

Circe1 (internal Version 2.2): Beam Spectra for Simulating Linear Collider Physics*

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Abstract

I describe parameterizations of realistic e^{\pm} - and γ -beam spectra at future linear e^+e^- -colliders. Emphasis is put on simplicity and reproducibility of the parameterizations, supporting reproducible physics simulations. The parameterizations are implemented in a library of distribution functions and event generators.

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Program Summary:

- **Title of program:** Circe1 (March 2014)
- **Program obtainable** by anonymous ftp from the host `crunch.ikp.physik.th-darmstadt.de` in the directory `pub/ohl/circe`.
- **Licensing provisions:** Free software under the GNU General Public License.
- **Programming language used:** Fortran77 originally, transferred to Fortran90
- **Number of program lines in distributed program, including test data, etc.:** ≈ 1100 (excluding comments)
- **Computer/Operating System:** Any with a Fortran90 programming environment.
- **Memory required to execute with typical data:** Negligible on the scale of typical applications calling the library.
- **Typical running time:** A small fraction (typically a few percent) of the running time of applications calling the library.
- **Purpose of program:** Provide simple and reproducible, yet realistic, parameterizations of the e^\pm - and γ -beam spectra for linear colliders.
- **Nature of physical problem:** The intricate beam dynamics in the interaction region of a high luminosity linear collider at $\sqrt{s} = 500\text{GeV}$ result in non-trivial energy spectra of the scattering electrons, positrons and photons. Physics simulations require simple and reproducible, yet realistic, parameterizations of these spectra.
- **Method of solution:** Parameterization, curve fitting, Monte Carlo event generation.
- **Keywords:** Event generation, beamstrahlung, linear colliders.

1 Introduction

Despite the enormous quantitative success of the electro-weak standard model up to energies of 200GeV, neither the nature of electro-weak symmetry breaking (EWSB) nor the origin of mass are understood.

From theoretical considerations, we know that clues to the answer of these open questions are hidden in the energy range below $\Lambda_{\text{EWSB}} = 4\pi v \approx 3.1\text{TeV}$. Either we will discover a Higgs particle in this energy range or signatures for a strongly interacting EWSB sector will be found. Experiments at CERN's Large Hadron Collider (LHC) will shed a first light on this regime in the next decade. In the past it has been very fruitful to complement experiments at high energy hadron colliders with experiments at e^+e^- -colliders. The simpler initial state allows more precise measurements with smaller theoretical errors. Lucid expositions of the physics opportunities of high energy e^+e^- colliders with references to the literature can be found in [1].

However, the power emitted by circular storage rings in form of synchrotron radiation scales like $(E/m)^4/R^2$ with the energy and mass of the particle and the radius of the ring. This cost becomes prohibitive after LEP2 and a Linear Collider (LC) has to be built instead.

Unfortunately, the “interesting” hard cross sections scale like $1/s$ with the square of the center of mass energy and a LC will have to operate at extremely high luminosities in excess of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. To achieve such luminosities, the bunches of electrons and positrons have to be very dense. Under these conditions, the electrons undergo acceleration from strong electromagnetic forces from the positron bunch (and vice versa). The resulting synchrotron radiation is called *beamstrahlung* [2] and has a strong effect on the energy spectrum $D(x_1, x_2)$ of the colliding particles. This changes the observable e^+e^- cross sections

$$\frac{d\sigma_0^{e^+e^-}}{d\Omega}(s) \rightarrow \frac{d\sigma^{e^+e^-}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^+e^-}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^+e^-}}{d\Omega'}(x_1 x_2 s) \quad (1a)$$

and produces luminosity for $e^\pm\gamma$ and $\gamma\gamma$ collisions:

$$\frac{d\sigma^{e^\pm\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^\pm\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^\pm\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1b)$$

$$\frac{d\sigma^{\gamma\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{\gamma\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{\gamma\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1c)$$

Therefore, simulations of the physics expected at a LC need to know the spectra of the e^\pm and γ beams precisely.

Microscopic simulations of the beam dynamics are available (e.g. ABEL[3], CAIN[4] and Guinea-Pig[5]) and their predictions are compatible with each other. But they require too much computer time and memory for direct use in physics programs. Circe1 provides a fast and simple parameterization of the results from these simulations. Furthermore, even if the computational cost of the simulations would be negligible, the input parameters for microscopic simulations are not convenient for particle physics applications. Due to the highly

| | SBAND | TESLA | XBAND | SBAND | TESLA | XBAND |
|---------------------------------|-------|-------|-------|--------|--------|-------|
| E/GeV | 250 | 250 | 250 | 500 | 500 | 500 |
| $N_{\text{particles}}/10^{10}$ | 1.1 | 3.63 | 0.65 | 2.9 | 1.8 | 0.95 |
| $\epsilon_x/10^{-6}\text{mrad}$ | 5 | 14 | 5 | 10 | 14 | 5 |
| $\epsilon_y/10^{-6}\text{mrad}$ | 0.25 | 0.25 | 0.08 | 0.1 | 0.06 | 0.1 |
| β_x^*/mm | 10.98 | 24.95 | 8.00 | 32 | 25 | 10.00 |
| β_y^*/mm | 0.45 | 0.70 | 0.13 | 0.8 | 0.7 | 0.12 |
| σ_x/nm | 335 | 845 | 286 | 571.87 | 598.08 | 226 |
| σ_y/nm | 15.1 | 18.9 | 4.52 | 9.04 | 6.55 | 3.57 |
| $\sigma_z/\mu\text{m}$ | 300 | 700 | 100 | 500 | 500 | 125 |
| f_{rep} | 50 | 5 | 180 | 50 | 5 | 180 |
| n_{bunch} | 333 | 1135 | 90 | 125 | 2270 | 90 |

Table 1: Accelerator parameters for three typical designs at $\sqrt{s} = 500\text{GeV}$ and $\sqrt{s} = 1\text{TeV}$. The resulting distributions are shown in figure 1. The design efforts are currently concentrated on a 350GeV-800GeV LC. Therefore the Tesla parameters for 1TeV are slightly out of date.

non-linear beam dynamics, the optimization of LC designs is a subtle art [6], that is best practiced by the experts. Furthermore, particle physics applications need benchmarking and easily reproducible parameterizations are required for this purpose.

The parameterizations in **Circe1** are not based on approximate solutions (cf. [7]) of the beamstrahlung dynamics. Instead, they provide a “phenomenological” description of the results from full simulations. The parameterizations are as simple as possible while remaining consistent with basic physical principles:

1. *positivity*: the distribution functions $D(x_1, x_2)$ *must not* be negative in the physical region $[0, 1] \times [0, 1]$.
2. *integrability*: the definite integral of the distribution functions over the physical region $[0, 1] \times [0, 1]$ *must* exist, even though the distributions can have singularities.

This paper is organized as follows: I start in section 2 with a discussion of the input for the microscopic simulations. In section 3 I describe the usage of the **Circe1** library and in section 4 I discuss some technical details of the implementation. After discussing the parameterizations available (in internal version 2.2) in section 5, I conclude in section 8.

2 Parameters

The microscopic simulation program **Guinea-Pig** [5] used for the current version of the parameterizations in **Circe1** simulates the passage of electrons through a



Figure 1: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions at $\sqrt{s} = 500 \text{ GeV}$ and $\sqrt{s} = 1 \text{ TeV}$ in a doubly logarithmic plot. The accelerator parameters are taken from table 1.

| | TESLA | TESLA | TESLA |
|---------------------------------|---------|-------|--------|
| E/GeV | 175 | 250 | 400 |
| $N_{\text{particles}}/10^{10}$ | 3.63 | 3.63 | 3.63 |
| $\epsilon_x/10^{-6}\text{mrad}$ | 14 | 14 | 14 |
| $\epsilon_y/10^{-6}\text{mrad}$ | 0.25 | 0.25 | 0.1 |
| β_x^*/mm | 25.00 | 24.95 | 15.00 |
| β_y^*/mm | 0.70 | 0.70 | 0.70 |
| σ_x/nm | 1010.94 | 845 | 668.67 |
| σ_y/nm | 22.6 | 18.9 | 9.46 |
| $\sigma_z/\mu\text{m}$ | 700 | 700 | 700 |
| f_{rep} | 5 | 5 | 5 |
| n_{bunch} | 1135 | 1135 | 1135 |

Table 2: Accelerator parameters for the Tesla design at three planned [8] energies. The resulting distributions are shown in figure 2.

| | High- \mathcal{L} | Low- \mathcal{L} | Low- ϵ_y |
|---------------------------------|---------------------|--------------------|-------------------|
| E/GeV | 400 | 400 | 400 |
| $N_{\text{particles}}/10^{10}$ | 3.63 | 3.63 | 1.800 |
| $\epsilon_x/10^{-6}\text{mrad}$ | 14 | 14 | 12 |
| $\epsilon_y/10^{-6}\text{mrad}$ | 0.1 | 0.25 | 0.025 |
| β_x^*/mm | 15.00 | 25.00 | 25.00 |
| β_y^*/mm | 0.70 | 0.70 | 0.50 |
| σ_x/nm | 668.67 | 700.00 | |
| σ_y/nm | 9.46 | | |
| $\sigma_z/\mu\text{m}$ | 700 | 700 | 500 |
| f_{rep} | 5 | 5 | 3 |
| n_{bunch} | 1135 | 1135 | 2260 |

Table 3: Variant accelerator parameters for the Tesla design at 800 GeV.



Figure 2: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions for Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 2.

| | TESLA | TESLA |
|----------------------------------|-------|-------|
| E/GeV | 250 | 400 |
| $N_{\text{particles}}/10^{10}$ | 2 | 1.40 |
| $\epsilon_x/10^{-6}\text{m rad}$ | 10 | 8 |
| $\epsilon_y/10^{-6}\text{m rad}$ | 0.03 | 0.01 |
| β_x^*/mm | 15.00 | 15.00 |
| β_y^*/mm | 0.40 | 0.30 |
| σ_x/nm | 553 | 391 |
| σ_y/nm | 5 | 2 |
| $\sigma_z/\mu\text{m}$ | 400 | 300 |
| f_{rep} | 5 | 3 |
| n_{bunch} | 2820 | 4500 |

Table 4: Accelerator parameters for a high luminosity Tesla design at two planned [8] energies. The resulting distributions are shown in figure 3.



Figure 3: Version 5, revision 1998 05 05 of the factorized e^{\pm} - and γ -distributions for a high luminosity Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 4.

bunch of electrons (and vice versa). It takes the following accelerator parameters as input:

E : the energy of the particles before the beam-beam interaction.

$N_{\text{particles}}$: the number of particles per bunch.

$\epsilon_{x,y}$: the normalized horizontal and vertical emittances.

$\beta_{x,y}^*$: the horizontal and vertical beta functions.

$\sigma_{x,y,z}$: the horizontal, vertical and longitudinal beam size. A Gaussian shape is used for the charge distribution in the bunches.

f_{rep} : the repetition rate.

n_{bunch} : the number of bunches per train.

The transversal beam sizes, beta functions and normalized emittances for relativistic particles are related by

$$\beta_{x,y}^* = \frac{\sigma_{x,y}^2}{\epsilon_{x,y}} \frac{E}{m_e} \quad (2)$$

The parameters used in the most recent revision of the parameterizations are collected in tables 1 and 2. The resulting factorized electron/positron and photon distributions in version 1 of the parameterizations are depicted in figures 1 and 2.

The most important purpose of **Circe1** is to map the manifold of possible beam spectra for the NLC to a *finite* number of *reproducible* parameterizations. The distributions

$$D_{p_1 p_2}^{\alpha \nu \rho}(x_1, x_2; \sqrt{s}) \quad (3)$$

provided by **Circe1** are indexed by three integers



Figure 4: *Experimental:* Version 1, revision 0 of the factorized e^- - and γ -distributions for Tesla- e^-e^- in a doubly logarithmic plot. The accelerator parameters are taken from table 2 and have *not* been endorsed for use in an e^-e^- -machine yet!.

α : the *accelerator design class*: currently there are three options: S-band [9], Tesla [8], X-band [10, 11]. More variety will be added later, in particular the e^-e^- mode and the $e^-\gamma$ and $\gamma\gamma$ laser backscattering modes of these designs.

ν : the *version of the parameterization*: over the years, the form of the parameterizations can change, either because better approximations are found or because new simulation programs become available. All versions will remain available in order to be able to reproduce calculations.

ρ : the *revision date for the parameterization*: a particular parameterization can contain bugs, which will be fixed in subsequent revisions. While only the most recent revision should be used for new calculations, old revisions will remain available in order to be able to reproduce calculations.

The continuous parameter \sqrt{s} in (3) is misleading, because accelerator parameters have been optimized for discrete values of the energy. Therefore the distributions are not available for all values of \sqrt{s} .

The usage of the distributions in application programs is discussed in section 3.1. `Circe1` provides for each of the distributions a non-uniform random variate generator, that generates energy fractions according to the distributions. The usage of these generators is discussed in section 3.2.

3 Usage

3.1 Distributions

A generic interface to all distributions $D_{p_1 p_2}(x_1, x_2)$ is given by the `circe` function

11a \langle API documentation 11a $\rangle \equiv$

```
function circe, d, x1, x2
    real(kind=double) :: circe
    integer :: p1, p2
    d = circe (x1, x2, p1, p2)
```

Uses `circe` 31b.

where the energy fractions are specified by $x_{1,2}$ and the particles $p_{1,2}$ are identified by their standard Monte Carlo codes (we use C1 as a prefix to avoid name clashes when using CIRCE1 inside WHIZARD):[13]

11b \langle Particle codes 11b $\rangle \equiv$

```
integer, parameter, public :: C1_ELECTRON = 11
integer, parameter, public :: C1_POSITRON = -11
integer, parameter, public :: C1_PHOTON = 22
```

Defines:

C1_ELECTRON, used in chunks 21e, 31b, 73b, 80c, and 81a.
C1_PHOTON, used in chunks 31b, 73b, 80c, 81a, and 87.
C1_POSITRON, used in chunks 22 and 81a.

The distributions can have integrable singularities at the end points, therefore the calling functions *must not* evaluate them at the endpoints 0 and 1. This is usually not a problem, since standard mapping techniques (cf. (10) below) will have to be used to take care of the singularity anyway. Nevertheless, all applications should favor open quadrature formulae (i.e. formulae not involving the endpoints) over closed formulae. The distributions are guaranteed to vanish unless $0 < x_{1,2} < 1$, with two exceptions. Firstly, the value -1 allows to pick up the integral of the continuum contribution:

$$D_{p_1 p_2}(-1, x_2) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 D_{p_1 p_2}(x_1, x_2) \quad (4a)$$

$$D_{p_1 p_2}(x_1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4b)$$

$$D_{p_1 p_2}(-1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4c)$$

The other exception is that the strength of δ -function contributions at the endpoint can be picked up from the value at this endpoint:

$$D_{e^+ e^-}(x_1, x_2) = D_{e^+ e^-}(1, 1) \delta(1 - x_1) \delta(1 - x_2) + \text{smooth and single } \delta \quad (5a)$$

$$D_{e^\pm \gamma}(x_1, x_2) = D_{e^\pm \gamma}(1, x_2) \delta(1 - x_1) + \text{smooth} \quad (5b)$$

$$D_{\gamma e^\pm}(x_1, x_2) = D_{\gamma e^\pm}(x_1, 1) \delta(1 - x_2) + \text{smooth} \quad (5c)$$

The use of these special values is demonstrated in an example in section 3.1.1 below.

The distributions are normalized such that

$$\lim_{\epsilon \rightarrow +0} \int_{-\epsilon}^{1+\epsilon} dx_1 dx_2 D_{e^+ e^-}(x_1, x_2) = 1. \quad (6)$$

and the nominal $e^+ e^-$ -luminosity of the currently active accelerator design can be retrieved from the database with the subroutine `circel`. The value is given in units of

$$\text{fb}^{-1} v^{-1} = 10^{32} \text{cm}^{-2} \text{sec}^{-1} \quad (7)$$

where $v = 10^7 \text{sec} \approx \text{year}/\pi$ is an “effective year” of running with about 30% up-time.

12a `<API documentation 11a>+≡`
`real(kind=dobule) :: lumi`
`call circel (lumi)`

Uses `circel 41e`.

A particular parameterization is selected by the `circses` function:

12b `<API documentation 11a>+≡`
`real(kind=double) :: x1m, x2m, roots`
`integer :: acc, ver, rev, chat`
`call circses (x1m, x2m, roots, acc, ver, rev, chat)`

Uses `circses 32a`.

The parameter **roots** corresponds to the nominal center of mass energy \sqrt{s} /GeV of the collider. Currently $\sqrt{s} = 350\text{GeV}, 500\text{GeV}, 800\text{GeV}, 1\text{TeV}$ (i.e. 350D0, 500D0, 800D0 and 1000D0) are supported. Application programs can *not* assume that energy values are interpolated. For convenience, e.g. in top threshold scans around 350GeV, a small interval around the supported values will be accepted as synonymous with the central value, but a warning will be printed. Section 5 should be consulted for the discrete values supported by a particular version of the parameterizations. Negative values of **roots** will keep the currently active value for \sqrt{s} .

The parameters **x1m** and **x2m** will set thresholds $x_{1,\min}$ and $x_{2,\min}$ for the event generation in the routines described in section 3.2.

The parameter **acc** selects the accelerator design. Currently the following accelerator codes are recognized:

13a $\langle \text{Accelerator codes 13a} \rangle \equiv$

```

integer, parameter :: SBAND = 1
integer, parameter :: TESLA = 2
integer, parameter :: XBAND = 3
integer, parameter :: JLCNLC = 3
integer, parameter :: SBNDEE = 4
integer, parameter :: TESLEE = 5
integer, parameter :: XBNDEE = 6
integer, parameter :: NLCH = 7
integer, parameter :: ILC = 8
integer, parameter :: CLIC = 9

```

Defines:

CLIC, used in chunk 35d.
 ILC, used in chunks 35d, 69–71, and 92.
 JLCNLC, used in chunks 17b, 18, 35d, 57a, 60–63, 66c, 67c, and 92.
 SBAND, used in chunks 35d, 40, 44–47, 49b, 50a, 92, 107c, and 108c.
 SBNDEE, used in chunks 34b, 35d, 41b, 44c, 46–49, and 92.
 TESLA, used in chunks 32, 35d, 40, 44–47, 49–54, 56, 57, 59d, 60b, 62, 66b, 67b, 91b, 92, 107c, 108c, and 110a.
 TESLEE, used in chunks 34b, 35d, 48, and 92.
 XBAND, used in chunks 40, 44–47, 49b, 50a, 57d, 107c, and 108c.
 XBNDEE, used in chunks 34b, 35d, 48, 49a, 91b, and 92.

The total number of accelerator codes

13b $\langle \text{Accelerator codes 13a} \rangle + \equiv$

```

integer, parameter :: NACC = 9

```

Defines:

NACC, used in chunks 17b, 34, 35, 40, 41, 44–47, 51d, 53e, 56a, 59c, 62a, and 66a.

The **ver** parameter is used to determine the version as follows:

ver > 0 : a frozen version which is documented in section 5. For example, version 1 is a family of factorized Beta distributions: $D(x_1, x_2) \propto x_1^{a_1} (1 - x_1)^{b_1} x_2^{a_2} (1 - x_2)^{b_2}$.

ver = 0 : the latest experimental version, which is usually not documented and can change at any time without announcement.

ver < 0 : keep the currently active version.

The **rev** parameter is used to determine the revision of a version as follows:

rev > 0 : a frozen revision which is documented in section 5. The integer **rev** is constructed from the date as follows: **rev** = $10^4 \cdot \text{year} + 10^2 \cdot \text{month} + \text{day}$, where the year is greater than 1995. Since Fortran77 ignored whitespace, it could be written like 1996 07 11 for readability. In Fortran90 the white space have been erased. If there is no exact match, the most recent revision before the specified date is chosen.

rev = 0 : the most recent revision.

rev < 0 : keep the currently active revision.

Finally, the parameter **chat** controls the “chattiness” of **circe**. If it is 0, only error messages are printed. If it is 1, the parameters in use are printed whenever they change. Higher values of **chat** can produce even more diagnostics.

In addition to the generic interface **circe**, there are specialized functions for particular particle distributions. Obviously

$$D_{e^\pm\gamma}^{\alpha\nu\rho}(x_1, x_2, s) = D_{\gamma e^\pm}^{\alpha\nu\rho}(x_2, x_1, s) \quad (8)$$

and there are three independent functions $D_{e^-e^+}$, $D_{e^-\gamma}$ and $D_{\gamma\gamma}$ for the e^+e^- colliders with reasonable mnemonics:

14 `<API documentation 11a>+≡`
`real(kind=double) :: circee, circeg, circgg`
`d = circee (x1, x2)`
`d = circeg (x1, x2)`
`d = circgg (x1, x2)`

Uses **circee** 41g, **circeg** 42c, and **circgg** 43c.

Calling the latter three functions is marginally faster in the current implementation, but this can change in the future.

3.1.1 Example

For clarification, let me give a simple example. Imagine we want to calculate the integrated production cross section

$$\sigma_X(s) = \int dx_1 dx_2 \sigma_{e^+e^- \rightarrow X}(x_1 x_2 s) D_{e^+e^-}(x_1, x_2, s) \quad (9)$$

Since the distributions are singular in the $x_{1,2} \rightarrow 1$ limit, we have to map away this singularity with

$$x \rightarrow t = (1 - x)^{1/\eta} \quad (10a)$$

Therefore

$$\int_0^1 dx f(x) = \int_0^1 dt \eta t^{\eta-1} f(1 - t^\eta) \quad (10b)$$

with η sufficiently large to give the integrand a finite limit at $x \rightarrow 1$. If f diverges like a power $f(x) \propto 1/(1 - x)^\beta$, this means $\eta > 1/(1 - \beta)$.

As a specific example, let us “measure” a one particle s -channel exchange cross section

$$\sigma(s) \propto \frac{1}{s} \quad (11)$$

```

15a <circe1_sample.f90: public 15a>≡
    public :: sigma
    Uses sigma 15b.

15b <circe1_sample.f90: subroutines 15b>≡
    function sigma (s)
    real(kind=double) :: s, sigma
    sigma = 1d0 / s
    end function sigma

```

Defines:
 sigma, used in chunks 15–18 and 20d.

I will present the example code in a bottom-up fashion, which should be intuitive and is described in some more detail in appendix A. Assuming the existence of a one- and a two-dimensional Gaussian integration function `gauss1` and `gauss2`,¹ we can perform the integral as follows:

```

15c <Gauss integration 15c>≡
    s = sigma (1d0) * circee (1d0, 1d0) &
    + gauss1 (d1, 0d0, 1d0, EPS) &
    + gauss1 (d2, 0d0, 1d0, EPS) &
    + gauss2 (d12, 0d0, 1d0, 0d0, 1d0, EPS)
    write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
    1000 format (1X, A22, 1X, F6.2, '%')

```

Uses `circee` 41g, `d1` 16a, `d12` 15e, `d2` 16c, `gauss1` 89f, `gauss2` 90d, and `sigma` 15b.

Note how the four combinations of continuum and δ -peak are integrated separately, where you have to use three auxiliary functions `d1`, `d2` and `d12`. The continuum contribution, including the Jacobian:

```

15d <circe1_sample.f90: public 15a>+≡
    public :: d12
    Uses d12 15e.

15e <circe1_sample.f90: subroutines 15b>+≡
    function d12 (t1, t2)
    real(kind=double) :: d12, t1, t2, x1, x2
    <EPS & PWR 16d>
    x1 = 1d0 - t1**PWR
    x2 = 1d0 - t2**PWR
    d12 = PWR*PWR * (t1*t2)**(PWR-1d0) &
    * sigma (x1*x2) * circee (x1, x2)
    end function d12

```

Defines:
 d12, used in chunk 15.
 Uses `circee` 41g and `sigma` 15b.

the first product of continuum and δ -peak:

```

15f <circe1_sample.f90: public 15a>+≡
    public :: d1
    Uses d1 16a.

```

¹They are provided in the example program `circe1_sample.f90`.

```

16a <circe1_sample.f90: subroutines 15b>+≡
      function d1 (t1)
        real(kind=double) :: t1, x1, d1
        <EPS & PWR 16d>
        x1 = 1d0 - t1**PWR
        d1 = PWR * t1**(PWR-1d0) * sigma (x1) * circee (x1, 1d0)
      end function d1

```

Defines:

d1, used in chunks 15, 16, 41–43, and 74–77.

Uses circee 41g and sigma 15b.

and the second one:

```

16b <circe1_sample.f90: public 15a>+≡
      public :: d2

```

Uses d2 16c.

```

16c <circe1_sample.f90: subroutines 15b>+≡
      function d2 (t2)
        real(kind=double) :: t2, x2, d2
        <EPS & PWR 16d>
        x2 = 1d0 - t2**PWR
        d2 = PWR * t2**(PWR-1d0) * sigma (x2) * circee (1d0, x2)
      end function d2

```

Defines:

d2, used in chunks 15, 16, 41–43, and 74–77.

Uses circee 41g and sigma 15b.

Below you will see that the power of the singularity of the e^+e^- distributions at $x \rightarrow 1$ is $\approx -2/3$. To be on the safe side, we choose the power η in (10) as 5. It is kept in the parameter PWR, while EPS is the desired accuracy of the Gaussian integration:

```

16d <EPS & PWR 16d>≡
      real(kind=double), parameter :: EPS = 1d-6, PWR = 5d0

```

The Gauss integration of the non-singular version converges to the cotrrect value only if the final bin is integrated separately:

```

16e <Second Gauss integration 16e>≡
      s = gauss2 (d12a, 0d0, 1d0-KIREPS, 0d0, 1d0-KIREPS, EPS) &
        + gauss2 (d12a, 0d0, 1d0-KIREPS, 1d0-KIREPS, 1d0, EPS) &
        + gauss2 (d12a, 1d0-KIREPS, 1d0, 0d0, 1d0-KIREPS, EPS) &
        + gauss2 (d12a, 1d0-KIREPS, 1d0, 1d0-KIREPS, 1d0, EPS)
      write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0

```

Uses d12a 17a, gauss2 90d, and sigma 15b.

```

16f <EPS & PWR 16d>+≡
      real(kind=double), parameter :: KIREPS = 1D-6

```

```

16g <circe1_sample.f90: public 15a>+≡
      public :: d12a

```

Uses d12a 17a.


```

17a <circe1_sample.f90: subroutines 15b>+=
      function d12a (x1, x2)
        real(kind=double) :: x1, x2, d12a
        d12a = sigma (x1*x2) * kirkee (x1, x2)
      end function d12a

```

Defines:

d12a, used in chunk 16.

Uses kirkee 74b and sigma 15b.

These code fragments can now be used in a main program that loops over energies and accelerator designs

```

17b <circe1_sample.f90 17b>≡
      ! circe1_sample.f90 -- canonical beam spectra for linear collider physics
      ! $Id: prelude.nw 6466 2015-01-10 16:06:40Z jr_reuter $
      <Copyleft notice 29b>
      module sample_routines
        use kinds
        use circe1 !NODEP!

        implicit none
        private

        <circe1_sample.f90: public 15a>

        contains

        <circe1_sample.f90: subroutines 15b>

      end module sample_routines

      program circe1_sample
        use kinds
        use sample_routines
        use circe1

        implicit none

        <Accelerator codes 13a>
        <EPS & PWR 16d>
        <Other variables in sample 19>
        integer :: acc, ver, i
        real(kind=double), dimension(9) :: roots(9) = &
          (/ 90D0, 170D0, 250D0, 350D0, 500D0, &
            800D0, 1000D0, 1200D0, 1500D0 /)
        do acc = 1, NACC
          ! do acc = JLCNLC, NLCH, NLCH-JLCNLC
          do ver = 9, 9
            do i = 1, 9
              call circes (0d0, 0d0, roots(i), acc, ver, 20020328, 1)
              <Gauss integration 15c>
            end do
          end do
        end do
      end program circe1_sample

```

```

        <Second Gauss integration 16e>
        <Monte Carlo integration 20d>
    end do
end do
end do
end program circe1_sample

```

Uses JLCNLC 13a, NACC 13b, and circes 32a.

with the following result

```

18 <Sample output 18>≡
    circe1:message: starting up ...
    circe1:message: $Id: prelude.nw 6466 2015-01-10 16:06:40Z jr_reuter $
    circe1:message: updating 'roots' to 90.0
    circe1:message: updating 'ver' to 7
    circe1:message: updating 'rev' to 20000501
    delta(sigma) (Gauss) = 0.11%
    delta(sigma) (MC)    = 0.11%
                        +/- 0.00%
    circe1:message: updating 'roots' to 170.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 0.38%
    delta(sigma) (MC)    = 0.38%
                        +/- 0.01%
    circe1:message: updating 'roots' to 350.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 1.67%
    delta(sigma) (MC)    = 1.66%
                        +/- 0.03%
    circe1:message: updating 'roots' to 500.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 3.66%
    delta(sigma) (MC)    = 3.58%
                        +/- 0.07%
    circe1:message: updating 'roots' to 800.0
    circe1:message: updating 'ver' to 7
    delta(sigma) (Gauss) = 5.21%
    delta(sigma) (MC)    = 5.19%
                        +/- 0.11%
    circe1:message: updating 'roots' to 1000.0
    circe1:message: updating 'ver' to 7
    circe1:message: energy 1000.0GeV too high, using spectrum for 800.0GeV
    delta(sigma) (Gauss) = 5.21%
    delta(sigma) (MC)    = 5.19%
                        +/- 0.11%
    circe1:message: updating 'roots' to 90.0
    circe1:message: updating 'acc' to JLCNLC
    circe1:message: updating 'ver' to 7
    circe1:message: energy 90.0GeV too low, using spectrum for 500.0GeV
    delta(sigma) (Gauss) = 4.74%

```

```

delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 170.0
circe1:message: updating 'ver' to 7
circe1:message: energy 170.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.68%
                    +/- 0.11%
circe1:message: updating 'roots' to 350.0
circe1:message: updating 'ver' to 7
circe1:message: energy 350.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 500.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 800.0
circe1:message: updating 'ver' to 7
circe1:message: energy 800.0GeV interpolated between 500.0 and 1000.0GeV
delta(sigma) (Gauss) = 8.37%
delta(sigma) (MC)    = 8.39%
                    +/- 0.21%
circe1:message: updating 'roots' to 1000.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 15.39%
delta(sigma) (MC)    = 14.68%
                    +/- 0.33%

```

Uses JLCNLC 13a and sigma 15b.

We almost forgot to declare the variables in the main program

```

19 <Other variables in sample 19>≡
    real(kind=double) :: s

```

This concludes the integration example. It should have made it obvious how to proceed in a realistic application.

In section 3.2.1 below, I will describe a Monte Carlo method for calculating such integrals efficiently.

3.2 Generators

The function `circe` and its companions are opaque to the user. Since they will in general contain singularities, applications will *not* be able to generate corresponding samples of random numbers efficiently. To fill this gap, four random number generators are provided. The subroutine `girce` will generate particle types $p_{1,2}$ and energy fractions $x_{1,2}$ in one step, according to the selected distribution.² Particle p_1 will be either a positron or a photon and p_2 will be

²The implementation of the flavor selection with non-vanishing thresholds $x_{1,\min}$ and $x_{2,\min}$ is moderately inefficient at the moment. It can be improved by a factor of two.

either an electron or a photon. The energy fractions are guaranteed to be above the currently active thresholds: $x_i \geq x_{i,\min}$. This can be used to cut on soft events—the photon distributions are rather soft—which might not be interesting in most simulations.

20a *⟨API documentation 11a⟩*+≡
`call girce (x1, x2, p1, p2, rng)`

Uses `girce 80c`.

The output parameters of `girce` are identical to the input parameters of `circe`, with the exception of `rng`. The latter is a subroutine with a single double precision argument, which will be assigned a uniform deviate from the interval $[0, 1]$ after each call:

20b *⟨API documentation 11a⟩*+≡
`subroutine rng (r)`
`real(kind=double) :: r`
`r = ⟨uniform deviate on [0, 1] (never defined)⟩`
`end subroutine rng`

Typically, it will be just a wrapper around the standard random number generator of the application program. For studies with a definite initial state, three generator functions are available.

20c *⟨API documentation 11a⟩*+≡
`call gircee (x1, x2, rng)`
`call girceg (x1, x2, rng)`
`call gircgg (x1, x2, rng)`

Uses `gircee 81e`, `girceg 82c`, and `gircgg 83c`.

3.2.1 Example

Returning to the example from section 3.2.1, I present a concise Monte Carlo algorithm for calculating the same integral:

20d *⟨Monte Carlo integration 20d⟩*≡
`s = 0d0`
`s2 = 0d0`
`do n = 1, NEVENT`
`call gircee (x1, x2, random)`
`w = sigma (x1*x2)`
`s = s + w`
`s2 = s2 + w*w`
`end do`
`s = s / dble(NEVENT)`
`s2 = s2 / dble(NEVENT)`
`write (*, 1000) 'delta(sigma) (MC) = ', (s-1d0)*100d0`
`write (*, 1000) ' +/- ', sqrt((s2-s*s)/dble(NEVENT))*100d0`

Uses `gircee 81e`, `random 21b`, and `sigma 15b`.

20e *⟨Other variables in sample 19⟩*+≡
`real(kind=double) :: w, s2, x1, x2`
`integer, parameter :: NEVENT = 10000`
`integer :: n`

Here is a simple linear congruential random number generator for the sample program. Real applications will use their more sophisticated generators instead.

```

21a <circe1_sample.f90: public 15a>+≡
    public :: random
    Uses random 21b.
21b <circe1_sample.f90: subroutines 15b>+≡
    subroutine random (r)
        real(kind=double), intent(out) :: r
        integer :: m = 259200, a = 7141, c = 54773
        integer, save :: n = 0
        ! data n /0/
        n = mod(n*a+c,m)
        r = real (n, kind=double) / real (m, kind=double)
    end subroutine random

```

Defines:

`random`, used in chunks 20 and 21.

If the cross section is slowly varying on the range where the $x_{1,2}$ distributions are non-zero, this algorithm is very efficient.

However, if this condition is not met, the explicit form of the parameterizations in section 5 should be consulted and appropriate mapping techniques should be applied. The typical example for this problem is a narrow resonance just below the nominal beam energy.

3.2.2 Event Generators

For Monte Carlo event generators that use the standard `/hepevt/` common block [14], the addition of the `Circe1` library is trivial. During the initialization of the event generator, the `circes` subroutine is called to set up `Circe1`'s internal state. For example:

```

21c <Initialize event generator 21c>≡
    call circes (0d0, 0d0, roots, acc, ver, 1996 07 11, 1)
    Uses circes 32a.

```

During event generation, before setting up the e^+e^- initial state, the `gircee` subroutine is called with the event generator's random number generator:

```

21d <Event generation 21d>≡
    call gircee (x1, x2, random)
    Uses gircee 81e and random 21b.

```

The resulting energy fractions x_1 and x_2 are now available for defining the initial state electron

```

21e <Event generation 21d>+≡
    isthep(1) = 101
    idhep(1) = C1_ELECTRON
    phep(1,1) = 0d0
    phep(2,1) = 0d0
    phep(3,1) = x1 * ebeam
    phep(4,1) = x1 * ebeam
    phep(5,1) = 0d0
    Uses C1_ELECTRON 11b.

```



Figure 5: Architecture of **Circe1**: `circes()` selects energy and accelerator and loads the parameterization. The function `circe()` calculates the values of the selected distribution function at the given energy fractions. The subroutine `girce()` generates energy fractions using a specified random number generator in accordance with the selected distribution.

and positron.

```

22 <Event generation 21d>+≡
    isthep(2) = 102
    idhep(2) = C1_POSITRON
    phep(1,2) = 0d0
    phep(2,2) = 0d0
    phep(3,2) = - x2 * ebeam
    phep(4,2) = x2 * ebeam
    phep(5,2) = 0d0

```

Uses C1_POSITRON 11b.

Using **Circe1** with other event generators should be straightforward as well.

4 Technical Notes

The structure of **Circe1** is extremely simple (cf. figure 5) and is mainly a book-keeping exercise. All that needs to be done is to maintain a database of available parameterizations and to evaluate the corresponding functions. The only non trivial algorithms are used for the efficient generation of random deviates.

| | SBAND | TESLA | TESLA' | XBAND |
|--------------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $\mathcal{L}/\text{fb}^{-1}\nu^{-1}$ | $31.38^{+0.22}_{-0.22}$ | $106.25^{+0.71}_{-0.71}$ | $95.24^{+0.73}_{-0.73}$ | $36.39^{+0.29}_{-0.29}$ |
| $\int d_{e\pm}$ | $0.4812^{+0.0041}_{-0.0041}$ | $0.5723^{+0.0046}_{-0.0045}$ | $0.3512^{+0.0048}_{-0.0048}$ | $0.3487^{+0.0040}_{-0.0040}$ |
| $x_{e\pm}^\alpha$ | $11.1534^{+0.0770}_{-0.0761}$ | $15.2837^{+0.0923}_{-0.0914}$ | $27.1032^{+0.3071}_{-0.3019}$ | $6.9853^{+0.0733}_{-0.0718}$ |
| $(1 - x_{e\pm})^\alpha$ | $-0.6302^{+0.0013}_{-0.0012}$ | $-0.6166^{+0.0011}_{-0.0011}$ | $-0.6453^{+0.0017}_{-0.0017}$ | $-0.6444^{+0.0017}_{-0.0017}$ |
| $\int d_\gamma$ | $0.6237^{+0.0033}_{-0.0033}$ | $0.7381^{+0.0036}_{-0.0036}$ | $0.3502^{+0.0034}_{-0.0034}$ | $0.4149^{+0.0031}_{-0.0031}$ |
| x_γ^α | $-0.6911^{+0.0006}_{-0.0006}$ | $-0.6921^{+0.0006}_{-0.0006}$ | $-0.6947^{+0.0011}_{-0.0011}$ | $-0.6876^{+0.0010}_{-0.0010}$ |
| $(1 - x_\gamma)^\alpha$ | $14.9355^{+0.0761}_{-0.0754}$ | $24.1647^{+0.1124}_{-0.1116}$ | $33.6576^{+0.3021}_{-0.2983}$ | $8.3227^{+0.0659}_{-0.0649}$ |

Table 5: Version 1, revision 1997 04 16 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

I have avoided the use of initialized `common` blocks (i.e. `block data` subroutines), because the Fortran77 standard does not provide a *portable* way of ensuring that `block data` subroutines are actually executed at loading time³. Instead, the `/circom/` common block is tagged by a “magic number” to check for initialization and its members are filled by the `circes` subroutine when necessary.

A more flexible method would be to replace the `data` statements by reading external files. This option causes portability problems, however, because I would have to make sure that the names of the external files are valid in all files systems of the target operating systems. More significantly, splitting the implementation into several parts forces the user to keep all files up to date. This can be a problem, because Fortran source files and data input files will typically be kept in different parts of the file system.

The option of implementing `Circe1` statelessly, i.e. with pure function calls and without `common` blocks, has been dismissed. While it would have been more straightforward on the side of the library, it would have placed the burden of maintaining state (accelerator, energy, etc.) on the application program, thereby complicating them considerably. Keeping an explicit state in `Circe1` has the additional benefit of allowing to precompute certain internal variables, resulting in a more efficient implementation.

5 Parameterizations

The internal Version 2.2 of `Circe11` supports just one version of the parameterizations. Future versions will provide additional parameterizations.

5.1 Version 1

The first version of the parameterization uses a simple factorized *ansatz*

$$D_{p_1 p_2}^{\alpha 1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha 1 \rho}(x_1) d_{p_2}^{\alpha 1 \rho}(x_2) \quad (12a)$$

³In Fortran90 the common blocks have been replaced by saved module variables.

| | SBAND | TESLA | TESLA' | XBAND |
|------------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $\mathcal{L}/\text{fb}^{-1}v^{-1}$ | $119.00^{+0.83}_{-0.83}$ | 214.33^{+0***}_{-0***} | 212.22^{+0***}_{-0***} | $118.99^{+0.91}_{-0.91}$ |
| $\int d_{e\pm}$ | $0.5604^{+0.0040}_{-0.0039}$ | $0.6686^{+0.0040}_{-0.0040}$ | $0.4448^{+0.0043}_{-0.0043}$ | $0.5001^{+0.0038}_{-0.0038}$ |
| $x_{e\pm}^\alpha$ | $4.2170^{+0.0258}_{-0.0255}$ | $5.5438^{+0.0241}_{-0.0239}$ | $9.6341^{+0.0814}_{-0.0803}$ | $2.6184^{+0.0192}_{-0.0190}$ |
| $(1 - x_{e\pm})^\alpha$ | $-0.6118^{+0.0013}_{-0.0013}$ | $-0.5847^{+0.0011}_{-0.0011}$ | $-0.6359^{+0.0014}_{-0.0014}$ | $-0.6158^{+0.0015}_{-0.0015}$ |
| $\int d_\gamma$ | $0.7455^{+0.0032}_{-0.0032}$ | $1.0112^{+0.0033}_{-0.0033}$ | $0.4771^{+0.0031}_{-0.0031}$ | $0.6741^{+0.0031}_{-0.0031}$ |
| x_γ^α | $-0.6870^{+0.0006}_{-0.0006}$ | $-0.6908^{+0.0004}_{-0.0004}$ | $-0.6936^{+0.0008}_{-0.0008}$ | $-0.6834^{+0.0007}_{-0.0007}$ |
| $(1 - x_\gamma)^\alpha$ | $6.7145^{+0.0310}_{-0.0308}$ | $9.9992^{+0.0342}_{-0.0340}$ | $13.1607^{+0.0896}_{-0.0886}$ | $3.8589^{+0.0215}_{-0.0213}$ |

Table 6: Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.

| | 350 GeV | 500 GeV | 800 GeV | 1600 GeV |
|------------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $\mathcal{L}/\text{fb}^{-1}v^{-1}$ | $97.45^{+0.67}_{-0.67}$ | $106.25^{+0.71}_{-0.71}$ | 170.86^{+0***}_{-0***} | 340.86^{+0***}_{-0***} |
| $\int d_{e\pm}$ | $0.6093^{+0.0049}_{-0.0049}$ | $0.5723^{+0.0046}_{-0.0045}$ | $0.6398^{+0.0042}_{-0.0041}$ | $0.5094^{+0.0040}_{-0.0040}$ |
| $x_{e\pm}^\alpha$ | $17.6137^{+0.1065}_{-0.1055}$ | $15.2837^{+0.0923}_{-0.0914}$ | $7.6221^{+0.0365}_{-0.0361}$ | $5.0550^{+0.0353}_{-0.0349}$ |
| $(1 - x_{e\pm})^\alpha$ | $-0.6061^{+0.0011}_{-0.0011}$ | $-0.6166^{+0.0011}_{-0.0011}$ | $-0.5944^{+0.0011}_{-0.0011}$ | $-0.6187^{+0.0013}_{-0.0013}$ |
| $\int d_\gamma$ | $0.7729^{+0.0039}_{-0.0039}$ | $0.7381^{+0.0036}_{-0.0036}$ | $0.9178^{+0.0034}_{-0.0034}$ | $0.5875^{+0.0031}_{-0.0031}$ |
| x_γ^α | $-0.6949^{+0.0006}_{-0.0006}$ | $-0.6921^{+0.0006}_{-0.0006}$ | $-0.6908^{+0.0005}_{-0.0005}$ | $-0.6892^{+0.0007}_{-0.0007}$ |
| $(1 - x_\gamma)^\alpha$ | $28.9399^{+0.1370}_{-0.1361}$ | $24.1647^{+0.1124}_{-0.1116}$ | $13.1167^{+0.0497}_{-0.0495}$ | $7.5514^{+0.0428}_{-0.0424}$ |

Table 7: Version 1, revision 1997 04 17 of the beam spectra for TESLA.

| | 500 GeV | 800 GeV |
|------------------------------------|-------------------------------|-------------------------------|
| $\mathcal{L}/\text{fb}^{-1}v^{-1}$ | $339.80^{+0.83}_{-0.83}$ | $359.36^{+0.93}_{-0.93}$ |
| $\int d_{e\pm}$ | $0.5019^{+0.0016}_{-0.0016}$ | $0.4125^{+0.0016}_{-0.0016}$ |
| $x_{e\pm}^\alpha$ | $12.2867^{+0.0318}_{-0.0316}$ | $13.3242^{+0.0442}_{-0.0440}$ |
| $(1 - x_{e\pm})^\alpha$ | $-0.6276^{+0.0005}_{-0.0005}$ | $-0.6401^{+0.0005}_{-0.0005}$ |
| $\int d_\gamma$ | $0.5114^{+0.0012}_{-0.0012}$ | $0.3708^{+0.0011}_{-0.0011}$ |
| x_γ^α | $-0.6912^{+0.0003}_{-0.0003}$ | $-0.6924^{+0.0004}_{-0.0004}$ |
| $(1 - x_\gamma)^\alpha$ | $17.0673^{+0.0375}_{-0.0375}$ | $16.8145^{+0.0482}_{-0.0480}$ |

Table 8: Version 5, revision 1998 05 05 of the beam spectra for high luminosity TESLA.

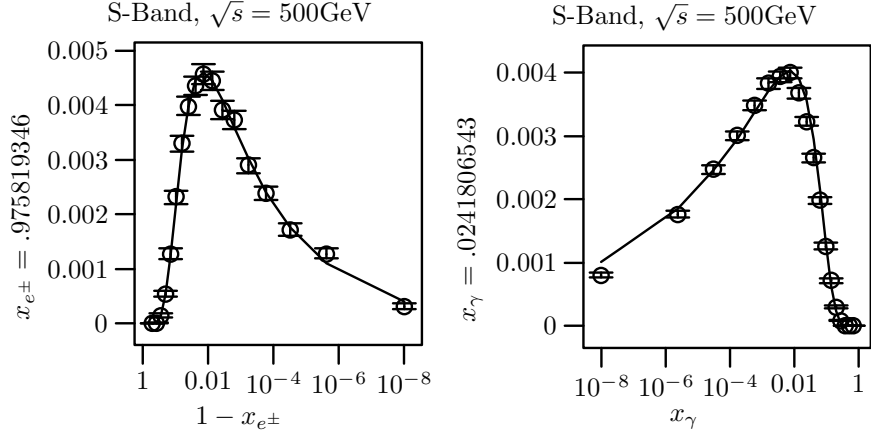


Figure 6: Fit of the e^\pm - and γ -distributions for the S-Band design at $\sqrt{s} = 500\text{GeV}$. The open circles with error bars are the result of the Guinea-Pig simulation. The full line is the fit.

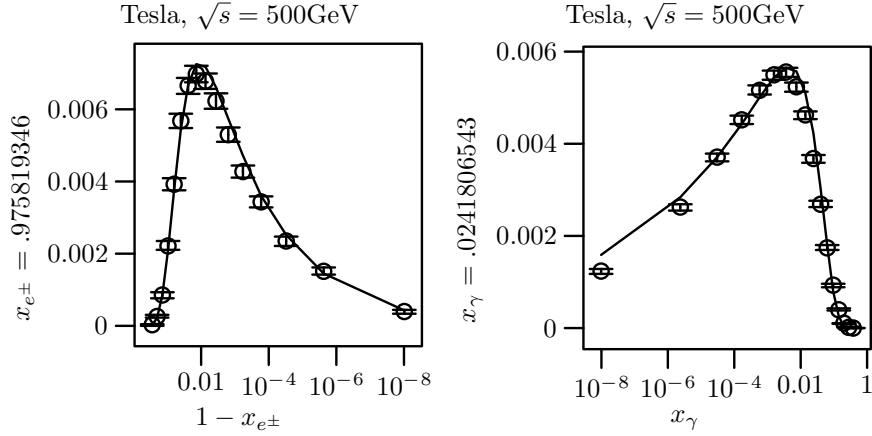


Figure 7: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 500\text{GeV}$.



Figure 8: Fit of the e^\pm - and γ -distributions for the X-Band design at $\sqrt{s} = 500 \text{ GeV}$.



Figure 9: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 1 \text{ TeV}$.

| | SBNDEE | TESLEE | XBNDEE |
|------------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $\mathcal{L}/\text{fb}^{-1}v^{-1}$ | $9.29^{+0.06}_{-0.06}$ | $21.62^{+0.17}_{-0.17}$ | $13.97^{+0.10}_{-0.10}$ |
| $\int d_{e^\pm}$ | $.6513^{+0.0059}_{-0.0059}$ | $.7282^{+0.0083}_{-0.0082}$ | $.5270^{+0.0049}_{-0.0049}$ |
| $x_{e^\pm}^\alpha$ | $10.3040^{+0.0601}_{-0.0593}$ | $14.8578^{+0.1047}_{-0.1034}$ | $5.8897^{+0.0455}_{-0.0448}$ |
| $(1 - x_{e^\pm})^\alpha$ | $-.5946^{+0.0015}_{-0.0015}$ | $-.5842^{+0.0018}_{-0.0018}$ | $-.6169^{+0.0016}_{-0.0015}$ |
| $\int d_\gamma$ | $.4727^{+0.0035}_{-0.0035}$ | $.5300^{+0.0046}_{-0.0046}$ | $.3746^{+0.0029}_{-0.0029}$ |
| x_γ^α | $-.6974^{+0.0009}_{-0.0009}$ | $-.7039^{+0.0009}_{-0.0009}$ | $-.6892^{+0.0010}_{-0.0010}$ |
| $(1 - x_\gamma)^\alpha$ | $20.6447^{+0.1513}_{-0.1497}$ | $36.1286^{+0.3027}_{-0.2991}$ | $10.0872^{+0.0822}_{-0.0815}$ |

Table 9: *Experimental* Version 1, revision 0 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

| | SBNDEE | TESLEE | XBNDEE |
|------------------------------------|-------------------------------|-------------------------------|------------------------------|
| $\mathcal{L}/\text{fb}^{-1}v^{-1}$ | $45.59^{+0.34}_{-0.34}$ | $25.47^{+0.20}_{-0.20}$ | $41.06^{+0.28}_{-0.28}$ |
| $\int d_{e^\pm}$ | $.7892^{+0.0075}_{-0.0074}$ | $.6271^{+0.0066}_{-0.0065}$ | $.7203^{+0.0058}_{-0.0058}$ |
| $x_{e^\pm}^\alpha$ | $5.4407^{+0.0285}_{-0.0281}$ | $8.7504^{+0.0669}_{-0.0658}$ | $2.7415^{+0.0121}_{-0.0119}$ |
| $(1 - x_{e^\pm})^\alpha$ | $-.5285^{+0.0020}_{-0.0020}$ | $-.6058^{+0.0017}_{-0.0017}$ | $-.5049^{+0.0020}_{-0.0020}$ |
| $\int d_\gamma$ | $.6403^{+0.0040}_{-0.0040}$ | $.4278^{+0.0038}_{-0.0038}$ | $.6222^{+0.0032}_{-0.0032}$ |
| x_γ^α | $-.6960^{+0.0008}_{-0.0008}$ | $-.6982^{+0.0010}_{-0.0010}$ | $-.6795^{+0.0008}_{-0.0008}$ |
| $(1 - x_\gamma)^\alpha$ | $12.4803^{+0.0839}_{-0.0831}$ | $18.5260^{+0.1674}_{-0.1655}$ | $4.7506^{+0.0262}_{-0.0260}$ |

Table 10: *Experimental* Version 1, revision 0 of the beam spectra at 1 TeV.

where the distributions are simple Beta distributions:

$$d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} \delta(1-x) + a_1^{\alpha\rho} x^{a_2^{\alpha\rho}} (1-x)^{a_3^{\alpha\rho}} \quad (12b)$$

$$d_\gamma^{\alpha 1\rho}(x) = a_4^{\alpha\rho} x^{a_5^{\alpha\rho}} (1-x)^{a_6^{\alpha\rho}} \quad (12c)$$

This form of the distributions is motivated by the observation [2] that the e^\pm distributions diverge like a power for $x \rightarrow 1$ and vanish at $x \rightarrow 0$. The behavior of the γ distributions is similar with the borders exchanged.

5.1.1 Fitting

The parameters a_i in (12) have been obtained by a least-square fit of (12) to histograms of simulation results from **Guinea-Pig**. Some care has to be taken when fitting singular distributions to histogrammed data. Obviously equidistant bins are not a good idea, because most bins will be almost empty (cf. figures 1 and 2) and consequently a lot of information will be wasted. One solution to this problem is the use of logarithmic bins. This, however, maps the compact region $[0, 1] \times [0, 1]$ to $[-\infty, 0] \times [-\infty, 0]$, which is inconvenient because of the missing lower bounds.

| | 350 GeV | 500 GeV | 800 GeV |
|------------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $\mathcal{L}/\text{fb}^{-1}v^{-1}$ | $15.18^{+0.13}_{-0.13}$ | $21.62^{+0.17}_{-0.17}$ | $43.98^{+0.38}_{-0.38}$ |
| $\int d_{e\pm}$ | $.6691^{+0.0083}_{-0.0083}$ | $.7282^{+0.0083}_{-0.0082}$ | $.7701^{+0.0090}_{-0.0089}$ |
| $x_{e\pm}^\alpha$ | $25.2753^{+0.2040}_{-0.2007}$ | $14.8578^{+0.1047}_{-0.1034}$ | $8.1905^{+0.0543}_{-0.0535}$ |
| $(1 - x_{e\pm})^\alpha$ | $-.5994^{+0.0017}_{-0.0017}$ | $-.5842^{+0.0018}_{-0.0018}$ | $-.5575^{+0.0021}_{-0.0021}$ |
| $\int d_\gamma$ | $.4464^{+0.0047}_{-0.0047}$ | $.5300^{+0.0046}_{-0.0046}$ | $.5839^{+0.0047}_{-0.0047}$ |
| x_γ^α | $-.7040^{+0.0011}_{-0.0011}$ | $-.7039^{+0.0009}_{-0.0009}$ | $-.7046^{+0.0009}_{-0.0009}$ |
| $(1 - x_\gamma)^\alpha$ | $60.1882^{+0.5882}_{-0.5797}$ | $36.1286^{+0.3027}_{-0.2991}$ | $19.3944^{+0.1681}_{-0.1660}$ |

Table 11: *Experimental* Version 1, revision 0 of the beam spectra for TESLEE.

The more appropriate solution is to use two maps

$$\begin{aligned}\phi : [0, 1] &\rightarrow [0, 1] \\ x &\mapsto y = x^{1/\eta}\end{aligned}\tag{13}$$

where $x = x_\gamma$ or $x = 1 - x_{e\pm}$, and to bin the result equidistantly. If η is chosen properly (cf. (10)), the bin contents will then fall off at the singularity. The fits in tables 5, 6, and 7 have been performed with $\eta = 5$ and the resulting bin contents can be read off from figures 6–9.

Using this procedure for binning the results of the simulations, the popular fitting package MINUIT [15] converges quickly in all cases considered. The resulting parameters are given in tables 5, 6, and 7. Plots of the corresponding distributions have been shown in figures 1 and 2. It is obvious that an *ansatz* like (12) is able to distinguish among the accelerator designs. Thus it can provide a solid basis for physics studies.

In figures 6–9 I give a graphical impression of the quality of the fit, which appears to be as good as one could reasonably expect for a simple *ansatz* like (12). Note that the histograms have non-equidistant bins and that the resulting Jacobians have not been removed. Therefore the bin contents falls off at the singularities, as discussed above.

The errors used for the least-square fit had to be taken from a Monte Carlo (MC) study. **Guinea-Pig** only provides the \sqrt{n} from Poissonian statistics for each bin, but the error accumulation during tracking the particles through phase space is not available. The MC studies shows that the latter error dominates the former, but appears to be reasonably Gaussian. A complete MC study of all parameter sets is computationally expensive (more than a week of processor time on a fast SGI). From an exemplary MC study of a few parameter sets, it appears that the errors can be described reasonably well by rescaling the Poissonian error in each bin with appropriate factors for electrons/positrons and photons and for continuum and delta. This procedure has been adopted.

The $\chi^2/\text{d.o.f.}$'s of the fits are less than $\mathcal{O}(10)$. The simple *ansatz* (12) is therefore very satisfactory. In fact, trying to improve the ad-hoc factorized Beta distributions by the better motivated approximations from [7] or [16], it turns out [17] that (12) provides a significantly better fit of the results of the simulations. The price to pay is that the parameters in (12) have no direct

physical interpretation.

5.1.2 Generators

For this version of the parameterizations we need a fast generator of Beta distributions:

$$\beta^{a,b}(x) \propto x^{a-1}(1-x)^{b-1} \quad (14)$$

This problem has been studied extensively and we can use a published algorithm [18] that is guaranteed to be very fast for all a, b such that $0 < a \leq 1 \leq b$, which turns out to be always the case (cf. tables 5, 6, and 7).

5.2 Future Versions

There are two ways in which the parameterizations can be improved:

more complicated functions: the factorized fits can only be improved marginally by adding more positive semi-definite factors to (12). More improvement is possible by using sums of functions, but in this case, the best fits violate the positivity requirement and have to be discarded.

correlations: the parameterization in section 5.1 is factorized. While this is a good approximation, the simulations nevertheless show correlations among x_1 and x_2 . These correlations can be included in a future version.

interpolation: the parameterization in section 5.1 is based on fitting the simulation results by simple functions. Again, this appears to be a good approximation. But such fits can not uncover any fine structure of the distributions. Therefore it will be worthwhile to study interpolations of the simulation results in the future. A proper interpolation of results with statistical errors is however far from trivial: straightforward polynomial or spline interpolations will be oscillatory and violate the positivity requirement. Smoothing algorithms have to be investigated in depth before such a parameterization can be released.

other simulations: besides [5], other simulation codes are invited to contribute their results for inclusion in the Circe1 library.

6 Implementation of circe1

```

29a <circe1.f90 29a>≡
    ! circe1.f90 -- canonical beam spectra for linear collider physics
    ! $Id: circe1.nw 6466 2015-01-10 16:06:40Z jr_reuter $
    <Copyleft notice 29b>
    <Main module 30b>

29b <Copyleft notice 29b>≡
    !
    ! Copyright (C) 1999-2015 by
    !     Wolfgang Kilian <kilian@physik.uni-siegen.de>
    !     Thorsten Ohl <ohl@physik.uni-wuerzburg.de>

```

```

!      Juergen Reuter <juergen.reuter@desy.de>
!      with contributions from
!      Christian Speckner <cnspeckn@googlemail.com>
!
! WHIZARD is free software; you can redistribute it and/or modify it
! under the terms of the GNU General Public License as published by
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! any later version.
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! WHIZARD is distributed in the hope that it will be useful, but
! WITHOUT ANY WARRANTY; without even the implied warranty of
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! You should have received a copy of the GNU General Public License
! along with this program; if not, write to the Free Software
! Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! This file has been stripped of most comments. For documentation, refer
! to the source 'circe1.nw'

```

Now we can move on to the implementation.

6.1 Symbolic Constants

The file `circe.h` contains symbolic names for various magic constants used by Circe1:

```

30a <circe.h 30a>≡
    c circe.h -- canonical beam spectra for linear collider physics
    c $Id: circe1.nw 6466 2015-01-10 16:06:40Z jr_reuter $
Uses circe 31b.

30b <Main module 30b>≡
    module circe1
    use kinds

    implicit none
    private
    <Public subroutines 31a>

    <Public types 79d>

    <Particle codes 11b>
    <Accelerator codes 13a>
    <Private parameters 37c>

    integer, parameter, public :: MAGIC0 = 19040616
    real(kind=double), parameter :: KIREPS = 1D-6

    <Declaration: circe1 parameters 32c>

```

```

type(circe1_params_t), public, save :: circe1_params

<Abstract types 79e>

<Abstract interfaces 79c>

contains

  <Module subroutines 31b>
end module circe1

```

6.2 Distributions

6.2.1 Version 1

We start with a convenience function which dispatches over the valid particle types. The hardest part is of course to avoid typos in such trivial functions ...

```

31a <Public subroutines 31a>≡
    public :: circe
    Uses circe 31b.

31b <Module subroutines 31b>≡
    function circe (x1, x2, p1, p2)
    real(kind=double) :: x1, x2
    integer :: p1, p2
    real(kind=double) :: circe
    <Initialization check 32g>
    circe = -1.0
    if (abs(p1) .eq. C1_ELECTRON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            circe = circee (x1, x2)
        else if (p2 .eq. C1_PHOTON) then
            circe = circeg (x1, x2)
        end if
    else if (p1 .eq. C1_PHOTON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            circe = circeg (x2, x1)
        else if (p2 .eq. C1_PHOTON) then
            circe = circgg (x1, x2)
        end if
    end if
end function circe

```

Defines:

circe, used in chunks 11a, 31, 30a, 31a, 87, 93b, 110b, and 115d.

Uses C1_ELECTRON 11b, C1_PHOTON 11b, circee 41g, circeg 42c, and circgg 43c.

```

31c <Public subroutines 31a>+≡
    public :: circses
    Uses circses 32a.

```

32a *<Module subroutines 31b>+≡*

```

subroutine circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
  real(kind=double) :: xx1m, xx2m, xroots
  integer :: xacc, xver, xrev, xchat
<Local variables for circes 33b>
<Initializations for circes 35b>
  if (circe1_params%magic .ne. 19040616) then
    circe1_params%magic = 19040616
    <Initialize circe1 parameters 32h>
  end if
<Update circe1 parameters 33a>
<formats for circes 38d>
end subroutine circes

```

Defines:

circes, used in chunks 32a, 12b, 17b, 32a, 21c, 31, 32, 35f, 87, and 91b.

32b *<Public subroutines 31a>+≡*

```

public :: circe1_params_t

```

32c *<Declaration: circe1 parameters 32c>≡*

```

type :: circe1_params_t
<8-byte aligned part of circe1 parameters 32d>
<4-byte aligned part of circe1 parameters 32e>
end type circe1_params_t

```

32d *<8-byte aligned part of circe1 parameters 32d>≡*

```

real(kind=double) :: x1m = 0d0
real(kind=double) :: x2m = 0d0
real(kind=double) :: roots = 500D0

```

32e *<4-byte aligned part of circe1 parameters 32e>≡*

```

integer :: acc = TESLA
integer :: ver = 0
integer :: rev = 0
integer :: chat = 1

```

Uses TESLA 13a.

Instead of using fragile `block data` subroutines, we use a magic number to tag `circe1_params` as initialized:

32f *<4-byte aligned part of circe1 parameters 32e>+≡*

```

integer :: magic

```

Since negative values are no updated, we can call `circes` with all negative variables to ensure initialization:

32g *<Initialization check 32g>≡*

```

if (circe1_params%magic .ne. MAGIC0) then
  call circes (-1d0, -1d0, -1d0, -1, -1, -1, -1)
endif

```

Uses `circes` 32a.

32h *<Initialize circe1 parameters 32h>≡*

```

circe1_params%x1m = 0d0
circe1_params%x2m = 0d0

```



```

circe1_params%roots = 500D0
circe1_params%acc = TESLA
circe1_params%ver = 0
circe1_params%rev = 0
circe1_params%chat = 1
if (xchat .ne. 0) then
  call circem ('MESSAGE', 'starting up ...')
  call circem ('MESSAGE', &
    '$Id: circe1.nw 6466 2015-01-10 16:06:40Z jr_reuter $')
endif

```

Uses TESLA 13a and circem 86e.

```

33a <Update circe1 parameters 33a>≡
  if ((xchat .ge. 0) .and. (xchat .ne. circe1_params%chat)) then
    circe1_params%chat = xchat
    if (circe1_params%chat .ge. 1) then
      write (msgbuf, 1000) 'chat', circe1_params%chat
1000   format ('updating ', A, ''' to ', I2)
      call circem ('MESSAGE', msgbuf)
    endif
  else
    if (circe1_params%chat .ge. 2) then
      write (msgbuf, 1100) 'chat', circe1_params%chat
1100   format ('keeping ', A, ''' at ', I2)
      call circem ('MESSAGE', msgbuf)
    endif
  endif
endif

```

Uses circem 86e.

```

33b <Local variables for circes 33b>≡
  character(len=60) :: msgbuf

```

```

33c <Update circe1 parameters 33a>+≡
  if ((xx1m .ge. 0d0) .and. (xx1m .ne. circe1_params%x1m)) then
    circe1_params%x1m = xx1m
    if (circe1_params%chat .ge. 1) then
      write (msgbuf, 1001) 'x1min', circe1_params%x1m
1001   format ('updating ', A, ''' to ', E12.4)
      call circem ('MESSAGE', msgbuf)
    endif
  else
    if (circe1_params%chat .ge. 2) then
      write (msgbuf, 1101) 'x1min', circe1_params%x1m
1101   format ('keeping ', A, ''' at ', E12.4)
      call circem ('MESSAGE', msgbuf)
    endif
  endif
endif

```

Uses circem 86e.

```

33d <Update circe1 parameters 33a>+≡
  if ((xx2m .ge. 0d0) .and. (xx2m .ne. circe1_params%x2m)) then
    circe1_params%x2m = xx2m

```

```

        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1001) 'x2min', circe1_params%x2m
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1101) 'x2min', circe1_params%x2m
            call circem ('MESSAGE', msgbuf)
        endif
    endif
    Uses circem 86e.
34a  <Update circe1 parameters 33a>+=
        if ((xroots .ge. 0d0) .and.(xroots .ne. circe1_params%roots)) then
            circe1_params%roots = xroots
            if (circe1_params%chat .ge. 1) then
                write (msgbuf, 1002) 'roots', circe1_params%roots
1002      format ('updating ', A, ''' to ', F6.1)
                call circem ('MESSAGE', msgbuf)
            endif
        else
            if (circe1_params%chat .ge. 2) then
                write (msgbuf, 1102) 'roots', circe1_params%roots
1102      format ('keeping ', A, ''' at ', F6.1)
                call circem ('MESSAGE', msgbuf)
            endif
        endif
    Uses circem 86e.
34b  <Update circe1 parameters 33a>+=
        if ((xacc .ge. 0) .and.(xacc .ne. circe1_params%acc)) then
            if ((xacc .ge. 1) .and. (xacc .le. NACC)) then
                circe1_params%acc = xacc
                if (circe1_params%chat .ge. 1) then
                    write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
1003      format ('updating ', A, ''' to ', A)
                    call circem ('MESSAGE', msgbuf)
                endif
            else
                write (msgbuf, 1203) xacc
1203      format ('invalid 'acc': ', I8)
                call circem ('ERROR', msgbuf)
                write (msgbuf, 1103) 'acc', accnam(circe1_params%acc)
1103      format ('keeping ', A, ''' at ', A)
                call circem ('MESSAGE', msgbuf)
            endif
        else
            if (circe1_params%chat .ge. 2) then
                write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
                call circem ('MESSAGE', msgbuf)
            endif
        endif

```

```

endif
if ((circe1_params%acc .eq. SBNDEE) .or. (circe1_params%acc .eq. TESLEE) &
    .or. (circe1_params%acc .eq. XBNDEE)) then
  <Warn that no parameter set has been endorsed for  $e^-e^-$  yet 36a>
endif
Uses NACC 13b, SBNDEE 13a, TESLEE 13a, XBNDEE 13a, and circem 86e.

35a <Local variables for circes 33b>+≡
    <Declaration of accnam 35c>

35b <Initializations for circes 35b>≡
    <Initialization of accnam 35d>

35c <Declaration of accnam 35c>≡
    character(len=6), dimension(NACC) :: accnam
Uses NACC 13b.

35d <Initialization of accnam 35d>≡
    data accnam(SBAND) /'SBAND'/
    data accnam(TESLA) /'TESLA'/
    data accnam(JLCNLC) /'JLCNLC'/
    data accnam(SBNDEE) /'SBNDEE'/
    data accnam(TESLEE) /'TESLEE'/
    data accnam(XBNDEE) /'XBNDEE'/
    data accnam(NLCH) /'NLC H'/
    data accnam(ILC) /'ILC'/
    data accnam(CLIC) /'CLIC'/
Uses CLIC 13a, ILC 13a, JLCNLC 13a, SBAND 13a, SBNDEE 13a, TESLA 13a, TESLEE 13a,
and XBNDEE 13a.

35e <Public subroutines 31a>+≡
    public :: circex
Uses circex 35f.

35f <Module subroutines 31b>+≡
    subroutine circex (xx1m, xx2m, xroots, cacc, xver, xrev, xchat)
    real(kind=double) :: xx1m, xx2m, xroots
    character(*) :: cacc
    integer :: xver, xrev, xchat
    integer :: xacc, i
    <Accelerator codes 13a>
    <Declaration of accnam 35c>
    <Initialization of accnam 35d>
    xacc = -1
    do i = 1, NACC
        if (trim (accnam(i)) == trim (cacc)) then
            xacc = i
        end if
    end do
    call circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
end subroutine circex
Defines:
    circex, used in chunk 35e.
Uses NACC 13b and circes 32a.

```

36a \langle Warn that no parameter set has been endorsed for e^-e^- yet 36a $\rangle \equiv$

```

call circem ('WARNING', '*****')
call circem ('WARNING', '* The accelerator parameters have *')
call circem ('WARNING', '* not been endorsed for use in *')
call circem ('WARNING', '* an e-e- collider yet!!! *')
call circem ('WARNING', '*****')

```

Uses circem 86e.

36b \langle Update circe1 parameters 33a $\rangle + \equiv$

```

if (xver .ge. 0) then
  circe1_params%ver = xver
  if (circe1_params%chat .ge. 1) then
    write (msgbuf, 1000) 'ver', circe1_params%ver
    call circem ('MESSAGE', msgbuf)
  endif
else
  if (circe1_params%chat .ge. 2) then
    write (msgbuf, 1100) 'ver', circe1_params%ver
    call circem ('MESSAGE', msgbuf)
  endif
endif
endif

```

Uses circem 86e.

36c \langle Update circe1 parameters 33a $\rangle + \equiv$

```

if ((xrev .ge. 0) .and. (xrev .ne. circe1_params%rev)) then
  circe1_params%rev = xrev
  if (circe1_params%chat .ge. 1) then
    write (msgbuf, 1004) 'rev', circe1_params%rev
1004    format ('updating ', A, ''' to ', I8)
    call circem ('MESSAGE', msgbuf)
  endif
else
  if (circe1_params%chat .ge. 2) then
    write (msgbuf, 1104) 'rev', circe1_params%rev
1104    format ('keeping ', A, ''' at ', I8)
    call circem ('MESSAGE', msgbuf)
  endif
endif
endif

```

Uses circem 86e.

Versions 3 and 4 are identical to version 1, except for TESLA at 800 GeV.

36d \langle Update circe1 parameters 33a $\rangle + \equiv$

```

ver34 = 0
if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
   $\langle$ Update version 1 derived parameters in circe1 parameters 37d $\rangle$ 
  else if ((circe1_params%ver .eq. 3) .or. (circe1_params%ver .eq. 4)) then
    ver34 = circe1_params%ver
    circe1_params%ver = 1
   $\langle$ Update version 3 and 4 derived parameters in circe1 parameters 50c $\rangle$ 
  else if (circe1_params%ver .eq. 5) then
    circe1_params%ver = 1

```

```

    <Update version 5 derived parameters in circe1 parameters 53a>
      else if (circe1_params%ver .eq. 6) then
        circe1_params%ver = 1
    <Update version 6 derived parameters in circe1 parameters 54c>
      else if (circe1_params%ver .eq. 7) then
        circe1_params%ver = 1
    <Update version 7 derived parameters in circe1 parameters 56d>
      else if (circe1_params%ver .eq. 8) then
        circe1_params%ver = 1
    <Update version 8 derived parameters in circe1 parameters 61a>
      else if (circe1_params%ver .eq. 9) then
        circe1_params%ver = 1
    <Update version 9 derived parameters in circe1 parameters 63a>
      else if (circe1_params%ver .eq. 10) then
        circe1_params%ver = 1
    <Update version 10 derived parameters in circe1 parameters 68b>
    <else handle invalid versions 37b>
37a <Local variables for circes 33b>+=
      integer :: ver34
37b <else handle invalid versions 37b>≡
      else if (circe1_params%ver .eq. 2) then
    <Version 2 has been retired 50b>
      else if (circe1_params%ver .gt. 10) then
        call circem ('PANIC', 'versions >10 not available yet')
        return
      else
        call circem ('PANIC', 'version must be positive')
        return
      end if
    Uses circem 86e.
37c <Private parameters 37c>≡
      integer :: e, r, ehi, elo
37d <Update version 1 derived parameters in circe1 parameters 37d>≡
      if (circe1_params%rev .eq. 0) then
        r = 0
      elseif (circe1_params%rev .ge. 19970417) then
        r = 5
      elseif (circe1_params%rev .ge. 19960902) then
        r = 4
      elseif (circe1_params%rev .ge. 19960729) then
        r = 3
      elseif (circe1_params%rev .ge. 19960711) then
        r = 2
      elseif (circe1_params%rev .ge. 19960401) then
        r = 1
      elseif (circe1_params%rev .lt. 19960401) then
        call circem ('ERROR', &
          'no revision of version 1 before 96/04/01 available')

```

```

        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    if (circe1_params%chat .ge. 2) then
        write (msgbuf, 2000) circe1_params%rev, r
2000    format ('mapping date ', I8, ' to revision index ', I2)
        call circem ('MESSAGE', msgbuf)
    endif
    Uses circem 86e.

38a  <Log revision mapping 38a>≡
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 2000) circe1_params%rev, r
            call circem ('MESSAGE', msgbuf)
        endif
    Uses circem 86e.

38b  <Update version 1 derived parameters in circe1 parameters 37d>+≡
        <Map roots to e 38c>

38c  <Map roots to e 38c>≡
        if (circe1_params%roots .eq. 350d0) then
            e = GEV350
        else if ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0))
            write (msgbuf, 2001) circe1_params%roots, 350d0
            call circem ('MESSAGE', msgbuf)
            e = GEV350
        Uses circem 86e.

38d  <formats for circes 38d>≡
        2001 format ('treating energy ', F6.1, 'GeV as ', F6.1, 'GeV')

38e  <Map roots to e 38c>+≡
        else if (circe1_params%roots .eq. 500d0) then
            e = GEV500
        else if ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0))
            write (msgbuf, 2001) circe1_params%roots, 500d0
            call circem ('MESSAGE', msgbuf)
            e = GEV500
        else if (circe1_params%roots .eq. 800d0) then
            e = GEV800
        else if ((circe1_params%roots .ge. 750d0) .and. (circe1_params%roots .le. 850d0))
            write (msgbuf, 2001) circe1_params%roots, 800d0
            call circem ('MESSAGE', msgbuf)
            e = GEV800
        else if (circe1_params%roots .eq. 1000d0) then
            e = TEV1
        else if ((circe1_params%roots .ge. 900d0) .and. (circe1_params%roots .le. 1100d0))
            write (msgbuf, 2001) circe1_params%roots, 1000d0
            call circem ('MESSAGE', msgbuf)
            e = TEV1
        else if (circe1_params%roots .eq. 1600d0) then

```

```

        e = TEV16
    else if ((circe1_params%roots .ge. 1500d0) .and. (circe1_params%roots .le. 1700d0)
        write (msgbuf, 2001) circe1_params%roots, 1600d0
        call circem ('MESSAGE', msgbuf)
        e = TEV16
    Uses circem 86e.
39a <Map roots to e 38c>+≡
    else
        call circem ('ERROR', &
            'only ROOTS = 350, 500, 800, 1000 and 1600GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    Uses circem 86e.
39b <Update version 1 derived parameters in circe1 parameters 37d>+≡
    if (xallum(e,circe1_params%acc,r) .lt. 0d0) then
        write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    end if
    <Log energy mapping 39d>
    Uses circem 86e.
39c <formats for circes 38d>+≡
    2002 format ('energy ', F6.1, ' not available for ', A6,' in revision ', I2)
39d <Log energy mapping 39d>≡
    if (circe1_params%chat .ge. 2) then
        if (e .ge. GEV090) then
            write (msgbuf, 2003) circe1_params%roots, e
            call circem ('MESSAGE', msgbuf)
        else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
            write (msgbuf, 2013) circe1_params%roots, elo, ehi
            call circem ('MESSAGE', msgbuf)
        end if
    endif
    Uses circem 86e.
39e <formats for circes 38d>+≡
    2003 format ('mapping energy ', F6.1, ' to energy index ', I2)
    2013 format ('mapping energy ', F6.1, ' to energy indices ', I2, ' and ', I2)
    The energies 250 GeV, 1.2 TeV and 1.5 TeV were entered late into the game by
    the SLAC people. And, of course, 200 GeV and 230 GeV only appeared even
    much later
39f <Local variables for circes 33b>+≡
    integer, parameter :: EINVAL = -2
    integer, parameter :: GEV090 = -1
    integer, parameter :: GEV170 = 0
    integer, parameter :: GEV350 = 1

```

```

integer, parameter :: GEV500 = 2
integer, parameter :: GEV800 = 3
integer, parameter :: TEV1   = 4
integer, parameter :: TEV16  = 5
integer, parameter :: GEV250 = 6
integer, parameter :: TEV12  = 7
integer, parameter :: TEV15  = 8
integer, parameter :: GEV200 = 9
integer, parameter :: GEV230 = 10
integer, parameter :: A1NEGY = 5
integer, parameter :: A1NREV = 5
integer :: i

```

40a *<8-byte aligned part of circe1 parameters 32d>+≡*

```

real(kind=double) :: lumi
real(kind=double) :: a1(0:7)

```

40b *<Update version 1 derived parameters in circe1 parameters 37d>+≡*

```

circe1_params%lumi = xallum(e,circe1_params%acc,r)
do i = 0, 7
  circe1_params%a1(i) = xa1(i,e,circe1_params%acc,r)
end do

```

40c *<Local variables for circes 33b>+≡*

```

real(kind=double), dimension(A1NEGY,NACC,0:A1NREV), save :: xallum = 0
real(kind=double), dimension(0:7,A1NEGY,NACC,0:A1NREV), save :: xa1 = 0

```

Uses NACC 13b.

Revision 1. The mother of all revisions.

40d *<Initializations for circes 35b>+≡*

```

xallum(GEV500,SBAND,1) = 5.212299E+01
xa1(0:7,GEV500,SBAND,1) = (/ &
  .39192E+00, .66026E+00, .11828E+02,-.62543E+00, &
  .52292E+00,-.69245E+00, .14983E+02, .65421E+00 /)
xallum(GEV500,TESLA,1) = 6.066178E+01
xa1(0:7,GEV500,TESLA,1) = (/ &
  .30196E+00, .12249E+01, .21423E+02,-.57848E+00, &
  .68766E+00,-.69788E+00, .23121E+02, .78399E+00 /)
xallum(GEV500,XBAND,1) = 5.884699E+01
xa1(0:7,GEV500,XBAND,1) = (/ &
  .48594E+00, .52435E+00, .83585E+01,-.61347E+00, &
  .30703E+00,-.68804E+00, .84109E+01, .44312E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

40e *<Initializations for circes 35b>+≡*

```

xallum(TEV1,SBAND,1) = 1.534650E+02
xa1(0:7,TEV1,SBAND,1) = (/ &
  .24399E+00, .87464E+00, .66751E+01,-.56808E+00, &
  .59295E+00,-.68921E+00, .94232E+01, .83351E+00 /)
xallum(TEV1,TESLA,1) = 1.253381E+03
xa1(0:7,TEV1,TESLA,1) = (/ &
  .39843E+00, .70097E+00, .11602E+02,-.61061E+00, &

```



```

        .40737E+00, -.69319E+00, .14800E+02, .51382E+00 /)
xa1lum(TEV1, XBAND, 1) = 1.901783E+02
xa1(0:7, TEV1, XBAND, 1) = (/ &
        .32211E+00, .61798E+00, .28298E+01, -.54644E+00, &
        .45674E+00, -.67301E+00, .41703E+01, .74536E+00 /)
Uses SBAND 13a, TESLA 13a, and XBAND 13a.
Unavailable
41a <Initializations for circes 35b>+≡
        xa1lum(GEV350, 1:NACC, 1) = NACC * (-1d0)
        xa1lum(GEV800, 1:NACC, 1) = NACC * (-1d0)
Uses NACC 13b.
Unavailable as well
41b <Initializations for circes 35b>+≡
        xa1lum(GEV500, SBNDEE:NACC, 1) = 4 * (-1d0)
        xa1lum(TEV1, SBNDEE:NACC, 1) = 4 * (-1d0)
Uses NACC 13b and SBNDEE 13a.
No 1.6TeV parameters in this revision
41c <Initializations for circes 35b>+≡
        xa1lum(TEV16, 1:NACC, 1) = 7 * (-1d0)
Uses NACC 13b.
41d <Public subroutines 31a>+≡
        public :: circel
Uses circel 41e.
41e <Module subroutines 31b>+≡
        subroutine circel (l)
            real(kind=double), intent(out) :: l
            l = circel_params%lumi
        end subroutine circel

Defines:
        circel, used in chunks 41e, 12a, and 41d.
41f <Public subroutines 31a>+≡
        public :: circee
Uses circee 41g.
41g <Module subroutines 31b>+≡
        function circee (x1, x2)
            real(kind=double) :: x1, x2
            real(kind=double) :: circee
            real(kind=double) :: d1, d2
        <Initialization check 32g>
            circee = -1.0
            if ((circel_params%ver .eq. 1) .or. (circel_params%ver .eq. 0)) then
        <Calculate version 1 of the  $e^+e^-$  distribution 42a>
        <else handle invalid versions 37b>
            end function circee

```

Defines:

`circee`, used in chunks 14–16, 31b, 41f, and 42a.

Uses d1 16a and d2 16c.

The first version of the parametrization is factorized

$$D_{p_1 p_2}^{\alpha 1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha 1 \rho}(x_1) d_{p_2}^{\alpha 1 \rho}(x_2) \quad (15)$$

where the distributions are

$$d_{e^\pm}^{\alpha 1 \rho}(x) = a_0^{\alpha \rho} \delta(1-x) + a_1^{\alpha \rho} x^{a_2^{\alpha \rho}} (1-x)^{a_3^{\alpha \rho}} \quad (16)$$

$$d_\gamma(x) = a_4^{\alpha \rho} x^{a_5^{\alpha \rho}} (1-x)^{a_6^{\alpha \rho}} \quad (17)$$

42a \langle Calculate version 1 of the e^+e^- distribution 42a $\rangle \equiv$

```

if (x1 .eq. 1d0) then
  d1 = circe1_params%a1(0)
elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
  d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
elseif (x1 .eq. -1d0) then
  d1 = 1d0 - circe1_params%a1(0)
else
  d1 = 0d0
endif
if (x2 .eq. 1d0) then
  d2 = circe1_params%a1(0)
elseif (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
  d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%
elseif (x2 .eq. -1d0) then
  d2 = 1d0 - circe1_params%a1(0)
else
  d2 = 0d0
endif
circee = d1 * d2

```

Uses `circee` 41g, d1 16a, and d2 16c.

42b \langle Public subroutines 31a $\rangle + \equiv$

```

public :: circeg

```

Uses `circeg` 42c.

42c \langle Module subroutines 31b $\rangle + \equiv$

```

function circeg (x1, x2)
  real(kind=double) :: x1, x2
  real(kind=double) :: circeg
  real(kind=double) :: d1, d2
 $\langle$ Initialization check 32g $\rangle$ 
  circeg = -1.0
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
 $\langle$ Calculate version 1 of the  $e^\pm\gamma$  distribution 43a $\rangle$ 
 $\langle$ else handle invalid versions 37b $\rangle$ 
  end function circeg

```

Defines:

`circeg`, used in chunks 14, 31b, 42b, and 43a.

Uses d1 16a and d2 16c.

43a \langle Calculate version 1 of the $e^\pm\gamma$ distribution 43a $\rangle\equiv$

```

    if (x1 .eq. 1d0) then
        d1 = circe1_params%a1(0)
    else if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%a1(3)
    else if (x1 .eq. -1d0) then
        d1 = 1d0 - circe1_params%a1(0)
    else
        d1 = 0d0
    end if
    if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%a1(6)
    else if (x2 .eq. -1d0) then
        d2 = circe1_params%a1(7)
    else
        d2 = 0d0
    end if
    circeg = d1 * d2

```

Uses circeg 42c, d1 16a, and d2 16c.

43b \langle Public subroutines 31a $\rangle+\equiv$

```

    public :: circgg

```

Uses circgg 43c.

43c \langle Module subroutines 31b $\rangle+\equiv$

```

    function circgg (x1, x2)
        real(kind=double) :: x1, x2
        real(kind=double) :: circgg
        real(kind=double) :: d1, d2
         $\langle$ Initialization check 32g $\rangle$ 
        circgg = -1.0
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
             $\langle$ Calculate version 1 of the  $\gamma\gamma$  distribution 43d $\rangle$ 
             $\langle$ else handle invalid versions 37b $\rangle$ 
        end function circgg

```

Defines:

circgg, used in chunks 14, 31b, 43, and 81a.

Uses d1 16a and d2 16c.

43d \langle Calculate version 1 of the $\gamma\gamma$ distribution 43d $\rangle\equiv$

```

    if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%a1(6)
    elseif (x1 .eq. -1d0) then
        d1 = circe1_params%a1(7)
    else
        d1 = 0d0
    endif
    if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%a1(6)
    elseif (x2 .eq. -1d0) then
        d2 = circe1_params%a1(7)
    else
        d2 = 0d0
    endif

```

```

else
    d2 = 0d0
endif
circgg = d1 * d2

```

Uses circgg 43c, d1 16a, and d2 16c.

Revision 2. New Tesla parameters, including 350 GeV and 800 GeV.

44a *⟨Initializations for circes 35b⟩*+≡

```

xa1lum(GEV500,SBAND,2) = .31057E+02
xa1(0:7,GEV500,SBAND,2) = (/ &
    .38504E+00, .79723E+00, .14191E+02,-.60456E+00, &
    .53411E+00,-.68873E+00, .15105E+02, .65151E+00 /)
xa1lum(TEV1,SBAND,2) = .24297E+03
xa1(0:7,TEV1,SBAND,2) = (/ &
    .24374E+00, .89466E+00, .70242E+01,-.56754E+00, &
    .60910E+00,-.68682E+00, .96083E+01, .83985E+00 /)
xa1lum(GEV350,TESLA,2) = .73369E+02
xa1(0:7,GEV350,TESLA,2) = (/ &
    .36083E+00, .12819E+01, .37880E+02,-.59492E+00, &
    .69109E+00,-.69379E+00, .40061E+02, .65036E+00 /)
xa1lum(GEV500,TESLA,2) = .10493E+03
xa1(0:7,GEV500,TESLA,2) = (/ &
    .29569E+00, .11854E+01, .21282E+02,-.58553E+00, &
    .71341E+00,-.69279E+00, .24061E+02, .77709E+00 /)
xa1lum(GEV800,TESLA,2) = .28010E+03
xa1(0:7,GEV800,TESLA,2) = (/ &
    .22745E+00, .11265E+01, .10483E+02,-.55711E+00, &
    .69579E+00,-.69068E+00, .13093E+02, .89605E+00 /)
xa1lum(TEV1,TESLA,2) = .10992E+03
xa1(0:7,TEV1,TESLA,2) = (/ &
    .40969E+00, .66105E+00, .11972E+02,-.62041E+00, &
    .40463E+00,-.69354E+00, .14669E+02, .51281E+00 /)
xa1lum(GEV500,XBAND,2) = .35689E+02
xa1(0:7,GEV500,XBAND,2) = (/ &
    .48960E+00, .46815E+00, .75249E+01,-.62769E+00, &
    .30341E+00,-.68754E+00, .85545E+01, .43453E+00 /)
xa1lum(TEV1,XBAND,2) = .11724E+03
xa1(0:7,TEV1,XBAND,2) = (/ &
    .31939E+00, .62415E+00, .30763E+01,-.55314E+00, &
    .45634E+00,-.67089E+00, .41529E+01, .73807E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Unavailable

44b *⟨Initializations for circes 35b⟩*+≡

```

xa1lum(GEV350,SBAND,2) = -1d0
xa1lum(GEV350,XBAND,2) = -1d0
xa1lum(GEV800,SBAND,2) = -1d0
xa1lum(GEV800,XBAND,2) = -1d0

```

Uses SBAND 13a and XBAND 13a.

Unavailable as well

44c *⟨Initializations for circes 35b⟩*+≡
`xallum(GEV350,SBNDDEE:NACC,2) = 4 * (-1d0)`
`xallum(GEV500,SBNDDEE:NACC,2) = 4 * (-1d0)`
`xallum(GEV800,SBNDDEE:NACC,2) = 4 * (-1d0)`
`xallum(TEV1,SBNDDEE:NACC,2) = 4 * (-1d0)`

Uses NACC 13b and SBNDDEE 13a.

No 1.6TeV parameters in this revision

45a *⟨Initializations for circes 35b⟩*+≡
`xallum(TEV16,1:NACC,2) = 7 * (-1d0)`

Uses NACC 13b.

Revision 3. Features:

- improved error estimates.
- cleaner fitting procedure, including delta function pieces.

45b *⟨Initializations for circes 35b⟩*+≡
`xallum(GEV500,SBAND,3) = .31469E+02`
`xa1(0:7,GEV500,SBAND,3) = (/ &`
`.38299E+00, .72035E+00, .12618E+02,-.61611E+00, &`
`.51971E+00,-.68960E+00, .15066E+02, .63784E+00 /)`
`xallum(TEV1,SBAND,3) = .24566E+03`
`xa1(0:7,TEV1,SBAND,3) = (/ &`
`.24013E+00, .95763E+00, .69085E+01,-.55151E+00, &`
`.59497E+00,-.68622E+00, .94494E+01, .82158E+00 /)`
`xallum(GEV350,TESLA,3) = .74700E+02`
`xa1(0:7,GEV350,TESLA,3) = (/ &`
`.34689E+00, .12484E+01, .33720E+02,-.59523E+00, &`
`.66266E+00,-.69524E+00, .38488E+02, .63775E+00 /)`
`xallum(GEV500,TESLA,3) = .10608E+03`
`xa1(0:7,GEV500,TESLA,3) = (/ &`
`.28282E+00, .11700E+01, .19258E+02,-.58390E+00, &`
`.68777E+00,-.69402E+00, .23638E+02, .75929E+00 /)`
`xallum(GEV800,TESLA,3) = .28911E+03`
`xa1(0:7,GEV800,TESLA,3) = (/ &`
`.21018E+00, .12039E+01, .96763E+01,-.54024E+00, &`
`.67220E+00,-.69083E+00, .12733E+02, .87355E+00 /)`
`xallum(TEV1,TESLA,3) = .10936E+03`
`xa1(0:7,TEV1,TESLA,3) = (/ &`
`.41040E+00, .68099E+00, .11610E+02,-.61237E+00, &`
`.40155E+00,-.69073E+00, .14698E+02, .49989E+00 /)`
`xallum(GEV500,XBAND,3) = .36145E+02`
`xa1(0:7,GEV500,XBAND,3) = (/ &`
`.51285E+00, .45812E+00, .75135E+01,-.62247E+00, &`
`.30444E+00,-.68530E+00, .85519E+01, .43062E+00 /)`
`xallum(TEV1,XBAND,3) = .11799E+03`
`xa1(0:7,TEV1,XBAND,3) = (/ &`
`.31241E+00, .61241E+00, .29938E+01,-.55848E+00, &`
`.44801E+00,-.67116E+00, .41119E+01, .72753E+00 /)`

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

46a $\langle \text{Initializations for circes 35b} \rangle + \equiv$
 xallum(GEV350,SBAND,3) = -1d0
 xallum(GEV350,XBAND,3) = -1d0
 xallum(GEV800,SBAND,3) = -1d0
 xallum(GEV800,XBAND,3) = -1d0

Uses SBAND 13a and XBAND 13a.

Unavailable as well

46b $\langle \text{Initializations for circes 35b} \rangle + \equiv$
 xallum(GEV350,SBNDDEE:NACC,3) = 4 * (-1d0)
 xallum(GEV500,SBNDDEE:NACC,3) = 4 * (-1d0)
 xallum(GEV800,SBNDDEE:NACC,3) = 4 * (-1d0)
 xallum(TEV1,SBNDDEE:NACC,3) = 4 * (-1d0)

Uses NACC 13b and SBNDDEE 13a.

No 1.6TeV parameters in this revision

46c $\langle \text{Initializations for circes 35b} \rangle + \equiv$
 xallum(TEV16,1:NACC,3) = 7 * (-1d0)

Uses NACC 13b.

Revision 4. Features:

- a bug in Guinea-Pig's synchrotron radiation spectrum has been fixed.

46d $\langle \text{Initializations for circes 35b} \rangle + \equiv$
 xallum(GEV500,SBAND,4) = .31528E+02
 xa1(0:7,GEV500,SBAND,4) = (/ &
 .38169E+00, .73949E+00, .12543E+02, -.61112E+00, &
 .51256E+00, -.69009E+00, .14892E+02, .63314E+00 /)
 xallum(TEV1,SBAND,4) = .24613E+03
 xa1(0:7,TEV1,SBAND,4) = (/ &
 .24256E+00, .94117E+00, .66775E+01, -.55160E+00, &
 .57484E+00, -.68891E+00, .92271E+01, .81162E+00 /)
 xallum(GEV350,TESLA,4) = .74549E+02
 xa1(0:7,GEV350,TESLA,4) = (/ &
 .34120E+00, .12230E+01, .32932E+02, -.59850E+00, &
 .65947E+00, -.69574E+00, .38116E+02, .63879E+00 /)
 xallum(GEV500,TESLA,4) = .10668E+03
 xa1(0:7,GEV500,TESLA,4) = (/ &
 .28082E+00, .11074E+01, .18399E+02, -.59118E+00, &
 .68880E+00, -.69375E+00, .23463E+02, .76073E+00 /)
 xallum(GEV800,TESLA,4) = .29006E+03
 xa1(0:7,GEV800,TESLA,4) = (/ &
 .21272E+00, .11443E+01, .92564E+01, -.54657E+00, &
 .66799E+00, -.69137E+00, .12498E+02, .87571E+00 /)
 xallum(TEV1,TESLA,4) = .11009E+03
 xa1(0:7,TEV1,TESLA,4) = (/ &
 .41058E+00, .64745E+00, .11271E+02, -.61996E+00, &
 .39801E+00, -.69150E+00, .14560E+02, .49924E+00 /)
 xallum(GEV500,XBAND,4) = .36179E+02
 xa1(0:7,GEV500,XBAND,4) = (/ &

```

        .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
        .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
xa1lum(TEV1,XBAND,4) = .11748E+03
xa1(0:7,TEV1,XBAND,4) = (/ &
        .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
        .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

47a *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBAND,4) = -1d0
        xa1lum(GEV350,XBAND,4) = -1d0
        xa1lum(GEV800,SBAND,4) = -1d0
        xa1lum(GEV800,XBAND,4) = -1d0

```

Uses SBAND 13a and XBAND 13a.

Unavailable as well

47b *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBNDDEE:NACC,4) = 4 * (-1d0)
        xa1lum(GEV500,SBNDDEE:NACC,4) = 4 * (-1d0)
        xa1lum(GEV800,SBNDDEE:NACC,4) = 4 * (-1d0)
        xa1lum(TEV1,SBNDDEE:NACC,4) = 4 * (-1d0)

```

Uses NACC 13b and SBNDDEE 13a.

No 1.6TeV parameters in this revision

47c *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(TEV16,1:NACC,4) = 7 * (-1d0)

```

Uses NACC 13b.

Revision 5. Features:

- a bug in Guinea-Pig has been fixed.
- updated parameter sets

47d *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBAND,5) = 0.21897E+02
        xa1(0:7,GEV350,SBAND,5) = (/ &
            0.57183E+00, 0.53877E+00, 0.19422E+02,-0.63064E+00, &
            0.49112E+00,-0.69109E+00, 0.24331E+02, 0.52718E+00 /)
        xa1lum(GEV500,SBAND,5) = 0.31383E+02
        xa1(0:7,GEV500,SBAND,5) = (/ &
            0.51882E+00, 0.49915E+00, 0.11153E+02,-0.63017E+00, &
            0.50217E+00,-0.69113E+00, 0.14935E+02, 0.62373E+00 /)
        xa1lum(GEV800,SBAND,5) = 0.95091E+02
        xa1(0:7,GEV800,SBAND,5) = (/ &
            0.47137E+00, 0.46150E+00, 0.56562E+01,-0.61758E+00, &
            0.46863E+00,-0.68897E+00, 0.85876E+01, 0.67577E+00 /)
        xa1lum(TEV1,SBAND,5) = 0.11900E+03
        xa1(0:7,TEV1,SBAND,5) = (/ &
            0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
            0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
        xa1lum(TEV16,SBAND,5) = 0.11900E+03

```

```

xa1(0:7,TEV16,SBAND,5) = (/ &
    0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
    0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
xa1lum(GEV350,TESLA,5) = 0.97452E+02
xa1(0:7,GEV350,TESLA,5) = (/ &
    0.39071E+00, 0.84996E+00, 0.17614E+02,-0.60609E+00, &
    0.73920E+00,-0.69490E+00, 0.28940E+02, 0.77286E+00 /)
xa1lum(GEV500,TESLA,5) = 0.10625E+03
xa1(0:7,GEV500,TESLA,5) = (/ &
    0.42770E+00, 0.71457E+00, 0.15284E+02,-0.61664E+00, &
    0.68166E+00,-0.69208E+00, 0.24165E+02, 0.73806E+00 /)
xa1lum(GEV800,TESLA,5) = 0.17086E+03
xa1(0:7,GEV800,TESLA,5) = (/ &
    0.36025E+00, 0.69118E+00, 0.76221E+01,-0.59440E+00, &
    0.71269E+00,-0.69077E+00, 0.13117E+02, 0.91780E+00 /)
xa1lum(TEV1,TESLA,5) = 0.21433E+03
xa1(0:7,TEV1,TESLA,5) = (/ &
    0.33145E+00, 0.67075E+00, 0.55438E+01,-0.58468E+00, &
    0.72503E+00,-0.69084E+00, 0.99992E+01, 0.10112E+01 /)
xa1lum(TEV16,TESLA,5) = 0.34086E+03
xa1(0:7,TEV16,TESLA,5) = (/ &
    0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
    0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)
xa1lum(GEV350,XBAND,5) = 0.31901E+02
xa1(0:7,GEV350,XBAND,5) = (/ &
    0.65349E+00, 0.31752E+00, 0.94342E+01,-0.64291E+00, &
    0.30364E+00,-0.68989E+00, 0.11446E+02, 0.40486E+00 /)
xa1lum(GEV500,XBAND,5) = 0.36386E+02
xa1(0:7,GEV500,XBAND,5) = (/ &
    0.65132E+00, 0.28728E+00, 0.69853E+01,-0.64440E+00, &
    0.28736E+00,-0.68758E+00, 0.83227E+01, 0.41492E+00 /)
xa1lum(GEV800,XBAND,5) = 0.10854E+03
xa1(0:7,GEV800,XBAND,5) = (/ &
    0.49478E+00, 0.36221E+00, 0.30116E+01,-0.61548E+00, &
    0.39890E+00,-0.68418E+00, 0.45183E+01, 0.67243E+00 /)
xa1lum(TEV1,XBAND,5) = 0.11899E+03
xa1(0:7,TEV1,XBAND,5) = (/ &
    0.49992E+00, 0.34299E+00, 0.26184E+01,-0.61584E+00, &
    0.38450E+00,-0.68342E+00, 0.38589E+01, 0.67408E+00 /)
xa1lum(TEV16,XBAND,5) = 0.13675E+03
xa1(0:7,TEV16,XBAND,5) = (/ &
    0.50580E+00, 0.30760E+00, 0.18339E+01,-0.61421E+00, &
    0.35233E+00,-0.68315E+00, 0.26708E+01, 0.67918E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Revision 0. Features:

- e^-e^- mode

48 *⟨Initializations for circes 35b⟩*₊₌
 xa1lum(GEV500,SBNDDEE,0) = .92914E+01


```

xa1(0:7,GEV500,SBNDDEE,0) = (/ &
    .34866E+00, .78710E+00, .10304E+02,-.59464E+00, &
    .40234E+00,-.69741E+00, .20645E+02, .47274E+00 /)
xa1lum(TEV1,SBNDDEE,0) = .45586E+02
xa1(0:7,TEV1,SBNDDEE,0) = (/ &
    .21084E+00, .99168E+00, .54407E+01,-.52851E+00, &
    .47493E+00,-.69595E+00, .12480E+02, .64027E+00 /)
xa1lum(GEV350,TESLEE,0) = .15175E+02
xa1(0:7,GEV350,TESLEE,0) = (/ &
    .33093E+00, .11137E+01, .25275E+02,-.59942E+00, &
    .49623E+00,-.70403E+00, .60188E+02, .44637E+00 /)
xa1lum(GEV500,TESLEE,0) = .21622E+02
xa1(0:7,GEV500,TESLEE,0) = (/ &
    .27175E+00, .10697E+01, .14858E+02,-.58418E+00, &
    .50824E+00,-.70387E+00, .36129E+02, .53002E+00 /)
xa1lum(GEV800,TESLEE,0) = .43979E+02
xa1(0:7,GEV800,TESLEE,0) = (/ &
    .22994E+00, .10129E+01, .81905E+01,-.55751E+00, &
    .46551E+00,-.70461E+00, .19394E+02, .58387E+00 /)
xa1lum(TEV1,TESLEE,0) = .25465E+02
xa1(0:7,TEV1,TESLEE,0) = (/ &
    .37294E+00, .67522E+00, .87504E+01,-.60576E+00, &
    .35095E+00,-.69821E+00, .18526E+02, .42784E+00 /)
xa1lum(GEV500,XBNDDEE,0) = .13970E+02
xa1(0:7,GEV500,XBNDDEE,0) = (/ &
    .47296E+00, .46800E+00, .58897E+01,-.61689E+00, &
    .27181E+00,-.68923E+00, .10087E+02, .37462E+00 /)
xa1lum(TEV1,XBNDDEE,0) = .41056E+02
xa1(0:7,TEV1,XBNDDEE,0) = (/ &
    .27965E+00, .74816E+00, .27415E+01,-.50491E+00, &
    .38320E+00,-.67945E+00, .47506E+01, .62218E+00 /)

```

Uses SBNDDEE 13a, TESLEE 13a, and XBNDDEE 13a.

Still unavailable

49a $\langle \text{Initializations for circes 35b} \rangle + \equiv$

```

xa1lum(GEV350,SBNDDEE,0) = -1d0
xa1lum(GEV350,XBNDDEE,0) = -1d0
xa1lum(GEV800,SBNDDEE,0) = -1d0
xa1lum(GEV800,XBNDDEE,0) = -1d0

```

Uses SBNDDEE 13a and XBNDDEE 13a.

49b $\langle \text{Initializations for circes 35b} \rangle + \equiv$

```

xa1lum(GEV500,SBAND,0) = .31528E+02
xa1(0:7,GEV500,SBAND,0) = (/ &
    .38169E+00, .73949E+00, .12543E+02,-.61112E+00, &
    .51256E+00,-.69009E+00, .14892E+02, .63314E+00 /)
xa1lum(TEV1,SBAND,0) = .24613E+03
xa1(0:7,TEV1,SBAND,0) = (/ &
    .24256E+00, .94117E+00, .66775E+01,-.55160E+00, &
    .57484E+00,-.68891E+00, .92271E+01, .81162E+00 /)
xa1lum(GEV350,TESLA,0) = .74549E+02

```

```

xa1(0:7,GEV350,TESLA,0) = (/ &
    .34120E+00, .12230E+01, .32932E+02,-.59850E+00, &
    .65947E+00,-.69574E+00, .38116E+02, .63879E+00 /)
xa1lum(GEV500,TESLA,0) = .10668E+03
xa1(0:7,GEV500,TESLA,0) = (/ &
    .28082E+00, .11074E+01, .18399E+02,-.59118E+00, &
    .68880E+00,-.69375E+00, .23463E+02, .76073E+00 /)
xa1lum(GEV800,TESLA,0) = .29006E+03
xa1(0:7,GEV800,TESLA,0) = (/ &
    .21272E+00, .11443E+01, .92564E+01,-.54657E+00, &
    .66799E+00,-.69137E+00, .12498E+02, .87571E+00 /)
xa1lum(TEV1,TESLA,0) = .11009E+03
xa1(0:7,TEV1,TESLA,0) = (/ &
    .41058E+00, .64745E+00, .11271E+02,-.61996E+00, &
    .39801E+00,-.69150E+00, .14560E+02, .49924E+00 /)
xa1lum(GEV500,XBAND,0) = .36179E+02
xa1(0:7,GEV500,XBAND,0) = (/ &
    .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
    .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
xa1lum(TEV1,XBAND,0) = .11748E+03
xa1(0:7,TEV1,XBAND,0) = (/ &
    .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
    .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

50a *⟨Initializations for circes 35b⟩*+≡

```

xa1lum(GEV350,SBAND,0) = -1d0
xa1lum(GEV350,XBAND,0) = -1d0
xa1lum(GEV800,SBAND,0) = -1d0
xa1lum(GEV800,XBAND,0) = -1d0

```

Uses SBAND 13a and XBAND 13a.

6.2.2 Version 2

50b *⟨Version 2 has been retired 50b⟩*≡

```

call circem ('PANIC', '*****')
call circem ('PANIC', '* version 2 has been retired, *')
call circem ('PANIC', '* please use version 1 instead! *')
call circem ('PANIC', '*****')
return

```

Uses circem 86e.

6.2.3 Versions 3 and 4

50c *⟨Update version 3 and 4 derived parameters in circe1 parameters 50c⟩*≡

```

if (circe1_params%rev .eq. 0) then
    r = 0
elseif (circe1_params%rev .ge. 19970417) then
    r = 5

```

```

        if (ver34 .eq. 3) then
            call circem ('WARNING', 'version 3 retired after 97/04/17')
            call circem ('MESSAGE', 'falling back to version 4')
        end if
    else if (circe1_params%rev .ge. 19961022) then
        r = ver34
        if ((circe1_params%roots .ne. 800d0) .or. (circe1_params%acc .ne. TESLA)) then
            call circem ('ERROR', 'versions 3 and 4 before 97/04/17')
            call circem ('ERROR', 'apply to TESLA at 800 GeV only')
            call circem ('MESSAGE', 'falling back to TESLA at 800GeV')
            circe1_params%acc = TESLA
            e = GEV800
        end if
    else if (circe1_params%rev .lt. 19961022) then
        call circem ('ERROR', &
            'no revision of versions 3 and 4 available before 96/10/22')
        call circem ('MESSAGE', 'falling back to default')
        r = 5
    end if
    <Log revision mapping 38a>
    Uses TESLA 13a and circem 86e.

51a <Update version 3 and 4 derived parameters in circe1 parameters 50c>+≡
    <Map roots to e 38c>
        if (xa3lum(e,circe1_params%acc,r) .lt. 0d0) then
            write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
            call circem ('ERROR', msgbuf)
            call circem ('MESSAGE', 'falling back to 500GeV')
            e = GEV500
        endif
    <Log energy mapping 39d>
    Uses circem 86e.

51b <Local variables for circes 33b>+≡
    integer, parameter :: A3NEGY = 5, A3NREV = 5

51c <Update version 3 and 4 derived parameters in circe1 parameters 50c>+≡
    circe1_params%lumi = xa3lum (e,circe1_params%acc,r)
    do i = 0, 7
        circe1_params%a1(i) = xa3(i,e,circe1_params%acc,r)
    end do

51d <Local variables for circes 33b>+≡
    real, dimension(A3NEGY,NACC,0:A3NREV), save :: xa3lum = -1
    real, dimension(0:7,A3NEGY,NACC,0:A3NREV), save :: xa3 = 0
    Uses NACC 13b.

Revisions 3 & 4. The mother of all revisions.

51e <Initializations for circes 35b>+≡
    xa3lum(GEV800,TESLA,3) = .17196E+03
    xa3(0:7,GEV800,TESLA,3) = (/ &
        .21633E+00, .11333E+01, .95928E+01,-.55095E+00, &

```

```

        .73044E+00,-.69101E+00, .12868E+02, .94737E+00 /)
xa3lum(GEV800,TESLA, 4) = .16408E+03
xa3(0:7,GEV800,TESLA, 4) = (/ &
        .41828E+00, .72418E+00, .14137E+02,-.61189E+00, &
        .36697E+00,-.69205E+00, .17713E+02, .43583E+00 /)

```

Uses TESLA 13a.

Revision 5.

52a *⟨Initializations for circes 35b⟩+≡*

```

xa3lum(GEV350,TESLA,5) = 0.66447E+02
xa3(0:7,GEV350,TESLA,5) = (/ &
        0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &
        0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)
xa3lum(GEV500,TESLA,5) = 0.95241E+02
xa3(0:7,GEV500,TESLA,5) = (/ &
        0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &
        0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)
xa3lum(GEV800,TESLA,5) = 0.16974E+03
xa3(0:7,GEV800,TESLA,5) = (/ &
        0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &
        0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)
xa3lum(TEV1,TESLA,5) = 0.21222E+03
xa3(0:7,TEV1,TESLA,5) = (/ &
        0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &
        0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)
xa3lum(TEV16,TESLA,5) = 0.34086E+03
xa3(0:7,TEV16,TESLA,5) = (/ &
        0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
        0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)

```

Uses TESLA 13a.

Revision 0. Currently identical to revision 5.

52b *⟨Initializations for circes 35b⟩+≡*

```

xa3lum(GEV350,TESLA,0) = 0.66447E+02
xa3(0:7,GEV350,TESLA,0) = (/ &
        0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &
        0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)
xa3lum(GEV500,TESLA,0) = 0.95241E+02
xa3(0:7,GEV500,TESLA,0) = (/ &
        0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &
        0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)
xa3lum(GEV800,TESLA,0) = 0.16974E+03
xa3(0:7,GEV800,TESLA,0) = (/ &
        0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &
        0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)
xa3lum(TEV1,TESLA,0) = 0.21222E+03
xa3(0:7,TEV1,TESLA,0) = (/ &
        0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &
        0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)
xa3lum(TEV16,TESLA,0) = 0.34086E+03
xa3(0:7,TEV16,TESLA,0) = (/ &

```

```

0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)

```

Uses TESLA 13a.

6.2.4 Version 5

53a *<Update version 5 derived parameters in circe1 parameters 53a>≡*

```

if (circe1_params%rev .eq. 0) then
  r = 0
elseif (circe1_params%rev .ge. 19980505) then
  r = 1
elseif (circe1_params%rev .lt. 19980505) then
  call circem ('ERROR', &
    'no revision of version 5 available before 98/05/05')
  call circem ('MESSAGE', 'falling back to default')
  r = 1
endif

```

<Log revision mapping 38a>

Uses circem 86e.

53b *<Update version 5 derived parameters in circe1 parameters 53a>+≡*

```

if (circe1_params%acc .ne. TESLA) then
  call circem ('ERROR', 'versions 5 applies to TESLA only')
  circe1_params%acc = TESLA
end if

```

<Map roots to e 38c>

```

if (xa5lum(e,circe1_params%acc,r) .lt. 0d0) then
  write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
  call circem ('ERROR', msgbuf)
  call circem ('MESSAGE', 'falling back to 500GeV')
  e = GEV500
endif

```

<Log energy mapping 39d>

Uses TESLA 13a and circem 86e.

53c *<Local variables for circes 33b>+≡*

```

integer, parameter :: A5NEGY = 5, A5NREV = 1

```

53d *<Update version 5 derived parameters in circe1 parameters 53a>+≡*

```

circe1_params%lumi = xa5lum (e,circe1_params%acc,r)
do i = 0, 7
  circe1_params%a1(i) = xa5(i,e,circe1_params%acc,r)
end do

```

53e *<Local variables for circes 33b>+≡*

```

real, dimension(A5NEGY,NACC,0:A5NREV), save :: xa5lum
real, dimension(0:7,A5NEGY,NACC,0:A5NREV), save :: xa5

```

Uses NACC 13b.

Revision 1. The mother of all revisions. Note that $3.3980 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.4099 \cdot 10^{34} \text{ m}^{-2} \cdot 2820 \cdot 5 \text{ s}^{-1}$ and $3.5936 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.6619 \cdot 10^{34} \text{ m}^{-2} \cdot 4500 \cdot 3 \text{ s}^{-1}$. This unit conversion is missing in *all* earlier versions, unfortunately.

54a $\langle \text{Initializations for circes 35b} \rangle + \equiv$
`xa5lum(GEV350, TESLA, 1) = -1.0`
`xa5lum(GEV500, TESLA, 1) = 0.33980E+03`
`xa5(0:7, GEV500, TESLA, 1) = (/ &`
`0.49808E+00, 0.54613E+00, 0.12287E+02, -0.62756E+00, &`
`0.42817E+00, -0.69120E+00, 0.17067E+02, 0.51143E+00 /)`
`xa5lum(GEV800, TESLA, 1) = 0.35936E+03`
`xa5(0:7, GEV800, TESLA, 1) = (/ &`
`0.58751E+00, 0.43128E+00, 0.13324E+02, -0.64006E+00, &`
`0.30682E+00, -0.69235E+00, 0.16815E+02, 0.37078E+00 /)`
`xa5lum(TEV1, TESLA, 1) = -1.0`
`xa5lum(TEV16, TESLA, 1) = -1.0`

Uses TESLA 13a.

Revision 0. Currently identical to revision 1.

54b $\langle \text{Initializations for circes 35b} \rangle + \equiv$
`xa5lum(GEV350, TESLA, 0) = -1.0`
`xa5lum(GEV500, TESLA, 0) = 0.33980E+03`
`xa5(0:7, GEV500, TESLA, 0) = (/ &`
`0.49808E+00, 0.54613E+00, 0.12287E+02, -0.62756E+00, &`
`0.42817E+00, -0.69120E+00, 0.17067E+02, 0.51143E+00 /)`
`xa5lum(GEV800, TESLA, 0) = 0.35936E+03`
`xa5(0:7, GEV800, TESLA, 0) = (/ &`
`0.58751E+00, 0.43128E+00, 0.13324E+02, -0.64006E+00, &`
`0.30682E+00, -0.69235E+00, 0.16815E+02, 0.37078E+00 /)`
`xa5lum(TEV1, TESLA, 0) = -1.0`
`xa5lum(TEV16, TESLA, 0) = -1.0`

Uses TESLA 13a.

6.2.5 Version 6

54c $\langle \text{Update version 6 derived parameters in circe1 parameters 54c} \rangle \equiv$
`if (circe1_params%rev .eq. 0) then`
`r = 0`
`else if (circe1_params%rev .ge. 19990415) then`
`r = 1`
`else if (circe1_params%rev .lt. 19990415) then`
`call circem ('ERROR', &`
`'no revision of version 6 available before 1999/04/15')`
`call circem ('MESSAGE', 'falling back to default')`
`r = 1`
`end if`

$\langle \text{Log revision mapping 38a} \rangle$

Uses circem 86e.

54d $\langle \text{Update version 6 derived parameters in circe1 parameters 54c} \rangle + \equiv$
`if (circe1_params%acc .ne. TESLA) then`

```

        call circem ('ERROR', 'versions 6 applies to TESLA only')
        circe1_params%acc = TESLA
    end if
    <Map roots to e at low energies 55a>
    if (xa6lum(e,circe1_params%acc,r) .lt. 0d0) then
        write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    <Log energy mapping 39d>
    Uses TESLA 13a and circem 86e.
55a <Map roots to e at low energies 55a>≡
    if (circe1_params%roots .eq. 90d0) then
        e = GEV090
    elseif ((circe1_params%roots .ge. 85d0) .and. (circe1_params%roots .le. 95d0)) then
        write (msgbuf, 2001) circe1_params%roots, 90d0
        call circem ('MESSAGE', msgbuf)
        e = GEV090
    elseif (circe1_params%roots .eq. 170d0) then
        e = GEV170
    elseif ((circe1_params%roots .ge. 160d0) .and. (circe1_params%roots .le. 180d0)) then
        write (msgbuf, 2001) circe1_params%roots, 170d0
        call circem ('MESSAGE', msgbuf)
        e = GEV170
    elseif (circe1_params%roots .eq. 350d0) then
        e = GEV350
    elseif ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0)) then
        write (msgbuf, 2001) circe1_params%roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350
    elseif (circe1_params%roots .eq. 500d0) then
        e = GEV500
    elseif ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0)) then
        write (msgbuf, 2001) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    else
        call circem ('ERROR', &
            'only ROOTS = 90, 170, 350, and 500GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    Uses circem 86e.
55b <Local variables for circses 33b>+≡
    integer, parameter :: A6NEGY = 2, A6NREV = 1
55c <Update version 6 derived parameters in circe1 parameters 54c>+≡
    circe1_params%lumi = xa6lum (e,circe1_params%acc,r)

```

```

do i = 0, 7
  circe1_params%a1(i) = xa6(i,e,circe1_params%acc,r)
end do

```

56a *<Local variables for circes 33b>+≡*

```

real, dimension(GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6lum
real, dimension(0:7,GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6

```

Uses NACC 13b.

Revision 1. The mother of all revisions.

56b *<Initializations for circes 35b>+≡*

```

xa6lum(GEV090,TESLA,1) = 0.62408E+02
xa6(0:7,GEV090,TESLA,1) = (/ &
  0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
  0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
xa6lum(GEV170,TESLA,1) = 0.11532E+02
xa6(0:7,GEV170,TESLA,1) = (/ &
  0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
  0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa6lum(GEV350,TESLA,1) = 0.24641E+03
xa6(0:7,GEV350,TESLA,1) = (/ &
  0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
  0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa6lum(GEV500,TESLA,1) = 0.30340E+03
xa6(0:7,GEV500,TESLA,1) = (/ &
  0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
  0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)

```

Uses TESLA 13a.

Revision 0. Currently identical to revision 1.

56c *<Initializations for circes 35b>+≡*

```

xa6lum(GEV090,TESLA,0) = 0.62408E+02
xa6(0:7,GEV090,TESLA,0) = (/ &
  0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
  0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
xa6lum(GEV170,TESLA,0) = 0.11532E+02
xa6(0:7,GEV170,TESLA,0) = (/ &
  0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
  0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa6lum(GEV350,TESLA,0) = 0.24641E+03
xa6(0:7,GEV350,TESLA,0) = (/ &
  0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
  0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa6lum(GEV500,TESLA,0) = 0.30340E+03
xa6(0:7,GEV500,TESLA,0) = (/ &
  0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
  0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)

```

Uses TESLA 13a.

6.2.6 Version 7

56d *<Update version 7 derived parameters in circe1 parameters 56d>≡*


```

        if (circe1_params%rev .eq. 0) then
            r = 0
        elseif (circe1_params%rev .ge. 20000426) then
            r = 1
        elseif (circe1_params%rev .lt. 20000426) then
            call circem ('ERROR', &
                'no revision of version 7 available before 2000/04/26')
            call circem ('MESSAGE', 'falling back to default')
            r = 1
        endif
    <Log revision mapping 38a>
    Uses circem 86e.

57a <Update version 7 derived parameters in circe1 parameters 56d>+=
        if (circe1_params%acc .ne. TESLA .and. circe1_params%acc .ne. JLCNLC) then
            call circem ('ERROR', &
                'version 7 applies to TESLA and JLCNLC only')
            call circem ('ERROR', 'falling back to TESLA')
            circe1_params%acc = TESLA
        end if
    <Linearly interpolate energies 57d>
    <Log energy mapping 39d>
    Uses JLCNLC 13a, TESLA 13a, and circem 86e.

57b <formats for circes 38d>+=
    2004 format ('energy ', F6.1, 'GeV too low, using spectrum for ', F6.1, 'GeV')
    2005 format ('energy ', F6.1, 'GeV too high, using spectrum for ', F6.1, 'GeV')
    2006 format ('energy ', F6.1, 'GeV interpolated between ', F6.1, ' and ', F6.1, 'GeV')

57c <Local variables for circes 33b>+=
    real(kind=double) :: eloval, ehival
    real(kind=double), parameter :: DELTAE = 0.5d0

    The rules are as follows: XBAND has 500 GeV and 1 TeV, TESLA has 500 GeV and
    800 TeV. Low energy TESLA will be added.

57d <Linearly interpolate energies 57d>=
    e = GEV090 - 1
    elo = e
    ehi = e
    if (circe1_params%acc .eq. TESLA) then
        if (circe1_params%roots .lt. 90d0 - DELTAE) then
            write (msgbuf, 2004) circe1_params%roots, 90d0
            call circem ('MESSAGE', msgbuf)
            e = GEV090
        elseif (abs (circe1_params%roots-090d0) .le. DELTAE) then
            e = GEV090
        elseif (circe1_params%roots .lt. 170d0 - DELTAE) then
            write (msgbuf, 2005) circe1_params%roots, 170d0
            call circem ('MESSAGE', msgbuf)
            e = GEV170
        elseif (abs (circe1_params%roots-170d0) .le. DELTAE) then
            e = GEV170

```

```

elseif (circe1_params%roots .lt. 350d0-DELTA_E) then
    write (msgbuf, 2006) circe1_params%roots, 170d0, 350d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV170
    ehi = GEV350
    eloal = 170d0
    ehival = 350d0
elseif (abs (circe1_params%roots-350d0) .le. DELTA_E) then
    e = GEV350
elseif (circe1_params%roots .lt. 500d0 - DELTA_E) then
    write (msgbuf, 2006) circe1_params%roots, 350d0, 500d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV350
    ehi = GEV500
    eloal = 350d0
    ehival = 500d0
elseif (abs (circe1_params%roots-500d0) .le. DELTA_E) then
    e = GEV500
elseif (circe1_params%roots .lt. 800d0 - DELTA_E) then
    write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV500
    ehi = GEV800
    eloal = 500d0
    ehival = 800d0
elseif (abs (circe1_params%roots-800d0) .le. DELTA_E) then
    e = GEV800
else
    write (msgbuf, 2005) circe1_params%roots, 800d0
    call circem ('MESSAGE', msgbuf)
    e = GEV800
endif
elseif (circe1_params%acc .eq. XBAND) then
    if (circe1_params%roots .lt. 500d0 - DELTA_E) then
        write (msgbuf, 2004) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    elseif (abs (circe1_params%roots-500d0) .le. DELTA_E) then
        e = GEV500
    elseif (circe1_params%roots .lt. 1000d0 - DELTA_E) then
        write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = TEV1
        eloal = 500d0
        ehival = 1000d0
    elseif (abs (circe1_params%roots-1000d0) .le. DELTA_E) then
        e = TEV1
    else
        write (msgbuf, 2005) circe1_params%roots, 1000d0

```

```

        call circem ('MESSAGE', msgbuf)
        e = TEV1
    endif
endif
Uses TESLA 13a, XBAND 13a, and circem 86e.
59a <Local variables for circes 33b>+≡
    integer, parameter :: A7NEGY = TEV1, A7NREV = 1
Note that ew must not interpolate a1(0) and a1(7) because they depend non-
linearly on the other parameters!
59b <Update version 7 derived parameters in circe1 parameters 56d>+≡
    if (e .ge. GEV090) then
        circe1_params%lumi = xa7lum(e,circe1_params%acc,r)
        do i = 0, 7
            circe1_params%a1(i) = xa7(i,e,circe1_params%acc,r)
        end do
    else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
        circe1_params%lumi = ((circe1_params%roots-eloval)*xa7lum(ehi,circe1_params%acc,
            + (ehival-circe1_params%roots)*xa7lum(elo,circe1_params%acc,r)) / (ehival -
        do i = 1, 6
            circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa7(i,ehi,circe1_params%
            + (ehival-circe1_params%roots)*xa7(i,elo,circe1_params%acc,r)) / (ehival -
        end do
        circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
        circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
    endif
Uses beta 105.
59c <Local variables for circes 33b>+≡
    real, dimension(GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7lum
    real, dimension(0:7,GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7
Uses NACC 13b.
Revision 1. The mother of all revisions.
59d <Initializations for circes 35b>+≡
    xa7lum(GEV090,TESLA,1) = 0.62408E+02
    xa7(0:7,GEV090,TESLA,1) = (/ &
        0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
        0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
    xa7lum(GEV170,TESLA,1) = 0.11532E+02
    xa7(0:7,GEV170,TESLA,1) = (/ &
        0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
        0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
    xa7lum(GEV350,TESLA,1) = 0.24641E+03
    xa7(0:7,GEV350,TESLA,1) = (/ &
        0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
        0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
    xa7lum(GEV500,TESLA,1) = 0.34704E+03
    xa7(0:7,GEV500,TESLA,1) = (/ &
        0.51288E+00, 0.49025E+00, 0.99716E+01,-0.62850E+00, &
        0.41048E+00,-0.69065E+00, 0.13922E+02, 0.51902E+00 /)

```

```

xa7lum(GEV800, TESLA, 1) = 0.57719E+03
xa7(0:7, GEV800, TESLA, 1) = (/ &
    0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00, &
    0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1, TESLA, 1) = -1.0

```

Uses TESLA 13a.

60a *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090, JLCNLC, 1) = -1.0
xa7lum(GEV170, JLCNLC, 1) = -1.0
xa7lum(GEV350, JLCNLC, 1) = -1.0
xa7lum(GEV500, JLCNLC, 1) = 0.63039E+02
xa7(0:7, GEV500, JLCNLC, 1) = (/ &
    0.58967E+00, 0.34035E+00, 0.63631E+01, -0.63683E+00, &
    0.33383E+00, -0.68803E+00, 0.81005E+01, 0.48702E+00 /)
xa7lum(TEV1, JLCNLC, 1) = 0.12812E+03
xa7(0:7, TEV1, JLCNLC, 1) = (/ &
    0.50222E+00, 0.33773E+00, 0.25681E+01, -0.61711E+00, &
    0.36826E+00, -0.68335E+00, 0.36746E+01, 0.65393E+00 /)

```

Uses JLCNLC 13a.

Revision 0.

60b *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090, TESLA, 0) = 0.62408E+02
xa7(0:7, GEV090, TESLA, 0) = (/ &
    0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00, &
    0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /)
xa7lum(GEV170, TESLA, 0) = 0.11532E+02
xa7(0:7, GEV170, TESLA, 0) = (/ &
    0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00, &
    0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa7lum(GEV350, TESLA, 0) = 0.24641E+03
xa7(0:7, GEV350, TESLA, 0) = (/ &
    0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00, &
    0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa7lum(GEV500, TESLA, 0) = 0.34704E+03
xa7(0:7, GEV500, TESLA, 0) = (/ &
    0.51288E+00, 0.49025E+00, 0.99716E+01, -0.62850E+00, &
    0.41048E+00, -0.69065E+00, 0.13922E+02, 0.51902E+00 /)
xa7lum(GEV800, TESLA, 0) = 0.57719E+03
xa7(0:7, GEV800, TESLA, 0) = (/ &
    0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00, &
    0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1, TESLA, 0) = -1.0

```

Uses TESLA 13a.

60c *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090, JLCNLC, 0) = -1.0
xa7lum(GEV170, JLCNLC, 0) = -1.0
xa7lum(GEV350, JLCNLC, 0) = -1.0
xa7lum(GEV500, JLCNLC, 0) = 0.63039E+02

```

```

xa7(0:7,GEV500,JLCNLC,0) = (/ &
    0.58967E+00, 0.34035E+00, 0.63631E+01,-0.63683E+00, &
    0.33383E+00,-0.68803E+00, 0.81005E+01, 0.48702E+00 /)
xa7lum(TEV1,JLCNLC,0) = 0.12812E+03
xa7(0:7,TEV1,JLCNLC,0) = (/ &
    0.50222E+00, 0.33773E+00, 0.25681E+01,-0.61711E+00, &
    0.36826E+00,-0.68335E+00, 0.36746E+01, 0.65393E+00 /)

```

Uses JLCNLC 13a.

6.2.7 Version 8

61a *Update version 8 derived parameters in circe1 parameters 61a* \equiv

```

if (circe1_params%rev .eq. 0) then
    r = 0
elseif (circe1_params%rev .ge. 20010617) then
    r = 1
elseif (circe1_params%rev .lt. 20010617) then
    call circem ('ERROR', &
        'no revision of version 8 available before 2001/06/17')
    call circem ('MESSAGE', 'falling back to default')
    r = 1
endif

```

Log revision mapping 38a

Uses circem 86e.

61b *Update version 8 derived parameters in circe1 parameters 61a* \equiv

```

if (circe1_params%acc .eq. NLCH) then
    circe1_params%acc = JLCNLC
end if
if (circe1_params%acc .ne. JLCNLC) then
    call circem ('ERROR', &
        'version 8 applies to JLCNLC (NLC H) only')
    call circem ('ERROR', 'falling back to JLCNLC')
    circe1_params%acc = JLCNLC
end if

```

Linearly interpolate energies 57d
Log energy mapping 39d

Uses JLCNLC 13a and circem 86e.

61c *Local variables for circes 33b* \equiv

```

integer, parameter :: A8NEGY = TEV1, A8NREV = 1

```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

61d *Update version 8 derived parameters in circe1 parameters 61a* \equiv

```

if (e .ge. GEV090) then
    circe1_params%lumi = xa8lum(e,circe1_params%acc,r)
    do i = 0, 7
        circe1_params%a1(i) = xa8(i,e,circe1_params%acc,r)
    end do
elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then

```

```

        circe1_params%lumi = ((circe1_params%roots-eloval)*xa8lum(ehi,circe1_params%acc
        + (ehival-circe1_params%roots)*xa8lum(elo,circe1_params%acc,r)) / (ehival-
do i = 1, 6
        circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa8(i,ehi,circe1_params%
        + (ehival-circe1_params%roots)*xa8(i,elo,circe1_params%acc,r)) / (ehival-
end do
        circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
        circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
endif

```

Uses beta 105.

62a *<Local variables for circes 33b>+≡*
 real, dimension(GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8lum
 real, dimension(0:7,GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8

Uses NACC 13b.

Revision 1. The mother of all revisions.

62b *<Initializations for circes 35b>+≡*
 xa8lum(GEV090,TESLA,1) = -1.0
 xa8lum(GEV170,TESLA,1) = -1.0
 xa8lum(GEV350,TESLA,1) = -1.0
 xa8lum(GEV500,TESLA,1) = -1.0
 xa8lum(GEV800,TESLA,1) = -1.0
 xa8lum(TEV1, TESLA,1) = -1.0

Uses TESLA 13a.

62c *<Initializations for circes 35b>+≡*
 xa8lum(GEV090,JLCNLC,1) = -1.0
 xa8lum(GEV170,JLCNLC,1) = -1.0
 xa8lum(GEV350,JLCNLC,1) = -1.0
 xa8lum(GEV500,JLCNLC,1) = 0.239924E+03
 xa8(0:7,GEV500,JLCNLC,1) = (/ &
 0.57025E+00, 0.34004E+00, 0.52864E+01,-0.63405E+00, &
 0.31627E+00,-0.68722E+00, 0.69629E+01, 0.47973E+00 /)
 xa8lum(TEV1,JLCNLC,1) = 0.40858E+03
 xa8(0:7,TEV1,JLCNLC,1) = (/ &
 0.52344E+00, 0.31536E+00, 0.25244E+01,-0.62215E+00, &
 0.31935E+00,-0.68424E+00, 0.35877E+01, 0.57315E+00 /)

Uses JLCNLC 13a.

Revision 0.

62d *<Initializations for circes 35b>+≡*
 xa8lum(GEV090,TESLA,0) = -1.0
 xa8lum(GEV170,TESLA,0) = -1.0
 xa8lum(GEV350,TESLA,0) = -1.0
 xa8lum(GEV500,TESLA,0) = -1.0
 xa8lum(GEV800,TESLA,0) = -1.0
 xa8lum(TEV1, TESLA,0) = -1.0

Uses TESLA 13a.

62e *<Initializations for circes 35b>+≡*
 xa8lum(GEV090,JLCNLC,0) = -1.0

```

xa8lum(GEV170,JLCNLC,0) = -1.0
xa8lum(GEV350,JLCNLC,0) = -1.0
xa8lum(GEV500,JLCNLC,0) = 0.239924E+03
xa8(0:7,GEV500,JLCNLC,0) = (/ &
    0.57025E+00, 0.34004E+00, 0.52864E+01,-0.63405E+00, &
    0.31627E+00,-0.68722E+00, 0.69629E+01, 0.47973E+00 /)
xa8lum(TEV1,JLCNLC,0) = 0.40858E+03
xa8(0:7,TEV1,JLCNLC,0) = (/ &
    0.52344E+00, 0.31536E+00, 0.25244E+01,-0.62215E+00, &
    0.31935E+00,-0.68424E+00, 0.35877E+01, 0.57315E+00 /)

```

Uses JLCNLC 13a.

6.2.8 Version 9

63a *<Update version 9 derived parameters in circe1 parameters 63a>≡*

```

    if (circe1_params%rev .eq. 0) then
        r = 0
    elseif (circe1_params%rev .ge. 20020328) then
        r = 1
    elseif (circe1_params%rev .lt. 20020328) then
        call circem ('ERROR', &
            'no revision of version 9 available before 2002/03/28')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 38a>

```

Uses circem 86e.

63b *<Update version 9 derived parameters in circe1 parameters 63a>+≡*

```

    if (circe1_params%acc .ne. JLCNLC .and. circe1_params%acc .ne. NLCH) then
        call circem ('ERROR', &
            'version 9 applies to JLCNLC and NLCH only')
        call circem ('ERROR', 'falling back to JLCNLC')
        circe1_params%acc = JLCNLC
    end if
    if (circe1_params%acc .eq. JLCNLC) then
        <Linearly interpolate energies for JLC/NLC 2002 63c>
    else if (circe1_params%acc .eq. NLCH) then
        <Linearly interpolate energies for NLC H 2002 65a>
    end if
    <Log energy mapping 39d>

```

Uses JLCNLC 13a and circem 86e.

63c *<Linearly interpolate energies for JLC/NLC 2002 63c>≡*

```

    e = GEV090 - 1
    elo = e
    ehi = e
    if (circe1_params%roots .lt. 250d0 - DELTAE) then
        write (msgbuf, 2004) circe1_params%roots, 250d0
        call circem ('MESSAGE', msgbuf)
        e = GEV250
    end if

```

```

elseif (abs (circe1_params%roots-250d0) .le. DELTAE) then
    e = GEV250
elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 250d0, 500d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV250
    ehi = GEV500
    eloal = 250d0
    ehival = 500d0
elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
    e = GEV500
elseif (circe1_params%roots .lt. 800d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV500
    ehi = GEV800
    eloal = 500d0
    ehival = 800d0
elseif (abs (circe1_params%roots-800d0) .le. DELTAE) then
    e = GEV800
elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 800d0, 1000d0
    call circem ('MESSAGE', msgbuf)
    elo = GEV800
    ehi = TEV1
    eloal = 800d0
    ehival = 1000d0
elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
    e = TEV1
elseif (circe1_params%roots .lt. 1200d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 1000d0, 1200d0
    call circem ('MESSAGE', msgbuf)
    elo = TEV1
    ehi = TEV12
    eloal = 1000d0
    ehival = 1200d0
elseif (abs (circe1_params%roots-1200d0) .le. DELTAE) then
    e = TEV12
elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 1200d0, 1500d0
    call circem ('MESSAGE', msgbuf)
    elo = TEV12
    ehi = TEV15
    eloal = 1200d0
    ehival = 1500d0
elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
    e = TEV15
else
    write (msgbuf, 2005) circe1_params%roots, 1500d0
    call circem ('MESSAGE', msgbuf)

```



```

        e = TEV15
    endif
    Uses circem 86e.
65a  <Linearly interpolate energies for NLC H 2002 65a>≡
        e = GEV090 - 1
        elo = e
        ehi = e
        if (circe1_params%roots .lt. 500d0 - DELTAE) then
            write (msgbuf, 2004) circe1_params%roots, 500d0
            call circem ('MESSAGE', msgbuf)
            e = GEV500
        elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
            e = GEV500
        elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
            write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
            call circem ('MESSAGE', msgbuf)
            elo = GEV500
            ehi = TEV1
            eloal = 500d0
            ehival = 1000d0
        elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
            e = TEV1
        elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
            write (msgbuf, 2006) circe1_params%roots, 1000d0, 1500d0
            call circem ('MESSAGE', msgbuf)
            elo = TEV1
            ehi = TEV15
            eloal = 1000d0
            ehival = 1500d0
        elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
            e = TEV15
        else
            write (msgbuf, 2005) circe1_params%roots, 1500d0
            call circem ('MESSAGE', msgbuf)
            e = TEV15
        endif
    Uses circem 86e.
65b  <Local variables for circses 33b>+≡
        integer, parameter :: A9NEGY = TEV15, A9NREV = 1
    Note that ew must not interpolate a1(0) and a1(7) because they depend non-
    linearly on the other parameters!
65c  <Update version 9 derived parameters in circe1 parameters 63a>+≡
        if (e .ge. GEV090) then
            circe1_params%lumi = xa9lum(e,circe1_params%acc,r)
            do i = 0, 7
                circe1_params%a1(i) = xa9(i,e,circe1_params%acc,r)
            end do
        else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then

```

```

circe1_params%lumi = ((circe1_params%roots-eloval)*xa9lum(ehi,circe1_params%acc
+ (ehival-circe1_params%roots)*xa9lum(elo,circe1_params%acc,r)) / (ehival
do i = 1, 6
    circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa9(i,ehi,circe1_params%
+ (ehival-circe1_params%roots)*xa9(i,elo,circe1_params%acc,r)) / (ehival
end do
circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
end if

```

Uses beta 105.

66a *<Local variables for circes 33b>+≡*
 real, dimension(GEV090:A9NEGY,NACC,0:A9NREV) :: xa9lum
 real, dimension(0:7,GEV090:A9NEGY,NACC,0:A9NREV) :: xa9

Uses NACC 13b.

Revision 1. The mother of all revisions.

66b *<Initializations for circes 35b>+≡*
 xa9lum(GEV090,TESLA,1) = -1.0
 xa9lum(GEV170,TESLA,1) = -1.0
 xa9lum(GEV350,TESLA,1) = -1.0
 xa9lum(GEV500,TESLA,1) = -1.0
 xa9lum(GEV800,TESLA,1) = -1.0
 xa9lum(TEV1, TESLA,1) = -1.0
 xa9lum(TEV12, TESLA,1) = -1.0
 xa9lum(TEV15, TESLA,1) = -1.0
 xa9lum(TEV16, TESLA,1) = -1.0

Uses TESLA 13a.

66c *<Initializations for circes 35b>+≡*
 xa9lum(GEV090,JLCNLC,1) = -1.0
 xa9lum(GEV170,JLCNLC,1) = -1.0
 xa9lum(GEV250,JLCNLC,1) = 109.886976
 xa9(0:7,GEV250,JLCNLC,1) = (/ &
 0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
 0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
 xa9lum(GEV350,JLCNLC,1) = -1.0
 xa9lum(GEV500,JLCNLC,1) = 220.806144
 xa9(0:7,GEV500,JLCNLC,1) = (/ &
 0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
 0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
 xa9lum(GEV800,JLCNLC,1) = 304.63488
 xa9(0:7,GEV800,JLCNLC,1) = (/ &
 0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
 0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)
 xa9lum(TEV1, JLCNLC,1) = 319.95648
 xa9(0:7,TEV1, JLCNLC,1) = (/ &
 0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
 0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
 xa9lum(TEV12,JLCNLC,1) = 349.90848
 xa9(0:7,TEV12,JLCNLC,1) = (/ &

```

0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
xa9lum(TEV15,JLCNLC,1) = 363.15648
xa9(0:7,TEV15,JLCNLC,1) = (/ &
0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
xa9lum(TEV16,JLCNLC,1) = -1.0

```

Uses JLCNLC 13a.

67a *<Initializations for circes 35b>+≡*

```

xa9lum(GEV090,NLCH,1) = -1.0
xa9lum(GEV170,NLCH,1) = -1.0
xa9lum(GEV250,NLCH,1) = -1.0
xa9lum(GEV350,NLCH,1) = -1.0
xa9lum(GEV500,NLCH,1) = 371.4624
xa9(0:7,GEV500,NLCH,1)= (/ &
0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
xa9lum(GEV800,NLCH,1) = -1.0
xa9lum(TEV1,NLCH,1) = 516.41856
xa9(0:7,TEV1,NLCH,1)= (/ &
0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
xa9lum(TEV12,NLCH,1) = -1.0
xa9lum(TEV15,NLCH,1) = 575.06688
xa9(0:7,TEV15,NLCH,1)= (/ &
0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
xa9lum(TEV16,NLCH, 1) = -1.0

```

Revision 0.

67b *<Initializations for circes 35b>+≡*

```

xa9lum(GEV090,TESLA,0) = -1.0
xa9lum(GEV170,TESLA,0) = -1.0
xa9lum(GEV350,TESLA,0) = -1.0
xa9lum(GEV500,TESLA,0) = -1.0
xa9lum(GEV800,TESLA,0) = -1.0
xa9lum(TEV1, TESLA,0) = -1.0
xa9lum(TEV12, TESLA,0) = -1.0
xa9lum(TEV15, TESLA,0) = -1.0
xa9lum(TEV16, TESLA,0) = -1.0

```

Uses TESLA 13a.

67c *<Initializations for circes 35b>+≡*

```

xa9lum(GEV090,JLCNLC,0) = -1.0
xa9lum(GEV170,JLCNLC,0) = -1.0
xa9lum(GEV250,JLCNLC,0) = 109.886976
xa9(0:7,GEV250,JLCNLC,0) = (/ &
0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
xa9lum(GEV350,JLCNLC,0) = -1.0

```

```

xa9lum(GEV500,JLCNLC,0) = 220.806144
xa9(0:7,GEV500,JLCNLC,0) = (/ &
    0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
    0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
xa9lum(GEV800,JLCNLC,0) = 304.63488
xa9(0:7,GEV800,JLCNLC,0) = (/ &
    0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
    0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)
xa9lum(TEV1, JLCNLC,0) = 319.95648
xa9(0:7,TEV1, JLCNLC,0) = (/ &
    0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
    0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
xa9lum(TEV12,JLCNLC,0) = 349.90848
xa9(0:7,TEV12,JLCNLC,0) = (/ &
    0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
    0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
xa9lum(TEV15,JLCNLC,0) = 363.15648
xa9(0:7,TEV15,JLCNLC,0) = (/ &
    0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
    0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
xa9lum(TEV16,JLCNLC,0) = -1.0

```

Uses JLCNLC 13a.

68a *⟨Initializations for circes 35b⟩*+≡

```

xa9lum(GEV090,NLCH,0) = -1.0
xa9lum(GEV170,NLCH,0) = -1.0
xa9lum(GEV250,NLCH,0) = -1.0
xa9lum(GEV350,NLCH,0) = -1.0
xa9lum(GEV500,NLCH,0) = 371.4624
xa9(0:7,GEV500,NLCH,0) = (/ &
    0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
    0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
xa9lum(GEV800,NLCH,0) = -1.0
xa9lum(TEV1,NLCH,0) = 516.41856
xa9(0:7,TEV1,NLCH,0) = (/ &
    0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
    0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
xa9lum(TEV12,NLCH,0) = -1.0
xa9lum(TEV15,NLCH,0) = 575.06688
xa9(0:7,TEV15,NLCH,0) = (/ &
    0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
    0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
xa9lum(TEV16,NLCH,0) = -1.0

```

6.2.9 Version 10

68b *⟨Update version 10 derived parameters in circe1 parameters 68b⟩*≡

```

if (circe1_params%rev .eq. 0) then
    r = 0
elseif (circe1_params%rev .ge. 20140305) then

```

```

        r = 1
    elseif (circe1_params%rev .lt. 20140305) then
        call circem ('ERROR', &
            'no revision of version 10 available before 2014/03/05')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 38a>

```

Uses circem 86e.

```

69a <Update version 10 derived parameters in circe1 parameters 68b>+≡
    if (circe1_params%acc .ne. ILC) then
        call circem ('ERROR', 'version 10 applies to ILC only')
        call circem ('ERROR', 'falling back to ILC')
        circe1_params%acc = ILC
    end if
    if (circe1_params%acc .eq. ILC) then
        <Linearly interpolate energies for ILC 2013 69b>
    end if
    <Log energy mapping 39d>

```

Uses ILC 13a and circem 86e.

```

69b <Linearly interpolate energies for ILC 2013 69b>≡
    e = -EINVAL
    elo = -EINVAL
    ehi = -EINVAL
    if (circe1_params%roots .lt. 200d0 - DELTAE) then
        write (msgbuf, 2004) circe1_params%roots, 200d0
        call circem ('MESSAGE', msgbuf)
        e = GEV200
    elseif (abs (circe1_params%roots-200d0) .le. DELTAE) then
        e = GEV200
    elseif (circe1_params%roots .lt. 230d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 200d0, 230d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV200
        ehi = GEV230
        eloval = 200d0
        ehival = 230d0
    elseif (abs (circe1_params%roots-230d0) .le. DELTAE) then
        e = GEV230
    elseif (circe1_params%roots .lt. 250d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 230d0, 250d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV230
        ehi = GEV250
        eloval = 230d0
        ehival = 250d0
    elseif (abs (circe1_params%roots-250d0) .le. DELTAE) then
        e = GEV250
    elseif (circe1_params%roots .lt. 350d0 - DELTAE) then

```

```

write (msgbuf, 2006) circe1_params%roots, 250d0, 350d0
call circem ('MESSAGE', msgbuf)
elo = GEV250
ehi = GEV350
eloval = 250d0
ehival = 350d0
elseif (abs (circe1_params%roots-350d0) .le. DELTAE) then
e = GEV350
elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
write (msgbuf, 2006) circe1_params%roots, 350d0, 500d0
call circem ('MESSAGE', msgbuf)
elo = GEV350
ehi = GEV500
eloval = 350d0
ehival = 500d0
elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
e = GEV500
else
write (msgbuf, 2005) circe1_params%roots, 500d0
call circem ('MESSAGE', msgbuf)
e = GEV500
endif

```

Uses circem 86e.

70a *<Local variables for circes 33b>+≡*

```
integer, parameter :: A1ONEGY = GEV230, A1ONREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

70b *<Update version 10 derived parameters in circe1 parameters 68b>+≡*

```

if (e .ne. EINVAL) then
circe1_params%lumi = xa10lum(e,circe1_params%acc,r)
do i = 0, 7
circe1_params%a1(i) = xa10(i,e,circe1_params%acc,r)
end do
else if (elo .ne. EINVAL .and. ehi .ne. EINVAL) then
circe1_params%lumi = ((circe1_params%roots-eloval)*xa10lum(ehi,circe1_params%acc,r)
+ (ehival-circe1_params%roots)*xa10lum(elo,circe1_params%acc,r)) / (ehival-elo)
do i = 1, 6
circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa10(i,ehi,circe1_params%acc,r)
+ (ehival-circe1_params%roots)*xa10(i,elo,circe1_params%acc,r)) / (ehival-elo)
end do
circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1_params%a1(7))
end if

```

Uses beta 105.

70c *<Local variables for circes 33b>+≡*

```

real, dimension(GEV090:A1ONEGY,ILC:ILC,0:A1ONREV) :: xa10lum
real, dimension(0:7,GEV090:A1ONEGY,ILC:ILC,0:A1ONREV) :: xa10

```

Uses ILC 13a.

Revision 1. The mother of all revisions.

71a *<Initializations for circes 35b>+≡*

```
xa10lum = -1
xa10 = -1
```

71b *<Initializations for circes 35b>+≡*

```
xa10lum(GEV200,ILC,1) = 56
xa10(:,GEV200,ILC,1) = (/ &
    0.66253E+00, 0.51646E+00, 0.43632E+02, -0.64508E+00, &
    0.35915E+00, -0.69716E+00, 0.51645E+02, 0.32097E+00 /)
xa10lum(GEV230,ILC,1) = 83
xa10(:,GEV230,ILC,1) = (/ &
    0.62360E+00, 0.52780E+00, 0.31915E+02, -0.64171E+00, &
    0.38375E+00, -0.69529E+00, 0.39717E+02, 0.36597E+00 /)
xa10lum(GEV250,ILC,1) = 97
xa10(:,GEV250,ILC,1) = (/ &
    0.59996E+00, 0.52141E+00, 0.26647E+02, -0.64331E+00, &
    0.39186E+00, -0.69687E+00, 0.33764E+02, 0.39669E+00 /)
xa10lum(GEV350,ILC,1) = 100
xa10(:,GEV350,ILC,1) = (/ &
    0.58875E+00, 0.50027E+00, 0.18594E+02, -0.63380E+00, &
    0.38659E+00, -0.69239E+00, 0.23964E+02, 0.42049E+00 /)
xa10lum(GEV500,ILC,1) = 180
xa10(:,GEV500,ILC,1) = (/ &
    0.46755E+00, 0.51768E+00, 0.83463E+01, -0.62311E+00, &
    0.45704E+00, -0.69165E+00, 0.12372E+02, 0.60192E+00 /)
```

Uses ILC **13a**.

71c *<Initializations for circes 35b>+≡*

Revision 0 The latest is the default:

71d *<Initializations for circes 35b>+≡*

```
xa10lum(:, :, 0) = xa10lum(:, :, A10NREV)
xa10(:, :, :, 0) = xa10(:, :, :, A10NREV)
```

6.3 Special Functions

71e *<Module subroutines 31b>+≡*

```
function beta (a, b)
    real(kind=double) :: a, b, beta
    beta = exp (dlogam(a) + dlogam(b) - dlogam(a+b))
end function beta
```

Uses beta **105**.

71f *<Module subroutines 31b>+≡*

```
!!! CERNLIB C304
```

```
function dlogam (x)
    real(kind=double) :: dlogam
    real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
```

```

real(kind=double), dimension(5) :: c, xl
real(kind=double) :: x, y, zero, one, two, half, ap, aq
integer :: i
data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
        +5.5584045723515D+1, -2.1513513573726D+2, &
        -2.4587261722292D+2, -5.7500893603041D+1, &
        -2.3359098949513D+0/
data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
        +1.9387784034377D+2, +3.0882954973424D+2, &
        +1.5006839064891D+2, +2.0106851344334D+1, &
        +4.5717420282503D-1/
data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
        +2.1797366058896D+3, +3.7975124011525D+3, &
        -1.9778070769842D+3, -3.6929834005591D+3, &
        -5.6017773537804D+2/
data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
        +1.5612045277929D+3, +7.2340087928948D+3, &
        +1.0459576594059D+4, +4.1699415153200D+3, &
        +2.7678583623804D+2/
data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
        -4.7504594653440D+6, -2.9423445930322D+6, &
        +3.6321804931543D+7, -3.3567782814546D+6, &
        -2.4804369488286D+7/
data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
        -1.5552890280854D+5, -3.4152517108011D+6, &
        -2.0969623255804D+7, -3.4544175093344D+7, &
        -9.1605582863713D+6/
data c / 1.1224921356561D-1, 7.9591692961204D-2, &
        -1.7087794611020D-3, 9.1893853320467D-1, &
        1.3469905627879D+0/
if (x .le. xl(1)) then
    print *, 'ERROR: DLOGAM non positive argument: ', X
    dlogam = zero
end if
if (x .le. xl(2)) then
    y = x + one
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
        ap = p1(i) + y * ap
        aq = q1(i) + y * aq
    end do
    y = - log(x) + x * ap / aq
else if (x .le. xl(3)) then
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
        ap = p1(i) + x * ap

```



```

        aq = q1(i) + x * aq
    end do
    y = (x - one) * ap / aq
else if (x .le. xl(4)) then
    ap = p2(1)
    aq = q2(1)
    do i = 2, 7
        ap = p2(i) + x * ap
        aq = q2(i) + x * aq
    end do
    y = (x-two) * ap / aq
else if (x .le. xl(5)) then
    ap = p3(1)
    aq = q3(1)
    do i = 2, 7
        ap = p3(i) + x * ap
        aq = q3(i) + x * aq
    end do
    y = ap / aq
else
    y = one / x**2
    y = (x-half) * log(x) - x + c(4) + &
        (c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
end if
dlogam = y
end function dlogam

```

6.4 Non-Singular Distributions

73a *(Public subroutines 31a)*+≡

```
public :: kirke
```

Uses kirke **73b**.

73b *(Module subroutines 31b)*+≡

```

function kirke (x1, x2, p1, p2)
    real(kind=double) :: x1, x2
    real(kind=double) :: kirke
    integer :: p1, p2
    (Initialization check 32g)
    kirke = -1.0
    if (abs(p1) .eq. C1_ELECTRON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            kirke = kirkee (x1, x2)
        else if (p2 .eq. C1_PHOTON) then
            kirke = kirkeg (x1, x2)
        end if
    else if (p1 .eq. C1_PHOTON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            kirke = kirkeg (x2, x1)

```

```

        else if (p2 .eq. C1_PHOTON) then
            kirke = kirkgg (x1, x2)
        end if
    endif
end function kirke

```

Defines:

kirke, used in chunk 73a.

Uses C1_ELECTRON 11b, C1_PHOTON 11b, kirkee 74b, kirkeg 76c, and kirkgg 77a.

74a \langle Public subroutines 31a $\rangle + \equiv$
 public :: kirkee

Uses kirkee 74b.

74b \langle Module subroutines 31b $\rangle + \equiv$
 function kirkee (x1, x2)
 real(kind=double) :: x1, x2
 real(kind=double) :: kirkee
 real(kind=double) :: d1, d2
 \langle Initialization check 32g \rangle
 kirkee = -1.0
 if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
 \langle Calculate version 1 of the non-singular e^+e^- distribution 75c \rangle
 \langle else handle invalid versions 37b \rangle
 end function kirkee

Defines:

kirkee, used in chunks 17a and 73–75.

Uses d1 16a and d2 16c.

74c \langle 8-byte aligned part of circe1 parameters 32d $\rangle + \equiv$
 real(kind=double) :: elect0, gamma0

$$\int_{1-\epsilon}^{1^+} dx d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} + a_1^{\alpha\rho} \int_{1-\epsilon}^{1^-} dx x^{a_2^{\alpha\rho}} (1-x)^{a_3^{\alpha\rho}} \quad (18)$$

Approximately

$$\int_{1-\epsilon}^{1^+} dx d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} + a_1^{\alpha\rho} \int_{1-\epsilon}^{1^-} dx (1-x)^{a_3^{\alpha\rho}} = a_0^{\alpha\rho} + a_1^{\alpha\rho} \int_{0^+}^{\epsilon} d\xi \xi^{a_3^{\alpha\rho}} \quad (19)$$

and therefore

$$\int_{1-\epsilon}^{1^+} dx d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} + a_1^{\alpha\rho} \frac{1 - \epsilon^{a_3^{\alpha\rho}+1}}{a_3^{\alpha\rho} + 1} \quad (20)$$

This simple approximation is good enough

74d \langle Update circe1 parameters 33a $\rangle + \equiv$

```

    circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * KIREPS**(circe1
    circe1_params%elect0 = circe1_params%elect0 / KIREPS
    circe1_params%gamma0 = circe1_params%a1(4) * KIREPS**(circe1_params%a1(5)+1) / (ci
    circe1_params%gamma0 = circe1_params%gamma0 / KIREPS

```

but we can also use incomplete Beta functions for the exact result:

75a *⟨Alternative: Update circe1 parameters 75a⟩*≡

```

circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * beta (circe1_pa
    * (1d0 - betinc (circe1_params%a1(2)+1, circe1_params%a1(3)+1, 1d0 -
circe1_params%elect0 = circe1_params%elect0 / KIREPS
circe1_params%gamma0 = circe1_params%a1(7) + circe1_params%a1(4) * beta (circe1_pa
    * betinc (circe1_params%a1(5)+1, circe1_params%a1(6)+1, KIREPS)
circe1_params%gamma0 = circe1_params%gamma0 / KIREPS

```

Uses beta **105**.

75b *⟨Alternative: Local variables for circes 75b⟩*≡

```

real(kind=double) :: betinc
external betinc

```

75c *⟨Calculate version 1 of the non-singular e^+e^- distribution 75c⟩*≡

```

if (x1 .gt. 1d0) then
    d1 = 0d0
elseif (x1 .ge. (1d0 - KIREPS)) then
    d1 = circe1_params%elect0
elseif (x1 .ge. 0d0) then
    d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
else
    d1 = 0d0
endif
if (x2 .gt. 1d0) then
    d2 = 0d0
elseif (x2 .ge. (1d0 - KIREPS)) then
    d2 = circe1_params%elect0
elseif (x2 .ge. 0d0) then
    d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%
else
    d2 = 0d0
endif
kirkee = d1 * d2

```

Uses d1 **16a**, d2 **16c**, and kirkee **74b**.

75d *⟨Calculate version 1 of the non-singular $e^\pm\gamma$ distribution 75d⟩*≡

```

if (x1 .gt. 1d0) then
    d1 = 0d0
elseif (x1 .ge. (1d0 - KIREPS)) then
    d1 = circe1_params%elect0
elseif (x1 .ge. 0d0) then
    d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
else
    d1 = 0d0
endif
if (x2 .gt. 1d0) then
    d2 = 0d0
elseif (x2 .gt. KIREPS) then
    d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
elseif (x2 .ge. 0d0) then

```

```

        d2 = circe1_params%gamma0
    else
        d2 = 0d0
    endif
    kirkeg = d1 * d2

```

Uses d1 16a, d2 16c, and kirkeg 76c.

76a \langle Calculate version 1 of the non-singular $\gamma\gamma$ distribution 76a $\rangle \equiv$

```

    if (x1 .gt. 1d0) then
        d1 = 0d0
    elseif (x1 .gt. KIREPS) then
        d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%
    elseif (x1 .ge. 0d0) then
        d1 = circe1_params%gamma0
    else
        d1 = 0d0
    endif
    if (x2 .gt. 1d0) then
        d2 = 0d0
    elseif (x2 .gt. KIREPS) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
    elseif (x2 .ge. 0d0) then
        d2 = circe1_params%gamma0
    else
        d2 = 0d0
    endif
    kirkgg = d1 * d2

```

Uses d1 16a, d2 16c, and kirkgg 77a.

76b \langle Public subroutines 31a $\rangle + \equiv$

```

    public :: kirkeg

```

Uses kirkeg 76c.

76c \langle Module subroutines 31b $\rangle + \equiv$

```

    function kirkeg (x1, x2)
        real(kind=double) :: x1, x2
        real(kind=double) :: kirkeg
        real(kind=double) :: d1, d2
         $\langle$ Initialization check 32g $\rangle$ 
        kirkeg = -1.0
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
             $\langle$ Calculate version 1 of the non-singular  $e^\pm\gamma$  distribution 75d $\rangle$ 
             $\langle$ else handle invalid versions 37b $\rangle$ 
        end function kirkeg

```

Defines:

kirkeg, used in chunks 73b, 75d, and 76b.

Uses d1 16a and d2 16c.

76d \langle Public subroutines 31a $\rangle + \equiv$

```

    public :: kirkgg

```

Uses kirkgg 77a.

77a *<Module subroutines 31b>+≡*

```

function kirkgg (x1, x2)
  real(kind=double) :: x1, x2
  real(kind=double) :: kirkgg
  real(kind=double) :: d1, d2
  <Initialization check 32g>
  kirkgg = -1.0
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Calculate version 1 of the non-singular  $\gamma\gamma$  distribution 76a>
  <else handle invalid versions 37b>
  end function kirkgg

```

Defines:

kirkgg, used in chunks 73b and 76.

Uses d1 16a and d2 16c.

77b *<Alternative: Subroutines 77b>≡*

```

function betinc (a, b, x)
  real(kind=double) :: x, a, b
  real(kind=double) :: betinc
  real(kind=double) :: bt
  if (x .lt. 0d0 .or. x .gt. 1d0) then
    betinc = 0d0
  else
    if (x .eq. 0d0 .or. x .eq. 1d0) then
      bt = 0d0
    else
      bt = exp(dlogam(a+b)-dlogam(a)-dlogam(b) &
        + a*log(x) + b*log(1d0-x))
    end if
    if (x .lt. (a+1d0)/(a+b+2d0)) then
      betinc = bt*betacf (a, b, x) / a
    else
      betinc = 1d0 - bt*betacf (b, a, 1d0-x) / b
    end if
  end if
end function betinc

```

77c *<Alternative: Subroutines 77b>+≡*

```

function betacf (a, b, x)
  real(kind=double) :: x, a, b
  real(kind=double) :: betacf
  integer, parameter :: itmax = 100
  real(kind=double), parameter = eps = 3d-7
  real(kind=double) :: am, bm, curr, prev, qab, qap, qam, bz, &
    ap, bp, app, bpp, em, tem, d

  integer :: m
  am = 1d0
  bm = 1d0
  curr = 1d0

```

```

qab = a + b
qap = a + 1d0
qam = a - 1d0
bz = 1d0 - qab * x / qap
do m = 1, ITMAX
  em = m
  tem = 2*em
  d = em * (b - m) * x / ((qam + tem) * (a + tem))
  ap = curr + d*am
  bp = bz + d*bm
  d = - (a + em) * (qab + em) * x / ((a + tem) * (qap + tem))
  app = ap + d * curr
  bpp = bp + d * bz
  prev = curr
  am = ap / bpp
  bm = bp / bpp
  curr = app / bpp
  bz = 1d0
  if (abs (curr - prev) .lt. EPS * abs (curr)) then
    betacf = curr
    return
  end if
end do
print *, 'betacf: failed to converge'
betacf = 0d0
end

```

6.5 Generators

6.5.1 Random-Number Generator

The generator routines do not fix or provide a random-number generator. The caller has to provide an implementation which is transferred to the subroutines in one of two possible forms:

1. as a subroutine which generates a single random number, working on an implicit external state
2. as an object with a method the generates a single random number, working on an internal state

These snippets should be used by the procedures that use a RNG:

78a *<RNG dummy arguments 78a>*≡
 rng, rng_obj

78b *<RNG dummy declarations 78b>*≡
 procedure(rng_proc), optional :: rng
 class(rng_type), intent(inout), optional :: rng_obj

Uses rng_proc **79c** and rng_type **79e**.

79a $\langle RNG: generate\ u\ 79a \rangle \equiv$
`call rng_call (u, $\langle RNG\ dummy\ arguments\ 78a \rangle$)`
 Uses `rng_call` **79b**.

79b $\langle Module\ subroutines\ 31b \rangle + \equiv$

```

subroutine rng_call (u,  $\langle RNG\ dummy\ arguments\ 78a \rangle$ )
  real(kind=double), intent(out) :: u
   $\langle RNG\ dummy\ declarations\ 78b \rangle$ 
  if (present (rng)) then
    call rng (u)
  else if (present (rng_obj)) then
    call rng_obj%generate (u)
  else
    call circem ('PANIC', &
      'generator requires either rng or rng_obj argument')
  end if
end subroutine rng_call

```

Defines:
`rng_call`, used in chunk **79a**.
 Uses `circem` **86e**.

This defines the procedure version of the RNG, corresponding to the traditional F77 `external` interface. The abstract interface enables the compiler to check conformance.

79c $\langle Abstract\ interfaces\ 79c \rangle \equiv$

```

abstract interface
  subroutine rng_proc (u)
    import :: double
    real(kind=double), intent(out) :: u
  end subroutine rng_proc
end interface

```

Defines:
`rng_proc`, used in chunks **78b** and **79c**.

Here we define the object version of the RNG. It has to implement a `generate` method which parallels the `rng_proc` procedure above.

79d $\langle Public\ types\ 79d \rangle \equiv$

```

public :: rng_type

```

 Uses `rng_type` **79e**.

79e $\langle Abstract\ types\ 79e \rangle \equiv$

```

type, abstract :: rng_type
contains
  procedure(rng_generate), deferred :: generate
end type rng_type

```

Defines:
`rng_type`, used in chunks **78–80**.
 Uses `rng_generate` **80a**.

80a \langle Abstract interfaces 79c $\rangle + \equiv$

```

abstract interface
  subroutine rng_generate (rng_obj, u)
    import :: rng_type, double
    class(rng_type), intent(inout) :: rng_obj
    real(kind=double), intent(out) :: u
  end subroutine rng_generate
end interface

```

Defines:

`rng_generate`, used in chunk 79e.

Uses `rng_type` 79e.

6.5.2 Version 1

Beta distributions have the practical advantage that they have been popular among mathematicians.[?]

80b \langle Public subroutines 31a $\rangle + \equiv$

```

public :: girce

```

Uses `girce` 80c.

80c \langle Module subroutines 31b $\rangle + \equiv$

```

subroutine girce (x1, x2, p1, p2,  $\langle$ RNG dummy arguments 78a $\rangle$ )
  real(kind=double), intent(out) :: x1, x2
  integer :: p1, p2
   $\langle$ RNG dummy declarations 78b $\rangle$ 
  real(kind=double) :: u, w
   $\langle$ Initialization check 32g $\rangle$ 
   $\langle$ x1m, x2m kludge, part 1 81b $\rangle$ 
   $\langle$ Select particles p1 and p2 81a $\rangle$ 
  if (abs(p1) .eq. C1_ELECTRON) then
    if (abs(p2) .eq. C1_ELECTRON) then
      call gircee (x1, x2,  $\langle$ RNG dummy arguments 78a $\rangle$ )
    else if (p2 .eq. C1_PHOTON) then
      call girceg (x1, x2,  $\langle$ RNG dummy arguments 78a $\rangle$ )
    end if
  else if (p1 .eq. C1_PHOTON) then
    if (abs(p2) .eq. C1_ELECTRON) then
      call girceg (x2, x1,  $\langle$ RNG dummy arguments 78a $\rangle$ )
    else if (p2 .eq. C1_PHOTON) then
      call gircgg (x1, x2,  $\langle$ RNG dummy arguments 78a $\rangle$ )
    end if
  end if
   $\langle$ x1m, x2m kludge, part 2 81c $\rangle$ 
end subroutine girce

```

Defines:

`girce`, used in chunks 80c, 20a, and 80.

Uses `C1_ELECTRON` 11b, `C1_PHOTON` 11b, `gircee` 81e, `girceg` 82c, and `gircgg` 83c.

81a *<Select particles p1 and p2 81a>*≡

```

w = 1d0 / (1d0 + circgg (-1d0, -1d0))
<RNG: generate u 79a>
if (u*u .le. w) then
  p1 = C1_POSITRON
else
  p1 = C1_PHOTON
end if
<RNG: generate u 79a>
if (u*u .le. w) then
  p2 = C1_ELECTRON
else
  p2 = C1_PHOTON
end if

```

Uses C1_ELECTRON 11b, C1_PHOTON 11b, C1_POSITRON 11b, and circgg 43c.

The flavor selection is incorrect, because the relative weights depend on the minimum energy fractions. We resort to a moderately inefficient kludge, because we don't have the distribution functions available yet. We'll have to implement incomplete Beta functions and other horrible things for this. Fortunately, the efficiency can not drop below the relative contribution of e^+e^- .

81b *<x1m, x2m kludge, part 1 81b>*≡
do

Crude rejection:

81c *<x1m, x2m kludge, part 2 81c>*≡

```

if ((x1 .ge. circe1_params%x1m) .and. (x2 .ge. circe1_params%x2m)) exit
end do

```

81d *<Public subroutines 31a>*+≡

```

public :: gircee

```

Uses gircee 81e.

81e *<Module subroutines 31b>*+≡

```

subroutine gircee (x1, x2, <RNG dummy arguments 78a>)
  real(kind=double), intent(out) :: x1, x2
  <RNG dummy declarations 78b>
  real(kind=double) :: u
  <Initialization check 32g>
  x1 = 1
  x2 = 1
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Generate version 1 of the e+e- distribution 82a>
    <else handle invalid versions 37b>
  end if
end subroutine gircee

```

Defines:

gircee, used in chunks 20, 81e, 21d, 80c, and 81d.

For version 1 of the parametrizations we rely on gircee, a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (21)$$

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1} (1 - \xi)^{b-1} \quad (22)$$

```

82a  <Generate version 1 of the  $e^+e^-$  distribution 82a>≡
      <RNG: generate u 79a>
      if (u .le. circe1_params%a1(0)) then
        x1 = 1d0
      else
        x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, &
                           circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                           <RNG dummy arguments 78a>)
      endif
      <RNG: generate u 79a>
      if (u .le. circe1_params%a1(0)) then
        x2 = 1d0
      else
        x2 = 1d0 - girceb (0d0, 1d0-circe1_params%x2m, &
                           circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                           <RNG dummy arguments 78a>)
      endif
      Uses girceb 84b.

82b  <Public subroutines 31a>+≡
      public :: girceg
      Uses girceg 82c.

82c  <Module subroutines 31b>+≡
      subroutine girceg (x1, x2, <RNG dummy arguments 78a>)
        real(kind=double), intent(out) :: x1, x2
        <RNG dummy declarations 78b>
        real(kind=double) :: u
        <Initialization check 32g>
        x1 = 1
        x2 = 1
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
          <Generate version 1 of the  $e^\pm\gamma$  distribution 82d>
          <else handle invalid versions 37b>
        end subroutine girceg

      Defines:
        girceg, used in chunks 20c, 80c, and 82b.

82d  <Generate version 1 of the  $e^\pm\gamma$  distribution 82d>≡
      <RNG: generate u 79a>
      if (u .le. circe1_params%a1(0)) then
        x1 = 1d0
      else
        x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, &
                           circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                           <RNG dummy arguments 78a>)
      endif
      x2 = girceb (circe1_params%x2m, 1d0, &

```

```

circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
<RNG dummy arguments 78a>)

```

Uses girceb 84b.

83a *<Public subroutines 31a>+≡*

```

public :: gircgg

```

Uses gircgg 83c.

83b *<Module subroutines 31b>+≡*

```

subroutine gircgg (x1, x2, <RNG dummy arguments 78a>)
  real(kind=double), intent(out) :: x1, x2
  <RNG dummy declarations 78b>
  <Initialization check 32g>
  x1 = 1
  x2 = 1
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Generate version 1 of the γγ distribution 83c>
  <else handle invalid versions 37b>
  end subroutine gircgg

```

Uses gircgg 83c.

83c *<Generate version 1 of the γγ distribution 83c>≡*

```

x1 = girceb (circe1_params%x1m, 1d0, &
             circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
             <RNG dummy arguments 78a>)
x2 = girceb (circe1_params%x2m, 1d0, &
             circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
             <RNG dummy arguments 78a>)

```

Defines:

gircgg, used in chunks 20c, 80c, and 83.

Uses girceb 84b.

6.5.3 Version 2

Retired.

6.5.4 Version 3 and 4

Identical to version 1.

6.6 Utilities

For version 1 of the parametrizations we need a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (23)$$

with the *incomplete Beta-function* I :

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1}(1-\xi)^{b-1} \quad (24)$$

$$B(a, b) = I(0, a, b) \quad (25)$$

This problem has been studied extensively [?] and we can use an algorithm [18] that is very fast for $0 < a \leq 1 \leq b$, which turns out to be the case in our application.

```

84a <Public subroutines 31a>+=
      public :: girceb
      Uses girceb 84b.
84b <Module subroutines 31b>+=
      function girceb (xmin, xmax, a, b, <RNG dummy arguments 78a>)
        real(kind=double) :: xmin, xmax, a, b
        real(kind=double) :: girceb
        <RNG dummy declarations 78b>
        real(kind=double) :: t, p, u, umin, umax, x, w
        <Check a and b 84c>
        <Set up girceb parameters 84d>
        do
          <Generate a trial x and calculate its weight w 85a>
          <RNG: generate u 79a>
          if (w .gt. u) exit
        end do
        girceb = x
      end function girceb

```

Defines:

girceb, used in chunks 84b, 82–84, and 86b.

In fact, this algorithm works for $0 < a \leq 1 \leq b$ only:

```

84c <Check a and b 84c>=
      if ((a .ge. 1d0) .or. (b .le. 1d0)) then
        girceb = -1d0
        call circem ('ERROR', 'beta-distribution expects a<1<b')
        return
      end if

```

Uses circem 86e and girceb 84b.

The trick is to split the interval $[0, 1]$ into two parts $[0, t]$ and $[t, 1]$. In these intervals we obviously have

$$x^{a-1}(1-x)^{b-1} \leq \begin{cases} x^{a-1} & \text{for } x \leq t \\ t^{a-1}(1-x)^{b-1} & \text{for } x \geq t \end{cases} \quad (26)$$

because we have assumed that $0 < a < 1 < b$. The integrals of the two dominating distributions are t^a/a and $t^{a-1}(1-t)^b/b$ respectively and therefore the probability for picking a random number from the first interval is

$$P(x \leq t) = \frac{bt}{bt + a(1-t)^b} \quad (27)$$

We postpone the discussion of the choice of t until later:

```

84d <Set up girceb parameters 84d>=
      <Set up best value for t 86c>
      p = b*t / (b*t + a * (1d0 - t)**b)

```

The dominating distributions can be generated by simple mappings

$$\phi : [0, 1] \rightarrow [0, 1] \quad (28)$$

$$u \mapsto \begin{cases} t \left(\frac{u}{p} \right)^{\frac{1}{a}} & < t \text{ for } u < p \\ t & = t \text{ for } u = p \\ 1 - (1 - t) \left(\frac{1-u}{1-p} \right)^{\frac{1}{b}} & > t \text{ for } u > p \end{cases} \quad (29)$$

The beauty of the algorithm is that we can use a single uniform deviate u for both intervals:

```
85a <Generate a trial x and calculate its weight w 85a>≡
    <RNG: generate u 79a>
    u = umin + (umax - umin) * u
    if (u .le. p) then
        x = t * (u/p)**(1d0/a)
        w = (1d0 - x)**(b-1d0)
    else
        x = 1d0 - (1d0 - t) * ((1d0 - u)/(1d0 - p))**(1d0/b)
        w = (x/t)**(a-1d0)
    end if
```

The weights that are derived by dividing the distribution by the dominating distributions are already normalized correctly:

$$w : [0, 1] \rightarrow [0, 1] \quad (30)$$

$$x \mapsto \begin{cases} (1 - x)^{b-1} & \in [(1 - t)^{b-1}, 1] \text{ for } x \leq t \\ \left(\frac{x}{t} \right)^{a-1} & \in [t^{1-a}, 1] \text{ for } x \geq t \end{cases} \quad (31)$$

To derive $u_{\min, \max}$ from $x_{\min, \max}$ we can use ϕ^{-1} :

$$\phi^{-1} : [0, 1] \rightarrow [0, 1] \quad (32)$$

$$x \mapsto \begin{cases} p \left(\frac{x}{t} \right)^a & < p \text{ for } x < t \\ p & = p \text{ for } x = t \\ 1 - (1 - p) \left(\frac{1-x}{1-t} \right)^b & > p \text{ for } x > t \end{cases} \quad (33)$$

We start with u_{\min} . For efficiency, we handle the most common cases (small x_{\min}) first:

```
85b <Set up girceb parameters 84d>+≡
    if (xmin .le. 0d0) then
        umin = 0d0
    elseif (xmin .lt. t) then
        umin = p * (xmin/t)**a
    elseif (xmin .eq. t) then
        umin = p
    elseif (xmin .lt. 1d0) then
        umin = 1d0 - (1d0 - p) * ((1d0 - xmin)/(1d0 - t))**b
    else
        umin = 1d0
    endif
```

Same procedure for u_{\max} ; again, handle the most common cases (large x_{\max}) first:

```
86a <Set up girceb parameters 84d>+≡
      if (xmax .ge. 1d0) then
        umax = 1d0
      elseif (xmax .gt. t) then
        umax = 1d0 - (1d0 - p) * ((1d0 - xmax)/(1d0 - t))**b
      elseif (xmax .eq. t) then
        umax = p
      elseif (xmax .gt. 0d0) then
        umax = p * (xmax/t)**a
      else
        umax = 0d0
      endif
```

Check for absurd cases.

```
86b <Set up girceb parameters 84d>+≡
      if (umax .lt. umin) then
        girceb = -1d0
        return
      endif
```

Uses girceb 84b.

It remains to choose the best value for t . The rejection efficiency ϵ of the algorithm is given by the ratio of the dominating distribution and the distribution

$$\frac{1}{\epsilon(t)} = \frac{B(a, b)}{ab} (bt^a + at^{a-1}(1-t)^b). \quad (34)$$

It is maximized for

$$bt - bt(1-t)^{b-1} + (a-1)(1-t)^b = 0 \quad (35)$$

This equation has a solution which can be determined numerically. While this determination is far too expensive compared to a moderate loss in efficiency, we could perform it once after fitting the coefficients a , b . Nevertheless, it has been shown,[\[18\]](#) that

$$t = \frac{1-a}{b+1-a} \quad (36)$$

results in non-vanishing efficiency for all values $1 < a \leq 1 \leq b$. Empirically we have found efficiencies of at least 80% for this choice, which is enough for our needs.

```
86c <Set up best value for t 86c>≡
      t = (1d0 - a) / (b + 1d0 - a)
```

```
86d <Public subroutines 31a>+≡
      public :: circem
```

Uses circem 86e.

```
86e <Module subroutines 31b>+≡
      subroutine circem (errlvl, errmsg)
        character(len=*) :: errlvl, errmsg
        integer, save :: errcnt = 0
```

```

if (errlvl .eq. 'MESSAGE') then
  print *, 'circe1:message: ', errmsg
else if (errlvl .eq. 'WARNING') then
  if (errcnt .lt. 100) then
    errcnt = errcnt + 1
    print *, 'circe1:warning: ', errmsg
  else if (errcnt .eq. 100) then
    errcnt = errcnt + 1
    print *, 'circe1:message: more than 100 messages'
    print *, 'circe1:message: turning warnings off'
  endif
else if (errlvl .eq. 'ERROR') then
  if (errcnt .lt. 200) then
    errcnt = errcnt + 1
    print *, 'circe1:error: ', errmsg
  else if (errcnt .eq. 200) then
    errcnt = errcnt + 1
    print *, 'circe1:message: more than 200 messages'
    print *, 'circe1:message: turning error messages off'
  endif
else if (errlvl .eq. 'PANIC') then
  if (errcnt .lt. 300) then
    errcnt = errcnt + 1
    print *, 'circe1:panic: ', errmsg
  else if (errcnt .eq. 300) then
    errcnt = errcnt + 1
    print *, 'circe1:message: more than 300 messages'
    print *, 'circe1:message: turning panic messages off'
  endif
else
  print *, 'circe1:panic: invalid error code ', errlvl
endif
end subroutine circem

```

Defines:

circem, used in chunks 32-34, 36-39, 50, 51a, 53-57, 61, 63, 65a, 68, 69, 79b, 84c, and 86d.

6.7 Examples

6.7.1 Distributions

```

87 <circe1_plot.f90 87>≡
  program circe1_plot
    use kinds
    use circe1

    implicit none

    real(kind=double) :: xmin, xmax, y, roots

```

```

integer :: xory, nstep, p1, p2, acc, ver, rev, i
real(kind=double) :: x, logx, d
read *, xory, xmin, xmax, nstep, y, p1, p2, roots, acc, ver, rev
call circes (0d0, 0d0, roots, acc, ver, rev, 0)
do i = 0, nstep
  logx = log (xmin) + i * log (xmax/xmin) / nstep
  x = exp (logx)
  d = 0d0
  if (xory .eq. 1) then
    if (p1 .eq. C1_PHOTON) then
      d = circe (x, y, p1, p2)
    else
      d = circe (1d0 - x, y, p1, p2)
    end if
  else if (xory .eq. 2) then
    if (p1 .eq. C1_PHOTON) then
      d = circe (y, x, p1, p2)
    else
      d = circe (y, 1d0 - x, p1, p2)
    end if
  end if
  if (d .gt. 1d-4) print *, x, d
end do
end program circe1_plot

```

Uses C1_PHOTON 11b, circe 31b, and circes 32a.

6.7.2 Library functions

If Fortran77 only had first class functions, then the following cruft would not be necessary. OK, here's the outline of the adaptive Gauss integration routine from CERNLIB:

```

88 <Part one of Gaussian integration 88>≡
  real(kind=double) :: f, a, b, eps
  external f
  real(kind=double), parameter :: Z1 = 1, HF = Z1/2, CST = 5*Z1/1000
  integer :: i
  real(kind=double) :: h, const, aa, bb, c1, c2, s8, s16, u
  <Gaussian weights 91a>
  h = 0
  if (b .eq. a) go to 99
  const = CST / dabs(b-a)
  bb = a
1 continue
  aa = bb
  bb = b
2 continue
  c1 = HF*(bb+aa)
  c2 = HF*(bb-aa)
  s8 = 0

```



```

do i = 1, 4
  u = c2*x(i)

```

Here are now the first two function calls that we have to fill in later in various ways:

```

89a <Function call stub 89a>≡
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))

```

Continuing

```

89b <Part two of Gaussian integration 89b>≡
      end do
      s16 = 0
      do i = 5, 12
        u = c2*x(i)

```

And here are the other two function calls:

```

89c <Function call stub 89a>+≡
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))

```

Terminating:

```

89d <Part three of Gaussian integration 89d>≡
      end do
      s16 = c2*s16
      if (dabs(s16-c2*s8) .le. eps*(1+dabs(s16))) then
        h = h + s16
        if (bb .ne. b) go to 1
      else
        bb = c1
        if (1 + const*dabs(c2) .ne. 1) go to 2
        h = 0
        print *, 'gauss: too high accuracy required'
        go to 99
      end if
      99 continue

```

This one is still reasonably straightforward

$$\text{gauss1} : (f, a, b) \mapsto \int_a^b dx f(x) \quad (37)$$

```

89e <circe1_sample.f90: public 15a>+≡
      public :: gauss1

```

Uses gauss1 89f.

```

89f <circe1_sample.f90: subroutines 15b>+≡
      function gauss1 (f, a, b, eps)
      real(kind=double) :: gauss1
      <Part one of Gaussian integration 88>
        s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
      <Part two of Gaussian integration 89b>
        s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
      <Part three of Gaussian integration 89d>
        gauss1 = h
      end function gauss1

```

Defines:

`gauss1`, used in chunks 89f, 15c, and 89.

But this almost identical repeat

$$\text{gaussx} : (f, a, b) \mapsto \left(y \mapsto \int_a^b dx f(y, x) \right) \quad (38)$$

would not be necessary in a modern programming language with currying:

```
90a <circe1_sample.f90: public 15a>+≡
    public :: gaussx
    Uses gaussx 90b.
90b <circe1_sample.f90: subroutines 15b>+≡
    function gaussx (f, y, a, b, eps)
        real(kind=double) :: y
        real(kind=double) :: gaussx
        <Part one of Gaussian integration 88>
        s8 = s8 + w(i) * (f (y, c1+u) + f (y, c1-u))
        <Part two of Gaussian integration 89b>
        s16 = s16 + w(i) * (f (y, c1+u) + f (y, c1-u))
        <Part three of Gaussian integration 89d>
        gaussx = h
    end function gaussx
```

Defines:

`gaussx`, used in chunk 90.

Fortunately, this is the last one we need

$$\begin{aligned} \text{gauss2} : (f, a, b, a_1, b_1) &\mapsto \int_a^b dx \int_{a_1}^{b_1} dy f(x, y) \\ &= \text{gauss1}(\text{gaussx}(f, a, b), a_1, b_1) \end{aligned} \quad (39)$$

```
90c <circe1_sample.f90: public 15a>+≡
    public :: gauss2
    Uses gauss2 90d.
90d <circe1_sample.f90: subroutines 15b>+≡
    function gauss2 (f, a, b, a1, b1, eps)
        real(kind=double) :: a1, b1
        real(kind=double) :: gauss2
        <Part one of Gaussian integration 88>
        s8 = s8 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &
                        + gaussx (f, c1-u, a1, b1, eps))
        <Part two of Gaussian integration 89b>
        s16 = s16 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &
                        + gaussx (f, c1-u, a1, b1, eps))
        <Part three of Gaussian integration 89d>
        gauss2 = h
    end function gauss2
```

Defines:

`gauss2`, used in chunks [90d](#), [15c](#), [16e](#), and [90](#).

Uses `gaussx` [90b](#).

```
91a  <Gaussian weights 91a>≡
      real(kind=double), dimension(12), parameter :: &
      x = (/ 9.6028985649753623d-1, &
             7.9666647741362674d-1, &
             5.2553240991632899d-1, &
             1.8343464249564980d-1, &
             9.8940093499164993d-1, &
             9.4457502307323258d-1, &
             8.6563120238783174d-1, &
             7.5540440835500303d-1, &
             6.1787624440264375d-1, &
             4.5801677765722739d-1, &
             2.8160355077925891d-1, &
             9.5012509837637440d-2 /), &
      w = (/ 1.0122853629037626d-1, &
             2.2238103445337447d-1, &
             3.1370664587788729d-1, &
             3.6268378337836198d-1, &
             2.7152459411754095d-2, &
             6.2253523938647893d-2, &
             9.5158511682492785d-2, &
             1.2462897125553387d-1, &
             1.4959598881657673d-1, &
             1.6915651939500254d-1, &
             1.8260341504492359d-1, &
             1.8945061045506850d-1 /)
```

6.7.3 Generators

6.8 Dumping Parameters

```
91b  <params.f90 91b>≡
      program params
      use kinds
      use circe1

      implicit none
      integer :: acc, ver, i
      real(kind=double), dimension(7), parameter :: roots = &
        (/ 90D0, 170D0, 350D0, 500D0, 800D0, 1000D0, 1500D0 /)
      do ver = 7, 8
        print *, "VERSION ", ver
        do acc = TESLA, XBNDDEE
          do 12 i = 1, 7
            print *, "====="
```

```

        call circes (0d0, 0d0, roots(i), acc, ver, 20020307, 0)
        call dump ()
    end do
end do
end do
end program params

```

Uses TESLA 13a, XBNDEE 13a, and circes 32a.

```

92 <params.f90 91b>+≡
    subroutine dump
    <Accelerator codes 13a>
    character(len=9) :: name
    select case (acc)
    case (SBAND)
        name = 'SBAND'
    case (TESLA)
        name = 'TESLA'
    case (JLCNLC)
        name = 'JLCNLC'
    case (SBNDEE)
        name = 'SBAND/EE'
    case (TESLEE)
        name = 'TESLA/EE'
    case (XBNDEE)
        name = 'JLCNLC/EE'
    case (ILC)
        name = 'ILC'
    case default
        print *, "Accelerator mode not recognized"
    end select
    write (*, 1000) name, circe1_params%roots
    write (*, 1001) 'e+/e-', circe1_params%lumi
    write (*, 1002) 'e+/e-', circe1_params%a1(0)
    write (*, 1003) 'e+/e-', 1 - circe1_params%a1(0)
    write (*, 1004) 'e+/e-', circe1_params%a1(1), circe1_params%a1(2), circe1_params%a1(3)
    write (*, 1003) 'gamma', circe1_params%a1(7)
    write (*, 1004) 'gamma', circe1_params%a1(4), circe1_params%a1(5), circe1_params%a1(6)
1000 format (A9, ' @ ', F5.0, ' GeV')
1001 format (4X, A7, ' lumi          = ', F7.2, ' * 1032 cm-2 sec-1')
1002 format (4X, A7, ' delta strength = ', F9.5)
1003 format (4X, A7, ' integral(cont.) = ', F9.5)
1004 format (4X, A7, ' distribution   = ', F9.5, ' * x{', F9.5, ' } * (1-x){', F9.5, ' }
    end subroutine dump

```

Uses ILC 13a, JLCNLC 13a, SBAND 13a, SBNDEE 13a, TESLA 13a, TESLEE 13a, and XBNDEE 13a.

7 Fitting

7.1 Version 1: Factorized Beta Distributions

```
93a  <Copyleft notice 29b>+≡
!
! Copyright (C) 1999-2015 by
!   Wolfgang Kilian <kilian@physik.uni-siegen.de>
!   Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
!   Juergen Reuter <juergen.reuter@desy.de>
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! Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! This file has been stripped of most comments.  For documentation, refer
! to the source 'minuit.nw'

93b  <circe1_fit.f90 93b>≡
! circe1_fit.f90 -- fitting for circe
<Copyleft notice 29b>

module fit_routines
  use kinds

  implicit none
  private

  <circe1_fit.f90: public 95a>

  contains
  <circe1_fit.f90: subroutines 95b>
end module fit_routines

program fit
  use kinds
  use fit_routines

  implicit none

  integer :: i, rcode
```

<Declare NPARAM 94a>
<Declare parameters 94b>
<Declare arguments 94c>

```

<Initialize parameters for circe1_fit.f90 94e>
call mninit (5, 6, 7)
<Load parameters 94d>
call mnseti ('CIRCE: fit version 1      ')
argv(1) = 1
call mnexcm (fct, 'SET PRINTOUT        ', argv, 1, rcode, 0d0)
argv(1) = 1
call mnexcm (fct, 'CALL FCT            ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'MIGRAD              ', argv, 0, rcode, 0d0)
call mnexcm (fct, 'MINOS               ', argv, 0, rcode, 0d0)
argv(1) = 3
call mnexcm (fct, 'CALL FCT            ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'STOP                ', argv, 0, rcode, 0d0)

```

end program fit

Defines:
 fit, used in chunks 94d, 111, and 115c.
 Uses circe 31b and fct 95b 111c.

94a *<Declare NPARAM 94a>*≡
 integer, parameter :: NPARAM = 6

Defines:
 NPARAM, used in chunks 94 and 101b.

94b *<Declare parameters 94b>*≡
 integer, dimension(NPARAM) :: pnum
 character(len=10), dimension(NPARAM) :: pname
 real(kind=double), dimension(NPARAM) :: pstart, pstep

Uses NPARAM 94a.

94c *<Declare arguments 94c>*≡
 integer, parameter :: ARGC = 10
 real(kind=double), dimension(ARGC) :: argv

94d *<Load parameters 94d>*≡
 do i = 1, NPARAM
 call mnparm (pnum(i), pname(i), pstart(i), pstep (i), 0d0, 0d0, rcode)
 if (rcode .ne. 0) then
 print *, "fit: MINUIT won't accept parameter ", pnum(i)
 stop
 endif
 end do

Uses NPARAM 94a and fit 93b.

94e *<Initialize parameters for circe1_fit.f90 94e>*≡
 data pnum / 1, 2, 3, 4, 5, 6 /
 data pname / '1_e', 'x_e', '1-x_e', '1_g', 'x_g', '1-x_g' /
 data pstart / -1.00, 20.00, 0.20, -1.00, 0.20, 20.00 /
 data pstep / 0.01, 0.01, 0.01, 0.01, 0.01, 0.01 /

95a `<circe1_fit.f90: public 95a>≡`
`public :: fct`

Uses fct 95b 111c.

95b `<circe1_fit.f90: subroutines 95b>≡`
`subroutine fct (nx, df, f, a, mode, g)`
`integer :: nx, mode`
`real(kind=double) :: f, g`
`real(kind=double), dimension(:) :: df, a`
`<Local variables for fct (v1) 95e>`
`if (mode .eq. 1) then`
`<Read input data (v1) 95c>`
`else if (mode .eq. 2) then`
`<Calculate ∇f 99a>`
`end if`
`<Calculate f (v1) 99b>`
`end if`
`if (mode .eq. 3) then`
`<Write output (v1) 101a>`
`end if`
`end subroutine fct`

Defines:

fct, used in chunks 93b, 95a, 110, 111, and 115f.

95c `<Read input data (v1) 95c>≡`
`<Read data from file 95d>`
`<Fixup errors 97a>`
`<Normalize 97d>`

95d `<Read data from file 95d>≡`
`call gethst ('ee', NDATA, xee, fee, dfec, see, tee, pwr)`
`call gethst ('eg', NDATA, xeg, feg, dfeg, seg, teg, pwr)`
`call gethst ('ge', NDATA, xge, fge, dfge, sge, tge, pwr)`
`call gethst ('gg', NDATA, xgg, fgg, dfgg, sgg, tgg, pwr)`

Uses gethst 96a.

95e `<Local variables for fct (v1) 95e>≡`
`integer, parameter :: NDATA = 20`
`real(kind=double) :: see, tee, dtee`
`real(kind=double) :: seg, teg, dteg`
`real(kind=double) :: sge, tge, dtge`
`real(kind=double) :: sgg, tgg, dtgg`
`real(kind=double), dimension(2,0:NDATA+1,0:NDATA+1) :: xee, xeg, &`
`xge, xgg`
`real(kind=double), dimension(0:NDATA+1,0:NDATA+1) :: fee, dfec, &`
`feg, dfeg, fge, dfge, fgg, dfgg`
`real(kind=double) :: pwr`

95f `<circe1_fit.f90: public 95a>+=`
`public :: gethst`

Uses gethst 96a.

```

96a <circe1_fit.f90: subroutines 95b>+≡
    subroutine gethst (tag, ndata, x, f, df, s, t, pwr)
        character(len=2) :: tag
        integer :: ndata
        real(kind=double) :: s, t, pwr
        real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
        integer :: i, j
        open (10, file = 'lumidiff-'//tag//'.dat')
        read (10, *) pwr
        s = 0d0
        <Read continuum, summing in s 96b>
        t = s
        <Read single  $\delta$ , summing in t 96c>
        <Read double  $\delta$ , summing in t 96e>
        close (10)
    end subroutine gethst

```

Defines:

gethst, used in chunk 95.

```

96b <Read continuum, summing in s 96b>≡
    do i = 1, ndata
        do j = 1, ndata
            read (10, *) x(1,i,j), x(2,i,j), f(i,j), df(i,j)
            s = s + f(i,j)
        end do
    end do

96c <Read single  $\delta$ , summing in t 96c>≡
    do i = 1, ndata
        read (10, *) x(1,i,0), f(i,0), df(i,0), &
            f(i,ndata+1), df(i,ndata+1)
        x(1,i,ndata+1) = x(1,i,0)
        t = t + f(i,0) + f(i,ndata+1)
    end do

96d <Read single  $\delta$ , summing in t 96c>+≡
    do i = 1, ndata
        read (10, *) x(2,0,i), f(0,i), df(0,i), &
            f(ndata+1,i), df(ndata+1,i)
        x(2,ndata+1,i) = x(2,0,i)
        t = t + f(0,i) + f(ndata+1,i)
    end do

96e <Read double  $\delta$ , summing in t 96e>≡
    read (10, *) f(0,0), df(0,0), f(0,ndata+1), df(0,ndata+1)
    t = t + f(0,0) + f(0,ndata+1)
    read (10, *) f(ndata+1,0), df(ndata+1,0), &
        f(ndata+1,ndata+1), df(ndata+1,ndata+1)
    t = t + f(ndata+1,0) + f(ndata+1,ndata+1)

```


Guinea-Pig does not provide the full error. A Monte Carlo study shows that it is a reasonable approximation to rescale the bin error by suitable factors. These factors are different for each distribution and the factors for the δ -pieces are bigger than those for the continuum parts. The follows factors are for the `slow` parameter set.

97a *<Fixup errors 97a>*≡

```
call fixerr (NDATA, dfee, 20d0, 30d0, 40d0)
call fixerr (NDATA, dfeg, 15d0, 20d0, 0d0)
call fixerr (NDATA, dfge, 15d0, 20d0, 0d0)
call fixerr (NDATA, dfgg, 10d0, 0d0, 0d0)
```

Uses `fixerr 97c`.

97b *<circe1_fit.f90: public 95a>*+≡

```
public :: fixerr
```

Uses `fixerr 97c`.

97c *<circe1_fit.f90: subroutines 95b>*+≡

```
subroutine fixerr (ndata, df, c, sd, dd)
  integer :: ndata
  real(kind=double) :: c, sd, dd
  real(kind=double), dimension(0:ndata+1,0:ndata+1) :: df
  integer :: i, j
  do i = 1, NDATA
    do j = 1, NDATA
      df(i,j) = c * df(i,j)
    end do
  end do
  do i = 1, NDATA
    df(0,i) = sd * df(0,i)
    df(i,0) = sd * df(i,0)
    df(ndata+1,i) = sd * df(ndata+1,i)
    df(i,ndata+1) = sd * df(i,ndata+1)
  end do
  df(0,0) = dd * df(0,0)
  df(ndata+1,0) = dd * df(ndata+1,0)
  df(0,ndata+1) = dd * df(0,ndata+1)
  df(ndata+1,ndata+1) = dd * df(ndata+1,ndata+1)
end subroutine fixerr
```

Defines:

`fixerr`, used in chunk **97**.

The error on the integrated luminosity is obtained from adding the error in channels in quadrature.

97d *<Normalize 97d>*≡

```
dtee = sumsqu (NDATA, dfee)
dteg = sumsqu (NDATA, dfeg)
dtge = sumsqu (NDATA, dfge)
dtgg = sumsqu (NDATA, dfgg)
```

Uses `sumsqu 98b`.

98a `<circe1_fit.f90: public 95a>+≡`
`public :: sumsqu`

Uses `sumsqu 98b`.

98b `<circe1_fit.f90: subroutines 95b>+≡`
`function sumsqu (ndata, f)`
`integer :: ndata`
`real(kind=double) :: sumsqu`
`real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f`
`integer :: i, j`
`real(kind=double) :: s2`
`s2 = 0`
`do i = 0, NDATA+1`
`do j = 0, NDATA+1`
`s2 = s2 + f(i,j)*f(i,j)`
`end do`
`end do`
`sumsqu = sqrt (s2)`
`end function sumsqu`

Defines:

`sumsqu`, used in chunks `97d` and `98a`.

98c `<Normalize 97d>+≡`
`call scale (NDATA, 1d0/tee, fee)`
`call scale (NDATA, 1d0/tee, dfec)`
`call scale (NDATA, 1d0/tee, feg)`
`call scale (NDATA, 1d0/tee, dfeg)`
`call scale (NDATA, 1d0/tee, fge)`
`call scale (NDATA, 1d0/tee, dfge)`
`call scale (NDATA, 1d0/tee, fgg)`
`call scale (NDATA, 1d0/tee, dfgg)`

Uses `scale 98e`.

98d `<circe1_fit.f90: public 95a>+≡`
`public :: scale`

Uses `scale 98e`.

98e `<circe1_fit.f90: subroutines 95b>+≡`
`subroutine scale (ndata, s, f)`
`integer :: ndata`
`real(kind=double) :: s`
`real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f`
`integer :: i, j`
`do i = 0, NDATA+1`
`do j = 0, NDATA+1`
`f(i,j) = s * f(i,j)`
`end do`
`end do`
`end subroutine scale`

Defines:

`scale`, used in chunk `98`.

```

99a <Calculate  $\nabla f$  99a>≡
      print *, "ERROR:  $\nabla f$  n.a."
      stop

```

Log-likelihood won't fly, because we can't normalize the likelihood function for an unbounded parameter range. Let's use good ole least-squares instead.

```

99b <Calculate  $f$  (v1) 99b>≡
      f = 0d0
      do i = 1, NDATA
        do j = 1, NDATA
          if (dfee(i,j) .gt. 0d0) then
            f = f + ((phie(xee(1,i,j),a) * phie(xee(2,i,j),a) &
              - fee(i,j)) / dfec(i,j))**2
          end if
          if (dfeg(i,j) .gt. 0d0) then
            f = f + ((phie(xeg(1,i,j),a) * phig(xeg(2,i,j),a) &
              - feg(i,j)) / dfeg(i,j))**2
          end if
          if (dfge(i,j) .gt. 0d0) then
            f = f + ((phig(xge(1,i,j),a) * phie(xge(2,i,j),a) &
              - fge(i,j)) / dfge(i,j))**2
          end if
          if (dfgg(i,j) .gt. 0d0) then
            f = f + ((phig(xgg(1,i,j),a) * phig(xgg(2,i,j),a) &
              - fgg(i,j)) / dfgg(i,j))**2
          end if
        end do
      end do

```

Uses phie 100c and phig 100e.

```

99c <Local variables for fct (v1) 95e>+≡
      integer :: i, j
      real(kind=double) :: delta

```

```

99d <Calculate  $f$  (v1) 99b>+≡
      if ((a(2) .le. -1d0) .or. (a(3) .le. -1d0/pwr)) then
        print *, "warning: discarding out-of-range a2/3: ", a(2), a(3)
        <Give up on f 99e>
      else
        delta = 1d0 - exp(a(1)) * beta(a(2)+1d0,a(3)+1d0/pwr) * dble(NDATA) / pwr
        if (delta .lt. 0d0) then
          print *, "warning: delta forced to 0 from ", delta
          delta = 0d0
        end if

```

Uses beta 105.

```

99e <Give up on f 99e>≡
      f = 1d100

```

```

99f <Calculate  $f$  (v1) 99b>+≡
      do i = 1, NDATA
        if (dfec(ndata+1,i) .gt. 0d0) then

```

```

        f = f + ((delta*phie(xee(2,ndata+1,i),a) &
                - fee(ndata+1,i)) / dfee(ndata+1,i))**2
    end if
    if (dfeg(ndata+1,i) .gt. 0d0) then
        f = f + ((delta*phig(xeg(2,ndata+1,i),a) &
                - feg(ndata+1,i)) / dfeg(ndata+1,i))**2
    end if
    if (dfee(i,ndata+1) .gt. 0d0) then
        f = f + ((delta*phie(xee(1,i,ndata+1),a) &
                - fee(i,ndata+1)) / dfee(i,ndata+1))**2
    end if
    if (dfge(i,ndata+1) .gt. 0d0) then
        f = f + ((delta*phig(xge(1,i,ndata+1),a) &
                - fge(i,ndata+1)) / dfge(i,ndata+1))**2
    end if
end do
Uses phie 100c and phig 100e.
100a <Calculate  $f(v1)$  99b>+≡
    if (dfee(ndata+1,ndata+1) .gt. 0d0) then
        f = f + ((delta*delta &
                - fee(ndata+1,ndata+1)) / dfee(ndata+1,ndata+1))**2
    end if
100b <circe1_fit.f90: public 95a>+≡
    public :: phie
Uses phie 100c.
100c <circe1_fit.f90: subroutines 95b>+≡
    function phie (x, a)
    real(kind=double) :: x, phie
    real(kind=double), dimension(6) :: a
    phie = exp (a(1) + a(2)*log(x) + a(3)*log(1d0-x))
end function phie

Defines:
    phie, used in chunks 99, 100b, and 103a.
100d <circe1_fit.f90: public 95a>+≡
    public :: phig
Uses phig 100e.
100e <circe1_fit.f90: subroutines 95b>+≡
    function phig (x, a)
    real(kind=double) :: x, phig
    real(kind=double), dimension(6) :: a
    phig = exp (a(4) + a(5)*log(x) + a(6)*log(1d0-x))
end function phig

Defines:
    phig, used in chunks 99, 100d, and 103a.

```

```

101a <Write output (v1) 101a>≡
      a1(1) = exp(a(1)) * dble(NDATA) / pwr
      a1(2) = a(2)
      a1(3) = a(3) - 1d0 + 1d0/pwr
      a1(4) = exp(a(4)) * dble(NDATA) / pwr
      a1(5) = a(5) - 1d0 + 1d0/pwr
      a1(6) = a(6)
      open (10, file = 'Parameters')
      write (10, 1000) REV, tee / 1D32
      write (10, 1001) REV, &
          1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0), &
          a1(1), a1(2), a1(3), a1(4), a1(5), a1(6), &
          a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
1000 format ( '      data xa5lum(@ENERGY@,@ACC@,', I2, ') / ', E12.5, ' /')
1001 format ( '      data (xa5(i,@ENERGY@,@ACC@,', I2, '),i=0,7) /', /, &
          '      $ ', 4(E12.5,', ', '), /, &
          '      $ ', 3(E12.5,', ', '), E12.5, ' /')
      close (10)

```

Uses beta 105.

```

101b <Local variables for fct (v1) 95e>+≡
      <Declare NPARAM 94a>
      real(kind=double), dimension(NPARAM) :: a1
      integer, parameter :: REV = 1

```

Uses NPARAM 94a.

The average elektron energy in the continuum can be calculated analytically:

$$\begin{aligned}
 \langle E_{e^\pm} \rangle_{\text{cont}} &= E_{\text{beam}} \langle x_{e^\pm} \rangle_{\text{cont}} = E_{\text{beam}} \frac{\int dx x^{a_2} (1-x)^{a_3} x}{B(a_2, a_3)} \\
 &= E_{\text{beam}} \frac{B(a_2 + 1, a_3)}{B(a_2, a_3)} = E_{\text{beam}} \frac{a_2 + 1}{a_2 + a_3 + 2} \quad (40)
 \end{aligned}$$

```

101c <Write output (v1) 101a>+≡
      delta = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
      print *, '< x_e > = ', delta + (1d0-delta)*(a1(2)+1d0)/(a1(2)+a1(3)+2d0)

```

Uses beta 105.

similarly:

$$\langle E_\gamma \rangle = E_{\text{beam}} \frac{a_5 + 1}{a_5 + a_6 + 2} \quad (41)$$

```

101d <Write output (v1) 101a>+≡
      print *, '< x_g > = ', (a1(5)+1d0)/(a1(5)+a1(6)+2d0)

```

Count the degrees of freedom in ndof:

```

101e <Write output (v1) 101a>+≡
      ndof = 0
      do i = 0, ndata+1
        do j = 0, ndata+1
          if (dfec(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfeg(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfge(i,j) .gt. 0d0) ndof = ndof + 1

```

```

        if (dfgg(i,j) .gt. 0d0) ndof = ndof + 1
    end do
end do
print *, 'CHI2 = ', f / ndof
102a <Local variables for fct (v1) 95e>+≡
    integer :: ndof

```

The error on the luminosity is just the (possibly rescaled) counting error:

```

102b <Write output (v1) 101a>+≡
    open (10, file = 'Errors.tex')
    write (10, 1099) tee / 1d32, dtee / 1d32, dtee / 1d32
    1099 format ('$ ', F8.2, '_{-', F4.2, '}^{+', F4.2, '} $')

```

After retrieving the error from MINUIT, we have to take care of the mapping of the parameters

$$a'_{1/4} = e^{a_{1/4}} B(a_{2/5} + 1, a_{3/6} + 1) N_{\text{bins}} \eta^{-1} \implies \delta a'_{1/4} = a'_{1/4} \delta a_{1/4} \quad (42)$$

ignoring the errors in the integral (i.e. the Beta function).

```

102c <Write output (v1) 101a>+≡
    call mnerrs (1, eplus, eminus, epara, corr)
    ab = a1(1) * beta(a1(2)+1d0, a1(3)+1d0)
    write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
    1100 format ('$ ', F8.4, '_{-', F6.4, '}^{+', F6.4, '} $')
    Uses beta 105.
102d <Local variables for fct (v1) 95e>+≡
    real(kind=double) :: ab

```

The other mappings are even more trivial:

$$a'_{2/6} = a_{2/6} - 1 + \eta^{-1} \implies \delta a'_{2/6} = \delta a_{2/6} \quad a'_{3/5} = a_{3/5} - 1 + \eta^{-1} \implies \delta a'_{3/5} = \delta a_{3/5} \quad (43)$$

```

102e <Write output (v1) 101a>+≡
    do i = 2, 3
        call mnerrs (i, eplus, eminus, epara, corr)
        write (10, 1100) a1(i), abs (eminus), abs (eplus)
    end do
    call mnerrs (4, eplus, eminus, epara, corr)
    ab = a1(4) * beta(a1(5)+1d0, a1(6)+1d0)
    write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
    do i = 5, 6
        call mnerrs (i, eplus, eminus, epara, corr)
        write (10, 1100) a1(i), abs (eminus), abs (eplus)
    end do
    close (10)
    Uses beta 105.
102f <Local variables for fct (v1) 95e>+≡
    real(kind=double) :: eplus, eminus, epara, corr
    integer :: n

```

```

103a <Write output (v1) 101a>+≡
      do n = 1, 10
        call pslice ('ee','x',n,NDATA,xee,fee,dfeg,phie,phie,a)
        call pslice ('eg','x',n,NDATA,xeg,feg,dfeg,phie,phig,a)
        call pslice ('ge','x',n,NDATA,xge,fge,dfeg,phig,phie,a)
        call pslice ('gg','x',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
        call pslice ('ee','y',n,NDATA,xee,fee,dfeg,phie,phie,a)
        call pslice ('eg','y',n,NDATA,xeg,feg,dfeg,phie,phig,a)
        call pslice ('ge','y',n,NDATA,xge,fge,dfeg,phig,phie,a)
        call pslice ('gg','y',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
      end do
      call pslice ('ee','x',21,NDATA,xee,fee,dfeg,phie,phie,a)
      call pslice ('eg','x',21,NDATA,xeg,feg,dfeg,phie,phig,a)
      call pslice ('ee','y',21,NDATA,xee,fee,dfeg,phie,phie,a)
      call pslice ('ge','y',21,NDATA,xge,fge,dfeg,phig,phie,a)

```

Uses phie 100c, phig 100e, and pslice 103d.

UNIX Fortran compiler want backslashes escaped:

```

103b <Write output (v1) 101a>+≡
      open (10, file = 'Slices.mp4')
      write (10,*) "picture eslice[], gslice[];"
      do n = 1, NDATA
        write (10,*) 'eslice[', n, ']' := ', &
          'btex $x_{e^{\backslash pm}} = ', xee(1,n,1), '$ etex;'
        write (10,*) 'gslice[', n, ']' := ', &
          'btex $x_{\backslash gamma} = ', xgg(1,n,1), '$ etex;'
      end do
      close (10)

```

```

103c <circe1_fit.f90: public 95a>+≡
      public :: pslice

```

Uses pslice 103d.

```

103d <circe1_fit.f90: subroutines 95b>+≡
      subroutine pslice (pp, xy, n, ndata, x, f, df, phi1, phi2, a)
        character(len=2) :: pp
        character(len=1) :: xy
        integer :: n, ndata
        real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
        real(kind=double), dimension(6) :: a
        real(kind=double) :: z
        real(kind=double) :: phi1, phi2, d, delta, pwr
        external phi1, phi2
        integer :: i
        character(len=2) digits
        write (digits, '(I2.2)') n
        open (10, file = 'lumidiff-'//pp//xy//digits//'.dat')
        open (11, file = 'lumidiff-'//pp//xy//digits//'.fit')
        open (12, file = 'lumidiff-'//pp//xy//digits//'.chi')
        if (n .eq. ndata+1) then

```

```

        pwr = 5d0
        delta = 1d0 - exp(a(1))*beta(a(2)+1d0,a(3)+1d0/pwr) &
            * dble(NDATA) / pwr
    else
        delta = 0
    end if
    if (xy .eq. 'x') then
        do i = 1, ndata
            if (df(n,i) .gt. 0d0) then
                if (pp(2:2) .eq. 'g') then
                    z = x(2,n,i)
                else
                    z = 1d0 - x(2,n,i)
                endif
            endif
            if (n .eq. ndata+1) then
                d = delta*phi2(x(2,n,i),a)
            else
                d = phi1(x(1,n,i),a)*phi2(x(2,n,i),a)
            endif
            write (10,*) z, f(n,i), df(n,i)
            write (11,*) z, d
            write (12,*) z, (f(n,i) - d) / df(n,i)
        endif
    end do
    else if (xy .eq. 'y') then
        do i = 1, ndata
            if (df(i,n) .gt. 0d0) then
                if (pp(1:1) .eq. 'g') then
                    z = x(1,i,n)
                else
                    z = 1d0 - x(1,i,n)
                endif
            endif
            if (n .eq. ndata+1) then
                d = phi1(x(1,i,n),a)*delta
            else
                d = phi1(x(1,i,n),a)*phi2(x(2,i,n),a)
            endif
            write (10,*) z, f(i,n), df(i,n)
            write (11,*) z, d
            write (12,*) z, (f(i,n) - d) / df(i,n)
        endif
    end do
endif
close (10)
close (11)
close (12)
end subroutine pslice

```

Defines:
 pslice, used in chunk 103.

Uses beta 105.

```
105 (circe1_fit.f90: subroutines 95b)+≡
function beta (a, b)
  real(kind=double) :: a, b, beta
  beta = exp (dlgama(a) + dlgamma(b) - dlgamma(a+b))
contains
  function dlgamma (x)
    real(kind=double) :: dlgamma
    real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
    real(kind=double), dimension(5) :: c, xl
    real(kind=double) :: x, y, zero, one, two, half, ap, aq
    integer :: i
    data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
    data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
    data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
      +5.5584045723515D+1, -2.1513513573726D+2, &
      -2.4587261722292D+2, -5.7500893603041D+1, &
      -2.3359098949513D+0/
    data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
      +1.9387784034377D+2, +3.0882954973424D+2, &
      +1.5006839064891D+2, +2.0106851344334D+1, &
      +4.5717420282503D-1/
    data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
      +2.1797366058896D+3, +3.7975124011525D+3, &
      -1.9778070769842D+3, -3.6929834005591D+3, &
      -5.6017773537804D+2/
    data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
      +1.5612045277929D+3, +7.2340087928948D+3, &
      +1.0459576594059D+4, +4.1699415153200D+3, &
      +2.7678583623804D+2/
    data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
      -4.7504594653440D+6, -2.9423445930322D+6, &
      +3.6321804931543D+7, -3.3567782814546D+6, &
      -2.4804369488286D+7/
    data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
      -1.5552890280854D+5, -3.4152517108011D+6, &
      -2.0969623255804D+7, -3.4544175093344D+7, &
      -9.1605582863713D+6/
    data c / 1.1224921356561D-1, 7.9591692961204D-2, &
      -1.7087794611020D-3, 9.1893853320467D-1, &
      1.3469905627879D+0/
    if (x .le. xl(1)) then
      print *, 'ERROR: DLGAMA non positive argument: ', x
      dlgamma = zero
    end if
    if (x .le. xl(2)) then
      y = x + one
      ap = p1(1)
      aq = q1(1)
```

```

do i = 2, 7
    ap = p1(i) + y * ap
    aq = q1(i) + y * aq
end do
y = - log(x) + x * ap / aq
else if (x .le. xl(3)) then
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
        ap = p1(i) + x * ap
        aq = q1(i) + x * aq
    end do
    y = (x - one) * ap / aq
else if (x .le. xl(4)) then
    ap = p2(1)
    aq = q2(1)
    do i = 2, 7
        ap = p2(i) + x * ap
        aq = q2(i) + x * aq
    end do
    y = (x-two) * ap / aq
else if (x .le. xl(5)) then
    ap = p3(1)
    aq = q3(1)
    do i = 2, 7
        ap = p3(i) + x * ap
        aq = q3(i) + x * aq
    end do
    y = ap / aq
else
    y = one / x**2
    y = (x-half) * log(x) - x + c(4) + &
        (c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
end if
dlgama = y
end function dlgamma
end function beta

```

Defines:

beta, used in chunks 59b, 61d, 65c, 70b, 71e, 75a, 99d, 101–103, and 108b.

```

106 <circe1_fit.sh 106>≡
    #! /bin/sh
    # mode=${2-slow}
    mode=${2-fast}
    root='pwd'
    indir=${root}/${3-input}
    tmpdir=${root}/tmp
    outdir=${root}/output
    acc="${1-sband350 sband500 sband800 sband1000 sband1600}

```

```

tesla350 tesla500 tesla800 tesla1000 tesla1600
tesla350-low tesla500-low tesla800-low tesla1000-low tesla1600-low
xband350 xband500 xband800 xband1000 xband1600}"

107a <circe1_fit.sh 106>+=
  xmkdir () {
    for d in "$@"; do
      mkdir $d 2>/dev/null || true
    done
  }
  rm -fr ${tmpdir}
  xmkdir ${outdir} ${tmpdir}

107b <circe1_fit.sh 106>+=
  cd ${tmpdir}
  cat /dev/null >${outdir}/Params.f90
  for a in $acc; do
    case "$a" in
      *1600*) energy=TEV16;;
      *1000*) energy=TEV1;;
      *800*) energy=GEV800;;
      *500*) energy=GEV500;;
      *3[56]0*) energy=GEV350;;
      *170*) energy=GEV170;;
      *90*) energy=GEV090;;
      *) energy=GEV500;;
    esac
    cp ${indir}/${a}_${mode}/lumidiff-???.dat .
    ${root}/circe1_fit.bin
    rm -fr ${outdir}/${a}_${mode}
    mkdir ${outdir}/${a}_${mode}
    cp Slices.mp4 ${outdir}
    cp Errors.tex lumidiff-??x[0-9][0-9].??? ${outdir}/${a}_${mode}
    sed -e "s/@ENERGY@/$energy/g" \
        -e "s/@ACC@/'echo $a | tr a-z A-Z | tr -cd A-Z'/g" Parameters \
        >>${outdir}/Params.f90
  done
  cd ${root}
  rm -fr ${tmpdir}

107c <circe1_fit.sh 106>+=
  cat >${outdir}/Params.tex <<'END'
  \begin{table}
  \begin{center}
  \renewcommand{\arraystretch}{1.3}
  \begin{tabular}{|c|c|c|c|c|}\hline
    & \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND} \\
    \\\hline\hline
  \end{tabular}
  \end{center}
  END

```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

```

108a <circe1_fit.sh 106>+=
line () {
    for a in $acc; do
        case $a in
            *350* | *800* | *1000* | *1600*)
                ;;
            *) echo -n ' & '
                sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                ;;
        esac
    done
    echo '\\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
echo '$\int d_e\text{pm}$'; line 2
echo '$x_{e\text{pm}}^{\alpha}$'; line 3
echo '$(1-x_{e\text{pm}})^{\alpha}$'; line 4
echo '$\int d_{\gamma}$'; line 5
echo '$x_{\gamma}^{\alpha}$'; line 6
echo '$(1-x_{\gamma})^{\alpha}$'; line 7
) >>${outdir}/Params.tex

108b <circe1_fit.sh 106>+=
cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
    \end{center}
    \caption{\label{tab:param}%
        Version 1, revision 1997 04 16 of the beam spectra at 500 GeV.
        The rows correspond to the luminosity per effective year, the
        integral over the continuum and the powers in the factorized Beta
        distributions~(\ref{eq:beta}).}
    \end{table}
END
Uses beta 105.

108c <circe1_fit.sh 106>+=
cat >>${outdir}/Params.tex <<'END'
\begin{table}
\begin{center}
\renewcommand{\arraystretch}{1.3}
\begin{tabular}{|c|c|c|c|c|}\hline
& \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND} \\
\\ \hline \hline
\end{tabular}
\end{center}
END
Uses SBAND 13a, TESLA 13a, and XBAND 13a.

108d <circe1_fit.sh 106>+=
line () {
    for a in $acc; do
        case $a in
            *1000*)

```

```

        echo -n ' & '
        sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
        ;;
    esac
done
echo '\\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\text{ }\epsilon^{-1}$'; line 1
echo '$\int d_e\text{ }\text{ }\text{ }$'; line 2
echo '$x_{e\text{ }\text{ }\text{ }}^{\alpha}$'; line 3
echo '$(1-x_{e\text{ }\text{ }\text{ }})^{\alpha}$'; line 4
echo '$\int d_{\gamma}$'; line 5
echo '$x_{\gamma}^{\alpha}$'; line 6
echo '$(1-x_{\gamma})^{\alpha}$'; line 7
) >>${outdir}/Params.tex

109a <circe1_fit.sh 106>+≡
cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
    \end{center}
    \caption{\label{tab:param/TeV}%
        Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.}
\end{table}
END

109b <circe1_fit.sh 106>+≡
cat >>${outdir}/Params.tex <<'END'
\begin{table}
    \begin{center}
        \renewcommand{\arraystretch}{1.3}
        \begin{tabular}{|c||c|c|c|c|}\hline
            & 350 GeV & 500 GeV & 800 GeV & 1600 GeV \\
            \\\hline\hline
        \end{tabular}
    \end{center}
END

109c <circe1_fit.sh 106>+≡
line () {
    for a in $acc; do
        case $a in
            tesla*-low)
                ;;
            tesla1000)
                ;;
            tesla*)
                echo -n ' & '
                sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                ;;
        esac
    done
    echo '\\\\hline'
}

```

```

(echo '$\mathcal{L}/\text{fb}^{-1}\text{upilon}^{-1}$'; line 1
echo '$\int d_{\text{e}^{\text{pm}}}$'; line 2
echo '$x_{\text{e}^{\text{pm}}}^{\alpha}$'; line 3
echo '$(1-x_{\text{e}^{\text{pm}}})^{\alpha}$'; line 4
echo '$\int d_{\text{gamma}}$'; line 5
echo '$x_{\text{gamma}}^{\alpha}$'; line 6
echo '$(1-x_{\text{gamma}})^{\alpha}$'; line 7
) >>${outdir}/Params.tex

110a <circe1_fit.sh 106>+=
cat >>${outdir}/Params.tex <<'END'
\end{tabular}
\end{center}
\caption{\label{tab:param/Tesla}%
Version 1, revision 1997 04 17 of the beam spectra for TESLA.}
\end{table}
END
exit 0
Uses TESLA 13a.

```

7.2 Experimental

7.2.1 Quasi One Dimensional

```

110b <circe1_minuit1.f90 110b>≡
! circe1_minuit1.f90 -- fitting for circe
<Copley notice 29b>
Uses circe 31b.

```

We're utilizing the familiar “MINUIT” package [15].

```

110c <circe1_minuit1.f90 110b>+=
<Minuit1 module 110d>
<Minuit1 main program 111a>

```

```

110d <Minuit1 module 110d>≡
module minuit1
use kinds

implicit none

public :: fct
public :: phi

contains

<Function to minimize 111c>

<Function phi1 112d>
end module minuit1

```

Defines:

minuit1, used in chunk 111a.

Uses fct 95b 111c and phi 112d 116.

111a *<Minuit1 main program 111a>*≡

```
program fit
  use kinds
  use minuit1

  implicit none

  call minuit (fct, 0d0)
end program fit
```

Uses fct 95b 111c, fit 93b, and minuit1 110d.

111b *<Minuit2 main program 111b>*≡

```
program fit
  use kinds
  use minuit2

  implicit none

  call minuit (fct, 0d0)
end program fit
```

Uses fct 95b 111c, fit 93b, and minuit2 115f.

111c *<Function to minimize 111c>*≡

```
subroutine fct (nx, df, f, a, mode, g)
  integer, intent(in) :: nx, mode
  real(kind=double) :: f, g
  real(kind=double), dimension(:) :: df, a
  <Local variables for fct 112a>
  if (mode .eq. 1) then
    <Read input data 111d>
  else if (mode .eq. 2) then
    <Calculate  $\nabla f$  99a>
  end if
  <Calculate f 112b>
  if (mode .eq. 3) then
    <Write output 112c>
  end if
end subroutine fct
```

Defines:

fct, used in chunks 93b, 95a, 110, 111, and 115f.

111d *<Read input data 111d>*≡

```
open (10, file = 'minuit.data')
do i = 1, NDATA
  do j = 1, NDATA
    read (10, *) xi(1,i,j), xi(2,i,j), fi(i,j), dfi(i,j)
    fi(i,j) = fi(i,j)/1d30
```

```

        dfi(i,j) = dfi(i,j)/1d30
    end do
end do
close (10)

112a <Local variables for fct 112a>≡
integer, parameter :: NDATA = 20
real(kind=double) :: chi, chi2
real(kind=double), dimension(2,NDATA,NDATA) :: xi
real(kind=double), dimension(NDATA,NDATA) :: fi, dfi
integer :: i, j, n

112b <Calculate f 112b>≡
f = 0d0
do i = 1, NDATA
    do j = 1, NDATA
        if (dfi(i,j).gt.0d0) then
            f = f + ((phi(xi(1,i,j),xi(2,i,j),a) &
                - fi(i,j)) / dfi(i,j))**2
        end if
    end do
end do
Uses phi 112d 116.

112c <Write output 112c>≡
chi2 = 0d0
n = 0
open (10, file = 'minuit.fit')
do i = 1, NDATA
    do j = 1, NDATA
        if (dfi(i,j).gt.0d0) then
            chi = (phi(xi(1,i,j),xi(2,i,j),a)-fi(i,j))/dfi(i,j)
            write (10,*) xi(1,i,j), xi(2,i,j), &
                1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
                1d30 * fi(i,j), &
                chi
            chi2 = chi2 + chi**2
            n = n + 1
        else
            write (10,*) xi(1,i,j), xi(2,i,j), &
                1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
                1d30 * fi(i,j)
        end if
    end do
end do
close (10)
print *, 'CHI2 = ', chi2/n
Uses phi 112d 116.

112d <Function phi1 112d>≡
function phi (e1, e2, a)
real(kind=double) :: e1, e2

```



```

real(kind=double), dimension(17) :: a
real(kind=double) :: phi
real(kind=double) :: y1, y2
y1 = e1 / 250d0
y2 = e2 / 250d0
phi = exp (
    + a( 1) * 1d0
    + a( 2) * log(y1)
    + a( 3) * log(1d0-y1)
    + a( 4) * log(-log(y1))
    + a( 5) * log(-log(1d0-y1))
    + a( 6) * y1
    + a( 7) * log(y1)**2
    + a( 8) * log(1d0-y1)**2
    + a( 9) * log(-log(y1))**2
    + a(10) * log(-log(1d0-y1))**2
    + a(11) * y1**2
    + a(12) / log(y1)
    + a(13) / log(1d0-y1)
    + a(14) / log(-log(y1))
    + a(15) / log(-log(1d0-y1))
    + a(16) / y1
    + a(17) / (1d0-y1)
    + a( 2) * log(y2)
    + a( 3) * log(1d0-y2)
    + a( 4) * log(-log(y2))
    + a( 5) * log(-log(1d0-y2))
    + a( 6) * y2
    + a( 7) * log(y2)**2
    + a( 8) * log(1d0-y2)**2
    + a( 9) * log(-log(y2))**2
    + a(10) * log(-log(1d0-y2))**2
    + a(11) * y2**2
    + a(12) / log(y2)
    + a(13) / log(1d0-y2)
    + a(14) / log(-log(y2))
    + a(15) / log(-log(1d0-y2))
    + a(16) / y2
    + a(17) / (1d0-y2)
)
end function phi

```

Defines:

phi, used in chunks 110d, 112, and 115f.

```

113 <circe1_minuit1.sh 113>≡
    #! /bin/sh
    minuit_bin='pwd'/circe1_minuit1.bin
    <Process arguments 114a>
    (

```

```

    <Define parameters 114d>
    <Fix parameters 114e>
    <Fix strategy 115a>
    <Run Minuit 115b>
) | eval "$minuit_bin $filter"
<Maybe plot results 115c>
exit 0

114a <Process arguments 114a>≡
    tmp="$IFS"
    IFS=:
    args=":$*:"
    IFS="$tmp"

114b <Process arguments 114a>+≡
    filter="| \
        awk '/STATUS=(CONVERGED|CALL LIMIT|FAILED)/ { p=1; print }; \
            /@.* \.00000 *fixed/ { next }; \
            /EDM=|CHI2|@/ && p { print }' "

114c <Process arguments 114a>+≡
    case "$args" in
        *:v:*) filter=;;
    esac

114d <Define parameters 114d>≡
    cat <<END
    set title
    CIRCE
    parameters
    1 '@ 1          ' 0.00 0.01
    2 '@ lx         ' 0.20 0.01
    3 '@ l(1-x)     ' 0.20 0.01
    4 '@ llx        ' 0.00 0.01
    5 '@ ll(1-x)    ' 0.00 0.01
    6 '@ x          ' 0.00 0.01
    7 '@ lx^2       ' 0.00 0.01
    8 '@ l(1-x)^2   ' 0.00 0.01
    9 '@ llx^2      ' 0.00 0.01
    10 '@ ll(1-x)^2 ' 0.00 0.01
    11 '@ x^2       ' 0.00 0.01
    12 '@ 1/lx      ' 0.00 0.01
    13 '@ 1/l(1-x)  ' 0.00 0.01
    14 '@ 1/llx     ' 0.00 0.01
    15 '@ 1/ll(1-x)' 0.00 0.01
    16 '@ 1/x       ' 0.00 0.01
    17 '@ 1/(1-x)   ' 0.00 0.01

    END

114e <Fix parameters 114e>≡
    for p in 1 2 3 4 5 6 7 8 9 10 \

```

```

        11 12 13 14 15 16 17; do
    case "$args" in
        *:$p=*) val='echo "$args" | sed 's/.*:'"$p"'=\\([0-9.-]*\\):.*\\/\\1/'';
                echo set parameter $p $val;
                echo fix $p;;
        *:$p:*) ;;
        *) echo fix $p;;
    esac
done
115a <Fix strategy 115a>≡
    case "$args" in
        *:S0:*) echo set strategy 0;;
        *:S1:*) echo set strategy 1;;
        *:S2:*) echo set strategy 2;;
    esac
115b <Run Minuit 115b>≡
    cat <<END
    migrat 10000 0.01
    stop
    END
115c <Maybe plot results 115c>≡
    case "$args" in
        *:p:*) awk '$5 != "" { print $1, $2, $5 }' minuit.fit > chi2
                awk '$5 != "" { print $1, $5 }' minuit.fit > chix
                awk '$5 != "" { print $2, $5 }' minuit.fit > chiy
                gnuplot -geometry -0+0 plot2 >/dev/null 2>&1
    esac
    Uses fit 93b.

```

7.2.2 Quasi Two Dimensional

```

115d <circe1_minuit2.f90 115d>≡
    ! minuit2.f90 -- fitting for circe
    <Copyleft notice 29b>
    Uses circe 31b and minuit2 115f.
115e <circe1_minuit2.f90 115d>+≡
    <Minuit2 module 115f>
    <Minuit2 main program 111b>
115f <Minuit2 module 115f>≡
    module minuit2
        use kinds

        implicit none

        public :: fct
        public :: phi

```

contains

```
<Function to minimize 111c>  
<Function phi2 116>  
end module minuit2
```

Defines:

minuit2, used in chunks 111b and 115d.

Uses fct 95b 111c and phi 112d 116.

```
116 <Function phi2 116>≡  
    function phi (e1, e2, a)  
        real(kind=double) :: e1, e2  
        real(kind=double), dimension(33) :: a  
        real(kind=double) :: phi  
        real(kind=double) :: y1, y2  
        y1 = e1 / 250d0  
        y2 = e2 / 250d0  
        phi = exp (  
            + a( 1) * 1d0  
            + a( 2) * log(y1)  
            + a( 3) * log(1d0-y1)  
            + a( 4) * log(-log(y1))  
            + a( 5) * log(-log(1d0-y1))  
            + a( 6) * y1  
            + a( 7) * log(y1)**2  
            + a( 8) * log(1d0-y1)**2  
            + a( 9) * log(-log(y1))**2  
            + a(10) * log(-log(1d0-y1))**2  
            + a(11) * y1**2  
            + a(12) / log(y1)  
            + a(13) / log(1d0-y1)  
            + a(14) / log(-log(y1))  
            + a(15) / log(-log(1d0-y1))  
            + a(16) / y1  
            + a(17) / (1d0-y1)  
            + a(18) * log(y2)  
            + a(19) * log(1d0-y2)  
            + a(20) * log(-log(y2))  
            + a(21) * log(-log(1d0-y2))  
            + a(22) * y2  
            + a(23) * log(y2)**2  
            + a(24) * log(1d0-y2)**2  
            + a(25) * log(-log(y2))**2  
            + a(26) * log(-log(1d0-y2))**2  
            + a(27) * y2**2  
            + a(28) / log(y2)  
            + a(29) / log(1d0-y2)  
            + a(30) / log(-log(y2))  
            + a(31) / log(-log(1d0-y2))  
            + a(32) / y2  
        )
```

```

        + a(33) / (1d0-y2)      &
    )
end function phi

```

Defines:

phi, used in chunks 110d, 112, and 115f.

```

117a <circe1_minuit2.sh 117a>≡
    #! /bin/sh
    minuit_bin='pwd'/circe1_minuit2.bin
    <Process arguments 114a>
    (
        <Define parameters (2dim) 117b>
        <Fix parameters (2dim) 118>
        <Fix strategy 115a>
        <Run Minuit 115b>
    ) | eval "$minuit_bin $filter"
    <Maybe plot results 115c>
    exit 0

```

```

117b <Define parameters (2dim) 117b>≡
    cat <<END
    set title
    CIRCE
    parameters
    1  '@ 1          ' 0.00 0.01
    2  '@ 1x         ' 0.20 0.01
    3  '@ 1(1-x)     ' 0.20 0.01
    4  '@ 11x        ' 0.00 0.01
    5  '@ 11(1-x)    ' 0.00 0.01
    6  '@ x          ' 0.00 0.01
    7  '@ 1x^2       ' 0.00 0.01
    8  '@ 1(1-x)^2   ' 0.00 0.01
    9  '@ 11x^2      ' 0.00 0.01
    10 '@ 11(1-x)^2  ' 0.00 0.01
    11 '@ x^2        ' 0.00 0.01
    12 '@ 1/1x       ' 0.00 0.01
    13 '@ 1/1(1-x)   ' 0.00 0.01
    14 '@ 1/11x      ' 0.00 0.01
    15 '@ 1/11(1-x)  ' 0.00 0.01
    16 '@ 1/x        ' 0.00 0.01
    17 '@ 1/(1-x)    ' 0.00 0.01
    18 '@ 1y         ' 0.20 0.01
    19 '@ 1(1-y)     ' 0.20 0.01
    20 '@ 11y        ' 0.00 0.01
    21 '@ 11(1-y)    ' 0.00 0.01
    22 '@ y          ' 0.00 0.01
    23 '@ 1y^2       ' 0.00 0.01
    24 '@ 1(1-y)^2   ' 0.00 0.01
    25 '@ 11y^2      ' 0.00 0.01
    26 '@ 11(1-y)^2  ' 0.00 0.01

```

```

27 '@ y^2      ' 0.00 0.01
28 '@ 1/ly     ' 0.00 0.01
29 '@ 1/l(1-y) ' 0.00 0.01
30 '@ 1/lly    ' 0.00 0.01
31 '@ 1/ll(1-y)' 0.00 0.01
32 '@ 1/y      ' 0.00 0.01
33 '@ 1/(1-y)  ' 0.00 0.01

END

118 <Fix parameters (2dim) 118>≡
for p in 1 2 3 4 5 6 7 8 9 10 \
      11 12 13 14 15 16 17 18 19 20 \
      21 22 23 24 25 26 27 28 29 30 \
      31 32 33; do
case "$args" in
*: $p=*) val='echo "$args" | sed 's/.*:.'"$p"'='\([0-9.-]*\)'.*/\1/''';
        echo set parameter $p $val;
        echo fix $p;;
*: $p:*) ;;
*) echo fix $p;;
esac
done

```

7.3 Version 2

8 Conclusions

I have presented a library of simple parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. The library can be used for integration and event generation. Emphasis is put on simplicity and reproducibility of the parameterizations for supporting reproducible physics simulations.

Acknowledgements

Daniel Schulte made his simulation code **Guinea-Pig** available and answered questions. Harald Anlauf and Torbjörn Sjöstrand have contributed useful suggestions. The Tesla group at DESY/Zeuthen made error estimates feasible by donating time on the multi-headed number cruncher **Hydra**. The 1996 ECFA/Desy Linear Collider Workshop got me started and provided support. Thanks to all of them.

Identifiers

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A Literate Programming

A.1 Paradigm

I have presented the sample code in this paper using the *literate programming* paradigm. This paradigm has been introduced by Donald Knuth [19] and his programs T_EX [20] and METAFONT [21] provide excellent examples of the virtues of literate programming. Knuth summarized his intention as follows ([19], p. 99)

“Let us change our traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a *computer* what to do, let us concentrate rather on explaining to *human beings* what we want a computer to do.”

Usually, literate programming uses two utility programs to produce two kinds of files from the source

tangle produces the computer program that is acceptable to an “illiterate” (Fortran, C, etc.) compiler. This process consists of stripping documentation and reordering code. Therefore it frees the author from having to present the code in the particular order enforced by a compiler for purely technical reasons. Instead, the author can present the code in the order that is most comprehensible.

weave produces a documents that describes the program. Extensive cross referencing of the code sections is usually provided, which has been suppressed in this paper. If a powerful typesetting system (such a T_EX) is used, the document can present the algorithms in clear mathematical notation alongside the code. These features improve readability and maintainability of scientific code immensely.

A.2 Practice

Circe1 uses the **noweb** [22] system. This system has the advantage to work with any traditional programming language and support the essential features described in section A.1 with minimal effort. **noweb**’s **tangle** program only reorders the code sections, but does not reformat them. Therefore its output can be used just like any other “illiterate” program.

The examples above should be almost self-explaining, but in order to avoid any ambiguities, I give another example:

```
125a <Literate programming example 125a>≡  
      <Code that has to be at the top 125c>  
      <Other code 125b>
```

I can start the presentation with the first line of the “other code”:

```
125b <Other code 125b>≡  
      line 1 of the other code
```

If appropriate, the first line of the code that has to appear *before* the other code can be presented later:

```
125c <Code that has to be at the top 125c>≡  
      line 1 of the code at the top
```

Now I can augment the sections:

```
126a  <Other code 125b>+≡  
      line 2 of the other code  
126b  <Code that has to be at the top 125c>+≡  
      line 2 of the code at the top
```

The complete “program” will be presented to the compiler as

```
line 1 of the code at the top  
line 2 of the code at the top  
line 1 of the other code  
line 2 of the other code
```

The examples in section 3.1.1 show that this reordering is particularly useful for declaring variables when they are first used (rather than at the beginning) and for zooming in on code inside of loops.

B Fortran Name Space

In addition to the ten procedures and one `common` block discussed in section 3

- `circe`, `circee`, `circeg`, `circgg`,
- `girce`, `gircee`, `girceg`, `gircgg`,
- `circes`, `circel`, `/circom/`,

there are two more globally visible functions which are used internally:

- `circem`: error message handler,
- `girceb`: efficient Beta distribution generator.

Even if the `/circom/` is globally visible, application programs *must not* manipulate it directly. The `circes` subroutine is provided for this purpose and updates some internal parameters as well.

With features from the current Fortran standard (Fortran90), I could have kept the last two functions and the `common` block private.

Application programs wishing to remain compatible with future versions of `Circe1` must not use `common` blocks or procedures starting with `circe` or `girce`.

C Updates

Information about updates can be obtained

- on the World Wide Web:

`http:http://projects.hepforge.org/whizard/`

Contributions of results from other simulation programs and updated accelerator designs are welcome at

`ohl@physik.uni-wuerzburg.de`