

Circe Version 1.02 β : Beam Spectra for Simulating Linear Collider Physics*

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Abstract

I describe parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. Emphasis is put on simplicity and reproducibility of the parameterizations, supporting reproducible physics simulations. The parameterizations are implemented in a library of distribution functions and event generators.

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Program Summary:

- **Title of program:** Circe, Version 1.02 β (September 1996)
- **Program obtainable** by anonymous ftp from the host `crunch.ikp.physik.th-darmstadt.de` in the directory `pub/ohl/circe`.
- **Licensing provisions:** Free software under the GNU General Public License.
- **Programming language used:** Fortran77
- **Number of program lines in distributed program, including test data, etc.:** ≈ 1100 (excluding comments)
- **Computer/Operating System:** Any with a Fortran77 programming environment.
- **Memory required to execute with typical data:** Negligible on the scale of typical applications calling the library.
- **Typical running time:** A small fraction (typically a few percent) of the running time of applications calling the library.
- **Purpose of program:** Provide simple and reproducible, yet realistic, parameterizations of the e^{\pm} - and γ -beam spectra for linear colliders.
- **Nature of physical problem:** The intricate beam dynamics in the interaction region of a high luminosity linear collider at $\sqrt{s} = 500\text{GeV}$ result in non-trivial energy spectra of the scattering electrons, positrons and photons. Physics simulations require simple and reproducible, yet realistic, parameterizations of these spectra.
- **Method of solution:** Parameterization, curve fitting, Monte Carlo event generation.
- **Keywords:** Event generation, beamstrahlung, linear colliders.

1 Introduction

Despite the enormous quantitative success of the electro-weak standard model up to energies of 200GeV, neither the nature of electro-weak symmetry breaking (EWSB) nor the origin of mass are understood.

From theoretical considerations, we know that clues to the answer of these open questions are hidden in the energy range below $\Lambda_{\text{EWSB}} = 4\pi v \approx 3.1\text{TeV}$. Either we will discover a Higgs particle in this energy range or signatures for a strongly interacting EWSB sector will be found. Experiments at CERN's Large Hadron Collider (LHC) will shed a first light on this regime in the next decade. In the past it has been very fruitful to complement experiments at high energy hadron colliders with experiments at e^+e^- -colliders. The simpler initial state allows more precise measurements with smaller theoretical errors. Lucid expositions of the physics opportunities of high energy e^+e^- colliders with references to the literature can be found in [1].

However, the power emitted by circular storage rings in form of synchrotron radiation scales like $(E/m)^4/R^2$ with the energy and mass of the particle and the radius of the ring. This cost becomes prohibitive after LEP2 and a Linear Collider (LC) has to be built instead.

Unfortunately, the “interesting” hard cross sections scale like $1/s$ with the square of the center of mass energy and a LC will have to operate at extremely high luminosities in excess of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. To achieve such luminosities, the bunches of electrons and positrons have to be very dense. Under these conditions, the electrons undergo acceleration from strong electromagnetic forces from the positron bunch (and vice versa). The resulting synchrotron radiation is called *beamstrahlung* [2] and has a strong effect on the energy spectrum $D(x_1, x_2)$ of the colliding particles. This changes the observable e^+e^- cross sections

$$\frac{d\sigma_0^{e^+e^-}}{d\Omega}(s) \rightarrow \frac{d\sigma^{e^+e^-}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^+e^-}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^+e^-}}{d\Omega'}(x_1 x_2 s) \quad (1a)$$

and produces luminosity for $e^\pm\gamma$ and $\gamma\gamma$ collisions:

$$\frac{d\sigma^{e^\pm\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^\pm\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^\pm\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1b)$$

$$\frac{d\sigma^{\gamma\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{\gamma\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{\gamma\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1c)$$

Therefore, simulations of the physics expected at a LC need to know the spectra of the e^\pm and γ beams precisely.

Microscopic simulations of the beam dynamics are available (e.g. ABEL[3], CAIN[4] and Guinea-Pig[5]) and their predictions are compatible with each other. But they require too much computer time and memory for direct use in physics programs. Circe provides a fast and simple parameterization of the results from these simulations. Furthermore, even if the computational cost of the simulations would be negligible, the input parameters for microscopic simulations are not convenient for particle physics applications. Due to the highly

	SBAND	TESLA	XBAND	SBAND	TESLA	XBAND
E/GeV	250	250	250	500	500	500
$N_{\text{particles}}/10^{10}$	1.1	3.63	0.65	2.9	1.8	0.95
$\epsilon_x/10^{-6}\text{mrad}$	5	14	5	10	14	5
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.08	0.1	0.06	0.1
β_x^*/mm	10.98	24.95	8.00	32	25	10.00
β_y^*/mm	0.45	0.70	0.13	0.8	0.7	0.12
σ_x/nm	335	845	286	571.87	598.08	226
σ_y/nm	15.1	18.9	4.52	9.04	6.55	3.57
$\sigma_z/\mu\text{m}$	300	700	100	500	500	125
f_{rep}	50	5	180	50	5	180
n_{bunch}	333	1135	90	125	2270	90

Table 1: Accelerator parameters for three typical designs at $\sqrt{s} = 500\text{GeV}$ and $\sqrt{s} = 1\text{TeV}$. The resulting distributions are shown in figure 1. The design efforts are currently concentrated on a 350GeV-800GeV LC. Therefore the Tesla parameters for 1TeV are slightly out of date.

non-linear beam dynamics, the optimization of LC designs is a subtle art [6], that is best practiced by the experts. Furthermore, particle physics applications need benchmarking and easily reproducible parameterizations are required for this purpose.

The parameterizations in **Circe** are not based on approximate solutions (cf. [7]) of the beamstrahlung dynamics. Instead, they provide a “phenomenological” description of the results from full simulations. The parameterizations are as simple as possible while remaining consistent with basic physical principles:

1. *positivity*: the distribution functions $D(x_1, x_2)$ *must not* be negative in the physical region $[0, 1] \times [0, 1]$.
2. *integrability*: the definite integral of the distribution functions over the physical region $[0, 1] \times [0, 1]$ *must* exist, even though the distributions can have singularities.

This paper is organized as follows: I start in section 2 with a discussion of the input for the microscopic simulations. In section 3 I describe the usage of the **Circe** library and in section 4 I discuss some technical details of the implementation. After discussing the parameterizations available in version 1.02 β in section 5, I conclude in section 8.

2 Parameters

The microscopic simulation program **Guinea-Pig** [5] used for the current version of the parameterizations in **Circe** simulates the passage of electrons through a

Figure 1: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions at $\sqrt{s} = 500\text{GeV}$ and $\sqrt{s} = 1\text{TeV}$ in a doubly logarithmic plot. The accelerator parameters are taken from table 1.

	TESLA	TESLA	TESLA
E/GeV	175	250	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	3.63
$\epsilon_x/10^{-6}\text{mrad}$	14	14	14
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.1
β_x^*/mm	25.00	24.95	15.00
β_y^*/mm	0.70	0.70	0.70
σ_x/nm	1010.94	845	668.67
σ_y/nm	22.6	18.9	9.46
$\sigma_z/\mu\text{m}$	700	700	700
f_{rep}	5	5	5
n_{bunch}	1135	1135	1135

Table 2: Accelerator parameters for the Tesla design at three planned [8] energies. The resulting distributions are shown in figure 2.

	High- \mathcal{L}	Low- \mathcal{L}	Low- ϵ_y
E/GeV	400	400	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	1.800
$\epsilon_x/10^{-6}\text{mrad}$	14	14	12
$\epsilon_y/10^{-6}\text{mrad}$	0.1	0.25	0.025
β_x^*/mm	15.00	25.00	25.00
β_y^*/mm	0.70	0.70	0.50
σ_x/nm	668.67	700.00	
σ_y/nm	9.46		
$\sigma_z/\mu\text{m}$	700	700	500
f_{rep}	5	5	3
n_{bunch}	1135	1135	2260

Table 3: Variant accelerator parameters for the Tesla design at 800 GeV.

Figure 2: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions for Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 2.

	TESLA	TESLA
E/GeV	250	400
$N_{\text{particles}}/10^{10}$	2	1.40
$\epsilon_x/10^{-6}\text{m rad}$	10	8
$\epsilon_y/10^{-6}\text{m rad}$	0.03	0.01
β_x^*/mm	15.00	15.00
β_y^*/mm	0.40	0.30
σ_x/nm	553	391
σ_y/nm	5	2
$\sigma_z/\mu\text{m}$	400	300
f_{rep}	5	3
n_{bunch}	2820	4500

Table 4: Accelerator parameters for a high luminosity Tesla design at two planned [8] energies. The resulting distributions are shown in figure 3.

Figure 3: Version 5, revision 1998 05 05 of the factorized e^\pm - and γ -distributions for a high luminosity Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 4.

Figure 4: *Experimental:* Version 1, revision 0 of the factorized e^- - and γ -distributions for Tesla- e^-e^- in a doubly logarithmic plot. The accelerator parameters are taken from table 2 and have *not* been endorsed for use in an e^-e^- -machine yet!.

bunch of electrons (and vice versa). It takes the following accelerator parameters as input:

E : the energy of the particles before the beam-beam interaction.

$N_{\text{particles}}$: the number of particles per bunch.

$\epsilon_{x,y}$: the normalized horizontal and vertical emittances.

$\beta_{x,y}^*$: the horizontal and vertical beta functions.

$\sigma_{x,y,z}$: the horizontal, vertical and longitudinal beam size. A Gaussian shape is used for the charge distribution in the bunches.

f_{rep} : the repetition rate.

n_{bunch} : the number of bunches per train.

The transversal beam sizes, beta functions and normalized emittances for relativistic particles are related by

$$\beta_{x,y}^* = \frac{\sigma_{x,y}^2}{\epsilon_{x,y}} \frac{E}{m_e} \quad (2)$$

The parameters used in the most recent revision of the parameterizations are collected in tables 1 and 2. The resulting factorized electron/positron and photon distributions in version 1 of the parameterizations are depicted in figures 1 and 2.

The most important purpose of **Circe** is to map the manifold of possible beam spectra for the NLC to a *finite* number of *reproducible* parameterizations. The distributions

$$D_{p_1 p_2}^{\alpha \nu \rho}(x_1, x_2; \sqrt{s}) \quad (3)$$

provided by **Circe** are indexed by three integers

α : the *accelerator design class*: currently there are three options: S-band [9], Tesla [8], X-band [10, 11]. More variety will be added later, in particular the e^-e^- mode and the $e^-\gamma$ and $\gamma\gamma$ laser backscattering modes of these designs.

ν : the *version of the parameterization*: over the years, the form of the parameterizations can change, either because better approximations are found or because new simulation programs become available. All versions will remain available in order to be able to reproduce calculations.

ρ : the *revision date for the parameterization*: a particular parameterization can contain bugs, which will be fixed in subsequent revisions. While only the most recent revision should be used for new calculations, old revisions will remain available in order to be able to reproduce calculations.

The continuous parameter \sqrt{s} in (3) is misleading, because accelerator parameters have been optimized for discrete values of the energy. Therefore the distributions are not available for all values of \sqrt{s} .

The usage of the distributions in application programs is discussed in section 3.1. **Circe** provides for each of the distributions a non-uniform random variate generator, that generates energy fractions according to the distributions. The usage of these generators is discussed in section 3.2.

3 Usage

3.1 Distributions

A generic interface to all distributions $D_{p_1 p_2}(x_1, x_2)$ is given by the `circe` function

```
9a  <API documentation 9a>≡
      double precision circe, d, x1, x2
      integer p1, p2
      d = circe (x1, x2, p1, p2)
```

Uses `circe` 24d.

where the energy fractions are specified by $x_{1,2}$ and the particles $p_{1,2}$ are identified by their standard Monte Carlo codes:[13]

```
9b  <Particle codes 9b>≡
      integer ELECTR, POSITR, PHOTON
      parameter (ELECTR = 11)
      parameter (POSITR = -11)
      parameter (PHOTON = 22)
```

Defines:

`ELECTR`, used in chunks 18b, 24d, 62, 67, and 68a.

`PHOTON`, used in chunks 24d, 62, 67, 68a, and 74.

`POSITR`, used in chunks 18c and 68a.

The distributions can have integrable singularities at the end points, therefore the calling functions *must not* evaluate them at the endpoints 0 and 1. This is usually not a problem, since standard mapping techniques (cf. (10) below) will have to be used to take care of the singularity anyway. Nevertheless, all applications should favor open quadrature formulae (i.e. formulae not involving the endpoints) over closed formulae. The distributions are guaranteed to vanish unless $0 < x_{1,2} < 1$, with two exceptions. Firstly, the value -1 allows to pick up the integral of the continuum contribution:

$$D_{p_1 p_2}(-1, x_2) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 D_{p_1 p_2}(x_1, x_2) \quad (4a)$$

$$D_{p_1 p_2}(x_1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4b)$$

$$D_{p_1 p_2}(-1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4c)$$

The other exception is that the strength of δ -function contributions at the endpoint can be picked up from the value at this endpoint:

$$D_{e^+ e^-}(x_1, x_2) = D_{e^+ e^-}(1, 1) \delta(1 - x_1) \delta(1 - x_2) + \text{smooth and single } \delta \quad (5a)$$

$$D_{e^{\pm} \gamma}(x_1, x_2) = D_{e^{\pm} \gamma}(1, x_2) \delta(1 - x_1) + \text{smooth} \quad (5b)$$

$$D_{\gamma e^{\pm}}(x_1, x_2) = D_{\gamma e^{\pm}}(x_1, 1) \delta(1 - x_2) + \text{smooth} \quad (5c)$$

The use of these special values is demonstrated in an example in section 3.1.1 below.

The distributions are normalized such that

$$\lim_{\epsilon \rightarrow +0} \int_{-\epsilon}^{1+\epsilon} dx_1 dx_2 D_{e^+e^-}(x_1, x_2) = 1. \quad (6)$$

and the nominal e^+e^- -luminosity of the currently active accelerator design can be retrieved from the database with the subroutine `circel`. The value is given in units of

$$\text{fb}^{-1} v^{-1} = 10^{32} \text{cm}^{-2} \text{sec}^{-1} \quad (7)$$

where $v = 10^7 \text{sec} \approx \text{year}/\pi$ is an “effective year” of running with about 30% up-time.

10a `<API documentation 9a>+≡`
`double precision lumi`
`call circel (lumi)`

Uses `circel` 33f.

A particular parameterization is selected by the `circes` function:

10b `<API documentation 9a>+≡`
`double precision x1m, x2m, roots`
`integer acc, ver, rev, chat`
`call circes (x1m, x2m, roots, acc, ver, rev, chat)`

Uses `circes` 25a.

The parameter `roots` corresponds to the nominal center of mass energy \sqrt{s}/GeV of the collider. Currently $\sqrt{s} = 350\text{GeV}, 500\text{GeV}, 800\text{GeV}, 1\text{TeV}$ (i.e. 350D0, 500D0, 800D0 and 1000D0) are supported. Application programs can *not* assume that energy values are interpolated. For convenience, e.g. in top threshold scans around 350GeV, a small interval around the supported values will be accepted as synonymous with the central value, but a warning will be printed. Section 5 should be consulted for the discrete values supported by a particular version of the parameterizations. Negative values of `roots` will keep the currently active value for \sqrt{s} .

The parameters `x1m` and `x2m` will set thresholds $x_{1,\min}$ and $x_{2,\min}$ for the event generation in the routines described in section 3.2.

The parameter `acc` selects the accelerator design. Currently the following accelerator codes are recognized:

10c `<Accelerator codes 10c>≡`
`integer SBAND, TESLA, XBAND`
`parameter (SBAND = 1, TESLA = 2, XBAND = 3)`
`integer JLCNLC`
`parameter (JLCNLC = 3)`
`integer SBNDEE, TESLEE, XBNDEE`
`parameter (SBNDEE = 4, TESLEE = 5, XBNDEE = 6)`
`integer NLCH`
`parameter (NLCH = 7)`
`integer NACC`
`parameter (NACC = 7)`

Defines:

`NACC`, used in chunks 28, 32, 33, 37–39, 43e, 45e, 48a, 51c, 54a, and 58a.

`SBAND`, used in chunks 28c, 33, 36–39, 41c, 42a, 78a, 90c, and 91c.

`TESLA`, used in chunks 26f, 28c, 33, 36–39, 41c, 43–46, 48, 49, 51d, 52b, 54, 58b, 59b, 77b, 78a, 90c, 91c, and 93a.

`XBAND`, used in chunks 33, 36–39, 41c, 42a, 49d, 90c, and 91c.

The **ver** parameter is used to determine the version as follows:

ver > 0 : a frozen version which is documented in section 5. For example, version 1 is a family of factorized Beta distributions: $D(x_1, x_2) \propto x_1^{a_1} (1 - x_1)^{b_1} x_2^{a_2} (1 - x_2)^{b_2}$.

ver = 0 : the latest experimental version, which is usually not documented and can change at any time without announcement.

ver < 0 : keep the currently active version.

The **rev** parameter is used to determine the revision of a version as follows:

rev > 0 : a frozen revision which is documented in section 5. The integer **rev** is constructed from the date as follows: $\text{rev} = 10^4 \cdot \text{year} + 10^2 \cdot \text{month} + \text{day}$, where the year is greater than 1995. Since Fortran77 ignores whitespace, it can be written like 1996 07 11 for readability. If there is no exact match, the most recent revision before the specified date is chosen.

rev = 0 : the most recent revision.

rev < 0 : keep the currently active revision.

Finally, the parameter **chat** controls the “chattiness” of **circe**. If it is 0, only error messages are printed. If it is 1, the parameters in use are printed whenever they change. Higher values of **chat** can produce even more diagnostics.

In addition to the generic interface **circe**, there are specialized functions for particular particle distributions. Obviously

$$D_{e^\pm\gamma}^{\alpha\nu\rho}(x_1, x_2, s) = D_{\gamma e^\pm}^{\alpha\nu\rho}(x_2, x_1, s) \quad (8)$$

and there are three independent functions $D_{e^-e^+}$, $D_{e^-\gamma}$ and $D_{\gamma\gamma}$ for the e^+e^- colliders with reasonable mnemonics:

```
11 <API documentation 9a>+≡
    double precision circee, circeg, circgg
    d = circee (x1, x2)
    d = circeg (x1, x2)
    d = circgg (x1, x2)
```

Uses **circee** 34a, **circeg** 35a, and **circgg** 35c.

Calling the latter three functions is marginally faster in the current implementation, but this can change in the future.

3.1.1 Example

For clarification, let me give a simple example. Imagine we want to calculate the integrated production cross section

$$\sigma_X(s) = \int dx_1 dx_2 \sigma_{e^+e^- \rightarrow X}(x_1 x_2 s) D_{e^+e^-}(x_1, x_2, s) \quad (9)$$

Since the distributions are singular in the $x_{1,2} \rightarrow 1$ limit, we have to map away this singularity with

$$x \rightarrow t = (1 - x)^{1/\eta} \quad (10a)$$

Therefore

$$\int_0^1 dx f(x) = \int_0^1 dt \eta t^{\eta-1} f(1-t^\eta) \quad (10b)$$

with η sufficiently large to give the integrand a finite limit at $x \rightarrow 1$. If f diverges like a power $f(x) \propto 1/(1-x)^\beta$, this means $\eta > 1/(1-\beta)$.

As a specific example, let us “measure” a one particle s -channel exchange cross section

$$\sigma(s) \propto \frac{1}{s} \quad (11)$$

```
12a <sample.f 12a>≡
      double precision function sigma (s)
      implicit none
      double precision s
      sigma = 1d0 / s
      end
```

I will present the example code in a bottom-up fashion, which should be intuitive and is described in some more detail in appendix A. Assuming the existence of a one- and a two-dimensional Gaussian integration function **gauss1** and **gauss2**,¹ we can perform the integral as follows:

```
12b <Gauss integration 12b>≡
      s = sigma (1d0) * circee (1d0, 1d0)
      $ + gauss1 (d1, 0d0, 1d0, EPS)
      $ + gauss1 (d2, 0d0, 1d0, EPS)
      $ + gauss2 (d12, 0d0, 1d0, 0d0, 1d0, EPS)
      write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
      1000 format (1X, A22, 1X, F6.2, '%')
```

Uses **circee** 34a, **gauss1** 76a, and **gauss2** 76c.

Note how the four combinations of continuum and δ -peak are integrated separately, where you have to use three auxiliary functions **d1**, **d2** and **d12**. The continuum contribution, including the Jacobian:

```
12c <sample.f 12a>+≡
      double precision function d12 (t1, t2)
      implicit none
      double precision t1, t2, x1, x2, sigma, circee
      <EPS & PWR 13b>
      x1 = 1d0 - t1**PWR
      x2 = 1d0 - t2**PWR
      d12 = PWR*PWR * (t1*t2)**(PWR-1d0)
      $      * sigma (x1*x2) * circee (x1, x2)
      end
```

Uses **circee** 34a.

the first product of continuum and δ -peak:

```
12d <sample.f 12a>+≡
      double precision function d1 (t1)
      implicit none
```

¹They are provided in the example program **sample.f**.

```

double precision t1, x1, sigma, circee
(EPS & PWR 13b)
x1 = 1d0 - t1**PWR
d1 = PWR * t1**(PWR-1d0) * sigma (x1) * circee (x1, 1d0)
end

```

Uses circee 34a.

and the second one:

```

13a (sample.f 12a)+≡
double precision function d2 (t2)
implicit none
double precision t2, x2, sigma, circee
(EPS & PWR 13b)
x2 = 1d0 - t2**PWR
d2 = PWR * t2**(PWR-1d0) * sigma (x2) * circee (1d0, x2)
end

```

Uses circee 34a.

Below you will see that the power of the singularity of the e^+e^- distributions at $x \rightarrow 1$ is $\approx -2/3$. To be on the safe side, we choose the power η in (10) as 5. It is kept in the parameter PWR, while EPS is the desired accuracy of the Gaussian integration:

```

13b (EPS & PWR 13b)≡
double precision EPS, PWR
parameter (EPS = 1d-6, PWR = 5d0)

```

The Gauss integration of the non-singular version converges to the cotrrect value only if the final bin is integrated separately:

```

13c (Second Gauss integration 13c)≡
s = gauss2 (d12a, 0d0, 1d0-KIREPS, 0d0, 1d0-KIREPS, EPS)
$ + gauss2 (d12a, 0d0, 1d0-KIREPS, 1d0-KIREPS, 1d0, EPS)
$ + gauss2 (d12a, 1d0-KIREPS, 1d0, 0d0, 1d0-KIREPS, EPS)
$ + gauss2 (d12a, 1d0-KIREPS, 1d0, 1d0-KIREPS, 1d0, EPS)
write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0

```

Uses gauss2 76c.

```

13d (EPS & PWR 13b)+≡
double precision KIREPS
parameter (KIREPS = 1D-6)

```

```

13e (sample.f 12a)+≡
double precision function d12a (x1, x2)
implicit none
double precision x1, x2, sigma, kirkee
d12a = sigma (x1*x2) * kirkee (x1, x2)
end

```

Uses kirkee 63a.

These code fragments can now be used in a main program that loops over energies and accelerator designs

```

13f (sample.f 12a)+≡
program sample
implicit none

```

```

    <Accelerator codes 10c>
    <EPS & PWR 13b>
    <Other variables in sample 15>
    integer acc, ver, i
    double precision roots(9)
    data roots / 90D0, 170D0, 250D0, 350D0, 500D0,
$      800D0, 1000D0, 1200D0, 1500D0 /
    do 10 acc = JLCNLC, NLCH, NLCH-JLCNLC
      do 11 ver = 9, 9
        do 12 i = 1, 9
          call circes (0d0, 0d0, roots(i), acc, ver, 20020328, 1)
          <Gauss integration 12b>
          <Second Gauss integration 13c>
          <Monte Carlo integration 16d>
14          continue
12          continue
13          continue
11          continue
10          continue
      end
end

```

Uses circes 25a.

with the following result

```

14 <Sample output 14>≡
  circe:message: starting up ...
  circe:message: $Id: prelude.nw 66 2002-03-28 17:00:10Z ohl $
  circe:message: updating 'roots' to 90.0
  circe:message: updating 'ver' to 7
  circe:message: updating 'rev' to 20000501
  delta(sigma) (Gauss) = 0.11%
  delta(sigma) (MC)    = 0.11%
                      +/- 0.00%

  circe:message: updating 'roots' to 170.0
  circe:message: updating 'ver' to 7
  delta(sigma) (Gauss) = 0.38%
  delta(sigma) (MC)    = 0.38%
                      +/- 0.01%

  circe:message: updating 'roots' to 350.0
  circe:message: updating 'ver' to 7
  delta(sigma) (Gauss) = 1.67%
  delta(sigma) (MC)    = 1.66%
                      +/- 0.03%

  circe:message: updating 'roots' to 500.0
  circe:message: updating 'ver' to 7
  delta(sigma) (Gauss) = 3.66%
  delta(sigma) (MC)    = 3.58%
                      +/- 0.07%

  circe:message: updating 'roots' to 800.0
  circe:message: updating 'ver' to 7
  delta(sigma) (Gauss) = 5.21%

```

```

delta(sigma) (MC)    = 5.19%
                    +/- 0.11%
circe:message: updating 'roots' to 1000.0
circe:message: updating 'ver' to 7
circe:message: energy 1000.0GeV too high, using spectrum for 800.0GeV
delta(sigma) (Gauss) = 5.21%
delta(sigma) (MC)    = 5.19%
                    +/- 0.11%
circe:message: updating 'roots' to 90.0
circe:message: updating 'acc' to JLCNLC
circe:message: updating 'ver' to 7
circe:message: energy 90.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe:message: updating 'roots' to 170.0
circe:message: updating 'ver' to 7
circe:message: energy 170.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.68%
                    +/- 0.11%
circe:message: updating 'roots' to 350.0
circe:message: updating 'ver' to 7
circe:message: energy 350.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe:message: updating 'roots' to 500.0
circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe:message: updating 'roots' to 800.0
circe:message: updating 'ver' to 7
circe:message: energy 800.0GeV interpolated between 500.0 and 1000.0GeV
delta(sigma) (Gauss) = 8.37%
delta(sigma) (MC)    = 8.39%
                    +/- 0.21%
circe:message: updating 'roots' to 1000.0
circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 15.39%
delta(sigma) (MC)    = 14.68%
                    +/- 0.33%

```

Uses circe 24d.

We almost forgot to declare the variables in the main program

```

15  <Other variables in sample 15>≡
      double precision s
      double precision gauss1, gauss2, circee, sigma, d1, d2, d12, d12a
      external d1, d2, d12, d12a

```

Uses `gircee` 34a, `gauss1` 76a, and `gauss2` 76c.

This concludes the integration example. It should have made it obvious how to proceed in a realistic application.

In section 3.2.1 below, I will describe a Monte Carlo method for calculating such integrals efficiently.

3.2 Generators

The function `circe` and its companions are opaque to the user. Since they will in general contain singularities, applications will *not* be able to generate corresponding samples of random numbers efficiently. To fill this gap, four random number generators are provided. The subroutine `girce` will generate particle types $p_{1,2}$ and energy fractions $x_{1,2}$ in one step, according to the selected distribution.² Particle p_1 will be either a positron or a photon and p_2 will be either an electron or a photon. The energy fractions are guaranteed to be above the currently active thresholds: $x_i \geq x_{i,\min}$. This can be used to cut on soft events—the photon distributions are rather soft—which might not be interesting in most simulations.

16a *<API documentation 9a>*+≡
`call girce (x1, x2, p1, p2, rng)`

Uses `girce` 67.

The output parameters of `girce` are identical to the input parameters of `circe`, with the exception of `rng`. The latter is a subroutine with a single double precision argument, which will be assigned a uniform deviate from the interval $[0, 1]$ after each call:

16b *<API documentation 9a>*+≡
`subroutine rng (r)`
`double precision r`
`r = <uniform deviate on [0, 1] (never defined)>`
`end`

Typically, it will be just a wrapper around the standard random number generator of the application program. For studies with a definite initial state, three generator functions are available.

16c *<API documentation 9a>*+≡
`call gircee (x1, x2, rng)`
`call girceg (x1, x2, rng)`
`call gircgg (x1, x2, rng)`

Uses `gircee` 68d, `girceg` 69b, and `gircgg` 70a.

3.2.1 Example

Returning to the example from section 3.2.1, I present a concise Monte Carlo algorithm for calculating the same integral:

16d *<Monte Carlo integration 16d>*≡
`s = 0d0`
`s2 = 0d0`

²The implementation of the flavor selection with non-vanishing thresholds $x_{1,\min}$ and $x_{2,\min}$ is moderately inefficient at the moment. It can be improved by a factor of two.


```

do 100 n = 1, NEVENT
  call gircee (x1, x2, random)
  w = sigma (x1*x2)
  s = s + w
  s2 = s2 + w*w
100 continue
s = s / dble(NEVENT)
s2 = s2 / dble(NEVENT)
write (*, 1000) 'delta(sigma) (MC)      =', (s-1d0)*100d0
write (*, 1000) '                        +/-',
$              sqrt((s2-s*s)/dble(NEVENT))*100d0

```

Uses gircee 68d.

17a *<Other variables in sample 15>+≡*

```

double precision w, s2, x1, x2
external random
integer NEVENT, n
parameter (NEVENT = 10000)

```

Here is a simple linear congruential random number generator for the sample program. Real applications will use their more sophisticated generators instead.

17b *<sample.f 12a>+≡*

```

subroutine random (r)
implicit none
double precision r
integer m, a, c
parameter (M = 259200, A = 7141, C = 54773)
integer n
save n
data n /0/
n = mod(n*a+c,m)
r = dble (n) / dble (m)
end

```

If the cross section is slowly varying on the range where the $x_{1,2}$ distributions are non-zero, this algorithm is very efficient.

However, if this condition is not met, the explicit form of the parameterizations in section 5 should be consulted and appropriate mapping techniques should be applied. The typical example for this problem is a narrow resonance just below the nominal beam energy.

3.2.2 Event Generators

For Monte Carlo event generators that use the standard `/hepevt/` common block [14], the addition of the `Circe` library is trivial. During the initialization of the event generator, the `circes` subroutine is called to set up `Circe`'s internal state. For example:

17c *<Initialize event generator 17c>≡*

```

call circes (0d0, 0d0, roots, acc, ver, 1996 07 11, 1)

```

Uses circes 25a.

Figure 5: Architecture of **Circe**: **circes()** selects energy and accelerator and loads the parameterization. The function **circe()** calculates the values of the selected distribution function at the given energy fractions. The subroutine **girce()** generates energy fractions using a specified random number generator in accordance with the selected distribution.

During event generation, before setting up the e^+e^- initial state, the **gircee** subroutine is called with the event generator’s random number generator:

18a $\langle \text{Event generation 18a} \rangle \equiv$
`call gircee (x1, x2, random)`

Uses **gircee** 68d.

The resulting energy fractions x_1 and x_2 are now available for defining the initial state electron

18b $\langle \text{Event generation 18a} \rangle + \equiv$
`isthep(1) = 101`
`idhep(1) = ELECTR`
`phep(1,1) = 0d0`
`phep(2,1) = 0d0`
`phep(3,1) = x1 * ebeam`
`phep(4,1) = x1 * ebeam`
`phep(5,1) = 0d0`

Uses **ELECTR** 9b.

and positron.

18c $\langle \text{Event generation 18a} \rangle + \equiv$
`isthep(2) = 102`
`idhep(2) = POSITR`
`phep(1,2) = 0d0`
`phep(2,2) = 0d0`
`phep(3,2) = - x2 * ebeam`
`phep(4,2) = x2 * ebeam`
`phep(5,2) = 0d0`

Uses **POSITR** 9b.

Using **Circe** with other event generators should be straightforward as well.

4 Technical Notes

The structure of **Circe** is extremely simple (cf. figure 5) and is mainly a book-keeping exercise. All that needs to be done is to maintain a database of available parameterizations and to evaluate the corresponding functions. The only non trivial algorithms are used for the efficient generation of random deviates.

I have avoided the use of initialized **common** blocks (i.e. **block data** subroutines), because the Fortran77 standard does not provide a *portable* way of ensuring that **block data** subroutines are actually executed at loading time. Instead, the **/circom/** common block is tagged by a “magic number” to check for initialization and its members are filled by the **circes** subroutine when necessary.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}\nu^{-1}$	$31.38^{+0.22}_{-0.22}$	$106.25^{+0.71}_{-0.71}$	$95.24^{+0.73}_{-0.73}$	$36.39^{+0.29}_{-0.29}$
$\int d_{e^\pm}$	$0.4812^{+0.0041}_{-0.0041}$	$0.5723^{+0.0046}_{-0.0045}$	$0.3512^{+0.0048}_{-0.0048}$	$0.3487^{+0.0040}_{-0.0040}$
$x_{e^\pm}^\alpha$	$11.1534^{+0.0770}_{-0.0761}$	$15.2837^{+0.0923}_{-0.0914}$	$27.1032^{+0.3071}_{-0.3019}$	$6.9853^{+0.0733}_{-0.0718}$
$(1 - x_{e^\pm})^\alpha$	$-0.6302^{+0.0013}_{-0.0012}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.6453^{+0.0017}_{-0.0017}$	$-0.6444^{+0.0017}_{-0.0017}$
$\int d_\gamma$	$0.6237^{+0.0033}_{-0.0033}$	$0.7381^{+0.0036}_{-0.0036}$	$0.3502^{+0.0034}_{-0.0034}$	$0.4149^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6911^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6947^{+0.0011}_{-0.0011}$	$-0.6876^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$14.9355^{+0.0761}_{-0.0754}$	$24.1647^{+0.1124}_{-0.1116}$	$33.6576^{+0.3021}_{-0.2983}$	$8.3227^{+0.0659}_{-0.0649}$

Table 5: Version 1, revision 1997 04 16 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

Figure 6: Fit of the e^\pm - and γ -distributions for the S-Band design at $\sqrt{s} = 500\text{GeV}$. The open circles with error bars are the result of the **Guinea-Pig** simulation. The full line is the fit.

Figure 7: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 500\text{GeV}$.

A more flexible method would be to replace the **data** statements by reading external files. This option causes portability problems, however, because I would have to make sure that the names of the external files are valid in all files systems of the target operating systems. More significantly, splitting the implementation into several parts forces the user to keep all files up to date. This can be a problem, because Fortran source files and data input files will typically be kept in different parts of the file system.

The option of implementing **Circe** statelessly, i.e. with pure function calls and without **common** blocks, has been dismissed. While it would have been more straightforward on the side of the library, it would have placed the burdon of maintaining state (accelerator, energy, etc.) on the application program, thereby complicating them considerably. Keeping an explicit state in **Circe** has the additional benefit of allowing to precompute certain internal variables, resulting in a more efficient implementation.

5 Parameterizations

Version 1.02 β of **Circe** supports just one version of the parameterizations. Future versions will provide additional parameterizations.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$119.00^{+0.83}_{-0.83}$	214.33^{+0***}_{-0***}	212.22^{+0***}_{-0***}	$118.99^{+0.91}_{-0.91}$
$\int d_{e^\pm}$	$0.5604^{+0.0040}_{-0.0039}$	$0.6686^{+0.0040}_{-0.0040}$	$0.4448^{+0.0043}_{-0.0043}$	$0.5001^{+0.0038}_{-0.0038}$
$x_{e^\pm}^\alpha$	$4.2170^{+0.0258}_{-0.0255}$	$5.5438^{+0.0241}_{-0.0239}$	$9.6341^{+0.0814}_{-0.0803}$	$2.6184^{+0.0192}_{-0.0190}$
$(1 - x_{e^\pm})^\alpha$	$-0.6118^{+0.0013}_{-0.0013}$	$-0.5847^{+0.0011}_{-0.0011}$	$-0.6359^{+0.0014}_{-0.0014}$	$-0.6158^{+0.0015}_{-0.0015}$
$\int d_\gamma$	$0.7455^{+0.0032}_{-0.0032}$	$1.0112^{+0.0033}_{-0.0033}$	$0.4771^{+0.0031}_{-0.0031}$	$0.6741^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6870^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0004}_{-0.0004}$	$-0.6936^{+0.0008}_{-0.0008}$	$-0.6834^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$6.7145^{+0.0310}_{-0.0308}$	$9.9992^{+0.0342}_{-0.0340}$	$13.1607^{+0.0896}_{-0.0886}$	$3.8589^{+0.0215}_{-0.0213}$

Table 6: Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.

	350 GeV	500 GeV	800 GeV	1600 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$97.45^{+0.67}_{-0.67}$	$106.25^{+0.71}_{-0.71}$	170.86^{+0***}_{-0***}	340.86^{+0***}_{-0***}
$\int d_{e^\pm}$	$0.6093^{+0.0049}_{-0.0049}$	$0.5723^{+0.0046}_{-0.0045}$	$0.6398^{+0.0042}_{-0.0041}$	$0.5094^{+0.0040}_{-0.0040}$
$x_{e^\pm}^\alpha$	$17.6137^{+0.1065}_{-0.1055}$	$15.2837^{+0.0923}_{-0.0914}$	$7.6221^{+0.0365}_{-0.0361}$	$5.0550^{+0.0353}_{-0.0349}$
$(1 - x_{e^\pm})^\alpha$	$-0.6061^{+0.0011}_{-0.0011}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.5944^{+0.0011}_{-0.0011}$	$-0.6187^{+0.0013}_{-0.0013}$
$\int d_\gamma$	$0.7729^{+0.0039}_{-0.0039}$	$0.7381^{+0.0036}_{-0.0036}$	$0.9178^{+0.0034}_{-0.0034}$	$0.5875^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6949^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0005}_{-0.0005}$	$-0.6892^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$28.9399^{+0.1370}_{-0.1361}$	$24.1647^{+0.1124}_{-0.1116}$	$13.1167^{+0.0497}_{-0.0495}$	$7.5514^{+0.0428}_{-0.0424}$

Table 7: Version 1, revision 1997 04 17 of the beam spectra for TESLA.

	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$339.80^{+0.83}_{-0.83}$	$359.36^{+0.93}_{-0.93}$
$\int d_{e^\pm}$	$0.5019^{+0.0016}_{-0.0016}$	$0.4125^{+0.0016}_{-0.0016}$
$x_{e^\pm}^\alpha$	$12.2867^{+0.0318}_{-0.0316}$	$13.3242^{+0.0442}_{-0.0440}$
$(1 - x_{e^\pm})^\alpha$	$-0.6276^{+0.0005}_{-0.0005}$	$-0.6401^{+0.0005}_{-0.0005}$
$\int d_\gamma$	$0.5114^{+0.0012}_{-0.0012}$	$0.3708^{+0.0011}_{-0.0011}$
x_γ^α	$-0.6912^{+0.0003}_{-0.0003}$	$-0.6924^{+0.0004}_{-0.0004}$
$(1 - x_\gamma)^\alpha$	$17.0673^{+0.0375}_{-0.0375}$	$16.8145^{+0.0482}_{-0.0480}$

Table 8: Version 5, revision 1998 05 05 of the beam spectra for high luminosity TESLA.

Figure 8: Fit of the e^\pm - and γ -distributions for the X-Band design at $\sqrt{s} = 500\text{GeV}$.

Figure 9: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 1\text{TeV}$.

	SBNDEE	TESLEE	XBNDDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$9.29^{+0.06}_{-0.06}$	$21.62^{+0.17}_{-0.17}$	$13.97^{+0.10}_{-0.10}$
$\int d_{e^\pm}$	$.6513^{+0.0059}_{-0.0059}$	$.7282^{+0.0083}_{-0.0082}$	$.5270^{+0.0049}_{-0.0049}$
$x_{e^\pm}^\alpha$	$10.3040^{+0.0601}_{-0.0593}$	$14.8578^{+0.1047}_{-0.1034}$	$5.8897^{+0.0455}_{-0.0448}$
$(1 - x_{e^\pm})^\alpha$	$-.5946^{+0.0015}_{-0.0015}$	$-.5842^{+0.0018}_{-0.0018}$	$-.6169^{+0.0016}_{-0.0015}$
$\int d_\gamma$	$.4727^{+0.0035}_{-0.0035}$	$.5300^{+0.0046}_{-0.0046}$	$.3746^{+0.0029}_{-0.0029}$
x_γ^α	$-.6974^{+0.0009}_{-0.0009}$	$-.7039^{+0.0009}_{-0.0009}$	$-.6892^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$20.6447^{+0.1513}_{-0.1497}$	$36.1286^{+0.3027}_{-0.2991}$	$10.0872^{+0.0822}_{-0.0815}$

Table 9: *Experimental* Version 1, revision 0 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

	SBNDEE	TESLEE	XBNDDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$45.59^{+0.34}_{-0.34}$	$25.47^{+0.20}_{-0.20}$	$41.06^{+0.28}_{-0.28}$
$\int d_{e^\pm}$	$.7892^{+0.0075}_{-0.0074}$	$.6271^{+0.0066}_{-0.0065}$	$.7203^{+0.0058}_{-0.0058}$
$x_{e^\pm}^\alpha$	$5.4407^{+0.0285}_{-0.0281}$	$8.7504^{+0.0669}_{-0.0658}$	$2.7415^{+0.0121}_{-0.0119}$
$(1 - x_{e^\pm})^\alpha$	$-.5285^{+0.0020}_{-0.0020}$	$-.6058^{+0.0017}_{-0.0017}$	$-.5049^{+0.0020}_{-0.0020}$
$\int d_\gamma$	$.6403^{+0.0040}_{-0.0040}$	$.4278^{+0.0038}_{-0.0038}$	$.6222^{+0.0032}_{-0.0032}$
x_γ^α	$-.6960^{+0.0008}_{-0.0008}$	$-.6982^{+0.0010}_{-0.0010}$	$-.6795^{+0.0008}_{-0.0008}$
$(1 - x_\gamma)^\alpha$	$12.4803^{+0.0839}_{-0.0831}$	$18.5260^{+0.1674}_{-0.1655}$	$4.7506^{+0.0262}_{-0.0260}$

Table 10: *Experimental* Version 1, revision 0 of the beam spectra at 1 TeV.

5.1 Version 1

The first version of the parameterization uses a simple factorized *ansatz*

$$D_{p_1 p_2}^{\alpha 1\rho}(x_1, x_2, s) = d_{p_1}^{\alpha 1\rho}(x_1) d_{p_2}^{\alpha 1\rho}(x_2) \quad (12a)$$

where the distributions are simple Beta distributions:

$$d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} \delta(1-x) + a_1^{\alpha\rho} x^{a_2^{\alpha\rho}} (1-x)^{a_3^{\alpha\rho}} \quad (12b)$$

$$d_\gamma^{\alpha 1\rho}(x) = a_4^{\alpha\rho} x^{a_5^{\alpha\rho}} (1-x)^{a_6^{\alpha\rho}} \quad (12c)$$

This form of the distributions is motivated by the observation [2] that the e^\pm distributions diverge like a power for $x \rightarrow 1$ and vanish at $x \rightarrow 0$. The behavior of the γ distributions is similar with the borders exchanged.

	350 GeV	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$15.18^{+0.13}_{-0.13}$	$21.62^{+0.17}_{-0.17}$	$43.98^{+0.38}_{-0.38}$
$\int d_{e^\pm}$	$.6691^{+0.0083}_{-0.0083}$	$.7282^{+0.0083}_{-0.0082}$	$.7701^{+0.0090}_{-0.0089}$
$x_{e^\pm}^\alpha$	$25.2753^{+0.2040}_{-0.2007}$	$14.8578^{+0.1047}_{-0.1034}$	$8.1905^{+0.0543}_{-0.0535}$
$(1 - x_{e^\pm})^\alpha$	$-.5994^{+0.0017}_{-0.0017}$	$-.5842^{+0.0018}_{-0.0018}$	$-.5575^{+0.0021}_{-0.0021}$
$\int d_\gamma$	$.4464^{+0.0047}_{-0.0047}$	$.5300^{+0.0046}_{-0.0046}$	$.5839^{+0.0047}_{-0.0047}$
x_γ^α	$-.7040^{+0.0011}_{-0.0011}$	$-.7039^{+0.0009}_{-0.0009}$	$-.7046^{+0.0009}_{-0.0009}$
$(1 - x_\gamma)^\alpha$	$60.1882^{+0.5882}_{-0.5797}$	$36.1286^{+0.3027}_{-0.2991}$	$19.3944^{+0.1681}_{-0.1660}$

Table 11: *Experimental* Version 1, revision 0 of the beam spectra for TESLEE.

5.1.1 Fitting

The parameters a_i in (12) have been obtained by a least-square fit of (12) to histograms of simulation results from **Guinea-Pig**. Some care has to be taken when fitting singular distributions to histogrammed data. Obviously equidistant bins are not a good idea, because most bins will be almost empty (cf. figures 1 and 2) and consequently a lot of information will be wasted. One solution to this problem is the use of logarithmic bins. This, however, maps the compact region $[0, 1] \times [0, 1]$ to $[-\infty, 0] \times [-\infty, 0]$, which is inconvenient because of the missing lower bounds.

The more appropriate solution is to use two maps

$$\begin{aligned} \phi : [0, 1] &\rightarrow [0, 1] \\ x &\mapsto y = x^{1/\eta} \end{aligned} \tag{13}$$

where $x = x_\gamma$ or $x = 1 - x_{e^\pm}$, and to bin the result equidistantly. If η is chosen properly (cf. (10)), the bin contents will then fall off at the singularity. The fits in tables 5, 6, and 7 have been performed with $\eta = 5$ and the resulting bin contents can be read off from figures 6–9.

Using this procedure for binning the results of the simulations, the popular fitting package **MINUIT** [15] converges quickly in all cases considered. The resulting parameters are given in tables 5, 6, and 7. Plots of the corresponding distributions have been shown in figures 1 and 2. It is obvious that an *ansatz* like (12) is able to distinguish among the accelerator designs. Thus it can provide a solid basis for physics studies.

In figures 6–9 I give a graphical impression of the quality of the fit, which appears to be as good as one could reasonably expect for a simple *ansatz* like (12). Note that the histograms have non-equidistant bins and that the resulting Jacobians have not been removed. Therefore the bin contents falls off at the singularities, as discussed above.

The errors used for the least-square fit had to be taken from a Monte Carlo (MC) study. **Guinea-Pig** only provides the \sqrt{n} from Poissonian statistics for each bin, but the error accumulation during tracking the particles through phase space is not available. The MC studies shows that the latter error dominates the former, but appears to be reasonably Gaussian. A complete MC study

of all parameter sets is computationally expensive (more than a week of processor time on a fast SGI). From an exemplary MC study of a few parameter sets, it appears that the errors can be described reasonably well by rescaling the Poissonian error in each bin with appropriate factors for electrons/positrons and photons and for continuum and delta. This procedure has been adopted.

The $\chi^2/\text{d.o.f.}$'s of the fits are less than $\mathcal{O}(10)$. The simple *ansatz* (12) is therefore very satisfactory. In fact, trying to improve the ad-hoc factorized Beta distributions by the better motivated approximations from [7] or [16], it turns out [17] that (12) provides a significantly better fit of the results of the simulations. The price to pay is that the parameters in (12) have no direct physical interpretation.

5.1.2 Generators

For this version of the parameterizations we need a fast generator of Beta distributions:

$$\beta^{a,b}(x) \propto x^{a-1}(1-x)^{b-1} \quad (14)$$

This problem has been studied extensively and we can use a published algorithm [18] that is guaranteed to be very fast for all a, b such that $0 < a \leq 1 \leq b$, which turns out to be always the case (cf. tables 5, 6, and 7).

5.2 Future Versions

There are two ways in which the parameterizations can be improved:

more complicated functions: the factorized fits can only be improved marginally by adding more positive semi-definite factors to (12). More improvement is possible by using sums of functions, but in this case, the best fits violate the positivity requirement and have to be discarded.

correlations: the parameterization in section 5.1 is factorized. While this is a good approximation, the simulations nevertheless show correlations among x_1 and x_2 . These correlations can be included in a future version.

interpolation: the parameterization in section 5.1 is based on fitting the simulation results by simple functions. Again, this appears to be a good approximation. But such fits can not uncover any fine structure of the distributions. Therefore it will be worthwhile to study interpolations of the simulation results in the future. A proper interpolation of results with statistical errors is however far from trivial: straightforward polynomial or spline interpolations will be oscillatory and violate the positivity requirement. Smoothing algorithms have to be investigated in depth before such a parameterization can be released.

other simulations: besides [5], other simulation codes are invited to contribute their results for inclusion in the `Circe` library.

6 Implementation of circe

24a `<circe1.f 24a>≡`
 `c circe1.f -- canonical beam spectra for linear collider physics`
 `c $Id: circe.nw 67 2002-03-28 17:13:06Z ohl $`
 `<Copleft notice 24b>`
 `<Subroutines 24d>`
Uses `circe` 24d.

The following is usually not needed for scientific programs. Nobody is going to hijack such code. But let us include it anyway to spread the gospel of free software:

24b `<Copleft notice 24b>≡`
 `c Copyright (C) 1996-2010 by Thorsten Ohl <ohl@physik.uni-wuerzburg.de>`
 `c`
 `c Circe is free software; you can redistribute it and/or modify it`
 `c under the terms of the GNU General Public License as published by`
 `c the Free Software Foundation; either version 2, or (at your option)`
 `c any later version.`
 `c`
 `c Circe is distributed in the hope that it will be useful, but`
 `c WITHOUT ANY WARRANTY; without even the implied warranty of`
 `c MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the`
 `c GNU General Public License for more details.`
 `c`
 `c You should have received a copy of the GNU General Public License`
 `c along with this program; if not, write to the Free Software`
 `c Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.`

Now we can move on to the implementation.

6.1 Symbolic Constants

The file `circe.h` contains symbolic names for various magic constants used by `Circe`:

24c `<circe.h 24c>≡`
 `c circe.h -- canonical beam spectra for linear collider physics`
 `c $Id: circe.nw 67 2002-03-28 17:13:06Z ohl $`
 `<Copleft notice 24b>`
 `<Particle codes 9b>`
 `<Accelerator codes 10c>`
Uses `circe` 24d.

6.2 Distributions

6.2.1 Version 1

We start with a convenience function which dispatches over the valid particle types. The hardest part is of course to avoid typos in such trivial functions ...

24d `<Subroutines 24d>≡`


```

double precision function circe (x1, x2, p1, p2)
implicit none
double precision x1, x2
integer p1, p2
double precision circee, circeg, circgg
<Particle codes 9b>
</circom/ 25b>
<Initialization check 26e>
circe = -1.0
if (abs(p1) .eq. ELECTR) then
  if (abs(p2) .eq. ELECTR) then
    circe = circee (x1, x2)
  elseif (p2 .eq. PHOTON) then
    circe = circeg (x1, x2)
  endif
elseif (p1 .eq. PHOTON) then
  if (abs(p2) .eq. ELECTR) then
    circe = circeg (x2, x1)
  elseif (p2 .eq. PHOTON) then
    circe = circgg (x1, x2)
  endif
endif
end
end

```

Defines:

`circe`, used in chunks 24d, 9a, 24, 14, 24, 26f, 74, 78b, 93b, and 97e.

Uses `ELECTR` 9b, `PHOTON` 9b, `circee` 34a, `circeg` 35a, and `circgg` 35c.

```

25a <Subroutines 24d>+≡
subroutine circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
implicit none
double precision xx1m, xx2m, xroots
double precision beta
integer xacc, xver, xrev, xchat
<Accelerator codes 10c>
</circom/ 25b>
<Local variables for circes 27a>
<Initializations for circes 28c>
if (magic .ne. 1904 06 16) then
  magic = 1904 06 16
  <Initialize /circom/ 26f>
endif
<Update /circom/ 26g>
<formats for circes 31c>
end

```

Defines:

`circes`, used in chunks 25a, 10b, 13f, 25a, 17c, 25, 26e, 74, and 77b.

Uses `beta` 89a.

```

25b </circom/ 25b>≡
<parameter part of /circom/ 26d>
<8-byte aligned part of /circom/ 26a>
<4-byte aligned part of /circom/ 26b>

```

```

        save /circom/
26a  <8-byte aligned part of /circom/ 26a>≡
        double precision x1m, x2m, roots
        common /circom/ x1m, x2m, roots
26b  <4-byte aligned part of /circom/ 26b>≡
        integer acc, ver, rev, chat
        common /circom/ acc, ver, rev, chat

Instead of using fragile block data subroutines, we use a magic number to tag
/circom/ as initialized:
26c  <4-byte aligned part of /circom/ 26b>+≡
        integer magic
        common /circom/ magic
26d  <parameter part of /circom/ 26d>≡
        integer MAGIC0
        parameter (MAGIC0 = 1904 06 16)

Since negative values are no updated, we can call circes with all negative
variables to ensure initialization:
26e  <Initialization check 26e>≡
        if (magic .ne. MAGIC0) then
            call circes (-1d0, -1d0, -1d0, -1, -1, -1, -1)
        endif

Uses circes 25a.
26f  <Initialize /circom/ 26f>≡
        x1m = 0d0
        x2m = 0d0
        roots = 500D0
        acc = TESLA
        ver = 0
        rev = 0
        chat = 1
        if (xchat .ne. 0) then
            call circem ('MESSAGE', 'starting up ...')
            call circem ('MESSAGE',
$              '$Id: circe.nw 67 2002-03-28 17:13:06Z ohl $')
        endif

Uses TESLA 10c and circe 24d.
26g  <Update /circom/ 26g>≡
        if ((xchat .ge. 0) .and. (xchat .ne. chat)) then
            chat = xchat
            if (chat .ge. 1) then
                write (msgbuf, 1000) 'chat', chat
1000        format ('updating ', A, ''' to ', I2)
                call circem ('MESSAGE', msgbuf)
            endif
        else
            if (chat .ge. 2) then

```

```

        write (msgbuf, 1100) 'chat', chat
1100      format ('keeping ', A, ''' at ', I2)
        call circem ('MESSAGE', msgbuf)
      endif
    endif
27a  <Local variables for circes 27a>≡
      character*60 msgbuf
27b  <Update /circom/ 26g>+≡
      if ((xx1m .ge. 0d0) .and. (xx1m .ne. x1m)) then
        x1m = xx1m
        if (chat .ge. 1) then
          write (msgbuf, 1001) 'x1min', x1m
1001      format ('updating ', A, ''' to ', E12.4)
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1101) 'x1min', x1m
1101      format ('keeping ', A, ''' at ', E12.4)
          call circem ('MESSAGE', msgbuf)
        endif
      endif
27c  <Update /circom/ 26g>+≡
      if ((xx2m .ge. 0d0) .and. (xx2m .ne. x2m)) then
        x2m = xx2m
        if (chat .ge. 1) then
          write (msgbuf, 1001) 'x2min', x2m
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1101) 'x2min', x2m
          call circem ('MESSAGE', msgbuf)
        endif
      endif
27d  <Update /circom/ 26g>+≡
      if ((xroots .ge. 0d0) .and. (xroots .ne. roots)) then
        roots = xroots
        if (chat .ge. 1) then
          write (msgbuf, 1002) 'roots', roots
1002      format ('updating ', A, ''' to ', F6.1)
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1102) 'roots', roots
1102      format ('keeping ', A, ''' at ', F6.1)
          call circem ('MESSAGE', msgbuf)

```

```

        endif
    endif
28a  <Update /circom/ 26g>+≡
        if ((xacc .ge. 0) .and.(xacc .ne. acc)) then
            if ((xacc .ge. 1) .and. (xacc .le. NACC)) then
                acc = xacc
                if (chat .ge. 1) then
                    write (msgbuf, 1003) 'acc', accnam(acc)
1003        format ('updating ', A, ''' to ', A)
                    call circem ('MESSAGE', msgbuf)
                endif
            else
                write (msgbuf, 1203) xacc
1203        format ('invalid 'acc': ', I8)
                call circem ('ERROR', msgbuf)
                write (msgbuf, 1103) 'acc', accnam(acc)
1103        format ('keeping ', A, ''' at ', A)
                call circem ('MESSAGE', msgbuf)
            endif
        else
            if (chat .ge. 2) then
                write (msgbuf, 1003) 'acc', accnam(acc)
                call circem ('MESSAGE', msgbuf)
            endif
        endif
        if ((acc .eq. SBNDEE) .or. (acc .eq. TESLEE)
$         .or. (acc .eq. XBNDEE)) then
        <Warn that no parameter set has been endorsed for e-e- yet 28d>
        endif
    Uses NACC 10c.

28b  <Local variables for circes 27a>+≡
        character*6 accnam(NACC)
    Uses NACC 10c.

28c  <Initializations for circes 28c>≡
        data accnam(SBAND)  /'SBAND'/
        data accnam(TESLA)  /'TESLA'/
        data accnam(JLCNLC) /'JLCNLC'/
        data accnam(SBNDEE) /'SBNDEE'/
        data accnam(TESLEE) /'TESLEE'/
        data accnam(XBNDEE) /'XBNDEE'/
        data accnam(NLCH)  /'NLC H'/
    Uses SBAND 10c and TESLA 10c.

28d  <Warn that no parameter set has been endorsed for e-e- yet 28d>≡
        call circem ('WARNING', '*****')
        call circem ('WARNING', '* The accelerator parameters have *')
        call circem ('WARNING', '* not been endorsed for use in      *')
        call circem ('WARNING', '* an e-e- collider yet!!!      *')
        call circem ('WARNING', '*****')

```

```

29a  <Update /circom/ 26g>+≡
      if (xver .ge. 0) then
        ver = xver
        if (chat .ge. 1) then
          write (msgbuf, 1000) 'ver', ver
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1100) 'ver', ver
          call circem ('MESSAGE', msgbuf)
        endif
      endif

29b  <Update /circom/ 26g>+≡
      if ((xrev .ge. 0) .and.(xrev .ne. rev)) then
        rev = xrev
        if (chat .ge. 1) then
          write (msgbuf, 1004) 'rev', rev
1004      format ('updating ', A, ''' to ', I8)
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1104) 'rev', rev
1104      format ('keeping ', A, ''' at ', I8)
          call circem ('MESSAGE', msgbuf)
        endif
      endif

```

Versions 3 and 4 are identical to version 1, except for TESLA at 800 GeV.

```

29c  <Update /circom/ 26g>+≡
      ver34 = 0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
        <Update version 1 derived parameters in /circom/ 30d>
      elseif ((ver .eq. 3) .or. (ver .eq. 4)) then
        ver34 = ver
        ver = 1
        <Update version 3 and 4 derived parameters in /circom/ 43a>
      elseif (ver .eq. 5) then
        ver = 1
        <Update version 5 derived parameters in /circom/ 45a>
      elseif (ver .eq. 6) then
        ver = 1
        <Update version 6 derived parameters in /circom/ 46c>
      elseif (ver .eq. 7) then
        ver = 1
        <Update version 7 derived parameters in /circom/ 48d>
      elseif (ver .eq. 8) then
        ver = 1
        <Update version 8 derived parameters in /circom/ 53a>

```

```

        elseif (ver .eq. 9) then
            ver = 1
            <Update version 9 derived parameters in /circom/ 55a>
            <else handle invalid versions 30b>
30a  <Local variables for circes 27a>+≡
            integer ver34
30b  <else handle invalid versions 30b>≡
            elseif (ver .eq. 2) then
                <Version 2 has been retired 42b>
                elseif (ver .gt. 9) then
                    call circem ('PANIC', 'versions >9 not available yet')
                    return
                else
                    call circem ('PANIC', 'version must be positive')
                    return
            endif
30c  <4-byte aligned part of /circom/ 26b>+≡
            integer e, r, ehi, elo
            common /circom/ e, r, ehi, elo
30d  <Update version 1 derived parameters in /circom/ 30d>≡
            if (rev .eq. 0) then
                r = 0
            elseif (rev .ge. 1997 04 17) then
                r = 5
            elseif (rev .ge. 1996 09 02) then
                r = 4
            elseif (rev .ge. 1996 07 29) then
                r = 3
            elseif (rev .ge. 1996 07 11) then
                r = 2
            elseif (rev .ge. 1996 04 01) then
                r = 1
            elseif (rev .lt. 1996 04 01) then
                call circem ('ERROR',
$          'no revision of version 1 before 96/04/01 available')
                call circem ('MESSAGE', 'falling back to default')
                r = 1
            endif
            if (chat .ge. 2) then
                write (msgbuf, 2000) rev, r
2000  format ('mapping date ', I8, ' to revision index ', I2)
                call circem ('MESSAGE', msgbuf)
            endif
30e  <Log revision mapping 30e>≡
            if (chat .ge. 2) then
                write (msgbuf, 2000) rev, r
                call circem ('MESSAGE', msgbuf)
            endif

```

```

31a  <Update version 1 derived parameters in /circom/ 30d>+=
      <Map roots to e 31b>
31b  <Map roots to e 31b>=
      if (roots .eq. 350d0) then
        e = GEV350
      elseif ((roots .ge. 340d0) .and. (roots .le. 370d0)) then
        write (msgbuf, 2001) roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350
31c  <formats for circses 31c>=
      2001 format ('treating energy ', F6.1, 'GeV as ', F6.1, 'GeV')
31d  <Map roots to e 31b>+=
      elseif (roots .eq. 500d0) then
        e = GEV500
      elseif ((roots .ge. 480d0) .and. (roots .le. 520d0)) then
        write (msgbuf, 2001) roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
      elseif (roots .eq. 800d0) then
        e = GEV800
      elseif ((roots .ge. 750d0) .and. (roots .le. 850d0)) then
        write (msgbuf, 2001) roots, 800d0
        call circem ('MESSAGE', msgbuf)
        e = GEV800
      elseif (roots .eq. 1000d0) then
        e = TEV1
      elseif ((roots .ge. 900d0) .and. (roots .le. 1100d0)) then
        write (msgbuf, 2001) roots, 1000d0
        call circem ('MESSAGE', msgbuf)
        e = TEV1
      elseif (roots .eq. 1600d0) then
        e = TEV16
      elseif ((roots .ge. 1500d0) .and. (roots .le. 1700d0)) then
        write (msgbuf, 2001) roots, 1600d0
        call circem ('MESSAGE', msgbuf)
        e = TEV16
31e  <Map roots to e 31b>+=
      else
        call circem ('ERROR',
$          'only ROOTS = 350, 500, 800, 1000 and 1600GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
      endif
31f  <Update version 1 derived parameters in /circom/ 30d>+=
      if (xallum(e,acc,r) .lt. 0d0) then
        write (msgbuf, 2002) roots, accnam(acc), r
        call circem ('ERROR', msgbuf)

```

```

        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    <Log energy mapping 32b>
32a <formats for circes 31c>+=
    2002 format ('energy ', F6.1, ' not available for ', A6,
        $          ' in revision ', I2)
32b <Log energy mapping 32b>=
    if (chat .ge. 2) then
        if (e .ge. GEV090) then
            write (msgbuf, 2003) roots, e
            call circem ('MESSAGE', msgbuf)
        else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
            write (msgbuf, 2013) roots, elo, ehi
            call circem ('MESSAGE', msgbuf)
        end if
    endif
32c <formats for circes 31c>+=
    2003 format ('mapping energy ', F6.1, ' to energy index ', I2)
    2013 format ('mapping energy ', F6.1, ' to energy indices ',
        $          I2, ' and ', I2)

    The energies 250 GeV, 1.2 TeV and 1.5 TeV were entered late into the game by
    teh SLAC people.
32d <Local variables for circes 27a>+=
    integer GEV090, GEV170, GEV350, GEV500, GEV800, TEV1, TEV16
    integer GEV250, TEV12, TEV15
    parameter (GEV090 = -1, GEV170 = 0, GEV350 = 1, GEV500 = 2,
        $          GEV800 = 3, TEV1 = 4, TEV16 = 5,
        $          GEV250 = 6, TEV12 = 7, TEV15 = 8)
    integer A1NEGY, A1NREV
    parameter (A1NEGY = 5, A1NREV = 5)
    integer i
32e <8-byte aligned part of /circom/ 26a>+=
    double precision lumi
    common /circom/ lumi
    double precision a1(0:7)
    common /circom/ a1
32f <Update version 1 derived parameters in /circom/ 30d>+=
    lumi = xallum(e,acc,r)
    do 10 i = 0, 7
        a1(i) = xa1(i,e,acc,r)
    10 continue
32g <Local variables for circes 27a>+=
    real xallum(A1NEGY,NACC,0:A1NREV)
    real xa1(0:7,A1NEGY,NACC,0:A1NREV)

    Uses NACC 10c.

```


Revision 1. The mother of all revisions.

33a *<Initializations for circes 28c>+≡*
 data xallum(GEV500,SBAND,1) / 5.212299E+01 /
 data (xa1(i,GEV500,SBAND,1),i=0,7) /
 \$.39192E+00, .66026E+00, .11828E+02, -.62543E+00,
 \$.52292E+00, -.69245E+00, .14983E+02, .65421E+00 /
 data xallum(GEV500,TESLA,1) / 6.066178E+01 /
 data (xa1(i,GEV500,TESLA,1),i=0,7) /
 \$.30196E+00, .12249E+01, .21423E+02, -.57848E+00,
 \$.68766E+00, -.69788E+00, .23121E+02, .78399E+00 /
 data xallum(GEV500,XBAND,1) / 5.884699E+01 /
 data (xa1(i,GEV500,XBAND,1),i=0,7) /
 \$.48594E+00, .52435E+00, .83585E+01, -.61347E+00,
 \$.30703E+00, -.68804E+00, .84109E+01, .44312E+00 /

Uses SBAND 10c, TESLA 10c, and XBAND 10c.

33b *<Initializations for circes 28c>+≡*
 data xallum(TEV1,SBAND,1) / 1.534650E+02 /
 data (xa1(i,TEV1,SBAND,1),i=0,7) /
 \$.24399E+00, .87464E+00, .66751E+01, -.56808E+00,
 \$.59295E+00, -.68921E+00, .94232E+01, .83351E+00 /
 data xallum(TEV1,TESLA,1) / 1.253381E+03 /
 data (xa1(i,TEV1,TESLA,1),i=0,7) /
 \$.39843E+00, .70097E+00, .11602E+02, -.61061E+00,
 \$.40737E+00, -.69319E+00, .14800E+02, .51382E+00 /
 data xallum(TEV1,XBAND,1) / 1.901783E+02 /
 data (xa1(i,TEV1,XBAND,1),i=0,7) /
 \$.32211E+00, .61798E+00, .28298E+01, -.54644E+00,
 \$.45674E+00, -.67301E+00, .41703E+01, .74536E+00 /

Uses SBAND 10c, TESLA 10c, and XBAND 10c.

Unavailable

33c *<Initializations for circes 28c>+≡*
 data (xallum(GEV350,i,1),i=1,NACC) / NACC*-1d0 /
 data (xallum(GEV800,i,1),i=1,NACC) / NACC*-1d0 /

Uses NACC 10c.

Unavailable as well

33d *<Initializations for circes 28c>+≡*
 data (xallum(GEV500,i,1),i=SBNDDEE,NACC) / 4*-1d0 /
 data (xallum(TEV1,i,1),i=SBNDDEE,NACC) / 4*-1d0 /

Uses NACC 10c.

No 1.6TeV parameters in this revision

33e *<Initializations for circes 28c>+≡*
 data (xallum(TEV16,i,1),i=1,NACC) / 7*-1d0 /

Uses NACC 10c.

33f *<Subroutines 24d>+≡*
 subroutine circel (1)
 implicit none
 double precision 1

```

</circom/ 25b>
  l = lumi
end

```

Defines:

`circel`, used in chunks 33f and 10a.

```

34a  <Subroutines 24d>+≡
      double precision function circee (x1, x2)
      implicit none
      double precision x1, x2
</circom/ 25b>
      double precision d1, d2
<Initialization check 26e>
      circee = -1.0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
<Calculate version 1 of the  $e^+e^-$  distribution 34b>
      <else handle invalid versions 30b>
      end

```

Defines:

`circee`, used in chunks 11–13, 15, 24d, and 34b.

The first version of the parametrization is factorized

$$D_{p_1 p_2}^{\alpha_1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha_1 \rho}(x_1) d_{p_2}^{\alpha_1 \rho}(x_2) \quad (15)$$

where the distributions are

$$d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha_1 \rho} \delta(1-x) + a_1^{\alpha_1 \rho} x^{a_2^{\alpha_1 \rho}} (1-x)^{a_3^{\alpha_1 \rho}} \quad (16)$$

$$d_\gamma(x) = a_4^{\alpha_1 \rho} x^{a_5^{\alpha_1 \rho}} (1-x)^{a_6^{\alpha_1 \rho}} \quad (17)$$

```

34b  <Calculate version 1 of the  $e^+e^-$  distribution 34b>≡
      if (x1 .eq. 1d0) then
        d1 = a1(0)
      elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
      elseif (x1 .eq. -1d0) then
        d1 = 1d0 - a1(0)
      else
        d1 = 0d0
      endif
      if (x2 .eq. 1d0) then
        d2 = a1(0)
      elseif (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = a1(1) * x2**a1(2) * (1d0 - x2)**a1(3)
      elseif (x2 .eq. -1d0) then
        d2 = 1d0 - a1(0)
      else
        d2 = 0d0
      endif
      circee = d1 * d2

```

Uses `circee` 34a.

```

35a  <Subroutines 24d>+≡
      double precision function circeg (x1, x2)
      implicit none
      double precision x1, x2
      </circom/ 25b>
      double precision d1, d2
      <Initialization check 26e>
      circeg = -1.0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
      <Calculate version 1 of the  $e^\pm\gamma$  distribution 35b>
      <else handle invalid versions 30b>
      end

```

Defines:

circeg, used in chunks 11, 24d, and 35b.

```

35b  <Calculate version 1 of the  $e^\pm\gamma$  distribution 35b>≡
      if (x1 .eq. 1d0) then
      d1 = a1(0)
      elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
      d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
      elseif (x1 .eq. -1d0) then
      d1 = 1d0 - a1(0)
      else
      d1 = 0d0
      endif
      if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
      d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
      elseif (x2 .eq. -1d0) then
      d2 = a1(7)
      else
      d2 = 0d0
      endif
      circeg = d1 * d2

```

Uses circeg 35a.

```

35c  <Subroutines 24d>+≡
      double precision function circgg (x1, x2)
      implicit none
      double precision x1, x2
      </circom/ 25b>
      double precision d1, d2
      <Initialization check 26e>
      circgg = -1.0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
      <Calculate version 1 of the  $\gamma\gamma$  distribution 35d>
      <else handle invalid versions 30b>
      end

```

Defines:

circgg, used in chunks 11, 24d, 35d, 67, and 68a.

```

35d  <Calculate version 1 of the  $\gamma\gamma$  distribution 35d>≡
      if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then

```

```

        d1 = a1(4) * x1**a1(5) * (1d0 - x1)**a1(6)
    elseif (x1 .eq. -1d0) then
        d1 = a1(7)
    else
        d1 = 0d0
    endif
    if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
    elseif (x2 .eq. -1d0) then
        d2 = a1(7)
    else
        d2 = 0d0
    endif
    circgg = d1 * d2

```

Uses circgg 35c.

Revision 2. New Tesla parameters, including 350 GeV and 800 GeV.

```

36  <Initializations for circes 28c>+≡
    data xallum(GEV500,SBAND,2) / .31057E+02 /
    data (xa1(i,GEV500,SBAND,2),i=0,7) /
    $ .38504E+00, .79723E+00, .14191E+02, -.60456E+00,
    $ .53411E+00, -.68873E+00, .15105E+02, .65151E+00 /
    data xallum(TEV1,SBAND,2) / .24297E+03 /
    data (xa1(i,TEV1,SBAND,2),i=0,7) /
    $ .24374E+00, .89466E+00, .70242E+01, -.56754E+00,
    $ .60910E+00, -.68682E+00, .96083E+01, .83985E+00 /
    data xallum(GEV350,TESLA,2) / .73369E+02 /
    data (xa1(i,GEV350,TESLA,2),i=0,7) /
    $ .36083E+00, .12819E+01, .37880E+02, -.59492E+00,
    $ .69109E+00, -.69379E+00, .40061E+02, .65036E+00 /
    data xallum(GEV500,TESLA,2) / .10493E+03 /
    data (xa1(i,GEV500,TESLA,2),i=0,7) /
    $ .29569E+00, .11854E+01, .21282E+02, -.58553E+00,
    $ .71341E+00, -.69279E+00, .24061E+02, .77709E+00 /
    data xallum(GEV800,TESLA,2) / .28010E+03 /
    data (xa1(i,GEV800,TESLA,2),i=0,7) /
    $ .22745E+00, .11265E+01, .10483E+02, -.55711E+00,
    $ .69579E+00, -.69068E+00, .13093E+02, .89605E+00 /
    data xallum(TEV1,TESLA,2) / .10992E+03 /
    data (xa1(i,TEV1,TESLA,2),i=0,7) /
    $ .40969E+00, .66105E+00, .11972E+02, -.62041E+00,
    $ .40463E+00, -.69354E+00, .14669E+02, .51281E+00 /
    data xallum(GEV500,XBAND,2) / .35689E+02 /
    data (xa1(i,GEV500,XBAND,2),i=0,7) /
    $ .48960E+00, .46815E+00, .75249E+01, -.62769E+00,
    $ .30341E+00, -.68754E+00, .85545E+01, .43453E+00 /
    data xallum(TEV1,XBAND,2) / .11724E+03 /
    data (xa1(i,TEV1,XBAND,2),i=0,7) /
    $ .31939E+00, .62415E+00, .30763E+01, -.55314E+00,
    $ .45634E+00, -.67089E+00, .41529E+01, .73807E+00 /

```

Uses SBAND 10c, TESLA 10c, and XBAND 10c.

Unavailable

```
37a <Initializations for circes 28c>+≡
      data xallum(GEV350,SBAND,2) / -1d0 /
      data xallum(GEV350,XBAND,2) / -1d0 /
      data xallum(GEV800,SBAND,2) / -1d0 /
      data xallum(GEV800,XBAND,2) / -1d0 /
```

Uses SBAND 10c and XBAND 10c.

Unavailable as well

```
37b <Initializations for circes 28c>+≡
      data (xallum(GEV350,i,2),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(GEV500,i,2),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(GEV800,i,2),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(TEV1,i,2),i=SBNDDEE,NACC) / 4*-1d0 /
```

Uses NACC 10c.

No 1.6TeV parameters in this revision

```
37c <Initializations for circes 28c>+≡
      data (xallum(TEV16,i,2),i=1,NACC) / 7*-1d0 /
```

Uses NACC 10c.

Revision 3. Features:

- improved error estimates.
- cleaner fitting procedure, including delta function pieces.

```
37d <Initializations for circes 28c>+≡
      data xallum(GEV500,SBAND, 3) / .31469E+02 /
      data (xa1(i,GEV500,SBAND, 3),i=0,7) /
$ .38299E+00, .72035E+00, .12618E+02, -.61611E+00,
$ .51971E+00, -.68960E+00, .15066E+02, .63784E+00 /
      data xallum(TEV1, SBAND, 3) / .24566E+03 /
      data (xa1(i,TEV1, SBAND, 3),i=0,7) /
$ .24013E+00, .95763E+00, .69085E+01, -.55151E+00,
$ .59497E+00, -.68622E+00, .94494E+01, .82158E+00 /
      data xallum(GEV350,TESLA, 3) / .74700E+02 /
      data (xa1(i,GEV350,TESLA, 3),i=0,7) /
$ .34689E+00, .12484E+01, .33720E+02, -.59523E+00,
$ .66266E+00, -.69524E+00, .38488E+02, .63775E+00 /
      data xallum(GEV500,TESLA, 3) / .10608E+03 /
      data (xa1(i,GEV500,TESLA, 3),i=0,7) /
$ .28282E+00, .11700E+01, .19258E+02, -.58390E+00,
$ .68777E+00, -.69402E+00, .23638E+02, .75929E+00 /
      data xallum(GEV800,TESLA, 3) / .28911E+03 /
      data (xa1(i,GEV800,TESLA, 3),i=0,7) /
$ .21018E+00, .12039E+01, .96763E+01, -.54024E+00,
$ .67220E+00, -.69083E+00, .12733E+02, .87355E+00 /
      data xallum(TEV1, TESLA, 3) / .10936E+03 /
      data (xa1(i,TEV1, TESLA, 3),i=0,7) /
$ .41040E+00, .68099E+00, .11610E+02, -.61237E+00,
```

```

$ .40155E+00, -.69073E+00, .14698E+02, .49989E+00 /
data xallum(GEV500,XBAND, 3) / .36145E+02 /
data (xa1(i,GEV500,XBAND, 3),i=0,7) /
$ .51285E+00, .45812E+00, .75135E+01, -.62247E+00,
$ .30444E+00, -.68530E+00, .85519E+01, .43062E+00 /
data xallum(TEV1, XBAND, 3) / .11799E+03 /
data (xa1(i,TEV1, XBAND, 3),i=0,7) /
$ .31241E+00, .61241E+00, .29938E+01, -.55848E+00,
$ .44801E+00, -.67116E+00, .41119E+01, .72753E+00 /

```

Uses SBAND 10c, TESLA 10c, and XBAND 10c.

Still unavailable

```

38a <Initializations for circes 28c>+≡
      data xallum(GEV350,SBAND,3) / -1d0 /
      data xallum(GEV350,XBAND,3) / -1d0 /
      data xallum(GEV800,SBAND,3) / -1d0 /
      data xallum(GEV800,XBAND,3) / -1d0 /

```

Uses SBAND 10c and XBAND 10c.

Unavailable as well

```

38b <Initializations for circes 28c>+≡
      data (xallum(GEV350,i,3),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(GEV500,i,3),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(GEV800,i,3),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(TEV1,i,3),i=SBNDDEE,NACC) / 4*-1d0 /

```

Uses NACC 10c.

No 1.6TeV parameters in this revision

```

38c <Initializations for circes 28c>+≡
      data (xallum(TEV16,i,3),i=1,NACC) / 7*-1d0 /

```

Uses NACC 10c.

Revision 4. Features:

- a bug in Guinea-Pig's synchrotron radiation spectrum has been fixed.

```

38d <Initializations for circes 28c>+≡
      data xallum(GEV500,SBAND, 4) / .31528E+02 /
      data (xa1(i,GEV500,SBAND, 4),i=0,7) /
$ .38169E+00, .73949E+00, .12543E+02, -.61112E+00,
$ .51256E+00, -.69009E+00, .14892E+02, .63314E+00 /
      data xallum(TEV1, SBAND, 4) / .24613E+03 /
      data (xa1(i,TEV1, SBAND, 4),i=0,7) /
$ .24256E+00, .94117E+00, .66775E+01, -.55160E+00,
$ .57484E+00, -.68891E+00, .92271E+01, .81162E+00 /
      data xallum(GEV350,TESLA, 4) / .74549E+02 /
      data (xa1(i,GEV350,TESLA, 4),i=0,7) /
$ .34120E+00, .12230E+01, .32932E+02, -.59850E+00,
$ .65947E+00, -.69574E+00, .38116E+02, .63879E+00 /
      data xallum(GEV500,TESLA, 4) / .10668E+03 /
      data (xa1(i,GEV500,TESLA, 4),i=0,7) /
$ .28082E+00, .11074E+01, .18399E+02, -.59118E+00,
$ .68880E+00, -.69375E+00, .23463E+02, .76073E+00 /

```

```

data xallum(GEV800, TESLA, 4) / .29006E+03 /
data (xa1(i, GEV800, TESLA, 4), i=0, 7) /
$ .21272E+00, .11443E+01, .92564E+01, -.54657E+00,
$ .66799E+00, -.69137E+00, .12498E+02, .87571E+00 /
data xallum(TEV1, TESLA, 4) / .11009E+03 /
data (xa1(i, TEV1, TESLA, 4), i=0, 7) /
$ .41058E+00, .64745E+00, .11271E+02, -.61996E+00,
$ .39801E+00, -.69150E+00, .14560E+02, .49924E+00 /
data xallum(GEV500, XBAND, 4) / .36179E+02 /
data (xa1(i, GEV500, XBAND, 4), i=0, 7) /
$ .51155E+00, .43313E+00, .70446E+01, -.63003E+00,
$ .29449E+00, -.68747E+00, .83489E+01, .42458E+00 /
data xallum(TEV1, XBAND, 4) / .11748E+03 /
data (xa1(i, TEV1, XBAND, 4), i=0, 7) /
$ .32917E+00, .54322E+00, .28493E+01, -.57959E+00,
$ .39266E+00, -.68217E+00, .38475E+01, .68478E+00 /

```

Uses SBAND 10c, TESLA 10c, and XBAND 10c.

Still unavailable

39a *<Initializations for circes 28c>+≡*

```

data xallum(GEV350, SBAND, 4) / -1d0 /
data xallum(GEV350, XBAND, 4) / -1d0 /
data xallum(GEV800, SBAND, 4) / -1d0 /
data xallum(GEV800, XBAND, 4) / -1d0 /

```

Uses SBAND 10c and XBAND 10c.

Unavailable as well

39b *<Initializations for circes 28c>+≡*

```

data (xallum(GEV350, i, 4), i=SBNDDEE, NACC) / 4*-1d0 /
data (xallum(GEV500, i, 4), i=SBNDDEE, NACC) / 4*-1d0 /
data (xallum(GEV800, i, 4), i=SBNDDEE, NACC) / 4*-1d0 /
data (xallum(TEV1, i, 4), i=SBNDDEE, NACC) / 4*-1d0 /

```

Uses NACC 10c.

No 1.6TeV parameters in this revision

39c *<Initializations for circes 28c>+≡*

```

data (xallum(TEV16, i, 4), i=1, NACC) / 7*-1d0 /

```

Uses NACC 10c.

Revision 5. Features:

- a bug in Guinea-Pig has been fixed.
- updated parameter sets

39d *<Initializations for circes 28c>+≡*

```

data xallum(GEV350, SBAND, 5) / 0.21897E+02 /
data (xa1(i, GEV350, SBAND, 5), i=0, 7) /
$ 0.57183E+00, 0.53877E+00, 0.19422E+02, -0.63064E+00,
$ 0.49112E+00, -0.69109E+00, 0.24331E+02, 0.52718E+00 /
data xallum(GEV500, SBAND, 5) / 0.31383E+02 /
data (xa1(i, GEV500, SBAND, 5), i=0, 7) /
$ 0.51882E+00, 0.49915E+00, 0.11153E+02, -0.63017E+00,

```

```

$ 0.50217E+00, -0.69113E+00, 0.14935E+02, 0.62373E+00 /
data xallum(GEV800,SBAND, 5) / 0.95091E+02 /
data (xa1(i,GEV800,SBAND, 5),i=0,7) /
$ 0.47137E+00, 0.46150E+00, 0.56562E+01, -0.61758E+00,
$ 0.46863E+00, -0.68897E+00, 0.85876E+01, 0.67577E+00 /
data xallum(TEV1,SBAND, 5) / 0.11900E+03 /
data (xa1(i,TEV1,SBAND, 5),i=0,7) /
$ 0.43956E+00, 0.45471E+00, 0.42170E+01, -0.61180E+00,
$ 0.48711E+00, -0.68696E+00, 0.67145E+01, 0.74551E+00 /
data xallum(TEV16,SBAND, 5) / 0.11900E+03 /
data (xa1(i,TEV16,SBAND, 5),i=0,7) /
$ 0.43956E+00, 0.45471E+00, 0.42170E+01, -0.61180E+00,
$ 0.48711E+00, -0.68696E+00, 0.67145E+01, 0.74551E+00 /
data xallum(GEV350,TESLA, 5) / 0.97452E+02 /
data (xa1(i,GEV350,TESLA, 5),i=0,7) /
$ 0.39071E+00, 0.84996E+00, 0.17614E+02, -0.60609E+00,
$ 0.73920E+00, -0.69490E+00, 0.28940E+02, 0.77286E+00 /
data xallum(GEV500,TESLA, 5) / 0.10625E+03 /
data (xa1(i,GEV500,TESLA, 5),i=0,7) /
$ 0.42770E+00, 0.71457E+00, 0.15284E+02, -0.61664E+00,
$ 0.68166E+00, -0.69208E+00, 0.24165E+02, 0.73806E+00 /
data xallum(GEV800,TESLA, 5) / 0.17086E+03 /
data (xa1(i,GEV800,TESLA, 5),i=0,7) /
$ 0.36025E+00, 0.69118E+00, 0.76221E+01, -0.59440E+00,
$ 0.71269E+00, -0.69077E+00, 0.13117E+02, 0.91780E+00 /
data xallum(TEV1,TESLA, 5) / 0.21433E+03 /
data (xa1(i,TEV1,TESLA, 5),i=0,7) /
$ 0.33145E+00, 0.67075E+00, 0.55438E+01, -0.58468E+00,
$ 0.72503E+00, -0.69084E+00, 0.99992E+01, 0.10112E+01 /
data xallum(TEV16,TESLA, 5) / 0.34086E+03 /
data (xa1(i,TEV16,TESLA, 5),i=0,7) /
$ 0.49058E+00, 0.42609E+00, 0.50550E+01, -0.61867E+00,
$ 0.39225E+00, -0.68916E+00, 0.75514E+01, 0.58754E+00 /
data xallum(GEV350,XBAND, 5) / 0.31901E+02 /
data (xa1(i,GEV350,XBAND, 5),i=0,7) /
$ 0.65349E+00, 0.31752E+00, 0.94342E+01, -0.64291E+00,
$ 0.30364E+00, -0.68989E+00, 0.11446E+02, 0.40486E+00 /
data xallum(GEV500,XBAND, 5) / 0.36386E+02 /
data (xa1(i,GEV500,XBAND, 5),i=0,7) /
$ 0.65132E+00, 0.28728E+00, 0.69853E+01, -0.64440E+00,
$ 0.28736E+00, -0.68758E+00, 0.83227E+01, 0.41492E+00 /
data xallum(GEV800,XBAND, 5) / 0.10854E+03 /
data (xa1(i,GEV800,XBAND, 5),i=0,7) /
$ 0.49478E+00, 0.36221E+00, 0.30116E+01, -0.61548E+00,
$ 0.39890E+00, -0.68418E+00, 0.45183E+01, 0.67243E+00 /
data xallum(TEV1,XBAND, 5) / 0.11899E+03 /
data (xa1(i,TEV1,XBAND, 5),i=0,7) /
$ 0.49992E+00, 0.34299E+00, 0.26184E+01, -0.61584E+00,
$ 0.38450E+00, -0.68342E+00, 0.38589E+01, 0.67408E+00 /
data xallum(TEV16,XBAND, 5) / 0.13675E+03 /

```



```

data (xa1(i,TEV16,XBAND, 5),i=0,7) /
$ 0.50580E+00, 0.30760E+00, 0.18339E+01, -0.61421E+00,
$ 0.35233E+00, -0.68315E+00, 0.26708E+01, 0.67918E+00 /

```

Uses SBAND 10c, TESLA 10c, and XBAND 10c.

Revision 0. Features:

- e^-e^- mode

41a *<Initializations for circes 28c>+≡*

```

data xa1lum(GEV500,SBNDDEE, 0) / .92914E+01 /
data (xa1(i,GEV500,SBNDDEE, 0),i=0,7) /
$ .34866E+00, .78710E+00, .10304E+02, -.59464E+00,
$ .40234E+00, -.69741E+00, .20645E+02, .47274E+00 /
data xa1lum(TEV1, SBNDDEE, 0) / .45586E+02 /
data (xa1(i,TEV1, SBNDDEE, 0),i=0,7) /
$ .21084E+00, .99168E+00, .54407E+01, -.52851E+00,
$ .47493E+00, -.69595E+00, .12480E+02, .64027E+00 /
data xa1lum(GEV350,TESLEE, 0) / .15175E+02 /
data (xa1(i,GEV350,TESLEE, 0),i=0,7) /
$ .33093E+00, .11137E+01, .25275E+02, -.59942E+00,
$ .49623E+00, -.70403E+00, .60188E+02, .44637E+00 /
data xa1lum(GEV500,TESLEE, 0) / .21622E+02 /
data (xa1(i,GEV500,TESLEE, 0),i=0,7) /
$ .27175E+00, .10697E+01, .14858E+02, -.58418E+00,
$ .50824E+00, -.70387E+00, .36129E+02, .53002E+00 /
data xa1lum(GEV800,TESLEE, 0) / .43979E+02 /
data (xa1(i,GEV800,TESLEE, 0),i=0,7) /
$ .22994E+00, .10129E+01, .81905E+01, -.55751E+00,
$ .46551E+00, -.70461E+00, .19394E+02, .58387E+00 /
data xa1lum(TEV1, TESLEE, 0) / .25465E+02 /
data (xa1(i,TEV1, TESLEE, 0),i=0,7) /
$ .37294E+00, .67522E+00, .87504E+01, -.60576E+00,
$ .35095E+00, -.69821E+00, .18526E+02, .42784E+00 /
data xa1lum(GEV500,XBNDDEE, 0) / .13970E+02 /
data (xa1(i,GEV500,XBNDDEE, 0),i=0,7) /
$ .47296E+00, .46800E+00, .58897E+01, -.61689E+00,
$ .27181E+00, -.68923E+00, .10087E+02, .37462E+00 /
data xa1lum(TEV1, XBNDDEE, 0) / .41056E+02 /
data (xa1(i,TEV1, XBNDDEE, 0),i=0,7) /
$ .27965E+00, .74816E+00, .27415E+01, -.50491E+00,
$ .38320E+00, -.67945E+00, .47506E+01, .62218E+00 /

```

Still unavailable

41b *<Initializations for circes 28c>+≡*

```

data xa1lum(GEV350,SBNDDEE,0) / -1d0 /
data xa1lum(GEV350,XBNDDEE,0) / -1d0 /
data xa1lum(GEV800,SBNDDEE,0) / -1d0 /
data xa1lum(GEV800,XBNDDEE,0) / -1d0 /

```

41c *<Initializations for circes 28c>+≡*

```

data xa1lum(GEV500,SBAND, 0) / .31528E+02 /

```

```

data (xa1(i,GEV500,SBAND, 0),i=0,7) /
$ .38169E+00, .73949E+00, .12543E+02, -.61112E+00,
$ .51256E+00, -.69009E+00, .14892E+02, .63314E+00 /
data xallum(TEV1, SBAND, 0) / .24613E+03 /
data (xa1(i,TEV1, SBAND, 0),i=0,7) /
$ .24256E+00, .94117E+00, .66775E+01, -.55160E+00,
$ .57484E+00, -.68891E+00, .92271E+01, .81162E+00 /
data xallum(GEV350, TESLA, 0) / .74549E+02 /
data (xa1(i,GEV350, TESLA, 0),i=0,7) /
$ .34120E+00, .12230E+01, .32932E+02, -.59850E+00,
$ .65947E+00, -.69574E+00, .38116E+02, .63879E+00 /
data xallum(GEV500, TESLA, 0) / .10668E+03 /
data (xa1(i,GEV500, TESLA, 0),i=0,7) /
$ .28082E+00, .11074E+01, .18399E+02, -.59118E+00,
$ .68880E+00, -.69375E+00, .23463E+02, .76073E+00 /
data xallum(GEV800, TESLA, 0) / .29006E+03 /
data (xa1(i,GEV800, TESLA, 0),i=0,7) /
$ .21272E+00, .11443E+01, .92564E+01, -.54657E+00,
$ .66799E+00, -.69137E+00, .12498E+02, .87571E+00 /
data xallum(TEV1, TESLA, 0) / .11009E+03 /
data (xa1(i,TEV1, TESLA, 0),i=0,7) /
$ .41058E+00, .64745E+00, .11271E+02, -.61996E+00,
$ .39801E+00, -.69150E+00, .14560E+02, .49924E+00 /
data xallum(GEV500, XBAND, 0) / .36179E+02 /
data (xa1(i,GEV500, XBAND, 0),i=0,7) /
$ .51155E+00, .43313E+00, .70446E+01, -.63003E+00,
$ .29449E+00, -.68747E+00, .83489E+01, .42458E+00 /
data xallum(TEV1, XBAND, 0) / .11748E+03 /
data (xa1(i,TEV1, XBAND, 0),i=0,7) /
$ .32917E+00, .54322E+00, .28493E+01, -.57959E+00,
$ .39266E+00, -.68217E+00, .38475E+01, .68478E+00 /

```

Uses SBAND 10c, TESLA 10c, and XBAND 10c.

Still unavailable

42a *<Initializations for circes 28c>+≡*

```

data xallum(GEV350,SBAND,0) / -1d0 /
data xallum(GEV350,XBAND,0) / -1d0 /
data xallum(GEV800,SBAND,0) / -1d0 /
data xallum(GEV800,XBAND,0) / -1d0 /

```

Uses SBAND 10c and XBAND 10c.

6.2.2 Version 2

42b *<Version 2 has been retired 42b>≡*

```

call circem ('PANIC', '*****')
call circem ('PANIC', '* version 2 has been retired, *')
call circem ('PANIC', '* please use version 1 instead! *')
call circem ('PANIC', '*****')
return

```

6.2.3 Versions 3 and 4

```

43a  <Update version 3 and 4 derived parameters in /circom/ 43a>≡
      if (rev .eq. 0) then
        r = 0
      elseif (rev .ge. 1997 04 17) then
        r = 5
        if (ver34 .eq. 3) then
          call circem ('WARNING', 'version 3 retired after 97/04/17')
          call circem ('MESSAGE', 'falling back to version 4')
        endif
      elseif (rev .ge. 1996 10 22) then
        r = ver34
        if ((roots .ne. 800d0) .or. (acc .ne. TESLA)) then
          call circem ('ERROR', 'versions 3 and 4 before 97/04/17')
          call circem ('ERROR', 'apply to TESLA at 800 GeV only')
          call circem ('MESSAGE', 'falling back to TESLA at 800GeV')
          acc = TESLA
          e = GEV800
        endif
      elseif (rev .lt. 1996 10 22) then
        call circem ('ERROR',
$      'no revision of versions 3 and 4 available before 96/10/22')
        call circem ('MESSAGE', 'falling back to default')
        r = 5
      endif
      <Log revision mapping 30e>
      Uses TESLA 10c.

43b  <Update version 3 and 4 derived parameters in /circom/ 43a>+≡
      <Map roots to e 31b>
        if (xa3lum(e,acc,r) .lt. 0d0) then
          write (msgbuf, 2002) roots, accnam(acc), r
          call circem ('ERROR', msgbuf)
          call circem ('MESSAGE', 'falling back to 500GeV')
          e = GEV500
        endif
      <Log energy mapping 32b>

43c  <Local variables for circes 27a>+≡
        integer A3NEGY, A3NREV
        parameter (A3NEGY = 5, A3NREV = 5)

43d  <Update version 3 and 4 derived parameters in /circom/ 43a>+≡
        lumi = xa3lum (e,acc,r)
        do 20 i = 0, 7
          a1(i) = xa3(i,e,acc,r)
        20 continue

43e  <Local variables for circes 27a>+≡
        real xa3lum(A3NEGY,NACC,0:A3NREV)
        real xa3(0:7,A3NEGY,NACC,0:A3NREV)
      Uses NACC 10c.

```

Revisions 3 & 4. The mother of all revisions.

44a *<Initializations for circes 28c>+≡*
data xa3lum(GEV800, TESLA, 3) / .17196E+03 /
data (xa3(i, GEV800, TESLA, 3), i=0, 7) /
\$.21633E+00, .11333E+01, .95928E+01, -.55095E+00,
\$.73044E+00, -.69101E+00, .12868E+02, .94737E+00 /
data xa3lum(GEV800, TESLA, 4) / .16408E+03 /
data (xa3(i, GEV800, TESLA, 4), i=0, 7) /
\$.41828E+00, .72418E+00, .14137E+02, -.61189E+00,
\$.36697E+00, -.69205E+00, .17713E+02, .43583E+00 /

Uses TESLA 10c.

Revision 5.

44b *<Initializations for circes 28c>+≡*
data xa3lum(GEV350, TESLA, 5) / 0.66447E+02 /
data (xa3(i, GEV350, TESLA, 5), i=0, 7) /
\$ 0.69418E+00, 0.50553E+00, 0.48430E+02, -0.63911E+00,
\$ 0.34074E+00, -0.69533E+00, 0.55502E+02, 0.29397E+00 /
data xa3lum(GEV500, TESLA, 5) / 0.95241E+02 /
data (xa3(i, GEV500, TESLA, 5), i=0, 7) /
\$ 0.64882E+00, 0.45462E+00, 0.27103E+02, -0.64535E+00,
\$ 0.35101E+00, -0.69467E+00, 0.33658E+02, 0.35024E+00 /
data xa3lum(GEV800, TESLA, 5) / 0.16974E+03 /
data (xa3(i, GEV800, TESLA, 5), i=0, 7) /
\$ 0.58706E+00, 0.43771E+00, 0.13422E+02, -0.63804E+00,
\$ 0.35541E+00, -0.69467E+00, 0.17528E+02, 0.43051E+00 /
data xa3lum(TEV1, TESLA, 5) / 0.21222E+03 /
data (xa3(i, TEV1, TESLA, 5), i=0, 7) /
\$ 0.55525E+00, 0.42577E+00, 0.96341E+01, -0.63587E+00,
\$ 0.36448E+00, -0.69365E+00, 0.13161E+02, 0.47715E+00 /
data xa3lum(TEV16, TESLA, 5) / 0.34086E+03 /
data (xa3(i, TEV16, TESLA, 5), i=0, 7) /
\$ 0.49058E+00, 0.42609E+00, 0.50550E+01, -0.61867E+00,
\$ 0.39225E+00, -0.68916E+00, 0.75514E+01, 0.58754E+00 /

Uses TESLA 10c.

Revision 0. Currently identical to revision 5.

44c *<Initializations for circes 28c>+≡*
data xa3lum(GEV350, TESLA, 0) / 0.66447E+02 /
data (xa3(i, GEV350, TESLA, 0), i=0, 7) /
\$ 0.69418E+00, 0.50553E+00, 0.48430E+02, -0.63911E+00,
\$ 0.34074E+00, -0.69533E+00, 0.55502E+02, 0.29397E+00 /
data xa3lum(GEV500, TESLA, 0) / 0.95241E+02 /
data (xa3(i, GEV500, TESLA, 0), i=0, 7) /
\$ 0.64882E+00, 0.45462E+00, 0.27103E+02, -0.64535E+00,
\$ 0.35101E+00, -0.69467E+00, 0.33658E+02, 0.35024E+00 /
data xa3lum(GEV800, TESLA, 0) / 0.16974E+03 /
data (xa3(i, GEV800, TESLA, 0), i=0, 7) /
\$ 0.58706E+00, 0.43771E+00, 0.13422E+02, -0.63804E+00,
\$ 0.35541E+00, -0.69467E+00, 0.17528E+02, 0.43051E+00 /
data xa3lum(TEV1, TESLA, 0) / 0.21222E+03 /

```

      data (xa3(i,TEV1,TESLA, 0),i=0,7) /
$    0.55525E+00,  0.42577E+00,  0.96341E+01, -0.63587E+00,
$    0.36448E+00, -0.69365E+00,  0.13161E+02,  0.47715E+00 /
      data xa3lum(TEV16,TESLA, 0) / 0.34086E+03 /
      data (xa3(i,TEV16,TESLA, 0),i=0,7) /
$    0.49058E+00,  0.42609E+00,  0.50550E+01, -0.61867E+00,
$    0.39225E+00, -0.68916E+00,  0.75514E+01,  0.58754E+00 /

```

Uses TESLA 10c.

6.2.4 Version 5

```

45a  <Update version 5 derived parameters in /circom/ 45a>≡
      if (rev .eq. 0) then
        r = 0
      elseif (rev .ge. 1998 05 05) then
        r = 1
      elseif (rev .lt. 1998 05 05) then
        call circem ('ERROR',
$      'no revision of version 5 available before 98/05/05')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
      endif
      <Log revision mapping 30e>

45b  <Update version 5 derived parameters in /circom/ 45a>+≡
      if (acc .ne. TESLA) then
        call circem ('ERROR', 'versions 5 applies to TESLA only')
        acc = TESLA
      end if
      <Map roots to e 31b>
      if (xa5lum(e,acc,r) .lt. 0d0) then
        write (msgbuf, 2002) roots, accnam(acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
      endif
      <Log energy mapping 32b>
      Uses TESLA 10c.

45c  <Local variables for circes 27a>+≡
      integer A5NEGY, A5NREV
      parameter (A5NEGY = 5, A5NREV = 1)

45d  <Update version 5 derived parameters in /circom/ 45a>+≡
      lumi = xa5lum (e,acc,r)
      do 30 i = 0, 7
        a1(i) = xa5(i,e,acc,r)
      30 continue

45e  <Local variables for circes 27a>+≡
      real xa5lum(A5NEGY,NACC,0:A5NREV)
      real xa5(0:7,A5NEGY,NACC,0:A5NREV)

```

Uses NACC 10c.

Revision 1. The mother of all revisions. Note that $3.3980 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.4099 \cdot 10^{34} \text{ m}^{-2} \cdot 2820.5 \text{ s}^{-1}$ and $3.5936 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.6619 \cdot 10^{34} \text{ m}^{-2} \cdot 4500.3 \text{ s}^{-1}$. This unit conversion is missing in *all* earlier versions, unfortunately.

```
46a <Initializations for circes 28c>+≡
      data xa5lum(GEV350, TESLA, 1) / -1.0 /
      data xa5lum(GEV500, TESLA, 1) / 0.33980E+03 /
      data (xa5(i, GEV500, TESLA, 1), i=0, 7) /
$    0.49808E+00, 0.54613E+00, 0.12287E+02, -0.62756E+00,
$    0.42817E+00, -0.69120E+00, 0.17067E+02, 0.51143E+00 /
      data xa5lum(GEV800, TESLA, 1) / 0.35936E+03 /
      data (xa5(i, GEV800, TESLA, 1), i=0, 7) /
$    0.58751E+00, 0.43128E+00, 0.13324E+02, -0.64006E+00,
$    0.30682E+00, -0.69235E+00, 0.16815E+02, 0.37078E+00 /
      data xa5lum(TEV1, TESLA, 1) / -1.0 /
      data xa5lum(TEV16, TESLA, 1) / -1.0 /
```

Uses TESLA 10c.

Revision 0. Currently identical to revision 1.

```
46b <Initializations for circes 28c>+≡
      data xa5lum(GEV350, TESLA, 0) / -1.0 /
      data xa5lum(GEV500, TESLA, 0) / 0.33980E+03 /
      data (xa5(i, GEV500, TESLA, 0), i=0, 7) /
$    0.49808E+00, 0.54613E+00, 0.12287E+02, -0.62756E+00,
$    0.42817E+00, -0.69120E+00, 0.17067E+02, 0.51143E+00 /
      data xa5lum(GEV800, TESLA, 0) / 0.35936E+03 /
      data (xa5(i, GEV800, TESLA, 0), i=0, 7) /
$    0.58751E+00, 0.43128E+00, 0.13324E+02, -0.64006E+00,
$    0.30682E+00, -0.69235E+00, 0.16815E+02, 0.37078E+00 /
      data xa5lum(TEV1, TESLA, 0) / -1.0 /
      data xa5lum(TEV16, TESLA, 0) / -1.0 /
```

Uses TESLA 10c.

6.2.5 Version 6

```
46c <Update version 6 derived parameters in /circom/ 46c>≡
      if (rev .eq. 0) then
        r = 0
      elseif (rev .ge. 1999 04 15) then
        r = 1
      elseif (rev .lt. 1999 04 15) then
        call circem ('ERROR',
$      'no revision of version 6 available before 1999/04/15')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
      endif
      <Log revision mapping 30e>

46d <Update version 6 derived parameters in /circom/ 46c>+≡
      if (acc .ne. TESLA) then
```

```

        call circem ('ERROR', 'versions 6 applies to TESLA only')
        acc = TESLA
    end if
    <Map roots to e at low energies 47a>
    if (xa6lum(e,acc,r) .lt. 0d0) then
        write (msgbuf, 2002) roots, accnam(acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    <Log energy mapping 32b>
    Uses TESLA 10c.
47a <Map roots to e at low energies 47a>≡
    if (roots .eq. 90d0) then
        e = GEV090
    elseif ((roots .ge. 85d0) .and. (roots .le. 95d0)) then
        write (msgbuf, 2001) roots, 90d0
        call circem ('MESSAGE', msgbuf)
        e = GEV090
    elseif (roots .eq. 170d0) then
        e = GEV170
    elseif ((roots .ge. 160d0) .and. (roots .le. 180d0)) then
        write (msgbuf, 2001) roots, 170d0
        call circem ('MESSAGE', msgbuf)
        e = GEV170
    elseif (roots .eq. 350d0) then
        e = GEV350
    elseif ((roots .ge. 340d0) .and. (roots .le. 370d0)) then
        write (msgbuf, 2001) roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350
    elseif (roots .eq. 500d0) then
        e = GEV500
    elseif ((roots .ge. 480d0) .and. (roots .le. 520d0)) then
        write (msgbuf, 2001) roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    else
        call circem ('ERROR',
$           'only ROOTS = 90, 170, 350, and 500GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
47b <Local variables for circses 27a>+≡
    integer A6NEGY, A6NREV
    parameter (A6NEGY = 2, A6NREV = 1)
47c <Update version 6 derived parameters in /circom/ 46c>+≡
    lumi = xa6lum (e,acc,r)

```

```

do 40 i = 0, 7
  a1(i) = xa6(i,e,acc,r)
40 continue
48a <Local variables for circes 27a>+≡
      real xa6lum(GEV090:A6NEGY,NACC,0:A6NREV)
      real xa6(0:7,GEV090:A6NEGY,NACC,0:A6NREV)

```

Uses NACC 10c.

Revision 1. The mother of all revisions.

```

48b <Initializations for circes 28c>+≡
      data xa6lum(GEV090,TESLA, 1) / 0.62408E+02 /
      data (xa6(i,GEV090,TESLA, 1),i=0,7) /
$    0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$    0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
      data xa6lum(GEV170,TESLA, 1) / 0.11532E+02 /
      data (xa6(i,GEV170,TESLA, 1),i=0,7) /
$    0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$    0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
      data xa6lum(GEV350,TESLA, 1) / 0.24641E+03 /
      data (xa6(i,GEV350,TESLA, 1),i=0,7) /
$    0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$    0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /
      data xa6lum(GEV500,TESLA, 1) / 0.30340E+03 /
      data (xa6(i,GEV500,TESLA, 1),i=0,7) /
$    0.52744E+00, 0.52573E+00, 0.13895E+02, -0.63145E+00,
$    0.40824E+00, -0.69150E+00, 0.18645E+02, 0.47585E+00 /

```

Uses TESLA 10c.

Revision 0. Currently identical to revision 1.

```

48c <Initializations for circes 28c>+≡
      data xa6lum(GEV090,TESLA, 0) / 0.62408E+02 /
      data (xa6(i,GEV090,TESLA, 0),i=0,7) /
$    0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$    0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
      data xa6lum(GEV170,TESLA, 0) / 0.11532E+02 /
      data (xa6(i,GEV170,TESLA, 0),i=0,7) /
$    0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$    0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
      data xa6lum(GEV350,TESLA, 0) / 0.24641E+03 /
      data (xa6(i,GEV350,TESLA, 0),i=0,7) /
$    0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$    0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /
      data xa6lum(GEV500,TESLA, 0) / 0.30340E+03 /
      data (xa6(i,GEV500,TESLA, 0),i=0,7) /
$    0.52744E+00, 0.52573E+00, 0.13895E+02, -0.63145E+00,
$    0.40824E+00, -0.69150E+00, 0.18645E+02, 0.47585E+00 /

```

Uses TESLA 10c.

6.2.6 Version 7

```

48d <Update version 7 derived parameters in /circom/ 48d>≡

```



```

        if (rev .eq. 0) then
            r = 0
        elseif (rev .ge. 2000 04 26) then
            r = 1
        elseif (rev .lt. 2000 04 26) then
            call circem ('ERROR',
$           'no revision of version 7 available before 2000/04/26')
            call circem ('MESSAGE', 'falling back to default')
            r = 1
        endif
        <Log revision mapping 30e>
49a <Update version 7 derived parameters in /circom/ 48d>+=
        if (acc .ne. TESLA .and. acc .ne. JLCNLC) then
            call circem ('ERROR',
$           'version 7 applies to TESLA and JLCNLC only')
            call circem ('ERROR', 'falling back to TESLA')
            acc = TESLA
        end if
        <Linearly interpolate energies 49d>
        <Log energy mapping 32b>
        Uses TESLA 10c.
49b <formats for circles 31c>+=
        2004 format ('energy ', F6.1, 'GeV too low, using spectrum for ',
$           *           F6.1, 'GeV')
        2005 format ('energy ', F6.1, 'GeV too high, using spectrum for ',
$           *           F6.1, 'GeV')
        2006 format ('energy ', F6.1, 'GeV interpolated between ',
$           *           F6.1, ' and ', F6.1, 'GeV')
49c <Local variables for circles 27a>+=
        double precision eloal, ehival
        double precision DELTAE
        parameter (DELTAE = 0.5d0)

        The rules are as follows: XBAND has 500 GeV and 1 TeV, TESLA has 500 GeV and
        800 TeV. Low energy TESLA will be added.
49d <Linearly interpolate energies 49d>=
        e = GEV090 - 1
        elo = e
        ehi = e
        if (acc .eq. TESLA) then
            if (roots .lt. 90d0 - DELTAE) then
                write (msgbuf, 2004) roots, 90d0
                call circem ('MESSAGE', msgbuf)
                e = GEV090
            elseif (abs (roots-090d0) .le. DELTAE) then
                e = GEV090
            elseif (roots .lt. 170d0 - DELTAE) then
                write (msgbuf, 2005) roots, 170d0
                call circem ('MESSAGE', msgbuf)

```

```

        e = GEV170
    elseif (abs (roots-170d0) .le. DELTAE) then
        e = GEV170
    elseif (roots .lt. 350d0-DELTAE) then
        write (msgbuf, 2006) roots, 170d0, 350d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV170
        ehi = GEV350
        eloal = 170d0
        ehival = 350d0
    elseif (abs (roots-350d0) .le. DELTAE) then
        e = GEV350
    elseif (roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2006) roots, 350d0, 500d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV350
        ehi = GEV500
        eloal = 350d0
        ehival = 500d0
    elseif (abs (roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (roots .lt. 800d0 - DELTAE) then
        write (msgbuf, 2006) roots, 500d0, 800d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = GEV800
        eloal = 500d0
        ehival = 800d0
    elseif (abs (roots-800d0) .le. DELTAE) then
        e = GEV800
    else
        write (msgbuf, 2005) roots, 800d0
        call circem ('MESSAGE', msgbuf)
        e = GEV800
    endif
elseif (acc .eq. XBAND) then
    if (roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2004) roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    elseif (abs (roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (roots .lt. 1000d0 - DELTAE) then
        write (msgbuf, 2006) roots, 500d0, 1000d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = TEV1
        eloal = 500d0
        ehival = 1000d0
    elseif (abs (roots-1000d0) .le. DELTAE) then

```

```

        e = TEV1
    else
        write (msgbuf, 2005) roots, 1000d0
        call circem ('MESSAGE', msgbuf)
        e = TEV1
    endif
endif

```

Uses TESLA 10c and XBAND 10c.

51a *<Local variables for circes 27a>+≡*

```

        integer A7NEGY, A7NREV
        parameter (A7NEGY = TEV1, A7NREV = 1)

```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

51b *<Update version 7 derived parameters in /circom/ 48d>+≡*

```

        if (e .ge. GEV090) then
            lumi = xa7lum(e,acc,r)
            do 50 i = 0, 7
                a1(i) = xa7(i,e,acc,r)
50         continue
            elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
                lumi = ((roots-eloval)*xa7lum(ehi,acc,r)
$              + (ehival-roots)*xa7lum(elo,acc,r)) / (ehival - eloal)
            do 51 i = 1, 6
                a1(i) = ((roots-eloal)*xa7(i,ehi,acc,r)
$              + (ehival-roots)*xa7(i,elo,acc,r)) / (ehival - eloal)
51         continue
                a1(0) = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
                a1(7) = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
            endif

```

Uses beta 89a.

51c *<Local variables for circes 27a>+≡*

```

        real xa7lum(GEV090:A7NEGY,NACC,0:A7NREV)
        real xa7(0:7,GEV090:A7NEGY,NACC,0:A7NREV)

```

Uses NACC 10c.

Revision 1. The mother of all revisions.

51d *<Initializations for circes 28c>+≡*

```

        data xa7lum(GEV090,TESLA,1) / 0.62408E+02 /
        data (xa7(i,GEV090,TESLA,1),i=0,7) /
$ 0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$ 0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
        data xa7lum(GEV170,TESLA,1) / 0.11532E+02 /
        data (xa7(i,GEV170,TESLA,1),i=0,7) /
$ 0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$ 0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
        data xa7lum(GEV350,TESLA,1) / 0.24641E+03 /
        data (xa7(i,GEV350,TESLA,1),i=0,7) /
$ 0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$ 0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /

```

```

data xa7lum(GEV500, TESLA, 1) / 0.34704E+03 /
data (xa7(i, GEV500, TESLA, 1), i=0, 7) /
$ 0.51288E+00, 0.49025E+00, 0.99716E+01, -0.62850E+00,
$ 0.41048E+00, -0.69065E+00, 0.13922E+02, 0.51902E+00 /
data xa7lum(GEV800, TESLA, 1) / 0.57719E+03 /
data (xa7(i, GEV800, TESLA, 1), i=0, 7) /
$ 0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00,
$ 0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /
data xa7lum(TEV1, TESLA, 1) / -1.0 /

```

Uses TESLA 10c.

52a *<Initializations for circes 28c>+≡*

```

data xa7lum(GEV090, JLCNLC, 1) / -1.0 /
data xa7lum(GEV170, JLCNLC, 1) / -1.0 /
data xa7lum(GEV350, JLCNLC, 1) / -1.0 /
data xa7lum(GEV500, JLCNLC, 1) / 0.63039E+02 /
data (xa7(i, GEV500, JLCNLC, 1), i=0, 7) /
$ 0.58967E+00, 0.34035E+00, 0.63631E+01, -0.63683E+00,
$ 0.33383E+00, -0.68803E+00, 0.81005E+01, 0.48702E+00 /
data xa7lum(TEV1, JLCNLC, 1) / 0.12812E+03 /
data (xa7(i, TEV1, JLCNLC, 1), i=0, 7) /
$ 0.50222E+00, 0.33773E+00, 0.25681E+01, -0.61711E+00,
$ 0.36826E+00, -0.68335E+00, 0.36746E+01, 0.65393E+00 /

```

Revision 0.

52b *<Initializations for circes 28c>+≡*

```

data xa7lum(GEV090, TESLA, 0) / 0.62408E+02 /
data (xa7(i, GEV090, TESLA, 0), i=0, 7) /
$ 0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$ 0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
data xa7lum(GEV170, TESLA, 0) / 0.11532E+02 /
data (xa7(i, GEV170, TESLA, 0), i=0, 7) /
$ 0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$ 0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
data xa7lum(GEV350, TESLA, 0) / 0.24641E+03 /
data (xa7(i, GEV350, TESLA, 0), i=0, 7) /
$ 0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$ 0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /
data xa7lum(GEV500, TESLA, 0) / 0.34704E+03 /
data (xa7(i, GEV500, TESLA, 0), i=0, 7) /
$ 0.51288E+00, 0.49025E+00, 0.99716E+01, -0.62850E+00,
$ 0.41048E+00, -0.69065E+00, 0.13922E+02, 0.51902E+00 /
data xa7lum(GEV800, TESLA, 0) / 0.57719E+03 /
data (xa7(i, GEV800, TESLA, 0), i=0, 7) /
$ 0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00,
$ 0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /
data xa7lum(TEV1, TESLA, 0) / -1.0 /

```

Uses TESLA 10c.

52c *<Initializations for circes 28c>+≡*

```

data xa7lum(GEV090, JLCNLC, 0) / -1.0 /

```

```

data xa7lum(GEV170,JLCNLC,0) / -1.0 /
data xa7lum(GEV350,JLCNLC,0) / -1.0 /
data xa7lum(GEV500,JLCNLC,0) / 0.63039E+02 /
data (xa7(i,GEV500,JLCNLC,0),i=0,7) /
$ 0.58967E+00, 0.34035E+00, 0.63631E+01, -0.63683E+00,
$ 0.33383E+00, -0.68803E+00, 0.81005E+01, 0.48702E+00 /
data xa7lum(TEV1,JLCNLC,0) / 0.12812E+03 /
data (xa7(i,TEV1,JLCNLC,0),i=0,7) /
$ 0.50222E+00, 0.33773E+00, 0.25681E+01, -0.61711E+00,
$ 0.36826E+00, -0.68335E+00, 0.36746E+01, 0.65393E+00 /

```

6.2.7 Version 8

- 53a *<Update version 8 derived parameters in /circom/ 53a>≡*
- ```

if (rev .eq. 0) then
 r = 0
elseif (rev .ge. 2001 06 17) then
 r = 1
elseif (rev .lt. 2001 06 17) then
 call circem ('ERROR',
$ 'no revision of version 8 available before 2001/06/17')
 call circem ('MESSAGE', 'falling back to default')
 r = 1
endif
<Log revision mapping 30e>

```
- 53b *<Update version 8 derived parameters in /circom/ 53a>+≡*
- ```

if (acc .eq. NLCH) then
  acc = JLCNLC
end if
if (acc .ne. JLCNLC) then
  call circem ('ERROR',
$ 'version 8 applies to JLCNLC (NLC H) only')
  call circem ('ERROR', 'falling back to JLCNLC')
  acc = JLCNLC
end if
<Linearly interpolate energies 49d>
<Log energy mapping 32b>

```
- 53c *<Local variables for circes 27a>+≡*
- ```

integer A8NEGY, A8NREV
parameter (A8NEGY = TEV1, A8NREV = 1)

```
- Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!
- 53d *<Update version 8 derived parameters in /circom/ 53a>+≡*
- ```

if (e .ge. GEV090) then
  lumi = xa8lum(e,acc,r)
  do 60 i = 0, 7
    a1(i) = xa8(i,e,acc,r)
60 continue

```

```

elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
  lumi = ((roots-eloval)*xa8lum(ehi,acc,r)
$      + (ehival-roots)*xa8lum(elo,acc,r)) / (ehival - eloal)
  do 61 i = 1, 6
    a1(i) = ((roots-eloal)*xa8(i,ehi,acc,r)
$          + (ehival-roots)*xa8(i,elo,acc,r)) / (ehival - eloal)
61  continue
    a1(0) = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
    a1(7) = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
  endif

```

Uses beta 89a.

54a *<Local variables for circes 27a>+≡*

```

      real xa8lum(GEV090:A8NEGY,NACC,0:A8NREV)
      real xa8(0:7,GEV090:A8NEGY,NACC,0:A8NREV)

```

Uses NACC 10c.

Revision 1. The mother of all revisions.

54b *<Initializations for circes 28c>+≡*

```

      data xa8lum(GEV090,TESLA,1) / -1.0 /
      data xa8lum(GEV170,TESLA,1) / -1.0 /
      data xa8lum(GEV350,TESLA,1) / -1.0 /
      data xa8lum(GEV500,TESLA,1) / -1.0 /
      data xa8lum(GEV800,TESLA,1) / -1.0 /
      data xa8lum(TEV1, TESLA,1) / -1.0 /

```

Uses TESLA 10c.

54c *<Initializations for circes 28c>+≡*

```

      data xa8lum(GEV090,JLCNLC,1) / -1.0 /
      data xa8lum(GEV170,JLCNLC,1) / -1.0 /
      data xa8lum(GEV350,JLCNLC,1) / -1.0 /
      data xa8lum(GEV500,JLCNLC,1) / 0.239924E+03 /
      data (xa8(i,GEV500,JLCNLC,1),i=0,7) /
$ 0.57025E+00, 0.34004E+00, 0.52864E+01, -0.63405E+00,
$ 0.31627E+00, -0.68722E+00, 0.69629E+01, 0.47973E+00 /
      data xa8lum(TEV1,JLCNLC,1) / 0.40858E+03 /
      data (xa8(i,TEV1,JLCNLC,1),i=0,7) /
$ 0.52344E+00, 0.31536E+00, 0.25244E+01, -0.62215E+00,
$ 0.31935E+00, -0.68424E+00, 0.35877E+01, 0.57315E+00 /

```

Revision 0.

54d *<Initializations for circes 28c>+≡*

```

      data xa8lum(GEV090,TESLA,0) / -1.0 /
      data xa8lum(GEV170,TESLA,0) / -1.0 /
      data xa8lum(GEV350,TESLA,0) / -1.0 /
      data xa8lum(GEV500,TESLA,0) / -1.0 /
      data xa8lum(GEV800,TESLA,0) / -1.0 /
      data xa8lum(TEV1, TESLA,0) / -1.0 /

```

Uses TESLA 10c.

54e *<Initializations for circes 28c>+≡*

```

      data xa8lum(GEV090,JLCNLC,0) / -1.0 /

```

```

data xa8lum(GEV170,JLCNLC,0) / -1.0 /
data xa8lum(GEV350,JLCNLC,0) / -1.0 /
data xa8lum(GEV500,JLCNLC,0) / 0.239924E+03 /
data (xa8(i,GEV500,JLCNLC,0),i=0,7) /
$ 0.57025E+00, 0.34004E+00, 0.52864E+01, -0.63405E+00,
$ 0.31627E+00, -0.68722E+00, 0.69629E+01, 0.47973E+00 /
data xa8lum(TEV1,JLCNLC,0) / 0.40858E+03 /
data (xa8(i,TEV1,JLCNLC,0),i=0,7) /
$ 0.52344E+00, 0.31536E+00, 0.25244E+01, -0.62215E+00,
$ 0.31935E+00, -0.68424E+00, 0.35877E+01, 0.57315E+00 /

```

6.2.8 Version 9

```

55a <Update version 9 derived parameters in /circom/ 55a>≡
      if (rev .eq. 0) then
        r = 0
      elseif (rev .ge. 2002 03 28) then
        r = 1
      elseif (rev .lt. 2002 03 28) then
        call circem ('ERROR',
$      'no revision of version 9 available before 2002/03/28')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
      endif
      <Log revision mapping 30e>

55b <Update version 9 derived parameters in /circom/ 55a>+≡
      if (acc .ne. JLCNLC .and. acc .ne. NLCH) then
        call circem ('ERROR',
$      'version 9 applies to JLCNLC and NLCH only')
        call circem ('ERROR', 'falling back to JLCNLC')
        acc = JLCNLC
      end if
      if (acc .eq. JLCNLC) then
        <Linearly interpolate energies for JLC/NLC 2002 55c>
      else if (acc .eq. NLCH) then
        <Linearly interpolate energies for NLC H 2002 56>
      end if
      <Log energy mapping 32b>

55c <Linearly interpolate energies for JLC/NLC 2002 55c>≡
      e = GEV090 - 1
      elo = e
      ehi = e
      if (roots .lt. 250d0 - DELTAE) then
        write (msgbuf, 2004) roots, 250d0
        call circem ('MESSAGE', msgbuf)
        e = GEV250
      elseif (abs (roots-250d0) .le. DELTAE) then
        e = GEV250
      elseif (roots .lt. 500d0 - DELTAE) then

```

```

write (msgbuf, 2006) roots, 250d0, 500d0
call circem ('MESSAGE', msgbuf)
elo = GEV250
ehi = GEV500
eloval = 250d0
ehival = 500d0
elseif (abs (roots-500d0) .le. DELTAE) then
e = GEV500
elseif (roots .lt. 800d0 - DELTAE) then
write (msgbuf, 2006) roots, 500d0, 800d0
call circem ('MESSAGE', msgbuf)
elo = GEV500
ehi = GEV800
eloval = 500d0
ehival = 800d0
elseif (abs (roots-800d0) .le. DELTAE) then
e = GEV800
elseif (roots .lt. 1000d0 - DELTAE) then
write (msgbuf, 2006) roots, 800d0, 1000d0
call circem ('MESSAGE', msgbuf)
elo = GEV800
ehi = TEV1
eloval = 800d0
ehival = 1000d0
elseif (abs (roots-1000d0) .le. DELTAE) then
e = TEV1
elseif (roots .lt. 1200d0 - DELTAE) then
write (msgbuf, 2006) roots, 1000d0, 1200d0
call circem ('MESSAGE', msgbuf)
elo = TEV1
ehi = TEV12
eloval = 1000d0
ehival = 1200d0
elseif (abs (roots-1200d0) .le. DELTAE) then
e = TEV12
elseif (roots .lt. 1500d0 - DELTAE) then
write (msgbuf, 2006) roots, 1200d0, 1500d0
call circem ('MESSAGE', msgbuf)
elo = TEV12
ehi = TEV15
eloval = 1200d0
ehival = 1500d0
elseif (abs (roots-1500d0) .le. DELTAE) then
e = TEV15
else
write (msgbuf, 2005) roots, 1500d0
call circem ('MESSAGE', msgbuf)
e = TEV15
endif

```


56 *<Linearly interpolate energies for NLC H 2002 56>≡*

```

e = GEV090 - 1
elo = e
ehi = e
if (roots .lt. 500d0 - DELTAE) then
  write (msgbuf, 2004) roots, 500d0
  call circem ('MESSAGE', msgbuf)
  e = GEV500
elseif (abs (roots-500d0) .le. DELTAE) then
  e = GEV500
elseif (roots .lt. 1000d0 - DELTAE) then
  write (msgbuf, 2006) roots, 500d0, 1000d0
  call circem ('MESSAGE', msgbuf)
  elo = GEV500
  ehi = TEV1
  eloal = 500d0
  ehival = 1000d0
elseif (abs (roots-1000d0) .le. DELTAE) then
  e = TEV1
elseif (roots .lt. 1500d0 - DELTAE) then
  write (msgbuf, 2006) roots, 1000d0, 1500d0
  call circem ('MESSAGE', msgbuf)
  elo = TEV1
  ehi = TEV15
  eloal = 1000d0
  ehival = 1500d0
elseif (abs (roots-1500d0) .le. DELTAE) then
  e = TEV15
else
  write (msgbuf, 2005) roots, 1500d0
  call circem ('MESSAGE', msgbuf)
  e = TEV15
endif

```

57a *<Local variables for circes 27a>+≡*

```

integer A9NEGY, A9NREV
parameter (A9NEGY = TEV15, A9NREV = 1)

```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

57b *<Update version 9 derived parameters in /circom/ 55a>+≡*

```

if (e .ge. GEV090) then
  lumi = xa9lum(e,acc,r)
  do 70 i = 0, 7
    a1(i) = xa9(i,e,acc,r)
70  continue
elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
  lumi = ((roots-eloal)*xa9lum(ehi,acc,r)
$      + (ehival-roots)*xa9lum(elo,acc,r)) / (ehival - eloal)
  do 71 i = 1, 6
    a1(i) = ((roots-eloal)*xa9(i,ehi,acc,r)

```

```

$          + (ehival-roots)*xa9(i,elo,acc,r)) / (ehival - eloal)
71  continue
    a1(0) = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
    a1(7) = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
  endif

```

Uses beta 89a.

```

58a  <Local variables for circes 27a>+=
      real xa9lum(GEV090:A9NEGY,NACC,0:A9NREV)
      real xa9(0:7,GEV090:A9NEGY,NACC,0:A9NREV)

```

Uses NACC 10c.

Revision 1. The mother of all revisions.

```

58b  <Initializations for circes 28c>+=
      data xa9lum(GEV090,TESLA,1) / -1.0 /
      data xa9lum(GEV170,TESLA,1) / -1.0 /
      data xa9lum(GEV350,TESLA,1) / -1.0 /
      data xa9lum(GEV500,TESLA,1) / -1.0 /
      data xa9lum(GEV800,TESLA,1) / -1.0 /
      data xa9lum(TEV1, TESLA,1) / -1.0 /
      data xa9lum(TEV12, TESLA,1) / -1.0 /
      data xa9lum(TEV15, TESLA,1) / -1.0 /
      data xa9lum(TEV16, TESLA,1) / -1.0 /

```

Uses TESLA 10c.

```

58c  <Initializations for circes 28c>+=
      data xa9lum(GEV090,JLCNLC, 1) / -1.0 /
      data xa9lum(GEV170,JLCNLC, 1) / -1.0 /
      data xa9lum(GEV250,JLCNLC, 1) / 109.886976 /
      data (xa9(i,GEV250,JLCNLC, 1),i=0,7) /
$    0.65598E+00,  0.34993E+00,  0.13766E+02, -0.64698E+00,
$    0.29984E+00, -0.69053E+00,  0.16444E+02,  0.36060E+00 /
      data xa9lum(GEV350,JLCNLC, 1) / -1.0 /
      data xa9lum(GEV500,JLCNLC, 1) / 220.806144 /
      data (xa9(i,GEV500,JLCNLC, 1),i=0,7) /
$    0.57022E+00,  0.33782E+00,  0.52811E+01, -0.63540E+00,
$    0.32035E+00, -0.68776E+00,  0.69552E+01,  0.48751E+00 /
      data xa9lum(GEV800,JLCNLC, 1) / 304.63488 /
      data (xa9(i,GEV800,JLCNLC, 1),i=0,7) /
$    0.54839E+00,  0.31823E+00,  0.33071E+01, -0.62671E+00,
$    0.31655E+00, -0.68468E+00,  0.45325E+01,  0.53449E+00 /
      data xa9lum(TEV1, JLCNLC, 1) / 319.95648 /
      data (xa9(i,TEV1, JLCNLC, 1),i=0,7) /
$    0.56047E+00,  0.29479E+00,  0.28820E+01, -0.62856E+00,
$    0.29827E+00, -0.68423E+00,  0.39138E+01,  0.52297E+00 /
      data xa9lum(TEV12, JLCNLC, 1) / 349.90848 /
      data (xa9(i,TEV12, JLCNLC, 1),i=0,7) /
$    0.56102E+00,  0.28503E+00,  0.24804E+01, -0.62563E+00,
$    0.29002E+00, -0.68376E+00,  0.33854E+01,  0.52736E+00 /
      data xa9lum(TEV15, JLCNLC, 1) / 363.15648 /
      data (xa9(i,TEV15, JLCNLC, 1),i=0,7) /

```

```

$ 0.57644E+00, 0.26570E+00, 0.22007E+01, -0.62566E+00,
$ 0.27102E+00, -0.68283E+00, 0.29719E+01, 0.50764E+00 /
data xa9lum(TEV16, JLCNLC, 1) / -1.0 /

```

59a *<Initializations for circes 28c>+≡*

```

data xa9lum(GEV090,NLCH, 1) / -1.0 /
data xa9lum(GEV170,NLCH, 1) / -1.0 /
data xa9lum(GEV250,NLCH, 1) / -1.0 /
data xa9lum(GEV350,NLCH, 1) / -1.0 /
data xa9lum(GEV500,NLCH, 1) / 371.4624 /
data (xa9(i,GEV500,NLCH, 1),i=0,7) /
$ 0.33933E+00, 0.55165E+00, 0.29138E+01, -0.57341E+00,
$ 0.54323E+00, -0.68590E+00, 0.51786E+01, 0.88956E+00 /
data xa9lum(GEV800,NLCH, 1) / -1.0 /
data xa9lum(TEV1, NLCH, 1) / 516.41856 /
data (xa9(i,TEV1, NLCH, 1),i=0,7) /
$ 0.35478E+00, 0.46474E+00, 0.17666E+01, -0.56949E+00,
$ 0.49269E+00, -0.68384E+00, 0.31781E+01, 0.91121E+00 /
data xa9lum(TEV12, NLCH, 1) / -1.0 /
data xa9lum(TEV15, NLCH, 1) / 575.06688 /
data (xa9(i,TEV15, NLCH, 1),i=0,7) /
$ 0.38183E+00, 0.40310E+00, 0.13704E+01, -0.57742E+00,
$ 0.44548E+00, -0.68341E+00, 0.24956E+01, 0.87448E+00 /
data xa9lum(TEV16, NLCH, 1) / -1.0 /

```

Revision 0.

59b *<Initializations for circes 28c>+≡*

```

data xa9lum(GEV090,TESLA,0) / -1.0 /
data xa9lum(GEV170,TESLA,0) / -1.0 /
data xa9lum(GEV350,TESLA,0) / -1.0 /
data xa9lum(GEV500,TESLA,0) / -1.0 /
data xa9lum(GEV800,TESLA,0) / -1.0 /
data xa9lum(TEV1, TESLA,0) / -1.0 /
data xa9lum(TEV12, TESLA,0) / -1.0 /
data xa9lum(TEV15, TESLA,0) / -1.0 /
data xa9lum(TEV16, TESLA,0) / -1.0 /

```

Uses TESLA 10c.

59c *<Initializations for circes 28c>+≡*

```

data xa9lum(GEV090,JLCNLC, 0) / -1.0 /
data xa9lum(GEV170,JLCNLC, 0) / -1.0 /
data xa9lum(GEV250,JLCNLC, 0) / 109.886976 /
data (xa9(i,GEV250,JLCNLC, 0),i=0,7) /
$ 0.65598E+00, 0.34993E+00, 0.13766E+02, -0.64698E+00,
$ 0.29984E+00, -0.69053E+00, 0.16444E+02, 0.36060E+00 /
data xa9lum(GEV350,JLCNLC, 0) / -1.0 /
data xa9lum(GEV500,JLCNLC, 0) / 220.806144 /
data (xa9(i,GEV500,JLCNLC, 0),i=0,7) /
$ 0.57022E+00, 0.33782E+00, 0.52811E+01, -0.63540E+00,
$ 0.32035E+00, -0.68776E+00, 0.69552E+01, 0.48751E+00 /
data xa9lum(GEV800,JLCNLC, 0) / 304.63488 /

```

```

      data (xa9(i,GEV800,JLCNLC, 0),i=0,7) /
$    0.54839E+00,  0.31823E+00,  0.33071E+01, -0.62671E+00,
$    0.31655E+00, -0.68468E+00,  0.45325E+01,  0.53449E+00 /
      data xa9lum(TEV1, JLCNLC, 0) / 319.95648 /
      data (xa9(i,TEV1, JLCNLC, 0),i=0,7) /
$    0.56047E+00,  0.29479E+00,  0.28820E+01, -0.62856E+00,
$    0.29827E+00, -0.68423E+00,  0.39138E+01,  0.52297E+00 /
      data xa9lum(TEV12, JLCNLC, 0) / 349.90848 /
      data (xa9(i,TEV12, JLCNLC, 0),i=0,7) /
$    0.56102E+00,  0.28503E+00,  0.24804E+01, -0.62563E+00,
$    0.29002E+00, -0.68376E+00,  0.33854E+01,  0.52736E+00 /
      data xa9lum(TEV15, JLCNLC, 0) / 363.15648 /
      data (xa9(i,TEV15, JLCNLC, 0),i=0,7) /
$    0.57644E+00,  0.26570E+00,  0.22007E+01, -0.62566E+00,
$    0.27102E+00, -0.68283E+00,  0.29719E+01,  0.50764E+00 /
      data xa9lum(TEV16, JLCNLC, 0) / -1.0 /

```

60a *<Initializations for circes 28c>+≡*

```

      data xa9lum(GEV090,NLCH,  0) / -1.0 /
      data xa9lum(GEV170,NLCH,  0) / -1.0 /
      data xa9lum(GEV250,NLCH,  0) / -1.0 /
      data xa9lum(GEV350,NLCH,  0) / -1.0 /
      data xa9lum(GEV500,NLCH,  0) / 371.4624 /
      data (xa9(i,GEV500,NLCH,  0),i=0,7) /
$    0.33933E+00,  0.55165E+00,  0.29138E+01, -0.57341E+00,
$    0.54323E+00, -0.68590E+00,  0.51786E+01,  0.88956E+00 /
      data xa9lum(GEV800,NLCH,  0) / -1.0 /
      data xa9lum(TEV1, NLCH,  0) / 516.41856 /
      data (xa9(i,TEV1, NLCH,  0),i=0,7) /
$    0.35478E+00,  0.46474E+00,  0.17666E+01, -0.56949E+00,
$    0.49269E+00, -0.68384E+00,  0.31781E+01,  0.91121E+00 /
      data xa9lum(TEV12, NLCH,  0) / -1.0 /
      data xa9lum(TEV15, NLCH,  0) / 575.06688 /
      data (xa9(i,TEV15, NLCH,  0),i=0,7) /
$    0.38183E+00,  0.40310E+00,  0.13704E+01, -0.57742E+00,
$    0.44548E+00, -0.68341E+00,  0.24956E+01,  0.87448E+00 /
      data xa9lum(TEV16, NLCH,  0) / -1.0 /

```

6.3 Special Functions

60b *<Subroutines 24d>+≡*

```

      double precision function beta (a, b)
      implicit none
      double precision a, b
      double precision dlogam
      beta = exp (dlogam(a) + dlogam(b) - dlogam(a+b))
      end

```

Uses beta 89a.

60c *<Subroutines 24d>+≡*

```

CERNLIB C304
      DOUBLE PRECISION FUNCTION DLOGAM(X)
      IMPLICIT NONE
      DOUBLE PRECISION P1(7),Q1(7),P2(7),Q2(7),P3(7),Q3(7),C(5),XL(5)
      DOUBLE PRECISION X,Y,ZERO,ONE,TWO,HALF,AP,AQ
      INTEGER I
      DATA ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
      DATA XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
      DATA P1
1/+3.84287 36567 460D+0, +5.27068 93753 010D+1,
2 +5.55840 45723 515D+1, -2.15135 13573 726D+2,
3 -2.45872 61722 292D+2, -5.75008 93603 041D+1,
4 -2.33590 98949 513D+0/
      DATA Q1
1/+1.00000 00000 000D+0, +3.37330 47907 071D+1,
2 +1.93877 84034 377D+2, +3.08829 54973 424D+2,
3 +1.50068 39064 891D+2, +2.01068 51344 334D+1,
4 +4.57174 20282 503D-1/
      DATA P2
1/+4.87402 01396 839D+0, +2.48845 25168 574D+2,
2 +2.17973 66058 896D+3, +3.79751 24011 525D+3,
3 -1.97780 70769 842D+3, -3.69298 34005 591D+3,
4 -5.60177 73537 804D+2/
      DATA Q2
1/+1.00000 00000 000D+0, +9.50999 17418 209D+1,
2 +1.56120 45277 929D+3, +7.23400 87928 948D+3,
3 +1.04595 76594 059D+4, +4.16994 15153 200D+3,
4 +2.76785 83623 804D+2/
      DATA P3
1/-6.88062 40094 594D+3, -4.30699 69819 571D+5,
2 -4.75045 94653 440D+6, -2.94234 45930 322D+6,
3 +3.63218 04931 543D+7, -3.35677 82814 546D+6,
4 -2.48043 69488 286D+7/
      DATA Q3
1/+1.00000 00000 000D+0, -1.42168 29839 651D+3,
2 -1.55528 90280 854D+5, -3.41525 17108 011D+6,
3 -2.09696 23255 804D+7, -3.45441 75093 344D+7,
4 -9.16055 82863 713D+6/
      DATA C
1/ 1.12249 21356 561D-1, 7.95916 92961 204D-2,
1 -1.70877 94611 020D-3, 9.18938 53320 467D-1,
2 1.34699 05627 879D+0/
      IF(X .LE. XL(1)) THEN
        print *, 'ERROR: DLOGAM non positive argument: ', X
        DLOGAM=ZERO
      ENDIF
      IF(X .LE. XL(2)) THEN
        Y=X+ONE
        AP=P1(1)
        AQ=Q1(1)

```

```

DO 2 I = 2,7
  AP=P1(I)+Y*AP
  AQ=Q1(I)+Y*AQ
  Y=-LOG(X)+X*AP/AQ
ELSEIF(X .LE. XL(3)) THEN
  AP=P1(1)
  AQ=Q1(1)
DO 3 I = 2,7
  AP=P1(I)+X*AP
  AQ=Q1(I)+X*AQ
  Y=(X-ONE)*AP/AQ
ELSEIF(X .LE. XL(4)) THEN
  AP=P2(1)
  AQ=Q2(1)
DO 4 I = 2,7
  AP=P2(I)+X*AP
  AQ=Q2(I)+X*AQ
  Y=(X-TWO)*AP/AQ
ELSEIF(X .LE. XL(5)) THEN
  AP=P3(1)
  AQ=Q3(1)
DO 5 I = 2,7
  AP=P3(I)+X*AP
  AQ=Q3(I)+X*AQ
  Y=AP/AQ
ELSE
  Y=ONE/X**2
  Y=(X-HALF)*LOG(X)-X+C(4)+(C(1)+Y*(C(2)+Y*C(3)))/
1      ((C(5)+Y)*X)
ENDIF
DLOGAM=Y
END

```

6.4 Non-Singular Distributions

```

62  <Subroutines 24d>+≡
      double precision function kirke (x1, x2, p1, p2)
      implicit none
      double precision x1, x2
      integer p1, p2
      double precision kirkee, kirkeg, kirkgg
      <Particle codes 9b>
      </circom/ 25b>
      <Initialization check 26e>
      kirke = -1.0
      if (abs(p1) .eq. ELECTR) then
        if (abs(p2) .eq. ELECTR) then
          kirke = kirkee (x1, x2)
        elseif (p2 .eq. PHOTON) then

```

```

        kirke = kirkeg (x1, x2)
    endif
elseif (p1 .eq. PHOTON) then
    if (abs(p2) .eq. ELECTR) then
        kirke = kirkeg (x2, x1)
    elseif (p2 .eq. PHOTON) then
        kirke = kirkgg (x1, x2)
    endif
endif
end
end

```

Defines:

kirke, never used.

Uses ELECTR 9b, PHOTON 9b, kirkee 63a, kirkeg 65b, and kirkgg 65c.

63a *<Subroutines 24d>+≡*

```

double precision function kirkee (x1, x2)
implicit none
double precision x1, x2
</circom/ 25b>
double precision d1, d2
<Initialization check 26e>
kirkee = -1.0
if ((ver .eq. 1) .or. (ver .eq. 0)) then
    <Calculate version 1 of the non-singular e+e- distribution 64c>
    <else handle invalid versions 30b>
end

```

Defines:

kirkee, used in chunks 13e, 62, and 64c.

63b *<parameter part of /circom/ 26d>+≡*

```

double precision KIREPS
parameter (KIREPS = 1D-6)

```

63c *<8-byte aligned part of /circom/ 26a>+≡*

```

double precision elect0, gamma0
common /circom/ elect0, gamma0

```

$$\int_{1-\epsilon}^{1+} dx d_{e^{\pm}}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{1-\epsilon}^{1-} dx x^{a_2^{\alpha \rho}} (1-x)^{a_3^{\alpha \rho}} \quad (18)$$

Approximately

$$\int_{1-\epsilon}^{1+} dx d_{e^{\pm}}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{1-\epsilon}^{1-} dx (1-x)^{a_3^{\alpha \rho}} = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{0+}^{\epsilon} d\xi \xi^{a_3^{\alpha \rho}} \quad (19)$$

and therefore

$$\int_{1-\epsilon}^{1+} dx d_{e^{\pm}}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \frac{1 - \epsilon^{a_3^{\alpha \rho} + 1}}{a_3^{\alpha \rho} + 1} \quad (20)$$

This simple approximation is good enough

63d *<Update /circom/ 26g>+≡*

```

elect0 = a1(0) + a1(1) * KIREPS**(a1(3)+1) / (a1(3)+1)
elect0 = elect0 / KIREPS
gamma0 = a1(4) * KIREPS**(a1(5)+1) / (a1(5)+1)
gamma0 = gamma0 / KIREPS

```

but we can also use incomplete Beta functions for the exact result:

```
64a  <Alternative: Update /circom/ 64a>≡
      elect0 = a1(0) + a1(1) * beta (a1(2)+1, a1(3)+1)
      $
      * (1d0 - betinc (a1(2)+1, a1(3)+1, 1d0 - KIREPS))
      elect0 = elect0 / KIREPS
      gamma0 = a1(7) + a1(4) * beta (a1(5)+1, a1(6)+1)
      $
      * betinc (a1(5)+1, a1(6)+1, KIREPS)
      gamma0 = gamma0 / KIREPS
```

Uses beta 89a.

```
64b  <Local variables for circes 27a>+≡
      double precision betinc
      external betinc
```

```
64c  <Calculate version 1 of the non-singular  $e^+e^-$  distribution 64c>≡
      if (x1 .gt. 1d0) then
        d1 = 0d0
      elseif (x1 .ge. (1d0 - KIREPS)) then
        d1 = elect0
      elseif (x1 .ge. 0d0) then
        d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
      else
        d1 = 0d0
      endif
      if (x2 .gt. 1d0) then
        d2 = 0d0
      elseif (x2 .ge. (1d0 - KIREPS)) then
        d2 = elect0
      elseif (x2 .ge. 0d0) then
        d2 = a1(1) * x2**a1(2) * (1d0 - x2)**a1(3)
      else
        d2 = 0d0
      endif
      kirkee = d1 * d2
```

Uses kirkee 63a.

```
64d  <Calculate version 1 of the non-singular  $e^\pm\gamma$  distribution 64d>≡
      if (x1 .gt. 1d0) then
        d1 = 0d0
      elseif (x1 .ge. (1d0 - KIREPS)) then
        d1 = elect0
      elseif (x1 .ge. 0d0) then
        d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
      else
        d1 = 0d0
      endif
      if (x2 .gt. 1d0) then
        d2 = 0d0
      elseif (x2 .gt. KIREPS) then
        d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
      elseif (x2 .ge. 0d0) then
```



```

        d2 = gamma0
    else
        d2 = 0d0
    endif
    kirkeg = d1 * d2

```

Uses kirkeg 65b.

```

65a  <Calculate version 1 of the non-singular  $\gamma\gamma$  distribution 65a>≡
      if (x1 .gt. 1d0) then
        d1 = 0d0
      elseif (x1 .gt. KIREPS) then
        d1 = a1(4) * x1**a1(5) * (1d0 - x1)**a1(6)
      elseif (x1 .ge. 0d0) then
        d1 = gamma0
      else
        d1 = 0d0
      endif
      if (x2 .gt. 1d0) then
        d2 = 0d0
      elseif (x2 .gt. KIREPS) then
        d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
      elseif (x2 .ge. 0d0) then
        d2 = gamma0
      else
        d2 = 0d0
      endif
      kirkgg = d1 * d2

```

Uses kirkgg 65c.

```

65b  <Subroutines 24d>+≡
      double precision function kirkeg (x1, x2)
      implicit none
      double precision x1, x2
      </circom/ 25b>
      double precision d1, d2
      <Initialization check 26e>
      kirkeg = -1.0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
        <Calculate version 1 of the non-singular  $e^\pm\gamma$  distribution 64d>
      <else handle invalid versions 30b>
      end

```

Defines:

kirkeg, used in chunks 62 and 64d.

```

65c  <Subroutines 24d>+≡
      double precision function kirkgg (x1, x2)
      implicit none
      double precision x1, x2
      </circom/ 25b>
      double precision d1, d2
      <Initialization check 26e>

```

```

kirkgg = -1.0
if ((ver .eq. 1) .or. (ver .eq. 0)) then
  <Calculate version 1 of the non-singular  $\gamma\gamma$  distribution 65a>
  <else handle invalid versions 30b>
end

```

Defines:

kirkgg, used in chunks 62 and 65a.

66a *<Alternative: Subroutines 66a>*≡

```

double precision function betinc (a, b, x)
implicit none
double precision x, a, b
double precision bt, betacf, dlogam
external betacf, dlogam
if (x .lt. 0d0 .or. x .gt. 1d0) then
  betinc = 0d0
else
  if (x .eq. 0d0 .or. x .eq. 1d0) then
    bt = 0d0
  else
    bt = exp(dlogam(a+b)-dlogam(a)-dlogam(b)
$      + a*log(x) + b*log(1d0-x))
  endif
  if (x .lt. (a+1d0)/(a+b+2d0)) then
    betinc = bt*betacf (a, b, x) / a
  else
    betinc = 1d0 - bt*betacf (b, a, 1d0-x) / b
  endif
endif
end

```

66b *<Alternative: Subroutines 66a>*+≡

```

double precision function betacf (a, b, x)
implicit none
double precision x, a, b
integer ITMAX
double precision EPS
parameter (ITMAX = 100, EPS = 3D-7)
double precision am, bm, curr, prev, qab, qap, qam, bz,
$   ap, bp, app, bpp, em, tem, d
integer m
am = 1d0
bm = 1d0
curr = 1d0
qab = a + b
qap = a + 1d0
qam = a - 1d0
bz = 1d0 - qab * x / qap
do 10 m = 1, ITMAX
  em = m
  tem = 2*em

```

```

        d = em * (b - m) * x / ((qam + tem) * (a + tem))
        ap = curr + d*am
        bp = bz + d*bm
        d = - (a + em) * (qab + em) * x / ((a + tem) * (qap + tem))
        app = ap + d * curr
        bpp = bp + d * bz
        prev = curr
        am = ap / bpp
        bm = bp / bpp
        curr = app / bpp
        bz = 1d0
        if (abs (curr - prev) .lt. EPS * abs (curr)) then
            betacf = curr
            return
        endif
10 continue
    print *, 'betacf: failed to converge'
    betacf = 0d0
end

```

6.5 Generators

6.5.1 Version 1

Beta distributions have the practical advantage that they have been popular among mathematicians.[?]

```

67  <Subroutines 24d>+≡
        subroutine girce (x1, x2, p1, p2, rng)
        implicit none
        double precision x1, x2
        integer p1, p2
        external rng
        </circom/ 25b>
        double precision u, w, circgg
        <Particle codes 9b>
        <Initialization check 26e>
        <x1m, x2m kludge, part 1 68b>
        <Select particles p1 and p2 68a>
        if (abs(p1) .eq. ELECTR) then
            if (abs(p2) .eq. ELECTR) then
                call gircee (x1, x2, rng)
            elseif (p2 .eq. PHOTON) then
                call girceg (x1, x2, rng)
            endif
        elseif (p1 .eq. PHOTON) then
            if (abs(p2) .eq. ELECTR) then
                call girceg (x2, x1, rng)
            elseif (p2 .eq. PHOTON) then
                call gircgg (x1, x2, rng)
            endif
        endif
    end

```

```

endif
endif
<x1m, x2m kludge, part 2 68c>
end

```

Defines:

`girce`, used in chunks 67, 16a, and 67.

Uses `ELECTR` 9b, `PHOTON` 9b, `circgg` 35c, `gircee` 68d, `girceg` 69b, and `gircgg` 70a.

```

68a <Select particles p1 and p2 68a>≡
      w = 1d0 / (1d0 + circgg (-1d0, -1d0))
      call rng (u)
      if (u*u .le. w) then
        p1 = POSITR
      else
        p1 = PHOTON
      endif
      call rng (u)
      if (u*u .le. w) then
        p2 = ELECTR
      else
        p2 = PHOTON
      endif

```

Uses `ELECTR` 9b, `PHOTON` 9b, `POSITR` 9b, and `circgg` 35c.

The flavor selection is incorrect, because the relative weights depend on the minimum energy fractions. We resort to a moderately inefficient kludge, because we don't have the distribution functions available yet. We'll have to implement incomplete Beta functions and other horrible things for this. Fortunately, the efficiency can not drop below the relative contribution of e^+e^- .

```

68b <x1m, x2m kludge, part 1 68b>≡
      99 continue

```

Crude rejection:

```

68c <x1m, x2m kludge, part 2 68c>≡
      if ((x1 .lt. x1m) .or. (x2 .lt. x2m)) goto 99

```

```

68d <Subroutines 24d>+≡
      subroutine gircee (x1, x2, rng)
      implicit none
      double precision x1, x2
      external rng
      </circom/ 25b>
      double precision u, girceb
      <Initialization check 26e>
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
        <Generate version 1 of the  $e^+e^-$  distribution 69a>
      <else handle invalid versions 30b>
      end

```

Defines:

`gircee`, used in chunks 16, 68d, 18a, and 67.

Uses `girceb` 70b.

For version 1 of the parametrizations we rely on `girceb`, a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (21)$$

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1}(1-\xi)^{b-1} \quad (22)$$

```
69a  <Generate version 1 of the  $e^+e^-$  distribution 69a>≡
      call rng (u)
      if (u .le. a1(0)) then
        x1 = 1d0
      else
        x1 = 1d0 - girceb (0d0, 1d0-x1m, a1(3)+1d0, a1(2)+1d0, rng)
      endif
      call rng (u)
      if (u .le. a1(0)) then
        x2 = 1d0
      else
        x2 = 1d0 - girceb (0d0, 1d0-x2m, a1(3)+1d0, a1(2)+1d0, rng)
      endif
```

Uses `girceb` 70b.

```
69b  <Subroutines 24d>+≡
      subroutine girceg (x1, x2, rng)
      implicit none
      double precision x1, x2
      external rng
      </circom/ 25b>
      double precision u, girceb
      <Initialization check 26e>
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
        <Generate version 1 of the  $e^\pm\gamma$  distribution 69c>
        <else handle invalid versions 30b>
      end
```

Defines:

`girceg`, used in chunks 16c and 67.

Uses `girceb` 70b.

```
69c  <Generate version 1 of the  $e^\pm\gamma$  distribution 69c>≡
      call rng (u)
      if (u .le. a1(0)) then
        x1 = 1d0
      else
        x1 = 1d0 - girceb (0d0, 1d0-x1m, a1(3)+1d0, a1(2)+1d0, rng)
      endif
      x2 = girceb (x2m, 1d0, a1(5)+1d0, a1(6)+1d0, rng)
```

Uses `girceb` 70b.

```
69d  <Subroutines 24d>+≡
      subroutine gircgg (x1, x2, rng)
      implicit none
```

```

double precision x1, x2
external rng
</circom/ 25b>
double precision girceb
<Initialization check 26e>
if ((ver .eq. 1) .or. (ver .eq. 0)) then
  <Generate version 1 of the  $\gamma\gamma$  distribution 70a>
<else handle invalid versions 30b>
end

```

Uses girceb 70b and girceg 70a.

```

70a <Generate version 1 of the  $\gamma\gamma$  distribution 70a>≡
      x1 = girceb (x1m, 1d0, a1(5)+1d0, a1(6)+1d0, rng)
      x2 = girceb (x2m, 1d0, a1(5)+1d0, a1(6)+1d0, rng)

```

Defines:

girceg, used in chunks 16c, 67, and 69d.

Uses girceb 70b.

6.5.2 Version 2

Retired.

6.5.3 Version 3 and 4

Identical to version 1.

6.6 Utilities

For version 1 of the parametrizations we need a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max} - x)\Theta(x - x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (23)$$

with the *incomplete Beta-function* I :

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1} (1-\xi)^{b-1} \quad (24)$$

$$B(a, b) = I(0, a, b) \quad (25)$$

This problem has been studied extensively [?] and we can use an algorithm [18] that is very fast for $0 < a \leq 1 \leq b$, which turns out to be the case in our application.

```

70b <Subroutines 24d>+≡
      double precision function girceb (xmin, xmax, a, b, rng)
      implicit none
      double precision xmin, xmax, a, b
      external rng
      double precision t, p, u, umin, umax, x, w
      <Check a and b 71a>
      <Set up girceb parameters 71b>
10 continue
      <Generate a trial x and calculate its weight w 71c>

```

```

      call rng (u)
      if (w .le. u) goto 10
      girceb = x
      end

```

Defines:

girceb, used in chunks 68d, 70b, and 69–72.

In fact, this algorithm works for $0 < a \leq 1 \leq b$ only:

```

71a  <Check a and b 71a>≡
      if ((a .gt. 1d0) .or. (b .lt. 1d0)) then
        girceb = -1d0
        call circem ('ERROR', 'beta-distribution expects a<=1<=b')
        return
      endif

```

Uses girceb 70b.

The trick is to split the interval $[0, 1]$ into two parts $[0, t]$ and $[t, 1]$. In these intervals we obviously have

$$x^{a-1}(1-x)^{b-1} \leq \begin{cases} x^{a-1} & \text{for } x \leq t \\ t^{a-1}(1-x)^{b-1} & \text{for } x \geq t \end{cases} \quad (26)$$

because we have assumed that $0 < a \leq 1 \leq b$. The integrals of the two dominating distributions are t^a/a and $t^{a-1}(1-t)^b/b$ respectively and therefore the probability for picking a random number from the first interval is

$$P(x \leq t) = \frac{bt}{bt + a(1-t)^b} \quad (27)$$

We postpone the discussion of the choice of t until later:

```

71b  <Set up girceb parameters 71b>≡
      <Set up best value for t 73a>
      p = b*t / (b*t + a * (1d0 - t)**b)

```

The dominating distributions can be generated by simple mappings

$$\phi : [0, 1] \rightarrow [0, 1] \quad (28)$$

$$u \mapsto \begin{cases} t \left(\frac{u}{p}\right)^{\frac{1}{a}} & < t \text{ for } u < p \\ t & = t \text{ for } u = p \\ 1 - (1-t) \left(\frac{1-u}{1-p}\right)^{\frac{1}{b}} & > t \text{ for } u > p \end{cases} \quad (29)$$

The beauty of the algorithm is that we can use a single uniform deviate u for both intervals:

```

71c  <Generate a trial x and calculate its weight w 71c>≡
      call rng (u)
      u = umin + (umax - umin) * u
      if (u .le. p) then
        x = t * (u/p)**(1d0/a)
        w = (1d0 - x)**(b-1d0)
      else
        x = 1d0 - (1d0 - t) * ((1d0 - u)/(1d0 - p))**(1d0/b)
        w = (x/t)**(a-1d0)
      endif

```

The weights that are derived by dividing the distribution by the dominating distributions are already normalized correctly:

$$w : [0, 1] \rightarrow [0, 1] \quad (30)$$

$$x \mapsto \begin{cases} (1-x)^{b-1} & \in [(1-t)^{b-1}, 1] \text{ for } x \leq t \\ \left(\frac{x}{t}\right)^{a-1} & \in [t^{1-a}, 1] \text{ for } x \geq t \end{cases} \quad (31)$$

To derive $u_{\min, \max}$ from $x_{\min, \max}$ we can use ϕ^{-1} :

$$\phi^{-1} : [0, 1] \rightarrow [0, 1] \quad (32)$$

$$x \mapsto \begin{cases} p \left(\frac{x}{t}\right)^a & < p \text{ for } x < t \\ p & = p \text{ for } x = t \\ 1 - (1-p) \left(\frac{1-x}{1-t}\right)^b & > p \text{ for } x > t \end{cases} \quad (33)$$

We start with u_{\min} . For efficiency, we handle the most common cases (small x_{\min}) first:

```
72a <Set up girceb parameters 71b>+≡
      if (xmin .le. 0d0) then
        umin = 0d0
      elseif (xmin .lt. t) then
        umin = p * (xmin/t)**a
      elseif (xmin .eq. t) then
        umin = p
      elseif (xmin .lt. 1d0) then
        umin = 1d0 - (1d0 - p) * ((1d0 - xmin)/(1d0 - t))**b
      else
        umin = 1d0
      endif
```

Same procedure for u_{\max} ; again, handle the most common cases (large x_{\max}) first:

```
72b <Set up girceb parameters 71b>+≡
      if (xmax .ge. 1d0) then
        umax = 1d0
      elseif (xmax .gt. t) then
        umax = 1d0 - (1d0 - p) * ((1d0 - xmax)/(1d0 - t))**b
      elseif (xmax .eq. t) then
        umax = p
      elseif (xmax .gt. 0d0) then
        umax = p * (xmax/t)**a
      else
        umax = 0d0
      endif
```

Check for absurd cases.

```
72c <Set up girceb parameters 71b>+≡
      if (umax .lt. umin) then
        girceb = -1d0
        return
      endif
```

Uses girceb 70b.

It remains to choose the best value for t . The rejection efficiency ϵ of the algorithm is given by the ratio of the dominating distribution and the distribution

$$\frac{1}{\epsilon(t)} = \frac{B(a, b)}{ab} (bt^a + at^{a-1}(1-t)^b). \quad (34)$$

It is maximized for

$$bt - bt(1-t)^{b-1} + (a-1)(1-t)^b = 0 \quad (35)$$

This equation has a solution which can be determined numerically. While this determination is far too expensive compared to a moderate loss in efficiency, we could perform it once after fitting the coefficients a, b . Nevertheless, it has been shown,[18] that

$$t = \frac{1-a}{b+1-a} \quad (36)$$

results in non-vanishing efficiency for all values $1 < a \leq 1 \leq b$. Empirically we have found efficiencies of at least 80% for this choice, which is enough for our needs.

```

73a  <Set up best value for t 73a>≡
      t = (1d0 - a) / (b + 1d0 - a)

73b  <Subroutines 24d>+≡
      subroutine circem (errlvl, errmsg)
      implicit none
      character*(*) errlvl, errmsg
      </circum/ 25b>
      integer errcnt
      save errcnt
      data errcnt /0/
      if (errlvl .eq. 'MESSAGE') then
        print *, 'circe:message: ', errmsg
      elseif (errlvl .eq. 'WARNING') then
        if (errcnt .lt. 100) then
          errcnt = errcnt + 1
          print *, 'circe:warning: ', errmsg
        elseif (errcnt .eq. 100) then
          errcnt = errcnt + 1
          print *, 'circe:message: more than 100 messages'
          print *, 'circe:message: turning warnings off'
        endif
      elseif (errlvl .eq. 'ERROR') then
        if (errcnt .lt. 200) then
          errcnt = errcnt + 1
          print *, 'circe:error: ', errmsg
        elseif (errcnt .eq. 200) then
          errcnt = errcnt + 1
          print *, 'circe:message: more than 200 messages'
          print *, 'circe:message: turning error messages off'
        endif
      elseif (errlvl .eq. 'PANIC') then

```

```

        if (errcnt .lt. 300) then
            errcnt = errcnt + 1
            print *, 'circe:panic:      ', errmsg
        elseif (errcnt .eq. 300) then
            errcnt = errcnt + 1
            print *, 'circe:message: more than 300 messages'
            print *, 'circe:message: turning panic messages off'
        endif
    else
        print *, 'circe:panic:      invalid error code ', errlvl
    endif
end

```

6.7 Examples

6.7.1 Distributions

```

74 <cplot.f 74>≡
    program cplot
    implicit none
    <Particle codes 9b>
    double precision xmin, xmax, y, roots
    integer xory, nstep, p1, p2, acc, ver, rev
    double precision x, logx, d, circe
    read *, xory, xmin, xmax, nstep, y, p1, p2, roots, acc, ver, rev
    call circes (0d0, 0d0, roots, acc, ver, rev, 0)
    do 10 logx = log (xmin), log (xmax), log (xmax/xmin) / nstep
        x = exp (logx)
        d = 0d0
        if (xory .eq. 1) then
            if (p1 .eq. PHOTON) then
                d = circe (x, y, p1, p2)
            else
                d = circe (1d0 - x, y, p1, p2)
            endif
        elseif (xory .eq. 2) then
            if (p1 .eq. PHOTON) then
                d = circe (y, x, p1, p2)
            else
                d = circe (y, 1d0 - x, p1, p2)
            endif
        endif
        if (d .gt. 1d-4) print *, x, d
    10 continue
    end

```

Uses PHOTON 9b, circe 24d, and circes 25a.

6.7.2 Library functions

If Fortran77 only had first class functions, then the following cruft would not be necessary. OK, here's the outline of the adaptive Gauss integration routine from CERNLIB:

```
75a  <Part one of Gaussian integration 75a>≡
      double precision f, a, b, eps
      external f
      double precision Z1, HF, CST
      parameter (Z1 = 1, HF = Z1/2, CST = 5*Z1/1000)
      integer i
      double precision h, const, aa, bb, c1, c2, s8, s16, u
      <Gaussian weights 77a>
      h = 0
      if (b .eq. a) go to 99
      const = CST / dabs(b-a)
      bb = a
1 continue
      aa = bb
      bb = b
2 continue
      c1 = HF*(bb+aa)
      c2 = HF*(bb-aa)
      s8 = 0
      do 3 i = 1, 4
          u = c2*x(i)
```

Here are now the first two function calls that we have to fill in later in various ways:

```
75b  <Function call stub 75b>≡
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
```

Continuing

```
75c  <Part two of Gaussian integration 75c>≡
      3 continue
      s16 = 0
      do 4 i = 5, 12
          u = c2*x(i)
```

And here are the other two function calls:

```
75d  <Function call stub 75b>+≡
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
```

Terminating:

```
75e  <Part three of Gaussian integration 75e>≡
      4 continue
      s16 = c2*s16
      if (dabs(s16-c2*s8) .le. eps*(1+dabs(s16))) then
          h = h + s16
          if (bb .ne. b) go to 1
      else
          bb = c1
```

```

        if (1 + const*dabs(c2) .ne. 1) go to 2
        h = 0
        print *, 'gauss: too high accuracy required'
        go to 99
    end if
99 continue

```

This one is still reasonably straightforward

$$\text{gauss1} : (f, a, b) \mapsto \int_a^b dx f(x) \quad (37)$$

```

76a  <sample.f 12a>+≡
      double precision function gauss1 (f, a, b, eps)
      implicit none
      <Part one of Gaussian integration 75a>
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
      <Part two of Gaussian integration 75c>
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
      <Part three of Gaussian integration 75e>
      gauss1 = h
      end

```

Defines:

`gauss1`, used in chunks 76a, 12b, 15, and 76.

But this almost identical repeat

$$\text{gaussx} : (f, a, b) \mapsto \left(y \mapsto \int_a^b dx f(y, x) \right) \quad (38)$$

would not be necessary in a modern programming language with currying:

```

76b  <sample.f 12a>+≡
      double precision function gaussx (f, y, a, b, eps)
      implicit none
      double precision y
      <Part one of Gaussian integration 75a>
      s8 = s8 + w(i) * (f (y, c1+u) + f (y, c1-u))
      <Part two of Gaussian integration 75c>
      s16 = s16 + w(i) * (f (y, c1+u) + f (y, c1-u))
      <Part three of Gaussian integration 75e>
      gaussx = h
      end

```

Defines:

`gaussx`, used in chunk 76.

Fortunately, this is the last one we need

$$\begin{aligned} \text{gauss2} : (f, a, b, a_1, b_1) &\mapsto \int_a^b dx \int_{a_1}^{b_1} dy f(x, y) \\ &= \text{gauss1}(\text{gaussx}(f, a, b), a_1, b_1) \end{aligned} \quad (39)$$

```

76c  <sample.f 12a>+≡

```

```

double precision function gauss2 (f, a, b, a1, b1, eps)
implicit none
double precision a1, b1, gaussx
<Part one of Gaussian integration 75a>
s8 = s8 + w(i) * (gaussx (f, c1+u, a1, b1, eps)
$      + gaussx (f, c1-u, a1, b1, eps))
<Part two of Gaussian integration 75c>
s16 = s16 + w(i) * (gaussx (f, c1+u, a1, b1, eps)
$      + gaussx (f, c1-u, a1, b1, eps))
<Part three of Gaussian integration 75e>
gauss2 = h
end

```

Defines:

gauss2, used in chunks 76c, 12b, 13c, 15, and 76c.

Uses gaussx 76b.

77a *<Gaussian weights 77a>*≡

```

double precision w(12), x(12)
data x( 1) /9.6028985649753623d-1/, w( 1) /1.0122853629037626d-1/
data x( 2) /7.9666647741362674d-1/, w( 2) /2.2238103445337447d-1/
data x( 3) /5.2553240991632899d-1/, w( 3) /3.1370664587788729d-1/
data x( 4) /1.8343464249564980d-1/, w( 4) /3.6268378337836198d-1/
data x( 5) /9.8940093499164993d-1/, w( 5) /2.7152459411754095d-2/
data x( 6) /9.4457502307323258d-1/, w( 6) /6.2253523938647893d-2/
data x( 7) /8.6563120238783174d-1/, w( 7) /9.5158511682492785d-2/
data x( 8) /7.5540440835500303d-1/, w( 8) /1.2462897125553387d-1/
data x( 9) /6.1787624440264375d-1/, w( 9) /1.4959598881657673d-1/
data x(10) /4.5801677765722739d-1/, w(10) /1.6915651939500254d-1/
data x(11) /2.8160355077925891d-1/, w(11) /1.8260341504492359d-1/
data x(12) /9.5012509837637440d-2/, w(12) /1.8945061045506850d-1/

```

6.7.3 Generators

6.8 Dumping Parameters

77b *<params.f 77b>*≡

```

program params
implicit none
<Accelerator codes 10c>
integer acc, ver, i
double precision roots(7)
data roots / 90D0, 170D0, 350D0, 500D0, 800D0, 1000D0, 1500D0 /
do 10 ver = 7, 8
  print *, 'VERSION ', ver
  do 11 acc = TESLA, XBNDEE
    do 12 i = 1, 7
      print *, '=====',
      call circes (0d0, 0d0, roots(i), acc, ver, 20020307, 0)
      call dump ( )
12    continue

```

```

11    continue
10 continue
end

```

Uses TESLA 10c and circes 25a.

```

78a  <params.f 77b>+≡
      subroutine dump
      implicit none
      </circom/ 25b>
      <Accelerator codes 10c>
      character*9 name
      if (acc .eq. SBAND) then
        name = 'SBAND'
      else if (acc .eq. TESLA) then
        name = 'TESLA'
      else if (acc .eq. JLCNLC) then
        name = 'JLCNLC'
      else if (acc .eq. SBNDEE) then
        name = 'SBAND/EE'
      else if (acc .eq. TESLEE) then
        name = 'TESLA/EE'
      else if (acc .eq. XBNDEE) then
        name = 'JLCNLC/EE'
      end if
      write (*, 1000) name, roots
      write (*, 1001) 'e+ e-', lumi
      write (*, 1002) 'e+/e-', a1(0)
      write (*, 1003) 'e+/e-', 1 - a1(0)
      write (*, 1004) 'e+/e-', a1(1), a1(2), a1(3)
      write (*, 1003) 'gamma', a1(7)
      write (*, 1004) 'gamma', a1(4), a1(5), a1(6)
1000 format (A9, ' @ ', F5.0, ' GeV')
1001 format (4X, A7, ' lumi          = ', F7.2,
$          ' * 1032 cm-2 sec-1')
1002 format (4X, A7, ' delta strength = ', F9.5)
1003 format (4X, A7, ' integral(cont.) = ', F9.5)
1004 format (4X, A7, ' distribution   = ',
$          F9.5, ' * x{', F9.5, '}' * (1-x){', F9.5, '}'')
      end

```

Uses SBAND 10c and TESLA 10c.

7 Fitting

7.1 Version 1: Factorized Beta Distributions

```

78b  <fit_v1.f90 78b>≡
      c fit_v1.f90 -- fitting for circe

```

<Copyleft notice 24b>

```

program fit
implicit none
external fct
integer i, rcode
<Declare NPARAM 79a>
<Declare parameters 79b>
<Declare arguments 79c>
<Initialize parameters for fit_v1.f90 80a>
call mninit (5, 6, 7)
<Load parameters 79d>
call mnseti ('CIRCE: fit version 1      ')
argv(1) = 1
call mnexcm (fct, 'SET PRINTOUT      ', argv, 1, rcode, 0d0)
argv(1) = 1
call mnexcm (fct, 'CALL FCT          ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'MIGRAD            ', argv, 0, rcode, 0d0)
call mnexcm (fct, 'MINOS              ', argv, 0, rcode, 0d0)
argv(1) = 3
call mnexcm (fct, 'CALL FCT          ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'STOP                ', argv, 0, rcode, 0d0)
end

```

Defines:

fit, used in chunks 93d and 97d.

Uses circe 24d and fct 80b.

79a *<Declare NPARAM 79a>*≡
integer NPARAM
parameter (NPARAM = 6)

Defines:

NPARAM, used in chunks 79 and 86a.

79b *<Declare parameters 79b>*≡
integer pnun(NPARAM)
character*10 pname(NPARAM)
double precision pstart(NPARAM), pstep(NPARAM)

Uses NPARAM 79a.

79c *<Declare arguments 79c>*≡
integer ARGC
parameter (ARGC = 10)
double precision argv(ARGC)

79d *<Load parameters 79d>*≡
do 10 i = 1, NPARAM
call mnparm (pnun(i), pname(i), pstart(i), pstep (i),
\$ 0d0, 0d0, rcode)
if (rcode .ne. 0) then
print *, 'fit: MINUIT won't accept parameter ', pnun(i)
stop
endif
10 continue

Uses NPARAM 79a.

```

80a  <Initialize parameters for fit_v1.f90 80a>≡
      data pnum   /      1,      2,      3,      4,      5,      6 /
      data pname / '1_e', 'x_e', '1-x_e', '1_g', 'x_g', '1-x_g' /
      data pstart / -1.00, 20.00,  0.20, -1.00,  0.20,  20.00 /
      data pstep  /  0.01,  0.01,  0.01,  0.01,  0.01,  0.01 /

80b  <fit_v1.f90 78b>+≡
      subroutine fct (nx, df, f, a, mode, g)
      implicit none
      integer nx, mode
      double precision f, df(*), a(*), g
      <Local variables for fct (v1) 80e>
      external scale
      if (mode .eq. 1) then
        <Read input data (v1) 80c>
      else if (mode .eq. 2) then
        <Calculate ∇f 83d>
      endif
      <Calculate f (v1) 83e>
999  continue
      if (mode .eq. 3) then
        <Write output (v1) 85d>
      endif
      end

Defines:
      fct, used in chunks 78b and 93.
Uses scale 83c.

80c  <Read input data (v1) 80c>≡
      <Read data from file 80d>
      <Fixup errors 82a>
      <Normalize 82c>

80d  <Read data from file 80d>≡
      call gethst ('ee', NDATA, xee, fee, dfec, see, tee, pwr)
      call gethst ('eg', NDATA, xeg, feg, dfeg, seg, teg, pwr)
      call gethst ('ge', NDATA, xge, fge, dfge, sge, tge, pwr)
      call gethst ('gg', NDATA, xgg, fgg, dfgg, sgg, tgg, pwr)

Uses gethst 81a.

80e  <Local variables for fct (v1) 80e>≡
      integer NDATA
      parameter (NDATA = 20)
      double precision see, tee, dtee, xee(2,0:NDATA+1,0:NDATA+1),
$      fee(0:NDATA+1,0:NDATA+1), dfec(0:NDATA+1,0:NDATA+1)
      double precision seg, teg, dteg, xeg(2,0:NDATA+1,0:NDATA+1),
$      feg(0:NDATA+1,0:NDATA+1), dfeg(0:NDATA+1,0:NDATA+1)
      double precision sge, tge, dtge, xge(2,0:NDATA+1,0:NDATA+1),
$      fge(0:NDATA+1,0:NDATA+1), dfge(0:NDATA+1,0:NDATA+1)
      double precision sgg, tgg, dtgg, xgg(2,0:NDATA+1,0:NDATA+1),
$      fgg(0:NDATA+1,0:NDATA+1), dfgg(0:NDATA+1,0:NDATA+1)
      double precision pwr

```



```

81a  <fit_v1.f90 78b>+≡
      subroutine gethst (tag, ndata, x, f, df, s, t, pwr)
      implicit none
      character*(2) tag
      integer ndata
      double precision s, t, pwr, x(2,0:ndata+1,0:ndata+1)
      double precision f(0:ndata+1,0:ndata+1), df(0:ndata+1,0:ndata+1)
      integer i, j
      open (10, file = 'lumidiff-'//tag//'.dat')
      read (10, *) pwr
      s = 0d0
      <Read continuum, summing in s 81b>
      t = s
      <Read single  $\delta$ , summing in t 81c>
      <Read double  $\delta$ , summing in t 81e>
      close (10)
      end

Defines:
      gethst, used in chunk 80d.

81b  <Read continuum, summing in s 81b>≡
      do 10 i = 1, ndata
      do 11 j = 1, ndata
      read (10, *) x(1,i,j), x(2,i,j), f(i,j), df(i,j)
      s = s + f(i,j)
11    continue
10    continue

81c  <Read single  $\delta$ , summing in t 81c>≡
      do 20 i = 1, ndata
      read (10, *) x(1,i,0), f(i,0), df(i,0),
$          f(i,ndata+1), df(i,ndata+1)
      x(1,i,ndata+1) = x(1,i,0)
      t = t + f(i,0) + f(i,ndata+1)
20    continue

81d  <Read single  $\delta$ , summing in t 81c>+≡
      do 21 i = 1, ndata
      read (10, *) x(2,0,i), f(0,i), df(0,i),
$          f(ndata+1,i), df(ndata+1,i)
      x(2,ndata+1,i) = x(2,0,i)
      t = t + f(0,i) + f(ndata+1,i)
21    continue

81e  <Read double  $\delta$ , summing in t 81e>≡
      read (10, *) f(0,0), df(0,0), f(0,ndata+1), df(0,ndata+1)
      t = t + f(0,0) + f(0,ndata+1)
      read (10, *) f(ndata+1,0), df(ndata+1,0),
$          f(ndata+1,ndata+1), df(ndata+1,ndata+1)
      t = t + f(ndata+1,0) + f(ndata+1,ndata+1)

```

Guinea-Pig does not provide the full error. A Monte Carlo study shows that it is a reasonable approximation to rescale the bin error by suitable factors. These factors are different for each distribution and the factors for the δ -pieces are bigger than those for the continuum parts. The follows factors are for the `slow` parameter set.

82a $\langle \text{Fixup errors 82a} \rangle \equiv$

```
call fixerr (NDATA, dfec, 20d0, 30d0, 40d0)
call fixerr (NDATA, dfeg, 15d0, 20d0, 0d0)
call fixerr (NDATA, dfge, 15d0, 20d0, 0d0)
call fixerr (NDATA, dfgg, 10d0, 0d0, 0d0)
```

Uses `fixerr` 82b.

82b $\langle \text{fit_v1.f90 78b} \rangle \equiv$

```
subroutine fixerr (ndata, df, c, sd, dd)
implicit none
integer ndata
double precision df(0:ndata+1,0:ndata+1), c, sd, dd
integer i, j
do 1 i = 1, NDATA
  do 2 j = 1, NDATA
    df(i,j) = c * df(i,j)
  2 continue
1 continue
do 3 i = 1, NDATA
  df(0,i) = sd * df(0,i)
  df(i,0) = sd * df(i,0)
  df(ndata+1,i) = sd * df(ndata+1,i)
  df(i,ndata+1) = sd * df(i,ndata+1)
3 continue
df(0,0) = dd * df(0,0)
df(ndata+1,0) = dd * df(ndata+1,0)
df(0,ndata+1) = dd * df(0,ndata+1)
df(ndata+1,ndata+1) = dd * df(ndata+1,ndata+1)
end
```

Defines:

`fixerr`, used in chunk 82a.

The error on the integrated luminosity is obtained from adding the error in channels in quadrature.

82c $\langle \text{Normalize 82c} \rangle \equiv$

```
dtee = sumsqu (NDATA, dfec)
dteg = sumsqu (NDATA, dfeg)
dtge = sumsqu (NDATA, dfge)
dtgg = sumsqu (NDATA, dfgg)
```

Uses `sumsqu` 83a.

82d $\langle \text{Local variables for fct (v1) 80e} \rangle \equiv$

```
double precision sumsqu
external sumsqu
```

Uses `sumsqu` 83a.

```

83a  <fit_v1.f90 78b>+≡
      double precision function sumsqu (ndata, f)
      implicit none
      integer ndata
      double precision f(0:ndata+1,0:ndata+1)
      integer i, j
      double precision s2
      s2 = 0
      do 100 i = 0, NDATA+1
        do 101 j = 0, NDATA+1
          s2 = s2 + f(i,j)*f(i,j)
101      continue
100      continue
      sumsqu = sqrt (s2)
      end

```

Defines:
 sumsqu, used in chunk 82.

```

83b  <Normalize 82c>+≡
      call scale (NDATA, 1d0/tee, fee)
      call scale (NDATA, 1d0/tee, dfec)
      call scale (NDATA, 1d0/tee, feg)
      call scale (NDATA, 1d0/tee, dfeg)
      call scale (NDATA, 1d0/tee, fge)
      call scale (NDATA, 1d0/tee, dfge)
      call scale (NDATA, 1d0/tee, fgg)
      call scale (NDATA, 1d0/tee, dfgg)

```

Uses scale 83c.

```

83c  <fit_v1.f90 78b>+≡
      subroutine scale (ndata, s, f)
      implicit none
      integer ndata
      double precision s, f(0:ndata+1,0:ndata+1)
      integer i, j
      do 100 i = 0, NDATA+1
        do 101 j = 0, NDATA+1
          f(i,j) = s * f(i,j)
101      continue
100      continue
      end

```

Defines:
 scale, used in chunks 80b and 83b.

```

83d  <Calculate  $\nabla f$  83d>≡
      print *, 'ERROR:  $\nabla f$  n.a.'
      stop

```

Log-likelihood won't fly, because we can't normalize the likelihood function for an unbounded parameter range. Let's use good ole least-squares instead.

```

83e  <Calculate  $f(v1)$  83e>≡
      f = 0d0

```

```

do 10 i = 1, NDATA
do 11 j = 1, NDATA
  if (dfee(i,j) .gt. 0d0) then
    f = f + ((phie(xee(1,i,j),a) * phie(xee(2,i,j),a)
$      - fee(i,j)) / dfee(i,j))**2
  endif
  if (dfeg(i,j) .gt. 0d0) then
    f = f + ((phie(xeg(1,i,j),a) * phig(xeg(2,i,j),a)
$      - feg(i,j)) / dfeg(i,j))**2
  endif
  if (dfge(i,j) .gt. 0d0) then
    f = f + ((phig(xge(1,i,j),a) * phie(xge(2,i,j),a)
$      - fge(i,j)) / dfge(i,j))**2
  endif
  if (dfgg(i,j) .gt. 0d0) then
    f = f + ((phig(xgg(1,i,j),a) * phig(xgg(2,i,j),a)
$      - fgg(i,j)) / dfgg(i,j))**2
  endif
11 continue
10 continue

```

Uses phie 85b and phig 85c.

84a *<Local variables for fct (v1) 80e>+≡*

```

integer i, j
double precision phie, phig, delta
external phie, phig

```

Uses phie 85b and phig 85c.

84b *<Calculate f (v1) 83e>+≡*

```

if ((a(2) .le. -1d0) .or. (a(3) .le. -1d0/pwr)) then
  print *, 'warning: discarding out-of-range a2/3: ', a(2), a(3)
  <Give up on f 84c>
endif
delta = 1d0 - exp(a(1)) * beta(a(2)+1d0,a(3)+1d0/pwr)
$      * dble(NDATA) / pwr
if (delta .lt. 0d0) then
  print *, 'warning: delta forced to 0 from ', delta
  delta = 0d0
endif

```

Uses beta 89a.

84c *<Give up on f 84c>≡*

```

f = 1d100
goto 999

```

84d *<Calculate f (v1) 83e>+≡*

```

do 12 i = 1, NDATA
  if (dfee(ndata+1,i) .gt. 0d0) then
    f = f + ((delta*phie(xee(2,ndata+1,i),a)
$      - fee(ndata+1,i)) / dfee(ndata+1,i))**2
  endif
  if (dfeg(ndata+1,i) .gt. 0d0) then

```

```

        f = f + ((delta*phig(xeg(2,ndata+1,i),a)
$           - feg(ndata+1,i)) / dfeg(ndata+1,i))*2
    endif
    if (dfee(i,ndata+1) .gt. 0d0) then
        f = f + ((delta*phie(xee(1,i,ndata+1),a)
$           - fee(i,ndata+1)) / dfee(i,ndata+1))*2
    endif
    if (dfge(i,ndata+1) .gt. 0d0) then
        f = f + ((delta*phig(xge(1,i,ndata+1),a)
$           - fge(i,ndata+1)) / dfge(i,ndata+1))*2
    endif
12    continue
Uses phie 85b and phig 85c.
85a  <Calculate f (v1) 83e>+≡
        if (dfee(ndata+1,ndata+1) .gt. 0d0) then
            f = f + ((delta*delta
$           - fee(ndata+1,ndata+1)) / dfee(ndata+1,ndata+1))*2
        endif
85b  <fit_v1.f90 78b>+≡
        double precision function phie (x, a)
        implicit none
        double precision x, a(6)
        phie = exp (a(1) + a(2)*log(x) + a(3)*log(1d0-x))
        end
Defines:
        phie, used in chunks 83, 84, and 87e.
85c  <fit_v1.f90 78b>+≡
        double precision function phig (x, a)
        implicit none
        double precision x, a(6)
        phig = exp (a(4) + a(5)*log(x) + a(6)*log(1d0-x))
        end
Defines:
        phig, used in chunks 83, 84, and 87e.
85d  <Write output (v1) 85d>≡
        a1(1) = exp(a(1)) * dble(NDATA) / pwr
        a1(2) = a(2)
        a1(3) = a(3) - 1d0 + 1d0/pwr
        a1(4) = exp(a(4)) * dble(NDATA) / pwr
        a1(5) = a(5) - 1d0 + 1d0/pwr
        a1(6) = a(6)
        open (10, file = 'Parameters')
        write (10, 1000) REV, tee / 1D32
1000 format ( '          data xa5lum(@ENERGY@,@ACC@,', I2, ') / ',
$           E12.5, ' /')
        write (10, 1001) REV,
$           1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0),
$           a1(1), a1(2), a1(3), a1(4), a1(5), a1(6),

```

```

$      a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
1001 format ('      data (xa5(i,@ENERGY@,@ACC@,', I2 ,'),i=0,7) /', /,
$      ', 4(E12.5,', ' '), /,
$      ', 3(E12.5,', ' '), E12.5, ' /')
close (10)

```

Uses **beta** 89a.

```

86a  <Local variables for fct (v1) 80e>+≡
      <Declare NPARAM 79a>
      double precision beta, a1(NPARAM)
      integer REV
      parameter (REV = 1)

```

Uses **NPARAM** 79a and **beta** 89a.

The average elektron energy in the continuum can be calculated analytically:

$$\begin{aligned}
\langle E_{e\pm} \rangle_{\text{cont}} &= E_{\text{beam}} \langle x_{e\pm} \rangle_{\text{cont}} = E_{\text{beam}} \frac{\int dx x^{a_2} (1-x)^{a_3} x}{B(a_2, a_3)} \\
&= E_{\text{beam}} \frac{B(a_2 + 1, a_3)}{B(a_2, a_3)} = E_{\text{beam}} \frac{a_2 + 1}{a_2 + a_3 + 2} \quad (40)
\end{aligned}$$

```

86b  <Write output (v1) 85d>+≡
      delta = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
      print *, '< x_e > = ',
$      delta + (1d0-delta)*(a1(2)+1d0)/(a1(2)+a1(3)+2d0)

```

Uses **beta** 89a.

similarly:

$$\langle E_{\gamma} \rangle = E_{\text{beam}} \frac{a_5 + 1}{a_5 + a_6 + 2} \quad (41)$$

```

86c  <Write output (v1) 85d>+≡
      print *, '< x_g > = ',
$      (a1(5)+1d0)/(a1(5)+a1(6)+2d0)

```

Count the degrees of freedom in **ndof**:

```

86d  <Write output (v1) 85d>+≡
      ndof = 0
      do 40 i = 0, ndata+1
        do 41 j = 0, ndata+1
          if (dfec(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfeg(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfge(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfgg(i,j) .gt. 0d0) ndof = ndof + 1
41      continue
40      continue
      print *, 'CHI2 = ', f / ndof

```

```

86e  <Local variables for fct (v1) 80e>+≡
      integer ndof

```

The error on the luminosity is just the (possibly rescaled) counting error:

```

86f  <Write output (v1) 85d>+≡
      open (10, file = 'Errors.tex')
      write (10, 1099) tee / 1d32, dtee / 1d32, dtee / 1d32
1099 format ('$ ', F8.2, ' _{-}', F4.2, ' }^{+}', F4.2, ' }$')

```

After retrieving the error from MINUIT, we have to take care of the mapping of the parameters

$$a'_{1/4} = e^{a_{1/4}} B(a_{2/5} + 1, a_{3/6} + 1) N_{\text{bins}} \eta^{-1} \implies \delta a'_{1/4} = a'_{1/4} \delta a_{1/4} \quad (42)$$

ignoring the errors in the integral (i.e. the Beta function).

```
87a  <Write output (v1) 85d>+≡
      call mnerrs (1,  eplus,  eminus,  epara,  corr)
      ab = a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
      write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
      1100 format ('$', F8.4, '_{-', F6.4, '}^{+', F6.4, '}$')
```

Uses beta 89a.

```
87b  <Local variables for fct (v1) 80e>+≡
      double precision ab
```

The other mappings are even more trivial:

$$a'_{2/6} = a_{2/6} - 1 + \eta^{-1} \implies \delta a'_{2/6} = \delta a_{2/6} \quad a'_{3/5} = a_{3/5} - 1 + \eta^{-1} \implies \delta a'_{3/5} = \delta a_{3/5} \quad (43)$$

```
87c  <Write output (v1) 85d>+≡
      do 110 i = 2, 3
        call mnerrs (i,  eplus,  eminus,  epara,  corr)
        write (10, 1100) a1(i), abs (eminus), abs (eplus)
110  continue
      call mnerrs (4,  eplus,  eminus,  epara,  corr)
      ab = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
      write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
      do 111 i = 5, 6
        call mnerrs (i,  eplus,  eminus,  epara,  corr)
        write (10, 1100) a1(i), abs (eminus), abs (eplus)
111  continue
      close (10)
```

Uses beta 89a.

```
87d  <Local variables for fct (v1) 80e>+≡
      double precision eplus, eminus, epara, corr
      integer n
```

```
87e  <Write output (v1) 85d>+≡
      do 30 n = 1, 10
        call pslice ('ee', 'x', n, NDATA, xee, fee, dfec, phie, phie, a)
        call pslice ('eg', 'x', n, NDATA, xeg, feg, dfeg, phie, phig, a)
        call pslice ('ge', 'x', n, NDATA, xge, fge, dfge, phig, phie, a)
        call pslice ('gg', 'x', n, NDATA, xgg, fgg, dfgg, phig, phig, a)
        call pslice ('ee', 'y', n, NDATA, xee, fee, dfec, phie, phie, a)
        call pslice ('eg', 'y', n, NDATA, xeg, feg, dfeg, phie, phig, a)
        call pslice ('ge', 'y', n, NDATA, xge, fge, dfge, phig, phie, a)
        call pslice ('gg', 'y', n, NDATA, xgg, fgg, dfgg, phig, phig, a)
30  continue
      call pslice ('ee', 'x', 21, NDATA, xee, fee, dfec, phie, phie, a)
```

```

call pslice ('eg','x',21,NDATA,xeg,feg,dfeg,phie,phig,a)
call pslice ('ee','y',21,NDATA,xee,fee,dfeg,phie,phie,a)
call pslice ('ge','y',21,NDATA,xge,fge,dfge,phig,phie,a)

```

Uses phie 85b, phig 85c, and pslice 88b.

UNIX Fortran compiler want backslashes escaped:

```

88a  <Write output (v1) 85d>+≡
      open (10, file = 'Slices.mp4')
      write (10,*) 'picture eslice[], gslice[];'
      do 31 n = 1, NDATA
        write (10,*) 'eslice[' , n, ']' := ' ,
$      'btex $x_{e^{\pm}} = ' , xee(1,n,1), '$ etex;'
        write (10,*) 'gslice[' , n, ']' := ' ,
$      'btex $x_{\gamma} = ' , xgg(1,n,1), '$ etex;'
31    continue
      close (10)

88b  <fit_v1.f90 78b>+≡
      subroutine pslice (pp, xy, n, ndata, x, f, df, phi1, phi2, a)
      implicit none
      character*2 pp
      character*1 xy
      integer n, ndata
      double precision x(2,0:ndata+1,0:ndata+1)
      double precision f(0:ndata+1,0:ndata+1), df(0:ndata+1,0:ndata+1)
      double precision a(6)
      double precision z
      double precision phi1, phi2, d, delta, pwr, beta
      external phi1, phi2, beta
      integer i
      character*2 digits
      write (digits, '(I2.2)') n
      open (10, file = 'lumidiff-'//pp//xy//digits//'.dat')
      open (11, file = 'lumidiff-'//pp//xy//digits//'.fit')
      open (12, file = 'lumidiff-'//pp//xy//digits//'.chi')
      if (n .eq. ndata+1) then
        pwr = 5d0
        delta = 1d0 - exp(a(1))*beta(a(2)+1d0,a(3)+1d0/pwr)
$      * dble(NDATA) / pwr
      else
        delta = 0
      endif
      if (xy .eq. 'x') then
        do 10 i = 1, ndata
          if (df(n,i) .gt. 0d0) then
            if (pp(2:2) .eq. 'g') then
              z = x(2,n,i)
            else
              z = 1d0 - x(2,n,i)
            endif
          endif
          if (n .eq. ndata+1) then

```



```

        d = delta*phi2(x(2,n,i),a)
    else
        d = phi1(x(1,n,i),a)*phi2(x(2,n,i),a)
    endif
    write (10,*) z, f(n,i), df(n,i)
    write (11,*) z, d
    write (12,*) z, (f(n,i) - d) / df(n,i)
endif
10    continue
    else if (xy .eq. 'y') then
        do 11 i = 1, ndata
            if (df(i,n) .gt. 0d0) then
                if (pp(1:1) .eq. 'g') then
                    z = x(1,i,n)
                else
                    z = 1d0 - x(1,i,n)
                endif
                if (n .eq. ndata+1) then
                    d = phi1(x(1,i,n),a)*delta
                else
                    d = phi1(x(1,i,n),a)*phi2(x(2,i,n),a)
                endif
                write (10,*) z, f(i,n), df(i,n)
                write (11,*) z, d
                write (12,*) z, (f(i,n) - d) / df(i,n)
            endif
        11    continue
    endif
    close (10)
    close (11)
    close (12)
end

```

Defines:

pslice, used in chunk 87e.

Uses beta 89a.

```

89a  <fit_v1.f90 78b>+≡
      double precision function beta (a, b)
      implicit none
      double precision a, b
      double precision dlgamma
      beta = exp (dlgamma(a) + dlgamma(b) - dlgamma(a+b))
      end

```

Defines:

beta, used in chunks 25a, 51b, 53d, 57b, 60b, 64a, 84-88, and 91b.

```

89b  <fit_v1 89b>≡
      #! /bin/sh
      # mode=${2-slow}
      mode=${2-fast}
      root='pwd'

```

```

indir=${root}/${3-input}
tmpdir=${root}/tmp
outdir=${root}/output
acc="{1-sband350 sband500 sband800 sband1000 sband1600
      tesla350 tesla500 tesla800 tesla1000 tesla1600
      tesla350-low tesla500-low tesla800-low tesla1000-low tesla1600-low
      xband350 xband500 xband800 xband1000 xband1600}"

90a (fit_v1 89b)+≡
    xmkdir () {
        for d in "$@"; do
            mkdir $d 2>/dev/null || true
        done
    }
    rm -fr ${tmpdir}
    xmkdir ${outdir} ${tmpdir}

90b (fit_v1 89b)+≡
    cd ${tmpdir}
    cat /dev/null >${outdir}/Params.f90
    for a in $acc; do
        case "$a" in
            *1600*) energy=TEV16;;
            *1000*) energy=TEV1;;
            *800*) energy=GEV800;;
            *500*) energy=GEV500;;
            *3[56]0*) energy=GEV350;;
            *170*) energy=GEV170;;
            *90*) energy=GEV090;;
            *) energy=GEV500;;
        esac
        cp ${indir}/${a}_${mode}/lumidiff-???.dat .
        ${root}/fit_v1.bin
        rm -fr ${outdir}/${a}_${mode}
        mkdir ${outdir}/${a}_${mode}
        cp Slices.mp4 ${outdir}
        cp Errors.tex lumidiff-??x[0-9][0-9].??? ${outdir}/${a}_${mode}
        sed -e "s/@ENERGY@/$energy/g" \
            -e "s/@ACC@/'echo $a | tr a-z A-Z | tr -cd A-Z'/g" Parameters \
            >>${outdir}/Params.f90
    done
    cd ${root}
    rm -fr ${tmpdir}

90c (fit_v1 89b)+≡
    cat >${outdir}/Params.tex <<'END'
    \begin{table}
    \begin{center}
    \renewcommand{\arraystretch}{1.3}
    \begin{tabular}{|c||c|c|c|c|}\hline
        & \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND}
    \end{tabular}
    \end{center}
    \end{table}

```

```

        \\hline\hline
    END
    Uses SBAND 10c, TESLA 10c, and XBAND 10c.
91a  <fit_v1 89b>+=
    line () {
        for a in $acc; do
            case $a in
                *350* | *800* | *1000* | *1600*)
                    ;;
                *) echo -n ' & '
                    sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                    ;;
            esac
        done
        echo '\\hline'
    }
    (echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
    echo '$\int d_{e\pm}$'; line 2
    echo '$x_{e\pm}^{\alpha}$'; line 3
    echo '$(1-x_{e\pm})^{\alpha}$'; line 4
    echo '$\int d_{\gamma}$'; line 5
    echo '$x_{\gamma}^{\alpha}$'; line 6
    echo '$(1-x_{\gamma})^{\alpha}$'; line 7
    ) >>${outdir}/Params.tex
91b  <fit_v1 89b>+=
    cat >>${outdir}/Params.tex <<'END'
        \end{tabular}
        \end{center}
        \caption{\label{tab:param}%
        Version 1, revision 1997 04 16 of the beam spectra at 500 GeV.
        The rows correspond to the luminosity per effective year, the
        integral over the continuum and the powers in the factorized Beta
        distributions~(\ref{eq:beta}).}
    \end{table}
    END
    Uses beta 89a.
91c  <fit_v1 89b>+=
    cat >>${outdir}/Params.tex <<'END'
    \begin{table}
        \begin{center}
            \renewcommand{\arraystretch}{1.3}
            \begin{tabular}{|c||c|c|c|c|}\hline
                & \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND} \\
                \hline\hline
            \end{tabular}
        \end{center}
    \end{table}
    END
    Uses SBAND 10c, TESLA 10c, and XBAND 10c.
91d  <fit_v1 89b>+=
    line () {

```

```

    for a in $acc; do
        case $a in
            *1000*)
                echo -n ' & '
                sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                ;;
            esac
        done
        echo '\\\\hline'
    }
    (echo '$\mathcal{L}/\text{fb}^{-1}\text{ }\epsilon^{-1}$'; line 1
    echo '$\int d_e\text{ }\text{ }\text{ }$'; line 2
    echo '$x_{e\text{ }\text{ }\text{ }}^{\alpha}$'; line 3
    echo '$(1-x_{e\text{ }\text{ }\text{ }})^{\alpha}$'; line 4
    echo '$\int d_{\gamma}$'; line 5
    echo '$x_{\gamma}^{\alpha}$'; line 6
    echo '$(1-x_{\gamma})^{\alpha}$'; line 7
    ) >>${outdir}/Params.tex

92a <fit_v1 89b>+=
    cat >>${outdir}/Params.tex <<'END'
        \end{tabular}
        \end{center}
        \caption{\label{tab:param/TeV}%
            Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.}
        \end{table}
    END

92b <fit_v1 89b>+=
    cat >>${outdir}/Params.tex <<'END'
        \begin{table}
        \begin{center}
            \renewcommand{\arraystretch}{1.3}
            \begin{tabular}{|c||c|c|c|c|c|}\hline
                & 350 GeV & 500 GeV & 800 GeV & 1600 GeV \\
                \\\hline\hline
            END

92c <fit_v1 89b>+=
    line () {
        for a in $acc; do
            case $a in
                tesla*-low)
                    ;;
                tesla1000)
                    ;;
                tesla*)
                    echo -n ' & '
                    sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                    ;;
            esac

```

```

done
echo '\\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\text{upsilon}^{-1}$'; line 1
echo '$\int d_{\text{e}^{\text{pm}}}$'; line 2
echo '$x_{\text{e}^{\text{pm}}}^{\alpha}$'; line 3
echo '$(1-x_{\text{e}^{\text{pm}}})^{\alpha}$'; line 4
echo '$\int d_{\text{gamma}}$'; line 5
echo '$x_{\text{gamma}}^{\alpha}$'; line 6
echo '$(1-x_{\text{gamma}})^{\alpha}$'; line 7
) >>${outdir}/Params.tex
93a <fit_v1 89b>+=
cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
    \end{center}
    \caption{\label{tab:param/Tesla}%
        Version 1, revision 1997 04 17 of the beam spectra for TESLA.}
\end{table}
END
exit 0
Uses TESLA 10c.

```

7.2 Experimental

7.2.1 Quasi One Dimensional

```

93b <minuit1.f90 93b>≡
c minuit1.f90 -- fitting for circe
<Copleft notice 24b>
Uses circe 24d.

We're utilizing the familiar "MINUIT" package [15].

93c <minuit1.f90 93b>+=≡
    <Minuit main program 93d>
    <Function to minimize 93e>

93d <Minuit main program 93d>≡
    program fit
    implicit none
    external fct
    call minuit (fct, 0d0)
    end

Uses fct 80b and fit 78b.

93e <Function to minimize 93e>≡
    subroutine fct (nx, df, f, a, mode, g)
    implicit none
    integer nx, mode
    double precision f, df(*), a(*), g
    <Local variables for fct 94b>

```

```

if (mode .eq. 1) then
  <Read input data 94a>
else if (mode .eq. 2) then
  <Calculate  $\nabla f$  83d>
endif
<Calculate  $f$  94c>
if (mode .eq. 3) then
  <Write output 94d>
endif
end

```

Uses fct 80b.

```

94a  <Read input data 94a>≡
      open (10, file = 'minuit.data')
      do 10 i = 1, NDATA
        do 11 j = 1, NDATA
          read (10, *) xi(1,i,j), xi(2,i,j), fi(i,j), dfi(i,j)
          fi(i,j) = fi(i,j)/1d30
          dfi(i,j) = dfi(i,j)/1d30
11      continue
10      continue
      close (10)

94b  <Local variables for fct 94b>≡
      integer NDATA
      parameter (NDATA = 20)
      double precision xi(2,NDATA,NDATA),
$      fi(NDATA,NDATA), dfi(NDATA,NDATA)
      integer i, j, n
      double precision phi, chi, chi2

94c  <Calculate  $f$  94c>≡
      f = 0d0
      do 110 i = 1, NDATA
        do 111 j = 1, NDATA
          if (dfi(i,j).gt.0d0) then
            f = f + ((phi(xi(1,i,j),xi(2,i,j),a)
$              - fi(i,j)) / dfi(i,j))**2
          endif
111      continue
110      continue

94d  <Write output 94d>≡
      chi2 = 0d0
      n = 0
      open (10, file = 'minuit.fit')
      do 210 i = 1, NDATA
        do 211 j = 1, NDATA
          if (dfi(i,j).gt.0d0) then
            chi = (phi(xi(1,i,j),xi(2,i,j),a)-fi(i,j))/dfi(i,j)
            write (10,*) xi(1,i,j), xi(2,i,j),
$              1d30 * phi(xi(1,i,j),xi(2,i,j),a),

```

```

$                1d30 * fi(i,j),
$                chi
$                chi2 = chi2 + chi**2
$                n = n + 1
$            else
$                write (10,*) xi(1,i,j), xi(2,i,j),
$                1d30 * phi(xi(1,i,j),xi(2,i,j),a),
$                1d30 * fi(i,j)
$            endif
211        continue
210    continue
        close (10)
        print *, 'CHI2 = ', chi2/n
95  (minuit1.f90 93b)+≡
        double precision function phi (e1, e2, a)
        implicit none
        double precision e1, e2, a(17)
        double precision y1, y2
        y1 = e1 / 250d0
        y2 = e2 / 250d0
        phi = exp (
$      + a( 1) * 1d0
$      + a( 2) * log(y1)
$      + a( 3) * log(1d0-y1)
$      + a( 4) * log(-log(y1))
$      + a( 5) * log(-log(1d0-y1))
$      + a( 6) * y1
$      + a( 7) * log(y1)**2
$      + a( 8) * log(1d0-y1)**2
$      + a( 9) * log(-log(y1))**2
$      + a(10) * log(-log(1d0-y1))**2
$      + a(11) * y1**2
$      + a(12) / log(y1)
$      + a(13) / log(1d0-y1)
$      + a(14) / log(-log(y1))
$      + a(15) / log(-log(1d0-y1))
$      + a(16) / y1
$      + a(17) / (1d0-y1)
$      + a( 2) * log(y2)
$      + a( 3) * log(1d0-y2)
$      + a( 4) * log(-log(y2))
$      + a( 5) * log(-log(1d0-y2))
$      + a( 6) * y2
$      + a( 7) * log(y2)**2
$      + a( 8) * log(1d0-y2)**2
$      + a( 9) * log(-log(y2))**2
$      + a(10) * log(-log(1d0-y2))**2
$      + a(11) * y2**2
$      + a(12) / log(y2)

```

```

$      + a(13) / log(1d0-y2)
$      + a(14) / log(-log(y2))
$      + a(15) / log(-log(1d0-y2))
$      + a(16) / y2
$      + a(17) / (1d0-y2)
$      )
end
96a <minuit1.sh 96a>≡
    #! /bin/sh
    minuit_bin='pwd'/minuit1.bin
    <Process arguments 96b>
    (
        <Define parameters 96e>
        <Fix parameters 97a>
        <Fix strategy 97b>
        <Run Minuit 97c>
    ) | eval "$minuit_bin $filter"
    <Maybe plot results 97d>
    exit 0
96b <Process arguments 96b>≡
    tmp="$IFS"
    IFS=:
    args=":$*:"
    IFS="$tmp"
96c <Process arguments 96b>+≡
    filter="| \
    awk '/STATUS=(CONVERGED|CALL LIMIT|FAILED)/ { p=1; print }; \
    /@.* \.00000 *fixed/ { next }; \
    /EDM=|CHI2|@/ && p { print }' "
96d <Process arguments 96b>+≡
    case "$args" in
        *:v:*) filter=;;
    esac
96e <Define parameters 96e>≡
    cat <<END
    set title
    CIRCE
    parameters
    1 '@ 1          ' 0.00 0.01
    2 '@ 1x         ' 0.20 0.01
    3 '@ 1(1-x)     ' 0.20 0.01
    4 '@ 11x        ' 0.00 0.01
    5 '@ 11(1-x)    ' 0.00 0.01
    6 '@ x          ' 0.00 0.01
    7 '@ 1x^2       ' 0.00 0.01
    8 '@ 1(1-x)^2   ' 0.00 0.01
    9 '@ 11x^2      ' 0.00 0.01

```



```

10 '@ 11(1-x)^2' 0.00 0.01
11 '@ x^2        ' 0.00 0.01
12 '@ 1/lx       ' 0.00 0.01
13 '@ 1/l(1-x)   ' 0.00 0.01
14 '@ 1/llx      ' 0.00 0.01
15 '@ 1/ll(1-x)' 0.00 0.01
16 '@ 1/x        ' 0.00 0.01
17 '@ 1/(1-x)    ' 0.00 0.01

```

END

```

97a <Fix parameters 97a>≡
for p in 1 2 3 4 5 6 7 8 9 10 \
      11 12 13 14 15 16 17; do
  case "$args" in
    *:$p=*) val='echo "$args" | sed 's/.*:"$p"'=\\([0-9.-]*\\):.*\\/1/'';
              echo set parameter $p $val;
              echo fix $p;;
    *:$p:*) ;;
    *) echo fix $p;;
  esac
done

```

```

97b <Fix strategy 97b>≡
case "$args" in
  *:S0:*) echo set strategy 0;;
  *:S1:*) echo set strategy 1;;
  *:S2:*) echo set strategy 2;;
esac

```

```

97c <Run Minuit 97c>≡
cat <<END
migrat 10000 0.01
stop
END

```

```

97d <Maybe plot results 97d>≡
case "$args" in
  *:p:*) awk '$5 != "" { print $1, $2, $5 }' minuit.fit > chi2
          awk '$5 != "" { print $1, $5 }' minuit.fit > chix
          awk '$5 != "" { print $2, $5 }' minuit.fit > chiy
          gnuplot -geometry -0+0 plot2 >/dev/null 2>&1
esac
Uses fit 78b.

```

7.2.2 Quasi Two Dimensional

```

97e <minuit2.f90 97e>≡
c minuit2.f90 -- fitting for circe
<Copyleft notice 24b>
Uses circe 24d.

```

```

98a <minuit2.f90 97e>+=
    <Minuit main program 93d>
    <Function to minimize 93e>

98b <minuit2.f90 97e>+=
    double precision function phi (e1, e2, a)
    implicit none
    double precision e1, e2, a(33)
    double precision y1, y2
    y1 = e1 / 250d0
    y2 = e2 / 250d0
    phi = exp (
$      + a( 1) * 1d0
$      + a( 2) * log(y1)
$      + a( 3) * log(1d0-y1)
$      + a( 4) * log(-log(y1))
$      + a( 5) * log(-log(1d0-y1))
$      + a( 6) * y1
$      + a( 7) * log(y1)**2
$      + a( 8) * log(1d0-y1)**2
$      + a( 9) * log(-log(y1))**2
$      + a(10) * log(-log(1d0-y1))**2
$      + a(11) * y1**2
$      + a(12) / log(y1)
$      + a(13) / log(1d0-y1)
$      + a(14) / log(-log(y1))
$      + a(15) / log(-log(1d0-y1))
$      + a(16) / y1
$      + a(17) / (1d0-y1)
$      + a(18) * log(y2)
$      + a(19) * log(1d0-y2)
$      + a(20) * log(-log(y2))
$      + a(21) * log(-log(1d0-y2))
$      + a(22) * y2
$      + a(23) * log(y2)**2
$      + a(24) * log(1d0-y2)**2
$      + a(25) * log(-log(y2))**2
$      + a(26) * log(-log(1d0-y2))**2
$      + a(27) * y2**2
$      + a(28) / log(y2)
$      + a(29) / log(1d0-y2)
$      + a(30) / log(-log(y2))
$      + a(31) / log(-log(1d0-y2))
$      + a(32) / y2
$      + a(33) / (1d0-y2)
$    )
    end

98c <minuit2.sh 98c>+=
    #! /bin/sh
    minuit_bin='pwd'/minuit2.bin

```

```

    <Process arguments 96b>
    (
      <Define parameters (2dim) 99>
      <Fix parameters (2dim) 100>
      <Fix strategy 97b>
      <Run Minuit 97c>
    ) | eval "$minuit_bin $filter"
    <Maybe plot results 97d>
    exit 0
99 <Define parameters (2dim) 99>≡
    cat <<END
    set title
    CIRCE
    parameters
    1 '@ 1          ' 0.00 0.01
    2 '@ 1x         ' 0.20 0.01
    3 '@ 1(1-x)     ' 0.20 0.01
    4 '@ 11x        ' 0.00 0.01
    5 '@ 11(1-x)    ' 0.00 0.01
    6 '@ x          ' 0.00 0.01
    7 '@ 1x^2       ' 0.00 0.01
    8 '@ 1(1-x)^2   ' 0.00 0.01
    9 '@ 11x^2      ' 0.00 0.01
    10 '@ 11(1-x)^2 ' 0.00 0.01
    11 '@ x^2       ' 0.00 0.01
    12 '@ 1/1x      ' 0.00 0.01
    13 '@ 1/1(1-x)  ' 0.00 0.01
    14 '@ 1/11x     ' 0.00 0.01
    15 '@ 1/11(1-x) ' 0.00 0.01
    16 '@ 1/x       ' 0.00 0.01
    17 '@ 1/(1-x)   ' 0.00 0.01
    18 '@ 1y        ' 0.20 0.01
    19 '@ 1(1-y)    ' 0.20 0.01
    20 '@ 11y       ' 0.00 0.01
    21 '@ 11(1-y)   ' 0.00 0.01
    22 '@ y         ' 0.00 0.01
    23 '@ 1y^2      ' 0.00 0.01
    24 '@ 1(1-y)^2  ' 0.00 0.01
    25 '@ 11y^2     ' 0.00 0.01
    26 '@ 11(1-y)^2 ' 0.00 0.01
    27 '@ y^2       ' 0.00 0.01
    28 '@ 1/1y      ' 0.00 0.01
    29 '@ 1/1(1-y)  ' 0.00 0.01
    30 '@ 1/11y     ' 0.00 0.01
    31 '@ 1/11(1-y) ' 0.00 0.01
    32 '@ 1/y       ' 0.00 0.01
    33 '@ 1/(1-y)   ' 0.00 0.01

    END

```

```

100  <Fix parameters (2dim) 100>≡
      for p in 1 2 3 4 5 6 7 8 9 10 \
          11 12 13 14 15 16 17 18 19 20 \
          21 22 23 24 25 26 27 28 29 30 \
          31 32 33; do
      case "$args" in
        *:$p=*) val='echo "$args" | sed 's/.*:.'"$p"'='\([0-9.-]*\)'.*/\1/''';
                echo set parameter $p $val;
                echo fix $p;;
        *:$p:*) ;;
        *) echo fix $p;;
      esac
    done

```

7.3 Version 2

8 Conclusions

I have presented a library of simple parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. The library can be used for integration and event generation. Emphasis is put on simplicity and reproducibility of the parameterizations for supporting reproducible physics simulations.

Acknowledgements

Daniel Schulte made his simulation code **Guinea-Pig** available and answered questions. Harald Anlauf and Torbjörn Sjöstrand have contributed useful suggestions. The Tesla group at DESY/Zeuthen made error estimates feasible by donating time on the multi-headed number cruncher **Hydra**. The 1996 ECFA/Desy Linear Collider Workshop got me started and provided support. Thanks to all of them.

Identifiers

ELECTR: 9b, 18b, 24d, 62, 67, 68a
NACC: 10c, 28a, 28b, 32g, 33c, 33d, 33e, 37b, 37c, 38b, 38c, 39b, 39c, 43e, 45e, 48a, 51c, 54a, 58a
NPARAM: 79a, 79b, 79d, 86a
PHOTON: 9b, 24d, 62, 67, 68a, 74
POSITR: 9b, 18c, 68a
SBAND: 10c, 28c, 33a, 33b, 36, 37a, 37d, 38a, 38d, 39a, 39d, 41c, 42a, 78a, 90c, 91c
TESLA: 10c, 26f, 28c, 33a, 33b, 36, 37d, 38d, 39d, 41c, 43a, 44a, 44b, 44c, 45b, 46a, 46b, 46d, 48b, 48c, 49a, 49d, 51d, 52b, 54b, 54d, 58b, 59b, 77b, 78a, 90c, 91c, 93a
XBAND: 10c, 33a, 33b, 36, 37a, 37d, 38a, 38d, 39a, 39d, 41c, 42a, 49d, 90c, 91c
beta: 25a, 51b, 53d, 57b, 60b, 64a, 84b, 85d, 86a, 86b, 87a, 87c, 88b, 89a, 91b

circe: 24d, 9a, 24d, 24d, 14, 24d, 24d, 24d, 24d, 24a, 24c, 24d, 26f, 74, 78b,
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References

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A Literate Programming

A.1 Paradigm

I have presented the sample code in this paper using the *literate programming* paradigm. This paradigm has been introduced by Donald Knuth [19] and his programs `TEX` [20] and `METAFONT` [21] provide excellent examples of the virtues of literate programming. Knuth summarized his intention as follows ([19], p. 99)

“Let us change our traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a *computer* what to do, let us concentrate rather on explaining to *human beings* what we want a computer to do.”

Usually, literate programming uses two utility programs to produce two kinds of files from the source

`tangle` produces the computer program that is acceptable to an “illiterate” (Fortran, C, etc.) compiler. This process consists of stripping documentation and reordering code. Therefore it frees the author from having to present the code in the particular order enforced by a compiler for purely technical reasons. Instead, the author can present the code in the order that is most comprehensible.

`weave` produces a documents that describes the program. Extensive cross referencing of the code sections is usually provided, which has been suppressed in this paper. If a powerful typesetting system (such a `TEX`) is used, the document can present the algorithms in clear mathematical notation alongside the code. These features improve readability and maintainability of scientific code immensely.

A.2 Practice

`Circe` uses the `noweb` [22] system. This system has the advantage to work with any traditional programming language and support the essential features described in section A.1 with minimal effort. `noweb`’s `tangle` program only reorders the code sections, but does not reformat them. Therefore its output can be used just like any other “illiterate” program.

The examples above should be almost self-explaining, but in order to avoid any ambiguities, I give another example:

106a `<Literate programming example 106a>≡`
 `<Code that has to be at the top 106c>`
 `<Other code 106b>`

I can start the presentation with the first line of the “other code”:

106b `<Other code 106b>≡`
 `line 1 of the other code`

If appropriate, the first line of the code that has to appear *before* the other code can be presented later:

106c `<Code that has to be at the top 106c>≡`
 `line 1 of the code at the top`

Now I can augment the sections:

```
107a  <Other code 106b>+≡  
      line 2 of the other code  
107b  <Code that has to be at the top 106c>+≡  
      line 2 of the code at the top
```

The complete “program” will be presented to the compiler as

```
line 1 of the code at the top  
line 2 of the code at the top  
line 1 of the other code  
line 2 of the other code
```

The examples in section 3.1.1 show that this reordering is particularly useful for declaring variables when they are first used (rather than at the beginning) and for zooming in on code inside of loops.

B Fortran Name Space

In addition to the ten procedures and one `common` block discussed in section 3

- `circe`, `circee`, `circeg`, `circgg`,
- `girce`, `gircee`, `girceg`, `gircgg`,
- `circes`, `circel`, `/circom/`,

there are two more globally visible functions which are used internally:

- `circem`: error message handler,
- `girceb`: efficient Beta distribution generator.

Even if the `/circom/` is globally visible, application programs *must not* manipulate it directly. The `circes`, subroutine is provided for this purpose and updates some internal parameters as well.

With features from the current Fortran standard (Fortran90), I could have kept the last two functions and the `common` block private. But since Fortran90 has only been adopted by a small fraction of the high energy physics community, I have decided to remain in the confines of Fortran77 (except for the ubiquitous `implicit none`).

Application programs wishing to remain compatible with future versions of `Circe` must not use `common` blocks or procedures starting with `circe` or `girce`.

C Updates

Information about updates can be obtained

- on the World Wide Web:
<http://crunch.ikp.physik.th-darmstadt.de/nlc/beam.html>
- by internet FTP:

host: crunch.ikp.physik.th-darmstadt.de

user: anonymous

password: your email address

directory: pub/ohl/circe

- from mailing lists:

circe-announce@crunch.ikp.physik.th-darmstadt.de

circe-bugs@crunch.ikp.physik.th-darmstadt.de

circe-discuss@crunch.ikp.physik.th-darmstadt.de

Subscriptions are available from

majordomo@crunch.ikp.physik.th-darmstadt.de

Contributions of results from other simulation programs and updated accelerator designs are welcome at

Thorsten.Ohl@Physik.TH-Darmstadt.de