

Circe Version 1.02 β : Beam Spectra for Simulating Linear Collider Physics*

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Abstract

I describe parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. Emphasis is put on simplicity and reproducibility of the parameterizations, supporting reproducible physics simulations. The parameterizations are implemented in a library of distribution functions and event generators.

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Program Summary:

- **Title of program:** Circe, Version 1.02 β (September 1996)
- **Program obtainable** by anonymous ftp from the host `crunch.ikp.physik.th-darmstadt.de` in the directory `pub/ohl/circe`.
- **Licensing provisions:** Free software under the GNU General Public License.
- **Programming language used:** Fortran77 originally, transferred to Fortran90
- **Number of program lines in distributed program, including test data, etc.:** ≈ 1100 (excluding comments)
- **Computer/Operating System:** Any with a Fortran90 programming environment.
- **Memory required to execute with typical data:** Negligible on the scale of typical applications calling the library.
- **Typical running time:** A small fraction (typically a few percent) of the running time of applications calling the library.
- **Purpose of program:** Provide simple and reproducible, yet realistic, parameterizations of the e^\pm - and γ -beam spectra for linear colliders.
- **Nature of physical problem:** The intricate beam dynamics in the interaction region of a high luminosity linear collider at $\sqrt{s} = 500\text{GeV}$ result in non-trivial energy spectra of the scattering electrons, positrons and photons. Physics simulations require simple and reproducible, yet realistic, parameterizations of these spectra.
- **Method of solution:** Parameterization, curve fitting, Monte Carlo event generation.
- **Keywords:** Event generation, beamstrahlung, linear colliders.

1 Introduction

Despite the enormous quantitative success of the electro-weak standard model up to energies of 200GeV, neither the nature of electro-weak symmetry breaking (EWSB) nor the origin of mass are understood.

From theoretical considerations, we know that clues to the answer of these open questions are hidden in the energy range below $\Lambda_{\text{EWSB}} = 4\pi v \approx 3.1\text{TeV}$. Either we will discover a Higgs particle in this energy range or signatures for a strongly interacting EWSB sector will be found. Experiments at CERN's Large Hadron Collider (LHC) will shed a first light on this regime in the next decade. In the past it has been very fruitful to complement experiments at high energy hadron colliders with experiments at e^+e^- -colliders. The simpler initial state allows more precise measurements with smaller theoretical errors. Lucid expositions of the physics opportunities of high energy e^+e^- colliders with references to the literature can be found in [1].

However, the power emitted by circular storage rings in form of synchrotron radiation scales like $(E/m)^4/R^2$ with the energy and mass of the particle and the radius of the ring. This cost becomes prohibitive after LEP2 and a Linear Collider (LC) has to be built instead.

Unfortunately, the “interesting” hard cross sections scale like $1/s$ with the square of the center of mass energy and a LC will have to operate at extremely high luminosities in excess of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. To achieve such luminosities, the bunches of electrons and positrons have to be very dense. Under these conditions, the electrons undergo acceleration from strong electromagnetic forces from the positron bunch (and vice versa). The resulting synchrotron radiation is called *beamstrahlung* [2] and has a strong effect on the energy spectrum $D(x_1, x_2)$ of the colliding particles. This changes the observable e^+e^- cross sections

$$\frac{d\sigma_0^{e^+e^-}}{d\Omega}(s) \rightarrow \frac{d\sigma^{e^+e^-}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^+e^-}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^+e^-}}{d\Omega'}(x_1 x_2 s) \quad (1a)$$

and produces luminosity for $e^\pm\gamma$ and $\gamma\gamma$ collisions:

$$\frac{d\sigma^{e^\pm\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^\pm\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^\pm\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1b)$$

$$\frac{d\sigma^{\gamma\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{\gamma\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{\gamma\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1c)$$

Therefore, simulations of the physics expected at a LC need to know the spectra of the e^\pm and γ beams precisely.

Microscopic simulations of the beam dynamics are available (e.g. ABEL[3], CAIN[4] and Guinea-Pig[5]) and their predictions are compatible with each other. But they require too much computer time and memory for direct use in physics programs. Circe provides a fast and simple parameterization of the results from these simulations. Furthermore, even if the computational cost of the simulations would be negligible, the input parameters for microscopic simulations are not convenient for particle physics applications. Due to the highly

	SBAND	TESLA	XBAND	SBAND	TESLA	XBAND
E/GeV	250	250	250	500	500	500
$N_{\text{particles}}/10^{10}$	1.1	3.63	0.65	2.9	1.8	0.95
$\epsilon_x/10^{-6}\text{mrad}$	5	14	5	10	14	5
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.08	0.1	0.06	0.1
β_x^*/mm	10.98	24.95	8.00	32	25	10.00
β_y^*/mm	0.45	0.70	0.13	0.8	0.7	0.12
σ_x/nm	335	845	286	571.87	598.08	226
σ_y/nm	15.1	18.9	4.52	9.04	6.55	3.57
$\sigma_z/\mu\text{m}$	300	700	100	500	500	125
f_{rep}	50	5	180	50	5	180
n_{bunch}	333	1135	90	125	2270	90

Table 1: Accelerator parameters for three typical designs at $\sqrt{s} = 500\text{GeV}$ and $\sqrt{s} = 1\text{TeV}$. The resulting distributions are shown in figure 1. The design efforts are currently concentrated on a 350GeV-800GeV LC. Therefore the Tesla parameters for 1TeV are slightly out of date.

non-linear beam dynamics, the optimization of LC designs is a subtle art [6], that is best practiced by the experts. Furthermore, particle physics applications need benchmarking and easily reproducible parameterizations are required for this purpose.

The parameterizations in **Circe** are not based on approximate solutions (cf. [7]) of the beamstrahlung dynamics. Instead, they provide a “phenomenological” description of the results from full simulations. The parameterizations are as simple as possible while remaining consistent with basic physical principles:

1. *positivity*: the distribution functions $D(x_1, x_2)$ *must not* be negative in the physical region $[0, 1] \times [0, 1]$.
2. *integrability*: the definite integral of the distribution functions over the physical region $[0, 1] \times [0, 1]$ *must* exist, even though the distributions can have singularities.

This paper is organized as follows: I start in section 2 with a discussion of the input for the microscopic simulations. In section 3 I describe the usage of the **Circe** library and in section 4 I discuss some technical details of the implementation. After discussing the parameterizations available in version 1.02 β in section 5, I conclude in section 8.

2 Parameters

The microscopic simulation program **Guinea-Pig** [5] used for the current version of the parameterizations in **Circe** simulates the passage of electrons through a

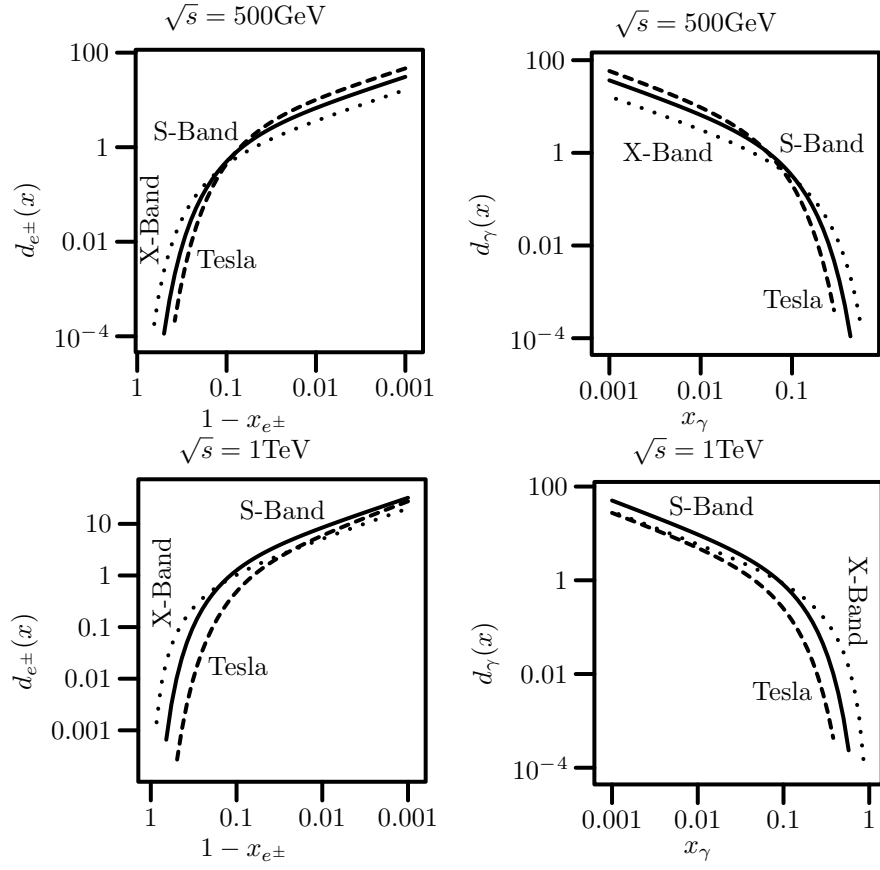


Figure 1: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions at $\sqrt{s} = 500 \text{ GeV}$ and $\sqrt{s} = 1 \text{ TeV}$ in a doubly logarithmic plot. The accelerator parameters are taken from table 1.

	TESLA	TESLA	TESLA
E/GeV	175	250	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	3.63
$\epsilon_x/10^{-6}\text{mrad}$	14	14	14
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.1
β_x^*/mm	25.00	24.95	15.00
β_y^*/mm	0.70	0.70	0.70
σ_x/nm	1010.94	845	668.67
σ_y/nm	22.6	18.9	9.46
$\sigma_z/\mu\text{m}$	700	700	700
f_{rep}	5	5	5
n_{bunch}	1135	1135	1135

Table 2: Accelerator parameters for the Tesla design at three planned [8] energies. The resulting distributions are shown in figure 2.

	High- \mathcal{L}	Low- \mathcal{L}	Low- ϵ_y
E/GeV	400	400	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	1.800
$\epsilon_x/10^{-6}\text{mrad}$	14	14	12
$\epsilon_y/10^{-6}\text{mrad}$	0.1	0.25	0.025
β_x^*/mm	15.00	25.00	25.00
β_y^*/mm	0.70	0.70	0.50
σ_x/nm	668.67	700.00	
σ_y/nm	9.46		
$\sigma_z/\mu\text{m}$	700	700	500
f_{rep}	5	5	3
n_{bunch}	1135	1135	2260

Table 3: Variant accelerator parameters for the Tesla design at 800 GeV.

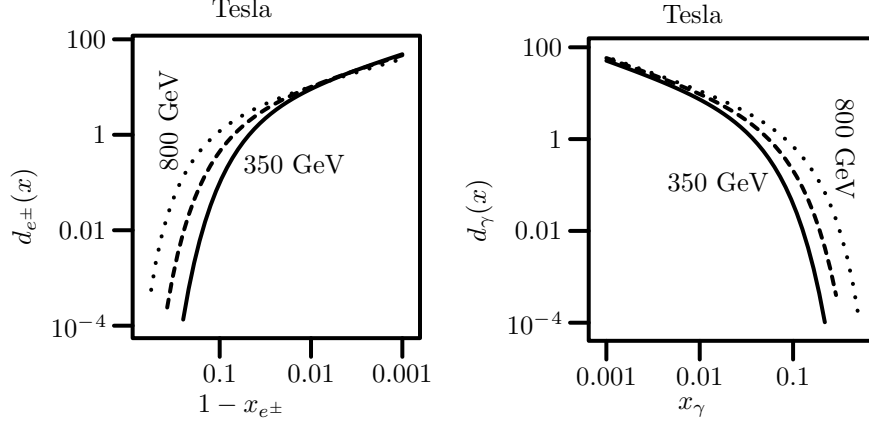


Figure 2: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions for Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 2.

	TESLA	TESLA
E/GeV	250	400
$N_{\text{particles}}/10^{10}$	2	1.40
$\epsilon_x/10^{-6}\text{m rad}$	10	8
$\epsilon_y/10^{-6}\text{m rad}$	0.03	0.01
β_x^*/mm	15.00	15.00
β_y^*/mm	0.40	0.30
σ_x/nm	553	391
σ_y/nm	5	2
$\sigma_z/\mu\text{m}$	400	300
f_{rep}	5	3
n_{bunch}	2820	4500

Table 4: Accelerator parameters for a high luminosity Tesla design at two planned [8] energies. The resulting distributions are shown in figure 3.



Figure 3: Version 5, revision 1998 05 05 of the factorized e^{\pm} - and γ -distributions for a high luminosity Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 4.

bunch of electrons (and vice versa). It takes the following accelerator parameters as input:

E : the energy of the particles before the beam-beam interaction.

$N_{\text{particles}}$: the number of particles per bunch.

$\epsilon_{x,y}$: the normalized horizontal and vertical emittances.

$\beta_{x,y}^*$: the horizontal and vertical beta functions.

$\sigma_{x,y,z}$: the horizontal, vertical and longitudinal beam size. A Gaussian shape is used for the charge distribution in the bunches.

f_{rep} : the repetition rate.

n_{bunch} : the number of bunches per train.

The transversal beam sizes, beta functions and normalized emittances for relativistic particles are related by

$$\beta_{x,y}^* = \frac{\sigma_{x,y}^2}{\epsilon_{x,y}} \frac{E}{m_e} \quad (2)$$

The parameters used in the most recent revision of the parameterizations are collected in tables 1 and 2. The resulting factorized electron/positron and photon distributions in version 1 of the parameterizations are depicted in figures 1 and 2.

The most important purpose of **Circe** is to map the manifold of possible beam spectra for the NLC to a *finite* number of *reproducible* parameterizations. The distributions

$$D_{p_1 p_2}^{\alpha \nu \rho}(x_1, x_2; \sqrt{s}) \quad (3)$$

provided by **Circe** are indexed by three integers

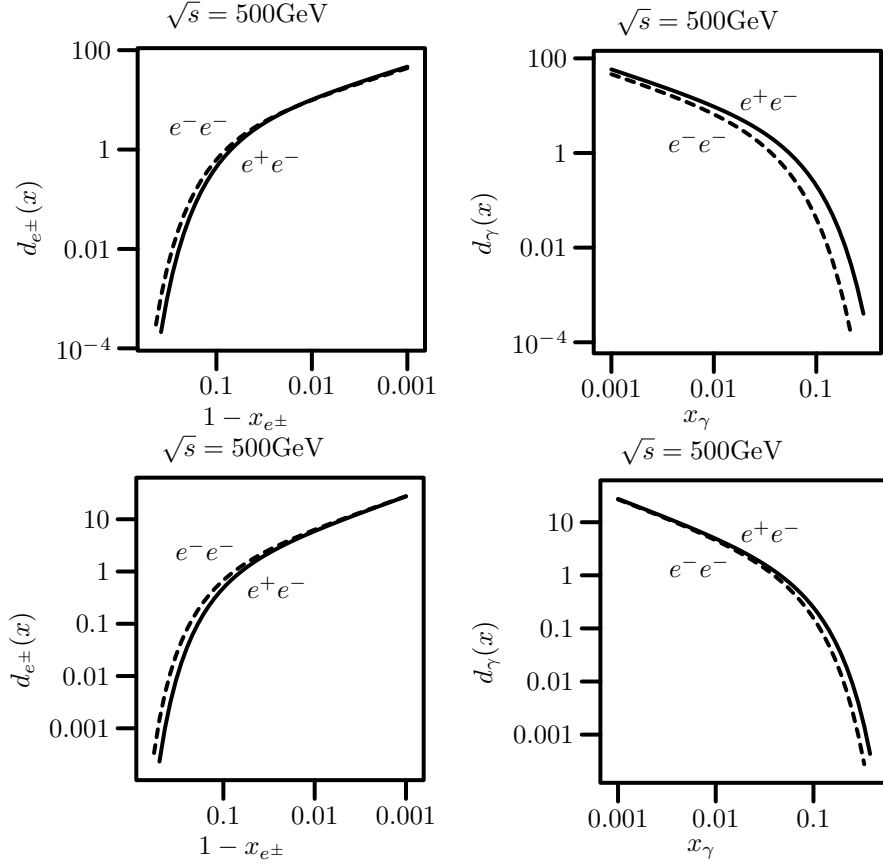


Figure 4: *Experimental:* Version 1, revision 0 of the factorized e^- - and γ -distributions for Tesla- e^-e^- in a doubly logarithmic plot. The accelerator parameters are taken from table 2 and have *not* been endorsed for use in an e^-e^- -machine yet!.

α : the *accelerator design class*: currently there are three options: S-band [9], Tesla [8], X-band [10, 11]. More variety will be added later, in particular the e^-e^- mode and the $e^-\gamma$ and $\gamma\gamma$ laser backscattering modes of these designs.

ν : the *version of the parameterization*: over the years, the form of the parameterizations can change, either because better approximations are found or because new simulation programs become available. All versions will remain available in order to be able to reproduce calculations.

ρ : the *revision date for the parameterization*: a particular parameterization can contain bugs, which will be fixed in subsequent revisions. While only the most recent revision should be used for new calculations, old revisions will remain available in order to be able to reproduce calculations.

The continuous parameter \sqrt{s} in (3) is misleading, because accelerator parameters have been optimized for discrete values of the energy. Therefore the distributions are not available for all values of \sqrt{s} .

The usage of the distributions in application programs is discussed in section 3.1. Circe provides for each of the distributions a non-uniform random variate generator, that generates energy fractions according to the distributions. The usage of these generators is discussed in section 3.2.

3 Usage

3.1 Distributions

A generic interface to all distributions $D_{p_1 p_2}(x_1, x_2)$ is given by the `circe` function

11a \langle API documentation 11a $\rangle \equiv$

```
function circe, d, x1, x2
    real(kind=double) :: circe
    integer :: p1, p2
    d = circe (x1, x2, p1, p2)
```

Uses `circe` 31b.

where the energy fractions are specified by $x_{1,2}$ and the particles $p_{1,2}$ are identified by their standard Monte Carlo codes (we use C1 as a prefix to avoid name clashes when using CIRCE1 inside WHIZARD):[13]

11b \langle Particle codes 11b $\rangle \equiv$

```
integer, parameter, public :: C1_ELECTRON = 11
integer, parameter, public :: C1_POSITRON = -11
integer, parameter, public :: C1_PHOTON = 22
```

Defines:

C1_ELECTRON, used in chunks 21d, 31b, 70b, and 75.
 C1_PHOTON, used in chunks 31b, 70b, 75, and 82.
 C1_POSITRON, used in chunks 22 and 75c.

The distributions can have integrable singularities at the end points, therefore the calling functions *must not* evaluate them at the endpoints 0 and 1. This is usually not a problem, since standard mapping techniques (cf. (10) below) will have to be used to take care of the singularity anyway. Nevertheless, all applications should favor open quadrature formulae (i.e. formulae not involving the endpoints) over closed formulae. The distributions are guaranteed to vanish unless $0 < x_{1,2} < 1$, with two exceptions. Firstly, the value -1 allows to pick up the integral of the continuum contribution:

$$D_{p_1 p_2}(-1, x_2) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 D_{p_1 p_2}(x_1, x_2) \quad (4a)$$

$$D_{p_1 p_2}(x_1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4b)$$

$$D_{p_1 p_2}(-1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4c)$$

The other exception is that the strength of δ -function contributions at the endpoint can be picked up from the value at this endpoint:

$$D_{e^+ e^-}(x_1, x_2) = D_{e^+ e^-}(1, 1) \delta(1 - x_1) \delta(1 - x_2) + \text{smooth and single } \delta \quad (5a)$$

$$D_{e^\pm \gamma}(x_1, x_2) = D_{e^\pm \gamma}(1, x_2) \delta(1 - x_1) + \text{smooth} \quad (5b)$$

$$D_{\gamma e^\pm}(x_1, x_2) = D_{\gamma e^\pm}(x_1, 1) \delta(1 - x_2) + \text{smooth} \quad (5c)$$

The use of these special values is demonstrated in an example in section 3.1.1 below.

The distributions are normalized such that

$$\lim_{\epsilon \rightarrow +0} \int_{-\epsilon}^{1+\epsilon} dx_1 dx_2 D_{e^+ e^-}(x_1, x_2) = 1. \quad (6)$$

and the nominal $e^+ e^-$ -luminosity of the currently active accelerator design can be retrieved from the database with the subroutine `circel`. The value is given in units of

$$\text{fb}^{-1} v^{-1} = 10^{32} \text{cm}^{-2} \text{sec}^{-1} \quad (7)$$

where $v = 10^7 \text{sec} \approx \text{year}/\pi$ is an “effective year” of running with about 30% up-time.

12a `<API documentation 11a>+≡`
`real(kind=dobule) :: lumi`
`call circel (lumi)`

Uses `circel` 40f.

A particular parameterization is selected by the `circses` function:

12b `<API documentation 11a>+≡`
`real(kind=double) :: x1m, x2m, roots`
`integer :: acc, ver, rev, chat`
`call circses (x1m, x2m, roots, acc, ver, rev, chat)`

Uses `circses` 32a.

The parameter **roots** corresponds to the nominal center of mass energy \sqrt{s} /GeV of the collider. Currently $\sqrt{s} = 350\text{GeV}, 500\text{GeV}, 800\text{GeV}, 1\text{TeV}$ (i.e. 350D0, 500D0, 800D0 and 1000D0) are supported. Application programs can *not* assume that energy values are interpolated. For convenience, e.g. in top threshold scans around 350GeV, a small interval around the supported values will be accepted as synonymous with the central value, but a warning will be printed. Section 5 should be consulted for the discrete values supported by a particular version of the parameterizations. Negative values of **roots** will keep the currently active value for \sqrt{s} .

The parameters **x1m** and **x2m** will set thresholds $x_{1,\min}$ and $x_{2,\min}$ for the event generation in the routines described in section 3.2.

The parameter **acc** selects the accelerator design. Currently the following accelerator codes are recognized:

13 $\langle \text{Accelerator codes 13} \rangle \equiv$

```
integer, parameter :: SBAND = 1
integer, parameter :: TESLA = 2
integer, parameter :: XBAND = 3
integer, parameter :: JLCNLC = 3
integer, parameter :: SBNDEE = 4
integer, parameter :: TESLEE = 5
integer, parameter :: XBNDEE = 6
integer, parameter :: NLCH = 7
integer, parameter :: NACC = 7
```

Defines:

JLCNLC, used in chunks 17, 18, 35b, 56b, 59–62, 65e, 67a, and 86b.

NACC, used in chunks 17, 34b, 35a, 39, 40, 44–46, 51b, 53c, 55c, 58c, 61c, and 65c.

SBAND, used in chunks 35b, 39f, 40a, 43–46, 49, 86b, 102a, and 103a.

SBNDEE, used in chunks 34b, 35b, 40c, 44–46, 48, and 86b.

TESLA, used in chunks 32, 35b, 39f, 40a, 43–46, 49–56, 58d, 59b, 61d, 62a, 65d, 66b, 86, and 102–104.

TESLEE, used in chunks 34b, 35b, 48a, and 86b.

XBAND, used in chunks 39f, 40a, 43–46, 49, 56e, 102a, and 103a.

XBNDEE, used in chunks 34b, 35b, 48, and 86.

The **ver** parameter is used to determine the version as follows:

ver > 0 : a frozen version which is documented in section 5. For example, version 1 is a family of factorized Beta distributions: $D(x_1, x_2) \propto x_1^{a_1} (1 - x_1)^{b_1} x_2^{a_2} (1 - x_2)^{b_2}$.

ver = 0 : the latest experimental version, which is usually not documented and can change at any time without announcement.

ver < 0 : keep the currently active version.

The **rev** parameter is used to determine the revision of a version as follows:

rev > 0 : a frozen revision which is documented in section 5. The integer **rev** is constructed from the date as follows: $\text{rev} = 10^4 \cdot \text{year} + 10^2 \cdot \text{month} + \text{day}$, where the year is greater than 1995. Since Fortran77 ignored whitespace, it could be written like 1996 07 11 for readability. In Fortran90 the white space have been erased. If there is no exact match, the most recent revision before the specified date is chosen.

`rev = 0` : the most recent revision.

`rev < 0` : keep the currently active revision.

Finally, the parameter `chat` controls the “chattiness” of `circe`. If it is 0, only error messages are printed. If it is 1, the parameters in use are printed whenever they change. Higher values of `chat` can produce even more diagnostics.

In addition to the generic interface `circe`, there are specialized functions for particular particle distributions. Obviously

$$D_{e^\pm\gamma}^{\alpha\nu\rho}(x_1, x_2, s) = D_{\gamma e^\pm}^{\alpha\nu\rho}(x_2, x_1, s) \quad (8)$$

and there are three independent functions $D_{e^-e^+}$, $D_{e^-\gamma}$ and $D_{\gamma\gamma}$ for the e^+e^- colliders with reasonable mnemonics:

14a `<API documentation 11a>+≡`
`real(kind=double) :: circee, circeg, circgg`
`d = circee (x1, x2)`
`d = circeg (x1, x2)`
`d = circgg (x1, x2)`

Uses `circee 41a`, `circeg 42a`, and `circgg 42d`.

Calling the latter three functions is marginally faster in the current implementation, but this can change in the future.

3.1.1 Example

For clarification, let me give a simple example. Imagine we want to calculate the integrated production cross section

$$\sigma_X(s) = \int dx_1 dx_2 \sigma_{e^+e^- \rightarrow X}(x_1 x_2 s) D_{e^+e^-}(x_1, x_2, s) \quad (9)$$

Since the distributions are singular in the $x_{1,2} \rightarrow 1$ limit, we have to map away this singularity with

$$x \rightarrow t = (1 - x)^{1/\eta} \quad (10a)$$

Therefore

$$\int_0^1 dx f(x) = \int_0^1 dt \eta t^{\eta-1} f(1 - t^\eta) \quad (10b)$$

with η sufficiently large to give the integrand a finite limit at $x \rightarrow 1$. If f diverges like a power $f(x) \propto 1/(1 - x)^\beta$, this means $\eta > 1/(1 - \beta)$.

As a specific example, let us “measure” a one particle s -channel exchange cross section

$$\sigma(s) \propto \frac{1}{s} \quad (11)$$

14b `<circe1_sample.f90: public 14b>≡`
`public :: sigma`
 Uses `sigma 15a`.

```

15a <circe1_sample.f90: subroutines 15a>≡
    function sigma (s)
        real(kind=double) :: s, sigma
        sigma = 1d0 / s
    end function sigma

```

Defines:

`sigma`, used in chunks 14–16, 18, and 20d.

I will present the example code in a bottom-up fashion, which should be intuitive and is described in some more detail in appendix A. Assuming the existence of a one- and a two-dimensional Gaussian integration function `gauss1` and `gauss2`,¹ we can perform the integral as follows:

```

15b <Gauss integration 15b>≡
    s = sigma (1d0) * circee (1d0, 1d0) &
        + gauss1 (d1, 0d0, 1d0, EPS) &
        + gauss1 (d2, 0d0, 1d0, EPS) &
        + gauss2 (d12, 0d0, 1d0, 0d0, 1d0, EPS)
    write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
    1000 format (1X, A22, 1X, F6.2, '%')

```

Uses `circee` 41a, `d1` 15f, `d12` 15d, `d2` 16b, `gauss1` 84b, `gauss2` 85b, and `sigma` 15a.

Note how the four combinations of continuum and δ -peak are integrated separately, where you have to use three auxiliary functions `d1`, `d2` and `d12`. The continuum contribution, including the Jacobian:

```

15c <circe1_sample.f90: public 14b>+≡
    public :: d12

```

Uses `d12` 15d.

```

15d <circe1_sample.f90: subroutines 15a>+≡
    function d12 (t1, t2)
        real(kind=double) :: d12, t1, t2, x1, x2
        <EPS & PWR 16c>
        x1 = 1d0 - t1**PWR
        x2 = 1d0 - t2**PWR
        d12 = PWR*PWR * (t1*t2)**(PWR-1d0) &
            * sigma (x1*x2) * circee (x1, x2)
    end function d12

```

Defines:

`d12`, used in chunk 15.

Uses `circee` 41a and `sigma` 15a.

the first product of continuum and δ -peak:

```

15e <circe1_sample.f90: public 14b>+≡
    public :: d1

```

Uses `d1` 15f.

```

15f <circe1_sample.f90: subroutines 15a>+≡
    function d1 (t1)
        real(kind=double) :: t1, x1, d1

```

¹They are provided in the example program `circe1_sample.f90`.

```

      <EPS & PWR 16c>
      x1 = 1d0 - t1**PWR
      d1 = PWR * t1**(PWR-1d0) * sigma (x1) * circee (x1, 1d0)
    end function d1

```

Defines:

d1, used in chunks 15, 41–43, and 70–73.

Uses circee 41a and sigma 15a.

and the second one:

```

16a <circe1_sample.f90: public 14b>+≡
      public :: d2
    Uses d2 16b.

16b <circe1_sample.f90: subroutines 15a>+≡
      function d2 (t2)
        real(kind=double) :: t2, x2, d2
        <EPS & PWR 16c>
        x2 = 1d0 - t2**PWR
        d2 = PWR * t2**(PWR-1d0) * sigma (x2) * circee (1d0, x2)
      end function d2

```

Defines:

d2, used in chunks 15, 16, 41–43, and 70–73.

Uses circee 41a and sigma 15a.

Below you will see that the power of the singularity of the e^+e^- distributions at $x \rightarrow 1$ is $\approx -2/3$. To be on the safe side, we choose the power η in (10) as 5. It is kept in the parameter PWR, while EPS is the desired accuracy of the Gaussian integration:

```

16c <EPS & PWR 16c>≡
      real(kind=double), parameter :: EPS = 1d-6, PWR = 5d0

```

The Gauss integration of the non-singular version converges to the cotrrect value only if the final bin is integrated separately:

```

16d <Second Gauss integration 16d>≡
      s = gauss2 (d12a, 0d0, 1d0-KIREPS, 0d0, 1d0-KIREPS, EPS) &
        + gauss2 (d12a, 0d0, 1d0-KIREPS, 1d0-KIREPS, 1d0, EPS) &
        + gauss2 (d12a, 1d0-KIREPS, 1d0, 0d0, 1d0-KIREPS, EPS) &
        + gauss2 (d12a, 1d0-KIREPS, 1d0, 1d0-KIREPS, 1d0, EPS)
      write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
    Uses d12a 16g, gauss2 85b, and sigma 15a.

```

```

16e <EPS & PWR 16c>+≡
      real(kind=double), parameter :: KIREPS = 1D-6

```

```

16f <circe1_sample.f90: public 14b>+≡
      public :: d12a
    Uses d12a 16g.

```

```

16g <circe1_sample.f90: subroutines 15a>+≡
      function d12a (x1, x2)
        real(kind=double) :: x1, x2, d12a
        d12a = sigma (x1*x2) * kirkee (x1, x2)

```



```
end function d12a
```

Defines:

d12a, used in chunk 16.

Uses kirkee 70d and sigma 15a.

These code fragments can now be used in a main program that loops over energies and accelerator designs

```
17 <circe1_sample.f90 17>≡
  ! circe1_sample.f90 -- canonical beam spectra for linear collider physics
  ! $Id: circe1.nw 67 2002-03-28 17:13:06Z ohl $
  <Copyleft notice 30a>
  module sample_routines
    use kinds
    use circe1 !NODEP!

    implicit none
    private

    <circe1_sample.f90: public 14b>

    contains

    <circe1_sample.f90: subroutines 15a>

  end module sample_routines

  program circe1_sample
    use kinds
    use sample_routines
    use circe1

    implicit none

    <Accelerator codes 13>
    <EPS & PWR 16c>
    <Other variables in sample 19>
    integer :: acc, ver, i
    real(kind=double), dimension(9) :: roots(9) = &
      (/ 90D0, 170D0, 250D0, 350D0, 500D0, &
        800D0, 1000D0, 1200D0, 1500D0 /)
    do acc = 1, NACC
      ! do acc = JLCNLC, NLCH, NLCH-JLCNLC
      do ver = 9, 9
        do i = 1, 9
          call circes (0d0, 0d0, roots(i), acc, ver, 20020328, 1)
          <Gauss integration 15b>
          <Second Gauss integration 16d>
          <Monte Carlo integration 20d>
        end do
      end do
    end do
```

```

        end do
    end program circe1_sample

```

Uses JLCNLC 13, NACC 13, and circes 32a.

with the following result

18 *<Sample output 18>*≡

```

circe1:message: starting up ...
circe1:message: $Id: prelude.nw 66 2002-03-28 17:00:10Z ohl $
circe1:message: updating 'roots' to 90.0
circe1:message: updating 'ver' to 7
circe1:message: updating 'rev' to 20000501
delta(sigma) (Gauss) = 0.11%
delta(sigma) (MC)    = 0.11%
                    +/- 0.00%

circe1:message: updating 'roots' to 170.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 0.38%
delta(sigma) (MC)    = 0.38%
                    +/- 0.01%

circe1:message: updating 'roots' to 350.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 1.67%
delta(sigma) (MC)    = 1.66%
                    +/- 0.03%

circe1:message: updating 'roots' to 500.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 3.66%
delta(sigma) (MC)    = 3.58%
                    +/- 0.07%

circe1:message: updating 'roots' to 800.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 5.21%
delta(sigma) (MC)    = 5.19%
                    +/- 0.11%

circe1:message: updating 'roots' to 1000.0
circe1:message: updating 'ver' to 7
circe1:message: energy 1000.0GeV too high, using spectrum for 800.0GeV
delta(sigma) (Gauss) = 5.21%
delta(sigma) (MC)    = 5.19%
                    +/- 0.11%

circe1:message: updating 'roots' to 90.0
circe1:message: updating 'acc' to JLCNLC
circe1:message: updating 'ver' to 7
circe1:message: energy 90.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%

circe1:message: updating 'roots' to 170.0
circe1:message: updating 'ver' to 7

```

```

circe1:message: energy 170.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.68%
                    +/- 0.11%
circe1:message: updating 'roots' to 350.0
circe1:message: updating 'ver' to 7
circe1:message: energy 350.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 500.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe1:message: updating 'roots' to 800.0
circe1:message: updating 'ver' to 7
circe1:message: energy 800.0GeV interpolated between 500.0 and 1000.0GeV
delta(sigma) (Gauss) = 8.37%
delta(sigma) (MC)    = 8.39%
                    +/- 0.21%
circe1:message: updating 'roots' to 1000.0
circe1:message: updating 'ver' to 7
delta(sigma) (Gauss) = 15.39%
delta(sigma) (MC)    = 14.68%
                    +/- 0.33%

```

Uses JLCNLC 13 and sigma 15a.

We almost forgot to declare the variables in the main program

```

19 <Other variables in sample 19>≡
    real(kind=double) :: s

```

This concludes the integration example. It should have made it obvious how to proceed in a realistic application.

In section 3.2.1 below, I will describe a Monte Carlo method for calculating such integrals efficiently.

3.2 Generators

The function `circe` and its companions are opaque to the user. Since they will in general contain singularities, applications will *not* be able to generate corresponding samples of random numbers efficiently. To fill this gap, four random number generators are provided. The subroutine `girce` will generate particle types $p_{1,2}$ and energy fractions $x_{1,2}$ in one step, according to the selected distribution.² Particle p_1 will be either a positron or a photon and p_2 will be either an electron or a photon. The energy fractions are guaranteed to be above the currently active thresholds: $x_i \geq x_{i,\min}$. This can be used to cut on soft events—the photon distributions are rather soft—which might not be interesting

²The implementation of the flavor selection with non-vanishing thresholds $x_{1,\min}$ and $x_{2,\min}$ is moderately inefficient at the moment. It can be improved by a factor of two.

in most simulations.

20a *<API documentation 11a>+≡*
`call girce (x1, x2, p1, p2, rng)`

Uses `girce 75b`.

The output parameters of `girce` are identical to the input parameters of `circe`, with the exception of `rng`. The latter is a subroutine with a single double precision argument, which will be assigned a uniform deviate from the interval $[0, 1]$ after each call:

20b *<API documentation 11a>+≡*
`subroutine rng (r)`
`real(kind=double) :: r`
`r = <uniform deviate on [0, 1] (never defined)>`
`end subroutine rng`

Typically, it will be just a wrapper around the standard random number generator of the application program. For studies with a definite initial state, three generator functions are available.

20c *<API documentation 11a>+≡*
`call gircee (x1, x2, rng)`
`call girceg (x1, x2, rng)`
`call gircgg (x1, x2, rng)`

Uses `gircee 76d`, `girceg 77b`, and `gircgg 77f`.

3.2.1 Example

Returning to the example from section 3.2.1, I present a concise Monte Carlo algorithm for calculating the same integral:

20d *<Monte Carlo integration 20d>≡*
`s = 0d0`
`s2 = 0d0`
`do n = 1, NEVENT`
`call gircee (x1, x2, random)`
`w = sigma (x1*x2)`
`s = s + w`
`s2 = s2 + w*w`
`end do`
`s = s / dble(NEVENT)`
`s2 = s2 / dble(NEVENT)`
`write (*, 1000) 'delta(sigma) (MC) =', (s-1d0)*100d0`
`write (*, 1000) ' +/-', sqrt((s2-s*s)/dble(NEVENT))*100d0`

Uses `gircee 76d`, `random 21a`, and `sigma 15a`.

20e *<Other variables in sample 19>+≡*
`real(kind=double) :: w, s2, x1, x2`
`integer, parameter :: NEVENT = 10000`
`integer :: n`

Here is a simple linear congruential random number generator for the sample program. Real applications will use their more sophisticated generators instead.

20f *<circe1_sample.f90: public 14b>+≡*

```
public :: random
```

Uses random 21a.

```
21a (circe1_sample.f90: subroutines 15a)+≡
subroutine random (r)
  real(kind=double), intent(out) :: r
  integer :: m = 259200, a = 7141, c = 54773
  integer, save :: n = 0
  ! data n /0/
  n = mod(n*a+c,m)
  r = real (n, kind=double) / real (m, kind=double)
end subroutine random
```

Defines:

random, used in chunks 20 and 21c.

If the cross section is slowly varying on the range where the $x_{1,2}$ distributions are non-zero, this algorithm is very efficient.

However, if this condition is not met, the explicit form of the parameterizations in section 5 should be consulted and appropriate mapping techniques should be applied. The typical example for this problem is a narrow resonance just below the nominal beam energy.

3.2.2 Event Generators

For Monte Carlo event generators that use the standard /hepevt/ common block [14], the addition of the Circe library is trivial. During the initialization of the event generator, the circes subroutine is called to set up Circe's internal state. For example:

```
21b (Initialize event generator 21b)≡
call circes (0d0, 0d0, roots, acc, ver, 1996 07 11, 1)
```

Uses circes 32a.

During event generation, before setting up the e^+e^- initial state, the gircee subroutine is called with the event generator's random number generator:

```
21c (Event generation 21c)≡
call gircee (x1, x2, random)
```

Uses gircee 76d and random 21a.

The resulting energy fractions x_1 and x_2 are now available for defining the initial state electron

```
21d (Event generation 21c)+≡
isthep(1) = 101
idhep(1) = C1_ELECTRON
phep(1,1) = 0d0
phep(2,1) = 0d0
phep(3,1) = x1 * ebeam
phep(4,1) = x1 * ebeam
phep(5,1) = 0d0
```

Uses C1_ELECTRON 11b.

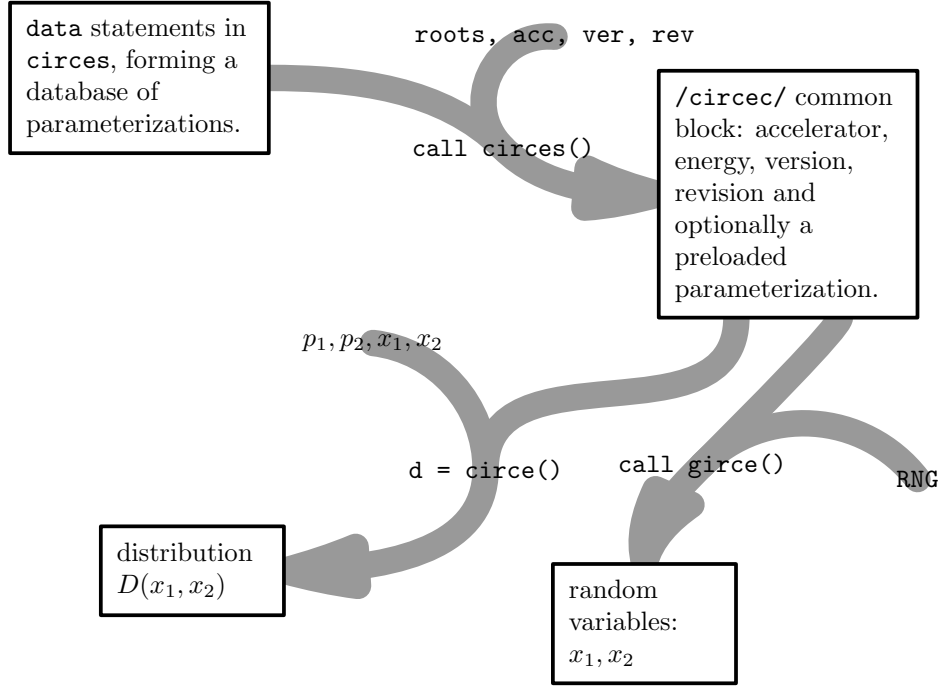


Figure 5: Architecture of **Circe**: `circes()` selects energy and accelerator and loads the parameterization. The function `circe()` calculates the values of the selected distribution function at the given energy fractions. The subroutine `girce()` generates energy fractions using a specified random number generator in accordance with the selected distribution.

and positron.

```

22 <Event generation 21c>+≡
    isthep(2) = 102
    idhep(2) = C1_POSITRON
    phep(1,2) = 0d0
    phep(2,2) = 0d0
    phep(3,2) = - x2 * ebeam
    phep(4,2) = x2 * ebeam
    phep(5,2) = 0d0

```

Uses C1_POSITRON 11b.

Using **Circe** with other event generators should be straightforward as well.

4 Technical Notes

The structure of **Circe** is extremely simple (cf. figure 5) and is mainly a book-keeping exercise. All that needs to be done is to maintain a database of available parameterizations and to evaluate the corresponding functions. The only non trivial algorithms are used for the efficient generation of random deviates.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}\nu^{-1}$	$31.38^{+0.22}_{-0.22}$	$106.25^{+0.71}_{-0.71}$	$95.24^{+0.73}_{-0.73}$	$36.39^{+0.29}_{-0.29}$
$\int d_{e\pm}$	$0.4812^{+0.0041}_{-0.0041}$	$0.5723^{+0.0046}_{-0.0045}$	$0.3512^{+0.0048}_{-0.0048}$	$0.3487^{+0.0040}_{-0.0040}$
$x_{e\pm}^\alpha$	$11.1534^{+0.0770}_{-0.0761}$	$15.2837^{+0.0923}_{-0.0914}$	$27.1032^{+0.3071}_{-0.3019}$	$6.9853^{+0.0733}_{-0.0718}$
$(1 - x_{e\pm})^\alpha$	$-0.6302^{+0.0013}_{-0.0012}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.6453^{+0.0017}_{-0.0017}$	$-0.6444^{+0.0017}_{-0.0017}$
$\int d_\gamma$	$0.6237^{+0.0033}_{-0.0033}$	$0.7381^{+0.0036}_{-0.0036}$	$0.3502^{+0.0034}_{-0.0034}$	$0.4149^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6911^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6947^{+0.0011}_{-0.0011}$	$-0.6876^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$14.9355^{+0.0761}_{-0.0754}$	$24.1647^{+0.1124}_{-0.1116}$	$33.6576^{+0.3021}_{-0.2983}$	$8.3227^{+0.0659}_{-0.0649}$

Table 5: Version 1, revision 1997 04 16 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

I have avoided the use of initialized `common` blocks (i.e. `block data` subroutines), because the Fortran77 standard does not provide a *portable* way of ensuring that `block data` subroutines are actually executed at loading time³. Instead, the `/circom/` common block is tagged by a “magic number” to check for initialization and its members are filled by the `circes` subroutine when necessary.

A more flexible method would be to replace the `data` statements by reading external files. This option causes portability problems, however, because I would have to make sure that the names of the external files are valid in all files systems of the target operating systems. More significantly, splitting the implementation into several parts forces the user to keep all files up to date. This can be a problem, because Fortran source files and data input files will typically be kept in different parts of the file system.

The option of implementing `Circe` statelessly, i.e. with pure function calls and without `common` blocks, has been dismissed. While it would have been more straightforward on the side of the library, it would have placed the burden of maintaining state (accelerator, energy, etc.) on the application program, thereby complicating them considerably. Keeping an explicit state in `Circe` has the additional benefit of allowing to precompute certain internal variables, resulting in a more efficient implementation.

5 Parameterizations

Version 1.02 β of `Circe` supports just one version of the parameterizations. Future versions will provide additional parameterizations.

5.1 Version 1

The first version of the parameterization uses a simple factorized *ansatz*

$$D_{p_1 p_2}^{\alpha 1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha 1 \rho}(x_1) d_{p_2}^{\alpha 1 \rho}(x_2) \quad (12a)$$

³In Fortran90 the common blocks have been replaced by saved module variables.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$119.00^{+0.83}_{-0.83}$	214.33^{+0***}_{-0***}	212.22^{+0***}_{-0***}	$118.99^{+0.91}_{-0.91}$
$\int d_{e^\pm}$	$0.5604^{+0.0040}_{-0.0039}$	$0.6686^{+0.0040}_{-0.0040}$	$0.4448^{+0.0043}_{-0.0043}$	$0.5001^{+0.0038}_{-0.0038}$
$x_{e^\pm}^\alpha$	$4.2170^{+0.0258}_{-0.0255}$	$5.5438^{+0.0241}_{-0.0239}$	$9.6341^{+0.0814}_{-0.0803}$	$2.6184^{+0.0192}_{-0.0190}$
$(1 - x_{e^\pm})^\alpha$	$-0.6118^{+0.0013}_{-0.0013}$	$-0.5847^{+0.0011}_{-0.0011}$	$-0.6359^{+0.0014}_{-0.0014}$	$-0.6158^{+0.0015}_{-0.0015}$
$\int d_\gamma$	$0.7455^{+0.0032}_{-0.0032}$	$1.0112^{+0.0033}_{-0.0033}$	$0.4771^{+0.0031}_{-0.0031}$	$0.6741^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6870^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0004}_{-0.0004}$	$-0.6936^{+0.0008}_{-0.0008}$	$-0.6834^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$6.7145^{+0.0310}_{-0.0308}$	$9.9992^{+0.0342}_{-0.0340}$	$13.1607^{+0.0896}_{-0.0886}$	$3.8589^{+0.0215}_{-0.0213}$

Table 6: Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.

	350 GeV	500 GeV	800 GeV	1600 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$97.45^{+0.67}_{-0.67}$	$106.25^{+0.71}_{-0.71}$	170.86^{+0***}_{-0***}	340.86^{+0***}_{-0***}
$\int d_{e^\pm}$	$0.6093^{+0.0049}_{-0.0049}$	$0.5723^{+0.0046}_{-0.0045}$	$0.6398^{+0.0042}_{-0.0041}$	$0.5094^{+0.0040}_{-0.0040}$
$x_{e^\pm}^\alpha$	$17.6137^{+0.1065}_{-0.1055}$	$15.2837^{+0.0923}_{-0.0914}$	$7.6221^{+0.0365}_{-0.0361}$	$5.0550^{+0.0353}_{-0.0349}$
$(1 - x_{e^\pm})^\alpha$	$-0.6061^{+0.0011}_{-0.0011}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.5944^{+0.0011}_{-0.0011}$	$-0.6187^{+0.0013}_{-0.0013}$
$\int d_\gamma$	$0.7729^{+0.0039}_{-0.0039}$	$0.7381^{+0.0036}_{-0.0036}$	$0.9178^{+0.0034}_{-0.0034}$	$0.5875^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6949^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0005}_{-0.0005}$	$-0.6892^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$28.9399^{+0.1370}_{-0.1361}$	$24.1647^{+0.1124}_{-0.1116}$	$13.1167^{+0.0497}_{-0.0495}$	$7.5514^{+0.0428}_{-0.0424}$

Table 7: Version 1, revision 1997 04 17 of the beam spectra for TESLA.

	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$339.80^{+0.83}_{-0.83}$	$359.36^{+0.93}_{-0.93}$
$\int d_{e^\pm}$	$0.5019^{+0.0016}_{-0.0016}$	$0.4125^{+0.0016}_{-0.0016}$
$x_{e^\pm}^\alpha$	$12.2867^{+0.0318}_{-0.0316}$	$13.3242^{+0.0442}_{-0.0440}$
$(1 - x_{e^\pm})^\alpha$	$-0.6276^{+0.0005}_{-0.0005}$	$-0.6401^{+0.0005}_{-0.0005}$
$\int d_\gamma$	$0.5114^{+0.0012}_{-0.0012}$	$0.3708^{+0.0011}_{-0.0011}$
x_γ^α	$-0.6912^{+0.0003}_{-0.0003}$	$-0.6924^{+0.0004}_{-0.0004}$
$(1 - x_\gamma)^\alpha$	$17.0673^{+0.0375}_{-0.0375}$	$16.8145^{+0.0482}_{-0.0480}$

Table 8: Version 5, revision 1998 05 05 of the beam spectra for high luminosity TESLA.

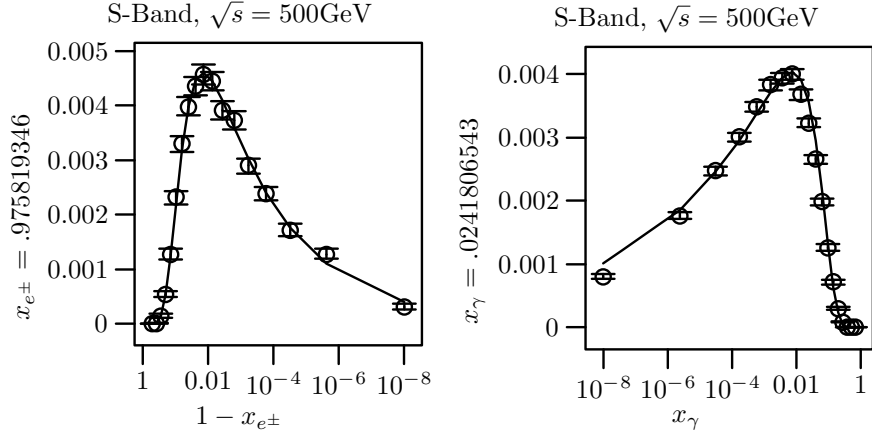


Figure 6: Fit of the e^\pm - and γ -distributions for the S-Band design at $\sqrt{s} = 500\text{GeV}$. The open circles with error bars are the result of the Guinea-Pig simulation. The full line is the fit.

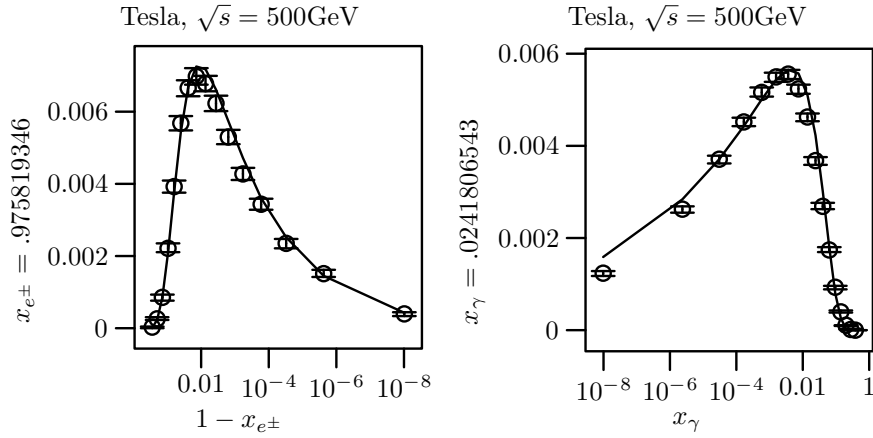


Figure 7: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 500\text{GeV}$.

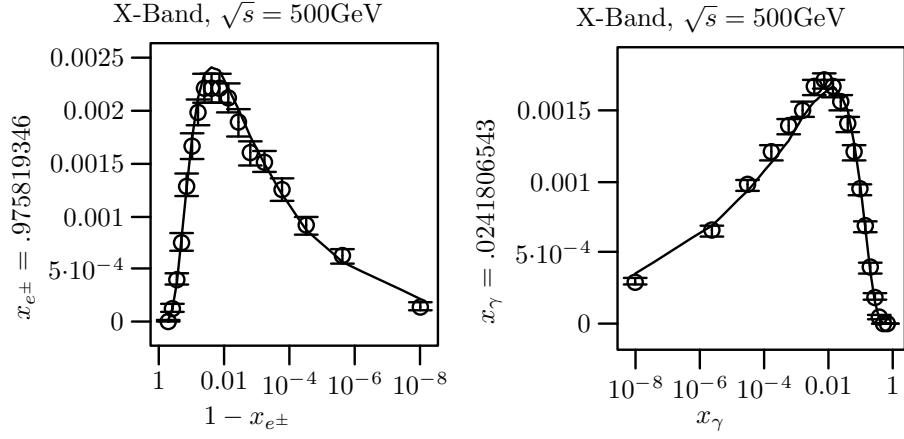


Figure 8: Fit of the e^\pm - and γ -distributions for the X-Band design at $\sqrt{s} = 500 \text{ GeV}$.

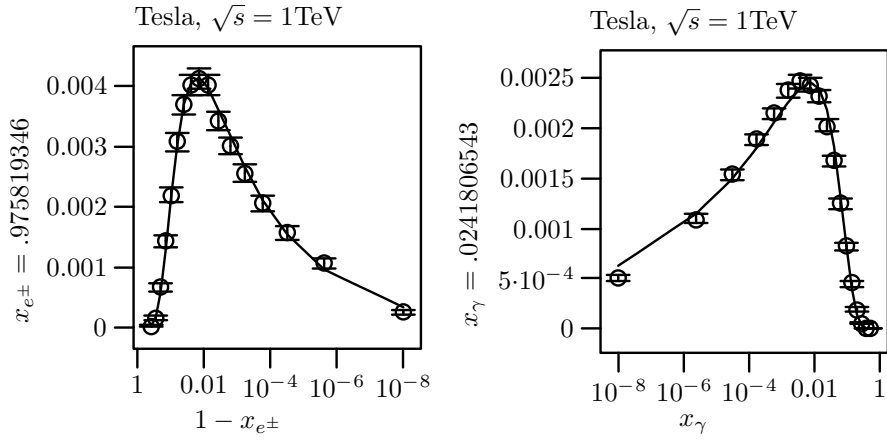


Figure 9: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 1 \text{ TeV}$.

	SBNDEE	TESLEE	XBNDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$9.29^{+0.06}_{-0.06}$	$21.62^{+0.17}_{-0.17}$	$13.97^{+0.10}_{-0.10}$
$\int d_{e^\pm}$	$.6513^{+0.0059}_{-0.0059}$	$.7282^{+0.0083}_{-0.0082}$	$.5270^{+0.0049}_{-0.0049}$
$x_{e^\pm}^\alpha$	$10.3040^{+0.0601}_{-0.0593}$	$14.8578^{+0.1047}_{-0.1034}$	$5.8897^{+0.0455}_{-0.0448}$
$(1 - x_{e^\pm})^\alpha$	$-.5946^{+0.0015}_{-0.0015}$	$-.5842^{+0.0018}_{-0.0018}$	$-.6169^{+0.0016}_{-0.0015}$
$\int d_\gamma$	$.4727^{+0.0035}_{-0.0035}$	$.5300^{+0.0046}_{-0.0046}$	$.3746^{+0.0029}_{-0.0029}$
x_γ^α	$-.6974^{+0.0009}_{-0.0009}$	$-.7039^{+0.0009}_{-0.0009}$	$-.6892^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$20.6447^{+0.1513}_{-0.1497}$	$36.1286^{+0.3027}_{-0.2991}$	$10.0872^{+0.0822}_{-0.0815}$

Table 9: *Experimental* Version 1, revision 0 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

	SBNDEE	TESLEE	XBNDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$45.59^{+0.34}_{-0.34}$	$25.47^{+0.20}_{-0.20}$	$41.06^{+0.28}_{-0.28}$
$\int d_{e^\pm}$	$.7892^{+0.0075}_{-0.0074}$	$.6271^{+0.0066}_{-0.0065}$	$.7203^{+0.0058}_{-0.0058}$
$x_{e^\pm}^\alpha$	$5.4407^{+0.0285}_{-0.0281}$	$8.7504^{+0.0669}_{-0.0658}$	$2.7415^{+0.0121}_{-0.0119}$
$(1 - x_{e^\pm})^\alpha$	$-.5285^{+0.0020}_{-0.0020}$	$-.6058^{+0.0017}_{-0.0017}$	$-.5049^{+0.0020}_{-0.0020}$
$\int d_\gamma$	$.6403^{+0.0040}_{-0.0040}$	$.4278^{+0.0038}_{-0.0038}$	$.6222^{+0.0032}_{-0.0032}$
x_γ^α	$-.6960^{+0.0008}_{-0.0008}$	$-.6982^{+0.0010}_{-0.0010}$	$-.6795^{+0.0008}_{-0.0008}$
$(1 - x_\gamma)^\alpha$	$12.4803^{+0.0839}_{-0.0831}$	$18.5260^{+0.1674}_{-0.1655}$	$4.7506^{+0.0262}_{-0.0260}$

Table 10: *Experimental* Version 1, revision 0 of the beam spectra at 1 TeV.

where the distributions are simple Beta distributions:

$$d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} \delta(1-x) + a_1^{\alpha\rho} x^{a_2^{\alpha\rho}} (1-x)^{a_3^{\alpha\rho}} \quad (12b)$$

$$d_\gamma^{\alpha 1\rho}(x) = a_4^{\alpha\rho} x^{a_5^{\alpha\rho}} (1-x)^{a_6^{\alpha\rho}} \quad (12c)$$

This form of the distributions is motivated by the observation [2] that the e^\pm distributions diverge like a power for $x \rightarrow 1$ and vanish at $x \rightarrow 0$. The behavior of the γ distributions is similar with the borders exchanged.

5.1.1 Fitting

The parameters a_i in (12) have been obtained by a least-square fit of (12) to histograms of simulation results from **Guinea-Pig**. Some care has to be taken when fitting singular distributions to histogrammed data. Obviously equidistant bins are not a good idea, because most bins will be almost empty (cf. figures 1 and 2) and consequently a lot of information will be wasted. One solution to this problem is the use of logarithmic bins. This, however, maps the compact region $[0, 1] \times [0, 1]$ to $[-\infty, 0] \times [-\infty, 0]$, which is inconvenient because of the missing lower bounds.

	350 GeV	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$15.18^{+0.13}_{-0.13}$	$21.62^{+0.17}_{-0.17}$	$43.98^{+0.38}_{-0.38}$
$\int d_{e\pm}$	$.6691^{+0.0083}_{-0.0083}$	$.7282^{+0.0083}_{-0.0082}$	$.7701^{+0.0090}_{-0.0089}$
$x_{e\pm}^\alpha$	$25.2753^{+0.2040}_{-0.2007}$	$14.8578^{+0.1047}_{-0.1034}$	$8.1905^{+0.0543}_{-0.0535}$
$(1 - x_{e\pm})^\alpha$	$-.5994^{+0.0017}_{-0.0017}$	$-.5842^{+0.0018}_{-0.0018}$	$-.5575^{+0.0021}_{-0.0021}$
$\int d_\gamma$	$.4464^{+0.0047}_{-0.0047}$	$.5300^{+0.0046}_{-0.0046}$	$.5839^{+0.0047}_{-0.0047}$
x_γ^α	$-.7040^{+0.0011}_{-0.0011}$	$-.7039^{+0.0009}_{-0.0009}$	$-.7046^{+0.0009}_{-0.0009}$
$(1 - x_\gamma)^\alpha$	$60.1882^{+0.5882}_{-0.5797}$	$36.1286^{+0.3027}_{-0.2991}$	$19.3944^{+0.1681}_{-0.1660}$

Table 11: *Experimental* Version 1, revision 0 of the beam spectra for TESLEE.

The more appropriate solution is to use two maps

$$\begin{aligned}\phi : [0, 1] &\rightarrow [0, 1] \\ x &\mapsto y = x^{1/\eta}\end{aligned}\tag{13}$$

where $x = x_\gamma$ or $x = 1 - x_{e\pm}$, and to bin the result equidistantly. If η is chosen properly (cf. (10)), the bin contents will then fall off at the singularity. The fits in tables 5, 6, and 7 have been performed with $\eta = 5$ and the resulting bin contents can be read off from figures 6–9.

Using this procedure for binning the results of the simulations, the popular fitting package MINUIT [15] converges quickly in all cases considered. The resulting parameters are given in tables 5, 6, and 7. Plots of the corresponding distributions have been shown in figures 1 and 2. It is obvious that an *ansatz* like (12) is able to distinguish among the accelerator designs. Thus it can provide a solid basis for physics studies.

In figures 6–9 I give a graphical impression of the quality of the fit, which appears to be as good as one could reasonably expect for a simple *ansatz* like (12). Note that the histograms have non-equidistant bins and that the resulting Jacobians have not been removed. Therefore the bin contents falls off at the singularities, as discussed above.

The errors used for the least-square fit had to be taken from a Monte Carlo (MC) study. **Guinea-Pig** only provides the \sqrt{n} from Poissonian statistics for each bin, but the error accumulation during tracking the particles through phase space is not available. The MC studies shows that the latter error dominates the former, but appears to be reasonably Gaussian. A complete MC study of all parameter sets is computationally expensive (more than a week of processor time on a fast SGI). From an exemplary MC study of a few parameter sets, it appears that the errors can be described reasonably well by rescaling the Poissonian error in each bin with appropriate factors for electrons/positrons and photons and for continuum and delta. This procedure has been adopted.

The $\chi^2/\text{d.o.f.}$'s of the fits are less than $\mathcal{O}(10)$. The simple *ansatz* (12) is therefore very satisfactory. In fact, trying to improve the ad-hoc factorized Beta distributions by the better motivated approximations from [7] or [16], it turns out [17] that (12) provides a significantly better fit of the results of the simulations. The price to pay is that the parameters in (12) have no direct

physical interpretation.

5.1.2 Generators

For this version of the parameterizations we need a fast generator of Beta distributions:

$$\beta^{a,b}(x) \propto x^{a-1}(1-x)^{b-1} \quad (14)$$

This problem has been studied extensively and we can use a published algorithm [18] that is guaranteed to be very fast for all a, b such that $0 < a \leq 1 \leq b$, which turns out to be always the case (cf. tables 5, 6, and 7).

5.2 Future Versions

There are two ways in which the parameterizations can be improved:

more complicated functions: the factorized fits can only be improved marginally by adding more positive semi-definite factors to (12). More improvement is possible by using sums of functions, but in this case, the best fits violate the positivity requirement and have to be discarded.

correlations: the parameterization in section 5.1 is factorized. While this is a good approximation, the simulations nevertheless show correlations among x_1 and x_2 . These correlations can be included in a future version.

interpolation: the parameterization in section 5.1 is based on fitting the simulation results by simple functions. Again, this appears to be a good approximation. But such fits can not uncover any fine structure of the distributions. Therefore it will be worthwhile to study interpolations of the simulation results in the future. A proper interpolation of results with statistical errors is however far from trivial: straightforward polynomial or spline interpolations will be oscillatory and violate the positivity requirement. Smoothing algorithms have to be investigated in depth before such a parameterization can be released.

other simulations: besides [5], other simulation codes are invited to contribute their results for inclusion in the Circe library.

6 Implementation of circe1

```
29 <circe1.f90 29>≡
    ! circe1.f90 -- canonical beam spectra for linear collider physics
    ! $Id: circe1.nw 67 2002-03-28 17:13:06Z ohl $
    <Copyleft notice 30a>
    <Main module 30c>
```

```

30a <Copyleft notice 30a>≡
!
! Copyright (C) 1999-2011 by
!   Wolfgang Kilian <kilian@physik.uni-siegen.de>
!   Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
!   Juergen Reuter <juergen.reuter@desy.de>
!   Christian Speckner <christian.speckner@physik.uni-freiburg.de>
!
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! any later version.
!
! WHIZARD is distributed in the hope that it will be useful, but
! WITHOUT ANY WARRANTY; without even the implied warranty of
! MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
! GNU General Public License for more details.
!
! You should have received a copy of the GNU General Public License
! along with this program; if not, write to the Free Software
! Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! This file has been stripped of most comments. For documentation, refer
! to the source 'circe1.nw'

```

Now we can move on to the implementation.

6.1 Symbolic Constants

The file `circe.h` contains symbolic names for various magic constants used by Circe:

```

30b <circe.h 30b>≡
c circe.h -- canonical beam spectra for linear collider physics
c $Id: circe1.nw 67 2002-03-28 17:13:06Z ohl $
Uses circe 31b.

30c <Main module 30c>≡
module circe1
  use kinds

  implicit none
  private
  <Public subroutines 31a>

  <Particle codes 11b>
  <Accelerator codes 13>
  <Private parameters 36d>

  integer, parameter, public :: MAGIC0 = 19040616
  real(kind=double), parameter :: KIREPS = 1D-6

```

```

    <Declaration: circe1 parameters 32c>

    type(circe1_params_t), public, save :: circe1_params

contains

    <Module subroutines 31b>
end module circe1

```

6.2 Distributions

6.2.1 Version 1

We start with a convenience function which dispatches over the valid particle types. The hardest part is of course to avoid typos in such trivial functions ...

```

31a <Public subroutines 31a>≡
    public :: circe
    Uses circe 31b.

31b <Module subroutines 31b>≡
    function circe (x1, x2, p1, p2)
    real(kind=double) :: x1, x2
    integer :: p1, p2
    real(kind=double) :: circe
    <Initialization check 32g>
    circe = -1.0
    if (abs(p1) .eq. C1_ELECTRON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            circe = circee (x1, x2)
        else if (p2 .eq. C1_PHOTON) then
            circe = circeg (x1, x2)
        end if
    else if (p1 .eq. C1_PHOTON) then
        if (abs(p2) .eq. C1_ELECTRON) then
            circe = circeg (x2, x1)
        else if (p2 .eq. C1_PHOTON) then
            circe = circgg (x1, x2)
        end if
    end if
end function circe

```

Defines:

 circe, used in chunks 11a, 31, 30–32, 82, 88a, 104c, and 110a.

Uses C1_ELECTRON 11b, C1_PHOTON 11b, circee 41a, circeg 42a, and circgg 42d.

```

31c <Public subroutines 31a>+≡
    public :: circes
    Uses circes 32a.

```

32a *<Module subroutines 31b>+≡*

```

subroutine circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
  real(kind=double) :: xx1m, xx2m, xroots
  integer :: xacc, xver, xrev, xchat
<Local variables for circes 33b>
<Initializations for circes 35b>
  if (circe1_params%magic .ne. 19040616) then
    circe1_params%magic = 19040616
    <Initialize circe1 parameters 32h>
  end if
<Update circe1 parameters 33a>
<formats for circes 37e>
end subroutine circes

```

Defines:

circes, used in chunks 32a, 12b, 17, 32a, 21b, 31, 32, 82, and 86a.

32b *<Public subroutines 31a>+≡*

```

public :: circe1_params_t

```

32c *<Declaration: circe1 parameters 32c>≡*

```

type :: circe1_params_t
<8-byte aligned part of circe1 parameters 32d>
<4-byte aligned part of circe1 parameters 32e>
end type circe1_params_t

```

32d *<8-byte aligned part of circe1 parameters 32d>≡*

```

real(kind=double) :: x1m = 0d0
real(kind=double) :: x2m = 0d0
real(kind=double) :: roots = 500D0

```

32e *<4-byte aligned part of circe1 parameters 32e>≡*

```

integer :: acc = TESLA
integer :: ver = 0
integer :: rev = 0
integer :: chat = 1

```

Uses TESLA 13.

Instead of using fragile `block data` subroutines, we use a magic number to tag `circe1_params` as initialized:

32f *<4-byte aligned part of circe1 parameters 32e>+≡*

```

integer :: magic

```

Since negative values are no updated, we can call `circes` with all negative variables to ensure initialization:

32g *<Initialization check 32g>≡*

```

if (circe1_params%magic .ne. MAGIC0) then
  call circes (-1d0, -1d0, -1d0, -1, -1, -1, -1)
endif

```

Uses `circes` 32a.

32h *<Initialize circe1 parameters 32h>≡*

```

circe1_params%x1m = 0d0
circe1_params%x2m = 0d0

```



```

circe1_params%roots = 500D0
circe1_params%acc = TESLA
circe1_params%ver = 0
circe1_params%rev = 0
circe1_params%chat = 1
if (xchat .ne. 0) then
  call circem ('MESSAGE', 'starting up ...')
  call circem ('MESSAGE', &
    '$Id: circe.nw 67 2002-03-28 17:13:06Z ohl $')
endif

```

Uses TESLA 13, circe 31b, and circem 81c.

```

33a <Update circe1 parameters 33a>≡
  if ((xchat .ge. 0) .and. (xchat .ne. circe1_params%chat)) then
    circe1_params%chat = xchat
    if (circe1_params%chat .ge. 1) then
      write (msgbuf, 1000) 'chat', circe1_params%chat
1000   format ('updating ', A, ''' to ', I2)
      call circem ('MESSAGE', msgbuf)
    endif
  else
    if (circe1_params%chat .ge. 2) then
      write (msgbuf, 1100) 'chat', circe1_params%chat
1100   format ('keeping ', A, ''' at ', I2)
      call circem ('MESSAGE', msgbuf)
    endif
  endif
endif

```

Uses circem 81c.

```

33b <Local variables for circes 33b>≡
  character(len=60) :: msgbuf

```

```

33c <Update circe1 parameters 33a>+≡
  if ((xx1m .ge. 0d0) .and. (xx1m .ne. circe1_params%x1m)) then
    circe1_params%x1m = xx1m
    if (circe1_params%chat .ge. 1) then
      write (msgbuf, 1001) 'x1min', circe1_params%x1m
1001   format ('updating ', A, ''' to ', E12.4)
      call circem ('MESSAGE', msgbuf)
    endif
  else
    if (circe1_params%chat .ge. 2) then
      write (msgbuf, 1101) 'x1min', circe1_params%x1m
1101   format ('keeping ', A, ''' at ', E12.4)
      call circem ('MESSAGE', msgbuf)
    endif
  endif
endif

```

Uses circem 81c.

```

33d <Update circe1 parameters 33a>+≡
  if ((xx2m .ge. 0d0) .and. (xx2m .ne. circe1_params%x2m)) then
    circe1_params%x2m = xx2m

```

```

        if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1001) 'x2min', circe1_params%x2m
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1101) 'x2min', circe1_params%x2m
            call circem ('MESSAGE', msgbuf)
        endif
    endif
    Uses circem 81c.
34a  <Update circe1 parameters 33a>+=
        if ((xroots .ge. 0d0) .and.(xroots .ne. circe1_params%roots)) then
            circe1_params%roots = xroots
            if (circe1_params%chat .ge. 1) then
                write (msgbuf, 1002) 'roots', circe1_params%roots
1002      format ('updating ', A, ''' to ', F6.1)
                call circem ('MESSAGE', msgbuf)
            endif
        else
            if (circe1_params%chat .ge. 2) then
                write (msgbuf, 1102) 'roots', circe1_params%roots
1102      format ('keeping ', A, ''' at ', F6.1)
                call circem ('MESSAGE', msgbuf)
            endif
        endif
    Uses circem 81c.
34b  <Update circe1 parameters 33a>+=
        if ((xacc .ge. 0) .and.(xacc .ne. circe1_params%acc)) then
            if ((xacc .ge. 1) .and. (xacc .le. NACC)) then
                circe1_params%acc = xacc
                if (circe1_params%chat .ge. 1) then
                    write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
1003      format ('updating ', A, ''' to ', A)
                    call circem ('MESSAGE', msgbuf)
                endif
            else
                write (msgbuf, 1203) xacc
1203      format ('invalid 'acc': ', I8)
                call circem ('ERROR', msgbuf)
                write (msgbuf, 1103) 'acc', accnam(circe1_params%acc)
1103      format ('keeping ', A, ''' at ', A)
                call circem ('MESSAGE', msgbuf)
            endif
        else
            if (circe1_params%chat .ge. 2) then
                write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
                call circem ('MESSAGE', msgbuf)
            endif
        endif

```

```

endif
if ((circe1_params%acc .eq. SBNDEE) .or. (circe1_params%acc .eq. TESLEE) &
    .or. (circe1_params%acc .eq. XBNDEE)) then
  <Warn that no parameter set has been endorsed for e-e- yet 35c>
endif
Uses NACC 13, SBNDEE 13, TESLEE 13, XBNDEE 13, and circem 81c.

35a <Local variables for circes 33b>+≡
      character(len=6), dimension(NACC) :: accnam
Uses NACC 13.

35b <Initializations for circes 35b>≡
      data accnam(SBAND)  /'SBAND'/
      data accnam(TESLA)  /'TESLA'/
      data accnam(JLCNLC) /'JLCNLC'/
      data accnam(SBNDEE) /'SBNDEE'/
      data accnam(TESLEE) /'TESLEE'/
      data accnam(XBNDEE) /'XBNDEE'/
      data accnam(NLCH)  /'NLC H'/
Uses JLCNLC 13, SBAND 13, SBNDEE 13, TESLA 13, TESLEE 13, and XBNDEE 13.

35c <Warn that no parameter set has been endorsed for e-e- yet 35c>≡
      call circem ('WARNING', '*****')
      call circem ('WARNING', '* The accelerator parameters have *')
      call circem ('WARNING', '* not been endorsed for use in      *')
      call circem ('WARNING', '* an e-e- collider yet!!!      *')
      call circem ('WARNING', '*****')
Uses circem 81c.

35d <Update circe1 parameters 33a>+≡
      if (xver .ge. 0) then
        circe1_params%ver = xver
        if (circe1_params%chat .ge. 1) then
          write (msgbuf, 1000) 'ver', circe1_params%ver
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (circe1_params%chat .ge. 2) then
          write (msgbuf, 1100) 'ver', circe1_params%ver
          call circem ('MESSAGE', msgbuf)
        endif
      endif
Uses circem 81c.

35e <Update circe1 parameters 33a>+≡
      if ((xrev .ge. 0) .and.(xrev .ne. circe1_params%rev)) then
        circe1_params%rev = xrev
        if (circe1_params%chat .ge. 1) then
          write (msgbuf, 1004) 'rev', circe1_params%rev
1004      format ('updating ', A, ''' to ', I8)
          call circem ('MESSAGE', msgbuf)
        endif

```

```

else
  if (circe1_params%chat .ge. 2) then
    write (msgbuf, 1104) 'rev', circe1_params%rev
1104    format ('keeping ', A, ''' at ', I8)
    call circem ('MESSAGE', msgbuf)
  endif
endif
Uses circem 81c.
Versions 3 and 4 are identical to version 1, except for TESLA at 800 GeV.
36a <Update circe1 parameters 33a>+≡
  ver34 = 0
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Update version 1 derived parameters in circe1 parameters 37a>
  else if ((circe1_params%ver .eq. 3) .or. (circe1_params%ver .eq. 4)) then
    ver34 = circe1_params%ver
    circe1_params%ver = 1
    <Update version 3 and 4 derived parameters in circe1 parameters 50a>
  else if (circe1_params%ver .eq. 5) then
    circe1_params%ver = 1
    <Update version 5 derived parameters in circe1 parameters 52a>
  else if (circe1_params%ver .eq. 6) then
    circe1_params%ver = 1
    <Update version 6 derived parameters in circe1 parameters 53f>
  else if (circe1_params%ver .eq. 7) then
    circe1_params%ver = 1
    <Update version 7 derived parameters in circe1 parameters 56a>
  else if (circe1_params%ver .eq. 8) then
    circe1_params%ver = 1
    <Update version 8 derived parameters in circe1 parameters 60b>
  else if (circe1_params%ver .eq. 9) then
    circe1_params%ver = 1
    <Update version 9 derived parameters in circe1 parameters 62c>
  <else handle invalid versions 36c>
36b <Local variables for circes 33b>+≡
  integer :: ver34
36c <else handle invalid versions 36c>≡
  else if (circe1_params%ver .eq. 2) then
    <Version 2 has been retired 49c>
  else if (circe1_params%ver .gt. 9) then
    call circem ('PANIC', 'versions >9 not available yet')
    return
  else
    call circem ('PANIC', 'version must be positive')
    return
  end if
Uses circem 81c.
36d <Private parameters 36d>≡
  integer :: e, r, ehi, elo

```

37a \langle Update version 1 derived parameters in circe1 parameters 37a $\rangle \equiv$

```

    if (circe1_params%rev .eq. 0) then
      r = 0
    elseif (circe1_params%rev .ge. 19970417) then
      r = 5
    elseif (circe1_params%rev .ge. 19960902) then
      r = 4
    elseif (circe1_params%rev .ge. 19960729) then
      r = 3
    elseif (circe1_params%rev .ge. 19960711) then
      r = 2
    elseif (circe1_params%rev .ge. 19960401) then
      r = 1
    elseif (circe1_params%rev .lt. 19960401) then
      call circem ('ERROR', &
        'no revision of version 1 before 96/04/01 available')
      call circem ('MESSAGE', 'falling back to default')
      r = 1
    endif
    if (circe1_params%chat .ge. 2) then
      write (msgbuf, 2000) circe1_params%rev, r
2000    format ('mapping date ', I8, ' to revision index ', I2)
      call circem ('MESSAGE', msgbuf)
    endif
  
```

Uses circem 81c.

37b \langle Log revision mapping 37b $\rangle \equiv$

```

    if (circe1_params%chat .ge. 2) then
      write (msgbuf, 2000) circe1_params%rev, r
      call circem ('MESSAGE', msgbuf)
    endif
  
```

Uses circem 81c.

37c \langle Update version 1 derived parameters in circe1 parameters 37a $\rangle + \equiv$
 \langle Map roots to e 37d \rangle

37d \langle Map roots to e 37d $\rangle \equiv$

```

    if (circe1_params%roots .eq. 350d0) then
      e = GEV350
    else if ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0))
      write (msgbuf, 2001) circe1_params%roots, 350d0
      call circem ('MESSAGE', msgbuf)
      e = GEV350
  
```

Uses circem 81c.

37e \langle formats for circes 37e $\rangle \equiv$

```

2001 format ('treating energy ', F6.1, 'GeV as ', F6.1, 'GeV')
  
```

37f \langle Map roots to e 37d $\rangle + \equiv$

```

    else if (circe1_params%roots .eq. 500d0) then
      e = GEV500
    else if ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0))
  
```

```

        write (msgbuf, 2001) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    else if (circe1_params%roots .eq. 800d0) then
        e = GEV800
    else if ((circe1_params%roots .ge. 750d0) .and. (circe1_params%roots .le. 850d0))
        write (msgbuf, 2001) circe1_params%roots, 800d0
        call circem ('MESSAGE', msgbuf)
        e = GEV800
    else if (circe1_params%roots .eq. 1000d0) then
        e = TEV1
    else if ((circe1_params%roots .ge. 900d0) .and. (circe1_params%roots .le. 1100d0))
        write (msgbuf, 2001) circe1_params%roots, 1000d0
        call circem ('MESSAGE', msgbuf)
        e = TEV1
    else if (circe1_params%roots .eq. 1600d0) then
        e = TEV16
    else if ((circe1_params%roots .ge. 1500d0) .and. (circe1_params%roots .le. 1700d0))
        write (msgbuf, 2001) circe1_params%roots, 1600d0
        call circem ('MESSAGE', msgbuf)
        e = TEV16

```

Uses circem 81c.

```

38a <Map roots to e 37d>+≡
    else
        call circem ('ERROR', &
            'only ROOTS = 350, 500, 800, 1000 and 1600GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif

```

Uses circem 81c.

```

38b <Update version 1 derived parameters in circe1 parameters 37a>+≡
    if (xallum(e,circe1_params%acc,r) .lt. 0d0) then
        write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    end if

```

<Log energy mapping 38d>

Uses circem 81c.

```

38c <formats for circes 37e>+≡
    2002 format ('energy ', F6.1, ' not available for ', A6,' in revision ', I2)

```

```

38d <Log energy mapping 38d>≡
    if (circe1_params%chat .ge. 2) then
        if (e .ge. GEV090) then
            write (msgbuf, 2003) circe1_params%roots, e
            call circem ('MESSAGE', msgbuf)
        else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
            write (msgbuf, 2013) circe1_params%roots, elo, ehi

```

```

        call circem ('MESSAGE', msgbuf)
    end if
endif
Uses circem 81c.
39a <formats for circes 37e>+≡
    2003 format ('mapping energy ', F6.1, ' to energy index ', I2)
    2013 format ('mapping energy ', F6.1, ' to energy indices ', I2, ' and ', I2)
The energies 250 GeV, 1.2 TeV and 1.5 TeV were entered late into the game by
teh SLAC people.
39b <Local variables for circes 33b>+≡
    integer, parameter :: GEV090 = -1
    integer, parameter :: GEV170 = 0
    integer, parameter :: GEV350 = 1
    integer, parameter :: GEV500 = 2
    integer, parameter :: GEV800 = 3
    integer, parameter :: TEV1  = 4
    integer, parameter :: TEV16 = 5
    integer, parameter :: GEV250 = 6
    integer, parameter :: TEV12 = 7
    integer, parameter :: TEV15 = 8
    integer, parameter :: A1NEGY = 5
    integer, parameter :: A1NREV = 5
    integer :: i
39c <8-byte aligned part of circe1 parameters 32d>+≡
    real(kind=double) :: lumi
    real(kind=double) :: a1(0:7)
39d <Update version 1 derived parameters in circe1 parameters 37a>+≡
    circe1_params%lumi = xallum (e,circe1_params%acc,r)
    do i = 0, 7
        circe1_params%a1(i) = xa1(i,e,circe1_params%acc,r)
    end do
39e <Local variables for circes 33b>+≡
    real(kind=double), dimension(A1NEGY,NACC,0:A1NREV), save :: xallum = 0
    real(kind=double), dimension(0:7,A1NEGY,NACC,0:A1NREV), save :: xa1 = 0
Uses NACC 13.
Revision 1. The mother of all revisions.
39f <Initializations for circes 35b>+≡
    xallum(GEV500,SBAND,1) = 5.212299E+01
    xa1(0:7,GEV500,SBAND,1) = (/ &
        .39192E+00, .66026E+00, .11828E+02, -.62543E+00, &
        .52292E+00, -.69245E+00, .14983E+02, .65421E+00 /)
    xallum(GEV500,TESLA,1) = 6.066178E+01
    xa1(0:7,GEV500,TESLA,1) = (/ &
        .30196E+00, .12249E+01, .21423E+02, -.57848E+00, &
        .68766E+00, -.69788E+00, .23121E+02, .78399E+00 /)
    xallum(GEV500,XBAND,1) = 5.884699E+01
    xa1(0:7,GEV500,XBAND,1) = (/ &

```

```
.48594E+00, .52435E+00, .83585E+01,-.61347E+00, &
.30703E+00,-.68804E+00, .84109E+01, .44312E+00 /)
```

Uses SBAND 13, TESLA 13, and XBAND 13.

40a *<Initializations for circes 35b>+≡*

```
xa1lum(TEV1,SBAND,1) = 1.534650E+02
xa1(0:7,TEV1,SBAND,1) = (/ &
.24399E+00, .87464E+00, .66751E+01,-.56808E+00, &
.59295E+00,-.68921E+00, .94232E+01, .83351E+00 /)
xa1lum(TEV1,TESLA,1) = 1.253381E+03
xa1(0:7,TEV1,TESLA,1) = (/ &
.39843E+00, .70097E+00, .11602E+02,-.61061E+00, &
.40737E+00,-.69319E+00, .14800E+02, .51382E+00 /)
xa1lum(TEV1,XBAND,1) = 1.901783E+02
xa1(0:7,TEV1,XBAND,1) = (/ &
.32211E+00, .61798E+00, .28298E+01, -.54644E+00, &
.45674E+00, -.67301E+00, .41703E+01, .74536E+00 /)
```

Uses SBAND 13, TESLA 13, and XBAND 13.

Unavailable

40b *<Initializations for circes 35b>+≡*

```
xa1lum(GEV350,1:NACC,1) = NACC * (-1d0)
xa1lum(GEV800,1:NACC,1) = NACC * (-1d0)
```

Uses NACC 13.

Unavailable as well

40c *<Initializations for circes 35b>+≡*

```
xa1lum(GEV500,SBNDDEE:NACC,1) = 4 * (-1d0)
xa1lum(TEV1,SBNDDEE:NACC,1) = 4 * (-1d0)
```

Uses NACC 13 and SBNDDEE 13.

No 1.6TeV parameters in this revision

40d *<Initializations for circes 35b>+≡*

```
xa1lum(TEV16,1:NACC,1) = 7 * (-1d0)
```

Uses NACC 13.

40e *<Public subroutines 31a>+≡*

```
public :: circel
```

Uses circel 40f.

40f *<Module subroutines 31b>+≡*

```
subroutine circel (l)
real(kind=double), intent(out) :: l
l = circel_params%lumi
end subroutine circel
```

Defines:

circel, used in chunks 40f, 12a, and 40e.

40g *<Public subroutines 31a>+≡*

```
public :: circee
```

Uses circee 41a.


```

41a <Module subroutines 31b>+≡
    function circee (x1, x2)
        real(kind=double) :: x1, x2
        real(kind=double) :: circee
        real(kind=double) :: d1, d2
    <Initialization check 32g>
        circee = -1.0
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Calculate version 1 of the e+e- distribution 41b>
    <else handle invalid versions 36c>
        end function circee

```

Defines:

circee, used in chunks 14–16, 31b, 40g, and 41b.

Uses d1 15f and d2 16b.

The first version of the parametrization is factorized

$$D_{p_1 p_2}^{\alpha_1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha_1 \rho}(x_1) d_{p_2}^{\alpha_1 \rho}(x_2) \quad (15)$$

where the distributions are

$$d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha_0 \rho} \delta(1-x) + a_1^{\alpha_1 \rho} x^{a_2^{\alpha_2 \rho}} (1-x)^{a_3^{\alpha_3 \rho}} \quad (16)$$

$$d_\gamma(x) = a_4^{\alpha_4 \rho} x^{a_5^{\alpha_5 \rho}} (1-x)^{a_6^{\alpha_6 \rho}} \quad (17)$$

```

41b <Calculate version 1 of the e+e- distribution 41b>≡
    if (x1 .eq. 1d0) then
        d1 = circe1_params%a1(0)
    elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
    elseif (x1 .eq. -1d0) then
        d1 = 1d0 - circe1_params%a1(0)
    else
        d1 = 0d0
    endif
    if (x2 .eq. 1d0) then
        d2 = circe1_params%a1(0)
    elseif (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%
    elseif (x2 .eq. -1d0) then
        d2 = 1d0 - circe1_params%a1(0)
    else
        d2 = 0d0
    endif
    circee = d1 * d2

```

Uses circee 41a, d1 15f, and d2 16b.

```

41c <Public subroutines 31a>+≡
    public :: circeg

```

Uses circeg 42a.

```

42a  <Module subroutines 31b>+≡
      function circeg (x1, x2)
        real(kind=double) :: x1, x2
        real(kind=double) :: circeg
        real(kind=double) :: d1, d2
      <Initialization check 32g>
        circeg = -1.0
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
      <Calculate version 1 of the  $e^\pm\gamma$  distribution 42b>
      <else handle invalid versions 36c>
        end function circeg

```

Defines:

 circeg, used in chunks 14a, 31b, 41c, and 42b.

Uses d1 15f and d2 16b.

```

42b  <Calculate version 1 of the  $e^\pm\gamma$  distribution 42b>≡
      if (x1 .eq. 1d0) then
        d1 = circe1_params%a1(0)
      else if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%a1(3)
      else if (x1 .eq. -1d0) then
        d1 = 1d0 - circe1_params%a1(0)
      else
        d1 = 0d0
      end if
      if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%a1(6)
      else if (x2 .eq. -1d0) then
        d2 = circe1_params%a1(7)
      else
        d2 = 0d0
      end if
      circeg = d1 * d2

```

Uses circeg 42a, d1 15f, and d2 16b.

```

42c  <Public subroutines 31a>+≡
      public :: circgg

```

Uses circgg 42d.

```

42d  <Module subroutines 31b>+≡
      function circgg (x1, x2)
        real(kind=double) :: x1, x2
        real(kind=double) :: circgg
        real(kind=double) :: d1, d2
      <Initialization check 32g>
        circgg = -1.0
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
      <Calculate version 1 of the  $\gamma\gamma$  distribution 43a>
      <else handle invalid versions 36c>
        end function circgg

```

Defines:

circgg, used in chunks 14a, 31b, 42c, 43a, and 75c.

Uses d1 15f and d2 16b.

43a \langle Calculate version 1 of the $\gamma\gamma$ distribution 43a $\rangle \equiv$

```

    if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%
    elseif (x1 .eq. -1d0) then
        d1 = circe1_params%a1(7)
    else
        d1 = 0d0
    endif
    if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
    elseif (x2 .eq. -1d0) then
        d2 = circe1_params%a1(7)
    else
        d2 = 0d0
    endif
    circgg = d1 * d2

```

Uses circgg 42d, d1 15f, and d2 16b.

Revision 2. New Tesla parameters, including 350 GeV and 800 GeV.

43b \langle Initializations for circes 35b $\rangle + \equiv$

```

xa1lum(GEV500,SBAND,2) = .31057E+02
xa1(0:7,GEV500,SBAND,2) = (/ &
    .38504E+00, .79723E+00, .14191E+02,-.60456E+00, &
    .53411E+00,-.68873E+00, .15105E+02, .65151E+00 /)
xa1lum(TEV1,SBAND,2) = .24297E+03
xa1(0:7,TEV1,SBAND,2) = (/ &
    .24374E+00, .89466E+00, .70242E+01,-.56754E+00, &
    .60910E+00,-.68682E+00, .96083E+01, .83985E+00 /)
xa1lum(GEV350,TESLA,2) = .73369E+02
xa1(0:7,GEV350,TESLA,2) = (/ &
    .36083E+00, .12819E+01, .37880E+02,-.59492E+00, &
    .69109E+00,-.69379E+00, .40061E+02, .65036E+00 /)
xa1lum(GEV500,TESLA,2) = .10493E+03
xa1(0:7,GEV500,TESLA,2) = (/ &
    .29569E+00, .11854E+01, .21282E+02,-.58553E+00, &
    .71341E+00,-.69279E+00, .24061E+02, .77709E+00 /)
xa1lum(GEV800,TESLA,2) = .28010E+03
xa1(0:7,GEV800,TESLA,2) = (/ &
    .22745E+00, .11265E+01, .10483E+02,-.55711E+00, &
    .69579E+00,-.69068E+00, .13093E+02, .89605E+00 /)
xa1lum(TEV1,TESLA,2) = .10992E+03
xa1(0:7,TEV1,TESLA,2) = (/ &
    .40969E+00, .66105E+00, .11972E+02,-.62041E+00, &
    .40463E+00,-.69354E+00, .14669E+02, .51281E+00 /)
xa1lum(GEV500,XBAND,2) = .35689E+02
xa1(0:7,GEV500,XBAND,2) = (/ &
    .48960E+00, .46815E+00, .75249E+01,-.62769E+00, &

```

```

        .30341E+00,-.68754E+00, .85545E+01, .43453E+00 /)
xa1lum(TEV1,XBAND,2) = .11724E+03
xa1(0:7,TEV1,XBAND,2) = (/ &
        .31939E+00, .62415E+00, .30763E+01,-.55314E+00, &
        .45634E+00,-.67089E+00, .41529E+01, .73807E+00 /)

```

Uses SBAND 13, TESLA 13, and XBAND 13.

Unavailable

44a *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBAND,2) = -1d0
        xa1lum(GEV350,XBAND,2) = -1d0
        xa1lum(GEV800,SBAND,2) = -1d0
        xa1lum(GEV800,XBAND,2) = -1d0

```

Uses SBAND 13 and XBAND 13.

Unavailable as well

44b *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350,SBNDDEE:NACC,2) = 4 * (-1d0)
        xa1lum(GEV500,SBNDDEE:NACC,2) = 4 * (-1d0)
        xa1lum(GEV800,SBNDDEE:NACC,2) = 4 * (-1d0)
        xa1lum(TEV1,SBNDDEE:NACC,2) = 4 * (-1d0)

```

Uses NACC 13 and SBNDDEE 13.

No 1.6TeV parameters in this revision

44c *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(TEV16,1:NACC,2) = 7 * (-1d0)

```

Uses NACC 13.

Revision 3. Features:

- improved error estimates.
- cleaner fitting procedure, including delta function pieces.

44d *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV500,SBAND,3) = .31469E+02
        xa1(0:7,GEV500,SBAND,3) = (/ &
        .38299E+00, .72035E+00, .12618E+02,-.61611E+00, &
        .51971E+00,-.68960E+00, .15066E+02, .63784E+00 /)
        xa1lum(TEV1,SBAND,3) = .24566E+03
        xa1(0:7,TEV1,SBAND,3) = (/ &
        .24013E+00, .95763E+00, .69085E+01,-.55151E+00, &
        .59497E+00,-.68622E+00, .94494E+01, .82158E+00 /)
        xa1lum(GEV350,TESLA,3) = .74700E+02
        xa1(0:7,GEV350,TESLA,3) = (/ &
        .34689E+00, .12484E+01, .33720E+02,-.59523E+00, &
        .66266E+00,-.69524E+00, .38488E+02, .63775E+00 /)
        xa1lum(GEV500,TESLA,3) = .10608E+03
        xa1(0:7,GEV500,TESLA,3) = (/ &
        .28282E+00, .11700E+01, .19258E+02,-.58390E+00, &
        .68777E+00,-.69402E+00, .23638E+02, .75929E+00 /)
        xa1lum(GEV800,TESLA,3) = .28911E+03
        xa1(0:7,GEV800,TESLA,3) = (/ &

```

```

        .21018E+00, .12039E+01, .96763E+01,-.54024E+00, &
        .67220E+00,-.69083E+00, .12733E+02, .87355E+00 /)
xa1lum(TEV1, TESLA, 3) = .10936E+03
xa1(0:7, TEV1, TESLA, 3) = (/ &
        .41040E+00, .68099E+00, .11610E+02,-.61237E+00, &
        .40155E+00,-.69073E+00, .14698E+02, .49989E+00 /)
xa1lum(GEV500, XBAND, 3) = .36145E+02
xa1(0:7, GEV500, XBAND, 3) = (/ &
        .51285E+00, .45812E+00, .75135E+01,-.62247E+00, &
        .30444E+00,-.68530E+00, .85519E+01, .43062E+00 /)
xa1lum(TEV1, XBAND, 3) = .11799E+03
xa1(0:7, TEV1, XBAND, 3) = (/ &
        .31241E+00, .61241E+00, .29938E+01,-.55848E+00, &
        .44801E+00,-.67116E+00, .41119E+01, .72753E+00 /)

```

Uses SBAND 13, TESLA 13, and XBAND 13.

Still unavailable

45a *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350, SBAND, 3) = -1d0
        xa1lum(GEV350, XBAND, 3) = -1d0
        xa1lum(GEV800, SBAND, 3) = -1d0
        xa1lum(GEV800, XBAND, 3) = -1d0

```

Uses SBAND 13 and XBAND 13.

Unavailable as well

45b *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV350, SBNDEE:NACC, 3) = 4 * (-1d0)
        xa1lum(GEV500, SBNDEE:NACC, 3) = 4 * (-1d0)
        xa1lum(GEV800, SBNDEE:NACC, 3) = 4 * (-1d0)
        xa1lum(TEV1, SBNDEE:NACC, 3) = 4 * (-1d0)

```

Uses NACC 13 and SBNDEE 13.

No 1.6TeV parameters in this revision

45c *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(TEV16, 1:NACC, 3) = 7 * (-1d0)

```

Uses NACC 13.

Revision 4. Features:

- a bug in Guinea-Pig's synchrotron radiation spectrum has been fixed.

45d *⟨Initializations for circes 35b⟩*+≡

```

        xa1lum(GEV500, SBAND, 4) = .31528E+02
        xa1(0:7, GEV500, SBAND, 4) = (/ &
        .38169E+00, .73949E+00, .12543E+02,-.61112E+00, &
        .51256E+00,-.69009E+00, .14892E+02, .63314E+00 /)
        xa1lum(TEV1, SBAND, 4) = .24613E+03
        xa1(0:7, TEV1, SBAND, 4) = (/ &
        .24256E+00, .94117E+00, .66775E+01,-.55160E+00, &
        .57484E+00,-.68891E+00, .92271E+01, .81162E+00 /)
        xa1lum(GEV350, TESLA, 4) = .74549E+02
        xa1(0:7, GEV350, TESLA, 4) = (/ &
        .34120E+00, .12230E+01, .32932E+02,-.59850E+00, &

```

```

        .65947E+00,-.69574E+00, .38116E+02, .63879E+00 /)
xaillum(GEV500,TESLA,4) = .10668E+03
xa1(0:7,GEV500,TESLA,4) = (/ &
        .28082E+00, .11074E+01, .18399E+02,-.59118E+00, &
        .68880E+00,-.69375E+00, .23463E+02, .76073E+00 /)
xaillum(GEV800,TESLA,4) = .29006E+03
xa1(0:7,GEV800,TESLA,4) = (/ &
        .21272E+00, .11443E+01, .92564E+01,-.54657E+00, &
        .66799E+00,-.69137E+00, .12498E+02, .87571E+00 /)
xaillum(TEV1,TESLA,4) = .11009E+03
xa1(0:7,TEV1,TESLA,4) = (/ &
        .41058E+00, .64745E+00, .11271E+02,-.61996E+00, &
        .39801E+00,-.69150E+00, .14560E+02, .49924E+00 /)
xaillum(GEV500,XBAND,4) = .36179E+02
xa1(0:7,GEV500,XBAND,4) = (/ &
        .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
        .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
xaillum(TEV1,XBAND,4) = .11748E+03
xa1(0:7,TEV1,XBAND,4) = (/ &
        .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
        .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)

```

Uses SBAND 13, TESLA 13, and XBAND 13.

Still unavailable

46a *⟨Initializations for circes 35b⟩*+≡

```

        xaillum(GEV350,SBAND,4) = -1d0
        xaillum(GEV350,XBAND,4) = -1d0
        xaillum(GEV800,SBAND,4) = -1d0
        xaillum(GEV800,XBAND,4) = -1d0

```

Uses SBAND 13 and XBAND 13.

Unavailable as well

46b *⟨Initializations for circes 35b⟩*+≡

```

        xaillum(GEV350,SBNDDEE:NACC,4) = 4 * (-1d0)
        xaillum(GEV500,SBNDDEE:NACC,4) = 4 * (-1d0)
        xaillum(GEV800,SBNDDEE:NACC,4) = 4 * (-1d0)
        xaillum(TEV1,SBNDDEE:NACC,4) = 4 * (-1d0)

```

Uses NACC 13 and SBNDDEE 13.

No 1.6TeV parameters in this revision

46c *⟨Initializations for circes 35b⟩*+≡

```

        xaillum(TEV16,1:NACC,4) = 7 * (-1d0)

```

Uses NACC 13.

Revision 5. Features:

- a bug in Guinea-Pig has been fixed.
- updated parameter sets

46d *⟨Initializations for circes 35b⟩*+≡

```

        xaillum(GEV350,SBAND,5) = 0.21897E+02
        xa1(0:7,GEV350,SBAND,5) = (/ &

```

```

0.57183E+00, 0.53877E+00, 0.19422E+02,-0.63064E+00, &
0.49112E+00,-0.69109E+00, 0.24331E+02, 0.52718E+00 /)
xa1lum(GEV500,SBAND,5) = 0.31383E+02
xa1(0:7,GEV500,SBAND,5) = (/ &
0.51882E+00, 0.49915E+00, 0.11153E+02,-0.63017E+00, &
0.50217E+00,-0.69113E+00, 0.14935E+02, 0.62373E+00 /)
xa1lum(GEV800,SBAND,5) = 0.95091E+02
xa1(0:7,GEV800,SBAND,5) = (/ &
0.47137E+00, 0.46150E+00, 0.56562E+01,-0.61758E+00, &
0.46863E+00,-0.68897E+00, 0.85876E+01, 0.67577E+00 /)
xa1lum(TEV1,SBAND,5) = 0.11900E+03
xa1(0:7,TEV1,SBAND,5) = (/ &
0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
xa1lum(TEV16,SBAND,5) = 0.11900E+03
xa1(0:7,TEV16,SBAND,5) = (/ &
0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
xa1lum(GEV350,TESLA,5) = 0.97452E+02
xa1(0:7,GEV350,TESLA,5) = (/ &
0.39071E+00, 0.84996E+00, 0.17614E+02,-0.60609E+00, &
0.73920E+00,-0.69490E+00, 0.28940E+02, 0.77286E+00 /)
xa1lum(GEV500,TESLA,5) = 0.10625E+03
xa1(0:7,GEV500,TESLA,5) = (/ &
0.42770E+00, 0.71457E+00, 0.15284E+02,-0.61664E+00, &
0.68166E+00,-0.69208E+00, 0.24165E+02, 0.73806E+00 /)
xa1lum(GEV800,TESLA,5) = 0.17086E+03
xa1(0:7,GEV800,TESLA,5) = (/ &
0.36025E+00, 0.69118E+00, 0.76221E+01,-0.59440E+00, &
0.71269E+00,-0.69077E+00, 0.13117E+02, 0.91780E+00 /)
xa1lum(TEV1,TESLA,5) = 0.21433E+03
xa1(0:7,TEV1,TESLA,5) = (/ &
0.33145E+00, 0.67075E+00, 0.55438E+01,-0.58468E+00, &
0.72503E+00,-0.69084E+00, 0.99992E+01, 0.10112E+01 /)
xa1lum(TEV16,TESLA,5) = 0.34086E+03
xa1(0:7,TEV16,TESLA,5) = (/ &
0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)
xa1lum(GEV350,XBAND,5) = 0.31901E+02
xa1(0:7,GEV350,XBAND,5) = (/ &
0.65349E+00, 0.31752E+00, 0.94342E+01,-0.64291E+00, &
0.30364E+00,-0.68989E+00, 0.11446E+02, 0.40486E+00 /)
xa1lum(GEV500,XBAND,5) = 0.36386E+02
xa1(0:7,GEV500,XBAND,5) = (/ &
0.65132E+00, 0.28728E+00, 0.69853E+01,-0.64440E+00, &
0.28736E+00,-0.68758E+00, 0.83227E+01, 0.41492E+00 /)
xa1lum(GEV800,XBAND,5) = 0.10854E+03
xa1(0:7,GEV800,XBAND,5) = (/ &
0.49478E+00, 0.36221E+00, 0.30116E+01,-0.61548E+00, &
0.39890E+00,-0.68418E+00, 0.45183E+01, 0.67243E+00 /)

```

```

xaillum(TEV1,XBAND,5) = 0.11899E+03
xa1(0:7,TEV1,XBAND,5) = (/ &
    0.49992E+00, 0.34299E+00, 0.26184E+01,-0.61584E+00, &
    0.38450E+00,-0.68342E+00, 0.38589E+01, 0.67408E+00 /)
xaillum(TEV16,XBAND,5) = 0.13675E+03
xa1(0:7,TEV16,XBAND,5) = (/ &
    0.50580E+00, 0.30760E+00, 0.18339E+01,-0.61421E+00, &
    0.35233E+00,-0.68315E+00, 0.26708E+01, 0.67918E+00 /)

```

Uses SBAND 13, TESLA 13, and XBAND 13.

Revision 0. Features:

- e^-e^- mode

48a *⟨Initializations for circes 35b⟩*+≡

```

xaillum(GEV500,SBNDDEE,0) = .92914E+01
xa1(0:7,GEV500,SBNDDEE,0) = (/ &
    .34866E+00, .78710E+00, .10304E+02,-.59464E+00, &
    .40234E+00,-.69741E+00, .20645E+02, .47274E+00 /)
xaillum(TEV1,SBNDDEE,0) = .45586E+02
xa1(0:7,TEV1,SBNDDEE,0) = (/ &
    .21084E+00, .99168E+00, .54407E+01,-.52851E+00, &
    .47493E+00,-.69595E+00, .12480E+02, .64027E+00 /)
xaillum(GEV350,TESLEE,0) = .15175E+02
xa1(0:7,GEV350,TESLEE,0) = (/ &
    .33093E+00, .11137E+01, .25275E+02,-.59942E+00, &
    .49623E+00,-.70403E+00, .60188E+02, .44637E+00 /)
xaillum(GEV500,TESLEE,0) = .21622E+02
xa1(0:7,GEV500,TESLEE,0) = (/ &
    .27175E+00, .10697E+01, .14858E+02,-.58418E+00, &
    .50824E+00,-.70387E+00, .36129E+02, .53002E+00 /)
xaillum(GEV800,TESLEE,0) = .43979E+02
xa1(0:7,GEV800,TESLEE,0) = (/ &
    .22994E+00, .10129E+01, .81905E+01,-.55751E+00, &
    .46551E+00,-.70461E+00, .19394E+02, .58387E+00 /)
xaillum(TEV1,TESLEE,0) = .25465E+02
xa1(0:7,TEV1,TESLEE,0) = (/ &
    .37294E+00, .67522E+00, .87504E+01,-.60576E+00, &
    .35095E+00,-.69821E+00, .18526E+02, .42784E+00 /)
xaillum(GEV500,XBNDDEE,0) = .13970E+02
xa1(0:7,GEV500,XBNDDEE,0) = (/ &
    .47296E+00, .46800E+00, .58897E+01,-.61689E+00, &
    .27181E+00,-.68923E+00, .10087E+02, .37462E+00 /)
xaillum(TEV1,XBNDDEE,0) = .41056E+02
xa1(0:7,TEV1,XBNDDEE,0) = (/ &
    .27965E+00, .74816E+00, .27415E+01,-.50491E+00, &
    .38320E+00,-.67945E+00, .47506E+01, .62218E+00 /)

```

Uses SBNDDEE 13, TESLEE 13, and XBNDDEE 13.

Still unavailable

48b *⟨Initializations for circes 35b⟩*+≡

```

xaillum(GEV350,SBNDDEE,0) = -1d0

```



```

xa1lum(GEV350,XBNDEE,0) = -1d0
xa1lum(GEV800,SBNDEE,0) = -1d0
xa1lum(GEV800,XBNDEE,0) = -1d0

```

Uses SBNDEE 13 and XBNDEE 13.

49a *<Initializations for circes 35b>+≡*

```

xa1lum(GEV500,SBAND,0) = .31528E+02
xa1(0:7,GEV500,SBAND,0) = (/ &
    .38169E+00, .73949E+00, .12543E+02,-.61112E+00, &
    .51256E+00,-.69009E+00, .14892E+02, .63314E+00 /)
xa1lum(TEV1,SBAND,0) = .24613E+03
xa1(0:7,TEV1,SBAND,0) = (/ &
    .24256E+00, .94117E+00, .66775E+01,-.55160E+00, &
    .57484E+00,-.68891E+00, .92271E+01, .81162E+00 /)
xa1lum(GEV350,TESLA,0) = .74549E+02
xa1(0:7,GEV350,TESLA,0) = (/ &
    .34120E+00, .12230E+01, .32932E+02,-.59850E+00, &
    .65947E+00,-.69574E+00, .38116E+02, .63879E+00 /)
xa1lum(GEV500,TESLA,0) = .10668E+03
xa1(0:7,GEV500,TESLA,0) = (/ &
    .28082E+00, .11074E+01, .18399E+02,-.59118E+00, &
    .68880E+00,-.69375E+00, .23463E+02, .76073E+00 /)
xa1lum(GEV800,TESLA,0) = .29006E+03
xa1(0:7,GEV800,TESLA,0) = (/ &
    .21272E+00, .11443E+01, .92564E+01,-.54657E+00, &
    .66799E+00,-.69137E+00, .12498E+02, .87571E+00 /)
xa1lum(TEV1,TESLA,0) = .11009E+03
xa1(0:7,TEV1,TESLA,0) = (/ &
    .41058E+00, .64745E+00, .11271E+02,-.61996E+00, &
    .39801E+00,-.69150E+00, .14560E+02, .49924E+00 /)
xa1lum(GEV500,XBAND,0) = .36179E+02
xa1(0:7,GEV500,XBAND,0) = (/ &
    .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
    .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
xa1lum(TEV1,XBAND,0) = .11748E+03
xa1(0:7,TEV1,XBAND,0) = (/ &
    .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
    .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)

```

Uses SBAND 13, TESLA 13, and XBAND 13.

Still unavailable

49b *<Initializations for circes 35b>+≡*

```

xa1lum(GEV350,SBAND,0) = -1d0
xa1lum(GEV350,XBAND,0) = -1d0
xa1lum(GEV800,SBAND,0) = -1d0
xa1lum(GEV800,XBAND,0) = -1d0

```

Uses SBAND 13 and XBAND 13.

6.2.2 Version 2

49c *<Version 2 has been retired 49c>≡*

```

call circem ('PANIC', '*****')
call circem ('PANIC', '* version 2 has been retired, *')
call circem ('PANIC', '* please use version 1 instead! *')
call circem ('PANIC', '*****')
return

```

Uses circem 81c.

6.2.3 Versions 3 and 4

50a \langle Update version 3 and 4 derived parameters in circe1 parameters 50a $\rangle \equiv$

```

if (circe1_params%rev .eq. 0) then
  r = 0
elseif (circe1_params%rev .ge. 19970417) then
  r = 5
  if (ver34 .eq. 3) then
    call circem ('WARNING', 'version 3 retired after 97/04/17')
    call circem ('MESSAGE', 'falling back to version 4')
  end if
else if (circe1_params%rev .ge. 19961022) then
  r = ver34
  if ((circe1_params%roots .ne. 800d0) .or. (circe1_params%acc .ne. TESLA)) then
    call circem ('ERROR', 'versions 3 and 4 before 97/04/17')
    call circem ('ERROR', 'apply to TESLA at 800 GeV only')
    call circem ('MESSAGE', 'falling back to TESLA at 800GeV')
    circe1_params%acc = TESLA
    e = GEV800
  end if
else if (circe1_params%rev .lt. 19961022) then
  call circem ('ERROR', &
    'no revision of versions 3 and 4 available before 96/10/22')
  call circem ('MESSAGE', 'falling back to default')
  r = 5
end if

```

\langle Log revision mapping 37b \rangle

Uses TESLA 13 and circem 81c.

50b \langle Update version 3 and 4 derived parameters in circe1 parameters 50a $\rangle + \equiv$

\langle Map roots to e 37d \rangle

```

if (xa3lum(e,circe1_params%acc,r) .lt. 0d0) then
  write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
  call circem ('ERROR', msgbuf)
  call circem ('MESSAGE', 'falling back to 500GeV')
  e = GEV500
endif

```

\langle Log energy mapping 38d \rangle

Uses circem 81c.

50c \langle Local variables for circes 33b $\rangle + \equiv$

```

integer, parameter :: A3NEGY = 5, A3NREV = 5

```

51a *<Update version 3 and 4 derived parameters in circe1 parameters 50a>+≡*
 circe1_params%lumi = xa3lum (e,circe1_params%acc,r)
 do i = 0, 7
 circe1_params%a1(i) = xa3(i,e,circe1_params%acc,r)
 end do

51b *<Local variables for circes 33b>+≡*
 real, dimension(A3NEGY,NACC,0:A3NREV), save :: xa3lum
 real, dimension(0:7,A3NEGY,NACC,0:A3NREV), save :: xa3

Uses NACC 13.

Revisions 3 & 4. The mother of all revisions.

51c *<Initializations for circes 35b>+≡*
 xa3lum(GEV800,TESLA,3) = .17196E+03
 xa3(0:7,GEV800,TESLA,3) = (/ &
 .21633E+00, .11333E+01, .95928E+01,-.55095E+00, &
 .73044E+00,-.69101E+00, .12868E+02, .94737E+00 /)
 xa3lum(GEV800,TESLA, 4) = .16408E+03
 xa3(0:7,GEV800,TESLA, 4) = (/ &
 .41828E+00, .72418E+00, .14137E+02,-.61189E+00, &
 .36697E+00,-.69205E+00, .17713E+02, .43583E+00 /)

Uses TESLA 13.

Revision 5.

51d *<Initializations for circes 35b>+≡*
 xa3lum(GEV350,TESLA,5) = 0.66447E+02
 xa3(0:7,GEV350,TESLA,5) = (/ &
 0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &
 0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)
 xa3lum(GEV500,TESLA,5) = 0.95241E+02
 xa3(0:7,GEV500,TESLA,5) = (/ &
 0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &
 0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)
 xa3lum(GEV800,TESLA,5) = 0.16974E+03
 xa3(0:7,GEV800,TESLA,5) = (/ &
 0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &
 0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)
 xa3lum(TEV1,TESLA,5) = 0.21222E+03
 xa3(0:7,TEV1,TESLA,5) = (/ &
 0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &
 0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)
 xa3lum(TEV16,TESLA,5) = 0.34086E+03
 xa3(0:7,TEV16,TESLA,5) = (/ &
 0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
 0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)

Uses TESLA 13.

Revision 0. Currently identical to revision 5.

51e *<Initializations for circes 35b>+≡*
 xa3lum(GEV350,TESLA,0) = 0.66447E+02
 xa3(0:7,GEV350,TESLA,0) = (/ &
 0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &

```

0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)
xa3lum(GEV500,TESLA,0) = 0.95241E+02
xa3(0:7,GEV500,TESLA,0) = (/ &
0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &
0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)
xa3lum(GEV800,TESLA,0) = 0.16974E+03
xa3(0:7,GEV800,TESLA,0) = (/ &
0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &
0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)
xa3lum(TEV1,TESLA,0) = 0.21222E+03
xa3(0:7,TEV1,TESLA,0) = (/ &
0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &
0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)
xa3lum(TEV16,TESLA,0) = 0.34086E+03
xa3(0:7,TEV16,TESLA,0) = (/ &
0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)

```

Uses TESLA 13.

6.2.4 Version 5

52a \langle Update version 5 derived parameters in circe1 parameters 52a $\rangle \equiv$

```

if (circe1_params%rev .eq. 0) then
  r = 0
elseif (circe1_params%rev .ge. 19980505) then
  r = 1
elseif (circe1_params%rev .lt. 19980505) then
  call circem ('ERROR', &
'no revision of version 5 available before 98/05/05')
  call circem ('MESSAGE', 'falling back to default')
  r = 1
endif

```

\langle Log revision mapping 37b \rangle

Uses circem 81c.

52b \langle Update version 5 derived parameters in circe1 parameters 52a $\rangle + \equiv$

```

if (circe1_params%acc .ne. TESLA) then
  call circem ('ERROR', 'versions 5 applies to TESLA only')
  circe1_params%acc = TESLA
end if

```

\langle Map roots to e 37d \rangle

```

if (xa5lum(e,circe1_params%acc,r) .lt. 0d0) then
  write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
  call circem ('ERROR', msgbuf)
  call circem ('MESSAGE', 'falling back to 500GeV')
  e = GEV500
endif

```

\langle Log energy mapping 38d \rangle

Uses TESLA 13 and circem 81c.

53a *<Local variables for circes 33b>+≡*
integer, parameter :: A5NEGY = 5, A5NREV = 1

53b *<Update version 5 derived parameters in circe1 parameters 52a>+≡*
circe1_params%lumi = xa5lum (e,circe1_params%acc,r)
do i = 0, 7
circe1_params%a1(i) = xa5(i,e,circe1_params%acc,r)
end do

53c *<Local variables for circes 33b>+≡*
real, dimension(A5NEGY,NACC,0:A5NREV), save :: xa5lum
real, dimension(0:7,A5NEGY,NACC,0:A5NREV), save :: xa5

Uses NACC 13.

Revision 1. The mother of all revisions. Note that $3.3980 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.4099 \cdot 10^{34} \text{ m}^{-2} \cdot 2820.5 \text{ s}^{-1}$ and $3.5936 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.6619 \cdot 10^{34} \text{ m}^{-2} \cdot 4500.3 \text{ s}^{-1}$. This unit conversion is missing in *all* earlier versions, unfortunately.

53d *<Initializations for circes 35b>+≡*
xa5lum(GEV350,TESLA,1) = -1.0
xa5lum(GEV500,TESLA,1) = 0.33980E+03
xa5(0:7,GEV500,TESLA,1) = (/ &
0.49808E+00, 0.54613E+00, 0.12287E+02,-0.62756E+00, &
0.42817E+00,-0.69120E+00, 0.17067E+02, 0.51143E+00 /)
xa5lum(GEV800,TESLA,1) = 0.35936E+03
xa5(0:7,GEV800,TESLA,1) = (/ &
0.58751E+00, 0.43128E+00, 0.13324E+02,-0.64006E+00, &
0.30682E+00,-0.69235E+00, 0.16815E+02, 0.37078E+00 /)
xa5lum(TEV1, TESLA,1) = -1.0
xa5lum(TEV16,TESLA,1) = -1.0

Uses TESLA 13.

Revision 0. Currently identical to revision 1.

53e *<Initializations for circes 35b>+≡*
xa5lum(GEV350,TESLA,0) = -1.0
xa5lum(GEV500,TESLA,0) = 0.33980E+03
xa5(0:7,GEV500,TESLA,0) = (/ &
0.49808E+00, 0.54613E+00, 0.12287E+02,-0.62756E+00, &
0.42817E+00,-0.69120E+00, 0.17067E+02, 0.51143E+00 /)
xa5lum(GEV800,TESLA,0) = 0.35936E+03
xa5(0:7,GEV800,TESLA,0) = (/ &
0.58751E+00, 0.43128E+00, 0.13324E+02,-0.64006E+00, &
0.30682E+00,-0.69235E+00, 0.16815E+02, 0.37078E+00 /)
xa5lum(TEV1, TESLA,0) = -1.0
xa5lum(TEV16,TESLA,0) = -1.0

Uses TESLA 13.

6.2.5 Version 6

53f *<Update version 6 derived parameters in circe1 parameters 53f>≡*
if (circe1_params%rev .eq. 0) then
r = 0
else if (circe1_params%rev .ge. 19990415) then

```

        r = 1
    else if (circe1_params%rev .lt. 19990415) then
        call circem ('ERROR', &
            'no revision of version 6 available before 1999/04/15')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    end if
    <Log revision mapping 37b>
    Uses circem 81c.

54a <Update version 6 derived parameters in circe1 parameters 53f>+=
    if (circe1_params%acc .ne. TESLA) then
        call circem ('ERROR', 'versions 6 applies to TESLA only')
        circe1_params%acc = TESLA
    end if
    <Map roots to e at low energies 54b>
    if (xa6lum(e,circe1_params%acc,r) .lt. 0d0) then
        write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    <Log energy mapping 38d>
    Uses TESLA 13 and circem 81c.

54b <Map roots to e at low energies 54b>=
    if (circe1_params%roots .eq. 90d0) then
        e = GEV090
    elseif ((circe1_params%roots .ge. 85d0) .and. (circe1_params%roots .le. 95d0)) then
        write (msgbuf, 2001) circe1_params%roots, 90d0
        call circem ('MESSAGE', msgbuf)
        e = GEV090
    elseif (circe1_params%roots .eq. 170d0) then
        e = GEV170
    elseif ((circe1_params%roots .ge. 160d0) .and. (circe1_params%roots .le. 180d0)) then
        write (msgbuf, 2001) circe1_params%roots, 170d0
        call circem ('MESSAGE', msgbuf)
        e = GEV170
    elseif (circe1_params%roots .eq. 350d0) then
        e = GEV350
    elseif ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0)) then
        write (msgbuf, 2001) circe1_params%roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350
    elseif (circe1_params%roots .eq. 500d0) then
        e = GEV500
    elseif ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0)) then
        write (msgbuf, 2001) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    else

```

```

        call circem ('ERROR', &
                     'only ROOTS = 90, 170, 350, and 500GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif
    Uses circem 81c.
55a <Local variables for circes 33b>+≡
    integer, parameter :: A6NEGY = 2, A6NREV = 1
55b <Update version 6 derived parameters in circe1 parameters 53f>+≡
    circe1_params%lumi = xa6lum (e,circe1_params%acc,r)
    do i = 0, 7
        circe1_params%a1(i) = xa6(i,e,circe1_params%acc,r)
    end do
55c <Local variables for circes 33b>+≡
    real, dimension(GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6lum
    real, dimension(0:7,GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6
    Uses NACC 13.

```

Revision 1. The mother of all revisions.

```

55d <Initializations for circes 35b>+≡
    xa6lum(GEV090,TESLA,1) = 0.62408E+02
    xa6(0:7,GEV090,TESLA,1) = (/ &
        0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
        0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
    xa6lum(GEV170,TESLA,1) = 0.11532E+02
    xa6(0:7,GEV170,TESLA,1) = (/ &
        0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
        0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
    xa6lum(GEV350,TESLA,1) = 0.24641E+03
    xa6(0:7,GEV350,TESLA,1) = (/ &
        0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
        0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
    xa6lum(GEV500,TESLA,1) = 0.30340E+03
    xa6(0:7,GEV500,TESLA,1) = (/ &
        0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
        0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)
    Uses TESLA 13.

```

Revision 0. Currently identical to revision 1.

```

55e <Initializations for circes 35b>+≡
    xa6lum(GEV090,TESLA,0) = 0.62408E+02
    xa6(0:7,GEV090,TESLA,0) = (/ &
        0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
        0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
    xa6lum(GEV170,TESLA,0) = 0.11532E+02
    xa6(0:7,GEV170,TESLA,0) = (/ &
        0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
        0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
    xa6lum(GEV350,TESLA,0) = 0.24641E+03

```

```

xa6(0:7,GEV350,TESLA,0) = (/ &
    0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
    0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa6lum(GEV500,TESLA,0) = 0.30340E+03
xa6(0:7,GEV500,TESLA,0) = (/ &
    0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
    0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)

```

Uses TESLA 13.

6.2.6 Version 7

56a *<Update version 7 derived parameters in circe1 parameters 56a>*≡

```

if (circe1_params%rev .eq. 0) then
    r = 0
elseif (circe1_params%rev .ge. 20000426) then
    r = 1
elseif (circe1_params%rev .lt. 20000426) then
    call circem ('ERROR', &
        'no revision of version 7 available before 2000/04/26')
    call circem ('MESSAGE', 'falling back to default')
    r = 1
endif

```

<Log revision mapping 37b>

Uses circem 81c.

56b *<Update version 7 derived parameters in circe1 parameters 56a>*+≡

```

if (circe1_params%acc .ne. TESLA .and. circe1_params%acc .ne. JLCNLC) then
    call circem ('ERROR', &
        'version 7 applies to TESLA and JLCNLC only')
    call circem ('ERROR', 'falling back to TESLA')
    circe1_params%acc = TESLA
end if

```

<Linearly interpolate energies 56e>

<Log energy mapping 38d>

Uses JLCNLC 13, TESLA 13, and circem 81c.

56c *<formats for circes 37e>*+≡

```

2004 format ('energy ', F6.1, 'GeV too low, using spectrum for ', F6.1, 'GeV')
2005 format ('energy ', F6.1, 'GeV too high, using spectrum for ', F6.1, 'GeV')
2006 format ('energy ', F6.1, 'GeV interpolated between ', F6.1, ' and ', F6.1, 'GeV')

```

56d *<Local variables for circes 33b>*+≡

```

real(kind=double) :: eloval, ehival
real(kind=double), parameter :: DELTAE = 0.5d0

```

The rules are as follows: XBAND has 500 GeV and 1 TeV, TESLA has 500 GeV and 800 TeV. Low energy TESLA will be added.

56e *<Linearly interpolate energies 56e>*≡

```

e = GEV090 - 1
elo = e
ehi = e
if (circe1_params%acc .eq. TESLA) then

```



```

if (circe1_params%roots .lt. 90d0 - DELTAE) then
  write (msgbuf, 2004) circe1_params%roots, 90d0
  call circem ('MESSAGE', msgbuf)
  e = GEV090
elseif (abs (circe1_params%roots-90d0) .le. DELTAE) then
  e = GEV090
elseif (circe1_params%roots .lt. 170d0 - DELTAE) then
  write (msgbuf, 2005) circe1_params%roots, 170d0
  call circem ('MESSAGE', msgbuf)
  e = GEV170
elseif (abs (circe1_params%roots-170d0) .le. DELTAE) then
  e = GEV170
elseif (circe1_params%roots .lt. 350d0-DELTAE) then
  write (msgbuf, 2006) circe1_params%roots, 170d0, 350d0
  call circem ('MESSAGE', msgbuf)
  elo = GEV170
  ehi = GEV350
  eloal = 170d0
  ehival = 350d0
elseif (abs (circe1_params%roots-350d0) .le. DELTAE) then
  e = GEV350
elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
  write (msgbuf, 2006) circe1_params%roots, 350d0, 500d0
  call circem ('MESSAGE', msgbuf)
  elo = GEV350
  ehi = GEV500
  eloal = 350d0
  ehival = 500d0
elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
  e = GEV500
elseif (circe1_params%roots .lt. 800d0 - DELTAE) then
  write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0
  call circem ('MESSAGE', msgbuf)
  elo = GEV500
  ehi = GEV800
  eloal = 500d0
  ehival = 800d0
elseif (abs (circe1_params%roots-800d0) .le. DELTAE) then
  e = GEV800
else
  write (msgbuf, 2005) circe1_params%roots, 800d0
  call circem ('MESSAGE', msgbuf)
  e = GEV800
endif
elseif (circe1_params%acc .eq. XBAND) then
  if (circe1_params%roots .lt. 500d0 - DELTAE) then
    write (msgbuf, 2004) circe1_params%roots, 500d0
    call circem ('MESSAGE', msgbuf)
    e = GEV500
  elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then

```

```

        e = GEV500
    elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = TEV1
        eloal = 500d0
        ehival = 1000d0
    elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
        e = TEV1
    else
        write (msgbuf, 2005) circe1_params%roots, 1000d0
        call circem ('MESSAGE', msgbuf)
        e = TEV1
    endif
endif

```

Uses TESLA 13, XBAND 13, and circem 81c.

58a *<Local variables for circes 33b>+≡*
integer, parameter :: A7NEGY = TEV1, A7NREV = 1

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

58b *<Update version 7 derived parameters in circe1 parameters 56a>+≡*
if (e .ge. GEV090) then
 circe1_params%lumi = xa7lum(e,circe1_params%acc,r)
 do i = 0, 7
 circe1_params%a1(i) = xa7(i,e,circe1_params%acc,r)
 end do
else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
 circe1_params%lumi = ((circe1_params%roots-eloal)*xa7lum(ehi,circe1_params%acc,r) + (ehival-circe1_params%roots)*xa7lum(elo,circe1_params%acc,r)) / (ehival - eloal)
 do i = 1, 6
 circe1_params%a1(i) = ((circe1_params%roots-eloal)*xa7(i,ehi,circe1_params%acc,r) + (ehival-circe1_params%roots)*xa7(i,elo,circe1_params%acc,r)) / (ehival - eloal)
 end do
 circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,circe1_params%a1(3)+1d0,circe1_params%a1(4)+1d0,circe1_params%a1(5)+1d0,circe1_params%a1(6)+1d0,circe1_params%a1(7)+1d0)
 circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1_params%a1(6)+1d0,circe1_params%a1(7)+1d0)
endif

Uses beta 99.

58c *<Local variables for circes 33b>+≡*
real, dimension(GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7lum
real, dimension(0:7,GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7

Uses NACC 13.

Revision 1. The mother of all revisions.

58d *<Initializations for circes 35b>+≡*
xa7lum(GEV090,TESLA,1) = 0.62408E+02
xa7(0:7,GEV090,TESLA,1) = (/ &
0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)

```

xa7lum(GEV170, TESLA, 1) = 0.11532E+02
xa7(0:7, GEV170, TESLA, 1) = (/ &
    0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00, &
    0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa7lum(GEV350, TESLA, 1) = 0.24641E+03
xa7(0:7, GEV350, TESLA, 1) = (/ &
    0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00, &
    0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa7lum(GEV500, TESLA, 1) = 0.34704E+03
xa7(0:7, GEV500, TESLA, 1) = (/ &
    0.51288E+00, 0.49025E+00, 0.99716E+01, -0.62850E+00, &
    0.41048E+00, -0.69065E+00, 0.13922E+02, 0.51902E+00 /)
xa7lum(GEV800, TESLA, 1) = 0.57719E+03
xa7(0:7, GEV800, TESLA, 1) = (/ &
    0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00, &
    0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1, TESLA, 1) = -1.0

```

Uses TESLA 13.

59a *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090, JLCNLC, 1) = -1.0
xa7lum(GEV170, JLCNLC, 1) = -1.0
xa7lum(GEV350, JLCNLC, 1) = -1.0
xa7lum(GEV500, JLCNLC, 1) = 0.63039E+02
xa7(0:7, GEV500, JLCNLC, 1) = (/ &
    0.58967E+00, 0.34035E+00, 0.63631E+01, -0.63683E+00, &
    0.33383E+00, -0.68803E+00, 0.81005E+01, 0.48702E+00 /)
xa7lum(TEV1, JLCNLC, 1) = 0.12812E+03
xa7(0:7, TEV1, JLCNLC, 1) = (/ &
    0.50222E+00, 0.33773E+00, 0.25681E+01, -0.61711E+00, &
    0.36826E+00, -0.68335E+00, 0.36746E+01, 0.65393E+00 /)

```

Uses JLCNLC 13.

Revision 0.

59b *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090, TESLA, 0) = 0.62408E+02
xa7(0:7, GEV090, TESLA, 0) = (/ &
    0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00, &
    0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /)
xa7lum(GEV170, TESLA, 0) = 0.11532E+02
xa7(0:7, GEV170, TESLA, 0) = (/ &
    0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00, &
    0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa7lum(GEV350, TESLA, 0) = 0.24641E+03
xa7(0:7, GEV350, TESLA, 0) = (/ &
    0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00, &
    0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa7lum(GEV500, TESLA, 0) = 0.34704E+03
xa7(0:7, GEV500, TESLA, 0) = (/ &
    0.51288E+00, 0.49025E+00, 0.99716E+01, -0.62850E+00, &
    0.41048E+00, -0.69065E+00, 0.13922E+02, 0.51902E+00 /)

```

```

xa7lum(GEV800, TESLA, 0) = 0.57719E+03
xa7(0:7, GEV800, TESLA, 0) = (/ &
    0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00, &
    0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1, TESLA, 0) = -1.0

```

Uses TESLA 13.

60a *<Initializations for circes 35b>+≡*

```

xa7lum(GEV090, JLCNLC, 0) = -1.0
xa7lum(GEV170, JLCNLC, 0) = -1.0
xa7lum(GEV350, JLCNLC, 0) = -1.0
xa7lum(GEV500, JLCNLC, 0) = 0.63039E+02
xa7(0:7, GEV500, JLCNLC, 0) = (/ &
    0.58967E+00, 0.34035E+00, 0.63631E+01, -0.63683E+00, &
    0.33383E+00, -0.68803E+00, 0.81005E+01, 0.48702E+00 /)
xa7lum(TEV1, JLCNLC, 0) = 0.12812E+03
xa7(0:7, TEV1, JLCNLC, 0) = (/ &
    0.50222E+00, 0.33773E+00, 0.25681E+01, -0.61711E+00, &
    0.36826E+00, -0.68335E+00, 0.36746E+01, 0.65393E+00 /)

```

Uses JLCNLC 13.

6.2.7 Version 8

60b *<Update version 8 derived parameters in circe1 parameters 60b>≡*

```

if (circe1_params%rev .eq. 0) then
    r = 0
elseif (circe1_params%rev .ge. 20010617) then
    r = 1
elseif (circe1_params%rev .lt. 20010617) then
    call circem ('ERROR', &
        'no revision of version 8 available before 2001/06/17')
    call circem ('MESSAGE', 'falling back to default')
    r = 1
endif
<Log revision mapping 37b>

```

Uses circem 81c.

60c *<Update version 8 derived parameters in circe1 parameters 60b>+≡*

```

if (circe1_params%acc .eq. NLCH) then
    circe1_params%acc = JLCNLC
end if
if (circe1_params%acc .ne. JLCNLC) then
    call circem ('ERROR', &
        'version 8 applies to JLCNLC (NLC H) only')
    call circem ('ERROR', 'falling back to JLCNLC')
    circe1_params%acc = JLCNLC
end if
<Linearly interpolate energies 56e>
<Log energy mapping 38d>

```

Uses JLCNLC 13 and circem 81c.

61a *<Local variables for circes 33b>+≡*

```
integer, parameter :: A8NEGY = TEV1, A8NREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

61b *<Update version 8 derived parameters in circe1 parameters 60b>+≡*

```
if (e .ge. GEV090) then
  circe1_params%lumi = xa8lum(e,circe1_params%acc,r)
  do i = 0, 7
    circe1_params%a1(i) = xa8(i,e,circe1_params%acc,r)
  end do
elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
  circe1_params%lumi = ((circe1_params%roots-eloval)*xa8lum(ehi,circe1_params%acc,
    + (ehival-circe1_params%roots)*xa8lum(elo,circe1_params%acc,r)) / (ehival -
  do i = 1, 6
    circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa8(i,ehi,circe1_params%
    + (ehival-circe1_params%roots)*xa8(i,elo,circe1_params%acc,r)) / (ehival -
  end do
  circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
  circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
endif
```

Uses beta 99.

61c *<Local variables for circes 33b>+≡*

```
real, dimension(GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8lum
real, dimension(0:7,GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8
```

Uses NACC 13.

Revision 1. The mother of all revisions.

61d *<Initializations for circes 35b>+≡*

```
xa8lum(GEV090,TESLA,1) = -1.0
xa8lum(GEV170,TESLA,1) = -1.0
xa8lum(GEV350,TESLA,1) = -1.0
xa8lum(GEV500,TESLA,1) = -1.0
xa8lum(GEV800,TESLA,1) = -1.0
xa8lum(TEV1, TESLA,1) = -1.0
```

Uses TESLA 13.

61e *<Initializations for circes 35b>+≡*

```
xa8lum(GEV090,JLCNLC,1) = -1.0
xa8lum(GEV170,JLCNLC,1) = -1.0
xa8lum(GEV350,JLCNLC,1) = -1.0
xa8lum(GEV500,JLCNLC,1) = 0.239924E+03
xa8(0:7,GEV500,JLCNLC,1) = (/ &
  0.57025E+00, 0.34004E+00, 0.52864E+01,-0.63405E+00, &
  0.31627E+00,-0.68722E+00, 0.69629E+01, 0.47973E+00 /)
xa8lum(TEV1,JLCNLC,1) = 0.40858E+03
xa8(0:7,TEV1,JLCNLC,1) = (/ &
  0.52344E+00, 0.31536E+00, 0.25244E+01,-0.62215E+00, &
  0.31935E+00,-0.68424E+00, 0.35877E+01, 0.57315E+00 /)
```

Uses JLCNLC 13.

Revision 0.

62a \langle Initializations for circes 35b $\rangle + \equiv$

```
xa8lum(GEV090, TESLA, 0) = -1.0
xa8lum(GEV170, TESLA, 0) = -1.0
xa8lum(GEV350, TESLA, 0) = -1.0
xa8lum(GEV500, TESLA, 0) = -1.0
xa8lum(GEV800, TESLA, 0) = -1.0
xa8lum(TEV1, TESLA, 0) = -1.0
```

Uses TESLA 13.

62b \langle Initializations for circes 35b $\rangle + \equiv$

```
xa8lum(GEV090, JLCNLC, 0) = -1.0
xa8lum(GEV170, JLCNLC, 0) = -1.0
xa8lum(GEV350, JLCNLC, 0) = -1.0
xa8lum(GEV500, JLCNLC, 0) = 0.239924E+03
xa8(0:7, GEV500, JLCNLC, 0) = (/ &
    0.57025E+00, 0.34004E+00, 0.52864E+01, -0.63405E+00, &
    0.31627E+00, -0.68722E+00, 0.69629E+01, 0.47973E+00 /)
xa8lum(TEV1, JLCNLC, 0) = 0.40858E+03
xa8(0:7, TEV1, JLCNLC, 0) = (/ &
    0.52344E+00, 0.31536E+00, 0.25244E+01, -0.62215E+00, &
    0.31935E+00, -0.68424E+00, 0.35877E+01, 0.57315E+00 /)
```

Uses JLCNLC 13.

6.2.8 Version 9

62c \langle Update version 9 derived parameters in circe1 parameters 62c $\rangle \equiv$

```
if (circe1_params%rev .eq. 0) then
    r = 0
elseif (circe1_params%rev .ge. 20020328) then
    r = 1
elseif (circe1_params%rev .lt. 20020328) then
    call circem ('ERROR', &
        'no revision of version 9 available before 2002/03/28')
    call circem ('MESSAGE', 'falling back to default')
    r = 1
endif
 $\langle$ Log revision mapping 37b $\rangle$ 
```

Uses circem 81c.

62d \langle Update version 9 derived parameters in circe1 parameters 62c $\rangle + \equiv$

```
if (circe1_params%acc .ne. JLCNLC .and. circe1_params%acc .ne. NLCH) then
    call circem ('ERROR', &
        'version 9 applies to JLCNLC and NLCH only')
    call circem ('ERROR', 'falling back to JLCNLC')
    circe1_params%acc = JLCNLC
end if
if (circe1_params%acc .eq. JLCNLC) then
     $\langle$ Linearly interpolate energies for JLC/NLC 2002 63 $\rangle$ 
else if (circe1_params%acc .eq. NLCH) then
```

⟨Linearly interpolate energies for NLC H 2002 64⟩

end if

⟨Log energy mapping 38d⟩

Uses JLCNLC 13 and circem 81c.

63 *⟨Linearly interpolate energies for JLC/NLC 2002 63⟩*≡

e = GEV090 - 1

elo = e

ehi = e

if (circe1_params%roots .lt. 250d0 - DELTAE) then

write (msgbuf, 2004) circe1_params%roots, 250d0

call circem ('MESSAGE', msgbuf)

e = GEV250

elseif (abs (circe1_params%roots-250d0) .le. DELTAE) then

e = GEV250

elseif (circe1_params%roots .lt. 500d0 - DELTAE) then

write (msgbuf, 2006) circe1_params%roots, 250d0, 500d0

call circem ('MESSAGE', msgbuf)

elo = GEV250

ehi = GEV500

eloval = 250d0

ehival = 500d0

elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then

e = GEV500

elseif (circe1_params%roots .lt. 800d0 - DELTAE) then

write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0

call circem ('MESSAGE', msgbuf)

elo = GEV500

ehi = GEV800

eloval = 500d0

ehival = 800d0

elseif (abs (circe1_params%roots-800d0) .le. DELTAE) then

e = GEV800

elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then

write (msgbuf, 2006) circe1_params%roots, 800d0, 1000d0

call circem ('MESSAGE', msgbuf)

elo = GEV800

ehi = TEV1

eloval = 800d0

ehival = 1000d0

elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then

e = TEV1

elseif (circe1_params%roots .lt. 1200d0 - DELTAE) then

write (msgbuf, 2006) circe1_params%roots, 1000d0, 1200d0

call circem ('MESSAGE', msgbuf)

elo = TEV1

ehi = TEV12

eloval = 1000d0

ehival = 1200d0

elseif (abs (circe1_params%roots-1200d0) .le. DELTAE) then

```

        e = TEV12
elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
    write (msgbuf, 2006) circe1_params%roots, 1200d0, 1500d0
    call circem ('MESSAGE', msgbuf)
    elo = TEV12
    ehi = TEV15
    eloal = 1200d0
    ehival = 1500d0
elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
    e = TEV15
else
    write (msgbuf, 2005) circe1_params%roots, 1500d0
    call circem ('MESSAGE', msgbuf)
    e = TEV15
endif

```

Uses circem 81c.

64 *⟨Linearly interpolate energies for NLC H 2002 64⟩*≡

```

    e = GEV090 - 1
    elo = e
    ehi = e
    if (circe1_params%roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2004) circe1_params%roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = TEV1
        eloal = 500d0
        ehival = 1000d0
    elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
        e = TEV1
    elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
        write (msgbuf, 2006) circe1_params%roots, 1000d0, 1500d0
        call circem ('MESSAGE', msgbuf)
        elo = TEV1
        ehi = TEV15
        eloal = 1000d0
        ehival = 1500d0
    elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
        e = TEV15
    else
        write (msgbuf, 2005) circe1_params%roots, 1500d0
        call circem ('MESSAGE', msgbuf)
        e = TEV15
    endif

```


Uses `circem 81c`.

65a *<Local variables for circes 33b>+≡*

```
integer, parameter :: A9NEGY = TEV15, A9NREV = 1
```

Note that *ew must not* interpolate `a1(0)` and `a1(7)` because they depend non-linearly on the other parameters!

65b *<Update version 9 derived parameters in circe1 parameters 62c>+≡*

```
if (e .ge. GEV090) then
  circe1_params%lumi = xa9lum(e,circe1_params%acc,r)
  do i = 0, 7
    circe1_params%a1(i) = xa9(i,e,circe1_params%acc,r)
  end do
else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
  circe1_params%lumi = ((circe1_params%roots-eloval)*xa9lum(ehi,circe1_params%acc,
    + (ehival-circe1_params%roots)*xa9lum(elo,circe1_params%acc,r)) / (ehival -
  do i = 1, 6
    circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa9(i,ehi,circe1_params%
    + (ehival-circe1_params%roots)*xa9(i,elo,circe1_params%acc,r)) / (ehival -
  end do
  circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
  circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
end if
```

Uses `beta 99`.

65c *<Local variables for circes 33b>+≡*

```
real, dimension(GEV090:A9NEGY,NACC,0:A9NREV) :: xa9lum
real, dimension(0:7,GEV090:A9NEGY,NACC,0:A9NREV) :: xa9
```

Uses `NACC 13`.

Revision 1. The mother of all revisions.

65d *<Initializations for circes 35b>+≡*

```
xa9lum(GEV090,TESLA,1) = -1.0
xa9lum(GEV170,TESLA,1) = -1.0
xa9lum(GEV350,TESLA,1) = -1.0
xa9lum(GEV500,TESLA,1) = -1.0
xa9lum(GEV800,TESLA,1) = -1.0
xa9lum(TEV1, TESLA,1) = -1.0
xa9lum(TEV12, TESLA,1) = -1.0
xa9lum(TEV15, TESLA,1) = -1.0
xa9lum(TEV16, TESLA,1) = -1.0
```

Uses `TESLA 13`.

65e *<Initializations for circes 35b>+≡*

```
xa9lum(GEV090,JLCNLC,1) = -1.0
xa9lum(GEV170,JLCNLC,1) = -1.0
xa9lum(GEV250,JLCNLC,1) = 109.886976
xa9(0:7,GEV250,JLCNLC,1) = (/ &
  0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
  0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
xa9lum(GEV350,JLCNLC,1) = -1.0
xa9lum(GEV500,JLCNLC,1) = 220.806144
```

```

xa9(0:7,GEV500,JLCNLC,1) = (/ &
    0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
    0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
xa9lum(GEV800,JLCNLC,1) = 304.63488
xa9(0:7,GEV800,JLCNLC,1) = (/ &
    0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
    0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)
xa9lum(TEV1, JLCNLC,1) = 319.95648
xa9(0:7,TEV1, JLCNLC,1) = (/ &
    0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
    0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
xa9lum(TEV12,JLCNLC,1) = 349.90848
xa9(0:7,TEV12,JLCNLC,1) = (/ &
    0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
    0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
xa9lum(TEV15,JLCNLC,1) = 363.15648
xa9(0:7,TEV15,JLCNLC,1) = (/ &
    0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
    0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
xa9lum(TEV16,JLCNLC,1) = -1.0

```

Uses JLCNLC 13.

66a *⟨Initializations for circes 35b⟩*+≡

```

xa9lum(GEV090,NLCH,1) = -1.0
xa9lum(GEV170,NLCH,1) = -1.0
xa9lum(GEV250,NLCH,1) = -1.0
xa9lum(GEV350,NLCH,1) = -1.0
xa9lum(GEV500,NLCH,1) = 371.4624
xa9(0:7,GEV500,NLCH,1)= (/ &
    0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
    0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
xa9lum(GEV800,NLCH,1) = -1.0
xa9lum(TEV1,NLCH,1) = 516.41856
xa9(0:7,TEV1,NLCH,1)= (/ &
    0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
    0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
xa9lum(TEV12,NLCH,1) = -1.0
xa9lum(TEV15,NLCH,1) = 575.06688
xa9(0:7,TEV15,NLCH,1)= (/ &
    0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
    0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
xa9lum(TEV16,NLCH, 1) = -1.0

```

Revision 0.

66b *⟨Initializations for circes 35b⟩*+≡

```

xa9lum(GEV090,TESLA,0) = -1.0
xa9lum(GEV170,TESLA,0) = -1.0
xa9lum(GEV350,TESLA,0) = -1.0
xa9lum(GEV500,TESLA,0) = -1.0
xa9lum(GEV800,TESLA,0) = -1.0
xa9lum(TEV1, TESLA,0) = -1.0

```

```

xa9lum(TEV12, TESLA,0) = -1.0
xa9lum(TEV15, TESLA,0) = -1.0
xa9lum(TEV16, TESLA,0) = -1.0

```

Uses TESLA 13.

67a *<Initializations for circes 35b>+≡*

```

xa9lum(GEV090,JLCNLC,0) = -1.0
xa9lum(GEV170,JLCNLC,0) = -1.0
xa9lum(GEV250,JLCNLC,0) = 109.886976
xa9(0:7,GEV250,JLCNLC,0) = (/ &
    0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
    0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
xa9lum(GEV350,JLCNLC,0) = -1.0
xa9lum(GEV500,JLCNLC,0) = 220.806144
xa9(0:7,GEV500,JLCNLC,0) = (/ &
    0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
    0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
xa9lum(GEV800,JLCNLC,0) = 304.63488
xa9(0:7,GEV800,JLCNLC,0) = (/ &
    0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
    0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)
xa9lum(TEV1, JLCNLC,0) = 319.95648
xa9(0:7,TEV1, JLCNLC,0) = (/ &
    0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
    0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
xa9lum(TEV12,JLCNLC,0) = 349.90848
xa9(0:7,TEV12,JLCNLC,0) = (/ &
    0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
    0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
xa9lum(TEV15,JLCNLC,0) = 363.15648
xa9(0:7,TEV15,JLCNLC,0) = (/ &
    0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
    0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
xa9lum(TEV16,JLCNLC,0) = -1.0

```

Uses JLCNLC 13.

67b *<Initializations for circes 35b>+≡*

```

xa9lum(GEV090,NLCH,0) = -1.0
xa9lum(GEV170,NLCH,0) = -1.0
xa9lum(GEV250,NLCH,0) = -1.0
xa9lum(GEV350,NLCH,0) = -1.0
xa9lum(GEV500,NLCH,0) = 371.4624
xa9(0:7,GEV500,NLCH,0) = (/ &
    0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
    0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
xa9lum(GEV800,NLCH,0) = -1.0
xa9lum(TEV1,NLCH,0) = 516.41856
xa9(0:7,TEV1,NLCH,0) = (/ &
    0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
    0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
xa9lum(TEV12,NLCH,0) = -1.0

```

```

xa9lum(TEV15,NLCH,0) = 575.06688
xa9(0:7,TEV15,NLCH,0) = (/ &
    0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
    0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
xa9lum(TEV16,NLCH,0) = -1.0

```

6.3 Special Functions

68a *<Module subroutines 31b>+≡*

```

function beta (a, b)
    real(kind=double) :: a, b, beta
    beta = exp (dlogam(a) + dlogam(b) - dlogam(a+b))
end function beta

```

Uses beta 99.

68b *<Module subroutines 31b>+≡*

```

!!! CERNLIB C304

function dlogam (x)
    real(kind=double) :: dlogam
    real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
    real(kind=double), dimension(5) :: c, xl
    real(kind=double) :: x, y, zero, one, two, half, ap, aq
    integer :: i
    data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
    data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
    data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
        +5.5584045723515D+1, -2.1513513573726D+2, &
        -2.4587261722292D+2, -5.7500893603041D+1, &
        -2.3359098949513D+0/
    data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
        +1.9387784034377D+2, +3.0882954973424D+2, &
        +1.5006839064891D+2, +2.0106851344334D+1, &
        +4.5717420282503D-1/
    data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
        +2.1797366058896D+3, +3.7975124011525D+3, &
        -1.9778070769842D+3, -3.6929834005591D+3, &
        -5.6017773537804D+2/
    data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
        +1.5612045277929D+3, +7.2340087928948D+3, &
        +1.0459576594059D+4, +4.1699415153200D+3, &
        +2.7678583623804D+2/
    data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
        -4.7504594653440D+6, -2.9423445930322D+6, &
        +3.6321804931543D+7, -3.3567782814546D+6, &
        -2.4804369488286D+7/
    data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
        -1.5552890280854D+5, -3.4152517108011D+6, &
        -2.0969623255804D+7, -3.4544175093344D+7, &

```

```

-9.1605582863713D+6/
data c / 1.1224921356561D-1, 7.9591692961204D-2, &
-1.7087794611020D-3, 9.1893853320467D-1, &
1.3469905627879D+0/
if (x .le. xl(1)) then
    print *, 'ERROR: DLOGAM non positive argument: ', X
    dlogam = zero
end if
if (x .le. xl(2)) then
    y = x + one
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
        ap = p1(i) + y * ap
        aq = q1(i) + y * aq
    end do
    y = - log(x) + x * ap / aq
else if (x .le. xl(3)) then
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
        ap = p1(i) + x * ap
        aq = q1(i) + x * aq
    end do
    y = (x - one) * ap / aq
else if (x .le. xl(4)) then
    ap = p2(1)
    aq = q2(1)
    do i = 2, 7
        ap = p2(i) + x * ap
        aq = q2(i) + x * aq
    end do
    y = (x-two) * ap / aq
else if (x .le. xl(5)) then
    ap = p3(1)
    aq = q3(1)
    do i = 2, 7
        ap = p3(i) + x * ap
        aq = q3(i) + x * aq
    end do
    y = ap / aq
else
    y = one / x**2
    y = (x-half) * log(x) - x + c(4) + &
(c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
end if
dlogam = y
end function dlogam

```

6.4 Non-Singular Distributions

70a *<Public subroutines 31a>+≡*

```
public :: kirke
```

Uses kirke 70b.

70b *<Module subroutines 31b>+≡*

```
function kirke (x1, x2, p1, p2)
  real(kind=double) :: x1, x2
  real(kind=double) :: kirke
  integer :: p1, p2
  <Initialization check 32g>
  kirke = -1.0
  if (abs(p1) .eq. C1_ELECTRON) then
    if (abs(p2) .eq. C1_ELECTRON) then
      kirke = kirkee (x1, x2)
    else if (p2 .eq. C1_PHOTON) then
      kirke = kirkeg (x1, x2)
    end if
  else if (p1 .eq. C1_PHOTON) then
    if (abs(p2) .eq. C1_ELECTRON) then
      kirke = kirkeg (x2, x1)
    else if (p2 .eq. C1_PHOTON) then
      kirke = kirkgg (x1, x2)
    end if
  endif
end function kirke
```

Defines:

kirke, used in chunk 70a.

Uses C1_ELECTRON 11b, C1_PHOTON 11b, kirkee 70d, kirkeg 73a, and kirkgg 73c.

70c *<Public subroutines 31a>+≡*

```
public :: kirkee
```

Uses kirkee 70d.

70d *<Module subroutines 31b>+≡*

```
function kirkee (x1, x2)
  real(kind=double) :: x1, x2
  real(kind=double) :: kirkee
  real(kind=double) :: d1, d2
  <Initialization check 32g>
  kirkee = -1.0
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    <Calculate version 1 of the non-singular  $e^+e^-$  distribution 71e>
  else
    <else handle invalid versions 36c>
  end function kirkee
```

Defines:

kirkee, used in chunks 16g, 70, and 71e.

Uses d1 15f and d2 16b.

71a \langle 8-byte aligned part of circe1 parameters 32d $\rangle + \equiv$
`real(kind=double) :: elect0, gamma0`

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha_\rho} + a_1^{\alpha_\rho} \int_{1-\epsilon}^{1-} dx x^{a_2^{\alpha_\rho}} (1-x)^{a_3^{\alpha_\rho}} \quad (18)$$

Approximately

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha_\rho} + a_1^{\alpha_\rho} \int_{1-\epsilon}^{1-} dx (1-x)^{a_3^{\alpha_\rho}} = a_0^{\alpha_\rho} + a_1^{\alpha_\rho} \int_{0+}^{\epsilon} d\xi \xi^{a_3^{\alpha_\rho}} \quad (19)$$

and therefore

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha_\rho} + a_1^{\alpha_\rho} \frac{1 - \epsilon^{a_3^{\alpha_\rho} + 1}}{a_3^{\alpha_\rho} + 1} \quad (20)$$

This simple approximation is good enough

71b \langle Update circe1 parameters 33a $\rangle + \equiv$
`circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * KIREPS**(circe1_params%elect0 / KIREPS)`
`circe1_params%gamma0 = circe1_params%a1(4) * KIREPS**(circe1_params%a1(5)+1) / (circe1_params%gamma0 / KIREPS)`

but we can also use incomplete Beta functions for the exact result:

71c \langle Alternative: Update circe1 parameters 71c $\rangle + \equiv$
`circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * beta (circe1_params%elect0 / KIREPS, (1d0 - betinc (circe1_params%a1(2)+1, circe1_params%a1(3)+1, 1d0 - betinc (circe1_params%a1(5)+1, circe1_params%a1(6)+1, KIREPS)), 1d0 - betinc (circe1_params%a1(2)+1, circe1_params%a1(3)+1, 1d0 - betinc (circe1_params%a1(5)+1, circe1_params%a1(6)+1, KIREPS)))`
`circe1_params%gamma0 = circe1_params%a1(7) + circe1_params%a1(4) * beta (circe1_params%gamma0 / KIREPS, (1d0 - betinc (circe1_params%a1(2)+1, circe1_params%a1(3)+1, 1d0 - betinc (circe1_params%a1(5)+1, circe1_params%a1(6)+1, KIREPS))), 1d0 - betinc (circe1_params%a1(2)+1, circe1_params%a1(3)+1, 1d0 - betinc (circe1_params%a1(5)+1, circe1_params%a1(6)+1, KIREPS)))`

Uses beta 99.

71d \langle Alternative: Local variables for circes 71d $\rangle + \equiv$
`real(kind=double) :: betinc`
`external betinc`

71e \langle Calculate version 1 of the non-singular e^+e^- distribution 71e $\rangle + \equiv$
`if (x1 .gt. 1d0) then`
`d1 = 0d0`
`elseif (x1 .ge. (1d0 - KIREPS)) then`
`d1 = circe1_params%elect0`
`elseif (x1 .ge. 0d0) then`
`d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%a1(3)`
`else`
`d1 = 0d0`
`endif`
`if (x2 .gt. 1d0) then`
`d2 = 0d0`
`elseif (x2 .ge. (1d0 - KIREPS)) then`
`d2 = circe1_params%elect0`
`elseif (x2 .ge. 0d0) then`
`d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%a1(3)`

```

else
    d2 = 0d0
endif
kirkee = d1 * d2

```

Uses d1 15f, d2 16b, and kirkee 70d.

72a \langle Calculate version 1 of the non-singular $e^\pm\gamma$ distribution 72a $\rangle \equiv$

```

if (x1 .gt. 1d0) then
    d1 = 0d0
elseif (x1 .ge. (1d0 - KIREPS)) then
    d1 = circe1_params%elect0
elseif (x1 .ge. 0d0) then
    d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
else
    d1 = 0d0
endif
if (x2 .gt. 1d0) then
    d2 = 0d0
elseif (x2 .gt. KIREPS) then
    d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
elseif (x2 .ge. 0d0) then
    d2 = circe1_params%gamma0
else
    d2 = 0d0
endif
kirkeg = d1 * d2

```

Uses d1 15f, d2 16b, and kirkeg 73a.

72b \langle Calculate version 1 of the non-singular $\gamma\gamma$ distribution 72b $\rangle \equiv$

```

if (x1 .gt. 1d0) then
    d1 = 0d0
elseif (x1 .gt. KIREPS) then
    d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%
elseif (x1 .ge. 0d0) then
    d1 = circe1_params%gamma0
else
    d1 = 0d0
endif
if (x2 .gt. 1d0) then
    d2 = 0d0
elseif (x2 .gt. KIREPS) then
    d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
elseif (x2 .ge. 0d0) then
    d2 = circe1_params%gamma0
else
    d2 = 0d0
endif
kirkgg = d1 * d2

```

Uses d1 15f, d2 16b, and kirkgg 73c.

72c \langle Public subroutines 31a $\rangle + \equiv$


```

    public :: kirkeg
    Uses kirkeg 73a.
73a  <Module subroutines 31b>+≡
    function kirkeg (x1, x2)
    real(kind=double) :: x1, x2
    real(kind=double) :: kirkeg
    real(kind=double) :: d1, d2
    <Initialization check 32g>
    kirkeg = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
        <Calculate version 1 of the non-singular  $e^{\pm\gamma}$  distribution 72a>
    <else handle invalid versions 36c>
    end function kirkeg

    Defines:
    kirkeg, used in chunks 70b and 72.
    Uses d1 15f and d2 16b.

73b  <Public subroutines 31a>+≡
    public :: kirkgg
    Uses kirkgg 73c.

73c  <Module subroutines 31b>+≡
    function kirkgg (x1, x2)
    real(kind=double) :: x1, x2
    real(kind=double) :: kirkgg
    real(kind=double) :: d1, d2
    <Initialization check 32g>
    kirkgg = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
        <Calculate version 1 of the non-singular  $\gamma\gamma$  distribution 72b>
    <else handle invalid versions 36c>
    end function kirkgg

    Defines:
    kirkgg, used in chunks 70b, 72b, and 73b.
    Uses d1 15f and d2 16b.

73d  <Alternative: Subroutines 73d>≡
    function betinc (a, b, x)
    real(kind=double) :: x, a, b
    real(kind=double) :: betinc
    real(kind=double) :: bt
    if (x .lt. 0d0 .or. x .gt. 1d0) then
        betinc = 0d0
    else
        if (x .eq. 0d0 .or. x .eq. 1d0) then
            bt = 0d0
        else
            bt = exp(dlogam(a+b)-dlogam(a)-dlogam(b) &
                + a*log(x) + b*log(1d0-x))
        end if
    end if
end function betinc

```

```

end if
if (x .lt. (a+1d0)/ (a+b+2d0)) then
  betinc = bt*betacf (a, b, x) / a
else
  betinc = 1d0 - bt*betacf (b, a, 1d0-x) / b
end if
end if
end function betinc

```

74 *⟨Alternative: Subroutines 73d⟩*+≡

```

function betacf (a, b, x)
  real(kind=double) :: x, a, b
  real(kind=double) :: betacf
  integer, parameter :: itmax = 100
  real(kind=double), parameter = eps = 3d-7
  real(kind=double) :: am, bm, curr, prev, qab, qap, qam, bz, &
    ap, bp, app, bpp, em, tem, d

  integer :: m
  am = 1d0
  bm = 1d0
  curr = 1d0
  qab = a + b
  qap = a + 1d0
  qam = a - 1d0
  bz = 1d0 - qab * x / qap
  do m = 1, ITMAX
    em = m
    tem = 2*em
    d = em * (b - m) * x / ((qam + tem) * (a + tem))
    ap = curr + d*am
    bp = bz + d*bm
    d = - (a + em) * (qab + em) * x / ((a + tem) * (qap + tem))
    app = ap + d * curr
    bpp = bp + d * bz
    prev = curr
    am = ap / bpp
    bm = bp / bpp
    curr = app / bpp
    bz = 1d0
    if (abs (curr - prev) .lt. EPS * abs (curr)) then
      betacf = curr
      return
    end if
  end do
  print *, 'betacf: failed to converge'
  betacf = 0d0
end

```

6.5 Generators

6.5.1 Version 1

Beta distributions have the practical advantage that they have been popular among mathematicians.[?]

75a *⟨Public subroutines 31a⟩*+≡

```
public :: girce
```

Uses *girce 75b*.

75b *⟨Module subroutines 31b⟩*+≡

```
subroutine girce (x1, x2, p1, p2, rng)
  real(kind=double), intent(out) :: x1, x2
  integer :: p1, p2
  external rng
  real(kind=double) :: u, w
```

⟨Initialization check 32g⟩

⟨x1m, x2m kludge, part 1 76a⟩

⟨Select particles p1 and p2 75c⟩

```
  if (abs(p1) .eq. C1_ELECTRON) then
    if (abs(p2) .eq. C1_ELECTRON) then
      call gircee (x1, x2, rng)
    else if (p2 .eq. C1_PHOTON) then
      call girceg (x1, x2, rng)
    end if
  else if (p1 .eq. C1_PHOTON) then
    if (abs(p2) .eq. C1_ELECTRON) then
      call girceg (x2, x1, rng)
    else if (p2 .eq. C1_PHOTON) then
      call gircgg (x1, x2, rng)
    end if
  end if
```

⟨x1m, x2m kludge, part 2 76b⟩

```
  end subroutine girce
```

Defines:

girce, used in chunks *75b*, *20a*, and *75*.

Uses *C1_ELECTRON 11b*, *C1_PHOTON 11b*, *gircee 76d*, *girceg 77b*, and *gircgg 77f*.

75c *⟨Select particles p1 and p2 75c⟩*≡

```
  w = 1d0 / (1d0 + circgg (-1d0, -1d0))
  call rng (u)
  if (u*u .le. w) then
    p1 = C1_POSITRON
  else
    p1 = C1_PHOTON
  end if
  call rng (u)
  if (u*u .le. w) then
    p2 = C1_ELECTRON
  else
```

```

        p2 = C1_PHOTON
    end if

```

Uses C1_ELECTRON 11b, C1_PHOTON 11b, C1_POSITRON 11b, and circgg 42d.

The flavor selection is incorrect, because the relative weights depend on the minimum energy fractions. We resort to a moderately inefficient kludge, because we don't have the distribution functions available yet. We'll have to implement incomplete Beta functions and other horrible things for this. Fortunately, the efficiency can not drop below the relative contribution of e^+e^- .

```

76a <x1m, x2m kludge, part 1 76a>≡
    do

```

Crude rejection:

```

76b <x1m, x2m kludge, part 2 76b>≡
        if ((x1 .ge. circe1_params%x1m) .and. (x2 .ge. circe1_params%x2m)) exit
    end do

```

```

76c <Public subroutines 31a>+≡
    public :: gircee

```

Uses gircee 76d.

```

76d <Module subroutines 31b>+≡
    subroutine gircee (x1, x2, rng)
        real(kind=double), intent(out) :: x1, x2
        external rng
        real(kind=double) :: u
        <Initialization check 32g>
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
            <Generate version 1 of the  $e^+e^-$  distribution 76e>
            <else handle invalid versions 36c>
        end subroutine gircee

```

Defines:

gircee, used in chunks 20, 76d, 21c, 75b, and 76c.

For version 1 of the parametrizations we rely on girceb, a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (21)$$

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1}(1-\xi)^{b-1} \quad (22)$$

```

76e <Generate version 1 of the  $e^+e^-$  distribution 76e>≡

```

```

    call rng (u)
    if (u .le. circe1_params%a1(0)) then
        x1 = 1d0
    else
        x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, circe1_params%a1(3)+1d0, circe1_
    endif
    call rng (u)
    if (u .le. circe1_params%a1(0)) then
        x2 = 1d0

```

```

        else
            x2 = 1d0 - girceb (0d0, 1d0-circe1_params%x2m, circe1_params%a1(3)+1d0, circe1_
        endif
    Uses girceb 78b.
77a  <Public subroutines 31a>+≡
        public :: girceg
    Uses girceg 77b.
77b  <Module subroutines 31b>+≡
        subroutine girceg (x1, x2, rng)
            real(kind=double), intent(out) :: x1, x2
            external rng
            real(kind=double) :: u
            <Initialization check 32g>
            if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
                <Generate version 1 of the  $e^\pm\gamma$  distribution 77c>
            <else handle invalid versions 36c>
            end subroutine girceg

Defines:
    girceg, used in chunks 20c, 75b, and 77a.
77c  <Generate version 1 of the  $e^\pm\gamma$  distribution 77c>≡
        call rng (u)
        if (u .le. circe1_params%a1(0)) then
            x1 = 1d0
        else
            x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, circe1_params%a1(3)+1d0, circe1_
        endif
        x2 = girceb (circe1_params%x2m, 1d0, circe1_params%a1(5)+1d0, circe1_params%a1(6)+
    Uses girceb 78b.
77d  <Public subroutines 31a>+≡
        public :: gircgg
    Uses gircgg 77f.
77e  <Module subroutines 31b>+≡
        subroutine gircgg (x1, x2, rng)
            real(kind=double), intent(out) :: x1, x2
            external rng
            <Initialization check 32g>
            if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
                <Generate version 1 of the  $\gamma\gamma$  distribution 77f>
            <else handle invalid versions 36c>
            end subroutine gircgg

    Uses gircgg 77f.
77f  <Generate version 1 of the  $\gamma\gamma$  distribution 77f>≡
        x1 = girceb (circe1_params%x1m, 1d0, circe1_params%a1(5)+1d0, circe1_params%a1(6)+
        x2 = girceb (circe1_params%x2m, 1d0, circe1_params%a1(5)+1d0, circe1_params%a1(6)+
    Defines:

```

gircegg, used in chunks 20c, 75b, and 77.
 Uses girceb 78b.

6.5.2 Version 2

Retired.

6.5.3 Version 3 and 4

Identical to version 1.

6.6 Utilities

For version 1 of the parametrizations we need a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max} - x)\Theta(x - x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (23)$$

with the *incomplete Beta-function* I :

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1} (1-\xi)^{b-1} \quad (24)$$

$$B(a, b) = I(0, a, b) \quad (25)$$

This problem has been studied extensively [?] and we can use an algorithm [18] that is very fast for $0 < a \leq 1 \leq b$, which turns out to be the case in our application.

78a \langle Public subroutines 31a $\rangle + \equiv$
 public :: girceb

Uses girceb 78b.

78b \langle Module subroutines 31b $\rangle + \equiv$
 function girceb (xmin, xmax, a, b, rng)
 real(kind=double) :: xmin, xmax, a, b
 real(kind=double) :: girceb
 external rng
 real(kind=double) :: t, p, u, umin, umax, x, w
 \langle Check a and b 79a \rangle
 \langle Set up girceb parameters 79b \rangle
 do
 \langle Generate a trial **x** and calculate its weight **w** 79c \rangle
 call rng (u)
 if (w .gt. u) exit
 end do
 girceb = x
 end function girceb

Defines:

girceb, used in chunks 78b and 76–80.

In fact, this algorithm works for $0 < a \leq 1 \leq b$ only:

```

79a <Check a and b 79a>≡
      if ((a .gt. 1d0) .or. (b .lt. 1d0)) then
        girceb = -1d0
        call circem ('ERROR', 'beta-distribution expects a<=1<=b')
        return
      end if

```

Uses `circem 81c` and `girceb 78b`.

The trick is to split the interval $[0, 1]$ into two parts $[0, t]$ and $[t, 1]$. In these intervals we obviously have

$$x^{a-1}(1-x)^{b-1} \leq \begin{cases} x^{a-1} & \text{for } x \leq t \\ t^{a-1}(1-x)^{b-1} & \text{for } x \geq t \end{cases} \quad (26)$$

because we have assumed that $0 < a \leq 1 \leq b$. The integrals of the two dominating distributions are t^a/a and $t^{a-1}(1-t)^b/b$ respectively and therefore the probability for picking a random number from the first interval is

$$P(x \leq t) = \frac{bt}{bt + a(1-t)^b} \quad (27)$$

We postpone the discussion of the choice of t until later:

```

79b <Set up girceb parameters 79b>≡
      <Set up best value for t 81a>
      p = b*t / (b*t + a * (1d0 - t)**b)

```

The dominating distributions can be generated by simple mappings

$$\phi : [0, 1] \rightarrow [0, 1] \quad (28)$$

$$u \mapsto \begin{cases} t \left(\frac{u}{p}\right)^{\frac{1}{a}} & < t \text{ for } u < p \\ t & = t \text{ for } u = p \\ 1 - (1-t) \left(\frac{1-u}{1-p}\right)^{\frac{1}{b}} & > t \text{ for } u > p \end{cases} \quad (29)$$

The beauty of the algorithm is that we can use a single uniform deviate u for both intervals:

```

79c <Generate a trial x and calculate its weight w 79c>≡
      call rng (u)
      u = umin + (umax - umin) * u
      if (u .le. p) then
        x = t * (u/p)**(1d0/a)
        w = (1d0 - x)**(b-1d0)
      else
        x = 1d0 - (1d0 - t) * ((1d0 - u)/(1d0 - p))**(1d0/b)
        w = (x/t)**(a-1d0)
      end if

```

The weights that are derived by dividing the distribution by the dominating distributions are already normalized correctly:

$$w : [0, 1] \rightarrow [0, 1] \quad (30)$$

$$x \mapsto \begin{cases} (1-x)^{b-1} & \in [(1-t)^{b-1}, 1] \text{ for } x \leq t \\ \left(\frac{x}{t}\right)^{a-1} & \in [t^{1-a}, 1] \text{ for } x \geq t \end{cases} \quad (31)$$

To derive $u_{\min, \max}$ from $x_{\min, \max}$ we can use ϕ^{-1} :

$$\phi^{-1} : [0, 1] \rightarrow [0, 1] \quad (32)$$

$$x \mapsto \begin{cases} p \left(\frac{x}{t}\right)^a & < p \text{ for } x < t \\ p & = p \text{ for } x = t \\ 1 - (1-p) \left(\frac{1-x}{1-t}\right)^b & > p \text{ for } x > t \end{cases} \quad (33)$$

We start with u_{\min} . For efficiency, we handle the most common cases (small x_{\min}) first:

```
80a <Set up girceb parameters 79b> +=
      if (xmin .le. 0d0) then
        umin = 0d0
      elseif (xmin .lt. t) then
        umin = p * (xmin/t)**a
      elseif (xmin .eq. t) then
        umin = p
      elseif (xmin .lt. 1d0) then
        umin = 1d0 - (1d0 - p) * ((1d0 - xmin)/(1d0 - t))**b
      else
        umin = 1d0
      endif
```

Same procedure for u_{\max} ; again, handle the most common cases (large x_{\max}) first:

```
80b <Set up girceb parameters 79b> +=
      if (xmax .ge. 1d0) then
        umax = 1d0
      elseif (xmax .gt. t) then
        umax = 1d0 - (1d0 - p) * ((1d0 - xmax)/(1d0 - t))**b
      elseif (xmax .eq. t) then
        umax = p
      elseif (xmax .gt. 0d0) then
        umax = p * (xmax/t)**a
      else
        umax = 0d0
      endif
```

Check for absurd cases.

```
80c <Set up girceb parameters 79b> +=
      if (umax .lt. umin) then
        girceb = -1d0
        return
      endif
```

Uses girceb 78b.

It remains to choose the best value for t . The rejection efficiency ϵ of the algorithm is given by the ratio of the dominating distribution and the distribution

$$\frac{1}{\epsilon(t)} = \frac{B(a,b)}{ab} (bt^a + at^{a-1}(1-t)^b). \quad (34)$$

It is maximized for

$$bt - bt(1-t)^{b-1} + (a-1)(1-t)^b = 0 \quad (35)$$

This equation has a solution which can be determined numerically. While this determination is far too expensive compared to a moderate loss in efficiency, we could perform it once after fitting the coefficients a , b . Nevertheless, it has been shown,[\[18\]](#) that

$$t = \frac{1-a}{b+1-a} \quad (36)$$

results in non-vanishing efficiency for all values $1 < a \leq 1 \leq b$. Empirically we have found efficiencies of at least 80% for this choice, which is enough for our needs.

```

81a <Set up best value for t 81a>≡
      t = (1d0 - a) / (b + 1d0 - a)

81b <Public subroutines 31a>+≡
      public :: circem
      Uses circem 81c.

81c <Module subroutines 31b>+≡
      subroutine circem (errlvl, errmsg)
      character(len=*) :: errlvl, errmsg
      integer, save :: errcnt = 0
      if (errlvl .eq. 'MESSAGE') then
        print *, 'circe1:message: ', errmsg
      else if (errlvl .eq. 'WARNING') then
        if (errcnt .lt. 100) then
          errcnt = errcnt + 1
          print *, 'circe1:warning: ', errmsg
        else if (errcnt .eq. 100) then
          errcnt = errcnt + 1
          print *, 'circe1:message: more than 100 messages'
          print *, 'circe1:message: turning warnings off'
        end if
      else if (errlvl .eq. 'ERROR') then
        if (errcnt .lt. 200) then
          errcnt = errcnt + 1
          print *, 'circe1:error: ', errmsg
        else if (errcnt .eq. 200) then
          errcnt = errcnt + 1
          print *, 'circe1:message: more than 200 messages'
          print *, 'circe1:message: turning error messages off'
        endif
      else if (errlvl .eq. 'PANIC') then
        if (errcnt .lt. 300) then
          errcnt = errcnt + 1
          print *, 'circe1:panic: ', errmsg

```

```

        else if (errcnt .eq. 300) then
            errcnt = errcnt + 1
            print *, 'circe1:message: more than 300 messages'
            print *, 'circe1:message: turning panic messages off'
        end if
    else
        print *, 'circe1:panic:    invalid error code ', errlvl
    end if
end subroutine circem

```

Defines:

circem, used in chunks 32-38, 49, 50, 52-54, 56, 60, 62-64, 79a, and 81b.

6.7 Examples

6.7.1 Distributions

```

82 <circe1_plot.f90 82>≡
    program circe1_plot
        use kinds
        use circe1

        implicit none

        real(kind=double) :: xmin, xmax, y, roots
        integer :: xory, nstep, p1, p2, acc, ver, rev, i
        real(kind=double) :: x, logx, d
        read *, xory, xmin, xmax, nstep, y, p1, p2, roots, acc, ver, rev
        call circses (0d0, 0d0, roots, acc, ver, rev, 0)
        do i = 0, nstep
            logx = log (xmin) + i * log (xmax/xmin) / nstep
            x = exp (logx)
            d = 0d0
            if (xory .eq. 1) then
                if (p1 .eq. C1_PHOTON) then
                    d = circe (x, y, p1, p2)
                else
                    d = circe (1d0 - x, y, p1, p2)
                end if
            else if (xory .eq. 2) then
                if (p1 .eq. C1_PHOTON) then
                    d = circe (y, x, p1, p2)
                else
                    d = circe (y, 1d0 - x, p1, p2)
                end if
            end if
            if (d .gt. 1d-4) print *, x, d
        end do
    end program circe1_plot

```

Uses C1_PHOTON 11b, circe 31b, and circes 32a.

6.7.2 Library functions

If Fortran77 only had first class functions, then the following cruft would not be necessary. OK, here's the outline of the adaptive Gauss integration routine from CERNLIB:

```
83a  <Part one of Gaussian integration 83a>≡
      real(kind=double) :: f, a, b, eps
      external f
      real(kind=double), parameter :: Z1 = 1, HF = Z1/2, CST = 5*Z1/1000
      integer :: i
      real(kind=double) :: h, const, aa, bb, c1, c2, s8, s16, u
      <Gaussian weights 85c>
      h = 0
      if (b .eq. a) go to 99
      const = CST / dabs(b-a)
      bb = a
1 continue
      aa = bb
      bb = b
2 continue
      c1 = HF*(bb+aa)
      c2 = HF*(bb-aa)
      s8 = 0
      do i = 1, 4
         u = c2*x(i)
```

Here are now the first two function calls that we have to fill in later in various ways:

```
83b  <Function call stub 83b>≡
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
```

Continuing

```
83c  <Part two of Gaussian integration 83c>≡
      end do
      s16 = 0
      do i = 5, 12
         u = c2*x(i)
```

And here are the other two function calls:

```
83d  <Function call stub 83b>+≡
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
```

Terminating:

```
83e  <Part three of Gaussian integration 83e>≡
      end do
      s16 = c2*s16
      if (dabs(s16-c2*s8) .le. eps*(1+dabs(s16))) then
         h = h + s16
         if (bb .ne. b) go to 1
```

```

else
  bb = c1
  if (1 + const*dabs(c2) .ne. 1) go to 2
  h = 0
  print *, 'gauss: too high accuracy required'
  go to 99
end if
99 continue

```

This one is still reasonably straightforward

$$\text{gauss1} : (f, a, b) \mapsto \int_a^b dx f(x) \quad (37)$$

```

84a <circe1_sample.f90: public 14b>+≡
    public :: gauss1
    Uses gauss1 84b.

84b <circe1_sample.f90: subroutines 15a>+≡
    function gauss1 (f, a, b, eps)
    real(kind=double) :: gauss1
    <Part one of Gaussian integration 83a>
    s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
    <Part two of Gaussian integration 83c>
    s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
    <Part three of Gaussian integration 83e>
    gauss1 = h
    end function gauss1

```

Defines:

`gauss1`, used in chunks 84b, 15b, and 84.

But this almost identical repeat

$$\text{gaussx} : (f, a, b) \mapsto \left(y \mapsto \int_a^b dx f(y, x) \right) \quad (38)$$

would not be necessary in a modern programming language with currying:

```

84c <circe1_sample.f90: public 14b>+≡
    public :: gaussx
    Uses gaussx 84d.

84d <circe1_sample.f90: subroutines 15a>+≡
    function gaussx (f, y, a, b, eps)
    real(kind=double) :: y
    real(kind=double) :: gaussx
    <Part one of Gaussian integration 83a>
    s8 = s8 + w(i) * (f (y, c1+u) + f (y, c1-u))
    <Part two of Gaussian integration 83c>
    s16 = s16 + w(i) * (f (y, c1+u) + f (y, c1-u))
    <Part three of Gaussian integration 83e>
    gaussx = h
    end function gaussx

```

Defines:

`gaussx`, used in chunks 84 and 85b.

Fortunately, this is the last one we need

$$\text{gauss2} : (f, a, b, a_1, b_1) \mapsto \int_a^b dx \int_{a_1}^{b_1} dy f(x, y) \\ = \text{gauss1}(\text{gaussx}(f, a, b), a_1, b_1) \quad (39)$$

```
85a <circe1_sample.f90: public 14b>+≡
    public :: gauss2
    Uses gauss2 85b.

85b <circe1_sample.f90: subroutines 15a>+≡
    function gauss2 (f, a, b, a1, b1, eps)
        real(kind=double) :: a1, b1
        real(kind=double) :: gauss2
        <Part one of Gaussian integration 83a>
        s8 = s8 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &
            + gaussx (f, c1-u, a1, b1, eps))
        <Part two of Gaussian integration 83c>
        s16 = s16 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &
            + gaussx (f, c1-u, a1, b1, eps))
        <Part three of Gaussian integration 83e>
        gauss2 = h
    end function gauss2
```

Defines:

`gauss2`, used in chunks 85b, 15b, 16d, and 85.

Uses `gaussx` 84d.

```
85c <Gaussian weights 85c>≡
    real(kind=double), dimension(12), parameter :: &
        x = (/ 9.6028985649753623d-1, &
            7.9666647741362674d-1, &
            5.2553240991632899d-1, &
            1.8343464249564980d-1, &
            9.8940093499164993d-1, &
            9.4457502307323258d-1, &
            8.6563120238783174d-1, &
            7.5540440835500303d-1, &
            6.1787624440264375d-1, &
            4.5801677765722739d-1, &
            2.8160355077925891d-1, &
            9.5012509837637440d-2 /), &
        w = (/ 1.0122853629037626d-1, &
            2.2238103445337447d-1, &
            3.1370664587788729d-1, &
            3.6268378337836198d-1, &
            2.7152459411754095d-2, &
            6.2253523938647893d-2, &
```

```

9.5158511682492785d-2, &
1.2462897125553387d-1, &
1.4959598881657673d-1, &
1.6915651939500254d-1, &
1.8260341504492359d-1, &
1.8945061045506850d-1 /)

```

6.7.3 Generators

6.8 Dumping Parameters

```

86a <params.f90 86a>≡
  program params
    use kinds
    use circe1

    implicit none
    integer :: acc, ver, i
    real(kind=double), dimension(7), parameter :: roots = &
      (/ 90D0, 170D0, 350D0, 500D0, 800D0, 1000D0, 1500D0 /)
    do ver = 7, 8
      print *, "VERSION ", ver
      do acc = TESLA, XBNDEE
        do 12 i = 1, 7
          print *, "=====
          call circes (0d0, 0d0, roots(i), acc, ver, 20020307, 0)
          call dump ()
        end do
      end do
    end do
  end program params

```

Uses TESLA 13, XBNDEE 13, and circes 32a.

```

86b <params.f90 86a>+≡
  subroutine dump
    <Accelerator codes 13>
    character(len=9) :: name
    select case (acc)
      case (SBAND)
        name = 'SBAND'
      case (TESLA)
        name = 'TESLA'
      case (JLCNLC)
        name = 'JLCNLC'
      case (SBNDEE)
        name = 'SBAND/EE'
      case (TESLEE)
        name = 'TESLA/EE'
    end select
  end subroutine dump

```

```

      case (XBNDEE)
        name = 'JLCNLC/EE'
      case default
        print *, "Accelerator mode not recognized"
      end select
      write (*, 1000) name, circe1_params%roots
      write (*, 1001) 'e+ e-', circe1_params%lumi
      write (*, 1002) 'e+/e-', circe1_params%a1(0)
      write (*, 1003) 'e+/e-', 1 - circe1_params%a1(0)
      write (*, 1004) 'e+/e-', circe1_params%a1(1), circe1_params%a1(2), circe1_params%a1(3)
      write (*, 1003) 'gamma', circe1_params%a1(7)
      write (*, 1004) 'gamma', circe1_params%a1(4), circe1_params%a1(5), circe1_params%a1(6)
1000 format (A9, ' @ ', F5.0, ' GeV')
1001 format (4X, A7, ' lumi          = ', F7.2, ' * 1032 cm-2 sec-1')
1002 format (4X, A7, ' delta strength = ', F9.5)
1003 format (4X, A7, ' integral(cont.) = ', F9.5)
1004 format (4X, A7, ' distribution   = ', F9.5, ' * x{', F9.5, ' } * (1-x){', F9.5, ' }')
      end subroutine dump

```

Uses JLCNLC 13, SBAND 13, SBNDEE 13, TESLA 13, TESLEE 13, and XBNDEE 13.

7 Fitting

7.1 Version 1: Factorized Beta Distributions

```

87  <Copyleft notice 30a>+≡
!
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!   Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
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!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

```

```

! This file has been stripped of most comments.  For documentation, refer
! to the source 'minuit.nw'

88a <circe1_fit.f90 88a>≡
! circe1_fit.f90 -- fitting for circe
<Copyleft notice 30a>

module fit_routines
  use kinds

  implicit none
  private

  <circe1_fit.f90: public 89e>

  contains
  <circe1_fit.f90: subroutines 89f>
end module fit_routines

program fit
  use kinds
  use fit_routines

  implicit none

  integer :: i, rcode
  <Declare NPARAM 88b>
  <Declare parameters 89a>
  <Declare arguments 89b>

  <Initialize parameters for circe1_fit.f90 89d>
  call mninit (5, 6, 7)
  <Load parameters 89c>
  call mnseti ('CIRCE: fit version 1      ')
  argv(1) = 1
  call mnexcm (fct, 'SET PRINTOUT          ', argv, 1, rcode, 0d0)
  argv(1) = 1
  call mnexcm (fct, 'CALL FCT              ', argv, 1, rcode, 0d0)
  call mnexcm (fct, 'MIGRAD                ', argv, 0, rcode, 0d0)
  call mnexcm (fct, 'MINOS                 ', argv, 0, rcode, 0d0)
  argv(1) = 3
  call mnexcm (fct, 'CALL FCT              ', argv, 1, rcode, 0d0)
  call mnexcm (fct, 'STOP                  ', argv, 0, rcode, 0d0)

end program fit

Defines:
  fit, used in chunks 89c, 105, and 109d.
Uses circe 31b and fct 89f 105e.

88b <Declare NPARAM 88b>≡

```



```
integer, parameter :: NPARAM = 6
```

Defines:

NPARAM, used in chunks 89 and 95f.

```
89a <Declare parameters 89a>≡
integer, dimension(NPARAM) :: pnum
character(len=10), dimension(NPARAM) :: pname
real(kind=double), dimension(NPARAM) :: pstart, pstep
```

Uses NPARAM 88b.

```
89b <Declare arguments 89b>≡
integer, parameter :: ARGC = 10
real(kind=double), dimension(ARGC) :: argv
```

```
89c <Load parameters 89c>≡
do i = 1, NPARAM
  call mnparm (pnum(i), pname(i), pstart(i), pstep(i), 0d0, 0d0, rcode)
  if (rcode .ne. 0) then
    print *, "fit: MINUIT won't accept parameter ", pnum(i)
    stop
  endif
end do
```

Uses NPARAM 88b and fit 88a.

```
89d <Initialize parameters for circle_fit.f90 89d>≡
data pnum / 1, 2, 3, 4, 5, 6 /
data pname / '1_e', 'x_e', '1-x_e', '1_g', 'x_g', '1-x_g' /
data pstart / -1.00, 20.00, 0.20, -1.00, 0.20, 20.00 /
data pstep / 0.01, 0.01, 0.01, 0.01, 0.01, 0.01 /
```

```
89e <circle_fit.f90: public 89e>≡
public :: fct
```

Uses fct 89f 105e.

```
89f <circle_fit.f90: subroutines 89f>≡
subroutine fct (nx, df, f, a, mode, g)
integer :: nx, mode
real(kind=double) :: f, g
real(kind=double), dimension(:) :: df, a
<Local variables for fct (v1) 90c>
if (mode .eq. 1) then
  <Read input data (v1) 90a>
else if (mode .eq. 2) then
  <Calculate ∇f 93c>
end if
<Calculate f (v1) 93d>
end if
if (mode .eq. 3) then
  <Write output (v1) 95e>
end if
end subroutine fct
```

Defines:

fct, used in chunks 88a, 89e, 105, and 110c.

```

90a  <Read input data (v1) 90a>≡
      <Read data from file 90b>
      <Fixup errors 91d>
      <Normalize 92a>

90b  <Read data from file 90b>≡
      call gethst ('ee', NDATA, xee, fee, dfec, see, tee, pwr)
      call gethst ('eg', NDATA, xeg, feg, dfeg, seg, teg, pwr)
      call gethst ('ge', NDATA, xge, fge, dfge, sge, tge, pwr)
      call gethst ('gg', NDATA, xgg, fgg, dfgg, sgg, tgg, pwr)

      Uses gethst 90e.

90c  <Local variables for fct (v1) 90c>≡
      integer, parameter :: NDATA = 20
      real(kind=double) :: see, tee, dtee
      real(kind=double) :: seg, teg, dteg
      real(kind=double) :: sge, tge, dtge
      real(kind=double) :: sgg, tgg, dtgg
      real(kind=double), dimension(2,0:NDATA+1,0:NDATA+1) :: xee, xeg, &
          xge, xgg
      real(kind=double), dimension(0:NDATA+1,0:NDATA+1) :: fee, dfec, &
          feg, dfeg, fge, dfge, fgg, dfgg
      real(kind=double) :: pwr

90d  <circe1_fit.f90: public 89e>+≡
      public :: gethst

      Uses gethst 90e.

90e  <circe1_fit.f90: subroutines 89f>+≡
      subroutine gethst (tag, ndata, x, f, df, s, t, pwr)
      character(len=2) :: tag
      integer :: ndata
      real(kind=double) :: s, t, pwr
      real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
      real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
      integer :: i, j
      open (10, file = 'lumidiff-'//tag//'.dat')
      read (10, *) pwr
      s = 0d0
      <Read continuum, summing in s 90f>
      t = s
      <Read single  $\delta$ , summing in t 91a>
      <Read double  $\delta$ , summing in t 91c>
      close (10)
      end subroutine gethst

      Defines:
      gethst, used in chunk 90.

90f  <Read continuum, summing in s 90f>≡
      do i = 1, ndata
      do j = 1, ndata

```

```

        read (10, *) x(1,i,j), x(2,i,j), f(i,j), df(i,j)
        s = s + f(i,j)
    end do
end do

91a <Read single  $\delta$ , summing in t 91a>≡
do i = 1, ndata
    read (10, *) x(1,i,0), f(i,0), df(i,0), &
        f(i,ndata+1), df(i,ndata+1)
    x(1,i,ndata+1) = x(1,i,0)
    t = t + f(i,0) + f(i,ndata+1)
end do

91b <Read single  $\delta$ , summing in t 91a>+≡
do i = 1, ndata
    read (10, *) x(2,0,i), f(0,i), df(0,i), &
        f(ndata+1,i), df(ndata+1,i)
    x(2,ndata+1,i) = x(2,0,i)
    t = t + f(0,i) + f(ndata+1,i)
end do

91c <Read double  $\delta$ , summing in t 91c>≡
read (10, *) f(0,0), df(0,0), f(0,ndata+1), df(0,ndata+1)
t = t + f(0,0) + f(0,ndata+1)
read (10, *) f(ndata+1,0), df(ndata+1,0), &
    f(ndata+1,ndata+1), df(ndata+1,ndata+1)
t = t + f(ndata+1,0) + f(ndata+1,ndata+1)

Guinea-Pig does not provide the full error. A Monte Carlo study shows that it
is a reasonable approximation to rescale the bin error by suitable factors. These
factors are different for each distribution and the factors for the  $\delta$ -pieces are
bigger than those for the continuum parts. The follows factors are for the slow
parameter set.

91d <Fixup errors 91d>≡
    call fixerr (NDATA, dfec, 20d0, 30d0, 40d0)
    call fixerr (NDATA, dfeg, 15d0, 20d0, 0d0)
    call fixerr (NDATA, dfge, 15d0, 20d0, 0d0)
    call fixerr (NDATA, dfgg, 10d0, 0d0, 0d0)

Uses fixerr 91f.

91e <circe1_fit.f90: public 89e>+≡
    public :: fixerr

Uses fixerr 91f.

91f <circe1_fit.f90: subroutines 89f>+≡
    subroutine fixerr (ndata, df, c, sd, dd)
        integer :: ndata
        real(kind=double) :: c, sd, dd
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: df
        integer :: i, j
        do i = 1, NDATA
            do j = 1, NDATA
                df(i,j) = c * df(i,j)
            end do
        end do
    end subroutine

```

```

        end do
    end do
    do i = 1, NDATA
        df(0,i) = sd * df(0,i)
        df(i,0) = sd * df(i,0)
        df(ndata+1,i) = sd * df(ndata+1,i)
        df(i,ndata+1) = sd * df(i,ndata+1)
    end do
    df(0,0) = dd * df(0,0)
    df(ndata+1,0) = dd * df(ndata+1,0)
    df(0,ndata+1) = dd * df(0,ndata+1)
    df(ndata+1,ndata+1) = dd * df(ndata+1,ndata+1)
end subroutine fixerr

```

Defines:

`fixerr`, used in chunk 91.

The error on the integrated luminosity is obtained from adding the error in channels in quadrature.

92a $\langle \text{Normalize } 92a \rangle \equiv$

```

    dtee = sumsqu (NDATA, dftee)
    dteg = sumsqu (NDATA, dfeg)
    dtge = sumsqu (NDATA, dfge)
    dtgg = sumsqu (NDATA, dfgg)

```

Uses `sumsqu` 92c.

92b $\langle \text{circe1_fit.f90: public } 89e \rangle + \equiv$
`public :: sumsqu`

Uses `sumsqu` 92c.

92c $\langle \text{circe1_fit.f90: subroutines } 89f \rangle + \equiv$

```

function sumsqu (ndata, f)
    integer :: ndata
    real(kind=double) :: sumsqu
    real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f
    integer :: i, j
    real(kind=double) :: s2
    s2 = 0
    do i = 0, NDATA+1
        do j = 0, NDATA+1
            s2 = s2 + f(i,j)*f(i,j)
        end do
    end do
    sumsqu = sqrt (s2)
end function sumsqu

```

Defines:

`sumsqu`, used in chunk 92.

92d $\langle \text{Normalize } 92a \rangle + \equiv$

```

    call scale (NDATA, 1d0/tee, fee)
    call scale (NDATA, 1d0/tee, dftee)

```

```

call scale (NDATA, 1d0/tee, feg)
call scale (NDATA, 1d0/tee, dfeg)
call scale (NDATA, 1d0/tee, fge)
call scale (NDATA, 1d0/tee, dfge)
call scale (NDATA, 1d0/tee, fgg)
call scale (NDATA, 1d0/tee, dfgg)

```

Uses scale 93b.

```

93a <circe1_fit.f90: public 89e>+≡
    public :: scale

```

Uses scale 93b.

```

93b <circe1_fit.f90: subroutines 89f>+≡
    subroutine scale (ndata, s, f)
        integer :: ndata
        real(kind=double) :: s
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f
        integer :: i, j
        do i = 0, NDATA+1
            do j = 0, NDATA+1
                f(i,j) = s * f(i,j)
            end do
        end do
    end subroutine scale

```

Defines:

scale, used in chunks 92d and 93a.

```

93c <Calculate  $\nabla f$  93c>≡
    print *, "ERROR:  $\nabla f$  n.a."
    stop

```

Log-likelihood won't fly, because we can't normalize the likelihood function for an unbounded parameter range. Let's use good ole least-squares instead.

```

93d <Calculate  $f(v1)$  93d>≡
    f = 0d0
    do i = 1, NDATA
        do j = 1, NDATA
            if (dfee(i,j) .gt. 0d0) then
                f = f + ((phie(xee(1,i,j),a) * phie(xee(2,i,j),a) &
                    - fee(i,j)) / dfee(i,j))**2
            end if
            if (dfeg(i,j) .gt. 0d0) then
                f = f + ((phig(xeg(1,i,j),a) * phig(xeg(2,i,j),a) &
                    - feg(i,j)) / dfeg(i,j))**2
            end if
            if (dfge(i,j) .gt. 0d0) then
                f = f + ((phig(xge(1,i,j),a) * phie(xge(2,i,j),a) &
                    - fge(i,j)) / dfge(i,j))**2
            end if
            if (dfgg(i,j) .gt. 0d0) then
                f = f + ((phig(xgg(1,i,j),a) * phig(xgg(2,i,j),a) &

```

```

- fgg(i,j)) / dfgg(i,j))**2
    end if
  end do
end do
Uses phie 95b and phig 95d.
94a <Local variables for fct (v1) 90c>+≡
    integer :: i, j
    real(kind=double) :: delta
94b <Calculate f (v1) 93d>+≡
    if ((a(2) .le. -1d0) .or. (a(3) .le. -1d0/pwr)) then
        print *, "warning: discarding out-of-range a2/3: ", a(2), a(3)
        <Give up on f 94c>
    else
        delta = 1d0 - exp(a(1)) * beta(a(2)+1d0,a(3)+1d0/pwr) * dble(NDATA) / pwr
        if (delta .lt. 0d0) then
            print *, "warning: delta forced to 0 from ", delta
            delta = 0d0
        end if
    end if
Uses beta 99.
94c <Give up on f 94c>≡
    f = 1d100
94d <Calculate f (v1) 93d>+≡
    do i = 1, NDATA
        if (dfee(ndata+1,i) .gt. 0d0) then
            f = f + ((delta*phie(xee(2,ndata+1,i),a) &
                - fee(ndata+1,i)) / dfee(ndata+1,i))**2
        end if
        if (dfeg(ndata+1,i) .gt. 0d0) then
            f = f + ((delta*phig(xeg(2,ndata+1,i),a) &
                - feg(ndata+1,i)) / dfeg(ndata+1,i))**2
        end if
        if (dfee(i,ndata+1) .gt. 0d0) then
            f = f + ((delta*phie(xee(1,i,ndata+1),a) &
                - fee(i,ndata+1)) / dfee(i,ndata+1))**2
        end if
        if (dfge(i,ndata+1) .gt. 0d0) then
            f = f + ((delta*phig(xge(1,i,ndata+1),a) &
                - fge(i,ndata+1)) / dfge(i,ndata+1))**2
        end if
    end do
Uses phie 95b and phig 95d.
94e <Calculate f (v1) 93d>+≡
    if (dfee(ndata+1,ndata+1) .gt. 0d0) then
        f = f + ((delta*delta &
            - fee(ndata+1,ndata+1)) / dfee(ndata+1,ndata+1))**2
    end if

```

95a \langle circe1_fit.f90: public 89e \rangle + \equiv

public :: phie

Uses phie 95b.

95b \langle circe1_fit.f90: subroutines 89f \rangle + \equiv

function phie (x, a)

real(kind=double) :: x, phie

real(kind=double), dimension(6) :: a

phie = exp (a(1) + a(2)*log(x) + a(3)*log(1d0-x))

end function phie

Defines:

phie, used in chunks 93–95 and 97d.

95c \langle circe1_fit.f90: public 89e \rangle + \equiv

public :: phig

Uses phig 95d.

95d \langle circe1_fit.f90: subroutines 89f \rangle + \equiv

function phig (x, a)

real(kind=double) :: x, phig

real(kind=double), dimension(6) :: a

phig = exp (a(4) + a(5)*log(x) + a(6)*log(1d0-x))

end function phig

Defines:

phig, used in chunks 93–95 and 97d.

95e \langle Write output (v1) 95e \rangle \equiv

a1(1) = exp(a(1)) * dble(NDATA) / pwr

a1(2) = a(2)

a1(3) = a(3) - 1d0 + 1d0/pwr

a1(4) = exp(a(4)) * dble(NDATA) / pwr

a1(5) = a(5) - 1d0 + 1d0/pwr

a1(6) = a(6)

open (10, file = 'Parameters')

write (10, 1000) REV, tee / 1D32

write (10, 1001) REV, &

1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0), &

a1(1), a1(2), a1(3), a1(4), a1(5), a1(6), &

a1(4) * beta(a1(5)+1d0,a1(6)+1d0)

1000 format (' data xa5lum(@ENERGY@,@ACC@,' , I2, ') / ', E12.5, ' /')

1001 format (' data (xa5(i,@ENERGY@,@ACC@,' , I2 ,'),i=0,7) / ', /, &

' \$ ', 4(E12.5,' , '), /, &

' \$ ', 3(E12.5,' , '), E12.5, ' /')

close (10)

Uses beta 99.

95f \langle Local variables for fct (v1) 90c \rangle + \equiv

\langle Declare NPARAM 88b \rangle

real(kind=double), dimension(NPARAM) :: a1

integer, parameter :: REV = 1

Uses NPARAM 88b.

The average elektron energy in the continuum can be calculated analytically:

$$\begin{aligned}\langle E_{e^\pm} \rangle_{\text{cont}} &= E_{\text{beam}} \langle x_{e^\pm} \rangle_{\text{cont}} = E_{\text{beam}} \frac{\int dx x^{a_2} (1-x)^{a_3} x}{B(a_2, a_3)} \\ &= E_{\text{beam}} \frac{B(a_2+1, a_3)}{B(a_2, a_3)} = E_{\text{beam}} \frac{a_2+1}{a_2+a_3+2} \quad (40)\end{aligned}$$

96a *<Write output (v1) 95e>+≡*
`delta = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)`
`print *, '< x_e > = ', delta + (1d0-delta)*(a1(2)+1d0)/(a1(2)+a1(3)+2d0)`
 Uses beta 99.
 similarly:

$$\langle E_\gamma \rangle = E_{\text{beam}} \frac{a_5+1}{a_5+a_6+2} \quad (41)$$

96b *<Write output (v1) 95e>+≡*
`print *, '< x_g > = ', (a1(5)+1d0)/(a1(5)+a1(6)+2d0)`

Count the degrees of freedom in ndof:

96c *<Write output (v1) 95e>+≡*
`ndof = 0`
`do i = 0, ndata+1`
`do j = 0, ndata+1`
`if (dfec(i,j) .gt. 0d0) ndof = ndof + 1`
`if (dfeg(i,j) .gt. 0d0) ndof = ndof + 1`
`if (dfge(i,j) .gt. 0d0) ndof = ndof + 1`
`if (dfgg(i,j) .gt. 0d0) ndof = ndof + 1`
`end do`
`end do`
`print *, 'CHI2 = ', f / ndof`

96d *<Local variables for fct (v1) 90c>+≡*
`integer :: ndof`

The error on the luminosity is just the (possibly rescaled) counting error:

96e *<Write output (v1) 95e>+≡*
`open (10, file = 'Errors.tex')`
`write (10, 1099) tee / 1d32, dtee / 1d32, dtee / 1d32`
`1099 format ('$ ', F8.2, ' _{-}', F4.2, ' ^{+}', F4.2, ' }$')`

After retrieving the error from MINUIT, we have to take care of the mapping of the parameters

$$a'_{1/4} = e^{a_{1/4}} B(a_{2/5} + 1, a_{3/6} + 1) N_{\text{bins}} \eta^{-1} \implies \delta a'_{1/4} = a'_{1/4} \delta a_{1/4} \quad (42)$$

ignoring the errors in the integral (i.e. the Beta function).

96f *<Write output (v1) 95e>+≡*
`call mnerrs (1, eplus, eminus, epara, corr)`
`ab = a1(1) * beta(a1(2)+1d0,a1(3)+1d0)`
`write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)`
`1100 format ('$ ', F8.4, ' _{-}', F6.4, ' ^{+}', F6.4, ' }$')`
 Uses beta 99.

97a \langle Local variables for fct (v1) 90c $\rangle + \equiv$
`real(kind=double) :: ab`

The other mappings are even more trivial:

$$a'_{2/6} = a_{2/6} - 1 + \eta^{-1} \implies \delta a'_{2/6} = \delta a_{2/6} \quad a'_{3/5} = a_{3/5} - 1 + \eta^{-1} \implies \delta a'_{3/5} = \delta a_{3/5} \quad (43)$$

97b \langle Write output (v1) 95e $\rangle + \equiv$
`do i = 2, 3`
`call mnerrs (i, eplus, eminus, epara, corr)`
`write (10, 1100) a1(i), abs (eminus), abs (eplus)`
`end do`
`call mnerrs (4, eplus, eminus, epara, corr)`
`ab = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)`
`write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)`
`do i = 5, 6`
`call mnerrs (i, eplus, eminus, epara, corr)`
`write (10, 1100) a1(i), abs (eminus), abs (eplus)`
`end do`
`close (10)`

Uses beta 99.

97c \langle Local variables for fct (v1) 90c $\rangle + \equiv$
`real(kind=double) :: eplus, eminus, epara, corr`
`integer :: n`

97d \langle Write output (v1) 95e $\rangle + \equiv$
`do n = 1, 10`
`call pslice ('ee','x',n,NDATA,xee,fee,dfee,phie,phie,a)`
`call pslice ('eg','x',n,NDATA,xeg,feg,dfeg,phie,phig,a)`
`call pslice ('ge','x',n,NDATA,xge,fge,dfge,phig,phie,a)`
`call pslice ('gg','x',n,NDATA,xgg,fgg,dfgg,phig,phig,a)`
`call pslice ('ee','y',n,NDATA,xee,fee,dfee,phie,phie,a)`
`call pslice ('eg','y',n,NDATA,xeg,feg,dfeg,phie,phig,a)`
`call pslice ('ge','y',n,NDATA,xge,fge,dfge,phig,phie,a)`
`call pslice ('gg','y',n,NDATA,xgg,fgg,dfgg,phig,phig,a)`
`end do`
`call pslice ('ee','x',21,NDATA,xee,fee,dfee,phie,phie,a)`
`call pslice ('eg','x',21,NDATA,xeg,feg,dfeg,phie,phig,a)`
`call pslice ('ee','y',21,NDATA,xee,fee,dfee,phie,phie,a)`
`call pslice ('ge','y',21,NDATA,xge,fge,dfge,phig,phie,a)`

Uses phie 95b, phig 95d, and pslice 98b.

UNIX Fortran compiler want backslashes escaped:

97e \langle Write output (v1) 95e $\rangle + \equiv$
`open (10, file = 'Slices.mp4')`
`write (10,*) "picture eslice[], gslice[];"`
`do n = 1, NDATA`
`write (10,*) 'eslice[' , n, ']' := ', &`
`'btex $x_{e^{\backslash\backslash pm}} = ', xee(1,n,1), '$ etex;'`
`write (10,*) 'gslice[' , n, ']' := ', &`

```

        'btex $x_\\gamma = ', xgg(1,n,1), '$ etex;'
    end do
    close (10)
98a  (circe1_fit.f90: public 89e)+≡
    public :: pslice
    Uses pslice 98b.
98b  (circe1_fit.f90: subroutines 89f)+≡
    subroutine pslice (pp, xy, n, ndata, x, f, df, phi1, phi2, a)
        character(len=2) :: pp
        character(len=1) :: xy
        integer :: n, ndata
        real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
        real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
        real(kind=double), dimension(6) :: a
        real(kind=double) :: z
        real(kind=double) :: phi1, phi2, d, delta, pwr
        external phi1, phi2
        integer :: i
        character(len=2) digits
        write (digits, '(I2.2)') n
        open (10, file = 'lumidiff-//pp//xy//digits//'.dat')
        open (11, file = 'lumidiff-//pp//xy//digits//'.fit')
        open (12, file = 'lumidiff-//pp//xy//digits//'.chi')
        if (n .eq. ndata+1) then
            pwr = 5d0
            delta = 1d0 - exp(a(1))*beta(a(2)+1d0,a(3)+1d0/pwr) &
                * dble(NDATA) / pwr
        else
            delta = 0
        end if
        if (xy .eq. 'x') then
            do i = 1, ndata
                if (df(n,i) .gt. 0d0) then
                    if (pp(2:2) .eq. 'g') then
                        z = x(2,n,i)
                    else
                        z = 1d0 - x(2,n,i)
                    endif
                endif
                if (n .eq. ndata+1) then
                    d = delta*phi2(x(2,n,i),a)
                else
                    d = phi1(x(1,n,i),a)*phi2(x(2,n,i),a)
                endif
                write (10,*) z, f(n,i), df(n,i)
                write (11,*) z, d
                write (12,*) z, (f(n,i) - d) / df(n,i)
            endif
        end do
        else if (xy .eq. 'y') then

```

```

do i = 1, ndata
  if (df(i,n) .gt. 0d0) then
    if (pp(1:1) .eq. 'g') then
      z = x(1,i,n)
    else
      z = 1d0 - x(1,i,n)
    endif
    if (n .eq. ndata+1) then
      d = phi1(x(1,i,n),a)*delta
    else
      d = phi1(x(1,i,n),a)*phi2(x(2,i,n),a)
    endif
    write (10,*) z, f(i,n), df(i,n)
    write (11,*) z, d
    write (12,*) z, (f(i,n) - d) / df(i,n)
  endif
end do
endif
close (10)
close (11)
close (12)
end subroutine pslice

```

Defines:

pslice, used in chunks 97d and 98a.

Uses beta 99.

```

99 <circe1_fit.f90: subroutines 89f>+≡
  function beta (a, b)
    real(kind=double) :: a, b, beta
    beta = exp (dlgamma(a) + dlgamma(b) - dlgamma(a+b))
  contains
    function dlgamma (x)
      real(kind=double) :: dlgamma
      real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
      real(kind=double), dimension(5) :: c, xl
      real(kind=double) :: x, y, zero, one, two, half, ap, aq
      integer :: i
      data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
      data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
      data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
        +5.5584045723515D+1, -2.1513513573726D+2, &
        -2.4587261722292D+2, -5.7500893603041D+1, &
        -2.3359098949513D+0/
      data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
        +1.9387784034377D+2, +3.0882954973424D+2, &
        +1.5006839064891D+2, +2.0106851344334D+1, &
        +4.5717420282503D-1/
      data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
        +2.1797366058896D+3, +3.7975124011525D+3, &
        -1.9778070769842D+3, -3.6929834005591D+3, &

```

```

-5.6017773537804D+2/
data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
+1.5612045277929D+3, +7.2340087928948D+3, &
+1.0459576594059D+4, +4.1699415153200D+3, &
+2.7678583623804D+2/
data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
-4.7504594653440D+6, -2.9423445930322D+6, &
+3.6321804931543D+7, -3.3567782814546D+6, &
-2.4804369488286D+7/
data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
-1.5552890280854D+5, -3.4152517108011D+6, &
-2.0969623255804D+7, -3.4544175093344D+7, &
-9.1605582863713D+6/
data c / 1.1224921356561D-1, 7.9591692961204D-2, &
-1.7087794611020D-3, 9.1893853320467D-1, &
1.3469905627879D+0/
if (x .le. xl(1)) then
    print *, 'ERROR: DLGAMA non positive argument: ', X
    dlgama = zero
end if
if (x .le. xl(2)) then
    y = x + one
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
        ap = p1(i) + y * ap
        aq = q1(i) + y * aq
    end do
    y = - log(x) + x * ap / aq
else if (x .le. xl(3)) then
    ap = p1(1)
    aq = q1(1)
    do i = 2, 7
        ap = p1(i) + x * ap
        aq = q1(i) + x * aq
    end do
    y = (x - one) * ap / aq
else if (x .le. xl(4)) then
    ap = p2(1)
    aq = q2(1)
    do i = 2, 7
        ap = p2(i) + x * ap
        aq = q2(i) + x * aq
    end do
    y = (x-two) * ap / aq
else if (x .le. xl(5)) then
    ap = p3(1)
    aq = q3(1)
    do i = 2, 7
        ap = p3(i) + x * ap

```

```

        aq = q3(i) + x * aq
    end do
    y = ap / aq
else
    y = one / x**2
    y = (x-half) * log(x) - x + c(4) + &
        (c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
end if
    dlgamma = y
end function dlgamma
end function beta

```

Defines:

beta, used in chunks 58b, 61b, 65b, 68a, 71c, 94-98, and 102c.

```

101a <circe1_fit.sh 101a>≡
    #! /bin/sh
    # mode=${2-slow}
    mode=${2-fast}
    root='pwd'
    indir=${root}/${3-input}
    tmpdir=${root}/tmp
    outdir=${root}/output
    acc="$1-sband350 sband500 sband800 sband1000 sband1600
        tesla350 tesla500 tesla800 tesla1000 tesla1600
        tesla350-low tesla500-low tesla800-low tesla1000-low tesla1600-low
        xband350 xband500 xband800 xband1000 xband1600}"

101b <circe1_fit.sh 101a>+≡
    xmkdir () {
        for d in "$@"; do
            mkdir $d 2>/dev/null || true
        done
    }
    rm -fr ${tmpdir}
    xmkdir ${outdir} ${tmpdir}

101c <circe1_fit.sh 101a>+≡
    cd ${tmpdir}
    cat /dev/null >${outdir}/Params.f90
    for a in $acc; do
        case "$a" in
            *1600*) energy=TEV16;;
            *1000*) energy=TEV1;;
            *800*) energy=GEV800;;
            *500*) energy=GEV500;;
            *3[56]0*) energy=GEV350;;
            *170*) energy=GEV170;;
            *90*) energy=GEV090;;
            *) energy=GEV500;;
        esac
        cp ${indir}/${a}_${mode}/lumidiff-?? .dat .
    done

```

```

    ${root}/circe1_fit.bin
    rm -fr ${outdir}/${a}_${mode}
    mkdir ${outdir}/${a}_${mode}
    cp Slices.mp4 ${outdir}
    cp Errors.tex lumidiff-??x[0-9][0-9].??? ${outdir}/${a}_${mode}
    sed -e "s/@ENERGY@/$energy/g" \
        -e "s/@ACC@/'echo $a | tr a-z A-Z | tr -cd A-Z'/g" Parameters \
        >>${outdir}/Params.f90
done
cd ${root}
rm -fr ${tmpdir}
102a <circe1_fit.sh 101a>+≡
    cat >${outdir}/Params.tex <<'END'
    \begin{table}
    \begin{center}
    \renewcommand{\arraystretch}{1.3}
    \begin{tabular}{|c|c|c|c|c|}\hline
        & \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND} \\
        \hline\hline
    END
    Uses SBAND 13, TESLA 13, and XBAND 13.
102b <circe1_fit.sh 101a>+≡
    line () {
        for a in $acc; do
            case $a in
                *350* | *800* | *1000* | *1600*)
                    ;;
                *) echo -n ' & '
                    sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                    ;;
            esac
        done
        echo '\\hline'
    }
    (echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
    echo '$\int d_{\text{e}}^{\text{pm}}$'; line 2
    echo '$x_{\text{e}}^{\text{pm}}{}^{\alpha}$'; line 3
    echo '$(1-x_{\text{e}}^{\text{pm}}) {}^{\alpha}$'; line 4
    echo '$\int d_{\text{gamma}}$'; line 5
    echo '$x_{\text{gamma}}^{\alpha}$'; line 6
    echo '$(1-x_{\text{gamma}}) {}^{\alpha}$'; line 7
    ) >>${outdir}/Params.tex
102c <circe1_fit.sh 101a>+≡
    cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
    \end{center}
    \caption{\label{tab:param}%
        Version 1, revision 1997 04 16 of the beam spectra at 500 GeV.

```

The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions~(\ref{eq:beta}).}

\end{table}

END

Uses beta 99.

```
103a <circe1_fit.sh 101a>+≡
cat >>${outdir}/Params.tex <<'END'
\begin{table}
\begin{center}
\renewcommand{\arraystretch}{1.3}
\begin{tabular}{|c||c|c|c|c|}\hline
& \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND} \\
\\ \hline \hline
```

END

Uses SBAND 13, TESLA 13, and XBAND 13.

```
103b <circe1_fit.sh 101a>+≡
line () {
for a in $acc; do
case $a in
*1000*)
echo -n ' & '
sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
;;
esac
done
echo '\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
echo '$\int d_{e^{\pm}}$'; line 2
echo '$x_{e^{\pm}}^{\alpha}$'; line 3
echo '$(1-x_{e^{\pm}})^{\alpha}$'; line 4
echo '$\int d_{\gamma}$'; line 5
echo '$x_{\gamma}^{\alpha}$'; line 6
echo '$(1-x_{\gamma})^{\alpha}$'; line 7
) >>${outdir}/Params.tex
```

```
103c <circe1_fit.sh 101a>+≡
cat >>${outdir}/Params.tex <<'END'
\end{tabular}
\end{center}
\caption{\label{tab:param/TeV}%
Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.}
\end{table}
END
```

```
103d <circe1_fit.sh 101a>+≡
cat >>${outdir}/Params.tex <<'END'
\begin{table}
\begin{center}
```

```

\renewcommand{\arraystretch}{1.3}
\begin{tabular}{|c||c|c|c|c|}\hline
& 350 GeV & 500 GeV & 800 GeV & 1600 GeV \\
\\ \hline \hline
END
104a <circe1_fit.sh 101a>+≡
line () {
for a in $acc; do
case $a in
tesla*-low)
;;
tesla1000)
;;
tesla*)
echo -n ' & '
sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
;;
esac
done
echo '\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\epsilon^{-1}$'; line 1
echo '$\int d_{e^{\pm}}$'; line 2
echo '$x_{e^{\pm}}^{\alpha}$'; line 3
echo '$(1-x_{e^{\pm}})^{\alpha}$'; line 4
echo '$\int d_{\gamma}$'; line 5
echo '$x_{\gamma}^{\alpha}$'; line 6
echo '$(1-x_{\gamma})^{\alpha}$'; line 7
) >>${outdir}/Params.tex
104b <circe1_fit.sh 101a>+≡
cat >>${outdir}/Params.tex <<'END'
\end{tabular}
\end{center}
\caption{\label{tab:param/Tesla}%
Version 1, revision 1997 04 17 of the beam spectra for TESLA.}
\end{table}
END
exit 0
Uses TESLA 13.

```

7.2 Experimental

7.2.1 Quasi One Dimensional

```

104c <circe1_minuit1.f90 104c>≡
! circe1_minuit1.f90 -- fitting for circe
<Copyleft notice 30a>
Uses circe 31b.

```


We’re utilizing the familiar “MINUIT” package [15].

105a $\langle \text{circe1_minuit1.f90 } 104c \rangle \equiv$
 $\langle \text{Minuit1 module } 105b \rangle$
 $\langle \text{Minuit1 main program } 105c \rangle$

105b $\langle \text{Minuit1 module } 105b \rangle \equiv$

```

module minuit1
  use kinds

  implicit none

  public :: fct
  public :: phi

contains

   $\langle \text{Function to minimize } 105e \rangle$ 

   $\langle \text{Function phi1 } 107 \rangle$ 
end module minuit1

```

Defines:

minuit1, used in chunk 105c.

Uses fct 89f 105e and phi 107 110d.

105c $\langle \text{Minuit1 main program } 105c \rangle \equiv$

```

program fit
  use kinds
  use minuit1

```

implicit none

call minuit (fct, 0d0)

end program fit

Uses fct 89f 105e, fit 88a, and minuit1 105b.

105d $\langle \text{Minuit2 main program } 105d \rangle \equiv$

```

program fit
  use kinds
  use minuit2

```

implicit none

call minuit (fct, 0d0)

end program fit

Uses fct 89f 105e, fit 88a, and minuit2 110c.

105e $\langle \text{Function to minimize } 105e \rangle \equiv$

```

subroutine fct (nx, df, f, a, mode, g)
  integer, intent(in) :: nx, mode
  real(kind=double) :: f, g
  real(kind=double), dimension(:) :: df, a

```

```

    <Local variables for fct 106b>
    if (mode .eq. 1) then
        <Read input data 106a>
    else if (mode .eq. 2) then
        <Calculate  $\nabla f$  93c>
    end if
    <Calculate f 106c>
    if (mode .eq. 3) then
        <Write output 106d>
    end if
end subroutine fct

```

Defines:

fct, used in chunks 88a, 89e, 105, and 110c.

```

106a <Read input data 106a>≡
    open (10, file = 'minuit.data')
    do i = 1, NDATA
        do j = 1, NDATA
            read (10, *) xi(1,i,j), xi(2,i,j), fi(i,j), dfi(i,j)
            fi(i,j) = fi(i,j)/1d30
            dfi(i,j) = dfi(i,j)/1d30
        end do
    end do
    close (10)

```

```

106b <Local variables for fct 106b>≡
    integer, parameter :: NDATA = 20
    real(kind=double) :: chi, chi2
    real(kind=double), dimension(2,NDATA,NDATA) :: xi
    real(kind=double), dimension(NDATA,NDATA) :: fi, dfi
    integer :: i, j, n

```

```

106c <Calculate f 106c>≡
    f = 0d0
    do i = 1, NDATA
        do j = 1, NDATA
            if (dfi(i,j).gt.0d0) then
                f = f + ((phi(xi(1,i,j),xi(2,i,j),a) &
                    - fi(i,j)) / dfi(i,j))**2
            end if
        end do
    end do

```

Uses phi 107 110d.

```

106d <Write output 106d>≡
    chi2 = 0d0
    n = 0
    open (10, file = 'minuit.fit')
    do i = 1, NDATA
        do j = 1, NDATA
            if (dfi(i,j).gt.0d0) then

```

```

        chi = (phi(xi(1,i,j),xi(2,i,j),a)-fi(i,j))/dfi(i,j)
        write (10,*) xi(1,i,j), xi(2,i,j), &
                    1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
                    1d30 * fi(i,j), &
                    chi
        chi2 = chi2 + chi**2
        n = n + 1
    else
        write (10,*) xi(1,i,j), xi(2,i,j), &
                    1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
                    1d30 * fi(i,j)

    end if
end do
close (10)
print *, 'CHI2 = ', chi2/n

```

Uses phi 107 110d.

107 *<Function phi1 107>*≡

```

function phi (e1, e2, a)
    real(kind=double) :: e1, e2
    real(kind=double), dimension(17) :: a
    real(kind=double) :: phi
    real(kind=double) :: y1, y2
    y1 = e1 / 250d0
    y2 = e2 / 250d0
    phi = exp (
        + a( 1) * 1d0
        + a( 2) * log(y1)
        + a( 3) * log(1d0-y1)
        + a( 4) * log(-log(y1))
        + a( 5) * log(-log(1d0-y1))
        + a( 6) * y1
        + a( 7) * log(y1)**2
        + a( 8) * log(1d0-y1)**2
        + a( 9) * log(-log(y1))**2
        + a(10) * log(-log(1d0-y1))**2
        + a(11) * y1**2
        + a(12) / log(y1)
        + a(13) / log(1d0-y1)
        + a(14) / log(-log(y1))
        + a(15) / log(-log(1d0-y1))
        + a(16) / y1
        + a(17) / (1d0-y1)
        + a( 2) * log(y2)
        + a( 3) * log(1d0-y2)
        + a( 4) * log(-log(y2))
        + a( 5) * log(-log(1d0-y2))
        + a( 6) * y2
        + a( 7) * log(y2)**2
    )

```

```

+ a( 8) * log(1d0-y2)**2      &
+ a( 9) * log(-log(y2))**2    &
+ a(10) * log(-log(1d0-y2))**2 &
+ a(11) * y2**2               &
+ a(12) / log(y2)             &
+ a(13) / log(1d0-y2)         &
+ a(14) / log(-log(y2))       &
+ a(15) / log(-log(1d0-y2))   &
+ a(16) / y2                  &
+ a(17) / (1d0-y2)           &
)
end function phi

```

Defines:

phi, used in chunks 105, 106, and 110c.

```

108a <circe1_minuit1.sh 108a>≡
    #! /bin/sh
    minuit_bin='pwd'/circe1_minuit1.bin
    <Process arguments 108b>
    (
        <Define parameters 108e>
        <Fix parameters 109a>
        <Fix strategy 109b>
        <Run Minuit 109c>
    ) | eval "$minuit_bin $filter"
    <Maybe plot results 109d>
    exit 0

108b <Process arguments 108b>≡
    tmp="$IFS"
    IFS=:
    args=":$*:"
    IFS="$tmp"

108c <Process arguments 108b>+≡
    filter="| \
    awk '/STATUS=(CONVERGED|CALL LIMIT|FAILED)/ { p=1; print }; \
    /@.* \.00000 *fixed/ { next }; \
    /EDM=|CHI2|@/ && p { print }' "

108d <Process arguments 108b>+≡
    case "$args" in
        *:v:*) filter=;;
    esac

108e <Define parameters 108e>≡
    cat <<END
    set title
    CIRCE
    parameters
    1 '@ 1          ' 0.00 0.01

```

```

2  '@ lx          ' 0.20 0.01
3  '@ l(1-x)     ' 0.20 0.01
4  '@ llx        ' 0.00 0.01
5  '@ ll(1-x)    ' 0.00 0.01
6  '@ x          ' 0.00 0.01
7  '@ lx^2       ' 0.00 0.01
8  '@ l(1-x)^2   ' 0.00 0.01
9  '@ llx^2      ' 0.00 0.01
10 '@ ll(1-x)^2  ' 0.00 0.01
11 '@ x^2        ' 0.00 0.01
12 '@ 1/lx       ' 0.00 0.01
13 '@ 1/l(1-x)   ' 0.00 0.01
14 '@ 1/llx      ' 0.00 0.01
15 '@ 1/ll(1-x)  ' 0.00 0.01
16 '@ 1/x        ' 0.00 0.01
17 '@ 1/(1-x)    ' 0.00 0.01

```

END

```

109a <Fix parameters 109a>≡
for p in 1 2 3 4 5 6 7 8 9 10 \
      11 12 13 14 15 16 17; do
  case "$args" in
    *:$p=*) val='echo "$args" | sed 's/.*:.'"$p"'=\([0-9.-]*\)::*/\1/'';
             echo set parameter $p $val;
             echo fix $p;;
    *:$p:*) ;;
    *) echo fix $p;;
  esac
done

```

```

109b <Fix strategy 109b>≡
case "$args" in
  *:S0:*) echo set strategy 0;;
  *:S1:*) echo set strategy 1;;
  *:S2:*) echo set strategy 2;;
esac

```

```

109c <Run Minuit 109c>≡
cat <<END
migrat 10000 0.01
stop
END

```

```

109d <Maybe plot results 109d>≡
case "$args" in
  *:p:*) awk '$5 != "" { print $1, $2, $5 }' minuit.fit > chi2
          awk '$5 != "" { print $1, $5 }' minuit.fit > chix
          awk '$5 != "" { print $2, $5 }' minuit.fit > chiy
          gnuplot -geometry -0+0 plot2 >/dev/null 2>&1
esac

```

Uses fit 88a.

7.2.2 Quasi Two Dimensional

```

110a <circe1_minuit2.f90 110a>≡
      ! minuit2.f90 -- fitting for circe
      <Copyleft notice 30a>
      Uses circe 31b and minuit2 110c.

110b <circe1_minuit2.f90 110a>+≡
      <Minuit2 module 110c>
      <Minuit2 main program 105d>

110c <Minuit2 module 110c>≡
      module minuit2
      use kinds

      implicit none

      public :: fct
      public :: phi

      contains

      <Function to minimize 105e>
      <Function phi2 110d>
      end module minuit2

Defines:
      minuit2, used in chunks 105d and 110a.
      Uses fct 89f 105e and phi 107 110d.

110d <Function phi2 110d>≡
      function phi (e1, e2, a)
      real(kind=double) :: e1, e2
      real(kind=double), dimension(33) :: a
      real(kind=double) :: phi
      real(kind=double) :: y1, y2
      y1 = e1 / 250d0
      y2 = e2 / 250d0
      phi = exp (
          + a( 1) * 1d0
          + a( 2) * log(y1)
          + a( 3) * log(1d0-y1)
          + a( 4) * log(-log(y1))
          + a( 5) * log(-log(1d0-y1))
          + a( 6) * y1
          + a( 7) * log(y1)**2
          + a( 8) * log(1d0-y1)**2
          + a( 9) * log(-log(y1))**2
          + a(10) * log(-log(1d0-y1))**2
          + a(11) * y1**2
          + a(12) / log(y1)
          + a(13) / log(1d0-y1)
      )

```

```

+ a(14) / log(-log(y1))      &
+ a(15) / log(-log(1d0-y1))  &
+ a(16) / y1                  &
+ a(17) / (1d0-y1)           &
+ a(18) * log(y2)             &
+ a(19) * log(1d0-y2)         &
+ a(20) * log(-log(y2))       &
+ a(21) * log(-log(1d0-y2))   &
+ a(22) * y2                   &
+ a(23) * log(y2)**2          &
+ a(24) * log(1d0-y2)**2      &
+ a(25) * log(-log(y2))**2    &
+ a(26) * log(-log(1d0-y2))**2 &
+ a(27) * y2**2               &
+ a(28) / log(y2)             &
+ a(29) / log(1d0-y2)         &
+ a(30) / log(-log(y2))       &
+ a(31) / log(-log(1d0-y2))   &
+ a(32) / y2                   &
+ a(33) / (1d0-y2)           &
)
end function phi

```

Defines:

phi, used in chunks 105, 106, and 110c.

```

111a <circe1_minuit2.sh 111a>≡
#! /bin/sh
minuit_bin='pwd'/circe1_minuit2.bin
<Process arguments 108b>
(
  <Define parameters (2dim) 111b>
  <Fix parameters (2dim) 112>
  <Fix strategy 109b>
  <Run Minuit 109c>
) | eval "$minuit_bin $filter"
<Maybe plot results 109d>
exit 0

```

```

111b <Define parameters (2dim) 111b>≡
cat <<END
set title
CIRCE
parameters
1  '@ 1      ' 0.00 0.01
2  '@ 1x     ' 0.20 0.01
3  '@ 1(1-x) ' 0.20 0.01
4  '@ 11x    ' 0.00 0.01
5  '@ 11(1-x) ' 0.00 0.01
6  '@ x      ' 0.00 0.01
7  '@ 1x^2   ' 0.00 0.01

```

```

8 '@ 1(1-x)^2 ' 0.00 0.01
9 '@ llx^2 ' 0.00 0.01
10 '@ ll(1-x)^2' 0.00 0.01
11 '@ x^2 ' 0.00 0.01
12 '@ 1/lx ' 0.00 0.01
13 '@ 1/l(1-x) ' 0.00 0.01
14 '@ 1/llx ' 0.00 0.01
15 '@ 1/ll(1-x)' 0.00 0.01
16 '@ 1/x ' 0.00 0.01
17 '@ 1/(1-x) ' 0.00 0.01
18 '@ ly ' 0.20 0.01
19 '@ l(1-y) ' 0.20 0.01
20 '@ lly ' 0.00 0.01
21 '@ ll(1-y) ' 0.00 0.01
22 '@ y ' 0.00 0.01
23 '@ ly^2 ' 0.00 0.01
24 '@ l(1-y)^2 ' 0.00 0.01
25 '@ lly^2 ' 0.00 0.01
26 '@ ll(1-y)^2' 0.00 0.01
27 '@ y^2 ' 0.00 0.01
28 '@ 1/ly ' 0.00 0.01
29 '@ 1/l(1-y) ' 0.00 0.01
30 '@ 1/lly ' 0.00 0.01
31 '@ 1/ll(1-y)' 0.00 0.01
32 '@ 1/y ' 0.00 0.01
33 '@ 1/(1-y) ' 0.00 0.01

```

END

112 $\langle \text{Fix parameters (2dim)} \rangle \equiv$

```

for p in 1 2 3 4 5 6 7 8 9 10 \
      11 12 13 14 15 16 17 18 19 20 \
      21 22 23 24 25 26 27 28 29 30 \
      31 32 33; do
  case "$args" in
    *:$p=*) val='echo "$args" | sed 's/.*:.'"$p"'=\\([0-9.-]*\\):.*\\/1/'';
             echo set parameter $p $val;
             echo fix $p;;
    *:$p:*) ;;
    *) echo fix $p;;
  esac
done

```

7.3 Version 2

8 Conclusions

I have presented a library of simple parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. The library can be used for integration and event generation. Emphasis is put on simplicity and reproducibility of the parameterizations for supporting reproducible physics simulations.

Acknowledgements

Daniel Schulte made his simulation code **Guinea-Pig** available and answered questions. Harald Anlauf and Torbjörn Sjöstrand have contributed useful suggestions. The Tesla group at DESY/Zeuthen made error estimates feasible by donating time on the multi-headed number cruncher **Hydra**. The 1996 ECFA/Desy Linear Collider Workshop got me started and provided support. Thanks to all of them.

Identifiers

C1_ELECTRON: [11b](#), [21d](#), [31b](#), [70b](#), [75b](#), [75c](#)
C1_PHOTON: [11b](#), [31b](#), [70b](#), [75b](#), [75c](#), [82](#)
C1_POSITRON: [11b](#), [22](#), [75c](#)
JLCNLC: [13](#), [17](#), [18](#), [35b](#), [56b](#), [59a](#), [60a](#), [60c](#), [61e](#), [62b](#), [62d](#), [65e](#), [67a](#), [86b](#)
NACC: [13](#), [17](#), [34b](#), [35a](#), [39e](#), [40b](#), [40c](#), [40d](#), [44b](#), [44c](#), [45b](#), [45c](#), [46b](#), [46c](#), [51b](#),
[53c](#), [55c](#), [58c](#), [61c](#), [65c](#)
NPARAM: [88b](#), [89a](#), [89c](#), [95f](#)
SBAND: [13](#), [35b](#), [39f](#), [40a](#), [43b](#), [44a](#), [44d](#), [45a](#), [45d](#), [46a](#), [46d](#), [49a](#), [49b](#), [86b](#),
[102a](#), [103a](#)
SBNDEE: [13](#), [34b](#), [35b](#), [40c](#), [44b](#), [45b](#), [46b](#), [48a](#), [48b](#), [86b](#)
TESLA: [13](#), [32e](#), [32h](#), [35b](#), [39f](#), [40a](#), [43b](#), [44d](#), [45d](#), [46d](#), [49a](#), [50a](#), [51c](#), [51d](#), [51e](#),
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TESLEE: [13](#), [34b](#), [35b](#), [48a](#), [86b](#)
XBAND: [13](#), [39f](#), [40a](#), [43b](#), [44a](#), [44d](#), [45a](#), [45d](#), [46a](#), [46d](#), [49a](#), [49b](#), [56e](#), [102a](#),
[103a](#)
XBNDEE: [13](#), [34b](#), [35b](#), [48a](#), [48b](#), [86a](#), [86b](#)
beta: [58b](#), [61b](#), [65b](#), [68a](#), [71c](#), [94b](#), [95e](#), [96a](#), [96f](#), [97b](#), [98b](#), [99](#), [102c](#)
circe: [11a](#), [31b](#), [31b](#), [31b](#), [31b](#), [30b](#), [31a](#), [31b](#), [32h](#), [82](#), [88a](#), [104c](#), [110a](#)
circee: [14a](#), [15b](#), [15d](#), [15f](#), [16b](#), [31b](#), [40g](#), [41a](#), [41b](#)
circeg: [14a](#), [31b](#), [41c](#), [42a](#), [42b](#)
circel: [40f](#), [12a](#), [40e](#), [40f](#)
circem: [32h](#), [33a](#), [33c](#), [33d](#), [34a](#), [34b](#), [35c](#), [35d](#), [35e](#), [36c](#), [37a](#), [37b](#), [37d](#), [37f](#),
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[62c](#), [62d](#), [63](#), [64](#), [79a](#), [81b](#), [81c](#)
circes: [32a](#), [12b](#), [17](#), [32a](#), [21b](#), [31c](#), [32a](#), [32a](#), [32g](#), [82](#), [86a](#)
circgg: [14a](#), [31b](#), [42c](#), [42d](#), [43a](#), [75c](#)
d1: [15b](#), [15f](#), [15e](#), [15f](#), [41a](#), [41b](#), [42a](#), [42b](#), [42d](#), [43a](#), [70d](#), [71e](#), [72a](#), [72b](#), [73a](#),
[73c](#)
d12: [15b](#), [15d](#), [15c](#), [15d](#)
d2: [15b](#), [16b](#), [16a](#), [16b](#), [41a](#), [41b](#), [42a](#), [42b](#), [42d](#), [43a](#), [70d](#), [71e](#), [72a](#), [72b](#), [73a](#),
[73c](#)

d12a: [16d](#), [16f](#), [16g](#)
fct: [88a](#), [89e](#), [89f](#), [105b](#), [105c](#), [105d](#), [105e](#), [110c](#)
fit: [88a](#), [89c](#), [105c](#), [105d](#), [109d](#)
fixerr: [91d](#), [91e](#), [91f](#)
gauss1: [84b](#), [15b](#), [84b](#), [84a](#), [84b](#), [84b](#)
gauss2: [85b](#), [15b](#), [16d](#), [85b](#), [85a](#), [85b](#)
gaussx: [84d](#), [84c](#), [84d](#), [84d](#), [85b](#)
gethst: [90b](#), [90d](#), [90e](#)
girce: [75b](#), [20a](#), [75b](#), [75a](#), [75b](#)
girceb: [78b](#), [76e](#), [77c](#), [77f](#), [78a](#), [78b](#), [79a](#), [80c](#)
gircee: [20c](#), [20d](#), [76d](#), [21c](#), [75b](#), [76c](#), [76d](#)
girceg: [20c](#), [75b](#), [77a](#), [77b](#)
gircgg: [20c](#), [75b](#), [77d](#), [77e](#), [77f](#)
kirke: [70a](#), [70b](#)
kirkee: [16g](#), [70b](#), [70c](#), [70d](#), [71e](#)
kirkeg: [70b](#), [72a](#), [72c](#), [73a](#)
kirkgg: [70b](#), [72b](#), [73b](#), [73c](#)
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minuit2: [105d](#), [110a](#), [110c](#)
phi: [105b](#), [106c](#), [106d](#), [107](#), [110c](#), [110d](#)
phie: [93d](#), [94d](#), [95a](#), [95b](#), [97d](#)
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sumsqu: [92a](#), [92b](#), [92c](#)

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*<Alternative: Update **circe1** parameters [71c](#)>*
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<Calculate version 1 of the $e^\pm\gamma$ distribution [42b](#)>
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A Literate Programming

A.1 Paradigm

I have presented the sample code in this paper using the *literate programming* paradigm. This paradigm has been introduced by Donald Knuth [19] and his programs T_EX [20] and METAFONT [21] provide excellent examples of the virtues of literate programming. Knuth summarized his intention as follows ([19], p. 99)

“Let us change our traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a *computer* what to do, let us concentrate rather on explaining to *human beings* what we want a computer to do.”

Usually, literate programming uses two utility programs to produce two kinds of files from the source

tangle produces the computer program that is acceptable to an “illiterate” (Fortran, C, etc.) compiler. This process consists of stripping documentation and reordering code. Therefore it frees the author from having to present the code in the particular order enforced by a compiler for purely technical reasons. Instead, the author can present the code in the order that is most comprehensible.

weave produces a documents that describes the program. Extensive cross referencing of the code sections is usually provided, which has been suppressed in this paper. If a powerful typesetting system (such a T_EX) is used, the document can present the algorithms in clear mathematical notation alongside the code. These features improve readability and maintainability of scientific code immensely.

A.2 Practice

Circe uses the **noweb** [22] system. This system has the advantage to work with any traditional programming language and support the essential features described in section A.1 with minimal effort. **noweb**’s **tangle** program only reorders the code sections, but does not reformat them. Therefore its output can be used just like any other “illiterate” program.

The examples above should be almost self-explaining, but in order to avoid any ambiguities, I give another example:

120a \langle Literate programming example 120a $\rangle \equiv$
 \langle Code that has to be at the top 120c \rangle
 \langle Other code 120b \rangle

I can start the presentation with the first line of the “other code”:

120b \langle Other code 120b $\rangle \equiv$
 line 1 of the other code

If appropriate, the first line of the code that has to appear *before* the other code can be presented later:

120c \langle Code that has to be at the top 120c $\rangle \equiv$
 line 1 of the code at the top

Now I can augment the sections:

```
121a <Other code 120b>+≡  
    line 2 of the other code  
121b <Code that has to be at the top 120c>+≡  
    line 2 of the code at the top
```

The complete “program” will be presented to the compiler as

```
line 1 of the code at the top  
line 2 of the code at the top  
line 1 of the other code  
line 2 of the other code
```

The examples in section 3.1.1 show that this reordering is particularly useful for declaring variables when they are first used (rather than at the beginning) and for zooming in on code inside of loops.

B Fortran Name Space

In addition to the ten procedures and one `common` block discussed in section 3

- `circe`, `circee`, `circeg`, `circgg`,
- `girce`, `gircee`, `girceg`, `gircgg`,
- `circes`, `circel`, `/circom/`,

there are two more globally visible functions which are used internally:

- `circem`: error message handler,
- `girceb`: efficient Beta distribution generator.

Even if the `/circom/` is globally visible, application programs *must not* manipulate it directly. The `circes` subroutine is provided for this purpose and updates some internal parameters as well.

With features from the current Fortran standard (Fortran90), I could have kept the last two functions and the `common` block private.

Application programs wishing to remain compatible with future versions of Circe must not use `common` blocks or procedures starting with `circe` or `girce`.

C Updates

Information about updates can be obtained

- on the World Wide Web:

`http:http://projects.hepforge.org/whizard/`

Contributions of results from other simulation programs and updated accelerator designs are welcome at

`ohl@physik.uni-wuerzburg.de`