();
printf "          s2 = 0"; nl ();
printf "       else"; nl ();
printf "          s2 = m**4/s**2 * (6*m**2 + 3*s) + &"; nl();
printf "              m**4/s**3 * (6*m**4 + 6*m**2*s + s**2) &"; nl();
printf "              * log(m**2/(s+m**2))"; nl ();
printf "       end if"; nl ();
printf "  end function s2stu"; nl();
nl ();
printf " !! %sfunction s3stu (s, m) result (s3)" pure; nl ();
printf " !!      real(kind=default), intent(in) :: s, m"; nl ();
printf " !!      real(kind=default) :: s3"; nl ();
printf " !!      if (m.ge.1.0e08) then"; nl ();
printf " !!         s3 = 0"; nl ();
printf " !!      else"; nl ();
printf " !!         s3 = m**4/s**3 * (60*m**4 + 60*m**2*s+11*s**2) + &"; nl();
printf " !!             m**4/s**4 * (2*m**2+s) (10*m**4 + 10*m**2*s + s**2) &"; nl();
printf " !!             * log(m**2/(s+m**2))"; nl ();
printf " !!      end if"; nl ();
printf " !!  end function s3stu"; nl();
nl ();
printf "  %sfunction p0stu (s, m) result (p0)" pure; nl ();
printf "       real(kind=default), intent(in) :: s, m"; nl ();
printf "       real(kind=default) :: p0"; nl ();
printf "       if (m.ge.1.0e08) then"; nl ();
printf "          p0 = 0"; nl ();
printf "       else"; nl ();
printf "          p0 = 1 + (2*s+m**2)*log(m**2/(s+m**2))/s"; nl ();
printf "       end if"; nl ();
printf "  end function p0stu"; nl();
nl ();
printf "  %sfunction p1stu (s, m) result (p1)" pure; nl ();
printf "       real(kind=default), intent(in) :: s, m"; nl ();
printf "       real(kind=default) :: p1"; nl ();
printf "       if (m.ge.1.0e08) then"; nl ();
printf "          p1 = 0"; nl ();
printf "       else"; nl ();
printf "          p1 = (m**2 + 2*s)/s**2 * (2*s+(2*m**2+s) &"; nl();
printf "                  * log(m**2/(s+m**2)))"; nl ();
printf "       end if"; nl ();
printf "  end function p1stu"; nl();
```

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nl ();
printf "␣␣%sfunction␣d0stu␣(s,␣m)␣result␣(d0)" *pure*; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣intent(in)␣::␣s,␣m"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣d0"; *nl* ();
printf "␣␣␣␣␣␣if␣(m.ge.1.0e08)␣then"; *nl* ();
printf "␣␣␣␣␣␣␣␣d0␣=␣0"; *nl* ();
printf "␣␣␣␣␣␣else"; *nl* ();
printf "␣␣␣␣␣␣␣␣d0␣=␣(2*m**2+11*s)/2␣+␣(m**4+6*m**2*s+6*s**2)␣&"; *nl*();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣/s␣*␣log(m**2/(s+m**2))"; *nl* ();
printf "␣␣␣␣␣␣end␣if"; *nl* ();
printf "␣␣end␣function␣d0stu"; *nl*();
nl ();
printf "␣␣%sfunction␣d1stu␣(s,␣m)␣result␣(d1)" *pure*; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣intent(in)␣::␣s,␣m"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣d1"; *nl* ();
printf "␣␣␣␣␣␣if␣(m.ge.1.0e08)␣then"; *nl* ();
printf "␣␣␣␣␣␣␣␣d1␣=␣0"; *nl* ();
printf "␣␣␣␣␣␣else"; *nl* ();
printf "␣␣␣␣␣␣␣␣d1␣=␣(s*(12*m**4␣+␣72*m**2*s␣+␣73*s**2)␣&"; *nl*();
printf "␣␣␣␣␣␣␣␣␣␣␣␣+␣6*(2*m**2␣+␣s)*(m**4␣+␣6*m**2*s␣+␣6*s**2)␣&"; *nl*();
printf "␣␣␣␣␣␣␣␣␣␣␣␣*␣log(m**2/(s+m**2)))/6/s**2"; *nl* ();
printf "␣␣␣␣␣␣end␣if"; *nl* ();
printf "␣␣end␣function␣d1stu"; *nl*();
nl ();
printf "␣␣%sfunction␣da00␣(cc,␣s,␣m)␣result␣(amp_00)" *pure*; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣intent(in)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣a00_0,␣a00_1,␣a00_a,␣a00_f"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default),␣dimension(1:7)␣::␣a00"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣ii,␣jj,␣amp_00"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣kappal,␣kappam,␣kappat"; *nl* ();
printf "␣␣␣␣␣␣ii␣=␣cmplx(0.0,1.0/32.0/Pi,default)"; *nl* ();
printf "␣␣␣␣␣␣jj␣=␣s**2/vev**4*ii"; *nl* ();
printf "␣␣␣␣␣␣kappal␣=␣cc(12)*((mass(23)**2+mass(24)**2)/m(4)**2-2*mass(23)**2*mass(24";
printf "␣␣␣␣␣␣kappam␣=␣cc(12)*((mass(23)**4+mass(24)**4)/m(4)**2/(mass(23)**2+mass(24)";
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣-␣2*mass(23)**2*mass(24)**2/m(4)**4)"; *nl* ();
printf "␣␣␣␣␣␣kappat␣=␣cc(12)*mass(23)**2*mass(24)**2/m(4)**4"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Longitudinal"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Scalar␣isosinglet"; *nl* ();
printf "␣␣␣␣␣␣a00(1)␣=␣-2.0␣*␣cc(1)**2/vev**2␣*␣s0stu(s,m(1))␣"; *nl* ();
printf "␣␣␣␣␣␣if␣(cc(1)␣/=␣0)␣then"; *nl* ();
printf "␣␣␣␣␣␣␣␣a00(1)␣=␣a00(1)␣-␣3.0*cc(1)**2/vev**2␣*␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣s**2/cmplx(s-m(1)**2,m(1)*wkm(1),default)␣"; *nl* ();
printf "␣␣␣␣␣␣end␣if"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Scalar␣isoquintet"; *nl* ();
printf "␣␣␣␣␣␣a00(2)␣=␣-5.0*cc(2)**2/vev**2␣*␣s0stu(s,m(2))␣/␣3.0"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Vector␣isotriplet"; *nl* ();
printf "␣␣␣␣␣␣a00(3)␣=␣-cc(3)**2*(4.0*p0stu(s,m(3))␣+␣6.0*s/m(3)**2)"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Tensor␣isosinglet"; *nl* ();

```
printf "       a00(4) = -cc(4)**2/vev**2/3 * (d0stu(s,m(4)) &"; nl ();
printf "                  - 2*kappal*s0stu(s,m(4)))"; nl ();
printf "       if ( (cc(4) /= 0).and.(kappal /= 0)) then"; nl ();
printf "         a00(4) = a00(4) - cc(4)**2/vev**2*kappal * &"; nl ();
printf "                  s**2/cmplx(s-m(4)**2,m(4) * wkm(4),default)"; nl ();
printf "       end if"; nl ();
printf "       !!! Tensor isoquintet"; nl ();
printf "       a00(5) = -5.0*cc(5)**2/vev**2*(d0stu(s,m(5)) &"; nl ();
printf "                  /3.0)/6.0"; nl ();
printf "       !!! Transversal"; nl ();
printf "       !!! Tensor isosinglet"; nl ();
printf "       a00(6) = - cc(9)**2/Pi/vev**6*mass(23)**2*mass(24)**2/4 * s**2 &"; nl ();
printf "                  * ((2-2*s/m(4)**2+s**2/m(4)**4)+kappat/2 )"; nl ();
printf "       if (a00(6) /= 0) then"; nl ();
printf "         a00(6) = a00(6)/cmplx(s-m(4)**2, - w_res/32/Pi * real(a00(6),default),
printf "       end if"; nl ();
printf "       a00(6) = a00(6) - cc(9)**2/Pi/vev**6*mass(23)**2*mass(24)**2/12 * (s0stu
printf "                  * (3*(1+2*s/m(4)**2+2*s**2/m(4)**4)+kappat ))"; nl ();
printf "       !!! Mixed"; nl ();
printf "       !!! Tensor isosinglet"; nl ();
printf "       a00(7) = - cc(11)*cc(9)*cc(4)/Pi/vev**4*(mass(23)**2+mass(24)**2)/4 * s*
printf "                  *  ((1-4*s/m(4)**2+2*s**2/m(4)**4)+kappam )"; nl ();
printf "       if (a00(7) /= 0) then"; nl ();
printf "         a00(7) = a00(7)/cmplx(s-m(4)**2, - w_res/32/Pi * real(a00(7),default),
printf "       end if"; nl ();
printf "       a00(7) = a00(7) - cc(11)*cc(9)*cc(4)/Pi/vev**4*(mass(23)**2+mass(24)**2)
printf "                  * (12*s/m(4)**2+12*s**2/m(4)**4+2*kappam ))"; nl ();
printf "       !!! Fudge-Higgs"; nl ();
printf "       a00_f = 2.*fudge_higgs*s/vev**2"; nl ();
printf "       a00_f = a00_f !!! - 0*5.*(1-ghvva)**2/vev**2*mass(25)**2"; nl ();
printf "       !!! Low energy theory alphas"; nl ();
printf "       a00_0 = 8.*(7.*a4 + 11.*a5)/3.*s**2/vev**4"; nl ();
printf "       a00_1 = (25.*log(lam_reg**2/s)/9 + 11./54.0_default)*s**2/vev**4"; nl ();
printf "       a00_a = a00_0 !!! + a00_1/16./Pi**2"; nl ();
printf "       !!! Unitarize"; nl ();
printf "       if (fudge_km /= 0) then"; nl ();
printf "         amp_00 = sum(a00)+a00_f+a00_a"; nl();
printf "         if (amp_00 /= 0) then"; nl ();
printf "           amp_00 = - a00_a - a00_f - part_r * (sum(a00) - a00(3)) + 1/(real(1/
printf "         end if"; nl ();
printf "       else"; nl ();
printf "         amp_00 = (1-part_r) * sum(a00) + part_r * a00(3)"; nl ();
printf "       end if"; nl ();
printf "       amp_00 = vev**4/s**2 * amp_00"; nl ();
printf "  end function da00"; nl();
nl ();
printf "  %sfunction da02 (cc, s, m) result (amp_02)" pure; nl ();
printf "       real(kind=default), intent(in) :: s"; nl ();
printf "       real(kind=default), dimension(1:12), intent(in) :: cc"; nl ();
printf "       real(kind=default), dimension(1:5), intent(in) :: m"; nl ();
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printf "      complex(kind=default) :: a02_0, a02_1, a02_a"; nl ();
printf "      complex(kind=default), dimension(1:7) :: a02"; nl ();
printf "      complex(kind=default) :: ii, jj, amp_02"; nl ();
printf "      real(kind=default) :: kappal, kappam, kappat"; nl ();
printf "      ii = cmplx(0.0,1.0/32.0/Pi,default)"; nl ();
printf "      jj = s**2/vev**4*ii"; nl ();
printf "      kappal = cc(12)*((mass(23)**2+mass(24)**2)/m(4)**2-2*mass(23)**2*mass(24
printf "      kappam = cc(12)*((mass(23)**4+mass(24)**4)/m(4)**2/(mass(23)**2+mass(24)
printf "              - 2*mass(23)**2*mass(24)**2/m(4)**4)"; nl ();
printf "      kappat = cc(12)*mass(23)**2*mass(24)**2/m(4)**4"; nl ();
printf "      !!! Longitudinal"; nl ();
printf "      !!! Scalar  isosinglet"; nl ();
printf "      a02(1) = -2.0*cc(1)**2/vev**2 * s2stu(s,m(1))"; nl ();
printf "      !!! Scalar isoquintet"; nl ();
printf "      a02(2) = -5.0*cc(2)**2/vev**2 * s2stu(s,m(2)) / 3.0"; nl ();
printf "      !!! Vector isotriplet"; nl ();
printf "      a02(3) = -4.0*cc(3)**2*(2*s+m(3)**2)*s2stu(s,m(3))/m(3)**4"; nl ();
printf "      !!! Tensor isosinglet"; nl ();
printf "      a02(4) = - cc(4)**2/vev**2/3 *  &"; nl ();
printf "                    ((1.+6.*s/m(4)**2+6.*s**2/m(4)**4)-2*kappal) * s2stu(s,m(4))
printf "      if (cc(4) /= 0) then"; nl ();
printf "         a02(4) = a02(4) - cc(4)**2/vev**2/10. &"; nl ();
printf "                     * s**2/cmplx(s-m(4)**2,m(4)*wkm(4),default)"; nl ();
printf "      end if"; nl ();
printf "      !!! Tensor isoquintet"; nl ();
printf "      a02(5) = - cc(5)**2/vev**2*(5.0*(1.0+6.0* &"; nl ();
printf "                    s/m(5)**2+6.0*s**2/m(5)**4)*s2stu(s,m(5))/3.0 &"; nl ();
printf "                    )/6.0"; nl ();
printf "      !!! Transversal"; nl ();
printf "      !!! Tensor isosinglet"; nl ();
printf "      a02(6) = - cc(9)**2/Pi/vev**6*mass(23)**2*mass(24)**2/40* s**2"; nl ();
printf "      if (a02(6) /= 0) then"; nl ();
printf "         a02(6) = a02(6)/cmplx(s-m(4)**2, - w_res/32/Pi * real(a02(6),default),
printf "      end if"; nl ();
printf "      a02(6) = a02(6) - cc(9)**2/Pi/vev**6*mass(23)**2*mass(24)**2/12 * (s2stu
printf "                    * (3*(1+2*s/m(4)**2+2*s**2/m(4)**4)+kappat ))"; nl ();
printf "      !!! Mixed"; nl ();
printf "      !!! Tensor isosinglet"; nl ();
printf "      a02(7) = - cc(11)*cc(9)*cc(4)/Pi/vev**4*(mass(23)**2+mass(24)**2)/20 &";
printf "                    * s**2"; nl ();
printf "      if (a02(7) /= 0) then"; nl ();
printf "         a02(7) = a02(7)/cmplx(s-m(4)**2, - w_res/32/Pi * real(a02(7),default),
printf "      end if"; nl ();
printf "      a02(7) = a02(7) - cc(11)*cc(9)*cc(4)/Pi/vev**4*(mass(23)**2+mass(24)**2)
printf "                    * (12*s/m(4)**2+12*s**2/m(4)**4+2*kappam ))"; nl ();
printf "      !!! Low energy theory alphas"; nl ();
printf "      a02_0 = (8.*(2.*a4 + a5)/15.)  * s**2/vev**4"; nl ();
printf "      a02_1 = (log(lam_reg**2/s)/9. - 7./135.0_default) *  s**2/vev**4"; nl ();
printf "      a02_a = a02_0 !!! + a02_1/16/Pi**2"; nl ();
printf "      !!! Unitarize"; nl ();
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printf "        if (fudge_km /= 0) then"; nl ();
printf "          amp_02 = sum(a02)+a02_a"; nl();
printf "          if (amp_02 /= 0) then"; nl ();
printf "            amp_02 =  a02_a - part_r * (sum(a02) - a02(3)) + 1/(real(1/amp_02,d
printf "          end if"; nl ();
printf "        else"; nl ();
printf "          amp_02 = (1-part_r) * sum(a02) + part_r * a02(3)"; nl ();
printf "        end if"; nl ();
printf "        amp_02 = vev**4/s**2 * amp_02"; nl ();
printf "  end function da02"; nl();
nl ();
printf "  %sfunction da11 (cc, s, m) result (amp_11)" pure; nl ();
printf "       real(kind=default), intent(in) :: s"; nl ();
printf "       real(kind=default), dimension(1:12), intent(in) :: cc"; nl ();
printf "       real(kind=default), dimension(1:5), intent(in) :: m"; nl ();
printf "       complex(kind=default) :: a11_0,a11_1,a11_a,a11_f"; nl ();
printf "       complex(kind=default), dimension(1:7) :: a11"; nl ();
printf "       complex(kind=default) :: ii, jj, amp_11"; nl ();
printf "       real(kind=default) :: kappal, kappam, kappat"; nl ();
printf "       ii = cmplx(0.0,1.0/32.0/Pi,default)"; nl ();
printf "       jj = s**2/vev**4*ii"; nl ();
printf "       kappal = cc(12)*((mass(23)**2+mass(24)**2)/m(4)**2-2*mass(23)**2*mass(24
printf "       kappam = cc(12)*((mass(23)**4+mass(24)**4)/m(4)**2/(mass(23)**2+mass(24)
printf "                - 2*mass(23)**2*mass(24)**2/m(4)**4)"; nl ();
printf "       kappat = cc(12)*mass(23)**2*mass(24)**2/m(4)**4"; nl ();
printf "       !!! Longitudinal"; nl ();
printf "       !!! Scalar isosinglet"; nl ();
printf "       a11(1) = - 2.0*cc(1)**2/vev**2 * s1stu(s,m(1))"; nl ();
printf "       !!! Scalar isoquintet"; nl ();
printf "       a11(2) = 5.0*cc(2)**2/vev**2 * s1stu(s,m(2)) / 6.0"; nl ();
printf "       !!! Vector isotriplet"; nl ();
printf "       a11(3) = - cc(3)**2 * &"; nl ();
printf "                    (s/m(3)**2 + 2. * p1stu(s,m(3)))"; nl ();
printf "       if (cc(3) /= 0) then"; nl ();
printf "         a11(3) = a11(3) -2./3. * cc(3)**2 * &"; nl ();
printf "                      s/cmplx(s-m(3)**2,m(3)*wkm(3),default) "; nl ();
printf "       end if"; nl ();
printf "       !!! Tensor isosinglet"; nl ();
printf "       a11(4) = - cc(4)**2/vev**2*(d1stu(s,m(4)-2*kappal*s1stu(s,m(4))) &"; nl ()
printf "                  /3.0)"; nl ();
printf "       !!! Tensor isoquintet"; nl ();
printf "       a11(5) =  5.0*cc(5)**2/vev**2*(d1stu(s,m(5)) &"; nl ();
printf "                  )/36.0"; nl ();
printf "       !!! Transversal"; nl ();
printf "       !!! Tensor isosinglet"; nl ();
printf "       a11(6) = -cc(9)**2/Pi/vev**6*mass(23)**2*mass(24)**2/12 * (s1stu(s,m(4))
printf "                (3*(1+2*s/m(4)**2+2*s**2/m(4)**4)+kappat ) - (s/m(4)**2+s**2/m(4)
printf "       !!! Mixed"; nl ();
printf "       !!! Tensor isosinglet"; nl ();
printf "       a11(7) = -cc(11)*cc(9)*cc(4)/Pi/vev**4*(mass(23)**2+mass(24)**2)/12 * (s
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printf "                  *  (12*s/m(4)**2+12*s**2/m(4)**4+2*kappam )  -  2*(s/m(4)**2+s**2/
printf "       !!! Fudge-Higgs"; nl ();
printf "       a11_f  =  fudge_higgs*s/3./vev**2"; nl ();
printf "       !!! Low energy theory alphas"; nl ();
printf "       a11_0  =  4.*(a4  -  2*a5)/3.  *  s**2/vev**4  "; nl ();
printf "       a11_1  =  -  1.0/54.0_default  *  s**2/vev**4"; nl ();
printf "       a11_a  =  a11_0  !!!  +  a11_1/16/Pi**2"; nl ();
printf "       !!! Unitarize"; nl ();
printf "       if  (fudge_km  /=  0)  then"; nl ();
printf "         amp_11  =  sum(a11)+a11_f+a11_a"; nl();
printf "         if  (amp_11  /=  0)  then"; nl ();
printf "           amp_11  =  -  a11_a  -  part_r  *  (sum(a11)  -  a11(3))  +  1/(real(1/amp_11,d
printf "         end  if"; nl ();
printf "       else"; nl ();
printf "         amp_11  =  (1-part_r)  *  sum(a11)  +  part_r  *  a11(3)"; nl ();
printf "       end  if"; nl ();
printf "     amp_11  =  vev**4/s**2  *  amp_11"; nl ();
printf "   end  function  da11"; nl();
nl ();
printf "   %sfunction  da20  (cc,  s,  m)  result  (amp_20)" pure; nl ();
printf "       real(kind=default),  intent(in)  ::  s"; nl ();
printf "       real(kind=default),  dimension(1:12),  intent(in)  ::  cc"; nl ();
printf "       real(kind=default),  dimension(1:5),  intent(in)  ::  m"; nl ();
printf "       complex(kind=default)  ::  a20_0,  a20_1,  a20_a,  a20_f"; nl ();
printf "       complex(kind=default),  dimension(1:7)  ::  a20"; nl ();
printf "       complex(kind=default)  ::  ii,  jj,  amp_20"; nl ();
printf "       real(kind=default)  ::  kappal,  kappam,  kappat"; nl ();
printf "       ii  =  cmplx(0.0,1.0/32.0/Pi,default)"; nl ();
printf "       jj  =  s**2/vev**4*ii"; nl ();
printf "       !!! Scalar isosinglet"; nl ();
printf "       kappal  =  cc(12)*((mass(23)**2+mass(24)**2)/m(4)**2-2*mass(23)**2*mass(24
printf "       kappam  =  cc(12)*((mass(23)**4+mass(24)**4)/m(4)**2/(mass(23)**2+mass(24)
printf "                   -  2*mass(23)**2*mass(24)**2/m(4)**4)"; nl ();
printf "       kappat  =  cc(12)*mass(23)**2*mass(24)**2/m(4)**4"; nl ();
printf "       !!! Longitudinal"; nl ();
printf "       a20(1)  =  -  2.0*cc(1)**2/vev**2  *  s0stu(s,m(1))"; nl ();
printf "       !!! Scalar isoquintet"; nl ();
printf "       a20(2)  =  -  cc(2)**2/vev**2/6.  *  s0stu(s,m(2))"; nl ();
printf "       if  (cc(2)  /=  0)  then"; nl ();
printf "         a20(2)  =  a20(2)  -  cc(2)**2/vev**2/2.  *&"; nl ();
printf "                     s**2/cmplx(s-m(2)**2,m(2)*wkm(2),default)"; nl ();
printf "       end  if"; nl ();
printf "       !!! Vector isotriplet"; nl ();
printf "       a20(3)  =  cc(3)**2*(2.0*p0stu(s,m(3))  +  3.0*s/m(3)**2)"; nl ();
printf "       !!! Tensor isosinglet"; nl ();
printf "       a20(4)  =  -  cc(4)**2/vev**2*(d0stu(s,m(4)-2*kappal*s0stu(s,m(4)))  &"; nl ()
printf "                     /3.0)"; nl ();
printf "       !!! Tensor isoquintet"; nl ();
printf "       a20(5)  =  -  cc(5)**2/vev**2*(d0stu(s,m(5))  &"; nl ();
printf "                     )/36.0"; nl ();
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printf "   !!! Transversal"; *nl* ();
printf "   !!! Tensor isosinglet"; *nl* ();
printf "   a20(6) = -cc(9)**2/Pi/vev**6*mass(23)**2*mass(24)**2/12 * (s0stu(s,m(4))&"; *nl* ();
printf "   * (3*(1+2*s/m(4)**2+2*s**2/m(4)**4)+kappat) - 3*(s/m(4)**2-s**2/&"; *nl* ();
printf "   !!! Mixed"; *nl* ();
printf "   !!! Tensor isosinglet"; *nl* ();
printf "   a20(7) = -cc(11)*cc(9)*cc(4)/Pi/vev**4*(mass(23)**2+mass(24)**2)/12 * (s&"; *nl* ();
printf "   * (12*s/m(4)**2+12*s**2/m(4)**4+2*kappam) - 6*(s/m(4)**2-s**2/&"; *nl* ();
printf "   !!! Fudge-Higgs"; *nl* ();
printf "   a20_f = - fudge_higgs*s/vev**2"; *nl* ();
printf "   a20_f = a20_f - 0*2*(1-ghvva)**2/vev**2*mass(25)**2"; *nl* ();
printf "   !!! Low energy theory alphas"; *nl* ();
printf "   a20_0 = 16*(2*a4 + a5)/3*s**2/vev**4"; *nl* ();
printf "   a20_1 = (10*log(lam_reg**2/s)/9 + 25/108.0_default) * s**2/vev**4"; *nl* ();
printf "   a20_a = a20_0 !!! + a20_1/16/Pi**2"; *nl* ();
printf "   !!! Unitarize"; *nl* ();
printf "   if (fudge_km /= 0) then"; *nl* ();
printf "   amp_20 = sum(a20)+a20_f+a20_a"; *nl*();
printf "   if (amp_20 /= 0) then"; *nl* ();
printf "   amp_20 = - a20_a - a20_f - part_r * (sum(a20) - a20(3)) + 1/(real(1/&"; *nl* ();
printf "   end if"; *nl* ();
printf "   else"; *nl* ();
printf "   amp_20 = (1-part_r) * sum(a20) + part_r * a20(3)"; *nl* ();
printf "   end if"; *nl* ();
printf "   amp_20 = vev**4/s**2 * amp_20"; *nl* ();
printf "   end function da20"; *nl*();
nl ();
printf " %sfunction da22 (cc, s, m) result (amp_22)" *pure*; *nl* ();
printf "   real(kind=default), intent(in) :: s"; *nl* ();
printf "   real(kind=default), dimension(1:12), intent(in) :: cc"; *nl* ();
printf "   real(kind=default), dimension(1:5), intent(in) :: m"; *nl* ();
printf "   complex(kind=default) :: a22_0, a22_1, a22_a, a22_r"; *nl* ();
printf "   complex(kind=default), dimension(1:7) :: a22"; *nl* ();
printf "   complex(kind=default) :: ii, jj, amp_22"; *nl* ();
printf "   real(kind=default) :: kappal, kappam, kappat"; *nl* ();
printf "   ii = cmplx(0.0,1.0/32.0/Pi,default)"; *nl* ();
printf "   jj = s**2/vev**4*ii"; *nl* ();
printf "   kappal = cc(12)*((mass(23)**2+mass(24)**2)/m(4)**2-2*mass(23)**2*mass(24&"; *nl* ();
printf "   kappam = cc(12)*((mass(23)**4+mass(24)**4)/m(4)**2/(mass(23)**2+mass(24)&"; *nl* ();
printf "   - 2*mass(23)**2*mass(24)**2/m(4)**4)"; *nl* ();
printf "   kappat = cc(12)*mass(23)**2*mass(24)**2/m(4)**4"; *nl* ();
printf "   !!! Longitudinal"; *nl* ();
printf "   !!! Scalar isosinglet"; *nl* ();
printf "   a22(1) = - 2.0*cc(1)**2/vev**2 * s2stu(s,m(1)))"; *nl* ();
printf "   !!! Scalar isoquintet"; *nl* ();
printf "   a22(2) = - cc(2)**2/vev**2 * s2stu(s,m(2)) / 6.0"; *nl* ();
printf "   !!! Vector triplet"; *nl* ();
printf "   a22(3) = 2.0*cc(3)**2*(2*s+m(3)**2)*s2stu(s,m(3))/m(3)**4"; *nl* ();
printf "   !!! Tensor isosinglet"; *nl* ();
printf "   a22(4) = - cc(4)**2/vev**2*((1.0 + 6.0*s/m(4)**2 &"; *nl* ();

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printf "␣␣␣␣␣␣␣␣␣␣␣␣+6.0*s**2/m(4)**4-2*kappal)*s2stu(s,m(4))/3.0)"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Tensor␣isoquintet"; *nl* ();
printf "␣␣␣␣␣␣a22(5)␣=␣-␣cc(5)**2/vev**2/36.␣*␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣((1.+6.*s/m(5)**2+6.*s**2/m(5)**4␣)␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*␣s2stu(s,m(5)))"; *nl* ();
printf "␣␣␣␣␣␣if␣(cc(5)␣/=␣0)␣then"; *nl* ();
printf "␣␣␣␣␣␣␣␣a22(5)␣=␣a22(5)␣-␣cc(5)**2/vev**2/60␣*␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣s**2/cmplx(s-m(5)**2,m(5)*wkm(5),default)"; *nl* ();
printf "␣␣␣␣␣␣end␣if"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Transversal"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Tensor␣isosinglet"; *nl* ();
printf "␣␣␣␣␣␣a22(6)␣=␣-cc(9)**2/Pi/vev**6*mass(23)**2*mass(24)**2/12␣*␣(s2stu(s,m(4))"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*␣(3*(1+2*s/m(4)**2+2*s**2/m(4)**4)+kappat␣))"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Mixed"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Tensor␣isosinglet"; *nl* ();
printf "␣␣␣␣␣␣a22(7)␣=␣-cc(11)*cc(9)*cc(4)/Pi/vev**4*(mass(23)**2+mass(24)**2)/12␣*␣(s"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣*␣(12*s/m(4)**2+12*s**2/m(4)**4+2*kappam␣))"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Low␣energy␣theory␣alphas"; *nl* ();
printf "␣␣␣␣␣␣a22_0␣=␣4*(a4␣+␣2*a5)/15*s**2/vev**4␣"; *nl* ();
printf "␣␣␣␣␣␣a22_1␣=␣(2*log(lam_reg**2/s)/45␣-␣247/5400.0_default)*s**2/vev**4"; *nl* ();
printf "␣␣␣␣␣␣a22_a␣=␣a22_0␣!!!␣+␣a22_1/16/Pi**2"; *nl* ();
printf "␣␣␣␣␣␣!!!␣Unitarize"; *nl* ();
printf "␣␣␣␣␣␣if␣(fudge_km␣/=␣0)␣then"; *nl* ();
printf "␣␣␣␣␣␣␣␣amp_22␣=␣sum(a22)+a22_a"; *nl*();
printf "␣␣␣␣␣␣␣␣if␣(amp_22␣/=␣0)␣then"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣amp_22␣=␣-␣a22_a␣-␣part_r␣*␣(sum(a22)␣-␣a22(3))␣+␣1/(real(1/amp_22,d"; *nl* ();
printf "␣␣␣␣␣␣␣␣end␣if"; *nl* ();
printf "␣␣␣␣␣␣else"; *nl* ();
printf "␣␣␣␣␣␣␣␣amp_22␣=␣(1-part_r)␣*␣sum(a22)␣+␣part_r␣*␣a22(3)"; *nl* ();
printf "␣␣␣␣␣␣end␣if"; *nl* ();
printf "␣␣␣␣␣amp_22␣=␣vev**4/s**2␣*␣amp_22"; *nl* ();
printf "␣␣end␣function␣da22"; *nl*();
nl ();
printf "␣␣%sfunction␣dalzz0_s␣(cc,m,k)␣result␣(alzz0_s)" *pure*; *nl* ();
printf "␣␣␣␣␣␣type(momentum),␣intent(in)␣::␣k"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣alzz0_s"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣s␣=␣k*k"; *nl* ();
printf "␣␣␣␣␣␣alzz0_s␣=␣2*g**4/costhw**2*((da00(cc,s,m)␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣-␣da20(cc,s,m))/24␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣-␣(5.)*(da02(cc,s,m)␣-␣da22(cc,s,m))/12)"; *nl* ();
printf "␣␣end␣function␣dalzz0_s"; *nl* ();
nl ();
printf "␣␣%sfunction␣dalzz0_t␣(cc,m,k)␣result␣(alzz0_t)" *pure*; *nl* ();
printf "␣␣␣␣␣␣type(momentum),␣intent(in)␣::␣k"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣alzz0_t"; *nl* ();

printf "␣␣␣␣␣␣real(kind=default)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣s␣=␣k*k"; *nl* ();
printf "␣␣␣␣␣␣alzz0_t␣=␣(5.)*g**4/costhw**2*(da02(cc,s,m)␣-␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣da22(cc,s,m))/4"; *nl* ();
printf "␣␣end␣function␣dalzz0_t"; *nl* ();
nl ();
printf "␣␣%sfunction␣dalzz1_s␣(cc,m,k)␣result␣(alzz1_s)" *pure*; *nl* ();
printf "␣␣␣␣␣␣type(momentum),␣intent(in)␣::␣k"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣alzz1_s"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣s␣=␣k*k"; *nl* ();
printf "␣␣␣␣␣␣alzz1_s␣=␣g**4/costhw**2*(da20(cc,s,m)/8␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣-␣(5.)*da22(cc,s,m)/4)"; *nl* ();
printf "␣␣end␣function␣dalzz1_s"; *nl* ();
nl ();
printf "␣␣%sfunction␣dalzz1_t␣(cc,m,k)␣result␣(alzz1_t)" *pure*; *nl* ();
printf "␣␣␣␣␣␣type(momentum),␣intent(in)␣::␣k"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣alzz1_t"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣s␣=␣k*k"; *nl* ();
printf "␣␣␣␣␣␣alzz1_t␣=␣g**4/costhw**2*(-␣(3.)*da11(cc,s,m)/8␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣+␣3*(5.)*da22(cc,s,m)/8)"; *nl* ();
printf "␣␣end␣function␣dalzz1_t"; *nl* ();
nl ();
printf "␣␣%sfunction␣dalzz1_u␣(cc,m,k)␣result␣(alzz1_u)" *pure*; *nl* ();
printf "␣␣␣␣␣␣type(momentum),␣intent(in)␣::␣k"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣alzz1_u"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣s␣=␣k*k"; *nl* ();
printf "␣␣␣␣␣␣alzz1_u␣=␣g**4/costhw**2*((3.)*da11(cc,s,m)/8␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣+␣3*(5.)*da22(cc,s,m)/8)"; *nl* ();
printf "␣␣end␣function␣dalzz1_u"; *nl* ();
nl ();
printf "␣␣%sfunction␣dalww0_s␣(cc,m,k)␣result␣(alww0_s)" *pure*; *nl* ();
printf "␣␣␣␣␣␣type(momentum),␣intent(in)␣::␣k"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣alww0_s"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣s␣=␣k*k"; *nl* ();
printf "␣␣␣␣␣␣alww0_s␣=␣g**4*((2*da00(cc,s,m)␣+␣da20(cc,s,m))/24␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣-␣(5.)*(2*da02(cc,s,m)␣+␣da22(cc,s,m))/12)"; *nl* ();
printf "␣␣end␣function␣dalww0_s"; *nl* ();
nl ();

```
printf "  %sfunction dalww0_t (cc,m,k) result (alww0_t)" pure; nl ();
printf "       type(momentum), intent(in) :: k"; nl ();
printf "       real(kind=default), dimension(1:12), intent(in) :: cc"; nl ();
printf "       real(kind=default), dimension(1:5), intent(in) :: m"; nl ();
printf "       complex(kind=default) :: alww0_t"; nl ();
printf "       real(kind=default) :: s"; nl ();
printf "       s = k*k"; nl ();
printf "       alww0_t = g**4*(2*(5.)*da02(cc,s,m) - (3.)*da11(cc,s,m) &"; nl ();
printf "                  + (5.)*da22(cc,s,m))/8"; nl ();
printf "  end function dalww0_t"; nl ();
nl ();
printf "  %sfunction dalww0_u (cc,m,k) result (alww0_u)" pure; nl ();
printf "       type(momentum), intent(in) :: k"; nl ();
printf "       real(kind=default), dimension(1:12), intent(in) :: cc"; nl ();
printf "       real(kind=default), dimension(1:5), intent(in) :: m"; nl ();
printf "       complex(kind=default) :: alww0_u"; nl ();
printf "       real(kind=default) :: s"; nl ();
printf "       s = k*k"; nl ();
printf "       alww0_u = g**4*(2*(5.)*da02(cc,s,m) + (3.)*da11(cc,s,m) &"; nl ();
printf "                  + (5.)*da22(cc,s,m))/8"; nl ();
printf "  end function dalww0_u"; nl ();
nl ();
printf "  %sfunction dalww2_s (cc,m,k) result (alww2_s)" pure; nl ();
printf "       type(momentum), intent(in) :: k"; nl ();
printf "       real(kind=default), dimension(1:12), intent(in) :: cc"; nl ();
printf "       real(kind=default), dimension(1:5), intent(in) :: m"; nl ();
printf "       complex(kind=default) :: alww2_s"; nl ();
printf "       real(kind=default) :: s"; nl ();
printf "       s = k*k"; nl ();
printf "       alww2_s = g**4*(da20(cc,s,m) - 2*(5.)*da22(cc,s,m))/4 "; nl ();
printf "  end function dalww2_s"; nl ();
nl ();
printf "  %sfunction dalww2_t (cc,m,k) result (alww2_t)" pure; nl ();
printf "       type(momentum), intent(in) :: k"; nl ();
printf "       real(kind=default), dimension(1:12), intent(in) :: cc"; nl ();
printf "       real(kind=default), dimension(1:5), intent(in) :: m"; nl ();
printf "       complex(kind=default) :: alww2_t"; nl ();
printf "       real(kind=default) :: s"; nl ();
printf "       s = k*k"; nl ();
printf "       alww2_t = 3*(5.)*g**4*da22(cc,s,m)/4"; nl ();
printf "  end function dalww2_t"; nl ();
nl ();
printf "  %sfunction dalz4_s (cc,m,k) result (alz4_s)" pure; nl ();
printf "       type(momentum), intent(in) :: k"; nl ();
printf "       real(kind=default), dimension(1:12), intent(in) :: cc"; nl ();
printf "       real(kind=default), dimension(1:5), intent(in) :: m"; nl ();
printf "       complex(kind=default) :: alz4_s"; nl ();
printf "       real(kind=default) :: s"; nl ();
printf "       s = k*k"; nl ();
printf "       alz4_s = g**4/costhw**4*((da00(cc,s,m) &"; nl ();
```

printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣+␣2*da20(cc,s,m))/12␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣-␣(5.)*(da02(cc,s,m)+2*da22(cc,s,m))/6)"; *nl* ();
printf "␣␣end␣function␣dalz4_s"; *nl* ();
nl ();
printf "␣␣@[<5>";
printf "␣␣%sfunction␣dalz4_t␣(cc,m,k)␣result␣(alz4_t)" *pure*; *nl* ();
printf "␣␣␣␣␣␣type(momentum),␣intent(in)␣::␣k"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:12),␣intent(in)␣::␣cc"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default),␣dimension(1:5),␣intent(in)␣::␣m"; *nl* ();
printf "␣␣␣␣␣␣complex(kind=default)␣::␣alz4_t"; *nl* ();
printf "␣␣␣␣␣␣real(kind=default)␣::␣s"; *nl* ();
printf "␣␣␣␣␣␣s␣=␣k*k"; *nl* ();
printf "␣␣␣␣␣␣alz4_t␣=␣g**4/costhw**4*(5.)*(da02(cc,s,m)␣&"; *nl* ();
printf "␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣+␣2*da22(cc,s,m))/4"; *nl* ();
printf "␣␣end␣function␣dalz4_t"; *nl* ();
nl ();
end

—16—
PHASE SPACE

16.1   Interface of Phasespace

module type T =
  sig
    type $momentum$

    type $\alpha\ t$
    type $\alpha\ decay$

Sort individual decays and complete phasespaces in a canonical order to determine topological equivalence classes.

    val $sort\ :\ (\alpha\ \rightarrow\ \alpha\ \rightarrow\ int)\ \rightarrow\ \alpha\ t\ \rightarrow\ \alpha\ t$
    val $sort\_decay\ :\ (\alpha\ \rightarrow\ \alpha\ \rightarrow\ int)\ \rightarrow\ \alpha\ decay\ \rightarrow\ \alpha\ decay$

Functionals:

    val $map\ :\ (\alpha\ \rightarrow\ \beta)\ \rightarrow\ \alpha\ t\ \rightarrow\ \beta\ t$
    val $map\_decay\ :\ (\alpha\ \rightarrow\ \beta)\ \rightarrow\ \alpha\ decay\ \rightarrow\ \beta\ decay$

    val $eval\ :\ (\alpha\ \rightarrow\ \beta)\ \rightarrow\ (\alpha\ \rightarrow\ \beta)\ \rightarrow\ (\alpha\ \rightarrow\ \beta\ \rightarrow\ \beta\ \rightarrow\ \beta)\ \rightarrow\ \alpha\ t\ \rightarrow\ \beta\ t$
    val $eval\_decay\ :\ (\alpha\ \rightarrow\ \beta)\ \rightarrow\ (\alpha\ \rightarrow\ \beta\ \rightarrow\ \beta\ \rightarrow\ \beta)\ \rightarrow\ \alpha\ decay\ \rightarrow$
$\beta\ decay$

of_momenta f1 f2 plist constructs the phasespace parameterization for a process $f_1 f_2\ \rightarrow\ X$ with flavor decoration from pairs of outgoing momenta and flavors *plist* and initial flavors $f1$ and $f2$

    val $of\_momenta\ :\ \alpha\ \rightarrow\ \alpha\ \rightarrow\ (momentum\ \times\ \alpha)\ list \rightarrow\ (momentum\ \times$
$\alpha)\ t$
    val $decay\_of\_momenta\ :\ (momentum\ \times \alpha)\ list \rightarrow\ (momentum\ \times \alpha)\ decay$

    exception $Duplicate$ of $momentum$
    exception $Unordered$ of $momentum$
    exception $Incomplete$ of $momentum$

  end

module $Make\ (M\ :\ Momentum.T)\ :\ T$ with type $momentum\ =\ M.t$

16.2   Implementation of Phasespace

16.2.1   Tools

These are candidates for *ThoList* and not specific to phase space.

let rec *first_match′ mismatch f* = function
| [] → *None*
| *x* :: *rest* →
    if *f x* then
        Some (*x*, *List.rev_append mismatch rest*)
    else
        first_match′ (*x* :: *mismatch*) *f rest*

Returns $(x, X \setminus \{x\})$ if $\exists x \in X : f(x)$.

let *first_match f l* = *first_match′* [] *f l*

let rec *first_pair′ mismatch1 f l1 l2* =
  match *l1* with
  | [] → *None*
  | *x1* :: *rest1* →
      begin match *first_match* (*f x1*) *l2* with
      | *None* → *first_pair′* (*x1* :: *mismatch1*) *f rest1 l2*
      | *Some* (*x2*, *rest2*) →
          Some ((*x1*, *x2*), (*List.rev_append mismatch1 rest1*, *rest2*))
      end

Returns $((x, y), (X \setminus \{x\}, Y \setminus \{y\}))$ if $\exists x \in X : \exists y \in Y : f(x, y)$.

let *first_pair f l1 l2* = *first_pair′* [] *f l1 l2*

16.2.2   Phase Space Parameterization Trees

module type T =
  sig
    type *momentum*
    type α *t*
    type α *decay*
    val *sort* : $(\alpha \rightarrow \alpha \rightarrow int) \rightarrow \alpha\ t \rightarrow \alpha\ t$
    val *sort_decay* : $(\alpha \rightarrow \alpha \rightarrow int) \rightarrow \alpha\ decay \rightarrow \alpha\ decay$
    val *map* : $(\alpha \rightarrow \beta) \rightarrow \alpha\ t \rightarrow \beta\ t$
    val *map_decay* : $(\alpha \rightarrow \beta) \rightarrow \alpha\ decay \rightarrow \beta\ decay$
    val *eval* : $(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta \rightarrow \beta \rightarrow \beta) \rightarrow \alpha\ t \rightarrow \beta\ t$
    val *eval_decay* : $(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta \rightarrow \beta \rightarrow \beta) \rightarrow \alpha\ decay \rightarrow \beta\ decay$
    val *of_momenta* : $\alpha \rightarrow \alpha \rightarrow (momentum \times \alpha)\ list \rightarrow (momentum \times \alpha)\ t$
    val *decay_of_momenta* : $(momentum \times \alpha)\ list \rightarrow (momentum \times \alpha)\ decay$
    exception *Duplicate* of *momentum*
    exception *Unordered* of *momentum*
    exception *Incomplete* of *momentum*
  end

```
module Make (M : Momentum.T) =
  struct

    type momentum = M.t
```

Finally, we came back to binary trees ...

<div align="center">

Cascade Decays

</div>

```
type α decay =
  | Leaf of α
  | Branch of α × α decay × α decay
```

Trees of type (*momentum* × α *option*) *decay* can be build easily and mapped to (*momentum* × α) *decay* later, once all the α slots are filled. A more elegant functor operating on β *decay* directly (with *Momentum* style functions defined for β) would not allow holes in the β *decay* during the construction.

```
let label = function
  | Leaf p → p
  | Branch (p, _, _) → p

let rec sort_decay cmp = function
  | Leaf _ as l → l
  | Branch (p, d1, d2) →
      let d1' = sort_decay cmp d1
      and d2' = sort_decay cmp d2 in
      if cmp (label d1') (label d2') ≤ 0 then
        Branch (p, d1', d2')
      else
        Branch (p, d2', d1')

let rec map_decay f = function
  | Leaf p → Leaf (f p)
  | Branch (p, d1, d2) → Branch (f p, map_decay f d1, map_decay f d2)

let rec eval_decay fl fb = function
  | Leaf p → Leaf (fl p)
  | Branch (p, d1, d2) →
      let d1' = eval_decay fl fb d1
      and d2' = eval_decay fl fb d2 in
      Branch (fb p (label d1') (label d2'), d1', d2')
```

Assuming that $p > p_D \lor p = p_D \lor p < p_D$, where p_D is the overall momentum of a decay tree D, we can add p to D at the top or somewhere in the middle. Note that '<' is not a total ordering and the operation can fail (raise exceptions) if the set of momenta does not correspond to a tree. Also note that a momentum can already be present without flavor as a complement in a branching entered earlier.

```
exception Duplicate of momentum
exception Unordered of momentum
```

```
let rec embed_in_decay (p, f as pf) = function
  | Leaf (p', f' as pf') as d' →
      if M.less p' p then
        Branch ((p, Some f), d', Leaf (M.sub p p', None))
      else if M.less p p' then
        Branch (pf', Leaf (p, Some f), Leaf (M.sub p' p, None))
      else if p = p' then
        begin match f' with
        | None → Leaf (p, Some f)
        | Some _ → raise (Duplicate p)
        end
      else
        raise (Unordered p)
  | Branch ((p', f' as pf'), d1, d2) as d' →
      let p1, _ = label d1
      and p2, _ = label d2 in
      if M.less p' p then
        Branch ((p, Some f), d', Leaf (M.sub p p', None))
      else if M.lesseq p p1 then
        Branch (pf', embed_in_decay pf d1, d2)
      else if M.lesseq p p2 then
        Branch (pf', d1, embed_in_decay pf d2)
      else if p = p' then
        begin match f' with
        | None → Branch ((p, Some f), d1, d2)
        | Some _ → raise (Duplicate p)
        end
      else
        raise (Unordered p)
```

Note that both *embed_in_decay* and *embed_in_decays* below do *not* commute, and should process 'bigger' momenta first, because disjoint sub-momenta will create disjoint subtrees in the latter and raise exceptions in the former.

```
exception Incomplete of momentum

let finalize1 = function
  | p, Some f → (p, f)
  | p, None → raise (Incomplete p)

let finalize_decay t = map_decay finalize1 t
```

Process the momenta starting in with the highest *M.rank*:

```
let sort_momenta plist =
  List.sort (fun (p1, _) (p2, _) → M.compare p1 p2) plist

let decay_of_momenta plist =
  match sort_momenta plist with
  | (p, f) :: rest →
      finalize_decay (List.fold_right embed_in_decay rest (Leaf (p, Some f)))
  | [] → invalid_arg "Phasespace.decay_of_momenta:␣empty"
```

Figure 16.1: Phasespace parameterization for $2 \to n$ scattering by a sequence of cascade decays.

$2 \to n$ Scattering

A general $2 \to n$ scattering process can be parameterized by a sequence of cascade decays. The most symmetric representation is a little bit redundant and enters each t-channel momentum twice.

> type α t $=$ $(\alpha \times \alpha$ decay $\times \alpha)$ list

let *topology* $=$ *map snd* has type $(momentum \times \alpha)$ t $\to$ α t and can be used to define topological equivalence classes "up to permutations of momenta," which are useful for calculating Whizard "groves"[1] [11].

> let *sort cmp* $=$ *List.map* (fun $(l,\ d,\ r)$ $\to$ $(l,\ sort\_decay\ cmp\ d,\ r))$
> let *map f* $=$ *List.map* (fun $(l,\ d,\ r)$ $\to$ $(f\ l,\ map\_decay\ f\ d,\ f\ r))$
> let *eval ft fl fb* $=$ *List.map* (fun $(l,\ d,\ r)$ $\to$ $(ft\ l,\ eval\_decay\ fl\ fb\ d,\ ft\ r))$

Find a tree with a defined ordering relation with respect to p or create a new one at the end of the list.

> let rec *embed_in_decays* $(p,\ f$ as $pf)$ $=$ function
> | [] $\to$ $[Leaf\ (p,\ Some\ f)]$
> | d' :: $rest$ $\to$
>   let $p',\ \_ =$ *label* d' in
>   if $M.lesseq\ p'\ p\ \lor\ M.less\ p\ p'$ then
>   *embed_in_decay* $pf\ d'$ :: $rest$
>   else
> d' :: *embed_in_decays* $pf\ rest$

Collecting Ingredients

> type α *unfinished_decays* $=$

---

[1]Not to be confused with gauge invariant classes of Feynman diagrams [12].

```
{ n : int;
    t_channel : (momentum × α option) list;
    decays : (momentum × α option) decay list }
```

let *empty* n = { n = n; t_channel = []; decays = [] }

let *insert_in_unfinished_decays* (p, f as pf) d =
  if M.Scattering.spacelike p then
    { d with t_channel = (p, Some f) :: d.t_channel }
  else
    { d with decays = embed_in_decays pf d.decays }

let *flip_incoming* plist =
  List.map (fun (p', f') → (M.Scattering.flip_s_channel_in p', f')) plist

let *unfinished_decays_of_momenta* n f2 p =
  List.fold_right insert_in_unfinished_decays
    (sort_momenta (flip_incoming ((M.of_ints n [2], f2) :: p))) (empty n)

Assembling Ingredients

let *sort3* compare x y z =
  let a = [| x; y; z |] in
  Array.sort compare a;
  (a.(0), a.(1), a.(2))

Take advantage of the fact that sorting with *M.compare* sorts with *rising* values
of *M.rank*:

let *allows_momentum_fusion* (p, _) (p1, _) (p2, _) =
  let p2', p1', p' = sort3 M.compare p p1 p2 in
  match M.try_fusion p' p1' p2' with
  | Some _ → true
  | None → false

let *allows_fusion* p1 p2 d = allows_momentum_fusion (label d) p1 p2

let rec *thread_unfinished_decays'* p acc tlist dlist =
  match first_pair (allows_fusion p) tlist dlist with
  | None → (p, acc, tlist, dlist)
  | Some ((t, _ as td), (tlist', dlist')) →
    thread_unfinished_decays' t (td :: acc) tlist' dlist'

let *thread_unfinished_decays* p c =
  match thread_unfinished_decays' p [] c.t_channel c.decays with
  | _, pairs, [], [] → pairs
  | _ → failwith "thread_unfinished_decays"

let rec *combine_decays* = function
  | [] → []
  | ((t, f as tf), d) :: rest →
    let p, _ = label d in
    begin match M.try_sub t p with
    | Some p' → (tf, d, (p', f)) :: combine_decays rest

```
            | None → (tf, d, (M.sub (M.neg t) p, f)) :: combine_decays rest
          end

let finalize t  =  map finalize1 t

let of_momenta f1 f2  =  function
  | (p, _) :: _ as l →
      let n  =  M.dim p in
      finalize (combine_decays
                    (thread_unfinished_decays (M.of_ints n [1], Some f1)
                        (unfinished_decays_of_momenta n f2 l)))
  | [] → []
```

<center>*Diagnostics*</center>

```
let p_to_string p  =
  String.concat "" (List.map string_of_int (M.to_ints (M.abs p)))

let rec to_string1  =  function
  | Leaf p → "(" ^ p_to_string p ^ ")"
  | Branch (_, d1, d2) → "(" ^ to_string1 d1 ^ to_string1 d2 ^ ")"

let to_string ps  =
  String.concat "/"
      (List.map (fun (p1, d, p2) →
          p_to_string p1 ^ to_string1 d ^ p_to_string p2) ps)
```

<center>*Examples*</center>

```
let try_thread_unfinished_decays p c =
  thread_unfinished_decays' p [] c.t_channel c.decays

let try_of_momenta f  =  function
  | (p, _) :: _ as l →
      let n  =  M.dim p in
      try_thread_unfinished_decays
        (M.of_ints n [1], None) (unfinished_decays_of_momenta n f l)
  | [] → invalid_arg "try_of_momenta"

end
```

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—17—
WHIZARD

Talk to [11].

17.1   Interface of Whizard

```
module type T =
  sig
    type t
    type amplitude
    val trees : amplitude → t
    val merge : t → t
    val write : out_channel → string → t → unit

  end

module Make (FM : Fusion.Maker) (P : Momentum.T)
    (PW : Momentum.Whizard with type t = P.t) (M : Model.T) :
    T with type amplitude = FM(P)(M).amplitude

val write_interface : out_channel → string list → unit
val write_makefile : out_channel → α → unit
val write_makefile_processes : out_channel → string list → unit
```

17.2   Implementation of Whizard

```
let rcs = RCS.parse "Whizard" ["Whizard␣Interface"]
    { RCS.revision = "$Revision:␣6465␣$";
      RCS.date = "$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";
      RCS.author = "$Author:␣jr␣reuter␣$";
      RCS.source
        = "$URL:␣svn+ssh://jr␣reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr

open Printf

module type T =
  sig
    type t
    type amplitude
```

```
    val trees : amplitude → t
    val merge : t → t
    val write : out_channel → string → t → unit

  end
module Make (FM : Fusion.Maker) (P : Momentum.T)
    (PW : Momentum.Whizard with type t = P.t) (M : Model.T) =
  struct
    module F = FM(P)(M)

    type tree = (P.t × F.flavor list) list

    module Poles = Map.Make
        (struct
          type t = int × int
          let compare (s1, t1) (s2, t2) =
            let c = compare s2 s1 in
            if c ≠ 0 then
              c
            else
              compare t1 t2
        end)

    let add_tree maps tree trees =
      Poles.add maps
        (try tree :: (Poles.find maps trees) with Not_found → [tree]) trees

    type t =
        { in1 : F.flavor;
          in2 : F.flavor;
          out : F.flavor list;
          trees : tree list Poles.t }

    type amplitude = F.amplitude
```

17.2.1   Building Trees

A singularity is to be mapped if it is timelike and not the overall *s*-channel.

```
    let timelike_map c = P.Scattering.timelike c ∧ ¬ (P.Scattering.s_channel c)

    let count_maps n clist =
      List.fold_left (fun (s, t as cnt) (c, _) →
        if timelike_map c then
          (succ s, t)
        else if P.Scattering.spacelike c then
          (s, succ t)
        else
          cnt) (0, 0) clist

    let poles_to_whizard n trees poles =
      let tree = List.map (fun wf →
        (P.Scattering.flip_s_channel_in (F.momentum wf), [F.flavor wf])) poles in
      add_tree (count_maps n tree) tree trees
```

⟳ I must reinstate the *conjugate* eventually!

```
let trees a =
  match F.externals a with
  | in1 :: in2 :: out →
      let n = List.length out + 2 in
      { in1 = F.flavor in1;
        in2 = F.flavor in2;
        out = List.map (fun f → (* M.conjugate *) (F.flavor f)) out;
        trees = List.fold_left
          (poles_to_whizard n) Poles.empty (F.poles a) }
  | _ → invalid_arg "Whizard().trees"
```

17.2.2 Merging Homomorphic Trees

```
module Pole_Map =
  Map.Make (struct type t = P.t list let compare = compare end)
module Flavor_Set =
  Set.Make (struct type t = F.flavor let compare = compare end)

let add_flavors flist fset =
  List.fold_right Flavor_Set.add flist fset

let set_of_flavors flist =
  List.fold_right Flavor_Set.add flist Flavor_Set.empty

let pack_tree map t =
  let c, f =
    List.split (List.sort (fun (c1, _) (c2, _) →
      compare (PW.of_momentum c2) (PW.of_momentum c1)) t) in
  let f' =
    try
      List.map2 add_flavors f (Pole_Map.find c map)
    with
    | Not_found → List.map set_of_flavors f in
  Pole_Map.add c f' map

let pack_map trees = List.fold_left pack_tree Pole_Map.empty trees

let merge_sets clist flist =
  List.map2 (fun c f → (c, Flavor_Set.elements f)) clist flist

let unpack_map map =
  Pole_Map.fold (fun c f l → (merge_sets c f) :: l) map []
```

If a singularity is to be mapped (i.e. if it is timelike and not the overall *s*-channel), expand merged particles again:

```
let unfold1 (c, f) =
  if timelike_map c then
    List.map (fun f' → (c, [f'])) f
  else
    [(c,f)]
```

let *unfold_tree tree* = *Product.list* (fun *x* → *x*) (*List.map unfold1 tree*)

let *unfold trees* = *ThoList.flatmap unfold_tree trees*

let *merge t* =
　{ *t* with *trees* = *Poles.map*
　　　(fun *t'* → *unfold* (*unpack_map* (*pack_map t'*))) *t.trees* }

17.2.3　Printing Trees

let *flavors_to_string f* =
　String.concat "/" (*List.map M.flavor_to_string f*)

let *whizard_tree t* =
　"tree␣" ^
　(*String.concat* "␣" (*List.rev_map* (fun (*c*, _) →
　　(*string_of_int* (*PW.of_momentum c*))) *t*)) ^
　"␣!␣" ^
　(*String.concat* ",␣" (*List.rev_map* (fun (_, *f*) → *flavors_to_string f*) *t*))

let *whizard_tree_debug t* =
　"tree␣" ^
　(*String.concat* "␣" (*List.rev_map* (fun (*c*, _) →
　　("[" ^ (*String.concat* "+" (*List.map string_of_int* (*P.to_ints c*))) ^ "]"))
　　　　　　　　　　(*List.sort* (fun (*t1*, _) (*t2*, _) →
　　　　　　　　　　　let *c* =
　　　　　　　　　　　　compare
　　　　　　　　　　　　　(*List.length* (*P.to_ints t2*))
　　　　　　　　　　　　　(*List.length* (*P.to_ints t1*)) in
　　　　　　　　　　　if *c* ≠ 0 then
　　　　　　　　　　　　c
　　　　　　　　　　　else
　　　　　　　　　　　　compare t1 t2) *t*))) ^
　"␣!␣" ^
　(*String.concat* ",␣" (*List.rev_map* (fun (_, *f*) → *flavors_to_string f*) *t*))

let *format_maps* = function
　| (0, 0) → "neither␣mapped␣timelike␣nor␣spacelike␣poles"
　| (0, 1) → "no␣mapped␣timelike␣poles,␣one␣spacelike␣pole"
　| (0, *n*) → "no␣mapped␣timelike␣poles,␣" ^
　　　string_of_int n ^ "␣spacelike␣poles"
　| (1, 0) → "one␣mapped␣timelike␣pole,␣no␣spacelike␣pole"
　| (1, 1) → "one␣mapped␣timelike␣and␣spacelike␣pole␣each"
　| (1, *n*) → "one␣mapped␣timelike␣and␣" ^
　　　string_of_int n ^ "␣spacelike␣poles"
　| (*n*, 0) → *string_of_int n* ^
　　　"␣mapped␣timelike␣poles␣and␣no␣spacelike␣pole"
　| (*n*, 1) → *string_of_int n* ^
　　　"␣mapped␣timelike␣poles␣and␣one␣spacelike␣pole"
　| (*n*, *n'*) → *string_of_int n* ^ "␣mapped␣timelike␣and␣" ^
　　　string_of_int n' ^ "␣spacelike␣poles"

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```
let format_flavor f =
  match flavors_to_string f with
  | "d" → "d" | "dbar" → "D"
  | "u" → "u" | "ubar" → "U"
  | "s" → "s" | "sbar" → "S"
  | "c" → "c" | "cbar" → "C"
  | "b" → "b" | "bbar" → "B"
  | "t" → "t" | "tbar" → "T"
  | "e-" → "e1" | "e+" → "E1"
  | "nue" → "n1" | "nuebar" → "N1"
  | "mu-" → "e2" | "mu+" → "E2"
  | "numu" → "n2" | "numubar" → "N2"
  | "tau-" → "e3" | "tau+" → "E3"
  | "nutau" → "n3" | "nutaubar" → "N3"
  | "g" → "G" | "A" → "A" | "Z" → "Z"
  | "W+" → "W+" | "W-" → "W-"
  | "H" → "H"
  | s → s ^ "␣(not␣translated)"

module Mappable =
  Set.Make (struct type t = string let compare = compare end)
let mappable =
  List.fold_right Mappable.add
    [ "T"; "Z"; "W+"; "W-"; "H" ] Mappable.empty

let analyze_tree ch t =
  List.iter (fun (c, f) →
    let f' = format_flavor f
    and c' = PW.of_momentum c in
    if P.Scattering.timelike c then begin
      if P.Scattering.s_channel c then
        fprintf ch "␣␣␣␣␣␣␣!␣overall␣s-channel␣%d␣%s␣not␣mapped\n" c' f'
      else if Mappable.mem f' mappable then
        fprintf ch "␣␣␣␣␣␣␣map␣%d␣s-channel␣%s\n" c' f'
      else
        fprintf ch
          "␣␣␣␣␣␣␣!␣%d␣s-channel␣%s␣can't␣be␣mapped␣by␣whizard\n"
          c' f'
    end else
      fprintf ch "␣␣␣␣␣␣␣!␣t-channel␣%d␣%s␣not␣mapped\n" c' f') t

let write ch pid t =
  failwith "Whizard.Make().write:␣incomplete"
  fprintf ch "process␣%s\n" pid;
  Poles.iter (fun maps ds →
    fprintf ch "\n␣␣␣␣␣!␣%d␣times␣%s:\n"
      (List.length ds) (format_maps maps);
    List.iter (fun d →
      fprintf ch "\n␣␣␣␣␣grove\n";
      fprintf ch "␣␣␣␣␣%s\n" (whizard_tree d);
      analyze_tree ch d) ds) t.trees;
  fprintf ch "\n!␣O'Mega␣revision␣control␣information:\n";
```

   List.iter (fun *s* → *fprintf ch* "!␣␣␣␣%s\n" *s*)
    (*ThoList.flatmap RCS.summary* (*rcs* :: *M.rcs* :: *F.rcs_list*));
   fprintf ch "\n"
i ×)

  end

17.2.4 Process Dispatcher

let *arguments* = function
 | [] → ("", "")
 | *args* →
   let *arg_list* = *String.concat* ",␣" (*List.map snd args*) in
   (*arg_list*, ",␣" ˆ *arg_list*)

let *import_prefixed ch pid name* =
 fprintf ch "␣␣␣␣␣use␣%s,␣only:␣%s_%s␣=>␣%s␣!NODEP!\n"
  pid pid name name

let *declare_argument ch* (*arg_type*, *arg*) =
 fprintf ch "␣␣␣␣␣%s,␣intent(in)␣::␣%s\n" *arg_type arg*

let *call_function ch pid result name args* =
 fprintf ch "␣␣␣␣␣␣␣case␣(pr_%s)\n" *pid*;
 fprintf ch "␣␣␣␣␣␣␣␣␣␣%s␣=␣%s_%s(%s)\n" *result pid name args*

let *default_function ch result default* =
 fprintf ch "␣␣␣␣␣␣␣case␣default\n";
 fprintf ch "␣␣␣␣␣␣␣␣␣␣call␣invalid_process␣(pid)\n";
 fprintf ch "␣␣␣␣␣␣␣␣␣␣%s␣=␣%s\n" *result default*

let *call_subroutine ch pid name args* =
 fprintf ch "␣␣␣␣␣␣␣case␣(pr_%s)\n" *pid*;
 fprintf ch "␣␣␣␣␣␣␣␣␣␣call␣%s_%s␣(%s)\n" *pid name args*

let *default_subroutine ch* =
 fprintf ch "␣␣␣␣␣␣␣case␣default\n";
 fprintf ch "␣␣␣␣␣␣␣␣␣␣call␣invalid_process␣(pid)\n"

let *write_interface_subroutine ch wrapper name args processes* =
 let *arg_list*, *arg_list*′ = *arguments args* in
 fprintf ch "␣␣subroutine␣%s␣(pid%s)\n" *wrapper arg_list*′;
 List.iter (fun *p* → *import_prefixed ch p name*) *processes*;
 List.iter (*declare_argument ch*) (("character(len=*)", "pid") :: *args*);
 fprintf ch "␣␣␣␣␣select␣case␣(pid)\n";
 List.iter (fun *p* → *call_subroutine ch p name arg_list*) *processes*;
 default_subroutine ch;
 fprintf ch "␣␣␣␣␣end␣select\n";
 fprintf ch "␣␣end␣subroutine␣%s\n" *wrapper*

let *write_interface_function ch wrapper name*
  (*result_type*, *result*, *default*) *args processes* =
 let *arg_list*, *arg_list*′ = *arguments args* in
 fprintf ch "␣␣function␣%s␣(pid%s)␣result␣(%s)\n" *wrapper arg_list*′ *result*;

    List.iter (fun $p \rightarrow$ *import_prefixed ch p name*) *processes*;
    List.iter (*declare_argument ch*) (("`character(len=*)`", "`pid`") :: *args*);
    fprintf ch "`     %s ::  %s\n`" *result_type result*;
    fprintf ch "`     select case  (pid)\n`";
    List.iter (fun $p \rightarrow$ *call_function ch p result name arg_list*) *processes*;
    default_function ch result default;
    fprintf ch "`     end select\n`";
    fprintf ch "`  end function %s\n`" *wrapper*

let *write_other_interface_functions ch* =
  fprintf ch "`  subroutine invalid_process  (pid)\n`";
  fprintf ch "`     character(len=*), intent(in):: pid\n`";
  fprintf ch "`     print *, \"PANIC:`";
  fprintf ch "` process '\"//trim(pid)//\"' not available!\"\n`";
  fprintf ch "`  end subroutine invalid_process\n`";
  fprintf ch "`  function n_tot  (pid) result  (n)\n`";
  fprintf ch "`     character(len=*), intent(in)  :: pid\n`";
  fprintf ch "`     integer  ::  n\n`";
  fprintf ch "`     n  =  n_in(pid)  +  n_out(pid)\n`";
  fprintf ch "`  end function n_tot\n`"

let *write_other_declarations ch* =
  fprintf ch "`  public ::  n_in,  n_out,  n_tot,  pdg_code\n`";
  fprintf ch "`  public ::  allow_helicities\n`";
  fprintf ch "`  public ::  create,  destroy\n`";
  fprintf ch "`  public ::  set_const,  sqme\n`";
  fprintf ch "`  interface  create\n`";
  fprintf ch "`     module procedure process_create\n`";
  fprintf ch "`  end interface\n`";
  fprintf ch "`  interface  destroy\n`";
  fprintf ch "`     module procedure process_destroy\n`";
  fprintf ch "`  end interface\n`";
  fprintf ch "`  interface  set_const\n`";
  fprintf ch "`     module procedure process_set_const\n`";
  fprintf ch "`  end interface\n`";
  fprintf ch "`  interface  sqme\n`";
  fprintf ch "`     module procedure process_sqme\n`";
  fprintf ch "`  end interface\n`"

let *write_interface ch names* =
  fprintf ch "`module process_interface\n`";
  fprintf ch "`  use kinds,  only: default   !NODEP!\n`";
  fprintf ch "`  use parameters,  only: parameter_set\n`";
  fprintf ch "`  implicit none\n`";
  fprintf ch "`  private\n`";
  List.iter (fun $p \rightarrow$
    fprintf ch
      "`  character(len=*), parameter, public ::  pr_%s = \"%s\"\n`" *p p*)
    names;
  write_other_declarations ch;
  fprintf ch "`contains\n`";
  write_interface_function ch "`n_in`" "`n_in`" ("`integer`", "`n`", "`0`") [] *names*;

write_interface_function ch `"n_out"` `"n_out"` (`"integer"`, `"n"`, `"0"`) [ ] *names*;
write_interface_function ch `"pdg_code"` `"pdg_code"`
  (`"integer"`, `"n"`, `"0"`) [`"integer"`, `"i"`] *names*;
write_interface_function ch `"allow_helicities"` `"allow_helicities"`
  (`"logical"`, `"yorn"`, `".false."`) [ ] *names*;
write_interface_subroutine ch `"process_create"` `"create"` [ ] *names*;
write_interface_subroutine ch `"process_destroy"` `"destroy"` [ ] *names*;
write_interface_subroutine ch `"process_set_const"` `"set_const"`
  [`"type(parameter_set)"`, `"par"`] *names*;
write_interface_function ch `"process_sqme"` `"sqme"`
  (`"real(kind=default)"`, `"sqme"`, `"0"`)
  [`"real(kind=default),␣dimension(0:,:)"`, `"p"`;
    `"integer,␣dimension(:),␣optional"`, `"h"`] *names*;
write_other_interface_functions ch;
fprintf ch `"end␣module␣process_interface\n"`

17.2.5 Makefile

let *write_makefile ch names* =
  fprintf ch `"KINDS␣=␣../@KINDS@\n"`;
  fprintf ch `"HELAS␣=␣../@HELAS@\n"`;
  fprintf ch `"F90␣=␣@F90@\n"`;
  fprintf ch `"F90FLAGS␣=␣@F90FLAGS@\n"`;
  fprintf ch `"F90INCL␣=␣-I$(KINDS)␣-I$(HELAS)\n"`;
  fprintf ch `"F90COMMON␣=␣omega_bundle_whizard.f90"`;
  fprintf ch `"␣file_utils.f90␣process_interface.f90\n"`;
  fprintf ch `"include␣Makefile.processes\n"`;
  fprintf ch `"F90SRC␣=␣$(F90COMMON)␣$(F90PROCESSES)\n"`;
  fprintf ch `"OBJ␣=␣$(F90SRC:.f90=.o)\n"`;
  fprintf ch `"MOD␣=␣$(F90SRC:.f90=.mod)\n"`;
  fprintf ch `"archive:␣processes.a\n"`;
  fprintf ch `"processes.a:␣$(OBJ)\n"`;
  fprintf ch `"\t$(AR)␣r␣$@␣$(OBJ)\n"`;
  fprintf ch `"\t@RANLIB@␣$@\n"`;
  fprintf ch `"clean:\n"`;
  fprintf ch `"\trm␣-f␣$(OBJ)\n"`;
  fprintf ch `"realclean:\n"`;
  fprintf ch `"\trm␣-f␣processes.a\n"`;
  fprintf ch `"parameters.o:␣file_utils.o\n"`;
  fprintf ch `"omega_bundle_whizard.o:␣parameters.o\n"`;
  fprintf ch `"process_interface.o:␣parameters.o\n"`;
  fprintf ch `"%%.o:␣%%.f90␣$(KINDS)/kinds.f90\n"`;
  fprintf ch `"\t$(F90)␣$(F90FLAGS)␣$(F90INCL)␣-c␣$<\n"`

let *write_makefile_processes ch names* =
  fprintf ch `"F90PROCESSES␣="`;
  List.iter (fun *f* → *fprintf ch* `"␣\\\n␣␣%s.f90"` *f*) *names*;
  fprintf ch `"\n"`;
  List.iter (fun *f* →
    fprintf ch `"%s.o:␣omega_bundle_whizard.o␣parameters.o\n"` *f*;

fprintf *ch* `"process_interface.o:␣%s.o\n"` *f*) *names*

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APPLICATIONS

18.1 Sample

18.2 Interface of Omega

```
module type T =
  sig
    val main : unit → unit
```

⌖ This used to be only intended for debugging O'Giga, but might live longer
...

```
    type flavor
    val diagrams : flavor → flavor → flavor list →
      ((flavor × Momentum.Default.t) ×
          (flavor × Momentum.Default.t,
            flavor × Momentum.Default.t) Tree.t) list
  end
module Make (FM : Fusion.Maker) (TM : Target.Maker) (M : Model.T) :
    T with type flavor = M.flavor
```

18.3 Implementation of Omega

```
let (<<) f g x = f (g x)
let (>>) f g x = g (f x)

module P = Momentum.Default
module P_Whizard = Momentum.DefaultW

module type T =
  sig
    val main : unit → unit
    type flavor
    val diagrams : flavor → flavor → flavor list →
      ((flavor × Momentum.Default.t) ×
          (flavor × Momentum.Default.t,
```

```
      flavor  ×  Momentum.Default.t) Tree.t) list
  end
```

module *Make* (*Fusion_Maker* : *Fusion.Maker*) (*Target_Maker* : *Target.Maker*) (*M* : *Model.T*) =
  struct

    module *CM* = *Colorize.It*(*M*)

    type *flavor* = *M.flavor*

    module *Proc* = *Process.Make*(*M*)

> ⚠ NB: this causes the constant initializers in *Fusion_Maker* more than once.
> Such side effects must be avoided if the initializers involve expensive computations. *Relying on the fact that the functor will be called only once is not a good idea!*

```
    module F  =  Fusion_Maker(P)(M)
    module CF  =  Fusion.Multi(Fusion_Maker)(P)(M)
    module T  =  Target_Maker(Fusion_Maker)(P)(M)
    module W  =  Whizard.Make(Fusion_Maker)(P)(P_Whizard)(M)
    module C  =  Cascade.Make(M)(P)
```

Form a *α list* from a *α option array*, containing the elements that are not *None* in order.

```
    let opt_array_to_list a  =
      let rec opt_array_to_list' acc i a  =
        if i < 0 then
          acc
        else
          begin match a.(i) with
          | None  →  opt_array_to_list' acc (pred i) a
          | Some x  →  opt_array_to_list' (x :: acc) (pred i) a
          end in
      opt_array_to_list' [] (Array.length a − 1) a
```

Return a list of *CF.amplitude list*s, corresponig to the diagrams for a specific color flow for each flavor combination.

```
    let amplitudes_by_flavor amplitudes  =
      List.map opt_array_to_list (Array.to_list (CF.process_table amplitudes))
```

> ⚠ If we plan to distiguish different couplings later on, we can no long map all instances of *coupling option* in the tree to *None*. In this case, we will need to normalize different fusion orders *Coupling.fuse2*, *Coupling.fuse3* or *Coupling.fusen*, because they would otherwise lead to inequivalent diagrams. Unfortunately, this stuff packaged deep in *Fusion.Tagged_Coupling*.

> ⚠ The *Tree.canonicalize* below should be necessary to remove topologically equivalent duplicates.

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Take a *CF.amplitude list* assumed to correspond to the same external states
after stripping the color and return a pair of the list of external particles and
the corresponding Feynman diagrams without color.

let *wf1 amplitude* =
  match *F.externals amplitude* with
  | *wf* :: _ → *wf*
  | [] → *failwith* "Omega.forest_sans_color:␣no␣external␣particles"

let *uniq l* =
  ThoList.uniq (*List.sort compare l*)

let *forest_sans_color* = function
  | *amplitude* :: _ as *amplitudes* →
    let *externals* = *F.externals amplitude* in
    let *prune_color wf* =
      (*F.flavor_sans_color wf*, *F.momentum_list wf*) in
    let *prune_color_and_couplings* (*wf*, *c*) =
      (*prune_color wf*, *None*) in
    (*List.map prune_color externals*,
     uniq
      (*List.map*
        (fun *t* →
          Tree.canonicalize
            (*Tree.map prune_color_and_couplings prune_color t*))
        (*ThoList.flatmap* (fun *a* → *F.forest* (*wf1 a*) *a*) *amplitudes*)))
  | [] → ([], [])

let *p2s p* =
  if *p* ≥ 0 ∧ *p* ≤ 9 then
    string_of_int p
  else if *p* ≤ 36 then
    String.make 1 (*Char.chr* (*Char.code* 'A' + *p* − 10))
  else
    "_"

let *format_p wf* =
  String.concat "" (*List.map p2s* (*F.momentum_list wf*))

let *variable wf* =
  M.flavor_to_string (*F.flavor_sans_color wf*) ^ "[" ^ *format_p wf* ^ "]"

let *variable' wf* =
  CM.flavor_to_TeX (*F.flavor wf*) ^ "(" ^ *format_p wf* ^ ")"

let *feynmf_style propagator color* =
  { *Tree.style* =
    begin match *propagator* with
    | *Coupling.Prop_Feynman*
    | *Coupling.Prop_Gauge* _ →
    begin match *color* with
    | *Color.AdjSUN* _ → *Some* ("gluon", "")
    | _ → *Some* ("boson", "")
    end

```
        |  Coupling.Prop_Col_Feynman  →  Some ("gluon", "")
        |  Coupling.Prop_Unitarity
        |  Coupling.Prop_Rxi _  →  Some ("dbl_wiggly", "")
        |  Coupling.Prop_Spinor
        |  Coupling.Prop_ConjSpinor  →  Some ("fermion", "")
        |  _  →  None
      end;
    Tree.rev  =
      begin match propagator with
      |  Coupling.Prop_Spinor  →  true
      |  Coupling.Prop_ConjSpinor  →  false
      |  _  →  false
      end;
    Tree.label  =  None;
    Tree.tension  =  None }
```

```
let header incoming outgoing  =
  "$ " ^
  String.concat " "
    (List.map (CM.flavor_to_TeX << F.flavor) incoming) ^
  " \\to " ^
  String.concat " "
    (List.map (CM.flavor_to_TeX << CM.conjugate << F.flavor) outgoing) ^
  " $"
```

```
let header_sans_color incoming outgoing  =
  "$ " ^
  String.concat " "
    (List.map (M.flavor_to_TeX << fst) incoming) ^
  " \\to " ^
  String.concat " "
    (List.map (M.flavor_to_TeX << M.conjugate << fst) outgoing) ^
  " $"
```

```
let diagram incoming tree  =
  let fmf wf  =
    let f  =  F.flavor wf in
    feynmf_style (CM.propagator f) (CM.color f) in
  Tree.map
    (fun (n, _)  →
      let n'  =  fmf n in
      if List.mem n incoming then
        { n' with Tree.rev  =  ¬ n'.Tree.rev }
      else
        n')
    (fun l  →
      if List.mem l incoming then
        l
      else
        F.conjugate l)
    tree
```

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```
let diagram_sans_color incoming (tree) =
  let fmf (f, p) =
    feynmf_style (M.propagator f) (M.color f) in
  Tree.map
    (fun (n, c) →
       let n' = fmf n in
       if List.mem n incoming then
         { n' with Tree.rev = ¬ n'.Tree.rev }
       else
         n')
    (fun (f, p) →
       if List.mem (f, p) incoming then
         (f, p)
       else
         (M.conjugate f, p))
    tree

let feynmf_set amplitude =
  match F.externals amplitude with
  | wf1 :: wf2 :: wfs →
    let incoming = [wf1; wf2] in
    { Tree.header = header incoming wfs;
      Tree.incoming = incoming;
      Tree.diagrams =
        List.map (diagram incoming) (F.forest wf1 amplitude) }
  | _ → failwith "less␣than␣two␣external␣particles"

let feynmf_set_sans_color (externals, trees) =
  match externals with
  | wf1 :: wf2 :: wfs →
    let incoming = [wf1; wf2] in
    { Tree.header = header_sans_color incoming wfs;
      Tree.incoming = incoming;
      Tree.diagrams =
        List.map (diagram_sans_color incoming) trees }
  | _ → failwith "less␣than␣two␣external␣particles"

let feynmf_set_sans_color_empty (externals, trees) =
  match externals with
  | wf1 :: wf2 :: wfs →
    let incoming = [wf1; wf2] in
    { Tree.header = header_sans_color incoming wfs;
      Tree.incoming = incoming;
      Tree.diagrams = [] }
  | _ → failwith "less␣than␣two␣external␣particles"

let uncolored_colored amplitudes =
  { Tree.outer = feynmf_set_sans_color (forest_sans_color amplitudes);
    Tree.inner = List.map feynmf_set amplitudes }

let uncolored_only amplitudes =
  { Tree.outer = feynmf_set_sans_color (forest_sans_color amplitudes);
    Tree.inner = [] }
```

```
let colored_only amplitudes =
  { Tree.outer = feynmf_set_sans_color_empty (forest_sans_color amplitudes);
    Tree.inner = List.map feynmf_set amplitudes }

let momentum_to_TeX (_, p) =
  String.concat "" (List.map p2s p)

let wf_to_TeX (f, _ as wf) =
  M.flavor_to_TeX f ^ "(" ^ momentum_to_TeX wf ^ ")"

let amplitudes_to_feynmf latex name amplitudes =
    Tree.feynmf_sets_wrapped latex name
      wf_to_TeX momentum_to_TeX variable' format_p
      (List.map uncolored_colored (amplitudes_by_flavor amplitudes))

let amplitudes_to_feynmf_sans_color latex name amplitudes =
    Tree.feynmf_sets_wrapped latex name
      wf_to_TeX momentum_to_TeX variable' format_p
      (List.map uncolored_only (amplitudes_by_flavor amplitudes))

let amplitudes_to_feynmf_color_only latex name amplitudes =
    Tree.feynmf_sets_wrapped latex name
      wf_to_TeX momentum_to_TeX variable' format_p
      (List.map colored_only (amplitudes_by_flavor amplitudes))

let version () =
  List.iter (fun s → prerr_endline ("RCS: " ^ s))
    (ThoList.flatmap RCS.summary (CM.rcs :: T.rcs_list @ F.rcs_list))

let debug (str, descr, opt, var) =
  [ "-warning:" ^ str, Arg.Unit (fun () → var := (opt, false) :: !var),
    "           check " ^ descr ^ " and print warning on error";
    "-error:" ^ str, Arg.Unit (fun () → var := (opt, true) :: !var),
    "            check " ^ descr ^ " and terminate on error" ]

let rec include_goldstones = function
  | [] → false
  | (T.Gauge, _) :: _ → true
  | _ :: rest → include_goldstones rest

let read_lines_rev file =
  let ic = open_in file in
  let rev_lines = ref [] in
  let rec slurp () =
    rev_lines := input_line ic :: !rev_lines;
    slurp () in
  try
    slurp ()
  with
  | End_of_file →
      close_in ic;
      !rev_lines

let read_lines file =
  List.rev (read_lines_rev file)
```

```
type cache_mode =
  | Cache_Default
  | Cache_Initialize of string

let cache_option =
  ref Cache_Default

let unphysical_polarization = ref None
```

18.3.1   Main Program

```
let main () =
  let usage =
    "usage:␣" ^ Sys.argv.(0) ^
    "␣[options]␣[" ^
      String.concat "|" (List.map M.flavor_to_string
                            (ThoList.flatmap snd
                               (M.external_flavors ())))) ^ "]"
  and rev_scatterings = ref []
  and rev_decays = ref []
  and cascades = ref []
  and checks = ref []
  and output_file = ref None
  and print_forest = ref false
  and template = ref false
  and diagrams_all = ref None
  and diagrams_sans_color = ref None
  and diagrams_color_only = ref None
  and diagrams_LaTeX = ref false
  and quiet = ref false
  and write = ref true
  and params = ref false
  and poles = ref false
  and dag_out = ref None
  and dag0_out = ref None in
  Arg.parse
    (Options.cmdline "-target:" T.options @
     Options.cmdline "-model:" M.options @
     Options.cmdline "-fusion:" CF.options @
     ThoList.flatmap debug
       ["a", "arguments", T.All, checks;
         "n", "#␣of␣input␣arguments", T.Arguments, checks;
         "m", "input␣momenta", T.Momenta, checks;
         "g", "internal␣Ward␣identities", T.Gauge, checks] @
     [("-o", Arg.String (fun s → output_file := Some s),
        "file␣␣␣␣␣␣␣␣␣␣␣␣␣␣write␣to␣given␣file␣instead␣of␣/dev/stdout");
      ("-scatter",
        Arg.String (fun s → rev_scatterings := s :: !rev_scatterings),
        "expr␣␣␣␣␣␣␣␣in1␣in2␣->␣out1␣out2␣...");
      ("-scatter_file",
```

$Arg.String$ (fun s → $rev\_scatterings$:= $read\_lines\_rev$ s @ !$rev\_scatterings$),
"name␣␣␣each␣line:␣in1␣in2␣->␣out1␣out2␣...");
("-decay", $Arg.String$ (fun s → rev_decays := s :: !rev_decays),
"expr␣␣␣␣␣␣␣␣␣␣␣in␣->␣out1␣out2␣...");
("-decay_file",
$Arg.String$ (fun s → rev_decays := $read\_lines\_rev$ s @ !rev_decays),
"name␣␣␣␣each␣line:␣in␣->␣out1␣out2␣...");
("-cascade", $Arg.String$ (fun s → $cascades$:= s :: !$cascades$),
"expr␣␣␣␣␣␣␣␣select␣diagrams");
("-initialize",
$Arg.String$ (fun s → $cache\_option$:= $Cache\_Initialize$ s),
"dir␣␣␣␣␣precompute␣lookup␣tables␣and␣store␣them␣in␣directory");
("-unphysical", $Arg.Int$ (fun i → $unphysical\_polarization$:= $Some$ i),
"n␣␣␣␣␣␣␣␣use␣unphysical␣polarization␣for␣n-th␣particle␣/␣test␣WIs");
("-template", $Arg.Set$ $template$,
"␣␣␣␣␣␣␣␣␣␣␣write␣a␣template␣for␣handwritten␣amplitudes");
("-forest", $Arg.Set$ $print\_forest$,
"␣␣␣␣␣␣␣␣␣␣␣␣␣Diagrammatic␣expansion");
("-diagrams", $Arg.String$ (fun s → $diagrams\_sans\_color$:= $Some$ s),
"file␣␣␣␣␣␣␣produce␣FeynMP␣output␣for␣Feynman␣diagrams");
("-diagrams:c", $Arg.String$ (fun s → $diagrams\_color\_only$:= $Some$ s),
"file␣␣␣␣␣produce␣FeynMP␣output␣for␣color␣flow␣diagrams");
("-diagrams:C", $Arg.String$ (fun s → $diagrams\_all$:= $Some$ s),
"file␣␣␣␣␣produce␣FeynMP␣output␣for␣Feynman␣and␣color␣flow␣diagrams");
("-diagrams_LaTeX", $Arg.Set$ $diagrams\_LaTeX$,
"␣␣␣␣␣enclose␣FeynMP␣output␣in␣LaTeX␣wrapper");
("-revision", $Arg.Unit$ $version$,
"␣␣␣␣␣␣␣␣␣␣␣print␣revision␣control␣information");
("-quiet", $Arg.Set$ $quiet$,
"␣␣␣␣␣␣␣␣␣␣␣␣␣␣don't␣print␣a␣summary");
("-summary", $Arg.Clear$ $write$,
"␣␣␣␣␣␣␣␣␣␣␣␣print␣only␣a␣summary");
("-params", $Arg.Set$ $params$,
"␣␣␣␣␣␣␣␣␣␣␣␣␣print␣the␣model␣parameters");
("-poles", $Arg.Set$ $poles$,
"␣␣␣␣␣␣␣␣␣␣␣␣␣␣print␣the␣Monte␣Carlo␣poles");
("-dag", $Arg.String$ (fun s → dag_out := $Some$ s),
"␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣print␣minimal␣DAG");
("-full_dag", $Arg.String$ (fun s → $dag0\_out$:= $Some$ s),
"␣␣␣␣␣␣␣␣␣␣␣print␣complete␣DAG")])
(fun _ → $prerr\_endline$ $usage$; $exit$ 1)
$usage$;

let $cmdline$ =
$String.concat$ "␣" ($List.map$ $ThoString.quote$ ($Array.to\_list$ $Sys.argv$)) in

let $output\_channel$ =
match !$output\_file$ with
| $None$ → $stdout$
| $Some$ $name$ → $open\_out$ $name$ in

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```
let processes =
  try
    ThoList.uniq
      (List.sort compare
        (match List.rev !rev_scatterings, List.rev !rev_decays with
        | [], [] → []
        | scatterings, [] →
            Proc.expand_scatterings (List.map Proc.parse_scattering scatterings)
        | [], decays →
            Proc.expand_decays (List.map Proc.parse_decay decays)
        | scatterings, decays →
            invalid_arg "mixed␣scattering␣and␣decay!"))
  with
  | Invalid_argument s →
      begin
        Printf.eprintf "O'Mega:␣invalid␣process␣specification:␣%s!\n" s;
        flush stderr;
        []
      end in
```

⚠ This is still crude. Eventually, we want to catch *all* exceptions and write an empty (but compilable) amplitude unless one of the special options is selected.

```
begin match processes, !cache_option, !params with
| [], Cache_Initialize dir, false →
    F.initialize_cache dir;
    exit 0
| _, _, true →
    T.parameters_to_channel output_channel;
    exit 0
| [], _, false →
    T.amplitudes_to_channel cmdline output_channel !checks CF.empty;
    exit 0
| _, _, false →

  let selectors =
    let fin, fout = List.hd processes in
    C.to_selectors (C.of_string_list (List.length fin + List.length fout) !cascades) in

  let amplitudes =
    try
      begin match F.check_charges () with
      | [] → ()
      | violators →
          let violator_strings =
            String.concat ",␣"
              (List.map
                (fun flist →
                  "(" ^ String.concat "," (List.map M.flavor_to_string flist) ^ ")")
                violators) in
```

```
            failwith ("charge␣violating␣vertices:␣" ^ violator_strings)
          end;
        CF.amplitudes (include_goldstones !checks) !unphysical_polarization selectors processes
      with
      | exc →
          begin
            Printf.eprintf
              "O'Mega:␣exception␣%s␣in␣amplitude␣construction!\n"
              (Printexc.to_string exc);
            flush stderr;
            CF.empty;
          end in

  if !write then
    T.amplitudes_to_channel cmdline output_channel !checks amplitudes;

  if ¬ !quiet then begin
    List.iter
      (fun amplitude →
        Printf.eprintf "SUMMARY:␣%d␣fusions,␣%d␣propagators"
          (F.count_fusions amplitude) (F.count_propagators amplitude);
        flush stderr;
        Printf.eprintf ",␣%d␣diagrams" (F.count_diagrams amplitude);
        Printf.eprintf "\n")
      (CF.processes amplitudes);
  end;

  if !poles then begin
    List.iter
      (fun amplitude →
        W.write output_channel "omega" (W.merge (W.trees amplitude)))
      (CF.processes amplitudes)
  end;

  begin match !dag0_out with
  | Some name →
      let ch = open_out name in
      List.iter (F.tower_to_dot ch) (CF.processes amplitudes);
      close_out ch
  | None → ()
  end;

  begin match !dag_out with
  | Some name →
      let ch = open_out name in
      List.iter (F.amplitude_to_dot ch) (CF.processes amplitudes);
      close_out ch
  | None → ()
  end;

  if !print_forest then
    List.iter
      (fun amplitude →
```

```
            List.iter (fun t  →  Printf.eprintf "%s\n"
                 (Tree.to_string
                     (Tree.map (fun (wf, _)  →   variable wf) (fun _  →
"") t)))
                 (F.forest (List.hd (F.externals amplitude)) amplitude))
            (CF.processes amplitudes);

      begin match !diagrams_all with
      | Some name →
        amplitudes_to_feynmf !diagrams_LaTeX name amplitudes
      | None → ()
      end;

      begin match !diagrams_sans_color with
      | Some name →
        amplitudes_to_feynmf_sans_color !diagrams_LaTeX name amplitudes
      | None → ()
      end;

      begin match !diagrams_color_only with
      | Some name →
        amplitudes_to_feynmf_color_only !diagrams_LaTeX name amplitudes
      | None → ()
      end;

      begin match !output_file with
      | None → ()
      | Some name → close_out output_channel
      end;

        exit 0

    end
```

⚠ This was only intended for debugging O'Giga . . .

```
  let decode wf =
    (F.flavor wf, (F.momentum wf : Momentum.Default.t))

  let diagrams in1 in2 out =
    match F.amplitudes false C.no_cascades [in1; in2] out with
    | a :: _ →
        let wf1 = List.hd (F.externals a)
        and wf2 = List.hd (List.tl (F.externals a)) in
        let wf2 = decode wf2 in
        List.map (fun t →
          (wf2,
            Tree.map (fun (wf, _) → decode wf) decode t))
          (F.forest wf1 a)
    | [] → []

  let diagrams in1 in2 out =
    failwith "Omega().diagrams:␣disabled"

end
```

18.4   Implementation of Omega_QED

module $O = Omega.Make(Fusion.Binary)(Targets.Fortran)(Modellib\_SM.QED)$
let $\_ = O.main$ ()

18.5   Implementation of Omega_SM

module $O = Omega.Make(Fusion.Mixed23)(Targets.Fortran)$
$(Modellib\_SM.SM(Modellib\_SM.SM\_no\_anomalous))$
let $\_ = O.main$ ()

18.6   Implementation of Omega_SYM

let $rcs\_file = RCS.parse$ `"omega_SYM"`
    `["Super␣Yang-Mills␣(incomplete,␣just␣for␣stress-testing␣Colorize.It())"]`
    { $RCS.revision =$ `"$Revision:␣6465␣$";`
      $RCS.date =$ `"$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣$";`
      $RCS.author =$ `"$Author:␣jr␣reuter␣$";`
      $RCS.source$
        `= "$URL:␣svn+ssh://jr␣reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/s`

module $SYM =$
  struct

    let $rcs = rcs\_file$

    open $Coupling$

    let $options = Options.empty$

    let $nc = 3$

    type $flavor =$
      | Q of int | SQ of int
      | G of int | SG of int
      | Phi

    let $generations = ThoList.range$ 1 1

    let $generations\_pairs =$
      $List.map$
        (function $[a; b] \rightarrow (a, b)$
          | $\_ \rightarrow failwith$ `"omega_SYM.generations_pairs"`)
        ($Product.power$ 2 $generations$)

    let $generations\_triples =$
      $List.map$
        (function $[a; b; c] \rightarrow (a, b, c)$
          | $\_ \rightarrow failwith$ `"omega_SYM.generations_triples"`)
        ($Product.power$ 3 $generations$)

    let $generations\_quadruples =$

List.map
  (function $[a; b; c; d]$ → $(a,\ b,\ c,\ d)$
    | _ → *failwith* `"omega_SYM.generations_quadruples"`)
  (*Product.power* 4 *generations*)

let *external_flavors* () =
  [`"Quarks"`, *List.map* (fun i → $Q\ i$) *generations*;
    `"Anti-Quarks"`, *List.map* (fun i → $Q\ (-i)$) *generations*;
    `"SQuarks"`, *List.map* (fun i → $SQ\ i$) *generations*;
    `"Anti-SQuarks"`, *List.map* (fun i → $SQ\ (-i)$) *generations*;
    `"Gluons"`, *List.map* (fun i → $G\ i$) *generations*;
    `"SGluons"`, *List.map* (fun i → $SG\ i$) *generations*;
    `"Other"`, $[Phi]$]

let *flavors* () =
  ThoList.flatmap snd (*external_flavors* ())

type *gauge* = *unit*
type *constant* =
  | *G_saa* of $int \times int$
  | *G_saaa* of $int \times int \times int$
  | *G3* of $int \times int \times int$
  | *I_G3* of $int \times int \times int$
  | *G4* of $int \times int \times int \times int$

type *orders* = *unit*
let *orders* = function
  | _ → ()

let *lorentz* = function
  | $Q\ i$ →
      if $i\ >\ 0$ then
        Spinor
      else if $i\ <\ 0$ then
        ConjSpinor
      else
        invalid_arg `"SYM.lorentz (Q 0)"`
  | SQ _ | Phi → *Scalar*
  | G _ → *Vector*
  | SG _ → *Majorana*

let *color* = function
  | $Q\ i$ | $SQ\ i$ →
      Color.SUN (if $i\ >\ 0$ then nc else if $i\ <\ 0$ then $-nc$ else *invalid_arg* `"SYM.color (Q 0)"`)
  | G _ | SG _ → *Color.AdjSUN nc*
  | Phi → *Color.Singlet*

let *propagator* = function
  | $Q\ i$ →
      if $i\ >\ 0$ then
        Prop_Spinor
      else if $i\ <\ 0$ then
        Prop_ConjSpinor
      else

```
            invalid_arg "SYM.lorentz␣(Q␣0)"
  | SQ _ | Phi → Prop_Scalar
  | G _ → Prop_Feynman
  | SG _ → Prop_Majorana

let width _ = Timelike
let goldstone _ = None

let conjugate = function
  | Q i → Q (−i)
  | SQ i → SQ (−i)
  | (G _ | SG _ | Phi) as p → p

let fermion = function
  | Q i →
      if i > 0 then
          1
      else if i < 0 then
          -1
      else
          invalid_arg "SYM.fermion␣(Q␣0)"
  | SQ _ | G _ | Phi → 0
  | SG _ → 2

module Ch = Charges.Null
let charges _ = ()

module F = Modeltools.Fusions (struct
  type f = flavor
  type c = constant
  let compare = compare
  let conjugate = conjugate
end)

let quark_current =
  List.map
      (fun (i, j, k) →
          ((Q (−i), G j, Q k), FBF (−1, Psibar, V, Psi), G3 (i, j, k)))
      generations_triples

let squark_current =
  List.map
      (fun (i, j, k) →
          ((G j, SQ i, SQ (−k)), Vector_Scalar_Scalar 1, G3 (i, j, k)))
      generations_triples

let three_gluon =
  List.map
      (fun (i, j, k) →
          ((G i, G j, G k), Gauge_Gauge_Gauge 1, I_G3 (i, j, k)))
      generations_triples

let gluon2_phi =
  List.map
      (fun (i, j) →
```

```
          ((Phi, G i, G j), Dim5_Scalar_Gauge2 1, G_saa (i, j)))
        generations_pairs
```

let *vertices3* =
  quark_current @ *squark_current* @ *three_gluon* @ *gluon2_phi*

let *gauge4* = *Vector4* [(2, *C_13_42*); (−1, *C_12_34*); (−1, *C_14_23*)]

let *squark_seagull* =
  List.map
    (fun (*i*, *j*, *k*, *l*) →
      ((*SQ i*, *SQ* (−*j*), *G k*, *G l*), *Scalar2_Vector2* 1, *G4* (*i*, *j*, *k*, *l*)))
    generations_quadruples

let *four_gluon* =
  List.map
    (fun (*i*, *j*, *k*, *l*) →
      ((*G i*, *G j*, *G k*, *G l*), *gauge4*, *G4* (*i*, *j*, *k*, *l*)))
    generations_quadruples

We need at least a *Dim6_Scalar_Gauge3* vertex to support this.

let *gluon3_phi* =
  [ ]

let *vertices4* =
  squark_seagull @ *four_gluon* @ *gluon3_phi*

let *vertices* () =
  (*vertices3*, *vertices4*, [ ])

let *table* = *F.of_vertices* (*vertices* ())
let *fuse2* = *F.fuse2 table*
let *fuse3* = *F.fuse3 table*
let *fuse* = *F.fuse table*
let *max_degree* () = 4

let *parameters* () = { *input* = []; *derived* = []; *derived_arrays* = [] }

let *invalid_flavor s* =
  invalid_arg ("omega_SYM.flavor_of_string:␣" ^ *s*)

let *flavor_of_string s* =
  let *l* = *String.length s* in
  if *l* < 2 then
    invalid_flavor s
  else if *l* = 2 then
    if *String.sub s* 0 1 = "q" then
      Q (*int_of_string* (*String.sub s* 1 1))
    else if *String.sub s* 0 1 = "Q" then
      Q (− (*int_of_string* (*String.sub s* 1 1)))
    else if *String.sub s* 0 1 = "g" then
      G (*int_of_string* (*String.sub s* 1 1))
    else
      invalid_flavor s

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```
        else if l = 3 then
          if s = "phi" then
            Phi
          else if String.sub s 0 2 = "sq" then
            SQ (int_of_string (String.sub s 2 1))
          else if String.sub s 0 2 = "sQ" then
            SQ (− (int_of_string (String.sub s 2 1)))
          else if String.sub s 0 2 = "sg" then
            SG (int_of_string (String.sub s 2 1))
          else
            invalid_flavor s
        else
          invalid_flavor s

let flavor_to_string = function
  | Q i →
      if i > 0 then
        "q" ^ string_of_int i
      else if i < 0 then
        "Q" ^ string_of_int (−i)
      else
        invalid_arg "SYM.flavor_to_string␣(Q␣0)"
  | SQ i →
      if i > 0 then
        "sq" ^ string_of_int i
      else if i < 0 then
        "sQ" ^ string_of_int (−i)
      else
        invalid_arg "SYM.flavor_to_string␣(SQ␣0)"
  | G i → "g" ^ string_of_int i
  | SG i → "sg" ^ string_of_int i
  | Phi → "phi"

let flavor_to_TeX = function
  | Q i →
      if i > 0 then
        "q_{" ^ string_of_int i ^ "}"
      else if i < 0 then
        "{\bar␣q}_{" ^ string_of_int (−i) ^ "}"
      else
        invalid_arg "SYM.flavor_to_string␣(Q␣0)"
  | SQ i →
      if i > 0 then
        "{\tilde␣q}_{" ^ string_of_int i ^ "}"
      else if i < 0 then
        "{\bar{\tilde␣q}}_{" ^ string_of_int (−i) ^ "}"
      else
        invalid_arg "SYM.flavor_to_string␣(SQ␣0)"
  | G i → "g_{" ^ string_of_int i ^ "}"
  | SG i → "{\tilde␣g}_{" ^ string_of_int i ^ "}"
  | Phi → "phi"
```

```
    let flavor_symbol = function
      | Q i →
          if i > 0 then
            "q" ^ string_of_int i
          else if i < 0 then
            "qbar" ^ string_of_int (−i)
          else
            invalid_arg "SYM.flavor_to_string␣(Q␣0)"
      | SQ i →
          if i > 0 then
            "sq" ^ string_of_int i
          else if i < 0 then
            "sqbar" ^ string_of_int (−i)
          else
            invalid_arg "SYM.flavor_to_string␣(SQ␣0)"
      | G i → "g" ^ string_of_int i
      | SG i → "sg" ^ string_of_int i
      | Phi → "phi"

  let gauge_symbol () =
    failwith "omega_SYM.gauge_symbol:␣internal␣error"

  let pdg _ = 0
  let mass_symbol _ = "0.0_default"
  let width_symbol _ = "0.0_default"

  let string_of_int_list int_list =
    "(" ^ String.concat "," (List.map string_of_int int_list) ^ ")"

  let constant_symbol = function
    | G_saa (i, j) → "g_saa" ^ string_of_int_list [i; j]
    | G_saaa (i, j, k) → "g_saaa" ^ string_of_int_list [i; j; k]
    | G3 (i, j, k) → "g3" ^ string_of_int_list [i; j; k]
    | I_G3 (i, j, k) → "ig3" ^ string_of_int_list [i; j; k]
    | G4 (i, j, k, l) → "g4" ^ string_of_int_list [i; j; k; l]

end

module O = Omega.Make(Fusion.Mixed23)(Targets.Fortran_Majorana)(SYM)
let _ = O.main ()
```

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—A—
Revision Control

A.1   Interface of RCS

This is a very simple library for exporting and accessing `RCS` and `CVS` revision control information. In addition, module names and short descriptions are supported as well.

If multiple applications are constructed by functors, the functions in this module can be used to identify the concrete implementations. In the context of O'Mega, this is particularly important for physics models and target languages. One structure of type *raw* has to be initialized in each file by the raw RCS keyword strings. It can remain private to the module, because it is only used as argument to the function *parse*.

type *raw* = { *revision* : *string*; *date* : *string*; *author* : *string*; *source* : *string* }

Parsed revision control info:

type *t*

parse name description keywords initializes revision control info:

val *parse* : *string* → *string list* → *raw* → *t*

rename rcs name description changes the name and description. This is useful if more than one module is defined in a file.

val *rename* : *t* → *string* → *string list* → *t*

Access individual parts of the revision control information:

val *name* : *t* → *string*
val *description* : *t* → *string list*
val *revision* : *t* → *string*
val *date* : *t* → *string*
val *author* : *t* → *string*

This one tries `URL` (svn), `Source` (CVS) and `Id`, in that order, for the filename.

val *source* : *t* → *string*

Return the formatted revision control info as a list of strings suitable for printing to the terminal or embedding in the output:

val *summary* : *t* → *string list*

A.2 Implementation of RCS

type *raw* = { *revision* : *string*; *date* : *string*; *author* : *string*; *source* : *string* }

type *t* =
    { *name* : *string*;
      description : *string list*;
      rcs_revision : *string*;
      rcs_date : *string*;
      rcs_author : *string*;
      rcs_source : *string* }

let *name r* = *r.name*
let *description r* = *r.description*
let *revision r* = *r.rcs_revision*
let *date r* = *r.rcs_date*
let *author r* = *r.rcs_author*
let *source r* = *r.rcs_source*

module *TS* = *ThoString*

let *strip_dollars s* =
  TS.strip_from_last '$' (*TS.strip_prefix* "$" *s*)

let *strip_keyword k s* =
  TS.strip_prefix_star ' ' (*TS.strip_prefix* ":" (*TS.strip_required_prefix k s*))

let *parse1 k s* =
  strip_keyword k (*strip_dollars s*)

let *strip_before_keyword k s* =
  try
    let *i* = *TS.index_string k s* in
    String.sub s i (*String.length s* − *i*)
  with
  | *Not_found* → *s*

let *strip_before_a_keyword k_list s* =
  let rec *strip_before_a_keyword′* = function
    | *k* :: *k_rest* →
        begin try
          let *i* = *TS.index_string k s* in
          String.sub s i (*String.length s* − *i*)
        with
        | *Not_found* → *strip_before_a_keyword′ k_rest*
        end
    | [] → *s* in
  strip_before_a_keyword′ k_list

Required for the transition from CVS to Subversion, because the latter doesn't support the Source keyword. URL is probably the way to go, but we leave in Id as a fallback option.

let *parse_source s* =
  let *s* = *strip_dollars s* in

try *strip_keyword* "URL" *s* with *Invalid_argument* _ →
   try *strip_keyword* "Source" *s* with *Invalid_argument* _ →
     TS.strip_from_first ' ' (*strip_keyword* "Id" *s*)

Assume that the SVN repository follows the recommended layout and that all files can be found beneath "/trunk/", "/branches/" or "/tags/". Strip everything before that.

let *strip_svn_repos s* =
  strip_before_a_keyword ["/trunk/"; "/branches/"; "/tags/"] *s*

let *parse name description r* =
  { *name* = *name*;
   description = *description*;
   rcs_revision = *parse1* "Revision" *r.revision*;
   rcs_date = *parse1* "Date" *r.date*;
   rcs_author = *parse1* "Author" *r.author*;
   rcs_source = *strip_svn_repos* (*parse_source r.source*) }

let *rename rcs name description* =
  { *rcs* with *name* = *name*; *description* = *description* }

let *summary rcs* =
  [*name rcs* ^ ":"] @
  List.map (fun *s* → "␣␣" ^ *s*) (*description rcs*) @
  ["␣␣Source:␣" ^ *source rcs*;
   "␣␣revision:␣" ^ *revision rcs* ^ "␣checked␣in␣by␣" ^
   author rcs ^ "␣at␣" ^ *date rcs*]

—B—
Autotools

B.1 Interface of Config

Cache writing is attempted in the order [$system\_cache\_dir$], [$user\_cache\_dir$], ["."] and cache reading in the opposite order.

val $system\_cache\_dir$: $string$
val $user\_cache\_dir$: $string$

val $cache\_prefix$: $string$
val $cache\_suffix$: $string$

val $openmp$: $bool$

B.2 Implementation of Config

let $system\_cache\_dir$ = "/Users/reuter/Physik/whizard/omega/inst/var/cache"
let $user\_cache\_dir$ = "/Users/reuter/.whizard/var/cache"

⚠ This relies on the fast that the executable names should be unique.

let $cache\_prefix$ =
  let $basename$ = $Filename.basename\ Sys.executable\_name$ in
  try $Filename.chop\_extension\ basename$ with | _ → $basename$

let $cache\_suffix$ = "vertices"

let $openmp$ = false

—C—
TEXTUAL OPTIONS

C.1   Interface of Options

```
type t
val empty  :  t
val merge  :  t  →  t  →  t
val extend  :  t  →  (string × Arg.spec  ×  string) list →  t
val create  :  (string ×  Arg.spec  ×  string) list →  t
val parse  :  t  →  string × string →  unit
val list : t  →  (string × string) list
val cmdline  :  string →  t  →  (string  ×  Arg.spec  ×  string) list

exception Invalid of  string  ×  string
```

C.2   Implementation of Options

```
module A  =  Map.Make (struct type t  =  string let compare  =  compare end)

type t  =
    { actions  :  Arg.spec A.t;
      raw  :  (string × Arg.spec  ×  string) list }

let empty  =  { actions  =  A.empty; raw  =  [] }

let extend old options  =
  { actions  =  List.fold_left
      (fun a (s,  f,  _)  →  A.add s f a) old.actions options;
    raw  =  options @ old.raw }

let create  =  extend empty

let merge o1  o2  =
  extend o1 o2.raw

exception Invalid of  string  ×  string

let parse options (name,  value)  =
  try
    match A.find name options.actions with
    | Arg.Unit f  →  f ()
```

```
      |  Arg.Set b  →  b  :=  true
      |  Arg.Clear b  →  b  :=  false
      |  Arg.String f  →  f value
      |  Arg.Int f  →  f (int_of_string value)
      |  Arg.Float f  →  f (float_of_string value)
      |  _  →  invalid_arg "Options.parse"
   with
   |  Not_found  →  raise (Invalid (name, value))

let list options  =
   List.map (fun (o, _, d)  →  (o, d)) options.raw

let cmdline prefix options  =
   List.map (fun (o, f, d)  →  (prefix ^ o, f, d)) options.raw
```

—D—
PROGRESS REPORTS

D.1   Interface of Progress

type t

val $dummy$: t
val $channel$: $out\_channel$ $\rightarrow$ int $\rightarrow$ t
val $file$: $string$ $\rightarrow$ int $\rightarrow$ t
val $open\_file$: $string$ $\rightarrow$ int $\rightarrow$ t
val $reset$: t $\rightarrow$ int $\rightarrow$ $string$ $\rightarrow$ $unit$
val $begin\_step$: t $\rightarrow$ $string$ $\rightarrow$ $unit$
val end_step : t $\rightarrow$ $string$ $\rightarrow$ $unit$
val $summary$: t $\rightarrow$ $string$ $\rightarrow$ $unit$

D.2   Implementation of Progress

type $channel$ =
  | $Channel$ of $out\_channel$
  | $File$ of $string$
  | $Open\_File$ of $string$ $\times$ $out\_channel$

type $state$ =
    { $channel$: $channel$;
      mutable $steps$: int;
      mutable $digits$: int;
      mutable $step$: int;
      $created$: $float$;
      mutable $last\_reset$: $float$;
      mutable $last\_begin$: $float$; }

type t = $state$ $option$

let $digits$ n =
  if $n > 0$ then
    $succ$ ($truncate$ ($log10$ ($float$ n)))
  else
    $invalid\_arg$ "Progress.digits:␣non-positive␣argument"

let mod_float2 a b =

```
    let modulus = mod_float a b in
    ((a − . modulus) /. b, modulus)

let time_to_string seconds =
  let minutes, seconds = mod_float2 seconds 60. in
  if minutes > 0.0 then
    let hours, minutes = mod_float2 minutes 60. in
    if hours > 0.0 then
      let days, hours = mod_float2 hours 24. in
      if days > 0.0 then
        Printf.sprintf "%.0f:%02.0f␣days" days hours
      else
        Printf.sprintf "%.0f:%02.0f␣hrs" hours minutes
    else
      Printf.sprintf "%.0f:%02.0f␣mins" minutes seconds
  else
    Printf.sprintf "%.2f␣secs" seconds

let create channel steps =
  let now = Sys.time () in
  Some { channel = channel;
         steps = steps;
         digits = digits steps;
         step = 0;
         created = now;
         last_reset = now;
         last_begin = now }

let dummy =
  None

let channel oc =
  create (Channel oc)

let file name =
  let oc = open_out name in
  close_out oc;
  create (File name)

let open_file name =
  let oc = open_out name in
  create (Open_File (name, oc))

let close_channel state =
  match state.channel with
  | Channel oc →
      flush oc
  | File _ → ()
  | Open_File (_, oc) →
      flush oc;
      close_out oc

let use_channel state f =
  match state.channel with
```

```
        |  Channel oc  |  Open_File (_, oc)  →
            f oc;
            flush oc
        |  File name  →
            let oc  =  open_out_gen [Open_append; Open_creat] 644₈ name in
            f oc;
            flush oc;
            close_out oc

  let reset state steps msg  =
      match state with
      |  None  →  ()
      |  Some state  →
            let now  =  Sys.time () in
            state.steps  ←  steps;
            state.digits  ←  digits steps;
            state.step  ←  0;
            state.last_reset  ←  now;
            state.last_begin  ←  now

  let begin_step state msg  =
      match state with
      |  None  →  ()
      |  Some state  →
            let now  =  Sys.time () in
            state.step  ←  succ state.step;
            state.last_begin  ←  now;
            use_channel state (fun oc  →
               Printf.fprintf oc "[%0*d/%0*d]␣%s␣..." state.digits state.step state.digits state.steps msg)

  let end_step state msg  =
      match state with
      |  None  →  ()
      |  Some state  →
            let now  =  Sys.time () in
            let last  =  now  −. state.last_begin in
            let elapsed  =  now  −. state.last_reset in
            let estimated  =  float state.steps ∗. elapsed /. float state.step in
            let remaining  =  estimated  −. elapsed in
            use_channel state (fun oc  →
               Printf.fprintf oc "␣%s.␣[time:␣%s,␣total:␣%s,␣remaining:␣%s]\n" msg
                  (time_to_string last) (time_to_string estimated) (time_to_string remaining))

  let summary state msg  =
      match state with
      |  None  →  ()
      |  Some state  →
            let now  =  Sys.time () in
            use_channel state (fun oc  →
               Printf.fprintf oc "%s.␣[total␣time:␣%s]\n" msg
                  (time_to_string (now  −. state.created)));
            close_channel state
```

—E—
More on Filenames

E.1   Interface of ThoFilename

val *split* : *string* → *string list*
val *join* : *string list* → *string*

val *expand_home* : *string* → *string*

E.2   Implementation of ThoFilename

let rec *split′ acc path* =
　match *Filename.dirname path*, *Filename.basename path* with
　| "/", *basename* → "/" :: *basename* :: *acc*
　| ".", *basename* → *basename* :: *acc*
　| *dirname*, *basename* → *split′* (*basename* :: *acc*) *dirname*

let *split path* =
　split′ [ ] *path*

let *join* = function
　| [ ] → "."
　| [*basename*] → *basename*
　| *dirname* :: *rest* → *List.fold_left Filename.concat dirname rest*

let *expand_home path* =
　match *split path* with
　| ("~" | "$HOME" | "${HOME}") :: *rest* →
　　　join ((try *Sys.getenv* "HOME" with *Not_found* → "/tmp") :: *rest*)
　| _ → *path*

—F—
Cache Files

F.1   Interface of Cache

```
module type T  =
  sig

    type key
    type hash  =  string
    type value

    type α result  =
      | Hit of α
      | Miss
      | Stale of string

    exception Mismatch of string × string × string

    val hash : key → hash
    val exists : hash → string → bool
    val find : hash → string → string option
    val write : hash → string → value → unit
    val write_dir : hash → string → string → value → unit
    val read : hash → string → value
    val maybe_read : hash → string → value result

  end

module type Key  =
  sig
    type t
  end

module type Value  =
  sig
    type t
  end

module Make (Key : Key) (Value : Value) :
    T with type key  =  Key.t and type value  =  Value.t
```

F.2   Implementation of Cache

```
let search_path  =
  [ Filename.current_dir_name;
    ThoFilename.expand_home Config.user_cache_dir;
    Config.system_cache_dir ]

module type T  =
  sig

    type key
    type hash  =  string
    type value

    type α result  =
      | Hit of α
      | Miss
      | Stale of string

    exception Mismatch of string × string × string

    val hash  :  key  →  hash
    val exists  :  hash  →  string →  bool
    val find  :  hash  →  string →  string option
    val write  :  hash  →  string →  value  →  unit
    val write_dir  :  hash  →  string →  string  →  value  →  unit
    val read  :  hash  →  string →  value
    val maybe_read  :  hash  →  string →  value result

  end

module type Key  =
  sig
    type t
  end

module type Value  =
  sig
    type t
  end

module Make (Key  :  Key) (Value  :  Value)  =
  struct

    type key  =  Key.t
    type hash  =  string
    type value  =  Value.t

    type tagged  =
        { tag  :  hash;
          value  :  value; }

    let hash value  =
      Digest.string (Marshal.to_string value [])

    let find_first path name  =
```

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```
let rec find_first′ = function
  | [] → raise Not_found
  | dir :: path →
      let f = Filename.concat dir name in
      if Sys.file_exists f then
        f
      else
        find_first′ path
in
find_first′ path

let find hash name =
  try Some (find_first search_path name) with Not_found → None

let exists hash name =
  match find hash name with
  | None → false
  | Some _ → true

let try_first f path name =
  let rec try_first′ = function
    | [] → raise Not_found
    | dir :: path →
        try (f (Filename.concat dir name), dir) with _ → try_first′ path
  in
  try_first′ path

let open_in_bin_first = try_first open_in_bin
let open_out_bin_last path = try_first open_out_bin (List.rev path)

let write hash name value =
  let oc, _ = open_out_bin_last search_path name in
  Marshal.to_channel oc { tag = hash; value = value } [];
  close_out oc

let write_dir hash dir name value =
  let oc = open_out_bin (Filename.concat dir name) in
  Marshal.to_channel oc { tag = hash; value = value } [];
  close_out oc

type α result =
  | Hit of α
  | Miss
  | Stale of string

exception Mismatch of string × string × string

let read hash name =
  let ic, dir = open_in_bin_first search_path name in
  let { tag = tag; value = value } = Marshal.from_channel ic in
  close_in ic;
  if tag = hash then
    value
  else
    raise (Mismatch (Filename.concat dir name, hash, tag))
```

```
let maybe_read hash name =
  try
    Hit (read hash name)
  with
  | Not_found → Miss
  | Mismatch (file, _, _) → Stale file
end
```

—G—
More On Lists

G.1   Interface of ThoList

splitn n l = (hdn l, tln l), but more efficient.

val *hdn : int → α list → α list*
val *tln : int → α list → α list*
val *splitn : int → α list → α list × α list*

chop n l chops *l* into pieces of size *n* (except for the last one, which contains th remainder).

val *chopn : int → α list → α list list*

of_subarray n m a is $[a.(n); a.(n + 1); \ldots; a.(m)]$. Values of *n* and *m* out of bounds are silently shifted towards these bounds.

val *of_subarray : int → int → α array → α list*

range s n m is $[n; n + s; n + 2s; \ldots; m - ((m - n) \mod s)]$

val *range : ?stride :int → int → int → int list*

enumerate s n $[a1; a2; \ldots]$ *is* $[(n, a1); (n + s, a2); \ldots]$

val *enumerate : ?stride :int → int → α list → (int × α) list*

Compress identical elements in a sorted list. Identity is determined using the polymorphic equality function *Pervasives.(=)*.

val *uniq : α list → α list*

Test if all members of a list are structurally identical (actually *homogeneous l* and *List.length (uniq l)* ≤ 1 are equivalent, but the former is more efficient if a mismatch comes early).

val *homogeneous : α list → bool*

compare cmp l1 l2 compare two lists *l1* and *l2* according to *cmp*. *cmp* defaults to the polymorphic *Pervasives.compare*.

val *compare : ?cmp :(α → α → int) → α list → α list → int*

Collect and count identical elements in a list. Identity is determined using the polymorphic equality function *Pervasives.(=)*. *classify* does not assume that

the list is sorted. However, it is $O(n)$ for sorted lists and $O(n^2)$ in the worst case.

val *classify* : α *list* $\rightarrow$ $(int \times \alpha)$ *list*

Collect the second factors with a common first factor in lists.

val *factorize* : $(\alpha \times \beta)$ *list* $\rightarrow$ $(\alpha \times \beta$ *list* $)$ *list*

flatmap f is equivalent to *flatten* $\circ$ $(map\ f)$, but more efficient, because no intermediate lists are built. Unfortunately, it is not tail recursive.

val *flatmap* : $(\alpha \rightarrow \beta$ *list* $)$ $\rightarrow$ α *list* $\rightarrow$ β *list*

rev_flatmap f is equivalent to *flatten* $\circ$ $(rev\_map\ (rev \circ f)) = rev \circ (flatmap\ f)$, but more efficient, because no intermediate lists are built. It is tail recursive.

val *rev_flatmap* : $(\alpha \rightarrow \beta$ *list* $)$ $\rightarrow$ α *list* $\rightarrow$ β *list*

val *clone* : $int \rightarrow \alpha \rightarrow \alpha$ *list*
val *multiply* : $int \rightarrow \alpha$ *list* $\rightarrow$ α *list*

⚠ Invent other names to avoid confusions with *List.fold_left2* and *List.fold_right2*.

val *fold_right2* : $(\alpha \rightarrow \beta \rightarrow \beta)$ $\rightarrow$ α *list list* $\rightarrow$ β $\rightarrow$ β
val *fold_left2* : $(\beta \rightarrow \alpha \rightarrow \beta)$ $\rightarrow$ β $\rightarrow$ α *list list* $\rightarrow$ β

iteri f n $[a; b; c]$ evaluates $f\ n\ a$, $f\ (n+1)\ b$ and $f\ (n+2)\ c$.

val *iteri* : $(int \rightarrow \alpha \rightarrow unit)$ $\rightarrow$ $int \rightarrow \alpha$ *list* $\rightarrow$ *unit*
val *mapi* : $(int \rightarrow \alpha \rightarrow \beta)$ $\rightarrow$ $int \rightarrow \alpha$ *list* $\rightarrow$ β *list*

iteri2 f n m $[[aa; ab]; [ba; bb]]$ evaluates $f\ n\ m\ aa$, $f\ n\ (m+1)\ ab$, $f\ (n+1)\ m\ ba$ and $f\ (n+1)\ (m+1)\ bb$. NB: the nested lists need not be rectangular.

val *iteri2* : $(int \rightarrow int \rightarrow \alpha \rightarrow unit)$ $\rightarrow$ $int \rightarrow int \rightarrow \alpha$ *list list* $\rightarrow$ *unit*

Transpose a *rectangular* list of lists like a matrix.

val *transpose* : α *list list* $\rightarrow$ α *list list*

interleave f list walks through *list* and inserts the result of f applied to the reversed list of elements before and the list of elements after. The empty lists at the beginning and end are included!

val *interleave* : $(\alpha$ *list* $\rightarrow$ α *list* $\rightarrow$ α *list* $)$ $\rightarrow$ α *list* $\rightarrow$ α *list*

interleave_nearest f list is like *interleave f list*, but f looks only at the nearest neighbors.

val *interleave_nearest* : $(\alpha \rightarrow \alpha \rightarrow \alpha$ *list* $)$ $\rightarrow$ α *list* $\rightarrow$ α *list*

partitioned_sort cmp index_sets list sorts the sublists of *list* specified by the *index_sets* and the complement of their union. **NB:** the sorting follows to order in the lists in *index_sets*. **NB:** the indices are 0-based.

val *partitioned_sort* : $(\alpha \rightarrow \alpha \rightarrow int)$ $\rightarrow$ int *list list* $\rightarrow$ α *list* $\rightarrow$ α *list*
exception *Overlapping_indices*
exception *Out_of_bounds*

ariadne_sort cmp list sorts *list* according to *cmp* (default *Pervasives.compare*) keeping track of the original order by a 0-based list of indices.

val *ariadne_sort* : ?*cmp* : ($\alpha \to \alpha \to int$) $\to \alpha$ *list* $\to \alpha$ *list* $\times$ *int list*

ariadne_unsort (*ariadne_sort cmp list*) returns *list*.

val *ariadne_unsort* : α *list* $\times$ *int list* $\to \alpha$ *list*

G.2   Implementation of ThoList

```
let rec hdn n l =
  if n ≤ 0 then
    []
  else
    match l with
    | x :: rest → x :: hdn (pred n) rest
    | [] → invalid_arg "ThoList.hdn"

let rec tln n l =
  if n ≤ 0 then
    l
  else
    match l with
    | _ :: rest → tln (pred n) rest
    | [] → invalid_arg "ThoList.tln"

let rec splitn' n l1_rev l2 =
  if n ≤ 0 then
    (List.rev l1_rev, l2)
  else
    match l2 with
    | x :: l2' → splitn' (pred n) (x :: l1_rev) l2'
    | [] → invalid_arg "ThoList.splitn␣n␣>␣len"

let splitn n l =
  if n < 0 then
    invalid_arg "ThoList.splitn␣n␣<␣0"
  else
    splitn' n [] l
```

This is *splitn'* all over again, but without the exception.

```
let rec chopn'' n l1_rev l2 =
  if n ≤ 0 then
    (List.rev l1_rev, l2)
  else
    match l2 with
    | x :: l2' → chopn'' (pred n) (x :: l1_rev) l2'
    | [] → (List.rev l1_rev, [])

let rec chopn' n ll_rev = function
  | [] → List.rev ll_rev
```

```
  | l →
      begin match chopn″ n [] l with
      | [], [] → List.rev ll_rev
      | l1, [] → List.rev (l1 :: ll_rev)
      | l1, l2 → chopn′ n (l1 :: ll_rev) l2
      end
```

let *chopn n l* =
  if $n \le 0$ then
    invalid_arg "ThoList.chopn␣n␣<=␣0"
  else
    chopn′ n [] l

let *of_subarray n1 n2 a* =
  let rec *of_subarray′ n1 n2* =
    if $n1 > n2$ then
      []
    else
      a.(n1) :: of_subarray′ (succ n1) n2 in
  of_subarray′ (max 0 n1) (min n2 (pred (Array.length a)))

let *range ?(stride = 1) n1 n2* =
  if $stride \le 0$ then
    invalid_arg "ThoList.range:␣stride␣<=␣0"
  else
    let rec *range′ n* =
      if $n > n2$ then
        []
      else
        n :: range′ (n + stride) in
    range′ n1

Tail recursive:

let *enumerate ?(stride = 1) n l* =
  let _, *l_rev* =
    List.fold_left
      (fun (i, acc) a → (i + stride, (i, a) :: acc))
      (n, []) l in
  List.rev l_rev

This is *not* tail recursive!

```
let rec flatmap f = function
  | [] → []
  | x :: rest → f x @ flatmap f rest
```

This is!

let *rev_flatmap f l* =
  let rec *rev_flatmap′ acc f* = function
    | [] → *acc*
    | *x :: rest → rev_flatmap′ (List.rev_append (f x) acc) f rest* in
  rev_flatmap′ [] f l

let *fold_left2 f acc lists* =
  List.fold_left (*List.fold_left f*) *acc lists*

let *fold_right2 f lists acc* =
  List.fold_right (*List.fold_right f*) *lists acc*

let *iteri f start list* =
  ignore (*List.fold_left* (fun *i a* → *f i a*; *succ i*) *start list*)

let *iteri2 f start_outer star_inner lists* =
  iteri (fun *j* → *iteri* (*f j*) *star_inner*) *start_outer lists*

let *mapi f start list* =
  let *next, list'* =
    List.fold_left (fun (*i, acc*) *a* → (*succ i, f i a* :: *acc*)) (*start,* []) *list* in
  List.rev list'

Is there a more efficient implementation?

let *transpose lists* =
  let rec *transpose' rest* =
    if *List.for_all* ((=) []) *rest* then
      []
    else
      List.map List.hd rest :: *transpose'* (*List.map List.tl rest*) in
  try
    transpose' lists
  with
  | *Failure* "tl" → *invalid_arg* "ThoList.transpose:␣not␣rectangular"

let *compare* ?(*cmp* = *Pervasives.compare*) *l1 l2* =
  let rec *compare' l1' l2'* =
    match *l1', l2'* with
    | [], [] → 0
    | [], _ → −1
    | _, [] → 1
    | *n1* :: *r1, n2* :: *r2* →
        let *c* = *cmp n1 n2* in
        if *c* ≠ 0 then
          c
        else
          compare' r1 r2
  in
  compare' l1 l2

let rec *uniq' x* = function
  | [] → []
  | *x'* :: *rest* →
      if *x'* = *x* then
        uniq' x rest
      else
        x' :: *uniq' x' rest*

let *uniq* = function
  | [] → []

```
    | x :: rest → x :: uniq' x rest

let rec homogeneous = function
  | [] | [_] → true
  | a1 :: (a2 :: _ as rest) →
      if a1 ≠ a2 then
        false
      else
        homogeneous rest
```

If we needed it, we could use a polymorphic version of *Set* to speed things up from $O(n^2)$ to $O(n \ln n)$. But not before it matters somewhere ...

```
let classify l =
  let rec add_to_class a = function
    | [] → [1, a]
    | (n, a') :: rest →
        if a = a' then
          (succ n, a) :: rest
        else
          (n, a') :: add_to_class a rest
  in
  let rec classify' cl = function
    | [] → cl
    | a :: rest → classify' (add_to_class a cl) rest
  in
  classify' [] l

let rec factorize l =
  let rec add_to_class x y = function
    | [] → [(x, [y])]
    | (x', ys) :: rest →
        if x = x' then
          (x, y :: ys) :: rest
        else
          (x', ys) :: add_to_class x y rest
  in
  let rec factorize' fl = function
    | [] → fl
    | (x, y) :: rest → factorize' (add_to_class x y fl) rest
  in
  List.map (fun (x, ys) → (x, List.rev ys)) (factorize' [] l)

let rec clone n x =
  if n < 0 then
    invalid_arg "ThoList.clone"
  else if n = 0 then
    []
  else
    x :: clone (pred n) x

let interleave f list =
  let rec interleave' rev_head tail =
```

```
      let rev_head′ = List.rev_append (f rev_head tail) rev_head in
      match tail with
      | [] → List.rev rev_head′
      | x :: tail′ → interleave′ (x :: rev_head′) tail′
    in
    interleave′ [] list

let interleave_nearest f list =
  interleave
    (fun head tail →
      match head, tail with
      | h :: _, t :: _ → f h t
      | _ → [])
    list

let rec rev_multiply n rl l =
  if n < 0 then
    invalid_arg "ThoList.multiply"
  else if n = 0 then
    []
  else
    List.rev_append rl (rev_multiply (pred n) rl l)

let multiply n l = rev_multiply n (List.rev l) l

module ISet = Set.Make (struct type t = int let compare = Pervasives.compare end)

exception Overlapping_indices
exception Out_of_bounds

let iset_of_list list =
  List.fold_right ISet.add list ISet.empty

let iset_list_union list =
  List.fold_right ISet.union list ISet.empty

let complement_index_sets n index_set_lists =
  let index_sets = List.map iset_of_list index_set_lists in
  let index_set = iset_list_union index_sets in
  let size_index_sets =
    List.fold_left (fun acc s → ISet.cardinal s + acc) 0 index_sets in
  if size_index_sets ≠ ISet.cardinal index_set then
    raise Overlapping_indices
  else if ISet.exists (fun i → i < 0 ∨ i ≥ n) index_set then
    raise Overlapping_indices
  else
    match ISet.elements (ISet.diff (iset_of_list (range 0 (pred n))) index_set) with
    | [] → index_set_lists
    | complement → complement :: index_set_lists

let sort_section cmp array index_set =
  List.iter2
    (Array.set array)
    index_set (List.sort cmp (List.map (Array.get array) index_set))
```

```
let partitioned_sort cmp index_sets list =
  let array = Array.of_list list in
  List.fold_left
    (fun () → sort_section cmp array)
    () (complement_index_sets (List.length list) index_sets);
  Array.to_list array

let ariadne_sort ?(cmp = Pervasives.compare) list =
  let sorted =
    List.sort (fun (n1, a1) (n2, a2) → cmp a1 a2) (enumerate 0 list) in
  (List.map snd sorted, List.map fst sorted)

let ariadne_unsort (sorted, indices) =
  List.map snd
    (List.sort
      (fun (n1, a1) (n2, a2) → Pervasives.compare n1 n2)
      (List.map2 (fun n a → (n, a)) indices sorted))
```

—H—

More On Arrays

H.1   Interface of ThoArray

Compressed arrays, i. e. arrays with only unique elements and an embedding that allows to recover the original array. NB: in the current implementation, compressing saves space, if *and only if* objects of type α require more storage than integers. The main use of α *compressed* is *not* for saving space, anyway, but for avoiding the repetition of hard calculations.

type α *compressed*
val *uniq* : α *compressed* $\rightarrow$ α *array*
val *embedding* : α *compressed* $\rightarrow$ *int array*

These two are inverses of each other:

val *compress* : α *array* $\rightarrow$ α *compressed*
val *uncompress* : α *compressed* $\rightarrow$ α *array*

One can play the same game for matrices.

type α *compressed2*
val *uniq2* : α *compressed2* $\rightarrow$ α *array array*
val *embedding1* : α *compressed2* $\rightarrow$ *int array*
val *embedding2* : α *compressed2* $\rightarrow$ *int array*

Again, these two are inverses of each other:

val *compress2* : α *array array* $\rightarrow$ α *compressed2*
val *uncompress2* : α *compressed2* $\rightarrow$ α *array array*

H.2   Implementation of ThoArray

type α *compressed* =
    { *uniq* : α *array*;
      embedding : *int array* }

let *uniq a* = *a.uniq*
let *embedding a* = *a.embedding*

type α *compressed2* =
    { *uniq2* : α *array array*;

```
        embedding1 :  int array;
        embedding2 :  int array }

let uniq2 a  =  a.uniq2
let embedding1 a  =  a.embedding1
let embedding2 a  =  a.embedding2

module PMap  =  Pmap.Tree

let compress a  =
  let last  =  Array.length a  −  1 in
  let embedding  =  Array.make (succ last) (−1) in
  let rec scan num_uniq uniq elements n  =
    if n  >  last then
      { uniq  =  Array.of_list (List.rev elements);
        embedding  =  embedding }
    else
      match PMap.find_opt compare a.(n) uniq with
      | Some n′  →
          embedding.(n)  ←  n′;
          scan num_uniq uniq elements (succ n)
      | None  →
          embedding.(n)  ←  num_uniq;
          scan
            (succ num_uniq)
            (PMap.add compare a.(n) num_uniq uniq)
            (a.(n) ::  elements)
            (succ n) in
  scan 0 PMap.empty [] 0

let uncompress a  =
  Array.map (Array.get a.uniq) a.embedding
```

Using *transpose* simplifies the algorithms, but can be inefficient. If this turns out to be the case, we should add special treatments for symmetric matrices.

```
let transpose a  =
  let dim1  =  Array.length a
  and dim2  =  Array.length a.(0) in
  let a′  =  Array.make_matrix dim2 dim1 a.(0).(0) in
  for i1  =  0 to pred dim1 do
    for i2  =  0 to pred dim2 do
      a′.(i2).(i1)  ←  a.(i1).(i2)
    done
  done;
  a′

let compress2 a  =
  let c2  =  compress a in
  let c12_transposed  =  compress (transpose c2.uniq) in
  { uniq2  =  transpose c12_transposed.uniq;
    embedding1  =  c12_transposed.embedding;
```

$embedding2 = c2.embedding$ }

let *uncompress2 a =*
　let $a2 = uncompress$ { $uniq = a.uniq2$; $embedding = a.embedding2$ } in
　$transpose$ ($uncompress$ { $uniq = transpose\ a2$; $embedding = a.embedding1$ })

—I—

More On Strings

I.1   Interface of ThoString

This is a very simple library if stroing manipulation functions missing in O'Caml's standard library.

strip_prefix prefix string returns *string* with 0 or 1 occurences of a leading *prefix* removed.

val *strip_prefix* : *string* → *string* → *string*

strip_prefix_star prefix string returns *string* with any number of leading occurences of *prefix* removed.

val *strip_prefix_star* : *char* → *string* → *string*

strip_prefix prefix string returns *string* with a leading *prefix* removed, raises *Invalid_argument* if there's no match.

val *strip_required_prefix* : *string* → *string* → *string*

strip_from_first c s returns *s* with everything starting from the first *c* removed.
strip_from_last c s returns *s* with everything starting from the last *c* removed.

val *strip_from_first* : *char* → *string* → *string*
val *strip_from_last* : *char* → *string* → *string*

index_string pattern string returns the index of the first occurence of *pattern* in *string*, if any. Raises *Not_found*, if *pattern* is not in *string*.

val *index_string* : *string* → *string* → *int*

This silently fails if the argument contains both single and double quotes!

val *quote* : *string* → *string*

I.2   Implementation of ThoString

```
let strip_prefix p s  =
  let lp  =  String.length p
  and ls  =  String.length s in
  if lp  >  ls then
    s
```

```
    else
      let rec strip_prefix' i =
        if i ≥ lp then
          String.sub s i (ls − i)
        else if p.[i] ≠ s.[i] then
          s
        else
          strip_prefix' (succ i)
      in
      strip_prefix' 0

let strip_prefix_star p s =
  let ls = String.length s in
  if ls < 1 then
    s
  else
    let rec strip_prefix_star' i =
      if i < ls then begin
        if p ≠ s.[i] then
          String.sub s i (ls − i)
        else
          strip_prefix_star' (succ i)
      end else
        ""
    in
    strip_prefix_star' 0

let strip_required_prefix p s =
  let lp = String.length p
  and ls = String.length s in
  if lp > ls then
    invalid_arg ("strip_required_prefix:␣expected␣'" ^ p ^ "'␣got␣'" ^ s ^ "'")
  else
    let rec strip_prefix' i =
      if i ≥ lp then
        String.sub s i (ls − i)
      else if p.[i] ≠ s.[i] then
        invalid_arg ("strip_required_prefix:␣expected␣'" ^ p ^ "'␣got␣'" ^ s ^ "'")
      else
        strip_prefix' (succ i)
    in
    strip_prefix' 0

let strip_from_first c s =
  try
    String.sub s 0 (String.index s c)
  with
  | Not_found → s

let strip_from_last c s =
  try
    String.sub s 0 (String.rindex s c)
```

```
  with
  | Not_found → s
let index_string pat s =
  let lpat = String.length pat
  and ls = String.length s in
  if lpat = 0 then
    0
  else
    let rec index_string' n =
      let i = String.index_from s n pat.[0] in
      if i + lpat > ls then
        raise Not_found
      else
        if String.compare pat (String.sub s i lpat) = 0 then
          i
        else
          index_string' (succ i)
    in
    index_string' 0
let quote s =
  if String.contains s ' ' ∨ String.contains s '\n' then begin
    if String.contains s '"' then
      "'" ^ s ^ "'"
    else
      "\"" ^ s ^ "\""
  end else
    s
```

—J—
Polymorphic Maps

From [9].

J.1 Interface of Pmap

Module *Pmap*: association tables over a polymorphic type[1].

```
module type T =
  sig
    type ('key, α) t
    val empty : ('key, α) t
    val is_empty : ('key, α) t → bool
    val singleton : 'key → α → ('key, α) t
    val add : ('key → 'key → int) → 'key → α → ('key, α) t →
('key, α) t
    val update : ('key → 'key → int) → (α → α → α) →
      'key → α → ('key, α) t → ('key, α) t
    val cons : ('key → 'key → int) → (α → α → α option) →
      'key → α → ('key, α) t → ('key, α) t
    val find : ('key → 'key → int) → 'key → ('key, α) t → α
    val find_opt : ('key → 'key → int) → 'key → ('key, α) t → α option
    val choose : ('key, α) t → 'key × α
    val choose_opt : ('key, α) t → ('key × α) option
    val uncons : ('key, α) t → 'key × α × ('key, α) t
    val uncons_opt : ('key, α) t → ('key × α × ('key, α) t) option
    val elements : ('key, α) t → ('key × α) list
    val mem : ('key → 'key → int) → 'key → ('key, α) t → bool
    val remove : ('key → 'key → int) → 'key → ('key, α) t → ('key, α) t
    val union : ('key → 'key → int) → (α → α → α) →
      ('key, α) t → ('key, α) t → ('key, α) t
    val compose : ('key → 'key → int) → (α → α → α option) →
      ('key, α) t → ('key, α) t → ('key, α) t
    val iter : ('key → α → unit) → ('key, α) t → unit
    val map : (α → β) → ('key, α) t → ('key, β) t
    val mapi : ('key → α → β) → ('key, α) t → ('key, β) t
    val fold : ('key → α → β → β) → ('key, α) t → β → β
```

---

[1] Extension of code © 1996 by Xavier Leroy

val *compare* : (*'key* → *'key* → *int*) → (α → α → *int*) →
  (*'key*, α) *t* → (*'key*, α) *t* → *int*
val *canonicalize* : (*'key* → *'key* → *int*) → (*'key*, α) *t* → (*'key*, α) *t*
  end

Balanced trees: logarithmic access, but representation not unique.

module *Tree* : *T*

Sorted lists: representation unique, but linear access.

module *List* : *T*

J.2   *Implementation of Pmap*

module type *T* =
  sig
    type (*'key*, α) *t*
    val *empty* : (*'key*, α) *t*
    val *is_empty* : (*'key*, α) *t* → *bool*
    val *singleton* : *'key* → α → (*'key*, α) *t*
    val *add* : (*'key* → *'key* → *int*) → *'key* → α → (*'key*, α) *t* →
(*'key*, α) *t*
    val *update* : (*'key* → *'key* → *int*) → (α → α → α) →
      'key → α → (*'key*, α) *t* → (*'key*, α) *t*
    val *cons* : (*'key* → *'key* → *int*) → (α → α → α *option*) →
      'key → α → (*'key*, α) *t* → (*'key*, α) *t*
    val *find* : (*'key* → *'key* → *int*) → *'key* → (*'key*, α) *t* → α
    val *find_opt* : (*'key* → *'key* → *int*) → *'key* → (*'key*, α) *t* → α *option*
    val *choose* : (*'key*, α) *t* → *'key* × α
    val *choose_opt* : (*'key*, α) *t* → (*'key* × α) *option*
    val *uncons* : (*'key*, α) *t* → *'key* × α × (*'key*, α) *t*
    val *uncons_opt* : (*'key*, α) *t* → (*'key* × α × (*'key*, α) *t*) *option*
    val *elements* : (*'key*, α) *t* → (*'key* × α) *list*
    val *mem* : (*'key* → *'key* → *int*) → *'key* → (*'key*, α) *t* → *bool*
    val *remove* : (*'key* → *'key* → *int*) → *'key* → (*'key*, α) *t* → (*'key*, α) *t*
    val *union* : (*'key* → *'key* → *int*) → (α → α → α) →
      (*'key*, α) *t* → (*'key*, α) *t* → (*'key*, α) *t*
    val *compose* : (*'key* → *'key* → *int*) → (α → α → α *option*) →
      (*'key*, α) *t* → (*'key*, α) *t* → (*'key*, α) *t*
    val *iter* : (*'key* → α → *unit*) → (*'key*, α) *t* → *unit*
    val *map* : (α → β) → (*'key*, α) *t* → (*'key*, β) *t*
    val *mapi* : (*'key* → α → β) → (*'key*, α) *t* → (*'key*, β) *t*
    val *fold* : (*'key* → α → β → β) → (*'key*, α) *t* → β → β
    val *compare* : (*'key* → *'key* → *int*) → (α → α → *int*) →
      (*'key*, α) *t* → (*'key*, α) *t* → *int*
    val *canonicalize* : (*'key* → *'key* → *int*) → (*'key*, α) *t* → (*'key*, α) *t*
  end

module *Tree* =
  struct

```
type ('key, α) t  =
  | Empty
  | Node of ('key, α) t  ×  'key  ×  α  ×  ('key, α) t  ×  int

let empty  =  Empty

let is_empty  =  function
  | Empty  →  true
  | _  →  false

let singleton k d  =
  Node (Empty, k, d, Empty, 1)

let height  =  function
  | Empty  →  0
  | Node (_, _, _, _, h)  →  h

let create l x d r  =
  let hl  =  height l and hr  =  height r in
  Node (l, x, d, r, (if hl  ≥  hr then hl  +  1 else hr  +  1))

let bal l x d r  =
  let hl  =  match l with Empty  →  0  |  Node (_, _, _, _, h)  →  h in
  let hr  =  match r with Empty  →  0  |  Node (_, _, _, _, h)  →  h in
  if hl  >  hr  +  2 then begin
    match l with
    | Empty  →  invalid_arg "Map.bal"
    | Node (ll, lv, ld, lr, _)  →
        if height ll  ≥  height lr then
          create ll lv ld (create lr x d r)
        else begin
          match lr with
          | Empty  →  invalid_arg "Map.bal"
          | Node (lrl, lrv, lrd, lrr, _) →
              create (create ll lv ld lrl) lrv lrd (create lrr x d r)
        end
  end else if hr  >  hl  +  2 then begin
    match r with
    | Empty  →  invalid_arg "Map.bal"
    | Node (rl, rv, rd, rr, _)  →
        if height rr  ≥  height rl then
          create (create l x d rl) rv rd rr
        else begin
          match rl with
          | Empty  →  invalid_arg "Map.bal"
          | Node (rll, rlv, rld, rlr, _)  →
              create (create l x d rll) rlv rld (create rlr rv rd rr)
        end
  end else
    Node (l, x, d, r, (if hl  ≥  hr then hl  +  1 else hr  +  1))

let rec join l x d r  =
  match bal l x d r with
  | Empty  →  invalid_arg "Pmap.join"
```

```
    | Node (l′, x′, d′, r′, _) as t′ →
        let d = height l′ − height r′ in
        if d < − 2 ∨ d > 2 then
          join l′ x′ d′ r′
        else
          t′
```

Merge two trees *t1* and *t2* into one. All elements of *t1* must precede the elements of *t2*. Assumes *height t1 − height t2 ≤ 2*.

```
    let rec merge t1 t2 =
      match t1, t2 with
      | Empty, t → t
      | t, Empty → t
      | Node (l1, v1, d1, r1, h1), Node (l2, v2, d2, r2, h2) →
          bal l1 v1 d1 (bal (merge r1 l2) v2 d2 r2)
```

Same as merge, but does not assume anything about *t1* and *t2*.

```
    let rec concat t1 t2 =
      match t1, t2 with
      | Empty, t → t
      | t, Empty → t
      | Node (l1, v1, d1, r1, h1), Node (l2, v2, d2, r2, h2) →
          join l1 v1 d1 (join (concat r1 l2) v2 d2 r2)
```

Splitting

```
    let rec split cmp x = function
      | Empty → (Empty, None, Empty)
      | Node (l, v, d, r, _) →
          let c = cmp x v in
          if c = 0 then
            (l, Some d, r)
          else if c < 0 then
            let ll, vl, rl = split cmp x l in
            (ll, vl, join rl v d r)
          else (∗ if c > 0 then ∗)
            let lr, vr, rr = split cmp x r in
            (join l v d lr, vr, rr)

    let rec find cmp x = function
      | Empty → raise Not_found
      | Node (l, v, d, r, _) →
          let c = cmp x v in
          if c = 0 then
            d
          else if c < 0 then
            find cmp x l
          else (∗ if c > 0 ∗)
            find cmp x r

    let rec find_opt cmp x = function
      | Empty → None
```

```
    | Node (l, v, d, r, _) →
        let c = cmp x v in
        if c = 0 then
          Some d
        else if c < 0 then
          find_opt cmp x l
        else (∗ if c > 0 ∗)
          find_opt cmp x r

let rec mem cmp x = function
  | Empty → false
  | Node (l, v, d, r, _) →
      let c = cmp x v in
      if c = 0 then
        true
      else if c < 0 then
        mem cmp x l
      else (∗ if c > 0 ∗)
        mem cmp x r

let choose = function
  | Empty → raise Not_found
  | Node (l, v, d, r, _) → (v, d)

let choose_opt = function
  | Empty → None
  | Node (l, v, d, r, _) → Some (v, d)

let uncons = function
  | Empty → raise Not_found
  | Node (l, v, d, r, h) → (v, d, merge l r)

let uncons_opt = function
  | Empty → None
  | Node (l, v, d, r, h) → Some (v, d, merge l r)

let rec remove cmp x = function
  | Empty → Empty
  | Node (l, v, d, r, h) →
      let c = cmp x v in
      if c = 0 then
        merge l r
      else if c < 0 then
        bal (remove cmp x l) v d r
      else (∗ if c > 0 ∗)
        bal l v d (remove cmp x r)

let rec cons cmp resolve x data' = function
  | Empty → Node (Empty, x, data', Empty, 1)
  | Node (l, v, data, r, h) →
      let c = cmp x v in
      if c = 0 then
        match resolve data' data with
        | Some data'' → Node (l, x, data'', r, h)
```

```
            |  None  →  merge l r
         else if  c  <  0 then
            bal (cons cmp resolve x data' l) v data r
         else (∗ if  c  >  0 ∗)
            bal l v data (cons cmp resolve x data' r)
let rec update cmp resolve x data'  =  function
  |  Empty  →  Node (Empty, x, data', Empty, 1)
  |  Node (l, v, data, r, h)  →
         let c  =  cmp x v in
         if  c  =  0 then
            Node (l, x, resolve data' data, r, h)
         else if  c  <  0 then
            bal (update cmp resolve x data' l) v data r
         else (∗ if  c  >  0 ∗)
            bal l v data (update cmp resolve x data' r)
let add cmp x data  =  update cmp (fun n o  →  n) x data
let rec compose cmp resolve s1 s2  =
  match s1, s2 with
  |  Empty,  t2  →  t2
  |  t1,  Empty  →  t1
  |  Node (l1, v1, d1, r1, h1), Node (l2, v2, d2, r2, h2)  →
         if  h1  ≥  h2 then
            if  h2  =  1 then
               cons cmp (fun o n  →  resolve n o) v2 d2 s1
            else begin
               match split cmp v1 s2 with
               |  l2', None, r2'  →
                  join (compose cmp resolve l1 l2') v1 d1
                     (compose cmp resolve r1 r2')
               |  l2', Some d, r2'  →
                  begin match resolve d1 d with
                  |  None  →
                     concat (compose cmp resolve l1 l2')
                        (compose cmp resolve r1 r2')
                  |  Some d  →
                     join (compose cmp resolve l1 l2') v1 d
                        (compose cmp resolve r1 r2')
                  end
            end
         else
            if  h1  =  1 then
               cons cmp resolve v1 d1 s2
            else begin
               match split cmp v2 s1 with
               |  l1', None, r1'  →
                  join (compose cmp resolve l1' l2) v2 d2
                     (compose cmp resolve r1' r2)
               |  l1', Some d, r1'  →
                  begin match resolve d d2 with
```

```
                | None →
                    concat (compose cmp resolve l1′ l2)
                      (compose cmp resolve r1′ r2)
                | Some d →
                    join (compose cmp resolve l1′ l2) v2 d
                      (compose cmp resolve r1′ r2)
              end
          end

let rec union cmp resolve s1 s2 =
  match s1, s2 with
  | Empty, t2 → t2
  | t1, Empty → t1
  | Node (l1, v1, d1, r1, h1), Node (l2, v2, d2, r2, h2) →
        if h1 ≥ h2 then
          if h2 = 1 then
            update cmp (fun o n → resolve n o) v2 d2 s1
          else begin
            match split cmp v1 s2 with
            | l2′, None, r2′ →
                join (union cmp resolve l1 l2′) v1 d1
                  (union cmp resolve r1 r2′)
            | l2′, Some d, r2′ →
                join (union cmp resolve l1 l2′) v1 (resolve d1 d)
                  (union cmp resolve r1 r2′)
          end
        else
          if h1 = 1 then
            update cmp resolve v1 d1 s2
          else begin
            match split cmp v2 s1 with
            | l1′, None, r1′ →
                join (union cmp resolve l1′ l2) v2 d2
                  (union cmp resolve r1′ r2)
            | l1′, Some d, r1′ →
                join (union cmp resolve l1′ l2) v2 (resolve d d2)
                  (union cmp resolve r1′ r2)
          end

let rec iter f = function
  | Empty → ()
  | Node (l, v, d, r, _) → iter f l; f v d; iter f r

let rec map f = function
  | Empty → Empty
  | Node (l, v, d, r, h) → Node (map f l, v, f d, map f r, h)

let rec mapi f = function
  | Empty → Empty
  | Node(l, v, d, r, h) → Node (mapi f l, v, f v d, mapi f r, h)

let rec fold f m accu =
```

```
      match m with
      | Empty → accu
      | Node (l, v, d, r, _) → fold f l (f v d (fold f r accu))

    let rec compare' cmp_k cmp_d l1 l2 =
      match l1, l2 with
      | [], [] → 0
      | [], _ → −1
      | _, [] → 1
      | Empty :: t1, Empty :: t2 → compare' cmp_k cmp_d t1 t2
      | Node (Empty, v1, d1, r1, _) :: t1,
          Node (Empty, v2, d2, r2, _) :: t2 →
          let cv = cmp_k v1 v2 in
          if cv ≠ 0 then begin
            cv
          end else begin
            let cd = cmp_d d1 d2 in
            if cd ≠ 0 then
              cd
            else
              compare' cmp_k cmp_d (r1 :: t1) (r2 :: t2)
          end
      | Node (l1, v1, d1, r1, _) :: t1, t2 →
          compare' cmp_k cmp_d (l1 :: Node (Empty, v1, d1, r1, 0) :: t1) t2
      | t1, Node (l2, v2, d2, r2, _) :: t2 →
          compare' cmp_k cmp_d t1 (l2 :: Node (Empty, v2, d2, r2, 0) :: t2)

    let compare cmp_k cmp_d m1 m2 = compare' cmp_k cmp_d [m1] [m2]

    let rec elements' accu = function
      | Empty → accu
      | Node (l, v, d, r, _) → elements' ((v, d) :: elements' accu r) l

    let elements s =
      elements' [] s

    let canonicalize cmp m =
      fold (add cmp) m empty

  end

module List =
  struct
    type ('key, α) t = ('key × α) list

    let empty = []

    let is_empty = function
      | [] → true
      | _ → false

    let singleton k d = [(k, d)]

    let rec cons cmp resolve k' d' = function
      | [] → [(k', d')]
      | ((k, d) as kd :: rest) as list →
```

```
        let c  =  cmp k' k in
        if c  =  0 then
           match resolve d' d with
           | None  →  rest
           | Some d''  →  (k', d'')  ::  rest
        else if c  <  0 then (∗ k'  <  k ∗)
           (k', d')  ::  list
        else (∗ if c  >  0, i. e. k  <  k' ∗)
           kd  ::  cons cmp resolve k' d' rest

let rec update cmp resolve k' d'  =  function
  | []  →  [(k', d')]
  | ((k, d) as kd  ::  rest) as list →
        let c  =  cmp k' k in
        if c  =  0 then
           (k', resolve d' d)  ::  rest
        else if c  <  0 then (∗ k'  <  k ∗)
           (k', d')  ::  list
        else (∗ if c  >  0, i. e. k  <  k' ∗)
           kd  ::  update cmp resolve k' d' rest

let add cmp k' d' list =
   update cmp (fun n o  →  n) k' d' list

let rec find cmp k'  =  function
  | []  →  raise Not_found
  | (k, d)  ::  rest  →
        let c  =  cmp k' k in
        if c  =  0 then
           d
        else if c  <  0 then (∗ k'  <  k ∗)
           raise Not_found
        else (∗ if c  >  0, i. e. k  <  k' ∗)
           find cmp k' rest

let rec find_opt cmp k'  =  function
  | []  →  None
  | (k, d)  ::  rest  →
        let c  =  cmp k' k in
        if c  =  0 then
           Some d
        else if c  <  0 then (∗ k'  <  k ∗)
           None
        else (∗ if c  >  0, i. e. k  <  k' ∗)
           find_opt cmp k' rest

let choose  =  function
  | []  →  raise Not_found
  | kd  ::  _  →  kd

let rec choose_opt  =  function
  | []  →  None
  | kd  ::  _  →  Some kd
```

```
let uncons = function
  | [] → raise Not_found
  | (k, d) :: rest → (k, d, rest)

let uncons_opt = function
  | [] → None
  | (k, d) :: rest → Some (k, d, rest)

let elements list = list

let rec mem cmp k' = function
  | [] → false
  | (k, d) :: rest →
      let c = cmp k' k in
      if c = 0 then
        true
      else if c < 0 then (* k' < k *)
        false
      else (* if c > 0, i. e. k < k' *)
        mem cmp k' rest

let rec remove cmp k' = function
  | [] → []
  | ((k, d) as kd :: rest) as list →
      let c = cmp k' k in
      if c = 0 then
        rest
      else if c < 0 then (* k' < k *)
        list
      else (* if c > 0, i. e. k < k' *)
        kd :: remove cmp k' rest

let rec compare cmp_k cmp_d m1 m2 =
  match m1, m2 with
  | [], [] → 0
  | [], _ → −1
  | _, [] → 1
  | (k1, d1) :: rest1, (k2, d2) :: rest2 →
      let c = cmp_k k1 k2 in
      if c = 0 then begin
        let c' = cmp_d d1 d2 in
        if c' = 0 then
          compare cmp_k cmp_d rest1 rest2
        else
          c'
      end else
        c

let rec iter f = function
  | [] → ()
  | (k, d) :: rest → f k d; iter f rest

let rec map f = function
  | [] → []
```

```
      | (k, d) :: rest → (k, f d) :: map f rest
  let rec mapi f = function
    | [] → []
    | (k, d) :: rest → (k, f k d) :: mapi f rest

  let rec fold f m accu =
    match m with
    | [] → accu
    | (k, d) :: rest → fold f rest (f k d accu)

  let rec compose cmp resolve m1 m2 =
    match m1, m2 with
    | [], [] → []
    | [], m → m
    | m, [] → m
    | ((k1, d1) as kd1 :: rest1), ((k2, d2) as kd2 :: rest2) →
        let c = cmp k1 k2 in
        if c = 0 then
          match resolve d1 d2 with
          | None → compose cmp resolve rest1 rest2
          | Some d → (k1, d) :: compose cmp resolve rest1 rest2
        else if c < 0 then (* k1 < k2 *)
          kd1 :: compose cmp resolve rest1 m2
        else (* if c > 0, i.e. k2 < k1 *)
          kd2 :: compose cmp resolve m1 rest2

  let rec union cmp resolve m1 m2 =
    match m1, m2 with
    | [], [] → []
    | [], m → m
    | m, [] → m
    | ((k1, d1) as kd1 :: rest1), ((k2, d2) as kd2 :: rest2) →
        let c = cmp k1 k2 in
        if c = 0 then
          (k1, resolve d1 d2) :: union cmp resolve rest1 rest2
        else if c < 0 then (* k1 < k2 *)
          kd1 :: union cmp resolve rest1 m2
        else (* if c > 0, i.e. k2 < k1 *)
          kd2 :: union cmp resolve m1 rest2

  let canonicalize cmp x = x

end
```

J.3 Interface of *Partial*

Partial maps that are constructed from assoc lists.

```
module type T =
  sig
```

The domain of the map. It needs to be compatible with *Map.OrderedType.t*

```
      type domain
```

The codomain α can be anything we want.

```
      type α t
```

A list of argument-value pairs is mapped to a partial map. If an argument appears twice, the later value takes precedence.

```
      val of_list : (domain × α) list → α t
```

Two lists of arguments and values (both must have the same length) are mapped to a partial map. Again the later value takes precedence.

```
      val of_lists : domain list → α list → α t
```

If domain and codomain disagree, we must raise an exception or provide a fallback.

```
      exception Undefined of domain
      val apply : α t → domain → α
      val apply_with_fallback : (domain → α) → α t → domain → α
```

Iff domain and codomain of the map agree, we can fall back to the identity map.

```
      val auto : domain t → domain → domain

  end

module Make : functor (D : Map.OrderedType) → T with type domain = D.t
module Test : sig val suite : OUnit.test end
```

J.4   Implementation of Partial

```
module type T =
  sig
    type domain
    type α t
    val of_list : (domain × α) list → α t
    val of_lists : domain list → α list → α t
    exception Undefined of domain
    val apply : α t → domain → α
    val apply_with_fallback : (domain → α) → α t → domain → α
    val auto : domain t → domain → domain
  end
module Make (D : Map.OrderedType) : T with type domain = D.t =
  struct

    module M = Map.Make (D)

    type domain = D.t
    type α t = α M.t

    let of_list l =
      List.fold_left (fun m (d, v) → M.add d v m) M.empty l
```

```
let of_lists domain values =
  of_list
    (try
        List.map2 (fun d v → (d, v)) domain values
      with
        | Invalid_argument "List.map2" →
          invalid_arg "Partial.of_lists:␣length␣mismatch")

let auto partial d =
  try
    M.find d partial
  with
  | Not_found → d

exception Undefined of domain

let apply partial d =
  try
    M.find d partial
  with
  | Not_found → raise (Undefined d)

let apply_with_fallback fallback partial d =
  try
    M.find d partial
  with
  | Not_found → fallback d
end
```

J.4.1   Unit Tests

```
module Test : sig val suite : OUnit.test end =
  struct

    open OUnit

    module P = Make (struct type t = int let compare = compare end)

    let apply_ok =
      "apply/ok" >::
        (fun () →
          let p = P.of_list [ (0,"a"); (1,"b"); (2,"c") ]
          and l = [ 0; 1; 2 ] in
          assert_equal [ "a"; "b"; "c" ] (List.map (P.apply p) l))

    let apply_ok2 =
      "apply/ok2" >::
        (fun () →
          let p = P.of_lists [0; 1; 2] ["a"; "b"; "c"]
          and l = [ 0; 1; 2 ] in
          assert_equal [ "a"; "b"; "c" ] (List.map (P.apply p) l))

    let apply_shadowed =
```

```
      "apply/shadowed" >::
        (fun () →
          let p = P.of_list [ (0,"a"); (1,"b"); (2,"c"); (1,"d") ]
          and l = [ 0; 1; 2 ] in
          assert_equal [ "a"; "d"; "c" ] (List.map (P.apply p) l))
  let apply_shadowed2 =
      "apply/shadowed2" >::
        (fun () →
          let p = P.of_lists [0; 1; 2; 1] ["a"; "b"; "c"; "d"]
          and l = [ 0; 1; 2 ] in
          assert_equal [ "a"; "d"; "c" ] (List.map (P.apply p) l))
  let apply_mismatch =
      "apply/mismatch" >::
        (fun () →
          assert_raises
            (Invalid_argument "Partial.of_lists:␣length␣mismatch")
            (fun () → P.of_lists [0; 1; 2] ["a"; "b"; "c"; "d"]))
  let suite_apply =
      "apply" >:::
        [apply_ok;
          apply_ok2;
          apply_shadowed;
          apply_shadowed2;
          apply_mismatch]
  let auto_ok =
      "auto/ok" >::
        (fun () →
          let p = P.of_list [ (0, 10); (1, 11)]
          and l = [ 0; 1; 2 ] in
          assert_equal [ 10; 11; 2 ] (List.map (P.auto p) l))
  let suite_auto =
      "auto" >:::
        [auto_ok]
  let apply_with_fallback_ok =
      "apply_with_fallback/ok" >::
        (fun () →
          let p = P.of_list [ (0, 10); (1, 11)]
          and l = [ 0; 1; 2 ] in
          assert_equal
            [ 10; 11; − 2 ] (List.map (P.apply_with_fallback (fun n →
 − n) p) l))
  let suite_apply_with_fallback =
      "apply_with_fallback" >:::
        [apply_with_fallback_ok]
  let suite =
      "Partial" >:::
```

```
      [suite_apply;
       suite_auto;
       suite_apply_with_fallback]
  let time () =
    ()
end
```

—K—
TRIES

From [4], extended for [9].

K.1 Interface of Trie

K.1.1 Monomorphically

```
module type T  =
  sig

    type key
    type (+α) t
    val empty : α t
    val is_empty : α t → bool
```

Standard trie interface:

```
    val add : key → α → α t → α t
    val find : key → α t → α
```

Functionals:

```
    val remove : key → α t → α t
    val mem : key → α t → bool
    val map : (α → β) → α t → β t
    val mapi : (key → α → β) → α t → β t
    val iter : (key → α → unit) → α t → unit
    val fold : (key → α → β → β) → α t → β → β
```

Try to match a longest prefix and return the unmatched rest.

```
    val longest : key → α t → α option × key
```

Try to match a shortest prefix and return the unmatched rest.

```
    val shortest : key → α t → α option × key
```

K.1.2 New in O'Caml 3.08

```
    val compare : (α → α → int) → α t → α t → int
    val equal : (α → α → bool) → α t → α t → bool
```

K.1.3   O'Mega customization

export f _ open f _ close f _ descend f _ match trie allows us to export the trie *trie* as source code to another programming language.

>   val *export* : (*int* → *unit*) → (*int* → *unit*) →
>   (*int* → *key* → *unit*) → (*int* → *key* → α → *unit*) → α *t* → *unit*

  end

O'Caml's *Map.S* prior to Version 3.12:

module type *Map_S* =
  sig
    type *key*
    type (+α) *t*
    val *empty* : α *t*
    val *is_empty* : α *t* → *bool*
    val *add* : *key* → α → α *t* → α *t*
    val *find* : *key* → α *t* → α
    val *remove* : *key* → α *t* → α *t*
    val *mem* : *key* → α *t* → *bool*
    val *iter* : (*key* → α → *unit*) → α *t* → *unit*
    val *map* : (α → β) → α *t* → β *t*
    val *mapi* : (*key* → α → β) → α *t* → β *t*
    val *fold* : (*key* → α → β → β) → α *t* → β → β
    val *compare* : (α → α → *int*) → α *t* → α *t* → *int*
    val *equal* : (α → α → *bool*) → α *t* → α *t* → *bool*
  end

module *Make* (*M* : *Map_S*) : *T* with type *key* = *M.key list*
module *MakeMap* (*M* : *Map_S*) : *Map_S* with type *key* = *M.key list*

K.1.4   Polymorphically

module type *Poly* =
  sig

    type (α, β) *t*
    val *empty* : (α, β) *t*

Standard trie interface:

>   val *add* : (α → α → *int*) → α *list* → β → (α, β) *t* → (α, β) *t*
>   val *find* : (α → α → *int*) → α *list* → (α, β) *t* → β

Functionals:

>   val *remove* : (α → α → *int*) → α *list* → (α, β) *t* → (α, β) *t*
>   val *mem* : (α → α → *int*) → α *list* → (α, β) *t* → *bool*
>   val *map* : (β → γ) → (α, β) *t* → (α, γ) *t*
>   val *mapi* : (α *list* → β → γ) → (α, β) *t* → (α, γ) *t*
>   val *iter* : (α *list* → β → *unit*) → (α, β) *t* → *unit*
>   val *fold* : (α *list* → β → γ → γ) → (α, β) *t* → γ → γ

Try to match a longest prefix and return the unmatched rest.

val *longest* : $(\alpha \to \alpha \to int) \to \alpha\ list \to (\alpha,\ \beta)\ t \to \beta\ option \times \alpha\ list$

Try to match a shortest prefix and return the unmatched rest.

val *shortest* : $(\alpha \to \alpha \to int) \to \alpha\ list \to (\alpha,\ \beta)\ t \to \beta\ option \times \alpha\ list$

K.1.5   O'Mega customization

export f ̲open f ̲close f ̲descend f ̲match trie allows us to export the trie *trie* as source code to another programming language.

val *export* : $(int \to unit) \to (int \to unit) \to$
$(int \to \alpha\ list \to unit) \to (int \to \alpha\ list \to \beta \to unit) \to (\alpha,\ \beta)\ t \to$
unit

   end

module *MakePoly* (M : *Pmap.T*) : *Poly*

K.2   Implementation of Trie

K.2.1   Monomorphically

module type T =
  sig
    type *key*
    type $(+\alpha)\ t$
    val *empty* : $\alpha\ t$
    val *is ̲empty* : $\alpha\ t \to bool$
    val *add* : $key \to \alpha \to \alpha\ t \to \alpha\ t$
    val *find* : $key \to \alpha\ t \to \alpha$
    val *remove* : $key \to \alpha\ t \to \alpha\ t$
    val *mem* : $key \to \alpha\ t \to bool$
    val *map* : $(\alpha \to \beta) \to \alpha\ t \to \beta\ t$
    val *mapi* : $(key \to \alpha \to \beta) \to \alpha\ t \to \beta\ t$
    val *iter* : $(key \to \alpha \to unit) \to \alpha\ t \to unit$
    val *fold* : $(key \to \alpha \to \beta \to \beta) \to \alpha\ t \to \beta \to \beta$
    val *longest* : $key \to \alpha\ t \to \alpha\ option \times key$
    val *shortest* : $key \to \alpha\ t \to \alpha\ option \times key$
    val *compare* : $(\alpha \to \alpha \to int) \to \alpha\ t \to \alpha\ t \to int$
    val *equal* : $(\alpha \to \alpha \to bool) \to \alpha\ t \to \alpha\ t \to bool$
    val *export* : $(int \to unit) \to (int \to unit) \to$
      $(int \to key \to unit) \to (int \to key \to \alpha \to unit) \to \alpha\ t \to unit$

   end

O'Caml's *Map.S* prior to Version 3.12:

module type $Map\text{ ̲}S$ =
  sig
    type *key*

```
    type (+α) t
    val empty :  α t
    val is_empty :  α t  →  bool
    val add :  key  →  α  →  α t  →  α t
    val find :  key  →  α t  →  α
    val remove :  key  →  α t  →  α t
    val mem :  key  →  α t  →  bool
    val iter :  (key  →  α  →  unit)  →  α t  →  unit
    val map :  (α  →  β)  →  α t  →  β t
    val mapi :  (key  →  α  →  β)  →  α t  →  β t
    val fold :  (key  →  α  →  β  →  β)  →  α t  →  β  →  β
    val compare :  (α  →  α  →  int)  →  α t  →  α t  →  int
    val equal :  (α  →  α  →  bool)  →  α t  →  α t  →  bool
  end
```

module *Make* (*M* : *Map_S*) : (*T* with type *key* = *M.key list*) =
  struct

Derived from SML code by Chris Okasaki [4].

```
    type key  =  M.key list

    type α t  =  Trie of α option × α t M.t

    let empty  =  Trie (None, M.empty)

    let is_empty  = function
      | Trie (None, m)  →
          m  =  M.empty (∗ after O'Caml 3.08: M.is_empty m ∗)
      | _  →  false

    let rec add key data trie  =
      match key, trie with
      | [], Trie (_, children)  →  Trie (Some data, children)
      | k :: rest, Trie (node, children)  →
          let t  =  try M.find k children with Not_found  →  empty in
          Trie (node, M.add k (add rest data t) children)

    let rec find key trie  =
      match key, trie with
      | [], Trie (None, _)  →  raise Not_found
      | [], Trie (Some data, _)  →  data
      | k :: rest, Trie (_, children)  →  find rest (M.find k children)
```

The rest is my own fault . . .

```
    let find1 k children  =
      try Some (M.find k children) with Not_found  →  None

    let add_non_empty k t children  =
      if t  =  empty then
        M.remove k children
      else
        M.add k t children

    let rec remove key trie  =
```

```
    match key, trie with
    | [], Trie (_, children) → Trie (None, children)
    | k :: rest, (Trie (node, children) as orig) →
        match find1 k children with
        | None → orig
        | Some t → Trie (node, add_non_empty k (remove rest t) children)
let rec mem key trie =
  match key, trie with
  | [], Trie (None, _) → false
  | [], Trie (Some data, _) → true
  | k :: rest, Trie (_, children) →
      match find1 k children with
      | None → false
      | Some t → mem rest t

let rec map f = function
  | Trie (Some data, children) →
      Trie (Some (f data), M.map (map f) children)
  | Trie (None, children) → Trie (None, M.map (map f) children)

let rec mapi' key f = function
  | Trie (Some data, children) →
      Trie (Some (f key data), descend key f children)
  | Trie (None, children) → Trie (None, descend key f children)
and descend key f = M.mapi (fun k → mapi' (key @ [k]) f)
let mapi f = mapi' [] f

let rec iter' key f = function
  | Trie (Some data, children) → f key data; descend key f children
  | Trie (None, children) → descend key f children
and descend key f = M.iter (fun k → iter' (key @ [k]) f)
let iter f = iter' [] f

let rec fold' key f t acc =
  match t with
  | Trie (Some data, children) → descend key f children (f key data acc)
  | Trie (None, children) → descend key f children acc
and descend key f = M.fold (fun k → fold' (key @ [k]) f)
let fold f t acc = fold' [] f t acc

let rec longest' partial partial_rest key trie =
  match key, trie with
  | [], Trie (data, _) → (data, [])
  | k :: rest, Trie (data, children) →
      match data, find1 k children with
      | None, None → (partial, partial_rest)
      | Some _, None → (data, key)
      | _, Some t → longest' partial partial_rest rest t
let longest key = longest' None key key

let rec shortest' partial partial_rest key trie =
  match key, trie with
  | [], Trie (data, _) → (data, [])
```

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```
      | k :: rest, Trie (Some _ as data, children) → (data, key)
      | k :: rest, Trie (None, children) →
          match find1 k children with
          | None → (partial, partial_rest)
          | Some t → shortest' partial partial_rest rest t
  let shortest key = shortest' None key key
```

K.2.2 O'Mega customization

```
  let rec export' n key f_open f_close f_descend f_match = function
    | Trie (Some data, children) →
        f_match n key data;
        if children ≠ M.empty then
          descend n key f_open f_close f_descend f_match children
    | Trie (None, children) →
        if children ≠ M.empty then begin
          f_descend n key;
          descend n key f_open f_close f_descend f_match children
        end
  and descend n key f_open f_close f_descend f_match children =
    f_open n;
    M.iter (fun k →
      export' (succ n) (k :: key) f_open f_close f_descend f_match) children;
    f_close n

  let export f_open f_close f_descend f_match =
    export' 0 [] f_open f_close f_descend f_match

  let compare _ _ _ =
    failwith "incomplete"

  let equal _ _ _ =
    failwith "incomplete"

  end

module MakeMap (M : Map_S) : (Map_S with type key = M.key list) = Make(M)
```

K.2.3 Polymorphically

```
module type Poly =
  sig
    type (α, β) t
    val empty : (α, β) t
    val add : (α → α → int) → α list → β → (α, β) t → (α, β) t
    val find : (α → α → int) → α list → (α, β) t → β
    val remove : (α → α → int) → α list → (α, β) t → (α, β) t
    val mem : (α → α → int) → α list → (α, β) t → bool
    val map : (β → γ) → (α, β) t → (α, γ) t
    val mapi : (α list → β → γ) → (α, β) t → (α, γ) t
    val iter : (α list → β → unit) → (α, β) t → unit
```

```
      val fold  :  (α list → β → γ → γ) → (α, β) t → γ → γ
      val longest  :  (α → α → int) → α list → (α, β) t → β option × α list
      val shortest  :  (α → α → int) → α list → (α, β) t → β option × α list
      val export  :  (int → unit) → (int → unit) →
        (int → α list → unit) → (int → α list → β → unit) → (α, β) t →
  unit
    end

module MakePoly (M  :  Pmap.T)  :  Poly  =
  struct
```

Derived from SML code by Chris Okasaki [4].

```
    type (α, β) t  =  Trie of β option × (α, (α, β) t) M.t

    let empty  =  Trie (None, M.empty)

    let rec add cmp key data trie  =
      match key, trie with
      | [], Trie (_, children) →  Trie (Some data, children)
      | k :: rest, Trie (node, children) →
          let t  =  try M.find cmp k children with Not_found →  empty in
          Trie (node, M.add cmp k (add cmp rest data t) children)

    let rec find cmp key trie  =
      match key, trie with
      | [], Trie (None, _) →  raise Not_found
      | [], Trie (Some data, _) →  data
      | k :: rest, Trie (_, children) →  find cmp rest (M.find cmp k children)
```

The rest is my own fault . . .

```
    let find1 cmp k children  =
      try Some (M.find cmp k children) with Not_found →  None

    let add_non_empty cmp k t children  =
      if t  =  empty then
        M.remove cmp k children
      else
        M.add cmp k t children

    let rec remove cmp key trie  =
      match key, trie with
      | [], Trie (_, children) →  Trie (None, children)
      | k :: rest, (Trie (node, children) as orig) →
          match find1 cmp k children with
          | None →  orig
          | Some t →  Trie (node, add_non_empty cmp k (remove cmp rest t) children)

    let rec mem cmp key trie  =
      match key, trie with
      | [], Trie (None, _) →  false
      | [], Trie (Some data, _) →  true
      | k :: rest, Trie (_, children) →
          match find1 cmp k children with
          | None →  false
```

       | *Some t* → *mem cmp rest t*

let rec *map f* = function
  | *Trie* (*Some data, children*) →
     Trie (*Some* (*f data*), *M.map* (*map f*) *children*)
  | *Trie* (*None, children*) → *Trie* (*None, M.map* (*map f*) *children*)

let rec *mapi′ key f* = function
  | *Trie* (*Some data, children*) →
     Trie (*Some* (*f key data*), *descend key f children*)
  | *Trie* (*None, children*) → *Trie* (*None, descend key f children*)
and *descend key f* = *M.mapi* (fun *k* → *mapi′* (*key* @ [*k*]) *f*)
let *mapi f* = *mapi′* [] *f*

let rec *iter′ key f* = function
  | *Trie* (*Some data, children*) → *f key data*; *descend key f children*
  | *Trie* (*None, children*) → *descend key f children*
and *descend key f* = *M.iter* (fun *k* → *iter′* (*key* @ [*k*]) *f*)
let *iter f* = *iter′* [] *f*

let rec *fold′ key f t acc* =
  match *t* with
  | *Trie* (*Some data, children*) → *descend key f children* (*f key data acc*)
  | *Trie* (*None, children*) → *descend key f children acc*
and *descend key f* = *M.fold* (fun *k* → *fold′* (*key* @ [*k*]) *f*)
let *fold f t acc* = *fold′* [] *f t acc*

let rec *longest′ cmp partial partial_rest key trie* =
  match *key*, *trie* with
  | [], *Trie* (*data*, _) → (*data*, [])
  | *k* :: *rest*, *Trie* (*data, children*) →
    match *data, find1 cmp k children* with
    | *None, None* → (*partial, partial_rest*)
    | *Some* _, *None* → (*data, key*)
    | _, *Some t* → *longest′ cmp partial partial_rest rest t*
let *longest cmp key* = *longest′ cmp None key key*

let rec *shortest′ cmp partial partial_rest key trie* =
  match *key*, *trie* with
  | [], *Trie* (*data*, _) → (*data*, [])
  | *k* :: *rest*, *Trie* (*Some* _ as *data, children*) → (*data, key*)
  | *k* :: *rest*, *Trie* (*None, children*) →
    match *find1 cmp k children* with
    | *None* → (*partial, partial_rest*)
    | *Some t* → *shortest′ cmp partial partial_rest rest t*
let *shortest cmp key* = *shortest′ cmp None key key*

K.2.4 O'Mega customization

let rec *export′ n key f_open f_close f_descend f_match* = function
  | *Trie* (*Some data, children*) →
    f_match n key data;

```
        if children  ≠  M.empty then
            descend n key f _open f _close f _descend f _match children
    |  Trie (None, children)  →
        if children  ≠  M.empty then begin
            f _descend n key;
            descend n key f _open f _close f _descend f _match children
        end
  and descend n key f _open f _close f _descend f _match children  =
    f _open n;
    M.iter (fun k  →
        export′ (succ n) (k  ::  key) f _open f _close f _descend f _match) children;
    f _close n

  let export f _open f _close f _descend f _match  =
    export′ 0 [] f _open f _close f _descend f _match

end
```

—L—
TENSOR PRODUCTS

From [9].

L.1   Interface of Product

L.1.1   Lists

Since April 2001, we preserve lexicographic ordering.

val $fold2$: $(\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \gamma) \rightarrow \alpha \; list \rightarrow \beta \; list \rightarrow \gamma \rightarrow \gamma$
val $fold3$: $(\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta \rightarrow \delta) \rightarrow \alpha \; list \rightarrow \beta \; list \rightarrow \gamma \; list \rightarrow \delta \rightarrow \delta$
val $fold$: $(\alpha \; list \rightarrow \beta \rightarrow \beta) \rightarrow \alpha \; list \; list \rightarrow \beta \rightarrow \beta$

val $list2$: $(\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \alpha \; list \rightarrow \beta \; list \rightarrow \gamma \; list$
val $list3$: $(\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta) \rightarrow \alpha \; list \rightarrow \beta \; list \rightarrow \gamma \; list \rightarrow \delta \; list$
val $list$: $(\alpha \; list \rightarrow \beta) \rightarrow \alpha \; list \; list \rightarrow \beta \; list$

val $power$: $int \rightarrow \alpha \; list \rightarrow \alpha \; list \; list$

val $thread$: $\alpha \; list \; list \rightarrow \alpha \; list \; list$

L.1.2   Sets

$'a\_set$ is actually $\alpha \; set$ for a suitable set, but this relation can not be expressed polymorphically (in set) in O'Caml. The two sets can be of different type, but we provide a symmetric version as syntactic sugar.

type $\alpha \; set$

type $(\alpha, \; 'a\_set, \; \beta) \; fold = (\alpha \rightarrow \beta \rightarrow \beta) \rightarrow \; 'a\_set \rightarrow \beta \rightarrow \beta$
type $(\alpha, \; 'a\_set, \; \beta, \; 'b\_set, \; \gamma) \; fold2 =$
$\qquad (\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \gamma) \rightarrow \; 'a\_set \rightarrow \; 'b\_set \rightarrow \gamma \rightarrow \gamma$

val $outer$: $(\alpha, \; 'a\_set, \; \gamma) \; fold \rightarrow (\beta, \; 'b\_set, \; \gamma) \; fold \rightarrow$
$\quad (\alpha, \; 'a\_set, \; \beta, \; 'b\_set, \; \gamma) \; fold2$
val $outer\_self$: $(\alpha, \; 'a\_set, \; \beta) \; fold \rightarrow (\alpha, \; 'a\_set, \; \alpha, \; 'a\_set, \; \beta) \; fold2$

L.2   Implementation of Product

L.2.1   Lists

We use the tail recursive *List.fold_left* over *List.fold_right* for efficiency, but revert the argument lists in order to preserve lexicographic ordering. The argument lists are much shorter than the results, so the cost of the *List.rev* is negligible.

```
let fold2_rev f l1 l2 acc =
   List.fold_left (fun acc1 x1 →
     List.fold_left (fun acc2 x2 → f x1 x2 acc2) acc1 l2) acc l1

let fold2 f l1 l2 acc =
   fold2_rev f (List.rev l1) (List.rev l2) acc

let fold3_rev f l1 l2 l3 acc =
   List.fold_left (fun acc1 x1 → fold2 (f x1) l2 l3 acc1) acc l1

let fold3 f l1 l2 l3 acc =
   fold3_rev f (List.rev l1) (List.rev l2) (List.rev l3) acc
```

If all lists have the same type, there's also

```
let rec fold_rev f ll acc =
   match ll with
   | [] → acc
   | [l] → List.fold_left (fun acc' x → f [x] acc') acc l
   | l :: rest →
       List.fold_left (fun acc' x → fold_rev (fun xr → f (x :: xr)) rest acc') acc l

let fold f ll acc = fold_rev f (List.map List.rev ll) acc

let list2 op l1 l2 =
   fold2 (fun x1 x2 c → op x1 x2 :: c) l1 l2 []

let list3 op l1 l2 l3 =
   fold3 (fun x1 x2 x3 c → op x1 x2 x3 :: c) l1 l2 l3 []

let list op ll =
   fold (fun l c → op l :: c) ll []

let power n l =
   list (fun x → x) (ThoList.clone n l)
```

Reshuffling lists:

$$[[a_1; \ldots; a_k]; [b_1; \ldots; b_k]; [c_1; \ldots; c_k]; \ldots] \to [[a_1; b_1; c_1; \ldots]; [a_2; b_2; c_2; \ldots]; \ldots]$$
(L.1)

⊘ *tho* : Is this really an optimal implementation?

```
let thread = function
   | head :: tail →
       List.map List.rev
         (List.fold_left (fun i acc → List.map2 (fun a b → b :: a) i acc)
           (List.map (fun i → [i]) head) tail)
   | [] → []
```

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L.2.2 Sets

The implementation is amazingly simple:

type α *set*

type $(\alpha,\ 'a\_set,\ \beta)\ fold\ =\ (\alpha\ \to\ \beta\ \to\ \beta)\ \to\ 'a\_set\ \to\ \beta\ \to\ \beta$
type $(\alpha,\ 'a\_set,\ \beta,\ 'b\_set,\ \gamma)\ fold2\ =$
    $(\alpha\ \to\ \beta\ \to\ \gamma\ \to\ \gamma)\ \to\ 'a\_set\ \to\ 'b\_set\ \to\ \gamma\ \to\ \gamma$

let *outer fold1 fold2 f l1 l2* $\ =\ $ *fold1* (fun $x1\ \to\ $ *fold2* (*f x1*) *l2*) *l1*
let *outer_self fold f l1 l2* $\ =\ $ *fold* (fun $x1\ \to\ $ *fold* (*f x1*) *l2*) *l1*

—M—
(Fiber) Bundles

M.1   Interface of Bundle

See figure M.1 for the geometric intuition behind the bundle structure.

> Does the current implementation support faithful projections with a forgetful comparison in the base?

```
module type Elt_Base  =
  sig
    type elt
    type base
    val compare_elt  :  elt  →  elt  →  int
```


Figure M.1:   The bundle structure implemented by *Bundle.T*

```
    val compare_base : base → base → int
  end
module type Projection =
  sig
    include Elt_Base
```
$\pi : E \to B$
```
    val pi : elt → base
  end
module type T =
  sig
    type t
    type elt
    type fiber = elt list
    type base
    val add : elt → t → t
    val of_list : elt list → t
```
$\pi : E \to B$
```
    val pi : elt → base
```
$\pi^{-1} : B \to E$
```
    val inv_pi : base → t → fiber
    val base : t → base list
```
$\pi^{-1} \circ \pi$
```
    val fiber : elt → t → fiber
    val fibers : t → (base × fiber) list
  end
module Make (P : Projection) : T with type elt = P.elt and type base = P.base
```

The same thing again, but with a projection that is not hardcoded, but passed as an argument at runtime.

```
module type Dyn =
  sig
    type t
    type elt
    type fiber = elt list
    type base
    val add : (elt → base) → elt → t → t
    val of_list : (elt → base) → elt list → t
    val inv_pi : base → t → fiber
    val base : t → base list
    val fiber : (elt → base) → elt → t → fiber
    val fibers : t → (base × fiber) list
  end
module Dyn (P : Elt_Base) : Dyn with type elt = P.elt and type base = P.base
```

M.2   Implementation of Bundle

```
module type Elt_Base =
  sig
    type elt
    type base
    val compare_elt  :  elt  →  elt  →  int
    val compare_base  :  base  →  base  →  int
  end
module type Dyn  =
  sig
    type t
    type elt
    type fiber  =  elt list
    type base
    val add  :  (elt  →  base)  →  elt  →  t  →  t
    val of_list  :  (elt  →  base)  →  elt list  →  t
    val inv_pi  :  base  →  t  →  fiber
    val base  :  t  →  base list
    val fiber  :  (elt  →  base)  →  elt  →  t  →  fiber
    val fibers  :  t  →  (base  ×  fiber) list
  end
module Dyn  (P  :  Elt_Base)  =
  struct
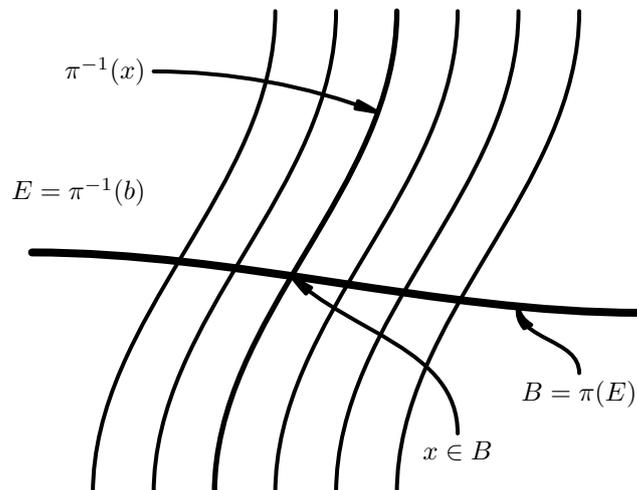
    type elt  =  P.elt
    type base  =  P.base

    type fiber  =  elt list

    module InvPi  =  Map.Make (struct type t  =  P.base let compare  =  P.compare_base end)
    module Fiber  =  Set.Make (struct type t  =  P.elt let compare  =  P.compare_elt end)

    type t  =  Fiber.t InvPi.t

    let add pi element fibers  =
      let base  =  pi element in
      let fiber  =
        try InvPi.find base fibers with Not_found  →  Fiber.empty in
      InvPi.add base (Fiber.add element fiber) fibers

    let of_list pi list =
      List.fold_right (add pi) list InvPi.empty

    let fibers bundle  =
      InvPi.fold
        (fun base fiber acc  →  (base, Fiber.elements fiber) ::  acc) bundle []

    let base bundle  =
      InvPi.fold
        (fun base fiber acc  →  base ::  acc) bundle []

    let inv_pi base bundle  =
```

```
      try
          Fiber.elements (InvPi.find base bundle)
      with
      | Not_found → [ ]

    let fiber pi elt bundle =
      inv_pi (pi elt) bundle

  end

module type Projection =
  sig
    include Elt_Base
    val pi : elt → base
  end

module type T =
  sig
    type t
    type elt
    type fiber = elt list
    type base
    val add : elt → t → t
    val of_list : elt list → t
    val pi : elt → base
    val inv_pi : base → t → fiber
    val base : t → base list
    val fiber : elt → t → fiber
    val fibers : t → (base × fiber) list
  end

module Make (P : Projection) =
  struct

    module D = Dyn (P)

    type elt = D.elt
    type base = D.base
    type fiber = D.fiber
    type t = D.t

    let pi = P.pi

    let add = D.add pi
    let of_list = D.of_list pi
    let base = D.base
    let inv_pi = D.inv_pi
    let fibers = D.fibers

    let fiber elt bundle =
      inv_pi (pi elt) bundle

  end
```

—N—
POWER SETS

N.1   Interface of PowSet

Manipulate the power set, i. e. the set of all subsets, of an set *Ordered_Type*. The concrete order is actually irrelevant, we just need it to construct *Set.S*s in the implementation. In fact, what we are implementating is the *free semilattice* generated from the set of subsets of *Ordered_Type*, where the join operation is the set union.

   The non trivial operation is *basis*, which takes a set of subsets and returns the smallest set of disjoint subsets from which the argument can be reconstructed by forming unions. It is used in O'Mega for finding coarsest partitions of sets of partiticles.

Eventually, this could be generalized from *power set* or *semi lattice* to *lattice* with a notion of subtraction.

module type *Ordered_Type* =
  sig
    type *t*
    val *compare* : *t* → *t* → *int*

Debugging . . .

    val *to_string* : *t* → *string*
  end

module type *T* =
  sig
    type *elt*
    type *t*

    val *empty* : *t*
    val *is_empty* : *t* → *bool*

Set union (a. k. a. join).

    val *union* : *t list* → *t*

Construct the abstract type from a list of subsets represented as lists and the inverse operation.

    val *of_lists* : *elt list list* → *t*

val *to_lists* : *t* → *elt list list*

The smallest set of disjoint subsets that generates the given subset.

val *basis* : *t* → *t*

Debugging ...

val *to_string* : *t* → *string*
  end

module *Make* (*E* : *Ordered_Type*) : *T* with type *elt* = *E.t*

N.2   Implementation of PowSet

module type *Ordered_Type* =
  sig
    type *t*
    val *compare* : *t* → *t* → *int*
    val *to_string* : *t* → *string*
  end

module type *T* =
  sig
    type *elt*
    type *t*
    val *empty* : *t*
    val *is_empty* : *t* → *bool*
    val *union* : *t list* → *t*
    val *of_lists* : *elt list list* → *t*
    val *to_lists* : *t* → *elt list list*
    val *basis* : *t* → *t*
    val *to_string* : *t* → *string*
  end

module *Make* (*E* : *Ordered_Type*) =
  struct

    type *elt* = *E.t*

    module *ESet* = *Set.Make* (*E*)
    type *set* = *ESet.t*

    module *EPowSet* = *Set.Make* (*ESet*)
    type *t* = *EPowSet.t*

    let *empty* = *EPowSet.empty*
    let *is_empty* = *EPowSet.is_empty*

    let *union s_list* =
      List.fold_right EPowSet.union s_list EPowSet.empty

    let *set_to_string set* =
      "{" ^ *String.concat* "," (*List.map E.to_string* (*ESet.elements set*)) ^ "}"

```
let to_string powset =
  "{" ^ String.concat "," (List.map set_to_string (EPowSet.elements powset)) ^ "}"
```

```
let set_of_list list =
  List.fold_right ESet.add list ESet.empty
```

```
let of_lists lists =
  List.fold_right
    (fun list acc → EPowSet.add (set_of_list list) acc)
    lists EPowSet.empty
```

```
let to_lists ps =
  List.map ESet.elements (EPowSet.elements ps)
```

$product\ (s_1, s_2) = s_1 \circ s_2 = \{s_1 \setminus s_2, s_1 \cap s_2, s_2 \setminus s_1\} \setminus \{\emptyset\}$

```
let product s1 s2 =
  List.fold_left
    (fun pset set → if ESet.is_empty set then pset else EPowSet.add set pset)
    EPowSet.empty [ESet.diff s1 s2; ESet.inter s1 s2; ESet.diff s2 s1]
```

```
let disjoint s1 s2 =
  ESet.is_empty (ESet.inter s1 s2)
```

In $augment\_basis\_overlapping\ (s, \{s_i\}_i)$, we are guaranteed that

$$\forall_i : s \cap s_i \neq \emptyset \qquad \text{(N.1a)}$$
$$\forall_{i \neq j} : s_i \cap s_j = \emptyset \,. \qquad \text{(N.1b)}$$

Therefore from (N.1b)

$$\forall_{i \neq j} : (s \cap s_i) \cap (s \cap s_j) = s \cap (s_i \cap s_j) = s \cap \emptyset = \emptyset \qquad \text{(N.2a)}$$
$$\forall_{i \neq j} : (s_i \setminus s) \cap (s_j \setminus s) \subset s_i \cap s_j = \emptyset \qquad \text{(N.2b)}$$
$$\forall_{i \neq j} : (s \setminus s_i) \cap (s_j \setminus s) \subset s \cap \bar{s} = \emptyset \qquad \text{(N.2c)}$$
$$\forall_{i \neq j} : (s \cap s_i) \cap (s_j \setminus s) \subset s \cap \bar{s} = \emptyset \,, \qquad \text{(N.2d)}$$

but in general

$$\exists_{i \neq j} : (s \setminus s_i) \cap (s \setminus s_j) \neq \emptyset \qquad \text{(N.3a)}$$
$$\exists_{i \neq j} : (s \setminus s_i) \cap (s \cap s_j) \neq \emptyset \,, \qquad \text{(N.3b)}$$

because, e.g., for $s_i = \{i\}$ and $s = \{1, 2, 3\}$

$$(s \setminus s_1) \cap (s \setminus s_2) = \{2, 3\} \cap \{1, 3\} = \{3\} \qquad \text{(N.4a)}$$
$$(s \setminus s_1) \cap (s \cap s_2) = \{2, 3\} \cap \{2\} = \{2\} \,. \qquad \text{(N.4b)}$$

Summarizing:

$\forall_{i \neq j} :\ A_i \cap A_j$	$s_j \setminus s$	$s \cap s_j$	$s \setminus s_j$
$s_i \setminus s$	$\emptyset$	$\emptyset$	$\emptyset$
$s \cap s_i$	$\emptyset$	$\emptyset$	$\neq \emptyset$
$s \setminus s_i$	$\emptyset$	$\neq \emptyset$	$\neq \emptyset$

Fortunately, we also know from (N.1a) that

$$\forall_i : |s \setminus s_i| < |s| \tag{N.5a}$$

$$\forall_i : |s \cap s_i| < \min(|s|, |s_i|) \tag{N.5b}$$

$$\forall_i : |s_i \setminus s| < |s_i| \tag{N.5c}$$

and can call *basis* recursively without risking non-termination.

```
let rec basis ps =
  EPowSet.fold augment_basis ps EPowSet.empty

and augment_basis s ps =
  if EPowSet.mem s ps then
    ps
  else
    let no_overlaps, overlaps = EPowSet.partition (disjoint s) ps in
    if EPowSet.is_empty overlaps then
      EPowSet.add s ps
    else
      EPowSet.union no_overlaps (augment_basis_overlapping s overlaps)

and augment_basis_overlapping s ps =
  basis (EPowSet.fold (fun s' → EPowSet.union (product s s')) ps EPowSet.empty)

end
```

—O—
Combinatorics

O.1 Interface of Combinatorics

This type is defined just for documentation. Below, most functions will construct a (possibly nested) *list* of partitions or permutations of a α *seq*.

type α *seq* $=$ α *list*

O.1.1 Simple Combinatorial Functions

The functions

$$factorial : n \to n! \tag{O.1a}$$

$$binomial : (n, k) \to \binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{O.1b}$$

$$multinomial : [n_1; n_2; \ldots; n_k] \to \binom{n_1 + n_2 + \ldots + n_k}{n_1, n_2, \ldots, n_k} = \frac{(n_1 + n_2 + \ldots + n_k)!}{n_1! n_2! \cdots n_k!} \tag{O.1c}$$

have not been optimized. They can quickly run out of the range of native integers.

val *factorial* : *int* $\to$ *int*
val *binomial* : *int* $\to$ *int* $\to$ *int*
val *multinomial* : *int list* $\to$ *int*

symmetry l returns the size of the symmetric group on *l*, i.e. the product of the factorials of the numbers of identical elements.

val *symmetry* : α *list* $\to$ *int*

O.1.2 Partitions

partitions $[n_1; n_2; \ldots; n_k] [x_1; x_2; \ldots; x_n]$, where $n = n_1 + n_2 + \ldots + n_k$, returns all inequivalent partitions of $[x_1; x_2; \ldots; x_n]$ into parts of size n_1, n_2, ..., n_k. The order of the n_i is not respected. There are

$$\frac{1}{S(n_1, n_2, \ldots, n_k)} \binom{n_1 + n_2 + \ldots + n_k}{n_1, n_2, \ldots, n_k} \tag{O.2}$$

such partitions, where the symmetry factor $S(n_1, n_2, \ldots, n_k)$ is the size of the permutation group of $[n_1; n_2; \ldots; n_k]$ as determined by the function *symmetry*.

val *partitions* : *int list* $\to$ α *seq* $\to$ α *seq list list*

ordered_partitions is identical to *partitions*, except that the order of the n_i is respected. There are

$$\binom{n_1 + n_2 + \ldots + n_k}{n_1, n_2, \ldots, n_k} \tag{O.3}$$

such partitions.

val *ordered_partitions* : *int list* $\to$ α *seq* $\to$ α *seq list list*

keystones m l is equivalent to *partitions m l*, except for the special case when the length of l is even and m contains a part that has exactly half the length of l. In this case only the half of the partitions is created that has the head of l in the longest part.

val *keystones* : *int list* $\to$ α *seq* $\to$ α *seq list list*

It can be beneficial to factorize a common part in the partitions and keystones:

val *factorized_partitions* : *int list* $\to$ α *seq* $\to$ (α *seq* $\times$ α *seq list list*) *list*
val *factorized_keystones* : *int list* $\to$ α *seq* $\to$ (α *seq* $\times$ α *seq list list*) *list*

Special Cases

partitions is built from components that can be convenient by themselves, even thepugh they are just special cases of *partitions*.

split k l returns the list of all inequivalent splits of the list l into one part of length k and the rest. There are

$$\frac{1}{S(|l| - k, k)} \binom{|l|}{k} \tag{O.4}$$

such splits. After replacing the pairs by two-element lists, *split k l* is equivalent to *partitions* $[k;\ length\ l\ -\ k]\ l$.

val *split* : *int* $\to$ α *seq* $\to$ (α *seq* $\times$ α *seq*) *list*

Create both equipartitions of lists of even length. There are

$$\binom{|l|}{k} \tag{O.5}$$

such splits. After replacing the pairs by two-element lists, the result of *ordered_split k l* is equivalent to *ordered_partitions* $[k;\ length\ l\ -\ k]\ l$.

val *ordered_split* : *int* $\to$ α *seq* $\to$ (α *seq* $\times$ α *seq*) *list*

multi_split n k l returns the list of all inequivalent splits of the list l into n parts of length k and the rest.

val *multi_split* : *int* $\to$ *int* $\to$ α *seq* $\to$ (α *seq list* $\times$ α *seq*) *list*
val *ordered_multi_split* : *int* $\to$ *int* $\to$ α *seq* $\to$ (α *seq list* $\times$ α *seq*) *list*

O.1.3   Choices

choose n $[x_1; x_2; \ldots; x_n]$ returns the list of all n-element subsets of $[x_1; x_2; \ldots; x_n]$.
choose n is equivalent to $(map\,fst) \circ (ordered\_split\, n)$.

val *choose* : $int \to \alpha\ seq \to \alpha\ seq\ list$

multi_choose $n\ k$ is equivalent to $(map\,fst) \circ (multi\_split\, n\, k)$.

val *multi_choose* : $int \to int \to \alpha\ seq \to \alpha\ seq\ list\ list$
val *ordered_multi_choose* : $int \to int \to \alpha\ seq \to \alpha\ seq\ list\ list$

O.1.4   Permutations

val *permute* : $\alpha\ seq \to \alpha\ seq\ list$

Graded Permutations

val *permute_signed* : $\alpha\ seq \to (int \times \alpha\ seq)\ list$
val *permute_even* : $\alpha\ seq \to \alpha\ seq\ list$
val *permute_odd* : $\alpha\ seq \to \alpha\ seq\ list$

Tensor Products of Permutations

In other words: permutations which respect compartmentalization.

val *permute_tensor* : $\alpha\ seq\ list \to \alpha\ seq\ list\ list$
val *permute_tensor_signed* : $\alpha\ seq\ list \to (int \times \alpha\ seq\ list)\ list$
val *permute_tensor_even* : $\alpha\ seq\ list \to \alpha\ seq\ list\ list$
val *permute_tensor_odd* : $\alpha\ seq\ list \to \alpha\ seq\ list\ list$

Sorting

val *sort_signed* : $(\alpha \to \alpha \to int) \to \alpha\ list \to int \times \alpha\ list$

O.2   Implementation of Combinatorics

type $\alpha\ seq\ =\ \alpha\ list$

O.2.1   Simple Combinatorial Functions

let rec *factorial′ fn n* $=$
  if $n\ <\ 1$ **then**
    fn
  else
    factorial′ $(n\ \times\ fn)\ (pred\ n)$

let *factorial n* $=$
  let *result* $=$ *factorial′* 1 *n* **in**

```
if result < 0 then
  invalid_arg "Combinatorics.factorial␣overflow"
else
  result
```

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1}$$

$$= \frac{n(n-1)\cdots(k+1)}{(n-k)(n-k-1)\cdots 1} = \begin{cases} B_{n-k+1}(n,k) & \text{for } k \leq \lfloor n/2 \rfloor \\ B_{k+1}(n,n-k) & \text{for } k > \lfloor n/2 \rfloor \end{cases} \quad \text{(O.6)}$$

where

$$B_{n_{\min}}(n,k) = \begin{cases} nB_{n_{\min}}(n-1,k) & \text{for } n \geq n_{\min} \\ \frac{1}{k}B_{n_{\min}}(n,k-1) & \text{for } k > 1 \\ 1 & \text{otherwise} \end{cases} \quad \text{(O.7)}$$

```
let rec binomial' n_min n k acc =
  if n ≥ n_min then
    binomial' n_min (pred n) k (n × acc)
  else if k > 1 then
    binomial' n_min n (pred k) (acc / k)
  else
    acc

let binomial n k =
  if k > n / 2 then
    binomial' (k + 1) n (n − k) 1
  else
    binomial' (n − k + 1) n k 1
```

Overflows later, but takes much more time:

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \quad \text{(O.8)}$$

```
let rec slow_binomial n k =
  if n < 0 ∨ k < 0 then
    invalid_arg "Combinatorics.binomial"
  else if k = 0 ∨ k = n then
    1
  else
    slow_binomial (pred n) k + slow_binomial (pred n) (pred k)

let multinomial n_list =
  List.fold_left (fun acc n → acc / (factorial n))
    (factorial (List.fold_left (+) 0 n_list)) n_list

let symmetry l =
  List.fold_left (fun s (n, _) → s × factorial n) 1 (ThoList.classify l)
```

O.2.2   Partitions

The inner steps of the recursion (i. e. $n = 1$) are expanded as follows

$$split'(1, [p_k; p_{k-1}; \ldots; p_1], [x_l; x_{l-1}; \ldots; x_1], [x_{l+1}; x_{l+2}; \ldots; x_m]) =$$
$$[([p_1; \ldots; p_k; x_{l+1}], [x_1; \ldots; x_l; x_{l+2}; \ldots; x_m]);$$
$$([p_1; \ldots; p_k; x_{l+2}], [x_1; \ldots; x_l; x_{l+1}; x_{l+3} \ldots; x_m]); \ldots;$$
$$([p_1; \ldots; p_k; x_m], [x_1; \ldots; x_l; x_{l+1}; \ldots; x_{m-1}])] \quad (O.9)$$

while the outer steps (i. e. $n > 1$) perform the same with one element moved from the last argument to the first argument. At the nth level we have

$$split'(n, [p_k; p_{k-1}; \ldots; p_1], [x_l; x_{l-1}; \ldots; x_1], [x_{l+1}; x_{l+2}; \ldots; x_m]) =$$
$$[([p_1; \ldots; p_k; x_{l+1}; x_{l+2}; \ldots; x_{l+n}], [x_1; \ldots; x_l; x_{l+n+1}; \ldots; x_m]); \ldots;$$
$$([p_1; \ldots; p_k; x_{m-n+1}; x_{m-n+2}; \ldots; x_m], [x_1; \ldots; x_l; x_{l+1}; \ldots; x_{m-n}])] \quad (O.10)$$

where the order of the $[x_1; x_2; \ldots; x_m]$ is maintained in the partitions. Variations on this multiple recursion idiom are used many times below.

```
let rec split' n rev_part rev_head = function
  | [] → []
  | x :: tail →
      let rev_part' = x :: rev_part
      and parts = split' n rev_part (x :: rev_head) tail in
      if n < 1 then
        failwith "Combinatorics.split':␣can't␣happen"
      else if n = 1 then
        (List.rev rev_part', List.rev_append rev_head tail) :: parts
      else
        split' (pred n) rev_part' rev_head tail @ parts
```

Kick off the recursion for $0 < n < |l|$ and handle the cases $n \in \{0, |l|\}$ explicitely. Use reflection symmetry for a small optimization.

```
let ordered_split_unsafe n abs_l l =
  let abs_l = List.length l in
  if n = 0 then
    [[], l]
  else if n = abs_l then
    [l, []]
  else if n ≤ abs_l / 2 then
    split' n [] [] l
  else
    List.rev_map (fun (a, b) → (b, a)) (split' (abs_l − n) [] [] l)
```

Check the arguments and call the workhorse:

```
let ordered_split n l =
  let abs_l = List.length l in
  if n < 0 ∨ n > abs_l then
    invalid_arg "Combinatorics.ordered_split"
  else
```

$ordered\_split\_unsafe\ n\ abs\_l\ l$

Handle equipartitions specially:

```
let split n l =
  let abs_l = List.length l in
  if n < 0 ∨ n > abs_l then
    invalid_arg "Combinatorics.split"
  else begin
    if 2 × n = abs_l then
      match l with
      | [] → failwith "Combinatorics.split:␣can't␣happen"
      | x :: tail →
          List.map (fun (p1, p2) → (x :: p1, p2)) (split' (pred n) [] [] tail)
    else
      ordered_split_unsafe n abs_l l
  end
```

If we chop off parts repeatedly, we can either keep permutations or suppress them. Generically, *attach_to_fst* has type

$$(\alpha \times \beta)\ list \to \alpha\ list \to (\alpha\ list \times \beta)\ list \to (\alpha\ list \times \beta)\ list$$

and semantics

$$attach\_to\_fst([(a_1, b_1), (a_2, b_2), \dots, (a_m, b_m)], [a'_1, a'_2, \dots]) =$$
$$[([a_1, a'_1, \dots], b_1), ([a_2, a'_1, \dots], b_2), \dots, ([a_m, a'_1, \dots], b_m)] \quad \text{(O.11)}$$

(where some of the result can be filtered out), assumed to be prepended to the final argument.

```
let rec multi_split' attach_to_fst n size splits =
  if n ≤ 0 then
    splits
  else
    multi_split' attach_to_fst (pred n) size
      (List.fold_left (fun acc (parts, tail) →
        attach_to_fst (ordered_split size tail) parts acc) [] splits)
```

```
let attach_to_fst_unsorted splits parts acc =
  List.fold_left (fun acc' (p, rest) → (p :: parts, rest) :: acc') acc splits
```

Similarly, if the secod argument is a list of lists:

```
let prepend_to_fst_unsorted splits parts acc =
  List.fold_left (fun acc' (p, rest) → (p @ parts, rest) :: acc') acc splits
```

```
let attach_to_fst_sorted splits parts acc =
  match parts with
  | [] → List.fold_left (fun acc' (p, rest) → ([p], rest) :: acc') acc splits
  | p :: _ as parts →
      List.fold_left (fun acc' (p', rest) →
        if p' > p then
          (p' :: parts, rest) :: acc'
        else
```

$$acc')\ acc\ splits$$

let *multi_split n size l* =
  multi_split' attach_to_fst_sorted n size [([], *l*)]

let *ordered_multi_split n size l* =
  multi_split' attach_to_fst_unsorted n size [([], *l*)]

let rec *partitions' splits* = function
  | [] → *List.map* (fun (*h, r*) → (*List.rev h, r*)) *splits*
  | (1, *size*) :: *more* →
      partitions'
        (*List.fold_left* (fun *acc* (*parts, rest*) →
          attach_to_fst_unsorted (*split size rest*) *parts acc*)
            [] *splits*) *more*
  | (*n, size*) :: *more* →
      partitions'
        (*List.fold_left* (fun *acc* (*parts, rest*) →
          prepend_to_fst_unsorted (*multi_split n size rest*) *parts acc*)
            [] *splits*) *more*

let *partitions multiplicities l* =
  if *List.fold_left* (+) 0 *multiplicities* ≠ *List.length l* then
    invalid_arg "Combinatorics.partitions"
  else
    List.map fst (*partitions'* [([], *l*)]
                      (*ThoList.classify* (*List.sort compare multiplicities*)))

let rec *ordered_partitions' splits* = function
  | [] → *List.map* (fun (*h, r*) → (*List.rev h, r*)) *splits*
  | *size* :: *more* →
      ordered_partitions'
        (*List.fold_left* (fun *acc* (*parts, rest*) →
          attach_to_fst_unsorted (*ordered_split size rest*) *parts acc*)
            [] *splits*) *more*

let *ordered_partitions multiplicities l* =
  if *List.fold_left* (+) 0 *multiplicities* ≠ *List.length l* then
    invalid_arg "Combinatorics.ordered_partitions"
  else
    List.map fst (*ordered_partitions'* [([], *l*)] *multiplicities*)

let *hdtl* = function
  | [] → *invalid_arg* "Combinatorics.hdtl"
  | *h* :: *t* → (*h, t*)

let *factorized_partitions multiplicities l* =
  ThoList.factorize (*List.map hdtl* (*partitions multiplicities l*))

In order to construct keystones (cf. chapter 3), we must eliminate reflectionsc
consistently. For this to work, the lengths of the parts *must not* be reordered
arbitrarily. Ordering with monotonously fallings lengths would be incorrect
however, because then some remainders could fake a reflection symmetry and
partitions would be dropped erroneously. Therefore we put the longest first and
order the remaining with rising lengths:

let *longest_first l* =
  match *ThoList.classify* (*List.sort* (fun *n1 n2* → *compare n2 n1*) *l*) with
  | [] → []
  | *longest* :: *rest* → *longest* :: *List.rev rest*

let *keystones multiplicities l* =
  if *List.fold_left* (+) 0 *multiplicities* ≠ *List.length l* then
    invalid_arg "Combinatorics.keystones"
  else
    List.map fst (*partitions'* [([], *l*)] (*longest_first multiplicities*))

let *factorized_keystones multiplicities l* =
  ThoList.factorize (*List.map hdtl* (*keystones multiplicities l*))

O.2.3   Choices

The implementation is very similar to *split'*, but here we don't have to keep
track of the complements of the chosen sets.

let rec *choose' n rev_choice* = function
  | [] → []
  | *x* :: *tail* →
    let *rev_choice'* = *x* :: *rev_choice*
    and *choices* = *choose' n rev_choice tail* in
    if *n* < 1 then
      failwith "Combinatorics.choose':␣can't␣happen"
    else if *n* = 1 then
      List.rev rev_choice' :: *choices*
    else
      choose' (*pred n*) *rev_choice' tail* @ *choices*

choose n is equivalent to (*List.map fst*) ∘ (*split_ordered n*), but more efficient.

let *choose n l* =
  let *abs_l* = *List.length l* in
  if *n* < 0 then
    invalid_arg "Combinatorics.choose"
  else if *n* > *abs_l* then
    []
  else if *n* = 0 then
    [[]]
  else if *n* = *abs_l* then
    [*l*]
  else
    choose' n [] *l*

let *multi_choose n size l* =
  List.map fst (*multi_split n size l*)

let *ordered_multi_choose n size l* =
  List.map fst (*ordered_multi_split n size l*)

O.2.4   Permutations

let rec *insert x* = function
  | [] → [[x]]
  | *h* :: *t* as *l* →
      (x :: l) :: *List.rev_map* (fun *l'* → *h* :: *l'*) (*insert x t*)

let *permute l* =
  List.fold_left (fun *acc x* → *ThoList.rev_flatmap* (*insert x*) *acc*) [[]] *l*

Graded Permutations

let rec *insert_signed x* = function
  | (*eps*, []) → [(*eps*, [x])]
  | (*eps*, *h* :: *t*) → (*eps*, *x* :: *h* :: *t*) ::
      (*List.map* (fun (*eps'*, *l'*) → (−*eps'*, *h* :: *l'*)) (*insert_signed x* (*eps*, *t*)))

let rec *permute_signed'* = function
  | (*eps*, []) → [(*eps*, [])]
  | (*eps*, *h* :: *t*) → *ThoList.flatmap* (*insert_signed h*) (*permute_signed'* (*eps*, *t*))

let *permute_signed l* =
  permute_signed' (1, *l*)

The following are wasting at most a factor of two and there's probably no point in improving on this ...

let *filter_sign s l* =
  List.map snd (*List.filter* (fun (*eps*, _) → *eps* = *s*) *l*)

let *permute_even l* =
  filter_sign 1 (*permute_signed l*)

let *permute_odd l* =
  filter_sign (−1) (*permute_signed l*)

Tensor Products of Permutations

let *permute_tensor ll* =
  Product.list (fun *l* → *l*) (*List.map permute ll*)

let *join_signs l* =
  let *el*, *pl* = *List.split l* in
  (*List.fold_left* (fun *acc x* → *x* × *acc*) 1 *el*, *pl*)

let *permute_tensor_signed ll* =
  Product.list join_signs (*List.map permute_signed ll*)

let *permute_tensor_even l* =
  filter_sign 1 (*permute_tensor_signed l*)

let *permute_tensor_odd l* =
  filter_sign (−1) (*permute_tensor_signed l*)

let *insert_inorder_signed order x* (*eps*, *l*) =

```
let rec insert eps' accu = function
  | [] → (eps × eps', List.rev_append accu [x])
  | h :: t →
      if order x h = 0 then
        invalid_arg
          "Combinatorics.insert_inorder_signed:␣identical␣elements"
      else if order x h < 0 then
        (eps × eps', List.rev_append accu (x :: h :: t))
      else
        insert (−eps') (h :: accu) t
in
insert 1 [] l
```

<div align="center">

Sorting

</div>

```
let sort_signed order l =
  List.fold_left (fun acc x → insert_inorder_signed order x acc) (1, []) l
```

O.3   Interface of *Permutation*

```
module type T =
  sig
    type t
    val of_list : int list → t
    val of_array : int array → t
    val inverse : t → t
    val compose : t → t → t
    val list : t → α list → α list
    val array : t → α array → α array
  end
module Using_Lists : T
module Using_Arrays : T

module Default : T

module Test : functor (P : T) →
  sig val suite : OUnit.test val time : unit → unit end
```

O.4   Implementation of *Permutation*

```
module type T =
  sig
    type t
    val of_list : int list → t
    val of_array : int array → t
    val inverse : t → t
    val compose : t → t → t
    val list : t → α list → α list
```

```
    val array : t → α array → α array
  end
module Using_Lists : T =
  struct

    type t = int list

    let of_list p =
      if List.sort compare p ≠ (ThoList.range 0 (List.length p − 1)) then
        invalid_arg "Permutation.of_list"
      else
        p

    let of_array p =
      try
        of_list (Array.to_list p)
      with
      | Invalid_argument "Permutation.of_list" →
        invalid_arg "Permutation.of_array"

    let inverse p = snd (ThoList.ariadne_sort p)

    let list p l =
      List.map snd
        (List.sort compare
          (try
              List.rev_map2 (fun i x → (i, x)) p l
            with
            | Invalid_argument "List.rev_map2" →
              invalid_arg "Permutation.list:␣length␣mismatch"))

    let array p a =
      try
        Array.of_list (list p (Array.to_list a))
      with
      | Invalid_argument "Permutation.list:␣length␣mismatch" →
        invalid_arg "Permutation.array:␣length␣mismatch"
```

Probably not optimal (or really inefficient), but correct by associativity.

```
    let compose p q =
      list (inverse q) p

  end
module Using_Arrays : T =
  struct

    type t = int array

    let of_list p =
      if List.sort compare p ≠ (ThoList.range 0 (List.length p − 1)) then
        invalid_arg "Permutation.of_list"
      else
        Array.of_list p
```

```
let of_array p =
  try
    of_list (Array.to_list p)
  with
  | Invalid_argument "Permutation.of_list" →
    invalid_arg "Permutation.of_array"

  let inverse p =
  let len_p = Array.length p in
  let p' = Array.make len_p p.(0) in
  for i = 0 to pred len_p do
    p'.(p.(i)) ← i
  done;
  p'

let array p a =
  let len_a = Array.length a
  and len_p = Array.length p in
  if len_a ≠ len_p then
    invalid_arg "Permutation.array:␣length␣mismatch";
  let a' = Array.make len_a a.(0) in
  for i = 0 to pred len_a do
    a'.(p.(i)) ← a.(i)
  done;
  a'

let list p l =
  try
    Array.to_list (array p (Array.of_list l))
  with
  | Invalid_argument "Permutation.array:␣length␣mismatch" →
    invalid_arg "Permutation.list:␣length␣mismatch"

let compose p q =
  array (inverse q) p

  end
```

module *Default* = *Using_Arrays*

To shuffle an array a of n elements (indices 0..n-1):
   for i from n 1 downto 1 do j random integer with 0 j i exchange aj and ai
   To initialize an array a of n elements to a randomly shuffled copy of source,
both 0-based:
   a0 source0 for i from 1 to n 1 do j random integer with 0 j i ai aj aj
source*i*

```
let shuffle l =
  let a = Array.of_list l in
  for n = Array.length a − 1 downto 1 do
    let k = Random.int (succ n) in
    if k ≠ n then
      let tmp = Array.get a n in
      Array.set a n (Array.get a k);
```

```
        Array.set a k tmp
   done;
   Array.to_list a

let time f x =
  let start = Sys.time () in
  let f_x = f x in
  let stop = Sys.time () in
  (f_x, stop − . start)

let print_time msg f x =
  let f_x, seconds = time f x in
  Printf.printf "%s␣took␣%10.2f␣ms\n" msg (seconds ∗. 1000.);
  f_x

module Test (P : T) : sig val suite : OUnit.test val time : unit → unit end =
  struct

    open OUnit
    open P

    let of_list_overlap =
      "overlap" >::
        (fun () →
          assert_raises (Invalid_argument "Permutation.of_list")
            (fun () →
              of_list [0; 1; 2; 2]))

    let of_list_gap =
      "gap" >::
        (fun () →
          assert_raises (Invalid_argument "Permutation.of_list")
            (fun () →
              of_list [0; 1; 2; 4; 5]))

    let of_list_ok =
      "ok" >::
        (fun () →
          let l = ThoList.range 0 10 in
          assert_equal (of_list l) (of_list l))

    let suite_of_list =
      "of_list" >:::
        [of_list_overlap;
         of_list_gap;
         of_list_ok]

    let apply_invalid_lengths =
      "invalid/lengths" >::
        (fun () →
          assert_raises
            (Invalid_argument "Permutation.list:␣length␣mismatch")
            (fun () →
              list (of_list [0; 1; 2; 3; 4]) [0; 1; 2; 3]))
```

```
let apply_ok =
  "ok" >::
    (fun () →
      assert_equal [2; 0; 1; 3; 5; 4]
        (list (of_list [1; 2; 0; 3; 5; 4]) [0; 1; 2; 3; 4; 5]))

let suite_apply =
  "apply" >:::
    [apply_invalid_lengths;
      apply_ok]

let inverse_ok =
  "ok" >::
    (fun () →
      let l = shuffle (ThoList.range 0 1000) in
      let p = of_list (shuffle l) in
      assert_equal l (list (inverse p) (list p l)))

let suite_inverse =
  "inverse" >:::
    [inverse_ok]

let compose_ok =
  "ok" >::
    (fun () →
      let id = ThoList.range 0 1000 in
      let p = of_list (shuffle id)
      and q = of_list (shuffle id)
      and l = id in
      assert_equal (list p (list q l)) (list (compose p q) l))

let compose_inverse_ok =
  "inverse/ok" >::
    (fun () →
      let id = ThoList.range 0 1000 in
      let p = of_list (shuffle id)
      and q = of_list (shuffle id) in
      assert_equal
        (compose (inverse p) (inverse q))
        (inverse (compose q p)))

let suite_compose =
  "compose" >:::
    [compose_ok;
      compose_inverse_ok]

let suite =
  "Permutations" >:::
    [suite_of_list;
      suite_apply;
      suite_inverse;
      suite_compose]

let repeat repetitions size =
```

```
    let id  =  ThoList.range 0 size in
    let p  =  of_list (shuffle id)
    and l  =  shuffle (List.map string_of_int id) in
    print_time (Printf.sprintf "reps=%d,␣len=%d" repetitions size)
      (fun ()  →
        for i  =  1 to repetitions do
          ignore (P.list p l)
        done)
      ()

  let time ()  =
    repeat 100000 10;
    repeat 10000 100;
    repeat 1000 1000;
    repeat 100 10000;
    repeat 10 100000;
    ()

end
```

—P—
PARTITIONS

P.1   Interface of Partition

pairs n n1 n2 returns all (unordered) pairs of integers with the sum n in the range from *n1* to *n2*.

val *pairs* : *int* → *int* → *int* → (*int* × *int*) *list*
val *triples* : *int* → *int* → *int* → (*int* × *int* × *int*) *list*

tuples d n n_min n_max returns all $[n_1; n_2; \ldots; n_d]$ with $n_{\min} \le n_1 \le n_2 \le \ldots \le n_d \le n_{\max}$ and

$$\sum_{i=1}^{d} n_i = n \qquad\qquad (\text{P.1})$$

val *tuples* : *int* → *int* → *int* → *int* → *int list list*

val *rcs* : *RCS.t*

P.2   Implementation of Partition

let *rcs* = *RCS.parse* "Partition" ["Partitions"]
   { *RCS.revision* = "\$Revision:␣6465␣\$";
     RCS.date = "\$Date:␣2015-01-10␣16:22:31␣+0100␣(Sat,␣10␣Jan␣2015)␣\$";
     RCS.author = "\$Author:␣jr_reuter␣\$";
     RCS.source
       = "\$URL:␣svn+ssh://jr_reuter@login.hepforge.org/hepforge/svn/whizard/trunk/omega/sr

All unordered pairs of integers with the same sum n in a given range $\{n_1, \ldots, n_2\}$:

$$pairs : (n, n_1, n_2) \rightarrow \big\{(i, j) \,|\, i + j = n \wedge n_1 \le i \le j \le n_2\big\} \qquad (\text{P.2})$$

let rec *pairs′ acc n1 n2* =
  if *n1* > *n2* then
    List.rev acc
  else
    pairs′ ((*n1*, *n2*) :: *acc*) (*succ n1*) (*pred n2*)

let *pairs sum min_n1 max_n2* =
  let *n1* = *max min_n1* (*sum* − *max_n2*) in

```
    let n2 = sum − n1 in
    if n2 ≤ max_n2 then
        pairs′ [] n1 n2
    else
        []

let rec tuples d sum n_min n_max =
    if d ≤ 0 then
        invalid_arg "tuples"
    else if d > 1 then
        tuples′ d sum n_min n_max n_min
    else if sum ≥ n_min ∧ sum ≤ n_max then
        [[sum]]
    else
        []

and tuples′ d sum n_min n_max n =
    if n > n_max then
        []
    else
        List.fold_right (fun l ll → (n :: l) :: ll)
            (tuples (pred d) (sum − n) (max n_min n) n_max)
            (tuples′ d sum n_min n_max (succ n))
```

> When I find a little spare time, I can provide a dedicated implementation,
> but we *know* that *Impossible* is *never* raised and the present approach is
> just as good (except for a possible tiny inefficiency).

```
exception Impossible of string
let impossible name = raise (Impossible name)

let triples sum n_min n_max =
    List.map (function [n1; n2; n3] → (n1, n2, n3) | _ → impossible "triples")
        (tuples 3 sum n_min n_max)
```

$$\text{---Q---}$$

TREES

From [10]: Trees with one root admit a straightforward recursive definition

$$T(N, L) = L \cup N \times T(N, L) \times T(N, L) \tag{Q.1}$$

that is very well adapted to mathematical reasoning. Such recursive definitions are useful because they allow us to prove properties of elements by induction

$$\forall l \in L : p(l) \wedge (\forall n \in N : \forall t_1, t_2 \in T(N, L) : p(t_1) \wedge p(t_2) \Rightarrow p(n \times t_1 \times t_2))$$
$$\implies \forall t \in T(N, L) : p(t) \tag{Q.2}$$

i. e. establishing a property for all leaves and showing that a node automatically satisfies the property if it is true for all children proves the property for *all* trees. This induction is of course modelled after standard mathematical induction

$$p(1) \wedge (\forall n \in \mathbf{N} : p(n) \Rightarrow p(n+1)) \implies \forall n \in \mathbf{N} : p(n) \tag{Q.3}$$

The recursive definition (Q.1) is mirrored by the two tree construction functions[1]

$$leaf : \nu \times \lambda \to (\nu, \lambda)T \tag{Q.4a}$$
$$node : \nu \times (\nu, \lambda)T \times (\nu, \lambda)T \to (\nu, \lambda)T \tag{Q.4b}$$

Renaming leaves and nodes leaves the structure of the tree invariant. Therefore, morphisms $L \to L'$ and $N \to N'$ of the sets of leaves and nodes induce natural homomorphisms $T(N, L) \to T(N', L')$ of trees

$$map : \ (\nu \to \nu') \times (\lambda \to \lambda') \times (\nu, \lambda)T \to (\nu', \lambda')T \tag{Q.5}$$

The homomorphisms constructed by *map* are trivial, but ubiquitous. More interesting are the morphisms

$$fold : (\nu \times \lambda \to \alpha) \times (\nu \times \alpha \times \alpha \to \alpha) \times (\nu, \lambda)T \to \alpha$$
$$(f_1, f_2, l \in L) \mapsto f_1(l) \tag{Q.6}$$
$$(f_1, f_2, (n, t_1, t_2)) \mapsto f_2(n, fold(f_1, f_2, t_1), fold(f_1, f_2, t_2))$$

---

[1]To make the introduction more accessible to non-experts, I avoid the 'curried' notation for functions with multiple arguments and use tuples instead. The actual implementation takes advantage of curried functions, however. Experts can read $\alpha \to \beta \to \gamma$ for $\alpha \times \beta \to \gamma$.

and

$$fan : (\nu \times \lambda \to \{\alpha\}) \times (\nu \times \alpha \times \alpha \to \{\alpha\}) \times (\nu, \lambda)T \to \{\alpha\}$$
$$(f_1, f_2, l \in L) \mapsto f_1(l) \qquad \text{(Q.7)}$$
$$(f_1, f_2, (n, t_1, t_2)) \mapsto f_2(n, fold(f_1, f_2, t_1) \otimes fold(f_1, f_2, t_2))$$

where the tensor product notation means that f_2 is applied to all combinations of list members in the argument:

$$\phi(\{x\} \otimes \{y\}) = \{\phi(x, y) | x \in \{x\} \wedge y \in \{y\}\} \qquad \text{(Q.8)}$$

But note that due to the recursive nature of trees, *fan* is *not* a morphism from $T(N, L)$ to $T(N \otimes N, L)$.

If we identify singleton sets with their members, *fold* could be viewed as a special case of *fan*, but that is probably more confusing than helpful. Also, using the special case $\alpha = (\nu', \lambda')T$, the homomorphism *map* can be expressed in terms of *fold* and the constructors

$$map : (\nu \to \nu') \times (\lambda \to \lambda') \times (\nu, \lambda)T \to (\nu', \lambda')T$$
$$(f, g, t) \mapsto fold(leaf \circ (f \times g), node \circ (f \times id \times id), t) \qquad \text{(Q.9)}$$

fold is much more versatile than *map*, because it can be used with constructors for other tree representations to translate among different representations. The target type can also be a mathematical expression. This is used extensively below for evaluating Feynman diagrams.

Using *fan* with $\alpha = (\nu', \lambda')T$ can be used to construct a multitude of homomorphic trees. In fact, below it will be used extensively to construct all Feynman diagrams $\{(\nu, \{p_1, \ldots, p_n\})T\}$ of a given topology $t \in (\emptyset, \{1, \ldots, n\})T$.

The physicist in me guesses that there is another morphism of trees that is related to *fan* like a Lie-algebra is related to the it's Lie-group. I have not been able to pin it down, but I guess that it is a generalization of *grow* below.

Q.1   Interface of *Tree*

This module provides utilities for generic decorated trees, such as FeynMF output.

Q.1.1   Abstract Data Type

type $(\nu,\ \lambda)\ t$

leaf n l returns a tree consisting of a single leaf node of type n with a label l.

val *leaf* $:\ \nu\ \to\ \lambda\ \to\ (\nu,\ \lambda)\ t$

cons n ch returns a tree node.

val *cons* $:\ \nu\ \to\ (\nu,\ \lambda)\ t\ list\ \to\ (\nu,\ \lambda)\ t$

Note that *cons node* [] constructs a terminal node, but *not* a leaf, since the latter *must* have a label!

⚡ This approach was probably tailored to Feynman diagrams, where we have external propagators as nodes with additional labels (cf. the function *to_feynmf* on page 875 below). I'm not so sure anymore that this was a good choice.

node t returns the top node of the tree *t*.

val *node* : $(\nu, \lambda)\ t\ \rightarrow\ \nu$

leafs t returns a list of all leaf labels *in order*.

val *leafs* : $(\nu, \lambda)\ t\ \rightarrow\ \lambda\ list$

nodes t returns a list of all nodes that are not leafs in post-order. This guarantees that the root node can be stripped from the result by *List.tl*.

val *nodes* : $(\nu, \lambda)\ t\ \rightarrow\ \nu\ list$

fuse conjg root contains_root trees joins the *trees*, using the leaf *root* in one of the trees as root of the new tree. *contains_root* guides the search for the subtree containing *root* as a leaf. fun $t\ \rightarrow\ List.mem\ root\ (leafs\ t)$ is acceptable, but more efficient solutions could be available in special circumstances.

val *fuse* : $(\nu\ \rightarrow\ \nu)\ \rightarrow\ \lambda\ \rightarrow\ ((\nu, \lambda)\ t\ \rightarrow\ bool)\ \rightarrow\ (\nu, \lambda)\ t\ list \rightarrow\ (\nu, \lambda)\ t$

sort lesseq t return a sorted copy of the tree *t*: node labels are ignored and nodes are according to the supremum of the leaf labels in the corresponding subtree.

val *sort* : $(\lambda\ \rightarrow\ \lambda\ \rightarrow\ bool)\ \rightarrow\ (\nu, \lambda)\ t\ \rightarrow\ (\nu, \lambda)\ t$
val *canonicalize* : $(\nu, \lambda)\ t\ \rightarrow\ (\nu, \lambda)\ t$

Q.1.2   Homomorphisms

val *map* : $('n1\ \rightarrow\ 'n2)\ \rightarrow\ ('l1\ \rightarrow\ 'l2)\ \rightarrow\ ('n1, 'l1)\ t\ \rightarrow\ ('n2, 'l2)\ t$
val *fold* : $(\nu\ \rightarrow\ \lambda\ \rightarrow\ \alpha)\ \rightarrow\ (\nu\ \rightarrow\ \alpha\ list\ \rightarrow\ \alpha)\ \rightarrow\ (\nu, \lambda)\ t\ \rightarrow\ \alpha$
val *fan* : $(\nu\ \rightarrow\ \lambda\ \rightarrow\ \alpha\ list)\ \rightarrow\ (\nu\ \rightarrow\ \alpha\ list\ \rightarrow\ \alpha\ list)\ \rightarrow$
 $(\nu, \lambda)\ t\ \rightarrow\ \alpha\ list$

Q.1.3   Output

val *to_string* : $(string, string)\ t\ \rightarrow\ string$

Feynmf

⚡ *style* : $(string \times string)\ option$ should be replaced by *style* : *string option*; *tex_label* : *string option*

type *feynmf* =
    { *style* : $(string \times string)\ option$;
      rev : *bool*;
      label : *string option*;
      tension : *float option* }

val *vanilla* : *feynmf*
val *sty* : (*string* × *string*) × *bool* × *string* → *feynmf*

to_feynmf file to_string incoming t write the trees in the list *t* to the file named *file*. The leaves *incoming* are used as incoming particles and *to_string* is use to convert leaf labels to L^AT_EX-strings.

type λ *feynmf_set* =
  { *header* : *string*;
    incoming : λ *list*;
    diagrams : (*feynmf*, λ) *t list* }

type (λ, μ) *feynmf_sets* =
  { *outer* : λ *feynmf_set*;
    inner : μ *feynmf_set list* }

val *feynmf_sets_plain* : *bool* → *int* → *string* →
  (λ → *string*) → (λ → *string*) →
  (μ → *string*) → (μ → *string*) → (λ, μ) *feynmf_sets list* → *unit*

val *feynmf_sets_wrapped* : *bool* → *string* →
  (λ → *string*) → (λ → *string*) →
  (μ → *string*) → (μ → *string*) → (λ, μ) *feynmf_sets list* → *unit*

If the diagrams at all levels are of the same type, we can recurse to arbitrary depth.

type λ *feynmf_levels* =
  { *this* : λ *feynmf_set*;
    lower : λ *feynmf_levels list* }

to_feynmf_levels_plain sections level file wf_to_TeX p_to_TeX levels ...

val *feynmf_levels_plain* : *bool* → *int* → *string* →
  (λ → *string*) → (λ → *string*) → λ *feynmf_levels list* → *unit*

to_feynmf_levels_wrapped file wf_to_TeX p_to_TeX levels ...

val *feynmf_levels_wrapped* : *string* →
  (λ → *string*) → (λ → *string*) → λ *feynmf_levels list* → *unit*

Least Squares Layout

A general graph with edges of type ε, internal nodes of type ν, and external nodes of type *'ext*.

type (ε, ν, *'ext*) *graph*
val *graph_of_tree* : (ν → ν → ε) → (ν → ν) →
  ν → (ν, ν) *t* → (ε, ν, ν) *graph*

A general graph with the layout of the external nodes fixed.

type (ε, ν, *'ext*) *ext_layout*
val *left_to_right* : *int* → (ε, ν, *'ext*) *graph* → (ε, ν, *'ext*) *ext_layout*

A general graph with the layout of all nodes fixed.

type (ε, ν, *'ext*) *layout*
val *layout* : (ε, ν, *'ext*) *ext_layout* $\rightarrow$ (ε, ν, *'ext*) *layout*

val *dump* : (ε, ν, *'ext*) *layout* $\rightarrow$ *unit*
val *iter_edges* : (ε $\rightarrow$ *float* $\times$ *float* $\rightarrow$ *float* $\times$ *float* $\rightarrow$ *unit*) $\rightarrow$
  (ε, ν, *'ext*) *layout* $\rightarrow$ *unit*
val *iter_internal* : (*float* $\times$ *float* $\rightarrow$ *unit*) $\rightarrow$
  (ε, ν, *'ext*) *layout* $\rightarrow$ *unit*
val *iter_incoming* : (*'ext* $\times$ *float* $\times$ *float* $\rightarrow$ *unit*) $\rightarrow$
  (ε, ν, *'ext*) *layout* $\rightarrow$ *unit*
val *iter_outgoing* : (*'ext* $\times$ *float* $\times$ *float* $\rightarrow$ *unit*) $\rightarrow$
  (ε, ν, *'ext*) *layout* $\rightarrow$ *unit*

Q.2   *Implementation of Tree*

Q.2.1   *Abstract Data Type*

type (ν, λ) *t* =
  | *Leaf* of ν $\times$ λ
  | *Node* of ν $\times$ (ν, λ) *t list*

let *leaf* *n* *l* = *Leaf* (*n*, *l*)

let *cons* *n* *children* = *Node* (*n*, *children*)

Presenting the leafs *in order* comes naturally, but will be useful below.

let rec *leafs* = function
  | *Leaf* (_, *l*) $\rightarrow$ [*l*]
  | *Node* (_, *ch*) $\rightarrow$ *ThoList.flatmap leafs ch*

let *node* = function
  | *Leaf* (*n*, _) $\rightarrow$ *n*
  | *Node* (*n*, _) $\rightarrow$ *n*

This guarantees that the root node can be stripped from the result by *List.tl*.

let rec *nodes* = function
  | *Leaf* _ $\rightarrow$ []
  | *Node* (*n*, *ch*) $\rightarrow$ *n* :: *ThoList.flatmap nodes ch*

first_match *p* *list* returns $(x, list')$, where x is the first element of *list* for which
p *x* = true and *list'* is *list* sans x.

let *first_match* *p* *list* =
  let rec *first_match'* *no_match* = function
    | [] $\rightarrow$ *invalid_arg* "Tree.fuse:␣prospective␣root␣not␣found"
    | *t* :: *rest* when *p* *t* $\rightarrow$ (*t*, *List.rev_append no_match rest*)
    | *t* :: *rest* $\rightarrow$ *first_match'* (*t* :: *no_match*) *rest* in
  first_match' [] *list*

One recursion step in *fuse′* rotates the topmost tree node, moving the prospective root up:

(Q.10)

let *fuse conjg root contains_root trees* =
  let rec *fuse′ subtrees* =
    match *first_match contains_root subtrees* with

If the prospective root is contained in a leaf, we have either found the root—in which case we're done—or have failed catastrophically:

    | *Leaf* (*n*, *l*), *children* →
      if *l* = *root* then
        Node (*conjg n*, *children*)
      else
        invalid_arg `"Tree.fuse:␣root␣predicate␣inconsistent"`

Otherwise, we perform a rotation as in (Q.10) and connect all nodes that do not contain the root to a new node. For efficiency, we append the new node at the end and prevent *first_match* from searching for the root in it in vain again. Since *root_children* is probably rather short, this should be a good strategy.

    | *Node* (*n*, *root_children*), *other_children* →
      fuse′ (*root_children* @ [*Node* (*conjg n*, *other_children*)]) in
  fuse′ trees

Sorting is also straightforward, we only have to keep track of the suprema of the subtrees:

type (*α*, *β*) *with_supremum* = { *sup* : *α*; *data* : *β* }

Since the lists are rather short, *Sort.list* could be replaced by an optimized version, but we're not (yet) dealing with the most important speed bottleneck here:

let rec *sort′ lesseq* = function
  | *Leaf* (_, *l*) as *e* → { *sup* = *l*; *data* = *e* }
  | *Node* (*n*, *ch*) →
    let *ch′* = *Sort.list*
       (fun *x y* → *lesseq x.sup y.sup*) (*List.map* (*sort′ lesseq*) *ch*) in
    { *sup* = (*List.hd* (*List.rev ch′*)).*sup*;
      data = *Node* (*n*, *List.map* (fun *x* → *x.data*) *ch′*) }

finally, throw away the overall supremum:

let *sort lesseq t* = (*sort′ lesseq t*).*data*

```
let rec canonicalize = function
  | Leaf (_, _) as l → l
  | Node (n, ch) →
    Node (n, List.sort compare (List.map canonicalize ch))
```

Q.2.2   Homomorphisms

Isomophisms are simple:

```
let rec map fn fl = function
  | Leaf (n, l) → Leaf (fn n, fl l)
  | Node (n, ch) → Node (fn n, List.map (map fn fl) ch)
```

homomorphisms are not more complicated:

```
let rec fold leaf node = function
  | Leaf (n, l) → leaf n l
  | Node (n, ch) → node n (List.map (fold leaf node) ch)
```

and tensor products are fun:

```
let rec fan leaf node = function
  | Leaf (n, l) → leaf n l
  | Node (n, ch) → Product.fold
        (fun ch' t → node n ch' @ t) (List.map (fan leaf node) ch) []
```

Q.2.3   Output

```
let leaf_to_string n l =
  if n = "" then
    l
  else if l = "" then
    n
  else
    n ^ "(" ^ l ^ ")"

let node_to_string n ch =
  "(" ^ (if n = "" then "" else n ^ ":") ^ (String.concat "," ch) ^ ")"

let to_string t =
  fold leaf_to_string node_to_string t
```

Feynmf

Add a value that is greater than all suprema

```
type α supremum_or_infinity = Infinity | Sup of α

type (α, β) with_supremum_or_infinity =
    { sup : α supremum_or_infinity; data : β }

let with_infinity lesseq x y =
  match x.sup, y.sup with
```

```
    | Infinity, _ → false
    | _, Infinity → true
    | Sup x', Sup y' → lesseq x' y'
```

Using this, we can sort the tree in another way that guarantees that a particular leaf (*i2*) is moved as far to the end as possible. We can then flip this leaf from outgoing to incoming without introducing a crossing:

```
let rec sort_2i' lesseq i2 = function
    | Leaf (_, l) as e →
        { sup = if l = i2 then Infinity else Sup l; data = e }
    | Node (n, ch) →
        let ch' = Sort.list (with_infinity lesseq)
                (List.map (sort_2i' lesseq i2) ch) in
        { sup = (List.hd (List.rev ch')).sup;
          data = Node (n, List.map (fun x → x.data) ch') }
```

again, throw away the overall supremum:

```
let sort_2i lesseq i2 t = (sort_2i' lesseq i2 t).data
```

```
type feynmf =
    { style : (string × string) option;
      rev : bool;
      label : string option;
      tension : float option }
```

```
open Printf
```

```
let style prop =
  match prop.style with
  | None → ("plain","")
  | Some s → s
```

```
let species prop = fst (style prop)
let tex_lbl prop = snd (style prop)
```

```
let leaf_label tex io leaf lab = function
    | None → fprintf tex "␣␣␣␣\\fmflabel{${%s}$}{%s%s}\n" lab io leaf
    | Some s →
        fprintf tex "␣␣␣␣\\fmflabel{${%s{}^{(%s)}}$}{%s%s}\n" s lab io leaf
```

```
let leaf_label tex io leaf lab label =
  ()
```

We try to draw diagrams more symmetrically by reducing the tension on the outgoing external lines.

⚠ This is insufficient for asymmetrical cascade decays.

```
let rec leaf_node tex to_label i2 n prop leaf =
  let io, tension, rev =
    if leaf = i2 then
      ("i", "", ¬ prop.rev)
    else
      ("o", ",tension=0.5", prop.rev) in
```

```
      leaf_label tex io (to_label leaf) (tex_lbl prop) prop.label ;
      fprintf tex "␣␣␣␣\\fmfdot{v%d}\n" n;
      if rev then
        fprintf tex "␣␣␣␣\\fmf{%s%s}{%s%s,v%d}\n"
          (species prop) tension io (to_label leaf) n
      else
        fprintf tex "␣␣␣␣\\fmf{%s%s}{v%d,%s%s}\n"
          (species prop) tension n io (to_label leaf)

and int_node tex to_label i2 n n' prop t =
  if prop.rev then
    fprintf tex
      "␣␣␣␣\\fmf{%s,label=\\begin{scriptsize}${%s}$\\end{scriptsize}}{v%d,v%d}\n"
      (species prop) (tex_lbl prop) n' n
  else
    fprintf tex
      "␣␣␣␣\\fmf{%s,label=\\begin{scriptsize}${%s}$\\end{scriptsize}}{v%d,v%d}\n"
      (species prop) (tex_lbl prop) n n';
  fprintf tex "␣␣␣␣\\fmfdot{v%d,v%d}\n" n n';
  edges_feynmf' tex to_label i2 n' t

and leaf_or_int_node tex to_label i2 n n' = function
  | Leaf (prop, l) → leaf_node tex to_label i2 n prop l
  | Node (prop, _) as t → int_node tex to_label i2 n n' prop t

and edges_feynmf' tex to_label i2 n = function
  | Leaf (prop, l) → leaf_node tex to_label i2 n prop l
  | Node (_, ch) →
      ignore (List.fold_right
                (fun t' n' →
                   leaf_or_int_node tex to_label i2 n n' t';
                   succ n') ch (4 × n))

let edges_feynmf tex to_label i1 i2 t =
  let n = 1 in
  begin match t with
  | Leaf _ → ()
  | Node (prop, _) →
      leaf_label tex "i" "1" (tex_lbl prop) prop.label;
      if prop.rev then
        fprintf tex "␣␣␣␣\\fmf{%s}{v%d,i%s}\n" (species prop) n (to_label i1)
      else
        fprintf tex "␣␣␣␣\\fmf{%s}{i%s,v%d}\n" (species prop) (to_label i1) n
  end;
  fprintf tex "␣␣␣␣\\fmfdot{v%d}\n" n;
  edges_feynmf' tex to_label i2 n t

let to_feynmf_channel tex to_TeX to_label incoming t =
  match incoming with
  | i1 :: i2 :: _ →
      let t' = sort_2i (≤ ) i2 t in
      let out = List.filter (fun a → i2 ≠ a) (leafs t') in
      fprintf tex "\\fmfframe(8,7)(8,6){%%\n";
```

Figure Q.1: Note that this is subtly different ...

```
    fprintf tex "␣␣\\begin{fmfgraph*}(35,30)\n";
    fprintf tex "␣␣␣\\fmfpen{thin}\n";
    fprintf tex "␣␣␣\\fmfset{arrow_len}{2mm}\n";
    fprintf tex "␣␣␣␣\\fmfleft{i%s,i%s}\n" (to_label i1) (to_label i2);
    fprintf tex "␣␣␣␣␣\\fmfright{o%s}\n"
      (String.concat ",o" (List.map to_label out));
    List.iter
      (fun s →
        fprintf tex "␣␣␣␣\\fmflabel{${%s}$}{i%s}\n"
          (to_TeX s) (to_label s))
      [i1; i2];
    List.iter
      (fun s →
        fprintf tex "␣␣␣␣\\fmflabel{${%s}$}{o%s}\n"
          (to_TeX s) (to_label s))
      out;
    edges_feynmf tex to_label i1 i2 t';
    fprintf tex "␣␣\\end{fmfgraph*}}\\hfil\\allowbreak\n"
  | _ → ()
```

```
let vanilla = { style = None; rev = false; label = None; tension = None }
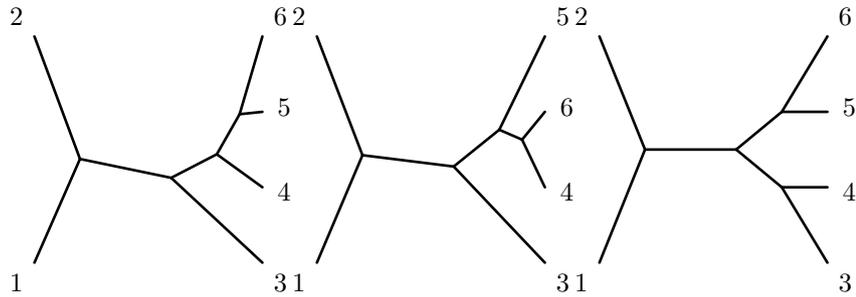
let sty (s, r, l) = { vanilla with style = Some s; rev = r; label = Some l }

type λ feynmf_set =
  { header : string;
    incoming : λ list;
    diagrams : (feynmf, λ) t list }

type (λ, μ) feynmf_sets =
  { outer : λ feynmf_set;
    inner : μ feynmf_set list }

type λ feynmf_levels =
  { this : λ feynmf_set;
    lower : λ feynmf_levels list }

let latex_section = function
```

```
    | level when level < 0 → "part"
    | 0 → "chapter"
    | 1 → "section"
    | 2 → "subsection"
    | 3 → "subsubsection"
    | 4 → "paragraph"
    | _ → "subparagraph"
```

let rec *feynmf_set tex sections level to_TeX to_label set* =
  fprintf tex "%s\\%s{%s}\n"
    (if *sections* then "" else "%%%␣")
    (*latex_section level*)
    set.header;
  List.iter
    (*to_feynmf_channel tex to_TeX to_label set.incoming*)
    set.diagrams

let *feynmf_sets tex sections level*
    to_TeX_outer to_label_outer to_TeX_inner to_label_inner set =
  feynmf_set tex sections level to_TeX_outer to_label_outer set.outer;
  List.iter
    (*feynmf_set tex sections* (*succ level*) *to_TeX_inner to_label_inner*)
    set.inner

let *feynmf_sets_plain sections level file*
    to_TeX_outer to_label_outer to_TeX_inner to_label_inner sets =
  let *tex* = *open_out* (*file* ^ ".tex") in
  List.iter
    (*feynmf_sets tex sections level*
        to_TeX_outer to_label_outer to_TeX_inner to_label_inner)
    sets;
  close_out tex

let *feynmf_header tex file* =
  fprintf tex "\\documentclass[10pt]{article}\n";
  fprintf tex "\\usepackage{ifpdf}\n";
  fprintf tex "\\usepackage[colorlinks]{hyperref}\n";
  fprintf tex "\\usepackage[a4paper,margin=1cm]{geometry}\n";
  fprintf tex "\\usepackage{feynmp}\n";
  fprintf tex "\\ifpdf\n";
  fprintf tex "␣␣␣\\DeclareGraphicsRule{*}{mps}{*}{}\n";
  fprintf tex "\\else\n";
  fprintf tex "␣␣␣\\DeclareGraphicsRule{*}{eps}{*}{}\n";
  fprintf tex "\\fi\n";
  fprintf tex "\\setlength{\\unitlength}{1mm}\n";
  fprintf tex "\\setlength{\\parindent}{0pt}\n";
  fprintf tex
    "\\renewcommand{\\mathstrut}{\\protect\\vphantom{\\hat{0123456789}}}\n";
  fprintf tex "\\begin{document}\n";
  fprintf tex "\\tableofcontents\n";
  fprintf tex "\\begin{fmffile}{%s-fmf}\n\n" *file*

let *feynmf_footer tex* =
  fprintf tex "\n";
  fprintf tex "\\end{fmffile}␣\n";
  fprintf tex "\\end{document}␣\n"

let *feynmf_sets_wrapped latex file*
    to_TeX_outer to_label_outer to_TeX_inner to_label_inner sets =
  let *tex* = *open_out* (*file* ^ ".tex") in
  if *latex* then *feynmf_header tex file*;
  List.iter
    (*feynmf_sets tex latex* 1
      to_TeX_outer to_label_outer to_TeX_inner to_label_inner)
    sets;
  if *latex* then *feynmf_footer tex*;
  close_out tex

let rec *feynmf_levels tex sections level to_TeX to_label set* =
  fprintf tex "%s\\%s{%s}\n"
    (if *sections* then "" else "%%␣")
    (*latex_section level*)
    set.this.header;
  List.iter
    (*to_feynmf_channel tex to_TeX to_label set.this.incoming*)
    set.this.diagrams;
  List.iter (*feynmf_levels tex sections* (*succ level*) *to_TeX to_label*) *set.lower*

let *feynmf_levels_plain sections level file to_TeX to_label sets* =
  let *tex* = *open_out* (*file* ^ ".tex") in
  List.iter (*feynmf_levels tex sections level to_TeX to_label*) *sets*;
  close_out tex

let *feynmf_levels_wrapped file to_TeX to_label sets* =
  let *tex* = *open_out* (*file* ^ ".tex") in
  feynmf_header tex file;
  List.iter (*feynmf_levels tex* true 1 *to_TeX to_label*) *sets*;
  feynmf_footer tex;
  close_out tex

Q.2.4   *Least Squares Layout*

$$L = \frac{1}{2}\sum_{i \neq i'} T_{ii'} \left(x_i - x_{i'}\right)^2 + \frac{1}{2}\sum_{i,j} T'_{ij} \left(x_i - e_j\right)^2 \qquad (\text{Q.11})$$

and thus

$$0 = \frac{\partial L}{\partial x_i} = \sum_{i' \neq i} T_{ii'} \left(x_i - x_{i'}\right) + \sum_{j} T'_{ij} \left(x_i - e_j\right) \qquad (\text{Q.12})$$

or

$$\left(\sum_{i' \neq i} T_{ii'} + \sum_{j} T'_{ij}\right) x_i - \sum_{i' \neq i} T_{ii'} x_{i'} = \sum_{j} T'_{ij} e_j \qquad (\text{Q.13})$$

where we can assume that

$$T_{ii'} = T_{i'i} \qquad\qquad (Q.14a)$$
$$T_{ii} = 0 \qquad\qquad (Q.14b)$$

type α *node_with_tension* $=$ { *node* : α; *tension* : *float* }

let *unit_tension* t $=$
  map (fun n $\rightarrow$ { *node* $=$ n; *tension* $=$ 1.0 }) (fun l $\rightarrow$ l) t

let *leafs_and_nodes* $i2$ t $=$
  let t' $=$ *sort_2i* ($\leq$) $i2$ t in
  match *nodes* t' with
  | [] $\rightarrow$ *failwith* `"Tree.nodes_and_leafs:␣impossible"`
  | $i1$ :: _ as n $\rightarrow$ ($i1$, $i2$, *List.filter* (fun l $\rightarrow$ l $\neq$ $i2$) (*leafs* t'), n)

Not tail recursive, but they're unlikely to meet any deep trees:

let rec *internal_edges_from* n $=$ function
  | *Leaf* _ $\rightarrow$ []
  | *Node* (n', *ch*) $\rightarrow$ (n', n) :: (*ThoList.flatmap* (*internal_edges_from* n') *ch*)

The root node of the tree represents a vertex (node) and an external line (leaf) of the Feynman diagram simultaneously. Thus it requires special treatment:

let *internal_edges* $=$ function
  | *Leaf* _ $\rightarrow$ []
  | *Node* (n, *ch*) $\rightarrow$ *ThoList.flatmap* (*internal_edges_from* n) *ch*

let rec *external_edges_from* n $=$ function
  | *Leaf* (n', _) $\rightarrow$ [(n', n)]
  | *Node* (n', *ch*) $\rightarrow$ *ThoList.flatmap* (*external_edges_from* n') *ch*

let *external_edges* $=$ function
  | *Leaf* (n, _) $\rightarrow$ [(n, n)]
  | *Node* (n, *ch*) $\rightarrow$ (n, n) :: *ThoList.flatmap* (*external_edges_from* n) *ch*

type ('*edge*, '*node*, '*ext*) *graph* $=$
    { *int_nodes* : '*node* *array*;
      ext_nodes : '*ext* *array*;
      int_edges : ('*edge* $\times$ *int* $\times$ *int*) *list*;
      ext_edges : ('*edge* $\times$ *int* $\times$ *int*) *list* }

module M $=$ *Pmap.Tree*

Invert an array, viewed as a map from non-negative integers into a set. The result is a map from the set to the integers: val *invert_array* : α *array* $\rightarrow$ (α, *int*) *M.t*

let *invert_array_unsafe* a $=$
  fst (*Array.fold_left* (fun (m, i) a_i $\rightarrow$
    (*M.add compare* a_i i m, *succ* i)) (*M.empty*, 0) a)

exception *Not_invertible*

let *add_unique key data map* $=$
  if *M.mem compare key map* then

```
        raise Not_invertible
      else
        M.add compare key data map

let invert_array a =
  fst (Array.fold_left (fun (m, i) a_i →
    (add_unique a_i i m, succ i)) (M.empty, 0) a)

let graph_of_tree nodes2edge conjugate i2 t =
  let i1, i2, out, vertices = leafs_and_nodes i2 t in
  let int_nodes = Array.of_list vertices
  and ext_nodes = Array.of_list (conjugate i1 :: i2 :: out) in
  let int_nodes_index_table = invert_array int_nodes
  and ext_nodes_index_table = invert_array ext_nodes in
  let int_nodes_index n = M.find compare n int_nodes_index_table
  and ext_nodes_index n = M.find compare n ext_nodes_index_table in
  { int_nodes = int_nodes;
    ext_nodes = ext_nodes;
    int_edges = List.map
      (fun (n1, n2) →
        (nodes2edge n1 n2, int_nodes_index n1, int_nodes_index n2))
      (internal_edges t);
    ext_edges = List.map
      (fun (e, n) →
        let e' =
          if e = i1 then
            conjugate e
          else
            e in
        (nodes2edge e' n, ext_nodes_index e', int_nodes_index n))
      (external_edges t) }

let int_incidence f null g =
  let n = Array.length g.int_nodes in
  let incidence = Array.make_matrix n n null in
  List.iter (fun (edge, n1, n2) →
    if n1 ≠ n2 then begin
      let edge' = f edge g.int_nodes.(n1) g.int_nodes.(n2) in
      incidence.(n1).(n2) ← edge';
      incidence.(n2).(n1) ← edge'
    end)
    g.int_edges;
  incidence

let ext_incidence f null g =
  let n_int = Array.length g.int_nodes
  and n_ext = Array.length g.ext_nodes in
  let incidence = Array.make_matrix n_int n_ext null in
  List.iter (fun (edge, e, n) →
    incidence.(n).(e) ← f edge g.ext_nodes.(e) g.int_nodes.(n))
    g.ext_edges;
  incidence
```

```
let division n =
  if n < 0 then
    []
  else if n = 1 then
    [0.5]
  else
    let n' = pred n in
    let d = 1.0 /. (float n') in
    let rec division' i acc =
      if i < 0 then
        acc
      else
        division' (pred i) (float i *. d :: acc) in
    division' n' []
```

type $(\varepsilon,\ \nu,\ 'ext)\ ext\_layout\ =\ (\varepsilon,\ \nu,\ 'ext\ \times\ float\ \times\ float)\ graph$
type $(\varepsilon,\ \nu,\ 'ext)\ layout\ =\ (\varepsilon,\ \nu\ \times\ float\ \times\ float,\ 'ext)\ ext\_layout$

```
let left_to_right num_in g =
  if num_in < 1 then
    invalid_arg "left_to_right"
  else
    let num_out = Array.length g.ext_nodes − num_in in
    if num_out < 1 then
      invalid_arg "left_to_right"
    else
      let incoming =
        List.map2 (fun e y → (e, 0.0, y))
          (Array.to_list (Array.sub g.ext_nodes 0 num_in))
          (division num_in)
      and outgoing =
        List.map2 (fun e y → (e, 1.0, y))
          (Array.to_list (Array.sub g.ext_nodes num_in num_out))
          (division num_out) in
      { g with ext_nodes = Array.of_list (incoming @ outgoing) }
```

Reformulating (Q.13)

$$Ax = b_x \tag{Q.15a}$$
$$Ay = b_y \tag{Q.15b}$$

with

$$A_{ii'} = \left(\sum_{i'' \neq i} T_{ii''} + \sum_{j} T'_{ij} \right) \delta_{ii'} - T_{ii'} \tag{Q.16a}$$

$$(b_{x/y})_i = \sum_{j} T'_{ij}(e_{x/y})_j \tag{Q.16b}$$

let $sum\ a\ =\ Array.fold\_left\ (+.)\ 0.0\ a$

let $tension\_to\_equation\ t\ t'\ e\ =$

```
  let xe, ye  =  List.split e in
  let bx  =  Linalg.matmulv t' (Array.of_list xe)
  and by  =  Linalg.matmulv t' (Array.of_list ye)
  and a  =  Array.init (Array.length t)
      (fun i  →
         let a_i  =  Array.map (~-.) t.(i) in
         a_i.(i)  ←  a_i.(i)  +.  sum t.(i)  +.  sum t'.(i);
         a_i) in
  (a,  bx,  by)

let layout g  =
  let ext_nodes  =
    List.map (fun (_,  x,  y)  →  (x,  y)) (Array.to_list g.ext_nodes) in
  let a,  bx,  by  =
    tension_to_equation
      (int_incidence (fun _ _ _  →  1.0) 0.0 g)
      (ext_incidence (fun _ _ _  →  1.0) 0.0 g) ext_nodes in
  match Linalg.solve_many a [bx; by] with
  | [x; y]  →  { g with int_nodes  =  Array.mapi
                    (fun i n  →  (n, x.(i), y.(i))) g.int_nodes }
  | _  →  failwith "impossible"

let iter_edges f g  =
  List.iter (fun (edge,  n1,  n2)  →
    let _, x1, y1  =  g.int_nodes.(n1)
    and _, x2, y2  =  g.int_nodes.(n2) in
    f edge (x1,  y1) (x2,  y2)) g.int_edges;
  List.iter (fun (edge,  e,  n)  →
    let _, x1, y1  =  g.ext_nodes.(e)
    and _, x2, y2  =  g.int_nodes.(n) in
    f edge (x1,  y1) (x2,  y2)) g.ext_edges

let iter_internal f g  =
  Array.iter (fun (node,  x,  y)  →  f (x,  y)) g.int_nodes

let iter_incoming f g  =
  f g.ext_nodes.(0);
  f g.ext_nodes.(1)

let iter_outgoing f g  =
  for i  =  2 to pred (Array.length g.ext_nodes) do
    f g.ext_nodes.(i)
  done

let dump g  =
  Array.iter (fun (_,  x,  y)  →  Printf.eprintf "(%g,%g) " x y) g.ext_nodes;
  Printf.eprintf "\n =>  ";
  Array.iter (fun (_,  x,  y)  →  Printf.eprintf "(%g,%g) " x y) g.int_nodes;
  Printf.eprintf "\n"
```

—R—
DEPENDENCY TREES

R.1   Interface of Tree2

Dependency trees for wavefunctions.

```
type (ν, ε) t
val cons : (ε × ν × (ν, ε) t list) list → (ν, ε) t
val leaf : ν → (ν, ε) t

val to_string : (ν → string) → (ε → string) → (ν, ε) t → string
```

R.2   Implementation of Tree2

Dependency trees for wavefunctions.

```
type (ν, ε) t =
  | Node of (ε × ν × (ν, ε) t list) list
  | Leaf of ν

let leaf node = Leaf node

let sort_children (edge, node, children) =
  (edge, node, List.sort compare children)

let cons fusions = Node (List.sort compare (List.map sort_children fusions))

let rec to_string n2s e2s = function
  | Leaf n → n2s n
  | Node children →
      "{" ^
      String.concat ","
        (List.map
          (fun (e, n, ch_list) →
            e2s e ^ ":" ^ n2s n ^
            "<(" ^ (String.concat ";" (List.map (to_string n2s e2s) ch_list)) ^ ")")
          children) ^
      "}"
```

—S—
Consistency Checks

⚠ *Application `count.ml` unavailable!*

—T—
Complex Numbers

⚠ *Interface `complex.mli` unavailable!*

⚠ *Implementation `complex.ml` unavailable!*

—U—
Algebra

U.1 Interface of Algebra

U.1.1 Coefficients

For our algebra, we need coefficient rings.

```
module type CRing =
  sig
    type t
    val null : t
    val unit : t
    val mul : t → t → t
    val add : t → t → t
    val sub : t → t → t
    val neg : t → t
    val to_string : t → string
  end
```

And rational numbers provide a particularly important example:

```
module type Rational =
  sig
    include CRing
    val is_null : t → bool
    val is_unit : t → bool
    val make : int → int → t
    val to_ratio : t → int × int
    val to_float : t → float
  end
```

U.1.2 Naive Rational Arithmetic

This *is* dangerous and will overflow even for simple applications. The production code will have to be linked to a library for large integer arithmetic.

```
module Small_Rational : Rational
```

U.1.3 Expressions: Terms, Rings and Linear Combinations

The tensor algebra will be spanned by an abelian monoid:

module type *Term* =
  sig
    type α t
    val *unit* : *unit* $\rightarrow$ α t
    val *is_unit* : α t $\rightarrow$ *bool*
    val *atom* : α $\rightarrow$ α t
    val *power* : *int* $\rightarrow$ α t $\rightarrow$ α t
    val *mul* : α t $\rightarrow$ α t $\rightarrow$ α t
    val *map* : $(\alpha \rightarrow \beta) \rightarrow \alpha$ t $\rightarrow$ β t
    val *to_string* : $(\alpha \rightarrow string) \rightarrow \alpha$ t $\rightarrow$ *string*

The derivative of a term is *not* a term, but a sum of terms instead:

$$D(f_1^{p_1} f_2^{p_2} \cdots f_n^{p_n}) = \sum_i (Df_i) p_i f_1^{p_1} f_2^{p_2} \cdots f_i^{p_i-1} \cdots f_n^{p_n} \qquad \text{(U.1)}$$

The function returns the sum as a list of triples $(Df_i, p_i, f_1^{p_1} f_2^{p_2} \cdots f_i^{p_i-1} \cdots f_n^{p_n})$. Summing the terms is left to the calling module and the Df_i are *not* guaranteed to be different. NB: The function implementating the inner derivative, is supposed to return *Some* Df_i and *None*, iff Df_i vanishes.

    val *derive* : $(\alpha \rightarrow \beta\ option) \rightarrow \alpha$ t $\rightarrow$ $(\beta \times int \times \alpha\ t)$ *list*

convenience function

    val *product* : α t *list* $\rightarrow$ α t
    val *atoms* : α t $\rightarrow$ α *list*

  end

module type *Ring* =
  sig
    module C : *Rational*
    type α t
    val *null* : *unit* $\rightarrow$ α t
    val *unit* : *unit* $\rightarrow$ α t
    val *is_null* : α t $\rightarrow$ *bool*
    val *is_unit* : α t $\rightarrow$ *bool*
    val *atom* : α $\rightarrow$ α t
    val *scale* : $C.t$ $\rightarrow$ α t $\rightarrow$ α t
    val *add* : α t $\rightarrow$ α t $\rightarrow$ α t
    val *sub* : α t $\rightarrow$ α t $\rightarrow$ α t
    val *mul* : α t $\rightarrow$ α t $\rightarrow$ α t
    val *neg* : α t $\rightarrow$ α t

Again

$$D(f_1^{p_1} f_2^{p_2} \cdots f_n^{p_n}) = \sum_i (Df_i) p_i f_1^{p_1} f_2^{p_2} \cdots f_i^{p_i-1} \cdots f_n^{p_n} \qquad \text{(U.2)}$$

but, iff Df_i can be identified with a f', we know how to perform the sum.

    val *derive_inner* : $(\alpha \rightarrow \alpha\ t) \rightarrow \alpha$ t $\rightarrow$ α t $(*$ this? $*)$

val *derive_inner′* : $(\alpha \rightarrow \alpha\ t\ option) \rightarrow \alpha\ t \rightarrow \alpha\ t$ (∗ or that? ∗)

Below, we will need partial derivatives that lead out of the ring: *derive_outer derive_atom term* returns a list of partial derivatives β with non-zero coefficients $\alpha\ t$:

val *derive_outer* : $(\alpha \rightarrow \beta\ option) \rightarrow \alpha\ t \rightarrow (\beta \times \alpha\ t)\ list$

convenience functions

val *sum* : $\alpha\ t\ list \rightarrow \alpha\ t$
val *product* : $\alpha\ t\ list \rightarrow \alpha\ t$

The list of all generators appearing in an expression:

val *atoms* : $\alpha\ t \rightarrow \alpha\ list$

val *to_string* : $(\alpha \rightarrow string) \rightarrow \alpha\ t \rightarrow string$

  end

module type *Linear* =
  sig
    module *C* : *Ring*
    type $(\alpha,\ \gamma)\ t$
    val *null* : $unit \rightarrow (\alpha,\ \gamma)\ t$
    val *atom* : $\alpha \rightarrow (\alpha,\ \gamma)\ t$
    val *singleton* : $\gamma\ C.t \rightarrow \alpha \rightarrow (\alpha,\ \gamma)\ t$
    val *scale* : $\gamma\ C.t \rightarrow (\alpha,\ \gamma)\ t \rightarrow (\alpha,\ \gamma)\ t$
    val *add* : $(\alpha,\ \gamma)\ t \rightarrow (\alpha,\ \gamma)\ t \rightarrow (\alpha,\ \gamma)\ t$
    val *sub* : $(\alpha,\ \gamma)\ t \rightarrow (\alpha,\ \gamma)\ t \rightarrow (\alpha,\ \gamma)\ t$

A partial derivative w. r. t. a vector maps from a coefficient ring to the dual vector space.

val *partial* : $(\gamma \rightarrow (\alpha,\ \gamma)\ t) \rightarrow \gamma\ C.t \rightarrow (\alpha,\ \gamma)\ t$

A linear combination of vectors

$$linear[(v_1, c_1); (v_2, c_2); \dots; (v_n, c_n)] = \sum_{i=1}^{n} c_i \cdot v_i \qquad \text{(U.3)}$$

val *linear* : $((\alpha,\ \gamma)\ t \times \gamma\ C.t)\ list \rightarrow (\alpha,\ \gamma)\ t$

Some convenience functions

val *map* : $(\alpha \rightarrow \gamma\ C.t \rightarrow (\beta,\ \delta)\ t) \rightarrow (\alpha,\ \gamma)\ t \rightarrow (\beta,\ \delta)\ t$
val *sum* : $(\alpha,\ \gamma)\ t\ list \rightarrow (\alpha,\ \gamma)\ t$

The list of all generators and the list of all generators of coefficients appearing in an expression:

val *atoms* : $(\alpha,\ \gamma)\ t \rightarrow \alpha\ list \times \gamma\ list$

val *to_string* : $(\alpha \rightarrow string) \rightarrow (\gamma \rightarrow string) \rightarrow (\alpha,\ \gamma)\ t \rightarrow string$

  end

module *Term* : *Term*

module *Make_Ring* $(C : Rational)\ (T : Term)$: *Ring*
module *Make_Linear* $(C : Ring)$: *Linear* with module *C* = *C*

U.2   Implementation of Algebra

The terms will be small and there's no need to be fancy and/or efficient. It's more important to have a unique representation.

module *PM* = *Pmap.List*

U.2.1   Coefficients

For our algebra, we need coefficient rings.

```
module type CRing =
  sig
    type t
    val null : t
    val unit : t
    val mul : t → t → t
    val add : t → t → t
    val sub : t → t → t
    val neg : t → t
    val to_string : t → string
  end
```

And rational numbers provide a particularly important example:

```
module type Rational =
  sig
    include CRing
    val is_null : t → bool
    val is_unit : t → bool
    val make : int → int → t
    val to_ratio : t → int × int
    val to_float : t → float
  end
```

U.2.2   Naive Rational Arithmetic

This *is* dangerous and will overflow even for simple applications. The production code will have to be linked to a library for large integer arithmetic.

Anyway, here's Euclid's algorithm:

```
let rec gcd i1 i2 =
  if i2 = 0 then
    abs i1
  else
    gcd i2 (i1 mod i2)
let lcm i1 i2 = (i1 / gcd i1 i2) × i2
module Small_Rational : Rational =
  struct
    type t = int × int
```

```
    let is_null (n, _) = (n = 0)
    let is_unit (n, d) = (n ≠ 0) ∧ (n = d)
    let null = (0, 1)
    let unit = (1, 1)
    let make n d =
      let c = gcd n d in
      (n / c, d / c)
    let mul (n1, d1) (n2, d2) = make (n1 × n2) (d1 × d2)
    let add (n1, d1) (n2, d2) = make (n1 × d2 + n2 × d1) (d1 × d2)
    let sub (n1, d1) (n2, d2) = make (n1 × d2 − n2 × d1) (d1 × d2)
    let neg (n, d) = (− n, d)
    let to_ratio (n, d) =
      if d < 0 then
        (−n, − d)
      else
        (n, d)
    let to_float (n, d) = float n /. float d
    let to_string (n, d) =
      if d = 1 then
        Printf.sprintf "%d" n
      else
        Printf.sprintf "(%d/%d)" n d
  end
```

U.2.3   Expressions: Terms, Rings and Linear Combinations

The tensor algebra will be spanned by an abelian monoid:

```
module type Term =
  sig
    type α t
    val unit : unit → α t
    val is_unit : α t → bool
    val atom : α → α t
    val power : int → α t → α t
    val mul : α t → α t → α t
    val map : (α → β) → α t → β t
    val to_string : (α → string) → α t → string
    val derive : (α → β option) → α t → (β × int × α t) list
    val product : α t list → α t
    val atoms : α t → α list
  end

module type Ring =
  sig
    module C : Rational
    type α t
    val null : unit → α t
    val unit : unit → α t
    val is_null : α t → bool
    val is_unit : α t → bool
```

```
      val atom  :  α  →  α t
      val scale  :  C.t  →  α t  →  α t
      val add  :  α t  →  α t  →  α t
      val sub  :  α t  →  α t  →  α t
      val mul  :  α t  →  α t  →  α t
      val neg  :  α t  →  α t
      val derive_inner  :  (α  →  α t)  →  α t  →  α t (∗ this? ∗)
      val derive_inner'  :  (α  →  α t option)  →  α t  →  α t (∗ or that? ∗)
      val derive_outer  :  (α  →  β option)  →  α t  →  (β  ×  α t) list
      val sum  :  α t list  →  α t
      val product  :  α t list  →  α t
      val atoms  :  α t  →  α list
      val to_string  :  (α  →  string)  →  α t  →  string
    end

module type Linear  =
  sig
      module C  :  Ring
      type (α,  γ) t
      val null  :  unit  →  (α,  γ) t
      val atom  :  α  →  (α,  γ) t
      val singleton  :  γ C.t  →  α  →  (α,  γ) t
      val scale  :  γ C.t  →  (α,  γ) t  →  (α,  γ) t
      val add  :  (α,  γ) t  →  (α,  γ) t  →  (α,  γ) t
      val sub  :  (α,  γ) t  →  (α,  γ) t  →  (α,  γ) t
      val partial  :  (γ  →  (α,  γ) t)  →  γ C.t  →  (α,  γ) t
      val linear  :  ((α,  γ) t  ×  γ C.t) list  →  (α,  γ) t
      val map  :  (α  →  γ C.t  →  (β,  δ) t)  →  (α,  γ) t  →  (β,  δ) t
      val sum  :  (α,  γ) t list  →  (α,  γ) t
      val atoms  :  (α,  γ) t  →  α list × γ list
      val to_string  :  (α  →  string)  →  (γ  →  string)  →  (α,  γ) t  →  string
    end

module Term  :  Term  =
  struct

      module M  =  PM

      type α t  =  (α,  int) M.t

      let unit ()  =  M.empty
      let is_unit  =  M.is_empty

      let atom f  =  M.singleton f 1

      let power p x  =  M.map (( × ) p) x

      let insert1 binop f p term  =
        let p'  =  binop (try M.find compare f term with Not_found  →  0) p in
        if p'  =  0 then
          M.remove compare f term
        else
          M.add compare f p' term
```

```
let mul1 f p term = insert1 (+) f p term
let mul x y = M.fold mul1 x y

let map f term = M.fold (fun t → mul1 (f t)) term M.empty

let to_string fmt term =
  String.concat "*"
    (M.fold (fun f p acc →
      (if p = 0 then
        "1"
      else if p = 1 then
        fmt f
      else
        "[" ^ fmt f ^ "]^" ^ string_of_int p) :: acc) term [])

let derive derive1 x =
  M.fold (fun f p dx →
    if p ≠ 0 then
      match derive1 f with
      | Some df → (df, p, mul1 f (pred p) (M.remove compare f x)) :: dx
      | None → dx
    else
      dx) x []

let product factors =
  List.fold_left mul (unit ()) factors

let atoms t =
  List.map fst (PM.elements t)
```

end

module *Make_Ring* (*C* : *Rational*) (*T* : *Term*) : *Ring* =
  struct

```
module C = C
let one = C.unit

module M = PM

type α t = (α T.t, C.t) M.t

let null () = M.empty
let is_null = M.is_empty

let power t p = M.singleton t p
let unit () = power (T.unit ()) one

let is_unit t = unit () = t
```

⚠ The following should be correct too, but produces to many false positives instead! What's going on?

```
let broken__is_unit t =
  match M.elements t with
  | [(t, p)] → T.is_unit t ∨ C.is_null p
```

```
    | _ → false

let atom t = power (T.atom t) one

let scale c x = M.map (C.mul c) x

let insert1 binop t c sum =
  let c′ = binop (try M.find compare t sum with Not_found → C.null) c in
  if C.is_null c′ then
    M.remove compare t sum
  else
    M.add compare t c′ sum

let add x y = M.fold (insert1 C.add) x y

let sub x y = M.fold (insert1 C.sub) y x
```

One might be tempted to use *Product.outer_self M.fold* instead, but this would require us to combine *tx* and *cx* to (*tx, cx*).

```
let fold2 f x y =
  M.fold (fun tx cx → M.fold (f tx cx) y) x

let mul x y =
  fold2 (fun tx cx ty cy → insert1 C.add (T.mul tx ty) (C.mul cx cy))
    x y (null ())

let neg x =
  sub (null ()) x

let neg x =
  scale (C.neg C.unit) x
```

Multiply the *derivatives* by *c* and add the result to *dx*.

```
let add_derivatives derivatives c dx =
  List.fold_left (fun acc (df, dt_c, dt_t) →
    add (mul df (power dt_t (C.mul c (C.make dt_c 1)))) acc) dx derivatives

let derive_inner derive1 x =
  M.fold (fun t →
    add_derivatives (T.derive (fun f → Some (derive1 f)) t)) x (null ())

let derive_inner′ derive1 x =
  M.fold (fun t → add_derivatives (T.derive derive1 t)) x (null ())

let collect_derivatives derivatives c dx =
  List.fold_left (fun acc (df, dt_c, dt_t) →
    (df, power dt_t (C.mul c (C.make dt_c 1))) :: acc) dx derivatives

let derive_outer derive1 x =
  M.fold (fun t → collect_derivatives (T.derive derive1 t)) x []

let sum terms =
  List.fold_left add (null ()) terms

let product factors =
  List.fold_left mul (unit ()) factors

let atoms t =
```

```
    ThoList.uniq (List.sort compare
                    (ThoList.flatmap (fun (t, _) → T.atoms t) (PM.elements t)))

let to_string fmt sum =
  "(" ^ String.concat "␣+␣"
        (M.fold (fun t c acc →
          if C.is_null c then
            acc
          else if C.is_unit c then
            T.to_string fmt t :: acc
          else if C.is_unit (C.neg c) then
            ("(-" ^ T.to_string fmt t ^ ")") :: acc
          else
            (C.to_string c ^ "*[" ^ T.to_string fmt t ^ "]") :: acc) sum []) ^ ")"

end

module Make_Linear (C : Ring) : Linear with module C = C =
  struct

    module C = C

    module M = PM

    type (α, γ) t = (α, γ C.t) M.t

    let null () = M.empty
    let is_null = M.is_empty
    let atom a = M.singleton a (C.unit ())
    let singleton c a = M.singleton a c

    let scale c x = M.map (C.mul c) x

    let insert1 binop t c sum =
      let c' = binop (try M.find compare t sum with Not_found → C.null ()) c in
      if C.is_null c' then
        M.remove compare t sum
      else
        M.add compare t c' sum

    let add x y = M.fold (insert1 C.add) x y
    let sub x y = M.fold (insert1 C.sub) y x

    let map f t =
      M.fold (fun a c → add (f a c)) t M.empty

    let sum terms =
      List.fold_left add (null ()) terms

    let linear terms =
      List.fold_left (fun acc (a, c) → add (scale c a) acc) (null ()) terms

    let partial derive t =
      let d t' =
        let dt' = derive t' in
        if is_null dt' then
          None
```

```
        else
            Some dt' in
      linear (C.derive_outer d t)

  let atoms t  =
    let a, c  =  List.split (PM.elements t) in
    (a,  ThoList.uniq (List.sort compare (ThoList.flatmap C.atoms c)))

  let to_string fmt cfmt sum  =
    "(" ^ String.concat " + "
              (M.fold (fun t c acc  →
                  if C.is_null c then
                      acc
                  else if C.is_unit c then
                      fmt t  ::  acc
                  else if C.is_unit (C.neg c) then
                      ("(-" ^ fmt t ^ ")")  ::  acc
                  else
                      (C.to_string cfmt c ^ "*" ^ fmt t)  ::  acc)
                  sum []) ^ ")"

end
```

—V—

SIMPLE LINEAR ALGEBRA

V.1   Interface of Linalg

exception *Singular*
exception *Not_Square*

val *copy_matrix* : *float array array* → *float array array*

val *matmul* : *float array array* → *float array array* → *float array array*
val *matmulv* : *float array array* → *float array* → *float array*

val *lu_decompose* : *float array array* → *float array array* × *float array array*
val *solve* : *float array array* → *float array* → *float array*
val *solve_many* : *float array array* → *float array list* → *float array list*

V.2   Implementation of Linalg

This is not a functional implementations, but uses imperative array in Fotran style for maximimum speed.

exception *Singular*
exception *Not_Square*

let *copy_matrix a* =
  Array.init (*Array.length a*)
    (fun *i* → *Array.copy a.(i)*)

let *matmul a b* =
  let *ni* = *Array.length a*
  and *nj* = *Array.length b.(0)*
  and *n* = *Array.length b* in
  let *ab* = *Array.make_matrix ni nj* 0.0 in
  for *i* = 0 to *pred ni* do
    for *j* = 0 to *pred nj* do
      for *k* = 0 to *pred n* do
        ab.(i).(j) ← *ab.(i).(j)* +. *a.(i).(k)* *. *b.(k).(j)*
      done
    done
  done;
  ab

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```
let matmulv a v  =
  let na  =  Array.length a in
  let nv  =  Array.length v in
  let v'  =  Array.make na 0.0 in
  for i  =  0 to pred na do
    for j  =  0 to pred nv do
      v'.(i)  ←  v'.(i)  +.  a.(i).(j)  *.  v.(j)
    done
  done;
  v'

let maxabsval a  :  float =
  let x  =  ref (abs_float a.(0)) in
  for i  =  1 to Array.length a  −  1 do
    x  :=  max !x (abs_float a.(i))
  done;
  !x
```

V.2.1   LU Decomposition

$$A = LU \tag{V.1a}$$

In more detail

$$
\begin{pmatrix}
a_{00} & a_{01} & \cdots & a_{0(n-1)} \\
a_{10} & a_{11} & \cdots & a_{1(n-1)} \\
\vdots & \vdots & \vdots & \vdots \\
a_{(n-1)0} & a_{(n-1)1} & \cdots & a_{(n-1)(n-1)}
\end{pmatrix} =
$$

$$
\begin{pmatrix}
1 & 0 & \cdots & 0 \\
l_{10} & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots \\
l_{(n-1)0} & l_{(n-1)1} & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
u_{00} & u_{01} & \cdots & u_{0(n-1)} \\
0 & u_{11} & \cdots & u_{1(n-1)} \\
\vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & u_{(n-1)(n-1)}
\end{pmatrix}
\tag{V.1b}
$$

Rewriting (V.1) in block matrix notation

$$
\begin{pmatrix} a_{00} & a_{0\cdot} \\ a_{\cdot 0} & A \end{pmatrix} =
\begin{pmatrix} 1 & 0 \\ l_{\cdot 0} & L \end{pmatrix}
\begin{pmatrix} u_{00} & u_{0\cdot} \\ 0 & U \end{pmatrix} =
\begin{pmatrix} u_{00} & u_{0\cdot} \\ l_{\cdot 0}u_{00} & l_{\cdot 0} \otimes u_{0\cdot} + LU \end{pmatrix}
\tag{V.2}
$$

we can solve it easily

$$u_{00} = a_{00} \tag{V.3a}$$

$$u_{0\cdot} = a_{0\cdot} \tag{V.3b}$$

$$l_{\cdot 0} = \frac{a_{\cdot 0}}{a_{00}} \tag{V.3c}$$

$$LU = A - \frac{a_{\cdot 0} \otimes a_{0\cdot}}{a_{00}} \tag{V.3d}$$

and (V.3c) and (V.3d) define a simple iterative algorithm if we work from the outside in. It just remains to add pivoting.

```
let swap a i j  =
```

```
    let a_i = a.(i) in
    a.(i) ← a.(j);
    a.(j) ← a_i
let pivot_column v a n =
  let n' = ref n
  and max_va = ref (v.(n) *. (abs_float a.(n).(n))) in
  for i = succ n to Array.length v − 1 do
    let va_i = v.(i) *. (abs_float a.(i).(n)) in
    if va_i > !max_va then begin
      n' := i;
      max_va := va_i
    end
  done;
  !n'

let lu_decompose_in_place a =
  let n = Array.length a in
  let eps = ref 1
  and pivots = Array.make n 0
  and v =
    try
      Array.init n (fun i →
        let a_i = a.(i) in
        if Array.length a_i ≠ n then
          raise Not_Square;
        1.0 /. (maxabsval a_i))
    with
    | Division_by_zero → raise Singular in
  for i = 0 to pred n do
    let pivot = pivot_column v a i in
    if pivot ≠ i then begin
      swap a pivot i;
      eps := − !eps;
      v.(pivot) ← v.(i)
    end;
    pivots.(i) ← pivot;
    let inv_a_ii =
      try 1.0 /. a.(i).(i) with Division_by_zero → raise Singular in
    for j = succ i to pred n do
      a.(j).(i) ← inv_a_ii *. a.(j).(i)
    done;
    for j = succ i to pred n do
      for k = succ i to pred n do
        a.(j).(k) ← a.(j).(k) −. a.(j).(i) *. a.(i).(k)
      done
    done
  done;
  (pivots, !eps)
let lu_decompose_split a pivots =
  let n = Array.length pivots in
```

```
    let l = Array.make_matrix n n 0.0 in
    let u = Array.make_matrix n n 0.0 in
    for i = 0 to pred n do
        l.(i).(i) ← 1.0;
        for j = succ i to pred n do
            l.(j).(i) ← a.(j).(i)
        done
    done;
    for i = pred n downto 0 do
        swap l i pivots.(i)
    done;
    for i = 0 to pred n do
        for j = 0 to i do
            u.(j).(i) ← a.(j).(i)
        done
    done;
    (l, u)

let lu_decompose a =
    let a = copy_matrix a in
    let pivots, _ = lu_decompose_in_place a in
    lu_decompose_split a pivots

let lu_backsubstitute a pivots b =
    let n = Array.length a in
    let nonzero = ref (−1) in
    let b = Array.copy b in
    for i = 0 to pred n do
        let ll = pivots.(i) in
        let b_i = ref (b.(ll)) in
        b.(ll) ← b.(i);
        if !nonzero ≥ 0 then
            for j = !nonzero to pred i do
                b_i := !b_i −. a.(i).(j) *. b.(j)
            done
        else if !b_i ≠ 0.0 then
            nonzero := i;
        b.(i) ← !b_i
    done;
    for i = pred n downto 0 do
        let b_i = ref (b.(i)) in
        for j = succ i to pred n do
            b_i := !b_i −. a.(i).(j) *. b.(j)
        done;
        b.(i) ← !b_i /. a.(i).(i)
    done;
    b

let solve_destructive a b =
    let pivot, _ = lu_decompose_in_place a in
    lu_backsubstitute a pivot b
```

let *solve_many_destructive a bs* =
  let *pivot*, _ = *lu_decompose_in_place a* in
  List.map (*lu_backsubstitute a pivot*) *bs*

let *solve a b* =
  solve_destructive (*copy_matrix a*) *b*

let *solve_many a bs* =
  solve_many_destructive (*copy_matrix a*) *bs*

—W—
Talk To The WHiZard . . .

Talk to [11].

Temporarily disabled, until, we implement some conditional weaving. . .

—X—
Fortran Libraries

X.1   Trivia

⟨omega_spinors.f90⟩≡
  ⟨*Copyleft*⟩
  module omega_spinors
    use kinds
    use constants
    implicit none
    private
    public :: operator (*), operator (+), operator (-)
    public :: abs
    ⟨intrinsic :: abs⟩
    type, public :: conjspinor
      ! private (omegalib needs access, but DON'T TOUCH IT!)
      complex(kind=default), dimension(4) :: a
    end type conjspinor
    type, public :: spinor
      ! private (omegalib needs access, but DON'T TOUCH IT!)
      complex(kind=default), dimension(4) :: a
    end type spinor
    ⟨*Declaration of operations for spinors*⟩
    integer, parameter, public :: omega_spinors_2010_01_A = 0
  contains
    ⟨*Implementation of operations for spinors*⟩
  end module omega_spinors

⟨intrinsic :: abs *(if working)*⟩≡
  intrinsic :: abs

⟨intrinsic :: conjg *(if working)*⟩≡
  intrinsic :: conjg

well, the Intel Fortran Compiler chokes on these with an internal error:

⟨intrinsic :: abs⟩≡

⟨intrinsic :: conjg⟩≡

To reenable the pure functions that have been removed for OpenMP, one should set this chunk to pure &

⟨pure *unless OpenMP*⟩≡

X.1.1   Inner Product

⟨*Declaration of operations for spinors*⟩≡
```
  interface operator (*)
     module procedure conjspinor_spinor
  end interface
  private :: conjspinor_spinor
```

$$\bar{\psi}\psi' \tag{X.1}$$

NB: `dot_product` conjugates its first argument, we can either cancel this or inline `dot_product`:

⟨*Implementation of operations for spinors*⟩≡
```
  pure function conjspinor_spinor (psibar, psi) result (psibarpsi)
    complex(kind=default) :: psibarpsi
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    psibarpsi = psibar%a(1)*psi%a(1) + psibar%a(2)*psi%a(2) &
              + psibar%a(3)*psi%a(3) + psibar%a(4)*psi%a(4)
  end function conjspinor_spinor
```

X.1.2   Spinor Vector Space

Scalar Multiplication

⟨*Declaration of operations for spinors*⟩+≡
```
  interface operator (*)
     module procedure integer_spinor, spinor_integer, &
         real_spinor, double_spinor, &
         complex_spinor, dcomplex_spinor, &
         spinor_real, spinor_double, &
         spinor_complex, spinor_dcomplex
  end interface
  private :: integer_spinor, spinor_integer, real_spinor, &
       double_spinor, complex_spinor, dcomplex_spinor, &
       spinor_real, spinor_double, spinor_complex, spinor_dcomplex
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function integer_spinor (x, y) result (xy)
    integer, intent(in) :: x
    type(spinor), intent(in) :: y
    type(spinor) :: xy
    xy%a = x * y%a
  end function integer_spinor
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function real_spinor (x, y) result (xy)
    real(kind=single), intent(in) :: x
    type(spinor), intent(in) :: y
    type(spinor) :: xy
    xy%a = x * y%a
  end function real_spinor
  pure function double_spinor (x, y) result (xy)
    real(kind=default), intent(in) :: x
    type(spinor), intent(in) :: y
```

```
      type(spinor) :: xy
      xy%a = x * y%a
    end function double_spinor
    pure function complex_spinor (x, y) result (xy)
      complex(kind=single), intent(in) :: x
      type(spinor), intent(in) :: y
      type(spinor) :: xy
      xy%a = x * y%a
    end function complex_spinor
    pure function dcomplex_spinor (x, y) result (xy)
      complex(kind=default), intent(in) :: x
      type(spinor), intent(in) :: y
      type(spinor) :: xy
      xy%a = x * y%a
    end function dcomplex_spinor
    pure function spinor_integer (y, x) result (xy)
      integer, intent(in) :: x
      type(spinor), intent(in) :: y
      type(spinor) :: xy
      xy%a = x * y%a
    end function spinor_integer
    pure function spinor_real (y, x) result (xy)
      real(kind=single), intent(in) :: x
      type(spinor), intent(in) :: y
      type(spinor) :: xy
      xy%a = x * y%a
    end function spinor_real
    pure function spinor_double (y, x) result (xy)
      real(kind=default), intent(in) :: x
      type(spinor), intent(in) :: y
      type(spinor) :: xy
      xy%a = x * y%a
    end function spinor_double
    pure function spinor_complex (y, x) result (xy)
      complex(kind=single), intent(in) :: x
      type(spinor), intent(in) :: y
      type(spinor) :: xy
      xy%a = x * y%a
    end function spinor_complex
    pure function spinor_dcomplex (y, x) result (xy)
      complex(kind=default), intent(in) :: x
      type(spinor), intent(in) :: y
      type(spinor) :: xy
      xy%a = x * y%a
    end function spinor_dcomplex
```

⟨*Declaration of operations for spinors*⟩+≡

```
    interface operator (*)
       module procedure integer_conjspinor, conjspinor_integer, &
            real_conjspinor, double_conjspinor, &
            complex_conjspinor, dcomplex_conjspinor, &
            conjspinor_real, conjspinor_double, &
            conjspinor_complex, conjspinor_dcomplex
    end interface
    private :: integer_conjspinor, conjspinor_integer, real_conjspinor, &
```

```
      double_conjspinor, complex_conjspinor, dcomplex_conjspinor, &
      conjspinor_real, conjspinor_double, conjspinor_complex, &
      conjspinor_dcomplex
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function integer_conjspinor (x, y) result (xy)
    integer, intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function integer_conjspinor
  pure function real_conjspinor (x, y) result (xy)
    real(kind=single), intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function real_conjspinor
  pure function double_conjspinor (x, y) result (xy)
    real(kind=default), intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function double_conjspinor
  pure function complex_conjspinor (x, y) result (xy)
    complex(kind=single), intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function complex_conjspinor
  pure function dcomplex_conjspinor (x, y) result (xy)
    complex(kind=default), intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function dcomplex_conjspinor
  pure function conjspinor_integer (y, x) result (xy)
    integer, intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function conjspinor_integer
  pure function conjspinor_real (y, x) result (xy)
    real(kind=single), intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function conjspinor_real
  pure function conjspinor_double (y, x) result (xy)
    real(kind=default), intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function conjspinor_double
  pure function conjspinor_complex (y, x) result (xy)
    complex(kind=single), intent(in) :: x
```

```
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function conjspinor_complex
  pure function conjspinor_dcomplex (y, x) result (xy)
    complex(kind=default), intent(in) :: x
    type(conjspinor), intent(in) :: y
    type(conjspinor) :: xy
    xy%a = x * y%a
  end function conjspinor_dcomplex
```

Unary Plus and Minus

⟨*Declaration of operations for spinors*⟩+≡
```
  interface operator (+)
     module procedure plus_spinor, plus_conjspinor
  end interface
  private :: plus_spinor, plus_conjspinor
  interface operator (-)
     module procedure neg_spinor, neg_conjspinor
  end interface
  private :: neg_spinor, neg_conjspinor
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function plus_spinor (x) result (plus_x)
    type(spinor), intent(in) :: x
    type(spinor) :: plus_x
    plus_x%a = x%a
  end function plus_spinor
  pure function neg_spinor (x) result (neg_x)
    type(spinor), intent(in) :: x
    type(spinor) :: neg_x
    neg_x%a = - x%a
  end function neg_spinor
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function plus_conjspinor (x) result (plus_x)
    type(conjspinor), intent(in) :: x
    type(conjspinor) :: plus_x
    plus_x%a = x%a
  end function plus_conjspinor
  pure function neg_conjspinor (x) result (neg_x)
    type(conjspinor), intent(in) :: x
    type(conjspinor) :: neg_x
    neg_x%a = - x%a
  end function neg_conjspinor
```

Addition and Subtraction

⟨*Declaration of operations for spinors*⟩+≡
```
  interface operator (+)
     module procedure add_spinor, add_conjspinor
  end interface
  private :: add_spinor, add_conjspinor
  interface operator (-)
```

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```
     module procedure sub_spinor, sub_conjspinor
  end interface
  private :: sub_spinor, sub_conjspinor
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function add_spinor (x, y) result (xy)
    type(spinor), intent(in) :: x, y
    type(spinor) :: xy
    xy%a = x%a + y%a
  end function add_spinor
  pure function sub_spinor (x, y) result (xy)
    type(spinor), intent(in) :: x, y
    type(spinor) :: xy
    xy%a = x%a - y%a
  end function sub_spinor
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function add_conjspinor (x, y) result (xy)
    type(conjspinor), intent(in) :: x, y
    type(conjspinor) :: xy
    xy%a = x%a + y%a
  end function add_conjspinor
  pure function sub_conjspinor (x, y) result (xy)
    type(conjspinor), intent(in) :: x, y
    type(conjspinor) :: xy
    xy%a = x%a - y%a
  end function sub_conjspinor
```

X.1.3   Norm

⟨*Declaration of operations for spinors*⟩+≡
```
  interface abs
     module procedure abs_spinor, abs_conjspinor
  end interface
  private :: abs_spinor, abs_conjspinor
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function abs_spinor (psi) result (x)
    type(spinor), intent(in) :: psi
    real(kind=default) :: x
    x = sqrt (real (dot_product (psi%a, psi%a)))
  end function abs_spinor
```

⟨*Implementation of operations for spinors*⟩+≡
```
  pure function abs_conjspinor (psibar) result (x)
    real(kind=default) :: x
    type(conjspinor), intent(in) :: psibar
    x = sqrt (real (dot_product (psibar%a, psibar%a)))
  end function abs_conjspinor
```

X.2   Spinors Revisited

⟨`omega_bispinors.f90`⟩≡
  ⟨*Copyleft*⟩

```
module omega_bispinors
  use kinds
  use constants
  implicit none
  private
  public :: operator (*), operator (+), operator (-)
  public :: abs
  type, public :: bispinor
     ! private (omegalib needs access, but DON'T TOUCH IT!)
     complex(kind=default), dimension(4) :: a
  end type bispinor
  ⟨Declaration of operations for bispinors⟩
  integer, parameter, public :: omega_bispinors_2010_01_A = 0
contains
  ⟨Implementation of operations for bispinors⟩
end module omega_bispinors
```

⟨*Declaration of operations for bispinors*⟩≡
```
  interface operator (*)
    module procedure spinor_product
  end interface
  private :: spinor_product
```

$$\bar{\psi}\psi' \qquad\qquad (X.2)$$

NB: `dot_product` conjugates its first argument, we have to cancel this.

⟨*Implementation of operations for bispinors*⟩≡
```
  pure function spinor_product (psil, psir) result (psilpsir)
    complex(kind=default) :: psilpsir
    type(bispinor), intent(in) :: psil, psir
    type(bispinor) :: psidum
    psidum%a(1) = psir%a(2)
    psidum%a(2) = - psir%a(1)
    psidum%a(3) = - psir%a(4)
    psidum%a(4) = psir%a(3)
    psilpsir = dot_product (conjg (psil%a), psidum%a)
  end function spinor_product
```

X.2.1   Spinor Vector Space

Scalar Multiplication

⟨*Declaration of operations for bispinors*⟩+≡
```
  interface operator (*)
     module procedure integer_bispinor, bispinor_integer, &
          real_bispinor, double_bispinor, &
          complex_bispinor, dcomplex_bispinor, &
          bispinor_real, bispinor_double, &
          bispinor_complex, bispinor_dcomplex
  end interface
  private :: integer_bispinor, bispinor_integer, real_bispinor, &
       double_bispinor, complex_bispinor, dcomplex_bispinor, &
       bispinor_real, bispinor_double, bispinor_complex, bispinor_dcomplex
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function integer_bispinor (x, y) result (xy)
    type(bispinor) :: xy
    integer, intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function integer_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function real_bispinor (x, y) result (xy)
    type(bispinor) :: xy
    real(kind=single), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function real_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function double_bispinor (x, y) result (xy)
    type(bispinor) :: xy
    real(kind=default), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function double_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function complex_bispinor (x, y) result (xy)
    type(bispinor) :: xy
    complex(kind=single), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function complex_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function dcomplex_bispinor (x, y) result (xy)
    type(bispinor) :: xy
    complex(kind=default), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function dcomplex_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function bispinor_integer (y, x) result (xy)
    type(bispinor) :: xy
    integer, intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function bispinor_integer
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function bispinor_real (y, x) result (xy)
    type(bispinor) :: xy
    real(kind=single), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function bispinor_real
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function bispinor_double (y, x) result (xy)
```

```
    type(bispinor) :: xy
    real(kind=default), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function bispinor_double
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function bispinor_complex (y, x) result (xy)
    type(bispinor) :: xy
    complex(kind=single), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function bispinor_complex
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function bispinor_dcomplex (y, x) result (xy)
    type(bispinor) :: xy
    complex(kind=default), intent(in) :: x
    type(bispinor), intent(in) :: y
    xy%a = x * y%a
  end function bispinor_dcomplex
```

Unary Plus and Minus

⟨*Declaration of operations for bispinors*⟩+≡
```
  interface operator (+)
     module procedure plus_bispinor
  end interface
  private :: plus_bispinor
  interface operator (-)
     module procedure neg_bispinor
  end interface
  private :: neg_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function plus_bispinor (x) result (plus_x)
    type(bispinor) :: plus_x
    type(bispinor), intent(in) :: x
    plus_x%a = x%a
  end function plus_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function neg_bispinor (x) result (neg_x)
    type(bispinor) :: neg_x
    type(bispinor), intent(in) :: x
    neg_x%a = - x%a
  end function neg_bispinor
```

Addition and Subtraction

⟨*Declaration of operations for bispinors*⟩+≡
```
  interface operator (+)
     module procedure add_bispinor
  end interface
  private :: add_bispinor
  interface operator (-)
```

```
    module procedure sub_bispinor
  end interface
  private :: sub_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function add_bispinor (x, y) result (xy)
    type(bispinor) :: xy
    type(bispinor), intent(in) :: x, y
    xy%a = x%a + y%a
  end function add_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function sub_bispinor (x, y) result (xy)
    type(bispinor) :: xy
    type(bispinor), intent(in) :: x, y
    xy%a = x%a - y%a
  end function sub_bispinor
```

X.2.2   Norm

⟨*Declaration of operations for bispinors*⟩+≡
```
  interface abs
     module procedure abs_bispinor
  end interface
  private :: abs_bispinor
```

⟨*Implementation of operations for bispinors*⟩+≡
```
  pure function abs_bispinor (psi) result (x)
    real(kind=default) :: x
    type(bispinor), intent(in) :: psi
    x = sqrt (real (dot_product (psi%a, psi%a)))
  end function abs_bispinor
```

X.3   Vectorspinors

⟨`omega_vectorspinors.f90`⟩≡
```
  ⟨Copyleft⟩
  module omega_vectorspinors
    use kinds
    use constants
    use omega_bispinors
    use omega_vectors
    implicit none
    private
    public :: operator (*), operator (+), operator (-)
    public :: abs
    type, public :: vectorspinor
       ! private (omegalib needs access, but DON'T TOUCH IT!)
       type(bispinor), dimension(4) :: psi
    end type vectorspinor
    ⟨Declaration of operations for vectorspinors⟩
    integer, parameter, public :: omega_vectorspinors_2010_01_A = 0
  contains
    ⟨Implementation of operations for vectorspinors⟩
```

```
      end module omega_vectorspinors
```

⟨*Declaration of operations for vectorspinors*⟩≡
```
  interface operator (*)
    module procedure vspinor_product
  end interface
  private :: vspinor_product
```

$$\bar{\psi}^\mu \psi'_\mu \qquad\qquad\qquad\text{(X.3)}$$

⟨*Implementation of operations for vectorspinors*⟩≡
```
  pure function vspinor_product (psil, psir) result (psilpsir)
    complex(kind=default) :: psilpsir
    type(vectorspinor), intent(in) :: psil, psir
    psilpsir = psil%psi(1) * psir%psi(1) &
             - psil%psi(2) * psir%psi(2) &
             - psil%psi(3) * psir%psi(3) &
             - psil%psi(4) * psir%psi(4)
  end function vspinor_product
```

X.3.1   Vectorspinor Vector Space

Scalar Multiplication

⟨*Declaration of operations for vectorspinors*⟩+≡
```
  interface operator (*)
    module procedure integer_vectorspinor, vectorspinor_integer, &
           real_vectorspinor, double_vectorspinor, &
           complex_vectorspinor, dcomplex_vectorspinor, &
           vectorspinor_real, vectorspinor_double, &
           vectorspinor_complex, vectorspinor_dcomplex, &
           momentum_vectorspinor, vectorspinor_momentum
  end interface
  private :: integer_vectorspinor, vectorspinor_integer, real_vectorspinor, &
       double_vectorspinor, complex_vectorspinor, dcomplex_vectorspinor, &
       vectorspinor_real, vectorspinor_double, vectorspinor_complex, &
       vectorspinor_dcomplex
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function integer_vectorspinor (x, y) result (xy)
    type(vectorspinor) :: xy
    integer, intent(in) :: x
    type(vectorspinor), intent(in) :: y
    integer :: k
    do k = 1,4
      xy%psi(k) = x * y%psi(k)
    end do
  end function integer_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function real_vectorspinor (x, y) result (xy)
    type(vectorspinor) :: xy
    real(kind=single), intent(in) :: x
    type(vectorspinor), intent(in) :: y
    integer :: k
```

```
      do k = 1,4
      xy%psi(k) = x * y%psi(k)
      end do
   end function real_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function double_vectorspinor (x, y) result (xy)
      type(vectorspinor) :: xy
      real(kind=default), intent(in) :: x
      type(vectorspinor), intent(in) :: y
      integer :: k
      do k = 1,4
      xy%psi(k) = x * y%psi(k)
      end do
   end function double_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function complex_vectorspinor (x, y) result (xy)
      type(vectorspinor) :: xy
      complex(kind=single), intent(in) :: x
      type(vectorspinor), intent(in) :: y
      integer :: k
      do k = 1,4
      xy%psi(k) = x * y%psi(k)
      end do
   end function complex_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function dcomplex_vectorspinor (x, y) result (xy)
      type(vectorspinor) :: xy
      complex(kind=default), intent(in) :: x
      type(vectorspinor), intent(in) :: y
      integer :: k
      do k = 1,4
      xy%psi(k) = x * y%psi(k)
      end do
   end function dcomplex_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function vectorspinor_integer (y, x) result (xy)
      type(vectorspinor) :: xy
      integer, intent(in) :: x
      type(vectorspinor), intent(in) :: y
      integer :: k
      do k = 1,4
      xy%psi(k) = y%psi(k) * x
      end do
   end function vectorspinor_integer
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function vectorspinor_real (y, x) result (xy)
      type(vectorspinor) :: xy
      real(kind=single), intent(in) :: x
      type(vectorspinor), intent(in) :: y
      integer :: k
      do k = 1,4
      xy%psi(k) = y%psi(k) * x
```

```
      end do
   end function vectorspinor_real
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function vectorspinor_double (y, x) result (xy)
    type(vectorspinor) :: xy
    real(kind=default), intent(in) :: x
    type(vectorspinor), intent(in) :: y
    integer :: k
    do k = 1,4
    xy%psi(k) = y%psi(k) * x
    end do
  end function vectorspinor_double
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function vectorspinor_complex (y, x) result (xy)
    type(vectorspinor) :: xy
    complex(kind=single), intent(in) :: x
    type(vectorspinor), intent(in) :: y
    integer :: k
    do k = 1,4
    xy%psi(k) = y%psi(k) * x
    end do
  end function vectorspinor_complex
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function vectorspinor_dcomplex (y, x) result (xy)
    type(vectorspinor) :: xy
    complex(kind=default), intent(in) :: x
    type(vectorspinor), intent(in) :: y
    integer :: k
    do k = 1,4
    xy%psi(k) = y%psi(k) * x
    end do
  end function vectorspinor_dcomplex
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function momentum_vectorspinor (y, x) result (xy)
    type(bispinor) :: xy
    type(momentum), intent(in) :: y
    type(vectorspinor), intent(in) :: x
    integer :: k
    do k = 1,4
    xy%a(k) = y%t     * x%psi(1)%a(k) - y%x(1) * x%psi(2)%a(k) - &
              y%x(2) * x%psi(3)%a(k) - y%x(3) * x%psi(4)%a(k)
    end do
  end function momentum_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function vectorspinor_momentum (y, x) result (xy)
    type(bispinor) :: xy
    type(momentum), intent(in) :: x
    type(vectorspinor), intent(in) :: y
    integer :: k
    do k = 1,4
    xy%a(k) = x%t     * y%psi(1)%a(k) - x%x(1) * y%psi(2)%a(k) - &
              x%x(2) * y%psi(3)%a(k) - x%x(3) * y%psi(4)%a(k)
```

```
      end do
   end function vectorspinor_momentum
```

Unary Plus and Minus

⟨*Declaration of operations for vectorspinors*⟩+≡
```
   interface operator (+)
      module procedure plus_vectorspinor
   end interface
   private :: plus_vectorspinor
   interface operator (-)
      module procedure neg_vectorspinor
   end interface
   private :: neg_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function plus_vectorspinor (x) result (plus_x)
      type(vectorspinor) :: plus_x
      type(vectorspinor), intent(in) :: x
      integer :: k
      do k = 1,4
      plus_x%psi(k) = + x%psi(k)
      end do
   end function plus_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function neg_vectorspinor (x) result (neg_x)
      type(vectorspinor) :: neg_x
      type(vectorspinor), intent(in) :: x
      integer :: k
      do k = 1,4
      neg_x%psi(k) = - x%psi(k)
      end do
   end function neg_vectorspinor
```

Addition and Subtraction

⟨*Declaration of operations for vectorspinors*⟩+≡
```
   interface operator (+)
      module procedure add_vectorspinor
   end interface
   private :: add_vectorspinor
   interface operator (-)
      module procedure sub_vectorspinor
   end interface
   private :: sub_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
   pure function add_vectorspinor (x, y) result (xy)
      type(vectorspinor) :: xy
      type(vectorspinor), intent(in) :: x, y
      integer :: k
      do k = 1,4
      xy%psi(k) = x%psi(k) + y%psi(k)
      end do
   end function add_vectorspinor
```

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⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function sub_vectorspinor (x, y) result (xy)
    type(vectorspinor) :: xy
    type(vectorspinor), intent(in) :: x, y
    integer :: k
    do k = 1,4
    xy%psi(k) = x%psi(k) - y%psi(k)
    end do
  end function sub_vectorspinor
```

X.3.2   Norm

⟨*Declaration of operations for vectorspinors*⟩+≡
```
  interface abs
     module procedure abs_vectorspinor
  end interface
  private :: abs_vectorspinor
```

⟨*Implementation of operations for vectorspinors*⟩+≡
```
  pure function abs_vectorspinor (psi) result (x)
    real(kind=default) :: x
    type(vectorspinor), intent(in) :: psi
    x = sqrt (real (dot_product (psi%psi(1)%a, psi%psi(1)%a) &
            - dot_product (psi%psi(2)%a, psi%psi(2)%a)   &
            - dot_product (psi%psi(3)%a, psi%psi(3)%a)   &
            - dot_product (psi%psi(4)%a, psi%psi(4)%a)))
  end function abs_vectorspinor
```

X.4   Vectors and Tensors

Condensed representation of antisymmetric rank-2 tensors:

$$\begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix} = \begin{pmatrix} 0 & T_e^1 & T_e^2 & T_e^3 \\ -T_e^1 & 0 & T_b^3 & -T_b^2 \\ -T_e^2 & -T_b^3 & 0 & T_b^1 \\ -T_e^3 & T_b^2 & -T_b^1 & 0 \end{pmatrix} \tag{X.4}$$

⟨`omega_vectors.f90`⟩≡
```
  ⟨Copyleft⟩
  module omega_vectors
    use kinds
    use constants
    implicit none
    private
    public :: assignment (=)
    public :: operator (*), operator (+), operator (-), operator (.wedge.)
    public :: abs, conjg
    public :: random_momentum
    ⟨intrinsic :: abs⟩
    ⟨intrinsic :: conjg⟩
    type, public :: momentum
       ! private (omegalib needs access, but DON'T TOUCH IT!)
       real(kind=default) :: t
```

```
        real(kind=default), dimension(3) :: x
     end type momentum
     type, public :: vector
        ! private (omegalib needs access, but DON'T TOUCH IT!)
        complex(kind=default) :: t
        complex(kind=default), dimension(3) :: x
     end type vector
     type, public :: tensor2odd
        ! private (omegalib needs access, but DON'T TOUCH IT!)
        complex(kind=default), dimension(3) :: e
        complex(kind=default), dimension(3) :: b
     end type tensor2odd
     ⟨Declaration of operations for vectors⟩
     integer, parameter, public :: omega_vectors_2010_01_A = 0
  contains
     ⟨Implementation of operations for vectors⟩
  end module omega_vectors
```

X.4.1 Constructors

⟨*Declaration of operations for vectors*⟩≡
```
  interface assignment (=)
     module procedure momentum_of_array, vector_of_momentum, &
           vector_of_array, vector_of_double_array, &
           array_of_momentum, array_of_vector
  end interface
  private :: momentum_of_array, vector_of_momentum, vector_of_array, &
        vector_of_double_array, array_of_momentum, array_of_vector
```

⟨*Implementation of operations for vectors*⟩≡
```
  pure subroutine momentum_of_array (m, p)
     type(momentum), intent(out) :: m
     real(kind=default), dimension(0:), intent(in) :: p
     m%t = p(0)
     m%x = p(1:3)
  end subroutine momentum_of_array
  pure subroutine array_of_momentum (p, v)
     real(kind=default), dimension(0:), intent(out) :: p
     type(momentum), intent(in) :: v
     p(0) = v%t
     p(1:3) = v%x
  end subroutine array_of_momentum
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure subroutine vector_of_array (v, p)
     type(vector), intent(out) :: v
     complex(kind=default), dimension(0:), intent(in) :: p
     v%t = p(0)
     v%x = p(1:3)
  end subroutine vector_of_array
  pure subroutine vector_of_double_array (v, p)
     type(vector), intent(out) :: v
     real(kind=default), dimension(0:), intent(in) :: p
     v%t = p(0)
     v%x = p(1:3)
```

```
    end subroutine vector_of_double_array
    pure subroutine array_of_vector (p, v)
      complex(kind=default), dimension(0:), intent(out) :: p
      type(vector), intent(in) :: v
      p(0) = v%t
      p(1:3) = v%x
    end subroutine array_of_vector
```

⟨*Implementation of operations for vectors*⟩+≡
```
    pure subroutine vector_of_momentum (v, p)
      type(vector), intent(out) :: v
      type(momentum), intent(in) :: p
      v%t = p%t
      v%x = p%x
    end subroutine vector_of_momentum
```

X.4.2   Inner Products

⟨*Declaration of operations for vectors*⟩+≡
```
    interface operator (*)
       module procedure momentum_momentum, vector_vector, &
            vector_momentum, momentum_vector, tensor2odd_tensor2odd
    end interface
    private :: momentum_momentum, vector_vector, vector_momentum, &
       momentum_vector, tensor2odd_tensor2odd
```

⟨*Implementation of operations for vectors*⟩+≡
```
    pure function momentum_momentum (x, y) result (xy)
      type(momentum), intent(in) :: x
      type(momentum), intent(in) :: y
      real(kind=default) :: xy
      xy = x%t*y%t - x%x(1)*y%x(1) - x%x(2)*y%x(2) - x%x(3)*y%x(3)
    end function momentum_momentum
    pure function momentum_vector (x, y) result (xy)
      type(momentum), intent(in) :: x
      type(vector), intent(in) :: y
      complex(kind=default) :: xy
      xy = x%t*y%t - x%x(1)*y%x(1) - x%x(2)*y%x(2) - x%x(3)*y%x(3)
    end function momentum_vector
    pure function vector_momentum (x, y) result (xy)
      type(vector), intent(in) :: x
      type(momentum), intent(in) :: y
      complex(kind=default) :: xy
      xy = x%t*y%t - x%x(1)*y%x(1) - x%x(2)*y%x(2) - x%x(3)*y%x(3)
    end function vector_momentum
    pure function vector_vector (x, y) result (xy)
      type(vector), intent(in) :: x
      type(vector), intent(in) :: y
      complex(kind=default) :: xy
      xy = x%t*y%t - x%x(1)*y%x(1) - x%x(2)*y%x(2) - x%x(3)*y%x(3)
    end function vector_vector
```

Just like classical electrodynamics:

$$\frac{1}{2}T_{\mu\nu}U^{\mu\nu} = \frac{1}{2}\left(-T^{0i}U^{0i} - T^{i0}U^{i0} + T^{ij}U^{ij}\right) = T_b^k U_b^k - T_e^k U_e^k \qquad \text{(X.5)}$$

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⟨*Implementation of operations for vectors*⟩+≡
```
pure function tensor2odd_tensor2odd (x, y) result (xy)
  type(tensor2odd), intent(in) :: x
  type(tensor2odd), intent(in) :: y
  complex(kind=default) :: xy
  xy = x%b(1)*y%b(1) + x%b(2)*y%b(2) + x%b(3)*y%b(3) &
      - x%e(1)*y%e(1) - x%e(2)*y%e(2) - x%e(3)*y%e(3)
end function tensor2odd_tensor2odd
```

X.4.3 Not Entirely Inner Products

⟨*Declaration of operations for vectors*⟩+≡
```
interface operator (*)
   module procedure momentum_tensor2odd, tensor2odd_momentum, &
        vector_tensor2odd, tensor2odd_vector
end interface
private :: momentum_tensor2odd, tensor2odd_momentum, vector_tensor2odd, &
     tensor2odd_vector
```

$$y^\nu = x_\mu T^{\mu\nu} : y^0 = -x^i T^{i0} = x^i T^{0i} \tag{X.6a}$$
$$y^1 = x^0 T^{01} - x^2 T^{21} - x^3 T^{31} \tag{X.6b}$$
$$y^2 = x^0 T^{02} - x^1 T^{12} - x^3 T^{32} \tag{X.6c}$$
$$y^3 = x^0 T^{03} - x^1 T^{13} - x^2 T^{23} \tag{X.6d}$$

⟨*Implementation of operations for vectors*⟩+≡
```
pure function vector_tensor2odd (x, t2) result (xt2)
  type(vector), intent(in) :: x
  type(tensor2odd), intent(in) :: t2
  type(vector) :: xt2
  xt2%t = x%x(1)*t2%e(1) + x%x(2)*t2%e(2) + x%x(3)*t2%e(3)
  xt2%x(1) = x%t*t2%e(1) + x%x(2)*t2%b(3) - x%x(3)*t2%b(2)
  xt2%x(2) = x%t*t2%e(2) + x%x(3)*t2%b(1) - x%x(1)*t2%b(3)
  xt2%x(3) = x%t*t2%e(3) + x%x(1)*t2%b(2) - x%x(2)*t2%b(1)
end function vector_tensor2odd
pure function momentum_tensor2odd (x, t2) result (xt2)
  type(momentum), intent(in) :: x
  type(tensor2odd), intent(in) :: t2
  type(vector) :: xt2
  xt2%t = x%x(1)*t2%e(1) + x%x(2)*t2%e(2) + x%x(3)*t2%e(3)
  xt2%x(1) = x%t*t2%e(1) + x%x(2)*t2%b(3) - x%x(3)*t2%b(2)
  xt2%x(2) = x%t*t2%e(2) + x%x(3)*t2%b(1) - x%x(1)*t2%b(3)
  xt2%x(3) = x%t*t2%e(3) + x%x(1)*t2%b(2) - x%x(2)*t2%b(1)
end function momentum_tensor2odd
```

$$y^\mu = T^{\mu\nu} x_\nu : y^0 = -T^{0i} x^i \tag{X.7a}$$
$$y^1 = T^{10} x^0 - T^{12} x^2 - T^{13} x^3 \tag{X.7b}$$
$$y^2 = T^{20} x^0 - T^{21} x^1 - T^{23} x^3 \tag{X.7c}$$
$$y^3 = T^{30} x^0 - T^{31} x^1 - T^{32} x^2 \tag{X.7d}$$

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function tensor2odd_vector (t2, x) result (t2x)
    type(tensor2odd), intent(in) :: t2
    type(vector), intent(in) :: x
    type(vector) :: t2x
    t2x%t = - t2%e(1)*x%x(1) - t2%e(2)*x%x(2) - t2%e(3)*x%x(3)
    t2x%x(1) = - t2%e(1)*x%t + t2%b(2)*x%x(3) - t2%b(3)*x%x(2)
    t2x%x(2) = - t2%e(2)*x%t + t2%b(3)*x%x(1) - t2%b(1)*x%x(3)
    t2x%x(3) = - t2%e(3)*x%t + t2%b(1)*x%x(2) - t2%b(2)*x%x(1)
  end function tensor2odd_vector
  pure function tensor2odd_momentum (t2, x) result (t2x)
    type(tensor2odd), intent(in) :: t2
    type(momentum), intent(in) :: x
    type(vector) :: t2x
    t2x%t = - t2%e(1)*x%x(1) - t2%e(2)*x%x(2) - t2%e(3)*x%x(3)
    t2x%x(1) = - t2%e(1)*x%t + t2%b(2)*x%x(3) - t2%b(3)*x%x(2)
    t2x%x(2) = - t2%e(2)*x%t + t2%b(3)*x%x(1) - t2%b(1)*x%x(3)
    t2x%x(3) = - t2%e(3)*x%t + t2%b(1)*x%x(2) - t2%b(2)*x%x(1)
  end function tensor2odd_momentum
```

X.4.4   Outer Products

⟨*Declaration of operations for vectors*⟩+≡
```
  interface operator (.wedge.)
     module procedure momentum_wedge_momentum, &
          momentum_wedge_vector, vector_wedge_momentum, vector_wedge_vector
  end interface
  private :: momentum_wedge_momentum, momentum_wedge_vector, &
       vector_wedge_momentum, vector_wedge_vector
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function momentum_wedge_momentum (x, y) result (t2)
    type(momentum), intent(in) :: x
    type(momentum), intent(in) :: y
    type(tensor2odd) :: t2
    t2%e = x%t * y%x - x%x * y%t
    t2%b(1) = x%x(2) * y%x(3) - x%x(3) * y%x(2)
    t2%b(2) = x%x(3) * y%x(1) - x%x(1) * y%x(3)
    t2%b(3) = x%x(1) * y%x(2) - x%x(2) * y%x(1)
  end function momentum_wedge_momentum
  pure function momentum_wedge_vector (x, y) result (t2)
    type(momentum), intent(in) :: x
    type(vector), intent(in) :: y
    type(tensor2odd) :: t2
    t2%e = x%t * y%x - x%x * y%t
    t2%b(1) = x%x(2) * y%x(3) - x%x(3) * y%x(2)
    t2%b(2) = x%x(3) * y%x(1) - x%x(1) * y%x(3)
    t2%b(3) = x%x(1) * y%x(2) - x%x(2) * y%x(1)
  end function momentum_wedge_vector
  pure function vector_wedge_momentum (x, y) result (t2)
    type(vector), intent(in) :: x
    type(momentum), intent(in) :: y
    type(tensor2odd) :: t2
    t2%e = x%t * y%x - x%x * y%t
```

```
      t2%b(1) = x%x(2) * y%x(3) - x%x(3) * y%x(2)
      t2%b(2) = x%x(3) * y%x(1) - x%x(1) * y%x(3)
      t2%b(3) = x%x(1) * y%x(2) - x%x(2) * y%x(1)
    end function vector_wedge_momentum
  pure function vector_wedge_vector (x, y) result (t2)
    type(vector), intent(in) :: x
    type(vector), intent(in) :: y
    type(tensor2odd) :: t2
    t2%e = x%t * y%x - x%x * y%t
    t2%b(1) = x%x(2) * y%x(3) - x%x(3) * y%x(2)
    t2%b(2) = x%x(3) * y%x(1) - x%x(1) * y%x(3)
    t2%b(3) = x%x(1) * y%x(2) - x%x(2) * y%x(1)
  end function vector_wedge_vector
```

X.4.5 Vector Space

Scalar Multiplication

⟨*Declaration of operations for vectors*⟩+≡
```
  interface operator (*)
    module procedure integer_momentum, real_momentum, double_momentum, &
        complex_momentum, dcomplex_momentum, &
        integer_vector, real_vector, double_vector, &
        complex_vector, dcomplex_vector, &
        integer_tensor2odd, real_tensor2odd, double_tensor2odd, &
        complex_tensor2odd, dcomplex_tensor2odd, &
        momentum_integer, momentum_real, momentum_double, &
        momentum_complex, momentum_dcomplex, &
        vector_integer, vector_real, vector_double, &
        vector_complex, vector_dcomplex, &
        tensor2odd_integer, tensor2odd_real, tensor2odd_double, &
        tensor2odd_complex, tensor2odd_dcomplex
  end interface
  private :: integer_momentum, real_momentum, double_momentum, &
      complex_momentum, dcomplex_momentum, integer_vector, real_vector, &
      double_vector, complex_vector, dcomplex_vector, &
      integer_tensor2odd, real_tensor2odd, double_tensor2odd, &
      complex_tensor2odd, dcomplex_tensor2odd, momentum_integer, &
      momentum_real, momentum_double, momentum_complex, &
      momentum_dcomplex, vector_integer, vector_real, vector_double, &
      vector_complex, vector_dcomplex, tensor2odd_integer, &
      tensor2odd_real, tensor2odd_double, tensor2odd_complex, &
      tensor2odd_dcomplex
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function integer_momentum (x, y) result (xy)
    integer, intent(in) :: x
    type(momentum), intent(in) :: y
    type(momentum) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function integer_momentum
  pure function real_momentum (x, y) result (xy)
    real(kind=single), intent(in) :: x
```

```
    type(momentum), intent(in) :: y
    type(momentum) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function real_momentum
  pure function double_momentum (x, y) result (xy)
    real(kind=default), intent(in) :: x
    type(momentum), intent(in) :: y
    type(momentum) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function double_momentum
  pure function complex_momentum (x, y) result (xy)
    complex(kind=single), intent(in) :: x
    type(momentum), intent(in) :: y
    type(vector) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function complex_momentum
  pure function dcomplex_momentum (x, y) result (xy)
    complex(kind=default), intent(in) :: x
    type(momentum), intent(in) :: y
    type(vector) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function dcomplex_momentum
⟨Implementation of operations for vectors⟩+≡
  pure function integer_vector (x, y) result (xy)
    integer, intent(in) :: x
    type(vector), intent(in) :: y
    type(vector) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function integer_vector
  pure function real_vector (x, y) result (xy)
    real(kind=single), intent(in) :: x
    type(vector), intent(in) :: y
    type(vector) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function real_vector
  pure function double_vector (x, y) result (xy)
    real(kind=default), intent(in) :: x
    type(vector), intent(in) :: y
    type(vector) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
  end function double_vector
  pure function complex_vector (x, y) result (xy)
    complex(kind=single), intent(in) :: x
    type(vector), intent(in) :: y
    type(vector) :: xy
    xy%t = x * y%t
    xy%x = x * y%x
```

```
    end function complex_vector
    pure function dcomplex_vector (x, y) result (xy)
      complex(kind=default), intent(in) :: x
      type(vector), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function dcomplex_vector
```

⟨*Implementation of operations for vectors*⟩+≡

```
    pure function integer_tensor2odd (x, t2) result (xt2)
      integer, intent(in) :: x
      type(tensor2odd), intent(in) :: t2
      type(tensor2odd) :: xt2
      xt2%e = x * t2%e
      xt2%b = x * t2%b
    end function integer_tensor2odd
    pure function real_tensor2odd (x, t2) result (xt2)
      real(kind=single), intent(in) :: x
      type(tensor2odd), intent(in) :: t2
      type(tensor2odd) :: xt2
      xt2%e = x * t2%e
      xt2%b = x * t2%b
    end function real_tensor2odd
    pure function double_tensor2odd (x, t2) result (xt2)
      real(kind=default), intent(in) :: x
      type(tensor2odd), intent(in) :: t2
      type(tensor2odd) :: xt2
      xt2%e = x * t2%e
      xt2%b = x * t2%b
    end function double_tensor2odd
    pure function complex_tensor2odd (x, t2) result (xt2)
      complex(kind=single), intent(in) :: x
      type(tensor2odd), intent(in) :: t2
      type(tensor2odd) :: xt2
      xt2%e = x * t2%e
      xt2%b = x * t2%b
    end function complex_tensor2odd
    pure function dcomplex_tensor2odd (x, t2) result (xt2)
      complex(kind=default), intent(in) :: x
      type(tensor2odd), intent(in) :: t2
      type(tensor2odd) :: xt2
      xt2%e = x * t2%e
      xt2%b = x * t2%b
    end function dcomplex_tensor2odd
```

⟨*Implementation of operations for vectors*⟩+≡

```
    pure function momentum_integer (y, x) result (xy)
      integer, intent(in) :: x
      type(momentum), intent(in) :: y
      type(momentum) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function momentum_integer
    pure function momentum_real (y, x) result (xy)
```

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```
      real(kind=single), intent(in) :: x
      type(momentum), intent(in) :: y
      type(momentum) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function momentum_real
    pure function momentum_double (y, x) result (xy)
      real(kind=default), intent(in) :: x
      type(momentum), intent(in) :: y
      type(momentum) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function momentum_double
    pure function momentum_complex (y, x) result (xy)
      complex(kind=single), intent(in) :: x
      type(momentum), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function momentum_complex
    pure function momentum_dcomplex (y, x) result (xy)
      complex(kind=default), intent(in) :: x
      type(momentum), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function momentum_dcomplex
```

⟨*Implementation of operations for vectors*⟩+≡

```
    pure function vector_integer (y, x) result (xy)
      integer, intent(in) :: x
      type(vector), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function vector_integer
    pure function vector_real (y, x) result (xy)
      real(kind=single), intent(in) :: x
      type(vector), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function vector_real
    pure function vector_double (y, x) result (xy)
      real(kind=default), intent(in) :: x
      type(vector), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function vector_double
    pure function vector_complex (y, x) result (xy)
      complex(kind=single), intent(in) :: x
      type(vector), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
```

```
      xy%x = x * y%x
    end function vector_complex
    pure function vector_dcomplex (y, x) result (xy)
      complex(kind=default), intent(in) :: x
      type(vector), intent(in) :: y
      type(vector) :: xy
      xy%t = x * y%t
      xy%x = x * y%x
    end function vector_dcomplex
```

⟨*Implementation of operations for vectors*⟩+≡

```
    pure function tensor2odd_integer (t2, x) result (t2x)
      type(tensor2odd), intent(in) :: t2
      integer, intent(in) :: x
      type(tensor2odd) :: t2x
      t2x%e = x * t2%e
      t2x%b = x * t2%b
    end function tensor2odd_integer
    pure function tensor2odd_real (t2, x) result (t2x)
      type(tensor2odd), intent(in) :: t2
      real(kind=single), intent(in) :: x
      type(tensor2odd) :: t2x
      t2x%e = x * t2%e
      t2x%b = x * t2%b
    end function tensor2odd_real
    pure function tensor2odd_double (t2, x) result (t2x)
      type(tensor2odd), intent(in) :: t2
      real(kind=default), intent(in) :: x
      type(tensor2odd) :: t2x
      t2x%e = x * t2%e
      t2x%b = x * t2%b
    end function tensor2odd_double
    pure function tensor2odd_complex (t2, x) result (t2x)
      type(tensor2odd), intent(in) :: t2
      complex(kind=single), intent(in) :: x
      type(tensor2odd) :: t2x
      t2x%e = x * t2%e
      t2x%b = x * t2%b
    end function tensor2odd_complex
    pure function tensor2odd_dcomplex (t2, x) result (t2x)
      type(tensor2odd), intent(in) :: t2
      complex(kind=default), intent(in) :: x
      type(tensor2odd) :: t2x
      t2x%e = x * t2%e
      t2x%b = x * t2%b
    end function tensor2odd_dcomplex
```

Unary Plus and Minus

⟨*Declaration of operations for vectors*⟩+≡

```
    interface operator (+)
       module procedure plus_momentum, plus_vector, plus_tensor2odd
    end interface
    private :: plus_momentum, plus_vector, plus_tensor2odd
```

```
   interface operator (-)
      module procedure neg_momentum, neg_vector, neg_tensor2odd
   end interface
   private :: neg_momentum, neg_vector, neg_tensor2odd
```

⟨*Implementation of operations for vectors*⟩+≡
```
   pure function plus_momentum (x) result (plus_x)
     type(momentum), intent(in) :: x
     type(momentum) :: plus_x
     plus_x = x
   end function plus_momentum
   pure function neg_momentum (x) result (neg_x)
     type(momentum), intent(in) :: x
     type(momentum) :: neg_x
     neg_x%t = - x%t
     neg_x%x = - x%x
   end function neg_momentum
```

⟨*Implementation of operations for vectors*⟩+≡
```
   pure function plus_vector (x) result (plus_x)
     type(vector), intent(in) :: x
     type(vector) :: plus_x
     plus_x = x
   end function plus_vector
   pure function neg_vector (x) result (neg_x)
     type(vector), intent(in) :: x
     type(vector) :: neg_x
     neg_x%t = - x%t
     neg_x%x = - x%x
   end function neg_vector
```

⟨*Implementation of operations for vectors*⟩+≡
```
   pure function plus_tensor2odd (x) result (plus_x)
     type(tensor2odd), intent(in) :: x
     type(tensor2odd) :: plus_x
     plus_x = x
   end function plus_tensor2odd
   pure function neg_tensor2odd (x) result (neg_x)
     type(tensor2odd), intent(in) :: x
     type(tensor2odd) :: neg_x
     neg_x%e = - x%e
     neg_x%b = - x%b
   end function neg_tensor2odd
```

Addition and Subtraction

⟨*Declaration of operations for vectors*⟩+≡
```
   interface operator (+)
      module procedure add_momentum, add_vector, &
           add_vector_momentum, add_momentum_vector, add_tensor2odd
   end interface
   private :: add_momentum, add_vector, add_vector_momentum, &
        add_momentum_vector, add_tensor2odd
   interface operator (-)
      module procedure sub_momentum, sub_vector, &
```

```
          sub_vector_momentum, sub_momentum_vector, sub_tensor2odd
    end interface
    private :: sub_momentum, sub_vector, sub_vector_momentum, &
          sub_momentum_vector, sub_tensor2odd
```

⟨*Implementation of operations for vectors*⟩+≡

```
    pure function add_momentum (x, y) result (xy)
      type(momentum), intent(in) :: x, y
      type(momentum) :: xy
      xy%t = x%t + y%t
      xy%x = x%x + y%x
    end function add_momentum
    pure function add_vector (x, y) result (xy)
      type(vector), intent(in) :: x, y
      type(vector) :: xy
      xy%t = x%t + y%t
      xy%x = x%x + y%x
    end function add_vector
    pure function add_momentum_vector (x, y) result (xy)
      type(momentum), intent(in) :: x
      type(vector), intent(in) :: y
      type(vector) :: xy
      xy%t = x%t + y%t
      xy%x = x%x + y%x
    end function add_momentum_vector
    pure function add_vector_momentum (x, y) result (xy)
      type(vector), intent(in) :: x
      type(momentum), intent(in) :: y
      type(vector) :: xy
      xy%t = x%t + y%t
      xy%x = x%x + y%x
    end function add_vector_momentum
    pure function add_tensor2odd (x, y) result (xy)
      type(tensor2odd), intent(in) :: x, y
      type(tensor2odd) :: xy
      xy%e = x%e + y%e
      xy%b = x%b + y%b
    end function add_tensor2odd
```

⟨*Implementation of operations for vectors*⟩+≡

```
    pure function sub_momentum (x, y) result (xy)
      type(momentum), intent(in) :: x, y
      type(momentum) :: xy
      xy%t = x%t - y%t
      xy%x = x%x - y%x
    end function sub_momentum
    pure function sub_vector (x, y) result (xy)
      type(vector), intent(in) :: x, y
      type(vector) :: xy
      xy%t = x%t - y%t
      xy%x = x%x - y%x
    end function sub_vector
    pure function sub_momentum_vector (x, y) result (xy)
      type(momentum), intent(in) :: x
      type(vector), intent(in) :: y
```

```
      type(vector) :: xy
      xy%t = x%t - y%t
      xy%x = x%x - y%x
    end function sub_momentum_vector
  pure function sub_vector_momentum (x, y) result (xy)
      type(vector), intent(in) :: x
      type(momentum), intent(in) :: y
      type(vector) :: xy
      xy%t = x%t - y%t
      xy%x = x%x - y%x
    end function sub_vector_momentum
  pure function sub_tensor2odd (x, y) result (xy)
      type(tensor2odd), intent(in) :: x, y
      type(tensor2odd) :: xy
      xy%e = x%e - y%e
      xy%b = x%b - y%b
    end function sub_tensor2odd
```

X.4.6   Norm

Not the covariant length!

⟨*Declaration of operations for vectors*⟩+≡
```
  interface abs
     module procedure abs_momentum, abs_vector, abs_tensor2odd
  end interface
  private :: abs_momentum, abs_vector, abs_tensor2odd
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function abs_momentum (x) result (absx)
    type(momentum), intent(in) :: x
    real(kind=default) :: absx
    absx = sqrt (real (x%t*x%t + dot_product (x%x, x%x)))
  end function abs_momentum
  pure function abs_vector (x) result (absx)
    type(vector), intent(in) :: x
    real(kind=default) :: absx
    absx = sqrt (real (conjg(x%t)*x%t + dot_product (x%x, x%x)))
  end function abs_vector
  pure function abs_tensor2odd (x) result (absx)
    type(tensor2odd), intent(in) :: x
    real(kind=default) :: absx
    absx = sqrt (real (dot_product (x%e, x%e) + dot_product (x%b, x%b)))
  end function abs_tensor2odd
```

X.4.7   Conjugation

⟨*Declaration of operations for vectors*⟩+≡
```
  interface conjg
     module procedure conjg_momentum, conjg_vector, conjg_tensor2odd
  end interface
  private :: conjg_momentum, conjg_vector, conjg_tensor2odd
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function conjg_momentum (x) result (conjg_x)
```

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```
    type(momentum), intent(in) :: x
    type(momentum) :: conjg_x
    conjg_x = x
  end function conjg_momentum
  pure function conjg_vector (x) result (conjg_x)
    type(vector), intent(in) :: x
    type(vector) :: conjg_x
    conjg_x%t = conjg (x%t)
    conjg_x%x = conjg (x%x)
  end function conjg_vector
  pure function conjg_tensor2odd (t2) result (conjg_t2)
    type(tensor2odd), intent(in) :: t2
    type(tensor2odd) :: conjg_t2
    conjg_t2%e = conjg (t2%e)
    conjg_t2%b = conjg (t2%b)
  end function conjg_tensor2odd
```

X.4.8 ϵ-Tensors

$$\epsilon_{0123} = 1 = -\epsilon^{0123} \tag{X.8}$$

in particular

$$\epsilon(p_1, p_2, p_3, p_4) = \epsilon_{\mu_1\mu_2\mu_3\mu_4} p_1^{\mu_1} p_2^{\mu_2} p_3^{\mu_3} p_4^{\mu_4} = p_1^0 p_2^1 p_3^2 p_4^3 \pm \dots \tag{X.9}$$

⟨*Declaration of operations for vectors*⟩+≡
```
  interface pseudo_scalar
    module procedure pseudo_scalar_momentum, pseudo_scalar_vector, &
        pseudo_scalar_vec_mom
  end interface
  public :: pseudo_scalar
  private :: pseudo_scalar_momentum, pseudo_scalar_vector
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function pseudo_scalar_momentum (p1, p2, p3, p4) result (eps1234)
    type(momentum), intent(in) :: p1, p2, p3, p4
    real(kind=default) :: eps1234
    eps1234 = &
        p1%t    * p2%x(1) * (p3%x(2) * p4%x(3) - p3%x(3) * p4%x(2)) &
      + p1%t    * p2%x(2) * (p3%x(3) * p4%x(1) - p3%x(1) * p4%x(3)) &
      + p1%t    * p2%x(3) * (p3%x(1) * p4%x(2) - p3%x(2) * p4%x(1)) &
      - p1%x(1) * p2%x(2) * (p3%x(3) * p4%t    - p3%t    * p4%x(3)) &
      - p1%x(1) * p2%x(3) * (p3%t    * p4%x(2) - p3%x(2) * p4%t   ) &
      - p1%x(1) * p2%t    * (p3%x(2) * p4%x(3) - p3%x(3) * p4%x(2)) &
      + p1%x(2) * p2%x(3) * (p3%t    * p4%x(1) - p3%x(1) * p4%t   ) &
      + p1%x(2) * p2%t    * (p3%x(1) * p4%x(3) - p3%x(3) * p4%x(1)) &
      + p1%x(2) * p2%x(1) * (p3%x(3) * p4%t    - p3%t    * p4%x(3)) &
      - p1%x(3) * p2%t    * (p3%x(1) * p4%x(2) - p3%x(2) * p4%x(1)) &
      - p1%x(3) * p2%x(1) * (p3%x(2) * p4%t    - p3%t    * p4%x(2)) &
      - p1%x(3) * p2%x(2) * (p3%t    * p4%x(1) - p3%x(1) * p4%t   )
  end function pseudo_scalar_momentum
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function pseudo_scalar_vector (p1, p2, p3, p4) result (eps1234)
    type(vector), intent(in) :: p1, p2, p3, p4
```

```
      complex(kind=default) :: eps1234
      eps1234 = &
          p1%t    * p2%x(1) * (p3%x(2) * p4%x(3) - p3%x(3) * p4%x(2)) &
        + p1%t    * p2%x(2) * (p3%x(3) * p4%x(1) - p3%x(1) * p4%x(3)) &
        + p1%t    * p2%x(3) * (p3%x(1) * p4%x(2) - p3%x(2) * p4%x(1)) &
        - p1%x(1) * p2%x(2) * (p3%x(3) * p4%t    - p3%t    * p4%x(3)) &
        - p1%x(1) * p2%x(3) * (p3%t    * p4%x(2) - p3%x(2) * p4%t   ) &
        - p1%x(1) * p2%t    * (p3%x(2) * p4%x(3) - p3%x(3) * p4%x(2)) &
        + p1%x(2) * p2%x(3) * (p3%t    * p4%x(1) - p3%x(1) * p4%t   ) &
        + p1%x(2) * p2%t    * (p3%x(1) * p4%x(3) - p3%x(3) * p4%x(1)) &
        + p1%x(2) * p2%x(1) * (p3%x(3) * p4%t    - p3%t    * p4%x(3)) &
        - p1%x(3) * p2%t    * (p3%x(1) * p4%x(2) - p3%x(2) * p4%x(1)) &
        - p1%x(3) * p2%x(1) * (p3%x(2) * p4%t    - p3%t    * p4%x(2)) &
        - p1%x(3) * p2%x(2) * (p3%t    * p4%x(1) - p3%x(1) * p4%t   )
    end function pseudo_scalar_vector
```

⟨*Implementation of operations for vectors*⟩+≡

```
  pure function pseudo_scalar_vec_mom (p1, v1, p2, v2) result (eps1234)
    type(momentum), intent(in)  :: p1, p2
    type(vector), intent(in) :: v1, v2
    complex(kind=default) :: eps1234
    eps1234 = &
        p1%t    * v1%x(1) * (p2%x(2) * v2%x(3) - p2%x(3) * v2%x(2)) &
      + p1%t    * v1%x(2) * (p2%x(3) * v2%x(1) - p2%x(1) * v2%x(3)) &
      + p1%t    * v1%x(3) * (p2%x(1) * v2%x(2) - p2%x(2) * v2%x(1)) &
      - p1%x(1) * v1%x(2) * (p2%x(3) * v2%t    - p2%t    * v2%x(3)) &
      - p1%x(1) * v1%x(3) * (p2%t    * v2%x(2) - p2%x(2) * v2%t   ) &
      - p1%x(1) * v1%t    * (p2%x(2) * v2%x(3) - p2%x(3) * v2%x(2)) &
      + p1%x(2) * v1%x(3) * (p2%t    * v2%x(1) - p2%x(1) * v2%t   ) &
      + p1%x(2) * v1%t    * (p2%x(1) * v2%x(3) - p2%x(3) * v2%x(1)) &
      + p1%x(2) * v1%x(1) * (p2%x(3) * v2%t    - p2%t    * v2%x(3)) &
      - p1%x(3) * v1%t    * (p2%x(1) * v2%x(2) - p2%x(2) * v2%x(1)) &
      - p1%x(3) * v1%x(1) * (p2%x(2) * v2%t    - p2%t    * v2%x(2)) &
      - p1%x(3) * v1%x(2) * (p2%t    * v2%x(1) - p2%x(1) * v2%t   )
  end function pseudo_scalar_vec_mom
```

$$\epsilon_\mu(p_1, p_2, p_3) = \epsilon_{\mu\mu_1\mu_2\mu_3} p_1^{\mu_1} p_2^{\mu_2} p_3^{\mu_3} \tag{X.10}$$

i. e.

$$\epsilon_0(p_1, p_2, p_3) = p_1^1 p_2^2 p_3^3 \pm \ldots \tag{X.11a}$$

$$\epsilon_1(p_1, p_2, p_3) = p_1^2 p_2^3 p_3^0 \pm \ldots \tag{X.11b}$$

$$\epsilon_2(p_1, p_2, p_3) = -p_1^3 p_2^0 p_3^1 \pm \ldots \tag{X.11c}$$

$$\epsilon_3(p_1, p_2, p_3) = p_1^0 p_2^1 p_3^2 \pm \ldots \tag{X.11d}$$

⟨*Declaration of operations for vectors*⟩+≡

```
  interface pseudo_vector
    module procedure pseudo_vector_momentum, pseudo_vector_vector, &
        pseudo_vector_vec_mom
  end interface
  public :: pseudo_vector
  private :: pseudo_vector_momentum, pseudo_vector_vector
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function pseudo_vector_momentum (p1, p2, p3) result (eps123)
    type(momentum), intent(in) :: p1, p2, p3
    type(momentum) :: eps123
    eps123%t = &
      + p1%x(1) * (p2%x(2) * p3%x(3) - p2%x(3) * p3%x(2)) &
      + p1%x(2) * (p2%x(3) * p3%x(1) - p2%x(1) * p3%x(3)) &
      + p1%x(3) * (p2%x(1) * p3%x(2) - p2%x(2) * p3%x(1))
    eps123%x(1) = &
      + p1%x(2) * (p2%x(3) * p3%t   - p2%t    * p3%x(3)) &
      + p1%x(3) * (p2%t    * p3%x(2) - p2%x(2) * p3%t   ) &
      + p1%t    * (p2%x(2) * p3%x(3) - p2%x(3) * p3%x(2))
    eps123%x(2) = &
      - p1%x(3) * (p2%t    * p3%x(1) - p2%x(1) * p3%t   ) &
      - p1%t    * (p2%x(1) * p3%x(3) - p2%x(3) * p3%x(1)) &
      - p1%x(1) * (p2%x(3) * p3%t   - p2%t    * p3%x(3))
    eps123%x(3) =   &
      + p1%t    * (p2%x(1) * p3%x(2) - p2%x(2) * p3%x(1)) &
      + p1%x(1) * (p2%x(2) * p3%t   - p2%t    * p3%x(2)) &
      + p1%x(2) * (p2%t    * p3%x(1) - p2%x(1) * p3%t   )
  end function pseudo_vector_momentum
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function pseudo_vector_vector (p1, p2, p3) result (eps123)
    type(vector), intent(in) :: p1, p2, p3
    type(vector) :: eps123
    eps123%t = &
      + p1%x(1) * (p2%x(2) * p3%x(3) - p2%x(3) * p3%x(2)) &
      + p1%x(2) * (p2%x(3) * p3%x(1) - p2%x(1) * p3%x(3)) &
      + p1%x(3) * (p2%x(1) * p3%x(2) - p2%x(2) * p3%x(1))
    eps123%x(1) = &
      + p1%x(2) * (p2%x(3) * p3%t   - p2%t    * p3%x(3)) &
      + p1%x(3) * (p2%t    * p3%x(2) - p2%x(2) * p3%t   ) &
      + p1%t    * (p2%x(2) * p3%x(3) - p2%x(3) * p3%x(2))
    eps123%x(2) = &
      - p1%x(3) * (p2%t    * p3%x(1) - p2%x(1) * p3%t   ) &
      - p1%t    * (p2%x(1) * p3%x(3) - p2%x(3) * p3%x(1)) &
      - p1%x(1) * (p2%x(3) * p3%t   - p2%t    * p3%x(3))
    eps123%x(3) =   &
      + p1%t    * (p2%x(1) * p3%x(2) - p2%x(2) * p3%x(1)) &
      + p1%x(1) * (p2%x(2) * p3%t   - p2%t    * p3%x(2)) &
      + p1%x(2) * (p2%t    * p3%x(1) - p2%x(1) * p3%t   )
  end function pseudo_vector_vector
```

⟨*Implementation of operations for vectors*⟩+≡
```
  pure function pseudo_vector_vec_mom (p1, p2, v) result (eps123)
    type(momentum), intent(in) :: p1, p2
    type(vector), intent(in)   :: v
    type(vector) :: eps123
    eps123%t = &
      + p1%x(1) * (p2%x(2) * v%x(3) - p2%x(3) * v%x(2)) &
      + p1%x(2) * (p2%x(3) * v%x(1) - p2%x(1) * v%x(3)) &
      + p1%x(3) * (p2%x(1) * v%x(2) - p2%x(2) * v%x(1))
    eps123%x(1) = &
      + p1%x(2) * (p2%x(3) * v%t   - p2%t    * v%x(3)) &
```

```
      + p1%x(3) * (p2%t    * v%x(2) - p2%x(2) * v%t    ) &
      + p1%t    * (p2%x(2) * v%x(3) - p2%x(3) * v%x(2))
   eps123%x(2) = &
      - p1%x(3) * (p2%t    * v%x(1) - p2%x(1) * v%t    ) &
      - p1%t    * (p2%x(1) * v%x(3) - p2%x(3) * v%x(1)) &
      - p1%x(1) * (p2%x(3) * v%t    - p2%t    * v%x(3))
   eps123%x(3) =  &
      + p1%t    * (p2%x(1) * v%x(2) - p2%x(2) * v%x(1)) &
      + p1%x(1) * (p2%x(2) * v%t    - p2%t    * v%x(2)) &
      + p1%x(2) * (p2%t    * v%x(1) - p2%x(1) * v%t    )
end function pseudo_vector_vec_mom
```

X.4.9   Utilities

⟨*Declaration of operations for vectors*⟩+≡

⟨*Implementation of operations for vectors*⟩+≡

```
subroutine random_momentum (p, pabs, m)
   type(momentum), intent(out) :: p
   real(kind=default), intent(in) :: pabs, m
   real(kind=default), dimension(2) :: r
   real(kind=default) :: phi, cos_th
   call random_number (r)
   phi = 2*PI * r(1)
   cos_th = 2 * r(2) - 1
   p%t = sqrt (pabs**2 + m**2)
   p%x = pabs * (/ cos_th * cos(phi), cos_th * sin(phi), sqrt (1 - cos_th**2) /)
end subroutine random_momentum
```

X.5   Polarization vectors

⟨`omega_polarizations.f90`⟩≡
```
  ⟨Copyleft⟩
  module omega_polarizations
    use kinds
    use constants
    use omega_vectors
    implicit none
    private
    ⟨Declaration of polarization vectors⟩
    integer, parameter, public :: omega_polarizations_2010_01_A = 0
  contains
    ⟨Implementation of polarization vectors⟩
  end module omega_polarizations
```

Here we use a phase convention for the polarization vectors compatible with the angular momentum coupling to spin 3/2 and spin 2.

$$\epsilon_1^\mu(k) = \frac{1}{|\vec{k}|\sqrt{k_x^2 + k_y^2}} \left(0; k_z k_x, k_y k_z, -k_x^2 - k_y^2\right) \qquad\text{(X.12a)}$$

$$\epsilon_2^\mu(k) = \frac{1}{\sqrt{k_x^2 + k_y^2}} \left(0; -k_y, k_x, 0\right) \qquad\text{(X.12b)}$$

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$$\epsilon_3^\mu(k) = \frac{k_0}{m|\vec{k}|} \left(\vec{k}^2/k_0; k_x, k_y, k_z \right) \tag{X.12c}$$

and

$$\epsilon_\pm^\mu(k) = \frac{1}{\sqrt{2}} (\epsilon_1^\mu(k) \pm \mathrm{i}\epsilon_2^\mu(k)) \tag{X.13a}$$

$$\epsilon_0^\mu(k) = \epsilon_3^\mu(k) \tag{X.13b}$$

i. e.

$$\epsilon_+^\mu(k) = \frac{1}{\sqrt{2}\sqrt{k_x^2 + k_y^2}} \left(0; \frac{k_z k_x}{|\vec{k}|} - \mathrm{i}k_y, \frac{k_y k_z}{|\vec{k}|} + \mathrm{i}k_x, -\frac{k_x^2 + k_y^2}{|\vec{k}|} \right) \tag{X.14a}$$

$$\epsilon_-^\mu(k) = \frac{1}{\sqrt{2}\sqrt{k_x^2 + k_y^2}} \left(0; \frac{k_z k_x}{|\vec{k}|} + \mathrm{i}k_y, \frac{k_y k_z}{|\vec{k}|} - \mathrm{i}k_x, -\frac{k_x^2 + k_y^2}{|\vec{k}|} \right) \tag{X.14b}$$

$$\epsilon_0^\mu(k) = \frac{k_0}{m|\vec{k}|} \left(\vec{k}^2/k_0; k_x, k_y, k_z \right) \tag{X.14c}$$

Determining the mass from the momenta is a numerically haphazardous for light particles. Therefore, we accept some redundancy and pass the mass explicitly.

⟨*Declaration of polarization vectors*⟩≡
```
  public :: eps
```

⟨*Implementation of polarization vectors*⟩≡
```
  pure function eps (m, k, s) result (e)
    type(vector) :: e
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: k
    integer, intent(in) :: s
    real(kind=default) :: kt, kabs, kabs2, sqrt2
    sqrt2 = sqrt (2.0_default)
    kabs2 = dot_product (k%x, k%x)
    e%t = 0
    e%x = 0
    if (kabs2 > 0) then
       kabs = sqrt (kabs2)
       select case (s)
       case (1)
          kt = sqrt (k%x(1)**2 + k%x(2)**2)
          if (abs(kt) <= epsilon(kt) * kabs) then
             if (k%x(3) > 0) then
                e%x(1) = cmplx (   1,   0, kind=default) / sqrt2
                e%x(2) = cmplx (   0,   1, kind=default) / sqrt2
             else
                e%x(1) = cmplx ( - 1,   0, kind=default) / sqrt2
                e%x(2) = cmplx (   0,   1, kind=default) / sqrt2
             end if
          else
             e%x(1) = cmplx (   k%x(3)*k%x(1)/kabs, &
                  - k%x(2), kind=default) / kt / sqrt2
             e%x(2) = cmplx (   k%x(2)*k%x(3)/kabs, &
                  k%x(1), kind=default) / kt / sqrt2
```

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```
             e%x(3) = - kt / kabs / sqrt2
          end if
       case (-1)
          kt = sqrt (k%x(1)**2 + k%x(2)**2)
          if (abs(kt) <= epsilon(kt) * kabs) then
             if (k%x(3) > 0) then
                e%x(1) = cmplx (   1,   0, kind=default) / sqrt2
                e%x(2) = cmplx (   0, - 1, kind=default) / sqrt2
             else
                e%x(1) = cmplx (  -1,   0, kind=default) / sqrt2
                e%x(2) = cmplx (   0, - 1, kind=default) / sqrt2
             end if
          else
             e%x(1) = cmplx (   k%x(3)*k%x(1)/kabs, &
                   k%x(2), kind=default) / kt / sqrt2
             e%x(2) = cmplx (   k%x(2)*k%x(3)/kabs, &
                   - k%x(1), kind=default) / kt / sqrt2
             e%x(3) = - kt / kabs / sqrt2
          end if
       case (0)
          if (m > 0) then
             e%t = kabs / m
             e%x = k%t / (m*kabs) * k%x
          end if
       case (3)
          e = (0,1) * k
       case (4)
          if (m > 0) then
             e = (1 / m) * k
          else
             e = (1 / k%t) * k
          end if
       end select
    else   !!! for particles in their rest frame defined to be
           !!! polarized along the 3-direction
       select case (s)
       case (1)
          e%x(1) = cmplx (   1,   0, kind=default) / sqrt2
          e%x(2) = cmplx (   0,   1, kind=default) / sqrt2
       case (-1)
          e%x(1) = cmplx (   1,   0, kind=default) / sqrt2
          e%x(2) = cmplx (   0, - 1, kind=default) / sqrt2
       case (0)
          if (m > 0) then
             e%x(3) = 1
          end if
       case (4)
          if (m > 0) then
             e = (1 / m) * k
          else
             e = (1 / k%t) * k
          end if
       end select
    end if
```

```
end function eps
```

X.6   *Polarization vectors revisited*

⟨omega_polarizations_madgraph.f90⟩≡
  ⟨*Copyleft*⟩
  module omega_polarizations_madgraph
    use kinds
    use constants
    use omega_vectors
    implicit none
    private
    ⟨*Declaration of polarization vectors for madgraph*⟩
    integer, parameter, public :: omega_pols_madgraph_2010_01_A = 0
  contains
    ⟨*Implementation of polarization vectors for madgraph*⟩
  end module omega_polarizations_madgraph

This set of polarization vectors is compatible with HELAS [5]:

$$\epsilon_1^\mu(k) = \frac{1}{|\vec{k}|\sqrt{k_x^2 + k_y^2}} \left(0; k_z k_x, k_y k_z, -k_x^2 - k_y^2\right) \tag{X.15a}$$

$$\epsilon_2^\mu(k) = \frac{1}{\sqrt{k_x^2 + k_y^2}} \left(0; -k_y, k_x, 0\right) \tag{X.15b}$$

$$\epsilon_3^\mu(k) = \frac{k_0}{m|\vec{k}|} \left(\vec{k}^2/k_0; k_x, k_y, k_z\right) \tag{X.15c}$$

and

$$\epsilon_\pm^\mu(k) = \frac{1}{\sqrt{2}}(\mp\epsilon_1^\mu(k) - \mathrm{i}\epsilon_2^\mu(k)) \tag{X.16a}$$

$$\epsilon_0^\mu(k) = \epsilon_3^\mu(k) \tag{X.16b}$$

i.e.

$$\epsilon_+^\mu(k) = \frac{1}{\sqrt{2}\sqrt{k_x^2 + k_y^2}} \left(0; -\frac{k_z k_x}{|\vec{k}|} + \mathrm{i}k_y, -\frac{k_y k_z}{|\vec{k}|} - \mathrm{i}k_x, \frac{k_x^2 + k_y^2}{|\vec{k}|}\right) \tag{X.17a}$$

$$\epsilon_-^\mu(k) = \frac{1}{\sqrt{2}\sqrt{k_x^2 + k_y^2}} \left(0; \frac{k_z k_x}{|\vec{k}|} + \mathrm{i}k_y, \frac{k_y k_z}{|\vec{k}|} - \mathrm{i}k_x, -\frac{k_x^2 + k_y^2}{|\vec{k}|}\right) \tag{X.17b}$$

$$\epsilon_0^\mu(k) = \frac{k_0}{m|\vec{k}|} \left(\vec{k}^2/k_0; k_x, k_y, k_z\right) \tag{X.17c}$$

Fortunately, for comparing with squared matrix generated by Madgraph we can also use the modified version, since the difference is only a phase and does *not* mix helicity states. Determining the mass from the momenta is a numerically haphazardous for light particles. Therefore, we accept some redundancy and pass the mass explicitely.

⟨*Declaration of polarization vectors for madgraph*⟩≡
  public :: eps

⟨Implementation of polarization vectors for madgraph⟩≡

```
pure function eps (m, k, s) result (e)
  type(vector) :: e
  real(kind=default), intent(in) :: m
  type(momentum), intent(in) :: k
  integer, intent(in) :: s
  real(kind=default) :: kt, kabs, kabs2, sqrt2
  sqrt2 = sqrt (2.0_default)
  kabs2 = dot_product (k%x, k%x)
  e%t = 0
  e%x = 0
  if (kabs2 > 0) then
     kabs = sqrt (kabs2)
     select case (s)
     case (1)
        kt = sqrt (k%x(1)**2 + k%x(2)**2)
        if (abs(kt) <= epsilon(kt) * kabs) then
           if (k%x(3) > 0) then
              e%x(1) = cmplx ( - 1,   0, kind=default) / sqrt2
              e%x(2) = cmplx (   0, - 1, kind=default) / sqrt2
           else
              e%x(1) = cmplx (   1,   0, kind=default) / sqrt2
              e%x(2) = cmplx (   0, - 1, kind=default) / sqrt2
           end if
        else
           e%x(1) = cmplx ( - k%x(3)*k%x(1)/kabs, &
                 k%x(2), kind=default) / kt / sqrt2
           e%x(2) = cmplx ( - k%x(2)*k%x(3)/kabs, &
                 - k%x(1), kind=default) / kt / sqrt2
           e%x(3) = kt / kabs / sqrt2
        end if
     case (-1)
        kt = sqrt (k%x(1)**2 + k%x(2)**2)
        if (abs(kt) <= epsilon(kt) * kabs) then
           if (k%x(3) > 0) then
              e%x(1) = cmplx (   1,   0, kind=default) / sqrt2
              e%x(2) = cmplx (   0, - 1, kind=default) / sqrt2
           else
              e%x(1) = cmplx (  -1,   0, kind=default) / sqrt2
              e%x(2) = cmplx (   0, - 1, kind=default) / sqrt2
           end if
        else
           e%x(1) = cmplx (   k%x(3)*k%x(1)/kabs, &
                 k%x(2), kind=default) / kt / sqrt2
           e%x(2) = cmplx (   k%x(2)*k%x(3)/kabs, &
                 - k%x(1), kind=default) / kt / sqrt2
           e%x(3) = - kt / kabs / sqrt2
        end if
     case (0)
        if (m > 0) then
           e%t = kabs / m
           e%x = k%t / (m*kabs) * k%x
        end if
     case (3)
```

```
          e = (0,1) * k
      case (4)
         if (m > 0) then
            e = (1 / m) * k
         else
            e = (1 / k%t) * k
         end if
      end select
   else   !!! for particles in their rest frame defined to be
          !!! polarized along the 3-direction
      select case (s)
      case (1)
         e%x(1) = cmplx ( - 1,    0, kind=default) / sqrt2
         e%x(2) = cmplx (    0, - 1, kind=default) / sqrt2
      case (-1)
         e%x(1) = cmplx (    1,    0, kind=default) / sqrt2
         e%x(2) = cmplx (    0, - 1, kind=default) / sqrt2
      case (0)
         if (m > 0) then
            e%x(3) = 1
         end if
      case (4)
         if (m > 0) then
            e = (1 / m) * k
         else
            e = (1 / k%t) * k
         end if
      end select
   end if
 end function eps
```

X.7   Symmetric Tensors

Spin-2 polarization tensors are symmetric, transversal and traceless

$$\epsilon_m^{\mu\nu}(k) = \epsilon_m^{\nu\mu}(k) \tag{X.18a}$$

$$k_\mu \epsilon_m^{\mu\nu}(k) = k_\nu \epsilon_m^{\mu\nu}(k) = 0 \tag{X.18b}$$

$$\epsilon_{m,\mu}^{\mu}(k) = 0 \tag{X.18c}$$

with $m = 1, 2, 3, 4, 5$. Our current representation is redundant and does *not* enforce symmetry or tracelessness.

⟨omega_tensors.f90⟩≡
  ⟨*Copyleft*⟩
  module omega_tensors
    use kinds
    use constants
    use omega_vectors
    implicit none
    private
    public :: operator (*), operator (+), operator (-), &
         operator (.tprod.)
    public :: abs, conjg

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```
⟨intrinsic :: abs⟩
⟨intrinsic :: conjg⟩
type, public :: tensor
   ! private (omegalib needs access, but DON'T TOUCH IT!)
   complex(kind=default), dimension(0:3,0:3) :: t
end type tensor
⟨Declaration of operations for tensors⟩
integer, parameter, public :: omega_tensors_2010_01_A = 0
contains
⟨Implementation of operations for tensors⟩
end module omega_tensors
```

X.7.1   Vector Space

Scalar Multliplication

⟨*Declaration of operations for tensors*⟩≡

```
interface operator (*)
   module procedure integer_tensor, real_tensor, double_tensor, &
        complex_tensor, dcomplex_tensor
end interface
private :: integer_tensor, real_tensor, double_tensor
private :: complex_tensor, dcomplex_tensor
```

⟨*Implementation of operations for tensors*⟩≡

```
pure function integer_tensor (x, y) result (xy)
  integer, intent(in) :: x
  type(tensor), intent(in) :: y
  type(tensor) :: xy
  xy%t = x * y%t
end function integer_tensor
pure function real_tensor (x, y) result (xy)
  real(kind=single), intent(in) :: x
  type(tensor), intent(in) :: y
  type(tensor) :: xy
  xy%t = x * y%t
end function real_tensor
pure function double_tensor (x, y) result (xy)
  real(kind=default), intent(in) :: x
  type(tensor), intent(in) :: y
  type(tensor) :: xy
  xy%t = x * y%t
end function double_tensor
pure function complex_tensor (x, y) result (xy)
  complex(kind=single), intent(in) :: x
  type(tensor), intent(in) :: y
  type(tensor) :: xy
  xy%t = x * y%t
end function complex_tensor
pure function dcomplex_tensor (x, y) result (xy)
  complex(kind=default), intent(in) :: x
  type(tensor), intent(in) :: y
  type(tensor) :: xy
  xy%t = x * y%t
end function dcomplex_tensor
```

Addition and Subtraction

⟨*Declaration of operations for tensors*⟩+≡

```
interface operator (+)
   module procedure plus_tensor
end interface
private :: plus_tensor
interface operator (-)
  module procedure neg_tensor
end interface
private :: neg_tensor
```

⟨*Implementation of operations for tensors*⟩+≡

```
pure function plus_tensor (t1) result (t2)
  type(tensor), intent(in) :: t1
  type(tensor) :: t2
  t2 = t1
end function plus_tensor
pure function neg_tensor (t1) result (t2)
  type(tensor), intent(in) :: t1
  type(tensor) :: t2
  t2%t = - t1%t
end function neg_tensor
```

⟨*Declaration of operations for tensors*⟩+≡

```
interface operator (+)
   module procedure add_tensor
end interface
private :: add_tensor
interface operator (-)
   module procedure sub_tensor
end interface
private :: sub_tensor
```

⟨*Implementation of operations for tensors*⟩+≡

```
pure function add_tensor (x, y) result (xy)
  type(tensor), intent(in) :: x, y
  type(tensor) :: xy
  xy%t = x%t + y%t
end function add_tensor
pure function sub_tensor (x, y) result (xy)
  type(tensor), intent(in) :: x, y
  type(tensor) :: xy
  xy%t = x%t - y%t
end function sub_tensor
```

⟨*Declaration of operations for tensors*⟩+≡

```
interface operator (.tprod.)
   module procedure out_prod_vv, out_prod_vm, &
       out_prod_mv, out_prod_mm
end interface
private :: out_prod_vv, out_prod_vm, &
    out_prod_mv, out_prod_mm
```

⟨*Implementation of operations for tensors*⟩+≡

```
pure function out_prod_vv (v, w) result (t)
  type(tensor) :: t
```

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```
      type(vector), intent(in) :: v, w
      integer :: i, j
      t%t(0,0) = v%t * w%t
      t%t(0,1:3) = v%t * w%x
      t%t(1:3,0) = v%x * w%t
      do i = 1, 3
         do j = 1, 3
            t%t(i,j) = v%x(i) * w%x(j)
         end do
      end do
   end function out_prod_vv
```

⟨*Implementation of operations for tensors*⟩+≡
```
   pure function out_prod_vm (v, m) result (t)
      type(tensor) :: t
      type(vector), intent(in) :: v
      type(momentum), intent(in) :: m
      integer :: i, j
      t%t(0,0) = v%t * m%t
      t%t(0,1:3) = v%t * m%x
      t%t(1:3,0) = v%x * m%t
      do i = 1, 3
         do j = 1, 3
            t%t(i,j) = v%x(i) * m%x(j)
         end do
      end do
   end function out_prod_vm
```

⟨*Implementation of operations for tensors*⟩+≡
```
   pure function out_prod_mv (m, v) result (t)
      type(tensor) :: t
      type(vector), intent(in) :: v
      type(momentum), intent(in) :: m
      integer :: i, j
      t%t(0,0) = m%t * v%t
      t%t(0,1:3) = m%t * v%x
      t%t(1:3,0) = m%x * v%t
      do i = 1, 3
         do j = 1, 3
            t%t(i,j) = m%x(i) * v%x(j)
         end do
      end do
   end function out_prod_mv
```

⟨*Implementation of operations for tensors*⟩+≡
```
   pure function out_prod_mm (m, n) result (t)
      type(tensor) :: t
      type(momentum), intent(in) :: m, n
      integer :: i, j
      t%t(0,0) = m%t * n%t
      t%t(0,1:3) = m%t * n%x
      t%t(1:3,0) = m%x * n%t
      do i = 1, 3
         do j = 1, 3
            t%t(i,j) = m%x(i) * n%x(j)
         end do
```

```
      end do
   end function out_prod_mm
```

⟨*Declaration of operations for tensors*⟩+≡
```
   interface abs
      module procedure abs_tensor
   end interface
   private :: abs_tensor
```

⟨*Implementation of operations for tensors*⟩+≡
```
   pure function abs_tensor (t) result (abs_t)
      type(tensor), intent(in) :: t
      real(kind=default) :: abs_t
      abs_t = sqrt (sum ((abs (t%t))**2))
   end function abs_tensor
```

⟨*Declaration of operations for tensors*⟩+≡
```
   interface conjg
      module procedure conjg_tensor
   end interface
   private :: conjg_tensor
```

⟨*Implementation of operations for tensors*⟩+≡
```
   pure function conjg_tensor (t) result (conjg_t)
      type(tensor), intent(in) :: t
      type(tensor) :: conjg_t
      conjg_t%t = conjg (t%t)
   end function conjg_tensor
```

⟨*Declaration of operations for tensors*⟩+≡
```
   interface operator (*)
      module procedure tensor_tensor, vector_tensor, tensor_vector, &
            momentum_tensor, tensor_momentum
   end interface
   private :: tensor_tensor, vector_tensor, tensor_vector, &
        momentum_tensor, tensor_momentum
```

⟨*Implementation of operations for tensors*⟩+≡
```
   pure function tensor_tensor (t1, t2) result (t1t2)
      type(tensor), intent(in) :: t1
      type(tensor), intent(in) :: t2
      complex(kind=default) :: t1t2
      integer :: i1, i2
      t1t2 = t1%t(0,0)*t2%t(0,0) &
          - dot_product (conjg (t1%t(0,1:)), t2%t(0,1:)) &
          - dot_product (conjg (t1%t(1:,0)), t2%t(1:,0))
      do i1 = 1, 3
         do i2 = 1, 3
            t1t2 = t1t2 + t1%t(i1,i2)*t2%t(i1,i2)
         end do
      end do
   end function tensor_tensor
```

⟨*Implementation of operations for tensors*⟩+≡
```
   pure function tensor_vector (t, v) result (tv)
      type(tensor), intent(in) :: t
      type(vector), intent(in) :: v
      type(vector) :: tv
```

```
   tv%t =    t%t(0,0) * v%t - dot_product (conjg (t%t(0,1:)), v%x)
   tv%x(1) = t%t(0,1) * v%t - dot_product (conjg (t%t(1,1:)), v%x)
   tv%x(2) = t%t(0,2) * v%t - dot_product (conjg (t%t(2,1:)), v%x)
   tv%x(3) = t%t(0,3) * v%t - dot_product (conjg (t%t(3,1:)), v%x)
 end function tensor_vector
```

⟨*Implementation of operations for tensors*⟩+≡
```
 pure function vector_tensor (v, t) result (vt)
   type(vector), intent(in) :: v
   type(tensor), intent(in) :: t
   type(vector) :: vt
   vt%t =    v%t * t%t(0,0) - dot_product (conjg (v%x), t%t(1:,0))
   vt%x(1) = v%t * t%t(0,1) - dot_product (conjg (v%x), t%t(1:,1))
   vt%x(2) = v%t * t%t(0,2) - dot_product (conjg (v%x), t%t(1:,2))
   vt%x(3) = v%t * t%t(0,3) - dot_product (conjg (v%x), t%t(1:,3))
 end function vector_tensor
```

⟨*Implementation of operations for tensors*⟩+≡
```
 pure function tensor_momentum (t, p) result (tp)
   type(tensor), intent(in) :: t
   type(momentum), intent(in) :: p
   type(vector) :: tp
   tp%t =    t%t(0,0) * p%t - dot_product (conjg (t%t(0,1:)), p%x)
   tp%x(1) = t%t(0,1) * p%t - dot_product (conjg (t%t(1,1:)), p%x)
   tp%x(2) = t%t(0,2) * p%t - dot_product (conjg (t%t(2,1:)), p%x)
   tp%x(3) = t%t(0,3) * p%t - dot_product (conjg (t%t(3,1:)), p%x)
 end function tensor_momentum
```

⟨*Implementation of operations for tensors*⟩+≡
```
 pure function momentum_tensor (p, t) result (pt)
   type(momentum), intent(in) :: p
   type(tensor), intent(in) :: t
   type(vector) :: pt
   pt%t =    p%t * t%t(0,0) - dot_product (p%x, t%t(1:,0))
   pt%x(1) = p%t * t%t(0,1) - dot_product (p%x, t%t(1:,1))
   pt%x(2) = p%t * t%t(0,2) - dot_product (p%x, t%t(1:,2))
   pt%x(3) = p%t * t%t(0,3) - dot_product (p%x, t%t(1:,3))
 end function momentum_tensor
```

X.8   Symmetric Polarization Tensors

$$\epsilon_{+2}^{\mu\nu}(k) = \epsilon_+^\mu(k)\epsilon_+^\nu(k) \tag{X.19a}$$

$$\epsilon_{+1}^{\mu\nu}(k) = \frac{1}{\sqrt{2}}\left(\epsilon_+^\mu(k)\epsilon_0^\nu(k) + \epsilon_0^\mu(k)\epsilon_+^\nu(k)\right) \tag{X.19b}$$

$$\epsilon_0^{\mu\nu}(k) = \frac{1}{\sqrt{6}}\left(\epsilon_+^\mu(k)\epsilon_-^\nu(k) + \epsilon_-^\mu(k)\epsilon_+^\nu(k) - 2\epsilon_0^\mu(k)\epsilon_0^\nu(k)\right) \tag{X.19c}$$

$$\epsilon_{-1}^{\mu\nu}(k) = \frac{1}{\sqrt{2}}\left(\epsilon_-^\mu(k)\epsilon_0^\nu(k) + \epsilon_0^\mu(k)\epsilon_-^\nu(k)\right) \tag{X.19d}$$

$$\epsilon_{-2}^{\mu\nu}(k) = \epsilon_-^\mu(k)\epsilon_-^\nu(k) \tag{X.19e}$$

Note that $\epsilon^{\mu}_{\pm 2, \mu}(k) = \epsilon^{\mu}_{\pm}(k)\epsilon_{\pm,\mu}(k) \propto \epsilon^{\mu}_{\pm}(k)\epsilon^{*}_{\mp,\mu}(k) = 0$ and that the sign in $\epsilon^{\mu\nu}_{0}(k)$ insures its tracelessness[1].

⟨omega_tensor_polarizations.f90⟩≡

  ⟨*Copyleft*⟩
```
module omega_tensor_polarizations
  use kinds
  use constants
  use omega_vectors
  use omega_tensors
  use omega_polarizations
  implicit none
  private
```
    ⟨*Declaration of polarization tensors*⟩
```
  integer, parameter, public :: omega_tensor_pols_2010_01_A = 0
contains
```
    ⟨*Implementation of polarization tensors*⟩
```
end module omega_tensor_polarizations
```

⟨*Declaration of polarization tensors*⟩≡
```
public :: eps2
```

⟨*Implementation of polarization tensors*⟩≡
```
pure function eps2 (m, k, s) result (t)
  type(tensor) :: t
  real(kind=default), intent(in) :: m
  type(momentum), intent(in) :: k
  integer, intent(in) :: s
  type(vector) :: ep, em, e0
  t%t = 0
  select case (s)
  case (2)
     ep = eps (m, k, 1)
     t = ep.tprod.ep
  case (1)
     ep = eps (m, k, 1)
     e0 = eps (m, k, 0)
     t = (1 / sqrt (2.0_default)) &
           * ((ep.tprod.e0) + (e0.tprod.ep))
  case (0)
     ep = eps (m, k, 1)
     e0 = eps (m, k, 0)
     em = eps (m, k, -1)
     t = (1 / sqrt (6.0_default)) &
           * ((ep.tprod.em) + (em.tprod.ep) - 2*(e0.tprod.e0))
  case (-1)
     e0 = eps (m, k, 0)
     em = eps (m, k, -1)
     t = (1 / sqrt (2.0_default)) &
           * ((em.tprod.e0) + (e0.tprod.em))
  case (-2)
```

---

[1] On the other hand, with the shift operator $L_- \left|+\right\rangle = \mathrm{e}^{\mathrm{i}\phi}\left|0\right\rangle$ and $L_- \left|0\right\rangle = \mathrm{e}^{\mathrm{i}\chi}\left|-\right\rangle$, we find
$$L^2_- \left|++\right\rangle = 2\mathrm{e}^{2\mathrm{i}\phi}\left|00\right\rangle + \mathrm{e}^{\mathrm{i}(\phi+\chi)}(\left|+-\right\rangle + \left|-+\right\rangle)$$
i.e. $\chi - \phi = \pi$, if we want to identify $\epsilon^{\mu}_{-,0,+}$ with $\left|-,0,+\right\rangle$.

```
      em = eps (m, k, -1)
      t = em.tprod.em
   end select
end function eps2
```

X.9   Couplings

⟨omega_couplings.f90⟩≡
  ⟨*Copyleft*⟩
  module omega_couplings
    use kinds
    use constants
    use omega_vectors
    use omega_tensors
    implicit none
    private
    ⟨*Declaration of couplings*⟩
    ⟨*Declaration of propagators*⟩
    integer, parameter, public :: omega_couplings_2010_01_A = 0
  contains
    ⟨*Implementation of couplings*⟩
    ⟨*Implementation of propagators*⟩
  end module omega_couplings

⟨*Declaration of propagators*⟩≡
  public :: wd_tl

⟨*Declaration of propagators*⟩+≡
  public :: gauss

$$\Theta(p^2)\Gamma \tag{X.20}$$

⟨*Implementation of propagators*⟩≡
  pure function wd_tl (p, w) result (width)
    real(kind=default) :: width
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: w
    if (p*p > 0) then
       width = w
    else
       width = 0
    end if
  end function wd_tl

⟨*Implementation of propagators*⟩+≡
  pure function gauss (x, mu, w) result (gg)
    real(kind=default) :: gg
    real(kind=default), intent(in) :: x, mu, w
    if (w > 0) then
       gg = exp(-(x - mu**2)**2/4.0_default/mu**2/w**2) * &
            sqrt(sqrt(PI/2)) / w / mu
       else
       gg = 1.0_default
    end if
  end function gauss
```

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⟨*Declaration of propagators*⟩+≡
```
  public :: pr_phi, pr_unitarity, pr_feynman, pr_gauge, pr_rxi
  public :: pr_vector_pure
  public :: pj_phi, pj_unitarity
  public :: pg_phi, pg_unitarity
```

$$\frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma}\phi \qquad\qquad (\mathrm{X}.21)$$

⟨*Implementation of propagators*⟩+≡
```
  pure function pr_phi (p, m, w, phi) result (pphi)
    complex(kind=default) :: pphi
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    complex(kind=default), intent(in) :: phi
    pphi = (1 / cmplx (p*p - m**2, m*w, kind=default)) * phi
  end function pr_phi
```

$$\sqrt{\frac{\pi}{M\Gamma}}\phi \qquad\qquad (\mathrm{X}.22)$$

⟨*Implementation of propagators*⟩+≡
```
  pure function pj_phi (m, w, phi) result (pphi)
    complex(kind=default) :: pphi
    real(kind=default), intent(in) :: m, w
    complex(kind=default), intent(in) :: phi
    pphi = (0, -1) * sqrt (PI / m / w) * phi
  end function pj_phi
```

⟨*Implementation of propagators*⟩+≡
```
  pure function pg_phi (p, m, w, phi) result (pphi)
    complex(kind=default) :: pphi
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    complex(kind=default), intent(in) :: phi
    pphi = ((0, 1) * gauss (p*p, m, w)) * phi
  end function pg_phi
```

$$\frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)\epsilon^\nu(p) \qquad\qquad (\mathrm{X}.23)$$

NB: the explicit cast to `vector` is required here, because a specific `complex_momentum` procedure for `operator (*)` would introduce ambiguities. NB: we used to use the constructor `vector (p%t, p%x)` instead of the temporary variable, but the Intel Fortran Compiler choked on it.

⟨*Implementation of propagators*⟩+≡
```
  pure function pr_unitarity (p, m, w, e) result (pe)
    type(vector) :: pe
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(vector), intent(in) :: e
    type(vector) :: pv
    pv = p
    pe = - (1 / cmplx (p*p - m**2, m*w, kind=default)) &
         * (e - (p*e / m**2) * pv)
  end function pr_unitarity
```

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$$\sqrt{\frac{\pi}{M\Gamma}}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m^2}\right)\epsilon^\nu(p) \tag{X.24}$$

⟨*Implementation of propagators*⟩+≡
```
  pure function pj_unitarity (p, m, w, e) result (pe)
    type(vector) :: pe
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(vector), intent(in) :: e
    type(vector) :: pv
    pv = p
    pe = (0, 1) * sqrt (PI / m / w) * (e - (p*e / m**2) * pv)
  end function pj_unitarity
```

⟨*Implementation of propagators*⟩+≡
```
  pure function pg_unitarity (p, m, w, e) result (pe)
    type(vector) :: pe
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(vector), intent(in) :: e
    type(vector) :: pv
    pv = p
    pe = - gauss (p*p, m, w) &
         * (e - (p*e / m**2) * pv)
  end function pg_unitarity
```

$$\frac{-i}{p^2}\epsilon^\nu(p) \tag{X.25}$$

⟨*Implementation of propagators*⟩+≡
```
  pure function pr_feynman (p, e) result (pe)
    type(vector) :: pe
    type(momentum), intent(in) :: p
    type(vector), intent(in) :: e
    pe = - (1 / (p*p)) * e
  end function pr_feynman
```

$$\frac{i}{p^2}\left(-g_{\mu\nu}+(1-\xi)\frac{p_\mu p_\nu}{p^2}\right)\epsilon^\nu(p) \tag{X.26}$$

⟨*Implementation of propagators*⟩+≡
```
  pure function pr_gauge (p, xi, e) result (pe)
    type(vector) :: pe
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: xi
    type(vector), intent(in) :: e
    real(kind=default) :: p2
    type(vector) :: pv
    p2 = p*p
    pv = p
    pe = - (1 / p2) * (e - ((1 - xi) * (p*e) / p2) * pv)
  end function pr_gauge
```

$$\frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma} \left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m^2} \right) \epsilon^\nu(p) \qquad \text{(X.27)}$$

⟨*Implementation of propagators*⟩+≡

```
pure function pr_rxi (p, m, w, xi, e) result (pe)
  type(vector) :: pe
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w, xi
  type(vector), intent(in) :: e
  real(kind=default) :: p2
  type(vector) :: pv
  p2 = p*p
  pv = p
  pe = - (1 / cmplx (p2 - m**2, m*w, kind=default)) &
       * (e - ((1 - xi) * (p*e) / (p2 - xi * m**2)) * pv)
end function pr_rxi
```

$$\frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}m\Gamma} \left(-g_{\mu\nu} \right) \epsilon^\nu(p) \qquad \text{(X.28)}$$

⟨*Implementation of propagators*⟩+≡

```
pure function pr_vector_pure (p, m, w, e) result (pe)
  type(vector) :: pe
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w
  type(vector), intent(in) :: e
  real(kind=default) :: p2
  type(vector) :: pv
  p2 = p*p
  pv = p
  pe = - (1 / cmplx (p2 - m**2, m*w, kind=default)) * e
end function pr_vector_pure
```

⟨*Declaration of propagators*⟩+≡

```
public :: pr_tensor, pr_tensor_pure
```

$$\frac{\mathrm{i}P^{\mu\nu,\rho\sigma}(p,m)}{p^2 - m^2 + \mathrm{i}m\Gamma} T_{\rho\sigma} \qquad \text{(X.29a)}$$

with

$$P^{\mu\nu,\rho\sigma}(p,m) = \frac{1}{2}\left(g^{\mu\rho} - \frac{p^\mu p^\nu}{m^2}\right)\left(g^{\nu\sigma} - \frac{p^\nu p^\sigma}{m^2}\right) + \frac{1}{2}\left(g^{\mu\sigma} - \frac{p^\mu p^\sigma}{m^2}\right)\left(g^{\nu\rho} - \frac{p^\nu p^\rho}{m^2}\right)$$
$$- \frac{1}{3}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right)\left(g^{\rho\sigma} - \frac{p^\rho p^\sigma}{m^2}\right) \quad \text{(X.29b)}$$

Be careful with raising and lowering of indices:

$$g^{\mu\nu} - \frac{k^\mu k^\nu}{m^2} = \begin{pmatrix} 1 - k^0 k^0/m^2 & -k^0\vec{k}/m^2 \\ -\vec{k}k^0/m^2 & -\mathbf{1} - \vec{k}\otimes\vec{k}/m^2 \end{pmatrix} \qquad \text{(X.30a)}$$

$$g^\mu_{\ \nu} - \frac{k^\mu k_\nu}{m^2} = \begin{pmatrix} 1 - k^0 k^0/m^2 & k^0\vec{k}/m^2 \\ -\vec{k}k^0/m^2 & \mathbf{1} + \vec{k}\otimes\vec{k}/m^2 \end{pmatrix} \qquad \text{(X.30b)}$$

⟨*Implementation of propagators*⟩+≡
```
pure function pr_tensor (p, m, w, t) result (pt)
  type(tensor) :: pt
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w
  type(tensor), intent(in) :: t
  complex(kind=default) :: p_dd_t
  real(kind=default), dimension(0:3,0:3) :: p_uu, p_ud, p_du, p_dd
  integer :: i, j
  p_uu(0,0) = 1 - p%t * p%t / m**2
  p_uu(0,1:3) = - p%t * p%x / m**2
  p_uu(1:3,0) = p_uu(0,1:3)
  do i = 1, 3
     do j = 1, 3
        p_uu(i,j) = - p%x(i) * p%x(j) / m**2
     end do
  end do
  do i = 1, 3
     p_uu(i,i) = - 1 + p_uu(i,i)
  end do
  p_ud(:,0) = p_uu(:,0)
  p_ud(:,1:3) = - p_uu(:,1:3)
  p_du = transpose (p_ud)
  p_dd(:,0) = p_du(:,0)
  p_dd(:,1:3) = - p_du(:,1:3)
  p_dd_t = 0
  do i = 0, 3
     do j = 0, 3
        p_dd_t = p_dd_t + p_dd(i,j) * t%t(i,j)
     end do
  end do
  pt%t = matmul (p_ud, matmul (0.5_default * (t%t + transpose (t%t)), p_du)) &
       - (p_dd_t / 3.0_default) * p_uu
  pt%t = pt%t / cmplx (p*p - m**2, m*w, kind=default)
end function pr_tensor
```

$$\frac{\mathrm{i}P_p^{\mu\nu,\rho\sigma}}{p^2 - m^2 + \mathrm{i}m\Gamma}T_{\rho\sigma} \qquad\qquad \text{(X.31a)}$$

with

$$P_p^{\mu\nu,\rho\sigma} \qquad = \qquad \frac{1}{2}g^{\mu\rho}g^{\nu\sigma} \quad + \quad \frac{1}{2}g^{\mu\sigma}g^{\nu\rho} \quad - \quad \frac{1}{2}g^{\mu\nu}g^{\rho\sigma} \quad \text{(X.31b)}$$

⟨*Implementation of propagators*⟩+≡
```
pure function pr_tensor_pure (p, m, w, t) result (pt)
  type(tensor) :: pt
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w
  type(tensor), intent(in) :: t
  complex(kind=default) :: p_dd_t
  real(kind=default), dimension(0:3,0:3) :: g_uu
  integer :: i, j
  g_uu(0,0) = 1
  g_uu(0,1:3) = 0
```

```
    g_uu(1:3,0) = g_uu(0,1:3)
    do i = 1, 3
       do j = 1, 3
          g_uu(i,j) = 0
       end do
    end do
    do i = 1, 3
       g_uu(i,i) = - 1
    end do
    p_dd_t = t%t(0,0) - t%t(1,1) - t%t(2,2) - t%t(3,3)
    pt%t =  0.5_default * ((t%t + transpose (t%t)) &
          - p_dd_t * g_uu )
    pt%t = pt%t / cmplx (p*p - m**2, m*w, kind=default)
  end function pr_tensor_pure
```

X.9.1 Triple Gauge Couplings

⟨*Declaration of couplings*⟩≡
```
  public :: g_gg
```

According to (9.6c)

$$A^{a,\mu}(k_1 + k_2) = -\mathrm{i}g\big((k_1^\mu - k_2^\mu)A^{a_1}(k_1) \cdot A^{a_2}(k_2)$$
$$+ (2k_2 + k_1) \cdot A^{a_1}(k_1)A^{a_2,\mu}(k_2) - A^{a_1,\mu}(k_1)A^{a_2}(k_2) \cdot (2k_1 + k_2)\big) \quad \text{(X.32)}$$

⟨*Implementation of couplings*⟩≡
```
  pure function g_gg (g, a1, k1, a2, k2) result (a)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: a1, a2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: a
    a = (0, -1) * g * ((k1 - k2) * (a1 * a2) &
                      + ((2*k2 + k1) * a1) * a2 - a1 * ((2*k1 + k2) * a2))
  end function g_gg
```

X.9.2 Quadruple Gauge Couplings

⟨*Declaration of couplings*⟩+≡
```
  public :: x_gg, g_gx
```

$$T^{a,\mu\nu}(k_1 + k_2) = g\big(A^{a_1,\mu}(k_1)A^{a_2,\nu}(k_2) - A^{a_1,\nu}(k_1)A^{a_2,\mu}(k_2)\big) \quad \text{(X.33)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function x_gg (g, a1, a2) result (x)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: a1, a2
    type(tensor2odd) :: x
    x = g * (a1 .wedge. a2)
  end function x_gg
```

$$A^{a,\mu}(k_1 + k_2) = gA_\nu^{a_1}(k_1)T^{a_2,\nu\mu}(k_2) \quad \text{(X.34)}$$

⟨*Implementation of couplings*⟩+≡

```
pure function g_gx (g, a1, x) result (a)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: a1
  type(tensor2odd), intent(in) :: x
  type(vector) :: a
  a = g * (a1 * x)
end function g_gx
```

X.9.3   Scalar Current

⟨*Declaration of couplings*⟩+≡
```
public :: v_ss, s_vs
```

$$V^\mu(k_1 + k_2) = g(k_1^\mu - k_2^\mu)\phi_1(k_1)\phi_2(k_2) \tag{X.35}$$

⟨*Implementation of couplings*⟩+≡
```
pure function v_ss (g, phi1, k1, phi2, k2) result (v)
  complex(kind=default), intent(in) :: g, phi1, phi2
  type(momentum), intent(in) :: k1, k2
  type(vector) :: v
  v = (k1 - k2) * (g * phi1 * phi2)
end function v_ss
```

$$\phi(k_1 + k_2) = g(k_1^\mu + 2k_2^\mu)V_\mu(k_1)\phi(k_2) \tag{X.36}$$

⟨*Implementation of couplings*⟩+≡
```
pure function s_vs (g, v1, k1, phi2, k2) result (phi)
  complex(kind=default), intent(in) :: g, phi2
  type(vector), intent(in) :: v1
  type(momentum), intent(in) :: k1, k2
  complex(kind=default) :: phi
  phi = g * ((k1 + 2*k2) * v1) * phi2
end function s_vs
```

X.9.4   Transversal Scalar-Vector Coupling

⟨*Declaration of couplings*⟩+≡
```
public :: s_vv_t, v_sv_t
```

$$phi(k_1 + k_2) = g((V_1(k_1)V_2(k_2))(k_1 k_2) - (V_1(k_1)k_2)(V_2(k_2)k_1)) \tag{X.37}$$

⟨*Implementation of couplings*⟩+≡
```
pure function s_vv_t (g, v1, k1, v2, k2) result (phi)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2
  type(momentum), intent(in) :: k1, k2
  complex(kind=default) :: phi
  phi = g * ((v1*v2) * (k1*k2) - (v1*k2) * (v2*k1))
end function s_vv_t
```

955

$$V_1^\mu(k_\phi + k_V) = gphi(((k_\phi + k_V)k_V)V_2^\mu - (k_\phi + k_V)V_2)k_V^\mu) \tag{X.38}$$

⟨*Implementation of couplings*⟩+≡
```
pure function v_sv_t (g, phi, kphi,v, kv) result (vout)
  complex(kind=default), intent(in) :: g, phi
  type(vector), intent(in) :: v
  type(momentum), intent(in) :: kv, kphi
  type(momentum) :: kout
  type(vector)  :: vout
  kout = - (kv + kphi)
  vout = g * phi * ((kout*kv) * v - (v * kout) * kv)
end function v_sv_t
```

X.9.5 Transversal TensorScalar-Vector Coupling

⟨*Declaration of couplings*⟩+≡
```
public :: tphi_vv, tphi_vv_cf, v_tphiv, v_tphiv_cf
```

$$phi(k_1 + k_2) = g(V_1(k_1)(k_1 + k_2)) * (V_2(k_2)(k_1 + k_2)) \tag{X.39}$$

⟨*Implementation of couplings*⟩+≡
```
pure function tphi_vv (g, v1, k1, v2, k2) result (phi)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2
  type(momentum), intent(in) :: k1, k2
  complex(kind=default) :: phi
  type(momentum) :: k
  k = - (k1 + k2)
  phi = 2 * g * (v1*k) * (v2*k)
end function tphi_vv
```

$$phi(k_1 + k_2) = g((V_1(k_1)V_2(k_2))(k_1 + k_2)^2) \tag{X.40}$$

⟨*Implementation of couplings*⟩+≡
```
pure function tphi_vv_cf (g, v1, k1, v2, k2) result (phi)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2
  type(momentum), intent(in) :: k1, k2
  complex(kind=default) :: phi
  type(momentum) :: k
  k = - (k1 + k2)
  phi = - g/2 * (v1*v2) * (k*k)
end function tphi_vv_cf
```

$$V_1^\mu(k_\phi + k_V) = gphi((k_\phi + k_V)V_2)(k_\phi + k_V)^\mu \tag{X.41}$$

⟨*Implementation of couplings*⟩+≡
```
pure function v_tphiv (g, phi, kphi,v, kv) result (vout)
  complex(kind=default), intent(in) :: g, phi
  type(vector), intent(in) :: v
  type(momentum), intent(in) :: kv, kphi
  type(momentum) :: kout
  type(vector)  :: vout
```

```
    kout = - (kv + kphi)
    vout = 2 * g * phi * ((v * kout) * kout)
  end function v_tphiv
```

$$V_1^\mu(k_\phi + k_V) = gphi((k_\phi + k_V)(k_\phi + k_V))V_2^\mu \qquad \text{(X.42)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function v_tphiv_cf (g, phi, kphi,v, kv) result (vout)
    complex(kind=default), intent(in) :: g, phi
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: kv, kphi
    type(momentum) :: kout
    type(vector)   :: vout
    kout = - (kv + kphi)
    vout = -g/2 * phi * (kout*kout) * v
  end function v_tphiv_cf
```

X.9.6   Triple Vector Couplings

⟨*Declaration of couplings*⟩+≡
```
  public :: tkv_vv, lkv_vv, tv_kvv, lv_kvv, kg_kgkg
  public :: t5kv_vv, l5kv_vv, t5v_kvv, l5v_kvv, kg5_kgkg, kg_kg5kg
  public :: dv_vv, v_dvv, dv_vv_cf, v_dvv_cf
```

$$V^\mu(k_1 + k_2) = ig(k_1 - k_2)^\mu V_1^\nu(k_1)V_{2,\nu}(k_2) \qquad \text{(X.43)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function tkv_vv (g, v1, k1, v2, k2) result (v)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: v
    v = (k1 - k2) * ((0, 1) * g * (v1*v2))
  end function tkv_vv
```

$$V^\mu(k_1 + k_2) = ig\epsilon^{\mu\nu\rho\sigma}(k_1 - k_2)_\nu V_{1,\rho}(k_1)V_{2,\sigma}(k_2) \qquad \text{(X.44)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function t5kv_vv (g, v1, k1, v2, k2) result (v)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: v
    type(vector) :: k
    k = k1 - k2
    v = (0, 1) * g * pseudo_vector (k, v1, v2)
  end function t5kv_vv
```

$$V^\mu(k_1 + k_2) = ig(k_1 + k_2)^\mu V_1^\nu(k_1)V_{2,\nu}(k_2) \qquad \text{(X.45)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function lkv_vv (g, v1, k1, v2, k2) result (v)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
```

```
   type(momentum), intent(in) :: k1, k2
   type(vector) :: v
   v = (k1 + k2) * ((0, 1) * g * (v1*v2))
 end function lkv_vv
```

$$V^\mu(k_1 + k_2) = \mathrm{i}g\epsilon^{\mu\nu\rho\sigma}(k_1 + k_2)_\nu V_{1,\rho}(k_1) V_{2,\sigma}(k_2) \tag{X.46}$$

⟨*Implementation of couplings*⟩+≡
```
 pure function l5kv_vv (g, v1, k1, v2, k2) result (v)
   complex(kind=default), intent(in) :: g
   type(vector), intent(in) :: v1, v2
   type(momentum), intent(in) :: k1, k2
   type(vector) :: v
   type(vector) :: k
   k = k1 + k2
   v = (0, 1) * g * pseudo_vector (k, v1, v2)
 end function l5kv_vv
```

$$V^\mu(k_1 + k_2) = \mathrm{i}g(k_2 - k)^\nu V_{1,\nu}(k_1) V_2^\mu(k_2) = \mathrm{i}g(2k_2 + k_1)^\nu V_{1,\nu}(k_1) V_2^\mu(k_2) \tag{X.47}$$

using $k = -k_1 - k_2$

⟨*Implementation of couplings*⟩+≡
```
 pure function tv_kvv (g, v1, k1, v2, k2) result (v)
   complex(kind=default), intent(in) :: g
   type(vector), intent(in) :: v1, v2
   type(momentum), intent(in) :: k1, k2
   type(vector) :: v
   v = v2 * ((0, 1) * g * ((2*k2 + k1)*v1))
 end function tv_kvv
```

$$V^\mu(k_1 + k_2) = \mathrm{i}g\epsilon^{\mu\nu\rho\sigma}(2k_2 + k_1)_\nu V_{1,\rho}(k_1) V_{2,\sigma}(k_2) \tag{X.48}$$

⟨*Implementation of couplings*⟩+≡
```
 pure function t5v_kvv (g, v1, k1, v2, k2) result (v)
   complex(kind=default), intent(in) :: g
   type(vector), intent(in) :: v1, v2
   type(momentum), intent(in) :: k1, k2
   type(vector) :: v
   type(vector) :: k
   k = k1 + 2*k2
   v = (0, 1) * g * pseudo_vector (k, v1, v2)
 end function t5v_kvv
```

$$V^\mu(k_1 + k_2) = -\mathrm{i}gk_1^\nu V_{1,\nu}(k_1) V_2^\mu(k_2) \tag{X.49}$$

using $k = -k_1 - k_2$

⟨*Implementation of couplings*⟩+≡
```
 pure function lv_kvv (g, v1, k1, v2) result (v)
   complex(kind=default), intent(in) :: g
   type(vector), intent(in) :: v1, v2
   type(momentum), intent(in) :: k1
   type(vector) :: v
   v = v2 * ((0, -1) * g * (k1*v1))
 end function lv_kvv
```

$$V^\mu(k_1 + k_2) = -\mathrm{i}g\epsilon^{\mu\nu\rho\sigma}k_{1,\nu}V_{1,\rho}(k_1)V_{2,\sigma}(k_2) \qquad \text{(X.50)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function l5v_kvv (g, v1, k1, v2) result (v)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1
    type(vector) :: v
    type(vector) :: k
    k = k1
    v = (0, -1) * g * pseudo_vector (k, v1, v2)
  end function l5v_kvv
```

$$A^\mu(k_1 + k_2) = \mathrm{i}gk^\nu\Big(F_{1,\nu}{}^\rho(k_1)F_{2,\rho\mu}(k_2) - F_{1,\mu}{}^\rho(k_1)F_{2,\rho\nu}(k_2)\Big) \qquad \text{(X.51)}$$

with $k = -k_1 - k_2$, i.e.

$$\begin{aligned}
A^\mu(k_1 + k_2) = -\mathrm{i}g\Big(&[(kk_2)(k_1A_2) - (k_1k_2)(kA_2)]A_1^\mu \\
&+ [(k_1k_2)(kA_1) - (kk_1)(k_2A_1)]A_2^\mu \\
&+ [(k_2A_1)(kA_2) - (kk_2)(A_1A_2)]k_1^\mu \\
&+ [(kk_1)(A_1A_2) - (kA_1)(k_1A_2)]k_2^\mu\Big) \quad \text{(X.52)}
\end{aligned}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function kg_kgkg (g, a1, k1, a2, k2) result (a)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: a1, a2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: a
    real(kind=default) :: k1k1, k2k2, k1k2, kk1, kk2
    complex(kind=default) :: a1a2, k2a1, ka1, k1a2, ka2
    k1k1 = k1 * k1
    k1k2 = k1 * k2
    k2k2 = k2 * k2
    kk1 = k1k1 + k1k2
    kk2 = k1k2 + k2k2
    k2a1 = k2 * a1
    ka1 = k2a1 + k1 * a1
    k1a2 = k1 * a2
    ka2 = k1a2 + k2 * a2
    a1a2 = a1 * a2
    a = (0, -1) * g * (   (kk2  * k1a2 - k1k2 * ka2 ) * a1 &
                        + (k1k2 * ka1  - kk1  * k2a1) * a2 &
                        + (ka2  * k2a1 - kk2  * a1a2) * k1 &
                        + (kk1  * a1a2 - ka1  * k1a2) * k2 )
  end function kg_kgkg
```

$$A^\mu(k_1 + k_2) = \mathrm{i}g\epsilon^{\mu\nu\rho\sigma}k_\nu F_{1,\rho}{}^\lambda(k_1)F_{2,\lambda\sigma}(k_2) \qquad \text{(X.53)}$$

with $k = -k_1 - k_2$, i.e.

$$A^\mu(k_1 + k_2) = -2\mathrm{i}g\epsilon^{\mu\nu\rho\sigma}k_\nu\Big((k_2A_1)k_{1,\rho}A_{2,\sigma} + (k_1A_2)A_{1,\rho}k_{2,\sigma}$$

$$- (A_1 A_2)k_{1,\rho}k_{2,\sigma} - (k_1 k_2)A_{1,\rho}A_{2,\sigma}\Big) \quad \text{(X.54)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function kg5_kgkg (g, a1, k1, a2, k2) result (a)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: a1, a2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: a
    type(vector) :: kv, k1v, k2v
    kv = - k1 - k2
    k1v = k1
    k2v = k2
    a = (0, -2) * g * (   (k2*A1) * pseudo_vector (kv, k1v, a2 ) &
                        + (k1*A2) * pseudo_vector (kv, A1 , k2v) &
                        - (A1*A2) * pseudo_vector (kv, k1v, k2v) &
                        - (k1*k2) * pseudo_vector (kv, a1 , a2 ) )
  end function kg5_kgkg
```

$$A^\mu(k_1 + k_2) = \mathrm{i}g k_\nu \left(\epsilon^{\mu\rho\lambda\sigma}F_{1,}{}^\nu{}_\rho - \epsilon^{\nu\rho\lambda\sigma}F_{1,}{}^\mu{}_\rho\right)\frac{1}{2}F_{1,\lambda\sigma} \quad \text{(X.55)}$$

with $k = -k_1 - k_2$, i. e.

$$A^\mu(k_1+k_2) = -\mathrm{i}g\left(\epsilon^{\mu\rho\lambda\sigma}(kk_2)A_{2,\rho} - \epsilon^{\mu\rho\lambda\sigma}(kA_2)k_{2,\rho} - k_2^\mu\epsilon^{\nu\rho\lambda\sigma}k_n u A_{2,\rho} + A_2^\mu\epsilon^{\nu\rho\lambda\sigma}k_n u k_{2,\rho}\right)k_{1,\lambda}A_{1,\sigma}$$
$$\text{(X.56)}$$

⧚ This is not the most efficienct way of doing it: $\epsilon^{\mu\nu\rho\sigma}F_{1,\rho\sigma}$ should be cached!

⟨*Implementation of couplings*⟩+≡
```
  pure function kg_kg5kg (g, a1, k1, a2, k2) result (a)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: a1, a2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: a
    type(vector) :: kv, k1v, k2v
    kv = - k1 - k2
    k1v = k1
    k2v = k2
    a = (0, -1) * g * (   (kv*k2v) * pseudo_vector (a2 , k1v, a1) &
                        - (kv*a2 ) * pseudo_vector (k2v, k1v, a1) &
                        -  k2v * pseudo_scalar (kv, a2,  k1v, a1) &
                        +  a2  * pseudo_scalar (kv, k2v, k1v, a1) )
  end function kg_kg5kg
```

$$V^\mu(k_1 + k_2) = -g((k_1 + k_2)V_1)V_2^\mu + ((k_1 + k_2)V_2)V_1^\mu \quad \text{(X.57)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function dv_vv (g, v1, k1, v2, k2) result (v)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: v
    type(vector) :: k
    k = -(k1 + k2)
    v = g * ((k * v1) * v2 + (k * v2) * v1)
  end function dv_vv
```

$$V^\mu(k_1 + k_2) = \frac{g}{2}(V_1(k_1)V_2(k_2))(k_1 + k_2)^\mu \qquad \text{(X.58)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function dv_vv_cf (g, v1, k1, v2, k2) result (v)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2
  type(momentum), intent(in) :: k1, k2
  type(vector) :: v
  type(vector) :: k
  k = -(k1 + k2)
  v = - g/2 * (v1 * v2) * k
end function dv_vv_cf
```

$$V_1^\mu = g * (kV_2)V(k) + (VV_2)k \qquad \text{(X.59)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function v_dvv (g, v, k, v2) result (v1)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v, v2
  type(momentum), intent(in) :: k
  type(vector) :: v1
  v1 = g * ((v * v2) * k + (k * v2) * v)
end function v_dvv
```

$$V_1^\mu = -\frac{g}{2}(V(k)k)V_2^\mu \qquad \text{(X.60)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function v_dvv_cf (g, v, k, v2) result (v1)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) ::  v, v2
  type(momentum), intent(in) :: k
  type(vector) :: v1
  v1 = - g/2 * (v * k) * v2
end function v_dvv_cf
```

X.10   Tensorvector - Scalar coupling

⟨*Declaration of couplings*⟩+≡
```
public :: dv_phi2,phi_dvphi, dv_phi2_cf, phi_dvphi_cf
```

$$V^\mu(k_1 + k_2) = g * ((k_1 k_2 + k_2 k_2)k_1^\mu + (k_1 k_2 + k_1 k_1)k_2^\mu) * phi_1(k_1)phi_2(k_2) \quad \text{(X.61)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function dv_phi2 (g, phi1, k1, phi2, k2) result (v)
  complex(kind=default), intent(in) :: g, phi1, phi2
  type(momentum), intent(in) :: k1, k2
  type(vector) :: v
  v = g * phi1 * phi2 * ( &
      (k1 * k2 + k2 * k2 ) * k1 + &
      (k1 * k2 + k1 * k1 ) * k2 )
end function dv_phi2
```

$$V^{\mu}(k_1 + k_2) = -\frac{g}{2} * (k_1 k_2) * (k_1 + k_2)^{\mu} * phi_1(k_1) phi_2(k_2) \qquad \text{(X.62)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function dv_phi2_cf (g, phi1, k1, phi2, k2) result (v)
    complex(kind=default), intent(in) :: g, phi1, phi2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: v
    v = - g/2 * phi1 * phi2 * (k1 * k2) * (k1 + k2)
  end function dv_phi2_cf
```

$$phi_1(k_1) = g*((k_1 k_2 + k_2 k_2)(k_1 * V(-k_1 - k_2)) + (k_1 k_2 + k_1 k_1)(k_2 * V(-k_1 - k_2))) * phi_2(k_2) \qquad \text{(X.63)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_dvphi (g, v, k, phi2, k2) result (phi1)
    complex(kind=default), intent(in) :: g, phi2
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k, k2
    complex(kind=default) :: phi1
    type(momentum) :: k1
    k1 = - (k + k2)
    phi1 = g * phi2 * ( &
        (k1 * k2 + k2 * k2 ) * ( k1 * V ) + &
        (k1 * k2 + k1 * k1 ) * ( k2 * V ) )
  end function phi_dvphi
```

$$phi_1(k_1) = -\frac{g}{2} * (k_1 k_2) * ((k_1 + k_2) V(-k_1 - k_2)) \qquad \text{(X.64)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_dvphi_cf (g, v, k, phi2, k2) result (phi1)
    complex(kind=default), intent(in) :: g, phi2
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k, k2
    complex(kind=default) :: phi1
    type(momentum) :: k1
    k1 = -(k + k2)
    phi1 = - g/2 * phi2 * (k1 * k2)  * ((k1 + k2) * v)
  end function phi_dvphi_cf
```

X.11   Scalar-Vector Dim-5 Couplings

⟨*Declaration of couplings*⟩+≡
```
  public :: phi_vv, v_phiv, phi_u_vv, v_u_phiv
```

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_vv (g, k1, k2, v1, v2) result (phi)
    complex(kind=default), intent(in) :: g
    type(momentum), intent(in) :: k1, k2
    type(vector), intent(in) :: v1, v2
    complex(kind=default) :: phi
    phi = g * pseudo_scalar (k1, v1, k2, v2)
  end function phi_vv
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_phiv (g, phi, k1, k2, v) result (w)
    complex(kind=default), intent(in) :: g, phi
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k1, k2
    type(vector) :: w
    w = g * phi * pseudo_vector (k1, k2, v)
  end function v_phiv
```

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_u_vv (g, k1, k2, v1, v2) result (phi)
    complex(kind=default), intent(in) :: g
    type(momentum), intent(in) :: k1, k2
    type(vector), intent(in) :: v1, v2
    complex(kind=default) :: phi
    phi = g * ((k1*v2)*((-(k1+k2))*v1) + &
               (k2*v1)*((-(k1+k2))*v2) + &
               (((k1+k2)*(k1+k2)) * (v1*v2)))
  end function phi_u_vv
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_u_phiv (g, phi, k1, k2, v) result (w)
    complex(kind=default), intent(in) :: g, phi
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k1, k2
    type(vector) :: w
    w = g * phi * ((k1*v)*k2 + &
         ((-(k1+k2))*v)*k1 + &
         ((k1*k1)*v))
  end function v_u_phiv
```

X.12   Dim-6 Anoumalous Couplings with Higgs

⟨*Declaration of couplings*⟩+≡
```
  public :: s_vv_6D, v_sv_6D, s_vv_6DP, v_sv_6DP, a_hz_D, h_az_D, z_ah_D, &
       a_hz_DP, h_az_DP, z_ah_DP, h_hh_6
```

⟨*Implementation of couplings*⟩+≡
```
  pure function s_vv_6D (g, v1, k1, v2, k2) result (phi)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    complex(kind=default) :: phi
    phi =  g * (-(k1 * v1) * (k1 * v2) - (k2 * v1) * (k2 * v2) &
         + ((k1 * k1) + (k2 * k2)) * (v1 * v2))
  end function s_vv_6D
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_sv_6D (g, phi, kphi, v, kv) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: kphi, kv
    type(vector) :: vout
    vout = g * ( - phi * (kv * v) * kv - phi * ((kphi + kv) * v) * (kphi + kv) &
```

```
              + phi * (kv * kv) * v + phi * ((kphi + kv)*(kphi + kv)) * v)
   end function v_sv_6D
```

⟨*Implementation of couplings*⟩+≡
```
   pure function s_vv_6DP (g, v1, k1, v2, k2) result (phi)
     complex(kind=default), intent(in) :: g
     type(vector), intent(in) :: v1, v2
     type(momentum), intent(in) :: k1, k2
     complex(kind=default) :: phi
     phi = g * ( (-(k1+k2)*v1) * (k1*v2) - ((k1+k2)*v2) * (k2*v1) + &
           ((k1+k2)*(k1+k2))*(v1*v2) )
   end function s_vv_6DP
```

⟨*Implementation of couplings*⟩+≡
```
   pure function v_sv_6DP (g, phi, kphi, v, kv) result (vout)
     complex(kind=default), intent(in) :: g
     complex(kind=default), intent(in) :: phi
     type(vector), intent(in) :: v
     type(momentum), intent(in) :: kphi, kv
     type(vector) :: vout
     vout = g * phi * ((-(kphi + kv)*v) * kphi + (kphi * v) * kv + &
           (kphi*kphi) * v )
   end function v_sv_6DP
```

⟨*Implementation of couplings*⟩+≡
```
   pure function a_hz_D (g, h1, k1, v2, k2) result (vout)
     complex(kind=default), intent(in) :: g
     complex(kind=default), intent(in) :: h1
     type(vector), intent(in) :: v2
     type(momentum), intent(in) :: k1, k2
     type(vector) :: vout
     vout = g * h1 * (((k1 + k2) * v2) * (k1 + k2) + &
           ((k1 + k2) * (k1 + k2)) * v2)
   end function a_hz_D
```

⟨*Implementation of couplings*⟩+≡
```
   pure function h_az_D (g, v1, k1, v2, k2) result (hout)
     complex(kind=default), intent(in) :: g
     type(vector), intent(in) :: v1, v2
     type(momentum), intent(in) :: k1, k2
     complex(kind=default) :: hout
     hout = g * ((k1 * v1) * (k1 * v2) + (k1 * k1) * (v1 * v2))
   end function h_az_D
```

⟨*Implementation of couplings*⟩+≡
```
   pure function z_ah_D (g, v1, k1, h2, k2) result (vout)
     complex(kind=default), intent(in) :: g
     complex(kind=default), intent(in) :: h2
     type(vector), intent(in) :: v1
     type(momentum), intent(in) :: k1, k2
     type(vector) :: vout
     vout = g * h2 * ((k1 * v1) * k1 + ((k1 * k1)) *v1)
   end function z_ah_D
```

⟨*Implementation of couplings*⟩+≡
```
   pure function a_hz_DP (g, h1, k1, v2, k2) result (vout)
     complex(kind=default), intent(in) :: g
     complex(kind=default), intent(in) :: h1
```

```
      type(vector), intent(in) :: v2
      type(momentum), intent(in) :: k1, k2
      type(vector) :: vout
      vout = g * ((- h1 * (k1 + k2) * v2) * (k1) &
            + h1 * ((k1 + k2) * (k1)) *v2)
    end function a_hz_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_az_DP (g, v1, k1, v2, k2) result (hout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    complex(kind=default) :: hout
    hout = g * (- (k1 * v2) * ((k1 + k2) * v1) + (k1 * (k1 + k2)) * (v1 * v2))
  end function h_az_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_ah_DP (g, v1, k1, h2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h2
    type(vector), intent(in) :: v1
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * h2* ((k2 * v1) * k1 - (k1 * k2) * v1)
  end function z_ah_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_hh_6 (g, h1, k1, h2, k2) result (hout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1, h2
    type(momentum), intent(in) :: k1, k2
    complex(kind=default) :: hout
    hout =  g * ((k1* k1) + (k2 * k2) + (k1* k2)) * h1 * h2
  end function h_hh_6
```

X.13 Dim-6 Anoumalous Couplings without Higgs

⟨*Declaration of couplings*⟩+≡
```
  public :: g_gg_13, g_gg_23, g_gg_6, kg_kgkg_i
```

⟨*Implementation of couplings*⟩+≡
```
  pure function g_gg_23 (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * (v1 * (-2*(k1*v2)) + v2 * (2*k2 * v1) + (k1 - k2) * (v1*v2))
  end function g_gg_23
```

⟨*Implementation of couplings*⟩+≡
```
  pure function g_gg_13 (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * (v1 * (2*(k1 + k2)*v2) - v2 * ((k1 + 2*k2) * v1) + 2*k2 * (v1 * v2))
```

```
      end function g_gg_13
```

⟨*Implementation of couplings*⟩+≡
```
  pure function g_gg_6 (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * &
        ( k1 * ((-(k1 + k2) * v2) * (k2 * v1) + ((k1 + k2) * k2) * (v1 * v2)) &
        + k2 * (((k1 + k2) * v1) * (k1 * v2) - ((k1 + k2) * k1) * (v1 * v2)) &
        + v1 * (-((k1 + k2) * k2) * (k1 * v2) + (k1 * k2) * ((k1 + k2) * v2)) &
        + v2 * (((k1 + k2) * k1) * (k2 * v1) - (k1 * k2) * ((k1 + k2) * v1)))
  end function g_gg_6
```

⟨*Implementation of couplings*⟩+≡
```
  pure function kg_kgkg_i (g, a1, k1, a2, k2) result (a)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: a1, a2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: a
    real(kind=default) :: k1k1, k2k2, k1k2, kk1, kk2
    complex(kind=default) :: a1a2, k2a1, ka1, k1a2, ka2
    k1k1 = k1 * k1
    k1k2 = k1 * k2
    k2k2 = k2 * k2
    kk1 = k1k1 + k1k2
    kk2 = k1k2 + k2k2
    k2a1 = k2 * a1
    ka1 = k2a1 + k1 * a1
    k1a2 = k1 * a2
    ka2 = k1a2 + k2 * a2
    a1a2 = a1 * a2
    a = (-1) * g * (   (kk2  * k1a2 - k1k2 * ka2 ) * a1 &
        + (k1k2 * ka1  - kk1  * k2a1) * a2 &
        + (ka2  * k2a1 - kk2  * a1a2) * k1 &
        + (kk1  * a1a2 - ka1  * k1a2) * k2 )
  end function kg_kgkg_i
```

X.14   Dim-6 Anoumalous Couplings with AWW

⟨*Declaration of couplings*⟩+≡
```
  public ::a_ww_DP, w_aw_DP, a_ww_DW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function a_ww_DP (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * ( - ((k1 + k2) * v2) * v1 + ((k1 + k2) * v1) * v2)
  end function a_ww_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_aw_DP (g, v1, k1, v2, k2) result (vout)
```

```
      complex(kind=default), intent(in) :: g
      type(vector), intent(in) :: v1, v2
      type(momentum), intent(in) :: k1, k2
      type(vector) :: vout
      vout = g * ((k1 * v2) * v1 - (v1 * v2) * k1)
    end function w_aw_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function a_ww_DW (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * (v1 * (- (4*k1 + 2*k2) * v2) &
          + v2 * ( (2*k1 + 4*k2) * v1) &
          + (k1 - k2) * (2*v1*v2))
  end function a_ww_DW
```

⟨*Declaration of couplings*⟩+≡
```
  public :: w_wz_DPW, z_ww_DPW, w_wz_DW, z_ww_DW, w_wz_D, z_ww_D
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_wz_DPW (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * (v1 * (-(k1+k2)*v2 - k1*v2) + v2 * ((k1+k2)*v1) + k1 * (v1*v2))
  end function w_wz_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_ww_DPW (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * (k1*(v1*v2) - k2*(v1*v2) - v1*(k1*v2) + v2*(k2*v1))
  end function z_ww_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_wz_DW (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * (v2 * (v1 * k2) - k2 * (v1 * v2))
  end function w_wz_DW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_ww_DW (g, v1, k1, v2, k2) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: vout
    vout = g * (v1 * ((-1)*(k1+k2) * v2) + v2 * ((k1+k2) * v1))
  end function z_ww_DW
```

⟨*Implementation of couplings*⟩+≡
```
pure function w_wz_D (g, v1, k1, v2, k2) result (vout)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2
  type(momentum), intent(in) :: k1, k2
  type(vector) :: vout
  vout = g * (v2 * (k2*v1) - k2 * (v1*v2))
end function w_wz_D
```

⟨*Implementation of couplings*⟩+≡
```
pure function z_ww_D (g, v1, k1, v2, k2) result (vout)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2
  type(momentum), intent(in) :: k1, k2
  type(vector) :: vout
  vout = g * (v1 * (- (k1 + k2) * v2) + v2 * ((k1 + k2) * v1))
end function z_ww_D
```

X.15 Dim-6 Quartic Couplings

⟨*Declaration of couplings*⟩+≡
```
public :: hhhh_p2, a_hww_DPB, h_aww_DPB, w_ahw_DPB, a_hww_DPW, h_aww_DPW, &
    w_ahw_DPW, a_hww_DW, h_aww_DW, w3_ahw_DW, w4_ahw_DW
```

⟨*Implementation of couplings*⟩+≡
```
pure function hhhh_p2 (g, h1, k1, h2, k2, h3, k3) result (hout)
  complex(kind=default), intent(in) :: g
  complex(kind=default), intent(in) :: h1, h2, h3
  type(momentum), intent(in) :: k1, k2, k3
  complex(kind=default) :: hout
  hout = g * h1*h2*h3* (k1*k1 + k2*k2 +k3*k3 + k1*k3 + k1*k2 + k2*k3)
end function hhhh_p2
```

⟨*Implementation of couplings*⟩+≡
```
pure function a_hww_DPB (g, h1, k1, v2, k2, v3, k3) result (vout)
  complex(kind=default), intent(in) :: g
  complex(kind=default), intent(in) :: h1
  type(vector), intent(in) :: v2, v3
  type(momentum), intent(in) :: k1, k2, k3
  type(vector) :: vout
  vout = g * h1 * (v3*((k1+k2+k3)*v2) - v2*((k1+k2+k3)*v3))
end function a_hww_DPB
```

⟨*Implementation of couplings*⟩+≡
```
pure function h_aww_DPB (g, v1, k1, v2, k2, v3, k3) result (hout)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2, v3
  type(momentum), intent(in) :: k1, k2, k3
  complex(kind=default) :: hout
  hout = g * ((k1 * v3) * (v1 * v2) - (k1 * v2) * (v1 * v3))
end function h_aww_DPB
```

⟨*Implementation of couplings*⟩+≡
```
pure function w_ahw_DPB (g, v1, k1, h2, k2, v3, k3) result (vout)
  complex(kind=default), intent(in) :: g
```

```
    complex(kind=default), intent(in) :: h2
    type(vector), intent(in) :: v1, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h2 * (v1 * (k1 * v3) - k1 * (v1 * v3))
  end function w_ahw_DPB
```

⟨*Implementation of couplings*⟩+≡
```
  pure function a_hww_DPW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * (v3 * ((2*k1+k2+k3)*v2) - v2 * ((2*k1+k2+k3)*v3))
  end function a_hww_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_aww_DPW (g, v1, k1, v2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * ((-(2*k1+k2+k3)*v2)*(v1*v3)+((2*k1+k2+k3)*v3)*(v1*v2))
  end function h_aww_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_ahw_DPW (g, v1, k1, h2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h2
    type(vector), intent(in) :: v1, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h2 * ((k2 - k1) * (v1 * v3) + v1 * ((k1 - k2) * v3))
  end function w_ahw_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function a_hww_DW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * ( v2 * (-(3*k1 + 4*k2 + 4*k3) * v3) &
          + v3 * ((3*k1 + 2*k2 + 4*k3) * v2)  &
          + (k2 - k3) *2*(v2 * v3))
  end function a_hww_DW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_aww_DW (g, v1, k1, v2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * ((v1*v2) * ((3*k1 - k2 - k3)*v3) &
          + (v1*v3) * ((-3*k1 - k2 + k3)*v2) &
          + (v2*v3) * (2*(k2-k3)*v1))
  end function h_aww_DW
```

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⟨*Implementation of couplings*⟩+≡
```
pure function w3_ahw_DW (g, v1, k1, h2, k2, v3, k3) result (vout)
  complex(kind=default), intent(in) :: g
  complex(kind=default), intent(in) :: h2
  type(vector), intent(in) :: v1, v3
  type(momentum), intent(in) :: k1, k2, k3
  type(vector) :: vout
  vout = g * h2 * (v1 * ((4*k1 + k2) * v3) &
      +v3 * (-2*(k1 + k2 + 2*k3) * v1) &
      +(-2*k1 + k2 + 2*k3) * (v1*v3))
end function w3_ahw_DW
```

⟨*Implementation of couplings*⟩+≡
```
pure function w4_ahw_DW (g, v1, k1, h2, k2, v3, k3) result (vout)
  complex(kind=default), intent(in) :: g
  complex(kind=default), intent(in) :: h2
  type(vector), intent(in) :: v1, v3
  type(momentum), intent(in) :: k1, k2, k3
  type(vector) :: vout
  vout = g * h2 * (v1 * (-(4*k1 + k2 + 2*k3) * v3) &
      + v3 * (2*(k1 + k2 + 2*k3) * v1) &
      +(4*k1 + k2) * (v1*v3))
end function w4_ahw_DW
```

⟨*Declaration of couplings*⟩+≡
```
public ::a_aww_DW, w_aaw_DW, a_aww_W, w_aaw_W
```

⟨*Implementation of couplings*⟩+≡
```
pure function a_aww_DW (g, v1, k1, v2, k2, v3, k3) result (vout)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2, v3
  type(momentum), intent(in) :: k1, k2, k3
  type(vector) :: vout
  vout = g * (2*v1*(v2*v3) - v2*(v1*v3) - v3*(v1*v2))
end function a_aww_DW
pure function w_aaw_DW (g, v1, k1, v2, k2, v3, k3) result (vout)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2, v3
  type(momentum), intent(in) :: k1, k2, k3
  type(vector) :: vout
  vout = g * (2*v3*(v1*v2) - v2*(v1*v3) - v1*(v2*v3))
end function w_aaw_DW
pure function a_aww_W (g, v1, k1, v2, k2, v3, k3) result (vout)
  complex(kind=default), intent(in) :: g
  type(vector), intent(in) :: v1, v2, v3
  type(momentum), intent(in) :: k1, k2, k3
  type(vector) :: vout
  vout = g * (v1*((-(k2+k3)*v2)*(k2*v3) + (-(k2+k3)*v3)*(k3*v2)) &
      +v2*((-((k2-k3)*v1)*(k1+k2+k3)*v3) - (k1*v3)*(k2*v1) &
      + ((k1+k2+k3)*v1)*(k2*v3)) &
      +v3*(((k2-k3)*v1)*((k1+k2+k3)*v2) - (k1*v2)*(k3*v1) &
      + ((k1+k2+k3)*v1)*(k3*v2)) &
      +(v1*v2)*(((2*k1+k2+k3)*v3)*k2 - (k2*v3)*k1 -(k1*v3)*k3) &
      +(v1*v3)*(((2*k1+k2+k3)*v2)*k3 - (k3*v2)*k1 - (k1*v2)*k3) &
      +(v2*v3)*((-(k1+k2+k3)*v1)*(k2+k3) + ((k2+k3)*v1)*k1) &
      +(-(k1+k2+k3)*k3 +k1*k2)*((v1*v3)*v2 - (v2*v3)*v1) &
```

```
          +(-(k1+k2+k3)*k2 + k1*k3)*((v1*v2)*v3 - (v2*v3)*v1))
    end function a_aww_W
    pure function w_aaw_W (g, v1, k1, v2, k2, v3, k3) result (vout)
      complex(kind=default), intent(in) :: g
      type(vector), intent(in) :: v1, v2, v3
      type(momentum), intent(in) :: k1, k2, k3
      type(vector) :: vout
      vout = g * (v1*((k1*v3)*(-(k1+k2+2*k3)*v2) + (k2*v3)*((k1+k2+k3)*v2) &
          + (k1*v2)*((k1+k2+k3)*v3)) &
          + v2*(((k1-k2)*v3)*((k1+k2+k3)*v1) - (k2*v3)*(k3*v1) &
          + (k2*v1)*((k1+k2+k3)*v3)) &
          + v3*((k1*v2)*(-(k1+k2)*v1) + (k2*v1)*(-(k1+k2)*v2)) &
          + (v1*v2)*((k1+k2)*(-(k1+k2+k3)*v3) + k3*((k1+k2)*v3))&
          + (v1*v3)*(-k2*(k3*v2) - k3*(k1*v2) + k1*((k1+k2+2*k3)*v2)) &
          + (v2*v3)*(-k1*(k3*v1) - k3*(k2*v1) + k2*((k1+k2+2*k3)*v1)) &
          + (-k2*(k1+k2+k3) + k1*k3)*(v1*(v2*v3) - v3*(v1*v2)) &
          + (-k1*(k1+k2+k3) + k2*k3)*(v2*(v1*v3) - v3*(v1*v2)) )
    end function w_aaw_W
```

⟨*Declaration of couplings*⟩+≡
```
    public :: h_hww_D, w_hhw_D, h_hww_DP, w_hhw_DP, h_hvv_PB, v_hhv_PB
```

⟨*Implementation of couplings*⟩+≡
```
    pure function h_hww_D (g, h1, k1, v2, k2, v3, k3) result (hout)
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h1
      type(vector), intent(in) :: v2, v3
      type(momentum), intent(in) :: k1, k2, k3
      complex(kind=default) :: hout
      hout = g * h1 * ((v2*v3)*((k2*k2)+(k3*k3)) - (k2*v2)*(k2*v3) &
          - (k3*v2)*(k3*v3))
    end function h_hww_D
```

⟨*Implementation of couplings*⟩+≡
```
    pure function w_hhw_D (g, h1, k1, h2, k2, v3, k3) result (vout)
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h1, h2
      type(vector), intent(in) :: v3
      type(momentum), intent(in) :: k1, k2, k3
      type(vector) :: vout
      vout = g * h1 * h2 * (v3 * ((k1+k2+k3)*(k1+k2+k3)+(k3*k3)) &
          - (k1+k2+k3) * ((k1+k2+k3)*v3) - k3 * (k3*v3))
    end function w_hhw_D
```

⟨*Implementation of couplings*⟩+≡
```
    pure function h_hww_DP (g, h1, k1, v2, k2, v3, k3) result (hout)
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h1
      type(vector), intent(in) :: v2, v3
      type(momentum), intent(in) :: k1, k2, k3
      complex(kind=default) :: hout
      hout = g * h1 * (-((k2+k3)*v2)*(k2*v3) - &
          ((k2+k3)*v3)*(k3*v2)+ (v2*v3)*((k2+k3)*(k2+k3)))
    end function h_hww_DP
```

⟨*Implementation of couplings*⟩+≡
```
    pure function w_hhw_DP (g, h1, k1, h2, k2, v3, k3) result (vout)
```

```
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h1, h2
      type(vector), intent(in) :: v3
      type(momentum), intent(in) :: k1, k2, k3
      type(vector) :: vout
      vout = g * h1 * h2 * (k3*((k1+k2)*v3) + (k1+k2)*(-(k1+k2+k3)*v3) &
           + v3*((k1+k2)*(k1+k2)))
    end function w_hhw_DP
```

⟨*Implementation of couplings*⟩+≡
```
    pure function h_hvv_PB (g, h1, k1, v2, k2, v3, k3) result (hout)
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h1
      type(vector), intent(in) :: v2, v3
      type(momentum), intent(in) :: k1, k2, k3
      complex(kind=default) :: hout
      hout = g * h1 * ((k2*v3)*(k3*v2) - (k2*k3)*(v2*v3))
    end function h_hvv_PB
```

⟨*Implementation of couplings*⟩+≡
```
    pure function v_hhv_PB (g, h1, k1, h2, k2, v3, k3) result (vout)
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h1, h2
      type(vector), intent(in) :: v3
      type(momentum), intent(in) :: k1, k2, k3
      type(vector) :: vout
      vout = g * h1 * h2 * ((-(k1+k2+k3)*v3)*k3 + ((k1+k2+k3)*k3)*v3)
    end function v_hhv_PB
```

⟨*Declaration of couplings*⟩+≡
```
    public :: a_hhz_D, h_ahz_D, z_ahh_D, a_hhz_DP, h_ahz_DP, z_ahh_DP, &
        a_hhz_PB, h_ahz_PB, z_ahh_PB
```

⟨*Implementation of couplings*⟩+≡
```
    pure function a_hhz_D (g, h1, k1, h2, k2, v3, k3) result (vout)
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h1, h2
      type(vector), intent(in) :: v3
      type(momentum), intent(in) :: k1, k2, k3
      type(vector) :: vout
      vout = g * h1 * h2 * ((k1+k2+k3) * ((k1+k2+k3)*v3) &
           - v3 * ((k1+k2+k3)*(k1+k2+k3)))
    end function a_hhz_D
```

⟨*Implementation of couplings*⟩+≡
```
    pure function h_ahz_D (g, v1, k1, h2, k2, v3, k3) result (hout)
      complex(kind=default), intent(in) :: g
      complex(kind=default), intent(in) :: h2
      type(vector), intent(in) :: v1, v3
      type(momentum), intent(in) :: k1, k2, k3
      complex(kind=default) :: hout
      hout = g * h2 * ((k1*v1)*(k1*v3) - (k1*k1)*(v1*v3))
    end function h_ahz_D
```

⟨*Implementation of couplings*⟩+≡
```
    pure function z_ahh_D (g, v1, k1, h2, k2, h3, k3) result (vout)
      complex(kind=default), intent(in) :: g
      type(vector), intent(in) :: v1
```

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```
      complex(kind=default), intent(in) :: h2, h3
      type(momentum), intent(in) :: k1, k2, k3
      type(vector) :: vout
      vout = g * h2 * h3 * ((k1*v1)*k1 - (k1*k1)*v1)
    end function z_ahh_D
```

⟨*Implementation of couplings*⟩+≡
```
  pure function a_hhz_DP (g, h1, k1, h2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1, h2
    type(vector), intent(in) :: v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * h2 * ((-(k1+k2+k3)*v3)*(k1+k2) + ((k1+k2+k3)*(k1+k2))*v3)
  end function a_hhz_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_ahz_DP (g, v1, k1, h2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h2
    type(vector), intent(in) :: v1, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * h2 * ( (k1*v3)*(-(k1+k3)*v1) + (k1*(k1+k3))*(v1*v3) )
  end function h_ahz_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_ahh_DP (g, v1, k1, h2, k2, h3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1
    complex(kind=default), intent(in) :: h2, h3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h2 * h3 * (k1*((k2+k3)*v1) - v1*(k1*(k2+k3)))
  end function z_ahh_DP
```

⟨*Implementation of couplings*⟩+≡
```
  pure function a_hhz_PB (g, h1, k1, h2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1, h2
    type(vector), intent(in) :: v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * h2 * (k3*((k1+k2+k3)*v3) - v3*((k1+k2+k3)*k3))
  end function a_hhz_PB
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_ahz_PB (g, v1, k1, h2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h2
    type(vector), intent(in) :: v1, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * h2 * ((-k1*v3)*(k3*v1) + (k1*k3)*(v1*v3))
  end function h_ahz_PB
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_ahh_PB (g, v1, k1, h2, k2, h3, k3) result (vout)
```

```
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1
    complex(kind=default), intent(in) :: h2, h3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h2 * h3 * (k1*((k1+k2+k3)*v1) - v1*(k1*(k1+k2+k3)))
  end function z_ahh_PB
```

⟨*Declaration of couplings*⟩+≡
```
  public :: h_wwz_DW, w_hwz_DW, z_hww_DW, h_wwz_DPB, w_hwz_DPB, z_hww_DPB
  public :: h_wwz_DDPW, w_hwz_DDPW, z_hww_DDPW, h_wwz_DPW, w_hwz_DPW, z_hww_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_wwz_DW (g, v1, k1, v2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * (((k1-k2)*v3)*(v1*v2)-((2*k1+k2)*v2)*(v1*v3) + &
          ((k1+2*k2)*v1)*(v2*v3))
  end function h_wwz_DW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_hwz_DW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * ( v2*(-(k1+2*k2+k3)*v3) + v3*((2*k1+k2+2*k3)*v2) - &
          (k1 - k2 + k3)*(v2*v3))
  end function w_hwz_DW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_hww_DW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * ((k2-k3)*(v2*v3) - v2*((2*k2+k3)*v3) + v3*((k2+2*k3)*v2))
  end function z_hww_DW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_wwz_DPB (g, v1, k1, v2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * ((k3*v1)*(v2*v3) - (k3*v2)*(v1*v3))
  end function h_wwz_DPB
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_hwz_DPB (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
```

```
    type(vector) :: vout
    vout = g * h1 * (k3*(v2*v3) - v3*(k3*v2))
  end function w_hwz_DPB
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_hww_DPB (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * (((k1+k2+k3)*v3)*v2 - ((k1+k2+k3)*v2)*v3)
  end function z_hww_DPB
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_wwz_DDPW (g, v1, k1, v2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * (((k1-k2)*v3)*(v1*v2)-((k1-k3)*v2)*(v1*v3)+((k2-k3)*v1)*(v2*v3))
  end function h_wwz_DDPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_hwz_DDPW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * ((-(k1+2*k2+k3)*v3)*v2 + ((k1+k2+2*k3)*v2)*v3 + &
         (v2*v3)*(k2-k3))
  end function w_hwz_DDPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_hww_DDPW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * ((v2*v3)*(k2-k3) - ((k1+2*k2+k3)*v3) *v2 + &
         ((k1+k2+2*k3)*v2)*v3 )
  end function z_hww_DDPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function h_wwz_DPW (g, v1, k1, v2, k2, v3, k3) result (hout)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    complex(kind=default) :: hout
    hout = g * (((k1-k2)*v3)*(v1*v2) + (-(2*k1+k2+k3)*v2)*(v1*v3) + &
         ((k1+2*k2+k3)*v1)*(v2*v3))
  end function h_wwz_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function w_hwz_DPW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
```

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```
      complex(kind=default), intent(in) :: h1
      type(vector), intent(in) :: v2, v3
      type(momentum), intent(in) :: k1, k2, k3
      type(vector) :: vout
      vout = g * h1 * ((-(k1+2*k2+k3)*v3)*v2 + ((2*k1+k2+k3)*v2)*v3 + &
             (v2*v3)*(k2-k1))
    end function w_hwz_DPW
```

⟨*Implementation of couplings*⟩+≡
```
  pure function z_hww_DPW (g, h1, k1, v2, k2, v3, k3) result (vout)
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: h1
    type(vector), intent(in) :: v2, v3
    type(momentum), intent(in) :: k1, k2, k3
    type(vector) :: vout
    vout = g * h1 * ((v2*v3)*(k2-k3) + ((k1-k2)*v3)*v2 + ((k3-k1)*v2)*v3)
  end function z_hww_DPW
```

X.16   Scalar3 Dim-5 Couplings

⟨*Declaration of couplings*⟩+≡
```
  public :: phi_dim5s2
```

$$\phi_1(k_1) = g(k_2 \cdot k_3)\phi_2(k_2)\phi_3(k_3) \tag{X.65}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_dim5s2 (g, phi2, k2, phi3, k3) result (phi1)
    complex(kind=default), intent(in) :: g, phi2, phi3
    type(momentum), intent(in) :: k2, k3
    complex(kind=default) :: phi1
    phi1 = g * phi2 * phi3 * (k2 * k3)
  end function phi_dim5s2
```

X.17   Tensorscalar-Scalar Couplings

⟨*Declaration of couplings*⟩+≡
```
  public :: tphi_ss, tphi_ss_cf, s_tphis, s_tphis_cf
```

$$\phi(k_1 + k_2) = 2g((k_1 \cdot k_2) + (k_1 \cdot k_1))((k_1 \cdot k_2) + (k_2 \cdot k_2))\phi_1(k_1)\phi_2(k_2) \tag{X.66}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function tphi_ss (g, phi1, k1, phi2, k2) result (phi)
    complex(kind=default), intent(in) :: g, phi1, phi2
    type(momentum), intent(in) :: k1, k2
    complex(kind=default) :: phi
    phi = 2 * g * phi1 * phi2 * &
            ((k1 * k2)+ (k1 * k1)) * &
            ((k1 * k2)+ (k2 * k2))
  end function tphi_ss
```

$$\phi(k_1 + k_2) = -g/2(k_1 \cdot k_2)((k_1 + k_2) \cdot (k_1 + k_2))\phi_1(k_1)\phi_2(k_2) \qquad \text{(X.67)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function tphi_ss_cf (g, phi1, k1, phi2, k2) result (phi)
  complex(kind=default), intent(in) :: g, phi1, phi2
  type(momentum), intent(in) :: k1, k2
  complex(kind=default) :: phi
  phi = - g/2 * phi1 * phi2 * &
          (k1 * k2) * &
          ((k1 + k2) * (k1 + k2))
end function tphi_ss_cf
```

$$\phi_1(k_1) = 2g((k_1 \cdot k_2) + (k_1 \cdot k_1))((k_1 \cdot k_2) + (k_2 \cdot k_2))\phi(k_2 - k_1)\phi_2(k_2) \quad \text{(X.68)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function s_tphis (g, phi, k, phi2, k2) result (phi1)
  complex(kind=default), intent(in) :: g, phi, phi2
  type(momentum), intent(in) :: k, k2
  complex(kind=default) :: phi1
  type(momentum) :: k1
  k1 = - ( k + k2)
  phi1 = 2 * g * phi * phi2 * &
          ((k1 * k2)+ (k1 * k1)) * &
          ((k1 * k2)+ (k2 * k2))
end function s_tphis
```

$$\phi_1(k_1) = -g/2(k_1 \cdot k_2)((k_1 + k_2) \cdot (k_1 + k_2))\phi(k_2 - k_1)\phi_2(k_2) \qquad \text{(X.69)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function s_tphis_cf (g, phi, k, phi2, k2) result (phi1)
  complex(kind=default), intent(in) :: g, phi, phi2
  type(momentum), intent(in) :: k, k2
  complex(kind=default) :: phi1
  type(momentum) :: k1
  k1 = - ( k + k2)
  phi1 = - g/2 * phi * phi2 * &
          (k1 * k2) * &
          ((k1 + k2) * (k1 + k2))
end function s_tphis_cf
```

X.18 Scalar2-Vector2 Dim-8 Couplings

⟨*Declaration of couplings*⟩+≡
```
public :: phi_phi2v_1, v_phi2v_1, phi_phi2v_2, v_phi2v_2
```

$$\phi_2(k_2) = g\left((k_1 \cdot V_1)(k_2 \cdot V_2) + (k_1 \cdot V_1)(k_1 \cdot V_2)\right)\phi_1(k_1) \qquad \text{(X.70)}$$

⟨*Implementation of couplings*⟩+≡
```
pure function phi_phi2v_1 (g, phi1, k1, v1, k_v1, v2, k_v2) result (phi2)
  complex(kind=default), intent(in) :: g, phi1
  type(momentum), intent(in) :: k1, k_v1, k_v2
```

```
      type(momentum) :: k2
      type(vector), intent(in) :: v1, v2
      complex(kind=default) :: phi2
      k2 = - k1 - k_v1 - k_v2
      phi2 = g * phi1 * &
           ( (k1 * v1) * (k2 * v2) + (k1 * v2) * (k2 * v1) )
    end function phi_phi2v_1
```

$$V_2^\mu = g \left(k_1^\mu \left(k_2 \cdot V_1 \right) + k_2^\mu \left(k_1 \cdot V_1 \right) \right) \phi_1(k_1) \phi_2(k_2) \qquad \text{(X.71)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function v_phi2v_1 (g, phi1, k1, phi2, k2, v1) result (v2)
    complex(kind=default), intent(in) :: g, phi1, phi2
    type(momentum), intent(in) :: k1, k2
    type(vector), intent(in) :: v1
    type(vector) :: v2
    v2 = g * phi1 * phi2 * &
         ( k1  * (k2 * v1) + k2 * (k1 * v1) )
  end function v_phi2v_1
```

$$\phi_2(k_2) = g \left(k_1 \cdot k_2 \right) \left(V_1 \cdot V_2 \right) \phi_1(k_1) \qquad \text{(X.72)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_phi2v_2 (g, phi1, k1, v1,k_v1, v2, k_v2) result (phi2)
    complex(kind=default), intent(in) :: g, phi1
    type(momentum), intent(in) :: k1, k_v1, k_v2
    type(vector), intent(in) :: v1, v2
    type(momentum) :: k2
    complex(kind=default) :: phi2
    k2 = - k1 - k_v1 - k_v2
    phi2 = g * phi1 * (k1 * k2) * (v1 * v2)
  end function phi_phi2v_2
```

$$V_2^\mu = g V_1^\mu \left(k_1 \cdot k_2 \right) \phi_1 \phi_2 \qquad \text{(X.73)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function v_phi2v_2 (g, phi1, k1, phi2, k2, v1) result (v2)
    complex(kind=default), intent(in) :: g, phi1, phi2
    type(momentum), intent(in) :: k1, k2
    type(vector), intent(in) :: v1
    type(vector) :: v2
    v2 = g * phi1 * phi2 * &
         ( k1  * k2 ) * v1
  end function v_phi2v_2
```

X.19   Scalar4 Dim-8 Couplings

⟨*Declaration of couplings*⟩+≡
```
  public :: s_dim8s3
```

$$\phi(k_1) = g \left[\left(k_1 \cdot k_2 \right) \left(k_3 \cdot k_4 \right) + \left(k_1 \cdot k_3 \right) \left(k_2 \cdot k_4 \right) + \left(k_1 \cdot k_4 \right) \left(k_2 \cdot k_3 \right) \right] \phi_2(k_2) \phi_3(k_3) \phi_4(k_4)$$
$$\text{(X.74)}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function s_dim8s3 (g, phi2, k2, phi3, k3, phi4, k4) result (phi1)
    complex(kind=default), intent(in) :: g, phi2, phi3, phi4
    type(momentum), intent(in) :: k2, k3, k4
    type(momentum) :: k1
    complex(kind=default) :: phi1
    k1 = - k2 - k3 - k4
    phi1 = g * ( (k1 * k2) * (k3 * k4) + (k1 * k3) * (k2 * k4) &
            + (k1 * k4) * (k2 * k3) ) * phi2 * phi3 * phi4
  end function s_dim8s3
```

X.20   Graviton Couplings

⟨*Declaration of couplings*⟩+≡
```
  public :: s_gravs, v_gravv, grav_ss, grav_vv
```

⟨*Implementation of couplings*⟩+≡
```
  pure function s_gravs (g, m, k1, k2, t, s) result (phi)
    complex(kind=default), intent(in) :: g, s
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: k1, k2
    type(tensor), intent(in) :: t
    complex(kind=default) :: phi, t_tr
    t_tr = t%t(0,0) - t%t(1,1) - t%t(2,2) - t%t(3,3)
    phi = g * s * (((t*k1)*k2) + ((t*k2)*k1) &
        - g * (m**2 + (k1*k2))*t_tr)/2.0_default
  end function s_gravs
```

⟨*Implementation of couplings*⟩+≡
```
  pure function grav_ss (g, m, k1, k2, s1, s2) result (t)
    complex(kind=default), intent(in) :: g, s1, s2
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: k1, k2
    type(tensor) :: t_metric, t
    t_metric%t = 0
    t_metric%t(0,0) = 1.0_default
    t_metric%t(1,1) = - 1.0_default
    t_metric%t(2,2) = - 1.0_default
    t_metric%t(3,3) = - 1.0_default
    t = g*s1*s2/2.0_default * (-(m**2 + (k1*k2)) * t_metric &
      + (k1.tprod.k2) + (k2.tprod.k1))
  end function grav_ss
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_gravv (g, m, k1, k2, t, v) result (vec)
    complex(kind=default), intent(in) :: g
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: k1, k2
    type(vector), intent(in) :: v
    type(tensor), intent(in) :: t
    complex(kind=default) :: t_tr
    real(kind=default) :: xi
    type(vector) :: vec
    xi = 1.0_default
```

```
    t_tr = t%t(0,0) - t%t(1,1) - t%t(2,2) - t%t(3,3)
    vec = (-g)/ 2.0_default * (((k1*k2) + m**2) * &
          (t*v + v*t - t_tr * v) + t_tr * (k1*v) * k2 &
          - (k1*v) * ((k2*t) + (t*k2)) &
          - ((k1*(t*v)) + (v*(t*k1))) * k2 &
          + ((k1*(t*k2)) + (k2*(t*k1))) * v)
!!!         Unitarity gauge: xi -> Infinity
!!!         + (1.0_default/xi) * (t_tr * ((k1*v)*k2) + &
!!!         (k2*v)*k2 + (k2*v)*k1 - (k1*(t*v))*k1 + &
!!!         (k2*v)*(k2*t) - (v*(t*k1))*k1 - (k2*v)*(t*k2)))
    end function v_gravv
```

⟨*Implementation of couplings*⟩+≡

```
  pure function grav_vv (g, m, k1, k2, v1, v2) result (t)
    complex(kind=default), intent(in) :: g
    type(momentum), intent(in) :: k1, k2
    real(kind=default), intent(in) :: m
    real(kind=default) :: xi
    type(vector), intent (in) :: v1, v2
    type(tensor) :: t_metric, t
    xi = 0.00001_default
    t_metric%t = 0
    t_metric%t(0,0) = 1.0_default
    t_metric%t(1,1) = - 1.0_default
    t_metric%t(2,2) = - 1.0_default
    t_metric%t(3,3) = - 1.0_default
    t = (-g)/2.0_default * ( &
          ((k1*k2) + m**2) * ( &
          (v1.tprod.v2) +  (v2.tprod.v1) - (v1*v2) * t_metric) &
          + (v1*k2)*(v2*k1)*t_metric &
          - (k2*v1)*((v2.tprod.k1) + (k1.tprod.v2)) &
          - (k1*v2)*((v1.tprod.k2) + (k2.tprod.v1)) &
          + (v1*v2)*((k1.tprod.k2) + (k2.tprod.k1)))
!!!         Unitarity gauge: xi -> Infinity
!!!         + (1.0_default/xi) * ( &
!!!         ((k1*v1)*(k1*v2) + (k2*v1)*(k2*v2) + (k1*v1)*(k2*v2))* &
!!!         t_metric) - (k1*v1) * ((k1.tprod.v2) + (v2.tprod.k1)) &
!!!         - (k2*v2) * ((k2.tprod.v1) + (v1.tprod.k2)))
    end function grav_vv
```

X.21 Tensor Couplings

⟨*Declaration of couplings*⟩+≡

```
  public :: t2_vv, v_t2v, t2_vv_cf, v_t2v_cf, &
        t2_vv_1, v_t2v_1, t2_vv_t, v_t2v_t, &
        t2_phi2, phi_t2phi, t2_phi2_cf, phi_t2phi_cf
```

$$T_{\mu\nu} = g * V_{1\,\mu}V_{2\,\nu} + V_{1\,\nu}V_{2\,\mu} \tag{X.75}$$

⟨*Implementation of couplings*⟩+≡

```
  pure function t2_vv (g, v1, v2) result (t)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
```

```
    type(tensor) :: t
    type(tensor) :: tmp
    tmp = v1.tprod.v2
    t%t = g * (tmp%t + transpose (tmp%t))
  end function t2_vv
```

$$V_{1\,\mu} = g * T_{\mu\nu} V_2^{\nu} + T_{\nu\mu} V_2^{\nu} \tag{X.76}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function v_t2v (g, t, v) result (tv)
    complex(kind=default), intent(in) :: g
    type(tensor), intent(in) :: t
    type(vector), intent(in) :: v
    type(vector) :: tv
    type(tensor) :: tmp
    tmp%t = t%t + transpose (t%t)
    tv = g * (tmp * v)
  end function v_t2v
```

$$T_{\mu\nu} = -\frac{g}{2} V_1^{\rho} V_{2\,\rho} \tag{X.77}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_vv_cf (g, v1, v2) result (t)
    complex(kind=default), intent(in) :: g
    complex(kind=default) :: tmp_s
    type(vector), intent(in) :: v1, v2
    type(tensor) :: t_metric, t
    t_metric%t = 0
    t_metric%t(0,0) =   1.0_default
    t_metric%t(1,1) = - 1.0_default
    t_metric%t(2,2) = - 1.0_default
    t_metric%t(3,3) = - 1.0_default
    tmp_s = v1 * v2
    t%t = - (g /2.0_default) * tmp_s * t_metric%t
  end function t2_vv_cf
```

$$V_{1\,\mu} = -\frac{g}{2} T_{\nu}^{\nu} V_2^{\mu} \tag{X.78}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function v_t2v_cf (g, t, v) result (tv)
    complex(kind=default), intent(in) :: g
    type(tensor), intent(in) :: t
    type(vector), intent(in) :: v
    type(vector) :: tv, tmp_tv
    tmp_tv =  ( t%t(0,0)-t%t(1,1)-t%t(2,2)-t%t(3,3) ) * v
    tv = - ( g /2.0_default) * tmp_tv
  end function v_t2v_cf
```

$$T_{\mu\nu} = g * (k_{1\,\mu} k_{2\,\nu} + k_{1\,\nu} k_{2\,\mu})\, \phi_1\,(k_1)\, \phi_1\,(k_2) \tag{X.79}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_phi2 (g, phi1, k1, phi2, k2) result (t)
    complex(kind=default), intent(in) :: g, phi1, phi2
    type(momentum), intent(in) :: k1, k2
```

```
      type(tensor) :: t
      type(tensor) :: tmp
      tmp = k1.tprod.k2
      t%t = g * (tmp%t + transpose (tmp%t)) * phi1 * phi2
    end function t2_phi2
```

$$\phi_1(k_1) = g * \left(T_{\mu\nu}k_1^\mu k_2^\nu + T_{\nu\mu}k_2^\mu k_1^\nu\right)\phi_2\left(k_2\right) \tag{X.80}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_t2phi (g, t, kt, phi2, k2) result (phi1)
    complex(kind=default), intent(in) :: g, phi2
    type(tensor), intent(in) :: t
    type(momentum), intent(in) :: kt, k2
    type(momentum) :: k1
    complex(kind=default) :: phi1
    type(tensor) :: tmp
    k1 = -kt - k2
    tmp%t = t%t + transpose (t%t)
    phi1 = g * ( (tmp * k2) * k1) * phi2
  end function phi_t2phi
```

$$T_{\mu\nu} = -\frac{g}{2}k_1^\rho k_{2\,\rho}\phi_1\left(k_1\right)\phi_2\left(k_2\right) \tag{X.81}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_phi2_cf (g, phi1, k1, phi2, k2) result (t)
    complex(kind=default), intent(in) :: g, phi1, phi2
    complex(kind=default) :: tmp_s
    type(momentum), intent(in) :: k1, k2
    type(tensor) :: t_metric, t
    t_metric%t = 0
    t_metric%t(0,0) =   1.0_default
    t_metric%t(1,1) = - 1.0_default
    t_metric%t(2,2) = - 1.0_default
    t_metric%t(3,3) = - 1.0_default
    tmp_s = (k1 * k2) * phi1 * phi2
    t%t = - (g /2.0_default) * tmp_s * t_metric%t
  end function t2_phi2_cf
```

$$\phi_1(k_1) = -\frac{g}{2}T_\nu^\nu\left(k_1 \cdot k_2\right)\phi_2(k_2) \tag{X.82}$$

⟨*Implementation of couplings*⟩+≡
```
  pure function phi_t2phi_cf (g, t, kt, phi2, k2) result (phi1)
    complex(kind=default), intent(in) :: g, phi2
    type(tensor), intent(in) :: t
    type(momentum), intent(in) :: kt, k2
    type(momentum) :: k1
    complex(kind=default) ::  tmp_ts, phi1
    k1 = - kt - k2
    tmp_ts =  ( t%t(0,0)-t%t(1,1)-t%t(2,2)-t%t(3,3) )
    phi1 = - ( g /2.0_default) * tmp_ts * (k1 * k2) * phi2
  end function phi_t2phi_cf
```

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_vv_1 (g, v1, v2) result (t)
    complex(kind=default), intent(in) :: g
    complex(kind=default) :: tmp_s
    type(vector), intent(in) :: v1, v2
    type(tensor) :: tmp
    type(tensor) :: t_metric, t
    t_metric%t = 0
    t_metric%t(0,0) =   1.0_default
    t_metric%t(1,1) = - 1.0_default
    t_metric%t(2,2) = - 1.0_default
    t_metric%t(3,3) = - 1.0_default
    tmp = v1.tprod.v2
    tmp_s = v1 * v2
    t%t = g * (tmp%t + transpose (tmp%t) - tmp_s * t_metric%t )
  end function t2_vv_1
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_t2v_1 (g, t, v) result (tv)
    complex(kind=default), intent(in) :: g
    type(tensor), intent(in) :: t
    type(vector), intent(in) :: v
    type(vector) :: tv, tmp_tv
    type(tensor) :: tmp
    tmp_tv =  ( t%t(0,0)-t%t(1,1)-t%t(2,2)-t%t(3,3) ) * v
    tmp%t = t%t + transpose (t%t)
    tv = g * (tmp * v - tmp_tv)
  end function v_t2v_1
```

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_vv_t (g, v1, k1, v2, k2) result (t)
    complex(kind=default), intent(in) :: g
    complex(kind=default) :: tmp_s
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(tensor) :: tmp, tmp_v1k2, tmp_v2k1, tmp_k1k2, tmp2
    type(tensor) :: t_metric, t
    t_metric%t = 0
    t_metric%t(0,0) =   1.0_default
    t_metric%t(1,1) = - 1.0_default
    t_metric%t(2,2) = - 1.0_default
    t_metric%t(3,3) = - 1.0_default
    tmp = v1.tprod.v2
    tmp_s = v1 * v2
    tmp_v1k2 = (v2 * k1) * (v1.tprod.k2)
    tmp_v2k1 = (v1 * k2) * (v2.tprod.k1)
    tmp_k1k2 = tmp_s * (k1.tprod.k2)
    tmp2%t = tmp_v1k2%t + tmp_v2k1%t - tmp_k1k2%t
    t%t = g * ( (k1*k2) * (tmp%t + transpose (tmp%t) - tmp_s * t_metric%t ) &
          + ((v1 * k2) * (v2 * k1)) * t_metric%t &
          - tmp2%t - transpose(tmp2%t))
  end function t2_vv_t
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_t2v_t (g, t, kt, v, kv) result (tv)
    complex(kind=default), intent(in) :: g
```

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```
      type(tensor), intent(in) :: t
      type(vector), intent(in) :: v
      type(momentum), intent(in) :: kt, kv
      type(momentum) :: kout
      type(vector) :: tv, tmp_tv
      type(tensor) :: tmp
      kout = - (kt + kv)
      tmp_tv =  ( t%t(0,0)-t%t(1,1)-t%t(2,2)-t%t(3,3) ) * v
      tmp%t = t%t + transpose (t%t)
      tv = g * ( (tmp * v - tmp_tv) * (kv * kout )&
           + ( t%t(0,0)-t%t(1,1)-t%t(2,2)-t%t(3,3) ) * (kout * v ) * kv &
           - (kout * v) * ( tmp * kv) &
           - (v* (t * kout) + kout * (t * v)) * kv &
           + (kout* (t * kv) + kv * (t * kout)) * v)
    end function v_t2v_t
```

⟨*Declaration of couplings*⟩+≡
```
  public :: t2_vv_d5_1, v_t2v_d5_1
```

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_vv_d5_1 (g, v1, k1, v2, k2) result (t)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(tensor) :: t
    t = (g * (v1 * v2)) * (k1-k2).tprod.(k1-k2)
  end function t2_vv_d5_1
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_t2v_d5_1 (g, t1, k1, v2, k2) result (tv)
    complex(kind=default), intent(in) :: g
    type(tensor), intent(in) :: t1
    type(vector), intent(in) :: v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: tv
    tv = (g * ((k1+2*k2).tprod.(k1+2*k2) * t1)) * v2
  end function v_t2v_d5_1
```

⟨*Declaration of couplings*⟩+≡
```
  public :: t2_vv_d5_2, v_t2v_d5_2
```

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_vv_d5_2 (g, v1, k1, v2, k2) result (t)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(tensor) :: t
    t = (g * (k2 * v1)) * (k2-k1).tprod.v2
    t%t = t%t + transpose (t%t)
  end function t2_vv_d5_2
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_t2v_d5_2 (g, t1, k1, v2, k2) result (tv)
    complex(kind=default), intent(in) :: g
    type(tensor), intent(in) :: t1
    type(vector), intent(in) :: v2
    type(momentum), intent(in) :: k1, k2
```

```
      type(vector) :: tv
      type(tensor) :: tmp
      type(momentum) :: k1_k2, k1_2k2
      k1_k2 = k1 + k2
      k1_2k2 = k1_k2 + k2
      tmp%t = t1%t + transpose (t1%t)
      tv = (g * (k1_k2 * v2)) * (k1_2k2 * tmp)
    end function v_t2v_d5_2
```

⟨*Declaration of couplings*⟩+≡
```
  public :: t2_vv_d7, v_t2v_d7
```

⟨*Implementation of couplings*⟩+≡
```
  pure function t2_vv_d7 (g, v1, k1, v2, k2) result (t)
    complex(kind=default), intent(in) :: g
    type(vector), intent(in) :: v1, v2
    type(momentum), intent(in) :: k1, k2
    type(tensor) :: t
    t = (g * (k2 * v1) * (k1 * v2)) * (k1-k2).tprod.(k1-k2)
  end function t2_vv_d7
```

⟨*Implementation of couplings*⟩+≡
```
  pure function v_t2v_d7 (g, t1, k1, v2, k2) result (tv)
    complex(kind=default), intent(in) :: g
    type(tensor), intent(in) :: t1
    type(vector), intent(in) :: v2
    type(momentum), intent(in) :: k1, k2
    type(vector) :: tv
    type(vector) :: k1_k2, k1_2k2
    k1_k2 = k1 + k2
    k1_2k2 = k1_k2 + k2
    tv = (- g * (k1_k2 * v2) * (k1_2k2.tprod.k1_2k2 * t1)) * k2
  end function v_t2v_d7
```

X.22 Spinor Couplings

⟨omega_spinor_couplings.f90⟩≡
```
  ⟨Copyleft⟩
  module omega_spinor_couplings
    use kinds
    use constants
    use omega_spinors
    use omega_vectors
    use omega_tensors
    use omega_couplings
    implicit none
    private
    ⟨Declaration of spinor on shell wave functions⟩
    ⟨Declaration of spinor off shell wave functions⟩
    ⟨Declaration of spinor currents⟩
    ⟨Declaration of spinor propagators⟩
    integer, parameter, public :: omega_spinor_cpls_2010_01_A = 0
  contains
    ⟨Implementation of spinor on shell wave functions⟩
    ⟨Implementation of spinor off shell wave functions⟩
```

⟨*Implementation of spinor currents*⟩
⟨*Implementation of spinor propagators*⟩
```
  end module omega_spinor_couplings
```

See table X.1 for the names of Fortran functions. We could have used long names instead, but this would increase the chance of running past continuation line limits without adding much to the legibility.

X.22.1   Fermionic Vector and Axial Couplings

There's more than one chiral representation. This one is compatible with HELAS [5].

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \; \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \; \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix} \tag{X.83}$$

Therefore

$$g_S + g_P\gamma_5 = \begin{pmatrix} g_S - g_P & 0 & 0 & 0 \\ 0 & g_S - g_P & 0 & 0 \\ 0 & 0 & g_S + g_P & 0 \\ 0 & 0 & 0 & g_S + g_P \end{pmatrix} \tag{X.84a}$$

$$g_V\gamma^0 - g_A\gamma^0\gamma_5 = \begin{pmatrix} 0 & 0 & g_V - g_A & 0 \\ 0 & 0 & 0 & g_V - g_A \\ g_V + g_A & 0 & 0 & 0 \\ 0 & g_V + g_A & 0 & 0 \end{pmatrix} \tag{X.84b}$$

$$g_V\gamma^1 - g_A\gamma^1\gamma_5 = \begin{pmatrix} 0 & 0 & 0 & g_V - g_A \\ 0 & 0 & g_V - g_A & 0 \\ 0 & -g_V - g_A & 0 & 0 \\ -g_V - g_A & 0 & 0 & 0 \end{pmatrix} \tag{X.84c}$$

$$g_V\gamma^2 - g_A\gamma^2\gamma_5 = \begin{pmatrix} 0 & 0 & 0 & -\mathrm{i}(g_V - g_A) \\ 0 & 0 & \mathrm{i}(g_V - g_A) & 0 \\ 0 & \mathrm{i}(g_V + g_A) & 0 & 0 \\ -\mathrm{i}(g_V + g_A) & 0 & 0 & 0 \end{pmatrix}$$
$$\tag{X.84d}$$

$$g_V\gamma^3 - g_A\gamma^3\gamma_5 = \begin{pmatrix} 0 & 0 & g_V - g_A & 0 \\ 0 & 0 & 0 & -g_V + g_A \\ -g_V - g_A & 0 & 0 & 0 \\ 0 & g_V + g_A & 0 & 0 \end{pmatrix} \tag{X.84e}$$

⟨*Declaration of spinor currents*⟩≡
```
  public :: va_ff, v_ff, a_ff, vl_ff, vr_ff, vlr_ff, grav_ff, va2_ff, &
            tva_ff, tlr_ff, trl_ff, tvam_ff, tlrm_ff, trlm_ff, va3_ff
```

⟨*Implementation of spinor currents*⟩≡
```
  pure function va_ff (gv, ga, psibar, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
```

$\bar{\psi}(g_V\gamma^\mu - g_A\gamma^\mu\gamma_5)\psi$	`va_ff`$(g_V, g_A, \bar{\psi}, \psi)$
$g_V\bar{\psi}\gamma^\mu\psi$	`v_ff`$(g_V, \bar{\psi}, \psi)$
$g_A\bar{\psi}\gamma_5\gamma^\mu\psi$	`a_ff`$(g_A, \bar{\psi}, \psi)$
$g_L\bar{\psi}\gamma^\mu(1 - \gamma_5)\psi$	`vl_ff`$(g_L, \bar{\psi}, \psi)$
$g_R\bar{\psi}\gamma^\mu(1 + \gamma_5)\psi$	`vr_ff`$(g_R, \bar{\psi}, \psi)$
$\slashed{V}(g_V - g_A\gamma_5)\psi$	`f_vaf`(g_V, g_A, V, ψ)
$g_V\slashed{V}\psi$	`f_vf`(g_V, V, ψ)
$g_A\gamma_5\slashed{V}\psi$	`f_af`(g_A, V, ψ)
$g_L\slashed{V}(1 - \gamma_5)\psi$	`f_vlf`(g_L, V, ψ)
$g_R\slashed{V}(1 + \gamma_5)\psi$	`f_vrf`(g_R, V, ψ)
$\bar{\psi}\slashed{V}(g_V - g_A\gamma_5)$	`f_fva`(g_V, g_A, ψ, V)
$g_V\bar{\psi}\slashed{V}$	`f_fv`$(g_V, \bar{\psi}, V)$
$g_A\bar{\psi}\gamma_5\slashed{V}$	`f_fa`$(g_A, \bar{\psi}, V)$
$g_L\bar{\psi}\slashed{V}(1 - \gamma_5)$	`f_fvl`$(g_L, \bar{\psi}, V)$
$g_R\bar{\psi}\slashed{V}(1 + \gamma_5)$	`f_fvr`$(g_R, \bar{\psi}, V)$

Table X.1: Mnemonically abbreviated names of Fortran functions implementing fermionic vector and axial currents.

$\bar{\psi}(g_S + g_P\gamma_5)\psi$	`sp_ff`$(g_S, g_P, \bar{\psi}, \psi)$
$g_S\bar{\psi}\psi$	`s_ff`$(g_S, \bar{\psi}, \psi)$
$g_P\bar{\psi}\gamma_5\psi$	`p_ff`$(g_P, \bar{\psi}, \psi)$
$g_L\bar{\psi}(1 - \gamma_5)\psi$	`sl_ff`$(g_L, \bar{\psi}, \psi)$
$g_R\bar{\psi}(1 + \gamma_5)\psi$	`sr_ff`$(g_R, \bar{\psi}, \psi)$
$\phi(g_S + g_P\gamma_5)\psi$	`f_spf`(g_S, g_P, ϕ, ψ)
$g_S\phi\psi$	`f_sf`(g_S, ϕ, ψ)
$g_P\phi\gamma_5\psi$	`f_pf`(g_P, ϕ, ψ)
$g_L\phi(1 - \gamma_5)\psi$	`f_slf`(g_L, ϕ, ψ)
$g_R\phi(1 + \gamma_5)\psi$	`f_srf`(g_R, ϕ, ψ)
$\bar{\psi}\phi(g_S + g_P\gamma_5)$	`f_fsp`$(g_S, g_P, \bar{\psi}, \phi)$
$g_S\bar{\psi}\phi$	`f_fs`$(g_S, \bar{\psi}, \phi)$
$g_P\bar{\psi}\phi\gamma_5$	`f_fp`$(g_P, \bar{\psi}, \phi)$
$g_L\bar{\psi}\phi(1 - \gamma_5)$	`f_fsl`$(g_L, \bar{\psi}, \phi)$
$g_R\bar{\psi}\phi(1 + \gamma_5)$	`f_fsr`$(g_R, \bar{\psi}, \phi)$

Table X.2: Mnemonically abbreviated names of Fortran functions implementing fermionic scalar and pseudo scalar "currents".

```
    complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
    gl = gv + ga
    gr = gv - ga
    g13 = psibar%a(1)*psi%a(3)
    g14 = psibar%a(1)*psi%a(4)
    g23 = psibar%a(2)*psi%a(3)
    g24 = psibar%a(2)*psi%a(4)
    g31 = psibar%a(3)*psi%a(1)
    g32 = psibar%a(3)*psi%a(2)
    g41 = psibar%a(4)*psi%a(1)
    g42 = psibar%a(4)*psi%a(2)
    j%t    = gr * (   g13 + g24) + gl * (   g31 + g42)
    j%x(1) =  gr * (   g14 + g23) - gl * (   g32 + g41)
    j%x(2) = (gr * ( - g14 + g23) + gl * (   g32 - g41)) * (0, 1)
    j%x(3) =  gr * (   g13 - g24) + gl * ( - g31 + g42)
  end function va_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function va2_ff (gva, psibar, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in), dimension(2) :: gva
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
    complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
    gl = gva(1) + gva(2)
    gr = gva(1) - gva(2)
    g13 = psibar%a(1)*psi%a(3)
    g14 = psibar%a(1)*psi%a(4)
    g23 = psibar%a(2)*psi%a(3)
    g24 = psibar%a(2)*psi%a(4)
    g31 = psibar%a(3)*psi%a(1)
    g32 = psibar%a(3)*psi%a(2)
    g41 = psibar%a(4)*psi%a(1)
    g42 = psibar%a(4)*psi%a(2)
    j%t    = gr * (   g13 + g24) + gl * (   g31 + g42)
    j%x(1) =  gr * (   g14 + g23) - gl * (   g32 + g41)
    j%x(2) = (gr * ( - g14 + g23) + gl * (   g32 - g41)) * (0, 1)
    j%x(3) =  gr * (   g13 - g24) + gl * ( - g31 + g42)
  end function va2_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function va3_ff (gv, ga, psibar, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j   = va_ff (gv, ga, psibar, psi)
    j%t = 0.0_default
  end function va3_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function tva_ff (gv, ga, psibar, psi) result (t)
    type(tensor2odd) :: t
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
```

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```
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
    complex(kind=default) :: g12, g21, g1m2, g34, g43, g3m4
    gr      = gv + ga
    gl      = gv - ga
    g12     = psibar%a(1)*psi%a(2)
    g21     = psibar%a(2)*psi%a(1)
    g1m2    = psibar%a(1)*psi%a(1) - psibar%a(2)*psi%a(2)
    g34     = psibar%a(3)*psi%a(4)
    g43     = psibar%a(4)*psi%a(3)
    g3m4    = psibar%a(3)*psi%a(3) - psibar%a(4)*psi%a(4)
    t%e(1) = (gl * ( - g12 - g21) + gr * (   g34 + g43)) * (0, 1)
    t%e(2) =  gl * ( - g12 + g21) + gr * (   g34 - g43)
    t%e(3) = (gl * ( - g1m2     ) + gr * (   g3m4     )) * (0, 1)
    t%b(1) =  gl * (   g12 + g21) + gr * (   g34 + g43)
    t%b(2) = (gl * ( - g12 + g21) + gr * ( - g34 + g43)) * (0, 1)
    t%b(3) =  gl * (   g1m2     ) + gr * (   g3m4     )
  end function tva_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function tlr_ff (gl, gr, psibar, psi) result (t)
    type(tensor2odd) :: t
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    t = tva_ff (gr+gl, gr-gl, psibar, psi)
  end function tlr_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function trl_ff (gr, gl, psibar, psi) result (t)
    type(tensor2odd) :: t
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    t = tva_ff (gr+gl, gr-gl, psibar, psi)
  end function trl_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function tvam_ff (gv, ga, psibar, psi, p) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    type(momentum), intent(in) :: p
    j = (tva_ff(gv, ga, psibar, psi) * p) * (0,1)
  end function tvam_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function tlrm_ff (gl, gr, psibar, psi, p) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    type(momentum), intent(in) :: p
    j = tvam_ff (gr+gl, gr-gl, psibar, psi, p)
  end function tlrm_ff
```

⟨*Implementation of spinor currents*⟩+≡

```
pure function trlm_ff (gr, gl, psibar, psi, p) result (j)
  type(vector) :: j
  complex(kind=default), intent(in) :: gl, gr
  type(conjspinor), intent(in) :: psibar
  type(spinor), intent(in) :: psi
  type(momentum), intent(in) :: p
  j = tvam_ff (gr+gl, gr-gl, psibar, psi, p)
end function trlm_ff
```

Special cases that avoid some multiplications

⟨*Implementation of spinor currents*⟩+≡

```
pure function v_ff (gv, psibar, psi) result (j)
  type(vector) :: j
  complex(kind=default), intent(in) :: gv
  type(conjspinor), intent(in) :: psibar
  type(spinor), intent(in) :: psi
  complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
  g13 = psibar%a(1)*psi%a(3)
  g14 = psibar%a(1)*psi%a(4)
  g23 = psibar%a(2)*psi%a(3)
  g24 = psibar%a(2)*psi%a(4)
  g31 = psibar%a(3)*psi%a(1)
  g32 = psibar%a(3)*psi%a(2)
  g41 = psibar%a(4)*psi%a(1)
  g42 = psibar%a(4)*psi%a(2)
  j%t    =   gv * (   g13 + g24 + g31 + g42)
  j%x(1) =   gv * (   g14 + g23 - g32 - g41)
  j%x(2) =   gv * ( - g14 + g23 + g32 - g41) * (0, 1)
  j%x(3) =   gv * (   g13 - g24 - g31 + g42)
end function v_ff
```

⟨*Implementation of spinor currents*⟩+≡

```
pure function a_ff (ga, psibar, psi) result (j)
  type(vector) :: j
  complex(kind=default), intent(in) :: ga
  type(conjspinor), intent(in) :: psibar
  type(spinor), intent(in) :: psi
  complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
  g13 = psibar%a(1)*psi%a(3)
  g14 = psibar%a(1)*psi%a(4)
  g23 = psibar%a(2)*psi%a(3)
  g24 = psibar%a(2)*psi%a(4)
  g31 = psibar%a(3)*psi%a(1)
  g32 = psibar%a(3)*psi%a(2)
  g41 = psibar%a(4)*psi%a(1)
  g42 = psibar%a(4)*psi%a(2)
  j%t    =   ga * ( - g13 - g24 + g31 + g42)
  j%x(1) = - ga * (   g14 + g23 + g32 + g41)
  j%x(2) =   ga * (   g14 - g23 + g32 - g41) * (0, 1)
  j%x(3) =   ga * ( - g13 + g24 - g31 + g42)
end function a_ff
```

⟨*Implementation of spinor currents*⟩+≡

```
pure function vl_ff (gl, psibar, psi) result (j)
  type(vector) :: j
```

```
    complex(kind=default), intent(in) :: gl
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl2
    complex(kind=default) :: g31, g32, g41, g42
    gl2 = 2 * gl
    g31 = psibar%a(3)*psi%a(1)
    g32 = psibar%a(3)*psi%a(2)
    g41 = psibar%a(4)*psi%a(1)
    g42 = psibar%a(4)*psi%a(2)
    j%t    =   gl2 * (   g31 + g42)
    j%x(1) = - gl2 * (   g32 + g41)
    j%x(2) =   gl2 * (   g32 - g41) * (0, 1)
    j%x(3) =   gl2 * ( - g31 + g42)
  end function vl_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function vr_ff (gr, psibar, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gr
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gr2
    complex(kind=default) :: g13, g14, g23, g24
    gr2 = 2 * gr
    g13 = psibar%a(1)*psi%a(3)
    g14 = psibar%a(1)*psi%a(4)
    g23 = psibar%a(2)*psi%a(3)
    g24 = psibar%a(2)*psi%a(4)
    j%t    = gr2 * (   g13 + g24)
    j%x(1) = gr2 * (   g14 + g23)
    j%x(2) = gr2 * ( - g14 + g23) * (0, 1)
    j%x(3) = gr2 * (   g13 - g24)
  end function vr_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function grav_ff (g, m, kb, k, psibar, psi) result (j)
    type(tensor) :: j
    complex(kind=default), intent(in) :: g
    real(kind=default), intent(in) :: m
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    type(momentum), intent(in) :: kb, k
    complex(kind=default) :: g2, g8, c_dum
    type(vector) :: v_dum
    type(tensor) :: t_metric
    t_metric%t = 0
    t_metric%t(0,0) = 1.0_default
    t_metric%t(1,1) = - 1.0_default
    t_metric%t(2,2) = - 1.0_default
    t_metric%t(3,3) = - 1.0_default
    g2 = g/2.0_default
    g8 = g/8.0_default
    v_dum = v_ff(g8, psibar, psi)
    c_dum = (- m) * s_ff (g2, psibar, psi) - (kb+k)*v_dum
```

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```
      j = c_dum*t_metric - (((kb+k).tprod.v_dum) + &
          (v_dum.tprod.(kb+k)))
  end function grav_ff
```

$$g_L\gamma_\mu(1-\gamma_5) + g_R\gamma_\mu(1+\gamma_5) = (g_L+g_R)\gamma_\mu - (g_L-g_R)\gamma_\mu\gamma_5 = g_V\gamma_\mu - g_A\gamma_\mu\gamma_5 \tag{X.85}$$

... give the compiler the benefit of the doubt that it will optimize the function all. If not, we could inline it ...

⟨*Implementation of spinor currents*⟩+≡

```
  pure function vlr_ff (gl, gr, psibar, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j = va_ff (gl+gr, gl-gr, psibar, psi)
  end function vlr_ff
```

and

$$\not{v} - \not{a}\gamma_5 = \begin{pmatrix} 0 & 0 & v_- - a_- & -v^* + a^* \\ 0 & 0 & -v + a & v_+ - a_+ \\ v_+ + a_+ & v^* + a^* & 0 & 0 \\ v + a & v_- + a_- & 0 & 0 \end{pmatrix} \tag{X.86}$$

with $v_\pm = v_0 \pm v_3$, $a_\pm = a_0 \pm a_3$, $v = v_1 + \mathrm{i}v_2$, $v^* = v_1 - \mathrm{i}v_2$, $a = a_1 + \mathrm{i}a_2$, and $a^* = a_1 - \mathrm{i}a_2$. But note that $\cdot^*$ is *not* complex conjugation for complex v_μ or a_μ.

⟨*Declaration of spinor currents*⟩+≡

```
  public :: f_vaf, f_vf, f_af, f_vlf, f_vrf, f_vlrf, f_va2f, &
            f_tvaf, f_tlrf, f_trlf, f_tvamf, f_tlrmf, f_trlmf, f_va3f
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_vaf (gv, ga, v, psi) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gv, ga
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
    complex(kind=default) :: vp, vm, v12, v12s
    gl = gv + ga
    gr = gv - ga
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = gr * (   vm  * psi%a(3) - v12s * psi%a(4))
    vpsi%a(2) = gr * ( - v12 * psi%a(3) + vp   * psi%a(4))
    vpsi%a(3) = gl * (   vp  * psi%a(1) + v12s * psi%a(2))
    vpsi%a(4) = gl * (   v12 * psi%a(1) + vm   * psi%a(2))
  end function f_vaf
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_va2f (gva, v, psi) result (vpsi)
    type(spinor) :: vpsi
```

```
      complex(kind=default), intent(in), dimension(2) :: gva
      type(vector), intent(in) :: v
      type(spinor), intent(in) :: psi
      complex(kind=default) :: gl, gr
      complex(kind=default) :: vp, vm, v12, v12s
      gl = gva(1) + gva(2)
      gr = gva(1) - gva(2)
      vp = v%t + v%x(3)
      vm = v%t - v%x(3)
      v12  =  v%x(1) + (0,1)*v%x(2)
      v12s =  v%x(1) - (0,1)*v%x(2)
      vpsi%a(1) = gr * (   vm  * psi%a(3) - v12s * psi%a(4))
      vpsi%a(2) = gr * ( - v12 * psi%a(3) + vp   * psi%a(4))
      vpsi%a(3) = gl * (   vp  * psi%a(1) + v12s * psi%a(2))
      vpsi%a(4) = gl * (   v12 * psi%a(1) + vm   * psi%a(2))
    end function f_va2f
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_va3f (gv, ga, v, psi) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gv, ga
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
    complex(kind=default) :: vp, vm, v12, v12s
    gl = gv + ga
    gr = gv - ga
    vp =    v%x(3) !+ v%t
    vm = -  v%x(3) !+ v%t
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = gr * (   vm  * psi%a(3) - v12s * psi%a(4))
    vpsi%a(2) = gr * ( - v12 * psi%a(3) + vp   * psi%a(4))
    vpsi%a(3) = gl * (   vp  * psi%a(1) + v12s * psi%a(2))
    vpsi%a(4) = gl * (   v12 * psi%a(1) + vm   * psi%a(2))
  end function f_va3f
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_tvaf (gv, ga, t, psi) result (tpsi)
    type(spinor) :: tpsi
    complex(kind=default), intent(in) :: gv, ga
    type(tensor2odd), intent(in) :: t
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
    complex(kind=default) :: e21, e21s, b12, b12s, be3, be3s
    gr   = gv + ga
    gl   = gv - ga
    e21  = t%e(2) + t%e(1)*(0,1)
    e21s = t%e(2) - t%e(1)*(0,1)
    b12  = t%b(1) + t%b(2)*(0,1)
    b12s = t%b(1) - t%b(2)*(0,1)
    be3  = t%b(3) + t%e(3)*(0,1)
    be3s = t%b(3) - t%e(3)*(0,1)
    tpsi%a(1) =   2*gl * (   psi%a(1) * be3  + psi%a(2) * ( e21 +b12s))
    tpsi%a(2) =   2*gl * ( - psi%a(2) * be3  + psi%a(1) * (-e21s+b12 ))
```

```
      tpsi%a(3) =   2*gr * (   psi%a(3) * be3s + psi%a(4) * (-e21 +b12s))
      tpsi%a(4) =   2*gr * ( - psi%a(4) * be3s + psi%a(3) * ( e21s+b12 ))
    end function f_tvaf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_tlrf (gl, gr, t, psi) result (tpsi)
    type(spinor) :: tpsi
    complex(kind=default), intent(in) :: gl, gr
    type(tensor2odd), intent(in) :: t
    type(spinor), intent(in) :: psi
    tpsi = f_tvaf (gr+gl, gr-gl, t, psi)
  end function f_tlrf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_trlf (gr, gl, t, psi) result (tpsi)
    type(spinor) :: tpsi
    complex(kind=default), intent(in) :: gl, gr
    type(tensor2odd), intent(in) :: t
    type(spinor), intent(in) :: psi
    tpsi = f_tvaf (gr+gl, gr-gl, t, psi)
  end function f_trlf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_tvamf (gv, ga, v, psi, k) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gv, ga
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    type(tensor2odd) :: t
    t = (v.wedge.k) * (0, 0.5)
    vpsi = f_tvaf(gv, ga, t, psi)
  end function f_tvamf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_tlrmf (gl, gr, v, psi, k) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gl, gr
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    vpsi = f_tvamf (gr+gl, gr-gl, v, psi, k)
  end function f_tlrmf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_trlmf (gr, gl, v, psi, k) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gl, gr
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    vpsi = f_tvamf (gr+gl, gr-gl, v, psi, k)
  end function f_trlmf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_vf (gv, v, psi) result (vpsi)
    type(spinor) :: vpsi
```

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```
      complex(kind=default), intent(in) :: gv
      type(vector), intent(in) :: v
      type(spinor), intent(in) :: psi
      complex(kind=default) :: vp, vm, v12, v12s
      vp = v%t + v%x(3)
      vm = v%t - v%x(3)
      v12  =  v%x(1) + (0,1)*v%x(2)
      v12s =  v%x(1) - (0,1)*v%x(2)
      vpsi%a(1) = gv * (   vm  * psi%a(3) - v12s * psi%a(4))
      vpsi%a(2) = gv * ( - v12 * psi%a(3) + vp   * psi%a(4))
      vpsi%a(3) = gv * (   vp  * psi%a(1) + v12s * psi%a(2))
      vpsi%a(4) = gv * (   v12 * psi%a(1) + vm   * psi%a(2))
    end function f_vf
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_af (ga, v, psi) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: ga
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    complex(kind=default) :: vp, vm, v12, v12s
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = ga * ( - vm  * psi%a(3) + v12s * psi%a(4))
    vpsi%a(2) = ga * (   v12 * psi%a(3) - vp   * psi%a(4))
    vpsi%a(3) = ga * (   vp  * psi%a(1) + v12s * psi%a(2))
    vpsi%a(4) = ga * (   v12 * psi%a(1) + vm   * psi%a(2))
  end function f_af
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_vlf (gl, v, psi) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gl
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    complex(kind=default) :: gl2
    complex(kind=default) :: vp, vm, v12, v12s
    gl2 = 2 * gl
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = 0
    vpsi%a(2) = 0
    vpsi%a(3) = gl2 * (   vp  * psi%a(1) + v12s * psi%a(2))
    vpsi%a(4) = gl2 * (   v12 * psi%a(1) + vm   * psi%a(2))
  end function f_vlf
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_vrf (gr, v, psi) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gr
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
```

```
    complex(kind=default) :: gr2
    complex(kind=default) :: vp, vm, v12, v12s
    gr2 = 2 * gr
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = gr2 * (   vm  * psi%a(3) - v12s * psi%a(4))
    vpsi%a(2) = gr2 * ( - v12 * psi%a(3) + vp   * psi%a(4))
    vpsi%a(3) = 0
    vpsi%a(4) = 0
  end function f_vrf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_vlrf (gl, gr, v, psi) result (vpsi)
    type(spinor) :: vpsi
    complex(kind=default), intent(in) :: gl, gr
    type(vector), intent(in) :: v
    type(spinor), intent(in) :: psi
    vpsi = f_vaf (gl+gr, gl-gr, v, psi)
  end function f_vlrf
```

⟨*Declaration of spinor currents*⟩+≡
```
  public :: f_fva, f_fv, f_fa, f_fvl, f_fvr, f_fvlr, f_fva2, &
            f_ftva, f_ftlr, f_ftrl, f_ftvam, f_ftlrm, f_ftrlm, f_fva3
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fva (gv, ga, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    complex(kind=default) :: gl, gr
    complex(kind=default) :: vp, vm, v12, v12s
    gl = gv + ga
    gr = gv - ga
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    psibarv%a(1) = gl * (   psibar%a(3) * vp   + psibar%a(4) * v12)
    psibarv%a(2) = gl * (   psibar%a(3) * v12s + psibar%a(4) * vm )
    psibarv%a(3) = gr * (   psibar%a(1) * vm   - psibar%a(2) * v12)
    psibarv%a(4) = gr * ( - psibar%a(1) * v12s + psibar%a(2) * vp )
  end function f_fva
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fva2 (gva, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in), dimension(2) :: gva
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    complex(kind=default) :: gl, gr
    complex(kind=default) :: vp, vm, v12, v12s
    gl = gva(1) + gva(2)
    gr = gva(1) - gva(2)
```

```
      vp = v%t + v%x(3)
      vm = v%t - v%x(3)
      v12  =  v%x(1) + (0,1)*v%x(2)
      v12s =  v%x(1) - (0,1)*v%x(2)
      psibarv%a(1) = gl * (   psibar%a(3) * vp   + psibar%a(4) * v12)
      psibarv%a(2) = gl * (   psibar%a(3) * v12s + psibar%a(4) * vm )
      psibarv%a(3) = gr * (   psibar%a(1) * vm   - psibar%a(2) * v12)
      psibarv%a(4) = gr * ( - psibar%a(1) * v12s + psibar%a(2) * vp )
    end function f_fva2
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_fva3 (gv, ga, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    complex(kind=default) :: gl, gr
    complex(kind=default) :: vp, vm, v12, v12s
    gl = gv + ga
    gr = gv - ga
    vp =    v%x(3) !+ v%t
    vm = -  v%x(3) !+ v%t
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    psibarv%a(1) = gl * (   psibar%a(3) * vp   + psibar%a(4) * v12)
    psibarv%a(2) = gl * (   psibar%a(3) * v12s + psibar%a(4) * vm )
    psibarv%a(3) = gr * (   psibar%a(1) * vm   - psibar%a(2) * v12)
    psibarv%a(4) = gr * ( - psibar%a(1) * v12s + psibar%a(2) * vp )
  end function f_fva3
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_ftva (gv, ga, psibar, t) result (psibart)
    type(conjspinor) :: psibart
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
    type(tensor2odd), intent(in) :: t
    complex(kind=default) :: gl, gr
    complex(kind=default) :: e21, e21s, b12, b12s, be3, be3s
    gr   = gv + ga
    gl   = gv - ga
    e21  = t%e(2) + t%e(1)*(0,1)
    e21s = t%e(2) - t%e(1)*(0,1)
    b12  = t%b(1) + t%b(2)*(0,1)
    b12s = t%b(1) - t%b(2)*(0,1)
    be3  = t%b(3) + t%e(3)*(0,1)
    be3s = t%b(3) - t%e(3)*(0,1)
    psibart%a(1) = 2*gl * (   psibar%a(1) * be3  + psibar%a(2) * (-e21s+b12 ))
    psibart%a(2) = 2*gl * ( - psibar%a(2) * be3  + psibar%a(1) * ( e21 +b12s))
    psibart%a(3) = 2*gr * (   psibar%a(3) * be3s + psibar%a(4) * ( e21s+b12 ))
    psibart%a(4) = 2*gr * ( - psibar%a(4) * be3s + psibar%a(3) * (-e21 +b12s))
  end function f_ftva
```

⟨*Implementation of spinor currents*⟩+≡

```
  pure function f_ftlr (gl, gr, psibar, t) result (psibart)
    type(conjspinor) :: psibart
    complex(kind=default), intent(in) :: gl, gr
```

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```
      type(conjspinor), intent(in) :: psibar
      type(tensor2odd), intent(in) :: t
      psibart = f_ftva (gr+gl, gr-gl, psibar, t)
    end function f_ftlr
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_ftrl (gr, gl, psibar, t) result (psibart)
    type(conjspinor) :: psibart
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(tensor2odd), intent(in) :: t
    psibart = f_ftva (gr+gl, gr-gl, psibar, t)
  end function f_ftrl
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_ftvam (gv, ga, psibar, v, k) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gv, ga
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k
    type(tensor2odd) :: t
    t = (v.wedge.k) * (0, 0.5)
    psibarv = f_ftva(gv, ga, psibar, t)
  end function f_ftvam
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_ftlrm (gl, gr, psibar, v, k) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k
    psibarv = f_ftvam (gr+gl, gr-gl, psibar, v, k)
  end function f_ftlrm
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_ftrlm (gr, gl, psibar, v, k) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k
    psibarv = f_ftvam (gr+gl, gr-gl, psibar, v, k)
  end function f_ftrlm
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fv (gv, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gv
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    complex(kind=default) :: vp, vm, v12, v12s
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
```

```
      psibarv%a(1) = gv * (   psibar%a(3) * vp   + psibar%a(4) * v12)
      psibarv%a(2) = gv * (   psibar%a(3) * v12s + psibar%a(4) * vm )
      psibarv%a(3) = gv * (   psibar%a(1) * vm   - psibar%a(2) * v12)
      psibarv%a(4) = gv * ( - psibar%a(1) * v12s + psibar%a(2) * vp )
    end function f_fv
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fa (ga, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: ga
    type(vector), intent(in) :: v
    type(conjspinor), intent(in) :: psibar
    complex(kind=default) :: vp, vm, v12, v12s
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    psibarv%a(1) = ga * (   psibar%a(3) * vp   + psibar%a(4) * v12)
    psibarv%a(2) = ga * (   psibar%a(3) * v12s + psibar%a(4) * vm )
    psibarv%a(3) = ga * ( - psibar%a(1) * vm   + psibar%a(2) * v12)
    psibarv%a(4) = ga * (   psibar%a(1) * v12s - psibar%a(2) * vp )
  end function f_fa
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fvl (gl, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gl
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    complex(kind=default) :: gl2
    complex(kind=default) :: vp, vm, v12, v12s
    gl2 = 2 * gl
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    psibarv%a(1) = gl2 * (   psibar%a(3) * vp   + psibar%a(4) * v12)
    psibarv%a(2) = gl2 * (   psibar%a(3) * v12s + psibar%a(4) * vm )
    psibarv%a(3) = 0
    psibarv%a(4) = 0
  end function f_fvl
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fvr (gr, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gr
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    complex(kind=default) :: gr2
    complex(kind=default) :: vp, vm, v12, v12s
    gr2 = 2 * gr
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    psibarv%a(1) = 0
```

```
      psibarv%a(2) = 0
      psibarv%a(3) = gr2 * (   psibar%a(1) * vm   - psibar%a(2) * v12)
      psibarv%a(4) = gr2 * ( - psibar%a(1) * v12s + psibar%a(2) * vp )
    end function f_fvr
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fvlr (gl, gr, psibar, v) result (psibarv)
    type(conjspinor) :: psibarv
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(vector), intent(in) :: v
    psibarv = f_fva (gl+gr, gl-gr, psibar, v)
  end function f_fvlr
```

X.22.2   Fermionic Scalar and Pseudo Scalar Couplings

⟨*Declaration of spinor currents*⟩+≡
```
  public :: sp_ff, s_ff, p_ff, sl_ff, sr_ff, slr_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function sp_ff (gs, gp, psibar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gs, gp
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j =    (gs - gp) * (psibar%a(1)*psi%a(1) + psibar%a(2)*psi%a(2)) &
         + (gs + gp) * (psibar%a(3)*psi%a(3) + psibar%a(4)*psi%a(4))
  end function sp_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function s_ff (gs, psibar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gs
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j = gs * (psibar * psi)
  end function s_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function p_ff (gp, psibar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gp
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j = gp * (  psibar%a(3)*psi%a(3) + psibar%a(4)*psi%a(4) &
              - psibar%a(1)*psi%a(1) - psibar%a(2)*psi%a(2))
  end function p_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function sl_ff (gl, psibar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j =  2 * gl * (psibar%a(1)*psi%a(1) + psibar%a(2)*psi%a(2))
  end function sl_ff
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function sr_ff (gr, psibar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gr
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j = 2 * gr * (psibar%a(3)*psi%a(3) + psibar%a(4)*psi%a(4))
  end function sr_ff
```

$$g_L(1 - \gamma_5) + g_R(1 + \gamma_5) = (g_R + g_L) + (g_R - g_L)\gamma_5 = g_S + g_P\gamma_5 \qquad (\text{X}.87)$$

⟨*Implementation of spinor currents*⟩+≡
```
  pure function slr_ff (gl, gr, psibar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    type(spinor), intent(in) :: psi
    j = sp_ff (gr+gl, gr-gl, psibar, psi)
  end function slr_ff
```

⟨*Declaration of spinor currents*⟩+≡
```
  public :: f_spf, f_sf, f_pf, f_slf, f_srf, f_slrf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_spf (gs, gp, phi, psi) result (phipsi)
    type(spinor) :: phipsi
    complex(kind=default), intent(in) :: gs, gp
    complex(kind=default), intent(in) :: phi
    type(spinor), intent(in) :: psi
    phipsi%a(1:2) = ((gs - gp) * phi) * psi%a(1:2)
    phipsi%a(3:4) = ((gs + gp) * phi) * psi%a(3:4)
  end function f_spf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_sf (gs, phi, psi) result (phipsi)
    type(spinor) :: phipsi
    complex(kind=default), intent(in) :: gs
    complex(kind=default), intent(in) :: phi
    type(spinor), intent(in) :: psi
    phipsi%a = (gs * phi) * psi%a
  end function f_sf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_pf (gp, phi, psi) result (phipsi)
    type(spinor) :: phipsi
    complex(kind=default), intent(in) :: gp
    complex(kind=default), intent(in) :: phi
    type(spinor), intent(in) :: psi
    phipsi%a(1:2) = (- gp * phi) * psi%a(1:2)
    phipsi%a(3:4) = (  gp * phi) * psi%a(3:4)
  end function f_pf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_slf (gl, phi, psi) result (phipsi)
    type(spinor) :: phipsi
    complex(kind=default), intent(in) :: gl
```

```
      complex(kind=default), intent(in) :: phi
      type(spinor), intent(in) :: psi
      phipsi%a(1:2) = (2 * gl * phi) * psi%a(1:2)
      phipsi%a(3:4) = 0
    end function f_slf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_srf (gr, phi, psi) result (phipsi)
    type(spinor) :: phipsi
    complex(kind=default), intent(in) :: gr
    complex(kind=default), intent(in) :: phi
    type(spinor), intent(in) :: psi
    phipsi%a(1:2) = 0
    phipsi%a(3:4) = (2 * gr * phi) * psi%a(3:4)
  end function f_srf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_slrf (gl, gr, phi, psi) result (phipsi)
    type(spinor) :: phipsi
    complex(kind=default), intent(in) :: gl, gr
    complex(kind=default), intent(in) :: phi
    type(spinor), intent(in) :: psi
    phipsi =  f_spf (gr+gl, gr-gl, phi, psi)
  end function f_slrf
```

⟨*Declaration of spinor currents*⟩+≡
```
  public :: f_fsp, f_fs, f_fp, f_fsl, f_fsr, f_fslr
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fsp (gs, gp, psibar, phi) result (psibarphi)
    type(conjspinor) :: psibarphi
    complex(kind=default), intent(in) :: gs, gp
    type(conjspinor), intent(in) :: psibar
    complex(kind=default), intent(in) :: phi
    psibarphi%a(1:2) = ((gs - gp) * phi) * psibar%a(1:2)
    psibarphi%a(3:4) = ((gs + gp) * phi) * psibar%a(3:4)
  end function f_fsp
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fs (gs, psibar, phi) result (psibarphi)
    type(conjspinor) :: psibarphi
    complex(kind=default), intent(in) :: gs
    type(conjspinor), intent(in) :: psibar
    complex(kind=default), intent(in) :: phi
    psibarphi%a = (gs * phi) * psibar%a
  end function f_fs
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fp (gp, psibar, phi) result (psibarphi)
    type(conjspinor) :: psibarphi
    complex(kind=default), intent(in) :: gp
    type(conjspinor), intent(in) :: psibar
    complex(kind=default), intent(in) :: phi
    psibarphi%a(1:2) = (- gp * phi) * psibar%a(1:2)
    psibarphi%a(3:4) = (  gp * phi) * psibar%a(3:4)
  end function f_fp
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fsl (gl, psibar, phi) result (psibarphi)
    type(conjspinor) :: psibarphi
    complex(kind=default), intent(in) :: gl
    type(conjspinor), intent(in) :: psibar
    complex(kind=default), intent(in) :: phi
    psibarphi%a(1:2) = (2 * gl * phi) * psibar%a(1:2)
    psibarphi%a(3:4) = 0
  end function f_fsl
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fsr (gr, psibar, phi) result (psibarphi)
    type(conjspinor) :: psibarphi
    complex(kind=default), intent(in) :: gr
    type(conjspinor), intent(in) :: psibar
    complex(kind=default), intent(in) :: phi
    psibarphi%a(1:2) = 0
    psibarphi%a(3:4) = (2 * gr * phi) * psibar%a(3:4)
  end function f_fsr
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fslr (gl, gr, psibar, phi) result (psibarphi)
    type(conjspinor) :: psibarphi
    complex(kind=default), intent(in) :: gl, gr
    type(conjspinor), intent(in) :: psibar
    complex(kind=default), intent(in) :: phi
    psibarphi = f_fsp (gr+gl, gr-gl, psibar, phi)
  end function f_fslr
```

⟨*Declaration of spinor currents*⟩+≡
```
  public :: f_gravf, f_fgrav
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_gravf (g, m, kb, k, t, psi) result (tpsi)
    type(spinor) :: tpsi
    complex(kind=default), intent(in) :: g
    real(kind=default), intent(in) :: m
    type(spinor), intent(in) :: psi
    type(tensor), intent(in) :: t
    type(momentum), intent(in) :: kb, k
    complex(kind=default) :: g2, g8, t_tr
    type(vector) :: kkb
    kkb = k + kb
    g2 = g / 2.0_default
    g8 = g / 8.0_default
    t_tr = t%t(0,0) - t%t(1,1) - t%t(2,2) - t%t(3,3)
    tpsi = (- f_sf (g2, cmplx (m,0.0, kind=default), psi) &
            - f_vf ((g8*m), kkb, psi)) * t_tr - &
    f_vf (g8,(t*kkb + kkb*t),psi)
  end function f_gravf
```

⟨*Implementation of spinor currents*⟩+≡
```
  pure function f_fgrav (g, m, kb, k, psibar, t) result (psibart)
    type(conjspinor) :: psibart
    complex(kind=default), intent(in) :: g
    real(kind=default), intent(in) :: m
    type(conjspinor), intent(in) :: psibar
```

```
    type(tensor), intent(in) :: t
    type(momentum), intent(in) :: kb, k
    type(vector) :: kkb
    complex(kind=default) :: g2, g8, t_tr
    kkb = k + kb
    g2 = g / 2.0_default
    g8 = g / 8.0_default
    t_tr = t%t(0,0) - t%t(1,1) - t%t(2,2) - t%t(3,3)
    psibart = (- f_fs (g2, psibar, cmplx (m, 0.0, kind=default)) &
        - f_fv ((g8 * m), psibar, kkb)) * t_tr - &
        f_fv (g8,psibar,(t*kkb + kkb*t))
  end function f_fgrav
```

X.22.3 On Shell Wave Functions

⟨*Declaration of spinor on shell wave functions*⟩≡
```
  public :: u, ubar, v, vbar
  private :: chi_plus, chi_minus
```

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_3)}} \begin{pmatrix} |\vec{p}| + p_3 \\ p_1 + \mathrm{i}p_2 \end{pmatrix} \tag{X.88a}$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_3)}} \begin{pmatrix} -p_1 + \mathrm{i}p_2 \\ |\vec{p}| + p_3 \end{pmatrix} \tag{X.88b}$$

⟨*Implementation of spinor on shell wave functions*⟩≡
```
  pure function chi_plus (p) result (chi)
    complex(kind=default), dimension(2) :: chi
    type(momentum), intent(in) :: p
    real(kind=default) :: pabs
    pabs = sqrt (dot_product (p%x, p%x))
    if (pabs + p%x(3) <= 1000 * epsilon (pabs) * pabs) then
       chi = (/ cmplx ( 0.0, 0.0, kind=default), &
                cmplx ( 1.0, 0.0, kind=default) /)
    else
       chi = 1 / sqrt (2*pabs*(pabs + p%x(3))) &
             * (/ cmplx (pabs + p%x(3), kind=default), &
                  cmplx (p%x(1), p%x(2), kind=default) /)
    end if
  end function chi_plus
```

⟨*Implementation of spinor on shell wave functions*⟩+≡
```
  pure function chi_minus (p) result (chi)
    complex(kind=default), dimension(2) :: chi
    type(momentum), intent(in) :: p
    real(kind=default) :: pabs
    pabs = sqrt (dot_product (p%x, p%x))
    if (pabs + p%x(3) <= 1000 * epsilon (pabs) * pabs) then
       chi = (/ cmplx (-1.0, 0.0, kind=default), &
                cmplx ( 0.0, 0.0, kind=default) /)
    else
       chi = 1 / sqrt (2*pabs*(pabs + p%x(3))) &
             * (/ cmplx (-p%x(1), p%x(2), kind=default), &
```

```
                cmplx (pabs + p%x(3), kind=default) /)
    end if
  end function chi_minus
```

$$u_\pm(p, |m|) = \begin{pmatrix} \sqrt{p_0 \mp |\vec{p}|} \cdot \chi_\pm(\vec{p}) \\ \sqrt{p_0 \pm |\vec{p}|} \cdot \chi_\pm(\vec{p}) \end{pmatrix} \qquad u_\pm(p, -|m|) = \begin{pmatrix} -i\sqrt{p_0 \mp |\vec{p}|} \cdot \chi_\pm(\vec{p}) \\ +i\sqrt{p_0 \pm |\vec{p}|} \cdot \chi_\pm(\vec{p}) \end{pmatrix}$$

(X.89)

Determining the mass from the momenta is a numerically haphazardous for light particles. Therefore, we accept some redundancy and pass the mass explicitly. Even if the mass is not used in the chiral representation, we do so for symmetry with polarization vectors and to be prepared for other representations.

⟨*Implementation of spinor on shell wave functions*⟩+≡

```
  pure function u (mass, p, s) result (psi)
    type(spinor) :: psi
    real(kind=default), intent(in) :: mass
    type(momentum), intent(in) :: p
    integer, intent(in) :: s
    complex(kind=default), dimension(2) :: chi
    real(kind=default) :: pabs, delta, m
    m = abs(mass)
    pabs = sqrt (dot_product (p%x, p%x))
    if (m < epsilon (m) * pabs) then
        delta = 0
    else
        delta = sqrt (max (p%t - pabs, 0._default))
    end if
    select case (s)
    case (1)
       chi = chi_plus (p)
       psi%a(1:2) = delta * chi
       psi%a(3:4) = sqrt (p%t + pabs) * chi
    case (-1)
       chi = chi_minus (p)
       psi%a(1:2) = sqrt (p%t + pabs) * chi
       psi%a(3:4) = delta * chi
    case default
       pabs = m ! make the compiler happy and use m
       psi%a = 0
    end select
    if (mass < 0) then
       psi%a(1:2) = - imago * psi%a(1:2)
       psi%a(3:4) = + imago * psi%a(3:4)
    end if
  end function u
```

⟨*Implementation of spinor on shell wave functions*⟩+≡

```
  pure function ubar (m, p, s) result (psibar)
    type(conjspinor) :: psibar
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: p
    integer, intent(in) :: s
    type(spinor) :: psi
    psi = u (m, p, s)
```

```
      psibar%a(1:2) = conjg (psi%a(3:4))
      psibar%a(3:4) = conjg (psi%a(1:2))
    end function ubar
```

$$v_{\pm}(p) = \begin{pmatrix} \mp\sqrt{p_0 \pm |\vec{p}|} \cdot \chi_{\mp}(\vec{p}) \\ \pm\sqrt{p_0 \mp |\vec{p}|} \cdot \chi_{\mp}(\vec{p}) \end{pmatrix} \qquad (\text{X}.90)$$

⟨*Implementation of spinor on shell wave functions*⟩+≡

```
  pure function v (mass, p, s) result (psi)
    type(spinor) :: psi
    real(kind=default), intent(in) :: mass
    type(momentum), intent(in) :: p
    integer, intent(in) :: s
    complex(kind=default), dimension(2) :: chi
    real(kind=default) :: pabs, delta, m
    m = abs(mass)
    pabs = sqrt (dot_product (p%x, p%x))
    if (m < epsilon (m) * pabs) then
        delta = 0
    else
        delta = sqrt (max (p%t - pabs, 0._default))
    end if
    select case (s)
    case (1)
       chi = chi_minus (p)
       psi%a(1:2) = - sqrt (p%t + pabs) * chi
       psi%a(3:4) =   delta * chi
    case (-1)
       chi = chi_plus (p)
       psi%a(1:2) =   delta * chi
       psi%a(3:4) = - sqrt (p%t + pabs) * chi
    case default
       pabs = m ! make the compiler happy and use m
       psi%a = 0
    end select
    if (mass < 0) then
       psi%a(1:2) = - imago * psi%a(1:2)
       psi%a(3:4) = + imago * psi%a(3:4)
     end if
  end function v
```

⟨*Implementation of spinor on shell wave functions*⟩+≡

```
  pure function vbar (m, p, s) result (psibar)
    type(conjspinor) :: psibar
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: p
    integer, intent(in) :: s
    type(spinor) :: psi
    psi = v (m, p, s)
    psibar%a(1:2) = conjg (psi%a(3:4))
    psibar%a(3:4) = conjg (psi%a(1:2))
  end function vbar
```

X.22.4 Off Shell Wave Functions

I've just taken this over from Christian Schwinn's version.

⟨*Declaration of spinor off shell wave functions*⟩≡
```
public :: brs_u, brs_ubar, brs_v, brs_vbar
```

The off-shell wave functions needed for gauge checking are obtained from the LSZ-formulas:

$$\langle \text{Out} | d^\dagger | \text{In} \rangle = i \int d^4x \, \bar{v} e^{-ikx} (i\slashed{\partial} - m) \langle \text{Out} | \psi | \text{In} \rangle \tag{X.91a}$$

$$\langle \text{Out} | b | \text{In} \rangle = -i \int d^4x \, \bar{u} e^{ikx} (i\slashed{\partial} - m) \langle \text{Out} | \psi | \text{In} \rangle \tag{X.91b}$$

$$\langle \text{Out} | d | \text{In} \rangle = i \int d^4x \, \langle \text{Out} | \bar{\psi} | \text{In} \rangle \, (-i\overleftarrow{\slashed{\partial}} - m) v e^{ikx} \tag{X.91c}$$

$$\langle \text{Out} | b^\dagger | \text{In} \rangle = -i \int d^4x \, \langle \text{Out} | \bar{\psi} | \text{In} \rangle \, (-i\overleftarrow{\slashed{\partial}} - m) u e^{-ikx} \tag{X.91d}$$

Since the relative sign between fermions and antifermions is ignored for on-shell amplitudes we must also ignore it here, so all wavefunctions must have a $(-i)$ factor. In momentum space we have:

$$brsu(p) = (-i)(\slashed{p} - m)u(p) \tag{X.92}$$

⟨*Implementation of spinor off shell wave functions*⟩≡
```
pure function brs_u (m, p, s) result (dpsi)
    type(spinor) :: dpsi,psi
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: p
    integer, intent(in) :: s
    type (vector)::vp
    complex(kind=default), parameter :: one = (1, 0)
    vp=p
    psi=u(m,p,s)
    dpsi=cmplx(0.0,-1.0)*(f_vf(one,vp,psi)-m*psi)
end function brs_u
```

$$brsv(p) = i(\slashed{p} + m)v(p) \tag{X.93}$$

⟨*Implementation of spinor off shell wave functions*⟩+≡
```
pure function brs_v (m, p, s) result (dpsi)
    type(spinor) :: dpsi, psi
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: p
    integer, intent(in) ::   s
    type (vector)::vp
    complex(kind=default), parameter :: one = (1, 0)
    vp=p
    psi=v(m,p,s)
    dpsi=cmplx(0.0,1.0)*(f_vf(one,vp,psi)+m*psi)
end function brs_v
```

$$brs\bar{u}(p) = (-i)\bar{u}(p)(\slashed{p} - m) \tag{X.94}$$

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```
pure function brs_ubar (m, p, s)result (dpsibar)
   type(conjspinor) :: dpsibar, psibar
   real(kind=default), intent(in) :: m
   type(momentum), intent(in) :: p
   integer, intent(in) :: s
   type (vector)::vp
   complex(kind=default), parameter :: one = (1, 0)
    vp=p
    psibar=ubar(m,p,s)
   dpsibar=cmplx(0.0,-1.0)*(f_fv(one,psibar,vp)-m*psibar)
 end function brs_ubar
```

$$brs\bar{v}(p) = (i)\bar{v}(p)(\not{p} + m) \tag{X.95}$$

⟨*Implementation of spinor off shell wave functions*⟩+≡
```
  pure function brs_vbar (m, p, s) result (dpsibar)
     type(conjspinor) :: dpsibar,psibar
     real(kind=default), intent(in) :: m
     type(momentum), intent(in) :: p
     integer, intent(in) :: s
     type(vector)::vp
     complex(kind=default), parameter :: one = (1, 0)
     vp=p
     psibar=vbar(m,p,s)
    dpsibar=cmplx(0.0,1.0)*(f_fv(one,psibar,vp)+m*psibar)
  end function brs_vbar
```

NB: The remarks on momentum flow in the propagators don't apply here since the incoming momenta are flipped for the wave functions.

X.22.5 Propagators

NB: the common factor of i is extracted:

⟨*Declaration of spinor propagators*⟩≡
```
  public :: pr_psi, pr_psibar
  public :: pj_psi, pj_psibar
  public :: pg_psi, pg_psibar
```

$$\frac{i(-\not{p} + m)}{p^2 - m^2 + im\Gamma}\psi \tag{X.96}$$

NB: the sign of the momentum comes about because all momenta are treated as *outgoing* and the particle charge flow is therefore opposite to the momentum.

⟨*Implementation of spinor propagators*⟩≡
```
  pure function pr_psi (p, m, w, psi) result (ppsi)
    type(spinor) :: ppsi
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(spinor), intent(in) :: psi
    type(vector) :: vp
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    ppsi = (1 / cmplx (p*p - m**2, m*w, kind=default)) &
         * (- f_vf (one, vp, psi) + m * psi)
  end function pr_psi
```

$$\sqrt{\frac{\pi}{M\Gamma}}(-\not{p}+m)\psi \qquad (\text{X.97})$$

⟨*Implementation of spinor propagators*⟩+≡
```
  pure function pj_psi (p, m, w, psi) result (ppsi)
    type(spinor) :: ppsi
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(spinor), intent(in) :: psi
    type(vector) :: vp
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    ppsi = (0, -1) * sqrt (PI / m / w) * (- f_vf (one, vp, psi) + m * psi)
  end function pj_psi
```

⟨*Implementation of spinor propagators*⟩+≡
```
  pure function pg_psi (p, m, w, psi) result (ppsi)
    type(spinor) :: ppsi
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(spinor), intent(in) :: psi
    type(vector) :: vp
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    ppsi = gauss(p*p, m, w) *  (- f_vf (one, vp, psi) + m * psi)
  end function pg_psi
```

$$\bar{\psi}\frac{i(\not{p}+m)}{p^2-m^2+im\Gamma} \qquad (\text{X.98})$$

NB: the sign of the momentum comes about because all momenta are treated as *outgoing* and the antiparticle charge flow is therefore parallel to the momentum.

⟨*Implementation of spinor propagators*⟩+≡
```
  pure function pr_psibar (p, m, w, psibar) result (ppsibar)
    type(conjspinor) :: ppsibar
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(conjspinor), intent(in) :: psibar
    type(vector) :: vp
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    ppsibar = (1 / cmplx (p*p - m**2, m*w, kind=default)) &
         * (f_fv (one, psibar, vp) + m * psibar)
  end function pr_psibar
```

$$\sqrt{\frac{\pi}{M\Gamma}}\bar{\psi}(\not{p}+m) \qquad (\text{X.99})$$

NB: the sign of the momentum comes about because all momenta are treated as *outgoing* and the antiparticle charge flow is therefore parallel to the momentum.

⟨*Implementation of spinor propagators*⟩+≡
```
  pure function pj_psibar (p, m, w, psibar) result (ppsibar)
    type(conjspinor) :: ppsibar
    type(momentum), intent(in) :: p
```

```
    real(kind=default), intent(in) :: m, w
    type(conjspinor), intent(in) :: psibar
    type(vector) :: vp
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    ppsibar = (0, -1) * sqrt (PI / m / w) * (f_fv (one, psibar, vp) + m * psibar)
  end function pj_psibar
```

⟨*Implementation of spinor propagators*⟩+≡
```
  pure function pg_psibar (p, m, w, psibar) result (ppsibar)
    type(conjspinor) :: ppsibar
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(conjspinor), intent(in) :: psibar
    type(vector) :: vp
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    ppsibar = gauss (p*p, m, w) * (f_fv (one, psibar, vp) + m * psibar)
  end function pg_psibar
```

$$\frac{i(-\not{p} + m)}{p^2 - m^2 + im\Gamma} \sum_n \psi_n \otimes \bar{\psi}_n \qquad (\text{X}.100)$$

NB: the temporary variables `psi(1:4)` are not nice, but the compilers should be able to optimize the unnecessary copies away. In any case, even if the copies are performed, they are (probably) negligible compared to the floating point multiplications anyway . . .

⟨*(Not used yet) Declaration of operations for spinors*⟩≡
```
  type, public :: spinordyad
     ! private (omegalib needs access, but DON'T TOUCH IT!)
     complex(kind=default), dimension(4,4) :: a
  end type spinordyad
```

⟨*(Not used yet) Implementation of spinor propagators*⟩≡
```
  pure function pr_dyadleft (p, m, w, psipsibar) result (psipsibarp)
    type(spinordyad) :: psipsibarp
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(spinordyad), intent(in) :: psipsibar
    integer :: i
    type(vector) :: vp
    type(spinor), dimension(4) :: psi
    complex(kind=default) :: pole
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    pole = 1 / cmplx (p*p - m**2, m*w, kind=default)
    do i = 1, 4
       psi(i)%a = psipsibar%a(:,i)
       psi(i) = pole * (- f_vf (one, vp, psi(i)) + m * psi(i))
       psipsibarp%a(:,i) = psi(i)%a
    end do
  end function pr_dyadleft
```

$$\sum_n \psi_n \otimes \bar{\psi}_n \frac{i(\not{p} + m)}{p^2 - m^2 + im\Gamma} \qquad (\text{X}.101)$$

⟨*(Not used yet) Implementation of spinor propagators*⟩+≡

```fortran
  pure function pr_dyadright (p, m, w, psipsibar) result (psipsibarp)
    type(spinordyad) :: psipsibarp
    type(momentum), intent(in) :: p
    real(kind=default), intent(in) :: m, w
    type(spinordyad), intent(in) :: psipsibar
    integer :: i
    type(vector) :: vp
    type(conjspinor), dimension(4) :: psibar
    complex(kind=default) :: pole
    complex(kind=default), parameter :: one = (1, 0)
    vp = p
    pole = 1 / cmplx (p*p - m**2, m*w, kind=default)
    do i = 1, 4
       psibar(i)%a = psipsibar%a(i,:)
       psibar(i) = pole * (f_fv (one, psibar(i), vp) + m * psibar(i))
       psipsibarp%a(i,:) = psibar(i)%a
    end do
  end function pr_dyadright
```

X.23   Spinor Couplings Revisited

⟨`omega_bispinor_couplings.f90`⟩≡

```fortran
  ⟨Copyleft⟩
  module omega_bispinor_couplings
    use kinds
    use constants
    use omega_bispinors
    use omega_vectorspinors
    use omega_vectors
    use omega_couplings
    implicit none
    private
    ⟨Declaration of bispinor on shell wave functions⟩
    ⟨Declaration of bispinor off shell wave functions⟩
    ⟨Declaration of bispinor currents⟩
    ⟨Declaration of bispinor propagators⟩
    integer, parameter, public :: omega_bispinor_cpls_2010_01_A = 0
  contains
    ⟨Implementation of bispinor on shell wave functions⟩
    ⟨Implementation of bispinor off shell wave functions⟩
    ⟨Implementation of bispinor currents⟩
    ⟨Implementation of bispinor propagators⟩
  end module omega_bispinor_couplings
```

See table X.1 for the names of Fortran functions. We could have used long names instead, but this would increase the chance of running past continuation line limits without adding much to the legibility.

X.23.1   Fermionic Vector and Axial Couplings

There's more than one chiral representation. This one is compatible with HELAS [5].

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \ \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \ \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}, \quad \text{(X.102a)}$$

$$C = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}, \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{X.102b}$$

Therefore

$$g_S + g_P\gamma_5 = \begin{pmatrix} g_S - g_P & 0 & 0 & 0 \\ 0 & g_S - g_P & 0 & 0 \\ 0 & 0 & g_S + g_P & 0 \\ 0 & 0 & 0 & g_S + g_P \end{pmatrix} \tag{X.103a}$$

$$g_V\gamma^0 - g_A\gamma^0\gamma_5 = \begin{pmatrix} 0 & 0 & g_V - g_A & 0 \\ 0 & 0 & 0 & g_V - g_A \\ g_V + g_A & 0 & 0 & 0 \\ 0 & g_V + g_A & 0 & 0 \end{pmatrix} \tag{X.103b}$$

$$g_V\gamma^1 - g_A\gamma^1\gamma_5 = \begin{pmatrix} 0 & 0 & 0 & g_V - g_A \\ 0 & 0 & g_V - g_A & 0 \\ 0 & -g_V - g_A & 0 & 0 \\ -g_V - g_A & 0 & 0 & 0 \end{pmatrix} \tag{X.103c}$$

$$g_V\gamma^2 - g_A\gamma^2\gamma_5 = \begin{pmatrix} 0 & 0 & 0 & -i(g_V - g_A) \\ 0 & 0 & i(g_V - g_A) & 0 \\ 0 & i(g_V + g_A) & 0 & 0 \\ -i(g_V + g_A) & 0 & 0 & 0 \end{pmatrix}$$
$$\text{(X.103d)}$$

$$g_V\gamma^3 - g_A\gamma^3\gamma_5 = \begin{pmatrix} 0 & 0 & g_V - g_A & 0 \\ 0 & 0 & 0 & -g_V + g_A \\ -g_V - g_A & 0 & 0 & 0 \\ 0 & g_V + g_A & 0 & 0 \end{pmatrix} \tag{X.103e}$$

and

$$C(g_S + g_P\gamma_5) = \begin{pmatrix} 0 & g_S - g_P & 0 & 0 \\ -g_S + g_P & 0 & 0 & 0 \\ 0 & 0 & 0 & -g_S - g_P \\ 0 & 0 & g_S + g_P & 0 \end{pmatrix} \tag{X.104a}$$

$$C(g_V\gamma^0 - g_A\gamma^0\gamma_5) = \begin{pmatrix} 0 & 0 & 0 & g_V - g_A \\ 0 & 0 & -g_V + g_A & 0 \\ 0 & -g_V - g_A & 0 & 0 \\ g_V + g_A & 0 & 0 & 0 \end{pmatrix}$$
$$\text{(X.104b)}$$

$$C(g_V\gamma^1 - g_A\gamma^1\gamma_5) = \begin{pmatrix} 0 & 0 & g_V - g_A & 0 \\ 0 & 0 & 0 & -g_V + g_A \\ g_V + g_A & 0 & 0 & 0 \\ 0 & -g_V - g_A & 0 & 0 \end{pmatrix}$$
$$\text{(X.104c)}$$

$$
C(g_V\gamma^2 - g_A\gamma^2\gamma_5) = \begin{pmatrix} 0 & 0 & \mathrm{i}(g_V - g_A) & 0 \\ 0 & 0 & 0 & \mathrm{i}(g_V - g_A) \\ \mathrm{i}(g_V + g_A) & 0 & 0 & 0 \\ 0 & \mathrm{i}(g_V + g_A) & 0 & 0 \end{pmatrix}
$$

$$\text{(X.104d)}$$

$$
C(g_V\gamma^3 - g_A\gamma^3\gamma_5) = \begin{pmatrix} 0 & 0 & 0 & -g_V + g_A \\ 0 & 0 & -g_V + g_A & 0 \\ 0 & -g_V - g_A & 0 & 0 \\ -g_V - g_A & 0 & 0 & 0 \end{pmatrix}
$$

$$\text{(X.104e)}$$

⟨*Declaration of bispinor currents*⟩≡
```
public :: va_ff, v_ff, a_ff, vl_ff, vr_ff, vlr_ff, va2_ff
```

⟨*Implementation of bispinor currents*⟩≡
```
pure function va_ff (gv, ga, psil, psir) result (j)
  type(vector) :: j
  complex(kind=default), intent(in) :: gv, ga
  type(bispinor), intent(in) :: psil, psir
  complex(kind=default) :: gl, gr
  complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
  gl = gv + ga
  gr = gv - ga
  g13 = psil%a(1)*psir%a(3)
  g14 = psil%a(1)*psir%a(4)
  g23 = psil%a(2)*psir%a(3)
  g24 = psil%a(2)*psir%a(4)
  g31 = psil%a(3)*psir%a(1)
  g32 = psil%a(3)*psir%a(2)
  g41 = psil%a(4)*psir%a(1)
  g42 = psil%a(4)*psir%a(2)
  j%t   =  gr * (   g14 - g23) + gl * ( - g32 + g41)
  j%x(1) =  gr * (   g13 - g24) + gl * (   g31 - g42)
  j%x(2) = (gr * (   g13 + g24) + gl * (   g31 + g42)) * (0, 1)
  j%x(3) =  gr * ( - g14 - g23) + gl * ( - g32 - g41)
end function va_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function va2_ff (gva, psil, psir) result (j)
  type(vector) :: j
  complex(kind=default), intent(in), dimension(2) :: gva
  type(bispinor), intent(in) :: psil, psir
  complex(kind=default) :: gl, gr
  complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
  gl = gva(1) + gva(2)
  gr = gva(1) - gva(2)
  g13 = psil%a(1)*psir%a(3)
  g14 = psil%a(1)*psir%a(4)
  g23 = psil%a(2)*psir%a(3)
  g24 = psil%a(2)*psir%a(4)
  g31 = psil%a(3)*psir%a(1)
  g32 = psil%a(3)*psir%a(2)
  g41 = psil%a(4)*psir%a(1)
  g42 = psil%a(4)*psir%a(2)
```

```
        j%t    =  gr * (   g14 - g23) + gl * ( - g32 + g41)
        j%x(1) =  gr * (   g13 - g24) + gl * (   g31 - g42)
        j%x(2) = (gr * (   g13 + g24) + gl * (   g31 + g42)) * (0, 1)
        j%x(3) =  gr * ( - g14 - g23) + gl * ( - g32 - g41)
      end function va2_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function v_ff (gv, psil, psir) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gv
    type(bispinor), intent(in) :: psil, psir
    complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
    g13 = psil%a(1)*psir%a(3)
    g14 = psil%a(1)*psir%a(4)
    g23 = psil%a(2)*psir%a(3)
    g24 = psil%a(2)*psir%a(4)
    g31 = psil%a(3)*psir%a(1)
    g32 = psil%a(3)*psir%a(2)
    g41 = psil%a(4)*psir%a(1)
    g42 = psil%a(4)*psir%a(2)
    j%t    =   gv * (   g14 - g23 - g32 + g41)
    j%x(1) =   gv * (   g13 - g24 + g31 - g42)
    j%x(2) =   gv * (   g13 + g24 + g31 + g42) * (0, 1)
    j%x(3) =   gv * ( - g14 - g23 - g32 - g41)
  end function v_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function a_ff (ga, psil, psir) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: ga
    type(bispinor), intent(in) :: psil, psir
    complex(kind=default) :: g13, g14, g23, g24, g31, g32, g41, g42
    g13 = psil%a(1)*psir%a(3)
    g14 = psil%a(1)*psir%a(4)
    g23 = psil%a(2)*psir%a(3)
    g24 = psil%a(2)*psir%a(4)
    g31 = psil%a(3)*psir%a(1)
    g32 = psil%a(3)*psir%a(2)
    g41 = psil%a(4)*psir%a(1)
    g42 = psil%a(4)*psir%a(2)
    j%t    =  -ga * (   g14 - g23 + g32 - g41)
    j%x(1) =  -ga * (   g13 - g24 - g31 + g42)
    j%x(2) =  -ga * (   g13 + g24 - g31 - g42) * (0, 1)
    j%x(3) =  -ga * ( - g14 - g23 + g32 + g41)
  end function a_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function vl_ff (gl, psil, psir) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl
    type(bispinor), intent(in) :: psil, psir
    complex(kind=default) :: gl2
    complex(kind=default) :: g31, g32, g41, g42
    gl2 = 2 * gl
    g31 = psil%a(3)*psir%a(1)
    g32 = psil%a(3)*psir%a(2)
```

```
      g41 = psil%a(4)*psir%a(1)
      g42 = psil%a(4)*psir%a(2)
      j%t    =   gl2 * ( - g32 + g41)
      j%x(1) =   gl2 * (   g31 - g42)
      j%x(2) =   gl2 * (   g31 + g42) * (0, 1)
      j%x(3) =   gl2 * ( - g32 - g41)
    end function vl_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function vr_ff (gr, psil, psir) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gr
    type(bispinor), intent(in) :: psil, psir
    complex(kind=default) :: gr2
    complex(kind=default) :: g13, g14, g23, g24
    gr2 = 2 * gr
    g13 = psil%a(1)*psir%a(3)
    g14 = psil%a(1)*psir%a(4)
    g23 = psil%a(2)*psir%a(3)
    g24 = psil%a(2)*psir%a(4)
    j%t    = gr2 * (   g14 - g23)
    j%x(1) = gr2 * (   g13 - g24)
    j%x(2) = gr2 * (   g13 + g24) * (0, 1)
    j%x(3) = gr2 * ( - g14 - g23)
  end function vr_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function vlr_ff (gl, gr, psibar, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(bispinor), intent(in) :: psibar
    type(bispinor), intent(in) :: psi
    j = va_ff (gl+gr, gl-gr, psibar, psi)
  end function vlr_ff
```

and

$$
\not{v} - \not{a}\gamma_5 = \begin{pmatrix} 0 & 0 & v_- - a_- & -v^* + a^* \\ 0 & 0 & -v + a & v_+ - a_+ \\ v_+ + a_+ & v^* + a^* & 0 & 0 \\ v + a & v_- + a_- & 0 & 0 \end{pmatrix} \tag{X.105}
$$

with $v_\pm = v_0 \pm v_3$, $a_\pm = a_0 \pm a_3$, $v = v_1 + iv_2$, $v^* = v_1 - iv_2$, $a = a_1 + ia_2$, and $a^* = a_1 - ia_2$. But note that $\cdot^*$ is *not* complex conjugation for complex v_μ or a_μ.

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: f_vaf, f_vf, f_af, f_vlf, f_vrf, f_vlrf, f_va2f
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vaf (gv, ga, v, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in) :: gv, ga
    type(vector), intent(in) :: v
    type(bispinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
    complex(kind=default) :: vp, vm, v12, v12s
```

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```
      gl = gv + ga
      gr = gv - ga
      vp = v%t + v%x(3)
      vm = v%t - v%x(3)
      v12  =  v%x(1) + (0,1)*v%x(2)
      v12s =  v%x(1) - (0,1)*v%x(2)
      vpsi%a(1) = gr * (   vm  * psi%a(3) - v12s * psi%a(4))
      vpsi%a(2) = gr * ( - v12 * psi%a(3) + vp   * psi%a(4))
      vpsi%a(3) = gl * (   vp  * psi%a(1) + v12s * psi%a(2))
      vpsi%a(4) = gl * (   v12 * psi%a(1) + vm   * psi%a(2))
    end function f_vaf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_va2f (gva, v, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in), dimension(2) :: gva
    type(vector), intent(in) :: v
    type(bispinor), intent(in) :: psi
    complex(kind=default) :: gl, gr
    complex(kind=default) :: vp, vm, v12, v12s
    gl = gva(1) + gva(2)
    gr = gva(1) - gva(2)
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = gr * (   vm  * psi%a(3) - v12s * psi%a(4))
    vpsi%a(2) = gr * ( - v12 * psi%a(3) + vp   * psi%a(4))
    vpsi%a(3) = gl * (   vp  * psi%a(1) + v12s * psi%a(2))
    vpsi%a(4) = gl * (   v12 * psi%a(1) + vm   * psi%a(2))
  end function f_va2f
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vf (gv, v, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in) :: gv
    type(vector), intent(in) :: v
    type(bispinor), intent(in) :: psi
    complex(kind=default) :: vp, vm, v12, v12s
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = gv * (   vm  * psi%a(3) - v12s * psi%a(4))
    vpsi%a(2) = gv * ( - v12 * psi%a(3) + vp   * psi%a(4))
    vpsi%a(3) = gv * (   vp  * psi%a(1) + v12s * psi%a(2))
    vpsi%a(4) = gv * (   v12 * psi%a(1) + vm   * psi%a(2))
  end function f_vf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_af (ga, v, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in) :: ga
    type(vector), intent(in) :: v
    type(bispinor), intent(in) :: psi
    complex(kind=default) :: vp, vm, v12, v12s
```

```
      vp = v%t + v%x(3)
      vm = v%t - v%x(3)
      v12  =  v%x(1) + (0,1)*v%x(2)
      v12s =  v%x(1) - (0,1)*v%x(2)
      vpsi%a(1) = ga * ( - vm  * psi%a(3) + v12s * psi%a(4))
      vpsi%a(2) = ga * (   v12 * psi%a(3) - vp   * psi%a(4))
      vpsi%a(3) = ga * (   vp  * psi%a(1) + v12s * psi%a(2))
      vpsi%a(4) = ga * (   v12 * psi%a(1) + vm   * psi%a(2))
    end function f_af
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vlf (gl, v, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in) :: gl
    type(vector), intent(in) :: v
    type(bispinor), intent(in) :: psi
    complex(kind=default) :: gl2
    complex(kind=default) :: vp, vm, v12, v12s
    gl2 = 2 * gl
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = 0
    vpsi%a(2) = 0
    vpsi%a(3) = gl2 * (   vp  * psi%a(1) + v12s * psi%a(2))
    vpsi%a(4) = gl2 * (   v12 * psi%a(1) + vm   * psi%a(2))
  end function f_vlf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vrf (gr, v, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in) :: gr
    type(vector), intent(in) :: v
    type(bispinor), intent(in) :: psi
    complex(kind=default) :: gr2
    complex(kind=default) :: vp, vm, v12, v12s
    gr2 = 2 * gr
    vp = v%t + v%x(3)
    vm = v%t - v%x(3)
    v12  =  v%x(1) + (0,1)*v%x(2)
    v12s =  v%x(1) - (0,1)*v%x(2)
    vpsi%a(1) = gr2 * (   vm  * psi%a(3) - v12s * psi%a(4))
    vpsi%a(2) = gr2 * ( - v12 * psi%a(3) + vp   * psi%a(4))
    vpsi%a(3) = 0
    vpsi%a(4) = 0
  end function f_vrf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vlrf (gl, gr, v, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in) :: gl, gr
    type(vector), intent(in) :: v
    type(bispinor), intent(in) :: psi
    vpsi = f_vaf (gl+gr, gl-gr, v, psi)
  end function f_vlrf
```

X.23.2   *Fermionic Scalar and Pseudo Scalar Couplings*

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: sp_ff, s_ff, p_ff, sl_ff, sr_ff, slr_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sp_ff (gs, gp, psil, psir) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gs, gp
    type(bispinor), intent(in) :: psil, psir
    j =    (gs - gp) * (psil%a(1)*psir%a(2) - psil%a(2)*psir%a(1)) &
         + (gs + gp) * (- psil%a(3)*psir%a(4) + psil%a(4)*psir%a(3))
  end function sp_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function s_ff (gs, psil, psir) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gs
    type(bispinor), intent(in) :: psil, psir
    j = gs * (psil * psir)
  end function s_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function p_ff (gp, psil, psir) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gp
    type(bispinor), intent(in) :: psil, psir
    j = gp * (- psil%a(1)*psir%a(2) + psil%a(2)*psir%a(1) &
              - psil%a(3)*psir%a(4) + psil%a(4)*psir%a(3))
  end function p_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sl_ff (gl, psil, psir) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl
    type(bispinor), intent(in) :: psil, psir
    j =  2 * gl * (psil%a(1)*psir%a(2) - psil%a(2)*psir%a(1))
  end function sl_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sr_ff (gr, psil, psir) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gr
    type(bispinor), intent(in) :: psil, psir
    j = 2 * gr * (- psil%a(3)*psir%a(4) + psil%a(4)*psir%a(3))
  end function sr_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slr_ff (gl, gr, psibar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(bispinor), intent(in) :: psibar
    type(bispinor), intent(in) :: psi
    j = sp_ff (gr+gl, gr-gl, psibar, psi)
  end function slr_ff
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: f_spf, f_sf, f_pf, f_slf, f_srf, f_slrf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_spf (gs, gp, phi, psi) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gs, gp
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    phipsi%a(1:2) = ((gs - gp) * phi) * psi%a(1:2)
    phipsi%a(3:4) = ((gs + gp) * phi) * psi%a(3:4)
  end function f_spf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_sf (gs, phi, psi) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gs
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    phipsi%a = (gs * phi) * psi%a
  end function f_sf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_pf (gp, phi, psi) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gp
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    phipsi%a(1:2) = (- gp * phi) * psi%a(1:2)
    phipsi%a(3:4) = (  gp * phi) * psi%a(3:4)
  end function f_pf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_slf (gl, phi, psi) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gl
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    phipsi%a(1:2) = (2 * gl * phi) * psi%a(1:2)
    phipsi%a(3:4) = 0
  end function f_slf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_srf (gr, phi, psi) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gr
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    phipsi%a(1:2) = 0
    phipsi%a(3:4) = (2 * gr * phi) * psi%a(3:4)
  end function f_srf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_slrf (gl, gr, phi, psi) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gl, gr
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    phipsi =  f_spf (gr+gl, gr-gl, phi, psi)
  end function f_slrf
```

X.23.3   Couplings for BRST Transformations

3-Couplings

The lists of needed gamma matrices can be found in the next subsection with the gravitino couplings.

⟨*Declaration of bispinor currents*⟩+≡
```
  private :: vv_ff, f_vvf
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: vmom_ff, mom_ff, mom5_ff, moml_ff, momr_ff, lmom_ff, rmom_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function vv_ff (psibar, psi, k) result (psibarpsi)
    type(vector) :: psibarpsi
    type(bispinor), intent(in) :: psibar, psi
    type(vector), intent(in) :: k
    complex(kind=default) :: kp, km, k12, k12s
    type(bispinor) :: kgpsi1, kgpsi2, kgpsi3, kgpsi4
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    kgpsi1%a(1) = -k%x(3) * psi%a(1) - k12s * psi%a(2)
    kgpsi1%a(2) = -k12 * psi%a(1) + k%x(3) * psi%a(2)
    kgpsi1%a(3) = k%x(3) * psi%a(3) + k12s * psi%a(4)
    kgpsi1%a(4) = k12 * psi%a(3) - k%x(3) * psi%a(4)
    kgpsi2%a(1) = ((0,-1) * k%x(2)) * psi%a(1) - km * psi%a(2)
    kgpsi2%a(2) = - kp * psi%a(1) + ((0,1) * k%x(2)) * psi%a(2)
    kgpsi2%a(3) = ((0,-1) * k%x(2)) * psi%a(3) + kp * psi%a(4)
    kgpsi2%a(4) = km * psi%a(3) + ((0,1) * k%x(2)) * psi%a(4)
    kgpsi3%a(1) = (0,1) * (k%x(1) * psi%a(1) + km * psi%a(2))
    kgpsi3%a(2) = (0,-1) * (kp * psi%a(1) + k%x(1) * psi%a(2))
    kgpsi3%a(3) = (0,1) * (k%x(1) * psi%a(3) - kp * psi%a(4))
    kgpsi3%a(4) = (0,1) * (km * psi%a(3) - k%x(1) * psi%a(4))
    kgpsi4%a(1) = -k%t * psi%a(1) - k12s * psi%a(2)
    kgpsi4%a(2) = k12 * psi%a(1) + k%t * psi%a(2)
    kgpsi4%a(3) = k%t * psi%a(3) - k12s * psi%a(4)
    kgpsi4%a(4) = k12 * psi%a(3) - k%t * psi%a(4)
    psibarpsi%t    = 2 * (psibar * kgpsi1)
    psibarpsi%x(1) = 2 * (psibar * kgpsi2)
    psibarpsi%x(2) = 2 * (psibar * kgpsi3)
    psibarpsi%x(3) = 2 * (psibar * kgpsi4)
  end function vv_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vvf (v, psi, k) result (kvpsi)
    type(bispinor) :: kvpsi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: k, v
      complex(kind=default) :: kv30, kv21, kv01, kv31, kv02, kv32
    complex(kind=default) :: ap, am, bp, bm, bps, bms
    kv30 = k%x(3) * v%t - k%t * v%x(3)
    kv21 = (0,1) * (k%x(2) * v%x(1) - k%x(1) * v%x(2))
    kv01 = k%t * v%x(1) - k%x(1) * v%t
    kv31 = k%x(3) * v%x(1) - k%x(1) * v%x(3)
```

```
      kv02 = (0,1) * (k%t * v%x(2) - k%x(2) * v%t)
      kv32 = (0,1) * (k%x(3) * v%x(2) - k%x(2) * v%x(3))
      ap  = 2 * (kv30 + kv21)
      am  = 2 * (-kv30 + kv21)
      bp  = 2 * (kv01 + kv31 + kv02 + kv32)
      bm  = 2 * (kv01 - kv31 + kv02 - kv32)
      bps = 2 * (kv01 + kv31 - kv02 - kv32)
      bms = 2 * (kv01 - kv31 - kv02 + kv32)
      kvpsi%a(1) = am * psi%a(1) + bms * psi%a(2)
      kvpsi%a(2) = bp * psi%a(1) - am * psi%a(2)
      kvpsi%a(3) = ap * psi%a(3) - bps * psi%a(4)
      kvpsi%a(4) = -bm * psi%a(3) - ap * psi%a(4)
    end function f_vvf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function vmom_ff (g, psibar, psi, k) result (psibarpsi)
    type(vector) :: psibarpsi
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psibar, psi
    type(momentum), intent(in) :: k
    type(vector) :: vk
    vk = k
    psibarpsi = g * vv_ff (psibar, psi, vk)
  end function vmom_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function mom_ff (g, m, psibar, psi, k) result (psibarpsi)
    complex(kind=default) :: psibarpsi
    type(bispinor), intent(in) :: psibar, psi
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: g, m
    type(bispinor) :: kmpsi
    complex(kind=default) :: kp, km, k12, k12s
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    kmpsi%a(1) = km * psi%a(3) - k12s * psi%a(4)
    kmpsi%a(2) = kp * psi%a(4) - k12 * psi%a(3)
    kmpsi%a(3) = kp * psi%a(1) + k12s * psi%a(2)
    kmpsi%a(4) = k12 * psi%a(1) + km * psi%a(2)
    psibarpsi = g * (psibar * kmpsi) + s_ff (m, psibar, psi)
  end function mom_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function mom5_ff (g, m, psibar, psi, k) result (psibarpsi)
    complex(kind=default) :: psibarpsi
    type(bispinor), intent(in) :: psibar, psi
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: g, m
    type(bispinor) :: g5psi
    g5psi%a(1:2) = - psi%a(1:2)
    g5psi%a(3:4) = psi%a(3:4)
    psibarpsi = mom_ff (g, m, psibar, g5psi, k)
  end function mom5_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function moml_ff (g, m, psibar, psi, k) result (psibarpsi)
    complex(kind=default) :: psibarpsi
    type(bispinor), intent(in) :: psibar, psi
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: g, m
    type(bispinor) :: leftpsi
    leftpsi%a(1:2) = 2 * psi%a(1:2)
    leftpsi%a(3:4) = 0
    psibarpsi = mom_ff (g, m, psibar, leftpsi, k)
  end function moml_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function momr_ff (g, m, psibar, psi, k) result (psibarpsi)
    complex(kind=default) :: psibarpsi
    type(bispinor), intent(in) :: psibar, psi
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: g, m
    type(bispinor) :: rightpsi
    rightpsi%a(1:2) = 0
    rightpsi%a(3:4) = 2 * psi%a(3:4)
    psibarpsi = mom_ff (g, m, psibar, rightpsi, k)
  end function momr_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function lmom_ff (g, m, psibar, psi, k) result (psibarpsi)
    complex(kind=default) :: psibarpsi
    type(bispinor), intent(in) :: psibar, psi
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: g, m
    psibarpsi = mom_ff  (g, m, psibar, psi, k) + &
                mom5_ff (g,-m, psibar, psi, k)
  end function lmom_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function rmom_ff (g, m, psibar, psi, k) result (psibarpsi)
    complex(kind=default) :: psibarpsi
    type(bispinor), intent(in) :: psibar, psi
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: g, m
    psibarpsi = mom_ff  (g, m, psibar, psi, k) - &
                mom5_ff (g,-m, psibar, psi, k)
  end function rmom_ff
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: f_vmomf, f_momf, f_mom5f, f_momlf, f_momrf, f_lmomf, f_rmomf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vmomf (g, v, psi, k) result (kvpsi)
    type(bispinor) :: kvpsi
    type(bispinor), intent(in) :: psi
    complex(kind=default), intent(in) :: g
    type(momentum), intent(in) :: k
    type(vector), intent(in) :: v
    type(vector) :: vk
    vk = k
    kvpsi = g * f_vvf (v, psi, vk)
```

```
      end function f_vmomf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_momf (g, m, phi, psi, k) result (kmpsi)
    type(bispinor) :: kmpsi
    type(bispinor), intent(in) :: psi
    complex(kind=default), intent(in) :: phi, g, m
    type(momentum), intent(in) :: k
    complex(kind=default) :: kp, km, k12, k12s
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    kmpsi%a(1) = km * psi%a(3) - k12s * psi%a(4)
    kmpsi%a(2) = -k12 * psi%a(3) + kp * psi%a(4)
    kmpsi%a(3) = kp * psi%a(1) + k12s * psi%a(2)
    kmpsi%a(4) = k12 * psi%a(1) + km * psi%a(2)
    kmpsi = g * (phi * kmpsi) + f_sf (m, phi, psi)
  end function f_momf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_mom5f (g, m, phi, psi, k) result (kmpsi)
    type(bispinor) :: kmpsi
    type(bispinor), intent(in) :: psi
    complex(kind=default), intent(in) :: phi, g, m
    type(momentum), intent(in) :: k
    type(bispinor) :: g5psi
    g5psi%a(1:2) = - psi%a(1:2)
    g5psi%a(3:4) =   psi%a(3:4)
    kmpsi = f_momf (g, m, phi, g5psi, k)
  end function f_mom5f
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_momlf (g, m, phi, psi, k) result (kmpsi)
    type(bispinor) :: kmpsi
    type(bispinor), intent(in) :: psi
    complex(kind=default), intent(in) :: phi, g, m
    type(momentum), intent(in) :: k
    type(bispinor) :: leftpsi
    leftpsi%a(1:2) = 2 * psi%a(1:2)
    leftpsi%a(3:4) = 0
    kmpsi = f_momf (g, m, phi, leftpsi, k)
  end function f_momlf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_momrf (g, m, phi, psi, k) result (kmpsi)
    type(bispinor) :: kmpsi
    type(bispinor), intent(in) :: psi
    complex(kind=default), intent(in) :: phi, g, m
    type(momentum), intent(in) :: k
    type(bispinor) :: rightpsi
    rightpsi%a(1:2) = 0
    rightpsi%a(3:4) = 2 * psi%a(3:4)
    kmpsi = f_momf (g, m, phi, rightpsi, k)
  end function f_momrf
```

⟨*Implementation of bispinor currents*⟩+≡

```
pure function f_lmomf (g, m, phi, psi, k) result (kmpsi)
  type(bispinor) :: kmpsi
  type(bispinor), intent(in) :: psi
  complex(kind=default), intent(in) :: phi, g, m
  type(momentum), intent(in) :: k
  kmpsi = f_momf  (g, m, phi, psi, k) + &
          f_mom5f (g,-m, phi, psi, k)
end function f_lmomf
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function f_rmomf (g, m, phi, psi, k) result (kmpsi)
  type(bispinor) :: kmpsi
  type(bispinor), intent(in) :: psi
  complex(kind=default), intent(in) :: phi, g, m
  type(momentum), intent(in) :: k
  kmpsi = f_momf  (g, m, phi, psi, k) - &
          f_mom5f (g,-m, phi, psi, k)
end function f_rmomf
```

4-Couplings

⟨*Declaration of bispinor currents*⟩+≡
```
public :: v2_ff, sv1_ff, sv2_ff, pv1_ff, pv2_ff, svl1_ff, svl2_ff, &
    svr1_ff, svr2_ff, svlr1_ff, svlr2_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function v2_ff (g, psibar, v, psi) result (v2)
  type(vector) :: v2
  complex (kind=default), intent(in) :: g
  type(bispinor), intent(in) :: psibar, psi
  type(vector), intent(in) :: v
  v2 = (-g) * vv_ff (psibar, psi, v)
end function v2_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function sv1_ff (g, psibar, v, psi) result (phi)
  complex(kind=default) :: phi
  type(bispinor), intent(in) :: psibar, psi
  type(vector), intent(in) :: v
  complex(kind=default), intent(in) :: g
  phi = psibar * f_vf (g, v, psi)
end function sv1_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function sv2_ff (g, psibar, phi, psi) result (v)
  type(vector) :: v
  complex(kind=default), intent(in) :: phi, g
  type(bispinor), intent(in) :: psibar, psi
  v = phi * v_ff (g, psibar, psi)
end function sv2_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function pv1_ff (g, psibar, v, psi) result (phi)
  complex(kind=default) :: phi
  type(bispinor), intent(in) :: psibar, psi
  type(vector), intent(in) :: v
```

```
    complex(kind=default), intent(in) :: g
    phi = - (psibar * f_af (g, v, psi))
  end function pv1_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function pv2_ff (g, psibar, phi, psi) result (v)
    type(vector) :: v
    complex(kind=default), intent(in) :: phi, g
    type(bispinor), intent(in) :: psibar, psi
    v = -(phi * a_ff (g, psibar, psi))
  end function pv2_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function svl1_ff (g, psibar, v, psi) result (phi)
    complex(kind=default) :: phi
    type(bispinor), intent(in) :: psibar, psi
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: g
    phi = psibar * f_vlf (g, v, psi)
  end function svl1_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function svl2_ff (g, psibar, phi, psi) result (v)
    type(vector) :: v
    complex(kind=default), intent(in) :: phi, g
    type(bispinor), intent(in) :: psibar, psi
    v = phi * vl_ff (g, psibar, psi)
  end function svl2_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function svr1_ff (g, psibar, v, psi) result (phi)
    complex(kind=default) :: phi
    type(bispinor), intent(in) :: psibar, psi
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: g
    phi = psibar * f_vrf (g, v, psi)
  end function svr1_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function svr2_ff (g, psibar, phi, psi) result (v)
    type(vector) :: v
    complex(kind=default), intent(in) :: phi, g
    type(bispinor), intent(in) :: psibar, psi
    v = phi * vr_ff (g, psibar, psi)
  end function svr2_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function svlr1_ff (gl, gr, psibar, v, psi) result (phi)
    complex(kind=default) :: phi
    type(bispinor), intent(in) :: psibar, psi
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: gl, gr
    phi = psibar * f_vlrf (gl, gr, v, psi)
  end function svlr1_ff
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function svlr2_ff (gl, gr, psibar, phi, psi) result (v)
    type(vector) :: v
```

```
      complex(kind=default), intent(in) :: phi, gl, gr
      type(bispinor), intent(in) :: psibar, psi
      v = phi * vlr_ff (gl, gr, psibar, psi)
    end function svlr2_ff
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: f_v2f, f_svf, f_pvf, f_svlf, f_svrf, f_svlrf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_v2f (g, v1, v2, psi) result (vpsi)
    type(bispinor) :: vpsi
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v1, v2
    vpsi = g * f_vvf (v2, psi, v1)
  end function f_v2f
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_svf (g, phi, v, psi) result (pvpsi)
    type(bispinor) :: pvpsi
    complex(kind=default), intent(in) :: g, phi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    pvpsi = phi * f_vf (g, v, psi)
  end function f_svf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_pvf (g, phi, v, psi) result (pvpsi)
    type(bispinor) :: pvpsi
    complex(kind=default), intent(in) :: g, phi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    pvpsi = -(phi * f_af (g, v, psi))
  end function f_pvf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_svlf (g, phi, v, psi) result (pvpsi)
    type(bispinor) :: pvpsi
    complex(kind=default), intent(in) :: g, phi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    pvpsi = phi * f_vlf (g, v, psi)
  end function f_svlf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_svrf (g, phi, v, psi) result (pvpsi)
    type(bispinor) :: pvpsi
    complex(kind=default), intent(in) :: g, phi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    pvpsi = phi * f_vrf (g, v, psi)
  end function f_svrf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_svlrf (gl, gr, phi, v, psi) result (pvpsi)
    type(bispinor) :: pvpsi
    complex(kind=default), intent(in) :: gl, gr, phi
    type(bispinor), intent(in) :: psi
```

```
      type(vector), intent(in) :: v
      pvpsi = phi * f_vlrf (gl, gr, v, psi)
    end function f_svlrf
```

X.23.4   Gravitino Couplings

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: pot_grf, pot_fgr, s_grf, s_fgr, p_grf, p_fgr, &
       sl_grf, sl_fgr, sr_grf, sr_fgr, slr_grf, slr_fgr
```

⟨*Declaration of bispinor currents*⟩+≡
```
  private :: fgvgr, fgvg5gr, fggvvgr, grkgf, grkggf, grkkggf, &
       fgkgr, fg5gkgr, grvgf, grg5vgf, grkgggf, fggkggr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function pot_grf (g, gravbar, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(vectorspinor) :: gamma_psi
    gamma_psi%psi(1)%a(1) = psi%a(3)
    gamma_psi%psi(1)%a(2) = psi%a(4)
    gamma_psi%psi(1)%a(3) = psi%a(1)
    gamma_psi%psi(1)%a(4) = psi%a(2)
    gamma_psi%psi(2)%a(1) = psi%a(4)
    gamma_psi%psi(2)%a(2) = psi%a(3)
    gamma_psi%psi(2)%a(3) = - psi%a(2)
    gamma_psi%psi(2)%a(4) = - psi%a(1)
    gamma_psi%psi(3)%a(1) = (0,-1) * psi%a(4)
    gamma_psi%psi(3)%a(2) = (0,1) * psi%a(3)
    gamma_psi%psi(3)%a(3) = (0,1) * psi%a(2)
    gamma_psi%psi(3)%a(4) = (0,-1) * psi%a(1)
    gamma_psi%psi(4)%a(1) = psi%a(3)
    gamma_psi%psi(4)%a(2) = - psi%a(4)
    gamma_psi%psi(4)%a(3) = - psi%a(1)
    gamma_psi%psi(4)%a(4) = psi%a(2)
    j = g * (gravbar * gamma_psi)
  end function pot_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function pot_fgr (g, psibar, grav) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(bispinor) :: gamma_grav
    gamma_grav%a(1) = grav%psi(1)%a(3) - grav%psi(2)%a(4) + &
          ((0,1)*grav%psi(3)%a(4)) - grav%psi(4)%a(3)
    gamma_grav%a(2) = grav%psi(1)%a(4) - grav%psi(2)%a(3) - &
          ((0,1)*grav%psi(3)%a(3)) + grav%psi(4)%a(4)
    gamma_grav%a(3) = grav%psi(1)%a(1) + grav%psi(2)%a(2) - &
          ((0,1)*grav%psi(3)%a(2)) + grav%psi(4)%a(1)
    gamma_grav%a(4) = grav%psi(1)%a(2) + grav%psi(2)%a(1) + &
          ((0,1)*grav%psi(3)%a(1)) - grav%psi(4)%a(2)
```

```
      j = g * (psibar * gamma_grav)
   end function pot_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function grvgf (gravbar, psi, k) result (j)
    complex(kind=default) :: j
    complex(kind=default) :: kp, km, k12, k12s
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: k
    type(vectorspinor) :: kg_psi
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    !!! Since we are taking the spinor product here, NO explicit
    !!! charge conjugation matrix is needed!
    kg_psi%psi(1)%a(1) = km * psi%a(1) - k12s * psi%a(2)
    kg_psi%psi(1)%a(2) = (-k12) * psi%a(1) + kp * psi%a(2)
    kg_psi%psi(1)%a(3) = kp * psi%a(3) + k12s * psi%a(4)
    kg_psi%psi(1)%a(4) = k12 * psi%a(3) + km * psi%a(4)
    kg_psi%psi(2)%a(1) = k12s * psi%a(1) - km * psi%a(2)
    kg_psi%psi(2)%a(2) = (-kp) * psi%a(1) + k12 * psi%a(2)
    kg_psi%psi(2)%a(3) = k12s * psi%a(3) + kp * psi%a(4)
    kg_psi%psi(2)%a(4) = km * psi%a(3) + k12 * psi%a(4)
    kg_psi%psi(3)%a(1) = (0,1) * (k12s * psi%a(1) + km * psi%a(2))
    kg_psi%psi(3)%a(2) = (0,1) * (- kp * psi%a(1) - k12 * psi%a(2))
    kg_psi%psi(3)%a(3) = (0,1) * (k12s * psi%a(3) - kp * psi%a(4))
    kg_psi%psi(3)%a(4) = (0,1) * (km * psi%a(3) - k12 * psi%a(4))
    kg_psi%psi(4)%a(1) = (-km) * psi%a(1) - k12s * psi%a(2)
    kg_psi%psi(4)%a(2) = k12 * psi%a(1) + kp * psi%a(2)
    kg_psi%psi(4)%a(3) = kp * psi%a(3) - k12s * psi%a(4)
    kg_psi%psi(4)%a(4) = k12 * psi%a(3) - km * psi%a(4)
    j = gravbar * kg_psi
  end function grvgf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function grg5vgf (gravbar, psi, k) result (j)
    complex(kind=default) :: j
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: k
    type(bispinor) :: g5_psi
    g5_psi%a(1:2) = - psi%a(1:2)
    g5_psi%a(3:4) =   psi%a(3:4)
    j = grvgf (gravbar, g5_psi, k)
  end function grg5vgf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function s_grf (g, gravbar, psi, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    type(vector) :: vk
```

```
      vk = k
      j = g * grvgf (gravbar, psi, vk)
    end function s_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sl_grf (gl, gravbar, psi, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l
    type(momentum), intent(in) :: k
    psi_l%a(1:2) = psi%a(1:2)
    psi_l%a(3:4) = 0
    j = s_grf (gl, gravbar, psi_l, k)
  end function sl_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sr_grf (gr, gravbar, psi, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gr
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_r
    type(momentum), intent(in) :: k
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    j = s_grf (gr, gravbar, psi_r, k)
  end function sr_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slr_grf (gl, gr, gravbar, psi, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    j = sl_grf (gl, gravbar, psi, k) + sr_grf (gr, gravbar, psi, k)
  end function slr_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function fgkgr (psibar, grav, k) result (j)
    complex(kind=default) :: j
    complex(kind=default) :: kp, km, k12, k12s
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: k
    type(bispinor) :: gk_grav
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    !!! Since we are taking the spinor product here, NO explicit
    !!! charge conjugation matrix is needed!
    gk_grav%a(1) =  kp * grav%psi(1)%a(1) + k12s * grav%psi(1)%a(2) &
                   - k12 * grav%psi(2)%a(1) - km * grav%psi(2)%a(2) &
```

```
                    + (0,1) * k12 * grav%psi(3)%a(1)   &
                    + (0,1) * km * grav%psi(3)%a(2) &
                    - kp * grav%psi(4)%a(1) - k12s * grav%psi(4)%a(2)
        gk_grav%a(2) = k12 * grav%psi(1)%a(1) + km * grav%psi(1)%a(2) &
                    - kp * grav%psi(2)%a(1) - k12s * grav%psi(2)%a(2) &
                    - (0,1) * kp * grav%psi(3)%a(1) &
                    - (0,1) * k12s * grav%psi(3)%a(2)   &
                    + k12 * grav%psi(4)%a(1) + km * grav%psi(4)%a(2)
        gk_grav%a(3) = km * grav%psi(1)%a(3) - k12s * grav%psi(1)%a(4) &
                    - k12 * grav%psi(2)%a(3) + kp * grav%psi(2)%a(4) &
                    + (0,1) * k12 * grav%psi(3)%a(3)   &
                    - (0,1) * kp * grav%psi(3)%a(4) &
                    + km * grav%psi(4)%a(3) - k12s * grav%psi(4)%a(4)
        gk_grav%a(4) = - k12 * grav%psi(1)%a(3) + kp * grav%psi(1)%a(4) &
                    + km * grav%psi(2)%a(3) - k12s * grav%psi(2)%a(4) &
                    + (0,1) * km * grav%psi(3)%a(3) &
                    - (0,1) * k12s * grav%psi(3)%a(4)   &
                    + k12 * grav%psi(4)%a(3) - kp * grav%psi(4)%a(4)
      j = psibar * gk_grav
    end function fgkgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function fg5gkgr (psibar, grav, k) result (j)
    complex(kind=default) :: j
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: k
    type(bispinor) :: psibar_g5
    psibar_g5%a(1:2) = - psibar%a(1:2)
    psibar_g5%a(3:4) =   psibar%a(3:4)
    j = fgkgr (psibar_g5, grav, k)
  end function fg5gkgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function s_fgr (g, psibar, grav, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(momentum), intent(in) :: k
    type(vector) :: vk
    vk = k
    j = g * fgkgr (psibar, grav, vk)
  end function s_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sl_fgr (gl, psibar, grav, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_l
    type(vectorspinor), intent(in) :: grav
    type(momentum), intent(in) :: k
    psibar_l%a(1:2) = psibar%a(1:2)
    psibar_l%a(3:4) = 0
    j = s_fgr (gl, psibar_l, grav, k)
```

```
      end function sl_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sr_fgr (gr, psibar, grav, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gr
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_r
    type(vectorspinor), intent(in) :: grav
    type(momentum), intent(in) :: k
    psibar_r%a(1:2) = 0
    psibar_r%a(3:4) = psibar%a(3:4)
    j = s_fgr (gr, psibar_r, grav, k)
  end function sr_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slr_fgr (gl, gr, psibar, grav, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(momentum), intent(in) :: k
    j = sl_fgr (gl, psibar, grav, k) + sr_fgr (gr, psibar, grav, k)
  end function slr_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function p_grf (g, gravbar, psi, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    type(vector) :: vk
    vk = k
    j = g * grg5vgf (gravbar, psi, vk)
  end function p_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function p_fgr (g, psibar, grav, k) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(momentum), intent(in) :: k
    type(vector) :: vk
    vk = k
    j = g * fg5gkgr (psibar, grav, vk)
  end function p_fgr
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: f_potgr, f_sgr, f_pgr, f_vgr, f_vlrgr, f_slgr, f_srgr, f_slrgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_potgr (g, phi, psi) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi
```

```
      type(vectorspinor), intent(in) :: psi
      phipsi%a(1) = (g * phi) * (psi%psi(1)%a(3) - psi%psi(2)%a(4) + &
                    ((0,1)*psi%psi(3)%a(4)) - psi%psi(4)%a(3))
      phipsi%a(2) = (g * phi) * (psi%psi(1)%a(4) - psi%psi(2)%a(3) - &
                    ((0,1)*psi%psi(3)%a(3)) + psi%psi(4)%a(4))
      phipsi%a(3) = (g * phi) * (psi%psi(1)%a(1) + psi%psi(2)%a(2) - &
                    ((0,1)*psi%psi(3)%a(2)) + psi%psi(4)%a(1))
      phipsi%a(4) = (g * phi) * (psi%psi(1)%a(2) + psi%psi(2)%a(1) + &
                    ((0,1)*psi%psi(3)%a(1)) - psi%psi(4)%a(2))
    end function f_potgr
```

The slashed notation:

$$
\slashed{k} = \begin{pmatrix} 0 & 0 & k_- & -k^* \\ 0 & 0 & -k & k_+ \\ k_+ & k^* & 0 & 0 \\ k & k_- & 0 & 0 \end{pmatrix}, \qquad
\slashed{k}\gamma_5 = \begin{pmatrix} 0 & 0 & k_- & -k^* \\ 0 & 0 & -k & k_+ \\ -k_+ & -k^* & 0 & 0 \\ -k & -k_- & 0 & 0 \end{pmatrix} \tag{X.106}
$$

with $k_\pm = k_0 \pm k_3$, $k = k_1 + \mathrm{i}k_2$, $k^* = k_1 - \mathrm{i}k_2$. But note that $\cdot^*$ is *not* complex conjugation for complex k_μ.

$$
\gamma^0\slashed{k} = \begin{pmatrix} k_+ & k^* & 0 & 0 \\ k & k_- & 0 & 0 \\ 0 & 0 & k_- & -k^* \\ 0 & 0 & -k & k_+ \end{pmatrix}, \qquad
\gamma^0\slashed{k}\gamma^5 = \begin{pmatrix} -k_+ & -k^* & 0 & 0 \\ -k & -k_- & 0 & 0 \\ 0 & 0 & k_- & -k^* \\ 0 & 0 & -k & k_+ \end{pmatrix}
$$
$$\tag{X.107a}$$

$$
\gamma^1\slashed{k} = \begin{pmatrix} k & k_- & 0 & 0 \\ k_+ & k^* & 0 & 0 \\ 0 & 0 & k & -k_+ \\ 0 & 0 & -k_- & k^* \end{pmatrix}, \qquad
\gamma^1\slashed{k}\gamma^5 = \begin{pmatrix} -k & -k_- & 0 & 0 \\ -k_+ & -k^* & 0 & 0 \\ 0 & 0 & k & -k_+ \\ 0 & 0 & -k_- & k^* \end{pmatrix}
$$
$$\tag{X.107b}$$

$$
\gamma^2\slashed{k} = \begin{pmatrix} -\mathrm{i}k & -\mathrm{i}k_- & 0 & 0 \\ \mathrm{i}k_+ & \mathrm{i}k^* & 0 & 0 \\ 0 & 0 & -\mathrm{i}k & \mathrm{i}k_+ \\ 0 & 0 & -\mathrm{i}k_- & \mathrm{i}k^* \end{pmatrix}, \qquad
\gamma^2\slashed{k}\gamma^5 = \begin{pmatrix} \mathrm{i}k & \mathrm{i}k_- & 0 & 0 \\ -\mathrm{i}k_+ & -\mathrm{i}k^* & 0 & 0 \\ 0 & 0 & -\mathrm{i}k & \mathrm{i}k_+ \\ 0 & 0 & -\mathrm{i}k_- & \mathrm{i}k^* \end{pmatrix}
$$
$$\tag{X.107c}$$

$$
\gamma^3\slashed{k} = \begin{pmatrix} k_+ & k^* & 0 & 0 \\ -k & -k_- & 0 & 0 \\ 0 & 0 & -k_- & k^* \\ 0 & 0 & -k & k_+ \end{pmatrix}, \qquad
\gamma^3\slashed{k}\gamma^5 = \begin{pmatrix} -k_+ & -k^* & 0 & 0 \\ k & k_- & 0 & 0 \\ 0 & 0 & -k_- & k^* \\ 0 & 0 & -k & k_+ \end{pmatrix}
$$
$$\tag{X.107d}$$

and

$$
\slashed{k}\gamma^0 = \begin{pmatrix} k_- & -k^* & 0 & 0 \\ -k & k_+ & 0 & 0 \\ 0 & 0 & k_+ & k^* \\ 0 & 0 & k & k_- \end{pmatrix}, \qquad
\slashed{k}\gamma^0\gamma^5 = \begin{pmatrix} -k_- & k^* & 0 & 0 \\ k & -k_+ & 0 & 0 \\ 0 & 0 & k_+ & k^* \\ 0 & 0 & k & k_- \end{pmatrix}
$$
$$\tag{X.108a}$$

$$\not{k}\gamma^1 = \begin{pmatrix} k^* & -k_- & 0 & 0 \\ -k_+ & k & 0 & 0 \\ 0 & 0 & k^* & k_+ \\ 0 & 0 & k_- & k \end{pmatrix}, \qquad \not{k}\gamma^1\gamma^5 = \begin{pmatrix} -k^* & k_- & 0 & 0 \\ k_+ & -k & 0 & 0 \\ 0 & 0 & k^* & k_+ \\ 0 & 0 & k_- & k \end{pmatrix}$$
$$(\text{X.108b})$$

$$\not{k}\gamma^2 = \begin{pmatrix} \mathrm{i}k^* & \mathrm{i}k_- & 0 & 0 \\ -\mathrm{i}k_+ & -\mathrm{i}k & 0 & 0 \\ 0 & 0 & \mathrm{i}k^* & -\mathrm{i}k_+ \\ 0 & 0 & \mathrm{i}k_- & -\mathrm{i}k \end{pmatrix}, \qquad \not{k}\gamma^2\gamma^5 = \begin{pmatrix} -\mathrm{i}k^* & -\mathrm{i}k_- & 0 & 0 \\ \mathrm{i}k_+ & \mathrm{i}k & 0 & 0 \\ 0 & 0 & \mathrm{i}k^* & -\mathrm{i}k_+ \\ 0 & 0 & \mathrm{i}k_- & -\mathrm{i}k \end{pmatrix}$$
$$(\text{X.108c})$$

$$\not{k}\gamma^3 = \begin{pmatrix} -k_- & -k^* & 0 & 0 \\ k & k_+ & 0 & 0 \\ 0 & 0 & k_+ & -k^* \\ 0 & 0 & k & -k_- \end{pmatrix}, \qquad \not{k}\gamma^3\gamma^5 = \begin{pmatrix} k_- & k^* & 0 & 0 \\ -k & -k_+ & 0 & 0 \\ 0 & 0 & k_+ & -k^* \\ 0 & 0 & k & -k_- \end{pmatrix}$$
$$(\text{X.108d})$$

and

$$C\gamma^0\not{k} = \begin{pmatrix} k & k_- & 0 & 0 \\ -k_+ & -k^* & 0 & 0 \\ 0 & 0 & k & -k_+ \\ 0 & 0 & k_- & -k^* \end{pmatrix}, \qquad C\gamma^0\not{k}\gamma^5 = \begin{pmatrix} -k & -k_- & 0 & 0 \\ k_+ & k^* & 0 & 0 \\ 0 & 0 & k & -k_+ \\ 0 & 0 & k_- & -k^* \end{pmatrix}$$
$$(\text{X.109a})$$

$$C\gamma^1\not{k} = \begin{pmatrix} k_+ & k^* & 0 & 0 \\ -k & -k_- & 0 & 0 \\ 0 & 0 & k_- & -k^* \\ 0 & 0 & k & -k_+ \end{pmatrix}, \qquad C\gamma^1\not{k}\gamma^5 = \begin{pmatrix} -k_+ & -k^* & 0 & 0 \\ k & k_- & 0 & 0 \\ 0 & 0 & k_- & -k^* \\ 0 & 0 & k & -k_+ \end{pmatrix}$$
$$(\text{X.109b})$$

$$C\gamma^2\not{k} = \begin{pmatrix} \mathrm{i}k_+ & \mathrm{i}k^* & 0 & 0 \\ \mathrm{i}k & \mathrm{i}k_- & 0 & 0 \\ 0 & 0 & \mathrm{i}k_- & -\mathrm{i}k^* \\ 0 & 0 & -\mathrm{i}k & \mathrm{i}k_+ \end{pmatrix}, \qquad C\gamma^2\not{k}\gamma^5 = \begin{pmatrix} -\mathrm{i}k_+ & -\mathrm{i}k^* & 0 & 0 \\ -\mathrm{i}k & -\mathrm{i}k_- & 0 & 0 \\ 0 & 0 & \mathrm{i}k_- & -\mathrm{i}k^* \\ 0 & 0 & -\mathrm{i}k & \mathrm{i}k_+ \end{pmatrix}$$
$$(\text{X.109c})$$

$$C\gamma^3\not{k} = \begin{pmatrix} -k & -k_- & 0 & 0 \\ -k_+ & -k^* & 0 & 0 \\ 0 & 0 & k & -k_+ \\ 0 & 0 & -k_- & k^* \end{pmatrix}, \qquad C\gamma^3\not{k}\gamma^5 = \begin{pmatrix} k & k_- & 0 & 0 \\ k_+ & k^* & 0 & 0 \\ 0 & 0 & k & -k_+ \\ 0 & 0 & -k_- & k^* \end{pmatrix}$$
$$(\text{X.109d})$$

and

$$C\not{k}\gamma^0 = \begin{pmatrix} -k & k^+ & 0 & 0 \\ -k_- & k^* & 0 & 0 \\ 0 & 0 & -k & -k_- \\ 0 & 0 & k_+ & k^* \end{pmatrix}, \qquad C\not{k}\gamma^0\gamma^5 = \begin{pmatrix} k & -k_+ & 0 & 0 \\ k_- & -k^* & 0 & 0 \\ 0 & 0 & -k & -k_- \\ 0 & 0 & k_+ & k^* \end{pmatrix}$$
$$(\text{X.110a})$$

$$C\not{k}\gamma^1 = \begin{pmatrix} -k_+ & k & 0 & 0 \\ -k^* & k_- & 0 & 0 \\ 0 & 0 & -k_- & -k \\ 0 & 0 & k^* & k_+ \end{pmatrix}, \qquad C\not{k}\gamma^1\gamma^5 = \begin{pmatrix} k_+ & -k & 0 & 0 \\ k^* & -k_- & 0 & 0 \\ 0 & 0 & -k_- & -k \\ 0 & 0 & k^* & k_+ \end{pmatrix}$$

$$\text{(X.110b)}$$

$$C\not{k}\gamma^2 = \begin{pmatrix} -\mathrm{i}k_+ & -\mathrm{i}k & 0 & 0 \\ -\mathrm{i}k^* & -\mathrm{i}k_- & 0 & 0 \\ 0 & 0 & -\mathrm{i}k_- & \mathrm{i}k \\ 0 & 0 & \mathrm{i}k^* & -\mathrm{i}k_+ \end{pmatrix}, \qquad C\not{k}\gamma^2\gamma^5 = \begin{pmatrix} \mathrm{i}k_+ & \mathrm{i}k & 0 & 0 \\ \mathrm{i}k^* & \mathrm{i}k_- & 0 & 0 \\ 0 & 0 & -\mathrm{i}k_- & \mathrm{i}k \\ 0 & 0 & \mathrm{i}k^* & -\mathrm{i}k_+ \end{pmatrix}$$

$$\text{(X.110c)}$$

$$C\not{k}\gamma^3 = \begin{pmatrix} k & k_+ & 0 & 0 \\ k_- & k^* & 0 & 0 \\ 0 & 0 & -k & k_- \\ 0 & 0 & k_+ & -k^* \end{pmatrix}, \qquad C\not{k}\gamma^3\gamma^5 = \begin{pmatrix} -k & -k_+ & 0 & 0 \\ -k_- & -k^* & 0 & 0 \\ 0 & 0 & -k & k_- \\ 0 & 0 & k_+ & -k^* \end{pmatrix}$$

$$\text{(X.110d)}$$

⟨*Implementation of bispinor currents*⟩+≡
```fortran
  pure function fgvgr (psi, k) result (kpsi)
    type(bispinor) :: kpsi
    complex(kind=default) :: kp, km, k12, k12s
    type(vector), intent(in) :: k
    type(vectorspinor), intent(in) :: psi
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    kpsi%a(1) = kp * psi%psi(1)%a(1) + k12s * psi%psi(1)%a(2) &
              - k12 * psi%psi(2)%a(1) - km * psi%psi(2)%a(2) &
              + (0,1) * k12 * psi%psi(3)%a(1) + (0,1) * km * psi%psi(3)%a(2) &
              - kp * psi%psi(4)%a(1) - k12s * psi%psi(4)%a(2)
    kpsi%a(2) = k12 * psi%psi(1)%a(1) + km * psi%psi(1)%a(2) &
              - kp * psi%psi(2)%a(1) - k12s * psi%psi(2)%a(2) &
              - (0,1) * kp * psi%psi(3)%a(1) - (0,1) * k12s * psi%psi(3)%a(2) &
              + k12 * psi%psi(4)%a(1) + km * psi%psi(4)%a(2)
    kpsi%a(3) = km * psi%psi(1)%a(3) - k12s * psi%psi(1)%a(4) &
              - k12 * psi%psi(2)%a(3) + kp * psi%psi(2)%a(4) &
              + (0,1) * k12 * psi%psi(3)%a(3) - (0,1) * kp * psi%psi(3)%a(4) &
              + km * psi%psi(4)%a(3) - k12s * psi%psi(4)%a(4)
    kpsi%a(4) = - k12 * psi%psi(1)%a(3) + kp * psi%psi(1)%a(4) &
              + km * psi%psi(2)%a(3) - k12s * psi%psi(2)%a(4) &
              + (0,1) * km * psi%psi(3)%a(3) - (0,1) * k12s * psi%psi(3)%a(4) &
              + k12 * psi%psi(4)%a(3) - kp * psi%psi(4)%a(4)
  end function fgvgr
```

⟨*Implementation of bispinor currents*⟩+≡
```fortran
  pure function f_sgr (g, phi, psi, k) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi
    type(momentum), intent(in) :: k
    type(vectorspinor), intent(in) :: psi
    type(vector) :: vk
    vk = k
```

```
      phipsi = (g * phi) * fgvgr (psi, vk)
    end function f_sgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_slgr (gl, phi, psi, k) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gl
    complex(kind=default), intent(in) :: phi
    type(momentum), intent(in) :: k
    type(vectorspinor), intent(in) :: psi
    phipsi = f_sgr (gl, phi, psi, k)
    phipsi%a(3:4) = 0
  end function f_slgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_srgr (gr, phi, psi, k) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: gr
    complex(kind=default), intent(in) :: phi
    type(momentum), intent(in) :: k
    type(vectorspinor), intent(in) :: psi
    phipsi = f_sgr (gr, phi, psi, k)
    phipsi%a(1:2) = 0
  end function f_srgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_slrgr (gl, gr, phi, psi, k) result (phipsi)
    type(bispinor) :: phipsi, phipsi_l, phipsi_r
    complex(kind=default), intent(in) :: gl, gr
    complex(kind=default), intent(in) :: phi
    type(momentum), intent(in) :: k
    type(vectorspinor), intent(in) :: psi
    phipsi_l = f_slgr (gl, phi, psi, k)
    phipsi_r = f_srgr (gr, phi, psi, k)
    phipsi%a(1:2) = phipsi_l%a(1:2)
    phipsi%a(3:4) = phipsi_r%a(3:4)
  end function f_slrgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function fgvg5gr (psi, k) result (kpsi)
    type(bispinor) :: kpsi
    type(vector), intent(in) :: k
    type(vectorspinor), intent(in) :: psi
    type(bispinor) :: kpsi_dum
    kpsi_dum = fgvgr (psi, k)
    kpsi%a(1:2) = - kpsi_dum%a(1:2)
    kpsi%a(3:4) =   kpsi_dum%a(3:4)
  end function fgvg5gr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_pgr (g, phi, psi, k) result (phipsi)
    type(bispinor) :: phipsi
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi
    type(momentum), intent(in) :: k
    type(vectorspinor), intent(in) :: psi
    type(vector) :: vk
```

```
      vk = k
      phipsi = (g * phi) * fgvg5gr (psi, vk)
    end function f_pgr
```

The needed construction of gamma matrices involving the commutator of two gamma matrices. For the slashed terms we use as usual the abbreviations $k_\pm = k_0 \pm k_3$, $k = k_1 + ik_2$, $k^* = k_1 - ik_2$ and analogous expressions for the vector v^μ. We remind you that $\cdot^*$ is *not* complex conjugation for complex k_μ. Furthermore we introduce (in what follows the brackets around the vector indices have the usual meaning of antisymmetrizing with respect to the indices inside the brackets, here without a factor two in the denominator)

$$a_+ = k_+ v_- + k v^* - k_- v_+ - k^* v = 2(k_{[3} v_{0]} + ik_{[2} v_{1]}) \tag{X.111a}$$

$$a_- = k_- v_+ + k v^* - k_+ v_- - k^* v = 2(-k_{[3} v_{0]} + ik_{[2} v_{1]}) \tag{X.111b}$$

$$b_+ = 2(k_+ v - k v_+) \qquad = 2(k_{[0} v_{1]} + k_{[3} v_{1]} + ik_{[0} v_{2]} + ik_{[3} v_{2]}) \tag{X.111c}$$

$$b_- = 2(k_- v - k v_-) \qquad = 2(k_{[0} v_{1]} - k_{[3} v_{1]} + ik_{[0} v_{2]} - ik_{[3} v_{2]}) \tag{X.111d}$$

$$b_{+*} = 2(k_+ v^* - k^* v_+) \qquad = 2(k_{[0} v_{1]} + k_{[3} v_{1]} - ik_{[0} v_{2]} - ik_{[3} v_{2]}) \tag{X.111e}$$

$$b_{-*} = 2(k_- v^* - k^* v_-) \qquad = 2(k_{[0} v_{1]} - k_{[3} v_{1]} - ik_{[0} v_{2]} + ik_{[3} v_{2]}) \tag{X.111f}$$

Of course, one could introduce a more advanced notation, but we don't want to become confused.

$$[\slashed{k}, \gamma^0] = \begin{pmatrix} -2k_3 & -2k^* & 0 & 0 \\ -2k & 2k_3 & 0 & 0 \\ 0 & 0 & 2k_3 & 2k^* \\ 0 & 0 & 2k & -2k_3 \end{pmatrix} \tag{X.112a}$$

$$[\slashed{k}, \gamma^1] = \begin{pmatrix} -2ik_2 & -2k_- & 0 & 0 \\ -2k_+ & 2ik_2 & 0 & 0 \\ 0 & 0 & -2ik_2 & 2k_+ \\ 0 & 0 & 2k_- & 2ik_2 \end{pmatrix} \tag{X.112b}$$

$$[\slashed{k}, \gamma^2] = \begin{pmatrix} 2ik_1 & 2ik_- & 0 & 0 \\ -2ik_+ & -2ik_1 & 0 & 0 \\ 0 & 0 & 2ik_1 & -2ik_+ \\ 0 & 0 & 2ik_- & -2ik_1 \end{pmatrix} \tag{X.112c}$$

$$[\slashed{k}, \gamma^3] = \begin{pmatrix} -2k_0 & -2k^* & 0 & 0 \\ 2k & 2k_0 & 0 & 0 \\ 0 & 0 & 2k_0 & -2k^* \\ 0 & 0 & 2k & -2k_0 \end{pmatrix} \tag{X.112d}$$

$$[\slashed{k}, \slashed{V}] = \begin{pmatrix} a_- & b_{-*} & 0 & 0 \\ b_+ & -a_- & 0 & 0 \\ 0 & 0 & a_+ & -b_{+*} \\ 0 & 0 & -b_- & -a_+ \end{pmatrix} \tag{X.112e}$$

$$\gamma^5\gamma^0[\not{k}, \not{V}] = \begin{pmatrix} 0 & 0 & -a_+ & b_{+*} \\ 0 & 0 & b_- & a_+ \\ a_- & b_{-*} & 0 & 0 \\ b_+ & -a_- & 0 & 0 \end{pmatrix} \tag{X.112f}$$

$$\gamma^5\gamma^1[\not{k}, \not{V}] = \begin{pmatrix} 0 & 0 & b_- & a_+ \\ 0 & 0 & -a_+ & b_{+*} \\ -b_+ & a_- & 0 & 0 \\ -a_- & -b_{-*} & 0 & 0 \end{pmatrix} \tag{X.112g}$$

$$\gamma^5\gamma^2[\not{k}, \not{V}] = \begin{pmatrix} 0 & 0 & -ib_- & -ia_+ \\ 0 & 0 & -ia_+ & ib_{+*} \\ ib_+ & -ia_- & 0 & 0 \\ -ia_- & -ib_{-*} & 0 & 0 \end{pmatrix} \tag{X.112h}$$

$$\gamma^5\gamma^3[\not{k}, \not{V}] = \begin{pmatrix} 0 & 0 & -a_+ & b_{+*} \\ 0 & 0 & -b_- & -a_+ \\ -a_- & -b_{-*} & 0 & 0 \\ b_+ & -a_- & 0 & 0 \end{pmatrix} \tag{X.112i}$$

and

$$[\not{k}, \not{V}]\gamma^0\gamma^5 = \begin{pmatrix} 0 & 0 & a_- & b_{-*} \\ 0 & 0 & b_+ & -a_- \\ -a_+ & b_{+*} & 0 & 0 \\ b_- & a_+ & 0 & 0 \end{pmatrix} \tag{X.113a}$$

$$[\not{k}, \not{V}]\gamma^1\gamma^5 = \begin{pmatrix} 0 & 0 & b_{-*} & a_- \\ 0 & 0 & -a_- & b_+ \\ -b_{+*} & a_+ & 0 & 0 \\ -a_+ & -b_- & 0 & 0 \end{pmatrix} \tag{X.113b}$$

$$[\not{k}, \not{V}]\gamma^2\gamma^5 = \begin{pmatrix} 0 & 0 & ib_{-*} & -ia_- \\ 0 & 0 & -ia_- & -ib_+ \\ -ib_{+*} & -ia_+ & 0 & 0 \\ -ia_+ & ib_- & 0 & 0 \end{pmatrix} \tag{X.113c}$$

$$[\not{k}, \not{V}]\gamma^3\gamma^5 = \begin{pmatrix} 0 & 0 & a_- & -b_{-*} \\ 0 & 0 & b_+ & a_- \\ a_+ & b_{+*} & 0 & 0 \\ -b_- & a_+ & 0 & 0 \end{pmatrix} \tag{X.113d}$$

In what follows l always means twice the value of k, e.g. $l_+ = 2k_+$. We use the abbreviation $C^{\mu\nu} \equiv C[\not{k}, \gamma^\mu]\gamma^\nu\gamma^5$.

$$C^{00} = \begin{pmatrix} 0 & 0 & -l & -l_3 \\ 0 & 0 & l_3 & l^* \\ l & -l_3 & 0 & 0 \\ -l_3 & -l^* & 0 & 0 \end{pmatrix}, \qquad C^{20} = \begin{pmatrix} 0 & 0 & -il_+ & -il_1 \\ 0 & 0 & -il_1 & -il_- \\ il_- & -il_1 & 0 & 0 \\ -il_1 & il_+ & 0 & 0 \end{pmatrix}$$
$$\tag{X.114a}$$

$$C^{01} = \begin{pmatrix} 0 & 0 & l_3 & -l \\ 0 & 0 & l^* & l_3 \\ l_3 & -l & 0 & 0 \\ l^* & l_3 & 0 & 0 \end{pmatrix}, \qquad C^{21} = \begin{pmatrix} 0 & 0 & -il_1 & -il_+ \\ 0 & 0 & -il_- & -il_1 \\ il_1 & -il_- & 0 & 0 \\ -il_+ & il_1 & 0 & 0 \end{pmatrix}$$
$$\tag{X.114b}$$

$$C^{02} = \begin{pmatrix} 0 & 0 & il_3 & il \\ 0 & 0 & il^* & -il_3 \\ il_3 & il & 0 & 0 \\ il^* & -il_3 & 0 & 0 \end{pmatrix}, \qquad C^{22} = \begin{pmatrix} 0 & 0 & l_1 & -l_+ \\ 0 & 0 & l_- & -l_1 \\ -l_1 & -l_- & 0 & 0 \\ l_+ & l_1 & 0 & 0 \end{pmatrix}$$

$$\text{(X.114c)}$$

$$C^{03} = \begin{pmatrix} 0 & 0 & -l & -l_3 \\ 0 & 0 & l_3 & -l^* \\ -l & -l_3 & 0 & 0 \\ l_3 & -l^* & 0 & 0 \end{pmatrix}, \qquad C^{23} = \begin{pmatrix} 0 & 0 & -il_+ & il_1 \\ 0 & 0 & -il_1 & il_- \\ -il_- & -il_1 & 0 & 0 \\ il_1 & il_+ & 0 & 0 \end{pmatrix}$$

$$\text{(X.114d)}$$

$$C^{10} = \begin{pmatrix} 0 & 0 & -l_+ & il_2 \\ 0 & 0 & il_2 & l_- \\ l_- & il_2 & 0 & 0 \\ il_2 & -l_+ & 0 & 0 \end{pmatrix}, \qquad C^{30} = \begin{pmatrix} 0 & 0 & l & l_0 \\ 0 & 0 & l_0 & l^* \\ l & -l_0 & 0 & 0 \\ -l_0 & l^* & 0 & 0 \end{pmatrix} \quad \text{(X.114e)}$$

$$C^{11} = \begin{pmatrix} 0 & 0 & il_2 & -l_+ \\ 0 & 0 & l_- & il_2 \\ -il_2 & -l_- & 0 & 0 \\ l_+ & -il_2 & 0 & 0 \end{pmatrix}, \qquad C^{31} = \begin{pmatrix} 0 & 0 & l_0 & l \\ 0 & 0 & l^* & l_0 \\ l_0 & -l & 0 & 0 \\ -l^* & l_0 & 0 & 0 \end{pmatrix} \quad \text{(X.114f)}$$

$$C^{12} = \begin{pmatrix} 0 & 0 & -l_2 & il_+ \\ 0 & 0 & il_- & l_2 \\ l_2 & il_- & 0 & 0 \\ il_+ & -l_2 & 0 & 0 \end{pmatrix}, \qquad C^{32} = \begin{pmatrix} 0 & 0 & il_0 & -il \\ 0 & 0 & il^* & -il_0 \\ il_0 & il & 0 & 0 \\ -il^* & -il_0 & 0 & 0 \end{pmatrix}$$

$$\text{(X.114g)}$$

$$C^{13} = \begin{pmatrix} 0 & 0 & -l_+ & -il_2 \\ 0 & 0 & il_2 & -l_- \\ -l_- & il_2 & 0 & 0 \\ -il_2 & -l_+ & 0 & 0 \end{pmatrix}, \qquad C^{33} = \begin{pmatrix} 0 & 0 & l & -l_0 \\ 0 & 0 & l_0 & -l^* \\ -l & -l_0 & 0 & 0 \\ l_0 & l^* & 0 & 0 \end{pmatrix}$$

$$\text{(X.114h)}$$

and, with the abbreviation $\tilde{C}^{\mu\nu} \equiv C\gamma^5\gamma^\nu[\not{k}, \gamma^\mu]$ (note the reversed order of the indices!)

$$\tilde{C}^{00} = \begin{pmatrix} 0 & 0 & -l & l_3 \\ 0 & 0 & l_3 & l^* \\ l & -l_3 & 0 & 0 \\ -l_3 & -l^* & 0 & 0 \end{pmatrix}, \qquad \tilde{C}^{20} = \begin{pmatrix} 0 & 0 & -il_- & il_1 \\ 0 & 0 & il_1 & -il_+ \\ il_+ & il_1 & 0 & 0 \\ il_1 & il_- & 0 & 0 \end{pmatrix}$$

$$\text{(X.115a)}$$

$$\tilde{C}^{01} = \begin{pmatrix} 0 & 0 & -l_3 & -l^* \\ 0 & 0 & l & -l_3 \\ -l_3 & -l^* & 0 & 0 \\ l & -l_3 & 0 & 0 \end{pmatrix}, \qquad \tilde{C}^{21} = \begin{pmatrix} 0 & 0 & -il_1 & il_+ \\ 0 & 0 & il_- & -il_1 \\ il_1 & il_- & 0 & 0 \\ il_+ & il_1 & 0 & 0 \end{pmatrix}$$

$$\text{(X.115b)}$$

$$\tilde{C}^{02} = \begin{pmatrix} 0 & 0 & -il_3 & -il^* \\ 0 & 0 & -il & il_3 \\ -il_3 & -il^* & 0 & 0 \\ -il & il_3 & 0 & 0 \end{pmatrix}, \qquad \tilde{C}^{22} = \begin{pmatrix} 0 & 0 & l_1 & -l_+ \\ 0 & 0 & l_- & -l_1 \\ -l_1 & -l_- & 0 & 0 \\ l_+ & l_1 & 0 & 0 \end{pmatrix}$$

$$\text{(X.115c)}$$

$$
\tilde{C}^{03} = \begin{pmatrix} 0 & 0 & l & -l_3 \\ 0 & 0 & l_3 & l^* \\ l & -l_3 & 0 & 0 \\ l_3 & l^* & 0 & 0 \end{pmatrix}, \qquad
\tilde{C}^{23} = \begin{pmatrix} 0 & 0 & \mathrm{i}l_- & -\mathrm{i}l_1 \\ 0 & 0 & \mathrm{i}l_1 & -\mathrm{i}l_+ \\ \mathrm{i}l_+ & \mathrm{i}l_1 & 0 & 0 \\ -\mathrm{i}l_1 & -\mathrm{i}l_- & 0 & 0 \end{pmatrix}
$$
$$\text{(X.115d)}$$

$$
\tilde{C}^{10} = \begin{pmatrix} 0 & 0 & -l_- & -\mathrm{i}l_2 \\ 0 & 0 & -\mathrm{i}l_2 & l_+ \\ l_+ & -\mathrm{i}l_2 & 0 & 0 \\ -\mathrm{i}l_2 & -l_- & 0 & 0 \end{pmatrix}, \qquad
\tilde{C}^{30} = \begin{pmatrix} 0 & 0 & -l & l_0 \\ 0 & 0 & l_0 & -l^* \\ -l & -l_0 & 0 & 0 \\ -l_0 & -l^* & 0 & 0 \end{pmatrix}
$$
$$\text{(X.115e)}$$

$$
\tilde{C}^{11} = \begin{pmatrix} 0 & 0 & \mathrm{i}l_2 & -l_+ \\ 0 & 0 & l_- & \mathrm{i}l_2 \\ -\mathrm{i}l_2 & -l_- & 0 & 0 \\ l_+ & -\mathrm{i}l_2 & 0 & 0 \end{pmatrix}, \qquad
\tilde{C}^{31} = \begin{pmatrix} 0 & 0 & -l_0 & l^* \\ 0 & 0 & l & -l_0 \\ -l_0 & -l^* & 0 & 0 \\ -l & -l_0 & 0 & 0 \end{pmatrix}
$$
$$\text{(X.115f)}$$

$$
\tilde{C}^{12} = \begin{pmatrix} 0 & 0 & -l_2 & -\mathrm{i}l_+ \\ 0 & 0 & -\mathrm{i}l_- & l_2 \\ l_2 & -\mathrm{i}l_- & 0 & 0 \\ -\mathrm{i}l_+ & -l_2 & 0 & 0 \end{pmatrix}, \qquad
\tilde{C}^{32} = \begin{pmatrix} 0 & 0 & -\mathrm{i}l_0 & \mathrm{i}l^* \\ 0 & 0 & -\mathrm{i}l & \mathrm{i}l_0 \\ -\mathrm{i}l_0 & -\mathrm{i}l^* & 0 & 0 \\ \mathrm{i}l & \mathrm{i}l_0 & 0 & 0 \end{pmatrix}
$$
$$\text{(X.115g)}$$

$$
\tilde{C}^{13} = \begin{pmatrix} 0 & 0 & l_- & \mathrm{i}l_2 \\ 0 & 0 & -\mathrm{i}l_2 & l_+ \\ l_+ & -\mathrm{i}l_2 & 0 & 0 \\ \mathrm{i}l_2 & l_- & 0 & 0 \end{pmatrix}, \qquad
\tilde{C}^{33} = \begin{pmatrix} 0 & 0 & l & -l_0 \\ 0 & 0 & l_0 & -l^* \\ -l & -l_0 & 0 & 0 \\ l_0 & l^* & 0 & 0 \end{pmatrix}
$$
$$\text{(X.115h)}$$

⟨*Implementation of bispinor currents*⟩+≡

```
pure function fggvvgr (v, psi, k) result (psikv)
  type(bispinor) :: psikv
  type(vectorspinor), intent(in) :: psi
  type(vector), intent(in) :: v, k
  complex(kind=default) :: kv30, kv21, kv01, kv31, kv02, kv32
  complex(kind=default) :: ap, am, bp, bm, bps, bms
  kv30 = k%x(3) * v%t - k%t * v%x(3)
  kv21 = (0,1) * (k%x(2) * v%x(1) - k%x(1) * v%x(2))
  kv01 = k%t * v%x(1) - k%x(1) * v%t
  kv31 = k%x(3) * v%x(1) - k%x(1) * v%x(3)
  kv02 = (0,1) * (k%t * v%x(2) - k%x(2) * v%t)
  kv32 = (0,1) * (k%x(3) * v%x(2) - k%x(2) * v%x(3))
  ap  = 2 * (kv30 + kv21)
  am  = 2 * (-kv30 + kv21)
  bp  = 2 * (kv01 + kv31 + kv02 + kv32)
  bm  = 2 * (kv01 - kv31 + kv02 - kv32)
  bps = 2 * (kv01 + kv31 - kv02 - kv32)
  bms = 2 * (kv01 - kv31 - kv02 + kv32)
  psikv%a(1) = (-ap) * psi%psi(1)%a(3) + bps * psi%psi(1)%a(4) &
          + (-bm) * psi%psi(2)%a(3) + (-ap) * psi%psi(2)%a(4) &
          + (0,1) * (bm * psi%psi(3)%a(3) + ap * psi%psi(3)%a(4)) &
          + ap * psi%psi(4)%a(3) + (-bps) * psi%psi(4)%a(4)
  psikv%a(2) =  bm * psi%psi(1)%a(3) + ap * psi%psi(1)%a(4) &
          + ap * psi%psi(2)%a(3) + (-bps) * psi%psi(2)%a(4) &
```

```
                 + (0,1) * (ap * psi%psi(3)%a(3) - bps * psi%psi(3)%a(4)) &
                 + bm * psi%psi(4)%a(3) + ap * psi%psi(4)%a(4)
       psikv%a(3) =  am * psi%psi(1)%a(1) + bms * psi%psi(1)%a(2) &
                 + bp * psi%psi(2)%a(1) + (-am) * psi%psi(2)%a(2) &
                 + (0,-1) * (bp * psi%psi(3)%a(1) + (-am) * psi%psi(3)%a(2)) &
                 + am * psi%psi(4)%a(1) + bms * psi%psi(4)%a(2)
       psikv%a(4) =  bp * psi%psi(1)%a(1) + (-am) * psi%psi(1)%a(2) &
                 + am * psi%psi(2)%a(1) + bms * psi%psi(2)%a(2) &
                 + (0,1) * (am * psi%psi(3)%a(1) + bms * psi%psi(3)%a(2)) &
                 + (-bp) * psi%psi(4)%a(1) + am * psi%psi(4)%a(2)
     end function fggvvgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vgr (g, v, psi, k) result (psikkkv)
    type(bispinor) :: psikkkv
    type(vectorspinor), intent(in) :: psi
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: g
    type(vector) :: vk
    vk = k
    psikkkv = g * (fggvvgr (v, psi, vk))
  end function f_vgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_vlrgr (gl, gr, v, psi, k) result (psikv)
    type(bispinor) :: psikv
    type(vectorspinor), intent(in) :: psi
    type(vector), intent(in) :: v
    type(momentum), intent(in) :: k
    complex(kind=default), intent(in) :: gl, gr
    type(vector) :: vk
    vk = k
    psikv = fggvvgr (v, psi, vk)
    psikv%a(1:2) = gl * psikv%a(1:2)
    psikv%a(3:4) = gr * psikv%a(3:4)
  end function f_vlrgr
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: gr_potf, gr_sf, gr_pf, gr_vf, gr_vlrf, gr_slf, gr_srf, gr_slrf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_potf (g, phi, psi) result (phipsi)
    type(vectorspinor) :: phipsi
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    phipsi%psi(1)%a(1) = (g * phi) * psi%a(3)
    phipsi%psi(1)%a(2) = (g * phi) * psi%a(4)
    phipsi%psi(1)%a(3) = (g * phi) * psi%a(1)
    phipsi%psi(1)%a(4) = (g * phi) * psi%a(2)
    phipsi%psi(2)%a(1) = (g * phi) * psi%a(4)
    phipsi%psi(2)%a(2) = (g * phi) * psi%a(3)
    phipsi%psi(2)%a(3) = ((-g) * phi) * psi%a(2)
    phipsi%psi(2)%a(4) = ((-g) * phi) * psi%a(1)
    phipsi%psi(3)%a(1) = ((0,-1) * g * phi) * psi%a(4)
```

```
      phipsi%psi(3)%a(2) = ((0,1) * g * phi) * psi%a(3)
      phipsi%psi(3)%a(3) = ((0,1) * g * phi) * psi%a(2)
      phipsi%psi(3)%a(4) = ((0,-1) * g * phi) * psi%a(1)
      phipsi%psi(4)%a(1) = (g * phi) * psi%a(3)
      phipsi%psi(4)%a(2) = ((-g) * phi) * psi%a(4)
      phipsi%psi(4)%a(3) = ((-g) * phi) * psi%a(1)
      phipsi%psi(4)%a(4) = (g * phi) * psi%a(2)
    end function gr_potf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function grkgf (psi, k) result (kpsi)
    type(vectorspinor) :: kpsi
    complex(kind=default) :: kp, km, k12, k12s
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: k
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    kpsi%psi(1)%a(1) = km * psi%a(1) - k12s * psi%a(2)
    kpsi%psi(1)%a(2) = (-k12) * psi%a(1) + kp * psi%a(2)
    kpsi%psi(1)%a(3) = kp * psi%a(3) + k12s * psi%a(4)
    kpsi%psi(1)%a(4) = k12 * psi%a(3) + km * psi%a(4)
    kpsi%psi(2)%a(1) = k12s * psi%a(1) - km * psi%a(2)
    kpsi%psi(2)%a(2) = (-kp) * psi%a(1) + k12 * psi%a(2)
    kpsi%psi(2)%a(3) = k12s * psi%a(3) + kp * psi%a(4)
    kpsi%psi(2)%a(4) = km * psi%a(3) + k12 * psi%a(4)
    kpsi%psi(3)%a(1) = (0,1) * (k12s * psi%a(1) + km * psi%a(2))
    kpsi%psi(3)%a(2) = (0,-1) * (kp * psi%a(1) + k12 * psi%a(2))
    kpsi%psi(3)%a(3) = (0,1) * (k12s * psi%a(3) - kp * psi%a(4))
    kpsi%psi(3)%a(4) = (0,1) * (km * psi%a(3) - k12 * psi%a(4))
    kpsi%psi(4)%a(1) = -(km * psi%a(1) + k12s * psi%a(2))
    kpsi%psi(4)%a(2) = k12 * psi%a(1) + kp * psi%a(2)
    kpsi%psi(4)%a(3) = kp * psi%a(3) - k12s * psi%a(4)
    kpsi%psi(4)%a(4) = k12 * psi%a(3) - km * psi%a(4)
  end function grkgf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_sf (g, phi, psi, k) result (phipsi)
    type(vectorspinor) :: phipsi
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    type(vector) :: vk
    vk = k
    phipsi = (g * phi) * grkgf (psi, vk)
  end function gr_sf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_slf (gl, phi, psi, k) result (phipsi)
    type(vectorspinor) :: phipsi
    complex(kind=default), intent(in) :: gl
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l
```

```
      type(momentum), intent(in) :: k
      psi_l%a(1:2) = psi%a(1:2)
      psi_l%a(3:4) = 0
      phipsi = gr_sf (gl, phi, psi_l, k)
    end function gr_slf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_srf (gr, phi, psi, k) result (phipsi)
    type(vectorspinor) :: phipsi
    complex(kind=default), intent(in) :: gr
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_r
    type(momentum), intent(in) :: k
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    phipsi = gr_sf (gr, phi, psi_r, k)
  end function gr_srf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_slrf (gl, gr, phi, psi, k) result (phipsi)
    type(vectorspinor) :: phipsi
    complex(kind=default), intent(in) :: gl, gr
    complex(kind=default), intent(in) :: phi
    type(bispinor), intent(in) :: psi
    type(momentum), intent(in) :: k
    phipsi = gr_slf (gl, phi, psi, k) + gr_srf (gr, phi, psi, k)
  end function gr_slrf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function grkggf (psi, k) result (kpsi)
    type(vectorspinor) :: kpsi
    complex(kind=default) :: kp, km, k12, k12s
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: k
    kp = k%t + k%x(3)
    km = k%t - k%x(3)
    k12  =  k%x(1) + (0,1)*k%x(2)
    k12s =  k%x(1) - (0,1)*k%x(2)
    kpsi%psi(1)%a(1) = (-km) * psi%a(1) + k12s * psi%a(2)
    kpsi%psi(1)%a(2) = k12 * psi%a(1) - kp * psi%a(2)
    kpsi%psi(1)%a(3) = kp * psi%a(3) + k12s * psi%a(4)
    kpsi%psi(1)%a(4) = k12 * psi%a(3) + km * psi%a(4)
    kpsi%psi(2)%a(1) = (-k12s) * psi%a(1) + km * psi%a(2)
    kpsi%psi(2)%a(2) = kp * psi%a(1) - k12 * psi%a(2)
    kpsi%psi(2)%a(3) = k12s * psi%a(3) + kp * psi%a(4)
    kpsi%psi(2)%a(4) = km * psi%a(3) + k12 * psi%a(4)
    kpsi%psi(3)%a(1) = (0,-1) * (k12s * psi%a(1) + km * psi%a(2))
    kpsi%psi(3)%a(2) = (0,1) * (kp * psi%a(1) + k12 * psi%a(2))
    kpsi%psi(3)%a(3) = (0,1) * (k12s * psi%a(3) - kp * psi%a(4))
    kpsi%psi(3)%a(4) = (0,1) * (km * psi%a(3) - k12 * psi%a(4))
    kpsi%psi(4)%a(1) = km * psi%a(1) + k12s * psi%a(2)
    kpsi%psi(4)%a(2) = -(k12 * psi%a(1) + kp * psi%a(2))
    kpsi%psi(4)%a(3) = kp * psi%a(3) - k12s * psi%a(4)
    kpsi%psi(4)%a(4) = k12 * psi%a(3) - km * psi%a(4)
  end function grkggf
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function gr_pf (g, phi, psi, k) result (phipsi)
  type(vectorspinor) :: phipsi
  complex(kind=default), intent(in) :: g
  complex(kind=default), intent(in) :: phi
  type(bispinor), intent(in) :: psi
  type(momentum), intent(in) :: k
  type(vector) :: vk
  vk = k
  phipsi = (g * phi) * grkggf (psi, vk)
end function gr_pf
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function grkkggf (v, psi, k) result (psikv)
  type(vectorspinor) :: psikv
  type(bispinor), intent(in) :: psi
  type(vector), intent(in) :: v, k
  complex(kind=default) :: kv30, kv21, kv01, kv31, kv02, kv32
  complex(kind=default) :: ap, am, bp, bm, bps, bms, imago
  imago = (0.0_default,1.0_default)
  kv30 = k%x(3) * v%t - k%t * v%x(3)
  kv21 = imago * (k%x(2) * v%x(1) - k%x(1) * v%x(2))
  kv01 = k%t * v%x(1) - k%x(1) * v%t
  kv31 = k%x(3) * v%x(1) - k%x(1) * v%x(3)
  kv02 = imago * (k%t * v%x(2) - k%x(2) * v%t)
  kv32 = imago * (k%x(3) * v%x(2) - k%x(2) * v%x(3))
  ap  = 2 * (kv30 + kv21)
  am  = 2 * ((-kv30) + kv21)
  bp  = 2 * (kv01 + kv31 + kv02 + kv32)
  bm  = 2 * (kv01 - kv31 + kv02 - kv32)
  bps = 2 * (kv01 + kv31 - kv02 - kv32)
  bms = 2 * (kv01 - kv31 - kv02 + kv32)
  psikv%psi(1)%a(1) = am * psi%a(3) + bms * psi%a(4)
  psikv%psi(1)%a(2) = bp * psi%a(3) + (-am) * psi%a(4)
  psikv%psi(1)%a(3) = (-ap) * psi%a(1) + bps * psi%a(2)
  psikv%psi(1)%a(4) = bm * psi%a(1) + ap * psi%a(2)
  psikv%psi(2)%a(1) = bms * psi%a(3) + am * psi%a(4)
  psikv%psi(2)%a(2) = (-am) * psi%a(3) + bp * psi%a(4)
  psikv%psi(2)%a(3) = (-bps) * psi%a(1) + ap * psi%a(2)
  psikv%psi(2)%a(4) = (-ap) * psi%a(1) + (-bm) * psi%a(2)
  psikv%psi(3)%a(1) = imago * (bms * psi%a(3) - am * psi%a(4))
  psikv%psi(3)%a(2) = (-imago) * (am * psi%a(3) + bp * psi%a(4))
  psikv%psi(3)%a(3) = (-imago) * (bps * psi%a(1) + ap * psi%a(2))
  psikv%psi(3)%a(4) = imago * ((-ap) * psi%a(1) + bm * psi%a(2))
  psikv%psi(4)%a(1) = am * psi%a(3) + (-bms) * psi%a(4)
  psikv%psi(4)%a(2) = bp * psi%a(3) + am * psi%a(4)
  psikv%psi(4)%a(3) = ap * psi%a(1) + bps * psi%a(2)
  psikv%psi(4)%a(4) = (-bm) * psi%a(1) + ap * psi%a(2)
end function grkkggf
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function gr_vf (g, v, psi, k) result (psikv)
  type(vectorspinor) :: psikv
  type(bispinor), intent(in) :: psi
  type(vector), intent(in) :: v
```

```
      type(momentum), intent(in) :: k
      complex(kind=default), intent(in) :: g
      type(vector) :: vk
      vk = k
      psikv = g * (grkkggf (v, psi, vk))
   end function gr_vf
```

⟨*Implementation of bispinor currents*⟩+≡
```
   pure function gr_vlrf (gl, gr, v, psi, k) result (psikv)
      type(vectorspinor) :: psikv
      type(bispinor), intent(in) :: psi
      type(bispinor) :: psi_l, psi_r
      type(vector), intent(in) :: v
      type(momentum), intent(in) :: k
      complex(kind=default), intent(in) :: gl, gr
      type(vector) :: vk
      vk = k
      psi_l%a(1:2) = psi%a(1:2)
      psi_l%a(3:4) = 0
      psi_r%a(1:2) = 0
      psi_r%a(3:4) = psi%a(3:4)
      psikv = gl * grkkggf (v, psi_l, vk) + gr * grkkggf (v, psi_r, vk)
   end function gr_vlrf
```

⟨*Declaration of bispinor currents*⟩+≡
```
   public :: v_grf, v_fgr
```

⟨*Declaration of bispinor currents*⟩+≡
```
   public :: vlr_grf, vlr_fgr
```

$V^\mu = \psi_\rho^T C^{\mu\rho} \psi$

⟨*Implementation of bispinor currents*⟩+≡
```
   pure function grkgggf (psil, psir, k) result (j)
      type(vector) :: j
      type(vectorspinor), intent(in) :: psil
      type(bispinor), intent(in) :: psir
      type(vector), intent(in) :: k
      type(vectorspinor) :: c_psir0, c_psir1, c_psir2, c_psir3
      complex(kind=default) :: kp, km, k12, k12s, ik2
      kp = k%t + k%x(3)
      km = k%t - k%x(3)
      k12  =  (k%x(1) + (0,1)*k%x(2))
      k12s =  (k%x(1) - (0,1)*k%x(2))
      ik2 = (0,1) * k%x(2)
      !!! New version:
      c_psir0%psi(1)%a(1) = (-k%x(3)) * psir%a(3) + (-k12s) * psir%a(4)
      c_psir0%psi(1)%a(2) = (-k12) * psir%a(3) + k%x(3) * psir%a(4)
      c_psir0%psi(1)%a(3) = (-k%x(3)) * psir%a(1) + (-k12s) * psir%a(2)
      c_psir0%psi(1)%a(4) = (-k12) * psir%a(1) + k%x(3) * psir%a(2)
      c_psir0%psi(2)%a(1) = (-k12s) * psir%a(3) + (-k%x(3)) * psir%a(4)
      c_psir0%psi(2)%a(2) = k%x(3) * psir%a(3) + (-k12) * psir%a(4)
      c_psir0%psi(2)%a(3) = k12s * psir%a(1) + k%x(3) * psir%a(2)
      c_psir0%psi(2)%a(4) = (-k%x(3)) * psir%a(1) + k12 * psir%a(2)
      c_psir0%psi(3)%a(1) = (0,1) * ((-k12s) * psir%a(3) + k%x(3) * psir%a(4))
      c_psir0%psi(3)%a(2) = (0,1) * (k%x(3) * psir%a(3) + k12 * psir%a(4))
      c_psir0%psi(3)%a(3) = (0,1) * (k12s * psir%a(1) + (-k%x(3)) * psir%a(2))
```

```
c_psir0%psi(3)%a(4) = (0,1) * ((-k%x(3)) * psir%a(1) + (-k12) * psir%a(2))
c_psir0%psi(4)%a(1) = (-k%x(3)) * psir%a(3) + k12s * psir%a(4)
c_psir0%psi(4)%a(2) = (-k12) * psir%a(3) + (-k%x(3)) * psir%a(4)
c_psir0%psi(4)%a(3) = k%x(3) * psir%a(1) + (-k12s) * psir%a(2)
c_psir0%psi(4)%a(4) = k12 * psir%a(1) + k%x(3) * psir%a(2)
!!!
c_psir1%psi(1)%a(1) = (-ik2) * psir%a(3) + (-km) * psir%a(4)
c_psir1%psi(1)%a(2) = (-kp) * psir%a(3) + ik2 * psir%a(4)
c_psir1%psi(1)%a(3) = ik2 * psir%a(1) + (-kp) * psir%a(2)
c_psir1%psi(1)%a(4) = (-km) * psir%a(1) + (-ik2) * psir%a(2)
c_psir1%psi(2)%a(1) = (-km) * psir%a(3) + (-ik2) * psir%a(4)
c_psir1%psi(2)%a(2) = ik2 * psir%a(3) + (-kp) * psir%a(4)
c_psir1%psi(2)%a(3) = kp * psir%a(1) + (-ik2) * psir%a(2)
c_psir1%psi(2)%a(4) = ik2 * psir%a(1) + km * psir%a(2)
c_psir1%psi(3)%a(1) = ((0,-1) * km) * psir%a(3) + (-k%x(2)) * psir%a(4)
c_psir1%psi(3)%a(2) = (-k%x(2)) * psir%a(3) + ((0,1) * kp) * psir%a(4)
c_psir1%psi(3)%a(3) = ((0,1) * kp) * psir%a(1) + (-k%x(2)) * psir%a(2)
c_psir1%psi(3)%a(4) = (-k%x(2)) * psir%a(1) + ((0,-1) * km) * psir%a(2)
c_psir1%psi(4)%a(1) = (-ik2) * psir%a(3) + km * psir%a(4)
c_psir1%psi(4)%a(2) = (-kp) * psir%a(3) + (-ik2) * psir%a(4)
c_psir1%psi(4)%a(3) = (-ik2) *  psir%a(1) + (-kp) * psir%a(2)
c_psir1%psi(4)%a(4) = km * psir%a(1) + (-ik2) * psir%a(2)
!!!
c_psir2%psi(1)%a(1) = (0,1) * (k%x(1) * psir%a(3) + km * psir%a(4))
c_psir2%psi(1)%a(2) = (0,-1) * (kp * psir%a(3) + k%x(1) * psir%a(4))
c_psir2%psi(1)%a(3) = (0,1) * ((-k%x(1)) * psir%a(1) + kp * psir%a(2))
c_psir2%psi(1)%a(4) = (0,1) * ((-km) * psir%a(1) + k%x(1) * psir%a(2))
c_psir2%psi(2)%a(1) = (0,1) * (km * psir%a(3) + k%x(1) * psir%a(4))
c_psir2%psi(2)%a(2) = (0,-1) * (k%x(1) * psir%a(3) + kp * psir%a(4))
c_psir2%psi(2)%a(3) = (0,-1) * (kp * psir%a(1) + (-k%x(1)) * psir%a(2))
c_psir2%psi(2)%a(4) = (0,-1) * (k%x(1) * psir%a(1) + (-km) * psir%a(2))
c_psir2%psi(3)%a(1) = (-km) * psir%a(3) + k%x(1) * psir%a(4)
c_psir2%psi(3)%a(2) = k%x(1) * psir%a(3) + (-kp) * psir%a(4)
c_psir2%psi(3)%a(3) = kp * psir%a(1) + k%x(1) * psir%a(2)
c_psir2%psi(3)%a(4) = k%x(1) * psir%a(1) + km * psir%a(2)
c_psir2%psi(4)%a(1) = (0,1) * (k%x(1) * psir%a(3) + (-km) * psir%a(4))
c_psir2%psi(4)%a(2) = (0,1) * ((-kp) * psir%a(3) + k%x(1) * psir%a(4))
c_psir2%psi(4)%a(3) = (0,1) * (k%x(1) * psir%a(1) + kp * psir%a(2))
c_psir2%psi(4)%a(4) = (0,1) * (km * psir%a(1) + k%x(1) * psir%a(2))
!!!
c_psir3%psi(1)%a(1) = (-k%t) * psir%a(3) - k12s * psir%a(4)
c_psir3%psi(1)%a(2) = k12 * psir%a(3) + k%t * psir%a(4)
c_psir3%psi(1)%a(3) = (-k%t) * psir%a(1) + k12s * psir%a(2)
c_psir3%psi(1)%a(4) = (-k12) * psir%a(1) + k%t * psir%a(2)
c_psir3%psi(2)%a(1) = (-k12s) * psir%a(3) + (-k%t) * psir%a(4)
c_psir3%psi(2)%a(2) = k%t * psir%a(3) + k12 * psir%a(4)
c_psir3%psi(2)%a(3) = (-k12s) * psir%a(1) + k%t * psir%a(2)
c_psir3%psi(2)%a(4) = (-k%t) * psir%a(1) + k12 * psir%a(2)
c_psir3%psi(3)%a(1) = (0,-1) * (k12s * psir%a(3) + (-k%t) * psir%a(4))
c_psir3%psi(3)%a(2) = (0,1) * (k%t * psir%a(3) + (-k12) * psir%a(4))
c_psir3%psi(3)%a(3) = (0,-1) * (k12s * psir%a(1) + k%t * psir%a(2))
c_psir3%psi(3)%a(4) = (0,-1) * (k%t * psir%a(1) + k12 * psir%a(2))
c_psir3%psi(4)%a(1) = (-k%t) * psir%a(3) + k12s * psir%a(4)
c_psir3%psi(4)%a(2) = k12 * psir%a(3) + (-k%t) * psir%a(4)
```

```
      c_psir3%psi(4)%a(3) = k%t * psir%a(1) + k12s * psir%a(2)
      c_psir3%psi(4)%a(4) = k12 * psir%a(1) + k%t * psir%a(2)
      j%t   =   2 * (psil * c_psir0)
      j%x(1) =   2 * (psil * c_psir1)
      j%x(2) =   2 * (psil * c_psir2)
      j%x(3) =   2 * (psil * c_psir3)
   end function grkgggf
```

⟨*Implementation of bispinor currents*⟩+≡
```
   pure function v_grf (g, psil, psir, k) result (j)
      type(vector) :: j
      complex(kind=default), intent(in) :: g
      type(vectorspinor), intent(in) :: psil
      type(bispinor), intent(in) :: psir
      type(momentum), intent(in) :: k
      type(vector) :: vk
      vk = k
      j = g * grkgggf (psil, psir, vk)
   end function v_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
   pure function vlr_grf (gl, gr, psil, psir, k) result (j)
      type(vector) :: j
      complex(kind=default), intent(in) :: gl, gr
      type(vectorspinor), intent(in) :: psil
      type(bispinor), intent(in) :: psir
      type(bispinor) :: psir_l, psir_r
      type(momentum), intent(in) :: k
      type(vector) :: vk
      vk = k
      psir_l%a(1:2) = psir%a(1:2)
      psir_l%a(3:4) = 0
      psir_r%a(1:2) = 0
      psir_r%a(3:4) = psir%a(3:4)
      j = gl * grkgggf (psil, psir_l, vk) + gr * grkgggf (psil, psir_r, vk)
   end function vlr_grf
```

$V^\mu = \psi^T \tilde{C}^{\mu\rho} \psi_\rho$; remember the reversed index order in $\tilde{C}$.

⟨*Implementation of bispinor currents*⟩+≡
```
   pure function fggkggr (psil, psir, k) result (j)
      type(vector) :: j
      type(vectorspinor), intent(in) :: psir
      type(bispinor), intent(in) :: psil
      type(vector), intent(in) :: k
      type(bispinor) :: c_psir0, c_psir1, c_psir2, c_psir3
      complex(kind=default) :: kp, km, k12, k12s, ik1, ik2
      kp = k%t + k%x(3)
      km = k%t - k%x(3)
      k12  =  k%x(1) + (0,1)*k%x(2)
      k12s =  k%x(1) - (0,1)*k%x(2)
      ik1 = (0,1) * k%x(1)
      ik2 = (0,1) * k%x(2)
      c_psir0%a(1) = k%x(3) * (psir%psi(1)%a(4) + psir%psi(4)%a(4) &
                   + psir%psi(2)%a(3) + (0,1) * psir%psi(3)%a(3)) &
                   - k12 * (psir%psi(1)%a(3) + psir%psi(4)%a(3)) &
                   + k12s * (psir%psi(2)%a(4) + (0,1) * psir%psi(3)%a(4))
```

```
c_psir0%a(2) = k%x(3) * (psir%psi(1)%a(3) - psir%psi(4)%a(3) + &
               psir%psi(2)%a(4) - (0,1) * psir%psi(3)%a(4)) + &
               k12s * (psir%psi(1)%a(4) - psir%psi(4)%a(4)) - &
               k12 * (psir%psi(2)%a(3) - (0,1) * psir%psi(3)%a(3))
c_psir0%a(3) = k%x(3) * (-psir%psi(1)%a(2) + psir%psi(4)%a(2) + &
               psir%psi(2)%a(1) + (0,1) * psir%psi(3)%a(1)) + &
               k12 * (psir%psi(1)%a(1) - psir%psi(4)%a(1)) + &
               k12s * (psir%psi(2)%a(2) + (0,1) * psir%psi(3)%a(2))
c_psir0%a(4) = k%x(3) * (-psir%psi(1)%a(1) - psir%psi(4)%a(1) + &
               psir%psi(2)%a(2) - (0,1) * psir%psi(3)%a(2)) - &
               k12s * (psir%psi(1)%a(2) + psir%psi(4)%a(2)) - &
               k12 * (psir%psi(2)%a(1) - (0,1) * psir%psi(3)%a(1))
!!!
c_psir1%a(1) = ik2 * (-psir%psi(1)%a(4) - psir%psi(4)%a(4) - &
               psir%psi(2)%a(3) - (0,1) * psir%psi(3)%a(3)) - &
               km * (psir%psi(1)%a(3) + psir%psi(4)%a(3)) + &
               kp * (psir%psi(2)%a(4) + (0,1) * psir%psi(3)%a(4))
c_psir1%a(2) = ik2 * (-psir%psi(1)%a(3) - psir%psi(2)%a(4) + &
               psir%psi(4)%a(3) + (0,1) * psir%psi(3)%a(4)) + &
               kp * (psir%psi(1)%a(4) - psir%psi(4)%a(4)) - &
               km * (psir%psi(2)%a(3) - (0,1) * psir%psi(3)%a(3))
c_psir1%a(3) = ik2 * (-psir%psi(1)%a(2) + psir%psi(2)%a(1) + &
               psir%psi(4)%a(2) + (0,1) * psir%psi(3)%a(1)) + &
               kp * (psir%psi(1)%a(1) - psir%psi(4)%a(1)) + &
               km * (psir%psi(2)%a(2) + (0,1) * psir%psi(3)%a(2))
c_psir1%a(4) = ik2 * (-psir%psi(1)%a(1) + psir%psi(2)%a(2) - &
               psir%psi(4)%a(1) - (0,1) * psir%psi(3)%a(2)) - &
               km * (psir%psi(1)%a(2) + psir%psi(4)%a(2)) - &
               kp * (psir%psi(2)%a(1) - (0,1) * psir%psi(3)%a(1))
!!!
c_psir2%a(1) = ik1 * (psir%psi(2)%a(3) + psir%psi(1)%a(4) &
               + psir%psi(4)%a(4) + (0,1) * psir%psi(3)%a(3)) - &
               ((0,1)*km) * (psir%psi(1)%a(3) + psir%psi(4)%a(3)) &
               + kp * (psir%psi(3)%a(4) - (0,1) * psir%psi(2)%a(4))
c_psir2%a(2) = ik1 * (psir%psi(1)%a(3) + psir%psi(2)%a(4) - &
               psir%psi(4)%a(3) - (0,1) * psir%psi(3)%a(4)) - &
               ((0,1)*kp) * (psir%psi(1)%a(4) - psir%psi(4)%a(4)) &
               - km * (psir%psi(3)%a(3) + (0,1) * psir%psi(2)%a(3))
c_psir2%a(3) = ik1 * (psir%psi(1)%a(2) - psir%psi(2)%a(1) - &
               psir%psi(4)%a(2) - (0,1) * psir%psi(3)%a(1)) + &
               ((0,1)*kp) * (psir%psi(1)%a(1) - psir%psi(4)%a(1)) &
               + km * (psir%psi(3)%a(2) - (0,1) * psir%psi(2)%a(2))
c_psir2%a(4) = ik1 * (psir%psi(1)%a(1) - psir%psi(2)%a(2) + &
               psir%psi(4)%a(1) + (0,1) * psir%psi(3)%a(2)) + &
               ((0,1)*km) * (psir%psi(1)%a(2) + psir%psi(4)%a(2)) - &
               kp * (psir%psi(3)%a(1) + (0,1) * psir%psi(2)%a(1))
!!!
c_psir3%a(1) = k%t * (psir%psi(1)%a(4) + psir%psi(4)%a(4) + &
               psir%psi(2)%a(3) + (0,1) * psir%psi(3)%a(3)) - &
               k12 * (psir%psi(1)%a(3) + psir%psi(4)%a(3)) - &
               k12s * (psir%psi(2)%a(4) + (0,1) * psir%psi(3)%a(4))
c_psir3%a(2) = k%t * (psir%psi(1)%a(3) - psir%psi(4)%a(3) + &
               psir%psi(2)%a(4) - (0,1) * psir%psi(3)%a(4)) - &
               k12s * (psir%psi(1)%a(4) - psir%psi(4)%a(4)) - &
```

```
                k12 * (psir%psi(2)%a(3) - (0,1) * psir%psi(3)%a(3))
    c_psir3%a(3) = k%t * (-psir%psi(1)%a(2) + psir%psi(2)%a(1) + &
                psir%psi(4)%a(2) + (0,1) * psir%psi(3)%a(1)) - &
                k12 * (psir%psi(1)%a(1) - psir%psi(4)%a(1)) + &
                k12s * (psir%psi(2)%a(2) + (0,1) * psir%psi(3)%a(2))
    c_psir3%a(4) = k%t * (-psir%psi(1)%a(1) + psir%psi(2)%a(2) - &
                psir%psi(4)%a(1) - (0,1) * psir%psi(3)%a(2)) - &
                k12s * (psir%psi(1)%a(2) + psir%psi(4)%a(2)) + &
                k12 * (psir%psi(2)%a(1) - (0,1) * psir%psi(3)%a(1))
    !!! Because we explicitly multiplied the charge conjugation matrix
    !!! we have to omit it from the spinor product and take the
    !!! ordinary product!
    j%t    =   2 * dot_product (conjg (psil%a), c_psir0%a)
    j%x(1) =   2 * dot_product (conjg (psil%a), c_psir1%a)
    j%x(2) =   2 * dot_product (conjg (psil%a), c_psir2%a)
    j%x(3) =   2 * dot_product (conjg (psil%a), c_psir3%a)
  end function fggkggr
```

⟨*Implementation of bispinor currents*⟩+≡

```
  pure function v_fgr (g, psil, psir, k) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: g
    type(vectorspinor), intent(in) :: psir
    type(bispinor), intent(in) :: psil
    type(momentum), intent(in) :: k
    type(vector) :: vk
    vk = k
    j = g * fggkggr (psil, psir, vk)
  end function v_fgr
```

⟨*Implementation of bispinor currents*⟩+≡

```
  pure function vlr_fgr (gl, gr, psil, psir, k) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(vectorspinor), intent(in) :: psir
    type(bispinor), intent(in) :: psil
    type(bispinor) :: psil_l
    type(bispinor) :: psil_r
    type(momentum), intent(in) :: k
    type(vector) :: vk
    vk = k
    psil_l%a(1:2) = psil%a(1:2)
    psil_l%a(3:4) = 0
    psil_r%a(1:2) = 0
    psil_r%a(3:4) = psil%a(3:4)
    j = gl * fggkggr (psil_l, psir, vk) + gr * fggkggr (psil_r, psir, vk)
  end function vlr_fgr
```

X.23.5 Gravitino 4-Couplings

⟨*Declaration of bispinor currents*⟩+≡

```
  public :: f_s2gr, f_svgr, f_slvgr, f_srvgr, f_slrvgr, f_pvgr, f_v2gr, f_v2lrgr
```

⟨*Implementation of bispinor currents*⟩+≡

```
  pure function f_s2gr (g, phi1, phi2, psi) result (phipsi)
```

```
    type(bispinor) :: phipsi
    type(vectorspinor), intent(in) :: psi
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi1, phi2
    phipsi = phi2 * f_potgr (g, phi1, psi)
  end function f_s2gr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_svgr (g, phi, v, grav) result (phigrav)
    type(bispinor) :: phigrav
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: g, phi
    phigrav = (g * phi) * fgvg5gr (grav, v)
  end function f_svgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_slvgr (gl, phi, v, grav) result (phigrav)
    type(bispinor) :: phigrav, phidum
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: gl, phi
    phidum = (gl * phi) * fgvg5gr (grav, v)
    phigrav%a(1:2) = phidum%a(1:2)
    phigrav%a(3:4) = 0
  end function f_slvgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_srvgr (gr, phi, v, grav) result (phigrav)
    type(bispinor) :: phigrav, phidum
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: gr, phi
    phidum = (gr * phi) * fgvg5gr (grav, v)
    phigrav%a(1:2) = 0
    phigrav%a(3:4) = phidum%a(3:4)
  end function f_srvgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_slrvgr (gl, gr, phi, v, grav) result (phigrav)
    type(bispinor) :: phigrav
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: gl, gr, phi
    phigrav = f_slvgr (gl, phi, v, grav) + f_srvgr (gr, phi, v, grav)
  end function f_slrvgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_pvgr (g, phi, v, grav) result (phigrav)
    type(bispinor) :: phigrav
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: g, phi
    phigrav = (g * phi) * fgvgr (grav, v)
  end function f_pvgr
```

⟨*Implementation of bispinor currents*⟩+≡

```
  pure function f_v2gr (g, v1, v2, grav) result (psi)
    type(bispinor) :: psi
    complex(kind=default), intent(in) :: g
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v1, v2
    psi = g * fggvvgr (v2, grav, v1)
  end function f_v2gr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function f_v2lrgr (gl, gr, v1, v2, grav) result (psi)
    type(bispinor) :: psi
    complex(kind=default), intent(in) :: gl, gr
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v1, v2
    psi = fggvvgr (v2, grav, v1)
    psi%a(1:2) = gl * psi%a(1:2)
    psi%a(3:4) = gr * psi%a(3:4)
  end function f_v2lrgr
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: gr_s2f, gr_svf, gr_pvf, gr_slvf, gr_srvf, gr_slrvf, gr_v2f, gr_v2lrf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_s2f (g, phi1, phi2, psi) result (phipsi)
    type(vectorspinor) :: phipsi
    type(bispinor), intent(in) :: psi
    complex(kind=default), intent(in) :: g
    complex(kind=default), intent(in) :: phi1, phi2
    phipsi = phi2 * gr_potf (g, phi1, psi)
  end function gr_s2f
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_svf (g, phi, v, psi) result (phipsi)
    type(vectorspinor) :: phipsi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: g, phi
    phipsi = (g * phi) * grkggf (psi, v)
  end function gr_svf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_slvf (gl, phi, v, psi) result (phipsi)
    type(vectorspinor) :: phipsi
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: gl, phi
    psi_l%a(1:2) = psi%a(1:2)
    psi_l%a(3:4) = 0
    phipsi = (gl * phi) * grkggf (psi_l, v)
  end function gr_slvf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_srvf (gr, phi, v, psi) result (phipsi)
    type(vectorspinor) :: phipsi
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_r
```

```
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: gr, phi
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    phipsi = (gr * phi) * grkggf (psi_r, v)
  end function gr_srvf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_slrvf (gl, gr, phi, v, psi) result (phipsi)
    type(vectorspinor) :: phipsi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: gl, gr, phi
    phipsi = gr_slvf (gl, phi, v, psi) + gr_srvf (gr, phi, v, psi)
  end function gr_slrvf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_pvf (g, phi, v, psi) result (phipsi)
    type(vectorspinor) :: phipsi
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    complex(kind=default), intent(in) :: g, phi
    phipsi = (g * phi) * grkgf (psi, v)
  end function gr_pvf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_v2f (g, v1, v2, psi) result (vvpsi)
    type(vectorspinor) :: vvpsi
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v1, v2
    vvpsi = g * grkkggf (v2, psi, v1)
  end function gr_v2f
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function gr_v2lrf (gl, gr, v1, v2, psi) result (vvpsi)
    type(vectorspinor) :: vvpsi
    complex(kind=default), intent(in) :: gl, gr
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l, psi_r
    type(vector), intent(in) :: v1, v2
    psi_l%a(1:2) = psi%a(1:2)
    psi_l%a(3:4) = 0
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    vvpsi = gl * grkkggf (v2, psi_l, v1) + gr * grkkggf (v2, psi_r, v1)
  end function gr_v2lrf
```

⟨*Declaration of bispinor currents*⟩+≡
```
  public :: s2_grf, s2_fgr, sv1_grf, sv2_grf, sv1_fgr, sv2_fgr, &
            slv1_grf, slv2_grf, slv1_fgr, slv2_fgr, &
            srv1_grf, srv2_grf, srv1_fgr, srv2_fgr, &
            slrv1_grf, slrv2_grf, slrv1_fgr, slrv2_fgr, &
            pv1_grf, pv2_grf, pv1_fgr, pv2_fgr, v2_grf, v2_fgr, &
            v2lr_grf, v2lr_fgr
```

⟨*Implementation of bispinor currents*⟩+≡

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```
    pure function s2_grf (g, gravbar, phi, psi) result (j)
      complex(kind=default) :: j
      complex(kind=default), intent(in) :: g, phi
      type(vectorspinor), intent(in) :: gravbar
      type(bispinor), intent(in) :: psi
      j = phi * pot_grf (g, gravbar, psi)
    end function s2_grf
```

⟨_Implementation of bispinor currents_⟩+≡
```
    pure function s2_fgr (g, psibar, phi, grav) result (j)
      complex(kind=default) :: j
      complex(kind=default), intent(in) :: g, phi
      type(bispinor), intent(in) :: psibar
      type(vectorspinor), intent(in) :: grav
      j = phi * pot_fgr (g, psibar, grav)
    end function s2_fgr
```

⟨_Implementation of bispinor currents_⟩+≡
```
    pure function sv1_grf (g, gravbar, v, psi) result (j)
      complex(kind=default) :: j
      complex(kind=default), intent(in) :: g
      type(vectorspinor), intent(in) :: gravbar
      type(bispinor), intent(in) :: psi
      type(vector), intent(in) :: v
      j = g * grg5vgf (gravbar, psi, v)
    end function sv1_grf
```

⟨_Implementation of bispinor currents_⟩+≡
```
    pure function slv1_grf (gl, gravbar, v, psi) result (j)
      complex(kind=default) :: j
      complex(kind=default), intent(in) :: gl
      type(vectorspinor), intent(in) :: gravbar
      type(bispinor), intent(in) :: psi
      type(bispinor) :: psi_l
      type(vector), intent(in) :: v
      psi_l%a(1:2) = psi%a(1:2)
      psi_l%a(3:4) = 0
      j = gl * grg5vgf (gravbar, psi_l, v)
    end function slv1_grf
```

⟨_Implementation of bispinor currents_⟩+≡
```
    pure function srv1_grf (gr, gravbar, v, psi) result (j)
      complex(kind=default) :: j
      complex(kind=default), intent(in) :: gr
      type(vectorspinor), intent(in) :: gravbar
      type(bispinor), intent(in) :: psi
      type(bispinor) :: psi_r
      type(vector), intent(in) :: v
      psi_r%a(1:2) = 0
      psi_r%a(3:4) = psi%a(3:4)
      j = gr * grg5vgf (gravbar, psi_r, v)
    end function srv1_grf
```

⟨_Implementation of bispinor currents_⟩+≡
```
    pure function slrv1_grf (gl, gr, gravbar, v, psi) result (j)
      complex(kind=default) :: j
      complex(kind=default), intent(in) :: gl, gr
```

```
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l, psi_r
    type(vector), intent(in) :: v
    psi_l%a(1:2) = psi%a(1:2)
    psi_l%a(3:4) = 0
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    j = gl * grg5vgf (gravbar, psi_l, v) + gr * grg5vgf (gravbar, psi_r, v)
  end function slrv1_grf
```

$$C\gamma^0\gamma^0 = -C\gamma^1\gamma^1 = -C\gamma^2\gamma^2 = C\gamma^3\gamma^3 = C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{(X.116a)}$$

$$C\gamma^0\gamma^1 = -C\gamma^1\gamma^0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{(X.116b)}$$

$$C\gamma^0\gamma^2 = -C\gamma^2\gamma^0 = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} \quad \text{(X.116c)}$$

$$C\gamma^0\gamma^3 = -C\gamma^3\gamma^0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{(X.116d)}$$

$$C\gamma^1\gamma^2 = -C\gamma^2\gamma^1 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix} \quad \text{(X.116e)}$$

$$C\gamma^1\gamma^3 = -C\gamma^3\gamma^1 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{(X.116f)}$$

$$C\gamma^2\gamma^3 = -C\gamma^3\gamma^2 = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} \quad \text{(X.116g)}$$

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sv2_grf (g, gravbar, phi, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: g, phi
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(vectorspinor) :: g0_psi, g1_psi, g2_psi, g3_psi
    g0_psi%psi(1)%a(1:2) = - psi%a(1:2)
    g0_psi%psi(1)%a(3:4) = psi%a(3:4)
```

```
    g0_psi%psi(2)%a(1) = psi%a(2)
    g0_psi%psi(2)%a(2) = psi%a(1)
    g0_psi%psi(2)%a(3) = psi%a(4)
    g0_psi%psi(2)%a(4) = psi%a(3)
    g0_psi%psi(3)%a(1) = (0,-1) * psi%a(2)
    g0_psi%psi(3)%a(2) = (0,1) * psi%a(1)
    g0_psi%psi(3)%a(3) = (0,-1) * psi%a(4)
    g0_psi%psi(3)%a(4) = (0,1) * psi%a(3)
    g0_psi%psi(4)%a(1) = psi%a(1)
    g0_psi%psi(4)%a(2) = - psi%a(2)
    g0_psi%psi(4)%a(3) = psi%a(3)
    g0_psi%psi(4)%a(4) = - psi%a(4)
    g1_psi%psi(1)%a(1:4) = - g0_psi%psi(2)%a(1:4)
    g1_psi%psi(2)%a(1:4) = - g0_psi%psi(1)%a(1:4)
    g1_psi%psi(3)%a(1) = (0,1) * psi%a(1)
    g1_psi%psi(3)%a(2) = (0,-1) * psi%a(2)
    g1_psi%psi(3)%a(3) = (0,-1) * psi%a(3)
    g1_psi%psi(3)%a(4) = (0,1) * psi%a(4)
    g1_psi%psi(4)%a(1) = - psi%a(2)
    g1_psi%psi(4)%a(2) = psi%a(1)
    g1_psi%psi(4)%a(3) = psi%a(4)
    g1_psi%psi(4)%a(4) = - psi%a(3)
    g2_psi%psi(1)%a(1:4) = - g0_psi%psi(3)%a(1:4)
    g2_psi%psi(2)%a(1:4) = - g1_psi%psi(3)%a(1:4)
    g2_psi%psi(3)%a(1:4) = - g0_psi%psi(1)%a(1:4)
    g2_psi%psi(4)%a(1) = (0,1) * psi%a(2)
    g2_psi%psi(4)%a(2) = (0,1) * psi%a(1)
    g2_psi%psi(4)%a(3) = (0,-1) * psi%a(4)
    g2_psi%psi(4)%a(4) = (0,-1) * psi%a(3)
    g3_psi%psi(1)%a(1:4) = - g0_psi%psi(4)%a(1:4)
    g3_psi%psi(2)%a(1:4) = - g1_psi%psi(4)%a(1:4)
    g3_psi%psi(3)%a(1:4) = - g2_psi%psi(4)%a(1:4)
    g3_psi%psi(4)%a(1:4) = - g0_psi%psi(1)%a(1:4)
    j%t    =   (g * phi) * (gravbar * g0_psi)
    j%x(1) =   (g * phi) * (gravbar * g1_psi)
    j%x(2) =   (g * phi) * (gravbar * g2_psi)
    j%x(3) =   (g * phi) * (gravbar * g3_psi)
  end function sv2_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slv2_grf (gl, gravbar, phi, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, phi
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l
    psi_l%a(1:2) = psi%a(1:2)
    psi_l%a(3:4) = 0
    j = sv2_grf (gl, gravbar, phi, psi_l)
  end function slv2_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function srv2_grf (gr, gravbar, phi, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gr, phi
```

```
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_r
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    j = sv2_grf (gr, gravbar, phi, psi_r)
  end function srv2_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slrv2_grf (gl, gr, gravbar, phi, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, gr, phi
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l, psi_r
    psi_l%a(1:2) = psi%a(1:2)
    psi_l%a(3:4) = 0
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    j = sv2_grf (gl, gravbar, phi, psi_l) + sv2_grf (gr, gravbar, phi, psi_r)
  end function slrv2_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function sv1_fgr (g, psibar, v, grav) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    j = g * fg5gkgr (psibar, grav, v)
  end function sv1_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slv1_fgr (gl, psibar, v, grav) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gl
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_l
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    psibar_l%a(1:2) = psibar%a(1:2)
    psibar_l%a(3:4) = 0
    j = gl * fg5gkgr (psibar_l, grav, v)
  end function slv1_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function srv1_fgr (gr, psibar, v, grav) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: gr
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_r
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    psibar_r%a(1:2) = 0
    psibar_r%a(3:4) = psibar%a(3:4)
    j = gr * fg5gkgr (psibar_r, grav, v)
  end function srv1_fgr
```

⟨*Implementation of bispinor currents*⟩+≡

```
pure function slrv1_fgr (gl, gr, psibar, v, grav) result (j)
  complex(kind=default) :: j
  complex(kind=default), intent(in) :: gl, gr
  type(bispinor), intent(in) :: psibar
  type(bispinor) :: psibar_l, psibar_r
  type(vectorspinor), intent(in) :: grav
  type(vector), intent(in) :: v
  psibar_l%a(1:2) = psibar%a(1:2)
  psibar_l%a(3:4) = 0
  psibar_r%a(1:2) = 0
  psibar_r%a(3:4) = psibar%a(3:4)
  j = gl * fg5gkgr (psibar_l, grav, v)  + gr * fg5gkgr (psibar_r, grav, v)
end function slrv1_fgr
```

⟨*Implementation of bispinor currents*⟩+≡

```
pure function sv2_fgr (g, psibar, phi, grav) result (j)
  type(vector) :: j
  complex(kind=default), intent(in) :: g, phi
  type(bispinor), intent(in) :: psibar
  type(vectorspinor), intent(in) :: grav
  type(bispinor) :: g0_grav, g1_grav, g2_grav, g3_grav
  g0_grav%a(1) = -grav%psi(1)%a(1) +  grav%psi(2)%a(2) - &
                (0,1) * grav%psi(3)%a(2) + grav%psi(4)%a(1)
  g0_grav%a(2) = -grav%psi(1)%a(2) + grav%psi(2)%a(1) + &
                (0,1) * grav%psi(3)%a(1) - grav%psi(4)%a(2)
  g0_grav%a(3) = grav%psi(1)%a(3) + grav%psi(2)%a(4) - &
                (0,1) * grav%psi(3)%a(4) + grav%psi(4)%a(3)
  g0_grav%a(4) = grav%psi(1)%a(4) + grav%psi(2)%a(3) + &
                (0,1) * grav%psi(3)%a(3) - grav%psi(4)%a(4)
  !!!
  g1_grav%a(1) = grav%psi(1)%a(2) - grav%psi(2)%a(1) + &
                (0,1) * grav%psi(3)%a(1) - grav%psi(4)%a(2)
  g1_grav%a(2) = grav%psi(1)%a(1) - grav%psi(2)%a(2) - &
                (0,1) * grav%psi(3)%a(2) + grav%psi(4)%a(1)
  g1_grav%a(3) = grav%psi(1)%a(4) + grav%psi(2)%a(3) - &
                (0,1) * grav%psi(3)%a(3) + grav%psi(4)%a(4)
  g1_grav%a(4) = grav%psi(1)%a(3) + grav%psi(2)%a(4) + &
                (0,1) * grav%psi(3)%a(4) - grav%psi(4)%a(3)
  !!!
  g2_grav%a(1) = (0,1) * (-grav%psi(1)%a(2) - grav%psi(2)%a(1) + &
                grav%psi(4)%a(2)) - grav%psi(3)%a(1)
  g2_grav%a(2) = (0,1) * (grav%psi(1)%a(1) + grav%psi(2)%a(2) + &
                grav%psi(4)%a(1)) - grav%psi(3)%a(2)
  g2_grav%a(3) = (0,1) * (-grav%psi(1)%a(4) + grav%psi(2)%a(3) - &
                grav%psi(4)%a(4)) + grav%psi(3)%a(3)
  g2_grav%a(4) = (0,1) * (grav%psi(1)%a(3) - grav%psi(2)%a(4) - &
                grav%psi(4)%a(3)) + grav%psi(3)%a(4)
  !!!
  g3_grav%a(1) = -grav%psi(1)%a(2) + grav%psi(2)%a(2) - &
                (0,1) * grav%psi(3)%a(2) - grav%psi(4)%a(1)
  g3_grav%a(2) = grav%psi(1)%a(1) - grav%psi(2)%a(1) - &
                (0,1) * grav%psi(3)%a(1) - grav%psi(4)%a(2)
  g3_grav%a(3) = -grav%psi(1)%a(2) - grav%psi(2)%a(4) + &
                (0,1) * grav%psi(3)%a(4) + grav%psi(4)%a(3)
```

```
      g3_grav%a(4) = -grav%psi(1)%a(4) + grav%psi(2)%a(3) + &
                      (0,1) * grav%psi(3)%a(3) + grav%psi(4)%a(4)
      j%t    =  (g * phi) * (psibar * g0_grav)
      j%x(1) =  (g * phi) * (psibar * g1_grav)
      j%x(2) =  (g * phi) * (psibar * g2_grav)
      j%x(3) =  (g * phi) * (psibar * g3_grav)
    end function sv2_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slv2_fgr (gl, psibar, phi, grav) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, phi
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_l
    type(vectorspinor), intent(in) :: grav
    psibar_l%a(1:2) = psibar%a(1:2)
    psibar_l%a(3:4) = 0
    j = sv2_fgr (gl, psibar_l, phi, grav)
  end function slv2_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function srv2_fgr (gr, psibar, phi, grav) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gr, phi
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_r
    type(vectorspinor), intent(in) :: grav
    psibar_r%a(1:2) = 0
    psibar_r%a(3:4) = psibar%a(3:4)
    j = sv2_fgr (gr, psibar_r, phi, grav)
  end function srv2_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function slrv2_fgr (gl, gr, psibar, phi, grav) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, gr, phi
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_l, psibar_r
    type(vectorspinor), intent(in) :: grav
    psibar_l%a(1:2) = psibar%a(1:2)
    psibar_l%a(3:4) = 0
    psibar_r%a(1:2) = 0
    psibar_r%a(3:4) = psibar%a(3:4)
    j = sv2_fgr (gl, psibar_l, phi, grav) + sv2_fgr (gr, psibar_r, phi, grav)
  end function slrv2_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function pv1_grf (g, gravbar, v, psi) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    j = g * grvgf (gravbar, psi, v)
  end function pv1_grf
```

⟨*Implementation of bispinor currents*⟩+≡

```
  pure function pv2_grf (g, gravbar, phi, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: g, phi
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: g5_psi
    g5_psi%a(1:2) = - psi%a(1:2)
    g5_psi%a(3:4) = psi%a(3:4)
    j = sv2_grf (g, gravbar, phi, g5_psi)
  end function pv2_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function pv1_fgr (g, psibar, v, grav) result (j)
    complex(kind=default) :: j
    complex(kind=default), intent(in) :: g
    type(bispinor), intent(in) :: psibar
    type(vectorspinor), intent(in) :: grav
    type(vector), intent(in) :: v
    j = g * fgkgr (psibar, grav, v)
  end function pv1_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function pv2_fgr (g, psibar, phi, grav) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: g, phi
    type(vectorspinor), intent(in) :: grav
    type(bispinor), intent(in) :: psibar
    type(bispinor) :: psibar_g5
    psibar_g5%a(1:2) = - psibar%a(1:2)
    psibar_g5%a(3:4) = psibar%a(3:4)
    j = sv2_fgr (g, psibar_g5, phi, grav)
  end function pv2_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function v2_grf (g, gravbar, v, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: g
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(vector), intent(in) :: v
    j = -g * grkgggf (gravbar, psi, v)
  end function v2_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
  pure function v2lr_grf (gl, gr, gravbar, v, psi) result (j)
    type(vector) :: j
    complex(kind=default), intent(in) :: gl, gr
    type(vectorspinor), intent(in) :: gravbar
    type(bispinor), intent(in) :: psi
    type(bispinor) :: psi_l, psi_r
    type(vector), intent(in) :: v
    psi_l%a(1:2) = psi%a(1:2)
    psi_l%a(3:4) = 0
    psi_r%a(1:2) = 0
    psi_r%a(3:4) = psi%a(3:4)
    j = -(gl * grkgggf (gravbar, psi_l, v) + gr * grkgggf (gravbar, psi_r, v))
  end function v2lr_grf
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function v2_fgr (g, psibar, v, grav) result (j)
  type(vector) :: j
  complex(kind=default), intent(in) :: g
  type(vectorspinor), intent(in) :: grav
  type(bispinor), intent(in) :: psibar
  type(vector), intent(in) :: v
  j = -g * fggkggr (psibar, grav, v)
end function v2_fgr
```

⟨*Implementation of bispinor currents*⟩+≡
```
pure function v2lr_fgr (gl, gr, psibar, v, grav) result (j)
  type(vector) :: j
  complex(kind=default), intent(in) :: gl, gr
  type(vectorspinor), intent(in) :: grav
  type(bispinor), intent(in) :: psibar
  type(bispinor) :: psibar_l, psibar_r
  type(vector), intent(in) :: v
  psibar_l%a(1:2) = psibar%a(1:2)
  psibar_l%a(3:4) = 0
  psibar_r%a(1:2) = 0
  psibar_r%a(3:4) = psibar%a(3:4)
  j = -(gl * fggkggr (psibar_l, grav, v) + gr * fggkggr (psibar_r, grav, v))
end function v2lr_fgr
```

X.23.6 On Shell Wave Functions

⟨*Declaration of bispinor on shell wave functions*⟩≡
```
public :: u, v, ghost
```

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_3)}} \begin{pmatrix} |\vec{p}| + p_3 \\ p_1 + \mathrm{i}p_2 \end{pmatrix} \tag{X.117a}$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_3)}} \begin{pmatrix} -p_1 + \mathrm{i}p_2 \\ |\vec{p}| + p_3 \end{pmatrix} \tag{X.117b}$$

$$u_\pm(p) = \begin{pmatrix} \sqrt{p_0 \mp |\vec{p}|} \cdot \chi_\pm(\vec{p}) \\ \sqrt{p_0 \pm |\vec{p}|} \cdot \chi_\pm(\vec{p}) \end{pmatrix} \tag{X.118}$$

⟨*Implementation of bispinor on shell wave functions*⟩≡
```
pure function u (mass, p, s) result (psi)
  type(bispinor) :: psi
  real(kind=default), intent(in) :: mass
  type(momentum), intent(in) :: p
  integer, intent(in) :: s
  complex(kind=default), dimension(2) :: chip, chim
  real(kind=default) :: pabs, norm, delta, m
  m = abs(mass)
  pabs = sqrt (dot_product (p%x, p%x))
  if (m < epsilon (m) * pabs) then
      delta = 0
  else
      delta = sqrt (max (p%t - pabs, 0._default))
```

```
      end if
      if (pabs + p%x(3) <= 1000 * epsilon (pabs) * pabs) then
         chip = (/ cmplx ( 0.0, 0.0, kind=default), &
                   cmplx ( 1.0, 0.0, kind=default) /)
         chim = (/ cmplx (-1.0, 0.0, kind=default), &
                   cmplx ( 0.0, 0.0, kind=default) /)
      else
         norm = 1 / sqrt (2*pabs*(pabs + p%x(3)))
         chip = norm * (/ cmplx (pabs + p%x(3), kind=default), &
                           cmplx (p%x(1), p%x(2), kind=default) /)
         chim = norm * (/ cmplx (-p%x(1), p%x(2), kind=default), &
                           cmplx (pabs + p%x(3), kind=default) /)
      end if
      if (s > 0) then
         psi%a(1:2) = delta * chip
         psi%a(3:4) = sqrt (p%t + pabs) * chip
      else
         psi%a(1:2) = sqrt (p%t + pabs) * chim
         psi%a(3:4) = delta * chim
      end if
      pabs = m ! make the compiler happy and use m
      if (mass < 0) then
         psi%a(1:2) = - imago * psi%a(1:2)
         psi%a(3:4) = + imago * psi%a(3:4)
      end if
   end function u
```

$$v_{\pm}(p) = \begin{pmatrix} \mp\sqrt{p_0 \pm |\vec{p}|} \cdot \chi_{\mp}(\vec{p}) \\ \pm\sqrt{p_0 \mp |\vec{p}|} \cdot \chi_{\mp}(\vec{p}) \end{pmatrix} \tag{X.119}$$

⟨*Implementation of bispinor on shell wave functions*⟩+≡

```
  pure function v (mass, p, s) result (psi)
    type(bispinor) :: psi
    real(kind=default), intent(in) :: mass
    type(momentum), intent(in) :: p
    integer, intent(in) :: s
    complex(kind=default), dimension(2) :: chip, chim
    real(kind=default) :: pabs, norm, delta, m
    pabs = sqrt (dot_product (p%x, p%x))
    m = abs(mass)
    if (m < epsilon (m) * pabs) then
       delta = 0
    else
       delta = sqrt (max (p%t - pabs, 0._default))
    end if
    if (pabs + p%x(3) <= 1000 * epsilon (pabs) * pabs) then
       chip = (/ cmplx ( 0.0, 0.0, kind=default), &
                 cmplx ( 1.0, 0.0, kind=default) /)
       chim = (/ cmplx (-1.0, 0.0, kind=default), &
                 cmplx ( 0.0, 0.0, kind=default) /)
    else
       norm = 1 / sqrt (2*pabs*(pabs + p%x(3)))
       chip = norm * (/ cmplx (pabs + p%x(3), kind=default), &
                         cmplx (p%x(1), p%x(2), kind=default) /)
```

```
        chim = norm * (/ cmplx (-p%x(1), p%x(2), kind=default), &
                          cmplx (pabs + p%x(3), kind=default) /)
     end if
     if (s > 0) then
        psi%a(1:2) = - sqrt (p%t + pabs) * chim
        psi%a(3:4) = delta * chim
     else
        psi%a(1:2) = delta * chip
        psi%a(3:4) = - sqrt (p%t + pabs) * chip
     end if
     pabs = m ! make the compiler happy and use m
     if (mass < 0) then
        psi%a(1:2) = - imago * psi%a(1:2)
        psi%a(3:4) = + imago * psi%a(3:4)
     end if
  end function v
```

⟨_Implementation of bispinor on shell wave functions_⟩+≡

```
  pure function ghost (m, p, s) result (psi)
     type(bispinor) :: psi
     real(kind=default), intent(in) :: m
     type(momentum), intent(in) :: p
     integer, intent(in) :: s
     psi%a(:) = 0
     select case (s)
     case (1)
        psi%a(1)   = 1
        psi%a(2:4) = 0
     case (2)
        psi%a(1)   = 0
        psi%a(2)   = 1
        psi%a(3:4) = 0
     case (3)
        psi%a(1:2) = 0
        psi%a(3)   = 1
        psi%a(4)   = 0
     case (4)
        psi%a(1:3) = 0
        psi%a(4)   = 1
     case (5)
        psi%a(1) =    1.4
        psi%a(2) = -  2.3
        psi%a(3) = - 71.5
        psi%a(4) =    0.1
     end select
  end function ghost
```

X.23.7   Off Shell Wave Functions

This is the same as for the Dirac fermions except that the expressions for [ubar] and [vbar] are missing.

⟨_Declaration of bispinor off shell wave functions_⟩≡

```
  public :: brs_u, brs_v
```

In momentum space we have:

$$brsu(p) = (-i)(\not{p} - m)u(p) \tag{X.120}$$

⟨*Implementation of bispinor off shell wave functions*⟩≡
```
pure function brs_u (m, p, s) result (dpsi)
    type(bispinor) :: dpsi, psi
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: p
    integer, intent(in) :: s
    type (vector)::vp
    complex(kind=default), parameter :: one = (1, 0)
    vp=p
    psi=u(m,p,s)
    dpsi=cmplx(0.0,-1.0)*(f_vf(one,vp,psi)-m*psi)
end function brs_u
```

$$brsv(p) = i(\not{p} + m)v(p) \tag{X.121}$$

⟨*Implementation of bispinor off shell wave functions*⟩+≡
```
pure function brs_v (m, p, s) result (dpsi)
    type(bispinor) :: dpsi, psi
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: p
    integer, intent(in) ::   s
    type (vector)::vp
    complex(kind=default), parameter :: one = (1, 0)
    vp=p
    psi=v(m,p,s)
    dpsi=cmplx(0.0,1.0)*(f_vf(one,vp,psi)+m*psi)
end function brs_v
```

X.23.8   Propagators

⟨*Declaration of bispinor propagators*⟩≡
```
public :: pr_psi, pr_grav
public :: pj_psi, pg_psi
```

$$\frac{\mathrm{i}(-\not{p} + m)}{p^2 - m^2 + im\Gamma}\psi \tag{X.122}$$

NB: the sign of the momentum comes about because all momenta are treated as *outgoing* and the particle charge flow is therefore opposite to the momentum.

⟨*Implementation of bispinor propagators*⟩≡
```
pure function pr_psi (p, m, w, psi) result (ppsi)
  type(bispinor) :: ppsi
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w
  type(bispinor), intent(in) :: psi
  type(vector) :: vp
  complex(kind=default), parameter :: one = (1, 0)
  vp = p
  ppsi = (1 / cmplx (p*p - m**2, m*w, kind=default)) &
       * (- f_vf (one, vp, psi) + m * psi)
end function pr_psi
```

$$\sqrt{\frac{\pi}{M\Gamma}}(-\not{p}+m)\psi \qquad \text{(X.123)}$$

⟨*Implementation of bispinor propagators*⟩+≡

```
pure function pj_psi (p, m, w, psi) result (ppsi)
  type(bispinor) :: ppsi
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w
  type(bispinor), intent(in) :: psi
  type(vector) :: vp
  complex(kind=default), parameter :: one = (1, 0)
  vp = p
  ppsi = (0, -1) * sqrt (PI / m / w) * (- f_vf (one, vp, psi) + m * psi)
end function pj_psi
```

⟨*Implementation of bispinor propagators*⟩+≡

```
pure function pg_psi (p, m, w, psi) result (ppsi)
  type(bispinor) :: ppsi
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w
  type(bispinor), intent(in) :: psi
  type(vector) :: vp
  complex(kind=default), parameter :: one = (1, 0)
  vp = p
  ppsi = gauss (p*p, m, w) * (- f_vf (one, vp, psi) + m * psi)
end function pg_psi
```

$$\frac{\mathrm{i}\left\{(-\not{p}+m)\left(-\eta_{\mu\nu}+\frac{p_\mu p_\nu}{m^2}\right)+\frac{1}{3}\left(\gamma_\mu-\frac{p_\mu}{m}\right)(\not{p}+m)\left(\gamma_\nu-\frac{p_\nu}{m}\right)\right\}}{p^2-m^2+\mathrm{i}m\Gamma}\,\psi^\nu \quad \text{(X.124)}$$

⟨*Implementation of bispinor propagators*⟩+≡

```
pure function pr_grav (p, m, w, grav) result (propgrav)
  type(vectorspinor) :: propgrav
  type(momentum), intent(in) :: p
  real(kind=default), intent(in) :: m, w
  type(vectorspinor), intent(in) :: grav
  type(vector) :: vp
  type(bispinor) :: pgrav, ggrav, ggrav1, ggrav2, ppgrav
  type(vectorspinor) :: etagrav_dum, etagrav, pppgrav, &
                        gg_grav_dum, gg_grav
  complex(kind=default), parameter :: one = (1, 0)
  real(kind=default) :: minv
  integer :: i
  vp = p
  minv = 1/m
  pgrav = p%t    * grav%psi(1) - p%x(1) * grav%psi(2) - &
          p%x(2) * grav%psi(3) - p%x(3) * grav%psi(4)
  ggrav%a(1) = grav%psi(1)%a(3) - grav%psi(2)%a(4) + (0,1) * &
               grav%psi(3)%a(4) - grav%psi(4)%a(3)
  ggrav%a(2) = grav%psi(1)%a(4) - grav%psi(2)%a(3) - (0,1) * &
               grav%psi(3)%a(3) + grav%psi(4)%a(4)
  ggrav%a(3) = grav%psi(1)%a(1) + grav%psi(2)%a(2) - (0,1) * &
```

```
                   grav%psi(3)%a(2) + grav%psi(4)%a(1)
     ggrav%a(4) = grav%psi(1)%a(2) + grav%psi(2)%a(1) + (0,1) * &
                   grav%psi(3)%a(1) - grav%psi(4)%a(2)
     ggrav1 = ggrav - minv * pgrav
     ggrav2 = f_vf (one, vp, ggrav1) + m * ggrav - pgrav
     ppgrav = (-minv**2) * f_vf (one, vp, pgrav) + minv * pgrav
     do i = 1, 4
     etagrav_dum%psi(i) = f_vf (one, vp, grav%psi(i))
     end do
     etagrav = etagrav_dum - m * grav
     pppgrav%psi(1) = p%t    * ppgrav
     pppgrav%psi(2) = p%x(1) * ppgrav
     pppgrav%psi(3) = p%x(2) * ppgrav
     pppgrav%psi(4) = p%x(3) * ppgrav
     gg_grav_dum%psi(1) = p%t    * ggrav2
     gg_grav_dum%psi(2) = p%x(1) * ggrav2
     gg_grav_dum%psi(3) = p%x(2) * ggrav2
     gg_grav_dum%psi(4) = p%x(3) * ggrav2
     gg_grav = gr_potf (one, one, ggrav2) - minv * gg_grav_dum
     propgrav = (1 / cmplx (p*p - m**2, m*w, kind=default)) * &
           (etagrav + pppgrav + (1/3.0_default) * gg_grav)
   end function pr_grav
```

X.24   *Polarization vectorspinors*

Here we construct the wavefunctions for (massive) gravitinos out of the wavefunctions of (massive) vectorbosons and (massive) Majorana fermions.

$$\psi^\mu_{(u;3/2)}(k) = \epsilon^\mu_+(k) \cdot u(k,+) \tag{X.125a}$$

$$\psi^\mu_{(u;1/2)}(k) = \sqrt{\frac{1}{3}}\, \epsilon^\mu_+(k) \cdot u(k,-) + \sqrt{\frac{2}{3}}\, \epsilon^\mu_0(k) \cdot u(k,+) \tag{X.125b}$$

$$\psi^\mu_{(u;-1/2)}(k) = \sqrt{\frac{2}{3}}\, \epsilon^\mu_0(k) \cdot u(k,-) + \sqrt{\frac{1}{3}}\, \epsilon^\mu_-(k) \cdot u(k,+) \tag{X.125c}$$

$$\psi^\mu_{(u;-3/2)}(k) = \epsilon^\mu_-(k) \cdot u(k,-) \tag{X.125d}$$

and in the same manner for $\psi^\mu_{(v;s)}$ with u replaced by v and with the conjugated polarization vectors. These gravitino wavefunctions obey the Dirac equation, they are transverse and they fulfill the irreducibility condition

$$\gamma_\mu \psi^\mu_{(u/v;s)} = 0. \tag{X.126}$$

⟨omega_vspinor_polarizations.f90⟩≡
  ⟨*Copyleft*⟩
  module omega_vspinor_polarizations
    use kinds
    use constants
    use omega_vectors
    use omega_bispinors
    use omega_bispinor_couplings
    use omega_vectorspinors
    implicit none

```
⟨Declaration of polarization vectorspinors⟩
integer, parameter, public :: omega_vspinor_pols_2010_01_A = 0
contains
⟨Implementation of polarization vectorspinors⟩
end module omega_vspinor_polarizations
```

⟨*Declaration of polarization vectorspinors*⟩≡
```
public :: ueps, veps
private :: eps
private :: outer_product
```

Here we implement the polarization vectors for vectorbosons with trigonometric functions, without the rotating of components done in HELAS [5]. These are only used for generating the polarization vectorspinors.

$$\epsilon_+^\mu(k) = \frac{-e^{+\mathrm{i}\phi}}{\sqrt{2}} \left(0; \cos\theta\cos\phi - \mathrm{i}\sin\phi, \cos\theta\sin\phi + \mathrm{i}\cos\phi, -\sin\theta\right) \quad \text{(X.127a)}$$

$$\epsilon_-^\mu(k) = \frac{e^{-\mathrm{i}\phi}}{\sqrt{2}} \left(0; \cos\theta\cos\phi + \mathrm{i}\sin\phi, \cos\theta\sin\phi - \mathrm{i}\cos\phi, -\sin\theta\right) \quad \text{(X.127b)}$$

$$\epsilon_0^\mu(k) = \frac{1}{m} \left(|\vec{k}|; k^0\sin\theta\cos\phi, k^0\sin\theta\sin\phi, k^0\cos\theta\right) \quad \text{(X.127c)}$$

Determining the mass from the momenta is a numerically haphazardous for light particles. Therefore, we accept some redundancy and pass the mass explicitly. For the case that the momentum lies totally in the z-direction we take the convention $\cos\phi = 1$ and $\sin\phi = 0$.

⟨*Implementation of polarization vectorspinors*⟩≡
```
pure function eps (mass, k, s) result (e)
  type(vector) :: e
  real(kind=default), intent(in) :: mass
  type(momentum), intent(in) :: k
  integer, intent(in) :: s
  real(kind=default) :: kabs, kabs2, sqrt2, m
  real(kind=default) :: cos_phi, sin_phi, cos_th, sin_th
  complex(kind=default) :: epiphi, emiphi
  sqrt2 = sqrt (2.0_default)
  kabs2 = dot_product (k%x, k%x)
  m = abs(mass)
  if (kabs2 > 0) then
     kabs = sqrt (kabs2)
     if ((k%x(1) == 0) .and. (k%x(2) == 0)) then
        cos_phi = 1
        sin_phi = 0
     else
        cos_phi = k%x(1) / sqrt(k%x(1)**2 + k%x(2)**2)
        sin_phi = k%x(2) / sqrt(k%x(1)**2 + k%x(2)**2)
     end if
     cos_th = k%x(3) / kabs
     sin_th = sqrt(1 - cos_th**2)
     epiphi = cos_phi + (0,1) * sin_phi
     emiphi = cos_phi - (0,1) * sin_phi
     e%t = 0
     e%x = 0
     select case (s)
```

```
        case (1)
            e%x(1) = epiphi * (-cos_th * cos_phi + (0,1) * sin_phi) / sqrt2
            e%x(2) = epiphi * (-cos_th * sin_phi - (0,1) * cos_phi) / sqrt2
            e%x(3) = epiphi * ( sin_th / sqrt2)
        case (-1)
            e%x(1) = emiphi * ( cos_th * cos_phi + (0,1) * sin_phi) / sqrt2
            e%x(2) = emiphi * ( cos_th * sin_phi - (0,1) * cos_phi) / sqrt2
            e%x(3) = emiphi * (-sin_th / sqrt2)
        case (0)
            if (m > 0) then
                e%t = kabs / m
                e%x = k%t / (m*kabs) * k%x
            end if
        case (4)
            if (m > 0) then
                e = (1 / m) * k
            else
                e = (1 / k%t) * k
            end if
        end select
    else   !!! for particles in their rest frame defined to be
           !!! polarized along the 3-direction
        e%t = 0
        e%x = 0
        select case (s)
        case (1)
            e%x(1) = cmplx ( - 1,   0, kind=default) / sqrt2
            e%x(2) = cmplx (   0,   1, kind=default) / sqrt2
        case (-1)
            e%x(1) = cmplx (   1,   0, kind=default) / sqrt2
            e%x(2) = cmplx (   0,   1, kind=default) / sqrt2
        case (0)
            if (m > 0) then
                e%x(3) = 1
            end if
        case (4)
            if (m > 0) then
                e = (1 / m) * k
            else
                e = (1 / k%t) * k
            end if
        end select
    end if
  end function eps
```

⟨*Implementation of polarization vectorspinors*⟩+≡

```
  pure function ueps (m, k, s) result (t)
    type(vectorspinor) :: t
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: k
    integer, intent(in) :: s
    integer :: i
    type(vector) :: ep, e0, em
    type(bispinor) :: up, um
    do i = 1, 4
```

```
      t%psi(i)%a = 0
    end do
    select case (s)
    case (2)
       ep = eps (m, k, 1)
       up = u (m, k, 1)
       t = outer_product (ep, up)
    case (1)
       ep = eps (m, k, 1)
       e0 = eps (m, k, 0)
       up = u (m, k, 1)
       um = u (m, k, -1)
       t = (1 / sqrt (3.0_default)) * (outer_product (ep, um) &
             + sqrt (2.0_default) * outer_product (e0, up))
    case (-1)
       e0 = eps (m, k, 0)
       em = eps (m, k, -1)
       up = u (m, k, 1)
       um = u (m, k, -1)
       t = (1 / sqrt (3.0_default)) * (sqrt (2.0_default) * &
             outer_product (e0, um) + outer_product (em, up))
    case (-2)
       em = eps (m, k, -1)
       um = u (m, k, -1)
       t = outer_product (em, um)
    end select
  end function ueps
```

⟨*Implementation of polarization vectorspinors*⟩+≡

```
  pure function veps (m, k, s) result (t)
    type(vectorspinor) :: t
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: k
    integer, intent(in) :: s
    integer :: i
    type(vector) :: ep, e0, em
    type(bispinor) :: vp, vm
    do i = 1, 4
      t%psi(i)%a = 0
    end do
    select case (s)
    case (2)
       ep = conjg(eps (m, k, 1))
       vp = v (m, k, 1)
       t = outer_product (ep, vp)
    case (1)
       ep = conjg(eps (m, k, 1))
       e0 = conjg(eps (m, k, 0))
       vp = v (m, k, 1)
       vm = v (m, k, -1)
       t = (1 / sqrt (3.0_default)) * (outer_product (ep, vm) &
             + sqrt (2.0_default) * outer_product (e0, vp))
    case (-1)
       e0 = conjg(eps (m, k,  0))
       em = conjg(eps (m, k, -1))
```

```
      vp = v (m, k, 1)
      vm = v (m, k, -1)
      t = (1 / sqrt (3.0_default)) * (sqrt (2.0_default) &
          * outer_product (e0, vm) + outer_product (em, vp))
   case (-2)
      em = conjg(eps (m, k, -1))
      vm = v (m, k, -1)
      t = outer_product (em, vm)
   end select
 end function veps
```

⟨*Implementation of polarization vectorspinors*⟩+≡
```
 pure function outer_product (ve, sp) result (vs)
   type(vectorspinor) :: vs
   type(vector), intent(in) :: ve
   type(bispinor), intent(in) :: sp
   integer :: i
   vs%psi(1)%a(1:4) = ve%t * sp%a(1:4)
   do i = 1, 3
      vs%psi((i+1))%a(1:4) = ve%x(i) * sp%a(1:4)
   end do
 end function outer_product
```

X.25   Color

⟨`omega_color.f90`⟩≡
```
 ⟨Copyleft⟩
 module omega_color
   use kinds
   implicit none
   private
   ⟨Declaration of color types⟩
   ⟨Declaration of color functions⟩
   integer, parameter, public :: omega_color_2010_01_A = 0
 contains
   ⟨Implementation of color functions⟩
 end module omega_color
```

X.25.1   Color Sum

⟨*Declaration of color types*⟩≡
```
 public :: omega_color_factor
 type omega_color_factor
    integer :: i1, i2
    real(kind=default) :: factor
 end type omega_color_factor
```

⟨*Declaration of color functions*⟩≡
```
 public :: omega_color_sum
```

The `!$omp` instruction will result in parallel code if compiled with support for OpenMP otherwise it is ignored.

⟨*Implementation of color functions*⟩≡
```
 ⟨pure unless OpenMP⟩
```

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```
function omega_color_sum (flv, hel, amp, cf) result (amp2)
  complex(kind=default) :: amp2
  integer, intent(in) :: flv, hel
  complex(kind=default), dimension(:,:,:), intent(in) :: amp
  type(omega_color_factor), dimension(:), intent(in) :: cf
  integer :: n
  amp2 = 0
  !$omp parallel do reduction(+:amp2)
  do n = 1, size (cf)
     amp2 = amp2 + cf(n)%factor * &
                 amp(flv,cf(n)%i1,hel) * conjg (amp(flv,cf(n)%i2,hel))
  end do
  !$omp end parallel do
end function omega_color_sum
```

In the bytecode for the OVM, we only save the symmetric part of the color factor table. This almost halves the size of n gluon amplitudes for $n > 6$. For $2 \to (5,6)\,g$ the reduced color factor table still amounts for $\sim (75, 93)\%$ of the bytecode, making it desirable to omit it completely by computing it dynamically to reduce memory requirements. Note that $2\mathrm{Re}(A_{i_1}A_{i_2}^*) = A_{i_1}A_{i_2}^* + A_{i_2}A_{i_1}^*$.

⟨*Declaration of color functions*⟩+≡
```
  public :: ovm_color_sum
```

⟨*Implementation of color functions*⟩+≡
```
  ⟨pure unless OpenMP⟩
  function ovm_color_sum (flv, hel, amp, cf) result (amp2)
    real(kind=default) :: amp2
    integer, intent(in) :: flv, hel
    complex(kind=default), dimension(:,:,:), intent(in) :: amp
    type(omega_color_factor), dimension(:), intent(in) :: cf
    integer :: n
    amp2 = 0
    !$omp parallel do reduction(+:amp2)
    do n = 1, size (cf)
       if (cf(n)%i1 == cf(n)%i2) then
          amp2 = amp2 + cf(n)%factor * &
                  real(amp(flv,cf(n)%i1,hel) * conjg(amp(flv,cf(n)%i2,hel)))
       else
          amp2 = amp2 + cf(n)%factor * 2 * &
                  real(amp(flv,cf(n)%i1,hel) * conjg(amp(flv,cf(n)%i2,hel)))
       end if
    end do
    !$omp end parallel do
  end function ovm_color_sum
```

X.26 Utilities

⟨omega_utils.f90⟩≡
```
  ⟨Copyleft⟩
  module omega_utils
    use kinds
    use omega_vectors
    use omega_polarizations
    implicit none
```

```
   private
   ⟨Declaration of utility functions⟩
   ⟨Numerical tolerances⟩
   integer, parameter, public :: omega_utils_2010_01_A = 0
 contains
   ⟨Implementation of utility functions⟩
 end module omega_utils
```

X.26.1 Helicity Selection Rule Heuristics

⟨*Declaration of utility functions*⟩≡
```
 public :: omega_update_helicity_selection
```

⟨*Implementation of utility functions*⟩≡
```
 pure subroutine omega_update_helicity_selection &
            (count, amp, max_abs, sum_abs, mask, threshold, cutoff, mask_dirty)
   integer, intent(inout) :: count
   complex(kind=default), dimension(:,:,:), intent(in) :: amp
   real(kind=default), dimension(:), intent(inout) :: max_abs
   real(kind=default), intent(inout) :: sum_abs
   logical, dimension(:), intent(inout) :: mask
   real(kind=default), intent(in) :: threshold
   integer, intent(in) :: cutoff
   logical, intent(out) :: mask_dirty
   integer :: h
   real(kind=default) :: avg
   mask_dirty = .false.
   if (threshold > 0) then
      count = count + 1
      if (count <= cutoff) then
         forall (h = lbound (amp, 3) : ubound (amp, 3))
            max_abs(h) = max (max_abs(h), maxval (abs (amp(:,:,h))))
         end forall
         sum_abs = sum_abs + sum (abs (amp))
         if (count == cutoff) then
            avg = sum_abs / size (amp) / cutoff
            mask = max_abs >= threshold * epsilon (avg) * avg
            mask_dirty = .true.
         end if
      end if
   end if
 end subroutine omega_update_helicity_selection
```

X.26.2 Diagnostics

⟨*Declaration of utility functions*⟩+≡
```
 public :: omega_report_helicity_selection
```

We shoul try to use `msg_message` from WHIZARD's `diagnostics` module, but this would spoil independent builds.

⟨*Implementation of utility functions*⟩+≡
```
 subroutine omega_report_helicity_selection (mask, spin_states, threshold, unit)
   logical, dimension(:), intent(in) :: mask
   integer, dimension(:,:), intent(in) :: spin_states
```

```
        real(kind=default), intent(in) :: threshold
        integer, intent(in), optional :: unit
        integer :: u
        integer :: h, i
        if (present(unit)) then
           u = unit
        else
           u = 6
        end if
        if (u >= 0) then
           write (unit = u, &
                fmt = "('| ','Contributing Helicity Combinations: ', I5, ' of ', I5)") &
                count (mask), size (mask)
           write (unit = u, &
                fmt = "('| ','Threshold: amp / avg > ', E9.2, ' = ', E9.2, ' * epsilon()')") &
                threshold * epsilon (threshold), threshold
           i = 0
           do h = 1, size (mask)
              if (mask(h)) then
                 i = i + 1
                 write (unit = u, fmt = "('| ',I4,': ',20I4)") i, spin_states (:, h)
              end if
           end do
        end if
     end subroutine omega_report_helicity_selection
```

⟨*Declaration of utility functions*⟩+≡
```
  public :: omega_ward_warn, omega_ward_panic
```

The O'Mega amplitudes have only one particle off shell and are the sum of *all*
possible diagrams with the other particles on-shell.

The problem with these gauge checks is that are numerically very small
amplitudes that vanish analytically and that violate transversality. The
hard part is to determine the thresholds that make threse tests usable.

⟨*Implementation of utility functions*⟩+≡
```
  subroutine omega_ward_warn (name, m, k, e)
    character(len=*), intent(in) :: name
    real(kind=default), intent(in) :: m
    type(momentum), intent(in) :: k
    type(vector), intent(in) :: e
    type(vector) :: ek
    real(kind=default) :: abs_eke, abs_ek_abs_e
    ek = eps (m, k, 4)
    abs_eke = abs (ek * e)
    abs_ek_abs_e = abs (ek) * abs (e)
    print *, name, ":", abs_eke / abs_ek_abs_e, abs (ek), abs (e)
    if (abs_eke > 1000 * epsilon (abs_ek_abs_e)) then
       print *, "O'Mega: warning: non-transverse vector field: ", &
            name, ":", abs_eke / abs_ek_abs_e, abs (e)
    end if
  end subroutine omega_ward_warn
```

⟨*Implementation of utility functions*⟩+≡
```
  subroutine omega_ward_panic (name, m, k, e)
```

```
      character(len=*), intent(in) :: name
      real(kind=default), intent(in) :: m
      type(momentum), intent(in) :: k
      type(vector), intent(in) :: e
      type(vector) :: ek
      real(kind=default) :: abs_eke, abs_ek_abs_e
      ek = eps (m, k, 4)
      abs_eke = abs (ek * e)
      abs_ek_abs_e = abs (ek) * abs (e)
      if (abs_eke > 1000 * epsilon (abs_ek_abs_e)) then
         print *, "O'Mega: panic: non-transverse vector field: ", &
              name, ":", abs_eke / abs_ek_abs_e, abs (e)
         stop
      end if
   end subroutine omega_ward_panic
```

⟨*Declaration of utility functions*⟩+≡
```
  public :: omega_slavnov_warn, omega_slavnov_panic
```

⟨*Implementation of utility functions*⟩+≡
```
   subroutine omega_slavnov_warn (name, m, k, e, phi)
      character(len=*), intent(in) :: name
      real(kind=default), intent(in) :: m
      type(momentum), intent(in) :: k
      type(vector), intent(in) :: e
      complex(kind=default), intent(in) :: phi
      type(vector) :: ek
      real(kind=default) :: abs_eke, abs_ek_abs_e
      ek = eps (m, k, 4)
      abs_eke = abs (ek * e - phi)
      abs_ek_abs_e = abs (ek) * abs (e)
      print *, name, ":", abs_eke / abs_ek_abs_e, abs (ek), abs (e)
      if (abs_eke > 1000 * epsilon (abs_ek_abs_e)) then
         print *, "O'Mega: warning: non-transverse vector field: ", &
              name, ":", abs_eke / abs_ek_abs_e, abs (e)
      end if
   end subroutine omega_slavnov_warn
```

⟨*Implementation of utility functions*⟩+≡
```
   subroutine omega_slavnov_panic (name, m, k, e, phi)
      character(len=*), intent(in) :: name
      real(kind=default), intent(in) :: m
      type(momentum), intent(in) :: k
      type(vector), intent(in) :: e
      complex(kind=default), intent(in) :: phi
      type(vector) :: ek
      real(kind=default) :: abs_eke, abs_ek_abs_e
      ek = eps (m, k, 4)
      abs_eke = abs (ek * e - phi)
      abs_ek_abs_e = abs (ek) * abs (e)
      if (abs_eke > 1000 * epsilon (abs_ek_abs_e)) then
         print *, "O'Mega: panic: non-transverse vector field: ", &
              name, ":", abs_eke / abs_ek_abs_e, abs (e)
         stop
      end if
   end subroutine omega_slavnov_panic
```

⟨*Declaration of utility functions*⟩+≡
```
  public :: omega_check_arguments_warn, omega_check_arguments_panic
```

⟨*Implementation of utility functions*⟩+≡
```
  subroutine omega_check_arguments_warn (n, k)
    integer, intent(in) :: n
    real(kind=default), dimension(0:,:), intent(in) :: k
    integer :: i
    i = size(k,dim=1)
    if (i /= 4) then
       print *, "O'Mega: warning: wrong # of dimensions:", i
    end if
    i = size(k,dim=2)
    if (i /= n) then
       print *, "O'Mega: warning: wrong # of momenta:", i, &
             ", expected", n
    end if
  end subroutine omega_check_arguments_warn
```

⟨*Implementation of utility functions*⟩+≡
```
  subroutine omega_check_arguments_panic (n, k)
    integer, intent(in) :: n
    real(kind=default), dimension(0:,:), intent(in) :: k
    logical :: error
    integer :: i
    error = .false.
    i = size(k,dim=1)
    if (i /= n) then
       print *, "O'Mega: warning: wrong # of dimensions:", i
       error = .true.
    end if
    i = size(k,dim=2)
    if (i /= n) then
       print *, "O'Mega: warning: wrong # of momenta:", i, &
             ", expected", n
       error = .true.
    end if
    if (error) then
       stop
    end if
  end subroutine omega_check_arguments_panic
```

⟨*Declaration of utility functions*⟩+≡
```
  public :: omega_check_helicities_warn, omega_check_helicities_panic
  private :: omega_check_helicity
```

⟨*Implementation of utility functions*⟩+≡
```
  function omega_check_helicity (m, smax, s) result (error)
    real(kind=default), intent(in) :: m
    integer, intent(in) :: smax, s
    logical :: error
    select case (smax)
    case (0)
       error = (s /= 0)
    case (1)
       error = (abs (s) /= 1)
```

```
    case (2)
       if (m == 0.0_default) then
          error = .not. (abs (s) == 1 .or. abs (s) == 4)
       else
          error = .not. (abs (s) <= 1 .or. abs (s) == 4)
       end if
    case (4)
       error = .true.
    case default
       error = .true.
    end select
  end function omega_check_helicity
```

⟨*Implementation of utility functions*⟩+≡
```
  subroutine omega_check_helicities_warn (m, smax, s)
    real(kind=default), dimension(:), intent(in) :: m
    integer, dimension(:), intent(in) :: smax, s
    integer :: i
    do i = 1, size (m)
       if (omega_check_helicity (m(i), smax(i), s(i))) then
          print *, "O'Mega: warning: invalid helicity", s(i)
       end if
    end do
  end subroutine omega_check_helicities_warn
```

⟨*Implementation of utility functions*⟩+≡
```
  subroutine omega_check_helicities_panic (m, smax, s)
    real(kind=default), dimension(:), intent(in) :: m
    integer, dimension(:), intent(in) :: smax, s
    logical :: error
    logical :: error1
    integer :: i
    error = .false.
    do i = 1, size (m)
       error1 = omega_check_helicity (m(i), smax(i), s(i))
       if (error1) then
          print *, "O'Mega: panic: invalid helicity", s(i)
          error = .true.
       end if
    end do
    if (error) then
       stop
    end if
  end subroutine omega_check_helicities_panic
```

⟨*Declaration of utility functions*⟩+≡
```
  public :: omega_check_momenta_warn, omega_check_momenta_panic
  private :: check_momentum_conservation, check_mass_shell
```

⟨*Numerical tolerances*⟩≡
```
  integer, parameter, private :: MOMENTUM_TOLERANCE = 10000
```

⟨*Implementation of utility functions*⟩+≡
```
  function check_momentum_conservation (k) result (error)
    real(kind=default), dimension(0:,:), intent(in) :: k
    logical :: error
    error = any (abs (sum (k(:,3:), dim = 2) - k(:,1) - k(:,2)) > &
```

```
           MOMENTUM_TOLERANCE * epsilon (maxval (abs (k), dim = 2)))
      if (error) then
         print *, sum (k(:,3:), dim = 2) - k(:,1) - k(:,2)
         print *, MOMENTUM_TOLERANCE * epsilon (maxval (abs (k), dim = 2)), &
               maxval (abs (k), dim = 2)
      end if
   end function check_momentum_conservation
```

⟨*Numerical tolerances*⟩+≡
```
   integer, parameter, private :: ON_SHELL_TOLERANCE = 1000000
```

⟨*Implementation of utility functions*⟩+≡
```
   function check_mass_shell (m, k) result (error)
      real(kind=default), intent(in) :: m
      real(kind=default), dimension(0:), intent(in) :: k
      real(kind=default) :: e2
      logical :: error
      e2 = k(1)**2 + k(2)**2 + k(3)**2 + m**2
      error = abs (k(0)**2 - e2) > ON_SHELL_TOLERANCE * epsilon (max (k(0)**2, e2))
      if (error) then
         print *, k(0)**2 - e2
         print *, ON_SHELL_TOLERANCE * epsilon (max (k(0)**2, e2)), max (k(0)**2, e2)
      end if
   end function check_mass_shell
```

⟨*Implementation of utility functions*⟩+≡
```
   subroutine omega_check_momenta_warn (m, k)
      real(kind=default), dimension(:), intent(in) :: m
      real(kind=default), dimension(0:,:), intent(in) :: k
      integer :: i
      if (check_momentum_conservation (k)) then
         print *, "O'Mega: warning: momentum not conserved"
      end if
      do i = 1, size(m)
         if (check_mass_shell (m(i), k(:,i))) then
            print *, "O'Mega: warning: particle #", i, "not on-shell"
         end if
      end do
   end subroutine omega_check_momenta_warn
```

⟨*Implementation of utility functions*⟩+≡
```
   subroutine omega_check_momenta_panic (m, k)
      real(kind=default), dimension(:), intent(in) :: m
      real(kind=default), dimension(0:,:), intent(in) :: k
      logical :: error
      logical :: error1
      integer :: i
      error = check_momentum_conservation (k)
      if (error) then
         print *, "O'Mega: panic: momentum not conserved"
      end if
      do i = 1, size(m)
         error1 = check_mass_shell (m(i), k(0:,i))
         if (error1) then
            print *, "O'Mega: panic: particle #", i, "not on-shell"
            error = .true.
```

```
      end if
    end do
    if (error) then
       stop
    end if
  end subroutine omega_check_momenta_panic
```

X.26.3   Obsolete Summation

Spin/Helicity Summation

⟨*Declaration of obsolete utility functions*⟩≡
```
  public :: omega_sum, omega_sum_nonzero, omega_nonzero
  private :: state_index
```

⟨*Implementation of obsolete utility functions*⟩≡
```
  pure function omega_sum (omega, p, states, fixed) result (sigma)
    real(kind=default) :: sigma
    real(kind=default), dimension(0:,:), intent(in) :: p
    integer, dimension(:), intent(in), optional :: states, fixed
    ⟨interface for O'Mega Amplitude⟩
    integer, dimension(size(p,dim=2)) :: s, nstates
    integer :: j
    complex(kind=default) :: a
    if (present (states)) then
       nstates = states
    else
       nstates = 2
    end if
    sigma = 0
    s = -1
    sum_spins: do
       if (present (fixed)) then
          !!! print *, 's = ', s, ', fixed = ', fixed, ', nstates = ', nstates, &
          !!!    ', fixed|s = ', merge (fixed, s, mask = nstates == 0)
          a = omega (p, merge (fixed, s, mask = nstates == 0))
       else
          a = omega (p, s)
       end if
       sigma = sigma + a * conjg(a)
       ⟨Step s like a n-ary number and terminate when all (s == -1)⟩
    end do sum_spins
    sigma = sigma / num_states (2, nstates(1:2))
  end function omega_sum
```

We're looping over all spins like a *n*-ary numbers $(-1, \ldots, -1, -1)$, $(-1, \ldots, -1, 0)$, $(-1, \ldots, -1, 1)$, $(-1, \ldots, 0, -1)$, ..., $(1, \ldots, 1, 0)$, $(1, \ldots, 1, 1)$:

⟨*Step* s *like a n-ary number and terminate when* all (s == -1)⟩≡
```
  do j = size (p, dim = 2), 1, -1
     select case (nstates (j))
     case (3) ! massive vectors
        s(j) = modulo (s(j) + 2, 3) - 1
     case (2) ! spinors, massless vectors
        s(j) = - s(j)
     case (1) ! scalars
```

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```
          s(j) = -1
      case (0) ! fized spin
          s(j) = -1
      case default ! ???
          s(j) = -1
      end select
      if (s(j) /= -1) then
          cycle sum_spins
      end if
   end do
   exit sum_spins
```

The dual operation evaluates an n-number:

⟨*Implementation of obsolete utility functions*⟩+≡
```
  pure function state_index (s, states) result (n)
    integer, dimension(:), intent(in) :: s
    integer, dimension(:), intent(in), optional :: states
    integer :: n
    integer :: j, p
    n = 1
    p = 1
    if (present (states)) then
       do j = size (s), 1, -1
          select case (states(j))
          case (3)
             n = n + p * (s(j) + 1)
          case (2)
             n = n + p * (s(j) + 1) / 2
          end select
          p = p * states(j)
       end do
    else
       do j = size (s), 1, -1
          n = n + p * (s(j) + 1) / 2
          p = p * 2
       end do
    end if
  end function state_index
```

⟨interface *for O'Mega Amplitude*⟩≡
```
  interface
     pure function omega (p, s) result (me)
       use kinds
       implicit none
       complex(kind=default) :: me
       real(kind=default), dimension(0:,:), intent(in) :: p
       integer, dimension(:), intent(in) :: s
     end function omega
  end interface
```

⟨*Declaration of obsolete utility functions*⟩+≡
```
  public :: num_states
```

⟨*Implementation of obsolete utility functions*⟩+≡
```
  pure function num_states (n, states) result (ns)
    integer, intent(in) :: n
```

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```
      integer, dimension(:), intent(in), optional :: states
      integer :: ns
      if (present (states)) then
          ns = product (states, mask = states == 2 .or. states == 3)
      else
          ns = 2**n
      end if
  end function num_states
```

X.27 *omega95*

⟨omega95.f90⟩≡
  ⟨*Copyleft*⟩
  module omega95
    use constants
    use omega_spinors
    use omega_vectors
    use omega_polarizations
    use omega_tensors
    use omega_tensor_polarizations
    use omega_couplings
    use omega_spinor_couplings
    use omega_color
    use omega_utils
    public
  end module omega95

X.28 *omega95* Revisited

⟨omega95_bispinors.f90⟩≡
  ⟨*Copyleft*⟩
  module omega95_bispinors
    use constants
    use omega_bispinors
    use omega_vectors
    use omega_vectorspinors
    use omega_polarizations
    use omega_vspinor_polarizations
    use omega_couplings
    use omega_bispinor_couplings
    use omega_color
    use omega_utils
    public
  end module omega95_bispinors

X.29 *Testing*

⟨omega_testtools.f90⟩≡
  ⟨*Copyleft*⟩
  module omega_testtools

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```
    use kinds
    implicit none
    private
    real(kind=default), parameter, private :: ABS_THRESHOLD_DEFAULT = 1E-17
    real(kind=default), parameter, private :: THRESHOLD_DEFAULT = 0.6
    real(kind=default), parameter, private :: THRESHOLD_WARN = 0.8
    ⟨Declaration of test support functions⟩
  contains
    ⟨Implementation of test support functions⟩
  end module omega_testtools
```

Quantify the agreement of two real or complex numbers

$$\text{agreement}(x, y) = \frac{\ln \Delta(x, y)}{\ln \epsilon} \in [0, 1] \qquad (\text{X.128})$$

with

$$\Delta(x, y) = \frac{|x - y|}{\max(|x|, |y|)} \qquad (\text{X.129})$$

and values outside $[0, 1]$ replaced the closed value in the interval. In other words

- 1 for $x - y = \max(|x|, |y|) \cdot \mathcal{O}(\epsilon)$ and

- 0 for $x - y = \max(|x|, |y|) \cdot \mathcal{O}(1)$

with logarithmic interpolation. The cases $x = 0$ and $y = 0$ must be treated separately.

⟨Declaration of test support functions⟩≡
```
  public :: agreement
  interface agreement
     module procedure agreement_real, agreement_complex, &
            agreement_real_complex, agreement_complex_real, &
            agreement_integer_complex, agreement_complex_integer, &
            agreement_integer_real, agreement_real_integer
  end interface
  private :: agreement_real, agreement_complex, &
       agreement_real_complex, agreement_complex_real, &
       agreement_integer_complex, agreement_complex_integer, &
       agreement_integer_real, agreement_real_integer
```

⟨Implementation of test support functions⟩≡
```
  elemental function agreement_real (x, y, base) result (a)
    real(kind=default) :: a
    real(kind=default), intent(in) :: x, y
    real(kind=default), intent(in), optional :: base
    real(kind=default) :: scale, dxy
    if (present (base)) then
       scale = max (abs (x), abs (y), abs (base))
    else
       scale = max (abs (x), abs (y))
    end if
    if (ieee_is_nan (x) .or. ieee_is_nan (y)) then
       a = 0
    else if (scale <= 0) then
       a = -1
```

```
      else
         dxy = abs (x - y) / scale
         if (dxy <= 0.0_default) then
            a = 1
         else
            a = log (dxy) / log (epsilon (scale))
            a = max (0.0_default, min (1.0_default, a))
            if (ieee_is_nan (a)) then
               a = 0
            end if
         end if
      end if
      if (ieee_is_nan (a)) then
         a = 0
      end if
   end function agreement_real
```

Poor man's replacement

⟨*Implementation of test support functions*⟩+≡
```
   elemental function ieee_is_nan (x) result (yorn)
      logical :: yorn
      real (kind=default), intent(in) :: x
      yorn = (x /= x)
   end function ieee_is_nan
```

⟨*Implementation of test support functions*⟩+≡
```
   elemental function agreement_complex (x, y, base) result (a)
      real(kind=default) :: a
      complex(kind=default), intent(in) :: x, y
      real(kind=default), intent(in), optional :: base
      real(kind=default) :: scale, dxy
      if (present (base)) then
         scale = max (abs (x), abs (y), abs (base))
      else
         scale = max (abs (x), abs (y))
      end if
      if (      ieee_is_nan (real (x, kind=default)) .or. ieee_is_nan (aimag (x)) &
           .or. ieee_is_nan (real (y, kind=default)) .or. ieee_is_nan (aimag (y))) then
         a = 0
      else if (scale <= 0) then
         a = -1
      else
         dxy = abs (x - y) / scale
         if (dxy <= 0.0_default) then
            a = 1
         else
            a = log (dxy) / log (epsilon (scale))
            a = max (0.0_default, min (1.0_default, a))
            if (ieee_is_nan (a)) then
               a = 0
            end if
         end if
      end if
      if (ieee_is_nan (a)) then
         a = 0
```

```
      end if
    end function agreement_complex
```

⟨*Implementation of test support functions*⟩+≡
```
    elemental function agreement_real_complex (x, y, base) result (a)
      real(kind=default) :: a
      real(kind=default), intent(in) :: x
      complex(kind=default), intent(in) :: y
      real(kind=default), intent(in), optional :: base
      a = agreement_complex (cmplx (x, kind=default), y, base)
    end function agreement_real_complex
```

⟨*Implementation of test support functions*⟩+≡
```
    elemental function agreement_complex_real (x, y, base) result (a)
      real(kind=default) :: a
      complex(kind=default), intent(in) :: x
      real(kind=default), intent(in) :: y
      real(kind=default), intent(in), optional :: base
      a = agreement_complex (x, cmplx (y, kind=default), base)
    end function agreement_complex_real
```

⟨*Implementation of test support functions*⟩+≡
```
    elemental function agreement_integer_complex (x, y, base) result (a)
      real(kind=default) :: a
      integer, intent(in) :: x
      complex(kind=default), intent(in) :: y
      real(kind=default), intent(in), optional :: base
      a = agreement_complex (cmplx (x, kind=default), y, base)
    end function agreement_integer_complex
```

⟨*Implementation of test support functions*⟩+≡
```
    elemental function agreement_complex_integer (x, y, base) result (a)
      real(kind=default) :: a
      complex(kind=default), intent(in) :: x
      integer, intent(in) :: y
      real(kind=default), intent(in), optional :: base
      a = agreement_complex (x, cmplx (y, kind=default), base)
    end function agreement_complex_integer
```

⟨*Implementation of test support functions*⟩+≡
```
    elemental function agreement_integer_real (x, y, base) result (a)
      real(kind=default) :: a
      integer, intent(in) :: x
      real(kind=default), intent(in) :: y
      real(kind=default), intent(in), optional :: base
      a = agreement_real (real(x, kind=default), y, base)
    end function agreement_integer_real
```

⟨*Implementation of test support functions*⟩+≡
```
    elemental function agreement_real_integer (x, y, base) result (a)
      real(kind=default) :: a
      real(kind=default), intent(in) :: x
      integer, intent(in) :: y
      real(kind=default), intent(in), optional :: base
      a = agreement_real (x, real (y, kind=default), base)
    end function agreement_real_integer
```

⟨*Declaration of test support functions*⟩+≡
```
  public:: vanishes
  interface vanishes
     module procedure vanishes_real, vanishes_complex
  end interface
  private :: vanishes_real, vanishes_complex
```

⟨*Implementation of test support functions*⟩+≡
```
  elemental function vanishes_real (x, scale) result (a)
    real(kind=default) :: a
    real(kind=default), intent(in) :: x
    real(kind=default), intent(in), optional :: scale
    real(kind=default) :: scaled_x
    if (x == 0.0_default) then
       a = 1
       return
    else if (ieee_is_nan (x)) then
       a = 0
       return
    end if
    scaled_x = x
    if (present (scale)) then
       if (scale /= 0) then
          scaled_x = x / abs (scale)
       else
          a = 0
          return
       end if
    else
    end if
    a = log (abs (scaled_x)) / log (epsilon (scaled_x))
    a = max (0.0_default, min (1.0_default, a))
    if (ieee_is_nan (a)) then
       a = 0
    end if
  end function vanishes_real
```

⟨*Implementation of test support functions*⟩+≡
```
  elemental function vanishes_complex (x, scale) result (a)
    real(kind=default) :: a
    complex(kind=default), intent(in) :: x
    real(kind=default), intent(in), optional :: scale
    a = vanishes_real (abs (x), scale)
  end function vanishes_complex
```

⟨*Declaration of test support functions*⟩+≡
```
  public :: expect
  interface expect
    module procedure expect_integer, expect_real, expect_complex, &
         expect_real_integer, expect_integer_real, &
         expect_complex_integer, expect_integer_complex, &
         expect_complex_real, expect_real_complex
  end interface
  private :: expect_integer, expect_real, expect_complex, &
       expect_real_integer, expect_integer_real, &
       expect_complex_integer, expect_integer_complex, &
```

```
        expect_complex_real, expect_real_complex
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_integer (x, x0, msg, passed, quiet, buffer, unit)
    integer, intent(in) :: x, x0
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    logical, intent(in), optional :: quiet
    character(len=*), intent(inout), optional :: buffer
    integer, intent(in), optional :: unit
    logical :: failed, verbose
    character(len=*), parameter :: fmt = "(1X,A,': ',A)"
    character(len=*), parameter :: &
        fmt_verbose = "(1X,A,': ',A,' [expected ',I6,', got ',I6,']')"
    failed = .false.
    verbose = .true.
    if (present (quiet)) then
       verbose = .not.quiet
    end if
    if (x == x0) then
       if (verbose) then
          if (.not. (present (buffer) .or. present (unit))) then
             write (unit = *, fmt = fmt) msg, "passed"
          end if
          if (present (unit)) then
             write (unit = unit, fmt = fmt) msg, "passed"
          end if
          if (present (buffer)) then
             write (unit = buffer, fmt = fmt) msg, "passed"
          end if
       end if
    else
       if (.not. (present (buffer) .or. present (unit))) then
          write (unit = *, fmt = fmt_verbose) msg, "failed", x0, x
       end if
       if (present (unit)) then
          write (unit = unit, fmt = fmt_verbose) msg, "failed", x0, x
       end if
       if (present (buffer)) then
          write (unit = buffer, fmt = fmt_verbose) msg, "failed", x0, x
       end if
       failed = .true.
    end if
    if (present (passed)) then
       passed = passed .and. .not.failed
    end if
  end subroutine expect_integer
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_real (x, x0, msg, passed, threshold, quiet, abs_threshold)
    real(kind=default), intent(in) :: x, x0
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    real(kind=default), intent(in), optional :: threshold
    real(kind=default), intent(in), optional :: abs_threshold
```

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```
      logical, intent(in), optional :: quiet
      logical :: failed, verbose
      real(kind=default) :: agreement_threshold, abs_agreement_threshold
      character(len=*), parameter :: fmt = "(1X,A,': ',A,' at ',I4,'%')"
      character(len=*), parameter :: fmt_verbose = "(1X,A,': ',A,' at ',I4,'%'," // &
            "' [expected ',E10.3,', got ',E10.3,']')"
      real(kind=default) :: a
      failed = .false.
      verbose = .true.
      if (present (quiet)) then
         verbose = .not.quiet
      end if
      if (x == x0) then
         if (verbose) then
            write (unit = *, fmt = fmt) msg, "passed", 100
         end if
      else
         if (x0 == 0) then
            a = vanishes (x)
         else
            a = agreement (x, x0)
         end if
         if (present (threshold)) then
            agreement_threshold = threshold
         else
            agreement_threshold = THRESHOLD_DEFAULT
         end if
         if (present (abs_threshold)) then
            abs_agreement_threshold = abs_threshold
         else
            abs_agreement_threshold = ABS_THRESHOLD_DEFAULT
         end if
         if (a >= agreement_threshold .or. &
              max(abs(x), abs(x0)) <= abs_agreement_threshold) then
            if (verbose) then
               if (a >= THRESHOLD_WARN) then
                  write (unit = *, fmt = fmt) msg, "passed", int (a * 100)
               else
                  write (unit = *, fmt = fmt_verbose) msg, "passed", int (a * 100), x0, x
               end if
            end if
         else
            failed = .true.
            write (unit = *, fmt = fmt_verbose) msg, "failed", int (a * 100), x0, x
         end if
      end if
      if (present (passed)) then
         passed = passed .and. .not. failed
      end if
   end subroutine expect_real
```

⟨*Implementation of test support functions*⟩+≡

```
   subroutine expect_complex (x, x0, msg, passed, threshold, quiet, abs_threshold)
      complex(kind=default), intent(in) :: x, x0
      character(len=*), intent(in) :: msg
```

```
logical, intent(inout), optional :: passed
real(kind=default), intent(in), optional :: threshold
real(kind=default), intent(in), optional :: abs_threshold
logical, intent(in), optional :: quiet
logical :: failed, verbose
real(kind=default) :: agreement_threshold, abs_agreement_threshold
character(len=*), parameter :: fmt = "(1X,A,': ',A,' at ',I4,'%')"
character(len=*), parameter :: fmt_verbose = "(1X,A,': ',A,' at ',I4,'%'," // &
     "' [expected (',E10.3,',',E10.3,'), got (',E10.3,',',E10.3,')]')"
character(len=*), parameter :: fmt_phase = "(1X,A,': ',A,' at ',I4,'%'," // &
     "' [modulus passed at ',I4,'%',', phases ',F5.3,' vs. ',F5.3,']')"
real(kind=default) :: a, a_modulus
failed = .false.
verbose = .true.
if (present (quiet)) then
   verbose = .not.quiet
end if
if (x == x0) then
   if (verbose) then
      write (unit = *, fmt = fmt) msg, "passed", 100
   end if
else
   if (x0 == 0) then
      a = vanishes (x)
   else
      a = agreement (x, x0)
   end if
   if (present (threshold)) then
      agreement_threshold = threshold
   else
      agreement_threshold = THRESHOLD_DEFAULT
   end if
   if (present (abs_threshold)) then
      abs_agreement_threshold = abs_threshold
   else
      abs_agreement_threshold = ABS_THRESHOLD_DEFAULT
   end if
   if (a >= agreement_threshold .or. &
       max(abs(x), abs(x0)) <= abs_agreement_threshold) then
      if (verbose) then
         if (a >= THRESHOLD_WARN) then
            write (unit = *, fmt = fmt) msg, "passed", int (a * 100)
         else
            write (unit = *, fmt = fmt_verbose) msg, "passed", int (a * 100), x0, x
         end if
      end if
   else
      a_modulus = agreement (abs (x), abs (x0))
      if (a_modulus >= agreement_threshold) then
         write (unit = *, fmt = fmt_phase) msg, "failed", int (a * 100), &
               int (a_modulus * 100), &
               atan2 (real (x, kind=default), aimag (x)), &
               atan2 (real (x0, kind=default), aimag (x0))
      else
```

```
                write (unit = *, fmt = fmt_verbose) msg, "failed", int (a * 100), x0, x
            end if
            failed = .true.
          end if
      end if
      if (present (passed)) then
          passed = passed .and. .not.failed
      end if
  end subroutine expect_complex
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_real_integer (x, x0, msg, passed, threshold, quiet)
    real(kind=default), intent(in) :: x
    integer, intent(in) :: x0
    character(len=*), intent(in) :: msg
    real(kind=default), intent(in), optional :: threshold
    logical, intent(inout), optional :: passed
    logical, intent(in), optional :: quiet
    call expect_real (x, real (x0, kind=default), msg, passed, threshold, quiet)
  end subroutine expect_real_integer
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_integer_real (x, x0, msg, passed, threshold, quiet)
    integer, intent(in) :: x
    real(kind=default), intent(in) :: x0
    character(len=*), intent(in) :: msg
    real(kind=default), intent(in), optional :: threshold
    logical, intent(inout), optional :: passed
    logical, intent(in), optional :: quiet
    call expect_real (real (x, kind=default), x0, msg, passed, threshold, quiet)
  end subroutine expect_integer_real
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_complex_integer (x, x0, msg, passed, threshold, quiet)
    complex(kind=default), intent(in) :: x
    integer, intent(in) :: x0
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    real(kind=default), intent(in), optional :: threshold
    logical, intent(in), optional :: quiet
    call expect_complex (x, cmplx (x0, kind=default), msg, passed, threshold, quiet)
  end subroutine expect_complex_integer
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_integer_complex (x, x0, msg, passed, threshold, quiet)
    integer, intent(in) :: x
    complex(kind=default), intent(in) :: x0
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    real(kind=default), intent(in), optional :: threshold
    logical, intent(in), optional :: quiet
    call expect_complex (cmplx (x, kind=default), x0, msg, passed, threshold, quiet)
  end subroutine expect_integer_complex
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_complex_real (x, x0, msg, passed, threshold, quiet)
    complex(kind=default), intent(in) :: x
```

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```
    real(kind=default), intent(in) :: x0
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    real(kind=default), intent(in), optional :: threshold
    logical, intent(in), optional :: quiet
    call expect_complex (x, cmplx (x0, kind=default), msg, passed, threshold, quiet)
  end subroutine expect_complex_real
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_real_complex (x, x0, msg, passed, threshold, quiet)
    real(kind=default), intent(in) :: x
    complex(kind=default), intent(in) :: x0
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    real(kind=default), intent(in), optional :: threshold
    logical, intent(in), optional :: quiet
    call expect_complex (cmplx (x, kind=default), x0, msg, passed, threshold, quiet)
  end subroutine expect_real_complex
```

⟨*Declaration of test support functions*⟩+≡
```
  public :: expect_zero
  interface expect_zero
     module procedure expect_zero_integer, expect_zero_real, expect_zero_complex
  end interface
  private :: expect_zero_integer, expect_zero_real, expect_zero_complex
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_zero_integer (x, msg, passed)
    integer, intent(in) :: x
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    call expect_integer (x, 0, msg, passed)
  end subroutine expect_zero_integer
```

⟨*Implementation of test support functions*⟩+≡
```
  subroutine expect_zero_real (x, scale, msg, passed, threshold, quiet)
    real(kind=default), intent(in) :: x, scale
    character(len=*), intent(in) :: msg
    logical, intent(inout), optional :: passed
    real(kind=default), intent(in), optional :: threshold
    logical, intent(in), optional :: quiet
    logical :: failed, verbose
    real(kind=default) :: agreement_threshold
    character(len=*), parameter :: fmt = "(1X,A,': ',A,' at ',I4,'%')"
    character(len=*), parameter :: fmt_verbose = "(1X,A,': ',A,' at ',I4,'%'," // &
         "' [expected 0 (relative to ',E10.3,') got ',E10.3,']')"
    real(kind=default) :: a
    failed = .false.
    verbose = .true.
    if (present (quiet)) then
       verbose = .not.quiet
    end if
    if (x == 0) then
       if (verbose) then
          write (unit = *, fmt = fmt) msg, "passed", 100
       end if
```

```
      else
         a = vanishes (x, scale = scale)
         if (present (threshold)) then
            agreement_threshold = threshold
         else
            agreement_threshold = THRESHOLD_DEFAULT
         end if
         if (a >= agreement_threshold) then
            if (verbose) then
               if (a >= THRESHOLD_WARN) then
                  write (unit = *, fmt = fmt) msg, "passed", int (a * 100)
               else
                  write (unit = *, fmt = fmt_verbose) msg, "passed", int (a * 100), scale, x
               end if
            end if
         else
            failed = .true.
            write (unit = *, fmt = fmt_verbose) msg, "failed", int (a * 100), scale, x
         end if
      end if
      if (present (passed)) then
         passed = passed .and. .not.failed
      end if
   end subroutine expect_zero_real
```

⟨*Implementation of test support functions*⟩+≡
```
   subroutine expect_zero_complex (x, scale, msg, passed, threshold, quiet)
     complex(kind=default), intent(in) :: x
     real(kind=default), intent(in) :: scale
     character(len=*), intent(in) :: msg
     logical, intent(inout), optional :: passed
     real(kind=default), intent(in), optional :: threshold
     logical, intent(in), optional :: quiet
     call expect_zero_real (abs (x), scale, msg, passed, threshold, quiet)
   end subroutine expect_zero_complex
```

⟨*Implementation of test support functions*⟩+≡
```
   subroutine print_matrix (a)
     complex(kind=default), dimension(:,:), intent(in) :: a
     integer :: row
     do row = 1, size (a, dim=1)
        write (unit = *, fmt = "(10(tr2, f5.2, '+', f5.2, 'I'))") a(row,:)
     end do
   end subroutine print_matrix
```

⟨*Declaration of test support functions*⟩+≡
```
   public :: print_matrix
```

⟨test_omega95.f90⟩≡
```
  ⟨Copyleft⟩
  program test_omega95
    use kinds
    use omega95
    use omega_testtools
    implicit none
    real(kind=default) :: m, pabs, qabs, w
```

```
      real(kind=default), dimension(0:3) :: r
      complex(kind=default) :: c_one, c_nil
      type(momentum) :: p, q, p0
      type(vector) :: vp, vq, vtest, v0
      type(tensor) :: ttest
      integer, dimension(8) :: date_time
      integer :: rsize
      logical :: passed
      call date_and_time (values = date_time)
      call random_seed (size = rsize)
      call random_seed (put = spread (product (date_time), dim = 1, ncopies = rsize))
      w = 1.4142
      c_one = 1.0_default
      c_nil = 0.0_default
      m = 13
      pabs = 42
      qabs = 137
      call random_number (r)
      vtest%t = cmplx (10.0_default * r(0), kind=default)
      vtest%x(1:3) = cmplx (10.0_default * r(1:3), kind=default)
      ttest = vtest.tprod.vtest
      call random_momentum (p, pabs, m)
      call random_momentum (q, qabs, m)
      call random_momentum (p0, 0.0_default, m)
      vp = p
      vq = q
      v0 = p0
      passed = .true.
      ⟨Test omega95⟩
      if (.not. passed) then
        stop 1
      end if
    end program test_omega95
⟨Test omega95⟩≡
  print *, "*** Checking the equations of motion ***:"
  call expect (abs(f_vf(c_one,vp,u(m,p,+1))-m*u(m,p,+1)), 0, "|[p-m]u(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,u(m,p,-1))-m*u(m,p,-1)), 0, "|[p-m]u(-)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(m,p,+1))+m*v(m,p,+1)), 0, "|[p+m]v(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(m,p,-1))+m*v(m,p,-1)), 0, "|[p+m]v(-)|=0", passed)
  call expect (abs(f_fv(c_one,ubar(m,p,+1),vp)-m*ubar(m,p,+1)), 0, "|ubar(+)[p-m]|=0", passed)
  call expect (abs(f_fv(c_one,ubar(m,p,-1),vp)-m*ubar(m,p,-1)), 0, "|ubar(-)[p-m]|=0", passed)
  call expect (abs(f_fv(c_one,vbar(m,p,+1),vp)+m*vbar(m,p,+1)), 0, "|vbar(+)[p+m]|=0", passed)
  call expect (abs(f_fv(c_one,vbar(m,p,-1),vp)+m*vbar(m,p,-1)), 0, "|vbar(-)[p+m]|=0", passed)
  print *, "*** Checking the equations of motion for negative mass***:"
  call expect (abs(f_vf(c_one,vp,u(-m,p,+1))+m*u(-m,p,+1)), 0, "|[p+m]u(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,u(-m,p,-1))+m*u(-m,p,-1)), 0, "|[p+m]u(-)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(-m,p,+1))-m*v(-m,p,+1)), 0, "|[p-m]v(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(-m,p,-1))-m*v(-m,p,-1)), 0, "|[p-m]v(-)|=0", passed)
  call expect (abs(f_fv(c_one,ubar(-m,p,+1),vp)+m*ubar(-m,p,+1)), 0, "|ubar(+)[p+m]|=0", passed)
  call expect (abs(f_fv(c_one,ubar(-m,p,-1),vp)+m*ubar(-m,p,-1)), 0, "|ubar(-)[p+m]|=0", passed)
  call expect (abs(f_fv(c_one,vbar(-m,p,+1),vp)-m*vbar(-m,p,+1)), 0, "|vbar(+)[p-m]|=0", passed)
  call expect (abs(f_fv(c_one,vbar(-m,p,-1),vp)-m*vbar(-m,p,-1)), 0, "|vbar(-)[p-m]|=0", passed)

⟨Test omega95⟩+≡
```

```
   print *, "*** Checking the normalization ***:"
   call expect (ubar(m,p,+1)*u(m,p,+1), +2*m, "ubar(+)*u(+)=+2m", passed)
   call expect (ubar(m,p,-1)*u(m,p,-1), +2*m, "ubar(-)*u(-)=+2m", passed)
   call expect (vbar(m,p,+1)*v(m,p,+1), -2*m, "vbar(+)*v(+)=-2m", passed)
   call expect (vbar(m,p,-1)*v(m,p,-1), -2*m, "vbar(-)*v(-)=-2m", passed)
   call expect (ubar(m,p,+1)*v(m,p,+1),    0, "ubar(+)*v(+)=0  ", passed)
   call expect (ubar(m,p,-1)*v(m,p,-1),    0, "ubar(-)*v(-)=0  ", passed)
   call expect (vbar(m,p,+1)*u(m,p,+1),    0, "vbar(+)*u(+)=0  ", passed)
   call expect (vbar(m,p,-1)*u(m,p,-1),    0, "vbar(-)*u(-)=0  ", passed)
   print *, "*** Checking the normalization for negative masses***:"
   call expect (ubar(-m,p,+1)*u(-m,p,+1), -2*m, "ubar(+)*u(+)=-2m", passed)
   call expect (ubar(-m,p,-1)*u(-m,p,-1), -2*m, "ubar(-)*u(-)=-2m", passed)
   call expect (vbar(-m,p,+1)*v(-m,p,+1), +2*m, "vbar(+)*v(+)=+2m", passed)
   call expect (vbar(-m,p,-1)*v(-m,p,-1), +2*m, "vbar(-)*v(-)=+2m", passed)
   call expect (ubar(-m,p,+1)*v(-m,p,+1),    0, "ubar(+)*v(+)=0  ", passed)
   call expect (ubar(-m,p,-1)*v(-m,p,-1),    0, "ubar(-)*v(-)=0  ", passed)
   call expect (vbar(-m,p,+1)*u(-m,p,+1),    0, "vbar(+)*u(+)=0  ", passed)
   call expect (vbar(-m,p,-1)*u(-m,p,-1),    0, "vbar(-)*u(-)=0  ", passed)
```

⟨*Test* omega95⟩+≡
```
  print *, "*** Checking the currents ***:"
  call expect (abs(v_ff(c_one,ubar(m,p,+1),u(m,p,+1))-2*vp), 0, "ubar(+).V.u(+)=2p", passed)
  call expect (abs(v_ff(c_one,ubar(m,p,-1),u(m,p,-1))-2*vp), 0, "ubar(-).V.u(-)=2p", passed)
  call expect (abs(v_ff(c_one,vbar(m,p,+1),v(m,p,+1))-2*vp), 0, "vbar(+).V.v(+)=2p", passed)
  call expect (abs(v_ff(c_one,vbar(m,p,-1),v(m,p,-1))-2*vp), 0, "vbar(-).V.v(-)=2p", passed)
  print *, "*** Checking the currents for negative masses***:"
  call expect (abs(v_ff(c_one,ubar(-m,p,+1),u(-m,p,+1))-2*vp), 0, "ubar(+).V.u(+)=2p", passed)
  call expect (abs(v_ff(c_one,ubar(-m,p,-1),u(-m,p,-1))-2*vp), 0, "ubar(-).V.u(-)=2p", passed)
  call expect (abs(v_ff(c_one,vbar(-m,p,+1),v(-m,p,+1))-2*vp), 0, "vbar(+).V.v(+)=2p", passed)
  call expect (abs(v_ff(c_one,vbar(-m,p,-1),v(-m,p,-1))-2*vp), 0, "vbar(-).V.v(-)=2p", passed)
```

⟨*Test* omega95⟩+≡
```
  print *, "*** Checking current conservation ***:"
  call expect ((vp-vq)*v_ff(c_one,ubar(m,p,+1),u(m,q,+1)), 0, "d(ubar(+).V.u(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,ubar(m,p,-1),u(m,q,-1)), 0, "d(ubar(-).V.u(-))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,vbar(m,p,+1),v(m,q,+1)), 0, "d(vbar(+).V.v(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,vbar(m,p,-1),v(m,q,-1)), 0, "d(vbar(-).V.v(-))=0", passed)
  print *, "*** Checking current conservation for negative masses***:"
  call expect ((vp-vq)*v_ff(c_one,ubar(-m,p,+1),u(-m,q,+1)), 0, "d(ubar(+).V.u(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,ubar(-m,p,-1),u(-m,q,-1)), 0, "d(ubar(-).V.u(-))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,vbar(-m,p,+1),v(-m,q,+1)), 0, "d(vbar(+).V.v(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,vbar(-m,p,-1),v(-m,q,-1)), 0, "d(vbar(-).V.v(-))=0", passed)
```

⟨*Test* omega95⟩+≡
```
  if (m == 0) then
     print *, "*** Checking axial current conservation ***:"
     call expect ((vp-vq)*a_ff(c_one,ubar(m,p,+1),u(m,q,+1)), 0, "d(ubar(+).A.u(+))=0", passed)
     call expect ((vp-vq)*a_ff(c_one,ubar(m,p,-1),u(m,q,-1)), 0, "d(ubar(-).A.u(-))=0", passed)
     call expect ((vp-vq)*a_ff(c_one,vbar(m,p,+1),v(m,q,+1)), 0, "d(vbar(+).A.v(+))=0", passed)
     call expect ((vp-vq)*a_ff(c_one,vbar(m,p,-1),v(m,q,-1)), 0, "d(vbar(-).A.v(-))=0", passed)
  end if
```

⟨*Test* omega95⟩+≡
```
  print *, "*** Checking implementation of the sigma vertex funktions ***:"
  call expect ((vp*tvam_ff(c_one,c_nil,ubar(m,p,+1),u(m,q,+1),q) - (p*q-m**2)*(ubar(m,p,+1)*u(m,q,+1
            "p*[ubar(p,+).(Isigma*q).u(q,+)] - (p*q-m^2)*ubar(p,+).u(q,+) = 0", passed)
```

```
    call expect ((vp*tvam_ff(c_one,c_nil,ubar(m,p,-1),u(m,q,-1),q) - (p*q-m**2)*(ubar(m,p,-1)*u(m,q,-1
                "p*[ubar(p,-).(Isigma*q).u(q,-)] - (p*q-m^2)*ubar(p,-).u(q,-) = 0", passed)
    call expect ((vp*tvam_ff(c_one,c_nil,vbar(m,p,+1),v(m,q,+1),q) - (p*q-m**2)*(vbar(m,p,+1)*v(m,q,+1
                "p*[vbar(p,+).(Isigma*q).v(q,+)] - (p*q-m^2)*vbar(p,+).v(q,+) = 0", passed)
    call expect ((vp*tvam_ff(c_one,c_nil,vbar(m,p,-1),v(m,q,-1),q) - (p*q-m**2)*(vbar(m,p,-1)*v(m,q,-1
                "p*[vbar(p,-).(Isigma*q).v(q,-)] - (p*q-m^2)*vbar(p,-).v(q,-) = 0", passed)
    call expect ((ubar(m,p,+1)*f_tvamf(c_one,c_nil,vp,u(m,q,+1),q) - (p*q-m**2)*(ubar(m,p,+1)*u(m,q,+1
                "ubar(p,+).[p*(Isigma*q).u(q,+)] - (p*q-m^2)*ubar(p,+).u(q,+) = 0", passed)
    call expect ((ubar(m,p,-1)*f_tvamf(c_one,c_nil,vp,u(m,q,-1),q) - (p*q-m**2)*(ubar(m,p,-1)*u(m,q,-1
                "ubar(p,-).[p*(Isigma*q).u(q,-)] - (p*q-m^2)*ubar(p,-).u(q,-) = 0", passed)
    call expect ((vbar(m,p,+1)*f_tvamf(c_one,c_nil,vp,v(m,q,+1),q) - (p*q-m**2)*(vbar(m,p,+1)*v(m,q,+1
                "vbar(p,+).[p*(Isigma*q).v(q,+)] - (p*q-m^2)*vbar(p,+).v(q,+) = 0", passed)
    call expect ((vbar(m,p,-1)*f_tvamf(c_one,c_nil,vp,v(m,q,-1),q) - (p*q-m**2)*(vbar(m,p,-1)*v(m,q,-1
                "vbar(p,-).[p*(Isigma*q).v(q,-)] - (p*q-m^2)*vbar(p,-).v(q,-) = 0", passed)
    call expect ((f_ftvam(c_one,c_nil,ubar(m,p,+1),vp,q)*u(m,q,+1) - (p*q-m**2)*(ubar(m,p,+1)*u(m,q,+1
                "[ubar(p,+).p*(Isigma*q)].u(q,+) - (p*q-m^2)*ubar(p,+).u(q,+) = 0", passed)
    call expect ((f_ftvam(c_one,c_nil,ubar(m,p,-1),vp,q)*u(m,q,-1) - (p*q-m**2)*(ubar(m,p,-1)*u(m,q,-1
                "[ubar(p,-).p*(Isigma*q)].u(q,-) - (p*q-m^2)*ubar(p,-).u(q,-) = 0", passed)
    call expect ((f_ftvam(c_one,c_nil,vbar(m,p,+1),vp,q)*v(m,q,+1) - (p*q-m**2)*(vbar(m,p,+1)*v(m,q,+1
                "[vbar(p,+).p*(Isigma*q)].v(q,+) - (p*q-m^2)*vbar(p,+).v(q,+) = 0", passed)
    call expect ((f_ftvam(c_one,c_nil,vbar(m,p,-1),vp,q)*v(m,q,-1) - (p*q-m**2)*(vbar(m,p,-1)*v(m,q,-1
                "[vbar(p,-).p*(Isigma*q)].v(q,-) - (p*q-m^2)*vbar(p,-).v(q,-) = 0", passed)

    call expect ((vp*tvam_ff(c_nil,c_one,ubar(m,p,+1),u(m,q,+1),q) - (p*q+m**2)*p_ff(c_one,ubar(m,p,+1
                "p*[ubar(p,+).(Isigma*q).g5.u(q,+)] - (p*q+m^2)*ubar(p,+).g5.u(q,+) = 0", passed)
    call expect ((vp*tvam_ff(c_nil,c_one,ubar(m,p,-1),u(m,q,-1),q) - (p*q+m**2)*p_ff(c_one,ubar(m,p,-1
                "p*[ubar(p,-).(Isigma*q).g5.u(q,-)] - (p*q+m^2)*ubar(p,-).g5.u(q,-) = 0", passed)
    call expect ((vp*tvam_ff(c_nil,c_one,vbar(m,p,+1),v(m,q,+1),q) - (p*q+m**2)*p_ff(c_one,vbar(m,p,+1
                "p*[vbar(p,+).(Isigma*q).g5.v(q,+)] - (p*q+m^2)*vbar(p,+).g5.v(q,+) = 0", passed)
    call expect ((vp*tvam_ff(c_nil,c_one,vbar(m,p,-1),v(m,q,-1),q) - (p*q+m**2)*p_ff(c_one,vbar(m,p,-1
                "p*[vbar(p,-).(Isigma*q).g5.v(q,-)] - (p*q+m^2)*vbar(p,-).g5.v(q,-) = 0", passed)
    call expect ((ubar(m,p,+1)*f_tvamf(c_nil,c_one,vp,u(m,q,+1),q) - (p*q+m**2)*p_ff(c_one,ubar(m,p,+1
                "p*[ubar(p,+).(Isigma*q).g5.u(q,+)] - (p*q+m^2)*ubar(p,+).g5.u(q,+) = 0", passed)
    call expect ((ubar(m,p,-1)*f_tvamf(c_nil,c_one,vp,u(m,q,-1),q) - (p*q+m**2)*p_ff(c_one,ubar(m,p,-1
                "p*[ubar(p,-).(Isigma*q).g5.u(q,-)] - (p*q+m^2)*ubar(p,-).g5.u(q,-) = 0", passed)
    call expect ((vbar(m,p,+1)*f_tvamf(c_nil,c_one,vp,v(m,q,+1),q) - (p*q+m**2)*p_ff(c_one,vbar(m,p,+1
                "p*[vbar(p,+).(Isigma*q).g5.v(q,+)] - (p*q+m^2)*vbar(p,+).g5.v(q,+) = 0", passed)
    call expect ((vbar(m,p,-1)*f_tvamf(c_nil,c_one,vp,v(m,q,-1),q) - (p*q+m**2)*p_ff(c_one,vbar(m,p,-1
                "p*[vbar(p,-).(Isigma*q).g5.v(q,-)] - (p*q+m^2)*vbar(p,-).g5.v(q,-) = 0", passed)
    call expect ((f_ftvam(c_nil,c_one,ubar(m,p,+1),vp,q)*u(m,q,+1) - (p*q+m**2)*p_ff(c_one,ubar(m,p,+1
                "p*[ubar(p,+).(Isigma*q).g5.u(q,+)] - (p*q+m^2)*ubar(p,+).g5.u(q,+) = 0", passed)
    call expect ((f_ftvam(c_nil,c_one,ubar(m,p,-1),vp,q)*u(m,q,-1) - (p*q+m**2)*p_ff(c_one,ubar(m,p,-1
                "p*[ubar(p,-).(Isigma*q).g5.u(q,-)] - (p*q+m^2)*ubar(p,-).g5.u(q,-) = 0", passed)
    call expect ((f_ftvam(c_nil,c_one,vbar(m,p,+1),vp,q)*v(m,q,+1) - (p*q+m**2)*p_ff(c_one,vbar(m,p,+1
                "p*[vbar(p,+).(Isigma*q).g5.v(q,+)] - (p*q+m^2)*vbar(p,+).g5.v(q,+) = 0", passed)
    call expect ((f_ftvam(c_nil,c_one,vbar(m,p,-1),vp,q)*v(m,q,-1) - (p*q+m**2)*p_ff(c_one,vbar(m,p,-1
                "p*[vbar(p,-).(Isigma*q).g5.v(q,-)] - (p*q+m^2)*vbar(p,-).g5.v(q,-) = 0", passed)
```

⟨*Test* omega95⟩+≡

```
  print *, "*** Checking polarisation vectors: ***"
  call expect (conjg(eps(m,p, 1))*eps(m,p, 1), -1, "e( 1).e( 1)=-1", passed)
  call expect (conjg(eps(m,p, 1))*eps(m,p,-1),  0, "e( 1).e(-1)= 0", passed)
  call expect (conjg(eps(m,p,-1))*eps(m,p, 1),  0, "e(-1).e( 1)= 0", passed)
  call expect (conjg(eps(m,p,-1))*eps(m,p,-1), -1, "e(-1).e(-1)=-1", passed)
  call expect (                 p*eps(m,p, 1),  0, "   p.e( 1)= 0", passed)
```

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```
      call expect (                    p*eps(m,p,-1),  0, "    p.e(-1)= 0", passed)
   if (m > 0) then
      call expect (conjg(eps(m,p, 1))*eps(m,p, 0),  0, "e( 1).e( 0)= 0", passed)
      call expect (conjg(eps(m,p, 0))*eps(m,p, 1),  0, "e( 0).e( 1)= 0", passed)
      call expect (conjg(eps(m,p, 0))*eps(m,p, 0), -1, "e( 0).e( 0)=-1", passed)
      call expect (conjg(eps(m,p, 0))*eps(m,p,-1),  0, "e( 0).e(-1)= 0", passed)
      call expect (conjg(eps(m,p,-1))*eps(m,p, 0),  0, "e(-1).e( 0)= 0", passed)
      call expect (                    p*eps(m,p, 0),  0, "    p.e( 0)= 0", passed)
   end if
```

⟨*Test* omega95⟩+≡
```
   print *, "*** Checking epsilon tensor: ***"
   call expect (  pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,p,0),eps(m,q,0)), &
                - pseudo_scalar(eps(m,q,1),eps(m,p,1),eps(m,p,0),eps(m,q,0)), "eps(1<->2)", passed)
   call expect (  pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,p,0),eps(m,q,0)), &
                - pseudo_scalar(eps(m,p,0),eps(m,q,1),eps(m,p,1),eps(m,q,0)), "eps(1<->3)", passed)
   call expect (  pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,p,0),eps(m,q,0)), &
                - pseudo_scalar(eps(m,q,0),eps(m,q,1),eps(m,p,0),eps(m,p,1)), "eps(1<->4)", passed)
   call expect (  pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,p,0),eps(m,q,0)), &
                - pseudo_scalar(eps(m,p,1),eps(m,p,0),eps(m,q,1),eps(m,q,0)), "eps(2<->3)", passed)
   call expect (  pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,p,0),eps(m,q,0)), &
                - pseudo_scalar(eps(m,p,1),eps(m,q,0),eps(m,p,0),eps(m,q,1)), "eps(2<->4)", passed)
   call expect (  pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,p,0),eps(m,q,0)), &
                - pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,q,0),eps(m,p,0)), "eps(3<->4)", passed)
   call expect (  pseudo_scalar(eps(m,p,1),eps(m,q,1),eps(m,p,0),eps(m,q,0)), &
                eps(m,p,1)*pseudo_vector(eps(m,q,1),eps(m,p,0),eps(m,q,0)), "eps'", passed)
```

$$\frac{1}{2}[x \wedge y]^*_{\mu\nu}[x \wedge y]^{\mu\nu} = \frac{1}{2}(x^*_\mu y^*_\nu - x^*_\nu y^*_\mu)(x^\mu y^\nu - x^\nu y^\mu) = (x^*x)(y^*y) - (x^*y)(y^*x) \tag{X.130}$$

⟨*Test* omega95⟩+≡
```
   print *, "*** Checking tensors: ***"
   call expect (conjg(p.wedge.q)*(p.wedge.q), (p*p)*(q*q)-(p*q)**2, &
        "[p,q].[q,p]=p.p*q.q-p.q^2", passed)
   call expect (conjg(p.wedge.q)*(q.wedge.p), (p*q)**2-(p*p)*(q*q), &
        "[p,q].[q,p]=p.q^2-p.p*q.q", passed)
```

i. e.
$$\frac{1}{2}[p \wedge \epsilon(p,i)]^*_{\mu\nu}[p \wedge \epsilon(p,j)]^{\mu\nu} = -p^2\delta_{ij} \tag{X.131}$$

⟨*Test* omega95⟩+≡
```
   call expect (conjg(p.wedge.eps(m,p, 1))*(p.wedge.eps(m,p, 1)), -p*p, &
        "[p,e( 1)].[p,e( 1)]=-p.p", passed)
   call expect (conjg(p.wedge.eps(m,p, 1))*(p.wedge.eps(m,p,-1)),    0, &
        "[p,e( 1)].[p,e(-1)]=0", passed)
   call expect (conjg(p.wedge.eps(m,p,-1))*(p.wedge.eps(m,p, 1)),    0, &
        "[p,e(-1)].[p,e( 1)]=0", passed)
   call expect (conjg(p.wedge.eps(m,p,-1))*(p.wedge.eps(m,p,-1)), -p*p, &
        "[p,e(-1)].[p,e(-1)]=-p.p", passed)
   if (m > 0) then
      call expect (conjg(p.wedge.eps(m,p, 1))*(p.wedge.eps(m,p, 0)),    0, &
           "[p,e( 1)].[p,e( 0)]=0", passed)
      call expect (conjg(p.wedge.eps(m,p, 0))*(p.wedge.eps(m,p, 1)),    0, &
           "[p,e( 0)].[p,e( 1)]=0", passed)
```

```
      call expect (conjg(p.wedge.eps(m,p, 0))*(p.wedge.eps(m,p, 0)), -p*p, &
           "[p,e( 0)].[p,e( 0)]=-p.p", passed)
      call expect (conjg(p.wedge.eps(m,p, 0))*(p.wedge.eps(m,p,-1)),    0, &
           "[p,e( 1)].[p,e(-1)]=0", passed)
      call expect (conjg(p.wedge.eps(m,p,-1))*(p.wedge.eps(m,p, 0)),    0, &
           "[p,e(-1)].[p,e( 0)]=0", passed)
   end if
also
```

$$[x \wedge y]_{\mu\nu}z^{\nu} = x_{\mu}(yz) - y_{\mu}(xz) \qquad (\text{X.132})$$
$$z_{\mu}[x \wedge y]^{\mu\nu} = (zx)y^{\nu} - (zy)x^{\nu} \qquad (\text{X.133})$$

$\langle$ *Test* omega95$\rangle$+$\equiv$
```
   call expect (abs ((p.wedge.eps(m,p, 1))*p + (p*p)*eps(m,p, 1)), 0, &
        "[p,e( 1)].p=-p.p*e( 1)]", passed)
   call expect (abs ((p.wedge.eps(m,p, 0))*p + (p*p)*eps(m,p, 0)), 0, &
        "[p,e( 0)].p=-p.p*e( 0)]", passed)
   call expect (abs ((p.wedge.eps(m,p,-1))*p + (p*p)*eps(m,p,-1)), 0, &
        "[p,e(-1)].p=-p.p*e(-1)]", passed)
   call expect (abs (p*(p.wedge.eps(m,p, 1)) - (p*p)*eps(m,p, 1)), 0, &
        "p.[p,e( 1)]=p.p*e( 1)]", passed)
   call expect (abs (p*(p.wedge.eps(m,p, 0)) - (p*p)*eps(m,p, 0)), 0, &
        "p.[p,e( 0)]=p.p*e( 0)]", passed)
   call expect (abs (p*(p.wedge.eps(m,p,-1)) - (p*p)*eps(m,p,-1)), 0, &
        "p.[p,e(-1)]=p.p*e(-1)]", passed)
```

$\langle$ *Test* omega95$\rangle$+$\equiv$
```
   print *, "*** Checking polarisation tensors: ***"
   call expect (conjg(eps2(m,p, 2))*eps2(m,p, 2), 1, "e2( 2).e2( 2)=1", passed)
   call expect (conjg(eps2(m,p, 2))*eps2(m,p,-2), 0, "e2( 2).e2(-2)=0", passed)
   call expect (conjg(eps2(m,p,-2))*eps2(m,p, 2), 0, "e2(-2).e2( 2)=0", passed)
   call expect (conjg(eps2(m,p,-2))*eps2(m,p,-2), 1, "e2(-2).e2(-2)=1", passed)
   if (m > 0) then
      call expect (conjg(eps2(m,p, 2))*eps2(m,p, 1), 0, "e2( 2).e2( 1)=0", passed)
      call expect (conjg(eps2(m,p, 2))*eps2(m,p, 0), 0, "e2( 2).e2( 0)=0", passed)
      call expect (conjg(eps2(m,p, 2))*eps2(m,p,-1), 0, "e2( 2).e2(-1)=0", passed)
      call expect (conjg(eps2(m,p, 1))*eps2(m,p, 2), 0, "e2( 1).e2( 2)=0", passed)
      call expect (conjg(eps2(m,p, 1))*eps2(m,p, 1), 1, "e2( 1).e2( 1)=1", passed)
      call expect (conjg(eps2(m,p, 1))*eps2(m,p, 0), 0, "e2( 1).e2( 0)=0", passed)
      call expect (conjg(eps2(m,p, 1))*eps2(m,p,-1), 0, "e2( 1).e2(-1)=0", passed)
      call expect (conjg(eps2(m,p, 1))*eps2(m,p,-2), 0, "e2( 1).e2(-2)=0", passed)
      call expect (conjg(eps2(m,p, 0))*eps2(m,p, 2), 0, "e2( 0).e2( 2)=0", passed)
      call expect (conjg(eps2(m,p, 0))*eps2(m,p, 1), 0, "e2( 0).e2( 1)=0", passed)
      call expect (conjg(eps2(m,p, 0))*eps2(m,p, 0), 1, "e2( 0).e2( 0)=1", passed)
      call expect (conjg(eps2(m,p, 0))*eps2(m,p,-1), 0, "e2( 0).e2(-1)=0", passed)
      call expect (conjg(eps2(m,p, 0))*eps2(m,p,-2), 0, "e2( 0).e2(-2)=0", passed)
      call expect (conjg(eps2(m,p,-1))*eps2(m,p, 2), 0, "e2(-1).e2( 2)=0", passed)
      call expect (conjg(eps2(m,p,-1))*eps2(m,p, 1), 0, "e2(-1).e2( 1)=0", passed)
      call expect (conjg(eps2(m,p,-1))*eps2(m,p, 0), 0, "e2(-1).e2( 0)=0", passed)
      call expect (conjg(eps2(m,p,-1))*eps2(m,p,-1), 1, "e2(-1).e2(-1)=1", passed)
      call expect (conjg(eps2(m,p,-1))*eps2(m,p,-2), 0, "e2(-1).e2(-2)=0", passed)
      call expect (conjg(eps2(m,p,-2))*eps2(m,p, 1), 0, "e2(-2).e2( 1)=0", passed)
      call expect (conjg(eps2(m,p,-2))*eps2(m,p, 0), 0, "e2(-2).e2( 0)=0", passed)
      call expect (conjg(eps2(m,p,-2))*eps2(m,p,-1), 0, "e2(-2).e2(-1)=0", passed)
   end if
```

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⟨*Test* omega95⟩+≡

```
  call expect (             abs(p*eps2(m,p, 2)  ), 0, " |p.e2( 2)|  =0", passed)
  call expect (              abs(eps2(m,p, 2)*p), 0, "    |e2( 2).p|=0", passed)
  call expect (             abs(p*eps2(m,p,-2)  ), 0, " |p.e2(-2)|  =0", passed)
  call expect (              abs(eps2(m,p,-2)*p), 0, "    |e2(-2).p|=0", passed)
  if (m > 0) then
     call expect (             abs(p*eps2(m,p, 1)  ), 0, " |p.e2( 1)|  =0", passed)
     call expect (              abs(eps2(m,p, 1)*p), 0, "    |e2( 1).p|=0", passed)
     call expect (             abs(p*eps2(m,p, 0)  ), 0, " |p.e2( 0)|  =0", passed)
     call expect (              abs(eps2(m,p, 0)*p), 0, "    |e2( 0).p|=0", passed)
     call expect (             abs(p*eps2(m,p,-1)  ), 0, " |p.e2(-1)|  =0", passed)
     call expect (              abs(eps2(m,p,-1)*p), 0, "    |e2(-1).p|=0", passed)
  end if
```

⟨*XXX Test* omega95⟩≡

```
  print *, " *** Checking the polarization tensors for massive gravitons:"
  call expect (abs(p * eps2(m,p,2)),  0, "p.e(+2)=0", passed)
  call expect (abs(p * eps2(m,p,1)),  0, "p.e(+1)=0", passed)
  call expect (abs(p * eps2(m,p,0)),  0, "p.e( 0)=0", passed)
  call expect (abs(p * eps2(m,p,-1)), 0, "p.e(-1)=0", passed)
  call expect (abs(p * eps2(m,p,-2)), 0, "p.e(-2)=0", passed)
  call expect (abs(trace(eps2 (m,p,2))),  0, "Tr[e(+2)]=0", passed)
  call expect (abs(trace(eps2 (m,p,1))),  0, "Tr[e(+1)]=0", passed)
  call expect (abs(trace(eps2 (m,p,0))),  0, "Tr[e( 0)]=0", passed)
  call expect (abs(trace(eps2 (m,p,-1))), 0, "Tr[e(-1)]=0", passed)
  call expect (abs(trace(eps2 (m,p,-2))), 0, "Tr[e(-2)]=0", passed)
  call expect (abs(eps2(m,p,2) * eps2(m,p,2)),   1, &
       "e(2).e(2)   = 1", passed)
  call expect (abs(eps2(m,p,2) * eps2(m,p,1)),   0, &
       "e(2).e(1)   = 0", passed)
  call expect (abs(eps2(m,p,2) * eps2(m,p,0)),   0, &
       "e(2).e(0)   = 0", passed)
  call expect (abs(eps2(m,p,2) * eps2(m,p,-1)),  0, &
       "e(2).e(-1)  = 0", passed)
  call expect (abs(eps2(m,p,2) * eps2(m,p,-2)),  0, &
       "e(2).e(-2)  = 0", passed)
  call expect (abs(eps2(m,p,1) * eps2(m,p,1)),   1, &
       "e(1).e(1)   = 1", passed)
  call expect (abs(eps2(m,p,1) * eps2(m,p,0)),   0, &
       "e(1).e(0)   = 0", passed)
  call expect (abs(eps2(m,p,1) * eps2(m,p,-1)),  0, &
       "e(1).e(-1)  = 0", passed)
  call expect (abs(eps2(m,p,1) * eps2(m,p,-2)),  0, &
       "e(1).e(-2)  = 0", passed)
  call expect (abs(eps2(m,p,0) * eps2(m,p,0)),   1, &
       "e(0).e(0)   = 1", passed)
  call expect (abs(eps2(m,p,0) * eps2(m,p,-1)),  0, &
       "e(0).e(-1)  = 0", passed)
  call expect (abs(eps2(m,p,0) * eps2(m,p,-2)),  0, &
       "e(0).e(-2)  = 0", passed)
  call expect (abs(eps2(m,p,-1) * eps2(m,p,-1)), 1, &
       "e(-1).e(-1) = 1", passed)
  call expect (abs(eps2(m,p,-1) * eps2(m,p,-2)), 0, &
       "e(-1).e(-2) = 0", passed)
  call expect (abs(eps2(m,p,-2) * eps2(m,p,-2)), 1, &
```

```
         "e(-2).e(-2) = 1", passed)
```

⟨*Test* omega95⟩+≡

```
  print *, " *** Checking the graviton propagator:"
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
              pr_tensor(p,m,w,eps2(m,p,-2)))), 0, "p.pr.e(-2)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
              pr_tensor(p,m,w,eps2(m,p,-1)))), 0, "p.pr.e(-1)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
              pr_tensor(p,m,w,eps2(m,p,0)))), 0, "p.pr.e(0)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
              pr_tensor(p,m,w,eps2(m,p,1)))), 0, "p.pr.e(1)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
              pr_tensor(p,m,w,eps2(m,p,2)))), 0, "p.pr.e(2)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
              pr_tensor(p,m,w,ttest))), 0, "p.pr.ttest", passed)
```

⟨test_omega95_bispinors.f90⟩≡

```
  ⟨Copyleft⟩
  program test_omega95_bispinors
    use kinds
    use omega95_bispinors
    use omega_vspinor_polarizations
    use omega_testtools
    implicit none
    integer :: i, j
    real(kind=default) :: m, pabs, qabs, tabs, zabs, w
    real(kind=default), dimension(4) :: r
    complex(kind=default) :: c_one, c_two
    type(momentum) :: p, q, t, z, p_0
    type(vector) :: vp, vq, vt, vz
    type(vectorspinor) :: testv
    type(bispinor) :: vv
    logical :: passed
    call random_seed ()
    c_one = 1
    c_two = 2
    w = 1.4142
    m = 13
    pabs = 42
    qabs = 137
    tabs = 84
    zabs = 3.1415
    p_0%t = m
    p_0%x = 0
    call random_momentum (p, pabs, m)
    call random_momentum (q, qabs, m)
    call random_momentum (t, tabs, m)
    call random_momentum (z, zabs, m)
    call random_number (r)
    do i = 1, 4
       testv%psi(1)%a(i) = (0.0_default, 0.0_default)
    end do
    do i = 2, 3
       do j = 1, 4
```

```
          testv%psi(i)%a(j) = cmplx (10.0_default * r(j))
        end do
      end do
      testv%psi(4)%a(1) = (1.0_default, 0.0_default)
      testv%psi(4)%a(2) = (0.0_default, 2.0_default)
      testv%psi(4)%a(3) = (1.0_default, 0.0_default)
      testv%psi(4)%a(4) = (3.0_default, 0.0_default)
      vp = p
      vq = q
      vt = t
      vz = z
      passed = .true.
      vv%a(1) = (1.0_default, 0.0_default)
      vv%a(2) = (0.0_default, 2.0_default)
      vv%a(3) = (1.0_default, 0.0_default)
      vv%a(4) = (3.0_default, 0.0_default)
      vv = pr_psi(p, m, w, vv)
```
⟨*Test* omega95_bispinors⟩
```
      if (.not. passed) then
        stop 1
      end if
    end program test_omega95_bispinors
```

⟨*Test* omega95_bispinors⟩≡
```
  print *, "*** Checking the equations of motion ***:"
  call expect (abs(f_vf(c_one,vp,u(m,p,+1))-m*u(m,p,+1)), 0, "|[p-m]u(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,u(m,p,-1))-m*u(m,p,-1)), 0, "|[p-m]u(-)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(m,p,+1))+m*v(m,p,+1)), 0, "|[p+m]v(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(m,p,-1))+m*v(m,p,-1)), 0, "|[p+m]v(-)|=0", passed)
  print *, "*** Checking the equations of motion for negative masses***:"
  call expect (abs(f_vf(c_one,vp,u(-m,p,+1))+m*u(-m,p,+1)), 0, "|[p+m]u(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,u(-m,p,-1))+m*u(-m,p,-1)), 0, "|[p+m]u(-)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(-m,p,+1))-m*v(-m,p,+1)), 0, "|[p-m]v(+)|=0", passed)
  call expect (abs(f_vf(c_one,vp,v(-m,p,-1))-m*v(-m,p,-1)), 0, "|[p-m]v(-)|=0", passed)
```

⟨*Test* omega95_bispinors⟩+≡
```
  print *, "*** Checking the normalization ***:"
  call expect (s_ff(c_one,v(m,p,+1),u(m,p,+1)), +2*m, "ubar(+)*u(+)=+2m", passed)
  call expect (s_ff(c_one,v(m,p,-1),u(m,p,-1)), +2*m, "ubar(-)*u(-)=+2m", passed)
  call expect (s_ff(c_one,u(m,p,+1),v(m,p,+1)), -2*m, "vbar(+)*v(+)=-2m", passed)
  call expect (s_ff(c_one,u(m,p,-1),v(m,p,-1)), -2*m, "vbar(-)*v(-)=-2m", passed)
  call expect (s_ff(c_one,v(m,p,+1),v(m,p,+1)),    0, "ubar(+)*v(+)=0  ", passed)
  call expect (s_ff(c_one,v(m,p,-1),v(m,p,-1)),    0, "ubar(-)*v(-)=0  ", passed)
  call expect (s_ff(c_one,u(m,p,+1),u(m,p,+1)),    0, "vbar(+)*u(+)=0  ", passed)
  call expect (s_ff(c_one,u(m,p,-1),u(m,p,-1)),    0, "vbar(-)*u(-)=0  ", passed)
  print *, "*** Checking the normalization for negative masses***:"
  call expect (s_ff(c_one,v(-m,p,+1),u(-m,p,+1)), -2*m, "ubar(+)*u(+)=-2m", passed)
  call expect (s_ff(c_one,v(-m,p,-1),u(-m,p,-1)), -2*m, "ubar(-)*u(-)=-2m", passed)
  call expect (s_ff(c_one,u(-m,p,+1),v(-m,p,+1)), +2*m, "vbar(+)*v(+)=+2m", passed)
  call expect (s_ff(c_one,u(-m,p,-1),v(-m,p,-1)), +2*m, "vbar(-)*v(-)=+2m", passed)
  call expect (s_ff(c_one,v(-m,p,+1),v(-m,p,+1)),    0, "ubar(+)*v(+)=0  ", passed)
  call expect (s_ff(c_one,v(-m,p,-1),v(-m,p,-1)),    0, "ubar(-)*v(-)=0  ", passed)
  call expect (s_ff(c_one,u(-m,p,+1),u(-m,p,+1)),    0, "vbar(+)*u(+)=0  ", passed)
  call expect (s_ff(c_one,u(-m,p,-1),u(-m,p,-1)),    0, "vbar(-)*u(-)=0  ", passed)
```

⟨*Test* omega95_bispinors⟩+≡

```
  print *, "*** Checking the currents ***:"
  call expect (abs(v_ff(c_one,v(m,p,+1),u(m,p,+1))-2*vp), 0, "ubar(+).V.u(+)=2p", passed)
  call expect (abs(v_ff(c_one,v(m,p,-1),u(m,p,-1))-2*vp), 0, "ubar(-).V.u(-)=2p", passed)
  call expect (abs(v_ff(c_one,u(m,p,+1),v(m,p,+1))-2*vp), 0, "vbar(+).V.v(+)=2p", passed)
  call expect (abs(v_ff(c_one,u(m,p,-1),v(m,p,-1))-2*vp), 0, "vbar(-).V.v(-)=2p", passed)
  print *, "*** Checking the currents for negative masses***:"
  call expect (abs(v_ff(c_one,v(-m,p,+1),u(-m,p,+1))-2*vp), 0, "ubar(+).V.u(+)=2p", passed)
  call expect (abs(v_ff(c_one,v(-m,p,-1),u(-m,p,-1))-2*vp), 0, "ubar(-).V.u(-)=2p", passed)
  call expect (abs(v_ff(c_one,u(-m,p,+1),v(-m,p,+1))-2*vp), 0, "vbar(+).V.v(+)=2p", passed)
  call expect (abs(v_ff(c_one,u(-m,p,-1),v(-m,p,-1))-2*vp), 0, "vbar(-).V.v(-)=2p", passed)
```

⟨*Test* omega95_bispinors⟩+≡
```
  print *, "*** Checking current conservation ***:"
  call expect ((vp-vq)*v_ff(c_one,v(m,p,+1),u(m,q,+1)), 0, "d(ubar(+).V.u(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,v(m,p,-1),u(m,q,-1)), 0, "d(ubar(-).V.u(-))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,u(m,p,+1),v(m,q,+1)), 0, "d(vbar(+).V.v(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,u(m,p,-1),v(m,q,-1)), 0, "d(vbar(-).V.v(-))=0", passed)
```

⟨*Test* omega95_bispinors⟩+≡
```
  print *, "*** Checking current conservation for negative masses***:"
  call expect ((vp-vq)*v_ff(c_one,v(-m,p,+1),u(-m,q,+1)), 0, "d(ubar(+).V.u(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,v(-m,p,-1),u(-m,q,-1)), 0, "d(ubar(-).V.u(-))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,u(-m,p,+1),v(-m,q,+1)), 0, "d(vbar(+).V.v(+))=0", passed)
  call expect ((vp-vq)*v_ff(c_one,u(-m,p,-1),v(-m,q,-1)), 0, "d(vbar(-).V.v(-))=0", passed)
```

⟨*Test* omega95_bispinors⟩+≡
```
  if (m == 0) then
     print *, "*** Checking axial current conservation ***:"
     call expect ((vp-vq)*a_ff(c_one,v(m,p,+1),u(m,q,+1)), 0, "d(ubar(+).A.u(+))=0", passed)
     call expect ((vp-vq)*a_ff(c_one,v(m,p,-1),u(m,q,-1)), 0, "d(ubar(-).A.u(-))=0", passed)
     call expect ((vp-vq)*a_ff(c_one,u(m,p,+1),v(m,q,+1)), 0, "d(vbar(+).A.v(+))=0", passed)
     call expect ((vp-vq)*a_ff(c_one,u(m,p,-1),v(m,q,-1)), 0, "d(vbar(-).A.v(-))=0", passed)
  end if
```

⟨*Test* omega95_bispinors⟩+≡
```
  print *, "*** Checking polarization vectors: ***"
  call expect (conjg(eps(m,p, 1))*eps(m,p, 1), -1, "e( 1).e( 1)=-1", passed)
  call expect (conjg(eps(m,p, 1))*eps(m,p,-1),  0, "e( 1).e(-1)= 0", passed)
  call expect (conjg(eps(m,p,-1))*eps(m,p, 1),  0, "e(-1).e( 1)= 0", passed)
  call expect (conjg(eps(m,p,-1))*eps(m,p,-1), -1, "e(-1).e(-1)=-1", passed)
  call expect (                  p*eps(m,p, 1),  0, "  p.e( 1)= 0", passed)
  call expect (                  p*eps(m,p,-1),  0, "  p.e(-1)= 0", passed)
  if (m > 0) then
     call expect (conjg(eps(m,p, 1))*eps(m,p, 0),  0, "e( 1).e( 0)= 0", passed)
     call expect (conjg(eps(m,p, 0))*eps(m,p, 1),  0, "e( 0).e( 1)= 0", passed)
     call expect (conjg(eps(m,p, 0))*eps(m,p, 0), -1, "e( 0).e( 0)=-1", passed)
     call expect (conjg(eps(m,p, 0))*eps(m,p,-1),  0, "e( 0).e(-1)= 0", passed)
     call expect (conjg(eps(m,p,-1))*eps(m,p, 0),  0, "e(-1).e( 0)= 0", passed)
     call expect (                  p*eps(m,p, 0),  0, "  p.e( 0)= 0", passed)
  end if
```

⟨*Test* omega95_bispinors⟩+≡
```
  print *, "*** Checking polarization vectorspinors: ***"
  call expect (abs(p * ueps(m, p,  2)), 0, "p.ueps ( 2)= 0", passed)
  call expect (abs(p * ueps(m, p,  1)), 0, "p.ueps ( 1)= 0", passed)
  call expect (abs(p * ueps(m, p, -1)), 0, "p.ueps (-1)= 0", passed)
  call expect (abs(p * ueps(m, p, -2)), 0, "p.ueps (-2)= 0", passed)
```

```
    call expect (abs(p * veps(m, p,  2)),  0, "p.veps ( 2)= 0", passed)
    call expect (abs(p * veps(m, p,  1)),  0, "p.veps ( 1)= 0", passed)
    call expect (abs(p * veps(m, p, -1)),  0, "p.veps (-1)= 0", passed)
    call expect (abs(p * veps(m, p, -2)),  0, "p.veps (-2)= 0", passed)
    print *, "*** Checking polarization vectorspinors (neg. masses): ***"
    call expect (abs(p * ueps(-m, p,  2)),  0, "p.ueps ( 2)= 0", passed)
    call expect (abs(p * ueps(-m, p,  1)),  0, "p.ueps ( 1)= 0", passed)
    call expect (abs(p * ueps(-m, p, -1)),  0, "p.ueps (-1)= 0", passed)
    call expect (abs(p * ueps(-m, p, -2)),  0, "p.ueps (-2)= 0", passed)
    call expect (abs(p * veps(-m, p,  2)),  0, "p.veps ( 2)= 0", passed)
    call expect (abs(p * veps(-m, p,  1)),  0, "p.veps ( 1)= 0", passed)
    call expect (abs(p * veps(-m, p, -1)),  0, "p.veps (-1)= 0", passed)
    call expect (abs(p * veps(-m, p, -2)),  0, "p.veps (-2)= 0", passed)
    print *, "*** in the rest frame ***"
    call expect (abs(p_0 * ueps(m, p_0,  2)),  0, "p0.ueps ( 2)= 0", passed)
    call expect (abs(p_0 * ueps(m, p_0,  1)),  0, "p0.ueps ( 1)= 0", passed)
    call expect (abs(p_0 * ueps(m, p_0, -1)),  0, "p0.ueps (-1)= 0", passed)
    call expect (abs(p_0 * ueps(m, p_0, -2)),  0, "p0.ueps (-2)= 0", passed)
    call expect (abs(p_0 * veps(m, p_0,  2)),  0, "p0.veps ( 2)= 0", passed)
    call expect (abs(p_0 * veps(m, p_0,  1)),  0, "p0.veps ( 1)= 0", passed)
    call expect (abs(p_0 * veps(m, p_0, -1)),  0, "p0.veps (-1)= 0", passed)
    call expect (abs(p_0 * veps(m, p_0, -2)),  0, "p0.veps (-2)= 0", passed)
    print *, "*** in the rest frame (neg. masses) ***"
    call expect (abs(p_0 * ueps(-m, p_0,  2)),  0, "p0.ueps ( 2)= 0", passed)
    call expect (abs(p_0 * ueps(-m, p_0,  1)),  0, "p0.ueps ( 1)= 0", passed)
    call expect (abs(p_0 * ueps(-m, p_0, -1)),  0, "p0.ueps (-1)= 0", passed)
    call expect (abs(p_0 * ueps(-m, p_0, -2)),  0, "p0.ueps (-2)= 0", passed)
    call expect (abs(p_0 * veps(-m, p_0,  2)),  0, "p0.veps ( 2)= 0", passed)
    call expect (abs(p_0 * veps(-m, p_0,  1)),  0, "p0.veps ( 1)= 0", passed)
    call expect (abs(p_0 * veps(-m, p_0, -1)),  0, "p0.veps (-1)= 0", passed)
    call expect (abs(p_0 * veps(-m, p_0, -2)),  0, "p0.veps (-2)= 0", passed)
```

⟨*Test* omega95_bispinors⟩+≡
```
  print *, "*** Checking the irreducibility condition: ***"
  call expect (abs(f_potgr (c_one, c_one, ueps(m, p,  2))),  0, "g.ueps ( 2)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(m, p,  1))),  0, "g.ueps ( 1)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(m, p, -1))),  0, "g.ueps (-1)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(m, p, -2))),  0, "g.ueps (-2)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(m, p,  2))),  0, "g.veps ( 2)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(m, p,  1))),  0, "g.veps ( 1)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(m, p, -1))),  0, "g.veps (-1)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(m, p, -2))),  0, "g.veps (-2)", passed)
  print *, "*** Checking the irreducibility condition (neg. masses): ***"
  call expect (abs(f_potgr (c_one, c_one, ueps(-m, p,  2))),  0, "g.ueps ( 2)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(-m, p,  1))),  0, "g.ueps ( 1)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(-m, p, -1))),  0, "g.ueps (-1)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(-m, p, -2))),  0, "g.ueps (-2)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(-m, p,  2))),  0, "g.veps ( 2)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(-m, p,  1))),  0, "g.veps ( 1)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(-m, p, -1))),  0, "g.veps (-1)", passed)
  call expect (abs(f_potgr (c_one, c_one, veps(-m, p, -2))),  0, "g.veps (-2)", passed)
  print *, "*** in the rest frame ***"
  call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0,  2))),  0, "g.ueps ( 2)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0,  1))),  0, "g.ueps ( 1)", passed)
  call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0, -1))),  0, "g.ueps (-1)", passed)
```

```
      call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0, -2))),  0, "g.ueps (-2)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0,  2))),  0, "g.veps ( 2)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0,  1))),  0, "g.veps ( 1)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0, -1))),  0, "g.veps (-1)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0, -2))),  0, "g.veps (-2)", passed)
      print *, "*** in the rest frame (neg. masses) ***"
      call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0,  2))),  0, "g.ueps ( 2)", passed)
      call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0,  1))),  0, "g.ueps ( 1)", passed)
      call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0, -1))),  0, "g.ueps (-1)", passed)
      call expect (abs(f_potgr (c_one, c_one, ueps(m, p_0, -2))),  0, "g.ueps (-2)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0,  2))),  0, "g.veps ( 2)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0,  1))),  0, "g.veps ( 1)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0, -1))),  0, "g.veps (-1)", passed)
      call expect (abs(f_potgr (c_one, c_one, veps(m, p_0, -2))),  0, "g.veps (-2)", passed)
```

⟨*Test* omega95_bispinors⟩+≡

```
  print *, "*** Testing vectorspinor normalization ***"
  call expect (veps(m,p, 2)*ueps(m,p, 2), -2*m, "ueps( 2).ueps( 2)= -2m", passed)
  call expect (veps(m,p, 1)*ueps(m,p, 1), -2*m, "ueps( 1).ueps( 1)= -2m", passed)
  call expect (veps(m,p,-1)*ueps(m,p,-1), -2*m, "ueps(-1).ueps(-1)= -2m", passed)
  call expect (veps(m,p,-2)*ueps(m,p,-2), -2*m, "ueps(-2).ueps(-2)= -2m", passed)
  call expect (ueps(m,p, 2)*veps(m,p, 2),  2*m, "veps( 2).veps( 2)= +2m", passed)
  call expect (ueps(m,p, 1)*veps(m,p, 1),  2*m, "veps( 1).veps( 1)= +2m", passed)
  call expect (ueps(m,p,-1)*veps(m,p,-1),  2*m, "veps(-1).veps(-1)= +2m", passed)
  call expect (ueps(m,p,-2)*veps(m,p,-2),  2*m, "veps(-2).veps(-2)= +2m", passed)
  call expect (ueps(m,p, 2)*ueps(m,p, 2),    0, "ueps( 2).veps( 2)=   0", passed)
  call expect (ueps(m,p, 1)*ueps(m,p, 1),    0, "ueps( 1).veps( 1)=   0", passed)
  call expect (ueps(m,p,-1)*ueps(m,p,-1),    0, "ueps(-1).veps(-1)=   0", passed)
  call expect (ueps(m,p,-2)*ueps(m,p,-2),    0, "ueps(-2).veps(-2)=   0", passed)
  call expect (veps(m,p, 2)*veps(m,p, 2),    0, "veps( 2).ueps( 2)=   0", passed)
  call expect (veps(m,p, 1)*veps(m,p, 1),    0, "veps( 1).ueps( 1)=   0", passed)
  call expect (veps(m,p,-1)*veps(m,p,-1),    0, "veps(-1).ueps(-1)=   0", passed)
  call expect (veps(m,p,-2)*veps(m,p,-2),    0, "veps(-2).ueps(-2)=   0", passed)
  print *, "*** Testing vectorspinor normalization (neg. masses) ***"
  call expect (veps(-m,p, 2)*ueps(-m,p, 2), +2*m, "ueps( 2).ueps( 2)= +2m", passed)
  call expect (veps(-m,p, 1)*ueps(-m,p, 1), +2*m, "ueps( 1).ueps( 1)= +2m", passed)
  call expect (veps(-m,p,-1)*ueps(-m,p,-1), +2*m, "ueps(-1).ueps(-1)= +2m", passed)
  call expect (veps(-m,p,-2)*ueps(-m,p,-2), +2*m, "ueps(-2).ueps(-2)= +2m", passed)
  call expect (ueps(-m,p, 2)*veps(-m,p, 2), -2*m, "veps( 2).veps( 2)= -2m", passed)
  call expect (ueps(-m,p, 1)*veps(-m,p, 1), -2*m, "veps( 1).veps( 1)= -2m", passed)
  call expect (ueps(-m,p,-1)*veps(-m,p,-1), -2*m, "veps(-1).veps(-1)= -2m", passed)
  call expect (ueps(-m,p,-2)*veps(-m,p,-2), -2*m, "veps(-2).veps(-2)= -2m", passed)
  call expect (ueps(-m,p, 2)*ueps(-m,p, 2),    0, "ueps( 2).veps( 2)=   0", passed)
  call expect (ueps(-m,p, 1)*ueps(-m,p, 1),    0, "ueps( 1).veps( 1)=   0", passed)
  call expect (ueps(-m,p,-1)*ueps(-m,p,-1),    0, "ueps(-1).veps(-1)=   0", passed)
  call expect (ueps(-m,p,-2)*ueps(-m,p,-2),    0, "ueps(-2).veps(-2)=   0", passed)
  call expect (veps(-m,p, 2)*veps(-m,p, 2),    0, "veps( 2).ueps( 2)=   0", passed)
  call expect (veps(-m,p, 1)*veps(-m,p, 1),    0, "veps( 1).ueps( 1)=   0", passed)
  call expect (veps(-m,p,-1)*veps(-m,p,-1),    0, "veps(-1).ueps(-1)=   0", passed)
  call expect (veps(-m,p,-2)*veps(-m,p,-2),    0, "veps(-2).ueps(-2)=   0", passed)
  print *, "*** in the rest frame ***"
  call expect (veps(m,p_0, 2)*ueps(m,p_0, 2), -2*m, "ueps( 2).ueps( 2)= -2m", passed)
  call expect (veps(m,p_0, 1)*ueps(m,p_0, 1), -2*m, "ueps( 1).ueps( 1)= -2m", passed)
  call expect (veps(m,p_0,-1)*ueps(m,p_0,-1), -2*m, "ueps(-1).ueps(-1)= -2m", passed)
  call expect (veps(m,p_0,-2)*ueps(m,p_0,-2), -2*m, "ueps(-2).ueps(-2)= -2m", passed)
```

```
    call expect (ueps(m,p_0, 2)*veps(m,p_0, 2),  2*m, "veps( 2).veps( 2)= +2m", passed)
    call expect (ueps(m,p_0, 1)*veps(m,p_0, 1),  2*m, "veps( 1).veps( 1)= +2m", passed)
    call expect (ueps(m,p_0,-1)*veps(m,p_0,-1),  2*m, "veps(-1).veps(-1)= +2m", passed)
    call expect (ueps(m,p_0,-2)*veps(m,p_0,-2),  2*m, "veps(-2).veps(-2)= +2m", passed)
    call expect (ueps(m,p_0, 2)*ueps(m,p_0, 2),    0, "ueps( 2).veps( 2)=   0", passed)
    call expect (ueps(m,p_0, 1)*ueps(m,p_0, 1),    0, "ueps( 1).veps( 1)=   0", passed)
    call expect (ueps(m,p_0,-1)*ueps(m,p_0,-1),    0, "ueps(-1).veps(-1)=   0", passed)
    call expect (ueps(m,p_0,-2)*ueps(m,p_0,-2),    0, "ueps(-2).veps(-2)=   0", passed)
    call expect (veps(m,p_0, 2)*veps(m,p_0, 2),    0, "veps( 2).ueps( 2)=   0", passed)
    call expect (veps(m,p_0, 1)*veps(m,p_0, 1),    0, "veps( 1).ueps( 1)=   0", passed)
    call expect (veps(m,p_0,-1)*veps(m,p_0,-1),    0, "veps(-1).ueps(-1)=   0", passed)
    call expect (veps(m,p_0,-2)*veps(m,p_0,-2),    0, "veps(-2).ueps(-2)=   0", passed)
    print *, "*** in the rest frame (neg. masses) ***"
    call expect (veps(-m,p_0, 2)*ueps(-m,p_0, 2), +2*m, "ueps( 2).ueps( 2)= +2m", passed)
    call expect (veps(-m,p_0, 1)*ueps(-m,p_0, 1), +2*m, "ueps( 1).ueps( 1)= +2m", passed)
    call expect (veps(-m,p_0,-1)*ueps(-m,p_0,-1), +2*m, "ueps(-1).ueps(-1)= +2m", passed)
    call expect (veps(-m,p_0,-2)*ueps(-m,p_0,-2), +2*m, "ueps(-2).ueps(-2)= +2m", passed)
    call expect (ueps(-m,p_0, 2)*veps(-m,p_0, 2), -2*m, "veps( 2).veps( 2)= -2m", passed)
    call expect (ueps(-m,p_0, 1)*veps(-m,p_0, 1), -2*m, "veps( 1).veps( 1)= -2m", passed)
    call expect (ueps(-m,p_0,-1)*veps(-m,p_0,-1), -2*m, "veps(-1).veps(-1)= -2m", passed)
    call expect (ueps(-m,p_0,-2)*veps(-m,p_0,-2), -2*m, "veps(-2).veps(-2)= -2m", passed)
    call expect (ueps(-m,p_0, 2)*ueps(-m,p_0, 2),    0, "ueps( 2).veps( 2)=   0", passed)
    call expect (ueps(-m,p_0, 1)*ueps(-m,p_0, 1),    0, "ueps( 1).veps( 1)=   0", passed)
    call expect (ueps(-m,p_0,-1)*ueps(-m,p_0,-1),    0, "ueps(-1).veps(-1)=   0", passed)
    call expect (ueps(-m,p_0,-2)*ueps(-m,p_0,-2),    0, "ueps(-2).veps(-2)=   0", passed)
    call expect (veps(-m,p_0, 2)*veps(-m,p_0, 2),    0, "veps( 2).ueps( 2)=   0", passed)
    call expect (veps(-m,p_0, 1)*veps(-m,p_0, 1),    0, "veps( 1).ueps( 1)=   0", passed)
    call expect (veps(-m,p_0,-1)*veps(-m,p_0,-1),    0, "veps(-1).ueps(-1)=   0", passed)
    call expect (veps(-m,p_0,-2)*veps(-m,p_0,-2),    0, "veps(-2).ueps(-2)=   0", passed)
```

⟨*Test* omega95_bispinors⟩+≡

```
  print *, "*** Majorana properties of gravitino vertices: ***"
  call expect (abs(u (m,q,1) * f_sgr (c_one, c_one, ueps(m,p,2), t) + &
    ueps(m,p,2) * gr_sf(c_one,c_one,u(m,q,1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  !!! call expect (abs(u (m,q,-1) * f_sgr (c_one, c_one, ueps(m,p,2), t) + &
  !!!    ueps(m,p,2) * gr_sf(c_one,c_one,u(m,q,-1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  !!! call expect (abs(u (m,q,1) * f_sgr (c_one, c_one, ueps(m,p,1), t) + &
  !!!    ueps(m,p,1) * gr_sf(c_one,c_one,u(m,q,1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  !!! call expect (abs(u (m,q,-1) * f_sgr (c_one, c_one, ueps(m,p,1), t) + &
  !!!    ueps(m,p,1) * gr_sf(c_one,c_one,u(m,q,-1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  !!! call expect (abs(u (m,q,1) * f_sgr (c_one, c_one, ueps(m,p,-1), t) + &
  !!!    ueps(m,p,-1) * gr_sf(c_one,c_one,u(m,q,1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  !!! call expect (abs(u (m,q,-1) * f_sgr (c_one, c_one, ueps(m,p,-1), t) + &
  !!!    ueps(m,p,-1) * gr_sf(c_one,c_one,u(m,q,-1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  !!! call expect (abs(u (m,q,1) * f_sgr (c_one, c_one, ueps(m,p,-2), t) + &
  !!!    ueps(m,p,-2) * gr_sf(c_one,c_one,u(m,q,1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  !!! call expect (abs(u (m,q,-1) * f_sgr (c_one, c_one, ueps(m,p,-2), t) + &
  !!!    ueps(m,p,-2) * gr_sf(c_one,c_one,u(m,q,-1),t)), 0, "f_sgr    + gr_sf    = 0", passed)
  call expect (abs(u (m,q,1) * f_slgr (c_one, c_one, ueps(m,p,2), t) + &
    ueps(m,p,2) * gr_slf(c_one,c_one,u(m,q,1),t)), 0, "f_slgr   + gr_slf   = 0", passed, thresho
  call expect (abs(u (m,q,1) * f_srgr (c_one, c_one, ueps(m,p,2), t) + &
    ueps(m,p,2) * gr_srf(c_one,c_one,u(m,q,1),t)), 0, "f_srgr   + gr_srf   = 0", passed, thresho
  call expect (abs(u (m,q,1) * f_slrgr (c_one, c_two, c_one, ueps(m,p,2), t) + &
    ueps(m,p,2) * gr_slrf(c_one,c_two,c_one,u(m,q,1),t)), 0, "f_slrgr  + gr_slrf  = 0", passed,
  call expect (abs(u (m,q,1) * f_pgr (c_one, c_one, ueps(m,p,2), t) + &
```

```
     ueps(m,p,2) * gr_pf(c_one,c_one,u(m,q,1),t)),  0, "f_pgr     + gr_pf     = 0", passed, threshol
call expect (abs(u (m,q,1) * f_vgr (c_one, vt, ueps(m,p,2), p+q) + &
     ueps(m,p,2) * gr_vf(c_one,vt,u(m,q,1),p+q)),  0, "f_vgr     + gr_vf = 0", passed, threshold = 0
call expect (abs(u (m,q,1) * f_vlrgr (c_one, c_two, vt, ueps(m,p,2), p+q) + &
     ueps(m,p,2) * gr_vlrf(c_one,c_two,vt,u(m,q,1),p+q)),  0, "f_vlrgr   + gr_vlrf  = 0", &
     passed, threshold = 0.5_default)
!!! call expect (abs(u (m,q,-1) * f_vgr (c_one, vt, ueps(m,p,2), p+q) + &
!!!      ueps(m,p,2) * gr_vf(c_one,vt,u(m,q,-1),p+q)),  0, "f_vgr     + gr_vf     = 0", passed)
!!! call expect (abs(u (m,q,1) * f_vgr (c_one, vt, ueps(m,p,1), p+q) + &
!!!      ueps(m,p,1) * gr_vf(c_one,vt,u(m,q,1),p+q)),  0, "f_vgr      + gr_vf     = 0", passed)
!!! call expect (abs(u (m,q,-1) * f_vgr (c_one, vt, ueps(m,p,1), p+q) + &
!!!      ueps(m,p,1) * gr_vf(c_one,vt,u(m,q,-1),p+q)),  0, "f_vgr      + gr_vf     = 0", passed)
!!! call expect (abs(u (m,q,1) * f_vgr (c_one, vt, ueps(m,p,-1), p+q) + &
!!!      ueps(m,p,-1) * gr_vf(c_one,vt,u(m,q,1),p+q)),  0, "f_vgr      + gr_vf     = 0", passed)
!!! call expect (abs(u (m,q,-1) * f_vgr (c_one, vt, veps(m,p,-1), p+q) + &
!!!      veps(m,p,-1) * gr_vf(c_one,vt,u(m,q,-1),p+q)),  0, "f_vgr      + gr_vf      = 0", passed)
!!! call expect (abs(v (m,q,1) * f_vgr (c_one, vt, ueps(m,p,-2), p+q) + &
!!!      ueps(m,p,-2) * gr_vf(c_one,vt,v(m,q,1),p+q)),  0, "f_vgr      + gr_vf      = 0", passed)
!!! call expect (abs(u (m,q,-1) * f_vgr (c_one, vt, ueps(m,p,-2), p+q) + &
!!!      ueps(m,p,-2) * gr_vf(c_one,vt,u(m,q,-1),p+q)),  0, "f_vgr      + gr_vf      = 0", passed)
call expect (abs(s_grf (c_one, ueps(m,p,2), u(m,q,1),t) + &
     s_fgr(c_one,u(m,q,1),ueps(m,p,2),t)),  0, "s_grf      + s_fgr     = 0", passed)
call expect (abs(sl_grf (c_one, ueps(m,p,2), u(m,q,1),t) + &
     sl_fgr(c_one,u(m,q,1),ueps(m,p,2),t)),  0, "sl_grf      + sl_fgr     = 0", passed)
call expect (abs(sr_grf (c_one, ueps(m,p,2), u(m,q,1),t) + &
     sr_fgr(c_one,u(m,q,1),ueps(m,p,2),t)),  0, "sr_grf      + sr_fgr     = 0", passed)
call expect (abs(slr_grf (c_one, c_two, ueps(m,p,2), u(m,q,1),t) + &
     slr_fgr(c_one,c_two,u(m,q,1),ueps(m,p,2),t)),  0, "slr_grf   + slr_fgr  = 0", passed)
call expect (abs(p_grf (c_one, ueps(m,p,2), u(m,q,1),t) + &
     p_fgr(c_one,u(m,q,1),ueps(m,p,2),t)),  0, "p_grf      + p_fgr      = 0", passed)
call expect (abs(v_grf (c_one, ueps(m,p,2), u(m,q,1),t) + &
     v_fgr(c_one,u(m,q,1),ueps(m,p,2),t)),  0, "v_grf      + v_fgr      = 0", passed)
call expect (abs(vlr_grf (c_one, c_two, ueps(m,p,2), u(m,q,1),t) + &
     vlr_fgr(c_one,c_two,u(m,q,1),ueps(m,p,2),t)),  0, "vlr_grf   + vlr_fgr  = 0", passed)
call expect (abs(u(m,p,1) * f_potgr (c_one,c_one,testv) - testv * gr_potf &
     (c_one,c_one,u (m,p,1))),  0, "f_potgr    - gr_potf  = 0", passed)
call expect (abs (pot_fgr (c_one,u(m,p,1),testv) - pot_grf(c_one, &
     testv,u(m,p,1))),  0, "pot_fgr   - pot_grf  = 0", passed)
call expect (abs(u(m,p,1) * f_s2gr (c_one,c_one,c_one,testv) - testv * gr_s2f &
     (c_one,c_one,c_one,u (m,p,1))),  0, "f_s2gr    - gr_s2f    = 0", passed)
call expect (abs (s2_fgr (c_one,u(m,p,1),c_one,testv) - s2_grf(c_one, &
     testv,c_one,u(m,p,1))),  0, "s2_fgr    - s2_grf    = 0", passed)
call expect (abs(u (m,q,1) * f_svgr (c_one, c_one, vt, ueps(m,p,2)) + &
     ueps(m,p,2) * gr_svf(c_one,c_one,vt,u(m,q,1))),  0, "f_svgr     + gr_svf     = 0", passed)
call expect (abs(u (m,q,1) * f_slvgr (c_one, c_one, vt, ueps(m,p,2)) + &
     ueps(m,p,2) * gr_slvf(c_one,c_one,vt,u(m,q,1))),  0, "f_slvgr   + gr_slvf   = 0", passed)
call expect (abs(u (m,q,1) * f_srvgr (c_one, c_one, vt, ueps(m,p,2)) + &
     ueps(m,p,2) * gr_srvf(c_one,c_one,vt,u(m,q,1))),  0, "f_srvgr    + gr_srvf   = 0", passed)
call expect (abs(u (m,q,1) * f_slrvgr (c_one, c_two, c_one, vt, ueps(m,p,2)) + &
     ueps(m,p,2) * gr_slrvf(c_one,c_two,c_one,vt,u(m,q,1))),  0, "f_slrvgr + gr_slrvf = 0", passed
call expect (abs (sv1_fgr (c_one,u(m,p,1),vt,ueps(m,q,2)) + sv1_grf(c_one, &
     ueps(m,q,2),vt,u(m,p,1))),  0, "sv1_fgr    + sv1_grf   = 0", passed)
call expect (abs (sv2_fgr (c_one,u(m,p,1),c_one,ueps(m,q,2)) + sv2_grf(c_one, &
     ueps(m,q,2),c_one,u(m,p,1))),  0, "sv2_fgr    + sv2_grf   = 0", passed)
```

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```
      call expect (abs (slv1_fgr (c_one,u(m,p,1),vt,ueps(m,q,2)) + slv1_grf(c_one, &
         ueps(m,q,2),vt,u(m,p,1))), 0, "slv1_fgr  + slv1_grf  = 0", passed)
      call expect (abs (srv2_fgr (c_one,u(m,p,1),c_one,ueps(m,q,2)) + srv2_grf(c_one, &
         ueps(m,q,2),c_one,u(m,p,1))), 0, "srv2_fgr  + srv2_grf  = 0", passed)
      call expect (abs (slrv1_fgr (c_one,c_two,u(m,p,1),vt,ueps(m,q,2)) + slrv1_grf(c_one,c_two, &
         ueps(m,q,2),vt,u(m,p,1))), 0, "slrv1_fgr + slrv1_grf = 0", passed)
      call expect (abs (slrv2_fgr (c_one,c_two,u(m,p,1),c_one,ueps(m,q,2)) + slrv2_grf(c_one, &
         c_two,ueps(m,q,2),c_one,u(m,p,1))), 0, "slrv2_fgr + slrv2_grf = 0", passed)
      call expect (abs(u (m,q,1) * f_pvgr (c_one, c_one, vt, ueps(m,p,2)) + &
         ueps(m,p,2) * gr_pvf(c_one,c_one,vt,u(m,q,1))),  0, "f_pvgr   + gr_pvf   = 0", passed)
      call expect (abs (pv1_fgr (c_one,u(m,p,1),vt,ueps(m,q,2)) + pv1_grf(c_one, &
         ueps(m,q,2),vt,u(m,p,1))), 0, "pv1_fgr   + pv1_grf   = 0", passed)
      call expect (abs (pv2_fgr (c_one,u(m,p,1),c_one,ueps(m,q,2)) + pv2_grf(c_one, &
         ueps(m,q,2),c_one,u(m,p,1))), 0, "pv2_fgr   + pv2_grf   = 0", passed)
      call expect (abs(u (m,q,1) * f_v2gr (c_one, vt, vz, ueps(m,p,2)) + &
         ueps(m,p,2) * gr_v2f(c_one,vt,vz,u(m,q,1))),  0, "f_v2gr    + gr_v2f    = 0", passed)
      call expect (abs(u (m,q,1) * f_v2lrgr (c_one, c_two, vt, vz, ueps(m,p,2)) + &
         ueps(m,p,2) * gr_v2lrf(c_one,c_two,vt,vz,u(m,q,1))),  0, "f_v2lrgr  + gr_v2lrf  = 0", passed)
      call expect (abs (v2_fgr (c_one,u(m,p,1),vt,ueps(m,q,2)) + v2_grf(c_one, &
         ueps(m,q,2),vt,u(m,p,1))), 0, "v2_fgr    + v2_grf    = 0", passed)
      call expect (abs (v2lr_fgr (c_one,c_two,u(m,p,1),vt,ueps(m,q,2)) + v2lr_grf(c_one, c_two, &
         ueps(m,q,2),vt,u(m,p,1))), 0, "v2lr_fgr  + v2lr_grf  = 0", passed)
```

⟨*Test* omega95_bispinors⟩+≡
```
  print *, "*** Testing the gravitino propagator: ***"
  print *, "Transversality:"
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,testv))), 0, "p.pr.test", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,ueps(m,p,2)))),  0, "p.pr.ueps ( 2)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,ueps(m,p,1)))),  0, "p.pr.ueps ( 1)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,ueps(m,p,-1)))), 0, "p.pr.ueps (-1)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,ueps(m,p,-2)))), 0, "p.pr.ueps (-2)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,veps(m,p,2)))),  0, "p.pr.veps ( 2)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,veps(m,p,1)))),  0, "p.pr.veps ( 1)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,veps(m,p,-1)))), 0, "p.pr.veps (-1)", passed)
  call expect (abs(p * (cmplx (p*p - m**2, m*w, kind=default) * &
                pr_grav(p,m,w,veps(m,p,-2)))), 0, "p.pr.veps (-2)", passed)
  print *, "Irreducibility:"
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                kind=default) * pr_grav(p,m,w,testv)))), 0, "g.pr.test", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                kind=default) * pr_grav(p,m,w,ueps(m,p,2))))), 0, &
                "g.pr.ueps ( 2)", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                kind=default) * pr_grav(p,m,w,ueps(m,p,1))))), 0, &
                "g.pr.ueps ( 1)", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                kind=default) * pr_grav(p,m,w,ueps(m,p,-1))))), 0, &
```

```
                    "g.pr.ueps (-1)", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                 kind=default) * pr_grav(p,m,w,ueps(m,p,-2))))), 0, &
                 "g.pr.ueps (-2)", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                 kind=default) * pr_grav(p,m,w,veps(m,p,2))))), 0, &
                 "g.pr.veps ( 2)", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                 kind=default) * pr_grav(p,m,w,veps(m,p,1))))), 0, &
                 "g.pr.veps ( 1)", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                 kind=default) * pr_grav(p,m,w,veps(m,p,-1))))), 0, &
                 "g.pr.veps (-1)", passed)
  call expect (abs(f_potgr (c_one, c_one, (cmplx (p*p - m**2, m*w, &
                 kind=default) * pr_grav(p,m,w,veps(m,p,-2))))), 0, &
                 "g.pr.veps (-2)", passed)
```

⟨omega_bundle.f90⟩≡
  ⟨omega_vectors.f90⟩
  ⟨omega_spinors.f90⟩
  ⟨omega_bispinors.f90⟩
  ⟨omega_vectorspinors.f90⟩
  ⟨omega_polarizations.f90⟩
  ⟨omega_tensors.f90⟩
  ⟨omega_tensor_polarizations.f90⟩
  ⟨omega_couplings.f90⟩
  ⟨omega_spinor_couplings.f90⟩
  ⟨omega_bispinor_couplings.f90⟩
  ⟨omega_vspinor_polarizations.f90⟩
  ⟨omega_utils.f90⟩
  ⟨omega95.f90⟩
  ⟨omega95_bispinors.f90⟩
  ⟨omega_parameters.f90⟩
  ⟨omega_parameters_madgraph.f90⟩

⟨omega_bundle_whizard.f90⟩≡
  ⟨omega_bundle.f90⟩
  ⟨omega_parameters_whizard.f90⟩

X.30   O'Mega Virtual Machine

This module defines the O'Mega Virtual Machine (OVM) completely, whereby all environmental dependencies like masses, widths and couplings have to be given to the constructor `vm%init` at runtime.

Support for Majorana particles and vectorspinors is only partially, especially all fusions are missing. Maybe it would be easier to make an additional `omegavm95_bispinors` to avoid namespace issues. Non-type specific chunks could be reused

⟨omegavm95.f90⟩≡
  ⟨Copyleft⟩
  module omegavm95
    use kinds, only: default
    use constants
```

```
    use iso_varying_string, string_t => varying_string
    use, intrinsic :: iso_fortran_env, only : input_unit, output_unit, error_unit
    use omega95
    use omega95_bispinors, only: bispinor, vectorspinor, veps, pr_grav
    use omega95_bispinors, only: bi_u => u
    use omega95_bispinors, only: bi_v => v
    use omega95_bispinors, only: bi_pr_psi => pr_psi
    use omega_bispinors, only: operator (*), operator (+)
    use omega_color, only: ovm_color_sum, OCF => omega_color_factor
    implicit none
    private
    ⟨Utilities Declarations⟩
    ⟨OVM Data Declarations⟩
    ⟨OVM Instructions⟩
  contains
    ⟨OVM Procedure Implementations⟩
    ⟨Utilities Procedure Implementations⟩
  end module omegavm95
```

This might not be the proper place but I don't know where to put it

⟨*Utilities Declarations*⟩≡
```
  integer, parameter, public :: stdin = input_unit
  integer, parameter, public :: stdout = output_unit
  integer, parameter, public :: stderr = error_unit
  integer, parameter :: MIN_UNIT = 11, MAX_UNIT = 99
```

⟨*OVM Procedure Implementations*⟩≡
```
  subroutine find_free_unit (u, iostat)
    integer, intent(out) :: u
    integer, intent(out), optional :: iostat
    logical :: exists, is_open
    integer :: i, status
    do i = MIN_UNIT, MAX_UNIT
       inquire (unit = i, exist = exists, opened = is_open, &
            iostat = status)
       if (status == 0) then
          if (exists .and. .not. is_open) then
             u = i
             if (present (iostat)) then
                iostat = 0
             end if
             return
          end if
       end if
    end do
    if (present (iostat)) then
       iostat = -1
    end if
    u = -1
  end subroutine find_free_unit
```

These abstract data types would ideally be the interface to communicate quantum numbers between O'Mega and Whizard. This gives full flexibility to change the representation at any time

⟨*Utilities Declarations*⟩+≡

```
    public :: color_t
    type color_t
    contains
      procedure :: write => color_write
    end type color_t

    public :: col_discrete
    type, extends(color_t) :: col_discrete
      integer :: i
    end type col_discrete

    public :: flavor_t
    type flavor_t
    contains
      procedure :: write => flavor_write
    end type flavor_t

    public :: flv_discrete
    type, extends(flavor_t) :: flv_discrete
      integer :: i
    end type flv_discrete

    public :: helicity_t
    type :: helicity_t
    contains
      procedure :: write => helicity_write
    end type helicity_t

    public :: hel_discrete
    type, extends(helicity_t) :: hel_discrete
      integer :: i
    end type hel_discrete

    public :: hel_trigonometric
    type, extends(helicity_t) :: hel_trigonometric
      real :: theta
    end type hel_trigonometric

    public :: hel_exponential
    type, extends(helicity_t) :: hel_exponential
      real :: phi
    end type hel_exponential

    public :: hel_spherical
    type, extends(helicity_t) :: hel_spherical
      real :: theta, phi
    end type hel_spherical
```

⟨*Utilities Procedure Implementations*⟩≡
```
  subroutine color_write (color, fh)
    class(color_t), intent(in) :: color
    integer, intent(in) :: fh
    select type(color)
    type is (col_discrete)
```

```
      write(fh, *) 'color_discrete%i            = ', color%i
    end select
  end subroutine color_write

  subroutine helicity_write (helicity, fh)
    class(helicity_t), intent(in) :: helicity
    integer, intent(in) :: fh
    select type(helicity)
    type is (hel_discrete)
       write(fh, *) 'helicity_discrete%i          = ', helicity%i
    type is (hel_trigonometric)
       write(fh, *) 'helicity_trigonometric%theta = ', helicity%theta
    type is (hel_exponential)
       write(fh, *) 'helicity_exponential%phi     = ', helicity%phi
    type is (hel_spherical)
       write(fh, *) 'helicity_spherical%phi       = ', helicity%phi
       write(fh, *) 'helicity_spherical%theta     = ', helicity%theta
    end select
  end subroutine helicity_write

  subroutine flavor_write (flavor, fh)
    class(flavor_t), intent(in) :: flavor
    integer, intent(in) :: fh
    select type(flavor)
    type is (flv_discrete)
       write(fh, *) 'flavor_discrete%i            = ', flavor%i
    end select
  end subroutine flavor_write
```

X.30.1 Memory Layout

Some internal parameters

⟨*OVM Data Declarations*⟩≡

```
  integer, parameter :: len_instructions = 8
  integer, parameter :: N_version_lines = 2
  ! Comment lines including the first header description line
  integer, parameter :: N_comments = 6
  ! Actual data lines plus intermediate description lines
  ! 'description \n 1 2 3 \n description \n 3 2 1' would count as 3
  integer, parameter :: N_header_lines = 5
  real(default), parameter, public :: N_ = three
```

This is the basic type of a VM

⟨*OVM Data Declarations*⟩+≡

```
  type :: basic_vm_t
     private
     logical :: verbose
     type(string_t) :: bytecode_file
     integer :: bytecode_fh, out_fh
     integer :: N_instructions, N_levels
     integer :: N_table_lines
     integer, dimension(:, :), allocatable :: instructions
```

```
      integer, dimension(:), allocatable :: levels
   end type
```

To allow for a lazy evaluation of amplitudes, we have to keep track whether a wave function has already been computed, to avoid multiple-computing that would arise when the bytecode has redundant fusions, which is necessary for flavor and color MC (and helicity MC when we use Weyl-van-der-Waerden-spinors)

⟨*OVM Data Declarations*⟩+≡

```
   type :: vm_scalar
      logical :: c
      complex(kind=default) :: v
   end type

   type :: vm_spinor
      logical :: c
      type(spinor) :: v
   end type

   type :: vm_conjspinor
      logical :: c
      type(conjspinor) :: v
   end type

   type :: vm_bispinor
      logical :: c
      type(bispinor) :: v
   end type

   type :: vm_vector
      logical :: c
      type(vector) :: v
   end type

   type :: vm_tensor_2
      logical :: c
      type(tensor) :: v
   end type

   type :: vm_tensor_1
      logical :: c
      type(tensor2odd) :: v
   end type

   type :: vm_vectorspinor
      logical :: c
      type(vectorspinor) :: v
   end type
```

We need a memory pool for all the intermediate results

⟨*OVM Data Declarations*⟩+≡

```
   type, public, extends (basic_vm_t) :: vm_t
      private
```

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```
    type(string_t) :: version
    type(string_t) :: model
    integer :: N_momenta, N_particles, N_prt_in, N_prt_out, N_amplitudes
    ! helicities = helicity combinations
    integer :: N_helicities, N_col_flows, N_col_indices, N_flavors, N_col_factors

    integer :: N_scalars, N_spinors, N_conjspinors, N_bispinors
    integer :: N_vectors, N_tensors_2, N_tensors_1, N_vectorspinors

    integer :: N_coupl_real, N_coupl_real2, N_coupl_cmplx, N_coupl_cmplx2

    integer, dimension(:, :), allocatable :: table_flavor
    integer, dimension(:, :, :), allocatable :: table_color_flows
    integer, dimension(:, :), allocatable :: table_spin
    logical, dimension(:, :), allocatable :: table_ghost_flags
    type(OCF), dimension(:), allocatable :: table_color_factors
    logical, dimension(:, :), allocatable :: table_flv_col_is_allowed

    real(default), dimension(:), allocatable :: coupl_real
    real(default), dimension(:, :), allocatable :: coupl_real2
    complex(default), dimension(:), allocatable :: coupl_cmplx
    complex(default), dimension(:, :), allocatable :: coupl_cmplx2
    real(default), dimension(:), allocatable :: mass
    real(default), dimension(:), allocatable :: width

    type(momentum), dimension(:), allocatable :: momenta
    complex(default), dimension(:), allocatable :: amplitudes
    complex(default), dimension(:, :, :), allocatable :: table_amplitudes
    class(flavor_t), dimension(:), allocatable :: flavor
    class(color_t), dimension(:), allocatable :: color
    ! gfortran 4.7
    !class(helicity_t), dimension(:), pointer :: helicity => null()
    integer, dimension(:), allocatable :: helicity

    type(vm_scalar), dimension(:), allocatable :: scalars
    type(vm_spinor), dimension(:), allocatable :: spinors
    type(vm_conjspinor), dimension(:), allocatable :: conjspinors
    type(vm_bispinor), dimension(:), allocatable :: bispinors
    type(vm_vector), dimension(:), allocatable :: vectors
    type(vm_tensor_2), dimension(:), allocatable :: tensors_2
    type(vm_tensor_1), dimension(:), allocatable :: tensors_1
    type(vm_vectorspinor), dimension(:), allocatable :: vectorspinors

    logical, dimension(:), allocatable :: hel_is_allowed
    real(default), dimension(:), allocatable :: hel_max_abs
    real(default) :: hel_sum_abs = 0, hel_threshold = 1E10
    integer :: hel_count = 0, hel_cutoff = 100
    integer, dimension(:), allocatable :: hel_map
    integer :: hel_finite

    logical :: openmp

contains
 ⟨VM: TBP⟩
```

```
    end type
```

⟨*OVM Procedure Implementations*⟩+≡
```
  subroutine alloc_arrays (vm)
    type(vm_t), intent(inout) :: vm
    integer :: i
    allocate (vm%table_flavor(vm%N_particles, vm%N_flavors))
    allocate (vm%table_color_flows(vm%N_col_indices, vm%N_particles, &
                                   vm%N_col_flows))
    allocate (vm%table_spin(vm%N_particles, vm%N_helicities))
    allocate (vm%table_ghost_flags(vm%N_particles, vm%N_col_flows))
    allocate (vm%table_color_factors(vm%N_col_factors))
    allocate (vm%table_flv_col_is_allowed(vm%N_flavors, vm%N_col_flows))
    allocate (vm%momenta(vm%N_momenta))
    allocate (vm%amplitudes(vm%N_amplitudes))
    allocate (vm%table_amplitudes(vm%N_flavors, vm%N_col_flows, &
                                  vm%N_helicities))
    vm%table_amplitudes = zero
    allocate (vm%scalars(vm%N_scalars))
    allocate (vm%spinors(vm%N_spinors))
    allocate (vm%conjspinors(vm%N_conjspinors))
    allocate (vm%bispinors(vm%N_bispinors))
    allocate (vm%vectors(vm%N_vectors))
    allocate (vm%tensors_2(vm%N_tensors_2))
    allocate (vm%tensors_1(vm%N_tensors_1))
    allocate (vm%vectorspinors(vm%N_vectorspinors))
    allocate (vm%hel_is_allowed(vm%N_helicities))
    vm%hel_is_allowed = .True.
    allocate (vm%hel_max_abs(vm%N_helicities))
    vm%hel_max_abs = 0
    allocate (vm%hel_map(vm%N_helicities))
    vm%hel_map = (/(i, i = 1, vm%N_helicities)/)
    vm%hel_finite = vm%N_helicities
  end subroutine alloc_arrays
```

X.30.2 Controlling the VM

These type-bound procedures steer the VM

⟨*VM: TBP*⟩≡
```
    procedure :: init => vm_init
    procedure :: write => vm_write
    procedure :: reset => vm_reset
    procedure :: run => vm_run
    procedure :: final => vm_final
```

The `init` completely sets the environment for the OVM. Parameters can be changed with `reset` without reloading the bytecode.

⟨*OVM Procedure Implementations*⟩+≡
```
  subroutine vm_init (vm, bytecode_file, version, model, &
      coupl_real, coupl_real2, coupl_cmplx, coupl_cmplx2, &
      mass, width, verbose, out_fh, openmp)
    class(vm_t), intent(out) :: vm
```

```
      type(string_t), intent(in) :: bytecode_file
      type(string_t), intent(in) :: version
      type(string_t), intent(in) :: model
      real(default), dimension(:), optional, intent(in) :: coupl_real
      real(default), dimension(:, :), optional, intent(in) :: coupl_real2
      complex(default), dimension(:), optional, intent(in) :: coupl_cmplx
      complex(default), dimension(:, :), optional, intent(in) :: coupl_cmplx2
      real(default), dimension(:), optional, intent(in) :: mass
      real(default), dimension(:), optional, intent(in) :: width
      logical, optional, intent(in) :: verbose
      integer, optional, intent(in) :: out_fh
      logical, optional, intent(in) :: openmp
      vm%bytecode_file = bytecode_file
      vm%version = version
      vm%model = model
      if (present (coupl_real)) then
         allocate (vm%coupl_real (size (coupl_real)), source=coupl_real)
      end if
      if (present (coupl_real2)) then
         allocate (vm%coupl_real2 (2, size (coupl_real2, 2)), source=coupl_real2)
      end if
      if (present (coupl_cmplx)) then
         allocate (vm%coupl_cmplx (size (coupl_cmplx)), source=coupl_cmplx)
      end if
      if (present (coupl_cmplx2)) then
         allocate (vm%coupl_cmplx2 (2, size (coupl_cmplx2, 2)), &
                   source=coupl_cmplx2)
      end if
      if (present (mass)) then
         allocate (vm%mass(size(mass)), source=mass)
      end if
      if (present (width)) then
         allocate (vm%width(size (width)), source=width)
      end if
      if (present (openmp)) then
         vm%openmp = openmp
      else
         vm%openmp = .false.
      end if

      call basic_init (vm, verbose, out_fh)
   end subroutine vm_init


⟨OVM Procedure Implementations⟩+≡
   subroutine vm_reset (vm, &
         coupl_real, coupl_real2, coupl_cmplx, coupl_cmplx2, &
         mass, width, verbose, out_fh)
      class(vm_t), intent(inout) :: vm
      real(default), dimension(:), optional, intent(in) :: coupl_real
      real(default), dimension(:, :), optional, intent(in) :: coupl_real2
      complex(default), dimension(:), optional, intent(in) :: coupl_cmplx
      complex(default), dimension(:, :), optional, intent(in) :: coupl_cmplx2
      real(default), dimension(:), optional, intent(in) :: mass
      real(default), dimension(:), optional, intent(in) :: width
```

```
      logical, optional, intent(in) :: verbose
      integer, optional, intent(in) :: out_fh
      if (present (coupl_real)) then
         vm%coupl_real = coupl_real
      end if
      if (present (coupl_real2)) then
         vm%coupl_real2 = coupl_real2
      end if
      if (present (coupl_cmplx)) then
         vm%coupl_cmplx = coupl_cmplx
      end if
      if (present (coupl_cmplx2)) then
         vm%coupl_cmplx2 = coupl_cmplx2
      end if
      if (present (mass)) then
         vm%mass = mass
      end if
      if (present (width)) then
         vm%width = width
      end if
      if (present (verbose)) then
         vm%verbose = verbose
      end if
      if (present (out_fh)) then
         vm%out_fh = out_fh
      end if
   end subroutine vm_reset
```

Mainly for debugging
⟨*OVM Procedure Implementations*⟩+≡

```
   subroutine vm_write (vm)
     class(vm_t), intent(in) :: vm
     integer :: i, j, k
     call basic_write (vm)
     write(vm%out_fh, *) 'table_flavor             = ', vm%table_flavor
     write(vm%out_fh, *) 'table_color_flows        = ', vm%table_color_flows
     write(vm%out_fh, *) 'table_spin               = ', vm%table_spin
     write(vm%out_fh, *) 'table_ghost_flags        = ', vm%table_ghost_flags
     write(vm%out_fh, *) 'table_color_factors      = '
     do i = 1, size(vm%table_color_factors)
        write(vm%out_fh, *)  vm%table_color_factors(i)%i1, &
              vm%table_color_factors(i)%i2, &
              vm%table_color_factors(i)%factor
     end do

     write(vm%out_fh, *) 'table_flv_col_is_allowed  = ', &
                     vm%table_flv_col_is_allowed
     do i = 1, vm%N_flavors
        do j = 1, vm%N_col_flows
           do k = 1, vm%N_helicities
              write(vm%out_fh, *) 'table_amplitudes(f,c,h), f, c, h = ', vm%table_amplitudes(i,j,k),
           end do
        end do
     end do
```

```
if (allocated(vm%coupl_real)) then
   write(vm%out_fh, *) 'coupl_real          = ', vm%coupl_real
end if
if (allocated(vm%coupl_real2)) then
   write(vm%out_fh, *) 'coupl_real2         = ', vm%coupl_real2
end if
if (allocated(vm%coupl_cmplx)) then
   write(vm%out_fh, *) 'coupl_cmplx         = ', vm%coupl_cmplx
end if
if (allocated(vm%coupl_cmplx2)) then
   write(vm%out_fh, *) 'coupl_cmplx2        = ', vm%coupl_cmplx2
end if
write(vm%out_fh, *) 'mass                = ', vm%mass
write(vm%out_fh, *) 'width               = ', vm%width
write(vm%out_fh, *) 'momenta             = ', vm%momenta
! gfortran 4.7
!do i = 1, size(vm%flavor)
   !call vm%flavor(i)%write (vm%out_fh)
!end do
!do i = 1, size(vm%color)
   !call vm%color(i)%write (vm%out_fh)
!end do
!do i = 1, size(vm%helicity)
   !call vm%helicity(i)%write (vm%out_fh)
!end do
write(vm%out_fh, *) 'helicity            = ', vm%helicity
write(vm%out_fh, *) 'amplitudes       = ', vm%amplitudes
write(vm%out_fh, *) 'scalars        = ', vm%scalars
write(vm%out_fh, *) 'spinors        = ', vm%spinors
write(vm%out_fh, *) 'conjspinors    = ', vm%conjspinors
write(vm%out_fh, *) 'bispinors      = ', vm%bispinors
write(vm%out_fh, *) 'vectors        = ', vm%vectors
write(vm%out_fh, *) 'tensors_2      = ', vm%tensors_2
write(vm%out_fh, *) 'tensors_1      = ', vm%tensors_1
!!! !!! !!! Regression with ifort 16.0.0
!!! write(vm%out_fh, *) 'vectorspinors = ', vm%vectorspinors
write(vm%out_fh, *) 'N_momenta      = ', vm%N_momenta
write(vm%out_fh, *) 'N_particles    = ', vm%N_particles
write(vm%out_fh, *) 'N_prt_in       = ', vm%N_prt_in
write(vm%out_fh, *) 'N_prt_out      = ', vm%N_prt_out
write(vm%out_fh, *) 'N_amplitudes   = ', vm%N_amplitudes
write(vm%out_fh, *) 'N_helicities   = ', vm%N_helicities
write(vm%out_fh, *) 'N_col_flows    = ', vm%N_col_flows
write(vm%out_fh, *) 'N_col_indices  = ', vm%N_col_indices
write(vm%out_fh, *) 'N_flavors      = ', vm%N_flavors
write(vm%out_fh, *) 'N_col_factors  = ', vm%N_col_factors
write(vm%out_fh, *) 'N_scalars      = ', vm%N_scalars
write(vm%out_fh, *) 'N_spinors      = ', vm%N_spinors
write(vm%out_fh, *) 'N_conjspinors  = ', vm%N_conjspinors
write(vm%out_fh, *) 'N_bispinors    = ', vm%N_bispinors
write(vm%out_fh, *) 'N_vectors      = ', vm%N_vectors
write(vm%out_fh, *) 'N_tensors_2    = ', vm%N_tensors_2
write(vm%out_fh, *) 'N_tensors_1    = ', vm%N_tensors_1
write(vm%out_fh, *) 'N_vectorspinors = ', vm%N_vectorspinors
```

```
      write(vm%out_fh, *) 'Overall size of VM: '
      ! GNU extension
      ! write(vm%out_fh, *) 'sizeof(wavefunctions) = ', &
      !   sizeof(vm%scalars) + sizeof(vm%spinors) + sizeof(vm%conjspinors) + &
      !   sizeof(vm%bispinors) + sizeof(vm%vectors) + sizeof(vm%tensors_2) + &
      !   sizeof(vm%tensors_1) +  sizeof(vm%vectorspinors)
      ! write(vm%out_fh, *) 'sizeof(mometa) = ', sizeof(vm%momenta)
      ! write(vm%out_fh, *) 'sizeof(amplitudes) = ', sizeof(vm%amplitudes)
      ! write(vm%out_fh, *) 'sizeof(tables) = ', &
      !   sizeof(vm%table_amplitudes) + sizeof(vm%table_spin) + &
      !   sizeof(vm%table_flavor) + sizeof(vm%table_flv_col_is_allowed) + &
      !   sizeof(vm%table_color_flows) + sizeof(vm%table_color_factors) + &
      !   sizeof(vm%table_ghost_flags)
  end subroutine vm_write
```

Most of this is redundant (Fortran will deallocate when we leave the scope) but when we change from `allocatable`s to `pointer`s, it is necessary to avoid leaks

⟨*OVM Procedure Implementations*⟩+≡

```
  subroutine vm_final (vm)
    class(vm_t), intent(inout) :: vm
    deallocate (vm%table_flavor)
    deallocate (vm%table_color_flows)
    deallocate (vm%table_spin)
    deallocate (vm%table_ghost_flags)
    deallocate (vm%table_color_factors)
    deallocate (vm%table_flv_col_is_allowed)
    if (allocated (vm%coupl_real)) then
       deallocate (vm%coupl_real)
    end if
    if (allocated (vm%coupl_real2)) then
       deallocate (vm%coupl_real2)
    end if
    if (allocated (vm%coupl_cmplx)) then
       deallocate (vm%coupl_cmplx)
    end if
    if (allocated (vm%coupl_cmplx2)) then
       deallocate (vm%coupl_cmplx2)
    end if
    if (allocated (vm%mass)) then
       deallocate (vm%mass)
    end if
    if (allocated (vm%width)) then
       deallocate (vm%width)
    end if
    deallocate (vm%momenta)
    deallocate (vm%flavor)
    deallocate (vm%color)
    deallocate (vm%helicity)
    deallocate (vm%amplitudes)
    deallocate (vm%table_amplitudes)
    deallocate (vm%scalars)
    deallocate (vm%spinors)
    deallocate (vm%conjspinors)
```

```
      deallocate (vm%bispinors)
      deallocate (vm%vectors)
      deallocate (vm%tensors_2)
      deallocate (vm%tensors_1)
      deallocate (vm%vectorspinors)
    end subroutine vm_final
```

Handing over the polymorph object helicity didn't work out as planned. A work-around is the use of `pointers`. `flavor` and `color` are not yet used but would have to be changed to `pointers` as well. At least this potentially avoids copying. Actually, neither the allocatable nor the pointer version works in `gfortran 4.7` due to the broken `select type`. Back to Stone Age, i.e. integers.

⟨*OVM Procedure Implementations*⟩+≡

```
  subroutine vm_run (vm, mom, flavor, color, helicity)
    class(vm_t), intent(inout) :: vm
    real(default), dimension(0:3, *), intent(in) :: mom
    class(flavor_t), dimension(:), optional, intent(in) :: flavor
    class(color_t), dimension(:), optional, intent(in) :: color
    ! gfortran 4.7
    !class(helicity_t), dimension(:), optional, target, intent(in) :: helicity
    integer, dimension(:), optional, intent(in) :: helicity
    integer :: i, h, hi
    do i = 1, vm%N_momenta
      if (i <= vm%N_prt_in) then
        vm%momenta(i) = - mom(:, i)          ! incoming, crossing symmetry
      else
        vm%momenta(i) = mom(:, i)            ! outgoing
      end if
    end do
    if (present (flavor)) then
      allocate(vm%flavor(size(flavor)), source=flavor)
    else
      if (.not. (allocated (vm%flavor))) then
        allocate(flv_discrete::vm%flavor(vm%N_particles))
      end if
    end if
    if (present (color)) then
      allocate(vm%color(size(color)), source=color)
    else
      if (.not. (allocated (vm%color))) then
        allocate(col_discrete::vm%color(vm%N_col_flows))
      end if
    end if
    ! gfortran 4.7
    if (present (helicity)) then
      !vm%helicity => helicity
      vm%helicity = helicity
      call vm_run_one_helicity (vm, 1)
    else
      !if (.not. (associated (vm%helicity))) then
        !allocate(hel_discrete::vm%helicity(vm%N_particles))
      !end if
      if (.not. (allocated (vm%helicity))) then
```

```
            allocate(vm%helicity(vm%N_particles))
         end if
         if (vm%hel_finite == 0) return
         do hi = 1, vm%hel_finite
            h = vm%hel_map(hi)
            !<Work around [[gfortran 4.7 Bug 56731]] Implementation>>
            vm%helicity = vm%table_spin(:,h)
            call vm_run_one_helicity (vm, h)
         end do
      end if
   end subroutine vm_run
```

This only removes the `ICE` but still leads to a segmentation fault in `gfortran`
`4.7`. I am running out of ideas how to make this compiler work with arrays of
polymorph datatypes.

⟨*Work around* `gfortran 4.7 Bug 56731` *Declarations*⟩≡
```
   integer :: hj
```

⟨*Work around* `gfortran 4.7 Bug 56731` *Implementation*⟩≡
```
   do hj = 1, size(vm%helicity)
      select type (hel => vm%helicity(hj))
      type is (hel_discrete)
         hel%i = vm%table_spin(hj,h)
      end select
   end do
```

⟨*Original version*⟩≡
```
   select type (hel => vm%helicity)
   type is (hel_discrete)
      hel(:)%i = vm%table_spin(:,h)
   end select
```

⟨*OVM Procedure Implementations*⟩+≡
```
   subroutine vm_run_one_helicity (vm, h)
      class(vm_t), intent(inout) :: vm
      integer, intent(in) :: h
      integer :: f, c, i
      vm%amplitudes = zero
      if (vm%N_levels > 0) then
         call null_all_wfs (vm)
         call iterate_instructions (vm)
      end if
      i = 1
      do c = 1, vm%N_col_flows
         do f = 1, vm%N_flavors
            if (vm%table_flv_col_is_allowed(f,c)) then
               vm%table_amplitudes(f,c,h) = vm%amplitudes(i)
               i = i + 1
            end if
         end do
      end do
   end subroutine
```

⟨*OVM Procedure Implementations*⟩+≡

```
subroutine null_all_wfs (vm)
  type(vm_t), intent(inout) :: vm
  integer :: i, j
  vm%scalars%c = .False.
  vm%scalars%v = zero
  vm%spinors%c = .False.
  vm%conjspinors%c = .False.
  vm%bispinors%c = .False.
  vm%vectorspinors%c = .False.
  do i = 1, 4
     vm%spinors%v%a(i) = zero
     vm%conjspinors%v%a(i) = zero
     vm%bispinors%v%a(i) = zero
     do j = 1, 4
        vm%vectorspinors%v%psi(i)%a(j) = zero
     end do
  end do
  vm%vectors%c = .False.
  vm%vectors%v%t = zero
  vm%tensors_1%c = .False.
  vm%tensors_2%c = .False.
  do i = 1, 3
     vm%vectors%v%x(i) = zero
     vm%tensors_1%v%e(i) = zero
     vm%tensors_1%v%b(i) = zero
     do j = 1, 3
        vm%tensors_2%v%t(i,j) = zero
     end do
  end do
end subroutine
```

X.30.3   Reading the bytecode

⟨*OVM Procedure Implementations*⟩+≡

```
subroutine load_header (vm, IO)
  type(vm_t), intent(inout) :: vm
  integer, intent(inout) :: IO
  integer, dimension(len_instructions) :: line
  read(vm%bytecode_fh, fmt = *, iostat = IO) line
  vm%N_momenta = line(1)
  vm%N_particles = line(2)
  vm%N_prt_in = line(3)
  vm%N_prt_out = line(4)
  vm%N_amplitudes = line(5)
  vm%N_helicities = line(6)
  vm%N_col_flows = line(7)
  if (vm%N_momenta == 0) then
     vm%N_col_indices = 2
  else
     vm%N_col_indices = line(8)
  end if
  read(vm%bytecode_fh, fmt = *, iostat = IO)
  read(vm%bytecode_fh, fmt = *, iostat = IO) line
```

```
      vm%N_flavors = line(1)
      vm%N_col_factors = line(2)
      vm%N_scalars = line(3)
      vm%N_spinors = line(4)
      vm%N_conjspinors = line(5)
      vm%N_bispinors = line(6)
      vm%N_vectors = line(7)
      vm%N_tensors_2 = line(8)
      read(vm%bytecode_fh, fmt = *, iostat = IO)
      read(vm%bytecode_fh, fmt = *, iostat = IO) line
      vm%N_tensors_1 = line(1)
      vm%N_vectorspinors = line(2)
      ! Add 1 for seperating label lines like 'Another table'
      vm%N_table_lines = vm%N_helicities + 1 + vm%N_flavors + 1 + vm%N_col_flows &
        + 1 + vm%N_col_flows + 1 + vm%N_col_factors + 1 + vm%N_col_flows
    end subroutine load_header

⟨OVM Procedure Implementations⟩+≡
  subroutine read_tables (vm, IO)
    type(vm_t), intent(inout) :: vm
    integer, intent(inout) :: IO
    integer :: i
    integer, dimension(2) :: tmpcf
    integer, dimension(3) :: tmpfactor
    integer, dimension(vm%N_flavors) :: tmpF
    integer, dimension(vm%N_particles) :: tmpP
    real(default) :: factor
    do i = 1, vm%N_helicities
      read(vm%bytecode_fh, fmt = *, iostat = IO) vm%table_spin(:, i)
    end do

    read(vm%bytecode_fh, fmt = *, iostat = IO)
    do i = 1, vm%N_flavors
      read(vm%bytecode_fh, fmt = *, iostat = IO) vm%table_flavor(:, i)
    end do

    read(vm%bytecode_fh, fmt = *, iostat = IO)
    do i = 1, vm%N_col_flows
      read(vm%bytecode_fh, fmt = *, iostat = IO) vm%table_color_flows(:, :, i)
    end do

    read(vm%bytecode_fh, fmt = *, iostat = IO)
    do i = 1, vm%N_col_flows
      read(vm%bytecode_fh, fmt = *, iostat = IO) tmpP
      vm%table_ghost_flags(:, i) = int_to_log(tmpP)
    end do

    read(vm%bytecode_fh, fmt = *, iostat = IO)
    do i = 1, vm%N_col_factors
      read(vm%bytecode_fh, fmt = '(2I9)', iostat = IO, advance='no') tmpcf
      factor = zero
      do
        read(vm%bytecode_fh, fmt = '(3I9)', iostat = IO, advance='no', EOR=10) tmpfactor
        factor = factor + color_factor(tmpfactor(1), tmpfactor(2), tmpfactor(3))
```

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```
      end do
      10 vm%table_color_factors(i) = OCF(tmpcf(1), tmpcf(2), factor)
    end do

    read(vm%bytecode_fh, fmt = *, iostat = IO)
    do i = 1, vm%N_col_flows
      read(vm%bytecode_fh, fmt = *, iostat = IO) tmpF
      vm%table_flv_col_is_allowed(:, i) = int_to_log(tmpF)
    end do
  end subroutine read_tables
```

This checking has proven useful more than once
⟨*OVM Procedure Implementations*⟩+≡
```
  subroutine extended_version_check (vm, IO)
    type(vm_t), intent(in) :: vm
    integer, intent(inout) :: IO
    character(256) :: buffer
    read(vm%bytecode_fh, fmt = *, iostat = IO) buffer
    if (vm%model /= buffer) then
      print *, "Bytecode has been generated for another/older model."
      stop 1
    else
      if (vm%verbose) then
         write (vm%out_fh, fmt = *) "Using the model: "
         write (vm%out_fh, fmt = *) char(vm%model)
      end if
    end if
  end subroutine extended_version_check
```

This chunk is copied verbatim from the `basic_vm`
⟨*OVM Procedure Implementations*⟩+≡
```
  subroutine basic_init (vm, verbose, out_fh)
    type(vm_t), intent(inout) :: vm
    logical, optional, intent(in) :: verbose
    integer, optional, intent(in) :: out_fh
    if (present (verbose)) then
       vm%verbose = verbose
    else
       vm%verbose = .true.
    end if
    if (present (out_fh)) then
       vm%out_fh = out_fh
     else
       vm%out_fh = stdout
    end if
    call set_stream (vm)
    call alloc_and_count (vm)
    if (vm%N_levels > 0) then
       call read_bytecode (vm)
       call sanity_check (vm)
    end if
    close (vm%bytecode_fh)
  end subroutine basic_init
```

```
subroutine basic_write (vm)
  type(vm_t), intent(in) :: vm
  integer :: i
  write (vm%out_fh, *) '=====> VM ', char(vm%version), ' <====='
  write (vm%out_fh, *) 'verbose           =     ', vm%verbose
  write (vm%out_fh, *) 'bytecode_file     =     ', char (vm%bytecode_file)
  write (vm%out_fh, *) 'N_instructions    =     ', vm%N_instructions
  write (vm%out_fh, *) 'N_levels          =     ', vm%N_levels
  write (vm%out_fh, *) 'instructions      =     '
  do i = 1, vm%N_instructions
     write (vm%out_fh, *) vm%instructions(:, i)
  end do
  write (vm%out_fh, *) 'levels            =     ', vm%levels
end subroutine basic_write

subroutine alloc_and_count (vm)
  type(vm_t), intent(inout) :: vm
  integer, dimension(len_instructions) :: line
  character(256) :: buffer
  integer :: i, IO
  read(vm%bytecode_fh, fmt = *, iostat = IO) buffer
  if (vm%version /= buffer) then
    print *, "Bytecode has been generated with another Omega.Targets."
    stop 1
  else
    if (vm%verbose) then
       write (vm%out_fh, fmt = *) "Bytecode version fits."
    end if
  end if
  call extended_version_check (vm, IO)
  if (vm%verbose) then
     write (vm%out_fh, fmt = *) "Trying to allocate."
  end if
  do i = 1, N_comments
    read(vm%bytecode_fh, fmt = *, iostat = IO)
  end do
  call load_header (vm, IO)
  call alloc_arrays (vm)
  if (vm%N_momenta /= 0) then
     do i = 1, vm%N_table_lines + 1
       read(vm%bytecode_fh, fmt = *, iostat = IO)
     end do
     vm%N_instructions = 0
     vm%N_levels = 0
     do
       read(vm%bytecode_fh, fmt = *, end = 42) line
       if (line(1) /= 0) then
         vm%N_instructions = vm%N_instructions + 1
       else
         vm%N_levels = vm%N_levels + 1
       end if
     end do
     42 rewind(vm%bytecode_fh, iostat = IO)
```

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```
      allocate (vm%instructions(len_instructions, vm%N_instructions))
      allocate (vm%levels(vm%N_levels))
      if (IO /= 0) then
        print *, "Error: vm.alloc : Couldn't load bytecode!"
        stop 1
      end if
    end if
  end subroutine alloc_and_count

  subroutine read_bytecode (vm)
    type(vm_t), intent(inout) :: vm
    integer, dimension(len_instructions) :: line
    integer :: i, j, IO
    ! Jump over version number, comments, header and first table description
    do i = 1, N_version_lines + N_comments + N_header_lines + 1
      read (vm%bytecode_fh, fmt = *, iostat = IO)
    end do
    call read_tables (vm, IO)
    read (vm%bytecode_fh, fmt = *, iostat = IO)
    i = 0; j = 0
    do
      read (vm%bytecode_fh, fmt = *, iostat = IO) line
      if (IO /= 0) exit
      if (line(1) == 0) then
        if (j <= vm%N_levels) then
          j = j + 1
          vm%levels(j) = i                    ! last index of a level is saved
        else
          print *, 'Error: vm.read_bytecode: File has more levels than anticipated!'
          stop 1
        end if
      else
        if (i <= vm%N_instructions) then
          i = i + 1                           ! A valid instruction line
          vm%instructions(:, i) = line
        else
          print *, 'Error: vm.read_bytecode: File is larger than anticipated!'
          stop 1
        end if
      end if
    end do
  end subroutine read_bytecode

  subroutine iterate_instructions (vm)
    type(vm_t), intent(inout) :: vm
    integer :: i, j
    if (vm%openmp) then
      !$omp parallel
      do j = 1, vm%N_levels - 1
        !$omp do schedule (static)
        do i = vm%levels (j) + 1, vm%levels (j + 1)
          call decode (vm, i)
        end do
        !$omp end do
```

```
      end do
      !$omp end parallel
   else
      do j = 1, vm%N_levels - 1
         do i = vm%levels (j) + 1, vm%levels (j + 1)
            call decode (vm, i)
         end do
      end do
   end if
end subroutine iterate_instructions

subroutine set_stream (vm)
   type(vm_t), intent(inout) :: vm
   integer :: IO
   call find_free_unit (vm%bytecode_fh, IO)
   open (vm%bytecode_fh, file = char (vm%bytecode_file), form = 'formatted', &
     access = 'sequential', status = 'old', position = 'rewind', iostat = IO, &
     action = 'read')
   if (IO /= 0) then
     print *, "Error: vm.set_stream: Bytecode file '", char(vm%bytecode_file), &
               "' not found!"
     stop 1
   end if
end subroutine set_stream

subroutine sanity_check (vm)
   type(vm_t), intent(in) :: vm
   if (vm%levels(1) /= 0) then
      print *, "Error: vm.vm_init: levels(1) != 0"
      stop 1
   end if
   if (vm%levels(vm%N_levels) /= vm%N_instructions) then
      print *, "Error: vm.vm_init: levels(N_levels) != N_instructions"
      stop 1
   end if
   if (vm%verbose) then
     write(vm%out_fh, *) "vm passed sanity check. Starting calculation."
   end if
end subroutine sanity_check
```

X.30.4 Main Decode Function

This is the heart of the OVM

⟨*OVM Procedure Implementations*⟩+≡

```
   ! pure & ! if no warnings
   subroutine decode (vm, instruction_index)
     type(vm_t), intent(inout) :: vm
     integer, intent(in) :: instruction_index
     integer, dimension(len_instructions) :: i, curr
     complex(default) :: braket
     integer :: tmp
     real(default) :: w
```

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```
        i = vm%instructions (:, instruction_index)
        select case (i(1))
        case ( : -1)          ! Jump over subinstructions

        ⟨cases of decode⟩
        case (0)
          print *, 'Error: Levelbreak put in decode! Line:', &
                   instruction_index
          stop 1
        case default
          print *, "Error: Decode has case not catched! Line: ", &
                   instruction_index
          stop 1
        end select
      end subroutine decode
```

Momenta

The most trivial instruction

⟨*OVM Instructions*⟩≡
```
  integer, parameter :: ovm_ADD_MOMENTA = 1
```

⟨*cases of* decode⟩≡
```
  case (ovm_ADD_MOMENTA)
     vm%momenta(i(4)) = vm%momenta(i(5)) + vm%momenta(i(6))
     if (i(7) > 0) then
        vm%momenta(i(4)) = vm%momenta(i(4)) + vm%momenta(i(7))
     end if
```

Loading External states

⟨*OVM Instructions*⟩+≡
```
  integer, parameter :: ovm_LOAD_SCALAR = 10
  integer, parameter :: ovm_LOAD_SPINOR_INC = 11
  integer, parameter :: ovm_LOAD_SPINOR_OUT = 12
  integer, parameter :: ovm_LOAD_CONJSPINOR_INC = 13
  integer, parameter :: ovm_LOAD_CONJSPINOR_OUT = 14
  integer, parameter :: ovm_LOAD_MAJORANA_INC = 15
  integer, parameter :: ovm_LOAD_MAJORANA_OUT = 16
  integer, parameter :: ovm_LOAD_VECTOR_INC = 17
  integer, parameter :: ovm_LOAD_VECTOR_OUT = 18
  integer, parameter :: ovm_LOAD_VECTORSPINOR_INC = 19
  integer, parameter :: ovm_LOAD_VECTORSPINOR_OUT = 20
  integer, parameter :: ovm_LOAD_TENSOR2_INC = 21
  integer, parameter :: ovm_LOAD_TENSOR2_OUT = 22
  integer, parameter :: ovm_LOAD_BRS_SCALAR = 30
  integer, parameter :: ovm_LOAD_BRS_SPINOR_INC = 31
  integer, parameter :: ovm_LOAD_BRS_SPINOR_OUT = 32
  integer, parameter :: ovm_LOAD_BRS_CONJSPINOR_INC = 33
  integer, parameter :: ovm_LOAD_BRS_CONJSPINOR_OUT = 34
  integer, parameter :: ovm_LOAD_BRS_VECTOR_INC = 37
```

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```
  integer, parameter :: ovm_LOAD_BRS_VECTOR_OUT = 38
  integer, parameter :: ovm_LOAD_MAJORANA_GHOST_INC = 23
  integer, parameter :: ovm_LOAD_MAJORANA_GHOST_OUT = 24
  integer, parameter :: ovm_LOAD_BRS_MAJORANA_INC = 35
  integer, parameter :: ovm_LOAD_BRS_MAJORANA_OUT = 36
```

⟨case*s of* decode⟩+≡
```
  case (ovm_LOAD_SCALAR)
    vm%scalars(i(4))%v = one
    vm%scalars(i(4))%c = .True.

  case (ovm_LOAD_SPINOR_INC)
     call load_spinor(vm%spinors(i(4)), - ⟨p⟩, ⟨m⟩, &
                      vm%helicity(i(5)), ovm_LOAD_SPINOR_INC)

  case (ovm_LOAD_SPINOR_OUT)
     call load_spinor(vm%spinors(i(4)), ⟨p⟩, ⟨m⟩, &
                      vm%helicity(i(5)), ovm_LOAD_SPINOR_OUT)

  case (ovm_LOAD_CONJSPINOR_INC)
     call load_conjspinor(vm%conjspinors(i(4)), - ⟨p⟩, &
       ⟨m⟩, vm%helicity(i(5)), ovm_LOAD_CONJSPINOR_INC)

  case (ovm_LOAD_CONJSPINOR_OUT)
     call load_conjspinor(vm%conjspinors(i(4)), ⟨p⟩, &
       ⟨m⟩, vm%helicity(i(5)), ovm_LOAD_CONJSPINOR_OUT)

  case (ovm_LOAD_MAJORANA_INC)
     call load_bispinor(vm%bispinors(i(4)), - ⟨p⟩, &
       ⟨m⟩, vm%helicity(i(5)), ovm_LOAD_MAJORANA_INC)

  case (ovm_LOAD_MAJORANA_OUT)
     call load_bispinor(vm%bispinors(i(4)), ⟨p⟩, ⟨m⟩, &
                      vm%helicity(i(5)), ovm_LOAD_MAJORANA_OUT)

  case (ovm_LOAD_VECTOR_INC)
     call load_vector(vm%vectors(i(4)), - ⟨p⟩, ⟨m⟩, &
                      vm%helicity(i(5)), ovm_LOAD_VECTOR_INC)

  case (ovm_LOAD_VECTOR_OUT)
     call load_vector(vm%vectors(i(4)), ⟨p⟩, ⟨m⟩, &
                      vm%helicity(i(5)), ovm_LOAD_VECTOR_OUT)

  case (ovm_LOAD_VECTORSPINOR_INC)
    !select type (h => vm%helicity(i(5)))
    !type is (hel_discrete)
       !vm%vectorspinors(i(4))%v = veps(⟨m⟩, - ⟨p⟩, &
                                        !h%i)
    !end select
    vm%vectorspinors(i(4))%v = veps(⟨m⟩, - ⟨p⟩, &
                                    vm%helicity(i(5)))
    vm%vectorspinors(i(4))%c = .True.

  case (ovm_LOAD_VECTORSPINOR_OUT)
```

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```
  !select type (h => vm%helicity(i(5)))
  !type is (hel_discrete)
     !vm%vectorspinors(i(4))%v = veps(⟨m⟩, ⟨p⟩, &
                                       !h%i)
  !end select
  vm%vectorspinors(i(4))%v = veps(⟨m⟩, ⟨p⟩, &
                                  vm%helicity(i(5)))
  vm%vectorspinors(i(4))%c = .True.

case (ovm_LOAD_TENSOR2_INC)
  !select type (h => vm%helicity(i(5)))
  !type is (hel_discrete)
     !vm%tensors_2(i(4))%v = eps2(⟨m⟩, - ⟨p⟩, &
                                  !h%i)
  !end select
  vm%tensors_2(i(4))%c = .True.

case (ovm_LOAD_TENSOR2_OUT)
  !select type (h => vm%helicity(i(5)))
  !type is (hel_discrete)
     !vm%tensors_2(i(4))%v = eps2(⟨m⟩, ⟨p⟩, h%i)
  !end select
  vm%tensors_2(i(4))%c = .True.

case (ovm_LOAD_BRS_SCALAR)
  vm%scalars(i(4))%v = (0, -1) * (⟨p⟩ * ⟨p⟩ - &
                                  ⟨m⟩**2)
  vm%scalars(i(4))%c = .True.

case (ovm_LOAD_BRS_SPINOR_INC)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_BRS_SPINOR_OUT)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_BRS_CONJSPINOR_INC)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_BRS_CONJSPINOR_OUT)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_BRS_VECTOR_INC)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_BRS_VECTOR_OUT)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_MAJORANA_GHOST_INC)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_MAJORANA_GHOST_OUT)
  print *, 'not implemented'
  stop 1
case (ovm_LOAD_BRS_MAJORANA_INC)
```

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```
    print *, 'not implemented'
    stop 1
  case (ovm_LOAD_BRS_MAJORANA_OUT)
    print *, 'not implemented'
    stop 1
```

Brakets and Fusions

NB: during, execution, the type of the coupling constant is implicit in the instruction

⟨*OVM Instructions*⟩+≡

```
  integer, parameter :: ovm_CALC_BRAKET = 2

  integer, parameter :: ovm_FUSE_V_FF = -1
  integer, parameter :: ovm_FUSE_F_VF = -2
  integer, parameter :: ovm_FUSE_F_FV = -3
  integer, parameter :: ovm_FUSE_VA_FF = -4
  integer, parameter :: ovm_FUSE_F_VAF = -5
  integer, parameter :: ovm_FUSE_F_FVA = -6
  integer, parameter :: ovm_FUSE_VA2_FF = -7
  integer, parameter :: ovm_FUSE_F_VA2F = -8
  integer, parameter :: ovm_FUSE_F_FVA2 = -9
  integer, parameter :: ovm_FUSE_A_FF = -10
  integer, parameter :: ovm_FUSE_F_AF = -11
  integer, parameter :: ovm_FUSE_F_FA = -12
  integer, parameter :: ovm_FUSE_VL_FF = -13
  integer, parameter :: ovm_FUSE_F_VLF = -14
  integer, parameter :: ovm_FUSE_F_FVL = -15
  integer, parameter :: ovm_FUSE_VR_FF = -16
  integer, parameter :: ovm_FUSE_F_VRF = -17
  integer, parameter :: ovm_FUSE_F_FVR = -18
  integer, parameter :: ovm_FUSE_VLR_FF = -19
  integer, parameter :: ovm_FUSE_F_VLRF = -20
  integer, parameter :: ovm_FUSE_F_FVLR = -21
  integer, parameter :: ovm_FUSE_SP_FF = -22
  integer, parameter :: ovm_FUSE_F_SPF = -23
  integer, parameter :: ovm_FUSE_F_FSP = -24
  integer, parameter :: ovm_FUSE_S_FF = -25
  integer, parameter :: ovm_FUSE_F_SF = -26
  integer, parameter :: ovm_FUSE_F_FS = -27
  integer, parameter :: ovm_FUSE_P_FF = -28
  integer, parameter :: ovm_FUSE_F_PF = -29
  integer, parameter :: ovm_FUSE_F_FP = -30
  integer, parameter :: ovm_FUSE_SL_FF = -31
  integer, parameter :: ovm_FUSE_F_SLF = -32
  integer, parameter :: ovm_FUSE_F_FSL = -33
  integer, parameter :: ovm_FUSE_SR_FF = -34
  integer, parameter :: ovm_FUSE_F_SRF = -35
  integer, parameter :: ovm_FUSE_F_FSR = -36
  integer, parameter :: ovm_FUSE_SLR_FF = -37
  integer, parameter :: ovm_FUSE_F_SLRF = -38
  integer, parameter :: ovm_FUSE_F_FSLR = -39
```

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```
integer, parameter :: ovm_FUSE_G_GG = -40
integer, parameter :: ovm_FUSE_V_SS = -41
integer, parameter :: ovm_FUSE_S_VV = -42
integer, parameter :: ovm_FUSE_S_VS = -43
integer, parameter :: ovm_FUSE_V_SV = -44
integer, parameter :: ovm_FUSE_S_SS = -45
integer, parameter :: ovm_FUSE_S_SVV = -46
integer, parameter :: ovm_FUSE_V_SSV = -47
integer, parameter :: ovm_FUSE_S_SSS = -48
integer, parameter :: ovm_FUSE_V_VVV = -49

integer, parameter :: ovm_FUSE_S_G2 = -50
integer, parameter :: ovm_FUSE_G_SG = -51
integer, parameter :: ovm_FUSE_G_GS = -52
integer, parameter :: ovm_FUSE_S_G2_SKEW = -53
integer, parameter :: ovm_FUSE_G_SG_SKEW = -54
integer, parameter :: ovm_FUSE_G_GS_SKEW = -55
```

Shorthands

⟨*p*⟩≡
```
  vm%momenta(i(5))
```

⟨*m*⟩≡
```
  vm%mass(i(2))
```

⟨*p1*⟩≡
```
  vm%momenta(curr(6))
```

⟨*p2*⟩≡
```
  vm%momenta(curr(8))
```

⟨*v1*⟩≡
```
  vm%vectors(curr(5))%v
```

⟨*v2*⟩≡
```
  vm%vectors(curr(7))%v
```

⟨*s1*⟩≡
```
  vm%scalars(curr(5))%v
```

⟨*s2*⟩≡
```
  vm%scalars(curr(7))%v
```

⟨*c*⟩≡
```
  sgn_coupl_cmplx(vm, curr(2))
```

⟨*c1*⟩≡
```
  sgn_coupl_cmplx2(vm, curr(2), 1)
```

⟨*c2*⟩≡
```
  sgn_coupl_cmplx2(vm, curr(2), 2)
```

⟨*check for matching color and flavor amplitude of braket (old)*⟩≡
```
  if ((i(4) == o%cols(1)) .or. (i(4) == o%cols(2)) .or. &
    ((mode%col_MC .eq. FULL_SUM) .or. (mode%col_MC .eq. DIAG_COL))) then
```

Just a stub for now. Will be reimplemented with the polymorph type `color` similar to the `select type(helicity)` when we need it.

⟨*check for matching color and flavor amplitude*⟩≡

⟨case*s of* decode⟩+≡
```
  case (ovm_CALC_BRAKET)
     ⟨check for matching color and flavor amplitude⟩
     tmp = instruction_index + 1
     do
       if (tmp > vm%N_instructions) exit
       curr = vm%instructions(:, tmp)
       if (curr(1) >= 0) exit                         ! End of fusions
       select case (curr(1))
       case (ovm_FUSE_V_FF, ovm_FUSE_VL_FF, ovm_FUSE_VR_FF)
         braket = vm%vectors(curr(4))%v * vec_ff(vm, curr)

       case (ovm_FUSE_F_VF, ovm_FUSE_F_VLF, ovm_FUSE_F_VRF)
         braket = vm%conjspinors(curr(4))%v * ferm_vf(vm, curr)

       case (ovm_FUSE_F_FV, ovm_FUSE_F_FVL, ovm_FUSE_F_FVR)
         braket = ferm_fv(vm, curr) * vm%spinors(curr(4))%v

       case (ovm_FUSE_VA_FF)
         braket = vm%vectors(curr(4))%v * vec_ff2(vm, curr)

       case (ovm_FUSE_F_VAF)
         braket = vm%conjspinors(curr(4))%v * ferm_vf2(vm, curr)

       case (ovm_FUSE_F_FVA)
         braket = ferm_fv2(vm, curr) * vm%spinors(curr(4))%v

       case (ovm_FUSE_S_FF, ovm_FUSE_SP_FF)
         braket = vm%scalars(curr(4))%v * scal_ff(vm, curr)

       case (ovm_FUSE_F_SF, ovm_FUSE_F_SPF)
         braket = vm%conjspinors(curr(4))%v * ferm_sf(vm, curr)

       case (ovm_FUSE_F_FS, ovm_FUSE_F_FSP)
         braket = ferm_fs(vm, curr) * vm%spinors(curr(4))%v

       case (ovm_FUSE_G_GG)
         braket = vm%vectors(curr(4))%v * &
           g_gg(⟨c⟩, &
                ⟨v1⟩, ⟨p1⟩, &
                ⟨v2⟩, ⟨p2⟩)

       case (ovm_FUSE_S_VV)
         braket = vm%scalars(curr(4))%v * ⟨c⟩ * &
                  (⟨v1⟩ * vm%vectors(curr(6))%v)

       case (ovm_FUSE_V_SS)
         braket = vm%vectors(curr(4))%v * &
                  v_ss(⟨c⟩, ⟨s1⟩, ⟨p1⟩, &
                             ⟨s2⟩, ⟨p2⟩)

       case (ovm_FUSE_S_G2, ovm_FUSE_S_G2_SKEW)
         braket = vm%scalars(curr(4))%v * scal_g2(vm, curr)
```

```fortran
      case (ovm_FUSE_G_SG, ovm_FUSE_G_GS, ovm_FUSE_G_SG_SKEW, ovm_FUSE_G_GS_SKEW)
        braket = vm%vectors(curr(4))%v * gauge_sg(vm, curr)

      case (ovm_FUSE_S_VS)
        braket = vm%scalars(curr(4))%v * &
          s_vs(⟨c⟩, &
               ⟨v1⟩, ⟨p1⟩, &
               ⟨s2⟩, ⟨p2⟩)

      case (ovm_FUSE_V_SV)
        braket = (vm%vectors(curr(4))%v * vm%vectors(curr(6))%v) * &
                 (⟨c⟩ * ⟨s1⟩)

      case (ovm_FUSE_S_SS)
        braket = vm%scalars(curr(4))%v * &
          ⟨c⟩ * &
          (⟨s1⟩ * vm%scalars(curr(6))%v)

      case (ovm_FUSE_S_SSS)
        braket = vm%scalars(curr(4))%v * &
          ⟨c⟩ * &
          (⟨s1⟩ * vm%scalars(curr(6))%v * &
           ⟨s2⟩)

      case (ovm_FUSE_S_SVV)
        braket = vm%scalars(curr(4))%v * &
          ⟨c⟩ * &
          ⟨s1⟩ * (vm%vectors(curr(6))%v * &
                             ⟨v2⟩)

      case (ovm_FUSE_V_SSV)
        braket = vm%vectors(curr(4))%v * &
          (⟨c⟩ * ⟨s1⟩ * &
           vm%scalars(curr(6))%v) * ⟨v2⟩

      case (ovm_FUSE_V_VVV)
        braket = ⟨c⟩ * &
          (⟨v1⟩ * vm%vectors(curr(6))%v) * &
          (vm%vectors(curr(4))%v * ⟨v2⟩)

      case default
        print *, 'Braket', curr(1), 'not implemented'
        stop 1

    end select
    vm%amplitudes(i(4)) = vm%amplitudes(i(4)) + curr(3) * braket
    tmp = tmp + 1
end do

vm%amplitudes(i(4)) = vm%amplitudes(i(4)) * i(2)
if (i(5) > 1) then
  vm%amplitudes(i(4)) = vm%amplitudes(i(4)) * &          ! Symmetry factor
                        (one / sqrt(real(i(5), kind=default)))
end if
```

Propagators

⟨*OVM Instructions*⟩+≡

```
integer, parameter :: ovm_PROPAGATE_SCALAR = 51
integer, parameter :: ovm_PROPAGATE_COL_SCALAR = 52
integer, parameter :: ovm_PROPAGATE_GHOST = 53
integer, parameter :: ovm_PROPAGATE_SPINOR = 54
integer, parameter :: ovm_PROPAGATE_CONJSPINOR = 55
integer, parameter :: ovm_PROPAGATE_MAJORANA = 56
integer, parameter :: ovm_PROPAGATE_COL_MAJORANA = 57
integer, parameter :: ovm_PROPAGATE_UNITARITY = 58
integer, parameter :: ovm_PROPAGATE_COL_UNITARITY = 59
integer, parameter :: ovm_PROPAGATE_FEYNMAN = 60
integer, parameter :: ovm_PROPAGATE_COL_FEYNMAN = 61
integer, parameter :: ovm_PROPAGATE_VECTORSPINOR = 62
integer, parameter :: ovm_PROPAGATE_TENSOR2 = 63
integer, parameter :: ovm_PROPAGATE_NONE = 64
```

⟨*check for matching color and flavor amplitude of propagator (old)*⟩≡

```
if ((mode%col_MC .eq. FULL_SUM) .or. (mode%col_MC .eq. DIAG_COL)) then
  select case(i(1))
  case (ovm_PROPAGATE_PSI)
    go = .not. vm%spinors%c(i(4))
  case (ovm_PROPAGATE_PSIBAR)
    go = .not. vm%conjspinors%c(i(4))
  case (ovm_PROPAGATE_UNITARITY, ovm_PROPAGATE_FEYNMAN, &
    ovm_PROPAGATE_COL_FEYNMAN)
    go = .not. vm%vectors%c(i(4))
  end select
else
  go = (i(8) == o%cols(1)) .or. (i(8) == o%cols(2))
end if
if (go) then
```

⟨*case*s *of* `decode`⟩+≡

```
  ⟨check for matching color and flavor amplitude⟩
  case (ovm_PROPAGATE_SCALAR : ovm_PROPAGATE_NONE)
    tmp = instruction_index + 1
    do
      curr = vm%instructions(:,tmp)
      if (curr(1) >= 0) exit                     ! End of fusions
      select case (curr(1))
      case (ovm_FUSE_V_FF, ovm_FUSE_VL_FF, ovm_FUSE_VR_FF)
        vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + curr(3) * &
                                vec_ff(vm, curr)

      case (ovm_FUSE_F_VF, ovm_FUSE_F_VLF, ovm_FUSE_F_VRF)
        vm%spinors(curr(4))%v = vm%spinors(curr(4))%v + curr(3) * &
                                ferm_vf(vm, curr)

      case (ovm_FUSE_F_FV, ovm_FUSE_F_FVL, ovm_FUSE_F_FVR)
        vm%conjspinors(curr(4))%v = vm%conjspinors(curr(4))%v + curr(3) * &
                                    ferm_fv(vm, curr)
```

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```
          case (ovm_FUSE_VA_FF)
            vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + curr(3) * &
                              vec_ff2(vm, curr)

          case (ovm_FUSE_F_VAF)
            vm%spinors(curr(4))%v = vm%spinors(curr(4))%v + curr(3) * &
                              ferm_vf2(vm, curr)

          case (ovm_FUSE_F_FVA)
            vm%conjspinors(curr(4))%v = vm%conjspinors(curr(4))%v + curr(3) * &
                                ferm_fv2(vm, curr)

          case (ovm_FUSE_S_FF, ovm_FUSE_SP_FF)
            vm%scalars(curr(4))%v = vm%scalars(curr(4))%v + curr(3) * &
                              scal_ff(vm, curr)

          case (ovm_FUSE_F_SF, ovm_FUSE_F_SPF)
            vm%spinors(curr(4))%v = vm%spinors(curr(4))%v + curr(3) * &
                              ferm_sf(vm, curr)

          case (ovm_FUSE_F_FS, ovm_FUSE_F_FSP)
            vm%conjspinors(curr(4))%v = vm%conjspinors(curr(4))%v + curr(3) * &
                                ferm_fs(vm, curr)

          case (ovm_FUSE_G_GG)
            vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + curr(3) * &
               g_gg(⟨c⟩, ⟨v1⟩, &
                    ⟨p1⟩, ⟨v2⟩, &
                    ⟨p2⟩)

          case (ovm_FUSE_S_VV)
            vm%scalars(curr(4))%v = vm%scalars(curr(4))%v + curr(3) * &
               ⟨c⟩ * &
               (⟨v1⟩ * vm%vectors(curr(6))%v)

          case (ovm_FUSE_V_SS)
            vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + curr(3) * &
                    v_ss(⟨c⟩, ⟨s1⟩, ⟨p1⟩, &
                              ⟨s2⟩, ⟨p2⟩)


          case (ovm_FUSE_S_G2, ovm_FUSE_S_G2_SKEW)
            vm%scalars(curr(4))%v = vm%scalars(curr(4))%v + &
                              scal_g2(vm, curr) * curr(3)

          case (ovm_FUSE_G_SG, ovm_FUSE_G_GS, ovm_FUSE_G_SG_SKEW, ovm_FUSE_G_GS_SKEW)
            vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + &
                              gauge_sg(vm, curr) * curr(3)

          case (ovm_FUSE_S_VS)
            vm%scalars(curr(4))%v = vm%scalars(curr(4))%v + &
               s_vs(⟨c⟩, &
                    ⟨v1⟩, ⟨p1⟩, &
```

```
                        ⟨s2⟩, ⟨p2⟩) * curr(3)

    case (ovm_FUSE_V_SV)
      vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + &
        vm%vectors(curr(6))%v * &
        (⟨c⟩ * ⟨s1⟩ * curr(3))

    case (ovm_FUSE_S_SS)
      vm%scalars(curr(4))%v = vm%scalars(curr(4))%v + &
        ⟨c⟩ * &
        (⟨s1⟩ * vm%scalars(curr(6))%v) * curr(3)

    case (ovm_FUSE_S_SSS)
      vm%scalars(curr(4))%v = vm%scalars(curr(4))%v + &
        ⟨c⟩ * &
        (⟨s1⟩ * vm%scalars(curr(6))%v * &
         ⟨s2⟩) * curr(3)

    case (ovm_FUSE_S_SVV)
      vm%scalars(curr(4))%v = vm%scalars(curr(4))%v + &
        ⟨c⟩ * &
        ⟨s1⟩ * (vm%vectors(curr(6))%v * &
                                ⟨v2⟩) * curr(3)

    case (ovm_FUSE_V_SSV)
      vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + &
        (⟨c⟩ * ⟨s1⟩ * &
         vm%scalars(curr(6))%v) * ⟨v2⟩ * curr(3)

    case (ovm_FUSE_V_VVV)
      vm%vectors(curr(4))%v = vm%vectors(curr(4))%v + &
        (⟨c⟩ * (⟨v1⟩ * &
         vm%vectors(curr(6))%v)) * curr(3) * ⟨v2⟩

    case default
      print *, 'Fusion', curr(1), 'not implemented'
      stop 1

    end select
    tmp = tmp + 1
  end do

  select case (i(3))
  case (0)
    w = zero

  case (1)
    w = vm%width(i(2))

  case (2)
    w = wd_tl(⟨p⟩, vm%width(i(2)))

  case default
     print *, 'not implemented'
```

```
      stop 1

    end select

    select case (i(1))
    ⟨propagator cases in decode⟩
    end select
```

⟨*propagator* cases *in* decode⟩≡
```
  case (ovm_PROPAGATE_SCALAR)
     vm%scalars(i(4))%v = pr_phi(⟨p⟩, ⟨m⟩, &
                                   w, vm%scalars(i(4))%v)
     vm%scalars(i(4))%c = .True.

  case (ovm_PROPAGATE_COL_SCALAR)
     vm%scalars(i(4))%v = - one / N_ * pr_phi(⟨p⟩, &
                           ⟨m⟩, w, vm%scalars(i(4))%v)
     vm%scalars(i(4))%c = .True.

  case (ovm_PROPAGATE_GHOST)
     vm%scalars(i(4))%v = imago * pr_phi(⟨p⟩, ⟨m⟩, &
                                   w, vm%scalars(i(4))%v)
     vm%scalars(i(4))%c = .True.

  case (ovm_PROPAGATE_SPINOR)
     vm%spinors(i(4))%v = pr_psi(⟨p⟩, ⟨m⟩, &
                                   w, vm%spinors(i(4))%v)
     vm%spinors(i(4))%c = .True.

  case (ovm_PROPAGATE_CONJSPINOR)
     vm%conjspinors(i(4))%v = pr_psibar(⟨p⟩, ⟨m⟩, &
                                   w, vm%conjspinors(i(4))%v)
     vm%conjspinors(i(4))%c = .True.

  case (ovm_PROPAGATE_MAJORANA)
     vm%bispinors(i(4))%v = bi_pr_psi(⟨p⟩, ⟨m⟩, &
                                   w, vm%bispinors(i(4))%v)
     vm%bispinors(i(4))%c = .True.

  case (ovm_PROPAGATE_COL_MAJORANA)
     vm%bispinors(i(4))%v = (- one / N_) * &
                           bi_pr_psi(⟨p⟩, ⟨m⟩, &
                                   w, vm%bispinors(i(4))%v)
     vm%bispinors(i(4))%c = .True.

  case (ovm_PROPAGATE_UNITARITY)
     vm%vectors(i(4))%v = pr_unitarity(⟨p⟩, ⟨m⟩, &
                                     w, vm%vectors(i(4))%v)
     vm%vectors(i(4))%c = .True.

  case (ovm_PROPAGATE_COL_UNITARITY)
     vm%vectors(i(4))%v = - one / N_ * pr_unitarity(⟨p⟩, &
                           ⟨m⟩, w, vm%vectors(i(4))%v)
     vm%vectors(i(4))%c = .True.
```

```
case (ovm_PROPAGATE_FEYNMAN)
  vm%vectors(i(4))%v = pr_feynman(⟨p⟩, vm%vectors(i(4))%v)
  vm%vectors(i(4))%c = .True.

case (ovm_PROPAGATE_COL_FEYNMAN)
  vm%vectors(i(4))%v = - one / N_ * &
                        pr_feynman(⟨p⟩, vm%vectors(i(4))%v)
  vm%vectors(i(4))%c = .True.

case (ovm_PROPAGATE_VECTORSPINOR)
  vm%vectorspinors(i(4))%v = pr_grav(⟨p⟩, ⟨m⟩, &
                                w, vm%vectorspinors(i(4))%v)
  vm%vectorspinors(i(4))%c = .True.

case (ovm_PROPAGATE_TENSOR2)
  vm%tensors_2(i(4))%v = pr_tensor(⟨p⟩, ⟨m⟩, &
                                w, vm%tensors_2(i(4))%v)
  vm%tensors_2(i(4))%c = .True.

case (ovm_PROPAGATE_NONE)
! This will not work with color MC. Appropriate type%c has to be set to
! .True.
```

X.30.5 Helper functions

Factoring out these parts helps a lot to keep sane but might hurt the performance of the VM noticably. In that case, we have to copy & paste to avoid the additional function calls. Note that with preprocessor macros, we could maintain this factorized form (and factor out even more since types don't have to match), in case we would decide to allow this

⟨load outer wave function⟩≡
```
  !select type (h)
  !type is (hel_trigonometric)
    !wf%v = (cos (h%theta) * load_wf (m, p, + 1) + &
            !sin (h%theta) * load_wf (m, p, - 1)) * sqrt2
  !type is (hel_exponential)
    !wf%v = exp (+ imago * h%phi) * load_wf (m, p, + 1) + &
            !exp (- imago * h%phi) * load_wf (m, p, - 1)
  !type is (hel_spherical)
    !wf%v = (exp (+ imago * h%phi) * cos (h%theta) * load_wf (m, p, + 1) + &
            !exp (- imago * h%phi) * sin (h%theta) * load_wf (m, p, - 1)) * &
            !sqrt2
  !type is(hel_discrete)
      !wf%v = load_wf (m, p, h%i)
  !end select
  wf%v = load_wf (m, p, h)
  wf%c = .True.
```
Caveat: Helicity MC not tested with Majorana particles but should be fine

⟨check for matching color and flavor amplitude of wf (old)⟩≡
```
  if ((mode%col_MC .eq. FULL_SUM) .or. (mode%col_MC .eq. DIAG_COL)) then
    go = .not. vm%spinors%c(i(4))
```

```
      else
        go = (i(8) == o%cols(1)) .or. (i(8) == o%cols(2))
      end if
      if (go) ..
```

⟨*OVM Procedure Implementations*⟩+≡

```
  subroutine load_bispinor(wf, p, m, h, opcode)
    type(vm_bispinor), intent(out) :: wf
    type(momentum), intent(in) :: p
    real(default), intent(in) :: m
    !class(helicity_t), intent(in) :: h
    integer, intent(in) :: h
    integer, intent(in) :: opcode
    procedure(bi_u), pointer :: load_wf
```
⟨*check for matching color and flavor amplitude*⟩
```
    select case (opcode)
    case (ovm_LOAD_MAJORANA_INC)
       load_wf => bi_u
    case (ovm_LOAD_MAJORANA_OUT)
       load_wf => bi_v
    case default
       load_wf => null()
    end select
```
⟨*load outer wave function*⟩
```
  end subroutine load_bispinor

  subroutine load_spinor(wf, p, m, h, opcode)
    type(vm_spinor), intent(out) :: wf
    type(momentum), intent(in) :: p
    real(default), intent(in) :: m
    !class(helicity_t), intent(in) :: h
    integer, intent(in) :: h
    integer, intent(in) :: opcode
    procedure(u), pointer :: load_wf
```
⟨*check for matching color and flavor amplitude*⟩
```
    select case (opcode)
    case (ovm_LOAD_SPINOR_INC)
       load_wf => u
    case (ovm_LOAD_SPINOR_OUT)
       load_wf => v
    case default
       load_wf => null()
    end select
```
⟨*load outer wave function*⟩
```
  end subroutine load_spinor

  subroutine load_conjspinor(wf, p, m, h, opcode)
    type(vm_conjspinor), intent(out) :: wf
    type(momentum), intent(in) :: p
    real(default), intent(in) :: m
    !class(helicity_t), intent(in) :: h
    integer, intent(in) :: h
    integer, intent(in) :: opcode
    procedure(ubar), pointer :: load_wf
```
⟨*check for matching color and flavor amplitude*⟩

```
    select case (opcode)
    case (ovm_LOAD_CONJSPINOR_INC)
       load_wf => vbar
    case (ovm_LOAD_CONJSPINOR_OUT)
       load_wf => ubar
    case default
       load_wf => null()
    end select
    ⟨load outer wave function⟩
  end subroutine load_conjspinor

  subroutine load_vector(wf, p, m, h, opcode)
    type(vm_vector), intent(out) :: wf
    type(momentum), intent(in) :: p
    real(default), intent(in) :: m
    !class(helicity_t), intent(in) :: h
    integer, intent(in) :: h
    integer, intent(in) :: opcode
    procedure(eps), pointer :: load_wf
    ⟨check for matching color and flavor amplitude⟩
    load_wf => eps
    ⟨load outer wave function⟩
    if (opcode == ovm_LOAD_VECTOR_OUT) then
       wf%v = conjg(wf%v)
    end if
  end subroutine load_vector
```

⟨OVM Procedure Implementations⟩+≡

```
  function ferm_vf(vm, curr) result (x)
    type(spinor) :: x
    class(vm_t), intent(in) :: vm
    integer, dimension(:), intent(in) :: curr
    procedure(f_vf), pointer :: load_wf
    select case (curr(1))
    case (ovm_FUSE_F_VF)
       load_wf => f_vf
    case (ovm_FUSE_F_VLF)
       load_wf => f_vlf
    case (ovm_FUSE_F_VRF)
       load_wf => f_vrf
    case default
       load_wf => null()
    end select
    x = load_wf(⟨c⟩, ⟨v1⟩, vm%spinors(curr(6))%v)
  end function ferm_vf

  function ferm_vf2(vm, curr) result (x)
    type(spinor) :: x
    class(vm_t), intent(in) :: vm
    integer, dimension(:), intent(in) :: curr
    procedure(f_vaf), pointer :: load_wf
    select case (curr(1))
    case (ovm_FUSE_F_VAF)
       load_wf => f_vaf
```

```
  case default
     load_wf => null()
  end select
  x = f_vaf(⟨c1⟩, ⟨c2⟩, ⟨v1⟩, vm%spinors(curr(6))%v)
end function ferm_vf2

function ferm_sf(vm, curr) result (x)
  type(spinor) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  select case (curr(1))
  case (ovm_FUSE_F_SF)
     x = f_sf(⟨c⟩, ⟨s1⟩, vm%spinors(curr(6))%v)
  case (ovm_FUSE_F_SPF)
     x = f_spf(⟨c1⟩, ⟨c2⟩, ⟨s1⟩, vm%spinors(curr(6))%v)
  case default
  end select
end function ferm_sf

function ferm_fv(vm, curr) result (x)
  type(conjspinor) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  procedure(f_fv), pointer :: load_wf
  select case (curr(1))
  case (ovm_FUSE_F_FV)
     load_wf => f_fv
  case (ovm_FUSE_F_FVL)
     load_wf => f_fvl
  case (ovm_FUSE_F_FVR)
     load_wf => f_fvr
  case default
     load_wf => null()
  end select
  x = load_wf(⟨c⟩, vm%conjspinors(curr(5))%v, vm%vectors(curr(6))%v)
end function ferm_fv

function ferm_fv2(vm, curr) result (x)
  type(conjspinor) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  procedure(f_fva), pointer :: load_wf
  select case (curr(1))
  case (ovm_FUSE_F_FVA)
     load_wf => f_fva
  case default
     load_wf => null()
  end select
  x = f_fva(⟨c1⟩, ⟨c2⟩, &
           vm%conjspinors(curr(5))%v, vm%vectors(curr(6))%v)
end function ferm_fv2

function ferm_fs(vm, curr) result (x)
  type(conjspinor) :: x
```

```
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  procedure(f_fs), pointer :: load_wf
  select case (curr(1))
  case (ovm_FUSE_F_FS)
     x = f_fs(⟨c⟩, vm%conjspinors(curr(5))%v, vm%scalars(curr(6))%v)
  case (ovm_FUSE_F_FSP)
     x = f_fsp(⟨c1⟩, ⟨c2⟩, &
          vm%conjspinors(curr(5))%v, vm%scalars(curr(6))%v)
  case default
     x%a = zero
  end select
end function ferm_fs

function vec_ff(vm, curr) result (x)
  type(vector) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  procedure(v_ff), pointer :: load_wf
  select case (curr(1))
  case (ovm_FUSE_V_FF)
     load_wf => v_ff
  case (ovm_FUSE_VL_FF)
     load_wf => vl_ff
  case (ovm_FUSE_VR_FF)
     load_wf => vr_ff
  case default
     load_wf => null()
  end select
  x = load_wf(⟨c⟩, vm%conjspinors(curr(5))%v, vm%spinors(curr(6))%v)
end function vec_ff

function vec_ff2(vm, curr) result (x)
  type(vector) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  procedure(va_ff), pointer :: load_wf
  select case (curr(1))
  case (ovm_FUSE_VA_FF)
     load_wf => va_ff
  case default
     load_wf => null()
  end select
  x = load_wf(⟨c1⟩, ⟨c2⟩, &
              vm%conjspinors(curr(5))%v, vm%spinors(curr(6))%v)
end function vec_ff2

function scal_ff(vm, curr) result (x)
  complex(default) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  select case (curr(1))
  case (ovm_FUSE_S_FF)
     x = s_ff(⟨c⟩, &
```

```
                 vm%conjspinors(curr(5))%v, vm%spinors(curr(6))%v)
    case (ovm_FUSE_SP_FF)
       x = sp_ff(⟨c1⟩, ⟨c2⟩, &
              vm%conjspinors(curr(5))%v, vm%spinors(curr(6))%v)
    case default
       x = zero
    end select
end function scal_ff

function scal_g2(vm, curr) result (x)
  complex(default) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  select case (curr(1))
  case (ovm_FUSE_S_G2)
     x = ⟨c⟩ * (((⟨p1⟩ * ⟨v2⟩) * &
                 (⟨p2⟩ * ⟨v1⟩) - &
                 (⟨p1⟩ * ⟨p2⟩) * &
                 (⟨v2⟩ * ⟨v1⟩)))
  case (ovm_FUSE_S_G2_SKEW)
     x = - phi_vv(⟨c⟩, ⟨p1⟩, ⟨p2⟩, &
                 ⟨v1⟩, ⟨v2⟩)
  case default
     x = zero
  end select
end function scal_g2

pure function gauge_sg(vm, curr) result (x)
  type(vector) :: x
  class(vm_t), intent(in) :: vm
  integer, dimension(:), intent(in) :: curr
  select case (curr(1))
  case (ovm_FUSE_G_SG)
     x = ⟨c⟩ * ⟨s1⟩ * ( &
         -((⟨p1⟩ + ⟨p2⟩) * &
           ⟨v2⟩) * ⟨p2⟩ - &
         (-(⟨p1⟩ + ⟨p2⟩) * &
           ⟨p2⟩) * ⟨v2⟩)
  case (ovm_FUSE_G_GS)
     x = ⟨c⟩ * ⟨s1⟩ * ( &
         -((⟨p1⟩ + ⟨p2⟩) * &
           ⟨v2⟩) * ⟨p2⟩ - &
         (-(⟨p1⟩ + ⟨p2⟩) * &
           ⟨p2⟩) * ⟨v2⟩)
  case (ovm_FUSE_G_SG_SKEW)
     x = - v_phiv(⟨c⟩, ⟨s1⟩, ⟨p1⟩, &
                 ⟨p2⟩, ⟨v2⟩)
  case (ovm_FUSE_G_GS_SKEW)
     x = - v_phiv(⟨c⟩, ⟨s2⟩, ⟨p1⟩, &
                 ⟨p2⟩, ⟨v1⟩)
  case default
     x = [zero, zero, zero, zero]
  end select
end function gauge_sg
```

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Some really tiny ones that hopefully get inlined by the compiler

⟨*OVM Procedure Implementations*⟩+≡

```
elemental function sgn_coupl_cmplx(vm, j) result (s)
  class(vm_t), intent(in) :: vm
  integer, intent(in) :: j
  complex(default) :: s
  s = isign(1, j) * vm%coupl_cmplx(abs(j))
end function sgn_coupl_cmplx

elemental function sgn_coupl_cmplx2(vm, j, i) result (s)
  class(vm_t), intent(in) :: vm
  integer, intent(in) :: j, i
  complex(default) :: s
  if (i == 1) then
     s = isign(1, j) * vm%coupl_cmplx2(i, abs(j))
   else
     s = isign(1, j) * vm%coupl_cmplx2(i, abs(j))
  end if
end function sgn_coupl_cmplx2

elemental function int_to_log(i) result(yorn)
  integer, intent(in) :: i
  logical :: yorn
  if (i /= 0) then
    yorn = .true.
  else
    yorn = .false.
  end if
end function

elemental function color_factor(num, den, pwr) result (cf)
  integer, intent(in) :: num, den, pwr
  real(kind=default) :: cf
  if (pwr == 0) then
    cf = (one * num) / den
  else
    cf = (one * num) / den * (N_**pwr)
  end if
end function color_factor
```

X.30.6 O'Mega Interface

We want to keep the interface close to the native Fortran code but of course one has to hand over the `vm` additionally

⟨*VM: TBP*⟩+≡

```
procedure :: number_particles_in => vm_number_particles_in
procedure :: number_particles_out => vm_number_particles_out
procedure :: number_color_indices => vm_number_color_indices
procedure :: reset_helicity_selection => vm_reset_helicity_selection
procedure :: new_event => vm_new_event
procedure :: color_sum => vm_color_sum
```

```
      procedure :: spin_states => vm_spin_states
      procedure :: number_spin_states => vm_number_spin_states
      procedure :: number_color_flows => vm_number_color_flows
      procedure :: flavor_states => vm_flavor_states
      procedure :: number_flavor_states => vm_number_flavor_states
      procedure :: color_flows => vm_color_flows
      procedure :: color_factors => vm_color_factors
      procedure :: number_color_factors => vm_number_color_factors
      procedure :: is_allowed => vm_is_allowed
      procedure :: get_amplitude => vm_get_amplitude
```

⟨*OVM Procedure Implementations*⟩+≡
```
  elemental function vm_number_particles_in (vm) result (n)
    class(vm_t), intent(in) :: vm
    integer :: n
    n = vm%N_prt_in
  end function vm_number_particles_in

  elemental function vm_number_particles_out (vm) result (n)
    class(vm_t), intent(in) :: vm
    integer :: n
    n = vm%N_prt_out
  end function vm_number_particles_out

  elemental function vm_number_spin_states (vm) result (n)
    class(vm_t), intent(in) :: vm
    integer :: n
    n = vm%N_helicities
  end function vm_number_spin_states

  pure subroutine vm_spin_states (vm, a)
    class(vm_t), intent(in) :: vm
    integer, dimension(:,:), intent(out) :: a
    a = vm%table_spin
  end subroutine vm_spin_states

  elemental function vm_number_flavor_states (vm) result (n)
    class(vm_t), intent(in) :: vm
    integer :: n
    n = vm%N_flavors
  end function vm_number_flavor_states

  pure subroutine vm_flavor_states (vm, a)
    class(vm_t), intent(in) :: vm
    integer, dimension(:,:), intent(out) :: a
    a = vm%table_flavor
  end subroutine vm_flavor_states

  elemental function vm_number_color_indices (vm) result (n)
    class(vm_t), intent(in) :: vm
    integer :: n
    n = vm%N_col_indices
  end function vm_number_color_indices

  elemental function vm_number_color_flows (vm) result (n)
```

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```
    class(vm_t), intent(in) :: vm
    integer :: n
    n = vm%N_col_flows
  end function vm_number_color_flows

  pure subroutine vm_color_flows (vm, a, g)
    class(vm_t), intent(in) :: vm
    integer, dimension(:,:,:), intent(out) :: a
    logical, dimension(:,:), intent(out) :: g
    a = vm%table_color_flows
    g = vm%table_ghost_flags
  end subroutine vm_color_flows

  elemental function vm_number_color_factors (vm) result (n)
    class(vm_t), intent(in) :: vm
    integer :: n
    n = vm%N_col_factors
  end function vm_number_color_factors

  pure subroutine vm_color_factors (vm, cf)
    class(vm_t), intent(in) :: vm
    type(OCF), dimension(:), intent(out) :: cf
    cf = vm%table_color_factors
  end subroutine vm_color_factors

  ! pure & ! pure unless OpenMp
  function vm_color_sum (vm, flv, hel) result (amp2)
    class(vm_t), intent(in) :: vm
    integer, intent(in) :: flv, hel
    real(default) :: amp2
    amp2 = ovm_color_sum (flv, hel, vm%table_amplitudes, vm%table_color_factors)
  end function vm_color_sum

  subroutine vm_new_event (vm, p)
    class(vm_t), intent(inout) :: vm
    real(default), dimension(0:3,*), intent(in) :: p
    logical :: mask_dirty
    integer :: hel
    call vm%run (p)
    if ((vm%hel_threshold .gt. 0) .and. (vm%hel_count .le. vm%hel_cutoff)) then
       call omega_update_helicity_selection (vm%hel_count, vm%table_amplitudes, &
         vm%hel_max_abs, vm%hel_sum_abs, vm%hel_is_allowed, vm%hel_threshold, &
         vm%hel_cutoff, mask_dirty)
       if (mask_dirty) then
         vm%hel_finite = 0
         do hel = 1, vm%N_helicities
            if (vm%hel_is_allowed(hel)) then
               vm%hel_finite = vm%hel_finite + 1
               vm%hel_map(vm%hel_finite) = hel
            end if
         end do
       end if
    end if
  end subroutine vm_new_event
```

```
pure subroutine vm_reset_helicity_selection (vm, threshold, cutoff)
  class(vm_t), intent(inout) :: vm
  real(kind=default), intent(in) :: threshold
  integer, intent(in) :: cutoff
  integer :: i
  vm%hel_is_allowed = .True.
  vm%hel_max_abs = 0
  vm%hel_sum_abs = 0
  vm%hel_count = 0
  vm%hel_threshold = threshold
  vm%hel_cutoff = cutoff
  vm%hel_map = (/(i, i = 1, vm%N_helicities)/)
  vm%hel_finite = vm%N_helicities
end subroutine vm_reset_helicity_selection

pure function vm_is_allowed (vm, flv, hel, col) result (yorn)
  class(vm_t), intent(in) :: vm
  logical :: yorn
  integer, intent(in) :: flv, hel, col
  yorn = vm%table_flv_col_is_allowed(flv,col) .and. vm%hel_is_allowed(hel)
end function vm_is_allowed

pure function vm_get_amplitude (vm, flv, hel, col) result (amp_result)
  class(vm_t), intent(in) :: vm
  complex(kind=default) :: amp_result
  integer, intent(in) :: flv, hel, col
  amp_result = vm%table_amplitudes(flv, col, hel)
end function vm_get_amplitude
```

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