

VAMP, Version 1.0: Vegas AMPlified: Anisotropy, Multi-channel sampling and Parallelization

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Abstract

We present an new implementation of the classic Vegas algorithm for adaptive multi-dimensional Monte Carlo integration in Fortran95. This implementation improves the performance for a large class of integrands, supporting stratified sampling in higher dimensions through automatic identification of the directions of largest variation. This implementation also supports multi channel sampling with individual adaptive grids. Sampling can be performed in parallel on workstation clusters and other parallel hardware. Note that for maintenance of the code, and especially its usage within the event generator WHIZARD, some features of Fortran2003 have been added.

Revision Control

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Program Summary:

- **Title of program:** VAMP, Version 1.0 (October 1999)
- **Program obtainable** by anonymous `ftp` from the host `crunch.ikp.physik.th-darmstadt.de` in the directory `pub/ohl/vamp`.
- **Licensing provisions:** Free software under the GNU General Public License.
- **Programming language used:** From version 2.2.0 of the program: Fortran2003 [8] Until version 2.1.x of the program: Fortran95 [9] (Fortran90 [7] and F [14] versions available as well)
- **Number of program lines in distributed program, including test data, etc.:** ≈ 4300 (excluding comments)
- **Computer/Operating System:** Any with a Fortran95 (or Fortran90 or F) programming environment.
- **Memory required to execute with typical data:** Negligible on the scale of typical applications calling the library.
- **Typical running time:** A small fraction (typically a few percent) of the running time of applications calling the library.
- **Purpose of program:**
- **Nature of physical problem:**
- **Method of solution:**
- **Keywords:** adaptive integration, event generation, parallel processing

—1—

INTRODUCTION

We present a reimplementation of the classic Vegas [1, 2] algorithm for adaptive multi-dimensional integration in Fortran95 [9, 13]¹ (Note that for the maintenance of the program and especially its usage within the event generator WHIZARD parts of the program have been adapted to Fortran2003). The purpose of this reimplementation is two-fold: for pedagogical reasons it is useful to employ Fortran95 features (in particular the array language) together with literate programming [4] for expressing the algorithm more concisely and more transparently. On the other hand we use a Fortran95 abstract type to separate the state from the functions. This allows multiple instances of Vegas with different adaptations to run in parallel and in paves the road for a more parallelizable implementation.

The variable names are more in line with [1] than with [2] or with [17, 18, 19], which is almost identical to [2].

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1 <Copyleft notice 1>≡ (37a 70b 167a 192a 202a 204b 219e 222a 245 247a 273d 274a 287a 289b 291a 294a 304a 307a 309a 311a 313a 315a 317a 319a 321a 323a 325a 327a 329a 331a 333a 335a 337a 339a 341a 343a 345a 347a 349a 351a 353a 355a 357a 359a 361a 363a 365a 367a 369a 371a 373a 375a 377a 379a 381a 383a 385a 387a 389a 391a 393a 395a 397a 399a 401a 403a 405a 407a 409a 411a 413a 415a 417a 419a 421a 423a 425a 427a 429a 431a 433a 435a 437a 439a 441a 443a 445a 447a 449a 451a 453a 455a 457a 459a 461a 463a 465a 467a 469a 471a 473a 475a 477a 479a 481a 483a 485a 487a 489a 491a 493a 495a 497a 499a 501a 503a 505a 507a 509a 511a 513a 515a 517a 519a 521a 523a 525a 527a 529a 531a 533a 535a 537a 539a 541a 543a 545a 547a 549a 551a 553a 555a 557a 559a 561a 563a 565a 567a 569a 571a 573a 575a 577a 579a 581a 583a 585a 587a 589a 591a 593a 595a 597a 599a 601a 603a 605a 607a 609a 611a 613a 615a 617a 619a 621a 623a 625a 627a 629a 631a 633a 635a 637a 639a 641a 643a 645a 647a 649a 651a 653a 655a 657a 659a 661a 663a 665a 667a 669a 671a 673a 675a 677a 679a 681a 683a 685a 687a 689a 691a 693a 695a 697a 699a 701a 703a 705a 707a 709a 711a 713a 715a 717a 719a 721a 723a 725a 727a 729a 731a 733a 735a 737a 739a 741a 743a 745a 747a 749a 751a 753a 755a 757a 759a 761a 763a 765a 767a 769a 771a 773a 775a 777a 779a 781a 783a 785a 787a 789a 791a 793a 795a 797a 799a 801a 803a 805a 807a 809a 811a 813a 815a 817a 819a 821a 823a 825a 827a 829a 831a 833a 835a 837a 839a 841a 843a 845a 847a 849a 851a 853a 855a 857a 859a 861a 863a 865a 867a 869a 871a 873a 875a 877a 879a 881a 883a 885a 887a 889a 891a 893a 895a 897a 899a 901a 903a 905a 907a 909a 911a 913a 915a 917a 919a 921a 923a 925a 927a 929a 931a 933a 935a 937a 939a 941a 943a 945a 947a 949a 951a 953a 955a 957a 959a 961a 963a 965a 967a 969a 971a 973a 975a 977a 979a 981a 983a 985a 987a 989a 991a 993a 995a 997a 999a)
```

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¹Fully functional versions conforming to preceeding Fortran standard [7], High Performance Fortran (HPF) [10, 11, 15], and to the Fortran90 subset F [14] are available as well. A translation to the obsolete FORTRAN77 standard [6] is possible in principle, but extremely tedious and error prone if the full functionality shall be preserved.

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!!

—2— ALGORITHMS

 The notation has to be synchronized with [3]!

We establish some notation to allow a concise discussion. Notation:

$$\text{expectation: } E(f) = \frac{1}{|\mathcal{D}|} \int_{\mathcal{D}} dx f(x) \quad (2.1a)$$

$$\text{variance: } V(f) = E(f^2) - (E(f))^2 \quad (2.1b)$$

$$\text{estimate of expectation (average): } \langle X|f \rangle = \frac{1}{|X|} \sum_{x \in X} f(x) \quad (2.1c)$$

$$\text{estimate of variance: } \sigma_X^2(f) = \frac{1}{|X| - 1} (\langle X|f^2 \rangle - \langle X|f \rangle^2) \quad (2.1d)$$

Where $|X|$ is the size of the point set and $|\mathcal{D}| = \int_{\mathcal{D}} dx$ the size of the integration region. If $\mathcal{E}(\langle f \rangle)$ denotes the ensemble average of $\langle X|f \rangle$ over random point sets X with $|X| = N$, we have for expectation and variance

$$\mathcal{E}(\langle f \rangle) = E(f) \quad (2.2a)$$

$$\mathcal{E}(\sigma^2(f)) = V(f) \quad (2.2b)$$

and the ensemble variance of the expectation is also given by the variance

$$\mathcal{V}(\langle f \rangle) = \frac{1}{N} V(f) \quad (2.2c)$$

Therefore, it can be estimated from $\sigma_X^2(f)$. Below, we will also use the notation \mathcal{E}_g for the ensemble average over random point sets X_g with probability distribution g . We will write $E_g(f) = E(fg)$ as well.

2.1 Importance Sampling

If, instead of uniformly distributed points X , we use points X_g distributed according to a probability density g , we can easily keep the expectation constant

$$\mathcal{E}_g(\langle f \rangle) = E_g \left(\frac{f}{g} \right) = E(f) \quad (2.3)$$

while the variance transforms non-trivially

$$\mathcal{V}_g(\langle f \rangle) = \frac{1}{N} V_g \left(\frac{f}{g} \right) = \frac{1}{N} \left(E_g \left(\frac{f^2}{g^2} \right) - \left(E_g \left(\frac{f}{g} \right) \right)^2 \right) \quad (2.4)$$

and the error is minimized when f/g is constant, i.e. g is a good approximation of f . The non-trivial problem is to find a g that can be generated efficiently and is a good approximation at the same time.

One of the more popular approaches is to use a mapping ϕ of the integration domain

$$\begin{aligned} \phi : \mathcal{D} &\rightarrow \Delta \\ x &\mapsto \xi = \phi(x) \end{aligned} \quad (2.5)$$

In the new coordinates, the distribution is multiplied by the Jacobian of the inverse map ϕ^{-1} :

$$\int_{\mathcal{D}} dx f(\phi(x)) = \int_{\Delta} d\xi J_{\phi^{-1}}(\xi) f(\xi) \quad (2.6)$$

A familiar example is given by the map

$$\begin{aligned} \phi : [0, 1] &\rightarrow \mathbf{R} \\ x &\mapsto \xi = x^0 + a \cdot \tan \left(\left(x - \frac{1}{2} \right) \pi \right) \end{aligned} \quad (2.7)$$

with the inverse $\phi^{-1}(\xi) = \text{atan}((\xi - x_0)/a)/\pi + 1/2$ and the corresponding Jacobian reproducing a resonance

$$J_{\phi^{-1}}(\xi) = \frac{d\phi^{-1}(\xi)}{d\xi} = \frac{a}{\pi} \frac{1}{(\xi - x^0)^2 + a^2} \quad (2.8)$$

Obviously, this works only for a few special distributions. Fortunately, we can combine several of these mappings to build efficient integration algorithms, as will be explained in section 2.4 below. Another approach is to construct the approximation numerically, by appropriate binning of the integration domain (cf. [1, 2, 20]). The most popular technique for this will be discussed below in section 2.3.

2.2 Stratified Sampling

The technique of importance sampling concentrates the sampling points in the region where the contribution to the integrand is largest. Alternatively we can also concentrate the sampling points in the region where the contribution to the variance is largest.

If we divide the sampling region \mathcal{D} into n disjoint subregions \mathcal{D}^i

$$\mathcal{D} = \bigcup_{i=1}^n \mathcal{D}^i, \quad \mathcal{D}^i \cap \mathcal{D}^j = \emptyset \quad (i \neq j) \quad (2.9)$$

a new estimator is

 Bzzzt! Wrong. These multi-channel formulae are incorrect for partitionings and must be fixed.

$$\overline{\langle X|f \rangle} = \sum_{i=1}^n \frac{N_i}{N} \langle X_{\theta_i}|f \rangle \quad (2.10)$$

where

$$\theta_i(x) = \begin{cases} 1 & \text{for } x \in \mathcal{D}^i \\ 0 & \text{for } x \notin \mathcal{D}^i \end{cases} \quad (2.11)$$

and

$$\sum_{i=1}^n N_i = N \quad (2.12)$$

since the expectation is linear

$$\mathcal{E}(\overline{\langle f \rangle}) = \sum_{i=1}^n \frac{N_i}{N} \mathcal{E}_{\theta_i}(\langle f \rangle) = \sum_{i=1}^n \frac{N_i}{N} E_{\theta_i}(f) = \sum_{i=1}^n \frac{N_i}{N} E(f\theta_i) = E(f) \quad (2.13)$$

On the other hand, the variance of the estimator $\overline{\langle X|f \rangle}$ is

$$\mathcal{V}(\overline{\langle f \rangle}) = \sum_{i=1}^n \frac{N_i}{N} \mathcal{V}_{\theta_i}(\langle f \rangle) \quad (2.14)$$

This is minimized for

$$N_i \propto \sqrt{V(f \cdot \theta_{\mathcal{D}^i})} \quad (2.15)$$

as a simple variation of $\mathcal{V}(\overline{\langle f \rangle})$ shows.

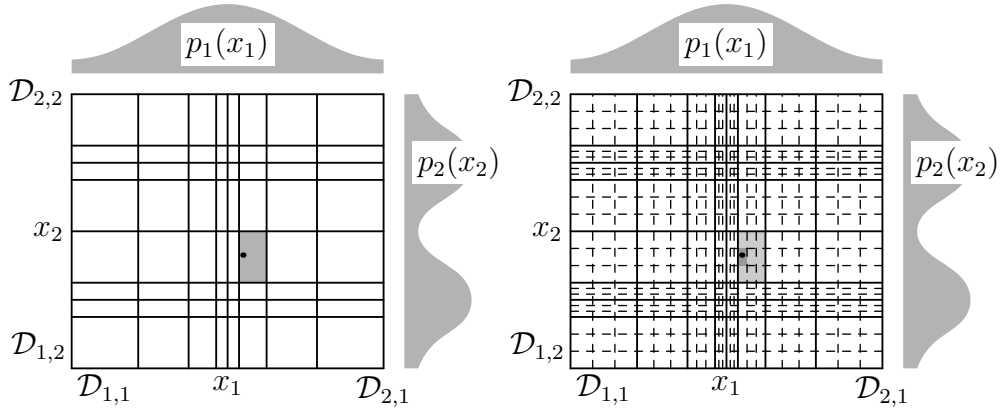


Figure 2.1: **vegas** grid structure for non-stratified sampling (left) and for genuinely stratified sampling (right), which is used in low dimensions. N.B.: the grid and the weight functions $p_{1,2}$ are only in qualitative agreement.



Figure 2.2: One-dimensional illustration of the **vegas** grid structure for pseudo stratified sampling, which is used in high dimensions.

2.3 Vegas



Under construction!

2.3.1 Vegas' Inflexibility

The classic implementation of the Vegas algorithm [1, 2] treats all dimensions alike. This constraint allows a very concise FORTRAN77-style coding of the algorithm, but there is no theoretical reason for having the same number of divisions in each direction. On the contrary, under these circumstances, even a dimension in which the integrand is rather smooth will contribute to the exponential blow-up of cells for stratified sampling. It is obviously beneficial to use a finer grid in those directions in which the fluctuations are stronger, while a coarser grid will suffice in the other directions.

One small step along this line is implemented in Version 5.0 of the package `BASES/SPRING` [20], where one set of “wild” variables is separated from “smooth” variables [21].

The present reimplementaion of the Vegas algorithm allows the application to choose the number of divisions in each direction freely. The routines that reshape the grid accept an integer array with the number of divisions as an optional argument `num_div`. It is easy to construct examples in which the careful use of this feature reduces the variance significantly.

Currently, no attempt is made for automatic optimization of the number of divisions. One reasonable approach is to monitor Vegas’ grid adjustments and to increase the number of division in those directions where Vegas’ keeps adjusting because of fluctuations. For each direction, a numerical measure of these fluctuations is given by the spread in the m_i . The total number of cells can be kept constant by reducing the number of divisions in the other directions appropriately. Thus

$$n_{\text{div},j} \rightarrow \frac{Q_j n_{\text{div},j}}{\left(\prod_j Q_j\right)^{1/n_{\text{dim}}}} \quad (2.16)$$

where we have used the damped standard deviation

$$Q_j = \left(\sqrt{\text{Var}(\{m\}_j)} \right)^\alpha \quad (2.17)$$

instead of the spread.

2.3.2 Vegas’ Dark Side



Under construction!

A partial solution of this problem will be presented in section 2.5.

2.4 Multi Channel Sampling

Even if Vegas performs well for a large class of integrands, many important applications do not lead to a factorizable distribution. The class of integrands that can be integrated efficiently by Vegas can be enlarged substantially by using multi channel methods. The new class will include almost all integrals appearing in high energy physics simulations.



The first version of this section is now obsolete. Consult [3] instead.

2.5 *Revolving*



Under construction!

2.6 *Parallelization*

Traditionally, parallel processing has not played a large rôle in simulations for high energy physics. A natural and trivial method of utilizing many processors will run many instances of the same (serial) program with different values of the input parameters in parallel. Typical matrix elements and phase space integrals offer few opportunities for small scale parallelization.

On the other hand, parameter fitting has become possible recently for observables involving a phase space integration. In this case, fast evaluation of the integral is essential and parallel execution becomes an interesting option.

A different approach to parallelizing Vegas has been presented recently [22].

2.6.1 *Multilinear Structure of the Sampling Algorithm*

In order to discuss the problems with parallelizing adaptive integration algorithms and to present solutions, it helps to introduce some mathematical notation. A sampling S is a map from the space π of point sets and the space F of functions to the real (or complex) numbers

$$\begin{aligned} S : \pi \times F &\rightarrow \mathbf{R} \\ (p, f) &\mapsto I = S(p, f) \end{aligned}$$

For our purposes, we have to be more specific about the nature of the point set. In general, the point set will be characterized by a sequence of pseudo random numbers $\rho \in R$ and by one or more grids $G \in \Gamma$ used for importance or stratified sampling. A simple sampling

$$\begin{aligned} S_0 : R \times \Gamma \times A \times F \times \mathbf{R} \times \mathbf{R} &\rightarrow R \times \Gamma \times A \times F \times \mathbf{R} \times \mathbf{R} \\ (\rho, G, a, f, \mu_1, \mu_2) &\mapsto (\rho', G, a', f, \mu'_1, \mu'_2) = S_0(\rho, G, a, f, \mu_1, \mu_2) \end{aligned} \tag{2.18}$$

estimates the n -th moments $\mu'_n \in \mathbf{R}$ of the function $f \in F$. The integral and its standard deviation can be derived easily from the moments

$$I = \mu_1 \tag{2.19a}$$

$$\sigma^2 = \frac{1}{N-1} (\mu_2 - \mu_1^2) \tag{2.19b}$$

while the latter are more convenient for the following discussion. In addition, S_0 collects auxiliary information to be used in the grid refinement, denoted by $a \in A$. The unchanged arguments G and f have been added to the result of S_0 in (2.18), so that S_0 has identical domain and codomain and can therefore be iterated. Previous estimates μ_n may be used in the estimation of μ'_n , but a particular S_0 is free to ignore them as well. Using a little notational freedom, we augment \mathbf{R} and A with a special value \cdot , which will always be discarded by S_0 .

In an adaptive integration algorithm, there is also a refinement operation $r : \Gamma \times A \rightarrow \Gamma$ that can be extended naturally to the codomain of S_0

$$\begin{aligned} r : R \times \Gamma \times A \times F \times \mathbf{R} \times \mathbf{R} &\rightarrow R \times \Gamma \times A \times F \times \mathbf{R} \times \mathbf{R} \\ (\rho, G, a, f, \mu_1, \mu_2) &\mapsto (\rho, G', a, f, \mu_1, \mu_2) = r(\rho, G, a, f, \mu_1, \mu_2) \end{aligned} \quad (2.20)$$

so that $S = rS_0$ is well defined and we can specify n -step adaptive sampling as

$$S_n = S_0(rS_0)^n \quad (2.21)$$

Since, in a typical application, only the estimate of the integral and the standard deviation are used, a projection can be applied to the result of S_n :

$$\begin{aligned} P : R \times \Gamma \times A \times F \times \mathbf{R} \times \mathbf{R} &\rightarrow \mathbf{R} \times \mathbf{R} \\ (\rho, G, a, f, \mu_1, \mu_2) &\mapsto (I, \sigma) \end{aligned} \quad (2.22)$$

Then

$$(I, \sigma) = PS_0(rS_0)^n(\rho, G_0, \cdot, f, \cdot, \cdot) \quad (2.23)$$

and a good refinement prescription r , such as Vegas, will minimize the σ .

For parallelization, it is crucial to find a division of S_n or any part of it into *independent* pieces that can be evaluated in parallel. In order to be effective, r has to be applied to *all* of a and therefore a synchronization of G before and after r is appropriately. Furthermore, r usually uses only a tiny fraction of the CPU time and it makes little sense to invest a lot of effort into parallelizing it beyond what the Fortran compiler can infer from array notation. On the other hand, S_0 can be parallelized naturally, because all operations are linear, including the computation of a . We only have to make sure that the cost of communicating the results of S_0 and r back and forth during the computation of S_n do not offset any performance gain from parallel processing.

When we construct a decomposition of S_0 and prove that it does not change the results, i.e.

$$S_0 = \iota S_0 \phi \quad (2.24)$$

where ϕ is a forking operation and ι is a joining operation, we are faced with the technical problem of a parallel random number source ρ . As made explicit in (2.18), S_0 changes the state of the random number general ρ , demanding *identical* results therefore imposes a strict ordering on the operations and defeats parallelization. It is possible to devise implementations of S_0 and ρ that circumvent this problem by distributing subsequences of ρ in such a way among processes that results do not depend on the number of parallel processes.

However, a reordering of the random number sequence will only change the result by the statistical error, as long as the scale of the allowed reorderings is *bounded* and much smaller than the period of the random number generator¹ Below, we will therefore use the notation $x \approx y$ for “equal for an appropriate finite reordering of the ρ used in calculating x and y ”. For our purposes, the relation $x \approx y$ is strong enough and allows simple and efficient implementations.

Since S_0 is essentially a summation, it is natural to expect a linear structure

$$\bigoplus_i S_0(\rho_i, G_i, a_i, f, \mu_{1,i}, \mu_{2,i}) \approx S_0(\rho, G, a, f, \mu_1, \mu_2) \quad (2.25a)$$

where

$$\rho = \bigoplus_i \rho_i \quad (2.25b)$$

$$G = \bigoplus_i G_i \quad (2.25c)$$

$$a = \bigoplus_i a_i \quad (2.25d)$$

$$\mu_n = \bigoplus_i \mu_{n,i} \quad (2.25e)$$

for appropriate definitions of “ \oplus ”. For the moments, we have standard addition

$$\mu_{n,1} \oplus \mu_{n,2} = \mu_{n,1} + \mu_{n,2} \quad (2.26)$$

and since we only demand equality up to reordering, we only need that the ρ_i are statistically independent. This leaves us with G and a and we have to discuss importance sampling and stratified sampling separately.

¹Arbitrary reorderings on the scale of the period of the random number generators could select constant sequences and have to be forbidden.

Importance Sampling

In the case of naive Monte Carlo and importance sampling the natural decomposition of G is to take j copies of the same grid G/j which is identical to G , each with one j -th of the total sampling points. As long as the a are linear themselves, we can add them up just like the moments

$$a_1 \oplus a_2 = a_1 + a_2 \quad (2.27)$$

and we have found a decomposition (2.25). In the case of Vegas, the a_i are sums of function values at the sampling points. Thus they are obviously linear and this approach is applicable to Vegas in the importance sampling mode.

Stratified Sampling

The situation is more complicated in the case of stratified sampling. The first complication is that in pure stratified sampling there are only two sampling points per cell. Splitting the grid in two pieces as above provide only a very limited amount of parallelization. The second complication is that the a are no longer linear, since they correspond to a sampling of the variance per cell and no longer of function values themselves.

However, as long as the samplings contribute to disjoint bins only, we can still “add” the variances by combining bins. The solution is therefore to divide the grid into disjoint bins along the divisions of the stratification grid and to assign a set of bins to each processor.

Finer decompositions will incur higher communications costs and other resource utilization. An implementation based on PVM is described in [22], which minimizes the overhead by running identical copies of the grid G on each processor. Since most of the time is usually spent in function evaluations, it makes sense to run a full S_0 on each processor, skipping function evaluations everywhere but in the region assigned to the processor. This is a neat trick, which is unfortunately tied to the computational model of message passing systems such as PVM and MPI [12]. More general paradigms can not be supported since the separation of the state for the processors is not explicit (it is implicit in the separated address space of the PVM or MPI processes).

However, it is possible to implement (2.25) directly in an efficient manner. This is based on the observation that the grid G used by Vegas is factorized into divisions D^j for each dimension

$$G = \bigotimes_{j=1}^{n_{\text{dim}}} D^j \quad (2.28)$$

and decompositions of the D^j induce decompositions of G

$$\begin{aligned} G_1 \oplus G_2 &= \left(\bigotimes_{j=1}^{i-1} D^j \otimes D_1^i \otimes \bigotimes_{i=j+1}^{n_{\text{dim}}} D^j \right) \oplus \left(\bigotimes_{j=1}^{i-1} D^j \otimes D_2^i \otimes \bigotimes_{i=j+1}^{n_{\text{dim}}} D^j \right) \\ &= \bigotimes_{j=1}^{i-1} D^j \otimes (D_1^i \oplus D_2^i) \otimes \bigotimes_{j=i+1}^{n_{\text{dim}}} D^j \quad (2.29) \end{aligned}$$

We can translate (2.29) directly to code that performs the decomposition $D^i = D_1^i \oplus D_2^i$ discussed below and simply duplicates the other divisions $D^{j \neq i}$. A decomposition along multiple dimensions is implemented by a recursive application of (2.29).

In Vegas, the auxiliary information a inherits a factorization similar to the grid (2.28)

$$a = (d^1, \dots, d^{n_{\text{dim}}}) \quad (2.30)$$

but not a multilinear structure. Instead, *as long as the decomposition respects the stratification grid*, we find the in place of (2.29)

$$a_1 \oplus a_2 = (d_1^1 + d_2^1, \dots, d_1^i \oplus d_2^i, \dots, d_1^{n_{\text{dim}}} + d_2^{n_{\text{dim}}}) \quad (2.31)$$

with “+” denoting the standard addition of the bin contents and “ \oplus ” denoting the aggregation of disjoint bins. If the decomposition of the division would break up cells of the stratification grid (2.31) would be incorrect, because, as discussed above, the variance is not linear.

Now it remains to find a decomposition

$$D^i = D_1^i \oplus D_2^i \quad (2.32)$$

for both the pure stratification mode and the pseudo stratification mode of vegas (cf. figure 2.1). In the pure stratification mode, the stratification grid is strictly finer than the adaptive grid and we can decompose along either of them immediately. Technically, a decomposition along the coarser of the two is straightforward. Since the adaptive grid already has more than 25 bins, a decomposition along the stratification grid makes no practical sense and the decomposition along the adaptive grid has been implemented. The sampling algorithm S_0 can be applied *unchanged* to the individual grids resulting from the decomposition.

For pseudo stratified sampling (cf. figure 2.2), the situation is more complicated, because the adaptive and the stratification grid do not share bin boundaries. Since Vegas does *not* use the variance in this mode, it would be theoretically possible to decompose along the adaptive grid and to mimic the



Figure 2.3: Forking one dimension d of a grid into three parts $ds(1)$, $ds(2)$, and $ds(3)$. The picture illustrates the most complex case of pseudo stratified sampling (cf. fig. 2.2).

incomplete bins of the stratification grid in the sampling algorithm. However, this would be a technical complication, destroying the universality of S_0 . Therefore, the adaptive grid is subdivided in a first step in

$$\text{lcm} \left(\frac{\text{lcm}(n_f, n_g)}{n_f}, n_x \right) \quad (2.33)$$

bins,² such that the adaptive grid is strictly finer than the stratification grid. This procedure is shown in figure 2.3.

2.6.2 State and Message Passing

2.6.3 Random Numbers

In the parallel example sitting on top of MPI [12] takes advantage of the ability of Knuth's generator [16] to generate statistically independent subse-

²The coarsest grid covering the division of n_g bins into n_f forks has $n_g / \text{gcd}(n_f, n_g) = \text{lcm}(n_f, n_g) / n_f$ bins per fork.

quences. However, since the state of the random number generator is explicit in all procedure calls, other means of obtaining subsequences can be implemented in a trivial wrapper.

The results of the parallel example will depend on the number of processors, because this effects the subsequences being used. Of course, the variation will be compatible with the statistical error. It must be stressed that the results are deterministic for a given number of processors and a given set of random number generator seeds. Since parallel computing environments allow to fix the number of processors, debugging of exceptional conditions is possible.

2.6.4 Practice

In this section we show three implementations of S_n : one serial, and two parallel, based on HPF [10, 11, 15] and MPI [12], respectively. From these examples, it should be obvious how to adapt VAMP to other parallel computing paradigms.

Serial

Here is a bare bones serial version of S_n , for comparison with the parallel versions below. The real implementation of `vamp_sample_grid` in the module `vamp` includes some error handling, diagnostics and the projection P (cf. (2.22)):

```
14  $\langle$ Serial implementation of  $S_n = S_0(rS_0)^n$  14 $\rangle \equiv$ 
  subroutine vamp_sample_grid (rng, g, iterations, func)
    type(tao_random_state), intent(inout) :: rng
    type(vamp_grid), intent(inout) :: g
    integer, intent(in) :: iterations
     $\langle$ Interface declaration for func 22 $\rangle$ 
    integer :: iteration
    iterate: do iteration = 1, iterations
      call vamp_sample_grid0 (rng, g, func)
      call vamp_refine_grid (g)
    end do iterate
  end subroutine vamp_sample_grid
```

HPF

The HPF version of S_n is based on decomposing the grid `g` as described in section 2.6.1 and lining up the components in an array `gs`. The elements of `gs` can then be processed in parallel. This version can be compiled with any

Fortran compiler and a more complete version of this procedure (including error handling, diagnostics and the projection P) is included with VAMP as `vamp_sample_grid_parallel` in the module `vamp`. This way, the algorithm can be tested on a serial machine, but there will obviously be no performance gain.

Instead of one random number generator state `rng`, it takes an array consisting of one state per processor. These `rng(:)` are assumed to be initialized, such that the resulting sequences are statistically independent. For this purpose, Knuth's random number generator [16] is most convenient and is included with VAMP (see the example on page 16). Before each S_0 , the procedure `vamp_distribute_work` determines a good decomposition of the grid `d` into `size(rng)` pieces. This decomposition is encoded in the array `d` where `d(1,:)` holds the dimensions along which to split the grid and `d(2,:)` holds the corresponding number of divisions. Using this information, the grid is decomposed by `vamp_fork_grid`. The HPF compiler will then distribute the `!hpf$ independent` loop among the processors. Finally, `vamp_join_grid` gathers the results.

```

15 <Parallel implementation of  $S_n = S_0(rS_0)^n$  (HPF) 15>≡
  subroutine vamp_sample_grid_hpf (rng, g, iterations, func)
    type(tao_random_state), dimension(:), intent(inout) :: rng
    type(vamp_grid), intent(inout) :: g
    integer, intent(in) :: iterations
    <Interface declaration for func 22>
    type(vamp_grid), dimension(:), allocatable :: gs, gx
    !hpf$ processors p(number_of_processors())
    !hpf$ distribute gs(cyclic(1)) onto p
    integer, dimension(:,:), pointer :: d
    integer :: iteration, num_workers
    iterate: do iteration = 1, iterations
      call vamp_distribute_work (size (rng), vamp_rigid_divisions (g), d)
      num_workers = max (1, product (d(2,:)))
      if (num_workers > 1) then
        allocate (gs(num_workers), gx(vamp_fork_grid_joints (d)))
        call vamp_create_empty_grid (gs)
        call vamp_fork_grid (g, gs, gx, d)
        !hpf$ independent
        do i = 1, num_workers
          call vamp_sample_grid0 (rng(i), gs(i), func)
        end do
        call vamp_join_grid (g, gs, gx, d)
        call vamp_delete_grid (gs)
        deallocate (gs, gx)
      end if
    end do
  end subroutine

```



```

        else
            call vamp_sample_grid0 (rng(1), g, func)
        end if
        call vamp_refine_grid (g)
    end do iterate
end subroutine vamp_sample_grid_hpf

```

Since `vamp_sample_grid0` performs the bulk of the computation, an almost linear speedup with the number of processors can be achieved, if `vamp_distribute_work` finds a good decomposition of the grid. The version of `vamp_distribute_work` distributed with VAMP does a good job in most cases, but will not be able to use all processors if their number is a prime number larger than the number of divisions in the stratification grid. Therefore it can be beneficial to tune `vamp_distribute_work` to specific hardware. Furthermore, using a finer stratification grid can improve performance.

For definiteness, here is an example of how to set up the array of random number generators for HPF. Note that this simple seeding procedure only guarantees statistically independent sequences with Knuth's random number generator [16] and will fail with other approaches.

```

16 <Parallel usage of  $S_n = S_0(rS_0)^n$  (HPF) 16>≡
    type(tao_random_state), dimension(:), allocatable :: rngs
    !hpf$ processors p(number_of_processors())
    !hpf$ distribute gs(cyclic(1)) onto p
    integer :: i, seed
    ! ...
    allocate (rngs(number_of_processors()))
    seed = 42 ! can be read from a file, of course ...
    !hpf$ independent
    do i = 1, size (rngs)
        call tao_random_create (rngs(i), seed + i)
    end do
    ! ...
    call vamp_sample_grid_hpf (rngs, g, 6, func)
    ! ...

```

MPI

The MPI version is more low level, because we have to keep track of message passing ourselves. Note that we have made this synchronization points explicit with three `if ... then ... else ... end if` blocks: forking, sampling, and joining. These blocks could be merged (without any performance gain) at the expense of readability. We assume that `rng` has been initialized

in each process such that the sequences are again statistically independent.

17 \langle Parallel implementation of $S_n = S_0(rS_0)^n$ (MPI) 17 $\rangle \equiv$

```

subroutine vamp_sample_grid_mpi (rng, g, iterations, func)
  type(tao_random_state), dimension(:), intent(inout) :: rng
  type(vamp_grid), intent(inout) :: g
  integer, intent(in) :: iterations
   $\langle$ Interface declaration for func 22 $\rangle$ 
  type(vamp_grid), dimension(:), allocatable :: gs, gx
  integer, dimension(:,:), pointer :: d
  integer :: num_proc, proc_id, iteration, num_workers
  call mpi90_size (num_proc)
  call mpi90_rank (proc_id)
  iterate: do iteration = 1, iterations
    if (proc_id == 0) then
      call vamp_distribute_work (num_proc, vamp_rigid_divisions (g), d)
      num_workers = max (1, product (d(2,:)))
    end if
    call mpi90_broadcast (num_workers, 0)
    if (proc_id == 0) then
      allocate (gs(num_workers), gx(vamp_fork_grid_joints (d)))
      call vamp_create_empty_grid (gs)
      call vamp_fork_grid (g, gs, gx, d)
      do i = 2, num_workers
        call vamp_send_grid (gs(i), i-1, 0)
      end do
    else if (proc_id < num_workers) then
      call vamp_receive_grid (g, 0, 0)
    end if
    if (proc_id == 0) then
      if (num_workers > 1) then
        call vamp_sample_grid0 (rng, gs(1), func)
      else
        call vamp_sample_grid0 (rng, g, func)
      end if
    else if (proc_id < num_workers) then
      call vamp_sample_grid0 (rng, g, func)
    end if
    if (proc_id == 0) then
      do i = 2, num_workers
        call vamp_receive_grid (gs(i), i-1, 0)
      end do
      call vamp_join_grid (g, gs, gx, d)
      call vamp_delete_grid (gs)
    end if
  end do
end subroutine vamp_sample_grid_mpi

```

```

        deallocate (gs, gx)
        call vamp_refine_grid (g)
    else if (proc_id < num_workers) then
        call vamp_send_grid (g, 0, 0)
    end if
end do iterate
end subroutine vamp_sample_grid_mpi

```

A more complete version of this procedure is included with VAMP as well, this time as `vamp_sample_grid` in the MPI support module `vampi`.

—3—

DESIGN TRADE OFFS

There have been three competing design goals for vegas, that are not fully compatible and had to be reconciled with compromises:

- *Ease-Of-Use*: few procedures, few arguments.
- *Parallelizability*: statelessness
- *Performance and Flexibility*: rich interface, functionality.

In fact, parallelizability and ease-of-use are complementary. A parallelizable implementation has to expose *all* the internal state. In our case, this includes the state of the random number generator and the adaptive grid. A simple interface would hide such details from the user.

The modern language features introduced to Fortran in 1990 [7] allows to reconcile these competing goals. Two abstract data types `vamp.state` and `tao.random.state` hide the details of the implementation from the user and encapsulate the two states in just two variables.

Another problem with parallelizability arised from the lack of a general exception mechanism in Fortran. The Fortran90 standard [9] forbids *any* input/output (even to the terminal) as well as `stop` statements in parallelizable (`pure`) procedures. This precludes simple approaches to monitoring and error handling. In Vegas we use a simple hand crafted exception mechanism (see chapter B) for communicating error conditions to the out layers of the applications. Unfortunately this requires the explicit passing of state in argument lists.

An unfortunate consequence of the similar approach to monitoring is that monitoring is *not* possible during execution. Instead, intermediate results can only be examined after a parallelized section of code has completed.

3.1 *Programming Language*

We have chosen to implement VAMP in Fortran90/95, which some might consider a questionable choice today. Nevertheless, we are convinced that Fortran90/95 (with all its weaknesses) is, by a wide margin, the right tool for the job.

Let us consider the alternatives

- FORTRAN77 is still the dominant language in high energy physics and all running experiment's software environments are based on it. However, the standard [6] is obsolete now and the successors [7, 9] have added many desirable features, while retaining almost all of FORTRAN77 as a subset.
- C/C++ appears to be the most popular programming language in industry and among young high energy physicists. Large experiments have taken a bold move and are basing their software environment on C++.
- Typed higher order functional programming languages (ML, Haskell, etc.) are a very promising development. Unfortunately, there is not yet enough industry support for high performance optimizing compilers. While the performance penalty of these languages is not as high as commonly believed (research compilers, which do not perform extensive processor specific optimizations, result in code that runs by a factor of two or three slower than equivalent Fortran code), it is relevant for long running, computing intensive applications. In addition, these languages are syntactically and idiomatically very different from Fortran and C. Another implementation of VAMP in ML will be undertaken for research purposes to investigate new algorithms that can only be expressed awkwardly in Fortran, but we do not expect it to gain immediate popularity.

—4— USAGE

4.1 Basic Usage

`type(vamp_grid)`

`subroutine vamp_create_grid (g, domain [, num_calls] [, exc])`

Create a fresh grid for the integration domain

$$\mathcal{D} = [D_{1,1}, D_{2,1}] \times [D_{1,2}, D_{2,2}] \times \dots \times [D_{1,n}, D_{2,n}] \quad (4.1)$$

dropping all accumulated results. This function *must not* be called twice on the first argument, without an intervening

`vamp_delete_grid`. Iff the variable `num_calls` is given, it will be the number of sampling points per iteration for the call to `vamp_sample_grid`.

`subroutine vamp_delete_grid (g [, exc])`

`subroutine vamp_discard_integral (g [, num_calls] [, exc])`

Keep the current optimized grid, but drop the accumulated results for the integral (value and errors). Iff the variable `num_calls` is given, it will be the new number of sampling points per iteration for the calls to `vamp_sample_grid`.

`subroutine vamp_reshape_grid (g [, num_calls] [, exc])`

Keep the current optimized grid and the accumulated results for the integral (value and errors). The variable `num_calls` is the new number of sampling points per iteration for the calls to `vamp_sample_grid`.

`subroutine vamp_sample_grid (rng, g, func, iterations
[, integral] [, std_dev] [, avg_chi2] [, exc] [, history])`

Sample the function `func` using the grid `g` for `iterations` iterations and optimize the grid after each iteration. The results are returned in `integral`, `std_dev` and `avg_chi2`. The random number generator uses and updates the state stored in `rng`. The explicit random number state is inconvenient, but required for parallelizability.

```
subroutine vamp_integrate (rng, g, func, calls [, integral]
  [, std_dev] [, avg_chi2] [, exc] [, history])
```

This is a wrapper around the above routines, that is steered by a integer, dimension(2,:) array `calls`. For each `i`, there will be `calls(1,i)` iterations with `calls(2,i)` sampling points.

```
subroutine vamp_integrate (rng, domain, func, calls
  [, integral] [, std_dev] [, avg_chi2] [, exc] [, history])
```

A second specific form of `vamp_integrate`. This one keeps a private grid and provides the shortest—and most inflexible—calling sequence.

```
22 <Interface declaration for func 22>≡ (14 15 17 86a 94c 103b 113 115 120b 135c 136c 139–42 169d 175c 182
interface
  function func (xi, data, weights, channel, grids) result (f)
    use kinds
    use vamp_grid_type !NODEP!
    import vamp_data_t
    real(kind=default), dimension(:), intent(in) :: xi
    class(vamp_data_t), intent(in) :: data
    real(kind=default), dimension(:), intent(in), optional :: weights
    integer, intent(in), optional :: channel
    type(vamp_grid), dimension(:), intent(in), optional :: grids
    real(kind=default) :: f
  end function func
end interface
```

4.1.1 Basic Example

In Fortran95, the function to be sampled *must* be pure, i.e. have no side effects to allow parallelization. The optional arguments `weights` and `channel` *must* be declared to allow the compiler to verify the interface, but they are ignored during basic use. Their use for multi channel sampling will be explained below. Here’s a Gaussian

$$f(x) = e^{-\frac{1}{2} \sum_i x_i^2} \quad (4.2)$$

```

23a <basic.f90 23a>≡
module basic_fct
  use kinds
  implicit none
  private
  public :: fct
contains
  function fct (x, weights, channel) result (f_x)
    real(kind=default), dimension(:), intent(in) :: x
    real(kind=default), dimension(:), intent(in), optional :: weights
    integer, intent(in), optional :: channel
    real(kind=default) :: f_x
    f_x = exp (-0.5 * sum (x*x))
  end function fct
end module basic_fct
23b >

```

In the main program, we need to import five modules. The customary module `kinds` defines `double` as the kind for double precision floating point numbers. The model `exceptions` provides simple error handling support (parallelizable routines are not allowed to issue error messages themselves, but must pass them along). The module `tao_random_numbers` hosts the random number generator used and `vamp` is the adaptive iteration module proper. Finally, the application module `basic_fct` has to be imported as well.

```

23b <basic.f90 23a>+≡
program basic
  use kinds
  use exceptions
  use tao_random_numbers
  use vamp
  use basic_fct
  implicit none
23c <23a 23c>

```

Then we define four variables for an error message, the random number generator state and the adaptive integration grid. We also declare a variable for holding the integration domain and variables for returning the result. In this case we integrate the 7-dimensional hypercube.

```

23c <basic.f90 23a>+≡
  type(exception) :: exc
  type(tao_random_state) :: rng
  type(vamp_grid) :: grid
  real(kind=default), dimension(2,7) :: domain
  real(kind=default) :: integral, error, chi2
  domain(1,:) = -1.0
  domain(2,:) = 1.0
23b <23b 24a>

```


Initialize and seed the random number generator. Initialize the grid for 10 000 sampling points.

```
24a <basic.f90 23a>+≡ <23c 24b>
    call tao_random_create (rng, seed=0)
    call clear_exception (exc)
    call vamp_create_grid (grid, domain, num_calls=10000, exc=exc)
    call handle_exception (exc)
```

Warm up the grid in six low statistics iterations. Clear the error status before and check it after the sampling.

```
24b <basic.f90 23a>+≡ <24a 24c>
    call clear_exception (exc)
    call vamp_sample_grid (rng, grid, fct, 6, exc=exc)
    call handle_exception (exc)
```

Throw away the intermediate results and reshape the grid for 100 000 sampling points—keeping the adapted grid—and do four iterations of a higher statistics integration

```
24c <basic.f90 23a>+≡ <24b>
    call clear_exception (exc)
    call vamp_discard_integral (grid, num_calls=100000, exc=exc)
    call handle_exception (exc)
    call clear_exception (exc)
    call vamp_sample_grid (rng, grid, fct, 4, integral, error, chi2, exc=exc)
    call handle_exception (exc)
    print *, "integral = ", integral, "+/-", error, " (chi^2 = ", chi2, ")"
end program basic
```

Since this is the most common use, there is a convenience routine available and the following code snippet is equivalent:

```
24d <Alternative to basic.f90 24d>≡
    integer, dimension(2,2) :: calls
    calls(:,1) = (/ 6, 10000 /)
    calls(:,2) = (/ 4, 100000 /)
    call clear_exception (exc)
    call vamp_integrate (rng, domain, fct, calls, integral, error, chi2, exc=exc)
    call handle_exception (exc)
```

4.2 Advanced Usage



Caveat emptor: no magic of literate programming can guarantee that the following remains in sync with the implementation. This has to be maintained manually.

All `real` variables are declared as `real(kind=default)` in the source and the variable `double` is imported from the module `kinds` (see appendix A.1). The representation of real numbers can therefore be changed by changing `double` in `kinds`.

4.2.1 Types

```
type(vamp_grid)
type(vamp_grids)
type(vamp_history)
type(exception)
  (from module exceptions)
```

4.2.2 Shared Arguments

Arguments keep their name across procedures, in order to make the Fortran90 keyword interface consistent.

```
real, intent(in) :: accuracy
```

Terminate S_n after $n' < n$ iterations, if relative error is smaller than `accuracy`. Specifically, the termination condition is

$$\frac{\text{std_dev}}{\text{integral}} < \text{accuracy} \quad (4.3)$$

```
real, intent(out) :: avg_chi2
```

The average χ^2 of the iterations.

```
integer, intent(in) :: channel
```

Call `func` with this optional argument. Multi channel sampling uses this to emulate arrays of functions

```
logical, intent(in) :: covariance
```

Collect covariance data.

```
type(exception), intent(inout) :: exc
```

Exceptional conditions are reported in `exc`.

```
type(vamp_grid), intent(inout) :: g
```

Unless otherwise noted, `g` denotes the active sampling grid in the documentation below.

```
type(vamp_histories), dimension(:), intent(inout) ::  
  histories
```

Diagnostic information for multi channel sampling.

```
type(vamp_history), dimension(:), intent(inout) ::  
  history
```

Diagnostic information for single channel sampling or summary of multi channel sampling.

```
real, intent(out) :: integral
```

The current best estimate of the integral.

```
integer, intent(in) :: iterations
```

```
real, dimension(:,:), intent(in) :: map
```

```
integer, intent(in) :: num_calls
```

The number of sampling points.

```
integer, dimension(:), intent(in) :: num_div
```

Number of divisions of the adaptive grid in each dimension.

```
logical, intent(in) :: quadrupole
```

Allow “quadrupole oscillations” of the sampling grid (cf. section 2.3.1).

```
type(tao_random_state), intent(inout) :: rng
```

Unless otherwise noted, `rng` denotes the source of random numbers used for sampling in the documentation below.

```
real, intent(out) :: std_dev
```

The current best estimate of the error on the integral.

```
logical, intent(in) :: stratified
```

Try to use stratified sampling.

```
real(kind=default), dimension(:), intent(in) :: weights
```

```
...
```

4.2.3 Single Channel Procedures

```
subroutine vamp_create_grid (g, domain, num_calls
    [, quadrupole] [, stratified] [, covariance] [, map] [, exc])

    real, dimension(:, :), intent(in) :: domain

subroutine vamp_create_empty_grid (g)

subroutine vamp_discard_integral (g [, num_calls]
    [, stratified] [, quadrupole] [, covariance] [, exc])

subroutine vamp_reshape_grid (g [, num_calls] [, num_div]
    [, stratified] [, quadrupole] [, covariance] [, exc])

subroutine vamp_sample_grid (rng, g, func, iterations
    [, integral] [, std_dev] [, avg_chi2] [, accuracy] [, channel]
    [, weights] [, exc] [, history])

    func

     $S_n$  with  $n = \text{iterations}$ 

subroutine vamp_sample_grid0 (rng, g, func, [, channel]
    [, weights] [, exc])

    func

     $S_0$ 

subroutine vamp_refine_grid (g, [, exc])

    r

subroutine vamp_average_iterations (g, iteration, integral,
    std_dev, avg_chi2)

    integer, intent(in) :: iteration
    Number of iterations so far (needed for  $\chi^2$ ).

subroutine vamp_integrate (g, func, calls [, integral]
    [, std_dev] [, avg_chi2] [, accuracy] [, covariance])

    type(vamp_grid), intent(inout) :: g
    func
```

```

integer, dimension(:,:), intent(in) :: calls

subroutine vamp_integratex (region, func, calls [, integral]
[, std_dev] [, avg_chi2] [, stratified] [, accuracy] [, pancake]
[, cigar])

real, dimension(:,:), intent(in) :: region
func
integer, dimension(:,:), intent(in) :: calls
integer, intent(in) :: pancake
integer, intent(in) :: cigar

subroutine vamp_copy_grid (lhs, rhs)

type(vamp_grid), intent(inout) :: lhs
type(vamp_grid), intent(in) :: rhs

subroutine vamp_delete_grid (g)

type(vamp_grid), intent(inout) :: g

```

4.2.4 *Inout/Output and Marshling*

```

subroutine vamp_write_grid (g, [, ...])

type(vamp_grid), intent(inout) :: g

subroutine vamp_read_grid (g, [, ...])

type(vamp_grid), intent(inout) :: g

subroutine vamp_write_grids (g, [, ...])

type(vamp_grids), intent(inout) :: g

subroutine vamp_read_grids (g, [, ...])

type(vamp_grids), intent(inout) :: g

pure subroutine vamp_marshall_grid (g, integer_buffer,
double_buffer)

```

```

type(vamp_grid), intent(in) :: g
integer, dimension(:), intent(inout) ::
    integer_buffer
real(kind=default), dimension(:), intent(inout)
    :: double_buffer

```

Marshal the grid `g` in the integer array `integer_buffer` and the real array `double_buffer`, which must have at least the sizes obtained from call `vamp_marshall_grid_size (g, integer_size, double_size)`.



Note that we can not use the `transfer` intrinsic function for marshalling types that contain pointers that substitute for allocatable array components. `transfer` would copy the pointers in this case and not where they point to!

```

pure subroutine vamp_marshall_grid_size (g, integer_size,
    double_size)

```

```

type(vamp_grid), intent(in) :: g
integer :: words

```

Compute the sizes of the arrays required for marshaling the grid `g`.

```

pure subroutine vamp_unmarshal_grid (g, integer_buffer,
    double_buffer)

```

```

type(vamp_grid), intent(inout) :: g
integer, dimension(:), intent(in) ::
    integer_buffer
real(kind=default), dimension(:), intent(in) ::
    double_buffer

```

Marshaling and unmarshaling need to use two separate buffers for integers and floating point numbers. In a homogeneous network, the intrinsic procedure `transfer` could be used to store the floating point numbers in the integer array. In a heterogeneous network this will fail. However, message passing environments provide methods for sending floating point numbers. For example, here's how to send a grid from process 0 to process 1 in MPI [12]

```

29 <MPI communication example 29>≡
    call vamp_marshall_grid_size (g, isize, dsize)

```

```

allocate (ibuf(isize), dbuf(dsize))
call mpi_comm_rank (MPI_COMM_WORLD, proc_id, errno)
select case (proc_id)
  case (0)
    call vamp_marshall_grid (g, ibuf, dbuf)
    call mpi_send (ibuf, size (ibuf), MPI_INTEGER, &
                  1, 1, MPI_COMM_WORLD, errno)
    call mpi_send (dbuf, size (dbuf), MPI_DOUBLE_PRECISION, &
                  1, 2, MPI_COMM_WORLD, errno)
  case (1)
    call mpi_recv (ibuf, size (ibuf), MPI_INTEGER, &
                  0, 1, MPI_COMM_WORLD, status, errno)
    call mpi_recv (dbuf, size (dbuf), MPI_DOUBLE_PRECISION, &
                  0, 2, MPI_COMM_WORLD, status, errno)
    call vamp_unmarshal_grid (g, ibuf, dbuf)
end select

```

assuming that double is such that MPI_DOUBLE_PRECISION corresponds to real(kind=default). The module `vampi` provides two high level functions `vamp_send_grid` and `vamp_receive_grid` that handle the low level details:

```

30  <MPI communication example' 30>≡
    call mpi_comm_rank (MPI_COMM_WORLD, proc_id, errno)
    select case (proc_id)
      case (0)
        call vamp_send_grid (g, 1, 0)
      case (1)
        call vamp_receive_grid (g, 0, 0)
    end select

    subroutine vamp_marshall_history_size (g, [, ...])

      type(vamp_grid), intent(inout) :: g

    subroutine vamp_marshall_history (g, [, ...])

      type(vamp_grid), intent(inout) :: g

    subroutine vamp_unmarshal_history (g, [, ...])

      type(vamp_grid), intent(inout) :: g

```

4.2.5 Multi Channel Procedures

$$g \circ \phi_i = \left| \frac{\partial \phi_i}{\partial x} \right|^{-1} \left(\alpha_i g_i + \sum_{\substack{j=1 \\ j \neq i}}^{N_c} \alpha_j (g_j \circ \pi_{ij}) \left| \frac{\partial \pi_{ij}}{\partial x} \right| \right). \quad (4.4)$$

31a \langle Interface declaration for phi 31a $\rangle \equiv$ (113 115 116b 136c 182c)

```
interface
  pure function phi (xi, channel) result (x)
    use kinds
    real(kind=default), dimension(:), intent(in) :: xi
    integer, intent(in) :: channel
    real(kind=default), dimension(size(xi)) :: x
  end function phi
end interface
```

31b \langle Interface declaration for ihp 31b $\rangle \equiv$ (113b)

```
interface
  pure function ihp (x, channel) result (xi)
    use kinds
    real(kind=default), dimension(:), intent(in) :: x
    integer, intent(in) :: channel
    real(kind=default), dimension(size(x)) :: xi
  end function ihp
end interface
```

31c \langle Interface declaration for jacobian 31c $\rangle \equiv$ (113)

```
interface
  pure function jacobian (x, data, channel) result (j)
    use kinds
    use vamp_grid_type !NODEP!
    import vamp_data_t
    real(kind=default), dimension(:), intent(in) :: x
    class(vamp_data_t), intent(in) :: data
    integer, intent(in) :: channel
    real(kind=default) :: j
  end function jacobian
end interface
```

```
function vamp_multi_channel (func, phi, ihp, jacobian, x,
  weights1, grids)
```

```
  real(kind=default), dimension(:), intent(in) :: x
```



```

real(kind=default), dimension(:), intent(in) ::
    weights
integer, intent(in) :: channel
type(vamp_grid), dimension(:), intent(in) :: grids

function vamp_multi_channel0 (func, phi, jacobian, x,
    weights1)

    real(kind=default), dimension(:), intent(in) :: x
    real(kind=default), dimension(:), intent(in) ::
        weights
    integer, intent(in) :: channel

subroutine vamp_check_jacobian (rng, n, channel, region,
    delta, [, x_delta])

    type(tao_random_state), intent(inout) :: rng
    integer, intent(in) :: n
    integer, intent(in) :: channel
    real(kind=default), dimension(:,:), intent(in) ::
        region
    real(kind=default), intent(out) :: delta
    real(kind=default), dimension(:), intent(out),
        optional :: x_delta

```

Verify that

$$g(\phi(x)) = \frac{1}{\left| \frac{\partial \phi}{\partial x} \right| (x)} \quad (4.5)$$

```

subroutine vamp_copy_grids (lhs, rhs)

    type(vamp_grids), intent(inout) :: lhs
    type(vamp_grids), intent(in) :: rhs

subroutine vamp_delete_grids (g)

    type(vamp_grids), intent(inout) :: g

```

```

subroutine vamp_create_grids (g, domain, num_calls, weights
[, maps] [, stratified])

    type(vamp_grids), intent(inout) :: g
    real, dimension(:,:), intent(in) :: domain
    integer, intent(in) :: num_calls
    real, dimension(:), intent(in) :: weights
    real, dimension(:,:,:), intent(in) :: maps

subroutine vamp_create_empty_grids (g)

    type(vamp_grids), intent(inout) :: g

subroutine vamp_discard_integrals (g [, num_calls]
[, stratified])

    type(vamp_grids), intent(inout) :: g
    integer, intent(in) :: num_calls

subroutine vamp_refine_weights (g [, power)

    type(vamp_grids), intent(inout) :: g
    real, intent(in) :: power

subroutine vamp_update_weights (g, weights [, num_calls]
[, stratified])

    type(vamp_grids), intent(inout) :: g
    real, dimension(:), intent(in) :: weights
    integer, intent(in) :: num_calls

subroutine vamp_reshape_grids (g, num_calls [, stratified])

    type(vamp_grids), intent(inout) :: g
    integer, intent(in) :: num_calls

subroutine vamp_reduce_channels (g, [, ...])

    type(vamp_grid), intent(inout) :: g

```

```

subroutine vamp_sample_grids (g, func, iterations [, integral]
    [, std_dev] [, accuracy] [, covariance] [, variance])

    type(vamp_grids), intent(inout) :: g
    func
    integer, intent(in) :: iterations

function vamp_sum_channels (x, weights, func)

    real, dimension(:), intent(in) :: x
    real, dimension(:), intent(in) :: weights
    func

```

4.2.6 Event Generation

```

subroutine vamp_next_event (g, [, ...])

subroutine vamp_warmup_grid (g, [, ...])

    type(vamp_grid), intent(inout) :: g
    func
    integer, intent(in) :: iterations

subroutine vamp_warmup_grids (g, [, ...])

    type(vamp_grids), intent(inout) :: g
    func
    integer, intent(in) :: iterations

```

4.2.7 Parallelization

```

subroutine vamp_fork_grid (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_join_grid (g, [, ...])

    type(vamp_grid), intent(inout) :: g

```

```

subroutine vamp_fork_grid_joints (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_sample_grid_parallel (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_distribute_work (g, [, ...])

    type(vamp_grid), intent(inout) :: g

```

4.2.8 Diagnostics

```

subroutine vamp_create_history (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_copy_history (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_delete_history (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_terminate_history (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_get_history (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_get_history_single (g, [, ...])

    type(vamp_grid), intent(inout) :: g

subroutine vamp_print_history (g, [, ...])

    type(vamp_grid), intent(inout) :: g

```



Discuss why the value of the integral in each channel differs.

4.2.9 *Other Procedures*

```
subroutine vamp_rigid_divisions (g, [, ...])  
    type(vamp_grid), intent(inout) :: g  
function vamp_get_covariance (g, [, ...])  
    type(vamp_grid), intent(inout) :: g  
subroutine vamp_nullify_covariance (g, [, ...])  
    type(vamp_grid), intent(inout) :: g  
function vamp_get_variance (g, [, ...])  
    type(vamp_grid), intent(inout) :: g  
subroutine vamp_nullify_variance (g, [, ...])  
    type(vamp_grid), intent(inout) :: g
```

4.2.10 *(Currently) Undocumented Procedures*

```
subroutine (... , [, ...])  
function (... , [, ...])
```

—5—

IMPLEMENTATION

5.1 *The Abstract Datatype `division`*

```
37a <divisions.f90 37a>≡
! divisions.f90 --
<Copyleft notice 1>
module divisions
  use kinds
  use exceptions
  use vamp_stat
  use utils
  use iso_fortran_env
  implicit none
  private
  <Declaration of divisions procedures 38a>
  <Interfaces of divisions procedures 61a>
  <Variables in divisions 46a>
  <Declaration of divisions types 37b>
  <Constants in divisions 64b>
contains
  <Implementation of divisions procedures 38b>
end module divisions
```

 `vamp_apply_equivalences` from `vamp` accesses %variance ...

```
37b <Declaration of divisions types 37b>≡ (37a) 57e▷
type, public :: division_t
! private
!!! Avoiding a g95 bug
real(kind=default), dimension(:), pointer :: x => null ()
real(kind=default), dimension(:), pointer :: integral => null ()
```

```

real(kind=default), dimension(:), pointer &
                                :: variance => null ()
!                                public :: variance => null ()
! real(kind=default), dimension(:), pointer :: efficiency => null ()
real(kind=default) :: x_min, x_max
real(kind=default) :: x_min_true, x_max_true
real(kind=default) :: dx, dxg
integer :: ng = 0
logical :: stratified = .true.
end type division_t

```

5.1.1 Creation, Manipulation & Injection

38a *<Declaration of divisions procedures 38a>*≡ (37a) 43a>

```

public :: create_division, create_empty_division
public :: copy_division, delete_division
public :: set_rigid_division, reshape_division

```

38b *<Implementation of divisions procedures 38b>*≡ (37a) 39a>

```

elemental subroutine create_division &
    (d, x_min, x_max, x_min_true, x_max_true)
type(division_t), intent(out) :: d
real(kind=default), intent(in) :: x_min, x_max
real(kind=default), intent(in), optional :: x_min_true, x_max_true
allocate (d%x(0:1), d%integral(1), d%variance(1))
! allocate (d%efficiency(1))
d%x(0) = 0.0
d%x(1) = 1.0
d%x_min = x_min
d%x_max = x_max
d%dx = d%x_max - d%x_min
d%stratified = .false.
d%ng = 1
d%dxg = 1.0 / d%ng
if (present (x_min_true)) then
    d%x_min_true = x_min_true
else
    d%x_min_true = x_min
end if
if (present (x_max_true)) then
    d%x_max_true = x_max_true
else
    d%x_max_true = x_max

```

```

end if
end subroutine create_division

```

39a *<Implementation of divisions procedures 38b>+≡* (37a) <38b 39b>

```

elemental subroutine create_empty_division (d)
  type(division_t), intent(out) :: d
  nullify (d%x, d%integral, d%variance)
! nullify (d%efficiency)
end subroutine create_empty_division

```

39b *<Implementation of divisions procedures 38b>+≡* (37a) <39a 39c>

```

elemental subroutine set_rigid_division (d, ng)
  type(division_t), intent(inout) :: d
  integer, intent(in) :: ng
  d%stratified = ng > 1
  d%ng = ng
  d%dxg = real (ubound (d%x, dim=1), kind=default) / d%ng
end subroutine set_rigid_division

```

$$dxg = \frac{n_{div}}{n_g} \quad (5.1)$$

such that $0 < \text{cell} \cdot dxg < n_{div}$

39c *<Implementation of divisions procedures 38b>+≡* (37a) <39b 43b>

```

elemental subroutine reshape_division (d, max_num_div, ng, use_variance)
  type(division_t), intent(inout) :: d
  integer, intent(in) :: max_num_div
  integer, intent(in), optional :: ng
  logical, intent(in), optional :: use_variance
  real(kind=default), dimension(:), allocatable :: old_x, m
  integer :: num_div, equ_per_adap
  if (present (ng)) then
    if (max_num_div > 1) then
      d%stratified = ng > 1
    else
      d%stratified = .false.
    end if
  else
    d%stratified = .false.
  end if
  if (d%stratified) then
    d%ng = ng
    <Initialize stratified sampling 42>

```



```

else
    num_div = max_num_div
    d%ng = 1
end if
d%dxg = real (num_div, kind=default) / d%ng
allocate (old_x(0:ubound(d%x,dim=1)), m(ubound(d%x,dim=1)))
old_x = d%x
⟨Set m to (1,1,...) or to rebinning weights from d%variance 40a⟩
⟨Resize arrays, iff necessary 40b⟩
d%x = rebin (m, old_x, num_div)
deallocate (old_x, m)
end subroutine reshape_division

```

40a $\langle \text{Set } m \text{ to } (1,1,\dots) \text{ or to rebinning weights from } d\%\text{variance } 40a \rangle \equiv$ (39c)

```

if (present (use_variance)) then
    if (use_variance) then
        m = rebinning_weights (d%variance)
    else
        m = 1.0
    end if
else
    m = 1.0
end if

```

40b $\langle \text{Resize arrays, iff necessary } 40b \rangle \equiv$ (39c)

```

if (ubound (d%x, dim=1) /= num_div) then
    deallocate (d%x, d%integral, d%variance)
    ! deallocate (d%efficiency)
    allocate (d%x(0:num_div), d%integral(num_div), d%variance(num_div))
    ! allocate (d%efficiency(num_div))
end if

```

Genuinely stratified sampling will superimpose an equidistant grid on the adaptive grid, as shown in figure 5.2. Obviously, this is only possible when the number of cells of the stratification grid is large enough, specifically when $n_g \geq n_{\text{div}}^{\min} = n_{\text{div}}^{\max}/2 = 25$. This condition can be met by a high number of sampling points or by a low dimensionality of the integration region (cf. table 5.1).

For a low number of sampling points and high dimensions, genuinely stratified sampling is impossible, because we would have to reduce the number n_{div} of adaptive divisions too far. Instead, we keep `stratified` false which will tell the integration routine not to concentrate the grid in the regions where the contribution to the error is largest, but to use importance sampling,

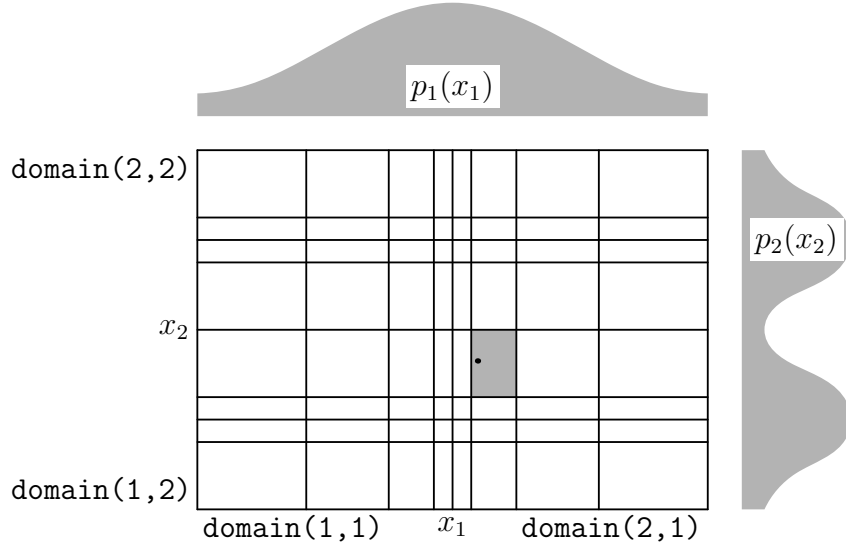


Figure 5.1: **vegas** grid structure for non-stratified sampling. N.B.: the grid and the weight functions $p_{1,2}$ are only in qualitative agreement.



Figure 5.2: **vegas** grid structure for genuinely stratified sampling, which is used in low dimensions. N.B.: the grid and the weight functions $p_{1,2}$ are only in qualitative agreement.

n_{dim}	$N_{\text{calls}}^{\text{max}}(n_g = 25)$
2	$1 \cdot 10^3$
3	$3 \cdot 10^4$
4	$8 \cdot 10^5$
5	$2 \cdot 10^7$
6	$5 \cdot 10^8$

Table 5.1: To stratify or not to stratify.

i. e. concentrating the grid in the regions where the contribution to the value is largest.

In this case, the rigid grid is much coarser than the adaptive grid and furthermore, the boundaries of the cells overlap in general. The interplay of the two grids during the sampling process is shown in figure 5.3. First we determine the (integer) number k of equidistant divisions of an adaptive cell for at most $n_{\text{div}}^{\text{max}}$ divisions of the adaptive grid

$$k = \left\lfloor \frac{n_g}{n_{\text{div}}^{\text{max}}} \right\rfloor + 1 \quad (5.2a)$$

and the corresponding number n_{div} of adaptive divisions

$$n_{\text{div}} = \left\lfloor \frac{n_g}{k} \right\rfloor \quad (5.2b)$$

Finally, adjust n_g to an exact multiple of n_{div}

$$n_g = k \cdot n_{\text{div}} \quad (5.2c)$$

42 *⟨Initialize stratified sampling 42⟩*≡ (39c)

```

if (d%ng >= max_num_div / 2) then
  d%stratified = .true.
  equ_per_adap = d%ng / max_num_div + 1
  num_div = d%ng / equ_per_adap
  if (num_div < 2) then
    d%stratified = .false.
    num_div = 2
    d%ng = 1
  else if (mod (num_div,2) == 1) then
    num_div = num_div - 1
    d%ng = equ_per_adap * num_div
  else
    d%ng = equ_per_adap * num_div

```



Figure 5.3: One-dimensional illustration of the **vegas** grid structure for pseudo stratified sampling, which is used in high dimensions.

```

    end if
  else
    d%stratified = .false.
    num_div = max_num_div
    d%ng = 1
  end if

```

Figure 5.3 on page 43 is a one-dimensional illustration of the sampling algorithm. In each cell of the rigid equidistant grid, two random points are selected (or N_{calls} in the not stratified case). For each point, the corresponding cell and relative coordinate in the adaptive grid is found, *as if the adaptive grid was equidistant* (upper arrow). Then this point is mapped according to the adapted grid (lower arrow) and the proper Jacobians are applied to the weight.

$$\prod_{j=1}^n (x_i^j - x_{i-1}^j) \cdot N^n = \text{Vol}(\text{cell}') \cdot \frac{1}{\text{Vol}(\text{cell})} = \frac{1}{p(x_i^j)} \quad (5.3)$$

```

43a  <Declaration of divisions procedures 38a>+≡ (37a) <38a 44c>
      public :: inject_division, inject_division_short

43b  <Implementation of divisions procedures 38b>+≡ (37a) <39c 44b>
      elemental subroutine inject_division (d, r, cell, x, x_mid, idx, wgt)
        type(division_t), intent(in) :: d
        real(kind=default), intent(in) :: r
        integer, intent(in) :: cell
        real(kind=default), intent(out) :: x, x_mid
        integer, intent(out) :: idx
        real(kind=default), intent(out) :: wgt
        real(kind=default) :: delta_x, xi
        integer :: i
        xi = (cell - r) * d%dxg + 1.0
        <Set i, delta_x, x, and wgt from xi 44a>

```

```

    idx = i
    x_mid = d%x_min + 0.5 * (d%x(i-1) + d%x(i)) * d%dx
end subroutine inject_division

```

44a \langle Set i, delta_x, x, and wgt from xi 44a $\rangle \equiv$ (43b 44b)

```

    i = max (min (int (xi), ubound (d%x, dim=1)), 1)
    delta_x = d%x(i) - d%x(i-1)
    x = d%x_min + (d%x(i-1) + (xi - i) * delta_x) * d%dx
    wgt = delta_x * ubound (d%x, dim=1)

```

44b \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 43b 44d \rangle

```

    elemental subroutine inject_division_short (d, r, x, idx, wgt)
        type(division_t), intent(in) :: d
        real(kind=default), intent(in) :: r
        integer, intent(out) :: idx
        real(kind=default), intent(out) :: x, wgt
        real(kind=default) :: delta_x, xi
        integer :: i
        xi = r * ubound (d%x, dim=1) + 1.0
         $\langle$ Set i, delta_x, x, and wgt from xi 44a $\rangle$ 
        idx = i
    end subroutine inject_division_short

```

5.1.2 Grid Refinement

44c \langle Declaration of divisions procedures 38a $\rangle + \equiv$ (37a) \langle 43a 45c \rangle

```

    public :: record_integral, record_variance, clear_integral_and_variance
    ! public :: record_efficiency

```

44d \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 44b 44e \rangle

```

    elemental subroutine record_integral (d, i, f)
        type(division_t), intent(inout) :: d
        integer, intent(in) :: i
        real(kind=default), intent(in) :: f
        d%integral(i) = d%integral(i) + f
        if (.not. d%stratified) then
            d%variance(i) = d%variance(i) + f*f
        end if
    end subroutine record_integral

```

44e \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 44d 45b \rangle

```

    elemental subroutine record_variance (d, i, var_f)

```

```

type(division_t), intent(inout) :: d
integer, intent(in) :: i
real(kind=default), intent(in) :: var_f
if (d%stratified) then
    d%variance(i) = d%variance(i) + var_f
end if
end subroutine record_variance

```

45a \langle Implementation of divisions procedures (removed from WHIZARD) 45a $\rangle \equiv$ 60b \rangle

```

elemental subroutine record_efficiency (d, i, eff)
    type(division_t), intent(inout) :: d
    integer, intent(in) :: i
    real(kind=default), intent(in) :: eff
    ! d%efficiency(i) = d%efficiency(i) + eff
end subroutine record_efficiency

```

45b \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 44e 45d \rangle

```

elemental subroutine clear_integral_and_variance (d)
    type(division_t), intent(inout) :: d
    d%integral = 0.0
    d%variance = 0.0
    ! d%efficiency = 0.0
end subroutine clear_integral_and_variance

```

45c \langle Declaration of divisions procedures 38a $\rangle + \equiv$ (37a) \langle 44c 47a \rangle

```

public :: refine_division

```

45d \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 45b 46b \rangle

```

elemental subroutine refine_division (d)
    type(division_t), intent(inout) :: d
    character(len=*), parameter :: FN = "refine_division"
    d%x = rebin (rebinning_weights (d%variance), d%x, size (d%variance))
end subroutine refine_division

```

Smooth the $d_i = \bar{f}_i \Delta x_i$

$$\begin{aligned}
 d_1 &\rightarrow \frac{1}{2}(d_1 + d_2) \\
 d_2 &\rightarrow \frac{1}{3}(d_1 + d_2 + d_3) \\
 &\dots \\
 d_{n-1} &\rightarrow \frac{1}{3}(d_{n-2} + d_{n-1} + d_n) \\
 d_n &\rightarrow \frac{1}{2}(d_{n-1} + d_n)
 \end{aligned} \tag{5.4}$$

As long as the initial `num_div` ≥ 6 , we know that `num_div` ≥ 3 .

46a *<Variables in divisions 46a>* \equiv (37a) 59a>
`integer, private, parameter :: MIN_NUM_DIV = 3`

Here the Fortran90 array notation really shines, but we have to handle the cases `nd` ≤ 2 specially, because the `quadrupole` option can lead to small `nds`. The equivalent Fortran77 code [2] is orders of magnitude less obvious ¹ Also protect against vanishing d_i that will blow up the logarithm.

$$m_i = \left(\frac{\frac{\bar{f}_i \Delta x_i}{\sum_j \bar{f}_j \Delta x_j} - 1}{\ln \left(\frac{\bar{f}_i \Delta x_i}{\sum_j \bar{f}_j \Delta x_j} \right)} \right)^\alpha \quad (5.5)$$

46b *<Implementation of divisions procedures 38b>* \equiv (37a) <45d 47c>
`pure function rebinning_weights (d) result (m)`
`real(kind=default), dimension(:), intent(in) :: d`
`real(kind=default), dimension(size(d)) :: m`
`real(kind=default), dimension(size(d)) :: smooth_d`
`real(kind=default), parameter :: ALPHA = 1.5`
`integer :: nd`
`<Bail out if any (d == NaN) 47b>`
`nd = size (d)`
`if (nd > 2) then`
`smooth_d(1) = (d(1) + d(2)) / 2.0`
`smooth_d(2:nd-1) = (d(1:nd-2) + d(2:nd-1) + d(3:nd)) / 3.0`
`smooth_d(nd) = (d(nd-1) + d(nd)) / 2.0`
`else`
`smooth_d = d`
`end if`
`if (all (smooth_d < tiny (1.0_default))) then`
`m = 1.0_default`
`else`
`smooth_d = smooth_d / sum (smooth_d)`
`where (smooth_d < tiny (1.0_default))`
`smooth_d = tiny (1.0_default)`
`end where`
`where (smooth_d /= 1._default)`
`m = ((smooth_d - 1.0) / (log (smooth_d)))**ALPHA`
`elsewhere`
`m = 1.0_default`
`endwhere`
`end if`
`end function rebinning_weights`

¹Some old timers call this a feature, however.

47a \langle Declaration of divisions procedures 38a $\rangle + \equiv$ (37a) \triangleleft 45c 47d \triangleright
 private :: rebinning_weights



The NaN test is probably not portable:

47b \langle Bail out if any (d == NaN) 47b $\rangle \equiv$ (46b)
 if (any (d /= d)) then
 m = 1.0
 return
 end if

Take a binning x and return a new binning with num_div bins with the m homogeneously distributed:

47c \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \triangleleft 46b 48c \triangleright
 pure function rebin (m, x, num_div) result (x_new)
 real(kind=default), dimension(:), intent(in) :: m
 real(kind=default), dimension(0:), intent(in) :: x
 integer, intent(in) :: num_div
 real(kind=default), dimension(0:num_div) :: x_new
 integer :: i, k
 real(kind=default) :: step, delta
 step = sum (m) / num_div
 k = 0
 delta = 0.0
 x_new(0) = x(0)
 do i = 1, num_div - 1
 \langle Increment k until $\sum m_k \geq \Delta$ and keep the surplus in δ 47e \rangle
 \langle Interpolate the new x_i from x_k and δ 48a \rangle
 end do
 x_new(num_div) = 1.0
 end function rebin

47d \langle Declaration of divisions procedures 38a $\rangle + \equiv$ (37a) \triangleleft 47a 48b \triangleright
 private :: rebin

We increment k until another Δ (a.k.a. step) of the integral has been accumulated (cf. figure 5.4). The mismatch will be corrected below.

47e \langle Increment k until $\sum m_k \geq \Delta$ and keep the surplus in δ 47e $\rangle \equiv$ (47c)
 do
 if (step <= delta) then
 exit
 end if
 k = k + 1
 delta = delta + m(k)
 end do
 delta = delta - step

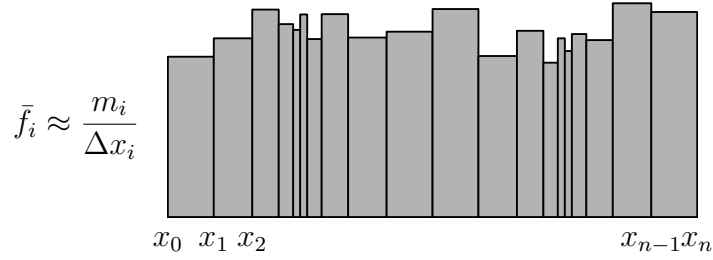


Figure 5.4: Typical weights used in the rebinning algorithm.

$$48a \quad \langle \text{Interpolate the new } x_i \text{ from } x_k \text{ and } \delta \text{ 48a} \rangle \equiv \quad (47c)$$

$$\mathbf{x_new}(i) = \mathbf{x}(k) - (\mathbf{x}(k) - \mathbf{x}(k-1)) * \mathbf{delta} / \mathbf{m}(k)$$

5.1.3 Probability Density

$$48b \quad \langle \text{Declaration of divisions procedures 38a} \rangle \equiv \quad (37a) \quad \langle 47d \quad 49a \rangle$$

$$\text{public} :: \text{probability}$$

$$\xi = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \in [0, 1] \quad (5.6)$$

and

$$\int_{x_{\min}}^{x_{\max}} dx p(x) = 1 \quad (5.7)$$

$$48c \quad \langle \text{Implementation of divisions procedures 38b} \rangle \equiv \quad (37a) \quad \langle 47c \quad 49b \rangle$$

```

elemental function probability (d, x) result (p)
  type(division_t), intent(in) :: d
  real(kind=default), intent(in) :: x
  real(kind=default) :: p
  real(kind=default) :: xi
  integer :: hi, mid, lo
  xi = (x - d%x_min) / d%dx
  if ((xi >= 0) .and. (xi <= 1)) then
    lo = lbound (d%x, dim=1)
    hi = ubound (d%x, dim=1)
    bracket: do
      if (lo >= hi - 1) then
        p = 1.0 / (ubound (d%x, dim=1) * d%dx * (d%x(hi) - d%x(hi-1)))
        return
      end if
      mid = (hi + lo) / 2
      if (xi > d%x(mid)) then

```

```

        lo = mid
    else
        hi = mid
    end if
end do bracket
else
    p = 0
end if
end function probability

```

5.1.4 *Quadrupole*

49a *<Declaration of divisions procedures 38a>+≡* (37a) <48b 49c>
 public :: **quadrupole_division**

49b *<Implementation of divisions procedures 38b>+≡* (37a) <48c 49d>
 elemental function **quadrupole_division** (d) result (q)
 type(**division_t**), intent(in) :: d
 real(kind=default) :: q
 !!! q = **value_spread_percent** (**rebinning_weights** (d%variance))
 q = **standard_deviation_percent** (**rebinning_weights** (d%variance))
 end function **quadrupole_division**

5.1.5 *Forking and Joining*

The goal is to split a division in such a way, that we can later sample the pieces separately and combine the results.

49c *<Declaration of divisions procedures 38a>+≡* (37a) <49a 54b>
 public :: **fork_division**, **join_division**, **sum_division**



Caveat emptor: splitting divisions can lead to **num_div** < 3 and the application *must not* try to refine such grids before merging them again!

49d *<Implementation of divisions procedures 38b>+≡* (37a) <49b 50>
 pure **subroutine fork_division** (d, ds, sum_calls, **num_calls**, exc)
 type(**division_t**), intent(in) :: d
 type(**division_t**), dimension(:), intent(inout) :: ds
 integer, intent(in) :: sum_calls
 integer, dimension(:), intent(inout) :: **num_calls**
 type(**exception**), intent(inout), optional :: exc
 character(len=*), parameter :: FN = "**fork_division**"
 integer, dimension(size(ds)) :: n0, n1
 integer, dimension(0:size(ds)) :: n, ds_ng

```

integer :: i, j, num_div, num_forks, nx
real(kind=default), dimension(:), allocatable :: d_x, d_integral, d_variance
! real(kind=default), dimension(:), allocatable :: d_efficiency
num_div = ubound (d%x, dim=1)
num_forks = size (ds)
if (d%ng == 1) then
    ⟨Fork an importance sampling division 51a⟩
else if (num_div >= num_forks) then
    if (modulo (d%ng, num_div) == 0) then
        ⟨Fork a pure stratified sampling division 51d⟩
    else
        ⟨Fork a pseudo stratified sampling division 53⟩
    end if
else
    if (present (exc)) then
        call raise_exception (exc, EXC_FATAL, FN, "internal error")
    end if
    num_calls = 0
end if
end subroutine fork_division

```

50 *⟨Implementation of divisions procedures 38b⟩*+≡ (37a) <49d 54c>

```

pure subroutine join_division (d, ds, exc)
type(division_t), intent(inout) :: d
type(division_t), dimension(:), intent(in) :: ds
type(exception), intent(inout), optional :: exc
character(len=*), parameter :: FN = "join_division"
integer, dimension(size(ds)) :: n0, n1
integer, dimension(0:size(ds)) :: n, ds_ng
integer :: i, j, num_div, num_forks, nx
real(kind=default), dimension(:), allocatable :: d_x, d_integral, d_variance
! real(kind=default), dimension(:), allocatable :: d_efficiency
num_div = ubound (d%x, dim=1)
num_forks = size (ds)
if (d%ng == 1) then
    ⟨Join importance sampling divisions 51b⟩
else if (num_div >= num_forks) then
    if (modulo (d%ng, num_div) == 0) then
        ⟨Join pure stratified sampling divisions 52a⟩
    else
        ⟨Join pseudo stratified sampling divisions 54a⟩
    end if
else
    if (present (exc)) then

```

```

        call raise_exception (exc, EXC_FATAL, FN, "internal error")
    end if
end if
end subroutine join_division

```

Importance Sampling

Importance sampling ($d\%ng == 1$) is trivial, since we can just sample `size(ds)` copies of the same grid with (almost) the same number of points

51a \langle Fork an importance sampling division 51a $\rangle \equiv$ (49d)

```

    if (d%stratified) then
        call raise_exception (exc, EXC_FATAL, FN, &
                               "ng == 1 incompatible w/ stratification")
    else
        call copy_division (ds, d)
        num_calls(2:) = ceiling (real (sum_calls) / num_forks)
        num_calls(1) = sum_calls - sum (num_calls(2:))
    end if

```

and sum up the results in the end:

51b \langle Join importance sampling divisions 51b $\rangle \equiv$ (50)

```

    call sum_division (d, ds)

```

Note, however, that this is only legitimate as long as $d\%ng == 1$ implies $d\%stratified == .false.$, because otherwise the sampling code would be incorrect (cf. `var_f` on page 89).

Stratified Sampling

For stratified sampling, we have to work a little harder, because there are just two points per cell and we have to slice along the lines of the stratification grid. Actually, we are slicing along the adaptive grid, since it has a reasonable size. Slicing along the stratification grid could be done using the method below. However, in this case *very* large adaptive grids would be shipped from one process to the other and the communication costs will outweigh the gains from parallel processing.

51c \langle Setup to fork a pure stratified sampling division 51c $\rangle \equiv$ (51d 52a)

```

    n = (num_div * (/ (j, j=0,num_forks) /)) / num_forks
    n0(1:num_forks) = n(0:num_forks-1)
    n1(1:num_forks) = n(1:num_forks)

```

51d \langle Fork a pure stratified sampling division 51d $\rangle \equiv$ (49d)

```

     $\langle$ Setup to fork a pure stratified sampling division 51c $\rangle$ 
    do i = 1, num_forks

```

```

    call copy_array_pointer (ds(i)%x, d%x(n0(i):n1(i)), lb = 0)
    call copy_array_pointer (ds(i)%integral, d%integral(n0(i)+1:n1(i)))
    call copy_array_pointer (ds(i)%variance, d%variance(n0(i)+1:n1(i)))
    ! call copy_array_pointer (ds(i)%efficiency, d%efficiency(n0(i)+1:n1(i)))
    ds(i)%x = (ds(i)%x - ds(i)%x(0)) / (d%x(n1(i)) - d%x(n0(i)))
end do
ds%x_min = d%x_min + d%dx * d%x(n0)
ds%x_max = d%x_min + d%dx * d%x(n1)
ds%dx = ds%x_max - ds%x_min
ds%x_min_true = d%x_min_true
ds%x_max_true = d%x_max_true
ds%stratified = d%stratified
ds%ng = (d%ng * (n1 - n0)) / num_div
num_calls = sum_calls ! this is a misnomer, it remains "calls per cell" here
ds%dxg = real (n1 - n0, kind=default) / ds%ng

```

Joining is the exact inverse, but we're only interested in `d%integral` and `d%variance` for the grid refinement:

52a $\langle \text{Join pure stratified sampling divisions 52a} \rangle \equiv$ (50)
 $\langle \text{Setup to fork a pure stratified sampling division 51c} \rangle$
do i = 1, num_forks
 d%integral(n0(i)+1:n1(i)) = ds(i)%integral
 d%variance(n0(i)+1:n1(i)) = ds(i)%variance
 ! d%efficiency(n0(i)+1:n1(i)) = ds(i)%efficiency
end do

Pseudo Stratified Sampling

The coarsest grid covering the division of n_g bins into n_f forks has $n_g / \gcd(n_f, n_g) = \text{lcm}(n_f, n_g) / n_f$ bins per fork. Therefore, we need

$$\text{lcm} \left(\frac{\text{lcm}(n_f, n_g)}{n_f}, n_x \right) \quad (5.8)$$

divisions of the adaptive grid (if n_x is the number of bins in the original adaptive grid).

Life would be much easier, if we knew that n_f divides n_g . However, this is hard to maintain in real life applications. We can try to achieve this if possible, but the algorithms must be prepared to handle the general case.

52b $\langle \text{Setup to fork a pseudo stratified sampling division 52b} \rangle \equiv$ (53 54a)
nx = lcm (d%ng / gcd (num_forks, d%ng), num_div)
ds_ng = (d%ng * (/ (j, j=0,num_forks) /)) / num_forks
n = (nx * ds_ng) / d%ng



Figure 5.5: Forking one dimension d of a grid into three parts $ds(1)$, $ds(2)$, and $ds(3)$. The picture illustrates the most complex case of pseudo stratified sampling (cf. fig. 5.3).

```

n0(1:num_forks) = n(0:num_forks-1)
n1(1:num_forks) = n(1:num_forks)

53  ⟨Fork a pseudo stratified sampling division 53⟩≡ (49d)
    ⟨Setup to fork a pseudo stratified sampling division 52b⟩
    allocate (d_x(0:nx), d_integral(nx), d_variance(nx))
    ! allocate (d_efficiency(nx))
    call subdivide (d_x, d%x)
    call distribute (d_integral, d%integral)
    call distribute (d_variance, d%variance)
    ! call distribute (d_efficiency, d%efficiency)
    do i = 1, num_forks
        call copy_array_pointer (ds(i)%x, d_x(n0(i):n1(i)), lb = 0)
        call copy_array_pointer (ds(i)%integral, d_integral(n0(i)+1:n1(i)))
        call copy_array_pointer (ds(i)%variance, d_variance(n0(i)+1:n1(i)))
        ! call copy_array_pointer (ds(i)%efficiency, d_efficiency(n0(i)+1:n1(i)))
        ds(i)%x = (ds(i)%x - ds(i)%x(0)) / (d_x(n1(i)) - d_x(n0(i)))
    end do
    ds%x_min = d%x_min + d%dx * d_x(n0)
    ds%x_max = d%x_min + d%dx * d_x(n1)

```

```

ds%dx = ds%x_max - ds%x_min
ds%x_min_true = d%x_min_true
ds%x_max_true = d%x_max_true
ds%stratified = d%stratified
ds%ng = ds_ng(1:num_forks) - ds_ng(0:num_forks-1)
num_calls = sum_calls ! this is a misnomer, it remains "calls per cell" here
ds%dxg = real (n1 - n0, kind=default) / ds%ng
deallocate (d_x, d_integral, d_variance)
! deallocate (d_efficiency)

54a <Join pseudo stratified sampling divisions 54a>≡ (50)
    <Setup to fork a pseudo stratified sampling division 52b>
    allocate (d_x(0:nx), d_integral(nx), d_variance(nx))
    ! allocate (d_efficiency(nx))
    do i = 1, num_forks
        d_integral(n0(i)+1:n1(i)) = ds(i)%integral
        d_variance(n0(i)+1:n1(i)) = ds(i)%variance
        ! d_efficiency(n0(i)+1:n1(i)) = ds(i)%efficiency
    end do
    call collect (d%integral, d_integral)
    call collect (d%variance, d_variance)
    ! call collect (d%efficiency, d_efficiency)
    deallocate (d_x, d_integral, d_variance)
    ! deallocate (d_efficiency)

54b <Declaration of divisions procedures 38a>+≡ (37a) <49c 55c>
    private :: subdivide
    private :: distribute
    private :: collect

54c <Implementation of divisions procedures 38b>+≡ (37a) <50 54d>
    pure subroutine subdivide (x, x0)
        real(kind=default), dimension(0:), intent(inout) :: x
        real(kind=default), dimension(0:), intent(in) :: x0
        integer :: i, n, n0
        n0 = ubound (x0, dim=1)
        n = ubound (x, dim=1) / n0
        x(0) = x0(0)
        do i = 1, n
            x(i:n) = x0(0:n0-1) * real (n - i) / n + x0(1:n0) * real (i) / n
        end do
    end subroutine subdivide

54d <Implementation of divisions procedures 38b>+≡ (37a) <54c 55a>
    pure subroutine distribute (x, x0)
        real(kind=default), dimension(:), intent(inout) :: x

```

```

    real(kind=default), dimension(:), intent(in) :: x0
    integer :: i, n
    n = ubound (x, dim=1) / ubound (x0, dim=1)
    do i = 1, n
        x(i:n) = x0 / n
    end do
end subroutine distribute

```

55a *<Implementation of divisions procedures 38b>+≡* (37a) <54d 55b>

```

pure subroutine collect (x0, x)
    real(kind=default), dimension(:), intent(inout) :: x0
    real(kind=default), dimension(:), intent(in) :: x
    integer :: i, n, n0
    n0 = ubound (x0, dim=1)
    n = ubound (x, dim=1) / n0
    do i = 1, n0
        x0(i) = sum (x((i-1)*n+1:i*n))
    end do
end subroutine collect

```

Trivia

55b *<Implementation of divisions procedures 38b>+≡* (37a) <55a 55d>

```

pure subroutine sum_division (d, ds)
    type(division_t), intent(inout) :: d
    type(division_t), dimension(:), intent(in) :: ds
    integer :: i
    d%integral = 0.0
    d%variance = 0.0
    ! d%efficiency = 0.0
    do i = 1, size (ds)
        d%integral = d%integral + ds(i)%integral
        d%variance = d%variance + ds(i)%variance
        ! d%efficiency = d%efficiency + ds(i)%efficiency
    end do
end subroutine sum_division

```

55c *<Declaration of divisions procedures 38a>+≡* (37a) <54b 56b>

```

public :: debug_division
public :: dump_division

```

55d *<Implementation of divisions procedures 38b>+≡* (37a) <55b 56a>

```

subroutine debug_division (d, prefix)
    type(division_t), intent(in) :: d
    character(len=*), intent(in) :: prefix

```



```

print "(1x,a,2(a,1x,i3,1x,f10.7))", prefix, ": d%x: ", &
    lbound(d%x,dim=1), d%x(lbound(d%x,dim=1)), &
    " ... ", &
    ubound(d%x,dim=1), d%x(ubound(d%x,dim=1))
print "(1x,a,2(a,1x,i3,1x,f10.7))", prefix, ": d%i: ", &
    lbound(d%integral,dim=1), d%integral(lbound(d%integral,dim=1)), &
    " ... ", &
    ubound(d%integral,dim=1), d%integral(ubound(d%integral,dim=1))
print "(1x,a,2(a,1x,i3,1x,f10.7))", prefix, ": d%v: ", &
    lbound(d%variance,dim=1), d%variance(lbound(d%variance,dim=1)), &
    " ... ", &
    ubound(d%variance,dim=1), d%variance(ubound(d%variance,dim=1))
! print "(1x,a,2(a,1x,i3,1x,f10.7))", prefix, ": d%e: ", &
!     lbound(d%efficiency,dim=1), d%efficiency(lbound(d%efficiency,dim=1)), &
!     " ... ", &
!     ubound(d%efficiency,dim=1), d%efficiency(ubound(d%efficiency,dim=1))
end subroutine debug_division

```

56a *<Implementation of divisions procedures 38b>+≡* (37a) *<55d 56c>*

```

subroutine dump_division (d, prefix)
    type(division_t), intent(in) :: d
    character(len=*), intent(in) :: prefix
! print "(2(1x,a),100(1x,f10.7))", prefix, ":x: ", d%x
    print "(2(1x,a),100(1x,f10.7))", prefix, ":x: ", d%x(1:)
    print "(2(1x,a),100(1x,e10.3))", prefix, ":i: ", d%integral
    print "(2(1x,a),100(1x,e10.3))", prefix, ":v: ", d%variance
! print "(2(1x,a),100(1x,e10.3))", prefix, ":e: ", d%efficiency
end subroutine dump_division

```

5.1.6 Inquiry

Trivial, but necessary for making divisions an abstract data type:

56b *<Declaration of divisions procedures 38a>+≡* (37a) *<55c 57f>*

```

public :: inside_division, stratified_division
public :: volume_division, rigid_division, adaptive_division

```

56c *<Implementation of divisions procedures 38b>+≡* (37a) *<56a 57a>*

```

elemental function inside_division (d, x) result (theta)
    type(division_t), intent(in) :: d
    real(kind=default), intent(in) :: x
    logical :: theta
    theta = (x >= d%x_min_true) .and. (x <= d%x_max_true)
end function inside_division

```

57a \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 56c 57b \rangle
 elemental function **stratified_division** (d) result (yorn)
 type(**division_t**), intent(in) :: d
 logical :: yorn
 yorn = d%**stratified**
 end function **stratified_division**

57b \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 57a 57c \rangle
 elemental function **volume_division** (d) result (vol)
 type(**division_t**), intent(in) :: d
 real(kind=default) :: vol
 vol = d%**dx**
 end function **volume_division**

57c \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 57b 57d \rangle
 elemental function **rigid_division** (d) result (n)
 type(**division_t**), intent(in) :: d
 integer :: n
 n = d%**ng**
 end function **rigid_division**

57d \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 57c 57g \rangle
 elemental function **adaptive_division** (d) result (n)
 type(**division_t**), intent(in) :: d
 integer :: n
 n = ubound (d%**x**, dim=1)
 end function **adaptive_division**

5.1.7 Diagnostics

57e \langle Declaration of divisions types 37b $\rangle + \equiv$ (37a) \langle 37b \rangle
 type, public :: **div_history**
 private
 logical :: **stratified**
 integer :: **ng**, **num_div**
 real(kind=default) :: x_min, x_max, x_min_true, x_max_true
 real(kind=default) :: &
 spread_f_p, **stddev_f_p**, **spread_p**, **stddev_p**, **spread_m**, **stddev_m**
 end type **div_history**

57f \langle Declaration of divisions procedures 38a $\rangle + \equiv$ (37a) \langle 56b 58a \rangle
 public :: **copy_history**, **summarize_division**

57g \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 57d 58b \rangle
 elemental function **summarize_division** (d) result (s)
 type(**division_t**), intent(in) :: d

```

type(div_history) :: s
real(kind=default), dimension(:), allocatable :: p, m
allocate (p(ubound(d%x,dim=1)), m(ubound(d%x,dim=1)))
p = probabilities (d%x)
m = rebinning_weights (d%variance)
s%ng = d%ng
s%num_div = ubound (d%x, dim=1)
s%stratified = d%stratified
s%x_min = d%x_min
s%x_max = d%x_max
s%x_min_true = d%x_min_true
s%x_max_true = d%x_max_true
s%spread_f_p = value_spread_percent (d%integral)
s%stddev_f_p = standard_deviation_percent (d%integral)
s%spread_p = value_spread_percent (p)
s%stddev_p = standard_deviation_percent (p)
s%spread_m = value_spread_percent (m)
s%stddev_m = standard_deviation_percent (m)
deallocate (p, m)
end function summarize_division

```

58a *<Declaration of divisions procedures 38a>+≡* (37a) <57f 59b>
private :: **probabilities**

58b *<Implementation of divisions procedures 38b>+≡* (37a) <57g 58c>
pure function **probabilities** (x) result (p)
real(kind=default), dimension(0:), intent(in) :: x
real(kind=default), dimension(ubound(x,dim=1)) :: p
integer :: num_div
num_div = ubound (x, dim=1)
p = 1.0 / (x(1:num_div) - x(0:num_div-1))
p = p / sum(p)
end function **probabilities**

58c *<Implementation of divisions procedures 38b>+≡* (37a) <58b 58d>
subroutine print_history (h, tag)
type(**div_history**), dimension(:), intent(in) :: h
character(len=*), intent(in), optional :: tag
call **write_history** (output_unit, h, tag)
flush (output_unit)
end **subroutine print_history**

58d *<Implementation of divisions procedures 38b>+≡* (37a) <58c 61b>
subroutine write_history (u, h, tag)
integer, intent(in) :: u
type(**div_history**), dimension(:), intent(in) :: h

```

character(len=*), intent(in), optional :: tag
character(len=BUFFER_SIZE) :: pfx
character(len=1) :: s
integer :: i
if (present (tag)) then
    pfx = tag
else
    pfx = "[vamp]"
end if
if ((minval (h%x_min) == maxval (h%x_min)) &
    .and. (minval (h%x_max) == maxval (h%x_max))) then
    write (u, "(1X,A11,1X,2X,1X,2(ES10.3,A4,ES10.3,A7))") pfx, &
        h(1)%x_min, " <= ", h(1)%x_min_true, &
        " < x < ", h(1)%x_max_true, " <= ", h(1)%x_max
else
    do i = 1, size (h)
        write (u, "(1X,A11,1X,I2,1X,2(ES10.3,A4,ES10.3,A7))") pfx, &
            i, h(i)%x_min, " <= ", h(i)%x_min_true, &
            " < x < ", h(i)%x_max_true, " <= ", h(i)%x_max
    end do
end if
write (u, "(1X,A11,1X,A2,2(1X,A3),A1,6(1X,A8))") pfx, &
    "it", "nd", "ng", "", &
    "spr(f/p)", "dev(f/p)", "spr(m)", "dev(m)", "spr(p)", "dev(p)"
iterations: do i = 1, size (h)
    if (h(i)%stratified) then
        s = "*"
    else
        s = ""
    end if
    write (u, "(1X,A11,1X,I2,2(1X,I3),A1,6(1X,F7.2,A1))") pfx, &
        i, h(i)%num_div, h(i)%ng, s, &
        h(i)%spread_f_p, "%", h(i)%stddev_f_p, "%", &
        h(i)%spread_m, "%", h(i)%stddev_m, "%", &
        h(i)%spread_p, "%", h(i)%stddev_p, "%"
end do iterations
flush (u)
end subroutine write_history

```

59a *<Variables in divisions 46a>*+≡ (37a) <46a 62a>

```
integer, private, parameter :: BUFFER_SIZE = 50
```

59b *<Declaration of divisions procedures 38a>*+≡ (37a) <58a 60f>

```
public :: print_history, write_history
```

- 60a *<Declaration of divisions procedures (removed from WHIZARD) 60a>*≡
 public :: **division_x**, **division_integral**
 public :: **division_variance**, **division_efficiency**
- 60b *<Implementation of divisions procedures (removed from WHIZARD) 45a>*+≡ <45a 60c>
 pure **subroutine division_x** (x, d)
 real(kind=default), dimension(:), pointer :: **x**
 type(**division_t**), intent(in) :: d
 call **copy_array_pointer** (x, d%x, 0)
 end **subroutine division_x**
- 60c *<Implementation of divisions procedures (removed from WHIZARD) 45a>*+≡ <60b 60d>
 pure **subroutine division_integral** (**integral**, d)
 real(kind=default), dimension(:), pointer :: **integral**
 type(**division_t**), intent(in) :: d
 call **copy_array_pointer** (**integral**, d%**integral**)
 end **subroutine division_integral**
- 60d *<Implementation of divisions procedures (removed from WHIZARD) 45a>*+≡ <60c 60e>
 pure **subroutine division_variance** (**variance**, d)
 real(kind=default), dimension(:), pointer :: **variance**
 type(**division_t**), intent(in) :: d
 call **copy_array_pointer** (**variance**, d%**variance**, 0)
 end **subroutine division_variance**
- 60e *<Implementation of divisions procedures (removed from WHIZARD) 45a>*+≡ <60d
 pure **subroutine division_efficiency** (**eff**, d)
 real(kind=default), dimension(:), pointer :: **eff**
 type(**division_t**), intent(in) :: d
 call **copy_array_pointer** (**eff**, d%**efficiency**, 0)
 end **subroutine division_efficiency**

5.1.8 I/O

- 60f *<Declaration of divisions procedures 38a>*+≡ (37a) <59b 66b>
 public :: **write_division**
 private :: **write_division_unit**, **write_division_name**
 public :: **read_division**
 private :: **read_division_unit**, **read_division_name**
 public :: **write_division_raw**
 private :: **write_division_raw_unit**, **write_division_raw_name**
 public :: **read_division_raw**
 private :: **read_division_raw_unit**, **read_division_raw_name**

61a \langle Interfaces of divisions procedures 61a $\rangle \equiv$ (37a)

```

interface write_division
    module procedure write_division_unit, write_division_name
end interface
interface read_division
    module procedure read_division_unit, read_division_name
end interface
interface write_division_raw
    module procedure write_division_raw_unit, write_division_raw_name
end interface
interface read_division_raw
    module procedure read_division_raw_unit, read_division_raw_name
end interface

```

It makes no sense to read or write d%integral, d%variance, and d%efficiency, because they are only used during sampling.

61b \langle Implementation of divisions procedures 38b $\rangle + \equiv$ (37a) \langle 58d 62b \rangle

```

subroutine write_division_unit (d, unit, write_integrals)
    type(division_t), intent(in) :: d
    integer, intent(in) :: unit
    logical, intent(in), optional :: write_integrals
    logical :: write_integrals0
    integer :: i
    write_integrals0 = .false.
    if (present(write_integrals)) write_integrals0 = write_integrals
    write (unit = unit, fmt = descr_fmt) "begin type(division_t) :: d"
    write (unit = unit, fmt = integer_fmt) "ubound(d%x,1) = ", ubound (d%x, dim=1)
    write (unit = unit, fmt = integer_fmt) "d%ng = ", d%ng
    write (unit = unit, fmt = logical_fmt) "d%stratified = ", d%stratified
    write (unit = unit, fmt = double_fmt) "d%dx = ", d%dx
    write (unit = unit, fmt = double_fmt) "d%dxg = ", d%dxg
    write (unit = unit, fmt = double_fmt) "d%x_min = ", d%x_min
    write (unit = unit, fmt = double_fmt) "d%x_max = ", d%x_max
    write (unit = unit, fmt = double_fmt) "d%x_min_true = ", d%x_min_true
    write (unit = unit, fmt = double_fmt) "d%x_max_true = ", d%x_max_true
    write (unit = unit, fmt = descr_fmt) "begin d%x"
    do i = 0, ubound (d%x, dim=1)
        if (write_integrals0 .and. i/=0) then
            write (unit = unit, fmt = double_array_fmt) &
                i, d%x(i), d%integral(i), d%variance(i)
        else
            write (unit = unit, fmt = double_array_fmt) i, d%x(i)
        end if
    end do
end do

```

```

        write (unit = unit, fmt = descr_fmt) "end d%x"
        write (unit = unit, fmt = descr_fmt) "end type(division_t)"
    end subroutine write_division_unit
62a  <Variables in divisions 46a>+≡ (37a) <59a
        character(len=*), parameter, private :: &
            descr_fmt = "(1x,a)", &
            integer_fmt = "(1x,a15,1x,i15)", &
            logical_fmt = "(1x,a15,1x,l1)", &
            double_fmt = "(1x,a15,1x,e30.22)", &
            double_array_fmt = "(1x,i15,1x,3(e30.22))"
62b  <Implementation of divisions procedures 38b>+≡ (37a) <61b 63b>
        subroutine read_division_unit (d, unit, read_integrals)
            type(division_t), intent(inout) :: d
            integer, intent(in) :: unit
            logical, intent(in), optional :: read_integrals
            logical :: read_integrals0
            integer :: i, idum, num_div
            character(len=80) :: chdum
            read_integrals0 = .false.
            if (present(read_integrals)) read_integrals0 = read_integrals
            read (unit = unit, fmt = descr_fmt) chdum
            read (unit = unit, fmt = integer_fmt) chdum, num_div
            <Insure that ubound (d%x, dim=1) == num_div 63a>
            read (unit = unit, fmt = integer_fmt) chdum, d%ng
            read (unit = unit, fmt = logical_fmt) chdum, d%stratified
            read (unit = unit, fmt = double_fmt) chdum, d%dx
            read (unit = unit, fmt = double_fmt) chdum, d%dxg
            read (unit = unit, fmt = double_fmt) chdum, d%x_min
            read (unit = unit, fmt = double_fmt) chdum, d%x_max
            read (unit = unit, fmt = double_fmt) chdum, d%x_min_true
            read (unit = unit, fmt = double_fmt) chdum, d%x_max_true
            read (unit = unit, fmt = descr_fmt) chdum
            do i = 0, ubound (d%x, dim=1)
                if (read_integrals0 .and. i/=0) then
                    read (unit = unit, fmt = double_array_fmt) &
                        & idum, d%x(i), d%integral(i), d%variance(i)
                else
                    read (unit = unit, fmt = double_array_fmt) idum, d%x(i)
                end if
            end do
            read (unit = unit, fmt = descr_fmt) chdum
            read (unit = unit, fmt = descr_fmt) chdum
            if (.not.read_integrals0) then

```

```

        d%integral = 0.0
        d%variance = 0.0
!       d%efficiency = 0.0
    end if
end subroutine read_division_unit

```



What happened to d%efficiency?

- 63a *<Insure that ubound (d%x, dim=1) == num_div 63a>≡* (62b 64c 67b)
- ```

 if (associated (d%x)) then
 if (ubound (d%x, dim=1) /= num_div) then
 deallocate (d%x, d%integral, d%variance)
! deallocate (d%efficiency)
 allocate (d%x(0:num_div), d%integral(num_div), d%variance(num_div))
! allocate (d%efficiency(num_div))
 end if
 else
 allocate (d%x(0:num_div), d%integral(num_div), d%variance(num_div))
! allocate (d%efficiency(num_div))
 end if

```
- 63b *<Implementation of divisions procedures 38b>+≡* (37a) <62b 63c>
- ```

subroutine write_division_name (d, name, write_integrals)
    type(division_t), intent(in) :: d
    character(len=*), intent(in) :: name
    logical, intent(in), optional :: write_integrals
    integer :: unit
    call find_free_unit (unit)
    open (unit = unit, action = "write", status = "replace", file = name)
    call write_division_unit (d, unit, write_integrals)
    close (unit = unit)
end subroutine write_division_name

```
- 63c *<Implementation of divisions procedures 38b>+≡* (37a) <63b 64a>
- ```

subroutine read_division_name (d, name, read_integrals)
 type(division_t), intent(inout) :: d
 character(len=*), intent(in) :: name
 logical, intent(in), optional :: read_integrals
 integer :: unit
 call find_free_unit (unit)
 open (unit = unit, action = "read", status = "old", file = name)
 call read_division_unit (d, unit, read_integrals)
 close (unit = unit)
end subroutine read_division_name

```



64a *<Implementation of divisions procedures 38b>+≡* (37a) <63c 64c>

```

subroutine write_division_raw_unit (d, unit, write_integrals)
 type(division_t), intent(in) :: d
 integer, intent(in) :: unit
 logical, intent(in), optional :: write_integrals
 logical :: write_integrals0
 integer :: i
 write_integrals0 = .false.
 if (present(write_integrals)) write_integrals0 = write_integrals
 write (unit = unit) MAGIC_DIVISION_BEGIN
 write (unit = unit) ubound (d%x, dim=1)
 write (unit = unit) d%ng
 write (unit = unit) d%stratified
 write (unit = unit) d%dx
 write (unit = unit) d%dxg
 write (unit = unit) d%x_min
 write (unit = unit) d%x_max
 write (unit = unit) d%x_min_true
 write (unit = unit) d%x_max_true
 do i = 0, ubound (d%x, dim=1)
 if (write_integrals0 .and. i/=0) then
 write (unit = unit) d%x(i), d%integral(i), d%variance(i)
 else
 write (unit = unit) d%x(i)
 end if
 end do
 write (unit = unit) MAGIC_DIVISION_END
end subroutine write_division_raw_unit

```

64b *<Constants in divisions 64b>≡* (37a)

```

integer, parameter, private :: MAGIC_DIVISION = 11111111
integer, parameter, private :: MAGIC_DIVISION_BEGIN = MAGIC_DIVISION + 1
integer, parameter, private :: MAGIC_DIVISION_END = MAGIC_DIVISION + 2

```

64c *<Implementation of divisions procedures 38b>+≡* (37a) <64a 65>

```

subroutine read_division_raw_unit (d, unit, read_integrals)
 type(division_t), intent(inout) :: d
 integer, intent(in) :: unit
 logical, intent(in), optional :: read_integrals
 logical :: read_integrals0
 integer :: i, num_div, magic
 character(len=*), parameter :: FN = "read_division_raw_unit"
 read_integrals0 = .false.
 if (present(read_integrals)) read_integrals0 = read_integrals
 read (unit = unit) magic

```

```

if (magic /= MAGIC_DIVISION_BEGIN) then
 print *, FN, " fatal: expecting magic ", MAGIC_DIVISION_BEGIN, &
 ", found ", magic
 stop
end if
read (unit = unit) num_div
<Insure that ubound (d%x, dim=1) == num_div 63a>
read (unit = unit) d%ng
read (unit = unit) d%stratified
read (unit = unit) d%dx
read (unit = unit) d%dxg
read (unit = unit) d%x_min
read (unit = unit) d%x_max
read (unit = unit) d%x_min_true
read (unit = unit) d%x_max_true
do i = 0, ubound (d%x, dim=1)
 if (read_integrals0 .and. i/=0) then
 read (unit = unit) d%x(i), d%integral(i), d%variance(i)
 else
 read (unit = unit) d%x(i)
 end if
end do
if (.not.read_integrals0) then
 d%integral = 0.0
 d%variance = 0.0
! d%efficiency = 0.0
end if
read (unit = unit) magic
if (magic /= MAGIC_DIVISION_END) then
 print *, FN, " fatal: expecting magic ", MAGIC_DIVISION_END, &
 ", found ", magic
 stop
end if
end subroutine read_division_raw_unit

```

65 <Implementation of divisions procedures 38b>+≡ (37a) <64c 66a>

```

subroutine write_division_raw_name (d, name, write_integrals)
 type(division_t), intent(in) :: d
 character(len=*), intent(in) :: name
 logical, intent(in), optional :: write_integrals
 integer :: unit
 call find_free_unit (unit)
 open (unit = unit, action = "write", status = "replace", &
 form = "unformatted", file = name)

```

```

 call write_division_unit (d, unit, write_integrals)
 close (unit = unit)
 end subroutine write_division_raw_name
66a <Implementation of divisions procedures 38b>+≡ (37a) <65 66c>
 subroutine read_division_raw_name (d, name, read_integrals)
 type(division_t), intent(inout) :: d
 character(len=*), intent(in) :: name
 logical, intent(in), optional :: read_integrals
 integer :: unit
 call find_free_unit (unit)
 open (unit = unit, action = "read", status = "old", &
 form = "unformatted", file = name)
 call read_division_unit (d, unit, read_integrals)
 close (unit = unit)
 end subroutine read_division_raw_name

```

### 5.1.9 Marshaling

Note that we can not use the transfer intrinsic function for marshalling types that contain pointers that substitute for allocatable array components. transfer will copy the pointers in this case and not where they point to!

```

66b <Declaration of divisions procedures 38a>+≡ (37a) <60f 67c>
 public :: marshal_division_size, marshal_division, unmarshal_division
66c <Implementation of divisions procedures 38b>+≡ (37a) <66a 67a>
 pure subroutine marshal_division (d, ibuf, dbuf)
 type(division_t), intent(in) :: d
 integer, dimension(:), intent(inout) :: ibuf
 real(kind=default), dimension(:), intent(inout) :: dbuf
 integer :: num_div
 num_div = ubound (d%x, dim=1)
 ibuf(1) = d%ng
 ibuf(2) = num_div
 if (d%stratified) then
 ibuf(3) = 1
 else
 ibuf(3) = 0
 end if
 dbuf(1) = d%x_min
 dbuf(2) = d%x_max
 dbuf(3) = d%x_min_true
 dbuf(4) = d%x_max_true
 dbuf(5) = d%dx
 end subroutine marshal_division

```

```

 dbuf(6) = d%dxg
 dbuf(7:7+num_div) = d%x
 dbuf(8+ num_div:7+2*num_div) = d%integral
 dbuf(8+2*num_div:7+3*num_div) = d%variance
 ! dbuf(8+3*num_div:7+4*num_div) = d%efficiency
end subroutine marshal_division

```

67a *<Implementation of divisions procedures 38b>+≡* (37a) <66c 67b>

```

pure subroutine marshal_division_size (d, iwords, dwords)
 type(division_t), intent(in) :: d
 integer, intent(out) :: iwords, dwords
 iwords = 3
 dwords = 7 + 3 * ubound (d%x, dim=1)
 ! dwords = 7 + 4 * ubound (d%x, dim=1)
end subroutine marshal_division_size

```

67b *<Implementation of divisions procedures 38b>+≡* (37a) <67a 67d>

```

pure subroutine unmarshal_division (d, ibuf, dbuf)
 type(division_t), intent(inout) :: d
 integer, dimension(:), intent(in) :: ibuf
 real(kind=default), dimension(:), intent(in) :: dbuf
 integer :: num_div
 d%ng = ibuf(1)
 num_div = ibuf(2)
 d%stratified = ibuf(3) /= 0
 d%x_min = dbuf(1)
 d%x_max = dbuf(2)
 d%x_min_true = dbuf(3)
 d%x_max_true = dbuf(4)
 d%dx = dbuf(5)
 d%dxg = dbuf(6)
 <Insure that ubound (d%x, dim=1) == num_div 63a>
 d%x = dbuf(7:7+num_div)
 d%integral = dbuf(8+ num_div:7+2*num_div)
 d%variance = dbuf(8+2*num_div:7+3*num_div)
 ! d%efficiency = dbuf(8+3*num_div:7+4*num_div)
end subroutine unmarshal_division

```

67c *<Declaration of divisions procedures 38a>+≡* (37a) <66b  
 public :: marshal\_div\_history\_size, marshal\_div\_history, unmarshal\_div\_history

67d *<Implementation of divisions procedures 38b>+≡* (37a) <67b 68a>

```

pure subroutine marshal_div_history (h, ibuf, dbuf)
 type(div_history), intent(in) :: h
 integer, dimension(:), intent(inout) :: ibuf
 real(kind=default), dimension(:), intent(inout) :: dbuf

```

```

 ibuf(1) = h%ng
 ibuf(2) = h%num_div
 if (h%stratified) then
 ibuf(3) = 1
 else
 ibuf(3) = 0
 end if
 dbuf(1) = h%x_min
 dbuf(2) = h%x_max
 dbuf(3) = h%x_min_true
 dbuf(4) = h%x_max_true
 dbuf(5) = h%spread_f_p
 dbuf(6) = h%stddev_f_p
 dbuf(7) = h%spread_p
 dbuf(8) = h%stddev_p
 dbuf(9) = h%spread_m
 dbuf(10) = h%stddev_m
end subroutine marshal_div_history
68a <Implementation of divisions procedures 38b>+≡ (37a) <67d 68b>
pure subroutine marshal_div_history_size (h, iwords, dwords)
 type(div_history), intent(in) :: h
 integer, intent(out) :: iwords, dwords
 iwords = 3
 dwords = 10
end subroutine marshal_div_history_size
68b <Implementation of divisions procedures 38b>+≡ (37a) <68a 69a>
pure subroutine unmarshal_div_history (h, ibuf, dbuf)
 type(div_history), intent(inout) :: h
 integer, dimension(:), intent(in) :: ibuf
 real(kind=default), dimension(:), intent(in) :: dbuf
 h%ng = ibuf(1)
 h%num_div = ibuf(2)
 h%stratified = ibuf(3) /= 0
 h%x_min = dbuf(1)
 h%x_max = dbuf(2)
 h%x_min_true = dbuf(3)
 h%x_max_true = dbuf(4)
 h%spread_f_p = dbuf(5)
 h%stddev_f_p = dbuf(6)
 h%spread_p = dbuf(7)
 h%stddev_p = dbuf(8)
 h%spread_m = dbuf(9)
 h%stddev_m = dbuf(10)

```

end subroutine unmarshal\_div\_history

### 5.1.10 Boring Copying and Deleting of Objects

69a  $\langle$ Implementation of divisions procedures 38b $\rangle + \equiv$  (37a)  $\langle$ 68b 69b $\rangle$

```

elemental subroutine copy_division (lhs, rhs)
 type(division_t), intent(inout) :: lhs
 type(division_t), intent(in) :: rhs
 if (associated (rhs%x)) then
 call copy_array_pointer (lhs%x, rhs%x, lb = 0)
 else if (associated (lhs%x)) then
 deallocate (lhs%x)
 end if
 if (associated (rhs%integral)) then
 call copy_array_pointer (lhs%integral, rhs%integral)
 else if (associated (lhs%integral)) then
 deallocate (lhs%integral)
 end if
 if (associated (rhs%variance)) then
 call copy_array_pointer (lhs%variance, rhs%variance)
 else if (associated (lhs%variance)) then
 deallocate (lhs%variance)
 end if
 ! if (associated (rhs%efficiency)) then
 ! call copy_array_pointer (lhs%efficiency, rhs%efficiency)
 ! else if (associated (lhs%efficiency)) then
 ! deallocate (lhs%efficiency)
 ! end if
 lhs%dx = rhs%dx
 lhs%dxg = rhs%dxg
 lhs%x_min = rhs%x_min
 lhs%x_max = rhs%x_max
 lhs%x_min_true = rhs%x_min_true
 lhs%x_max_true = rhs%x_max_true
 lhs%ng = rhs%ng
 lhs%stratified = rhs%stratified
end subroutine copy_division

```

69b  $\langle$ Implementation of divisions procedures 38b $\rangle + \equiv$  (37a)  $\langle$ 69a 70a $\rangle$

```

elemental subroutine delete_division (d)
 type(division_t), intent(inout) :: d
 if (associated (d%x)) then
 deallocate (d%x, d%integral, d%variance)
 end if

```

```

! deallocate (d%efficiency)
 end if
end subroutine delete_division
70a <Implementation of divisions procedures 38b>+≡ (37a) <69b
 elemental subroutine copy_history (lhs, rhs)
 type(div_history), intent(out) :: lhs
 type(div_history), intent(in) :: rhs
 lhs%stratified = rhs%stratified
 lhs%ng = rhs%ng
 lhs%num_div = rhs%num_div
 lhs%x_min = rhs%x_min
 lhs%x_max = rhs%x_max
 lhs%x_min_true = rhs%x_min_true
 lhs%x_max_true = rhs%x_max_true
 lhs%spread_f_p = rhs%spread_f_p
 lhs%stddev_f_p = rhs%stddev_f_p
 lhs%spread_p = rhs%spread_p
 lhs%stddev_p = rhs%stddev_p
 lhs%spread_m = rhs%spread_m
 lhs%stddev_m = rhs%stddev_m
 end subroutine copy_history


```

## 5.2 The Abstract Datatype *vamp\_grid*

```

70b <vamp.f90 70b>≡ 70c>
! vamp.f90 --
<Copyleft notice 1>

```

 NAG f95 requires this split. Check with the Fortran community, if it is really necessary, or a bug! The problem is that this split forces us to expose the components of **vamp\_grid**.

**NB:** with the introduction of **vamp\_equivalences**, this question has (probably) become academic.

```

70c <vamp.f90 70b>+≡ <70b 71a>
 module vamp_grid_type
 use kinds
 use divisions
 private
 <Declaration of vamp_grid_type types 76a>
 end module vamp_grid_type

```



By WK for WHIZARD.

```

71a <vamp.f90 70b>+≡ <70c 75a>
 module vamp_equivalences
 use kinds
 use divisions
 use vamp_grid_type !NODEP!
 implicit none
 private
 <Declaration of vamp_equivalences procedures 72a>
 <Constants in vamp_equivalences 71d>
 <Declaration of vamp_equivalences types 71b>
 contains
 <Implementation of vamp_equivalences procedures 71e>
 end module vamp_equivalences

71b <Declaration of vamp_equivalences types 71b>≡ (71a) 71c>
 type, public :: vamp_equivalence_t
 integer :: left, right
 integer, dimension(:), allocatable :: permutation
 integer, dimension(:), allocatable :: mode
 end type vamp_equivalence_t

71c <Declaration of vamp_equivalences types 71b>+≡ (71a) <71b
 type, public :: vamp_equivalences_t
 type(vamp_equivalence_t), dimension(:), allocatable :: eq
 integer :: n_eq, n_ch
 integer, dimension(:), allocatable :: pointer
 logical, dimension(:), allocatable :: independent
 integer, dimension(:), allocatable :: equivalent_to_ch
 integer, dimension(:), allocatable :: multiplicity
 integer, dimension(:), allocatable :: symmetry
 logical, dimension(:,,:), allocatable :: div_is_invariant
 end type vamp_equivalences_t

71d <Constants in vamp_equivalences 71d>≡ (71a)
 integer, parameter, public :: &
 VEQ_IDENTITY = 0, VEQ_INVERT = 1, VEQ_SYMMETRIC = 2, VEQ_INVARIANT = 3

71e <Implementation of vamp_equivalences procedures 71e>≡ (71a) 72b>
 subroutine vamp_equivalence_init (eq, n_dim)
 type(vamp_equivalence_t), intent(inout) :: eq
 integer, intent(in) :: n_dim
 allocate (eq%permutation(n_dim), eq%mode(n_dim))
 end subroutine vamp_equivalence_init

```



72a  $\langle$ Declaration of vamp\_equivalences procedures 72a $\rangle \equiv$  (71a) 72d $\rangle$   
public :: vamp\_equivalences\_init

72b  $\langle$ Implementation of vamp\_equivalences procedures 71e $\rangle + \equiv$  (71a)  $\langle$ 71e 72c $\rangle$   
subroutine vamp\_equivalences\_init (eq, n\_eq, n\_ch, n\_dim)  
type(vamp\_equivalences\_t), intent(inout) :: eq  
integer, intent(in) :: n\_eq, n\_ch, n\_dim  
integer :: i  
eq%n\_eq = n\_eq  
eq%n\_ch = n\_ch  
allocate (eq%eq(n\_eq))  
allocate (eq%pointer(n\_ch+1))  
do i=1, n\_eq  
call vamp\_equivalence\_init (eq%eq(i), n\_dim)  
end do  
allocate (eq%independent(n\_ch), eq%equivalent\_to\_ch(n\_ch))  
allocate (eq%multiplicity(n\_ch), eq%symmetry(n\_ch))  
allocate (eq%div\_is\_invariant(n\_ch, n\_dim))  
eq%independent = .true.  
eq%equivalent\_to\_ch = 0  
eq%multiplicity = 0  
eq%symmetry = 0  
eq%div\_is\_invariant = .false.  
end subroutine vamp\_equivalences\_init

72c  $\langle$ Implementation of vamp\_equivalences procedures 71e $\rangle + \equiv$  (71a)  $\langle$ 72b 72e $\rangle$   
subroutine vamp\_equivalence\_final (eq)  
type(vamp\_equivalence\_t), intent(inout) :: eq  
deallocate (eq%permutation, eq%mode)  
end subroutine vamp\_equivalence\_final

72d  $\langle$ Declaration of vamp\_equivalences procedures 72a $\rangle + \equiv$  (71a)  $\langle$ 72a 73b $\rangle$   
public :: vamp\_equivalences\_final

72e  $\langle$ Implementation of vamp\_equivalences procedures 71e $\rangle + \equiv$  (71a)  $\langle$ 72c 73a $\rangle$   
subroutine vamp\_equivalences\_final (eq)  
type(vamp\_equivalences\_t), intent(inout) :: eq  
! integer :: i  
! do i=1, eq%n\_eq  
! call vamp\_equivalence\_final (eq%eq(i))  
! end do  
if (allocated (eq%eq)) deallocate (eq%eq)  
if (allocated (eq%pointer)) deallocate (eq%pointer)  
if (allocated (eq%multiplicity)) deallocate (eq%multiplicity)  
if (allocated (eq%symmetry)) deallocate (eq%symmetry)  
if (allocated (eq%independent)) deallocate (eq%independent)

```

 if (allocated (eq%equivalent_to_ch)) deallocate (eq%equivalent_to_ch)
 if (allocated (eq%div_is_invariant)) deallocate (eq%div_is_invariant)
 eq%n_eq = 0
 eq%n_ch = 0
 end subroutine vamp_equivalences_final
73a <Implementation of vamp_equivalences procedures 71e>+≡ (71a) <72e 73c>
 subroutine vamp_equivalence_write (eq, unit)
 integer, intent(in), optional :: unit
 integer :: u
 type(vamp_equivalence_t), intent(in) :: eq
 u = 6; if (present (unit)) u = unit
 write (u, "(3x,A,2(1x,I0))") "Equivalent channels:", eq%left, eq%right
 write (u, "(5x,A,99(1x,I0))") "Permutation:", eq%permutation
 write (u, "(5x,A,99(1x,I0))") "Mode: ", eq%mode
 end subroutine vamp_equivalence_write
73b <Declaration of vamp_equivalences procedures 72a>+≡ (71a) <72d 74a>
 public :: vamp_equivalences_write
73c <Implementation of vamp_equivalences procedures 71e>+≡ (71a) <73a 74b>
 subroutine vamp_equivalences_write (eq, unit)
 type(vamp_equivalences_t), intent(in) :: eq
 integer, intent(in), optional :: unit
 integer :: u
 integer :: ch, i
 u = 6; if (present (unit)) u = unit
 write (u, "(1x,A)") "Inequivalent channels:"
 if (allocated (eq%independent)) then
 do ch=1, eq%n_ch
 if (eq%independent(ch)) then
 write (u, "(3x,A,1x,I0,A,4x,A,I0,4x,A,I0,4x,A,999(L1))") &
 "Channel", ch, ":", &
 "Mult. = ", eq%multiplicity(ch), &
 "Symm. = ", eq%symmetry(ch), &
 "Invar.: ", eq%div_is_invariant(ch,:)
 end if
 end do
 else
 write (u, "(3x,A)") "[not allocated]"
 end if
 write (u, "(1x,A)") "Equivalence list:"
 if (allocated (eq%eq)) then
 do i=1, size (eq%eq)
 call vamp_equivalence_write (eq%eq(i), u)
 end do
 end if
 end subroutine vamp_equivalences_write

```

```

 end do
 else
 write (u, "(3x,A)") "[not allocated]"
 end if
end subroutine vamp_equivalences_write

74a <Declaration of vamp_equivalences procedures 72a>+≡ (71a) <73b 74c>
 public :: vamp_equivalence_set

74b <Implementation of vamp_equivalences procedures 71e>+≡ (71a) <73c 74d>
 subroutine vamp_equivalence_set (eq, i, left, right, perm, mode)
 type(vamp_equivalences_t), intent(inout) :: eq
 integer, intent(in) :: i
 integer, intent(in) :: left, right
 integer, dimension(:), intent(in) :: perm, mode
 eq%eq(i)%left = left
 eq%eq(i)%right = right
 eq%eq(i)%permutation = perm
 eq%eq(i)%mode = mode
 end subroutine vamp_equivalence_set

74c <Declaration of vamp_equivalences procedures 72a>+≡ (71a) <74a>
 public :: vamp_equivalences_complete

74d <Implementation of vamp_equivalences procedures 71e>+≡ (71a) <74b>
 subroutine vamp_equivalences_complete (eq)
 type(vamp_equivalences_t), intent(inout) :: eq
 integer :: i, ch
 ch = 0
 do i=1, eq%n_eq
 if (ch /= eq%eq(i)%left) then
 ch = eq%eq(i)%left
 eq%pointer(ch) = i
 end if
 end do
 eq%pointer(ch+1) = eq%n_eq + 1
 do ch=1, eq%n_ch
 call set_multiplicities (eq%eq(eq%pointer(ch):eq%pointer(ch+1)-1))
 end do
 ! call write (6, eq)
contains
 subroutine set_multiplicities (eq_ch)
 type(vamp_equivalence_t), dimension(:), intent(in) :: eq_ch
 integer :: i
 if (.not. all(eq_ch%left == ch) .or. eq_ch(1)%right > ch) then
 do i = 1, size (eq_ch)

```

```

 call vamp_equivalence_write (eq_ch(i))
 end do
 stop "VAMP: Equivalences: Something's wrong with equivalence ordering"
end if
eq%symmetry(ch) = count (eq_ch%right == ch)
if (mod (size(eq_ch), eq%symmetry(ch)) /= 0) then
 do i = 1, size (eq_ch)
 call vamp_equivalence_write (eq_ch(i))
 end do
 stop "VAMP: Equivalences: Something's wrong with permutation count"
end if
eq%multiplicity(ch) = size (eq_ch) / eq%symmetry(ch)
eq%independent(ch) = all (eq_ch%right >= ch)
eq%equivalent_to_ch(ch) = eq_ch(1)%right
eq%div_is_invariant(ch,:) = eq_ch(1)%mode == VEQ_INVARIANT
end subroutine set_multiplicities
end subroutine vamp_equivalences_complete
75a <vamp.f90 70b>+≡ <71a 75b>
module vamp_rest
 use kinds
 use utils
 use exceptions
 use divisions
 use tao_random_numbers
 use vamp_stat
 use linalg
 use iso_fortran_env
 use vamp_grid_type !NODEP!
 use vamp_equivalences !NODEP!
 implicit none
 private
 <Declaration of vamp procedures 76b>
 <Interfaces of vamp procedures 95c>
 <Constants in vamp 152>
 <Declaration of vamp types 77a>
 <Variables in vamp 78a>
contains
 <Implementation of vamp procedures 77d>
end module vamp_rest
75b <vamp.f90 70b>+≡ <75a>
module vamp
 use vamp_grid_type !NODEP!
 use vamp_rest !NODEP!

```

```

 use vamp_equivalences !NODEP!
 public
end module vamp

```

N.B.: In Fortran95 we will be able to give default initializations to components of the type. In particular, we can use the `null ()` intrinsic to initialize the pointers to a disassociated state. Until then, the user *must* call the initializer `vamp_create_grid` himself of herself, because we can't check for the allocation status of the pointers in Fortran90 or F.

⚡ Augment this datatype by `real(kind=default), dimension(2) :: mu_plus, mu_minus` to record positive and negative weight separately, so that we can estimate the efficiency for reweighting from indefinite weights to  $\{+1, -1\}$ . [WK 2015/11/06: done. Those values are recorded but not used inside `vamp`. They can be retrieved by the caller.]

⚡ WK 2015/11/06: `f_min` and `f_max` work with the absolute value of the matrix element, so they record the minimum and maximum absolute value.

76a  $\langle$ Declaration of `vamp_grid_type` types 76a $\rangle \equiv$  (70c)

```

type, public :: vamp_grid
! private ! forced by use association in interface
type(division_t), dimension(:), pointer :: div => null ()
real(kind=default), dimension(:, :), pointer :: map => null ()
real(kind=default), dimension(:), pointer :: mu_x => null ()
real(kind=default), dimension(:), pointer :: sum_mu_x => null ()
real(kind=default), dimension(:, :), pointer :: mu_xx => null ()
real(kind=default), dimension(:, :), pointer :: sum_mu_xx => null ()
real(kind=default), dimension(2) :: mu
real(kind=default), dimension(2) :: mu_plus, mu_minus
real(kind=default) :: sum_integral, sum_weights, sum_chi2
real(kind=default) :: calls, dv2g, jacobi
real(kind=default) :: f_min, f_max
real(kind=default) :: mu_gi, sum_mu_gi
integer, dimension(:), pointer :: num_div => null ()
integer :: num_calls, calls_per_cell
logical :: stratified = .true.
logical :: all_stratified = .true.
logical :: quadrupole = .false.
logical :: independent
integer :: equivalent_to_ch, multiplicity
end type vamp_grid

```

76b  $\langle$ Declaration of `vamp` procedures 76b $\rangle \equiv$  (75a) 77c $\triangleright$

```

public :: vamp_copy_grid, vamp_delete_grid

```

### 5.2.1 Container for application data



By WK for WHIZARD. We define an empty data type that the application can extend according to its needs. The purpose is to hold all sorts of data that are predefined and accessed during the call of the sampling function.

The actual interface for the sampling function is PURE. Nevertheless, we can implement side effects via pointer components of a `vamp_data_t` extension.

```
77a <Declaration of vamp types 77a>≡ (75a) 77b>
 type, public :: vamp_data_t
end type vamp_data_t
```

This is the object to be passed if we want nothing else:

```
77b <Declaration of vamp types 77a>+≡ (75a) <77a 106a>
 type(vamp_data_t), parameter, public :: NO_DATA = vamp_data_t ()
```

### 5.2.2 Initialization

```
77c <Declaration of vamp procedures 76b>+≡ (75a) <76b 78b>
 public :: vamp_create_grid, vamp_create_empty_grid
```

Create a fresh grid for the integration domain

$$\mathcal{D} = [D_{1,1}, D_{2,1}] \times [D_{1,2}, D_{2,2}] \times \dots \times [D_{1,n}, D_{2,n}] \quad (5.9)$$

dropping all accumulated results. This function *must not* be called twice on the first argument, without an intervening `vamp_delete_grid`. If the second variable is given, it will be the number of sampling points for the call to `vamp_sample_grid`.

```
77d <Implementation of vamp procedures 77d>≡ (75a) 79a>
 pure subroutine vamp_create_grid &
 (g, domain, num_calls, num_div, &
 stratified, quadrupole, covariance, map, exc)
 type(vamp_grid), intent(inout) :: g
 real(kind=default), dimension(:, :), intent(in) :: domain
 integer, intent(in) :: num_calls
 integer, dimension(:), intent(in), optional :: num_div
 logical, intent(in), optional :: stratified, quadrupole, covariance
 real(kind=default), dimension(:, :), intent(in), optional :: map
 type(exception), intent(inout), optional :: exc
```

```

character(len=*), parameter :: FN = "vamp_create_grid"
real(kind=default), dimension(size(domain,dim=2)) :: &
 x_min, x_max, x_min_true, x_max_true
integer :: ndim
ndim = size (domain, dim=2)
allocate (g%div(ndim), g%num_div(ndim))
x_min = domain(1,:)
x_max = domain(2,:)
if (present (map)) then
 allocate (g%map(ndim,ndim))
 g%map = map
 x_min_true = x_min
 x_max_true = x_max
 call map_domain (g%map, x_min_true, x_max_true, x_min, x_max)
 call create_division (g%div, x_min, x_max, x_min_true, x_max_true)
else
 nullify (g%map)
 call create_division (g%div, x_min, x_max)
end if
g%num_calls = num_calls
if (present (num_div)) then
 g%num_div = num_div
else
 g%num_div = NUM_DIV_DEFAULT
end if
g%stratified = .true.
g%quadrupole = .false.
g%independent = .true.
g%equivalent_to_ch = 0
g%multiplicity = 1
nullify (g%mu_x, g%mu_xx, g%sum_mu_x, g%sum_mu_xx)
call vamp_discard_integral &
 (g, num_calls, num_div, stratified, quadrupole, covariance, exc)
end subroutine vamp_create_grid

```

Below, we assume that  $\text{NUM\_DIV\_DEFAULT} \geq 6$ , but we will never go that low anyway.

78a  $\langle \text{Variables in vamp 78a} \rangle \equiv$  (75a) 94b  $\triangleright$   
integer, private, parameter :: NUM\_DIV\_DEFAULT = 20

Given a linear map  $M$ , find a domain  $\mathcal{D}_0$  such that

$$\mathcal{D} \subset M\mathcal{D}_0 \quad (5.10)$$

78b  $\langle \text{Declaration of vamp procedures 76b} \rangle + \equiv$  (75a)  $\langle 77c \ 79c \rangle$   
private :: map\_domain

If we can assume that  $M$  is orthogonal  $M^{-1} = M^T$ , then we just have to rotate  $\mathcal{D}$  and determine the maximal and minimal extension of the corners:

$$\mathcal{D}_0^T = \overline{\mathcal{D}^T M} \quad (5.11)$$

The corners are just the powerset of the maximal and minimal extension in each coordinate. It is determined most easily with binary counting:

- 79a *<Implementation of vamp procedures 77d>+≡* (75a) *<77d 79b>*  

```

pure subroutine map_domain (map, true_xmin, true_xmax, xmin, xmax)
 real(kind=default), dimension(:,:), intent(in) :: map
 real(kind=default), dimension(:), intent(in) :: true_xmin, true_xmax
 real(kind=default), dimension(:), intent(out) :: xmin, xmax
 real(kind=default), dimension(2**size(xmin),size(xmin)) :: corners
 integer, dimension(size(xmin)) :: zero_to_n
 integer :: j, ndim, perm
 ndim = size (xmin)
 zero_to_n = (/ (j, j=0,ndim-1) /)
 do perm = 1, 2**ndim
 corners (perm,:) = &
 merge (true_xmin, true_xmax, btest (perm-1, zero_to_n))
 end do
 corners = matmul (corners, map)
 xmin = minval (corners, dim=1)
 xmax = maxval (corners, dim=1)
end subroutine map_domain

```

79b *<Implementation of vamp procedures 77d>+≡* (75a) *<79a 79d>*  

```

elemental subroutine vamp_create_empty_grid (g)
 type(vamp_grid), intent(inout) :: g
 nullify (g%div, g%num_div, g%map, g%mu_x, g%mu_xx, g%sum_mu_x, g%sum_mu_xx)
end subroutine vamp_create_empty_grid

```

79c *<Declaration of vamp procedures 76b>+≡* (75a) *<78b 80a>*  

```

public :: vamp_discard_integral

```

Keep the current optimized grid, but drop the accumulated results for the integral (value and errors). Iff the second variable is given, it will be the new number of sampling points for the next call to **vamp\_sample\_grid**.

79d *<Implementation of vamp procedures 77d>+≡* (75a) *<79b 80b>*  

```

pure subroutine vamp_discard_integral &
 (g, num_calls, num_div, stratified, quadrupole, covariance, exc, &
 & independent, equivalent_to_ch, multiplicity)
 type(vamp_grid), intent(inout) :: g
 integer, intent(in), optional :: num_calls
 integer, dimension(:), intent(in), optional :: num_div

```



```

logical, intent(in), optional :: stratified, quadrupole, covariance
type(exception), intent(inout), optional :: exc
logical, intent(in), optional :: independent
integer, intent(in), optional :: equivalent_to_ch, multiplicity
character(len=*), parameter :: FN = "vamp_discard_integral"
g%mu = 0.0
g%mu_plus = 0.0
g%mu_minus = 0.0
g%mu_gi = 0.0
g%sum_integral = 0.0
g%sum_weights = 0.0
g%sum_chi2 = 0.0
g%sum_mu_gi = 0.0
if (associated (g%sum_mu_x)) then
 g%sum_mu_x = 0.0
 g%sum_mu_xx = 0.0
end if
call set_grid_options (g, num_calls, num_div, stratified, quadrupole, &
 independent, equivalent_to_ch, multiplicity)
if ((present (num_calls)) &
 .or. (present (num_div)) &
 .or. (present (stratified)) &
 .or. (present (quadrupole)) &
 .or. (present (covariance))) then
 call vamp_reshape_grid &
 (g, g%num_calls, g%num_div, &
 g%stratified, g%quadrupole, covariance, exc)
end if
end subroutine vamp_discard_integral

```

80a  $\langle$ Declaration of vamp procedures 76b $\rangle + \equiv$  (75a)  $\triangleleft$ 79c 82a $\triangleright$

private :: set\_grid\_options

80b  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\triangleleft$ 79d 81 $\triangleright$

```

pure subroutine set_grid_options &
 (g, num_calls, num_div, stratified, quadrupole, &
 independent, equivalent_to_ch, multiplicity)
type(vamp_grid), intent(inout) :: g
integer, intent(in), optional :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole
logical, intent(in), optional :: independent
integer, intent(in), optional :: equivalent_to_ch, multiplicity
if (present (num_calls)) then
 g%num_calls = num_calls

```

```

end if
if (present (num_div)) then
 g%num_div = num_div
end if
if (present (stratified)) then
 g%stratified = stratified
end if
if (present (quadrupole)) then
 g%quadrupole = quadrupole
end if
if (present (independent)) then
 g%independent = independent
end if
if (present (equivalent_to_ch)) then
 g%equivalent_to_ch = equivalent_to_ch
end if
if (present (multiplicity)) then
 g%multiplicity = multiplicity
end if
end subroutine set_grid_options

```

### *Setting Up the Initial Grid*

Keep the current optimized grid and the accumulated results for the integral (value and errors). The second variable will be the new number of sampling points for the next call to `vamp_sample_grid`.

81 *<Implementation of vamp procedures 77d>+≡* (75a) <80b 82b>

```

pure subroutine vamp_reshape_grid_internal &
 (g, num_calls, num_div, &
 stratified, quadrupole, covariance, exc, use_variance, &
 independent, equivalent_to_ch, multiplicity)
type(vamp_grid), intent(inout) :: g
integer, intent(in), optional :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole, covariance
type(exception), intent(inout), optional :: exc
logical, intent(in), optional :: use_variance
logical, intent(in), optional :: independent
integer, intent(in), optional :: equivalent_to_ch, multiplicity
integer :: ndim, num_cells
integer, dimension(size(g%div)) :: ng
character(len=*), parameter :: FN = "vamp_reshape_grid_internal"
ndim = size (g%div)

```

```

call set_grid_options &
 (g, num_calls, num_div, stratified, quadrupole, &
 & independent, equivalent_to_ch, multiplicity)
<Adjust grid and other state for new num_calls 83>
g%all_stratified = all (stratified_division (g%div))
if (present (covariance)) then
 ndim = size (g%div)
 if (covariance .and. (.not. associated (g%mu_x))) then
 allocate (g%mu_x(ndim), g%mu_xx(ndim,ndim))
 allocate (g%sum_mu_x(ndim), g%sum_mu_xx(ndim,ndim))
 g%sum_mu_x = 0.0
 g%sum_mu_xx = 0.0
 else if ((.not. covariance) .and. (associated (g%mu_x))) then
 deallocate (g%mu_x, g%mu_xx, g%sum_mu_x, g%sum_mu_xx)
 end if
end if
end subroutine vamp_reshape_grid_internal

```

The `use_variance` argument is too dangerous for careless users, because the variance in the divisions will contain garbage before sampling and after reshaping. Build a fence with another routine.

- 82a <Declaration of `vamp` procedures 76b>+≡ (75a) <80a 84c>
- ```

private :: vamp_reshape_grid_internal
public :: vamp_reshape_grid

```
- 82b <Implementation of `vamp` procedures 77d>+≡ (75a) <81 84d>
- ```

pure subroutine vamp_reshape_grid &
 (g, num_calls, num_div, stratified, quadrupole, covariance, exc, &
 independent, equivalent_to_ch, multiplicity)
type(vamp_grid), intent(inout) :: g
integer, intent(in), optional :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole, covariance
type(exception), intent(inout), optional :: exc
logical, intent(in), optional :: independent
integer, intent(in), optional :: equivalent_to_ch, multiplicity
call vamp_reshape_grid_internal &
 (g, num_calls, num_div, stratified, quadrupole, covariance, &
 exc, use_variance = .false., &
 independent=independent, equivalent_to_ch=equivalent_to_ch, &
 multiplicity=multiplicity)
end subroutine vamp_reshape_grid

```

`vegas` operates in three different modes, which are chosen according to explicit user requests and to the relation of the requested number of sampling

points to the dimensionality of the integration domain.

The simplest case is when the user has overwritten the default of stratified sampling with the optional argument `stratified` in the call to `vamp_create_grid`. Then sample points will be chosen randomly with equal probability in each cell of the adaptive grid, as displayed in figure 5.1.

The implementation is actually shared with the stratified case described below, by pretending that there is just a single stratification cell. The number of divisions for the adaptive grid is set to a compile time maximum value.

If the user has agreed on stratified sampling then there are two cases, depending on the dimensionality of the integration region and the number of sample points. First we determine the number of divisions  $n_g$  (i. e. `ng`) of the rigid grid such that there will be two sampling points per cell.

$$N_{\text{calls}} = 2 \cdot (n_g)^{n_{\text{dim}}} \quad (5.12)$$

The additional optional argument  $\hat{n}_g$  specifies an anisotropy in the shape

$$n_{g,j} = \frac{\hat{n}_{g,j}}{\left(\prod_j \hat{n}_{g,j}\right)^{1/n_{\text{dim}}}} \left(\frac{N}{2}\right)^{1/n_{\text{dim}}} \quad (5.13)$$

NB:

$$\prod_j n_{g,j} = \frac{N}{2} \quad (5.14)$$

83 *Adjust grid and other state for new num\_calls 83* (81) 84a

```

if (g%stratified) then
 ng = (g%num_calls / 2.0 + 0.25)**(1.0/ndim)
! ng = ng * real (g%num_div, kind=default) &
! / (product (real (g%num_div, kind=default)))** (1.0/ndim)
else
 ng = 1
end if
call reshape_division (g%div, g%num_div, ng, use_variance)
call clear_integral_and_variance (g%div)
num_cells = product (rigid_division (g%div))
g%calls_per_cell = max (g%num_calls / num_cells, 2)
g%calls = real (g%calls_per_cell) * real (num_cells)
jacobi = J = Volume / N_calls

```

(5.15)

and

$$\text{dv2g} = \frac{N_{\text{calls}}^2 ((\Delta x)^{n_{\text{dim}}})^2}{N_{\text{calls/cell}}^2 (N_{\text{calls/cell}} - 1)} = \frac{\left(\frac{N_{\text{calls}}}{N_{\text{cells}}}\right)^2}{N_{\text{calls/cell}}^2 (N_{\text{calls/cell}} - 1)} \quad (5.16)$$

84a *<Adjust grid and other state for new num\_calls 83>+≡ (81) <83 84b>*  
`g%jacobi = product (volume_division (g%div)) / g%calls  
g%dv2g = (g%calls / num_cells)**2 &  
/ g%calls_per_cell / g%calls_per_cell / (g%calls_per_cell - 1.0)`

84b *<Adjust grid and other state for new num\_calls 83>+≡ (81) <84a*  
`call vamp_nullify_f_limits (g)`

When the grid is refined or reshaped, the recorded minimum and maximum of the sampling function should be nullified:

84c *<Declaration of vamp procedures 76b>+≡ (75a) <82a 84e>*  
`public :: vamp_nullify_f_limits`

84d *<Implementation of vamp procedures 77d>+≡ (75a) <82b 84f>*  
`elemental subroutine vamp_nullify_f_limits (g)  
type(vamp_grid), intent(inout) :: g  
g%f_min = 1.0  
g%f_max = 0.0  
end subroutine vamp_nullify_f_limits`

84e *<Declaration of vamp procedures 76b>+≡ (75a) <84c 85d>*  
`public :: vamp_rigid_divisions  
public :: vamp_get_covariance, vamp_nullify_covariance  
public :: vamp_get_variance, vamp_nullify_variance`

84f *<Implementation of vamp procedures 77d>+≡ (75a) <84d 84g>*  
`pure function vamp_rigid_divisions (g) result (ng)  
type(vamp_grid), intent(in) :: g  
integer, dimension(size(g%div)) :: ng  
ng = rigid_division (g%div)  
end function vamp_rigid_divisions`

84g *<Implementation of vamp procedures 77d>+≡ (75a) <84f 85a>*  
`pure function vamp_get_covariance (g) result (cov)  
type(vamp_grid), intent(in) :: g  
real(kind=default), dimension(size(g%div),size(g%div)) :: cov  
if (associated (g%mu_x)) then  
if (abs (g%sum_weights) <= tiny (cov(1,1))) then  
where (g%sum_mu_xx == 0.0_default)  
cov = 0.0  
elsewhere  
cov = huge (cov(1,1))  
endwhere  
else  
cov = g%sum_mu_xx / g%sum_weights &  
- outer_product (g%sum_mu_x, g%sum_mu_x) / g%sum_weights**2  
end if`

```

 else
 cov = 0.0
 end if
 end function vamp_get_covariance
85a <Implementation of vamp procedures 77d>+≡ (75a) <84g 85b>
 elemental subroutine vamp_nullify_covariance (g)
 type(vamp_grid), intent(inout) :: g
 if (associated (g%mu_x)) then
 g%sum_mu_x = 0
 g%sum_mu_xx = 0
 end if
 end subroutine vamp_nullify_covariance
85b <Implementation of vamp procedures 77d>+≡ (75a) <85a 85c>
 elemental function vamp_get_variance (g) result (v)
 type(vamp_grid), intent(in) :: g
 real(kind=default) :: v
 if (abs (g%sum_weights) <= tiny (v)) then
 if (g%sum_mu_gi == 0.0_default) then
 v = 0.0
 else
 v = huge (v)
 end if
 else
 v = g%sum_mu_gi / g%sum_weights
 end if
 end function vamp_get_variance
85c <Implementation of vamp procedures 77d>+≡ (75a) <85b 86a>
 elemental subroutine vamp_nullify_variance (g)
 type(vamp_grid), intent(inout) :: g
 g%sum_mu_gi = 0
 end subroutine vamp_nullify_variance

```

### 5.2.3 Sampling

```

85d <Declaration of vamp procedures 76b>+≡ (75a) <84e 91b>
 public :: vamp_sample_grid
 public :: vamp_sample_grid0
 public :: vamp_refine_grid
 public :: vamp_refine_grids

```

*Simple Non-Adaptive Sampling:  $S_0$*

86a *⟨Implementation of vamp procedures 77d⟩*+≡ (75a) <85c 92a>

```

subroutine vamp_sample_grid0 &
 (rng, g, func, data, channel, weights, grids, exc, &
 negative_weights)
 type(tao_random_state), intent(inout) :: rng
 type(vamp_grid), intent(inout) :: g
 class(vamp_data_t), intent(in) :: data
 integer, intent(in), optional :: channel
 real(kind=default), dimension(:), intent(in), optional :: weights
 type(vamp_grid), dimension(:), intent(in), optional :: grids
 type(exception), intent(inout), optional :: exc
 ⟨Interface declaration for func 22⟩
 character(len=*), parameter :: FN = "vamp_sample_grid0"
 logical, intent(in), optional :: negative_weights
 ⟨Local variables in vamp_sample_grid0 87b⟩
 integer :: ndim
 logical :: neg_w
 ndim = size (g%div)
 neg_w = .false.
 if (present (negative_weights)) neg_w = negative_weights
 ⟨Check optional arguments in vamp_sample_grid0 91a⟩
 ⟨Reset counters in vamp_sample_grid0 87a⟩
 loop_over_cells: do
 ⟨Sample calls_per_cell points in the current cell 87d⟩
 ⟨Collect integration and grid optimization data for current cell 89b⟩
 ⟨Count up cell, exit if done 86b⟩
 end do loop_over_cells
 ⟨Collect results of vamp_sample_grid0 90a⟩
end subroutine vamp_sample_grid0

```

Count cells like a  $n_g$ -ary number—i.e.  $(1, \dots, 1, 1)$ ,  $(1, \dots, 1, 2)$ ,  $\dots$ ,  $(1, \dots, 1, n_g)$ ,  $(1, \dots, 2, 1)$ ,  $\dots$ ,  $(n_g, \dots, n_g, n_g - 1)$ ,  $(n_g, \dots, n_g, n_g)$ —and terminate when all (`cell == 1`) again.

86b *⟨Count up cell, exit if done 86b⟩*≡ (86a)

```

do j = ndim, 1, -1
 cell(j) = modulo (cell(j), rigid_division (g%div(j))) + 1
 if (cell(j) /= 1) then
 cycle loop_over_cells
 end if
end do
exit loop_over_cells

```

```

87a <Reset counters in vamp_sample_grid0 87a>≡ (86a)
 g%mu = 0.0
 g%mu_plus = 0.0
 g%mu_minus = 0.0
 cell = 1
 call clear_integral_and_variance (g%div)
 if (associated (g%mu_x)) then
 g%mu_x = 0.0
 g%mu_xx = 0.0
 end if
 if (present (channel)) then
 g%mu_gi = 0.0
 end if

87b <Local variables in vamp_sample_grid0 87b>≡ (86a) 87c>
 real(kind=default), parameter :: &
 eps = tiny (1._default) / epsilon (1._default)
 character(len=6) :: buffer

87c <Local variables in vamp_sample_grid0 87b>+≡ (86a) <87b 89a>
 integer :: j, k
 integer, dimension(size(g%div)) :: cell

87d <Sample calls_per_cell points in the current cell 87d>≡ (86a)
 sum_f = 0.0
 sum_f_plus = 0.0
 sum_f_minus = 0.0
 sum_f2 = 0.0
 sum_f2_plus = 0.0
 sum_f2_minus = 0.0
 do k = 1, g%calls_per_cell
 <Get x in the current cell 87e>
 <f = wgt * func (x, weights, channel), iff x inside true_domain 88a>
 <Collect integration and grid optimization data for x from f 88b>
 end do

```

We are using the generic procedure `tao_random_number` from the `tao_random_numbers` module for generating an array of uniform deviates. A better alternative would be to pass the random number generator as an argument to `vamp_sample_grid`. Unfortunately, it is not possible to pass *generic* procedures in Fortran90, Fortran95, or F. While we could export a specific procedure from `tao_random_numbers`, a more serious problem is that we have to pass the state `rng` of the random number generator as a `tao_random_state` anyway and we have to hardcode the random number generator anyway.

```

87e <Get x in the current cell 87e>≡ (87d)

```



```

call tao_random_number (rng, r)
call inject_division (g%div, real (r, kind=default), &
 cell, x, x_mid, ia, wgts)
wgt = g%jacobi * product (wgts)
if (associated (g%map)) then
 x = matmul (g%map, x)
end if

```

This somewhat contorted nested if constructs allow to minimize the number of calls to `func`. This is useful, since `func` is the most expensive part of real world applications. Also `func` might be singular outside of `true_domain`.

The original `vegas` used to call `f = wgt * func (x, wgt)` below to allow `func` to use `wgt` (i.e.  $1/p(x)$ ) for integrating another function at the same time. This form of “parallelism” relies on side effects and is therefore impossible with pure functions. Consequently, it is not supported in the current implementation.

```

88a <f = wgt * func (x, weights, channel), iff x inside true_domain 88a>≡ (87d 135d)
 if (associated (g%map)) then
 if (all (inside_division (g%div, x))) then
 f = wgt * func (x, data, weights, channel, grids)
 else
 f = 0.0
 end if
 else
 f = wgt * func (x, data, weights, channel, grids)
 end if

88b <Collect integration and grid optimization data for x from f 88b>≡ (87d) 88c▷
 if (g%f_min > g%f_max) then
 g%f_min = abs (f) * g%calls
 g%f_max = abs (f) * g%calls
 else if (abs (f) * g%calls < g%f_min) then
 g%f_min = abs (f) * g%calls
 else if (abs (f) * g%calls > g%f_max) then
 g%f_max = abs (f) * g%calls
 end if

88c <Collect integration and grid optimization data for x from f 88b>+≡ (87d) <88b
 f2 = f * f
 sum_f = sum_f + f
 sum_f2 = sum_f2 + f2
 if (f > 0) then
 sum_f_plus = sum_f_plus + f
 sum_f2_plus = sum_f2_plus + f * f
 else if (f < 0) then

```

```

 sum_f_minus = sum_f_minus + f
 sum_f2_minus = sum_f2_minus + f * f
 end if
 call record_integral (g%div, ia, f)
 ! call record_efficiency (g%div, ia, f/g%f_max)
 if ((associated (g%mu_x)) .and. (.not. g%all_stratified)) then
 g%mu_x = g%mu_x + x * f
 g%mu_xx = g%mu_xx + outer_product (x, x) * f
 end if
 if (present (channel)) then
 g%mu_gi = g%mu_gi + f2
 end if

```

89a  $\langle$ Local variables in vamp\_sample\_grid0 87b $\rangle \equiv$  (86a)  $\triangleleft$  87c

```

real(kind=default) :: wgt, f, f2
real(kind=default) :: sum_f, sum_f2, var_f
real(kind=default) :: sum_f_plus, sum_f2_plus, var_f_plus
real(kind=default) :: sum_f_minus, sum_f2_minus, var_f_minus
real(kind=default), dimension(size(g%div)):: x, x_mid, wgts
real(kind=default), dimension(size(g%div)):: r
integer, dimension(size(g%div)) :: ia

```

$$\sigma^2 \cdot N_{\text{calls/cell}}^2 (N_{\text{calls/cell}} - 1) = \text{var}(f) = N^2 \sigma^2 \left( \left\langle \frac{f^2}{p} \right\rangle - \langle f \rangle^2 \right) \quad (5.17)$$

89b  $\langle$ Collect integration and grid optimization data for current cell 89b $\rangle \equiv$  (86a)

```

var_f = sum_f2 * g%calls_per_cell - sum_f**2
var_f_plus = sum_f2_plus * g%calls_per_cell - sum_f_plus**2
var_f_minus = sum_f2_minus * g%calls_per_cell - sum_f_minus**2
if (var_f <= 0.0) then
 var_f = tiny (1.0_default)
end if
if (sum_f_plus /= 0 .and. var_f_plus <= 0) then
 var_f_plus = tiny (1.0_default)
end if
if (sum_f_minus /= 0 .and. var_f_minus <= 0) then
 var_f_minus = tiny (1.0_default)
end if
g%mu = g%mu + (/ sum_f, var_f /)
g%mu_plus = g%mu_plus + (/ sum_f_plus, var_f_plus /)
g%mu_minus = g%mu_minus + (/ sum_f_minus, var_f_minus /)
call record_variance (g%div, ia, var_f)
if ((associated (g%mu_x)) .and. g%all_stratified) then
 if (associated (g%map)) then

```

```

 x_mid = matmul (g%map, x_mid)
end if
g%mu_x = g%mu_x + x_mid * var_f
g%mu_xx = g%mu_xx + outer_product (x_mid, x_mid) * var_f
end if

```

$$\sigma^2 = \frac{\left(\frac{N_{\text{calls}}}{N_{\text{cells}}}\right)^2}{N_{\text{calls/cell}}^2(N_{\text{calls/cell}} - 1)} \sum_{\text{cells}} \sigma_{\text{cell}}^2 \cdot N_{\text{calls/cell}}^2 (N_{\text{calls/cell}} - 1) \quad (5.18)$$

where the  $N_{\text{calls}}^2$  cancels the corresponding factor in the Jacobian and the  $N_{\text{cells}}^{-2}$  is the result of stratification. In order to avoid numerical noise for some OS when using 80bit precision, we wrap the numerical resetting into a negative weights-only if-clause.

```

90a <Collect results of vamp_sample_grid0 90a>≡ (86a) 90b>
 g%mu(2) = g%mu(2) * g%dv2g
 if (g%mu(2) < eps * max (g%mu(1)**2, 1._default)) then
 g%mu(2) = eps * max (g%mu(1)**2, 1._default)
 end if
 if (neg_w) then
 g%mu_plus(2) = g%mu_plus(2) * g%dv2g
 if (g%mu_plus(2) < eps * max (g%mu_plus(1)**2, 1._default)) then
 g%mu_plus(2) = eps * max (g%mu_plus(1)**2, 1._default)
 end if
 g%mu_minus(2) = g%mu_minus(2) * g%dv2g
 if (g%mu_minus(2) < eps * max (g%mu_minus(1)**2, 1._default)) then
 g%mu_minus(2) = eps * max (g%mu_minus(1)**2, 1._default)
 end if
 end if
end if

90b <Collect results of vamp_sample_grid0 90a>+≡ (86a) <90a
 if (g%mu(1)>0) then
 g%sum_integral = g%sum_integral + g%mu(1) / g%mu(2)
 g%sum_weights = g%sum_weights + 1.0 / g%mu(2)
 g%sum_chi2 = g%sum_chi2 + g%mu(1)**2 / g%mu(2)
 if (associated (g%mu_x)) then
 if (g%all_stratified) then
 g%mu_x = g%mu_x / g%mu(2)
 g%mu_xx = g%mu_xx / g%mu(2)
 else
 g%mu_x = g%mu_x / g%mu(1)
 g%mu_xx = g%mu_xx / g%mu(1)
 end if
 g%sum_mu_x = g%sum_mu_x + g%mu_x / g%mu(2)
 g%sum_mu_xx = g%sum_mu_xx + g%mu_xx / g%mu(2)
 end if
 end if

```

```

end if
if (present (channel)) then
 g%sum_mu_gi = g%sum_mu_gi + g%mu_gi / g%mu(2)
end if
else if (neg_w) then
 g%sum_integral = g%sum_integral + g%mu(1) / g%mu(2)
 g%sum_weights = g%sum_weights + 1.0 / g%mu(2)
 g%sum_chi2 = g%sum_chi2 + g%mu(1)**2 / g%mu(2)
 if (associated (g%mu_x)) then
 if (g%all_stratified) then
 g%mu_x = g%mu_x / g%mu(2)
 g%mu_xx = g%mu_xx / g%mu(2)
 else
 g%mu_x = g%mu_x / g%mu(1)
 g%mu_xx = g%mu_xx / g%mu(1)
 end if
 g%sum_mu_x = g%sum_mu_x + g%mu_x / g%mu(2)
 g%sum_mu_xx = g%sum_mu_xx + g%mu_xx / g%mu(2)
 end if
 if (present (channel)) then
 g%sum_mu_gi = g%sum_mu_gi + g%mu_gi / g%mu(2)
 end if
else
 if (present(channel) .and. g%mu(1)==0) then
 write (buffer, "(I6)" channel
 call raise_exception (exc, EXC_WARN, "! vamp", &
 "Function identically zero in channel " // buffer)
 else if (present(channel) .and. g%mu(1)<0) then
 write (buffer, "(I6)" channel
 call raise_exception (exc, EXC_ERROR, "! vamp", &
 "Negative integral in channel " // buffer)
 end if
 g%sum_integral = 0
 g%sum_chi2 = 0
 g%sum_weights = 0
end if
91a <Check optional arguments in vamp_sample_grid0 91a>≡ (86a)
 if (present (channel) .neqv. present (weights)) then
 call raise_exception (exc, EXC_FATAL, FN, &
 "channel and weights required together")
 return
 end if
91b <Declaration of vamp procedures 76b>+≡ (75a) <85d 95b>

```

```
public :: vamp_probability
```

92a *<Implementation of vamp procedures 77d>+≡* (75a) <86a 92b>

```
pure function vamp_probability (g, x) result (p)
 type(vamp_grid), intent(in) :: g
 real(kind=default), dimension(:), intent(in) :: x
 real(kind=default) :: p
 p = product (probability (g%div, x))
end function vamp_probability
```



%variance should be private to division

92b *<Implementation of vamp procedures 77d>+≡* (75a) <92a 93>

```
subroutine vamp_apply_equivalences (g, eq)
 type(vamp_grids), intent(inout) :: g
 type(vamp_equivalences_t), intent(in) :: eq
 integer :: n_ch, n_dim, nb, i, ch, ch_src, dim, dim_src
 integer, dimension(:,:), allocatable :: n_bin
 real(kind=default), dimension(:,:,:), allocatable :: var_tmp
 n_ch = size (g%grids)
 if (n_ch == 0) return
 n_dim = size (g%grids(1)%div)
 allocate (n_bin(n_ch, n_dim))
 do ch = 1, n_ch
 do dim = 1, n_dim
 n_bin(ch, dim) = size (g%grids(ch)%div(dim)%variance)
 end do
 end do
 allocate (var_tmp (maxval(n_bin), n_dim, n_ch))
 var_tmp = 0
 do i=1, eq%n_eq
 ch = eq%eq(i)%left
 ch_src = eq%eq(i)%right
 do dim=1, n_dim
 nb = n_bin(ch_src, dim)
 dim_src = eq%eq(i)%permutation(dim)
 select case (eq%eq(i)%mode(dim))
 case (VEQ_IDENTITY)
 var_tmp(:,nb,dim,ch) = var_tmp(:,nb,dim,ch) &
 & + g%grids(ch_src)%div(dim_src)%variance
 case (VEQ_INVERT)
 var_tmp(:,nb,dim,ch) = var_tmp(:,nb,dim,ch) &
 & + g%grids(ch_src)%div(dim_src)%variance(nb:1:-1)
 case (VEQ_SYMMETRIC)
```

```

 var_tmp(:nb,dim,ch) = var_tmp(:nb,dim,ch) &
 & + g%grids(ch_src)%div(dim_src)%variance / 2 &
 & + g%grids(ch_src)%div(dim_src)%variance(nb:1:-1)/2
 case (VEQ_INVARIANT)
 var_tmp(:nb,dim,ch) = 1
 end select
end do
end do
do ch=1, n_ch
 do dim=1, n_dim
 g%grids(ch)%div(dim)%variance = var_tmp(:n_bin(ch, dim),dim,ch)
 end do
end do
deallocate (var_tmp)
deallocate (n_bin)
end subroutine vamp_apply_equivalences

```

*Grid Refinement:  $r$*

$$n_{\text{div},j} \rightarrow \frac{Q_j n_{\text{div},j}}{\left(\prod_j Q_j\right)^{1/n_{\text{dim}}}} \quad (5.19)$$

where

$$Q_j = \left( \sqrt{\text{Var}(\{m\}_j)} \right)^\alpha \quad (5.20)$$

93  $\langle$ Implementation of `vamp` procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 92b 94a $\rangle$

```

pure subroutine vamp_refine_grid (g, exc)
 type(vamp_grid), intent(inout) :: g
 type(exception), intent(inout), optional :: exc
 real(kind=default), dimension(size(g%div)) :: quad
 integer :: ndim
 if (g%quadrupole) then
 ndim = size (g%div)
 quad = (quadrupole_division (g%div))**QUAD_POWER
 call vamp_reshape_grid_internal &
 (g, use_variance = .true., exc = exc, &
 num_div = int (quad / product (quad)**(1.0/ndim) * g%num_div))
 else
 call refine_division (g%div)
 call vamp_nullify_f_limits (g)
 end if
end subroutine vamp_refine_grid

```

94a  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 93 94c $\rangle$

```
subroutine vamp_refine_grids (g)
 type(vamp_grids), intent(inout) :: g
 integer :: ch
 do ch=1, size(g%grids)
 call refine_division (g%grids(ch)%div)
 call vamp_nullify_f_limits (g%grids(ch))
 end do
end subroutine vamp_refine_grids
```

94b  $\langle$ Variables in vamp 78a $\rangle + \equiv$  (75a)  $\langle$ 78a 109a $\rangle$

```
real(kind=default), private, parameter :: QUAD_POWER = 0.5_default
```

$$\text{Adaptive Sampling: } S_n = S_0(rS_0)^n$$

94c  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 94a 95a $\rangle$

```
subroutine vamp_sample_grid &
 (rng, g, func, data, iterations, &
 integral, std_dev, avg_chi2, accuracy, &
 channel, weights, grids, exc, history)
 type(tao_random_state), intent(inout) :: rng
 type(vamp_grid), intent(inout) :: g
 class(vamp_data_t), intent(in) :: data
 integer, intent(in) :: iterations
 real(kind=default), intent(out), optional :: integral, std_dev, avg_chi2
 real(kind=default), intent(in), optional :: accuracy
 integer, intent(in), optional :: channel
 real(kind=default), dimension(:), intent(in), optional :: weights
 type(vamp_grid), dimension(:), intent(in), optional :: grids
 type(exception), intent(inout), optional :: exc
 type(vamp_history), dimension(:), intent(inout), optional :: history
 \langle Interface declaration for func 22 \rangle
 character(len=*), parameter :: FN = "vamp_sample_grid"
 real(kind=default) :: local_integral, local_std_dev, local_avg_chi2
 integer :: iteration, ndim
 ndim = size (g%div)
 iterate: do iteration = 1, iterations
 call vamp_sample_grid0 &
 (rng, g, func, data, channel, weights, grids, exc)
 call vamp_average_iterations &
 (g, iteration, local_integral, local_std_dev, local_avg_chi2)
 \langle Trace results of vamp-sample.grid 106b \rangle
 \langle Exit iterate if accuracy has been reached 96b \rangle
 if (iteration < iterations) call vamp_refine_grid (g)
```

```

end do iterate
 <Copy results of vamp_sample_grid to dummy variables 96a>
end subroutine vamp_sample_grid

```

Assuming that the iterations have been statistically independent, we can combine them with the usual formulae.

$$\bar{I} = \sigma_I^2 \sum_i \frac{I_i}{\sigma_i^2} \quad (5.21a)$$

$$\frac{1}{\sigma_I^2} = \sum_i \frac{1}{\sigma_i^2} \quad (5.21b)$$

$$\chi^2 = \sum_i \frac{(I_i - \bar{I})^2}{\sigma_i^2} = \sum_i \frac{I_i^2}{\sigma_i^2} - \bar{I} \sum_i \frac{I_i}{\sigma_i^2} \quad (5.21c)$$

```

95a <Implementation of vamp procedures 77d>+≡ (75a) <94c 97a>
 elemental subroutine vamp_average_iterations_grid &
 (g, iteration, integral, std_dev, avg_chi2)
 type(vamp_grid), intent(in) :: g
 integer, intent(in) :: iteration
 real(kind=default), intent(out) :: integral, std_dev, avg_chi2
 real(kind=default), parameter :: eps = 1000 * epsilon (1._default)
 if (g%sum_weights>0) then
 integral = g%sum_integral / g%sum_weights
 std_dev = sqrt (1.0 / g%sum_weights)
 avg_chi2 = &
 max ((g%sum_chi2 - g%sum_integral * integral) / (iteration-0.99), &
 0.0_default)
 if (avg_chi2 < eps * g%sum_chi2) avg_chi2 = 0
 else
 integral = 0
 std_dev = 0
 avg_chi2 = 0
 end if
 end subroutine vamp_average_iterations_grid

95b <Declaration of vamp procedures 76b>+≡ (75a) <91b 96c>
 public :: vamp_average_iterations
 private :: vamp_average_iterations_grid

95c <Interfaces of vamp procedures 95c>≡ (75a) 96d>
 interface vamp_average_iterations
 module procedure vamp_average_iterations_grid
 end interface

```



Lepage suggests [1] to reweight the contributions as in the following improved formulae, which we might implement as an option later.

$$\bar{I} = \frac{1}{\left(\sum_i \frac{I_i^2}{\sigma_i^2}\right)^2} \sum_i I_i \frac{I_i^2}{\sigma_i^2} \quad (5.22a)$$

$$\frac{1}{\sigma_I^2} = \frac{1}{(\bar{I})^2} \sum_i \frac{I_i^2}{\sigma_i^2} \quad (5.22b)$$

$$\chi^2 = \sum_i \frac{(I_i - \bar{I})^2}{(\bar{I})^2} \frac{I_i^2}{\sigma_i^2} \quad (5.22c)$$

Iff possible, copy the result to the caller's variables:

```

96a <Copy results of vamp_sample_grid to dummy variables 96a>≡ (94c 103b 120b)
 if (present (integral)) then
 integral = local_integral
 end if
 if (present (std_dev)) then
 std_dev = local_std_dev
 end if
 if (present (avg_chi2)) then
 avg_chi2 = local_avg_chi2
 end if

96b <Exit iterate if accuracy has been reached 96b>≡ (94c 103b 120b)
 if (present (accuracy)) then
 if (local_std_dev <= accuracy * local_integral) then
 call raise_exception (exc, EXC_INFO, FN, &
 "requested accuracy reached")
 exit iterate
 end if
 end if

```

#### 5.2.4 Forking and Joining

```

96c <Declaration of vamp procedures 76b>+≡ (75a) <95b 102a>
 public :: vamp_fork_grid
 private :: vamp_fork_grid_single, vamp_fork_grid_multi
 public :: vamp_join_grid
 private :: vamp_join_grid_single, vamp_join_grid_multi

96d <Interfaces of vamp procedures 95c>+≡ (75a) <95c 106d>
 interface vamp_fork_grid
 module procedure vamp_fork_grid_single, vamp_fork_grid_multi

```

```

end interface
interface vamp_join_grid
 module procedure vamp_join_grid_single, vamp_join_grid_multi
end interface

```

Caveat emptor: splitting divisions can lead to `num_div < 3` an the application must not try to refine such grids before merging them again! `d == 0` is special.

97a *<Implementation of vamp procedures 77d>+≡* (75a) <95a 99e>

```

pure subroutine vamp_fork_grid_single (g, gs, d, exc)
 type(vamp_grid), intent(in) :: g
 type(vamp_grid), dimension(:), intent(inout) :: gs
 integer, intent(in) :: d
 type(exception), intent(inout), optional :: exc
 character(len=*), parameter :: FN = "vamp_fork_grid_single"
 type(division_t), dimension(:), allocatable :: d_tmp
 integer :: i, j, num_grids, num_div, ndim, num_cells
 num_grids = size (gs)
 ndim = size (g%div)
 <Allocate or resize the divisions 99d>
 do j = 1, ndim
 if (j == d) then
 <call fork_division (g%div(j), gs%div(j), g%calls_per_cell, ...) 99a>
 else
 <call copy_division (gs%div(j), g%div(j)) 99c>
 end if
 end do
 if (d == 0) then
 <Handle g%calls_per_cell for d == 0 97b>
 end if
 <Copy the rest of g to the gs 97c>
end subroutine vamp_fork_grid_single

```

Divide the sampling points among identical grids

97b *<Handle g%calls\_per\_cell for d == 0 97b>≡* (97a)

```

if (any (stratified_division (g%div))) then
 call raise_exception (exc, EXC_FATAL, FN, &
 "d == 0 incompatible w/ stratification")
else
 gs(2:)%calls_per_cell = ceiling (real (g%calls_per_cell) / num_grids)
 gs(1)%calls_per_cell = g%calls_per_cell - sum (gs(2:)%calls_per_cell)
end if

```

97c *<Copy the rest of g to the gs 97c>≡* (97a) 98a>

```

do i = 1, num_grids

```

```

call copy_array_pointer (gs(i)%num_div, g%num_div)
if (associated (g%map)) then
 call copy_array_pointer (gs(i)%map, g%map)
end if
if (associated (g%mu_x)) then
 call create_array_pointer (gs(i)%mu_x, ndim)
 call create_array_pointer (gs(i)%sum_mu_x, ndim)
 call create_array_pointer (gs(i)%mu_xx, (/ ndim, ndim /))
 call create_array_pointer (gs(i)%sum_mu_xx, (/ ndim, ndim /))
end if
end do

```

Reset results

98a  $\langle$ Copy the rest of g to the gs 97c $\rangle + \equiv$

(97a)  $\langle$ 97c 98b $\rangle$

```

gs%mu(1) = 0.0
gs%mu(2) = 0.0
gs%mu_plus(1) = 0.0
gs%mu_plus(2) = 0.0
gs%mu_minus(1) = 0.0
gs%mu_minus(2) = 0.0
gs%sum_integral = 0.0
gs%sum_weights = 0.0
gs%sum_chi2 = 0.0
gs%mu_gi = 0.0
gs%sum_mu_gi = 0.0

```

98b  $\langle$ Copy the rest of g to the gs 97c $\rangle + \equiv$

(97a)  $\langle$ 98a 98c $\rangle$

```

gs%stratified = g%stratified
gs%all_stratified = g%all_stratified
gs%quadrupole = g%quadrupole

```

98c  $\langle$ Copy the rest of g to the gs 97c $\rangle + \equiv$

(97a)  $\langle$ 98b

```

do i = 1, num_grids
 num_cells = product (rigid_division (gs(i)%div))
 gs(i)%calls = gs(i)%calls_per_cell * num_cells
 gs(i)%num_calls = gs(i)%calls
 gs(i)%jacobi = product (volume_division (gs(i)%div)) / gs(i)%calls
 gs(i)%dv2g = (gs(i)%calls / num_cells)**2 &
 / gs(i)%calls_per_cell / gs(i)%calls_per_cell / (gs(i)%calls_per_cell - 1.0)
end do
gs%f_min = g%f_min * (gs%jacobi * gs%calls) / (g%jacobi * g%calls)
gs%f_max = g%f_max * (gs%jacobi * gs%calls) / (g%jacobi * g%calls)

```

This could be self-explaining, if the standard would allow ... Note that we can get away with copying just the pointers, because `fork_division` does

the dirty work for the memory management.

```

99a <call fork_division (g%div(j), gs%div(j), g%calls_per_cell, ...) 99a>≡ (97a)
 allocate (d_tmp(num_grids))
 do i = 1, num_grids
 d_tmp(i) = gs(i)%div(j)
 end do
 call fork_division (g%div(j), d_tmp, g%calls_per_cell, gs%calls_per_cell, exc)
 do i = 1, num_grids
 gs(i)%div(j) = d_tmp(i)
 end do
 deallocate (d_tmp)
 <Bail out if exception exc raised 99b>

```

```

99b <Bail out if exception exc raised 99b>≡ (99a 100a 103b 140c 142b)
 if (present (exc)) then
 if (exc%level > EXC_WARN) then
 return
 end if
 end if

```

We have to do a deep copy ( $gs(i)\%div(j) = g\%div(j)$  does not suffice), because `copy_division` handles the memory management.

```

99c <call copy_division (gs%div(j), g%div(j)) 99c>≡ (97a)
 do i = 1, num_grids
 call copy_division (gs(i)%div(j), g%div(j))
 end do

```

```

99d <Allocate or resize the divisions 99d>≡ (97a)
 num_div = size (g%div)
 do i = 1, size (gs)
 if (associated (gs(i)%div)) then
 if (size (gs(i)%div) /= num_div) then
 allocate (gs(i)%div(num_div))
 call create_empty_division (gs(i)%div)
 end if
 else
 allocate (gs(i)%div(num_div))
 call create_empty_division (gs(i)%div)
 end if
 end do

```

```

99e <Implementation of vamp procedures 77d>+≡ (75a) <97a 101b>
 pure subroutine vamp_join_grid_single (g, gs, d, exc)
 type(vamp_grid), intent(inout) :: g
 type(vamp_grid), dimension(:), intent(inout) :: gs

```

```

integer, intent(in) :: d
type(exception), intent(inout), optional :: exc
type(division_t), dimension(:), allocatable :: d_tmp
integer :: i, j, num_grids
num_grids = size (gs)
do j = 1, size (g%div)
 if (j == d) then
 <call join_division (g%div(j), gs%div(j)) 100a>
 else
 <call sum_division (g%div(j), gs%div(j)) 100b>
 end if
end do
<Combine the rest of gs onto g 100c>
end subroutine vamp_join_grid_single
100a <call join_division (g%div(j), gs%div(j)) 100a>≡ (99e)
 allocate (d_tmp(num_grids))
 do i = 1, num_grids
 d_tmp(i) = gs(i)%div(j)
 end do
 call join_division (g%div(j), d_tmp, exc)
 deallocate (d_tmp)
 <Bail out if exception exc raised 99b>
100b <call sum_division (g%div(j), gs%div(j)) 100b>≡ (99e)
 allocate (d_tmp(num_grids))
 do i = 1, num_grids
 d_tmp(i) = gs(i)%div(j)
 end do
 call sum_division (g%div(j), d_tmp)
 deallocate (d_tmp)
100c <Combine the rest of gs onto g 100c>≡ (99e)
 g%f_min = minval (gs%f_min * (g%jacobi * g%calls) / (gs%jacobi * gs%calls))
 g%f_max = maxval (gs%f_max * (g%jacobi * g%calls) / (gs%jacobi * gs%calls))
 g%mu(1) = sum (gs%mu(1))
 g%mu(2) = sum (gs%mu(2))
 g%mu_plus(1) = sum (gs%mu_plus(1))
 g%mu_plus(2) = sum (gs%mu_plus(2))
 g%mu_minus(1) = sum (gs%mu_minus(1))
 g%mu_minus(2) = sum (gs%mu_minus(2))
 g%mu_gi = sum (gs%mu_gi)
 g%sum_mu_gi = g%sum_mu_gi + g%mu_gi / g%mu(2)
 g%sum_integral = g%sum_integral + g%mu(1) / g%mu(2)
 g%sum_chi2 = g%sum_chi2 + g%mu(1)**2 / g%mu(2)

```

```

g%sum_weights = g%sum_weights + 1.0 / g%mu(2)
if (associated (g%mu_x)) then
 do i = 1, num_grids
 g%mu_x = g%mu_x + gs(i)%mu_x
 g%mu_xx = g%mu_xx + gs(i)%mu_xx
 end do
 g%sum_mu_x = g%sum_mu_x + g%mu_x / g%mu(2)
 g%sum_mu_xx = g%sum_mu_xx + g%mu_xx / g%mu(2)
end if

```

The following is made a little bit hairy by the fact that `vamp_fork_grid` can't join grids onto a non-existing grid<sup>2</sup> therefore we have to keep a tree of joints. Maybe it would be the right thing to handle this tree of joints as a tree with pointers, but since we need the leaves flattened anyway (as food for multiple `vamp_sample_grid`) we use a similar storage layout for the joints.

```

101a <Idioms 101a>≡ 250>
 type(vamp_grid), dimension(:), allocatable :: gx
 integer, dimension(:,:), allocatable :: dim
 ...
 allocate (gx(vamp_fork_grid_joints (dim)))
 call vamp_fork_grid (g, gs, gx, dim, exc)
 ...
 call vamp_join_grid (g, gs, gx, dim, exc)

101b <Implementation of vamp procedures 77d>+≡ (75a) <99e 102b>
 pure recursive subroutine vamp_fork_grid_multi (g, gs, gx, d, exc)
 type(vamp_grid), intent(in) :: g
 type(vamp_grid), dimension(:), intent(inout) :: gs, gx
 integer, dimension(:,:), intent(in) :: d
 type(exception), intent(inout), optional :: exc
 character(len=*), parameter :: FN = "vamp_fork_grid_multi"
 integer :: i, offset, stride, joints_offset, joints_stride
 select case (size (d, dim=2))
 case (0)
 return
 case (1)
 call vamp_fork_grid_single (g, gs, d(1,1), exc)
 case default
 offset = 1
 stride = product (d(2,2:))
 joints_offset = 1 + d(2,1)
 joints_stride = vamp_fork_grid_joints (d(:,2:))

```

---

<sup>2</sup>It would be possible to make it possible by changing many things under the hood, but it doesn't really make sense, anyway.

```

call vamp_create_empty_grid (gx(1:d(2,1)))
call vamp_fork_grid_single (g, gx(1:d(2,1)), d(1,1), exc)
do i = 1, d(2,1)
 call vamp_fork_grid_multi &
 (gx(i), gs(offset:offset+stride-1), &
 gx(joints_offset:joints_offset+joints_stride-1), &
 d(:,2:), exc)
 offset = offset + stride
 joints_offset = joints_offset + joints_stride
end do
end select
end subroutine vamp_fork_grid_multi

```

102a  $\langle$ Declaration of vamp procedures 76b $\rangle + \equiv$  (75a)  $\langle$ 96c 103a $\rangle$

```

public :: vamp_fork_grid_joints

```

$$\sum_{n=1}^{N-1} \prod_{i_n=1}^n d_{i_n} = d_1(1 + d_2(1 + d_3(1 + \dots(1 + d_{N-1}) \dots))) \quad (5.23)$$

102b  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 101b 102c $\rangle$

```

pure function vamp_fork_grid_joints (d) result (s)
 integer, dimension(:,,:), intent(in) :: d
 integer :: s
 integer :: i
 s = 0
 do i = size (d, dim=2) - 1, 1, -1
 s = (s + 1) * d(2,i)
 end do
end function vamp_fork_grid_joints

```

102c  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 102b 103b $\rangle$

```

pure recursive subroutine vamp_join_grid_multi (g, gs, gx, d, exc)
 type(vamp_grid), intent(inout) :: g
 type(vamp_grid), dimension(:), intent(inout) :: gs, gx
 integer, dimension(:,,:), intent(in) :: d
 type(exception), intent(inout), optional :: exc
 character(len=*), parameter :: FN = "vamp_join_grid_multi"
 integer :: i, offset, stride, joints_offset, joints_stride
 select case (size (d, dim=2))
 case (0)
 return
 case (1)
 call vamp_join_grid_single (g, gs, d(1,1), exc)
 case default
 offset = 1

```

```

stride = product (d(2,2:))
joints_offset = 1 + d(2,1)
joints_stride = vamp_fork_grid_joints (d(:,2:))
do i = 1, d(2,1)
 call vamp_join_grid_multi &
 (gx(i), gs(offset:offset+stride-1), &
 gx(joints_offset:joints_offset+joints_stride-1), &
 d(:,2:), exc)
 offset = offset + stride
 joints_offset = joints_offset + joints_stride
end do
call vamp_join_grid_single (g, gx(1:d(2,1)), d(1,1), exc)
call vamp_delete_grid (gx(1:d(2,1)))
end select
end subroutine vamp_join_grid_multi

```

### 5.2.5 Parallel Execution

103a  $\langle$ Declaration of vamp procedures 76b $\rangle + \equiv$  (75a)  $\langle$ 102a 106c $\rangle$

```

public :: vamp_sample_grid_parallel
public :: vamp_distribute_work

```

HPF [10, 11, 15]:

103b  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 102c 105a $\rangle$

```

subroutine vamp_sample_grid_parallel &
 (rng, g, func, data, iterations, &
 integral, std_dev, avg_chi2, accuracy, &
 channel, weights, grids, exc, history)
type(tao_random_state), dimension(:), intent(inout) :: rng
type(vamp_grid), intent(inout) :: g
class(vamp_data_t), intent(in) :: data
integer, intent(in) :: iterations
real(kind=default), intent(out), optional :: integral, std_dev, avg_chi2
real(kind=default), intent(in), optional :: accuracy
integer, intent(in), optional :: channel
real(kind=default), dimension(:), intent(in), optional :: weights
type(vamp_grid), dimension(:), intent(in), optional :: grids
type(exception), intent(inout), optional :: exc
type(vamp_history), dimension(:), intent(inout), optional :: history
 \langle Interface declaration for func 22 \rangle
character(len=*), parameter :: FN = "vamp_sample_grid_parallel"
real(kind=default) :: local_integral, local_std_dev, local_avg_chi2
type(exception), dimension(size(rng)) :: excs

```



```

type(vamp_grid), dimension(:), allocatable :: gs, gx
!hpf$ processors p(number_of_processors())
!hpf$ distribute gs(cyclic(1)) onto p
integer, dimension(:,:), pointer :: d
integer :: iteration, i
integer :: num_workers
nullify (d)
call clear_exception (excs)
iterate: do iteration = 1, iterations
 call vamp_distribute_work (size (rng), vamp_rigid_divisions (g), d)
 num_workers = max (1, product (d(2,:)))
 if (num_workers > 1) then
 allocate (gs(num_workers), gx(vamp_fork_grid_joints (d)))
 call vamp_create_empty_grid (gs)
 ! vamp_fork_grid is certainly not local. Speed freaks might
 ! want to tune it to the processor topology, but the gain will be small.
 call vamp_fork_grid (g, gs, gx, d, exc)
 !hpf$ independent
 do i = 1, num_workers
 call vamp_sample_grid0 &
 (rng(i), gs(i), func, data, &
 channel, weights, grids, exc)
 end do
 <Gather exceptions in vamp_sample_grid_parallel 104>
 call vamp_join_grid (g, gs, gx, d, exc)
 call vamp_delete_grid (gs)
 deallocate (gs, gx)
 else
 call vamp_sample_grid0 &
 (rng(1), g, func, data, channel, weights, grids, exc)
 end if
 <Bail out if exception exc raised 99b>
 call vamp_average_iterations &
 (g, iteration, local_integral, local_std_dev, local_avg_chi2)
 <Trace results of vamp_sample_grid 106b>
 <Exit iterate if accuracy has been reached 96b>
 if (iteration < iterations) call vamp_refine_grid (g)
end do iterate
deallocate (d)
<Copy results of vamp_sample_grid to dummy variables 96a>
end subroutine vamp_sample_grid_parallel
104 <Gather exceptions in vamp_sample_grid_parallel 104>≡ (103b)
 if ((present (exc)) .and. (any (excs(1:num_workers)%level > 0))) then

```

```

 call gather_exceptions (exc, excs(1:num_workers))
end if

```

We could sort  $d$  such that (5.23) is minimized

$$d_1 \leq d_2 \leq \dots \leq d_N \quad (5.24)$$

but the gain will be negligible.

```

105a <Implementation of vamp procedures 77d>+≡ (75a) <103b 107a>
 pure subroutine vamp_distribute_work (num_workers, ng, d)
 integer, intent(in) :: num_workers
 integer, dimension(:), intent(in) :: ng
 integer, dimension(:, :), pointer :: d
 integer, dimension(32) :: factors
 integer :: n, num_factors, i, j
 integer, dimension(size(ng)) :: num_forks
 integer :: nfork
 try: do n = num_workers, 1, -1
 call factorize (n, factors, num_factors)
 num_forks = 1
 do i = num_factors, 1, -1
 j = sum (maxloc (ng / num_forks))
 nfork = num_forks(j) * factors(i)
 if (nfork <= ng(j)) then
 num_forks(j) = nfork
 else
 cycle try
 end if
 end do
 <Accept distribution among n workers 105b>
 end do try
 end subroutine vamp_distribute_work

105b <Accept distribution among n workers 105b>≡ (105a) 105c>
 j = count (num_forks > 1)
 if (associated (d)) then
 if (size (d, dim = 2) /= j) then
 deallocate (d)
 allocate (d(2,j))
 end if
 else
 allocate (d(2,j))
 end if

105c <Accept distribution among n workers 105b>+≡ (105a) <105b

```

```

j = 1
do i = 1, size (ng)
 if (num_forks(i) > 1) then
 d(:,j) = (/ i, num_forks(i) /)
 j = j + 1
 end if
end do
return

```

### 5.2.6 Diagnostics

- 106a  $\langle$ Declaration of vamp types 77a $\rangle + \equiv$  (75a)  $\langle$ 77b 112 $\rangle$
- ```

type, public :: vamp_history
  private
  real(kind=default) :: &
    integral, std_dev, avg_integral, avg_std_dev, avg_chi2, f_min, f_max
  integer :: calls
  logical :: stratified
  logical :: verbose
  type(div_history), dimension(:), pointer :: div => null ()
end type vamp_history

```
- 106b \langle Trace results of vamp_sample_grid 106b $\rangle \equiv$ (94c 103b)
- ```

if (present (history)) then
 if (iteration <= size (history)) then
 call vamp_get_history &
 (history(iteration), g, local_integral, local_std_dev, &
 local_avg_chi2)
 else
 call raise_exception (exc, EXC_WARN, FN, "history too short")
 end if
 call vamp_terminate_history (history(iteration+1:))
end if

```
- 106c  $\langle$ Declaration of vamp procedures 76b $\rangle + \equiv$  (75a)  $\langle$ 103a 108b $\rangle$
- ```

public :: vamp_create_history, vamp_copy_history, vamp_delete_history
public :: vamp_terminate_history
public :: vamp_get_history, vamp_get_history_single

```
- 106d \langle Interfaces of vamp procedures 95c $\rangle + \equiv$ (75a) \langle 96d 108c \rangle
- ```

interface vamp_get_history
 module procedure vamp_get_history_single
end interface

```

```

107a <Implementation of vamp procedures 77d>+≡ (75a) <105a 107b>
 elemental subroutine vamp_create_history (h, ndim, verbose)
 type(vamp_history), intent(out) :: h
 integer, intent(in), optional :: ndim
 logical, intent(in), optional :: verbose
 if (present (verbose)) then
 h%verbose = verbose
 else
 h%verbose = .false.
 end if
 h%calls = 0.0
 if (h%verbose .and. (present (ndim))) then
 if (associated (h%div)) then
 deallocate (h%div)
 end if
 allocate (h%div(ndim))
 end if
 end subroutine vamp_create_history

107b <Implementation of vamp procedures 77d>+≡ (75a) <107a 107c>
 elemental subroutine vamp_terminate_history (h)
 type(vamp_history), intent(inout) :: h
 h%calls = 0.0
 end subroutine vamp_terminate_history

107c <Implementation of vamp procedures 77d>+≡ (75a) <107b 108d>
 pure subroutine vamp_get_history_single (h, g, integral, std_dev, avg_chi2)
 type(vamp_history), intent(inout) :: h
 type(vamp_grid), intent(in) :: g
 real(kind=default), intent(in) :: integral, std_dev, avg_chi2
 h%calls = g%calls
 h%stratified = g%all_stratified
 h%integral = g%mu(1)
 h%std_dev = sqrt (g%mu(2))
 h%avg_integral = integral
 h%avg_std_dev = std_dev
 h%avg_chi2 = avg_chi2
 h%f_min = g%f_min
 h%f_max = g%f_max
 if (h%verbose) then
 <Adjust h%div iff necessary 108a>
 call copy_history (h%div, summarize_division (g%div))
 end if
 end subroutine vamp_get_history_single

```

108a *<Adjust h%div iff necessary 108a>*≡ (107c)

```

 if (associated (h%div)) then
 if (size (h%div) /= size (g%div)) then
 deallocate (h%div)
 allocate (h%div(size(g%div)))
 end if
 else
 allocate (h%div(size(g%div)))
 end if

```

108b *<Declaration of vamp procedures 76b>*+≡ (75a) <106c 113a>

```

 public :: vamp_print_history, vamp_write_history
 private :: vamp_print_one_history, vamp_print_histories
 ! private :: vamp_write_one_history, vamp_write_histories

```

108c *<Interfaces of vamp procedures 95c>*+≡ (75a) <106d 124b>

```

 interface vamp_print_history
 module procedure vamp_print_one_history, vamp_print_histories
 end interface
 interface vamp_write_history
 module procedure vamp_write_one_history_unit, vamp_write_histories_unit
 end interface

```

108d *<Implementation of vamp procedures 77d>*+≡ (75a) <107c 109b>

```

 subroutine vamp_print_one_history (h, tag)
 type(vamp_history), dimension(:), intent(in) :: h
 character(len=*), intent(in), optional :: tag
 type(div_history), dimension(:), allocatable :: h_tmp
 character(len=BUFFER_SIZE) :: pfx
 character(len=1) :: s
 integer :: i, imax, j
 if (present (tag)) then
 pfx = tag
 else
 pfx = "[vamp]"
 end if
 print "(1X,A78)", repeat("-", 78)
 print "(1X,A8,1X,A2,A9,A1,1X,A11,1X,8X,1X," &
 // "1X,A13,1X,8X,1X,A5,1X,A5)", &
 pfx, "it", "#calls", "", "integral", "average", "chi2", "eff."
 imax = size (h)
 iterations: do i = 1, imax
 if (h(i)%calls <= 0) then
 imax = i - 1
 end if
 end do iterations
 end subroutine

```

```

end if
! *JR: Skip zero channel
if (h(i)%f_max==0) cycle
if (h(i)%stratified) then
 s = "*"
else
 s = ""
end if
print "(1X,A8,1X,I2,I9,A1,1X,E11.4,A1,E8.2,A1," &
// "1X,E13.6,A1,E8.2,A1,F5.1,1X,F5.3)", pfx, &
i, h(i)%calls, s, h(i)%integral, "(", h(i)%std_dev, ")", &
h(i)%avg_integral, "(", h(i)%avg_std_dev, ")", h(i)%avg_chi2, &
h(i)%integral / h(i)%f_max
end do iterations
print "(1X,A78)", repeat("-", 78)
if (all (h%verbose) .and. (imax >= 1)) then
 if (associated (h(1)%div)) then
 allocate (h_tmp(imax))
 dimensions: do j = 1, size (h(1)%div)
 do i = 1, imax
 call copy_history (h_tmp(i), h(i)%div(j))
 end do
 if (present (tag)) then
 write (unit = pfx, fmt = "(A,A1,I2.2)") &
trim (tag(1:min(len_trim(tag),8))), "#", j
 else
 write (unit = pfx, fmt = "(A,A1,I2.2)") "[vamp]", "#", j
 end if
 call print_history (h_tmp, tag = pfx)
 print "(1X,A78)", repeat("-", 78)
 end do dimensions
 deallocate (h_tmp)
 end if
end if
flush (output_unit)
end subroutine vamp_print_one_history

```

109a *<Variables in vamp 78a>+≡* (75a) <94b 146a>  
integer, private, parameter :: BUFFER\_SIZE = 50

109b *<Implementation of vamp procedures 77d>+≡* (75a) <108d 110>  
subroutine vamp\_print\_histories (h, tag)  
type(vamp\_history), dimension(:,,:), intent(in) :: h  
character(len=\*), intent(in), optional :: tag  
character(len=BUFFER\_SIZE) :: pfx

```

integer :: i
print "(1X,A78)", repeat ("=", 78)
channels: do i = 1, size (h, dim=2)
 if (present (tag)) then
 write (unit = pfx, fmt = "(A4,A1,I3.3)") tag, "#", i
 else
 write (unit = pfx, fmt = "(A4,A1,I3.3)") "chan", "#", i
 end if
 call vamp_print_one_history (h(:,i), pfx)
end do channels
print "(1X,A78)", repeat ("=", 78)
flush (output_unit)
end subroutine vamp_print_histories

```



110 *<Implementation of vamp procedures 77d>+≡ (75a) <109b 113b>*

```

subroutine vamp_write_one_history_unit (u, h, tag)
 integer, intent(in) :: u
 type(vamp_history), dimension(:), intent(in) :: h
 character(len=*), intent(in), optional :: tag
 type(div_history), dimension(:), allocatable :: h_tmp
 character(len=BUFFER_SIZE) :: pfx
 character(len=1) :: s
 integer :: i, imax, j
 if (present (tag)) then
 pfx = tag
 else
 pfx = "[vamp]"
 end if
 write (u, "(1X,A78)") repeat ("-", 78)
 write (u, "(1X,A8,1X,A2,A9,A1,1X,A11,1X,8X,1X," &
 // "1X,A13,1X,8X,1X,A5,1X,A5)") &
 pfx, "it", "#calls", "", "integral", "average", "chi2", "eff."
 imax = size (h)
 iterations: do i = 1, imax
 if (h(i)%calls <= 0) then
 imax = i - 1
 exit iterations
 end if
 ! *WK: Skip zero channel
 if (h(i)%f_max==0) cycle
 if (h(i)%stratified) then
 s = "*"
 end if
 end do

```

```

else
 s = ""
end if
write (u, "(1X,A8,1X,I2,I9,A1,1X,ES11.4,A1,ES8.2,A1," &
// "1X,ES13.6,A1,ES8.2,A1,F5.1,1X,F5.3)") pfx, &
i, h(i)%calls, s, h(i)%integral, "(", h(i)%std_dev, ")", &
h(i)%avg_integral, "(", h(i)%avg_std_dev, ")", h(i)%avg_chi2, &
h(i)%integral / h(i)%f_max
end do iterations
write (u, "(1X,A78)") repeat ("-", 78)
if (all (h%verbose) .and. (imax >= 1)) then
 if (associated (h(1)%div)) then
 allocate (h_tmp(imax))
 dimensions: do j = 1, size (h(1)%div)
 do i = 1, imax
 call copy_history (h_tmp(i), h(i)%div(j))
 end do
 if (present (tag)) then
 write (unit = pfx, fmt = "(A,A1,I2.2)") &
 trim (tag(1:min(len_trim(tag),8))), "#", j
 else
 write (unit = pfx, fmt = "(A,A1,I2.2)") "[vamp]", "#", j
 end if
 call write_history (u, h_tmp, tag = pfx)
 print "(1X,A78)", repeat ("-", 78)
 end do dimensions
 deallocate (h_tmp)
 end if
end if
flush (u)
end subroutine vamp_write_one_history_unit
subroutine vamp_write_histories_unit (u, h, tag)
 integer, intent(in) :: u
 type(vamp_history), dimension(:,,:), intent(in) :: h
 character(len=*), intent(in), optional :: tag
 character(len=BUFFER_SIZE) :: pfx
 integer :: i
 write (u, "(1X,A78)") repeat ("=", 78)
 channels: do i = 1, size (h, dim=2)
 if (present (tag)) then
 write (unit = pfx, fmt = "(A4,A1,I3.3)") tag, "#", i
 else
 write (unit = pfx, fmt = "(A4,A1,I3.3)") "chan", "#", i
 end if
 end do
end subroutine

```



```

 end if
 call vamp_write_one_history_unit (u, h(:,i), pfx)
 end do channels
 write (u, "(1X,A78)") repeat ("=", 78)
 flush (u)
end subroutine vamp_write_histories_unit

```

### 5.2.7 Multi Channel

[23]

$$g(x) = \sum_i \alpha_i g_i(x) \quad (5.25a)$$

$$w(x) = \frac{f(x)}{g(x)} \quad (5.25b)$$

We want to minimize the variance  $W(\alpha)$  with the subsidiary condition  $\sum_i \alpha_i = 1$ . We introduce a Lagrange multiplier  $\lambda$ :

$$\tilde{W}(\alpha) = W(\alpha) + \lambda \left( \sum_i \alpha_i - 1 \right) \quad (5.26)$$

Therefore...

$$W_i(\alpha) = -\frac{\partial}{\partial \alpha_i} W(\alpha) = \int dx g_i(x) (w(x))^2 \approx \left\langle \frac{g_i(x)}{g(x)} (w(x))^2 \right\rangle \quad (5.27)$$

⚡ Here it *really* hurts that **Fortran** has no *first-class* functions. The following can be expressed much more elegantly in a functional programming language with *first-class* functions, currying and closures. **Fortran** makes it extra painful since not even procedure pointers are supported. This puts extra burden on the users of this library.

Note that the components of **vamp\_grids** are not protected. However, this is not a license for application programs to access it. Only Other libraries (e.g. for parallel processing, like **vampi**) should do so.

112  $\langle$ Declaration of **vamp** types 77a $\rangle + \equiv$  (75a)  $\triangleleft$ 106a

```

type, public :: vamp_grids
 !!! private ! used by vampi
 real(kind=default), dimension(:), pointer :: weights => null ()
 type(vamp_grid), dimension(:), pointer :: grids => null ()
 integer, dimension(:), pointer :: num_calls => null ()
 real(kind=default) :: sum_chi2, sum_integral, sum_weights
end type vamp_grids

```

$$g \circ \phi_i = \left| \frac{\partial \phi_i}{\partial x} \right|^{-1} \left( \alpha_i g_i + \sum_{\substack{j=1 \\ j \neq i}}^{N_c} \alpha_j (g_j \circ \pi_{ij}) \left| \frac{\partial \pi_{ij}}{\partial x} \right| \right). \quad (5.28)$$

113a  $\langle$ Declaration of vamp procedures 76b $\rangle + \equiv$  (75a)  $\langle$ 108b 114a $\rangle$   
`public :: vamp_multi_channel, vamp_multi_channel0`

113b  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 110 113c $\rangle$   
`function vamp_multi_channel &`  
`(func, data, phi, ihp, jacobian, x, weights, channel, grids) result (w_x)`  
`class(vamp_data_t), intent(in) :: data`  
`real(kind=default), dimension(:), intent(in) :: x`  
`real(kind=default), dimension(:), intent(in) :: weights`  
`integer, intent(in) :: channel`  
`type(vamp_grid), dimension(:), intent(in) :: grids`  
 $\langle$ Interface declaration for func 22 $\rangle$   
 $\langle$ Interface declaration for phi 31a $\rangle$   
 $\langle$ Interface declaration for ihp 31b $\rangle$   
 $\langle$ Interface declaration for jacobian 31c $\rangle$   
`real(kind=default) :: w_x`  
`integer :: i`  
`real(kind=default), dimension(size(x)) :: phi_x`  
`real(kind=default), dimension(size(weights)) :: g_phi_x, g_pi_x`  
`phi_x = phi(x, channel)`  
`do i = 1, size(weights)`  
`if (i == channel) then`  
`g_pi_x(i) = vamp_probability(grids(i), x)`  
`else`  
`g_pi_x(i) = vamp_probability(grids(i), ihp(phi_x, i))`  
`end if`  
`end do`  
`do i = 1, size(weights)`  
`g_phi_x(i) = g_pi_x(i) / g_pi_x(channel) * jacobian(phi_x, data, i)`  
`end do`  
`w_x = func(phi_x, data, weights, channel, grids) &`  
`/ dot_product(weights, g_phi_x)`  
`end function vamp_multi_channel`

113c  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 113b 114b $\rangle$   
`function vamp_multi_channel0 &`  
`(func, data, phi, jacobian, x, weights, channel) result (w_x)`  
`class(vamp_data_t), intent(in) :: data`  
`real(kind=default), dimension(:), intent(in) :: x`  
`real(kind=default), dimension(:), intent(in) :: weights`

```

integer, intent(in) :: channel
<Interface declaration for func 22>
<Interface declaration for phi 31a>
<Interface declaration for jacobian 31c>
real(kind=default) :: w_x
real(kind=default), dimension(size(x)) :: x_prime
real(kind=default), dimension(size(weights)) :: g_phi_x
integer :: i
x_prime = phi (x, channel)
do i = 1, size (weights)
 g_phi_x(i) = jacobian (x_prime, data, i)
end do
w_x = func (x_prime, data) / dot_product (weights, g_phi_x)
end function vamp_multi_channel0

```



WK

114a <Declaration of vamp procedures 76b>+≡ (75a) <113a 117a>  
 public :: vamp\_jacobian, vamp\_check\_jacobian

114b <Implementation of vamp procedures 77d>+≡ (75a) <113c 115>  
 pure subroutine vamp\_jacobian (phi, channel, x, region, jacobian, delta\_x)  
 integer, intent(in) :: channel  
 real(kind=default), dimension(:), intent(in) :: x  
 real(kind=default), dimension(:, :), intent(in) :: region  
 real(kind=default), intent(out) :: jacobian  
 real(kind=default), intent(in), optional :: delta\_x  
 interface  
 pure function phi (xi, channel) result (x)  
 use kinds  
 real(kind=default), dimension(:), intent(in) :: xi  
 integer, intent(in) :: channel  
 real(kind=default), dimension(size(xi)) :: x  
 end function phi  
 end interface  
 real(kind=default), dimension(size(x)) :: x\_min, x\_max  
 real(kind=default), dimension(size(x)) :: x\_plus, x\_minus  
 real(kind=default), dimension(size(x), size(x)) :: d\_phi  
 real(kind=default), parameter :: &  
 dx\_default = 10.0\_default\*\*(-precision(jacobian)/3)  
 real(kind=default) :: dx  
 integer :: j  
 if (present (delta\_x)) then  
 dx = delta\_x

```

else
 dx = dx_default
end if
x_min = region(1,:)
x_max = region(2,:)
x_minus = max (x_min, x)
x_plus = min (x_max, x)
do j = 1, size (x)
 x_minus(j) = max (x_min(j), x(j) - dx)
 x_plus(j) = min (x_max(j), x(j) + dx)
 d_phi(:,j) = (phi (x_plus, channel) - phi (x_minus, channel)) &
 / (x_plus(j) - x_minus(j))
 x_minus(j) = max (x_min(j), x(j))
 x_plus(j) = min (x_max(j), x(j))
end do
call determinant (d_phi, jacobian)
jacobian = abs (jacobian)
end subroutine vamp_jacobian

```

$$g(\phi(x)) = \frac{1}{\left| \frac{\partial \phi}{\partial x} \right| (x)} \quad (5.29)$$

115 *<Implementation of vamp procedures 77d>+≡ (75a) <114b 117b>*

```

subroutine vamp_check_jacobian &
 (rng, n, func, data, phi, channel, region, delta, x_delta)
type(tao_random_state), intent(inout) :: rng
integer, intent(in) :: n
class(vamp_data_t), intent(in) :: data
integer, intent(in) :: channel
real(kind=default), dimension(:, :), intent(in) :: region
real(kind=default), intent(out) :: delta
real(kind=default), dimension(:), intent(out), optional :: x_delta
<Interface declaration for func 22>
<Interface declaration for phi 31a>
real(kind=default), dimension(size(region,dim=2)) :: x, r
real(kind=default) :: jac, d
real(kind=default), dimension(0) :: wgts
integer :: i
delta = 0.0
do i = 1, max (1, n)
 call tao_random_number (rng, r)
 x = region(1,:) + (region(2,:) - region(1,:)) * r
 call vamp_jacobian (phi, channel, x, region, jac)
 d = func (phi (x, channel), data, wgts, channel) * jac &
 - 1.0_default

```

```

 if (abs (d) >= abs (delta)) then
 delta = d
 if (present (x_delta)) then
 x_delta = x
 end if
 end if
end do
end subroutine vamp_check_jacobian

```

This is a subroutine to comply with F's rules, otherwise, we would code it as a function.

116a *<Declaration of vamp procedures (removed from WHIZARD) 116a>≡*  
 private :: numeric\_jacobian

116b *<Implementation of vamp procedures (removed from WHIZARD) 116b>≡*  
 pure subroutine numeric\_jacobian (phi, channel, x, region, jacobian, delta\_x)  
 integer, intent(in) :: channel  
 real(kind=default), dimension(:), intent(in) :: x  
 real(kind=default), dimension(:, :), intent(in) :: region  
 real(kind=default), intent(out) :: jacobian  
 real(kind=default), intent(in), optional :: delta\_x  
*<Interface declaration for phi 31a>*  
 real(kind=default), dimension(size(x)) :: x\_min, x\_max  
 real(kind=default), dimension(size(x)) :: x\_plus, x\_minus  
 real(kind=default), dimension(size(x), size(x)) :: d\_phi  
 real(kind=default), parameter :: &  
 dx\_default = 10.0\_default\*(-precision(jacobian)/3)  
 real(kind=default) :: dx  
 integer :: j  
 if (present (delta\_x)) then  
 dx = delta\_x  
 else  
 dx = dx\_default  
 end if  
 x\_min = region(1, :)  
 x\_max = region(2, :)  
 x\_minus = max (x\_min, x)  
 x\_plus = min (x\_max, x)  
 do j = 1, size (x)  
 x\_minus(j) = max (x\_min(j), x(j) - dx)  
 x\_plus(j) = min (x\_max(j), x(j) + dx)  
 d\_phi(:, j) = (phi (x\_plus, channel) - phi (x\_minus, channel)) &  
 / (x\_plus(j) - x\_minus(j))  
 x\_minus(j) = max (x\_min(j), x(j))  
 end do

```

 x_plus(j) = min (x_max(j), x(j))
 end do
 call determinant (d_phi, jacobian)
 jacobian = abs (jacobian)
end subroutine numeric_jacobian

```

117a *<Declaration of vamp procedures 76b>+≡* (75a) *<114a 118b>*  
 public :: **vamp\_create\_grids**, **vamp\_create\_empty\_grids**  
 public :: **vamp\_copy\_grids**, **vamp\_delete\_grids**

The rules for optional arguments forces us to handle special cases, because we can't just pass a array section of an optional array as an actual argument (cf. 12.4.1.5(4) in [9]) even if the dummy argument is optional itself.

117b *<Implementation of vamp procedures 77d>+≡* (75a) *<115 118a>*  
 pure subroutine **vamp\_create\_grids** &  
 (g, domain, num\_calls, weights, maps, num\_div, &  
 stratified, quadrupole, exc)  
 type(**vamp\_grids**), intent(inout) :: g  
 real(kind=default), dimension(:,,:), intent(in) :: domain  
 integer, intent(in) :: num\_calls  
 real(kind=default), dimension(:), intent(in) :: weights  
 real(kind=default), dimension(:,:), intent(in), optional :: maps  
 integer, dimension(:), intent(in), optional :: num\_div  
 logical, intent(in), optional :: stratified, quadrupole  
 type(**exception**), intent(inout), optional :: exc  
 character(len=\*), parameter :: FN = "**vamp\_create\_grids**"  
 integer :: ch, nch  
 nch = size (weights)  
 allocate (g%grids(nch), g%weights(nch), g%num\_calls(nch))  
 g%weights = weights / sum (weights)  
 g%num\_calls = g%weights \* num\_calls  
 do ch = 1, size (g%grids)  
 if (present (maps)) then  
 call **vamp\_create\_grid** &  
 (g%grids(ch), domain, g%num\_calls(ch), num\_div, &  
 stratified, quadrupole, map = maps(:, :, ch), exc = exc)  
 else  
 call **vamp\_create\_grid** &  
 (g%grids(ch), domain, g%num\_calls(ch), num\_div, &  
 stratified, quadrupole, exc = exc)  
 end if  
 end do  
 g%sum\_integral = 0.0  
 g%sum\_chi2 = 0.0

```

 g%sum_weights = 0.0
end subroutine vamp_create_grids
118a <Implementation of vamp procedures 77d>+≡ (75a) <117b 118c>
 pure subroutine vamp_create_empty_grids (g)
 type(vamp_grids), intent(inout) :: g
 nullify (g%grids, g%weights, g%num_calls)
 end subroutine vamp_create_empty_grids
118b <Declaration of vamp procedures 76b>+≡ (75a) <117a 118d>
 public :: vamp_discard_integrals
118c <Implementation of vamp procedures 77d>+≡ (75a) <118a 118e>
 pure subroutine vamp_discard_integrals &
 (g, num_calls, num_div, stratified, quadrupole, exc, eq)
 type(vamp_grids), intent(inout) :: g
 integer, intent(in), optional :: num_calls
 integer, dimension(:), intent(in), optional :: num_div
 logical, intent(in), optional :: stratified, quadrupole
 type(exception), intent(inout), optional :: exc
 type(vamp_equivalences_t), intent(in), optional :: eq
 integer :: ch
 character(len=*), parameter :: FN = "vamp_discard_integrals"
 g%sum_integral = 0.0
 g%sum_weights = 0.0
 g%sum_chi2 = 0.0
 do ch = 1, size (g%grids)
 call vamp_discard_integral (g%grids(ch))
 end do
 if (present (num_calls)) then
 call vamp_reshape_grids &
 (g, num_calls, num_div, stratified, quadrupole, exc, eq)
 end if
 end subroutine vamp_discard_integrals
118d <Declaration of vamp procedures 76b>+≡ (75a) <118b 119a>
 public :: vamp_update_weights
 We must discard the accumulated integrals, because the weight function $w = f / \sum_i \alpha_i g_i$ changes:
118e <Implementation of vamp procedures 77d>+≡ (75a) <118c 119b>
 pure subroutine vamp_update_weights &
 (g, weights, num_calls, num_div, stratified, quadrupole, exc)
 type(vamp_grids), intent(inout) :: g
 real(kind=default), dimension(:), intent(in) :: weights
 integer, intent(in), optional :: num_calls

```

```

integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole
type(exception), intent(inout), optional :: exc
character(len=*), parameter :: FN = "vamp_update_weights"
if (sum (weights) > 0) then
 g%weights = weights / sum (weights)
else
 g%weights = 1._default / size(g%weights)
end if
if (present (num_calls)) then
 call vamp_discard_integrals (g, num_calls, num_div, &
 stratified, quadrupole, exc)
else
 call vamp_discard_integrals (g, sum (g%num_calls), num_div, &
 stratified, quadrupole, exc)
end if
end subroutine vamp_update_weights
119a <Declaration of vamp procedures 76b>+≡ (75a) <118d 120a>
public :: vamp_reshape_grids
119b <Implementation of vamp procedures 77d>+≡ (75a) <118e 120b>
pure subroutine vamp_reshape_grids &
 (g, num_calls, num_div, stratified, quadrupole, exc, eq)
type(vamp_grids), intent(inout) :: g
integer, intent(in) :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole
type(exception), intent(inout), optional :: exc
type(vamp_equivalences_t), intent(in), optional :: eq
integer, dimension(size(g%grids(1)%num_div)) :: num_div_new
integer :: ch
character(len=*), parameter :: FN = "vamp_reshape_grids"
g%num_calls = g%weights * num_calls
do ch = 1, size (g%grids)
 if (g%num_calls(ch) >= 2) then
 if (present (eq)) then
 if (present (num_div)) then
 num_div_new = num_div
 else
 num_div_new = g%grids(ch)%num_div
 end if
 where (eq%div_is_invariant(ch,:))
 num_div_new = 1
 end where

```



```

 call vamp_reshape_grid (g%grids(ch), g%num_calls(ch), &
 num_div_new, stratified, quadrupole, exc = exc, &
 independent = eq%independent(ch), &
 equivalent_to_ch = eq%equivalent_to_ch(ch), &
 multiplicity = eq%multiplicity(ch))
 else
 call vamp_reshape_grid (g%grids(ch), g%num_calls(ch), &
 num_div, stratified, quadrupole, exc = exc)
 end if
else
 g%num_calls(ch) = 0
end if
end do
end subroutine vamp_reshape_grids

```

120a *<Declaration of vamp procedures 76b>+≡* (75a) *<119a 122b>*  
 public :: vamp\_sample\_grids

Even if `g%num_calls` is derived from `g%weights`, we must *not* use the latter, allow for integer arithmetic in `g%num_calls`.

120b *<Implementation of vamp procedures 77d>+≡* (75a) *<119b 122c>*  
 subroutine vamp\_sample\_grids &  
 (rng, g, func, data, iterations, integral, std\_dev, avg\_chi2, &  
 accuracy, history, histories, exc, eq, warn\_error, negative\_weights)  
 type(tao\_random\_state), intent(inout) :: rng  
 type(vamp\_grids), intent(inout) :: g  
 class(vamp\_data\_t), intent(in) :: data  
 integer, intent(in) :: iterations  
 real(kind=default), intent(out), optional :: integral, std\_dev, avg\_chi2  
 real(kind=default), intent(in), optional :: accuracy  
 type(vamp\_history), dimension(:), intent(inout), optional :: history  
 type(vamp\_history), dimension(:, :), intent(inout), optional :: histories  
 type(exception), intent(inout), optional :: exc  
 type(vamp\_equivalences\_t), intent(in), optional :: eq  
 logical, intent(in), optional :: warn\_error, negative\_weights  
*<Interface declaration for func 22>*  
 integer :: ch, iteration  
 logical :: neg\_w  
 type(exception), dimension(size(g%grids)) :: excs  
 logical, dimension(size(g%grids)) :: active  
 real(kind=default), dimension(size(g%grids)) :: weights, integrals, std\_devs  
 real(kind=default) :: local\_integral, local\_std\_dev, local\_avg\_chi2  
 character(len=\*), parameter :: FN = "vamp\_sample\_grids"  
 integrals = 0

```

std_devs = 0
neg_w = .false.
if (present (negative_weights)) neg_w = negative_weights
active = (g%num_calls >= 2)
where (active)
 weights = g%num_calls
elsewhere
 weights = 0.0
endwhere
if (sum (weights) /= 0) weights = weights / sum (weights)
call clear_exception (excs)
iterate: do iteration = 1, iterations
 do ch = 1, size (g%grids)
 if (active(ch)) then
 call vamp_discard_integral (g%grids(ch))
 <Sample the grid g%grids(ch) 122a>
 else
 call vamp_nullify_variance (g%grids(ch))
 call vamp_nullify_covariance (g%grids(ch))
 end if
 end do
 if (present(eq)) call vamp_apply_equivalences (g, eq)
 if (iteration < iterations) then
 do ch = 1, size (g%grids)
 active(ch) = (integrals(ch) /= 0)
 if (active(ch)) then
 call vamp_refine_grid (g%grids(ch))
 end if
 end do
 end if
 if (present (exc) .and. (any (excs%level > 0))) then
 call gather_exceptions (exc, excs)
! return
 end if
 call vamp_reduce_channels (g, integrals, std_devs, active)
 call vamp_average_iterations &
 (g, iteration, local_integral, local_std_dev, local_avg_chi2)
 <Trace results of vamp_sample_grids 124d>
 <Exit iterate if accuracy has been reached 96b>
end do iterate
<Copy results of vamp_sample_grid to dummy variables 96a>
end subroutine vamp_sample_grids

```

We must refine the grids after *all* grids have been sampled, therefore we use `vamp_sample_grid0` instead of `vamp_sample_grid`:

```

122a <Sample the grid g%grids(ch) 122a>≡ (120b)
 call vamp_sample_grid0 &
 (rng, g%grids(ch), func, data, &
 ch, weights, g%grids, excs(ch), neg_w)
 if (present (exc) .and. present (warn_error)) then
 if (warn_error) call handle_exception (excs(ch))
 end if
 call vamp_average_iterations &
 (g%grids(ch), iteration, integrals(ch), std_devs(ch), local_avg_chi2)
 if (present (histories)) then
 if (iteration <= ubound (histories, dim=1)) then
 call vamp_get_history &
 (histories(iteration,ch), g%grids(ch), &
 integrals(ch), std_devs(ch), local_avg_chi2)
 else
 call raise_exception (exc, EXC_WARN, FN, "history too short")
 end if
 call vamp_terminate_history (histories(iteration+1:,ch))
 end if
122b <Declaration of vamp procedures 76b>+≡ (75a) <120a 123a>
 public :: vamp_reduce_channels

```

$$I = \frac{1}{N} \sum_c N_c I_c \quad (5.30a)$$

$$\sigma^2 = \frac{1}{N^2} \sum_c N_c^2 \sigma_c^2 \quad (5.30b)$$

$$N = \sum_c N_c \quad (5.30c)$$

where (5.30b) is actually

$$\sigma^2 = \frac{1}{N} (\mu_2 - \mu_1^1) = \frac{1}{N} \left( \frac{1}{N} \sum_c N_c \mu_{2,c} - I^2 \right) = \frac{1}{N} \left( \frac{1}{N} \sum_c (N_c^2 \sigma_c^2 + N_c I_c^2) - I^2 \right)$$

but the latter form suffers from numerical instability and (5.30b) is thus preferred.

```

122c <Implementation of vamp procedures 77d>+≡ (75a) <120b 123b>
 pure subroutine vamp_reduce_channels (g, integrals, std_devs, active)

```

```

type(vamp_grids), intent(inout) :: g
real(kind=default), dimension(:), intent(in) :: integrals, std_devs
logical, dimension(:), intent(in) :: active
real(kind=default) :: this_integral, this_weight, total_calls
real(kind=default) :: total_variance
if (.not.any(active)) return
total_calls = sum (g%num_calls, mask=active)
if (total_calls > 0) then
 this_integral = sum (g%num_calls * integrals, mask=active) / total_calls
else
 this_integral = 0
end if
total_variance = sum ((g%num_calls*std_devs)**2, mask=active)
if (total_variance > 0) then
 this_weight = total_calls**2 / total_variance
else
 this_weight = 0
end if
g%sum_weights = g%sum_weights + this_weight
g%sum_integral = g%sum_integral + this_weight * this_integral
g%sum_chi2 = g%sum_chi2 + this_weight * this_integral**2
end subroutine vamp_reduce_channels

```

123a *<Declaration of vamp procedures 76b>+≡* (75a) <122b 124a>  
public :: vamp\_refine\_weights

123b *<Implementation of vamp procedures 77d>+≡* (75a) <122c 124c>  
elemental subroutine vamp\_average\_iterations\_grids &  
(g, iteration, integral, std\_dev, avg\_chi2)  
type(vamp\_grids), intent(in) :: g  
integer, intent(in) :: iteration  
real(kind=default), intent(out) :: integral, std\_dev, avg\_chi2  
real(kind=default), parameter :: eps = 1000 \* epsilon (1.\_default)  
if (g%sum\_weights>0) then  
 integral = g%sum\_integral / g%sum\_weights  
 std\_dev = sqrt (1.0 / g%sum\_weights)  
 avg\_chi2 = &  
 max ((g%sum\_chi2 - g%sum\_integral \* integral) / (iteration-0.99), &  
 0.0\_default)  
 if (avg\_chi2 < eps \* g%sum\_chi2) avg\_chi2 = 0  
else  
 integral = 0  
 std\_dev = 0  
 avg\_chi2 = 0  
end if

```

end subroutine vamp_average_iterations_grids

124a <Declaration of vamp procedures 76b>+≡ (75a) <123a 124e>
private :: vamp_average_iterations_grids

124b <Interfaces of vamp procedures 95c>+≡ (75a) <108c 124f>
interface vamp_average_iterations
module procedure vamp_average_iterations_grids
end interface

$$\alpha_i \rightarrow \alpha_i \sqrt{V_i} \quad (5.31)$$

124c <Implementation of vamp procedures 77d>+≡ (75a) <123b 125a>
pure subroutine vamp_refine_weights (g, power)
type(vamp_grids), intent(inout) :: g
real(kind=default), intent(in), optional :: power
real(kind=default) :: local_power
real(kind=default), parameter :: DEFAULT_POWER = 0.5_default
if (present (power)) then
local_power = power
else
local_power = DEFAULT_POWER
end if
call vamp_update_weights &
(g, g%weights * vamp_get_variance (g%grids) ** local_power)
end subroutine vamp_refine_weights

124d <Trace results of vamp_sample_grids 124d>≡ (120b)
if (present (history)) then
if (iteration <= size (history)) then
call vamp_get_history &
(history(iteration), g, local_integral, local_std_dev, &
local_avg_chi2)
else
call raise_exception (exc, EXC_WARN, FN, "history too short")
end if
call vamp_terminate_history (history(iteration+1:))
end if

124e <Declaration of vamp procedures 76b>+≡ (75a) <124a 125b>
private :: vamp_get_history_multi

124f <Interfaces of vamp procedures 95c>+≡ (75a) <124b 130c>
interface vamp_get_history
module procedure vamp_get_history_multi
end interface

```

125a *<Implementation of vamp procedures 77d>+≡* (75a) <124c 125c>

```

pure subroutine vamp_get_history_multi (h, g, integral, std_dev, avg_chi2)
 type(vamp_history), intent(inout) :: h
 type(vamp_grids), intent(in) :: g
 real(kind=default), intent(in) :: integral, std_dev, avg_chi2
 h%calls = sum (g%grids%calls)
 h%stratified = all (g%grids%all_stratified)
 h%integral = 0.0
 h%std_dev = 0.0
 h%avg_integral = integral
 h%avg_std_dev = std_dev
 h%avg_chi2 = avg_chi2
 h%f_min = 0.0
 h%f_max = huge (h%f_max)
 if (h%verbose) then
 h%verbose = .false.
 if (associated (h%div)) then
 deallocate (h%div)
 end if
 end if
end subroutine vamp_get_history_multi

```



125b *<Declaration of vamp procedures 76b>+≡* (75a) <124e 126>

```

public :: vamp_sum_channels

```

125c *<Implementation of vamp procedures 77d>+≡* (75a) <125a 127b>

```

function vamp_sum_channels (x, weights, func, data, grids) result (g)
 real(kind=default), dimension(:), intent(in) :: x, weights
 class(vamp_data_t), intent(in) :: data
 type(vamp_grid), dimension(:), intent(in), optional :: grids
 interface
 function func (xi, data, weights, channel, grids) result (f)
 use kinds
 use vamp_grid_type !NODEP!
 import vamp_data_t
 real(kind=default), dimension(:), intent(in) :: xi
 class(vamp_data_t), intent(in) :: data
 real(kind=default), dimension(:), intent(in), optional :: weights
 integer, intent(in), optional :: channel
 type(vamp_grid), dimension(:), intent(in), optional :: grids
 real(kind=default) :: f
 end function func
 end interface


```

```

end interface
real(kind=default) :: g
integer :: ch
g = 0.0
do ch = 1, size (weights)
 g = g + weights(ch) * func (x, data, weights, ch, grids)
end do
end function vamp_sum_channels

```

### 5.2.8 Mapping

 This section is still under construction. The basic algorithm is in place, but the heuristics have not been developed yet.

The most naive approach is to use the rotation matrix  $R$  that diagonalizes the covariance  $C$ :

$$R_{ij} = (v_j)_i \quad (5.32)$$

where

$$Cv_j = \lambda_j v_j \quad (5.33)$$

with the eigenvalues  $\{\lambda_j\}$  and eigenvectors  $\{v_j\}$ . Then

$$R^T C R = \text{diag}(\lambda_1, \dots) \quad (5.34)$$

After call `diagonalize_real_symmetric (cov, evals, vecs)`, we have `evals(j) =  $\lambda_j$`  and `vecs(:,j) =  $v_j$` . This is equivalent with `vecs(i,j) =  $R_{ij}$` .

This approach will not work in high dimensions, however. In general,  $R$  will *not* leave most of the axes invariant, even if the covariance matrix is almost isotropic in these directions. In this case the benefit from the rotation is rather small and offset by the negative effects from the misalignment of the integration region.

A better strategy is to find the axis of the original coordinate system around which a rotation is most beneficial. There are two extreme cases:

- “pancake”: one eigenvalue much smaller than the others
- “cigar”: one eigenvalue much larger than the others

Actually, instead of rotating around a specific axis, we can as well diagonalize in a subspace. Empirically, rotation around an axis is better than diagonalizing in a two-dimensional subspace, but diagonalizing in a three-dimensional subspace can be even better.

126  $\langle \text{Declaration of } \text{vamp} \text{ procedures } 76b \rangle + \equiv \quad (75a) \quad \langle 125b \ 127c \rangle$

```

public :: select_rotation_axis
public :: select_rotation_subspace

127a <Set iv to the index of the optimal eigenvector 127a>≡ (129a 130a)
 if (num_pancake > 0) then
 print *, "FORCED PANCAKE: ", num_pancake
 iv = sum (minloc (evals))
 else if (num_cigar > 0) then
 print *, "FORCED CIGAR: ", num_cigar
 iv = sum (maxloc (evals))
 else
 call more_pancake_than_cigar (evals, like_pancake)
 if (like_pancake) then
 iv = sum (minloc (evals))
 else
 iv = sum (maxloc (evals))
 end if
 end if

127b <Implementation of vamp procedures 77d>+≡ (75a) <125c 129a>
 subroutine more_pancake_than_cigar (eval, yes_or_no)
 real(kind=default), dimension(:), intent(in) :: eval
 logical, intent(out) :: yes_or_no
 integer, parameter :: N_CL = 2
 real(kind=default), dimension(size(eval)) :: evals
 real(kind=default), dimension(N_CL) :: cluster_pos
 integer, dimension(N_CL,2) :: clusters
 evals = eval
 call sort (evals)
 call condense (evals, cluster_pos, clusters)
 print *, clusters(1,2) - clusters(1,1) + 1, "small EVs: ", &
 evals(clusters(1,1):clusters(1,2))
 print *, clusters(2,2) - clusters(2,1) + 1, "large EVs: ", &
 evals(clusters(2,1):clusters(2,2))
 if ((clusters(1,2) - clusters(1,1)) &
 < (clusters(2,2) - clusters(2,1))) then
 print *, " => PANCAKE!"
 yes_or_no = .true.
 else
 print *, " => CIGAR!"
 yes_or_no = .false.
 end if
 end subroutine more_pancake_than_cigar

127c <Declaration of vamp procedures 76b>+≡ (75a) <126 130d>

```



private :: more\_pancake\_than\_cigar

In both cases, we can rotate in the plane  $P_{ij}$  closest to eigenvector corresponding to the the singled out eigenvalue. This plane is given by

$$\max_{i \neq i'} \sqrt{(v_j)_i^2 + (v_j)_{i'}^2} \quad (5.35)$$

which is simply found by looking for the two largest  $|(v_j)_i|$ :<sup>3</sup>

128a *<Set i(1), i(2) to the axes of the optimal plane 128a>*≡ (129a) 128c>  
`abs_evec = abs (evecs(:,iv))`  
`i(1) = sum (maxloc (abs_evec))`  
`abs_evec(i(1)) = -1.0`  
`i(2) = sum (maxloc (abs_evec))`

The following is cute, but unfortunately broken, since it fails for degenerate eigenvalues:

128b *<Set i(1), i(2) to the axes of the optimal plane (broken!) 128b>*≡  
`abs_evec = abs (evecs(:,iv))`  
`i(1) = sum (maxloc (abs_evec))`  
`i(2) = sum (maxloc (abs_evec, mask = abs_evec < abs_evec(i(1))))`  
128c *<Set i(1), i(2) to the axes of the optimal plane 128a>*+≡ (129a) <128a  
`print *, iv, evals(iv), " => ", evecs(:,iv)`  
`print *, i(1), abs_evec(i(1)), ", ", i(2), abs_evec(i(2))`  
`print *, i(1), evecs(i(1),iv), ", ", i(2), evecs(i(2),iv)`  
128d *<Get cos θ and sin θ from evecs 128d>*≡ (129a)  
`cos_theta = evecs(i(1),iv)`  
`sin_theta = evecs(i(2),iv)`  
`norm = 1.0 / sqrt (cos_theta**2 + sin_theta**2)`  
`cos_theta = cos_theta * norm`  
`sin_theta = sin_theta * norm`

$$\hat{R}(\theta; i, j) = \begin{pmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & \cos \theta & \cdots & -\sin \theta & \\ & & \vdots & 1 & \vdots & \\ & & \sin \theta & \cdots & \cos \theta & \\ & & & & & \ddots & \\ & & & & & & 1 \end{pmatrix} \quad (5.36)$$

128e *<Construct  $\hat{R}(\theta; i, j)$  128e>*≡ (129a)

---

<sup>3</sup>The `sum` intrinsic is a convenient Fortran90 trick for turning the rank-one array with one element returned by `maxloc` into its value. It has no semantic significance.

```

call unit (r)
r(i(1),i) = (/ cos_theta, - sin_theta /)
r(i(2),i) = (/ sin_theta, cos_theta /)
129a <Implementation of vamp procedures 77d>+≡ (75a) <127b 129c>
subroutine select_rotation_axis (cov, r, pancake, cigar)
 real(kind=default), dimension(:, :), intent(in) :: cov
 real(kind=default), dimension(:, :), intent(out) :: r
 integer, intent(in), optional :: pancake, cigar
 integer :: num_pancake, num_cigar
 logical :: like_pancake
 real(kind=default), dimension(size(cov,dim=1),size(cov,dim=2)) :: evecs
 real(kind=default), dimension(size(cov,dim=1)) :: evals, abs_evec
 integer :: iv
 integer, dimension(2) :: i
 real(kind=default) :: cos_theta, sin_theta, norm
 <Handle optional pancake and cigar 129b>
 call diagonalize_real_symmetric (cov, evals, evecs)
 <Set iv to the index of the optimal eigenvector 127a>
 <Set i(1), i(2) to the axes of the optimal plane 128a>
 <Get cos θ and sin θ from evecs 128d>
 <Construct $\hat{R}(\theta; i, j)$ 128e>
end subroutine select_rotation_axis
129b <Handle optional pancake and cigar 129b>≡ (129a 130a)
 if (present (pancake)) then
 num_pancake = pancake
 else
 num_pancake = -1
 endif
 if (present (cigar)) then
 num_cigar = cigar
 else
 num_cigar = -1
 endif

```

Here's a less efficient version that can be easily generalized to more than two dimension, however:

```

129c <Implementation of vamp procedures 77d>+≡ (75a) <129a 130a>
subroutine select_subspace_explicit (cov, r, subspace)
 real(kind=default), dimension(:, :), intent(in) :: cov
 real(kind=default), dimension(:, :), intent(out) :: r
 integer, dimension(:), intent(in) :: subspace
 real(kind=default), dimension(size(subspace)) :: eval_sub
 real(kind=default), dimension(size(subspace),size(subspace)) :: &

```

```

 cov_sub, evec_sub
 cov_sub = cov(subspace,subspace)
 call diagonalize_real_symmetric (cov_sub, eval_sub, evec_sub)
 call unit (r)
 r(subspace,subspace) = evec_sub
end subroutine select_subspace_explicit
130a <Implementation of vamp procedures 77d>+≡ (75a) <129c 131a>
 subroutine select_subspace_guess (cov, r, ndim, pancake, cigar)
 real(kind=default), dimension(:,,:), intent(in) :: cov
 real(kind=default), dimension(:,,:), intent(out) :: r
 integer, intent(in) :: ndim
 integer, intent(in), optional :: pancake, cigar
 integer :: num_pancake, num_cigar
 logical :: like_pancake
 real(kind=default), dimension(size(cov,dim=1),size(cov,dim=2)) :: evecs
 real(kind=default), dimension(size(cov,dim=1)) :: evals, abs_evec
 integer :: iv, i
 integer, dimension(ndim) :: subspace
 <Handle optional pancake and cigar 129b>
 call diagonalize_real_symmetric (cov, evals, evecs)
 <Set iv to the index of the optimal eigenvector 127a>
 <Set subspace to the axes of the optimal plane 130b>
 call select_subspace_explicit (cov, r, subspace)
 end subroutine select_subspace_guess
130b <Set subspace to the axes of the optimal plane 130b>≡ (130a)
 abs_evec = abs (evecs(:,iv))
 subspace(1) = sum (maxloc (abs_evec))
 do i = 2, ndim
 abs_evec(subspace(i-1)) = -1.0
 subspace(i) = sum (maxloc (abs_evec))
 end do
130c <Interfaces of vamp procedures 95c>+≡ (75a) <124f 135a>
 interface select_rotation_subspace
 module procedure select_subspace_explicit, select_subspace_guess
 end interface
130d <Declaration of vamp procedures 76b>+≡ (75a) <127c 130e>
 private :: select_subspace_explicit
 private :: select_subspace_guess
130e <Declaration of vamp procedures 76b>+≡ (75a) <130d 131b>
 public :: vamp_print_covariance

```

131a  $\langle$ Implementation of vamp procedures 77d $\rangle + \equiv$  (75a)  $\langle$ 130a 133b $\rangle$

```

subroutine vamp_print_covariance (cov)
 real(kind=default), dimension(:, :), intent(in) :: cov
 real(kind=default), dimension(size(cov,dim=1)) :: &
 evals, abs_evals, tmp
 real(kind=default), dimension(size(cov,dim=1),size(cov,dim=2)) :: &
 evecs, abs_evecs
 integer, dimension(size(cov,dim=1)) :: idx
 integer :: i, i_max, j
 i_max = size (evals)
 call diagonalize_real_symmetric (cov, evals, evecs)
 call sort (evals, evecs)
 abs_evals = abs (evals)
 abs_evecs = abs (evecs)
 print "(1X,A78)", repeat ("-", 78)
 print "(1X,A)", "Eigenvalues and eigenvectors:"
 print "(1X,A78)", repeat ("-", 78)
 do i = 1, i_max
 print "(1X,I2,A1,1X,E11.4,1X,A1,10(1X,F5.2)/,18X))", &
 i, ":", evals(i), "|", evecs(:,i)
 end do
 print "(1X,A78)", repeat ("-", 78)
 print "(1X,A)", "Approximate subspaces:"
 print "(1X,A78)", repeat ("-", 78)
 do i = 1, i_max
 idx = (/ (j, j=1,i_max) /)
 tmp = abs_evecs(:,i)
 call sort (tmp, idx, reverse = .true.)
 print "(1X,I2,A1,1X,E11.4,1X,A1,10(1X,I5))", &
 i, ":", evals(i), "|", idx(1:min(10,size(idx)))
 print "(17X,A1,10(1X,F5.2))", &
 "|", evecs(idx(1:min(10,size(idx))),i)
 end do
 print "(1X,A78)", repeat ("-", 78)
end subroutine vamp_print_covariance

```

### Condensing Eigenvalues

In order to decide whether we have a “pancake” or a “cigar”, we have to classify the eigenvalues of the covariance matrix. We do this by condensing the  $n_{\text{dim}}$  eigenvalues into  $n_{\text{cl}} \ll n_{\text{dim}}$  clusters.

131b  $\langle$ Declaration of vamp procedures 76b $\rangle + \equiv$  (75a)  $\langle$ 130e 133c $\rangle$

```

! private :: condense

```

```
public :: condense
```

The rough description is as follows: in each step, combine the nearest neighbours (according to an appropriate metric) to form a smaller set. This is an extremely simplified, discretized modeling of molecules condensing under the influence of some potential.

⚡ If there's not a clean separation, this algorithm is certainly chaotic and we need to apply some form of damping!

```
132a <Initialize clusters 132a>≡ (133b)
 cl_pos = x
 cl_num = size (cl_pos)
 cl = spread ((/ (i, i=1,cl_num) /), dim = 2, ncopies = 2)
```

It appears that the logarithmic metric

$$d_0(x, y) = \left| \log \left( \frac{x}{y} \right) \right| \quad (5.37a)$$

performs better than the linear metric

$$d_1(x, y) = |x - y| \quad (5.37b)$$

since the latter won't separate very small eigenvalues from the bulk. Another option is

$$d_\alpha(x, y) = |x^\alpha - y^\alpha| \quad (5.37c)$$

with  $\alpha \neq 1$ , in particular  $\alpha \approx -1$ . I haven't studied it yet, though.

⚡ but I should perform more empirical studies to determine whether the logarithmic or the linear metric is more appropriate in realistic cases.

```
132b <Join closest clusters 132b>≡ (133b) 133a>
 if (linear_metric) then
 gap = sum (minloc (cl_pos(2:cl_num) - cl_pos(1:cl_num-1)))
 else
 gap = sum (minloc (cl_pos(2:cl_num) / cl_pos(1:cl_num-1)))
 end if
 wgt0 = cl(gap,2) - cl(gap,1) + 1
 wgt1 = cl(gap+1,2) - cl(gap+1,1) + 1
 cl_pos(gap) = (wgt0 * cl_pos(gap) + wgt1 * cl_pos(gap+1)) / (wgt0 + wgt1)
 cl(gap,2) = cl(gap+1,2)
```

133a *Join closest clusters 132b*+≡ (133b) <132b

```
cl_pos(gap+1:cl_num-1) = cl_pos(gap+2:cl_num)
cl(gap+1:cl_num-1,:) = cl(gap+2:cl_num,:)
```

133b *Implementation of vamp procedures 77d*+≡ (75a) <131a 133d>

```
subroutine condense (x, cluster_pos, clusters, linear)
 real(kind=default), dimension(:), intent(in) :: x
 real(kind=default), dimension(:), intent(out) :: cluster_pos
 integer, dimension(:,:), intent(out) :: clusters
 logical, intent(in), optional :: linear
 logical :: linear_metric
 real(kind=default), dimension(size(x)) :: cl_pos
 real(kind=default) :: wgt0, wgt1
 integer :: cl_num
 integer, dimension(size(x),2) :: cl
 integer :: i, gap
 linear_metric = .false.
 if (present (linear)) then
 linear_metric = linear
 end if
 Initialize clusters 132a
 do cl_num = size (cl_pos), size (cluster_pos) + 1, -1
 Join closest clusters 132b
 print *, cl_num, ": action = ", condense_action (x, cl)
 end do
 cluster_pos = cl_pos(1:cl_num)
 clusters = cl(1:cl_num,:)
end subroutine condense
```

133c *Declaration of vamp procedures 76b*+≡ (75a) <131b 134b>

```
! private :: condense_action
public :: condense_action
```

$$S = \sum_{c \in \text{clusters}} \text{var}^{\frac{\alpha}{2}}(c) \quad (5.38)$$

133d *Implementation of vamp procedures 77d*+≡ (75a) <133b 135c>

```
function condense_action (positions, clusters) result (s)
 real(kind=default), dimension(:), intent(in) :: positions
 integer, dimension(:,:), intent(in) :: clusters
 real(kind=default) :: s
 integer :: i
 integer, parameter :: POWER = 2
 s = 0
 do i = 1, size (clusters, dim = 1)
 s = s + standard_deviation (positions(clusters(i,1) &
```

```

 :clusters(i,2))) ** POWER
 end do
 end function condense_action
134a <ctest.f90 134a>≡
 program ctest
 use kinds
 use utils
 use vamp_stat
 use tao_random_numbers
 use vamp
 implicit none
 integer, parameter :: N = 16, NC = 2
 real(kind=default), dimension(N) :: eval
 real(kind=default), dimension(NC) :: cluster_pos
 integer, dimension(NC,2) :: clusters
 integer :: i
 call tao_random_number (eval)
 call sort (eval)
 print *, eval
 eval(1:N/2) = 0.95*eval(1:N/2)
 eval(N/2+1:N) = 1.0 - 0.95*(1.0 - eval(N/2+1:N))
 print *, eval
 call condense (eval, cluster_pos, clusters, linear=.true.)
 do i = 1, NC
 print "(I2,A,F5.2,A,I2,A,I2,A,A,F5.2,A,F5.2,A,32F5.2)", &
 i, ":", cluster_pos(i), &
 " [", clusters(i,1), "-", clusters(i,2), "]", &
 " [", eval(clusters(i,1)), " - ", eval(clusters(i,2)), "]", &
 eval(clusters(i,1)+1:clusters(i,2)) &
 - eval(clusters(i,1):clusters(i,2)-1)
 print *, average (eval(clusters(i,1):clusters(i,2))), "+/-", &
 standard_deviation (eval(clusters(i,1):clusters(i,2)))
 end do
 end program ctest

```

### 5.2.9 Event Generation

Automagically adaptive tools are not always appropriate for unweighted event generation, but we can give it a try.

```

134b <Declaration of vamp procedures 76b>+≡ (75a) <133c 135b>
 public :: vamp_next_event

```

135a *<Interfaces of vamp procedures 95c>+≡* (75a) <130c 140b>

```
interface vamp_next_event
 module procedure vamp_next_event_single, vamp_next_event_multi
end interface
```

135b *<Declaration of vamp procedures 76b>+≡* (75a) <134b 138d>

```
private :: vamp_next_event_single, vamp_next_event_multi
```

Both event generation routines operate in two modes, depending on whether the optional argument `weight` is present.

135c *<Implementation of vamp procedures 77d>+≡* (75a) <133d 136c>

```
subroutine vamp_next_event_single &
 (x, rng, g, func, data, &
 weight, channel, weights, grids, exc)
 real(kind=default), dimension(:), intent(out) :: x
 type(tao_random_state), intent(inout) :: rng
 type(vamp_grid), intent(inout) :: g
 real(kind=default), intent(out), optional :: weight
 class(vamp_data_t), intent(in) :: data
 integer, intent(in), optional :: channel
 real(kind=default), dimension(:), intent(in), optional :: weights
 type(vamp_grid), dimension(:), intent(in), optional :: grids
 type(exception), intent(inout), optional :: exc
 <Interface declaration for func 22>
 character(len=*), parameter :: FN = "vamp_next_event_single"
 real(kind=default), dimension(size(g%div)):: wgts
 real(kind=default), dimension(size(g%div)):: r
 integer, dimension(size(g%div)):: ia
 real(kind=default) :: f, wgt
 real(kind=default) :: r0
 rejection: do
 <Choose a x and calculate f(x) 135d>
 if (present (weight)) then
 <Unconditionally accept weighted event 136a>
 else
 <Maybe accept unweighted event 136b>
 end if
 end do rejection
end subroutine vamp_next_event_single
```

135d *<Choose a x and calculate f(x) 135d>≡* (135c)

```
call tao_random_number (rng, r)
call inject_division_short (g%div, real(r, kind=default), x, ia, wgts)
wgt = g%jacobi * product (wgts)
wgt = g%calls * wgt ! the calling procedure will divide by #calls
```



```

 if (associated (g%map)) then
 x = matmul (g%map, x)
 end if
 <f = wgt * func (x, weights, channel), iff x inside true_domain 88a>
 ! call record_efficiency (g%div, ia, f/g%f_max)
136a <Unconditionally accept weighted event 136a>≡ (135c)
 weight = f
 exit rejection
136b <Maybe accept unweighted event 136b>≡ (135c)
 if (abs(f) > g%f_max) then
 g%f_max = f
 call raise_exception (exc, EXC_WARN, FN, "weight > 1")
 exit rejection
 end if
 call tao_random_number (rng, r0)
 if (r0 * g%f_max <= abs(f)) then
 exit rejection
 end if

```

We know that `g%weights` are normalized: `sum (g%weights) == 1.0`. The basic formula for multi channel sampling is

$$f(x) = \sum_i \alpha_i g_i(x) w(x) \quad (5.39)$$

with  $w(x) = f(x)/g(x) = f(x)/\sum_i \alpha_i g_i(x)$  and  $\sum_i \alpha_i = 1$ . The non-trivial problem is that the adaptive grid is different in each channel, so we can't just reject on  $w(x)$ .

```

136c <Implementation of vamp procedures 77d>+≡ (75a) <135c 139a>
 subroutine vamp_next_event_multi &
 (x, rng, g, func, data, phi, weight, excess, positive, exc)
 real(kind=default), dimension(:), intent(out) :: x
 type(tao_random_state), intent(inout) :: rng
 type(vamp_grids), intent(inout) :: g
 class(vamp_data_t), intent(in) :: data
 real(kind=default), intent(out), optional :: weight
 real(kind=default), intent(out), optional :: excess
 logical, intent(out), optional :: positive
 type(exception), intent(inout), optional :: exc
 <Interface declaration for func 22>
 <Interface declaration for phi 31a>
 character(len=*), parameter :: FN = "vamp_next_event_multi"
 real(kind=default), dimension(size(x)) :: xi

```

```

real(kind=default) :: r, wgt
real(kind=default), dimension(size(g%weights)) :: weights
integer :: channel
⟨weights: $\alpha_i \rightarrow w_{\max,i}\alpha_i$ 137a⟩
rejection: do
 ⟨Select channel from weights 137b⟩
 call vamp_next_event_single &
 (xi, rng, g%grids(channel), func, data, wgt, &
 channel, g%weights, g%grids, exc)
 if (present (weight)) then
 ⟨Unconditionally accept weighted multi channel event 138a⟩
 else
 ⟨Maybe accept unweighted multi channel event 138b⟩
 end if
end do rejection
x = phi (xi, channel)
end subroutine vamp_next_event_multi

```

We can either reject with the weights

$$\frac{w_i(x)}{\max_i \max_x w_i(x)} \quad (5.40)$$

after using the apriori weights  $\alpha_i$  to select a channel  $i$  or we can reject with the weights

$$\frac{w_i(x)}{\max_x w_i(x)} \quad (5.41)$$

after using the apriori weights  $\alpha_i(\max_x w_i(x))/(\max_i \max_x w_i(x))$ . The latter method is more efficient if the  $\max_x w_i(x)$  have a wide spread.

137a ⟨weights:  $\alpha_i \rightarrow w_{\max,i}\alpha_i$  137a⟩≡ (136c 138c)

```

if (any (g%grids%f_max > 0)) then
 weights = g%weights * g%grids%f_max
else
 weights = g%weights
end if
weights = weights / sum (weights)

```

137b ⟨Select channel from weights 137b⟩≡ (136c)

```

call tao_random_number (rng, r)
select_channel: do channel = 1, size (g%weights)
 r = r - weights(channel)
 if (r <= 0.0) then
 exit select_channel
 end if
end do

```

```

end do select_channel
channel = min (channel, size (g%weights)) ! for $r = 1$ and rounding errors
138a <Unconditionally accept weighted multi channel event 138a>≡ (136c)
weight = wgt * g%weights(channel) / weights(channel)
exit rejection
138b <Maybe accept unweighted multi channel event 138b>≡ (136c)
if (abs (wgt) > g%grids(channel)%f_max) then
 if (present(excess)) then
 excess = abs (wgt) / g%grids(channel)%f_max - 1
 else
 call raise_exception (exc, EXC_WARN, FN, "weight > 1")
 ! print *, "weight > 1 (", wgt/g%grids(channel)%f_max, &
 ! & ") in channel ", channel

 end if
 ! exit rejection
 else
 if (present(excess)) excess = 0
 end if
 call tao_random_number (rng, r)
 if (r * g%grids(channel)%f_max <= abs (wgt)) then
 if (present (positive)) positive = wgt >= 0
 exit rejection
 end if
138c <Maybe accept unweighted multi channel event (old version) 138c>≡
 if (wgt > g%grids(channel)%f_max) then
 g%grids(channel)%f_max = wgt
 <weights: $\alpha_i \rightarrow w_{\max,i}\alpha_i$ 137a>
 call raise_exception (exc, EXC_WARN, FN, "weight > 1")
 exit rejection
 end if
 call tao_random_number (rng, r)
 if (r * g%grids(channel)%f_max <= wgt) then
 exit rejection
 end if

```

Using `vamp_sample_grid (g, ...)` to warm up the grid `g` has a somewhat subtle problem: the minimum and maximum weights `g%f_min` and `g%f_max` refer to the grid *before* the final refinement. One could require an additional `vamp_sample_grid0 (g, ...)`, but users are likely to forget such technical details. A better solution is a wrapper `vamp_warmup_grid (g, ...)` that drops the final refinement transparently.

138d <Declaration of vamp procedures 76b>+≡ (75a) <135b 140a>

```
public :: vamp_warmup_grid, vamp_warmup_grids
```

139a *<Implementation of vamp procedures 77d>+≡* (75a) <136c 139b>

```
subroutine vamp_warmup_grid &
 (rng, g, func, data, iterations, exc, history)
 type(tao_random_state), intent(inout) :: rng
 type(vamp_grid), intent(inout) :: g
 class(vamp_data_t), intent(in) :: data
 integer, intent(in) :: iterations
 type(exception), intent(inout), optional :: exc
 type(vamp_history), dimension(:), intent(inout), optional :: history
 <Interface declaration for func 22>
 call vamp_sample_grid &
 (rng, g, func, data, &
 iterations - 1, exc = exc, history = history)
 call vamp_sample_grid0 (rng, g, func, data, exc = exc)
end subroutine vamp_warmup_grid
```



WHERE ... END WHERE alert!

139b *<Implementation of vamp procedures 77d>+≡* (75a) <139a 140c>

```
subroutine vamp_warmup_grids &
 (rng, g, func, data, iterations, history, histories, exc)
 type(tao_random_state), intent(inout) :: rng
 type(vamp_grids), intent(inout) :: g
 class(vamp_data_t), intent(in) :: data
 integer, intent(in) :: iterations
 type(vamp_history), dimension(:), intent(inout), optional :: history
 type(vamp_history), dimension(:, :), intent(inout), optional :: histories
 type(exception), intent(inout), optional :: exc
 <Interface declaration for func 22>
 integer :: ch
 logical, dimension(size(g%grids)) :: active
 real(kind=default), dimension(size(g%grids)) :: weights
 active = (g%num_calls >= 2)
 where (active)
 weights = g%num_calls
 elsewhere
 weights = 0.0
 end where
 weights = weights / sum (weights)
 call vamp_sample_grids (rng, g, func, data, iterations - 1, &
 exc = exc, history = history, histories = histories)
 do ch = 1, size (g%grids)
```

```

 if (g%grids(ch)%num_calls >= 2) then
 call vamp_sample_grid0 &
 (rng, g%grids(ch), func, data, &
 ch, weights, g%grids, exc = exc)
 end if
 end do
end subroutine vamp_warmup_grids

```

### 5.2.10 Convenience Routines

- 140a *<Declaration of vamp procedures 76b>+≡* (75a) <138d 141b>  

```

public :: vamp_integrate
private :: vamp_integrate_grid, vamp_integrate_region

```
- 140b *<Interfaces of vamp procedures 95c>+≡* (75a) <135a 142a>  

```

interface vamp_integrate
 module procedure vamp_integrate_grid, vamp_integrate_region
end interface

```
- 140c *<Implementation of vamp procedures 77d>+≡* (75a) <139b 141a>  

```

subroutine vamp_integrate_grid &
 (rng, g, func, data, calls, integral, std_dev, avg_chi2, num_div, &
 stratified, quadrupole, accuracy, exc, history)
type(tao_random_state), intent(inout) :: rng
type(vamp_grid), intent(inout) :: g
class(vamp_data_t), intent(in) :: data
integer, dimension(:,:), intent(in) :: calls
real(kind=default), intent(out), optional :: integral, std_dev, avg_chi2
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole
real(kind=default), intent(in), optional :: accuracy
type(exception), intent(inout), optional :: exc
type(vamp_history), dimension(:), intent(inout), optional :: history
<Interface declaration for func 22>
character(len=*), parameter :: FN = "vamp_integrate_grid"
integer :: step, last_step, it
last_step = size(calls, dim = 2)
it = 1
do step = 1, last_step - 1
 call vamp_discard_integral (g, calls(2,step), num_div, &
 stratified, quadrupole, exc = exc)
 call vamp_sample_grid (rng, g, func, data, calls(1,step), &
 exc = exc, history = history(it:))
<Bail out if exception exc raised 99b>

```

```

 it = it + calls(1,step)
 end do
 call vamp_discard_integral (g, calls(2,last_step), exc = exc)
 call vamp_sample_grid (rng, g, func, data, calls(1,last_step), &
 integral, std_dev, avg_chi2, accuracy, exc = exc, &
 history = history(it:))
end subroutine vamp_integrate_grid
141a <Implementation of vamp procedures 77d>+≡ (75a) <140c 142b>
subroutine vamp_integrate_region &
 (rng, region, func, data, calls, &
 integral, std_dev, avg_chi2, num_div, &
 stratified, quadrupole, accuracy, map, covariance, exc, history)
type(tao_random_state), intent(inout) :: rng
real(kind=default), dimension(:,:), intent(in) :: region
class(vamp_data_t), intent(in) :: data
integer, dimension(:,:), intent(in) :: calls
real(kind=default), intent(out), optional :: integral, std_dev, avg_chi2
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole
real(kind=default), intent(in), optional :: accuracy
real(kind=default), dimension(:,:), intent(in), optional :: map
real(kind=default), dimension(:,:), intent(out), optional :: covariance
type(exception), intent(inout), optional :: exc
type(vamp_history), dimension(:), intent(inout), optional :: history
<Interface declaration for func 22>
character(len=*), parameter :: FN = "vamp_integrate_region"
type(vamp_grid) :: g
call vamp_create_grid &
 (g, region, calls(2,1), num_div, &
 stratified, quadrupole, present (covariance), map, exc)
call vamp_integrate_grid &
 (rng, g, func, data, calls, &
 integral, std_dev, avg_chi2, num_div, &
 accuracy = accuracy, exc = exc, history = history)
if (present (covariance)) then
 covariance = vamp_get_covariance (g)
end if
call vamp_delete_grid (g)
end subroutine vamp_integrate_region
141b <Declaration of vamp procedures 76b>+≡ (75a) <140a 143a>
public :: vamp_integratex
private :: vamp_integratex_region

```

142a *<Interfaces of vamp procedures 95c>+≡* (75a) <140b 143c>  
 interface vamp\_integratex  
   module procedure vamp\_integratex\_region  
end interface

142b *<Implementation of vamp procedures 77d>+≡* (75a) <141a 144b>  
 subroutine vamp\_integratex\_region &  
   (rng, region, func, data, calls, integral, std\_dev, avg\_chi2, &  
   num\_div, stratified, quadrupole, accuracy, pancake, cigar, &  
   exc, history)  
type(tao\_random\_state), intent(inout) :: rng  
real(kind=default), dimension(:,:), intent(in) :: region  
class(vamp\_data\_t), intent(in) :: data  
integer, dimension(:,:,:), intent(in) :: calls  
real(kind=default), intent(out), optional :: integral, std\_dev, avg\_chi2  
integer, dimension(:), intent(in), optional :: num\_div  
logical, intent(in), optional :: stratified, quadrupole  
real(kind=default), intent(in), optional :: accuracy  
integer, intent(in), optional :: pancake, cigar  
type(exception), intent(inout), optional :: exc  
type(vamp\_history), dimension(:), intent(inout), optional :: history  
*<Interface declaration for func 22>*  
real(kind=default), dimension(size(region,dim=2)) :: eval  
real(kind=default), dimension(size(region,dim=2),size(region,dim=2)) :: evec  
type(vamp\_grid) :: g  
integer :: step, last\_step, it  
it = 1  
call vamp\_create\_grid &  
   (g, region, calls(2,1,1), num\_div, &  
   stratified, quadrupole, covariance = .true., exc = exc)  
call vamp\_integrate\_grid &  
   (rng, g, func, data, calls(:, :, 1), num\_div = num\_div, &  
   exc = exc, history = history(it:))  
*<Bail out if exception exc raised 99b>*  
it = it + sum (calls(1, :, 1))  
last\_step = size (calls, dim = 3)  
do step = 2, last\_step - 1  
  call diagonalize\_real\_symmetric (vamp\_get\_covariance(g), eval, evec)  
  call sort (eval, evec)  
  call select\_rotation\_axis (vamp\_get\_covariance(g), evec, pancake, cigar)  
  call vamp\_delete\_grid (g)  
  call vamp\_create\_grid &  
   (g, region, calls(2,1,step), num\_div, stratified, quadrupole, &  
   covariance = .true., map = evec, exc = exc)

```

call vamp_integrate_grid &
 (rng, g, func, data, calls(:, :, step), num_div = num_div, &
 exc = exc, history = history(it:))
 <Bail out if exception exc raised 99b>
 it = it + sum (calls(1, :, step))
end do
call diagonalize_real_symmetric (vamp_get_covariance(g), eval, evec)
call sort (eval, evec)
call select_rotation_axis (vamp_get_covariance(g), evec, pancake, cigar)
call vamp_delete_grid (g)
call vamp_create_grid &
 (g, region, calls(2, 1, last_step), num_div, stratified, quadrupole, &
 covariance = .true., map = evec, exc = exc)
call vamp_integrate_grid &
 (rng, g, func, data, calls(:, :, last_step), &
 integral, std_dev, avg_chi2, &
 num_div = num_div, exc = exc, history = history(it:))
call vamp_delete_grid (g)
end subroutine vamp_integratex_region

```

### 5.2.11 I/O

- 143a <Declaration of vamp procedures 76b>+≡ (75a) <141b 143b>
- ```

public :: vamp_write_grid
private :: write_grid_unit, write_grid_name
public :: vamp_read_grid
private :: read_grid_unit, read_grid_name
public :: vamp_write_grids
private :: write_grids_unit, write_grids_name
public :: vamp_read_grids
private :: read_grids_unit, read_grids_name

```
- 143b <Declaration of vamp procedures 76b>+≡ (75a) <143a 157c>
- ```

public :: vamp_read_grids_raw
private :: read_grids_raw_unit, read_grids_raw_name
public :: vamp_read_grid_raw
private :: read_grid_raw_unit, read_grid_raw_name
public :: vamp_write_grids_raw
private :: write_grids_raw_unit, write_grids_raw_name
public :: vamp_write_grid_raw
private :: write_grid_raw_unit, write_grid_raw_name

```
- 143c <Interfaces of vamp procedures 95c>+≡ (75a) <142a 144a>
- ```

interface vamp_write_grid

```



```

        module procedure write_grid_unit, write_grid_name
    end interface
    interface vamp_read_grid
        module procedure read_grid_unit, read_grid_name
    end interface
    interface vamp_write_grids
        module procedure write_grids_unit, write_grids_name
    end interface
    interface vamp_read_grids
        module procedure read_grids_unit, read_grids_name
    end interface
144a <Interfaces of vamp procedures 95c>+≡ (75a) <143c
    interface vamp_write_grid_raw
        module procedure write_grid_raw_unit, write_grid_raw_name
    end interface
    interface vamp_read_grid_raw
        module procedure read_grid_raw_unit, read_grid_raw_name
    end interface
    interface vamp_write_grids_raw
        module procedure write_grids_raw_unit, write_grids_raw_name
    end interface
    interface vamp_read_grids_raw
        module procedure read_grids_raw_unit, read_grids_raw_name
    end interface
144b <Implementation of vamp procedures 77d>+≡ (75a) <142b 146b>
    subroutine write_grid_unit (g, unit, write_integrals)
        type(vamp_grid), intent(in) :: g
        integer, intent(in) :: unit
        logical, intent(in), optional :: write_integrals
        integer :: i, j
        write (unit = unit, fmt = descr_fmt) "begin type(vamp_grid) :: g"
        write (unit = unit, fmt = integer_fmt) "size (g%div) = ", size (g%div)
        write (unit = unit, fmt = integer_fmt) "num_calls = ", g%num_calls
        write (unit = unit, fmt = integer_fmt) "calls_per_cell = ", g%calls_per_cell
        write (unit = unit, fmt = logical_fmt) "stratified = ", g%stratified
        write (unit = unit, fmt = logical_fmt) "all_stratified = ", g%all_stratified
        write (unit = unit, fmt = logical_fmt) "quadrupole = ", g%quadrupole
        write (unit = unit, fmt = double_fmt) "mu(1) = ", g%mu(1)
        write (unit = unit, fmt = double_fmt) "mu(2) = ", g%mu(2)
        write (unit = unit, fmt = double_fmt) "mu_plus(1) = ", g%mu_plus(1)
        write (unit = unit, fmt = double_fmt) "mu_plus(2) = ", g%mu_plus(2)
        write (unit = unit, fmt = double_fmt) "mu_minus(1) = ", g%mu_minus(1)
        write (unit = unit, fmt = double_fmt) "mu_minus(2) = ", g%mu_minus(2)

```

```

write (unit = unit, fmt = double_fmt) "sum_integral = ", g%sum_integral
write (unit = unit, fmt = double_fmt) "sum_weights = ", g%sum_weights
write (unit = unit, fmt = double_fmt) "sum_chi2 = ", g%sum_chi2
write (unit = unit, fmt = double_fmt) "calls = ", g%calls
write (unit = unit, fmt = double_fmt) "dv2g = ", g%dv2g
write (unit = unit, fmt = double_fmt) "jacobi = ", g%jacobi
write (unit = unit, fmt = double_fmt) "f_min = ", g%f_min
write (unit = unit, fmt = double_fmt) "f_max = ", g%f_max
write (unit = unit, fmt = double_fmt) "mu_gi = ", g%mu_gi
write (unit = unit, fmt = double_fmt) "sum_mu_gi = ", g%sum_mu_gi
write (unit = unit, fmt = descr_fmt) "begin g%num_div"
do i = 1, size (g%div)
    write (unit = unit, fmt = integer_array_fmt) i, g%num_div(i)
end do
write (unit = unit, fmt = descr_fmt) "end g%num_div"
write (unit = unit, fmt = descr_fmt) "begin g%div"
do i = 1, size (g%div)
    call write_division (g%div(i), unit, write_integrals)
end do
write (unit = unit, fmt = descr_fmt) "end g%div"
if (associated (g%map)) then
    write (unit = unit, fmt = descr_fmt) "begin g%map"
    do i = 1, size (g%div)
        do j = 1, size (g%div)
            write (unit = unit, fmt = double_array2_fmt) i, j, g%map(i,j)
        end do
    end do
    write (unit = unit, fmt = descr_fmt) "end g%map"
else
    write (unit = unit, fmt = descr_fmt) "empty g%map"
end if
if (associated (g%mu_x)) then
    write (unit = unit, fmt = descr_fmt) "begin g%mu_x"
    do i = 1, size (g%div)
        write (unit = unit, fmt = double_array_fmt) i, g%mu_x(i)
        write (unit = unit, fmt = double_array_fmt) i, g%sum_mu_x(i)
        do j = 1, size (g%div)
            write (unit = unit, fmt = double_array2_fmt) i, j, g%mu_xx(i,j)
            write (unit = unit, fmt = double_array2_fmt) i, j, g%sum_mu_xx(i,j)
        end do
    end do
    write (unit = unit, fmt = descr_fmt) "end g%mu_x"
else

```

```

        write (unit = unit, fmt = descr_fmt) "empty g%mu_x"
    end if
    write (unit = unit, fmt = descr_fmt) "end type(vamp_grid)"
end subroutine write_grid_unit

146a  <Variables in vamp 78a>+≡ (75a) <109a
character(len=*), parameter, private :: &
    descr_fmt = "(1x,a)", &
    integer_fmt = "(1x,a17,1x,i15)", &
    integer_array_fmt = "(1x,i17,1x,i15)", &
    logical_fmt = "(1x,a17,1x,l1)", &
    double_fmt = "(1x,a17,1x,e30.22e4)", &
    double_array_fmt = "(1x,i17,1x,e30.22e4)", &
    double_array2_fmt = "(2(1x,i8),1x,e30.22e4)"

146b  <Implementation of vamp procedures 77d>+≡ (75a) <144b 148d>
subroutine read_grid_unit (g, unit, read_integrals)
    type(vamp_grid), intent(inout) :: g
    integer, intent(in) :: unit
    logical, intent(in), optional :: read_integrals
    character(len=*), parameter :: FN = "vamp_read_grid"
    character(len=80) :: chdum
    integer :: ndim, i, j, idum, jdum
    read (unit = unit, fmt = descr_fmt) chdum
    read (unit = unit, fmt = integer_fmt) chdum, ndim
    <Insure that size (g%div) == ndim 148a>
    call create_array_pointer (g%num_div, ndim)
    read (unit = unit, fmt = integer_fmt) chdum, g%num_calls
    read (unit = unit, fmt = integer_fmt) chdum, g%calls_per_cell
    read (unit = unit, fmt = logical_fmt) chdum, g%stratified
    read (unit = unit, fmt = logical_fmt) chdum, g%all_stratified
    read (unit = unit, fmt = logical_fmt) chdum, g%quadrupole
    read (unit = unit, fmt = double_fmt) chdum, g%mu(1)
    read (unit = unit, fmt = double_fmt) chdum, g%mu(2)
    read (unit = unit, fmt = double_fmt) chdum, g%mu_plus(1)
    read (unit = unit, fmt = double_fmt) chdum, g%mu_plus(2)
    read (unit = unit, fmt = double_fmt) chdum, g%mu_minus(1)
    read (unit = unit, fmt = double_fmt) chdum, g%mu_minus(2)
    read (unit = unit, fmt = double_fmt) chdum, g%sum_integral
    read (unit = unit, fmt = double_fmt) chdum, g%sum_weights
    read (unit = unit, fmt = double_fmt) chdum, g%sum_chi2
    read (unit = unit, fmt = double_fmt) chdum, g%calls
    read (unit = unit, fmt = double_fmt) chdum, g%dv2g
    read (unit = unit, fmt = double_fmt) chdum, g%jacobi
    read (unit = unit, fmt = double_fmt) chdum, g%f_min

```

```

read (unit = unit, fmt = double_fmt) chdum, g%f_max
read (unit = unit, fmt = double_fmt) chdum, g%mu_gi
read (unit = unit, fmt = double_fmt) chdum, g%sum_mu_gi
read (unit = unit, fmt = descr_fmt) chdum
do i = 1, size (g%div)
    read (unit = unit, fmt = integer_array_fmt) idum, g%num_div(i)
end do
read (unit = unit, fmt = descr_fmt) chdum
read (unit = unit, fmt = descr_fmt) chdum
do i = 1, size (g%div)
    call read_division (g%div(i), unit, read_integrals)
end do
read (unit = unit, fmt = descr_fmt) chdum
read (unit = unit, fmt = descr_fmt) chdum
if (chdum == "begin g%map") then
    call create_array_pointer (g%map, (/ ndim, ndim /))
    do i = 1, size (g%div)
        do j = 1, size (g%div)
            read (unit = unit, fmt = double_array2_fmt) idum, jdum, g%map(i,j)
        end do
    end do
    read (unit = unit, fmt = descr_fmt) chdum
else
    <Insure that associated (g%map) == .false. 148b>
end if
read (unit = unit, fmt = descr_fmt) chdum
if (chdum == "begin g%mu_x") then
    call create_array_pointer (g%mu_x, ndim )
    call create_array_pointer (g%sum_mu_x, ndim)
    call create_array_pointer (g%mu_xx, (/ ndim, ndim /))
    call create_array_pointer (g%sum_mu_xx, (/ ndim, ndim /))
    do i = 1, size (g%div)
        read (unit = unit, fmt = double_array_fmt) idum, jdum, g%mu_x(i)
        read (unit = unit, fmt = double_array_fmt) idum, jdum, g%sum_mu_x(i)
        do j = 1, size (g%div)
            read (unit = unit, fmt = double_array2_fmt) &
                idum, jdum, g%mu_xx(i,j)
            read (unit = unit, fmt = double_array2_fmt) &
                idum, jdum, g%sum_mu_xx(i,j)
        end do
    end do
    read (unit = unit, fmt = descr_fmt) chdum
else

```

```

        <Insure that associated (g%mu_x) == .false. 148c>
    end if
    read (unit = unit, fmt = descr_fmt) chdum
end subroutine read_grid_unit

148a <Insure that size (g%div) == ndim 148a>≡ (146b 153 160)
    if (associated (g%div)) then
        if (size (g%div) /= ndim) then
            call delete_division (g%div)
            deallocate (g%div)
            allocate (g%div(ndim))
            call create_empty_division (g%div)
        end if
    else
        allocate (g%div(ndim))
        call create_empty_division (g%div)
    end if

148b <Insure that associated (g%map) == .false. 148b>≡ (146b 153 160)
    if (associated (g%map)) then
        deallocate (g%map)
    end if

148c <Insure that associated (g%mu_x) == .false. 148c>≡ (146b 153 160)
    if (associated (g%mu_x)) then
        deallocate (g%mu_x)
    end if
    if (associated (g%mu_xx)) then
        deallocate (g%mu_xx)
    end if
    if (associated (g%sum_mu_x)) then
        deallocate (g%sum_mu_x)
    end if
    if (associated (g%sum_mu_xx)) then
        deallocate (g%sum_mu_xx)
    end if

148d <Implementation of vamp procedures 77d>+≡ (75a) <146b 149a>
    subroutine write_grid_name (g, name, write_integrals)
        type(vamp_grid), intent(inout) :: g
        character(len=*), intent(in) :: name
        logical, intent(in), optional :: write_integrals
        integer :: unit
        call find_free_unit (unit)
        open (unit = unit, action = "write", status = "replace", file = name)
        call write_grid_unit (g, unit, write_integrals)

```

```

        close (unit = unit)
    end subroutine write_grid_name
149a  <Implementation of vamp procedures 77d>+≡ (75a) <148d 149b>
    subroutine read_grid_name (g, name, read_integrals)
        type(vamp_grid), intent(inout) :: g
        character(len=*), intent(in) :: name
        logical, intent(in), optional :: read_integrals
        integer :: unit
        call find_free_unit (unit)
        open (unit = unit, action = "read", status = "old", file = name)
        call read_grid_unit (g, unit, read_integrals)
        close (unit = unit)
    end subroutine read_grid_name
149b  <Implementation of vamp procedures 77d>+≡ (75a) <149a 149c>
    subroutine write_grids_unit (g, unit, write_integrals)
        type(vamp_grids), intent(in) :: g
        integer, intent(in) :: unit
        logical, intent(in), optional :: write_integrals
        integer :: i
        write (unit = unit, fmt = descr_fmt) "begin type(vamp_grids) :: g"
        write (unit = unit, fmt = integer_fmt) "size (g%grids) = ", size (g%grids)
        write (unit = unit, fmt = double_fmt) "sum_integral = ", g%sum_integral
        write (unit = unit, fmt = double_fmt) "sum_weights = ", g%sum_weights
        write (unit = unit, fmt = double_fmt) "sum_chi2 = ", g%sum_chi2
        write (unit = unit, fmt = descr_fmt) "begin g%weights"
        do i = 1, size (g%grids)
            write (unit = unit, fmt = double_array_fmt) i, g%weights(i)
        end do
        write (unit = unit, fmt = descr_fmt) "end g%weights"
        write (unit = unit, fmt = descr_fmt) "begin g%num_calls"
        do i = 1, size (g%grids)
            write (unit = unit, fmt = integer_array_fmt) i, g%num_calls(i)
        end do
        write (unit = unit, fmt = descr_fmt) "end g%num_calls"
        write (unit = unit, fmt = descr_fmt) "begin g%grids"
        do i = 1, size (g%grids)
            call write_grid_unit (g%grids(i), unit, write_integrals)
        end do
        write (unit = unit, fmt = descr_fmt) "end g%grids"
        write (unit = unit, fmt = descr_fmt) "end type(vamp_grids)"
    end subroutine write_grids_unit
149c  <Implementation of vamp procedures 77d>+≡ (75a) <149b 150>

```

```

subroutine read_grids_unit (g, unit, read_integrals)
  type(vamp_grids), intent(inout) :: g
  integer, intent(in) :: unit
  logical, intent(in), optional :: read_integrals
  character(len=*), parameter :: FN = "vamp_read_grids"
  character(len=80) :: chdum
  integer :: i, nch, idum
  read (unit = unit, fmt = descr_fmt) chdum
  read (unit = unit, fmt = integer_fmt) chdum, nch
  if (associated (g%grids)) then
    if (size (g%grids) /= nch) then
      call vamp_delete_grid (g%grids)
      deallocate (g%grids, g%weights, g%num_calls)
      allocate (g%grids(nch), g%weights(nch), g%num_calls(nch))
      call vamp_create_empty_grid (g%grids)
    end if
  else
    allocate (g%grids(nch), g%weights(nch), g%num_calls(nch))
    call vamp_create_empty_grid (g%grids)
  end if
  read (unit = unit, fmt = double_fmt) chdum, g%sum_integral
  read (unit = unit, fmt = double_fmt) chdum, g%sum_weights
  read (unit = unit, fmt = double_fmt) chdum, g%sum_chi2
  read (unit = unit, fmt = descr_fmt) chdum
  do i = 1, nch
    read (unit = unit, fmt = double_array_fmt) idum, g%weights(i)
  end do
  read (unit = unit, fmt = descr_fmt) chdum
  read (unit = unit, fmt = descr_fmt) chdum
  do i = 1, nch
    read (unit = unit, fmt = integer_array_fmt) idum, g%num_calls(i)
  end do
  read (unit = unit, fmt = descr_fmt) chdum
  read (unit = unit, fmt = descr_fmt) chdum
  do i = 1, nch
    call read_grid_unit (g%grids(i), unit, read_integrals)
  end do
  read (unit = unit, fmt = descr_fmt) chdum
  read (unit = unit, fmt = descr_fmt) chdum
end subroutine read_grids_unit

```

150 *<Implementation of vamp procedures 77d>+≡* (75a) *<149c 151a>*

```

subroutine write_grids_name (g, name, write_integrals)
  type(vamp_grids), intent(inout) :: g

```

```

character(len=*), intent(in) :: name
logical, intent(in), optional :: write_integrals
integer :: unit
call find_free_unit (unit)
open (unit = unit, action = "write", status = "replace", file = name)
call write_grids_unit (g, unit, write_integrals)
close (unit = unit)
end subroutine write_grids_name
151a <Implementation of vamp procedures 77d>+≡ (75a) <150 151b>
subroutine read_grids_name (g, name, read_integrals)
type(vamp_grids), intent(inout) :: g
character(len=*), intent(in) :: name
logical, intent(in), optional :: read_integrals
integer :: unit
call find_free_unit (unit)
open (unit = unit, action = "read", status = "old", file = name)
call read_grids_unit (g, unit, read_integrals)
close (unit = unit)
end subroutine read_grids_name
151b <Implementation of vamp procedures 77d>+≡ (75a) <151a 153>
subroutine write_grid_raw_unit (g, unit, write_integrals)
type(vamp_grid), intent(in) :: g
integer, intent(in) :: unit
logical, intent(in), optional :: write_integrals
integer :: i, j
write (unit = unit) MAGIC_GRID_BEGIN
write (unit = unit) size (g%div)
write (unit = unit) g%num_calls
write (unit = unit) g%calls_per_cell
write (unit = unit) g%stratified
write (unit = unit) g%all_stratified
write (unit = unit) g%quadrupole
write (unit = unit) g%mu(1)
write (unit = unit) g%mu(2)
write (unit = unit) g%mu_plus(1)
write (unit = unit) g%mu_plus(2)
write (unit = unit) g%mu_minus(1)
write (unit = unit) g%mu_minus(2)
write (unit = unit) g%sum_integral
write (unit = unit) g%sum_weights
write (unit = unit) g%sum_chi2
write (unit = unit) g%calls
write (unit = unit) g%dv2g

```



```

write (unit = unit) g%jacobi
write (unit = unit) g%f_min
write (unit = unit) g%f_max
write (unit = unit) g%mu_gi
write (unit = unit) g%sum_mu_gi
do i = 1, size (g%div)
  write (unit = unit) g%num_div(i)
end do
do i = 1, size (g%div)
  call write_division_raw (g%div(i), unit, write_integrals)
end do
if (associated (g%map)) then
  write (unit = unit) MAGIC_GRID_MAP
  do i = 1, size (g%div)
    do j = 1, size (g%div)
      write (unit = unit) g%map(i,j)
    end do
  end do
else
  write (unit = unit) MAGIC_GRID_EMPTY
end if
if (associated (g%mu_x)) then
  write (unit = unit) MAGIC_GRID_MU_X
  do i = 1, size (g%div)
    write (unit = unit) g%mu_x(i)
    write (unit = unit) g%sum_mu_x(i)
    do j = 1, size (g%div)
      write (unit = unit) g%mu_xx(i,j)
      write (unit = unit) g%sum_mu_xx(i,j)
    end do
  end do
else
  write (unit = unit) MAGIC_GRID_EMPTY
end if
write (unit = unit) MAGIC_GRID_END
end subroutine write_grid_raw_unit

```

152 *<Constants in vamp 152>*≡ (75a) 156a▷

```

integer, parameter, private :: MAGIC_GRID = 22222222
integer, parameter, private :: MAGIC_GRID_BEGIN = MAGIC_GRID + 1
integer, parameter, private :: MAGIC_GRID_END = MAGIC_GRID + 2
integer, parameter, private :: MAGIC_GRID_EMPTY = MAGIC_GRID + 3
integer, parameter, private :: MAGIC_GRID_MAP = MAGIC_GRID + 4
integer, parameter, private :: MAGIC_GRID_MU_X = MAGIC_GRID + 5

```

```

153  <Implementation of vamp procedures 77d>+≡ (75a) <151b 155a>
      subroutine read_grid_raw_unit (g, unit, read_integrals)
        type(vamp_grid), intent(inout) :: g
        integer, intent(in) :: unit
        logical, intent(in), optional :: read_integrals
        character(len=*), parameter :: FN = "vamp_read_raw_grid"
        integer :: ndim, i, j, magic
        read (unit = unit) magic
        if (magic /= MAGIC_GRID_BEGIN) then
          print *, FN, " fatal: expecting magic ", MAGIC_GRID_BEGIN, &
            ", found ", magic
          stop
        end if
        read (unit = unit) ndim
        <Insure that size (g%div) == ndim 148a>
        call create_array_pointer (g%num_div, ndim)
        read (unit = unit) g%num_calls
        read (unit = unit) g%calls_per_cell
        read (unit = unit) g%stratified
        read (unit = unit) g%all_stratified
        read (unit = unit) g%quadrupole
        read (unit = unit) g%mu(1)
        read (unit = unit) g%mu(2)
        read (unit = unit) g%mu_plus(1)
        read (unit = unit) g%mu_plus(2)
        read (unit = unit) g%mu_minus(1)
        read (unit = unit) g%mu_minus(2)
        read (unit = unit) g%sum_integral
        read (unit = unit) g%sum_weights
        read (unit = unit) g%sum_chi2
        read (unit = unit) g%calls
        read (unit = unit) g%dv2g
        read (unit = unit) g%jacobi
        read (unit = unit) g%f_min
        read (unit = unit) g%f_max
        read (unit = unit) g%mu_gi
        read (unit = unit) g%sum_mu_gi
        do i = 1, size (g%div)
          read (unit = unit) g%num_div(i)
        end do
        do i = 1, size (g%div)
          call read_division_raw (g%div(i), unit, read_integrals)
        end do

```

```

read (unit = unit) magic
if (magic == MAGIC_GRID_MAP) then
  call create_array_pointer (g%map, (/ ndim, ndim /))
  do i = 1, size (g%div)
    do j = 1, size (g%div)
      read (unit = unit) g%map(i,j)
    end do
  end do
else if (magic == MAGIC_GRID_EMPTY) then
  <Insure that associated (g%map) == .false. 148b>
else
  print *, FN, " fatal: expecting magic ", MAGIC_GRID_EMPTY, &
    " or ", MAGIC_GRID_MAP, ", found ", magic
  stop
end if
read (unit = unit) magic
if (magic == MAGIC_GRID_MU_X) then
  call create_array_pointer (g%mu_x, ndim )
  call create_array_pointer (g%sum_mu_x, ndim)
  call create_array_pointer (g%mu_xx, (/ ndim, ndim /))
  call create_array_pointer (g%sum_mu_xx, (/ ndim, ndim /))
  do i = 1, size (g%div)
    read (unit = unit) g%mu_x(i)
    read (unit = unit) g%sum_mu_x(i)
    do j = 1, size (g%div)
      read (unit = unit) g%mu_xx(i,j)
      read (unit = unit) g%sum_mu_xx(i,j)
    end do
  end do
else if (magic == MAGIC_GRID_EMPTY) then
  <Insure that associated (g%mu_x) == .false. 148c>
else
  print *, FN, " fatal: expecting magic ", MAGIC_GRID_EMPTY, &
    " or ", MAGIC_GRID_MU_X, ", found ", magic
  stop
end if
read (unit = unit) magic
if (magic /= MAGIC_GRID_END) then
  print *, FN, " fatal: expecting magic ", MAGIC_GRID_END, &
    " found ", magic
  stop
end if
end subroutine read_grid_raw_unit

```

```

155a  <Implementation of vamp procedures 77d>+≡ (75a) <153 155b>
      subroutine write_grid_raw_name (g, name, write_integrals)
        type(vamp_grid), intent(inout) :: g
        character(len=*), intent(in) :: name
        logical, intent(in), optional :: write_integrals
        integer :: unit
        call find_free_unit (unit)
        open (unit = unit, action = "write", status = "replace", &
              form = "unformatted", file = name)
        call write_grid_raw_unit (g, unit, write_integrals)
        close (unit = unit)
      end subroutine write_grid_raw_name

155b  <Implementation of vamp procedures 77d>+≡ (75a) <155a 155c>
      subroutine read_grid_raw_name (g, name, read_integrals)
        type(vamp_grid), intent(inout) :: g
        character(len=*), intent(in) :: name
        logical, intent(in), optional :: read_integrals
        integer :: unit
        call find_free_unit (unit)
        open (unit = unit, action = "read", status = "old", &
              form = "unformatted", file = name)
        call read_grid_raw_unit (g, unit, read_integrals)
        close (unit = unit)
      end subroutine read_grid_raw_name

155c  <Implementation of vamp procedures 77d>+≡ (75a) <155b 156b>
      subroutine write_grids_raw_unit (g, unit, write_integrals)
        type(vamp_grids), intent(in) :: g
        integer, intent(in) :: unit
        logical, intent(in), optional :: write_integrals
        integer :: i
        write (unit = unit) MAGIC_GRIDS_BEGIN
        write (unit = unit) size (g%grids)
        write (unit = unit) g%sum_integral
        write (unit = unit) g%sum_weights
        write (unit = unit) g%sum_chi2
        do i = 1, size (g%grids)
          write (unit = unit) g%weights(i)
        end do
        do i = 1, size (g%grids)
          write (unit = unit) g%num_calls(i)
        end do
        do i = 1, size (g%grids)
          call write_grid_raw_unit (g%grids(i), unit, write_integrals)
        end do
      end subroutine write_grids_raw_unit

```

```

        end do
        write (unit = unit) MAGIC_GRIDS_END
    end subroutine write_grids_raw_unit
156a  <Constants in vamp 152>+≡ (75a) <152
        integer, parameter, private :: MAGIC_GRIDS = 33333333
        integer, parameter, private :: MAGIC_GRIDS_BEGIN = MAGIC_GRIDS + 1
        integer, parameter, private :: MAGIC_GRIDS_END = MAGIC_GRIDS + 2
156b  <Implementation of vamp procedures 77d>+≡ (75a) <155c 157a>
        subroutine read_grids_raw_unit (g, unit, read_integrals)
            type(vamp_grids), intent(inout) :: g
            integer, intent(in) :: unit
            logical, intent(in), optional :: read_integrals
            character(len=*), parameter :: FN = "vamp_read_grids_raw"
            integer :: i, nch, magic
            read (unit = unit) magic
            if (magic /= MAGIC_GRIDS_BEGIN) then
                print *, FN, " fatal: expecting magic ", MAGIC_GRIDS_BEGIN, &
                    " found ", magic
                stop
            end if
            read (unit = unit) nch
            if (associated (g%grids)) then
                if (size (g%grids) /= nch) then
                    call vamp_delete_grid (g%grids)
                    deallocate (g%grids, g%weights, g%num_calls)
                    allocate (g%grids(nch), g%weights(nch), g%num_calls(nch))
                    call vamp_create_empty_grid (g%grids)
                end if
            else
                allocate (g%grids(nch), g%weights(nch), g%num_calls(nch))
                call vamp_create_empty_grid (g%grids)
            end if
            read (unit = unit) g%sum_integral
            read (unit = unit) g%sum_weights
            read (unit = unit) g%sum_chi2
            do i = 1, nch
                read (unit = unit) g%weights(i)
            end do
            do i = 1, nch
                read (unit = unit) g%num_calls(i)
            end do
            do i = 1, nch
                call read_grid_raw_unit (g%grids(i), unit, read_integrals)

```

```

        end do
        read (unit = unit) magic
        if (magic /= MAGIC_GRIDS_END) then
            print *, FN, " fatal: expecting magic ", MAGIC_GRIDS_END, &
                " found ", magic
            stop
        end if
    end subroutine read_grids_raw_unit
157a  <Implementation of vamp procedures 77d>+≡ (75a) <156b 157b>
    subroutine write_grids_raw_name (g, name, write_integrals)
        type(vamp_grids), intent(inout) :: g
        character(len=*), intent(in) :: name
        logical, intent(in), optional :: write_integrals
        integer :: unit
        call find_free_unit (unit)
        open (unit = unit, action = "write", status = "replace", &
            form = "unformatted", file = name)
        call write_grids_raw_unit (g, unit, write_integrals)
        close (unit = unit)
    end subroutine write_grids_raw_name
157b  <Implementation of vamp procedures 77d>+≡ (75a) <157a 157d>
    subroutine read_grids_raw_name (g, name, read_integrals)
        type(vamp_grids), intent(inout) :: g
        character(len=*), intent(in) :: name
        logical, intent(in), optional :: read_integrals
        integer :: unit
        call find_free_unit (unit)
        open (unit = unit, action = "read", status = "old", &
            form = "unformatted", file = name)
        call read_grids_raw_unit (g, unit, read_integrals)
        close (unit = unit)
    end subroutine read_grids_raw_name

```

5.2.12 Marshaling

[WK] Note: mu_plus and mu_minus not transferred (hard-coded buffer indices)!

```

157c  <Declaration of vamp procedures 76b>+≡ (75a) <143b 161a>
    public :: vamp_marshall_grid_size, vamp_marshall_grid, vamp_unmarshal_grid
157d  <Implementation of vamp procedures 77d>+≡ (75a) <157b 159>
    pure subroutine vamp_marshall_grid (g, ibuf, dbuf)

```

```

type(vamp_grid), intent(in) :: g
integer, dimension(:), intent(inout) :: ibuf
real(kind=default), dimension(:), intent(inout) :: dbuf
integer :: i, iwords, dwords, iidx, didx, ndim
ndim = size (g%div)
ibuf(1) = g%num_calls
ibuf(2) = g%calls_per_cell
ibuf(3) = ndim
if (g%stratified) then
    ibuf(4) = 1
else
    ibuf(4) = 0
end if
if (g%all_stratified) then
    ibuf(5) = 1
else
    ibuf(5) = 0
end if
if (g%quadrupole) then
    ibuf(6) = 1
else
    ibuf(6) = 0
end if
dbuf(1:2) = g%mu
dbuf(3) = g%sum_integral
dbuf(4) = g%sum_weights
dbuf(5) = g%sum_chi2
dbuf(6) = g%calls
dbuf(7) = g%dv2g
dbuf(8) = g%jacobi
dbuf(9) = g%f_min
dbuf(10) = g%f_max
dbuf(11) = g%mu_gi
dbuf(12) = g%sum_mu_gi
ibuf(7:6+ndim) = g%num_div
iidx = 7 + ndim
didx = 13
do i = 1, ndim
    call marshal_division_size (g%div(i), iwords, dwords)
    ibuf(iidx) = iwords
    ibuf(iidx+1) = dwords
    iidx = iidx + 2
    call marshal_division (g%div(i), ibuf(iidx:iidx-1+iwords), &

```

```

                                dbuf(didx:didx-1+dwords))

    iidx = iidx + iwords
    didx = didx + dwords
end do
if (associated (g%map)) then
    ibuf(iidx) = 1
    dbuf(didx:didx-1+ndim**2) = reshape (g%map, (/ ndim**2 /))
    didx = didx + ndim**2
else
    ibuf(iidx) = 0
end if
iidx = iidx + 1
if (associated (g%mu_x)) then
    ibuf(iidx) = 1
    dbuf(didx:didx-1+ndim) = g%mu_x
    didx = didx + ndim
    dbuf(didx:didx-1+ndim) = g%sum_mu_x
    didx = didx + ndim
    dbuf(didx:didx-1+ndim**2) = reshape (g%mu_xx, (/ ndim**2 /))
    didx = didx + ndim**2
    dbuf(didx:didx-1+ndim**2) = reshape (g%sum_mu_xx, (/ ndim**2 /))
    didx = didx + ndim**2
else
    ibuf(iidx) = 0
end if
iidx = iidx + 1
end subroutine vamp_marshall_grid

```

159 *Implementation of vamp procedures 77d* \equiv (75a) \triangleleft 157d 160 \triangleright

```

pure subroutine vamp_marshall_grid_size (g, iwords, dwords)
  type(vamp_grid), intent(in) :: g
  integer, intent(out) :: iwords, dwords
  integer :: i, ndim, iw, dw
  ndim = size (g%div)
  iwords = 6 + ndim
  dwords = 12
  do i = 1, ndim
    call marshal_division_size (g%div(i), iw, dw)
    iwords = iwords + 2 + iw
    dwords = dwords + dw
  end do
  iwords = iwords + 1
  if (associated (g%map)) then
    dwords = dwords + ndim**2
  end if
end subroutine

```



```

end if
iwords = iwords + 1
if (associated (g%mu_x)) then
    dwords = dwords + 2 * (ndim + ndim**2)
end if
end subroutine vamp_marshall_grid_size
160 <Implementation of vamp procedures 77d>+≡ (75a) <159 161b>
pure subroutine vamp_unmarshal_grid (g, ibuf, dbuf)
    type(vamp_grid), intent(inout) :: g
    integer, dimension(:), intent(in) :: ibuf
    real(kind=default), dimension(:), intent(in) :: dbuf
    integer :: i, iwords, dwords, iidx, didx, ndim
    g%num_calls = ibuf(1)
    g%calls_per_cell = ibuf(2)
    ndim = ibuf(3)
    g%stratified = ibuf(4) /= 0
    g%all_stratified = ibuf(5) /= 0
    g%quadrupole = ibuf(6) /= 0
    g%mu = dbuf(1:2)
    g%sum_integral = dbuf(3)
    g%sum_weights = dbuf(4)
    g%sum_chi2 = dbuf(5)
    g%calls = dbuf(6)
    g%dv2g = dbuf(7)
    g%jacobi = dbuf(8)
    g%f_min = dbuf(9)
    g%f_max = dbuf(10)
    g%mu_gi = dbuf(11)
    g%sum_mu_gi = dbuf(12)
    call copy_array_pointer (g%num_div, ibuf(7:6+ndim))
    <Insure that size (g%div) == ndim 148a>
    iidx = 7 + ndim
    didx = 13
    do i = 1, ndim
        iwords = ibuf(iidx)
        dwords = ibuf(iidx+1)
        iidx = iidx + 2
        call unmarshal_division (g%div(i), ibuf(iidx:iidx-1+iwords), &
                                dbuf(didx:didx-1+dwords))

        iidx = iidx + iwords
        didx = didx + dwords
    end do
    if (ibuf(iidx) > 0) then

```

```

        call copy_array_pointer &
            (g%map, reshape (dbuf(didx:didx-1+ibuf(iidx)), (/ ndim, ndim /)))
        didx = didx + ibuf(iidx)
    else
        <Insure that associated (g%map) == .false. 148b>
    end if
    iidx = iidx + 1
    if (ibuf(iidx) > 0) then
        call copy_array_pointer (g%mu_x, dbuf(didx:didx-1+ndim))
        didx = didx + ndim
        call copy_array_pointer (g%sum_mu_x, dbuf(didx:didx-1+ndim))
        didx = didx + ndim
        call copy_array_pointer &
            (g%mu_xx, reshape (dbuf(didx:didx-1+ndim**2), (/ ndim, ndim /)))
        didx = didx + ndim**2
        call copy_array_pointer &
            (g%sum_mu_xx, reshape (dbuf(didx:didx-1+ndim**2), (/ ndim, ndim /)))
        didx = didx + ndim**2
    else
        <Insure that associated (g%mu_x) == .false. 148c>
    end if
    iidx = iidx + 1
end subroutine vamp_unmarshal_grid

161a <Declaration of vamp procedures 76b>+≡ (75a) <157c>
    public :: vamp_marshal_history_size, vamp_marshal_history
    public :: vamp_unmarshal_history

161b <Implementation of vamp procedures 77d>+≡ (75a) <160 162a>
    pure subroutine vamp_marshal_history (h, ibuf, dbuf)
        type(vamp_history), intent(in) :: h
        integer, dimension(:), intent(inout) :: ibuf
        real(kind=default), dimension(:), intent(inout) :: dbuf
        integer :: j, ndim, iidx, didx, iwords, dwords
        if (h%verbose .and. (associated (h%div))) then
            ndim = size (h%div)
        else
            ndim = 0
        end if
        ibuf(1) = ndim
        ibuf(2) = h%calls
        if (h%stratified) then
            ibuf(3) = 1
        else
            ibuf(3) = 0

```

```

end if
dbuf(1) = h%integral
dbuf(2) = h%std_dev
dbuf(3) = h%avg_integral
dbuf(4) = h%avg_std_dev
dbuf(5) = h%avg_chi2
dbuf(6) = h%f_min
dbuf(7) = h%f_max
iidx = 4
didx = 8
do j = 1, ndim
  call marshal_div_history_size (h%div(j), iwords, dwords)
  ibuf(iidx) = iwords
  ibuf(iidx+1) = dwords
  iidx = iidx + 2
  call marshal_div_history (h%div(j), ibuf(iidx:iidx-1+iwords), &
                           dbuf(didx:didx-1+dwords))

  iidx = iidx + iwords
  didx = didx + dwords
end do
end subroutine vamp_marshal_history

```

162a \langle Implementation of vamp procedures 77d $\rangle + \equiv$ (75a) \triangleleft 161b 162b \triangleright

```

pure subroutine vamp_marshal_history_size (h, iwords, dwords)
  type(vamp_history), intent(in) :: h
  integer, intent(out) :: iwords, dwords
  integer :: i, ndim, iw, dw
  if (h%verbose .and. (associated (h%div))) then
    ndim = size (h%div)
  else
    ndim = 0
  end if
  iwords = 3
  dwords = 7
  do i = 1, ndim
    call marshal_div_history_size (h%div(i), iw, dw)
    iwords = iwords + 2 + iw
    dwords = dwords + dw
  end do
end subroutine vamp_marshal_history_size

```

162b \langle Implementation of vamp procedures 77d $\rangle + \equiv$ (75a) \triangleleft 162a 163 \triangleright

```

pure subroutine vamp_unmarshal_history (h, ibuf, dbuf)
  type(vamp_history), intent(inout) :: h
  integer, dimension(:), intent(in) :: ibuf

```

```

real(kind=default), dimension(:), intent(in) :: dbuf
integer :: j, ndim, iidx, didx, iwords, dwords
ndim = ibuf(1)
h%calls = ibuf(2)
h%stratified = ibuf(3) /= 0
h%integral = dbuf(1)
h%std_dev = dbuf(2)
h%avg_integral = dbuf(3)
h%avg_std_dev = dbuf(4)
h%avg_chi2 = dbuf(5)
h%f_min = dbuf(6)
h%f_max = dbuf(7)
if (ndim > 0) then
  if (associated (h%div)) then
    if (size (h%div) /= ndim) then
      deallocate (h%div)
      allocate (h%div(ndim))
    end if
  else
    allocate (h%div(ndim))
  end if
  iidx = 4
  didx = 8
  do j = 1, ndim
    iwords = ibuf(iidx)
    dwords = ibuf(iidx+1)
    iidx = iidx + 2
    call unmarshal_div_history (h%div(j), ibuf(iidx:iidx-1+iwords), &
                               dbuf(didx:didx-1+dwords))

    iidx = iidx + iwords
    didx = didx + dwords
  end do
end if
end subroutine vamp_unmarshal_history

```

5.2.13 Boring Copying and Deleting of Objects

163 \langle Implementation of vamp procedures 77d $\rangle + \equiv$ (75a) \triangleleft 162b 165a \triangleright

```

elemental subroutine vamp_copy_grid (lhs, rhs)
  type(vamp_grid), intent(inout) :: lhs
  type(vamp_grid), intent(in) :: rhs
  integer :: ndim
  ndim = size (rhs%div)

```

```

lhs%mu = rhs%mu
lhs%mu_plus = rhs%mu_plus
lhs%mu_minus = rhs%mu_minus
lhs%sum_integral = rhs%sum_integral
lhs%sum_weights = rhs%sum_weights
lhs%sum_chi2 = rhs%sum_chi2
lhs%calls = rhs%calls
lhs%num_calls = rhs%num_calls
call copy_array_pointer (lhs%num_div, rhs%num_div)
lhs%dv2g = rhs%dv2g
lhs%jacobi = rhs%jacobi
lhs%f_min = rhs%f_min
lhs%f_max = rhs%f_max
lhs%mu_gi = rhs%mu_gi
lhs%sum_mu_gi = rhs%sum_mu_gi
lhs%calls_per_cell = rhs%calls_per_cell
lhs%stratified = rhs%stratified
lhs%all_stratified = rhs%all_stratified
lhs%quadrupole = rhs%quadrupole
if (associated (lhs%div)) then
  if (size (lhs%div) /= ndim) then
    call delete_division (lhs%div)
    deallocate (lhs%div)
    allocate (lhs%div(ndim))
  end if
else
  allocate (lhs%div(ndim))
end if
call copy_division (lhs%div, rhs%div)
if (associated (rhs%map)) then
  call copy_array_pointer (lhs%map, rhs%map)
else if (associated (lhs%map)) then
  deallocate (lhs%map)
end if
if (associated (rhs%mu_x)) then
  call copy_array_pointer (lhs%mu_x, rhs%mu_x)
  call copy_array_pointer (lhs%mu_xx, rhs%mu_xx)
  call copy_array_pointer (lhs%sum_mu_x, rhs%sum_mu_x)
  call copy_array_pointer (lhs%sum_mu_xx, rhs%sum_mu_xx)
else if (associated (lhs%mu_x)) then
  deallocate (lhs%mu_x, lhs%mu_xx, lhs%sum_mu_x, lhs%sum_mu_xx)
end if
end subroutine vamp_copy_grid

```

165a \langle Implementation of vamp procedures 77d $\rangle + \equiv$ (75a) \triangleleft 163 165b \triangleright

```

elemental subroutine vamp_delete_grid (g)
  type(vamp_grid), intent(inout) :: g
  if (associated (g%div)) then
    call delete_division (g%div)
    deallocate (g%div, g%num_div)
  end if
  if (associated (g%map)) then
    deallocate (g%map)
  end if
  if (associated (g%mu_x)) then
    deallocate (g%mu_x, g%mu_xx, g%sum_mu_x, g%sum_mu_xx)
  end if
end subroutine vamp_delete_grid

```

165b \langle Implementation of vamp procedures 77d $\rangle + \equiv$ (75a) \triangleleft 165a 165c \triangleright

```

elemental subroutine vamp_copy_grids (lhs, rhs)
  type(vamp_grids), intent(inout) :: lhs
  type(vamp_grids), intent(in) :: rhs
  integer :: nch
  nch = size (rhs%grids)
  lhs%sum_integral = rhs%sum_integral
  lhs%sum_chi2 = rhs%sum_chi2
  lhs%sum_weights = rhs%sum_weights
  if (associated (lhs%grids)) then
    if (size (lhs%grids) /= nch) then
      deallocate (lhs%grids)
      allocate (lhs%grids(nch))
      call vamp_create_empty_grid (lhs%grids(nch))
    end if
  else
    allocate (lhs%grids(nch))
    call vamp_create_empty_grid (lhs%grids(nch))
  end if
  call vamp_copy_grid (lhs%grids, rhs%grids)
  call copy_array_pointer (lhs%weights, rhs%weights)
  call copy_array_pointer (lhs%num_calls, rhs%num_calls)
end subroutine vamp_copy_grids

```

165c \langle Implementation of vamp procedures 77d $\rangle + \equiv$ (75a) \triangleleft 165b 166a \triangleright

```

elemental subroutine vamp_delete_grids (g)
  type(vamp_grids), intent(inout) :: g
  if (associated (g%grids)) then
    call vamp_delete_grid (g%grids)
    deallocate (g%weights, g%grids, g%num_calls)
  end if
end subroutine vamp_delete_grids

```

```

        end if
    end subroutine vamp_delete_grids

166a  <Implementation of vamp procedures 77d>+≡ (75a) <165c 166b>
        elemental subroutine vamp_copy_history (lhs, rhs)
            type(vamp_history), intent(inout) :: lhs
            type(vamp_history), intent(in) :: rhs
            lhs%calls = rhs%calls
            lhs%stratified = rhs%stratified
            lhs%verbose = rhs%verbose
            lhs%integral = rhs%integral
            lhs%std_dev = rhs%std_dev
            lhs%avg_integral = rhs%avg_integral
            lhs%avg_std_dev = rhs%avg_std_dev
            lhs%avg_chi2 = rhs%avg_chi2
            lhs%f_min = rhs%f_min
            lhs%f_max = rhs%f_max
            if (rhs%verbose) then
                if (associated (lhs%div)) then
                    if (size (lhs%div) /= size (rhs%div)) then
                        deallocate (lhs%div)
                        allocate (lhs%div(size(rhs%div)))
                    end if
                else
                    allocate (lhs%div(size(rhs%div)))
                end if
                call copy_history (lhs%div, rhs%div)
            end if
        end subroutine vamp_copy_history

166b  <Implementation of vamp procedures 77d>+≡ (75a) <166a
        elemental subroutine vamp_delete_history (h)
            type(vamp_history), intent(inout) :: h
            if (associated (h%div)) then
                deallocate (h%div)
            end if
        end subroutine vamp_delete_history

```

5.3 Interface to MPI

The module `vamp` makes no specific assumptions about the hardware and software supporting parallel execution. In this section, we present a specific

example of a parallel implementation of multi channel sampling using the message passing paradigm.

The modules `vamp_serial_mpi` and `vamp_parallel_mpi` are not intended to be used directly by application programs. For this purpose, the module `vampi` is provided. `vamp_serial_mpi` is identical to `vamp`, but some types, procedures and variables are renamed so that `vamp_parallel_mpi` can redefine them:

```
167a <vampi.f90 167a>≡ 167b>
! vampi.f90 --
<Copleft notice 1>
module vamp_serial_mpi
  use vamp, &
    <vamp0_* => vamp.* 168b>
  public
end module vamp_serial_mpi
```

`vamp_parallel_mpi` contains the non trivial MPI code and will be discussed in detail below.

```
167b <vampi.f90 167a>+≡ <167a 167c>
module vamp_parallel_mpi
  use kinds
  use utils
  use tao_random_numbers
  use exceptions
  use mpi90
  use divisions
  use vamp_serial_mpi !NODEP!
  use iso_fortran_env
  implicit none
  private
  <Declaration of vampi procedures 168a>
  <Interfaces of vampi procedures 172d>
  <Parameters in vampi 169a>
  <Declaration of vampi types 173a>
contains
  <Implementation of vampi procedures 168c>
end module vamp_parallel_mpi
```

`vampi` is now a plug-in replacement for `vamp` and *must not* be used together with `vamp`:

```
167c <vampi.f90 167a>+≡ <167b>
module vampi
  use vamp_serial_mpi !NODEP!
  use vamp_parallel_mpi !NODEP!
```



```

public
end module vampi

```

5.3.1 Parallel Execution

Single Channel

```

168a <Declaration of vampi procedures 168a>≡ (167b) 172b>
public :: vamp_create_grid
public :: vamp_discard_integral
public :: vamp_reshape_grid
public :: vamp_sample_grid
public :: vamp_delete_grid

168b <vamp0.* => vamp.* 168b>≡ (167a) 172c>
vamp0_create_grid => vamp_create_grid, &
vamp0_discard_integral => vamp_discard_integral, &
vamp0_reshape_grid => vamp_reshape_grid, &
vamp0_sample_grid => vamp_sample_grid, &
vamp0_delete_grid => vamp_delete_grid, &

168c <Implementation of vampi procedures 168c>≡ (167b) 169b>
subroutine vamp_create_grid &
  (g, domain, num_calls, num_div, &
   stratified, quadrupole, covariance, map, exc)
type(vamp_grid), intent(inout) :: g
real(kind=default), dimension(:,,:), intent(in) :: domain
integer, intent(in) :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole, covariance
real(kind=default), dimension(:,,:), intent(in), optional :: map
type(exception), intent(inout), optional :: exc
integer :: proc_id
call mpi90_rank (proc_id)
if (proc_id == VAMP_ROOT) then
  call vamp0_create_grid &
    (g, domain, num_calls, num_div, &
     stratified, quadrupole, covariance, map, exc)
else
  call vamp_create_empty_grid (g)
end if
end subroutine vamp_create_grid

```

169a *<Parameters in vampi 169a>*≡ (167b) 176a>
integer, public, parameter :: VAMP_ROOT = 0

169b *<Implementation of vampi procedures 168c>*+≡ (167b) <168c 169c>
subroutine vamp_discard_integral &
(g, num_calls, num_div, stratified, quadrupole, covariance, exc)
type(vamp_grid), intent(inout) :: g
integer, intent(in), optional :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole, covariance
type(exception), intent(inout), optional :: exc
integer :: proc_id
call mpi90_rank (proc_id)
if (proc_id == VAMP_ROOT) then
call vamp0_discard_integral &
(g, num_calls, num_div, stratified, quadrupole, covariance, exc)
end if
end subroutine vamp_discard_integral

169c *<Implementation of vampi procedures 168c>*+≡ (167b) <169b 169d>
subroutine vamp_reshape_grid &
(g, num_calls, num_div, stratified, quadrupole, covariance, exc)
type(vamp_grid), intent(inout) :: g
integer, intent(in), optional :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole, covariance
type(exception), intent(inout), optional :: exc
integer :: proc_id
call mpi90_rank (proc_id)
if (proc_id == VAMP_ROOT) then
call vamp0_reshape_grid &
(g, num_calls, num_div, stratified, quadrupole, covariance, exc)
end if
end subroutine vamp_reshape_grid

NB: grids has to have intent(inout) because we will call vamp_broadcast_grid on it.

169d *<Implementation of vampi procedures 168c>*+≡ (167b) <169c 172a>
subroutine vamp_sample_grid &
(rng, g, func, iterations, integral, std_dev, avg_chi2, accuracy, &
channel, weights, grids, exc, history)
type(tao_random_state), intent(inout) :: rng
type(vamp_grid), intent(inout) :: g
integer, intent(in) :: iterations
real(kind=default), intent(out), optional :: integral, std_dev, avg_chi2

```

real(kind=default), intent(in), optional :: accuracy
integer, intent(in), optional :: channel
real(kind=default), dimension(:), intent(in), optional :: weights
type(vamp_grid), dimension(:), intent(inout), optional :: grids
type(exception), intent(inout), optional :: exc
type(vamp_history), dimension(:), intent(inout), optional :: history
<Interface declaration for func 22>
character(len=*), parameter :: FN = "vamp_sample_grid"
real(kind=default) :: local_integral, local_std_dev, local_avg_chi2
type(vamp_grid), dimension(:), allocatable :: gs, gx
integer, dimension(:,:), pointer :: d
integer :: iteration, i
integer :: num_proc, proc_id, num_workers
nullify (d)
call mpi90_size (num_proc)
call mpi90_rank (proc_id)
iterate: do iteration = 1, iterations
    if (proc_id == VAMP_ROOT) then
        call vamp_distribute_work (num_proc, vamp_rigid_divisions (g), d)
        num_workers = max (1, product (d(2,:)))
    end if
    call mpi90_broadcast (num_workers, VAMP_ROOT)
    if ((present (grids)) .and. (num_workers > 1)) then
        call vamp_broadcast_grid (grids, VAMP_ROOT)
    end if
    if (proc_id == VAMP_ROOT) then
        allocate (gs(num_workers), gx(vamp_fork_grid_joints (d)))
        call vamp_create_empty_grid (gs)
        call vamp_fork_grid (g, gs, gx, d, exc)
        do i = 2, num_workers
            call vamp_send_grid (gs(i), i-1, 0)
        end do
    else if (proc_id < num_workers) then
        call vamp_receive_grid (g, VAMP_ROOT, 0)
    end if
    if (proc_id == VAMP_ROOT) then
        if (num_workers > 1) then
            call vamp_sample_grid0 &
                (rng, gs(1), func, channel, weights, grids, exc)
        else
            call vamp_sample_grid0 &
                (rng, g, func, channel, weights, grids, exc)
        end if
    end if
end do

```

```

else if (proc_id < num_workers) then
    call vamp_sample_grid0 &
        (rng, g, func, channel, weights, grids, exc)
end if
if (proc_id == VAMP_ROOT) then
    do i = 2, num_workers
        call vamp_receive_grid (gs(i), i-1, 0)
    end do
    call vamp_join_grid (g, gs, gx, d, exc)
    call vamp0_delete_grid (gs)
    deallocate (gs, gx)
    call vamp_refine_grid (g)
    call vamp_average_iterations &
        (g, iteration, local_integral, local_std_dev, local_avg_chi2)
    if (present (history)) then
        if (iteration <= size (history)) then
            call vamp_get_history &
                (history(iteration), g, &
                    local_integral, local_std_dev, local_avg_chi2)
        else
            call raise_exception (exc, EXC_WARN, FN, "history too short")
        end if
        call vamp_terminate_history (history(iteration+1:))
    end if
    if (present (accuracy)) then
        if (local_std_dev <= accuracy * local_integral) then
            call raise_exception (exc, EXC_INFO, FN, &
                "requested accuracy reached")
            exit iterate
        end if
    end if
else if (proc_id < num_workers) then
    call vamp_send_grid (g, VAMP_ROOT, 0)
end if
end do iterate
if (proc_id == VAMP_ROOT) then
    deallocate (d)
    if (present (integral)) then
        integral = local_integral
    end if
    if (present (std_dev)) then
        std_dev = local_std_dev
    end if
end if

```

```

        if (present (avg_chi2)) then
            avg_chi2 = local_avg_chi2
        end if
    end if
end subroutine vamp_sample_grid

172a <Implementation of vampi procedures 168c>+≡ (167b) <169d 172e>
    subroutine vamp_delete_grid (g)
        type(vamp_grid), intent(inout) :: g
        integer :: proc_id
        call mpi90_rank (proc_id)
        if (proc_id == VAMP_ROOT) then
            call vamp0_reshape_grid (g)
        end if
    end subroutine vamp_delete_grid

172b <Declaration of vampi procedures 168a>+≡ (167b) <168a 173c>
    public :: vamp_print_history
    private :: vamp_print_one_history, vamp_print_histories

172c <vamp0_* => vamp_* 168b>+≡ (167a) <168b 173b>
    vamp0_print_history => vamp_print_history, &

172d <Interfaces of vampi procedures 172d>≡ (167b) 182a>
    interface vamp_print_history
        module procedure vamp_print_one_history, vamp_print_histories
    end interface

172e <Implementation of vampi procedures 168c>+≡ (167b) <172a 172f>
    subroutine vamp_print_one_history (h, tag)
        type(vamp_history), dimension(:), intent(in) :: h
        character(len=*), intent(in), optional :: tag
        integer :: proc_id
        call mpi90_rank (proc_id)
        if (proc_id == VAMP_ROOT) then
            call vamp0_print_history (h, tag)
        end if
    end subroutine vamp_print_one_history

172f <Implementation of vampi procedures 168c>+≡ (167b) <172e 173e>
    subroutine vamp_print_histories (h, tag)
        type(vamp_history), dimension(:, :), intent(in) :: h
        character(len=*), intent(in), optional :: tag
        integer :: proc_id
        call mpi90_rank (proc_id)
        if (proc_id == VAMP_ROOT) then
            call vamp0_print_history (h, tag)

```

```

end if
end subroutine vamp_print_histories

```

Multi Channel

173a *<Declaration of vampi types 173a>*≡ (167b)

```

type, public :: vamp_grids
  !!! private
  type(vamp0_grids) :: g0
  logical, dimension(:), pointer :: active
  integer, dimension(:), pointer :: proc
  real(kind=default), dimension(:), pointer :: integrals, std_devs
end type vamp_grids

```

173b *<vamp0_* => vamp_* 168b>*+≡ (167a) <172c 173d>

```

vamp0_grids => vamp_grids, &

```

Partially duplicate the API of **vamp**:

173c *<Declaration of vampi procedures 168a>*+≡ (167b) <172b 178d>

```

public :: vamp_create_grids
public :: vamp_discard_integrals
public :: vamp_update_weights
public :: vamp_refine_weights
public :: vamp_delete_grids
public :: vamp_sample_grids

```

173d *<vamp0_* => vamp_* 168b>*+≡ (167a) <173b 181b>

```

vamp0_create_grids => vamp_create_grids, &
vamp0_discard_integrals => vamp_discard_integrals, &
vamp0_update_weights => vamp_update_weights, &
vamp0_refine_weights => vamp_refine_weights, &
vamp0_delete_grids => vamp_delete_grids, &
vamp0_sample_grids => vamp_sample_grids, &

```

Call **vamp_create_grids** just like the serial version. It will create the actual grids on the root processor and create stubs on the other processors

173e *<Implementation of vampi procedures 168c>*+≡ (167b) <172f 174a>

```

subroutine vamp_create_grids (g, domain, num_calls, weights, maps, &
                             num_div, stratified, quadrupole, exc)
  type(vamp_grids), intent(inout) :: g
  real(kind=default), dimension(:, :), intent(in) :: domain
  integer, intent(in) :: num_calls
  real(kind=default), dimension(:), intent(in) :: weights
  real(kind=default), dimension(:, :, :), intent(in), optional :: maps
  integer, dimension(:), intent(in), optional :: num_div

```

```

logical, intent(in), optional :: stratified, quadrupole
type(exception), intent(inout), optional :: exc
integer :: proc_id, nch
call mpi90_rank (proc_id)
nch = size (weights)
allocate (g%active(nch), g%proc(nch), g%integrals(nch), g%std_devs(nch))
if (proc_id == VAMP_ROOT) then
    call vamp0_create_grids (g%g0, domain, num_calls, weights, maps, &
                           num_div, stratified, quadrupole, exc)
else
    allocate (g%g0%grids(nch), g%g0%weights(nch), g%g0%num_calls(nch))
    call vamp_create_empty_grid (g%g0%grids)
end if
end subroutine vamp_create_grids
174a <Implementation of vampi procedures 168c>+≡ (167b) <173e 174b>
subroutine vamp_discard_integrals &
    (g, num_calls, num_div, stratified, quadrupole, exc)
type(vamp_grids), intent(inout) :: g
integer, intent(in), optional :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole
type(exception), intent(inout), optional :: exc
integer :: proc_id
call mpi90_rank (proc_id)
if (proc_id == VAMP_ROOT) then
    call vamp0_discard_integrals &
        (g%g0, num_calls, num_div, stratified, quadrupole, exc)
end if
end subroutine vamp_discard_integrals
174b <Implementation of vampi procedures 168c>+≡ (167b) <174a 175a>
subroutine vamp_update_weights &
    (g, weights, num_calls, num_div, stratified, quadrupole, exc)
type(vamp_grids), intent(inout) :: g
real(kind=default), dimension(:), intent(in) :: weights
integer, intent(in), optional :: num_calls
integer, dimension(:), intent(in), optional :: num_div
logical, intent(in), optional :: stratified, quadrupole
type(exception), intent(inout), optional :: exc
integer :: proc_id
call mpi90_rank (proc_id)
if (proc_id == VAMP_ROOT) then
    call vamp0_update_weights &
        (g%g0, weights, num_calls, num_div, stratified, quadrupole, exc)

```

```

        end if
    end subroutine vamp_update_weights

175a <Implementation of vampi procedures 168c>+≡ (167b) <174b 175b>
    subroutine vamp_refine_weights (g, power)
        type(vamp_grids), intent(inout) :: g
        real(kind=default), intent(in), optional :: power
        integer :: proc_id
        call mpi90_rank (proc_id)
        if (proc_id == VAMP_ROOT) then
            call vamp0_refine_weights (g%g0, power)
        end if
    end subroutine vamp_refine_weights

175b <Implementation of vampi procedures 168c>+≡ (167b) <175a 175c>
    subroutine vamp_delete_grids (g)
        type(vamp_grids), intent(inout) :: g
        character(len=*), parameter :: FN = "vamp_delete_grids"
        deallocate (g%active, g%proc, g%integrals, g%std_devs)
        call vamp0_delete_grids (g%g0)
    end subroutine vamp_delete_grids

    Call vamp_sample_grids just like vamp0_sample_grids.

175c <Implementation of vampi procedures 168c>+≡ (167b) <175b 179a>
    subroutine vamp_sample_grids &
        (rng, g, func, iterations, integral, std_dev, avg_chi2, &
         accuracy, history, histories, exc)
        type(tao_random_state), intent(inout) :: rng
        type(vamp_grids), intent(inout) :: g
        integer, intent(in) :: iterations
        real(kind=default), intent(out), optional :: integral, std_dev, avg_chi2
        real(kind=default), intent(in), optional :: accuracy
        type(vamp_history), dimension(:), intent(inout), optional :: history
        type(vamp_history), dimension(:, :), intent(inout), optional :: histories
        type(exception), intent(inout), optional :: exc
        <Interface declaration for func 22>
        character(len=*), parameter :: FN = "vamp_sample_grids"
        integer :: num_proc, proc_id, nch, ch, iteration
        real(kind=default), dimension(size(g%g0%weights)) :: weights
        real(kind=default) :: local_integral, local_std_dev, local_avg_chi2
        real(kind=default) :: current_accuracy, waste
        logical :: distribute_complete_grids
        call mpi90_size (num_proc)
        call mpi90_rank (proc_id)
        nch = size (g%g0%weights)

```



```

if (proc_id == VAMP_ROOT) then
  g%active = (g%g0%num_calls >= 2)
  where (g%active)
    weights = g%g0%num_calls
  elsewhere
    weights = 0.0
  endwhere
  weights = weights / sum (weights)
  call schedule (weights, num_proc, g%proc, waste)
  distribute_complete_grids = (waste <= VAMP_MAX_WASTE)
end if
call mpi90_broadcast (weights, VAMP_ROOT)
call mpi90_broadcast (g%active, VAMP_ROOT)
call mpi90_broadcast (distribute_complete_grids, VAMP_ROOT)
if (distribute_complete_grids) then
  call mpi90_broadcast (g%proc, VAMP_ROOT)
end if
iterate: do iteration = 1, iterations
  if (distribute_complete_grids) then
    call vamp_broadcast_grid (g%g0%grids, VAMP_ROOT)
    <Distribute complete grids among processes 176b>
  else
    <Distribute each grid among processes 180c>
  end if
  <Exit iterate if accuracy has been reached (MPI) 179d>
end do iterate
<Copy results of vamp_sample_grids to dummy variables 179c>
end subroutine vamp_sample_grids

```

Setting `VAMP_MAX_WASTE` to 1 disables the splitting of grids, which doesn't work yet.

```

176a <Parameters in vampi 169a>+≡ (167b) <169a 179b>
  real(kind=default), private, parameter :: VAMP_MAX_WASTE = 1.0
  ! real(kind=default), private, parameter :: VAMP_MAX_WASTE = 0.3

176b <Distribute complete grids among processes 176b>≡ (175c) 177a>
  do ch = 1, nch
    if (g%active(ch)) then
      if (proc_id == g%proc(ch)) then
        call vamp0_discard_integral (g%g0%grids(ch))
        <Sample g%g0%grids(ch) 177b>
      end if
    else
      call vamp_nullify_variance (g%g0%grids(ch))
    end if
  end do

```

```

        call vamp_nullify_covariance (g%g0%grids(ch))
    end if
end do
Refine the grids after all grids have been sampled:
177a <Distribute complete grids among processes 176b>+≡ (175c) <176b 177c>
    do ch = 1, nch
        if (g%active(ch) .and. (proc_id == g%proc(ch))) then
            call vamp_refine_grid (g%g0%grids(ch))
            if (proc_id /= VAMP_ROOT) then
                <Ship the result for channel #ch back to the root 178b>
            end if
        end if
    end do
therefore we use vamp_sample_grid0 instead of vamp0_sample_grid:
177b <Sample g%g0%grids(ch) 177b>≡ (176b)
    call vamp_sample_grid0 &
        (rng, g%g0%grids(ch), func, ch, weights, g%g0%grids, exc)
    call vamp_average_iterations &
        (g%g0%grids(ch), iteration, g%integrals(ch), g%std_devs(ch), local_avg_chi2)
    if (present (histories)) then
        if (iteration <= ubound (histories, dim=1)) then
            call vamp_get_history &
                (histories(iteration,ch), g%g0%grids(ch), &
                 g%integrals(ch), g%std_devs(ch), local_avg_chi2)
        else
            call raise_exception (exc, EXC_WARN, FN, "history too short")
        end if
        call vamp_terminate_history (histories(iteration+1:,ch))
    end if
177c <Distribute complete grids among processes 176b>+≡ (175c) <177a
    if (proc_id == VAMP_ROOT) then
        do ch = 1, nch
            if (g%active(ch) .and. (g%proc(ch) /= proc_id)) then
                <Receive the result for channel #ch at the root 178c>
            end if
        end do
        call vamp_reduce_channels (g%g0, g%integrals, g%std_devs, g%active)
        call vamp_average_iterations &
            (g%g0, iteration, local_integral, local_std_dev, local_avg_chi2)
    if (present (history)) then
        if (iteration <= size (history)) then
            call vamp_get_history &

```

```

        (history(iteration), g%g0, local_integral, local_std_dev, &
         local_avg_chi2)
    else
        call raise_exception (exc, EXC_WARN, FN, "history too short")
    end if
    call vamp_terminate_history (history(iteration+1:))
end if
end if

```

This would be cheaper than `vamp.broadcast_grid`, but we need the latter to support the adaptive multi channel sampling:

- 178a *⟨Ship g%g0%grids from the root to the assigned processor 178a⟩≡*
- ```

do ch = 1, nch
 if (g%active(ch) .and. (g%proc(ch) /= VAMP_ROOT)) then
 if (proc_id == VAMP_ROOT) then
 call vamp_send_grid &
 (g%g0%grids(ch), g%proc(ch), object (ch, TAG_GRID))
 else if (proc_id == g%proc(ch)) then
 call vamp_receive_grid &
 (g%g0%grids(ch), VAMP_ROOT, object (ch, TAG_GRID))
 end if
 end if
end if
end do

```
- 178b *⟨Ship the result for channel #ch back to the root 178b⟩≡* (177a)
- ```

call mpi90_send (g%integrals(ch), VAMP_ROOT, object (ch, TAG_INTEGRAL))
call mpi90_send (g%std_devs(ch), VAMP_ROOT, object (ch, TAG_STD_DEV))
call vamp_send_grid (g%g0%grids(ch), VAMP_ROOT, object (ch, TAG_GRID))
if (present (histories)) then
    call vamp_send_history &
        (histories(iteration,ch), VAMP_ROOT, object (ch, TAG_HISTORY))
end if

```
- 178c *⟨Receive the result for channel #ch at the root 178c⟩≡* (177c)
- ```

call mpi90_receive (g%integrals(ch), g%proc(ch), object (ch, TAG_INTEGRAL))
call mpi90_receive (g%std_devs(ch), g%proc(ch), object (ch, TAG_STD_DEV))
call vamp_receive_grid (g%g0%grids(ch), g%proc(ch), object (ch, TAG_GRID))
if (present (histories)) then
 call vamp_receive_history &
 (histories(iteration,ch), g%proc(ch), object (ch, TAG_HISTORY))
end if

```
- 178d *⟨Declaration of vampi procedures 168a⟩+≡* (167b) <173c 179e>
- ```

private :: object

```

```

179a  <Implementation of vampi procedures 168c>+≡ (167b) <175c 180a>
      pure function object (ch, obj) result (tag)
        integer, intent(in) :: ch, obj
        integer :: tag
        tag = 100 * ch + obj
      end function object

179b  <Parameters in vampi 169a>+≡ (167b) <176a
      integer, public, parameter :: &
        TAG_INTEGRAL = 1, &
        TAG_STD_DEV = 2, &
        TAG_GRID = 3, &
        TAG_HISTORY = 6, &
        TAG_NEXT_FREE = 9

179c  <Copy results of vamp_sample_grids to dummy variables 179c>≡ (175c)
      if (present (integral)) then
        call mpi90_broadcast (local_integral, VAMP_ROOT)
        integral = local_integral
      end if
      if (present (std_dev)) then
        call mpi90_broadcast (local_std_dev, VAMP_ROOT)
        std_dev = local_std_dev
      end if
      if (present (avg_chi2)) then
        call mpi90_broadcast (local_avg_chi2, VAMP_ROOT)
        avg_chi2 = local_avg_chi2
      end if

179d  <Exit iterate if accuracy has been reached (MPI) 179d>≡ (175c)
      if (present (accuracy)) then
        if (proc_id == VAMP_ROOT) then
          current_accuracy = local_std_dev / local_integral
        end if
        call mpi90_broadcast (current_accuracy, VAMP_ROOT)
        if (current_accuracy <= accuracy) then
          call raise_exception (exc, EXC_INFO, FN, &
            "requested accuracy reached")
          exit iterate
        end if
      end if

      A very simple minded scheduler: maximizes processor utilization and, does
      not pay attention to communication costs.

179e  <Declaration of vampi procedures 168a>+≡ (167b) <178d 181a>
      private :: schedule

```

We disfavor the root process a little bit (by starting up with a fake filling ratio of 10%) so that it is likely to be ready to answer all communication requests.

180a \langle Implementation of vampi procedures 168c $\rangle + \equiv$ (167b) \langle 179a 182b \rangle

```

pure subroutine schedule (jobs, num_procs, assign, waste)
  real(kind=default), dimension(:), intent(in) :: jobs
  integer, intent(in) :: num_procs
  integer, dimension(:), intent(out) :: assign
  real(kind=default), intent(out), optional :: waste
  integer, dimension(size(jobs)) :: idx
  real(kind=default), dimension(size(jobs)) :: sjobs
  real(kind=default), dimension(num_procs) :: fill
  integer :: job, proc
  sjobs = jobs / sum (jobs) * num_procs
  idx = (/ (job, job = 1, size(jobs)) /)
  call sort (sjobs, idx, reverse = .true.)
  fill = 0.0
  fill(VAMP_ROOT+1) = 0.1
  do job = 1, size (sjobs)
    proc = sum (minloc (fill))
    fill(proc) = fill(proc) + sjobs(job)
    assign(idx(job)) = proc - 1
  end do
  Estimate waste of processor time 180b
end subroutine schedule

```

Assuming equivalent processors and uniform computation costs, the waste is given by the fraction of the time that it spent by the other processors waiting for the processor with the biggest assignment:

180b \langle Estimate waste of processor time 180b $\rangle \equiv$ (180a)

```

if (present (waste)) then
  waste = 1.0 - sum (fill) / (num_procs * maxval (fill))
end if

```

Accordingly, if the waste caused by distributing only complete grids, we switch to splitting the grids, just like in single channel sampling. This is *not* the default, because the communication costs are measurably higher for many grids and many processors.

 This version is broken!

180c \langle Distribute each grid among processes 180c $\rangle \equiv$ (175c)

```

do ch = 1, size (g%g0%grids)
  if (g%active(ch)) then
    call vamp_discard_integral (g%g0%grids(ch))
  end if
end do

```

```

    if (present (histories)) then
        call vamp_sample_grid &
            (rng, g%g0%grids(ch), func, 1, g%integrals(ch), g%std_devs(ch), &
             channel = ch, weights = weights, grids = g%g0%grids, &
             history = histories(iteration:iteration,ch))
    else
        call vamp_sample_grid &
            (rng, g%g0%grids(ch), func, 1, g%integrals(ch), g%std_devs(ch), &
             channel = ch, weights = weights, grids = g%g0%grids)
    end if
else
    if (proc_id == VAMP_ROOT) then
        call vamp_nullify_variance (g%g0%grids(ch))
        call vamp_nullify_covariance (g%g0%grids(ch))
    end if
end if
end do
if (proc_id == VAMP_ROOT) then
    call vamp_reduce_channels (g%g0, g%integrals, g%std_devs, g%active)
    call vamp_average_iterations &
        (g%g0, iteration, local_integral, local_std_dev, local_avg_chi2)
    if (present (history)) then
        if (iteration <= size (history)) then
            call vamp_get_history &
                (history(iteration), g%g0, local_integral, local_std_dev, &
                 local_avg_chi2)
        else
            call raise_exception (exc, EXC_WARN, FN, "history too short")
        end if
        call vamp_terminate_history (history(iteration+1:))
    end if
end if
end if

```

5.3.2 Event Generation

This is currently only a syntactical translation ...

181a	\langle Declaration of vampi procedures 168a $\rangle + \equiv$ public :: vamp_warmup_grid public :: vamp_warmup_grids public :: vamp_next_event private :: vamp_next_event_single, vamp_next_event_multi	(167b) \triangleleft 179e 183c \triangleright
181b	\langle vamp0_* \Rightarrow vamp_* 168b $\rangle + \equiv$	(167a) \triangleleft 173d 183d \triangleright

```
vamp0_warmup_grid => vamp_warmup_grid, &
vamp0_warmup_grids => vamp_warmup_grids, &
vamp0_next_event => vamp_next_event, &
```

182a *<Interfaces of vampi procedures 172d>+≡ (167b) <172d 184a>*

```
interface vamp_next_event
  module procedure vamp_next_event_single, vamp_next_event_multi
end interface
```

182b *<Implementation of vampi procedures 168c>+≡ (167b) <180a 182c>*

```
subroutine vamp_next_event_single &
  (x, rng, g, func, weight, channel, weights, grids, exc)
  real(kind=default), dimension(:), intent(out) :: x
  type(tao_random_state), intent(inout) :: rng
  type(vamp_grid), intent(inout) :: g
  real(kind=default), intent(out), optional :: weight
  integer, intent(in), optional :: channel
  real(kind=default), dimension(:), intent(in), optional :: weights
  type(vamp_grid), dimension(:), intent(in), optional :: grids
  type(exception), intent(inout), optional :: exc
  <Interface declaration for func 22>
  integer :: proc_id
  call mpi90_rank (proc_id)
  if (proc_id == VAMP_ROOT) then
    call vamp0_next_event &
      (x, rng, g, func, weight, channel, weights, grids, exc)
  end if
end subroutine vamp_next_event_single
```

182c *<Implementation of vampi procedures 168c>+≡ (167b) <182b 183a>*

```
subroutine vamp_next_event_multi (x, rng, g, func, phi, weight, exc)
  real(kind=default), dimension(:), intent(out) :: x
  type(tao_random_state), intent(inout) :: rng
  type(vamp_grids), intent(inout) :: g
  real(kind=default), intent(out), optional :: weight
  type(exception), intent(inout), optional :: exc
  <Interface declaration for func 22>
  <Interface declaration for phi 31a>
  integer :: proc_id
  call mpi90_rank (proc_id)
  if (proc_id == VAMP_ROOT) then
    call vamp0_next_event (x, rng, g%g0, func, phi, weight, exc)
  end if
end subroutine vamp_next_event_multi
```

183a *<Implementation of vampi procedures 168c>+≡* (167b) <182c 183b>
`subroutine vamp_warmup_grid (rng, g, func, iterations, exc, history)`
`type(tao_random_state), intent(inout) :: rng`
`type(vamp_grid), intent(inout) :: g`
`integer, intent(in) :: iterations`
`type(exception), intent(inout), optional :: exc`
`type(vamp_history), dimension(:), intent(inout), optional :: history`
`<Interface declaration for func 22>`
`call vamp_sample_grid &`
`(rng, g, func, iterations - 1, exc = exc, history = history)`
`call vamp_sample_grid0 (rng, g, func, exc = exc)`
`end subroutine vamp_warmup_grid`

183b *<Implementation of vampi procedures 168c>+≡* (167b) <183a 184b>
`subroutine vamp_warmup_grids &`
`(rng, g, func, iterations, history, histories, exc)`
`type(tao_random_state), intent(inout) :: rng`
`type(vamp_grids), intent(inout) :: g`
`integer, intent(in) :: iterations`
`type(vamp_history), dimension(:), intent(inout), optional :: history`
`type(vamp_history), dimension(:, :), intent(inout), optional :: histories`
`type(exception), intent(inout), optional :: exc`
`<Interface declaration for func 22>`
`integer :: ch`
`call vamp0_sample_grids (rng, g%g0, func, iterations - 1, exc = exc, &`
`history = history, histories = histories)`
`do ch = 1, size (g%g0%grids)`
`! if (g%g0%grids(ch)%num_calls >= 2) then`
`call vamp_sample_grid0 (rng, g%g0%grids(ch), func, exc = exc)`
`! end if`
`end do`
`end subroutine vamp_warmup_grids`

5.3.3 I/O

183c *<Declaration of vampi procedures 168a>+≡* (167b) <181a 185a>
`public :: vamp_write_grid, vamp_read_grid`
`private :: write_grid_unit, write_grid_name`
`private :: read_grid_unit, read_grid_name`

183d *<vamp0.* => vamp.* 168b>+≡* (167a) <181b 185b>
`vamp0_write_grid => vamp_write_grid, &`
`vamp0_read_grid => vamp_read_grid, &`

184a *<Interfaces of vampi procedures 172d>+≡* (167b) <182a 185c>
 interface **vamp_write_grid**
 module procedure **write_grid_unit**, **write_grid_name**
end interface
 interface **vamp_read_grid**
 module procedure **read_grid_unit**, **read_grid_name**
end interface

184b *<Implementation of vampi procedures 168c>+≡* (167b) <183b 184c>
 subroutine **write_grid_unit** (g, unit)
 type(**vamp_grid**), intent(in) :: g
 integer, intent(in) :: unit
 integer :: proc_id
 call **mpi90_rank** (proc_id)
 if (proc_id == **VAMP_ROOT**) then
 call **vamp0_write_grid** (g, unit)
 end if
end subroutine **write_grid_unit**

184c *<Implementation of vampi procedures 168c>+≡* (167b) <184b 184d>
 subroutine **read_grid_unit** (g, unit)
 type(**vamp_grid**), intent(inout) :: g
 integer, intent(in) :: unit
 integer :: proc_id
 call **mpi90_rank** (proc_id)
 if (proc_id == **VAMP_ROOT**) then
 call **vamp0_read_grid** (g, unit)
 end if
end subroutine **read_grid_unit**

184d *<Implementation of vampi procedures 168c>+≡* (167b) <184c 184e>
 subroutine **write_grid_name** (g, name)
 type(**vamp_grid**), intent(inout) :: g
 character(len=*), intent(in) :: name
 integer :: proc_id
 call **mpi90_rank** (proc_id)
 if (proc_id == **VAMP_ROOT**) then
 call **vamp0_write_grid** (g, name)
 end if
end subroutine **write_grid_name**

184e *<Implementation of vampi procedures 168c>+≡* (167b) <184d 185d>
 subroutine **read_grid_name** (g, name)
 type(**vamp_grid**), intent(inout) :: g
 character(len=*), intent(in) :: name
 integer :: proc_id

```

        call mpi90_rank (proc_id)
        if (proc_id == VAMP_ROOT) then
            call vamp0_read_grid (g, name)
        end if
    end subroutine read_grid_name
185a  <Declaration of vampi procedures 168a>+≡ (167b) <183c 186c>
        public :: vamp_write_grids, vamp_read_grids
        private :: write_grids_unit, write_grids_name
        private :: read_grids_unit, read_grids_name
185b  <vamp0.* => vamp.* 168b>+≡ (167a) <183d
        vamp0_write_grids => vamp_write_grids, &
        vamp0_read_grids => vamp_read_grids, &
185c  <Interfaces of vampi procedures 172d>+≡ (167b) <184a 188b>
        interface vamp_write_grids
            module procedure write_grids_unit, write_grids_name
        end interface
        interface vamp_read_grids
            module procedure read_grids_unit, read_grids_name
        end interface

185d  <Implementation of vampi procedures 168c>+≡ (167b) <184e 185e>
        subroutine write_grids_unit (g, unit)
            type(vamp_grids), intent(in) :: g
            integer, intent(in) :: unit
            integer :: proc_id
            call mpi90_rank (proc_id)
            if (proc_id == VAMP_ROOT) then
                call vamp0_write_grids (g%g0, unit)
            end if
        end subroutine write_grids_unit

185e  <Implementation of vampi procedures 168c>+≡ (167b) <185d 186a>
        subroutine read_grids_unit (g, unit)
            type(vamp_grids), intent(inout) :: g
            integer, intent(in) :: unit
            integer :: proc_id
            call mpi90_rank (proc_id)
            if (proc_id == VAMP_ROOT) then
                call vamp0_read_grids (g%g0, unit)
            end if
        end subroutine read_grids_unit

```

186a *<Implementation of vampi procedures 168c>+≡* (167b) <185e 186b>

```

subroutine write_grids_name (g, name)
  type(vamp_grids), intent(inout) :: g
  character(len=*), intent(in) :: name
  integer :: proc_id
  call mpi90_rank (proc_id)
  if (proc_id == VAMP_ROOT) then
    call vamp0_write_grids (g%g0, name)
  end if
end subroutine write_grids_name

```

186b *<Implementation of vampi procedures 168c>+≡* (167b) <186a 186d>

```

subroutine read_grids_name (g, name)
  type(vamp_grids), intent(inout) :: g
  character(len=*), intent(in) :: name
  integer :: proc_id
  call mpi90_rank (proc_id)
  if (proc_id == VAMP_ROOT) then
    call vamp0_read_grids (g%g0, name)
  end if
end subroutine read_grids_name

```


5.3.4 Communicating Grids

186c *<Declaration of vampi procedures 168a>+≡* (167b) <185a 190a>

```

public :: vamp_send_grid
public :: vamp_receive_grid
public :: vamp_broadcast_grid
public :: vamp_broadcast_grids

```

 The next two are still kludged. Nicer implementations with one message less per call below, but MPICH does funny things during `mpi_get_count`, which is called by `mpi90_receive_pointer`.

Caveat: this `vamp_send_grid` uses *three* tags: `tag`, `tag+1` and `tag+2`:

186d *<Implementation of vampi procedures 168c>+≡* (167b) <186b 187a>

```

subroutine vamp_send_grid (g, target, tag, domain, error)
  type(vamp_grid), intent(in) :: g
  integer, intent(in) :: target, tag
  integer, intent(in), optional :: domain
  integer, intent(out), optional :: error
  integer, dimension(2) :: words

```

```

integer, dimension(:), allocatable :: ibuf
real(kind=default), dimension(:), allocatable :: dbuf
call vamp_marshal_grid_size (g, words(1), words(2))
allocate (ibuf(words(1)), dbuf(words(2)))
call vamp_marshal_grid (g, ibuf, dbuf)
call mpi90_send (words, target, tag, domain, error)
call mpi90_send (ibuf, target, tag+1, domain, error)
call mpi90_send (dbuf, target, tag+2, domain, error)
deallocate (ibuf, dbuf)
end subroutine vamp_send_grid

```

187a *<Implementation of vampi procedures 168c>+≡ (167b) <186d 188c>*

```

subroutine vamp_receive_grid (g, source, tag, domain, status, error)
  type(vamp_grid), intent(inout) :: g
  integer, intent(in) :: source, tag
  integer, intent(in), optional :: domain
  type(mpi90_status), intent(out), optional :: status
  integer, intent(out), optional :: error
  integer, dimension(2) :: words
  integer, dimension(:), allocatable :: ibuf
  real(kind=default), dimension(:), allocatable :: dbuf
  call mpi90_receive (words, source, tag, domain, status, error)
  allocate (ibuf(words(1)), dbuf(words(2)))
  call mpi90_receive (ibuf, source, tag+1, domain, status, error)
  call mpi90_receive (dbuf, source, tag+2, domain, status, error)
  call vamp_unmarshal_grid (g, ibuf, dbuf)
  deallocate (ibuf, dbuf)
end subroutine vamp_receive_grid

```

Caveat: the real `vamp_send_grid` uses *two* tags: `tag` and `tag+1`:

187b *<Implementation of vampi procedures (doesn't work with MPICH yet) 187b>≡ 188a>*

```

subroutine vamp_send_grid (g, target, tag, domain, error)
  type(vamp_grid), intent(in) :: g
  integer, intent(in) :: target, tag
  integer, intent(in), optional :: domain
  integer, intent(out), optional :: error
  integer :: iwwords, dwwords
  integer, dimension(:), allocatable :: ibuf
  real(kind=default), dimension(:), allocatable :: dbuf
  call vamp_marshal_grid_size (g, iwwords, dwwords)
  allocate (ibuf(iwwords), dbuf(dwwords))
  call vamp_marshal_grid (g, ibuf, dbuf)

```

```

    call mpi90_send (ibuf, target, tag, domain, error)
    call mpi90_send (dbuf, target, tag+1, domain, error)
    deallocate (ibuf, dbuf)
end subroutine vamp_send_grid

```

⚠ There's something wrong with MPICH: if I call `mpi90_receive_pointer` in the opposite order, the low level call to `mpi_get_count` bombs for no apparent reason!

⚠ There are also funky things going on with tag: `mpi90_receive_pointer` should leave it alone, but ...

188a *<Implementation of vampi procedures (doesn't work with MPICH yet) 187b>+≡ <187b*

```

subroutine vamp_receive_grid (g, source, tag, domain, status, error)
  type(vamp_grid), intent(inout) :: g
  integer, intent(in) :: source, tag
  integer, intent(in), optional :: domain
  type(mpi90_status), intent(out), optional :: status
  integer, intent(out), optional :: error
  integer, dimension(:), pointer :: ibuf
  real(kind=default), dimension(:), pointer :: dbuf
  nullify (ibuf, dbuf)
  call mpi90_receive_pointer (dbuf, source, tag+1, domain, status, error)
  call mpi90_receive_pointer (ibuf, source, tag, domain, status, error)
  call vamp_unmarshal_grid (g, ibuf, dbuf)
  deallocate (ibuf, dbuf)
end subroutine vamp_receive_grid

```

This is not a good idea, with respect to communication costs. For SMP machines, it appears to be negligible however.

188b *<Interfaces of vampi procedures 172d>+≡ (167b) <185c*

```

interface vamp_broadcast_grid
  module procedure &
    vamp_broadcast_one_grid, vamp_broadcast_many_grids
end interface

```

188c *<Implementation of vampi procedures 168c>+≡ (167b) <187a 189a>*

```

subroutine vamp_broadcast_one_grid (g, root, domain, error)
  type(vamp_grid), intent(inout) :: g
  integer, intent(in) :: root
  integer, intent(in), optional :: domain
  integer, intent(out), optional :: error
  integer, dimension(:), allocatable :: ibuf

```

```

real(kind=default), dimension(:), allocatable :: dbuf
integer :: iwords, dwords, me
call mpi90_rank (me)
if (me == root) then
    call vamp_marshall_grid_size (g, iwords, dwords)
end if
call mpi90_broadcast (iwords, root, domain, error)
call mpi90_broadcast (dwords, root, domain, error)
allocate (ibuf(iwords), dbuf(dwords))
if (me == root) then
    call vamp_marshall_grid (g, ibuf, dbuf)
end if
call mpi90_broadcast (ibuf, root, domain, error)
call mpi90_broadcast (dbuf, root, domain, error)
if (me /= root) then
    call vamp_unmarshal_grid (g, ibuf, dbuf)
end if
deallocate (ibuf, dbuf)
end subroutine vamp_broadcast_one_grid

```

189a *<Implementation of vampi procedures 168c>+≡ (167b) <188c 189b>*

```

subroutine vamp_broadcast_many_grids (g, root, domain, error)
    type(vamp_grid), dimension(:), intent(inout) :: g
    integer, intent(in) :: root
    integer, intent(in), optional :: domain
    integer, intent(out), optional :: error
    integer :: i
    do i = 1, size(g)
        call vamp_broadcast_one_grid (g(i), root, domain, error)
    end do
end subroutine vamp_broadcast_many_grids

```

189b *<Implementation of vampi procedures 168c>+≡ (167b) <189a 190b>*

```

subroutine vamp_broadcast_grids (g, root, domain, error)
    type(vamp0_grids), intent(inout) :: g
    integer, intent(in) :: root
    integer, intent(in), optional :: domain
    integer, intent(out), optional :: error
    integer :: nch, me
    call mpi90_broadcast (g%sum_chi2, root, domain, error)
    call mpi90_broadcast (g%sum_integral, root, domain, error)
    call mpi90_broadcast (g%sum_weights, root, domain, error)
    call mpi90_rank (me)

```

```

if (me == root) then
    nch = size (g%grids)
end if
call mpi90_broadcast (nch, root, domain, error)
if (me /= root) then
    if (associated (g%grids)) then
        if (size (g%grids) /= nch) then
            call vamp0_delete_grid (g%grids)
            deallocate (g%grids, g%weights, g%num_calls)
            allocate (g%grids(nch), g%weights(nch), g%num_calls(nch))
            call vamp_create_empty_grid (g%grids)
        end if
    else
        allocate (g%grids(nch), g%weights(nch), g%num_calls(nch))
        call vamp_create_empty_grid (g%grids)
    end if
end if
call vamp_broadcast_grid (g%grids, root, domain, error)
call mpi90_broadcast (g%weights, root, domain, error)
call mpi90_broadcast (g%num_calls, root, domain, error)
end subroutine vamp_broadcast_grids

```

190a \langle Declaration of vampi procedures 168a $\rangle + \equiv$ (167b) \triangleleft 186c

```

public :: vamp_send_history
public :: vamp_receive_history

```

190b \langle Implementation of vampi procedures 168c $\rangle + \equiv$ (167b) \triangleleft 189b 191 \triangleright

```

subroutine vamp_send_history (g, target, tag, domain, error)
    type(vamp_history), intent(in) :: g
    integer, intent(in) :: target, tag
    integer, intent(in), optional :: domain
    integer, intent(out), optional :: error
    integer, dimension(2) :: words
    integer, dimension(:), allocatable :: ibuf
    real(kind=default), dimension(:), allocatable :: dbuf
    call vamp_marshall_history_size (g, words(1), words(2))
    allocate (ibuf(words(1)), dbuf(words(2)))
    call vamp_marshall_history (g, ibuf, dbuf)
    call mpi90_send (words, target, tag, domain, error)
    call mpi90_send (ibuf, target, tag+1, domain, error)
    call mpi90_send (dbuf, target, tag+2, domain, error)
    deallocate (ibuf, dbuf)
end subroutine vamp_send_history

```

191 *⟨Implementation of vampi procedures 168c⟩+≡* (167b) <190b

```

subroutine vamp_receive_history (g, source, tag, domain, status, error)
  type(vamp_history), intent(inout) :: g
  integer, intent(in) :: source, tag
  integer, intent(in), optional :: domain
  type(mpi90_status), intent(out), optional :: status
  integer, intent(out), optional :: error
  integer, dimension(2) :: words
  integer, dimension(:), allocatable :: ibuf
  real(kind=default), dimension(:), allocatable :: dbuf
  call mpi90_receive (words, source, tag, domain, status, error)
  allocate (ibuf(words(1)), dbuf(words(2)))
  call mpi90_receive (ibuf, source, tag+1, domain, status, error)
  call mpi90_receive (dbuf, source, tag+2, domain, status, error)
  call vamp_unmarshal_history (g, ibuf, dbuf)
  deallocate (ibuf, dbuf)
end subroutine vamp_receive_history

```


—6—

SELF TEST

6.1 No Mapping Mode

In this chapter we perform a test of the major features of Vamp. A function with many peaks is integrated with the traditional Vegas algorithm, using a multi-channel approach and in parallel. The function is constructed to have a known analytical integral (which is chosen to be one) in order to be able to gauge the accuracy of the result and error estimate.

6.1.1 Serial Test

```

192a <vamp_test.f90 192a>≡
! vamp_test.f90 --
<Copyleft notice 1>
<Module vamp_test_functions 192b>
<Module vamp_tests 196b>
200c>

192b <Module vamp_test_functions 192b>≡
module vamp_test_functions
  use kinds
  use constants, only: PI
  use coordinates
  use vamp, only: vamp_grid, vamp_multi_channel
  use vamp, only: vamp_data_t
  implicit none
  private
  public :: f, j, phi, ihp, w
  public :: lorentzian
  private :: lorentzian_normalized
  real(kind=default), public :: width
contains
  <Implementation of vamp_test_functions procedures 193a>

```

$$\begin{aligned} &\text{end module vamp_test_functions} \\ &\int_{x_1}^{x_2} dx \frac{1}{(x-x_0)^2 + a^2} = \frac{1}{a} \left(\text{atan} \left(\frac{x_2-x_0}{a} \right) - \text{atan} \left(\frac{x_1-x_0}{a} \right) \right) = N(x_0, x_1, x_2, a) \end{aligned} \quad (6.1)$$

193a \langle Implementation of vamp_test_functions procedures 193a $\rangle \equiv$ (192b) 193b \rangle

```
pure function lorentzian_normalized (x, x0, x1, x2, a) result (f)
  real(kind=default), intent(in) :: x, x0, x1, x2, a
  real(kind=default) :: f
  if (x1 <= x .and. x <= x2) then
    f = 1 / ((x - x0)**2 + a**2) &
      * a / (atan2 (x2 - x0, a) - atan2 (x1 - x0, a))
  else
    f = 0
  end if
end function lorentzian_normalized
```

$$\int d^n x f(x) = \int d\Omega_n r^{n-1} dr f(x) = 1 \quad (6.2)$$

193b \langle Implementation of vamp_test_functions procedures 193a $\rangle + \equiv$ (192b) \langle 193a 193c \rangle

```
pure function lorentzian (x, x0, x1, x2, r0, a) result (f)
  real(kind=default), dimension(:), intent(in) :: x, x0, x1, x2
  real(kind=default), intent(in) :: r0, a
  real(kind=default) :: f
  real(kind=default) :: r, r1, r2
  integer :: n
  n = size (x)
  if (n > 1) then
    r = sqrt (dot_product (x-x0, x-x0))
    r1 = 0.4_default
    r2 = min (minval (x2-x0), minval (x0-x1))
    if (r1 <= r .and. r <= r2) then
      f = lorentzian_normalized (r, r0, r1, r2, a) * r**(1-n) / surface (n)
    else
      f = 0
    end if
  else
    f = lorentzian_normalized (x(1), x0(1), x1(1), x2(1), a)
  endif
end function lorentzian
```

193c \langle Implementation of vamp_test_functions procedures 193a $\rangle + \equiv$ (192b) \langle 193b 194 \rangle

```
pure function f (x, data, weights, channel, grids) result (f_x)
  real(kind=default), dimension(:), intent(in) :: x
  class(vamp_data_t), intent(in) :: data
```

```

real(kind=default), dimension(:), intent(in), optional :: weights
integer, intent(in), optional :: channel
type(vamp_grid), dimension(:), intent(in), optional :: grids
real(kind=default) :: f_x
real(kind=default), dimension(size(x)) :: minus_one, plus_one, zero, w_i, f_i
integer :: n, i
n = size(x)
minus_one = -1
zero = 0
plus_one = 1
w_i = 1
do i = 1, n
  if (all (abs (x(i+1:)) <= 1)) then
    f_i = lorentzian (x(1:i), zero(1:i), minus_one(1:i), plus_one(1:i), &
      0.7_default, width) &
      / 2.0_default**(n-i)
  else
    f_i = 0
  end if
end do
f_x = dot_product (w_i, f_i) / sum (w_i)
end function f

```

194 \langle Implementation of vamp_test functions procedures 193a $\rangle + \equiv$ (192b) \triangleleft 193c 195a \triangleright

```

pure function phi (xi, channel) result (x)
  real(kind=default), dimension(:), intent(in) :: xi
  integer, intent(in) :: channel
  real(kind=default), dimension(size(xi)) :: x
  real(kind=default) :: r
  real(kind=default), dimension(0) :: dummy
  integer :: n
  n = size(x)
  if (channel == 1) then
    x = xi
  else if (channel == 2) then
    r = (xi(1) + 1) / 2 * sqrt (2.0_default)
    x(1:2) = spherical_cos_to_cartesian (r, PI * xi(2), dummy)
    x(3:) = xi(3:)
  else if (channel < n) then
    r = (xi(1) + 1) / 2 * sqrt (real (channel, kind=default))
    x(1:channel) = spherical_cos_to_cartesian (r, PI * xi(2), xi(3:channel))
    x(channel+1:) = xi(channel+1:)
  else if (channel == n) then
    r = (xi(1) + 1) / 2 * sqrt (real (channel, kind=default))

```

```

        x = spherical_cos_to_cartesian (r, PI * xi(2), xi(3:))
    else
        x = 0
    end if
end function phi
195a <Implementation of vamp_test_functions procedures 193a>+≡ (192b) <194 195b>
pure function ihp (x, channel) result (xi)
    real(kind=default), dimension(:), intent(in) :: x
    integer, intent(in) :: channel
    real(kind=default), dimension(size(x)) :: xi
    real(kind=default) :: r, phi
    integer :: n
    n = size(x)
    if (channel == 1) then
        xi = x
    else if (channel == 2) then
        call cartesian_to_spherical_cos (x(1:2), r, phi)
        xi(1) = 2 * r / sqrt (2.0_default) - 1
        xi(2) = phi / PI
        xi(3:) = x(3:)
    else if (channel < n) then
        call cartesian_to_spherical_cos (x(1:channel), r, phi, xi(3:channel))
        xi(1) = 2 * r / sqrt (real (channel, kind=default)) - 1
        xi(2) = phi / PI
        xi(channel+1:) = x(channel+1:)
    else if (channel == n) then
        call cartesian_to_spherical_cos (x, r, phi, xi(3:))
        xi(1) = 2 * r / sqrt (real (channel, kind=default)) - 1
        xi(2) = phi / PI
    else
        xi = 0
    end if
end function ihp
195b <Implementation of vamp_test_functions procedures 193a>+≡ (192b) <195a 196a>
pure function j (x, data, channel) result (j_x)
    real(kind=default), dimension(:), intent(in) :: x
    class(vamp_data_t), intent(in) :: data
    integer, intent(in) :: channel
    real(kind=default) :: j_x
    if (channel == 1) then
        j_x = 1
    else if (channel > 1) then
        j_x = 2 / sqrt (real (channel, kind=default)) ! 1/|dr/dξ1|
    end if
end function j

```

```

        j_x = j_x / PI ! 1/|dφ/dξ2|
        j_x = j_x * cartesian_to_spherical_cos_j (x(1:channel))
    else
        j_x = 0
    end if
end function j

```

196a \langle Implementation of `vamp_test_functions` procedures 193a $\rangle \equiv$ (192b) \triangleleft 195b

```

function w (x, data, weights, channel, grids) result (w_x)
    real(kind=default), dimension(:), intent(in) :: x
    class(vamp_data_t), intent(in) :: data
    real(kind=default), dimension(:), intent(in), optional :: weights
    integer, intent(in), optional :: channel
    type(vamp_grid), dimension(:), intent(in), optional :: grids
    real(kind=default) :: w_x
    w_x = vamp_multi_channel (f, data, phi, ihp, j, x, weights, channel, grids)
end function w

```

196b \langle Module `vamp_tests` 196b $\rangle \equiv$ (192a)

```

module vamp_tests
    use kinds
    use exceptions
    use histograms
    use tao_random_numbers
    use coordinates
    use vamp
    use vamp_test_functions !NODEP!
    implicit none
    private
     $\langle$ Declaration of procedures in vamp_tests 196c $\rangle$ 
contains
     $\langle$ Implementation of procedures in vamp_tests 197a $\rangle$ 
end module vamp_tests

```

Verification

196c \langle Declaration of procedures in `vamp_tests` 196c $\rangle \equiv$ (196b 202b) 198a \triangleright

```

! public :: check_jacobians, check_inverses, check_inverses3
public :: check_inverses, check_inverses3

```

196d \langle Implementation of procedures in `vamp_tests` (broken?) 196d $\rangle \equiv$

```

subroutine check_jacobians (rng, region, weights, samples)
    type( tao_random_state ), intent(inout) :: rng
    real(kind=default), dimension(:, :), intent(in) :: region
    real(kind=default), dimension(:), intent(in) :: weights

```

```

integer, intent(in) :: samples
real(kind=default), dimension(size(region,dim=2)) :: x
real(kind=default) :: d
integer :: ch
do ch = 1, size(weights)
    call vamp_check_jacobian (rng, samples, j, NO_DATA, phi, ch, region, d, x)
    print *, "channel", ch, ": delta(j)/j=", real(d), ", @x=", real (x)
end do
end subroutine check_jacobians

```

197a \langle Implementation of procedures in vamp_tests 197a $\rangle \equiv$ (196b 202b) 197b \triangleright

```

subroutine check_inverses (rng, region, weights, samples)
    type(tao_random_state), intent(inout) :: rng
    real(kind=default), dimension(:,,:), intent(in) :: region
    real(kind=default), dimension(:), intent(in) :: weights
    integer, intent(in) :: samples
    real(kind=default), dimension(size(region,dim=2)) :: x1, x2, x_dx
    real(kind=default) :: dx, dx_max
    integer :: ch, i
    dx_max = 0
    x_dx = 0
    do ch = 1, size(weights)
        do i = 1, samples
            call tao_random_number (rng, x1)
            x2 = ihp (phi (x1, ch), ch)
            dx = sqrt (dot_product (x1-x2, x1-x2))
            if (dx > dx_max) then
                dx_max = dx
                x_dx = x1
            end if
        end do
        print *, "channel", ch, ": |x-x|=", real(dx), ", @x=", real (x_dx)
    end do
end subroutine check_inverses

```

197b \langle Implementation of procedures in vamp_tests 197a $\rangle + \equiv$ (196b 202b) \triangleleft 197a 198b \triangleright

```

subroutine check_inverses3 (rng, region, samples)
    type(tao_random_state), intent(inout) :: rng
    real(kind=default), dimension(:,,:), intent(in) :: region
    integer, intent(in) :: samples
    real(kind=default), dimension(size(region,dim=2)) :: x1, x2, x_dx, x_dj
    real(kind=default) :: r, phi, jac, caj, dx, dx_max, dj, dj_max
    real(kind=default), dimension(size(x1)-2) :: cos_theta
    integer :: i
    dx_max = 0

```

```

x_dx = 0
dj_max = 0
x_dj = 0
do i = 1, samples
  call tao_random_number (rng, x1)
  call cartesian_to_spherical_cos_2 (x1, r, phi, cos_theta, jac)
  call spherical_cos_to_cartesian_2 (r, phi, cos_theta, x2, caj)
  dx = sqrt (dot_product (x1-x2, x1-x2))
  dj = jac*caj - 1
  if (dx > dx_max) then
    dx_max = dx
    x_dx = x1
  end if
  if (dj > dj_max) then
    dj_max = dj
    x_dj = x1
  end if
end do
print *, "channel 3 : j*j-1=", real(dj), ", @x=", real (x_dj)
print *, "channel 3 : |x-x|=", real(dx), ", @x=", real (x_dx)
end subroutine check_inverses3

```

Integration

- 198a \langle Declaration of procedures in `vamp_tests` 196c $\rangle + \equiv$ (196b 202b) \triangleleft 196c 200a \triangleright
 public :: `single_channel`, `multi_channel`
- 198b \langle Implementation of procedures in `vamp_tests` 197a $\rangle + \equiv$ (196b 202b) \triangleleft 197b 199a \triangleright
 subroutine `single_channel` (rng, region, samples, iterations, &
 integral, standard_dev, chi_squared)
 type(`tao_random_state`), intent(inout) :: rng
 real(kind=default), dimension(:, :), intent(in) :: region
 integer, dimension(:), intent(in) :: samples, iterations
 real(kind=default), intent(out) :: integral, standard_dev, chi_squared
 type(`vamp_grid`) :: gr
 type(`vamp_history`), dimension(iterations(1)+iterations(2)) :: history
 call `vamp_create_history` (history)
 call `vamp_create_grid` (gr, region, samples(1))
 call `vamp_sample_grid` (rng, gr, f, NO_DATA, iterations(1), history = history)
 call `vamp_discard_integral` (gr, samples(2))
 call `vamp_sample_grid` &
 (rng, gr, f, NO_DATA, iterations(2), &
 integral, standard_dev, chi_squared, &
 history = history(iterations(1)+1:))

```

    call vamp_write_grid (gr, "vamp_test.grid")
    call vamp_delete_grid (gr)
    call vamp_print_history (history, "single")
    call vamp_delete_history (history)
end subroutine single_channel
199a <Implementation of procedures in vamp_tests 197a>+≡ (196b 202b) <198b 200b>
subroutine multi_channel (rng, region, weights, samples, iterations, powers, &
    integral, standard_dev, chi_squared)
type(tao_random_state), intent(inout) :: rng
real(kind=default), dimension(:,:), intent(in) :: region
real(kind=default), dimension(:), intent(inout) :: weights
integer, dimension(:), intent(in) :: samples, iterations
real(kind=default), dimension(:), intent(in) :: powers
real(kind=default), intent(out) :: integral, standard_dev, chi_squared
type(vamp_grids) :: grs
<Body of multi_channel 199b>
end subroutine multi_channel
199b <Body of multi_channel 199b>≡ (199a 212b) 213>
type(vamp_history), dimension(iterations(1)+iterations(2)+size(powers)-1) :: &
    history
type(vamp_history), dimension(size(history),size(weights)) :: histories
integer :: it, nit
nit = size (powers)
call vamp_create_history (history)
call vamp_create_history (histories)
call vamp_create_grids (grs, region, samples(1), weights)
call vamp_sample_grids (rng, grs, w, NO_DATA, iterations(1) - 1, &
    history = history, histories = histories)
call vamp_print_history (history, "multi")
call vamp_print_history (histories, "multi")
do it = 1, nit
    call vamp_sample_grids (rng, grs, w, NO_DATA, 1, &
        history = history(iterations(1)+it-1:), &
        histories = histories(iterations(1)+it-1:,:))
    call vamp_print_history (history(iterations(1)+it-1:), "multi")
    call vamp_print_history (histories(iterations(1)+it-1:,:), "multi")
    call vamp_refine_weights (grs, powers(it))
end do
call vamp_discard_integrals (grs, samples(2))
call vamp_sample_grids &
    (rng, grs, w, NO_DATA, iterations(2), &
    integral, standard_dev, chi_squared, &
    history = history(iterations(1)+nit:), &

```



```

        histories = histories(iterations(1)+nit:, :)
call vamp_print_history (history(iterations(1)+nit:), "multi")
call vamp_print_history (histories(iterations(1)+nit:, :), "multi")
call vamp_write_grids (grs, "vamp_test.grids")
call vamp_delete_grids (grs)
call vamp_print_history (history, "multi")
call vamp_print_history (histories, "multi")
call vamp_delete_history (history)
call vamp_delete_history (histories)

```

Input/Output

- 200a *<Declaration of procedures in vamp_tests 196c>+≡ (196b 202b) <198a*
public :: print_results
- 200b *<Implementation of procedures in vamp_tests 197a>+≡ (196b 202b) <199a*
subroutine print_results (prefix, prev_ticks, &
integral, std_dev, chi2, acceptable, failures)
character(len=*), intent(in) :: prefix
integer, intent(in) :: prev_ticks
real(kind=default), intent(in) :: integral, std_dev, chi2, acceptable
integer, intent(inout) :: failures
integer :: ticks, ticks_per_second
real(kind=default) :: pull
call system_clock (ticks, ticks_per_second)
pull = (integral - 1) / std_dev
print "(1X,A,A,F6.2,A)", prefix, &
": time = ", real (ticks - prev_ticks) / ticks_per_second, " secs"
print *, prefix, ": int, err, chi2: ", &
real (integral), real (std_dev), real (chi2)
if (abs (pull) > acceptable) then
failures = failures + 1
print *, prefix, ": unacceptable pull:", real (pull)
else
print *, prefix, ": acceptable pull:", real (pull)
end if
end subroutine print_results

Main Program

- 200c *<vamp_test.f90 192a>+≡ <192a*
program vamp_test
use kinds
use tao_random_numbers

```

use coordinates
use vamp
use vamp_test_functions !NODEP!
use vamp_tests !NODEP!
implicit none
integer :: start_ticks, status
integer, dimension(2) :: iterations, samples
real(kind=default), dimension(2,5) :: region
real(kind=default), dimension(5) :: weight_vector
real(kind=default), dimension(10) :: powers
real(kind=default) :: single_integral, single_standard_dev, single_chi_squared
real(kind=default) :: multi_integral, multi_standard_dev, multi_chi_squared
type(tao_random_state) :: rng
real(kind=default), parameter :: ACCEPTABLE = 4
integer :: failures
failures = 0
call tao_random_create (rng, 0)
call get_environment_variable (name="VAMP_RANDOM_TESTS", status=status)
if (status == 0) then
    call system_clock (start_ticks)
else
    start_ticks = 42
end if
call tao_random_seed (rng, start_ticks)
iterations = (/ 4, 3 /)
samples = (/ 20000, 200000 /)
region(1,:) = -1.0
region(2,:) = 1.0
width = 0.0001
print *, "Starting VAMP 1.0 self test..."
print *, "serial code"
call system_clock (start_ticks)
call single_channel (rng, region, samples, iterations, &
    single_integral, single_standard_dev, single_chi_squared)
call print_results ("SINGLE", start_ticks, &
    single_integral, single_standard_dev, single_chi_squared, &
    10*ACCEPTABLE, failures)
weight_vector = 1
powers = 0.25_default
call system_clock (start_ticks)
call multi_channel (rng, region, weight_vector, samples, iterations, &
    powers, multi_integral, multi_standard_dev, multi_chi_squared)
call print_results ("MULTI", start_ticks, &

```

```

        multi_integral, multi_standard_dev, multi_chi_squared, &
        ACCEPTABLE, failures)
    call system_clock (start_ticks)
! call check_jacobians (rng, region, weight_vector, samples(1))
    call check_inverses (rng, region, weight_vector, samples(1))
    call check_inverses3 (rng, region, samples(1))
    if (failures == 0) then
        stop 0
    else if (failures == 1) then
        stop 1
    else
        stop 2
    end if
end program vamp_test

```

6.1.2 Parallel Test

202a \langle vampi_test.f90 202a $\rangle \equiv$ 202b \triangleright

```

! vampi_test.f90 --
\langleCopyleft notice 1\rangle
\langleModule vamp_test_functions 192b\rangle

```

The following is identical to **vamp_tests**, except for use **vampi**:

202b \langle vampi_test.f90 202a $\rangle + \equiv$ \triangleleft 202a 202c \triangleright

```

module vampi_tests
    use kinds
    use exceptions
    use histograms
    use tao_random_numbers
    use coordinates
    use vampi
    use vamp_test_functions !NODEP!
    implicit none
    private
    \langleDeclaration of procedures in vamp_tests 196c\rangle
contains
    \langleImplementation of procedures in vamp_tests 197a\rangle
end module vampi_tests

```

202c \langle vampi_test.f90 202a $\rangle + \equiv$ \triangleleft 202b

```

program vampi_test
    use kinds
    use tao_random_numbers
    use coordinates

```

```

use vampi
use mpi90
use vamp_test_functions !NODEP!
use vampi_tests !NODEP!
implicit none
integer :: num_proc, proc_id, start_ticks
logical :: perform_io
integer, dimension(2) :: iterations, samples
real(kind=default), dimension(2,5) :: region
real(kind=default), dimension(5) :: weight_vector
real(kind=default), dimension(10) :: powers
real(kind=default) :: single_integral, single_standard_dev, single_chi_squared
real(kind=default) :: multi_integral, multi_standard_dev, multi_chi_squared
type(tao_random_state) :: rng
integer :: iostat, command
character(len=72) :: command_line
integer, parameter :: &
    CMD_ERROR = -1, CMD_END = 0, &
    CMD_NOP = 1, CMD_SINGLE = 2, CMD_MULTI = 3, CMD_CHECK = 4
call tao_random_create (rng, 0)
call mpi90_init ()
call mpi90_size (num_proc)
call mpi90_rank (proc_id)
perform_io = (proc_id == 0)
call system_clock (start_ticks)
call tao_random_seed (rng, start_ticks + proc_id)
iterations = (/ 4, 3 /)
samples = (/ 20000, 200000 /)
samples = (/ 200000, 2000000 /)
region(1,:) = -1.0
region(2,:) = 1.0
width = 0.0001
if (perform_io) then
    print *, "Starting VAMP 1.0 self test..."
    if (num_proc > 1) then
        print *, "parallel code running on ", num_proc, " processors"
    else
        print *, "parallel code running serially"
    end if
end if
command_loop: do
    Parse the commandline in vamp_test and set command (never defined)
    call mpi90_broadcast (command, 0)

```

```

    call system_clock (start_ticks)
    select case (command)
    <Execute command in vamp_test (never defined)>
    case (CMD_END)
        exit command_loop
    case (CMD_NOP)
        ! do nothing
    case (CMD_ERROR)
        ! do nothing
    end select
end do command_loop
call mpi90_finalize ()
end program vampi_test

```

6.1.3 Output

204a <vamp_test.out 204a>≡

6.2 Mapped Mode

In this chapter we perform a test of the major features of Vamp. A function with many peaks is integrated with the traditional Vegas algorithm, using a multi-channel approach and in parallel. The function is constructed to have a known analytical integral (which is chosen to be one) in order to be able to gauge the accuracy of the result and error estimate.

6.2.1 Serial Test

204b <vamp_test0.f90 204b>≡ 211a>
 ! vamp_test0.f90 --
 <Copyleft notice 1>
 <Module vamp_test0_functions 204c>

Single Channel

The functions to be integrated are shared by the serial and the parallel incarnation of the code.

204c <Module vamp_test0_functions 204c>≡ (204b 219e)
 module vamp_test0_functions
 use kinds
 use vamp, only: vamp_grid, vamp_multi_channel0

```

use vamp, only: vamp_data_t
implicit none
private
public :: f, g, phi, w
public :: create_sample, delete_sample
private :: f0, psi, g0, f_norm
real(kind=default), dimension(:), allocatable, private :: c, x_min, x_max
real(kind=default), dimension(:, :, :), allocatable, public :: x0, gamma
contains
  <Implementation of vamp_test0_functions procedures 205>
end module vamp_test0_functions

```

We start from a model of n_p interfering resonances in one variable (cf. section ??)

$$f_0(x|x_{\min}, x_{\max}, x_0, \gamma) = \frac{1}{N(x_{\min}, x_{\max}, x_0, \gamma)} \left| \sum_{p=1}^{n_p} \frac{1}{x - x_{0,p} + i\gamma_p} \right|^2 \quad (6.3)$$

where

$$N(x_{\min}, x_{\max}, x_0, \gamma) = \int_{x_{\min}}^{x_{\max}} dx \left| \sum_{p=1}^{n_p} \frac{1}{x - x_{0,p} + i\gamma_p} \right|^2 \quad (6.4)$$

such that

$$\int_{x_{\min}}^{x_{\max}} dx f_0(x|x_{\min}, x_{\max}, x_0, \gamma) = 1 \quad (6.5)$$

NB: the $N(x_{\min}, x_{\max}, x_0, \gamma)$ should be calculated once and tabulated to save processing time, but we are lazy here.

$$\begin{aligned}
N(x_{\min}, x_{\max}, x_0, \gamma) &= \sum_{p=1}^{n_p} \int_{x_{\min}}^{x_{\max}} dx \left| \frac{1}{x - x_{0,p} + i\gamma_p} \right|^2 \\
&\quad + 2 \operatorname{Re} \sum_{p=1}^{n_p} \sum_{q=1}^{n_p} \int_{x_{\min}}^{x_{\max}} dx \frac{1}{x - x_{0,p} + i\gamma_p} \frac{1}{x - x_{0,q} - i\gamma_q} \quad (6.6)
\end{aligned}$$

```

205 <Implementation of vamp_test0_functions procedures 205>≡ (204c) 206>
pure function f0(x, x_min, x_max, x0, g) result(f_x)
  real(kind=default), intent(in) :: x, x_min, x_max
  real(kind=default), dimension(:), intent(in) :: x0, g
  real(kind=default) :: f_x
  complex(kind=default) :: amp

```

```

real(kind=default) :: norm
integer :: i, j
amp = sum (1.0 / cmplx (x - x0, g, kind=default))
norm = 0
do i = 1, size (x0)
    norm = norm + f_norm (x_min, x_max, x0(i), g(i), x0(i), g(i))
    do j = i + 1, size (x0)
        norm = norm + 2 * f_norm (x_min, x_max, x0(i), g(i), x0(j), g(j))
    end do
end do
f_x = amp * conjg (amp) / norm
end function f0

```

$$\int_{x_{\min}}^{x_{\max}} dx \frac{1}{x - x_{0,p} + i\gamma_p} \frac{1}{x - x_{0,q} - i\gamma_q} = \frac{1}{x_{0,p} - x_{0,q} - i\gamma_p - i\gamma_q} \left(\ln \left(\frac{x_{\max} - x_{0,p} + i\gamma_p}{x_{\min} - x_{0,p} + i\gamma_p} \right) - \ln \left(\frac{x_{\max} - x_{0,q} - i\gamma_q}{x_{\min} - x_{0,q} - i\gamma_q} \right) \right) \quad (6.7)$$

Don't even think of merging the logarithms: it will screw up the Riemann sheet.

206 \langle Implementation of `vamp_test0_functions` procedures 205 $\rangle + \equiv$ (204c) \langle 205 207a \rangle

```

pure function f_norm (x_min, x_max, x0p, gp, x0q, gq) &
    result (norm)
real(kind=default), intent(in) :: x_min, x_max, x0p, gp, x0q, gq
real(kind=default) :: norm
norm = real (( log ( cmplx (x_max - x0p, gp, kind=default) &
                    / cmplx (x_min - x0p, gp, kind=default)) &
              - log ( cmplx (x_max - x0q, - gq, kind=default) &
                    / cmplx (x_min - x0q, - gq, kind=default))) &
            / cmplx (x0p - x0q, - gp - gq, kind=default), &
            kind=default)
end function f_norm

```

Since we want to be able to do the integral of f analytically, it is most convenient to take a weighted sum of products:

$$f(x_1, \dots, x_{n_d} | x_{\min}, x_{\max}, x_0, \gamma) = \frac{1}{\sum_{i=1}^{n_c} c_i} \sum_{i=1}^{n_c} c_i \prod_{j=1}^{n_d} f_0(x_j | x_{\min,j}, x_{\max,j}, x_{0,ij}, \gamma_{ij}) \quad (6.8)$$

Each summand is factorized and therefore very easily integrated by Vegas.
A non-trivial sum is more realistic in this respect.

207a \langle Implementation of vamp_test0.functions procedures 205 $\rangle + \equiv$ (204c) \langle 206 207b \rangle
pure function f (x, data, weights, channel, grids) result (f_x)

```

    real(kind=default), dimension(:), intent(in) :: x
    class(vamp_data_t), intent(in) :: data
    real(kind=default), dimension(:), intent(in), optional :: weights
    integer, intent(in), optional :: channel
    type(vamp_grid), dimension(:), intent(in), optional :: grids
    real(kind=default) :: f_x
    real(kind=default) :: fi_x
    integer :: i, j
    f_x = 0.0
    do i = 1, size (c)
        fi_x = 1.0
        do j = 1, size (x)
            if (all (gamma(:,i,j) > 0)) then
                fi_x = fi_x * f0 (x(j), x_min(j), x_max(j), &
                                x0(:,i,j), gamma(:,i,j))
            else
                fi_x = fi_x / (x_max(j) - x_min(j))
            end if
        end do
        f_x = f_x + c(i) * fi_x
    end do
    f_x = f_x / sum (c)
end function f
```

207b \langle Implementation of vamp_test0.functions procedures 205 $\rangle + \equiv$ (204c) \langle 207a 207c \rangle
subroutine delete_sample ()
deallocate (c, x_min, x_max, x0, gamma)
end subroutine delete_sample

207c \langle Implementation of vamp_test0.functions procedures 205 $\rangle + \equiv$ (204c) \langle 207b 208 \rangle
subroutine create_sample (num_poles, weights, region)
integer, intent(in) :: num_poles
real(kind=default), dimension(:), intent(in) :: weights
real(kind=default), dimension(:, :), intent(in) :: region
integer :: nd, nc
nd = size (region, dim=2)
nc = size (weights)
allocate (c(nc), x_min(nd), x_max(nd))
allocate (x0(num_poles,nc,nd), gamma(num_poles,nc,nd))
x_min = region(1,:)


```

x_max = region(2,:)
c = weights
end subroutine create_sample

```

Multi Channel

We start from the usual mapping for Lorentzian peaks

$$\begin{aligned} \psi(x_{\min}, x_{\max}, x_0, \gamma) : [x_{\min}, x_{\max}] &\rightarrow [x_{\min}, x_{\max}] \\ \xi \mapsto x = \psi(\xi|x_{\min}, x_{\max}, x_0, \gamma) \end{aligned} \quad (6.9)$$

where

$$\begin{aligned} \psi(\xi|x_{\min}, x_{\max}, x_0, \gamma) = x_0 + \\ \gamma \cdot \tan \left(\frac{\xi - x_{\min}}{x_{\max} - x_{\min}} \cdot \operatorname{atan} \frac{x_{\max} - x_0}{\gamma} - \frac{x_{\max} - \xi}{x_{\max} - x_{\min}} \cdot \operatorname{atan} \frac{x_0 - x_{\min}}{\gamma} \right) \end{aligned} \quad (6.10)$$

208 \langle Implementation of `vamp_test0_functions` procedures 205 $\rangle + \equiv$ (204c) \triangleleft 207c 209a \triangleright

```

pure function psi (xi, x_min, x_max, x0, gamma) result (x)
  real(kind=default), intent(in) :: xi, x_min, x_max, x0, gamma
  real(kind=default) :: x
  x = x0 + gamma &
    * tan (((xi - x_min) * atan ((x_max - x0) / gamma) &
      - (x_max - xi) * atan ((x0 - x_min) / gamma)) &
    / (x_max - x_min))
end function psi

```

The inverse mapping is

$$\begin{aligned} \psi^{-1}(x_{\min}, x_{\max}, x_0, \gamma) : [x_{\min}, x_{\max}] &\rightarrow [x_{\min}, x_{\max}] \\ x \mapsto \xi = \psi^{-1}(x|x_{\min}, x_{\max}, x_0, \gamma) \end{aligned} \quad (6.11)$$

with

$$\begin{aligned} \psi^{-1}(x|x_{\min}, x_{\max}, x_0, \gamma) = \\ \frac{x_{\max}(\operatorname{atan} \frac{x_0 - x_{\min}}{\gamma} + \operatorname{atan} \frac{x - x_0}{\gamma}) + x_{\min}(\operatorname{atan} \frac{x_{\max} - x_0}{\gamma} + \operatorname{atan} \frac{x_0 - x}{\gamma})}{\operatorname{atan} \frac{x_{\max} - x_0}{\gamma} + \operatorname{atan} \frac{x_0 - x_{\min}}{\gamma}} \end{aligned} \quad (6.12)$$

with Jacobian

$$\frac{d(\psi^{-1}(x|x_{\min}, x_{\max}, x_0, \gamma))}{dx} = \frac{x_{\max} - x_{\min}}{\operatorname{atan} \frac{x_{\max} - x_0}{\gamma} + \operatorname{atan} \frac{x_0 - x_{\min}}{\gamma}} \frac{\gamma}{(x - x_0)^2 + \gamma^2} \quad (6.13)$$

209a \langle Implementation of `vamp_test0_functions` procedures 205 $\rangle + \equiv$ (204c) \langle 208 209c \rangle

```

pure function g0 (x, x_min, x_max, x0, gamma) result (g_x)
  real(kind=default), intent(in) :: x, x_min, x_max, x0, gamma
  real(kind=default) :: g_x
  g_x = gamma / (atan ((x_max - x0) / gamma) - atan ((x_min - x0) / gamma)) &
    * (x_max - x_min) / ((x - x0)**2 + gamma**2)
end function g0

```

The function f has $n_c n_p^{n_d}$ peaks and we need a channel for each one, plus a constant function for the background. We encode the position on the grid linearly:

209b \langle Decode channel into `ch` and `p(:)` 209b $\rangle \equiv$ (209c 210a)

```

ch = channel - 1
do j = 1, size (x)
  p(j) = 1 + modulo (ch, np)
  ch = ch / np
end do
ch = ch + 1

```

The map ϕ is the direct product of ψ s:

209c \langle Implementation of `vamp_test0_functions` procedures 205 $\rangle + \equiv$ (204c) \langle 209a 210a \rangle

```

pure function phi (xi, channel) result (x)
  real(kind=default), dimension(:), intent(in) :: xi
  integer, intent(in) :: channel
  real(kind=default), dimension(size(xi)) :: x
  integer, dimension(size(xi)) :: p
  integer :: j, ch, np, nch, nd, channels
  np = size (x0, dim = 1)
  nch = size (x0, dim = 2)
  nd = size (x0, dim = 3)
  channels = nch * np**nd
  if (channel >= 1 .and. channel <= channels) then
     $\langle$ Decode channel into ch and p(:) 209b $\rangle$ 
    do j = 1, size (xi)
      if (all (gamma(:,ch,j) > 0)) then
        x(j) = psi (xi(j), x_min(j), x_max(j), &
          x0(p(j),ch,j), gamma(p(j),ch,j))
      else
        x = xi
      end if
    end do
  else if (channel == channels + 1) then
    x = xi
  else

```

```

        x = 0
    end if
end function phi

```

similarly for the Jacobians:

210a \langle Implementation of vamp_test0_functions procedures 205 $\rangle + \equiv$ (204c) \triangleleft 209c 210b \triangleright

```

pure recursive function g (x, data, channel) result (g_x)
    real(kind=default), dimension(:), intent(in) :: x
    class(vamp_data_t), intent(in) :: data
    integer, intent(in) :: channel
    real(kind=default) :: g_x
    integer, dimension(size(x)) :: p
    integer :: j, ch, np, nch, nd, channels
    np = size (x0, dim = 1)
    nch = size (x0, dim = 2)
    nd = size (x0, dim = 3)
    channels = nch * np**nd
    if (channel >= 1 .and. channel <= channels) then
         $\langle$ Decode channel into ch and p(:) 209b $\rangle$ 
        g_x = 1.0
        do j = 1, size (x)
            if (all (gamma(:,ch,j) > 0)) then
                g_x = g_x * g0 (x(j), x_min(j), x_max(j), &
                    x0(p(j),ch,j), gamma(p(j),ch,j))
            end if
        end do
    else if (channel == channels + 1) then
        g_x = 1.0
    else
        g_x = 0
    end if
end function g

```

210b \langle Implementation of vamp_test0_functions procedures 205 $\rangle + \equiv$ (204c) \triangleleft 210a

```

function w (x, data, weights, channel, grids) result (w_x)
    real(kind=default), dimension(:), intent(in) :: x
    class(vamp_data_t), intent(in) :: data
    real(kind=default), dimension(:), intent(in), optional :: weights
    integer, intent(in), optional :: channel
    type(vamp_grid), dimension(:), intent(in), optional :: grids
    real(kind=default) :: w_x
    w_x = vamp_multi_channel0 (f, data, phi, g, x, weights, channel)
end function w

```

Driver Routines

211a \langle vamp_test0.f90 204b \rangle + \equiv \langle 204b 217 \rangle
 module **vamp_tests0**
 \langle Modules used by vamp_tests0 211b \rangle
 use **vamp**
 implicit none
 private
 \langle Declaration of procedures in vamp_tests0 211e \rangle
 contains
 \langle Implementation of procedures in vamp_tests0 212a \rangle
 end module **vamp_tests0**

211b \langle Modules used by vamp_tests0 211b \rangle \equiv (211a 219e)
 use kinds
 use **exceptions**
 use **histograms**
 use **tao_random_numbers**
 use **vamp_test0_functions** !NODEP!

Verification

211c \langle Declaration of procedures in vamp_tests0 (broken?) 211c \rangle \equiv
 public :: **check_jacobians**

211d \langle Implementation of procedures in vamp_tests0 (broken?) 211d \rangle \equiv
subroutine check_jacobians (do_print, region, samples, rng)
 logical, intent(in) :: do_print
 real(kind=default), dimension(:,,:), intent(in) :: region
 integer, dimension(:), intent(in) :: samples
 type(**tao_random_state**), intent(inout) :: rng
 real(kind=default), dimension(size(region,dim=2)) :: **x**
 real(kind=default) :: d
 integer :: ch
 do ch = 1, size(x0,dim=2) * size(x0,dim=1)**size(x0,dim=3) + 1
 call **vamp_check_jacobian** (rng, samples(1), **g**, **phi**, ch, region, d, **x**)
 if (do_print) then
 print *, ch, ": ", d, ", ", **x** = ", real (**x**)
 end if
 end do
end **subroutine check_jacobians**

Integration

211e \langle Declaration of procedures in vamp_tests0 211e \rangle \equiv (211a 219e) 214a \rangle
 public :: **single_channel**, **multi_channel**

212a *⟨Implementation of procedures in vamp_tests0 212a⟩*≡ (211a 219e) 212b▷

```

subroutine single_channel (do_print, region, iterations, samples, rng, &
    acceptable, failures)
    logical, intent(in) :: do_print
    real(kind=default), dimension(:,:), intent(in) :: region
    integer, dimension(:), intent(in) :: iterations, samples
    type(tao_random_state), intent(inout) :: rng
    real(kind=default), intent(in) :: acceptable
    integer, intent(inout) :: failures
    type(vamp_grid) :: gr
    type(vamp_history), dimension(iterations(1)+iterations(2)) :: history
    real(kind=default) :: integral, standard_dev, chi_squared, pull
    call vamp_create_history (history)
    call vamp_create_grid (gr, region, samples(1))
    call vamp_sample_grid (rng, gr, f, NO_DATA, iterations(1), history = history)
    call vamp_discard_integral (gr, samples(2))
    call vamp_sample_grid &
        (rng, gr, f, NO_DATA, iterations(2), &
            integral, standard_dev, chi_squared, &
            history = history(iterations(1)+1:))
    call vamp_write_grid (gr, "vamp_test0.grid")
    call vamp_delete_grid (gr)
    call vamp_print_history (history, "single")
    call vamp_delete_history (history)
    pull = (integral - 1) / standard_dev
    if (do_print) then
        print *, "    int, err, chi2:", integral, standard_dev, chi_squared
    end if
    if (abs (pull) > acceptable) then
        failures = failures + 1
        print *, " unacceptable pull:", pull
    else
        print *, "    acceptable pull:", pull
    end if
end subroutine single_channel

```

212b *⟨Implementation of procedures in vamp_tests0 212a⟩*+≡ (211a 219e) ◁212a 214b▷

```

subroutine multi_channel (do_print, region, iterations, samples, rng, &
    acceptable, failures)
    logical, intent(in) :: do_print
    real(kind=default), dimension(:,:), intent(in) :: region
    integer, dimension(:), intent(in) :: iterations, samples
    type(tao_random_state), intent(inout) :: rng
    real(kind=default), intent(in) :: acceptable

```

```

    type(vamp_grids) :: grs
    integer, intent(inout) :: failures
    <Body of multi_channel 199b>
end subroutine multi_channel
213 <Body of multi_channel 199b>+≡ (199a 212b) <199b
    real(kind=default), &
        dimension(size(x0,dim=2)*size(x0,dim=1)**size(x0,dim=3)+1) :: &
            weight_vector
    type(vamp_history), dimension(iterations(1)+iterations(2)+4) :: history
    type(vamp_history), dimension(size(history),size(weight_vector)) :: histories
    real(kind=default) :: integral, standard_dev, chi_squared, pull
    integer :: it
    weight_vector = 1.0
    call vamp_create_history (history)
    call vamp_create_history (histories)
    call vamp_create_grids (grs, region, samples(1), weight_vector)
    call vamp_sample_grids (rng, grs, w, NO_DATA, iterations(1) - 1, &
        history = history, histories = histories)
do it = 1, 5
    call vamp_sample_grids (rng, grs, w, NO_DATA, 1, &
        history = history(iterations(1)+it-1:), &
        histories = histories(iterations(1)+it-1:,:))
    call vamp_refine_weights (grs)
end do
call vamp_discard_integrals (grs, samples(2))
call vamp_sample_grids &
    (rng, grs, w, NO_DATA, iterations(2), &
    integral, standard_dev, chi_squared, &
    history = history(iterations(1)+5:), &
    histories = histories(iterations(1)+5:,:))
call vamp_write_grids (grs, "vamp_test0.grids")
call vamp_delete_grids (grs)
call vamp_print_history (history, "multi")
call vamp_print_history (histories, "multi")
call vamp_delete_history (history)
call vamp_delete_history (histories)
if (do_print) then
    print *, integral, standard_dev, chi_squared
end if
pull = (integral - 1) / standard_dev
if (abs(pull) > acceptable) then
    failures = failures + 1
    print *, " unacceptable pull:", pull

```

```

else
  print *, "    acceptable pull:", pull
end if

```

Event Generation

```

214a <Declaration of procedures in vamp_tests0 211e>+≡ (211a 219e) <211e
      public :: single_channel_generator, multi_channel_generator

214b <Implementation of procedures in vamp_tests0 212a>+≡ (211a 219e) <212b 215>
      subroutine single_channel_generator (do_print, region, iterations, samples, rng)
        logical, intent(in) :: do_print
        real(kind=default), dimension(:,:), intent(in) :: region
        integer, dimension(:), intent(in) :: iterations, samples
        type(tao_random_state), intent(inout) :: rng
        type(vamp_grid) :: gr
        type(vamp_history), dimension(iterations(1)+iterations(2)) :: history
        type(histogram) :: unweighted, reweighted, weighted, weights
        type(exception) :: exc
        real(kind=default) :: weight, integral, standard_dev
        integer :: i
        real(kind=default), dimension(size(region,dim=2)) :: x
        call vamp_create_grid (gr, region, samples(1))
        call vamp_sample_grid (rng, gr, f, NO_DATA, iterations(1), history = history)
        call vamp_discard_integral (gr, samples(2))
        call vamp_warmup_grid &
          (rng, gr, f, NO_DATA, iterations(2), history = history(iterations(1)+1:))
        call vamp_print_history (history, "single")
        call vamp_delete_history (history)
        call create_histogram (unweighted, region(1,1), region(2,1), 100)
        call create_histogram (reweighted, region(1,1), region(2,1), 100)
        call create_histogram (weighted, region(1,1), region(2,1), 100)
        call create_histogram (weights, 0.0_default, 10.0_default, 100)
        ! do i = 1, 1000000
        do i = 1, 100
          call clear_exception (exc)
          call vamp_next_event (x, rng, gr, f, NO_DATA, exc = exc)
          call handle_exception (exc)
          call fill_histogram (unweighted, x(1))
          call fill_histogram (reweighted, x(1), 1.0_default / f (x, NO_DATA))
        end do
        integral = 0.0
        standard_dev = 0.0
        do i = 1, 10000

```

```

    call clear_exception (exc)
    call vamp_next_event (x, rng, gr, f, NO_DATA, weight, exc = exc)
    call handle_exception (exc)
    call fill_histogram (weighted, x(1), weight / f (x, NO_DATA))
    call fill_histogram (weights, x(1), weight)
    integral = integral + weight
    standard_dev = standard_dev + weight**2
end do
if (do_print) then
    print *, integral / (i-1), sqrt (standard_dev) / (i-1)
    call write_histogram (unweighted, "u_s.d")
    call write_histogram (reweighted, "r_s.d")
    call write_histogram (weighted, "w_s.d")
    call write_histogram (weights, "ws_s.d")
end if
call delete_histogram (unweighted)
call delete_histogram (reweighted)
call delete_histogram (weighted)
call delete_histogram (weights)
call vamp_delete_grid (gr)
end subroutine single_channel_generator

```

215 *<Implementation of procedures in vamp_tests0 212a>+≡ (211a 219e) <214b*

```

subroutine multi_channel_generator (do_print, region, iterations, samples, rng)
    logical, intent(in) :: do_print
    real(kind=default), dimension(:, :), intent(in) :: region
    integer, dimension(:), intent(in) :: iterations, samples
    type(tao_random_state), intent(inout) :: rng
    type(vamp_grids) :: grs
    real(kind=default), &
        dimension(size(x0,dim=2)*size(x0,dim=1)**size(x0,dim=3)+1) :: &
        weight_vector
    type(vamp_history), dimension(iterations(1)+iterations(2)+4) :: history
    type(vamp_history), dimension(size(history),size(weight_vector)) :: histories
    type(histogram) :: unweighted, reweighted, weighted, weights
    type(exception) :: exc
    real(kind=default) :: weight, integral, standard_dev
    real(kind=default), dimension(size(region,dim=2)) :: x
    character(len=5) :: pfx
    integer :: it, i, j
    weight_vector = 1.0
    call vamp_create_history (history)
    call vamp_create_history (histories)
    call vamp_create_grids (grs, region, samples(1), weight_vector)

```



```

call vamp_sample_grids (rng, grs, w, NO_DATA, iterations(1) - 1, &
                        history = history, histories = histories)
do it = 1, 5
    call vamp_sample_grids (rng, grs, w, NO_DATA, 1, &
                            history = history(iterations(1)+it-1:), &
                            histories = histories(iterations(1)+it-1:,:))
    call vamp_refine_weights (grs)
end do
call vamp_discard_integrals (grs, samples(2))
call vamp_warmup_grids &
    (rng, grs, w, NO_DATA, iterations(2), &
     history = history(iterations(1)+5:), &
     histories = histories(iterations(1)+5:,:))
call vamp_print_history (history, "multi")
call vamp_print_history (histories, "multi")
call vamp_delete_history (history)
call vamp_delete_history (histories)
!!! do i = 1, size (grs%grids)
!!!     do j = 1, size (grs%grids(i)%div)
!!!         write (pfx, "(I2.2,'/',I2.2)") i, j
!!!         call dump_division (grs%grids(i)%div(j), pfx)
!!!     end do
!!! end do
call create_histogram (unweighted, region(1,1), region(2,1), 100)
call create_histogram (reweighted, region(1,1), region(2,1), 100)
call create_histogram (weighted, region(1,1), region(2,1), 100)
call create_histogram (weights, 0.0_default, 10.0_default, 100)
! do i = 1, 1000000
do i = 1, 100
    call clear_exception (exc)
    call vamp_next_event (x, rng, grs, f, NO_DATA, phi, exc = exc)
    call handle_exception (exc)
    call fill_histogram (unweighted, x(1))
    call fill_histogram (reweighted, x(1), 1.0_default / f (x, NO_DATA))
end do
integral = 0.0
standard_dev = 0.0
do i = 1, 10000
    call clear_exception (exc)
    call vamp_next_event (x, rng, grs, f, NO_DATA, phi, weight, exc = exc)
    call handle_exception (exc)
    call fill_histogram (weighted, x(1), weight / f (x, NO_DATA))
    call fill_histogram (weights, x(1), weight)

```

```

        integral = integral + weight
        standard_dev = standard_dev + weight**2
    end do
    if (do_print) then
        print *, integral / (i-1), sqrt (standard_dev) / (i-1)
        call write_histogram (unweighted, "u_m.d")
        call write_histogram (reweighted, "r_m.d")
        call write_histogram (weighted, "w_m.d")
        call write_histogram (weights, "ws_m.d")
    end if
    call delete_histogram (unweighted)
    call delete_histogram (reweighted)
    call delete_histogram (weighted)
    call delete_histogram (weights)
    call vamp_delete_grids (grs)
end subroutine multi_channel_generator

```

Main Program

```

217 <vamp_test0.f90 204b>+≡ <211a
    program vamp_test0
        <Modules used by vamp_test0 219a>
        implicit none
        <Variables in vamp_test0 218f>
        do_print = .true.
        print *, "Starting VAMP 1.0 self test..."
        print *, "serial code"
        call tao_random_create (rng, 0)
        call get_environment_variable (name="VAMP_RANDOM_TESTS", status=status)
        if (status == 0) then
            call system_clock (ticks0)
        else
            ticks0 = 42
        end if
        call tao_random_seed (rng, ticks0)
        <Set up integrand and region in vamp_test0 219c>
        <Execute tests in vamp_test0 218a>
        <Cleanup in vamp_test0 219d>
        if (failures == 0) then
            stop 0
        else if (failures == 1) then
            stop 1
        else

```

```

        stop 2
    end if
end program vamp_test0

218a <Execute tests in vamp_test0 218a>≡ (217) 218b>
    failures = 0
    call system_clock (ticks0)
    call single_channel (do_print, region, iterations, samples, rng, 10*ACCEPTABLE, failures)
    call system_clock (ticks, ticks_per_second)
    print "(1X,A,F6.2,A)", &
        "time = ", real (ticks - ticks0) / ticks_per_second, " secs"

218b <Execute tests in vamp_test0 218a>+≡ (217) <218a 218c>
    call system_clock (ticks0)
    call single_channel_generator &
        (do_print, region, iterations, samples, rng)
    call system_clock (ticks, ticks_per_second)
    print "(1X,A,F6.2,A)", &
        "time = ", real (ticks - ticks0) / ticks_per_second, " secs"

218c <Execute tests in vamp_test0 218a>+≡ (217) <218b 218d>
    call system_clock (ticks0)
    call multi_channel (do_print, region, iterations, samples, rng, ACCEPTABLE, failures)
    call system_clock (ticks, ticks_per_second)
    print "(1X,A,F6.2,A)", &
        "time = ", real (ticks - ticks0) / ticks_per_second, " secs"

218d <Execute tests in vamp_test0 218a>+≡ (217) <218c 218e>
    call system_clock (ticks0)
    call multi_channel_generator &
        (do_print, region, iterations, samples, rng)
    call system_clock (ticks, ticks_per_second)
    print "(1X,A,F6.2,A)", &
        "time = ", real (ticks - ticks0) / ticks_per_second, " secs"

218e <Execute tests in vamp_test0 218a>+≡ (217) <218d
    call system_clock (ticks0)
    ! call check_jacobians (do_print, region, samples, rng)
    call system_clock (ticks, ticks_per_second)
    print "(1X,A,F6.2,A)", &
        "time = ", real (ticks - ticks0) / ticks_per_second, " secs"

218f <Variables in vamp_test0 218f>≡ (217 220) 219b>
    logical :: do_print

218g <Execute command 218g>≡ (220)

```

```

219a  <Modules used by vamp_test0 219a>≡ (217 220)
      use kinds
      use tao_random_numbers
      use vamp_test0_functions !NODEP!
      use vamp_tests0 !NODEP!

219b  <Variables in vamp_test0 218f>+≡ (217 220) <218f
      integer :: i, j, ticks, ticks_per_second, ticks0, status
      integer, dimension(2) :: iterations, samples
      real(kind=default), dimension(:,,:), allocatable :: region
      type(tao_random_state) :: rng
      real(kind=default), parameter :: ACCEPTABLE = 4
      integer :: failures

219c  <Set up integrand and region in vamp_test0 219c>≡ (217 220)
      iterations = (/ 4, 3 /)
      samples = (/ 10000, 50000 /)
      allocate (region(2,2))
      region(1,:) = -1.0
      region(2,:) = 2.0
      call create_sample &
           (num_poles = 2, weights = (/ 1.0_default, 2.0_default /), region = region)
      do i = 1, size (x0, dim=2)
        do j = 1, size (x0, dim=3)
          call tao_random_number (rng, x0(:,i,j))
        end do
      end do
      gamma = 0.001
      x0(1, :, :) = 0.2
      x0(2, :, :) = 0.8

219d  <Cleanup in vamp_test0 219d>≡ (217 220)
      call delete_sample ()
      deallocate (region)

```

6.2.2 Parallel Test

```

219e  <vampi_test0.f90 219e>≡ 220>
      ! vampi_test0.f90 --
      <Copyleft notice 1>
      <Module vamp_test0_functions 204c>
      module vamp_tests0
        <Modules used by vamp_tests0 211b>
        use vampi
        use mpi90

```

```

        implicit none
        private
        <Declaration of procedures in vamp_tests0 211e>
contains
        <Implementation of procedures in vamp_tests0 212a>
end module vamp_tests0

220 <vampi_test0.f90 219e>+≡ <219e
    program vampi_test0
        <Modules used by vamp_test0 219a>
        use mpi90
        implicit none
        <Variables in vamp_test0 218f>
        integer :: num_proc, proc_id
        call mpi90_init ()
        call mpi90_size (num_proc)
        call mpi90_rank (proc_id)
        if (proc_id == 0) then
            do_print = .true.
            print *, "Starting VAMP 1.0 self test..."
            if (num_proc > 1) then
                print *, "parallel code running on ", num_proc, " processors"
            else
                print *, "parallel code running serially"
            end if
        else
            do_print = .false.
        end if
        call tao_random_create (rng, 0)
        call system_clock (ticks0)
        call tao_random_seed (rng, ticks0 + proc_id)
        <Set up integrand and region in vamp_test0 219c>
        call mpi90_broadcast (x0, 0)
        call mpi90_broadcast (gamma, 0)
        command_loop: do
            if (proc_id == 0) then
                <Read command line and decode it as command (never defined)>
            end if
            call mpi90_broadcast (command, 0)
            call system_clock (ticks0)
            <Execute command 218g>
            call system_clock (ticks, ticks_per_second)
            if (proc_id == 0) then
                print "(1X,A,F6.2,A)", &

```

```

        "time = ", real (ticks - ticks0) / ticks_per_second, " secs"
    end if
end do command_loop
<Cleanup in vamp_test0 219d>
call mpi90_finalize ()
if (proc_id == 0) then
    print *, "bye."
end if
end program vampi_test0

```

6.2.3 Output

221 <vamp_test0.out 221>≡

—7—

APPLICATION

7.1 *Cross section*

```
222a <application.f90 222a>≡ 239▷
! application.f90 --
<Copyleft notice 1>
module cross_section
  use kinds
  use constants
  use utils
  use kinematics
  use tao_random_numbers
  use products, only: dot
  use helicity
  use vamp, only: vamp_grid, vamp_probability
  implicit none
  private
  <Declaration of cross_section procedures 223d>
  <Types in cross_section 228c>
  <Variables in cross_section 222b>
contains
  <Implementation of cross_section procedures 224a>
end module cross_section

222b <Variables in cross_section 222b>≡ (222a) 223c▷
  real(kind=default), private, parameter :: &
    MA_0 = 0.0, &
    MB_0 = 0.0, &
    M1_0 = 0.0, &
    M2_0 = 0.0, &
    M3_0 = 0.0, &
```

```
S_0 = 200.0 ** 2
```

223a \langle XXX Variables in cross_section 223a $\rangle \equiv$ 223b \triangleright

```
real(kind=default), private, parameter :: &
  MA_0 = 0.01, &
  MB_0 = 0.01, &
  M1_0 = 0.01, &
  M2_0 = 0.01, &
  M3_0 = 0.01, &
  S_0 = 200.0 ** 2
```

223b \langle XXX Variables in cross_section 223a $\rangle + \equiv$ \triangleleft 223a

```
real(kind=default), private, parameter :: &
  S1_MIN_0 = 0.0 ** 2, &
  S2_MIN_0 = 0.0 ** 2, &
  S3_MIN_0 = 0.0 ** 2, &
  T1_MIN_0 = 0.0 ** 2, &
  T2_MIN_0 = 0.0 ** 2
```

223c \langle Variables in cross_section 222b $\rangle + \equiv$ (222a) \triangleleft 222b 223f \triangleright

```
real(kind=default), private, parameter :: &
  S1_MIN_0 = 1.0 ** 2, &
  S2_MIN_0 = 1.0 ** 2, &
  S3_MIN_0 = 1.0 ** 2, &
  T1_MIN_0 = 10.0 ** 2, &
  T2_MIN_0 = 10.0 ** 2
```

223d \langle Declaration of cross_section procedures 223d $\rangle \equiv$ (222a) 225a \triangleright

```
private :: cuts
```

223e \langle XXX Implementation of cross_section procedures 223e $\rangle \equiv$

```
pure function cuts (k1, k2, p1, p2, q) result (inside)
  real(kind=default), dimension(0:), intent(in) :: k1, k2, p1, p2, q
  logical :: inside
  inside = (abs (dot (k1 - q, k1 - q)) >= T1_MIN_0) &
    .and. (abs (dot (k2 - q, k2 - q)) >= T2_MIN_0) &
    .and. (abs (dot (p1 + q, p1 + q)) >= S1_MIN_0) &
    .and. (abs (dot (p2 + q, p2 + q)) >= S2_MIN_0) &
    .and. (abs (dot (p1 + p2, p1 + p2)) >= S3_MIN_0)
end function cuts
```

223f \langle Variables in cross_section 222b $\rangle + \equiv$ (222a) \triangleleft 223c

```
real(kind=default), private, parameter :: &
```



```

E_MIN = 1.0, &
COSTH_SEP_MAX = 0.99, &
COSTH_BEAM_MAX = 0.99

```

224a *⟨Implementation of cross_section procedures 224a⟩*≡ (222a) 224b>

```

pure function cuts (k1, k2, p1, p2, q) result (inside)
  real(kind=default), dimension(0:), intent(in) :: k1, k2, p1, p2, q
  logical :: inside
  real(kind=default), dimension(3) :: p1n, p2n, qn
  inside = .false.
  if ((p1(0) < E_MIN) .or. (p2(0) < E_MIN) .or. (q(0) < E_MIN)) then
    return
  end if
  p1n = p1(1:3) / sqrt (dot_product (p1(1:3), p1(1:3)))
  p2n = p2(1:3) / sqrt (dot_product (p2(1:3), p2(1:3)))
  qn = q(1:3) / sqrt (dot_product (q(1:3), q(1:3)))
  if ((abs (qn(3)) > COSTH_BEAM_MAX) &
      .or. (abs (p1n(3)) > COSTH_BEAM_MAX)&
      .or. (abs (p2n(3)) > COSTH_BEAM_MAX)) then
    return
  end if
  if (dot_product (p1n, qn) > COSTH_SEP_MAX) then
    return
  end if
  if (dot_product (p2n, qn) > COSTH_SEP_MAX) then
    return
  end if
  if (dot_product (p1n, p2n) > COSTH_SEP_MAX) then
    return
  end if
  inside = .true.
end function cuts

```

224b *⟨Implementation of cross_section procedures 224a⟩*+≡ (222a) <224a 226b>

```

function xsect (k1, k2, p1, p2, q) result (xs)
  real(kind=default), dimension(0:), intent(in) :: k1, k2, p1, p2, q
  real(kind=default) :: xs
  complex(kind=default), dimension(-1:1,-1:1,-1:1,-1:1,-1:1) :: amp
  !!! xs = 1.0_double / phase_space_volume (3, k1(0) + k2(0))
  !!! xs = 1.0_double / dot (p1 + q, p1 + q) &
  !!!      + 1.0_double / dot (p2 + q, p2 + q)
  !!! return
  amp = nneeg (k1, k2, p1, p2, q)

```

```

xs = sum (amp(-1:1:2,-1:1:2,-1:1:2,-1:1:2,-1:1:2) &
          * conjg (amp(-1:1:2,-1:1:2,-1:1:2,-1:1:2,-1:1:2)))
end function xsect

```

225a \langle Declaration of cross_section procedures 223d $\rangle + \equiv$ (222a) \langle 223d 227b \rangle

```

private :: xsect

$$\phi : [0, 1]^{\otimes 5} \rightarrow [(m_2 + m_3)^2, (\sqrt{s} - m_1)^2] \otimes [t_1^{\min}(s_2), t_1^{\max}(s_2)]$$


$$\otimes [0, 2\pi] \otimes [-1, 1] \otimes [0, 2\pi]$$


$$(x_1, \dots, x_5) \mapsto (s_2, t_1, \phi, \cos \theta_3, \phi_3)$$


$$= (s_2(x_1), x_2 t_1^{\max}(s_2) + (1 - x_2) t_1^{\min}(s_2), 2\pi x_3, 2x_4 - 1, 2\pi x_5)$$

(7.1)

```

where

$$t_1^{\max/\min}(s_2)$$

$$= m_a^2 + m_1^2 - \frac{(s + m_a^2 - m_b^2)(s - s_2 + m_1^2) \mp \sqrt{\lambda(s, m_a^2, m_b^2)\lambda(s, s_2, m_1^2)}}{2s}$$
(7.2)

225b \langle Set $(s_2, t_1, \phi, \cos \theta_3, \phi_3)$ from (x_1, \dots, x_5) 225b $\rangle \equiv$ (226b)

```

! s2_min = S1_MIN_0
s2_min = (m2 + m3)**2
s2_max = (sqrt(s) - m1)**2
s2 = s2_max * x(1) + s2_min * (1 - x(1))
t1_min = ma**2 + m1**2 - ((s + ma**2 - mb**2) * (s - s2 + m1**2) &
+ sqrt(lambda(s, ma**2, mb**2) * lambda(s, s2, m1**2))) / (2*s)
t1_max = ma**2 + m1**2 - ((s + ma**2 - mb**2) * (s - s2 + m1**2) &
- sqrt(lambda(s, ma**2, mb**2) * lambda(s, s2, m1**2))) / (2*s)
t1 = t1_max * x(2) + t1_min * (1 - x(2))
phi = 2*PI * x(3)
cos_theta3 = 2 * x(4) - 1
phi3 = 2*PI * x(5)

```

225c \langle Set $(s_2, t_1, \phi, \cos \theta_3, \phi_3)$ from (x_1, \dots, x_5) (massless case) 225c $\rangle \equiv$ (228b)

```

! s2_min = S1_MIN_0
s2_min = 0
s2_max = s
s2 = s2_max * x(1) + s2_min * (1 - x(1))
t1_min = - (s - s2)
t1_max = 0
t1 = t1_max * x(2) + t1_min * (1 - x(2))
phi = 2*PI * x(3)
cos_theta3 = 2 * x(4) - 1
phi3 = 2*PI * x(5)

```

$$J_\phi(x_1, \dots, x_5) = \left| \begin{array}{cc} \frac{\partial s_2}{\partial x_1} & \frac{\partial t_1}{\partial x_1} \\ \frac{\partial s_2}{\partial x_2} & \frac{\partial t_1}{\partial x_2} \end{array} \right| \cdot 8\pi^2 \quad (7.3)$$

i.e.

$$J_\phi(x_1, \dots, x_5) = 8\pi^2 \cdot \left| \frac{ds_2}{dx_1} \right| \cdot (t_1^{\max}(s_2) - t_1^{\min}(s_2)) \quad (7.4)$$

226a $\langle \text{Adjust Jacobian 226a} \rangle \equiv$ (226b 228b)

```
p%jacobian = p%jacobian &
* (8.0 * PI**2 * (s2_max - s2_min) * (t1_max - t1_min))
```

226b $\langle \text{Implementation of cross_section procedures 224a} \rangle + \equiv$ (222a) $\langle 224b \ 227c \rangle$

```
pure function phase_space(x, channel) result(p)
  real(kind=default), dimension(:), intent(in) :: x
  integer, intent(in) :: channel
  type(LIPS3) :: p
  real(kind=default) :: &
    ma, mb, m1, m2, m3, s, t1, s2, phi, cos_theta3, phi3
  real(kind=default) :: s2_min, s2_max, t1_min, t1_max
  s = S_0
   $\langle m_a \leftrightarrow m_b, m_1 \leftrightarrow m_2 \text{ for channel \#1 226c} \rangle$ 
   $\langle \text{Set } (s_2, t_1, \phi, \cos \theta_3, \phi_3) \text{ from } (x_1, \dots, x_5) \text{ 225b} \rangle$ 
  p = two_to_three(s, t1, s2, phi, cos_theta3, phi3, ma, mb, m1, m2, m3)
   $\langle \text{Adjust Jacobian 226a} \rangle$ 
   $\langle p_1 \leftrightarrow p_2 \text{ for channel \#2 227a} \rangle$ 
end function phase_space
```

226c $\langle m_a \leftrightarrow m_b, m_1 \leftrightarrow m_2 \text{ for channel \#1 226c} \rangle \equiv$ (226b)

```
select case (channel)
case (1)
  ma = MA_0
  mb = MB_0
  m1 = M1_0
  m2 = M2_0
  m3 = M3_0
case (2)
  ma = MB_0
  mb = MA_0
  m1 = M2_0
  m2 = M1_0
  m3 = M3_0
case (3)
  ma = MA_0
  mb = MB_0
  m1 = M3_0
```

```

        m2 = M2_0
        m3 = M1_0
    case default
        ma = MA_0
        mb = MB_0
        m1 = M1_0
        m2 = M2_0
        m3 = M3_0
    end select

```

227a $\langle p_1 \leftrightarrow p_2 \text{ for channel \#2 227a} \rangle \equiv$ (226b 228b)

```

    select case (channel)
    case (1)
        ! OK
    case (2)
        call swap (p%p(1,:), p%p(2,:))
    case (3)
        call swap (p%p(1,:), p%p(3,:))
    case default
        ! OK
    end select

```

227b $\langle \text{Declaration of cross_section procedures 223d} \rangle + \equiv$ (222a) \langle 225a 228a \rangle
 private :: jacobian

227c $\langle \text{Implementation of cross_section procedures 224a} \rangle + \equiv$ (222a) \langle 226b 228b \rangle

```

pure function jacobian (k1, k2, p1, p2, q) result (jac)
    real(kind=default), dimension(0:), intent(in) :: k1, k2, p1, p2, q
    real(kind=default) :: jac
    real(kind=default) :: ma_2, mb_2, m1_2, m2_2, m3_2
    real(kind=default) :: s, s2, s2_min, s2_max, t1_min, t1_max
    ma_2 = max (dot (k1, k1), 0.0_double)
    mb_2 = max (dot (k2, k2), 0.0_double)
    m1_2 = max (dot (p1, p1), 0.0_double)
    m2_2 = max (dot (p2, p2), 0.0_double)
    m3_2 = max (dot (q, q), 0.0_double)
    s = dot (k1 + k2, k1 + k2)
    s2 = dot (p2 + q, p2 + q)
    ! s2_min = S1_MIN_0
    s2_min = (sqrt (m2_2) + sqrt (m3_2))**2
    s2_max = (sqrt (s) - sqrt (m1_2))**2
    t1_min = ma_2 + m1_2 - ((s + ma_2 - mb_2) * (s - s2 + m1_2) &
        + sqrt (lambda (s, ma_2, mb_2) * lambda (s, s2, m1_2))) / (2*s)
    t1_max = ma_2 + m1_2 - ((s + ma_2 - mb_2) * (s - s2 + m1_2) &
        - sqrt (lambda (s, ma_2, mb_2) * lambda (s, s2, m1_2))) / (2*s)

```

```

    jac = 1.0 / ((2*PI)**5 * 32 * s2) &
        * sqrt (lambda (s2, m2_2, m3_2) / lambda (s, ma_2, mb_2)) &
        * (8.0 * PI**2 * (s2_max - s2_min) * (t1_max - t1_min))
end function jacobian

```

228a \langle Declaration of cross_section procedures 223d $\rangle + \equiv$ (222a) \langle 227b 228e \rangle
 private :: phase_space, phase_space_massless

228b \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \langle 227c 228f \rangle
 pure function phase_space_massless (x, channel) result (p)
 real(kind=default), dimension(:), intent(in) :: x
 integer, intent(in) :: channel
 type(LIPS3) :: p
 real(kind=default) :: s, t1, s2, phi, cos_theta3, phi3
 real(kind=default) :: s2_min, s2_max, t1_min, t1_max
 s = S_0
 \langle Set $(s_2, t_1, \phi, \cos \theta_3, \phi_3)$ from (x_1, \dots, x_5) (massless case) 225c \rangle
 p = two_to_three (s, t1, s2, phi, cos_theta3, phi3)
 \langle Adjust Jacobian 226a \rangle
 \langle $p_1 \leftrightarrow p_2$ for channel #2 227a \rangle
 end function phase_space_massless

228c \langle Types in cross_section 228c $\rangle \equiv$ (222a) 228d \rangle
 type, public :: LIPS3_m5i2a3
 ! private
 real(kind=default) :: ma, mb, m1, m2, m3
 real(kind=default) :: s, s2, t1
 real(kind=default) :: phi, cos_theta3, phi3
 real(kind=default) :: jacobian
 end type LIPS3_m5i2a3

228d \langle Types in cross_section 228c $\rangle + \equiv$ (222a) \langle 228c \rangle
 type, public :: x5
 ! private
 real(kind=default), dimension(5) :: x
 real(kind=default) :: jacobian
 end type x5

228e \langle Declaration of cross_section procedures 223d $\rangle + \equiv$ (222a) \langle 228a 231a \rangle
 private :: invariants_from_p, invariants_to_p
 private :: invariants_from_x, invariants_to_x

228f \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \langle 228b 229a \rangle
 pure function invariants_from_p (p, k1, k2) result (q)
 type(LIPS3), intent(in) :: p

```

real(kind=default), dimension(0:), intent(in) :: k1, k2
type(LIPS3_m5i2a3) :: q
real(kind=default) :: ma_2, mb_2, m1_2, m2_2, m3_2
real(kind=default), dimension(0:3) :: k1k2, p2p3, k1p1, p3_23
k1k2 = k1 + k2
k1p1 = - k1 + p%p(1,:)
p2p3 = p%p(2,:) + p%p(3,:)
ma_2 = max (dot (k1, k1), 0.0_double)
mb_2 = max (dot (k2, k2), 0.0_double)
m1_2 = max (dot (p%p(1,:), p%p(1:)), 0.0_double)
m2_2 = max (dot (p%p(2,:), p%p(2:)), 0.0_double)
m3_2 = max (dot (p%p(3,:), p%p(3:)), 0.0_double)
q%ma = sqrt (ma_2)
q%mb = sqrt (mb_2)
q%m1 = sqrt (m1_2)
q%m2 = sqrt (m2_2)
q%m3 = sqrt (m3_2)
q%s = dot (k1k2, k1k2)
q%s2 = dot (p2p3, p2p3)
q%t1 = dot (k1p1, k1p1)
q%phi = atan2 (p%p(1,2), p%p(1,1))
if (q%phi < 0) then
    q%phi = q%phi + 2*PI
end if
p3_23 = boost_momentum (p%p(3,:), p2p3)
q%cos_theta3 = p3_23(3) / sqrt (dot_product (p3_23(1:3), p3_23(1:3)))
q%phi3 = atan2 (p3_23(2), p3_23(1))
if (q%phi3 < 0) then
    q%phi3 = q%phi3 + 2*PI
end if
q%jacobian = 1.0 / ((2*PI)**5 * 32 * q%s2) &
    * sqrt (lambda (q%s2, m2_2, m3_2) / lambda (q%s, ma_2, mb_2))
end function invariants_from_p

```

229a \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \triangleleft 228f 229b \triangleright

```

pure function invariants_to_p (p) result (q)
type(LIPS3_m5i2a3), intent(in) :: p
type(LIPS3) :: q
q = two_to_three (p%s, p%t1, p%s2, p%phi, p%cos_theta3, p%phi3)
q%jacobian = q%jacobian * p%jacobian
end function invariants_to_p

```

229b \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \triangleleft 229a 230 \triangleright

```

pure function invariants_from_x (x, s, ma, mb, m1, m2, m3) result (p)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default), intent(in) :: s, ma, mb, m1, m2, m3
  type(LIPS3_m5i2a3) :: p
  real(kind=default) :: s2_min, s2_max, t1_min, t1_max
  p%ma = ma
  p%mb = mb
  p%m1 = m1
  p%m2 = m2
  p%m3 = m3
  p%s = s
  s2_min = (p%m2 + p%m3)**2
  s2_max = (sqrt (p%s) - p%m1)**2
  p%s2 = s2_max * x(1) + s2_min * (1 - x(1))
  t1_min = p%ma**2 + p%m1**2 &
    - ((p%s + p%ma**2 - p%mb**2) * (p%s - p%s2 + p%m1**2) &
      + sqrt (lambda (p%s, p%ma**2, p%mb**2) &
        * lambda (p%s, p%s2, p%m1**2))) / (2*p%s)
  t1_max = p%ma**2 + p%m1**2 &
    - ((p%s + p%ma**2 - p%mb**2) * (p%s - p%s2 + p%m1**2) &
      - sqrt (lambda (p%s, p%ma**2, p%mb**2) &
        * lambda (p%s, p%s2, p%m1**2))) / (2*p%s)
  p%t1 = t1_max * x(2) + t1_min * (1 - x(2))
  p%phi = 2*PI * x(3)
  p%cos_theta3 = 2 * x(4) - 1
  p%phi3 = 2*PI * x(5)
  p%jacobian = 8*PI**2 * (s2_max - s2_min) * (t1_max - t1_min)
end function invariants_from_x

```

230 \langle Implementation of cross-section procedures 224a $\rangle + \equiv$ (222a) \langle 229b 231b \rangle

```

pure function invariants_to_x (p) result (x)
  type(LIPS3_m5i2a3), intent(in) :: p
  type(x5) :: x
  real(kind=default) :: s2_min, s2_max, t1_min, t1_max
  s2_min = (p%m2 + p%m3)**2
  s2_max = (sqrt (p%s) - p%m1)**2
  t1_min = p%ma**2 + p%m1**2 &
    - ((p%s + p%ma**2 - p%mb**2) * (p%s - p%s2 + p%m1**2) &
      + sqrt (lambda (p%s, p%ma**2, p%mb**2) &
        * lambda (p%s, p%s2, p%m1**2))) / (2*p%s)
  t1_max = p%ma**2 + p%m1**2 &
    - ((p%s + p%ma**2 - p%mb**2) * (p%s - p%s2 + p%m1**2) &
      - sqrt (lambda (p%s, p%ma**2, p%mb**2) &

```

```

        * lambda (p%s, p%s2, p%m1**2))) / (2*p%s)
x%x(1) = (p%s2 - s2_min) / (s2_max - s2_min)
x%x(2) = (p%t1 - t1_min) / (t1_max - t1_min)
x%x(3) = p%phi / (2*PI)
x%x(4) = (p%cos_theta3 + 1) / 2
x%x(5) = p%phi3 / (2*PI)
x%jacobian = p%jacobian * 8*PI**2 * (s2_max - s2_min) * (t1_max - t1_min)
end function invariants_to_x

```

231a *<Declaration of cross_section procedures 223d>+≡ (222a) <228e 232b>*
public :: sigma, sigma_raw, sigma_massless

231b *<Implementation of cross_section procedures 224a>+≡ (222a) <230 231c>*
function sigma (x, weights, channel, grids) result (xs)
 real(kind=default), dimension(:), intent(in) :: x
 real(kind=default), dimension(:), intent(in), optional :: weights
 integer, intent(in), optional :: channel
 type(vamp_grid), dimension(:), intent(in), optional :: grids
 real(kind=default) :: xs
 real(kind=default), dimension(2,0:3) :: k
 type(LIPS3) :: p
 k(1,:) = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
 k(2,:) = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
 if (present (channel)) then
 p = phase_space (x, channel)
 else
 p = phase_space (x, 0)
 end if
 if (cuts (k(1,:), k(2,:), p%p(1,:), p%p(2,:), p%p(3,:))) then
 xs = xsect (k(1,:), k(2,:), p%p(1,:), p%p(2,:), p%p(3,:)) &
 * jacobian (k(1,:), k(2,:), p%p(1,:), p%p(2,:), p%p(3,:))
 !!! * p%jacobian
 else
 xs = 0.0
 end if
end function sigma

231c *<Implementation of cross_section procedures 224a>+≡ (222a) <231b 232a>*
function sigma_raw (k1, k2, p1, p2, q) result (xs)
 real(kind=default), dimension(0:), intent(in) :: k1, k2, p1, p2, q
 real(kind=default) :: xs
 if (cuts (k1, k2, p1, p2, q)) then
 xs = xsect (k1, k2, p1, p2, q)
 end if
end function sigma_raw


```

else
  xs = 0.0
end if
end function sigma_raw

```

232a \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \langle 231c 232c \rangle

```

function sigma_massless (x, weights, channel, grids) result (xs)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default), dimension(:), intent(in), optional :: weights
  integer, intent(in), optional :: channel
  type(vamp_grid), dimension(:), intent(in), optional :: grids
  real(kind=default) :: xs
  real(kind=default), dimension(2,0:3) :: k
  type(LIPS3) :: p
  k(1,:) = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
  k(2,:) = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
  p = phase_space_massless (x, 0)
  if (cuts (k(1,:), k(2,:), p%p(1,:), p%p(2,:), p%p(3,:))) then
    xs = xsect (k(1,:), k(2,:), p%p(1,:), p%p(2,:), p%p(3,:)) &
      * p%jacobian
  else
    xs = 0.0
  end if
end function sigma_massless

```

232b \langle Declaration of cross_section procedures 223d $\rangle + \equiv$ (222a) \langle 231a 234a \rangle

```

public :: w

```



232c \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \langle 232a 233 \rangle

```

function w (x, weights, channel, grids) result (w_x)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default), dimension(:), intent(in), optional :: weights
  integer, intent(in), optional :: channel
  type(vamp_grid), dimension(:), intent(in), optional :: grids
  real(kind=default) :: w_x
  real(kind=default), dimension(size(weights)) :: g_x

```

```

real(kind=default), dimension(2,0:3) :: k
type(LIPS3) :: p
integer :: ch
if (present (channel)) then
    ch = channel
else
    ch = 0
end if
k(1,:) = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
k(2,:) = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
p = phase_space (x, abs (ch))
g_x(1) = 1.0_double / jacobian (k(1,:), k(2,:), p%p(1,:), p%p(2,:), p%p(3,:))
g_x(2) = 1.0_double / jacobian (k(1,:), k(2,:), p%p(2,:), p%p(1,:), p%p(3,:))
g_x(3) = 1.0_double / jacobian (k(1,:), k(2,:), p%p(3,:), p%p(2,:), p%p(1,:))
if (ch > 0) then
    w_x = sigma_raw (k(1,:), k(2,:), p%p(1,:), p%p(2,:), p%p(3,:)) &
        / sum (weights * g_x)
else if (ch < 0) then
    w_x = g_x(-ch) / sum (weights * g_x)
else
    w_x = -1
end if
end function w

```

233 \langle Implementation of cross-section procedures 224a $\rangle + \equiv$ (222a) \langle 232c 234c \rangle

```

function sigma_rambo (x, weights, channel, grids) result (xs)
    real(kind=default), dimension(:), intent(in) :: x
    real(kind=default), dimension(:), intent(in), optional :: weights
    integer, intent(in), optional :: channel
    type(vamp_grid), dimension(:), intent(in), optional :: grids
    real(kind=default) :: xs
    real(kind=default), dimension(2,0:3) :: k
    real(kind=default), dimension(3,0:3) :: p
    k(1,:) = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
    k(2,:) = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
    p = massless_isotropic_decay (sum (k(:,0)), reshape (x, (/ 3, 4 /)))
    if (cuts (k(1,:), k(2,:), p(1,:), p(2,:), p(3,:))) then
        xs = xsect (k(1,:), k(2,:), p(1,:), p(2,:), p(3,:)) &
            * phase_space_volume (size (p, dim = 1), sum (k(:,0)))
    else
        xs = 0.0
    end if
end function sigma_rambo

```

234a *<Declaration of cross_section procedures 223d>+≡ (222a) <232b 234b>*
public :: sigma_rambo

234b *<Declaration of cross_section procedures 223d>+≡ (222a) <234a 235a>*
public :: check_kinematics
private :: print_LIPS3_m5i2a3

234c *<Implementation of cross_section procedures 224a>+≡ (222a) <233 234d>*
subroutine check_kinematics (rng)
type(tao_random_state), intent(inout) :: rng
real(kind=default), dimension(5) :: x
real(kind=default), dimension(0:3) :: k1, k2
type(x5) :: x1, x2
type(LIPS3) :: p1, p2
type(LIPS3_m5i2a3) :: q, q1, q2
k1 = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
k2 = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
call tao_random_number (rng, x)
q = invariants_from_x (x, S_0, MA_0, MB_0, M1_0, M2_0, M3_0)
p1 = invariants_to_p (q)
q1 = invariants_from_p (p1, k1, k2)
p2 = phase_space (x, 1)
q2 = invariants_from_p (p2, k1, k2)
x1 = invariants_to_x (q1)
x2 = invariants_to_x (q2)
print *, p1%jacobian, p2%jacobian, x1%jacobian, x2%jacobian
call print_lips3_m5i2a3 (q)
call print_lips3_m5i2a3 (q1)
call print_lips3_m5i2a3 (q2)
end subroutine check_kinematics

234d *<Implementation of cross_section procedures 224a>+≡ (222a) <234c 235b>*
subroutine print_LIPS3_m5i2a3 (p)
type(LIPS3_m5i2a3), intent(in) :: p
print "(1x,5('m',a1,'=',e9.2,' '))", &
'a', p%ma, 'b', p%mb, '1', p%m1, '2', p%m2, '3', p%m3
print "(1x,'s=',e9.2,' s2=',e9.2,' t1=',e9.2)", &
p%s, p%s2, p%t1
print "(1x,'phi=',e9.2,' cos(th3)=',e9.2,' phi2=',e9.2)", &
p%phi, p%cos_theta3, p%phi3
print "(1x,'j=',e9.2)", &
p%jacobian
end subroutine print_LIPS3_m5i2a3

235a *<Declaration of cross_section procedures 223d>+≡ (222a) <234b 238a>*

```
public :: phi12, phi21, phi1, phi2
public :: g12, g21, g1, g2
```

235b *<Implementation of cross_section procedures 224a>+≡ (222a) <234d 235c>*

```
pure function phi12 (x1, dummy) result (x2)
  real(kind=default), dimension(:), intent(in) :: x1
  integer, intent(in) :: dummy
  real(kind=default), dimension(size(x1)) :: x2
  type(LIPS3) :: p1, p2
  type(LIPS3_m5i2a3) :: q1, q2
  type(x5) :: x52
  real(kind=default), dimension(0:3) :: k1, k2
  k1 = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
  k2 = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
  q1 = invariants_from_x (x1, S_0, MA_0, MB_0, M1_0, M2_0, M3_0)
  p1 = invariants_to_p (q1)
  p2%p(1,:) = p1%p(2,:)
  p2%p(2,:) = p1%p(1,:)
  p2%p(3,:) = p1%p(3,:)
  if (dummy < 0) then
    q2 = invariants_from_p (p2, k2, k1)
  else
    q2 = invariants_from_p (p2, k1, k2)
  end if
  x52 = invariants_to_x (q2)
  x2 = x52%x
end function phi12
```

235c *<Implementation of cross_section procedures 224a>+≡ (222a) <235b 236a>*

```
pure function phi21 (x2, dummy) result (x1)
  real(kind=default), dimension(:), intent(in) :: x2
  integer, intent(in) :: dummy
  real(kind=default), dimension(size(x2)) :: x1
  type(LIPS3) :: p1, p2
  type(LIPS3_m5i2a3) :: q1, q2
  type(x5) :: x51
  real(kind=default), dimension(0:3) :: k1, k2
  k1 = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
  k2 = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
  q2 = invariants_from_x (x2, S_0, MA_0, MB_0, M2_0, M1_0, M3_0)
  p2 = invariants_to_p (q2)
  p1%p(1,:) = p2%p(2,:)
  p1%p(2,:) = p2%p(1,:)
```

```

p1%p(3,:) = p2%p(3,:)
if (dummy < 0) then
    q1 = invariants_from_p (p1, k2, k1)
else
    q1 = invariants_from_p (p1, k1, k2)
end if
x51 = invariants_to_x (q1)
x1 = x51%x
end function phi21

```

236a \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \langle 235c 236b \rangle

```

pure function phi1 (x1) result (p1)
    real(kind=default), dimension(:), intent(in) :: x1
    type(LIPS3) :: p1
    type(LIPS3_m5i2a3) :: q1
    q1 = invariants_from_x (x1, S_0, MA_0, MB_0, M1_0, M2_0, M3_0)
    p1 = invariants_to_p (q1)
end function phi1

```

236b \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \langle 236a 236c \rangle

```

pure function phi2 (x2) result (p2)
    real(kind=default), dimension(:), intent(in) :: x2
    type(LIPS3) :: p2
    type(LIPS3_m5i2a3) :: q2
    q2 = invariants_from_x (x2, S_0, MA_0, MB_0, M2_0, M1_0, M3_0)
    p2 = invariants_to_p (q2)
end function phi2

```

236c \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \langle 236b 237a \rangle

```

pure function g12 (x1) result (g)
    real(kind=default), dimension(:), intent(in) :: x1
    real(kind=default) :: g
    type(LIPS3) :: p1, p2
    type(LIPS3_m5i2a3) :: q1, q2
    type(x5) :: x52
    real(kind=default), dimension(0:3) :: k1, k2
    k1 = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
    k2 = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
    q1 = invariants_from_x (x1, S_0, MA_0, MB_0, M1_0, M2_0, M3_0)
    p1 = invariants_to_p (q1)
    p2%p(1,:) = p1%p(2,:)
    p2%p(2,:) = p1%p(1,:)

```

```

p2%p(3,:) = p1%p(3,:)
q2 = invariants_from_p (p2, k2, k1)
x52 = invariants_to_x (q2)
g = x52%jacobian / p1%jacobian
end function g12

```

237a \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \triangleleft 236c 237b \triangleright

```

pure function g21 (x2) result (g)
  real(kind=default), dimension(:), intent(in) :: x2
  real(kind=default) :: g
  type(LIPS3) :: p1, p2
  type(LIPS3_m5i2a3) :: q1, q2
  type(x5) :: x51
  real(kind=default), dimension(0:3) :: k1, k2
  k1 = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
  k2 = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
  q2 = invariants_from_x (x2, S_0, MA_0, MB_0, M2_0, M1_0, M3_0)
  p2 = invariants_to_p (q2)
  p1%p(1,:) = p2%p(2,:)
  p1%p(2,:) = p2%p(1,:)
  p1%p(3,:) = p2%p(3,:)
  q1 = invariants_from_p (p1, k2, k1)
  x51 = invariants_to_x (q1)
  g = x51%jacobian / p2%jacobian
end function g21

```

237b \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \triangleleft 237a 237c \triangleright

```

pure function g1 (x1) result (g)
  real(kind=default), dimension(:), intent(in) :: x1
  real(kind=default) :: g
  type(LIPS3) :: p1
  type(LIPS3_m5i2a3) :: q1
  q1 = invariants_from_x (x1, S_0, MA_0, MB_0, M1_0, M2_0, M3_0)
  p1 = invariants_to_p (q1)
  g = 1 / p1%jacobian
end function g1

```

237c \langle Implementation of cross_section procedures 224a $\rangle + \equiv$ (222a) \triangleleft 237b 238b \triangleright

```

pure function g2 (x2) result (g)
  real(kind=default), dimension(:), intent(in) :: x2
  real(kind=default) :: g
  type(LIPS3) :: p2

```

```

type(LIPS3_m5i2a3) :: q2
q2 = invariants_from_x (x2, S_0, MA_0, MB_0, M2_0, M1_0, M3_0)
p2 = invariants_to_p (q2)
g = 1 / p2%jacobian
end function g2

```

238a *<Declaration of cross_section procedures 223d>+≡* (222a) <235a
public :: wx

238b *<Implementation of cross_section procedures 224a>+≡* (222a) <237c
function wx (x, weights, channel, grids) result (w_x)
 real(kind=default), dimension(:), intent(in) :: x
 real(kind=default), dimension(:), intent(in) :: weights
 integer, intent(in) :: channel
 type(vamp_grid), dimension(:), intent(in) :: grids
 real(kind=default) :: w_x
 real(kind=default), dimension(size(weights)) :: g_x, p_q
 real(kind=default), dimension(size(x)) :: x1, x2
 real(kind=default), dimension(2,0:3) :: k
 type(LIPS3) :: q
 k(1,:) = (/ 100.0_double, 0.0_double, 0.0_double, 100.0_double /)
 k(2,:) = (/ 100.0_double, 0.0_double, 0.0_double, -100.0_double /)
 select case (abs (channel))
 case (1)
 x1 = x
 x2 = phi12 (x, 0)
 q = phi1 (x1)
 case (2)
 x1 = phi21 (x, 0)
 x2 = x
 q = phi2 (x2)
 end select
 p_q(1) = vamp_probability (grids(1), x1)
 p_q(2) = vamp_probability (grids(2), x2)
 g_x(1) = p_q(1) * g1 (x1)
 g_x(2) = p_q(2) * g2 (x2)
 g_x = g_x / p_q(abs(channel))
 if (channel > 0) then
 w_x = sigma_raw (k(1,:), k(2,:), q%p(1,:), q%p(2,:), q%p(3,:)) &
 / dot_product (weights, g_x)
 else if (channel < 0) then
 w_x = vamp_probability (grids(-channel), x) / dot_product (weights, g_x)
 else
 w_x = 0
 end if
end function wx

```

        end if
    end function wx

```

```

239  <application.f90 222a>+≡                                     <222a
    program application
        use kinds
        use utils
        use vampi
        use mpi90
        use linalg
        use exceptions
        use kinematics, only: phase_space_volume
        use cross_section !NODEP!
        use tao_random_numbers
        implicit none
        type(vamp_grid) :: gr
        type(vamp_grids) :: grs
        real(kind=default), dimension(:,:), allocatable :: region
        real(kind=default) :: integral, standard_dev, chi_squared
        real(kind=default) :: &
            single_integral, single_standard_dev, &
            rambo_integral, rambo_standard_dev
        real(kind=default), dimension(2) :: weight_vector
        integer, dimension(2) :: calls, iterations
        type(vamp_history), dimension(100) :: history
        type(vamp_history), dimension(100,size(weight_vector)) :: histories
        type(exception) :: exc
        type( tao_random_state ) :: rng
        real(kind=default), dimension(5) :: x
        real(kind=default) :: jac
        integer :: i
        integer :: num_proc, proc_id, ticks, ticks0, ticks_per_second, command
        character(len=72) :: command_line
        integer, parameter :: &
            CMD_SINGLE = 1, &
            CMD_MULTI = 2, &
            CMD_ROTATING = 3, &
            CMD_RAMBO = 4, &
            CMD_COMPARE = 5, &
            CMD_MASSLESS = 6, &
            CMD_ERROR = 0
        call mpi90_init ()
        call mpi90_size (num_proc)

```



```

call mpi90_rank (proc_id)
call system_clock (ticks0)
call tao_random_create (rng, 0)
call tao_random_seed (rng, ticks0 + proc_id)
!!! call tao_random_seed (rng, proc_id)
call vamp_create_history (history, verbose = .true.)
call vamp_create_history (histories, verbose = .true.)
iterations = (/ 3, 4 /)
calls = (/ 10000, 100000 /)
if (proc_id == 0) then
  read *, command_line
  if (command_line == "single") then
    command = CMD_SINGLE
  else if (command_line == "multi") then
    command = CMD_MULTI
  else if (command_line == "rotating") then
    command = CMD_ROTATING
  else if (command_line == "rambo") then
    command = CMD_RAMBO
  else if (command_line == "compare") then
    command = CMD_COMPARE
  else if (command_line == "massless") then
    command = CMD_MASSLESS
  else
    command = CMD_ERROR
  end if
end if
call mpi90_broadcast (command, 0)
call system_clock (ticks0)
select case (command)
case (CMD_SINGLE)
  Application in single channel mode 242a
case (CMD_MASSLESS)
  Application in massless single channel mode 242b
case (CMD_MULTI)
  Application in multi channel mode 243
case (CMD_ROTATING)
  allocate (region(2,5))
  region(1,:) = 0.0
  region(2,:) = 1.0
  if (proc_id == 0) then
    print *, "rotating N/A yet ..."
  end if

```

```

case (CMD_RAMBO)
  <Application in Rambo mode 244>
case (CMD_COMPARE)
  <Application in single channel mode 242a>
  single_integral = integral
  single_standard_dev = standard_dev
  <Application in Rambo mode 244>
  if (proc_id == 0) then
    rambo_integral = integral
    rambo_standard_dev = standard_dev
    integral = &
      (single_integral / single_standard_dev**2 &
       + rambo_integral / rambo_standard_dev**2) &
      / (1.0_double / single_standard_dev**2 &
       + 1.0_double / rambo_standard_dev**2)
    standard_dev = 1.0_double &
      / sqrt (1.0_double / single_standard_dev**2 &
       + 1.0_double / rambo_standard_dev**2)
    chi_squared = &
      ((single_integral - integral)**2 / single_standard_dev**2) &
      + ((rambo_integral - integral)**2 / rambo_standard_dev**2)
    print *, "S&R:   ", integral, standard_dev, chi_squared
  end if
case default
  if (proc_id == 0) then
    print *, "???: ", command
    !!! TO BE REMOVED !!!
    call check_kinematics (rng)
    allocate (region(2,5))
    region(1,:) = 0
    region(2,:) = 1
    do i = 1, 10
      call tao_random_number (rng, x)
      call vamp_jacobian (phi12, 0, x, region, jac)
      print *, "12:  ", jac, 1 / g12 (x), jac * g12 (x) - 1
      call vamp_jacobian (phi21, 0, x, region, jac)
      print *, "21:  ", jac, 1 / g21 (x), jac * g21 (x) - 1
      print *, "1:   ", real(x)
      print *, "2:   ", real(phi12(phi21(x,0),0))
      print *, "2':  ", real(phi12(phi21(x,-1),-1))
      print *, "3:   ", real(phi21(phi12(x,0),0))
      print *, "3':  ", real(phi21(phi12(x,-1),-1))
      print *, "2-1: ", real(phi12(phi21(x,0),0) - x)
    end do
  end if
end

```

```

        print *, "3-1: ", real(phi21(phi12(x,0),0) - x)
        print *, "a:   ", real(phi12(x,0))
        print *, "a':  ", real(phi12(x,-1))
        print *, "b:   ", real(phi21(x,0))
        print *, "b':  ", real(phi21(x,-1))
    end do
    deallocate (region)
    ! do i = 2, 5
    !     print *, i, phase_space_volume (i, 200.0_double)
    ! end do
end if
end select
if (proc_id == 0) then
    call system_clock (ticks, ticks_per_second)
    print "(1X,A,F8.2,A)", &
        "time = ", real (ticks - ticks0) / ticks_per_second, " secs"
end if
call mpi90_finalize ()
end program application
242a  <Application in single channel mode 242a>≡ (239)
    allocate (region(2,5))
    region(1,:) = 0.0
    region(2,:) = 1.0
    call vamp_create_grid (gr, region, calls(1))
    call clear_exception (exc)
    call vamp_sample_grid &
        (rng, gr, sigma, iterations(1), history = history, exc = exc)
    call handle_exception (exc)
    call vamp_discard_integral (gr, calls(2))
    call vamp_sample_grid &
        (rng, gr, sigma, iterations(2), &
            integral, standard_dev, chi_squared, &
            history = history(iterations(1)+1:), exc = exc)
    call handle_exception (exc)
    call vamp_print_history (history, "single")
    if (proc_id == 0) then
        print *, "SINGLE: ", integral, standard_dev, chi_squared
    end if
    call vamp_write_grid (gr, "application.grid")
    call vamp_delete_grid (gr)
    deallocate (region)
242b  <Application in massless single channel mode 242b>≡ (239)
    allocate (region(2,5))

```

```

region(1,:) = 0.0
region(2,:) = 1.0
call vamp_create_grid (gr, region, calls(1))
call clear_exception (exc)
call vamp_sample_grid &
    (rng, gr, sigma_massless, iterations(1), history = history, exc = exc)
call handle_exception (exc)
call vamp_discard_integral (gr, calls(2))
call vamp_sample_grid &
    (rng, gr, sigma_massless, iterations(2), &
     integral, standard_dev, chi_squared, &
     history = history(iterations(1)+1:), exc = exc)
call handle_exception (exc)
call vamp_print_history (history, "single")
if (proc_id == 0) then
    print *, "M=0:   ", integral, standard_dev, chi_squared
end if
call vamp_write_grid (gr, "application.grid")
call vamp_delete_grid (gr)
deallocate (region)

```

243 *Application in multi channel mode 243* ≡ (239)

```

allocate (region(2,5))
region(1,:) = 0.0
region(2,:) = 1.0
weight_vector = 1.0
if (proc_id == 0) then
    read *, weight_vector
end if
call mpi90_broadcast (weight_vector, 0)
weight_vector = weight_vector / sum (weight_vector)
call vamp_create_grids (grs, region, calls(1), weight_vector)
do i = 1, 3
    call clear_exception (exc)
    call vamp_sample_grids &
        (rng, grs, wx, iterations(1), &
         history = history(1+(i-1)*iterations(1):), &
         histories = histories(1+(i-1)*iterations(1):,:), exc = exc)
    call handle_exception (exc)
    call vamp_refine_weights (grs)
end do
call vamp_discard_integrals (grs, calls(2))
call vamp_sample_grids &
    (rng, grs, wx, iterations(2), &

```

```

        integral, standard_dev, chi_squared, &
        history = history(3*iterations(1)+1:), &
        histories = histories(3*iterations(1)+1:,:), exc = exc)
call handle_exception (exc)
call vamp_print_history (history, "multi")
call vamp_print_history (histories, "multi")
if (proc_id == 0) then
    print *, "MULTI: ", integral, standard_dev, chi_squared
end if
call vamp_write_grids (grs, "application.grids")
call vamp_delete_grids (grs)
deallocate (region)

```

244 *Application in Rambo mode 244*≡

(239)

```

allocate (region(2,12))
region(1,:) = 0.0
region(2,:) = 1.0
call vamp_create_grid (gr, region, calls(1))
call clear_exception (exc)
call vamp_sample_grid &
    (rng, gr, sigma_rambo, iterations(1), history = history, exc = exc)
call handle_exception (exc)
call vamp_discard_integral (gr, calls(2))
call vamp_sample_grid &
    (rng, gr, sigma_rambo, iterations(2), &
    integral, standard_dev, chi_squared, &
    history = history(iterations(1)+1:), exc = exc)
call handle_exception (exc)
call vamp_print_history (history, "rambo")
if (proc_id == 0) then
    print *, "RAMBO: ", integral, standard_dev, chi_squared
end if
call vamp_delete_grid (gr)
deallocate (region)

```

—A—

CONSTANTS

A.1 Kinds

This borders on overkill, but it is the most portable way to get double precision in standard Fortran without relying on `kind (1.0D0)`. Currently, it is possible to change `double` to any other supported real kind. The MPI interface is a potential trouble source for such things, however.

```
245a <vamp_kinds.f90 245a>≡
! vamp_kinds.f90 --
<Copyleft notice 1>
module kinds
  implicit none
  integer, parameter, private :: single = &
    & selected_real_kind (precision(1.0), range(1.0))
  integer, parameter, private :: double = &
    & selected_real_kind (precision(1.0_single) + 1, range(1.0_single) + 1)
  integer, parameter, private :: extended = &
    & selected_real_kind (precision (1.0_double) + 1, range (1.0_double))
  integer, parameter, public :: default = double
end module kinds
```

A.2 Mathematical and Physical Constants

```
245b <constants.f90 245b>≡
! constants.f90 --
<Copyleft notice 1>
module constants
  use kinds
  implicit none
```

```
private
real(kind=default), public, parameter :: &
    PI = 3.1415926535897932384626433832795028841972_default
end module constants
```

—B—

ERRORS AND EXCEPTIONS

Fortran95 does not allow *any* I/O in pure and elemental procedures, not even output to the unit *. A `stop` statement is verboten as well. Therefore we have to use condition codes

```

247a <exceptions.f90 247a>≡
    ! exceptions.f90 --
    <Copyleft notice 1>
    module exceptions
        use kinds
        implicit none
        private
        <Declaration of exceptions procedures 248b>
        <Interfaces of exceptions procedures (never defined)>
        <Variables in exceptions 247c>
        <Declaration of exceptions types 247b>
    contains
        <Implementation of exceptions procedures 248c>
    end module exceptions

247b <Declaration of exceptions types 247b>≡                                     (247a)
    type, public :: exception
        integer :: level = EXC_NONE
        character(len=NAME_LENGTH) :: message = ""
        character(len=NAME_LENGTH) :: origin = ""
    end type exception

247c <Variables in exceptions 247c>≡                                           (247a) 248a▷
    integer, public, parameter :: &
        EXC_NONE = 0, &
        EXC_INFO = 1, &
        EXC_WARN = 2, &
        EXC_ERROR = 3, &
        EXC_FATAL = 4

```


248a *<Variables in exceptions 247c>+≡* (247a) *<247c*
integer, private, parameter :: EXC_DEFAULT = EXC_ERROR
integer, private, parameter :: NAME_LENGTH = 64

248b *<Declaration of exceptions procedures 248b>≡* (247a) 248d*>*
public :: handle_exception

248c *<Implementation of exceptions procedures 248c>≡* (247a) 248e*>*
subroutine handle_exception (exc)
type(exception), intent(inout) :: exc
character(len=10) :: name
if (exc%level > 0) then
select case (exc%level)
case (EXC_NONE)
name = "(none)"
case (EXC_INFO)
name = "info"
case (EXC_WARN)
name = "warning"
case (EXC_ERROR)
name = "error"
case (EXC_FATAL)
name = "fatal"
case default
name = "invalid"
end select
print *, trim (exc%origin), ": ", trim(name), ": ", trim (exc%message)
if (exc%level >= EXC_FATAL) then
print *, "terminated."
stop
end if
end if
end subroutine handle_exception

248d *<Declaration of exceptions procedures 248b>+≡* (247a) *<248b*
public :: raise_exception, clear_exception, gather_exceptions

Raise an exception, but don't overwrite the messages in **exc** if it holds a more severe exception. This way we can accumulate error codes across procedure calls. We have **exc** optional to simplify life for the cslling procedures, which might have it optional themselves.

248e *<Implementation of exceptions procedures 248c>+≡* (247a) *<248c 249a>*
elemental subroutine raise_exception (exc, level, origin, message)
type(exception), intent(inout), optional :: exc
integer, intent(in), optional :: level

```

character(len=*), intent(in), optional :: origin, message
integer :: local_level
if (present (exc)) then
  if (present (level)) then
    local_level = level
  else
    local_level = EXC_DEFAULT
  end if
  if (exc%level < local_level) then
    exc%level = local_level
    if (present (origin)) then
      exc%origin = origin
    else
      exc%origin = "[vamp]"
    end if
    if (present (message)) then
      exc%message = message
    else
      exc%message = "[vamp]"
    end if
  end if
end if
end subroutine raise_exception

```

249a *<Implementation of exceptions procedures 248c>+≡* (247a) <248e 249b>

```

elemental subroutine clear_exception (exc)
  type(exception), intent(inout) :: exc
  exc%level = 0
  exc%message = ""
  exc%origin = ""
end subroutine clear_exception

```

249b *<Implementation of exceptions procedures 248c>+≡* (247a) <249a

```

pure subroutine gather_exceptions (exc, excs)
  type(exception), intent(inout) :: exc
  type(exception), dimension(:), intent(in) :: excs
  integer :: i
  i = sum (maxloc (excs%level))
  if (exc%level < excs(i)%level) then
    call raise_exception (exc, excs(i)%level, excs(i)%origin, &
      excs(i)%message)
  end if
end subroutine gather_exceptions

```

Here's how to use `gather_exceptions`. elemental_procedure

250 $\langle \textit{Idioms } 101a \rangle + \equiv$

$\triangleleft 101a$

```
call clear_exception (excs)
call elemental_procedure_1 (y, x, excs)
call elemental_procedure_2 (b, a, excs)
if (any (excs%level > 0)) then
  call gather_exceptions (exc, excs)
  return
end if
```

—C—

THE ART OF RANDOM NUMBERS

Volume two of Donald E. Knuth’ *The Art of Computer Programming* [16] has always been celebrated as a prime reference for random number generation. Recently, the third edition has been published and it contains a gem of a *portable* random number generator. It generates 30-bit integers with the following desirable properties

- they pass all the tests from George Marsaglia’s “diehard” suite of tests for random number generators [24] (but see [16] for a caveat regarding the “birthday-spacing” test)
- they can be generated with portable signed 32-bit arithmetic (Fortran can’t do unsigned arithmetic)
- it is faster than other lagged Fibonacci generators
- it can create at least $2^{30} - 2$ independent sequences

We implement the improved versions available as FORTRAN77 code from

<http://www-cs-faculty.stanford.edu/~uno/programs.html#rng>

that contain a streamlined seeding algorithm with better independence of substreams.

C.1 Application Program Interface

A function returning single reals and integers. Note that the static version without the `tao_random_state` argument does not require initialization. It will behave as if call `tao_random_seed(0)` had been executed. On the other hand, the parallelizable version with the explicit `tao_random_state` will fail

if none of the `tao_random_create` have been called for the state. (This is a deficiency of Fortran90 that can be fixed in Fortran95).

252a *<API documentation 252a>*≡ 252b>
 call `tao_random_number` (`r`)
 call `tao_random_number` (`s`, `r`)

The state of the random number generator comes in two varieties: buffered and raw. The former is much more efficient, but it can be beneficial to flush the buffers and to pass only the raw state in order to save of interprocess communication (IPC) costs.

252b *<API documentation 252a>*+≡ <252a 252c>
 type(`tao_random_state`) :: `s`
 type(`tao_random_raw_state`) :: `rs`

Subroutines filling arrays of reals and integers:

252c *<API documentation 252a>*+≡ <252b 252d>
 call `tao_random_number` (`a`, `num` = `n`)
 call `tao_random_number` (`s`, `a`, `num` = `n`)

Subroutine for changing the seed:

252d *<API documentation 252a>*+≡ <252c 252e>
 call `tao_random_seed` (`seed` = `seed`)
 call `tao_random_seed` (`s`, `seed` = `seed`)

Subroutine for changing the luxury. Per default, use all random numbers:

252e *<API documentation 252a>*+≡ <252d 252f>
 call `tao_random_luxury` (`)`
 call `tao_random_luxury` (`s`)

With an integer argument, use the first `n` of each fill of the buffer:

252f *<API documentation 252a>*+≡ <252e 252g>
 call `tao_random_luxury` (`n`)
 call `tao_random_luxury` (`s`, `n`)

With a floating point argument, use that fraction of each fill of the buffer:

252g *<API documentation 252a>*+≡ <252f 252h>
 call `tao_random_luxury` (`x`)
 call `tao_random_luxury` (`s`, `x`)

Create a `tao_random_state`

252h *<API documentation 252a>*+≡ <252g 253a>
 call `tao_random_create` (`s`, `seed`, `buffer_size` = `buffer_size`)
 call `tao_random_create` (`s`, `raw_state`, `buffer_size` = `buffer_size`)
 call `tao_random_create` (`s`, `state`)

Create a `tao_random_raw_state`

253a *<API documentation 252a>+≡* <252h 253b>
 call `tao_random_create` (rs, seed)
 call `tao_random_create` (rs, raw_state)
 call `tao_random_create` (rs, state)

Destroy a `tao_random_state` or `tao_random_raw_state`

253b *<API documentation 252a>+≡* <253a 253c>
 call `tao_random_destroy` (s)

Copy `tao_random_state` and `tao_random_raw_state` in all four combinations

253c *<API documentation 252a>+≡* <253b 253d>
 call `tao_random_copy` (lhs, rhs)
 lhs = rhs

253d *<API documentation 252a>+≡* <253c 253e>
 call `tao_random_flush` (s)

253e *<API documentation 252a>+≡* <253d 253f>
 call `tao_random_read` (s, unit)
 call `tao_random_write` (s, unit)

253f *<API documentation 252a>+≡* <253e 253g>
 call `tao_random_test` (name = name)

Here is a sample application of random number states:

253g *<API documentation 252a>+≡* <253f 253h>
 subroutine threads (args, y, state)
 real, dimension(:), intent(in) :: args
 real, dimension(:), intent(out) :: y
 type(`tao_random_state`) :: state
 integer :: seed
 type(`tao_random_raw_state`), dimension(size(y)) :: states
 integer :: s
 call `tao_random_number` (state, seed)
 call `tao_random_create` (states, (/ (s, s=seed,size(y)-1) /))
 y = thread (args, states)
 end function thread

In this example, we could equivalently pass an integer seed, instead of `raw_state`. But in more complicated cases it can be beneficial to have the option of reusing `raw_state` in the calling routine.

253h *<API documentation 252a>+≡* <253g
 elemental function thread (arg, raw_state) result (y)
 real, dimension, intent(in) :: arg
 type(`tao_random_raw_state`) :: raw_state
 real :: y

```

type(tao_random_state) :: state
real :: r
call tao_random_create (state, raw_state)
do
  ...
  call tao_random_number (state, r)
  ...
end do
end function thread

```

C.2 Low Level Routines

Here the low level routines are *much* more interesting than the high level routines. The latter contain a lot of duplication (made necessary by Fortran's lack of parametric polymorphism) and consist mostly of bookkeeping. We wil therefore start with the former.

C.2.1 Generation of 30-bit Random Numbers

The generator is a subtractive lagged Fibonacci

$$X_j = (X_{j-K} - X_{j-L}) \mod 2^{30} \quad (\text{C.1})$$

with lags $K = 100$ and $L = 37$.

254a \langle Parameters in tao_random_numbers 254a $\rangle \equiv$ (273d 274a) 255a \triangleright

```
integer, parameter, private :: K = 100, L = 37
```

Other good choices for K and L are (cf. [16], table 1 in section 3.2.2, p. 29)

254b \langle Parameters in tao_random_numbers (alternatives) 254b $\rangle \equiv$

```
integer, parameter, private :: K = 55, L = 24
integer, parameter, private :: K = 89, L = 38
integer, parameter, private :: K = 100, L = 37
integer, parameter, private :: K = 127, L = 30
integer, parameter, private :: K = 258, L = 83
integer, parameter, private :: K = 378, L = 107
integer, parameter, private :: K = 607, L = 273
```

A modulus of 2^{30} is the largest we can handle in *portable* (i.e. *signed*) 32-bit arithmetic

254c \langle Variables in 30-bit tao_random_numbers 254c $\rangle \equiv$ (273d) 256b \triangleright

```
integer(kind=tao_i32), parameter, private :: M = 2**30
```

generate fills the array a_1, \dots, a_n with random integers $0 \leq a_i < 2^{30}$. We *must* have at least $n \geq K$. Higher values don't change the results, but make **generate** more efficient (about a factor of two, asymptotically). For $K = 100$, DEK recommends $n \geq 1000$. Best results are obtained using the first 100 random numbers out of 1009. Let's therefore use 1009 as a default buffer size. The user can call **tao_random_luxury** (100) him/herself:

255a \langle Parameters in **tao_random_numbers** 254a $\rangle + \equiv$ (273d 274a) \triangleleft 254a
integer, parameter, private :: **DEFAULT_BUFFER_SIZE** = 1009

Since users are not expected to call **generate** directly, we do *not* check for $n \geq K$ and assume that the caller knows what (s)he's doing ...

255b \langle Implementation of 30-bit **tao_random_numbers** 255b $\rangle \equiv$ (273d) 255e \triangleright
pure subroutine **generate** (a, state)
integer(kind=tao_i32), dimension(:), intent(inout) :: a, state
integer :: j, n
n = size (a)
 \langle Load a and refresh state 255d \rangle
end subroutine **generate**

255c \langle Declaration of **tao_random_numbers** 255c $\rangle \equiv$ (273d 274a) 259b \triangleright
private :: **generate**

state(1:K) is already set up properly:

255d \langle Load a and refresh state 255d $\rangle \equiv$ (255b) 255e \triangleright
a(1:K) = state(1:K)

The remaining $n - K$ random numbers can be gotten directly from the recursion (C.1). Note that Fortran90's **modulo** intrinsic does the right thing, since it guarantees (unlike Fortran77's **mod**) that $0 \leq \text{modulo}(a, m) < a$ if $m > 0$).

255e \langle Load a and refresh state 255d $\rangle + \equiv$ (255b) \triangleleft 255d 255f \triangleright
do j = K+1, n
a(j) = modulo (a(j-K) - a(j-L), M)
end do

Do the recursion (C.1) K more times to prepare **state(1:K)** for the next invocation of **generate**.

255f \langle Load a and refresh state 255d $\rangle + \equiv$ (255b) \triangleleft 255e
state(1:L) = modulo (a(n+1-K:n+L-K) - a(n+1-L:n), M)
do j = L+1, K
state(j) = modulo (a(n+j-K) - state(j-L), M)
end do

C.2.2 Initialization of 30-bit Random Numbers

The non-trivial and most beautiful part is the algorithm to initialize the random number generator state `state` with the first K numbers. I haven't studied algebra over finite fields in sufficient depth to consider the mathematics behind it straightforward. The commentary below is rather verbose and reflects my understanding of DEK's rather terse remarks (solution to exercise 3.6-9 [16]).

256a *<Implementation of tao_random_numbers 256a>*≡ (273d 274a) 256c>
`subroutine seed_static (seed)`
`integer, optional, intent(in) :: seed`
`call seed_stateless (s_state, seed)`
`s_virginal = .false.`
`s_last = size (s_buffer)`
`end subroutine seed_static`

The static version of `tao_random_raw_state`:

256b *<Variables in 30-bit tao_random_numbers 254c>*+≡ (273d) <254c 275c>
`integer(kind=tao_i32), dimension(K), save, private :: s_state`
`logical, save, private :: s_virginal = .true.`

256c *<Implementation of tao_random_numbers 256a>*+≡ (273d 274a) <256a 256d>
`elemental subroutine seed_raw_state (s, seed)`
`type(tao_random_raw_state), intent(inout) :: s`
`integer, optional, intent(in) :: seed`
`call seed_stateless (s%x, seed)`
`end subroutine seed_raw_state`

256d *<Implementation of tao_random_numbers 256a>*+≡ (273d 274a) <256c 262d>
`elemental subroutine seed_state (s, seed)`
`type(tao_random_state), intent(inout) :: s`
`integer, optional, intent(in) :: seed`
`call seed_raw_state (s%state, seed)`
`s%last = size (s%buffer)`
`end subroutine seed_state`

This incarnation of the procedure is pure.

256e *<Implementation of 30-bit tao_random_numbers 255b>*+≡ (273d) <255b 267c>
`pure subroutine seed_stateless (state, seed)`
`integer(kind=tao_i32), dimension(:), intent(out) :: state`
`integer, optional, intent(in) :: seed`
<Parameters local to tao_random_seed 257a>
`integer :: seed_value, j, s, t`
`integer(kind=tao_i32), dimension(2*K-1) :: x`
<Set up seed_value from seed or DEFAULT_SEED 257c>

```

    <Bootstrap the x buffer 257d>
    <Set up s and t 257f>
    do
        < $p(z) \rightarrow p(z)^2 \pmod{z^K + z^L + 1}$  258a>
        < $p(z) \rightarrow zp(z) \pmod{z^K + z^L + 1}$  258c>
        <Shift s or t and exit if  $t \leq 0$  258d>
    end do
    <Fill state from x 258e>
    <Warm up state 258f>
end subroutine seed_stateless

```

Any default will do

```

257a <Parameters local to tao_random_seed 257a>≡ (256e 260a) 257b>
    integer, parameter :: DEFAULT_SEED = 0

```

These must not be changed:

```

257b <Parameters local to tao_random_seed 257a>+≡ (256e 260a) <257a
    integer, parameter :: MAX_SEED = 2**30 - 3
    integer, parameter :: TT = 70

```

```

257c <Set up seed_value from seed or DEFAULT_SEED 257c>≡ (256e 260a)
    if (present (seed)) then
        seed_value = modulo (seed, MAX_SEED + 1)
    else
        seed_value = DEFAULT_SEED
    end if

```

Fill the array x_1, \dots, x_K with even integers, shifted cyclically by 29 bits.

```

257d <Bootstrap the x buffer 257d>≡ (256e) 257e>
    s = seed_value - modulo (seed_value, 2) + 2
    do j = 1, K
        x(j) = s
        s = 2*s
        if (s >= M) then
            s = s - M + 2
        end if
    end do
    x(K+1:2*K-1) = 0

```

Make x_2 (and only x_2) odd:

```

257e <Bootstrap the x buffer 257d>+≡ (256e) <257d
    x(2) = x(2) + 1

```

```

257f <Set up s and t 257f>≡ (256e 260a)
    s = seed_value
    t = TT - 1

```

Consider the polynomial

$$p(z) = \sum_{n=1}^K x_n z^{n-1} = x_K z^{K-1} + \dots + x_2 z + x_1 \quad (\text{C.2})$$

We have $p(z)^2 = p(z^2) \pmod{2}$ because cross terms have an even coefficient and $x_n^2 = x_n \pmod{2}$. Therefore we can square the polynomial by shifting the coefficients. The coefficients for $n > K$ will be reduced.

258a $\langle p(z) \rightarrow p(z)^2 \pmod{z^K + z^L + 1} \text{ 258a} \rangle \equiv$ (256e) 258b \triangleright
 $\mathbf{x}(3:2*K-1:2) = \mathbf{x}(2:K)$
 $\mathbf{x}(2:2*K-2:2) = 0$

Let's return to the coefficients for $n > K$ generated by the shifting above. Subtract $z^n(z^K + z^L + 1) = z^n z^K (1 + z^{-(K-L)} + z^{-K})$. The coefficient of $z^n z^K$ is left alone, because it doesn't belong to $p(z)$ anyway.

258b $\langle p(z) \rightarrow p(z)^2 \pmod{z^K + z^L + 1} \text{ 258a} \rangle + \equiv$ (256e) \triangleleft 258a
do $\mathbf{j} = 2*K-1, K+1, -1$
 $\mathbf{x}(\mathbf{j}-(K-L)) = \text{modulo } (\mathbf{x}(\mathbf{j}-(K-L))-\mathbf{x}(\mathbf{j}), \mathbf{M})$
 $\mathbf{x}(\mathbf{j}-K) = \text{modulo } (\mathbf{x}(\mathbf{j}-K)-\mathbf{x}(\mathbf{j}), \mathbf{M})$
end do

258c $\langle p(z) \rightarrow zp(z) \pmod{z^K + z^L + 1} \text{ 258c} \rangle \equiv$ (256e)
if (modulo (s, 2) == 1) then
 $\mathbf{x}(2:K+1) = \mathbf{x}(1:K)$
 $\mathbf{x}(1) = \mathbf{x}(K+1)$
 $\mathbf{x}(L+1) = \text{modulo } (\mathbf{x}(L+1) - \mathbf{x}(K+1), \mathbf{M})$
end if

258d $\langle \text{Shift s or t and exit if } t \leq 0 \text{ 258d} \rangle \equiv$ (256e 260a)
if (s /= 0) then
 $\mathbf{s} = \mathbf{s} / 2$
else
 $\mathbf{t} = \mathbf{t} - 1$
end if
if (t <= 0) then
exit
end if

258e $\langle \text{Fill state from x 258e} \rangle \equiv$ (256e 260a)
 $\text{state}(1:K-L) = \mathbf{x}(L+1:K)$
 $\text{state}(K-L+1:K) = \mathbf{x}(1:L)$

258f $\langle \text{Warm up state 258f} \rangle \equiv$ (256e 260a)
do $\mathbf{j} = 1, 10$
call generate (x, state)
end do

259a *⟨Interfaces of tao_random_numbers 259a⟩*≡ (273d 274a) 262a>
 interface **tao_random_seed**
 module procedure *⟨Specific procedures for tao_random_seed 259c⟩*
 end interface

259b *⟨Declaration of tao_random_numbers 255c⟩*+≡ (273d 274a) <255c 260b>
 private :: *⟨Specific procedures for tao_random_seed 259c⟩*

259c *⟨Specific procedures for tao_random_seed 259c⟩*≡ (259)
seed_static, seed_state, seed_raw_state

C.2.3 Generation of 52-bit Random Numbers

$$X_j = (X_{j-K} + X_{j-L}) \mod 1 \quad (\text{C.3})$$

259d *⟨Variables in 52-bit tao_random_numbers 259d⟩*≡ (274a) 259e>
 real(kind=tao_r64), parameter, private :: **M = 1.0_tao_r64**

The state of the internal routines

259e *⟨Variables in 52-bit tao_random_numbers 259d⟩*+≡ (274a) <259d 276b>
 real(kind=tao_r64), dimension(**K**), save, private :: **s_state**
 logical, save, private :: **s_virginal = .true.**

259f *⟨Implementation of 52-bit tao_random_numbers 259f⟩*≡ (274a) 260a>
 pure **subroutine generate** (a, state)
 real(kind=tao_r64), dimension(:), intent(inout) :: a
 real(kind=tao_r64), dimension(:), intent(inout) :: state
 integer :: **j, n**
 n = size (a)
 ⟨Load 52-bit a and refresh state 259g⟩
 end **subroutine generate**

That's almost identical to the 30-bit version, except that the relative sign is flipped:

259g *⟨Load 52-bit a and refresh state 259g⟩*≡ (259f)
 a(1:**K**) = state(1:**K**)
 do **j = K+1, n**
 a(**j**) = modulo (a(**j-K**) + a(**j-L**), **M**)
 end do
 state(1:**L**) = modulo (a(n+1-**K**:n+**L-K**) + a(n+1-**L**:n), **M**)
 do **j = L+1, K**
 state(**j**) = modulo (a(n+**j-K**) + state(**j-L**), **M**)
 end do

C.2.4 Initialization of 52-bit Random Numbers

This incarnation of the procedure is pure.

```

260a  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <259f 268a>
      pure subroutine seed_stateless (state, seed)
        real(kind=tao_r64), dimension(:), intent(out) :: state
        integer, optional, intent(in) :: seed
        <Parameters local to tao_random_seed 257a>
        <Variables local to 52-bit tao_random_seed 260c>
        <Set up seed_value from seed or DEFAULT_SEED 257c>
        <Bootstrap the 52-bit x buffer 260e>
        <Set up s and t 257f>
        do
          <52-bit  $p(z) \rightarrow p(z)^2$  (modulo  $z^K + z^L + 1$ ) 261a>
          <52-bit  $p(z) \rightarrow zp(z)$  (modulo  $z^K + z^L + 1$ ) 261c>
          <Shift s or t and exit if t ≤ 0 258d>
        end do
        <Fill state from x 258e>
        <Warm up state 258f>
      end subroutine seed_stateless

260b  <Declaration of tao_random_numbers 255c>+≡      (273d 274a) <259b 262b>
      private :: seed_stateless

260c  <Variables local to 52-bit tao_random_seed 260c>≡      (260a) 260d>
      real(kind=tao_r64), parameter :: ULP = 2.0_tao_r64**(-52)

260d  <Variables local to 52-bit tao_random_seed 260c>+≡      (260a) <260c
      real(kind=tao_r64), dimension(2*K-1) :: x
      real(kind=tao_r64) :: ss
      integer :: seed_value, t, s, j

260e  <Bootstrap the 52-bit x buffer 260e>≡      (260a) 260f>
      ss = 2*ULP * (seed_value + 2)
      do j = 1, K
        x(j) = ss
        ss = 2*ss
        if (ss >= 1) then
          ss = ss - 1 + 2*ULP
        end if
      end do
      x(K+1:2*K-1) = 0.0

260f  <Bootstrap the 52-bit x buffer 260e>+≡      (260a) <260e
      x(2) = x(2) + ULP

```

261a $\langle 52\text{-bit } p(z) \rightarrow p(z)^2 \text{ (modulo } z^K + z^L + 1) \text{ 261a)} \equiv$ (260a) 261b \triangleright
 $\mathbf{x}(3:2*K-1:2) = \mathbf{x}(2:K)$
 $\mathbf{x}(2:2*K-2:2) = 0$

This works because $2*K-1$ is odd

261b $\langle 52\text{-bit } p(z) \rightarrow p(z)^2 \text{ (modulo } z^K + z^L + 1) \text{ 261a)} + \equiv$ (260a) \triangleleft 261a
do $\mathbf{j} = 2*K-1, K+1, -1$
 $\mathbf{x}(\mathbf{j}-(K-L)) = \text{modulo } (\mathbf{x}(\mathbf{j}-(K-L)) + \mathbf{x}(\mathbf{j}), M)$
 $\mathbf{x}(\mathbf{j}-K) = \text{modulo } (\mathbf{x}(\mathbf{j}-K) + \mathbf{x}(\mathbf{j}), M)$
end do

261c $\langle 52\text{-bit } p(z) \rightarrow zp(z) \text{ (modulo } z^K + z^L + 1) \text{ 261c)} \equiv$ (260a)
if (modulo (s, 2) == 1) THEN
 $\mathbf{x}(2:K+1) = \mathbf{x}(1:K)$
 $\mathbf{x}(1) = \mathbf{x}(K+1)$
 $\mathbf{x}(L+1) = \text{modulo } (\mathbf{x}(L+1) + \mathbf{x}(K+1), M)$
end if

C.3 The State

261d $\langle \text{Declaration of 30-bit tao_random_numbers types 261d)} \equiv$ (273d) 261e \triangleright
type, public :: tao_random_raw_state
private
integer(kind=tao_i32), dimension(K) :: x
end type tao_random_raw_state

261e $\langle \text{Declaration of 30-bit tao_random_numbers types 261d)} + \equiv$ (273d) \triangleleft 261d
type, public :: tao_random_state
private
type(tao_random_raw_state) :: state
integer(kind=tao_i32), dimension(:), pointer :: buffer => null ()
integer :: buffer_end, last
end type tao_random_state

261f $\langle \text{Declaration of 52-bit tao_random_numbers types 261f)} \equiv$ (274a) 261g \triangleright
type, public :: tao_random_raw_state
private
real(kind=tao_r64), dimension(K) :: x
end type tao_random_raw_state

261g $\langle \text{Declaration of 52-bit tao_random_numbers types 261f)} + \equiv$ (274a) \triangleleft 261f
type, public :: tao_random_state
private
type(tao_random_raw_state) :: state
real(kind=tao_r64), dimension(:), pointer :: buffer => null ()

```

        integer :: buffer_end, last
end type tao_random_state

```

C.3.1 Creation

262a *<Interfaces of tao_random_numbers 259a>+≡ (273d 274a) <259a 263e>*

```

        interface tao_random_create
            module procedure <Specific procedures for tao_random_create 262c>
        end interface

```

262b *<Declaration of tao_random_numbers 255c>+≡ (273d 274a) <260b 264a>*

```

        private :: <Specific procedures for tao_random_create 262c>

```

262c *<Specific procedures for tao_random_create 262c>≡ (262)*

```

        create_state_from_seed, create_raw_state_from_seed, &
        create_state_from_state, create_raw_state_from_state, &
        create_state_from_raw_state, create_raw_state_from_raw_st

```

There are no procedures for copying the state of the static generator to or from an explicit `tao_random_state`. Users needing this functionality can be expected to handle explicit states anyway. Since the direction of the copying can not be obvious from the type of the argument, such functions would spoil the simplicity of the generic procedure interface.

262d *<Implementation of tao_random_numbers 256a>+≡ (273d 274a) <256d 262e>*

```

        elemental subroutine create_state_from_seed (s, seed, buffer_size)
            type(tao_random_state), intent(out) :: s
            integer, intent(in) :: seed
            integer, intent(in), optional :: buffer_size
            call create_raw_state_from_seed (s%state, seed)
            if (present (buffer_size)) then
                s%buffer_end = max (buffer_size, K)
            else
                s%buffer_end = DEFAULT_BUFFER_SIZE
            end if
            allocate (s%buffer(s%buffer_end))
            call tao_random_flush (s)
        end subroutine create_state_from_seed

```

262e *<Implementation of tao_random_numbers 256a>+≡ (273d 274a) <262d 263a>*

```

        elemental subroutine create_state_from_state (s, state)
            type(tao_random_state), intent(out) :: s
            type(tao_random_state), intent(in) :: state
            call create_raw_state_from_raw_st (s%state, state%state)
            allocate (s%buffer(size(state%buffer)))
            call tao_random_copy (s, state)

```

```

end subroutine create_state_from_state

263a <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <262e 263b>
elemental subroutine create_state_from_raw_state &
    (s, raw_state, buffer_size)
    type(tao_random_state), intent(out) :: s
    type(tao_random_raw_state), intent(in) :: raw_state
    integer, intent(in), optional :: buffer_size
    call create_raw_state_from_raw_st (s%state, raw_state)
    if (present (buffer_size)) then
        s%buffer_end = max (buffer_size, K)
    else
        s%buffer_end = DEFAULT_BUFFER_SIZE
    end if
    allocate (s%buffer(s%buffer_end))
    call tao_random_flush (s)
end subroutine create_state_from_raw_state

263b <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <263a 263c>
elemental subroutine create_raw_state_from_seed (s, seed)
    type(tao_random_raw_state), intent(out) :: s
    integer, intent(in) :: seed
    call seed_raw_state (s, seed)
end subroutine create_raw_state_from_seed

263c <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <263b 263d>
elemental subroutine create_raw_state_from_state (s, state)
    type(tao_random_raw_state), intent(out) :: s
    type(tao_random_state), intent(in) :: state
    call copy_state_to_raw_state (s, state)
end subroutine create_raw_state_from_state

263d <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <263c 264b>
elemental subroutine create_raw_state_from_raw_st (s, raw_state)
    type(tao_random_raw_state), intent(out) :: s
    type(tao_random_raw_state), intent(in) :: raw_state
    call copy_raw_state (s, raw_state)
end subroutine create_raw_state_from_raw_st

```

C.3.2 Destruction

```

263e <Interfaces of tao_random_numbers 259a>+≡ (273d 274a) <262a 264d>
interface tao_random_destroy
    module procedure destroy_state, destroy_raw_state
end interface

```


264a *<Declaration of tao_random_numbers 255c>+≡ (273d 274a) <262b 264f>*
 private :: destroy_state, destroy_raw_state

264b *<Implementation of tao_random_numbers 256a>+≡ (273d 274a) <263d 264c>*
 elemental subroutine destroy_state (s)
 type(tao_random_state), intent(inout) :: s
 deallocate (s%buffer)
 end subroutine destroy_state

Currently, this is a no-op, but we might need a non-trivial destruction method in the future

264c *<Implementation of tao_random_numbers 256a>+≡ (273d 274a) <264b 264h>*
 elemental subroutine destroy_raw_state (s)
 type(tao_random_raw_state), intent(inout) :: s
 end subroutine destroy_raw_state

C.3.3 Copying

264d *<Interfaces of tao_random_numbers 259a>+≡ (273d 274a) <263e 264e>*
 interface tao_random_copy
 module procedure *<Specific procedures for tao_random_copy 264g>*
 end interface

264e *<Interfaces of tao_random_numbers 259a>+≡ (273d 274a) <264d 265e>*
 interface assignment(=)
 module procedure *<Specific procedures for tao_random_copy 264g>*
 end interface

264f *<Declaration of tao_random_numbers 255c>+≡ (273d 274a) <264a 266a>*
 public :: assignment(=)
 private :: *<Specific procedures for tao_random_copy 264g>*

264g *<Specific procedures for tao_random_copy 264g>≡ (264)*
 copy_state, copy_raw_state, &
 copy_raw_state_to_state, copy_state_to_raw_state

264h *<Implementation of tao_random_numbers 256a>+≡ (273d 274a) <264c 265a>*
 elemental subroutine copy_state (lhs, rhs)
 type(tao_random_state), intent(inout) :: lhs
 type(tao_random_state), intent(in) :: rhs
 call copy_raw_state (lhs%state, rhs%state)
 if (size (lhs%buffer) /= size (rhs%buffer)) then
 deallocate (lhs%buffer)
 allocate (lhs%buffer(size(rhs%buffer)))
 end if
 lhs%buffer = rhs%buffer

```

    lhs%buffer_end = rhs%buffer_end
    lhs%last = rhs%last
end subroutine copy_state
265a <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <264h 265b>
    elemental subroutine copy_raw_state (lhs, rhs)
        type(tao_random_raw_state), intent(out) :: lhs
        type(tao_random_raw_state), intent(in) :: rhs
        lhs%x = rhs%x
    end subroutine copy_raw_state
265b <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <265a 265c>
    elemental subroutine copy_raw_state_to_state (lhs, rhs)
        type(tao_random_state), intent(inout) :: lhs
        type(tao_random_raw_state), intent(in) :: rhs
        call copy_raw_state (lhs%state, rhs)
        call tao_random_flush (lhs)
    end subroutine copy_raw_state_to_state
265c <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <265b 265d>
    elemental subroutine copy_state_to_raw_state (lhs, rhs)
        type(tao_random_raw_state), intent(out) :: lhs
        type(tao_random_state), intent(in) :: rhs
        call copy_raw_state (lhs, rhs%state)
    end subroutine copy_state_to_raw_state

```

C.3.4 Flushing

```

265d <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <265c 266d>
    elemental subroutine tao_random_flush (s)
        type(tao_random_state), intent(inout) :: s
        s%last = size (s%buffer)
    end subroutine tao_random_flush

```

C.3.5 Input and Output

```

265e <Interfaces of tao_random_numbers 259a>+≡ (273d 274a) <264e 266b>
    interface tao_random_write
        module procedure &
            write_state_unit, write_state_name, &
            write_raw_state_unit, write_raw_state_name
    end interface

```

```

266a  <Declaration of tao_random_numbers 255c>+≡ (273d 274a) <264f 266c>
      private :: write_state_unit, write_state_name
      private :: write_raw_state_unit, write_raw_state_name

266b  <Interfaces of tao_random_numbers 259a>+≡ (273d 274a) <265e 270b>
      interface tao_random_read
        module procedure &
          read_state_unit, read_state_name, &
          read_raw_state_unit, read_raw_state_name
      end interface

266c  <Declaration of tao_random_numbers 255c>+≡ (273d 274a) <266a 269b>
      private :: read_state_unit, read_state_name
      private :: read_raw_state_unit, read_raw_state_name

266d  <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <265d 266e>
      subroutine write_state_unit (s, unit)
        type(tao_random_state), intent(in) :: s
        integer, intent(in) :: unit
        write (unit = unit, fmt = *) "BEGIN TAO_RANDOM_STATE"
        call write_raw_state_unit (s%state, unit)
        write (unit = unit, fmt = "(2(1x,a16,1x,i10/),1x,a16,1x,i10)") &
          "BUFFER_SIZE", size (s%buffer), &
          "BUFFER_END", s%buffer_end, &
          "LAST", s%last
        write (unit = unit, fmt = *) "BEGIN BUFFER"
        call write_state_array (s%buffer, unit)
        write (unit = unit, fmt = *) "END BUFFER"
        write (unit = unit, fmt = *) "END TAO_RANDOM_STATE"
      end subroutine write_state_unit

266e  <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <266d 267a>
      subroutine read_state_unit (s, unit)
        type(tao_random_state), intent(inout) :: s
        integer, intent(in) :: unit
        integer :: buffer_size
        read (unit = unit, fmt = *)
        call read_raw_state_unit (s%state, unit)
        read (unit = unit, fmt = "(2(1x,16x,1x,i10/),1x,16x,1x,i10)") &
          buffer_size, s%buffer_end, s%last
        read (unit = unit, fmt = *)
        if (buffer_size /= size (s%buffer)) then
          deallocate (s%buffer)
          allocate (s%buffer(buffer_size))
        end if
        call read_state_array (s%buffer, unit)

```

```

        read (unit = unit, fmt = *)
        read (unit = unit, fmt = *)
    end subroutine read_state_unit
267a <Implementation of tao_random_numbers 256a>+≡      (273d 274a) <266e 267b>
    subroutine write_raw_state_unit (s, unit)
        type(tao_random_raw_state), intent(in) :: s
        integer, intent(in) :: unit
        write (unit = unit, fmt = *) "BEGIN TAO_RANDOM_RAW_STATE"
        call write_state_array (s%x, unit)
        write (unit = unit, fmt = *) "END TAO_RANDOM_RAW_STATE"
    end subroutine write_raw_state_unit
267b <Implementation of tao_random_numbers 256a>+≡      (273d 274a) <267a 268e>
    subroutine read_raw_state_unit (s, unit)
        type(tao_random_raw_state), intent(inout) :: s
        integer, intent(in) :: unit
        read (unit = unit, fmt = *)
        call read_state_array (s%x, unit)
        read (unit = unit, fmt = *)
    end subroutine read_raw_state_unit
267c <Implementation of 30-bit tao_random_numbers 255b>+≡      (273d) <256e 267e>
    subroutine write_state_array (a, unit)
        integer(kind=tao_i32), dimension(:), intent(in) :: a
        integer, intent(in) :: unit
        integer :: i
        do i = 1, size (a)
            write (unit = unit, fmt = "(1x,i10,1x,i10)") i, a(i)
        end do
    end subroutine write_state_array
267d <Declaration of 30-bit tao_random_numbers 267d>≡      (273d) 267f>
    private :: write_state_array
267e <Implementation of 30-bit tao_random_numbers 255b>+≡      (273d) <267c 270d>
    subroutine read_state_array (a, unit)
        integer(kind=tao_i32), dimension(:), intent(inout) :: a
        integer, intent(in) :: unit
        integer :: i, idum
        do i = 1, size (a)
            read (unit = unit, fmt = *) idum, a(i)
        end do
    end subroutine read_state_array
267f <Declaration of 30-bit tao_random_numbers 267d>+≡      (273d) <267d 281b>
    private :: read_state_array

```

Reading and writing 52-bit floating point numbers accurately is beyond most Fortran runtime libraries. Their job is simplified considerably if we rescale by 2^{52} before writing. Then the temptation to truncate will not be as overwhelming as before ...

- 268a \langle Implementation of 52-bit tao_random_numbers 259f $\rangle + \equiv$ (274a) \langle 260a 268c \rangle

```

subroutine write_state_array (a, unit)
  real(kind=tao_r64), dimension(:), intent(in) :: a
  integer, intent(in) :: unit
  integer :: i
  do i = 1, size (a)
    write (unit = unit, fmt = "(1x,i10,1x,f30.0)") i, 2.0_tao_r64**52 * a(i)
  end do
end subroutine write_state_array

```

268b \langle Declaration of 52-bit tao_random_numbers 268b $\rangle \equiv$ (274a) 268d \rangle

```

private :: write_state_array

```

268c \langle Implementation of 52-bit tao_random_numbers 259f $\rangle + \equiv$ (274a) \langle 268a 272b \rangle

```

subroutine read_state_array (a, unit)
  real(kind=tao_r64), dimension(:), intent(inout) :: a
  integer, intent(in) :: unit
  real(kind=tao_r64) :: x
  integer :: i, idum
  do i = 1, size (a)
    read (unit = unit, fmt = *) idum, x
    a(i) = 2.0_tao_r64**(-52) * x
  end do
end subroutine read_state_array

```

268d \langle Declaration of 52-bit tao_random_numbers 268b $\rangle + \equiv$ (274a) \langle 268b 281f \rangle

```

private :: read_state_array

```

268e \langle Implementation of tao_random_numbers 256a $\rangle + \equiv$ (273d 274a) \langle 267b 269c \rangle

```

subroutine find_free_unit (u, iostat)
  integer, intent(out) :: u
  integer, intent(out), optional :: iostat
  logical :: exists, is_open
  integer :: i, status
  do i = MIN_UNIT, MAX_UNIT
    inquire (unit = i, exist = exists, opened = is_open, &
      iostat = iostat)
    if (status == 0) then
      if (exists .and. .not. is_open) then
        u = i
        if (present (iostat)) then

```

```

        iostat = 0
    end if
    return
end if
end do
if (present (iostat)) then
    iostat = -1
end if
u = -1
end subroutine find_free_unit

```

269a *<Variables in tao_random_numbers 269a>*≡ (273d 274a)

```
integer, parameter, private :: MIN_UNIT = 11, MAX_UNIT = 99
```

269b *<Declaration of tao_random_numbers 255c>*+≡ (273d 274a) <266c 270c>

```
private :: find_free_unit
```

269c *<Implementation of tao_random_numbers 256a>*+≡ (273d 274a) <268e 269d>

```

subroutine write_state_name (s, name)
    type(tao_random_state), intent(in) :: s
    character(len=*), intent(in) :: name
    integer :: unit
    call find_free_unit (unit)
    open (unit = unit, action = "write", status = "replace", file = name)
    call write_state_unit (s, unit)
    close (unit = unit)
end subroutine write_state_name

```

269d *<Implementation of tao_random_numbers 256a>*+≡ (273d 274a) <269c 269e>

```

subroutine write_raw_state_name (s, name)
    type(tao_random_raw_state), intent(in) :: s
    character(len=*), intent(in) :: name
    integer :: unit
    call find_free_unit (unit)
    open (unit = unit, action = "write", status = "replace", file = name)
    call write_raw_state_unit (s, unit)
    close (unit = unit)
end subroutine write_raw_state_name

```

269e *<Implementation of tao_random_numbers 256a>*+≡ (273d 274a) <269d 270a>

```

subroutine read_state_name (s, name)
    type(tao_random_state), intent(inout) :: s
    character(len=*), intent(in) :: name
    integer :: unit

```

```

        call find_free_unit (unit)
        open (unit = unit, action = "read", status = "old", file = name)
        call read_state_unit (s, unit)
        close (unit = unit)
    end subroutine read_state_name
270a <Implementation of tao_random_numbers 256a>+≡      (273d 274a) <269e 281h>
    subroutine read_raw_state_name (s, name)
        type(tao_random_raw_state), intent(inout) :: s
        character(len=*), intent(in) :: name
        integer :: unit
        call find_free_unit (unit)
        open (unit = unit, action = "read", status = "old", file = name)
        call read_raw_state_unit (s, unit)
        close (unit = unit)
    end subroutine read_raw_state_name

```

C.3.6 Marshaling and Unmarshaling

Note that we can not use the transfer intrinsic function for marshalling types that contain pointers that substitute for allocatable array components. transfer will copy the pointers in this case and not where they point to!

```

270b <Interfaces of tao_random_numbers 259a>+≡      (273d 274a) <266b>
    interface tao_random_marshall_size
        module procedure marshal_state_size, marshal_raw_state_size
    end interface
    interface tao_random_marshall
        module procedure marshal_state, marshal_raw_state
    end interface
    interface tao_random_unmarshal
        module procedure unmarshal_state, unmarshal_raw_state
    end interface
270c <Declaration of tao_random_numbers 255c>+≡      (273d 274a) <269b 274b>
    public :: tao_random_marshall
    private :: marshal_state, marshal_raw_state
    public :: tao_random_marshall_size
    private :: marshal_state_size, marshal_raw_state_size
    public :: tao_random_unmarshal
    private :: unmarshal_state, unmarshal_raw_state
270d <Implementation of 30-bit tao_random_numbers 255b>+≡      (273d) <267e 271a>
    pure subroutine marshal_state (s, ibuf, dbuf)
        type(tao_random_state), intent(in) :: s

```

```

integer, dimension(:), intent(inout) :: ibuf
real(kind=tao_r64), dimension(:), intent(inout) :: dbuf
integer :: buf_size
buf_size = size (s%buffer)
ibuf(1) = s%buffer_end
ibuf(2) = s%last
ibuf(3) = buf_size
ibuf(4:3+buf_size) = s%buffer
call marshal_raw_state (s%state, ibuf(4+buf_size:), dbuf)
end subroutine marshal_state
271a <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <270d 271b>
pure subroutine marshal_state_size (s, iwords, dwords)
type(tao_random_state), intent(in) :: s
integer, intent(out) :: iwords, dwords
call marshal_raw_state_size (s%state, iwords, dwords)
iwords = iwords + 3 + size (s%buffer)
end subroutine marshal_state_size
271b <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <271a 271c>
pure subroutine unmarshal_state (s, ibuf, dbuf)
type(tao_random_state), intent(inout) :: s
integer, dimension(:), intent(in) :: ibuf
real(kind=tao_r64), dimension(:), intent(in) :: dbuf
integer :: buf_size
s%buffer_end = ibuf(1)
s%last = ibuf(2)
buf_size = ibuf(3)
s%buffer = ibuf(4:3+buf_size)
call unmarshal_raw_state (s%state, ibuf(4+buf_size:), dbuf)
end subroutine unmarshal_state
271c <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <271b 271d>
pure subroutine marshal_raw_state (s, ibuf, dbuf)
type(tao_random_raw_state), intent(in) :: s
integer, dimension(:), intent(inout) :: ibuf
real(kind=tao_r64), dimension(:), intent(inout) :: dbuf
ibuf(1) = size (s%x)
ibuf(2:1+size(s%x)) = s%x
end subroutine marshal_raw_state
271d <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <271c 272a>
pure subroutine marshal_raw_state_size (s, iwords, dwords)
type(tao_random_raw_state), intent(in) :: s
integer, intent(out) :: iwords, dwords
iwords = 1 + size (s%x)

```



```

        dwords = 0
    end subroutine marshal_raw_state_size
272a  <Implementation of 30-bit tao_random_numbers 255b>+≡      (273d) <271d 275a>
    pure subroutine unmarshal_raw_state (s, ibuf, dbuf)
        type(tao_random_raw_state), intent(inout) :: s
        integer, dimension(:), intent(in) :: ibuf
        real(kind=tao_r64), dimension(:), intent(in) :: dbuf
        integer :: buf_size
        buf_size = ibuf(1)
        s%x = ibuf(2:1+buf_size)
    end subroutine unmarshal_raw_state

272b  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <268c 272c>
    pure subroutine marshal_state (s, ibuf, dbuf)
        type(tao_random_state), intent(in) :: s
        integer, dimension(:), intent(inout) :: ibuf
        real(kind=tao_r64), dimension(:), intent(inout) :: dbuf
        integer :: buf_size
        buf_size = size (s%buffer)
        ibuf(1) = s%buffer_end
        ibuf(2) = s%last
        ibuf(3) = buf_size
        dbuf(1:buf_size) = s%buffer
        call marshal_raw_state (s%state, ibuf(4:), dbuf(buf_size+1:))
    end subroutine marshal_state

272c  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <272b 272d>
    pure subroutine marshal_state_size (s, iwords, dwords)
        type(tao_random_state), intent(in) :: s
        integer, intent(out) :: iwords, dwords
        call marshal_raw_state_size (s%state, iwords, dwords)
        iwords = iwords + 3
        dwords = dwords + size(s%buffer)
    end subroutine marshal_state_size

272d  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <272c 273a>
    pure subroutine unmarshal_state (s, ibuf, dbuf)
        type(tao_random_state), intent(inout) :: s
        integer, dimension(:), intent(in) :: ibuf
        real(kind=tao_r64), dimension(:), intent(in) :: dbuf
        integer :: buf_size
        s%buffer_end = ibuf(1)
        s%last = ibuf(2)
        buf_size = ibuf(3)
        s%buffer = dbuf(1:buf_size)

```

```

        call unmarshal_raw_state (s%state, ibuf(4:), dbuf(buf_size+1:))
    end subroutine unmarshal_state
273a  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <272d 273b>
    pure subroutine marshal_raw_state (s, ibuf, dbuf)
        type(tao_random_raw_state), intent(in) :: s
        integer, dimension(:), intent(inout) :: ibuf
        real(kind=tao_r64), dimension(:), intent(inout) :: dbuf
        ibuf(1) = size (s%x)
        dbuf(1:size(s%x)) = s%x
    end subroutine marshal_raw_state
273b  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <273a 273c>
    pure subroutine marshal_raw_state_size (s, iwords, dwords)
        type(tao_random_raw_state), intent(in) :: s
        integer, intent(out) :: iwords, dwords
        iwords = 1
        dwords = size (s%x)
    end subroutine marshal_raw_state_size
273c  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <273b 276a>
    pure subroutine unmarshal_raw_state (s, ibuf, dbuf)
        type(tao_random_raw_state), intent(inout) :: s
        integer, dimension(:), intent(in) :: ibuf
        real(kind=tao_r64), dimension(:), intent(in) :: dbuf
        integer :: buf_size
        buf_size = ibuf(1)
        s%x = dbuf(1:buf_size)
    end subroutine unmarshal_raw_state

```

C.4 High Level Routines

```

273d  <tao_random_numbers.f90 273d>≡
    ! tao_random_numbers.f90 --
    <Copyleft notice 1>
    module tao_random_numbers
        use kinds
        implicit none
        integer, parameter, private :: tao_i32 = selected_int_kind (9)
        integer, parameter, private :: tao_r64 = selected_real_kind (15)
        <Declaration of tao_random_numbers 255c>
        <Declaration of 30-bit tao_random_numbers 267d>
        <Interfaces of tao_random_numbers 259a>
        <Interfaces of 30-bit tao_random_numbers 280f>
    end module tao_random_numbers

```

```

    <Parameters in tao_random_numbers 254a>
    <Variables in tao_random_numbers 269a>
    <Variables in 30-bit tao_random_numbers 254c>
    <Declaration of 30-bit tao_random_numbers types 261d>
contains
    <Implementation of tao_random_numbers 256a>
    <Implementation of 30-bit tao_random_numbers 255b>
end module tao_random_numbers

274a <tao52_random_numbers.f90 274a>≡
    ! tao52_random_numbers.f90 --
    <Copyleft notice 1>
module tao52_random_numbers
    use kinds
    implicit none
    integer, parameter, private :: tao_i32 = selected_int_kind (9)
    integer, parameter, private :: tao_r64 = selected_real_kind (15)
    <Declaration of tao_random_numbers 255c>
    <Declaration of 52-bit tao_random_numbers 268b>
    <Interfaces of tao_random_numbers 259a>
    <Interfaces of 52-bit tao_random_numbers 281d>
    <Parameters in tao_random_numbers 254a>
    <Variables in tao_random_numbers 269a>
    <Variables in 52-bit tao_random_numbers 259d>
    <Declaration of 52-bit tao_random_numbers types 261f>
contains
    <Implementation of tao_random_numbers 256a>
    <Implementation of 52-bit tao_random_numbers 259f>
end module tao52_random_numbers

Ten functions are exported

274b <Declaration of tao_random_numbers 255c>+≡ (273d 274a) <270c
    public :: tao_random_number
    public :: tao_random_seed
    public :: tao_random_create
    public :: tao_random_destroy
    public :: tao_random_copy
    public :: tao_random_read
    public :: tao_random_write
    public :: tao_random_flush
    ! public :: tao_random_luxury
    public :: tao_random_test

```

C.4.1 Single Random Numbers

A random integer r with $0 \leq r < 2^{30} = 1073741824$:

275a \langle Implementation of 30-bit tao_random_numbers 255b $\rangle + \equiv$ (273d) \triangleleft 272a 275e \triangleright
 pure subroutine integer_stateless &
 (state, buffer, buffer_end, last, r)
 integer(kind=tao_i32), dimension(:), intent(inout) :: state, buffer
 integer, intent(in) :: buffer_end
 integer, intent(inout) :: last
 integer, intent(out) :: r
 integer, parameter :: NORM = 1
 \langle Body of tao_random. 275b \rangle*
 end subroutine integer_stateless

275b \langle Body of tao_random.* 275b $\rangle \equiv$ (275 276a)
 \langle Step last and reload buffer iff necessary 275d \rangle
 r = NORM * buffer(last)

The low level routine **generate** will fill an array a_1, \dots, a_n , which will be consumed and refilled like an input buffer. We need at least $n \geq K$ for the call to **generate**.

275c \langle Variables in 30-bit tao_random_numbers 254c $\rangle + \equiv$ (273d) \triangleleft 256b
 integer(kind=tao_i32), dimension(DEFAULT_BUFFER_SIZE), save, private :: s_buffer
 integer, save, private :: s_buffer_end = size(s_buffer)
 integer, save, private :: s_last = size(s_buffer)

Increment the index **last** and reload the array **buffer**, iff this buffer is exhausted. Throughout these routines, **last** will point to random number that has just been consumed. For the array filling routines below, this is simpler than pointing to the next waiting number.

275d \langle Step last and reload buffer iff necessary 275d $\rangle \equiv$ (275b)
 last = last + 1
 if (last > buffer_end) then
 call generate(buffer, state)
 last = 1
 end if

A random real $r \in [0, 1)$. This is almost identical to **tao_random_integer**, but we duplicate the code to avoid the function call overhead for speed.

275e \langle Implementation of 30-bit tao_random_numbers 255b $\rangle + \equiv$ (273d) \triangleleft 275a 276c \triangleright
 pure subroutine real_stateless (state, buffer, buffer_end, last, r)
 integer(kind=tao_i32), dimension(:), intent(inout) :: state, buffer
 integer, intent(in) :: buffer_end
 integer, intent(inout) :: last
 real(kind=default), intent(out) :: r

```

    real(kind=default), parameter :: NORM = 1.0_default / M
    <Body of tao_random.* 275b>
end subroutine real_stateless

```

A random real $r \in [0, 1)$.

276a <Implementation of 52-bit tao_random_numbers 259f>+≡ (274a) <273c 278a>
 pure subroutine real_stateless (state, buffer, buffer_end, last, r)
 real(kind=tao_r64), dimension(:), intent(inout) :: state, buffer
 integer, intent(in) :: buffer_end
 integer, intent(inout) :: last
 real(kind=default), intent(out) :: r
 integer, parameter :: NORM = 1
 <Body of tao_random.* 275b>
end subroutine real_stateless

The low level routine **generate** will fill an array a_1, \dots, a_N , which will be consumed and refilled like an input buffer.

276b <Variables in 52-bit tao_random_numbers 259d>+≡ (274a) <259e
 real(kind=tao_r64), dimension(DEFAULT_BUFFER_SIZE), save, private :: s_buffer
 integer, save, private :: s_buffer_end = size (s_buffer)
 integer, save, private :: s_last = size (s_buffer)

C.4.2 Arrays of Random Numbers

Fill the array j_1, \dots, j_ν with random integers $0 \leq j_i < 2^{30} = 1073741824$. This has to be done such that the underlying array length in **generate** is transparent to the user. At the same time we want to avoid the overhead of calling `tao_random_real` ν times.

276c <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <275e 277e>
 pure subroutine integer_array_stateless &
 (state, buffer, buffer_end, last, v, num)
 integer(kind=tao_i32), dimension(:), intent(inout) :: state, buffer
 integer, intent(in) :: buffer_end
 integer, intent(inout) :: last
 integer, dimension(:), intent(out) :: v
 integer, optional, intent(in) :: num
 integer, parameter :: NORM = 1
 <Body of tao_random.*_array 276d>
end subroutine integer_array_stateless

276d <Body of tao_random.*_array 276d>≡ (276–78)
 integer :: nu, done, todo, chunk
 <Set nu to num or size(v) 277a>
 <Prepare array buffer and done, todo, chunk 277b>

```

v(1:chunk) = NORM * buffer(last+1:last+chunk)
do
  <Update last, done and todo and set new chunk 277c>
  <Reload buffer or exit 277d>
  v(done+1:done+chunk) = NORM * buffer(1:chunk)
end do

```

277a <Set nu to num or size(v) 277a>≡ (276d)

```

if (present (num)) then
  nu = num
else
  nu = size (v)
end if

```

`last` is used as an offset into the buffer `buffer`, as usual. `done` is an offset into the target. We still have to process all `nu` numbers. The first chunk can only use what's left in the buffer.

277b <Prepare array buffer and done, todo, chunk 277b>≡ (276d)

```

if (last >= buffer_end) then
  call generate (buffer, state)
  last = 0
end if
done = 0
todo = nu
chunk = min (todo, buffer_end - last)

```

This logic is a bit weird, but after the first chunk, `todo` will either vanish (in which case we're done) or we have consumed all of the buffer and must reload. In any case we can pretend that the next chunk can use the whole buffer.

277c <Update last, done and todo and set new chunk 277c>≡ (276d)

```

last = last + chunk
done = done + chunk
todo = todo - chunk
chunk = min (todo, buffer_end)

```

277d <Reload buffer or exit 277d>≡ (276d)

```

if (chunk <= 0) then
  exit
end if
call generate (buffer, state)
last = 0

```

277e <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <276c 278b>

```

pure subroutine real_array_stateless &
  (state, buffer, buffer_end, last, v, num)

```

```

integer(kind=tao_i32), dimension(:), intent(inout) :: state, buffer
integer, intent(in) :: buffer_end
integer, intent(inout) :: last
real(kind=default), dimension(:), intent(out) :: v
integer, optional, intent(in) :: num
real(kind=default), parameter :: NORM = 1.0_default / M
  <Body of tao_random*_array 276d>
end subroutine real_array_stateless

```

Fill the array v_1, \dots, v_ν with uniform deviates $v_i \in [0, 1)$.

278a <Implementation of 52-bit tao_random_numbers 259f>+≡ (274a) <276a 278d>
 pure **subroutine real_array_stateless** &
 (state, buffer, **buffer_end**, **last**, v, num)
 real(kind=tao_r64), dimension(:), intent(inout) :: state, buffer
 integer, intent(in) :: **buffer_end**
 integer, intent(inout) :: **last**
 real(kind=default), dimension(:), intent(out) :: v
 integer, optional, intent(in) :: num
 integer, parameter :: NORM = 1
 <Body of tao_random*_array 276d>
end **subroutine real_array_stateless**

C.4.3 Procedures With Explicit *tao_random_state*

Unfortunately, this is very boring, but Fortran's lack of parametric polymorphism forces this duplication on us:

278b <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <277e 278c>
 elemental **subroutine integer_state** (s, r)
 type(**tao_random_state**), intent(inout) :: s
 integer, intent(out) :: r
 call **integer_stateless** (s%state%x, s%buffer, s%**buffer_end**, s%**last**, r)
end **subroutine integer_state**

278c <Implementation of 30-bit tao_random_numbers 255b>+≡ (273d) <278b 279a>
 elemental **subroutine real_state** (s, r)
 type(**tao_random_state**), intent(inout) :: s
 real(kind=default), intent(out) :: r
 call **real_stateless** (s%state%x, s%buffer, s%**buffer_end**, s%**last**, r)
end **subroutine real_state**

278d <Implementation of 52-bit tao_random_numbers 259f>+≡ (274a) <278a 279c>
 elemental **subroutine real_state** (s, r)
 type(**tao_random_state**), intent(inout) :: s
 real(kind=default), intent(out) :: r

```

        call real_stateless (s%state%x, s%buffer, s%buffer_end, s%last, r)
    end subroutine real_state

279a  <Implementation of 30-bit tao_random_numbers 255b>+≡      (273d) <278c 279b>
    pure subroutine integer_array_state (s, v, num)
        type(tao_random_state), intent(inout) :: s
        integer, dimension(:), intent(out) :: v
        integer, optional, intent(in) :: num
        call integer_array_stateless &
            (s%state%x, s%buffer, s%buffer_end, s%last, v, num)
    end subroutine integer_array_state

279b  <Implementation of 30-bit tao_random_numbers 255b>+≡      (273d) <279a 279e>
    pure subroutine real_array_state (s, v, num)
        type(tao_random_state), intent(inout) :: s
        real(kind=default), dimension(:), intent(out) :: v
        integer, optional, intent(in) :: num
        call real_array_stateless &
            (s%state%x, s%buffer, s%buffer_end, s%last, v, num)
    end subroutine real_array_state

279c  <Implementation of 52-bit tao_random_numbers 259f>+≡      (274a) <278d 280b>
    pure subroutine real_array_state (s, v, num)
        type(tao_random_state), intent(inout) :: s
        real(kind=default), dimension(:), intent(out) :: v
        integer, optional, intent(in) :: num
        call real_array_stateless &
            (s%state%x, s%buffer, s%buffer_end, s%last, v, num)
    end subroutine real_array_state

```

C.4.4 Static Procedures

First make sure that `tao_random_seed` has been called to initialize the generator state:

```

279d  <Initialize a virginal random number generator 279d>≡      (279 280 282)
    if (s_virginal) then
        call tao_random_seed ()
    end if

279e  <Implementation of 30-bit tao_random_numbers 255b>+≡      (273d) <279b 280a>
    subroutine integer_static (r)
        integer, intent(out) :: r
        <Initialize a virginal random number generator 279d>
        call integer_stateless (s_state, s_buffer, s_buffer_end, s_last, r)
    end subroutine integer_static

```


280a *<Implementation of 30-bit tao_random_numbers 255b>+≡* (273d) *<279e 280c>*
`subroutine real_static (r)`
`real(kind=default), intent(out) :: r`
<Initialize a virginal random number generator 279d>
`call real_stateless (s_state, s_buffer, s_buffer_end, s_last, r)`
`end subroutine real_static`

280b *<Implementation of 52-bit tao_random_numbers 259f>+≡* (274a) *<279c 280e>*
`subroutine real_static (r)`
`real(kind=default), intent(out) :: r`
<Initialize a virginal random number generator 279d>
`call real_stateless (s_state, s_buffer, s_buffer_end, s_last, r)`
`end subroutine real_static`

280c *<Implementation of 30-bit tao_random_numbers 255b>+≡* (273d) *<280a 280d>*
`subroutine integer_array_static (v, num)`
`integer, dimension(:), intent(out) :: v`
`integer, optional, intent(in) :: num`
<Initialize a virginal random number generator 279d>
`call integer_array_stateless &`
`(s_state, s_buffer, s_buffer_end, s_last, v, num)`
`end subroutine integer_array_static`

280d *<Implementation of 30-bit tao_random_numbers 255b>+≡* (273d) *<280c 283d>*
`subroutine real_array_static (v, num)`
`real(kind=default), dimension(:), intent(out) :: v`
`integer, optional, intent(in) :: num`
<Initialize a virginal random number generator 279d>
`call real_array_stateless &`
`(s_state, s_buffer, s_buffer_end, s_last, v, num)`
`end subroutine real_array_static`

280e *<Implementation of 52-bit tao_random_numbers 259f>+≡* (274a) *<280b 285b>*
`subroutine real_array_static (v, num)`
`real(kind=default), dimension(:), intent(out) :: v`
`integer, optional, intent(in) :: num`
<Initialize a virginal random number generator 279d>
`call real_array_stateless &`
`(s_state, s_buffer, s_buffer_end, s_last, v, num)`
`end subroutine real_array_static`

C.4.5 Generic Procedures

280f *<Interfaces of 30-bit tao_random_numbers 280f>≡* (273d)
`interface tao_random_number`

```

        module procedure <Specific procedures for 30-bit tao_random_number 281a>
    end interface

281a <Specific procedures for 30-bit tao_random_number 281a>≡ (280f 281c)
    integer_static, integer_state, &
    integer_array_static, integer_array_state, &
    real_static, real_state, real_array_static, real_array_state
    These are not exported
281b <Declaration of 30-bit tao_random_numbers 267d>+≡ (273d) <267f 281c>
    private :: &
        integer_stateless, integer_array_stateless, &
        real_stateless, real_array_stateless
281c <Declaration of 30-bit tao_random_numbers 267d>+≡ (273d) <281b>
    private :: <Specific procedures for 30-bit tao_random_number 281a>
281d <Interfaces of 52-bit tao_random_numbers 281d>≡ (274a)
    interface tao_random_number
        module procedure <Specific procedures for 52-bit tao_random_number 281e>
    end interface
281e <Specific procedures for 52-bit tao_random_number 281e>≡ (281)
    real_static, real_state, real_array_static, real_array_state
    Thes are not exported
281f <Declaration of 52-bit tao_random_numbers 268b>+≡ (274a) <268d 281g>
    private :: real_stateless, real_array_stateless
281g <Declaration of 52-bit tao_random_numbers 268b>+≡ (274a) <281f>
    private :: <Specific procedures for 52-bit tao_random_number 281e>

```

C.4.6 *Luxury*

```

281h <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <270a 282a>
    pure subroutine luxury_stateless &
        (buffer_size, buffer_end, last, consumption)
        integer, intent(in) :: buffer_size
        integer, intent(inout) :: buffer_end
        integer, intent(inout) :: last
        integer, intent(in) :: consumption
        if (consumption >= 1 .and. consumption <= buffer_size) then
            buffer_end = consumption
            last = min (last, buffer_end)
        else
            !!! print *, "tao_random_luxury: ", "invalid consumption ", &
            !!!      consumption, ", not in [ 1,", buffer_size, "]"

```

```

        buffer_end = buffer_size
    end if
end subroutine luxury_stateless

282a <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <281h 282b>
    elemental subroutine luxury_state (s)
        type(tao_random_state), intent(inout) :: s
        call luxury_state_integer (s, size (s%buffer))
    end subroutine luxury_state

282b <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <282a 282c>
    elemental subroutine luxury_state_integer (s, consumption)
        type(tao_random_state), intent(inout) :: s
        integer, intent(in) :: consumption
        call luxury_stateless (size (s%buffer), s%buffer_end, s%last, consumption)
    end subroutine luxury_state_integer

282c <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <282b 282d>
    elemental subroutine luxury_state_real (s, consumption)
        type(tao_random_state), intent(inout) :: s
        real(kind=default), intent(in) :: consumption
        call luxury_state_integer (s, int (consumption * size (s%buffer)))
    end subroutine luxury_state_real

282d <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <282c 282e>
    subroutine luxury_static ()
        <Initialize a virginal random number generator 279d>
        call luxury_static_integer (size (s_buffer))
    end subroutine luxury_static

282e <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <282d 282f>
    subroutine luxury_static_integer (consumption)
        integer, intent(in) :: consumption
        <Initialize a virginal random number generator 279d>
        call luxury_stateless (size (s_buffer), s_buffer_end, s_last, consumption)
    end subroutine luxury_static_integer

282f <Implementation of tao_random_numbers 256a>+≡ (273d 274a) <282e
    subroutine luxury_static_real (consumption)
        real(kind=default), intent(in) :: consumption
        <Initialize a virginal random number generator 279d>
        call luxury_static_integer (int (consumption * size (s_buffer)))
    end subroutine luxury_static_real

282g <Interfaces of tao_random_numbers (unused luxury) 282g>≡
    interface tao_random_luxury
        module procedure <Specific procedures for tao_random_luxury 283c>
    end interface

```

283a \langle Declaration of `tao_random_numbers` (unused *luxury*) 283a $\rangle \equiv$ 283b \triangleright
`private :: luxury_stateless`

283b \langle Declaration of `tao_random_numbers` (unused *luxury*) 283a $\rangle + \equiv$ \triangleleft 283a
`private :: \langle Specific procedures for tao_random_luxury 283c \rangle`

283c \langle Specific procedures for `tao_random_luxury` 283c $\rangle \equiv$ (282g 283b)
`luxury_static, luxury_state, &`
`luxury_static_integer, luxury_state_integer, &`
`luxury_static_real, luxury_state_real`

C.5 Testing

C.5.1 30-bit

283d \langle Implementation of 30-bit `tao_random_numbers` 255b $\rangle + \equiv$ (273d) \triangleleft 280d
`subroutine tao_random_test (name)`
`character(len=*), optional, intent(in) :: name`
`character (len = *), parameter :: &`
`OK = "(1x,i10,' is ok. ')", &`
`NOT_OK = "(1x,i10,' is not ok, (expected ',i10,')!')"`
 \langle Parameters in `tao_random_test` 283e \rangle
`integer, parameter :: &`
`A_2027082 = 995235265`
`integer, dimension(N) :: a`
`type(tao_random_state) :: s, t`
`integer, dimension(:), allocatable :: ibuf`
`real(kind=tao_r64), dimension(:), allocatable :: dbuf`
`integer :: i, ibuf_size, dbuf_size`
`print *, "testing the 30-bit tao_random_numbers ..."`
 \langle Perform simple tests of `tao_random_numbers` 283f \rangle
 \langle Perform more tests of `tao_random_numbers` 284c \rangle
`end subroutine tao_random_test`

283e \langle Parameters in `tao_random_test` 283e $\rangle \equiv$ (283d 285b)
`integer, parameter :: &`
`SEED = 310952, &`
`N = 2009, M = 1009, &`
`N_SHORT = 1984`
 DEK's "official" test expects $a_{1009 \cdot 2009 + 1} = a_{2027082} = 995235265$:

283f \langle Perform simple tests of `tao_random_numbers` 283f $\rangle \equiv$ (283d 285b) 284b \triangleright
`! call tao_random_luxury ()`

```

call tao_random_seed (SEED)
do i = 1, N+1
    call tao_random_number (a, M)
end do
⟨Test a(1) = A_2027082 284a⟩
284a ⟨Test a(1) = A_2027082 284a⟩≡ (283-85)
    if (a(1) == A_2027082) then
        print OK, a(1)
    else
        print NOT_OK, a(1), A_2027082
        stop 1
    end if

```

Deja vu all over again, but 2027081 is factored the other way around this time

```

284b ⟨Perform simple tests of tao_random_numbers 283f⟩+≡ (283d 285b) <283f
    call tao_random_seed (SEED)
    do i = 1, M+1
        call tao_random_number (a)
    end do
    ⟨Test a(1) = A_2027082 284a⟩

```

Now checkpoint the random number generator after $N_{\text{short}} \cdot M$ numbers

```

284c ⟨Perform more tests of tao_random_numbers 284c⟩≡ (283d 285b) 284d>
    print *, "testing the stateless stuff ..."
    call tao_random_create (s, SEED)
    do i = 1, N_SHORT
        call tao_random_number (s, a, M)
    end do
    call tao_random_create (t, s)
    do i = 1, N+1 - N_SHORT
        call tao_random_number (s, a, M)
    end do
    ⟨Test a(1) = A_2027082 284a⟩

```

and restart the saved generator

```

284d ⟨Perform more tests of tao_random_numbers 284c⟩+≡ (283d 285b) <284c 284e>
    do i = 1, N+1 - N_SHORT
        call tao_random_number (t, a, M)
    end do
    ⟨Test a(1) = A_2027082 284a⟩

```

The same story again, but this time saving the copy to a file

```

284e ⟨Perform more tests of tao_random_numbers 284c⟩+≡ (283d 285b) <284d 285a>
    if (present (name)) then

```

```

print *, "testing I/O ..."
call tao_random_seed (s, SEED)
do i = 1, N_SHORT
    call tao_random_number (s, a, M)
end do
call tao_random_write (s, name)
do i = 1, N+1 - N_SHORT
    call tao_random_number (s, a, M)
end do
<Test a(1) = A_2027082 284a>
call tao_random_read (s, name)
do i = 1, N+1 - N_SHORT
    call tao_random_number (s, a, M)
end do
<Test a(1) = A_2027082 284a>
end if

```

And finally using marshaling/unmarshaling:

```

285a <Perform more tests of tao_random_numbers 284c>+≡ (283d 285b) <284e
print *, "testing marshaling/unmarshaling ..."
call tao_random_seed (s, SEED)
do i = 1, N_SHORT
    call tao_random_number (s, a, M)
end do
call tao_random_marshal_size (s, ibuf_size, dbuf_size)
allocate (ibuf(ibuf_size), dbuf(dbuf_size))
call tao_random_marshal (s, ibuf, dbuf)
do i = 1, N+1 - N_SHORT
    call tao_random_number (s, a, M)
end do
<Test a(1) = A_2027082 284a>
call tao_random_unmarshal (s, ibuf, dbuf)
do i = 1, N+1 - N_SHORT
    call tao_random_number (s, a, M)
end do
<Test a(1) = A_2027082 284a>

```

C.5.2 52-bit

DEK's "official" test expects $x_{1009 \cdot 2009 + 1} = x_{2027082} = 0.36410514377569680455$:

```

285b <Implementation of 52-bit tao_random_numbers 259f>+≡ (274a) <280e
subroutine tao_random_test (name)
character(len=*), optional, intent(in) :: name

```

```

character(len=*), parameter :: &
    OK = "(1x,f22.20,' is ok. ')", &
    NOT_OK = "(1x,f22.20,' is not ok, (A_2027082 ',f22.20,')!')'"
<Parameters in tao_random_test 283e>
real(kind=default), parameter :: &
    A_2027082 = 0.36410514377569680455_tao_r64
real(kind=default), dimension(N) :: a
type(tao_random_state) :: s, t
integer, dimension(:), allocatable :: ibuf
real(kind=tao_r64), dimension(:), allocatable :: dbuf
integer :: i, ibuf_size, dbuf_size
print *, "testing the 52-bit tao_random_numbers ..."
<Perform simple tests of tao_random_numbers 283f>
<Perform more tests of tao_random_numbers 284c>
end subroutine tao_random_test

```

C.5.3 Test Program

```

286 <tao_test.f90 286>≡
  program tao_test
    use tao_random_numbers, only: test30 => tao_random_test
    use tao52_random_numbers, only: test52 => tao_random_test
    implicit none
    call test30 ("tmp.tao")
    call test52 ("tmp.tao")
    stop 0
  end program tao_test

```

—D—

SPECIAL FUNCTIONS

```

287a  <specfun.f90 287a>≡
      ! specfun.f90 --
      <Copyleft notice 1>
      module specfun
        use kinds
        ! use constants
        implicit none
        private
        <Declaration of specfun procedures 287b>
        real(kind=default), public, parameter :: &
            PI = 3.1415926535897932384626433832795028841972_default
        contains
        <Implementation of specfun procedures 288b>
      end module specfun

```

The algorithm is stolen from the FORTRAN version in routine C303 of the CERN library [25]. It has an accuracy which is approximately one digit less than machine precision.

```

287b  <Declaration of specfun procedures 287b>≡ (287a)
      public :: gamma

```

The so-called reflection formula is used for negative arguments:

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x} \quad (\text{D.1})$$

Here's the identity transformation that pulls the argument of Γ into $[3, 4]$:

$$\Gamma(u) = \begin{cases} (u-1)\Gamma(u-1) & \text{for } u > 4 \\ \frac{1}{u}\Gamma(u+1) & \text{for } u < 3 \end{cases} \quad (\text{D.2})$$

```

287c  <Pull u into the intervall [3, 4] 287c>≡ (288b)
      f = 1

```



```

if (u < 3) then
  do i = 1, int (4 - u)
    f = f / u
    u = u + 1
  end do
else
  do i = 1, int (u - 3)
    u = u - 1
    f = f * u
  end do
end if

```

A Chebyshev approximation for $\Gamma(x)$ is used after mapping $x \in [3, 4]$ linearly to $h \in [-1, 1]$. The series is evaluated by Clenshaw's recurrence formula:

$$\begin{aligned}
 d_m &= d_{m+1} = 0 \\
 d_j &= 2xd_{j+1} - d_{j+2} + c_j \text{ for } 0 < j < m - 1 \\
 f(x) &= d_0 = xd_1 - d_2 + \frac{1}{2}c_0
 \end{aligned} \tag{D.3}$$

288a \langle Clenshaw's recurrence formula 288a $\rangle \equiv$ (288b)

```

alpha = 2*g
b1 = 0
b2 = 0
do i = 15, 0, -1
  b0 = c(i) + alpha * b1 - b2
  b2 = b1
  b1 = b0
end do
g = f * (b0 - g * b2)

```

Note that we're assuming that $c(0)$ is in fact $c_0/2$. This is for compatibility with the CERN library routines.

288b \langle Implementation of specfun procedures 288b $\rangle \equiv$ (287a)

```

pure function gamma (x) result (g)
  real(kind=default), intent(in) :: x
  real(kind=default) :: g
  integer :: i
  real(kind=default) :: u, f, alpha, b0, b1, b2
  real(kind=default), dimension(0:15), parameter :: &
    c =  $\langle c_0/2, c_1, c_2, \dots, c_{15} \text{ for } \Gamma(x) \text{ 289a} \rangle$ 
  u = x
  if (u <= 0.0) then
    if (u == int (u)) then
      g = huge (g)
    end if
  end if

```

```

        return
    else
        u = 1 - u
    end if
endif
⟨Pull u into the intervall [3, 4] 287c⟩
g = 2*u - 7
⟨Clenshaw's recurrence formula 288a⟩
if (x < 0) then
    g = PI / (sin (PI * x) * g)
end if
end function gamma
289a ⟨c0/2, c1, c2, ..., c15 for Γ(x) 289a⟩≡ (288b)
(/ 3.65738772508338244_default, &
  1.95754345666126827_default, &
  0.33829711382616039_default, &
  0.04208951276557549_default, &
  0.00428765048212909_default, &
  0.00036521216929462_default, &
  0.00002740064222642_default, &
  0.00000181240233365_default, &
  0.00000010965775866_default, &
  0.00000000598718405_default, &
  0.00000000030769081_default, &
  0.00000000001431793_default, &
  0.00000000000065109_default, &
  0.00000000000002596_default, &
  0.00000000000000111_default, &
  0.00000000000000004_default /)

```

D.1 Test

```

289b ⟨stest.f90 289b⟩≡ (290c)
! stest.f90 --
⟨Copyleft notice 1⟩
module stest_functions
  use kinds
  use constants
  use specfun
  private
  ⟨Declaration of stest_functions procedures 290a⟩

```

```

contains
  <Implementation of stest_functions procedures 290b>
end module stest_functions

290a <Declaration of stest_functions procedures 290a>≡ (289b)
  public :: gauss_multiplication
  Gauss' multiplication fomula can serve as a non-trivial test


$$\Gamma(nx) = (2\pi)^{(1-n)/2} n^{nx-1/2} \prod_{k=0}^{n-1} \Gamma(x + k/n) \quad (\text{D.4})$$


290b <Implementation of stest_functions procedures 290b>≡ (289b)
  pure function gauss_multiplication (x, n) result (delta)
    real(kind=default), intent(in) :: x
    integer, intent(in) :: n
    real(kind=default) :: delta
    real(kind=default) :: gxn
    integer :: k
    gxn = (2*PI)**(0.5_double*(1-n)) * n**(n*x-0.5_double)
    do k = 0, n - 1
      gxn = gxn * gamma (x + real (k, kind=default) / n)
    end do
    delta = abs ((gamma (n*x) - gxn) / gamma (n*x))
  end function gauss_multiplication

290c <stest.f90 289b>+≡ <289b
  program stest
    use kinds
    use specfun
    use stest_functions !NODEP!
    implicit none
    integer :: i, steps
    real(kind=default) :: x, g, xmin, xmax
    xmin = -4.5
    xmax = 4.5
    steps = 100 ! 9
    do i = 0, steps
      x = xmin + ((xmax - xmin) / real (steps)) * i
      print "(f6.3,4(1x,e9.2))", x, &
        gauss_multiplication (x, 2), &
        gauss_multiplication (x, 3), &
        gauss_multiplication (x, 4), &
        gauss_multiplication (x, 5)
    end do
  end program stest

```

—E—

STATISTICS

291a $\langle \text{vamp_stat.f90 291a} \rangle \equiv$
`! vamp_stat.f90 --`
 $\langle \text{Copyleft notice 1} \rangle$
`module vamp_stat`
`use kinds`
`implicit none`
`private`
 $\langle \text{Declaration of vamp_stat procedures 291b} \rangle$
`contains`
 $\langle \text{Implementation of vamp_stat procedures 291c} \rangle$
`end module vamp_stat`

291b $\langle \text{Declaration of vamp_stat procedures 291b} \rangle \equiv$ (291a) 292c \triangleright
`public :: average, standard_deviation, value_spread`

$$\text{avg}(X) = \frac{1}{|X|} \sum_{x \in X} x \quad (\text{E.1})$$

291c $\langle \text{Implementation of vamp_stat procedures 291c} \rangle \equiv$ (291a) 292a \triangleright
`pure function average (x) result (a)`
`real(kind=default), dimension(:), intent(in) :: x`
`real(kind=default) :: a`
`integer :: n`
`n = size (x)`
`if (n == 0) then`
`a = 0.0`
`else`
`a = sum (x) / n`
`end if`
`end function average`

$$\text{stddev}(X) = \frac{1}{|X| - 1} \sum_{x \in X} (x - \text{avg}(X))^2 = \frac{1}{|X| - 1} \left(\frac{1}{|X|} \sum_{x \in X} x^2 - (\text{avg}(X))^2 \right) \quad (\text{E.2})$$

292a \langle Implementation of vamp_stat procedures 291c $\rangle + \equiv$ (291a) \triangleleft 291c 292b \triangleright

```

pure function standard_deviation (x) result (s)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default) :: s
  integer :: n
  n = size (x)
  if (n < 2) then
    s = huge (s)
  else
    s = sqrt (max ((sum (x**2) / n - (average (x))**2) / (n - 1), &
                  0.0_default))
  end if
end function standard_deviation

```

$$\text{spread}(X) = \max_{x \in X}(x) - \min_{x \in X}(x) \quad (\text{E.3})$$

292b \langle Implementation of vamp_stat procedures 291c $\rangle + \equiv$ (291a) \triangleleft 292a 292d \triangleright

```

pure function value_spread (x) result (s)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default) :: s
  s = maxval(x) - minval(x)
end function value_spread

```

292c \langle Declaration of vamp_stat procedures 291b $\rangle + \equiv$ (291a) \triangleleft 291b

```

public :: standard_deviation_percent, value_spread_percent

```

292d \langle Implementation of vamp_stat procedures 291c $\rangle + \equiv$ (291a) \triangleleft 292b 292e \triangleright

```

pure function standard_deviation_percent (x) result (s)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default) :: s
  real(kind=default) :: abs_avg
  abs_avg = abs (average (x))
  if (abs_avg <= tiny (abs_avg)) then
    s = huge (s)
  else
    s = 100.0 * standard_deviation (x) / abs_avg
  end if
end function standard_deviation_percent

```

292e \langle Implementation of vamp_stat procedures 291c $\rangle + \equiv$ (291a) \triangleleft 292d

```

pure function value_spread_percent (x) result (s)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default) :: s
  real(kind=default) :: abs_avg
  abs_avg = abs (average (x))
  if (abs_avg <= tiny (abs_avg)) then

```

```
        s = huge (s)
    else
        s = 100.0 * value_spread (x) / abs_avg
    end if
end function value_spread_percent
```

—F—

HISTOGRAMMING

⚠ Merged WK's improvements for WHIZARD. TODO *after* merging:

1. bins3 is a bad undescriptive name
2. bins3 should be added to `histogram2`
3. `write_histogram2_unit` for symmetry.

⚠ There's almost no sanity checking. If you call one of these functions on a histogram that has not been initialized, you loose. — *Big time.*

```

294a <histograms.f90 294a>≡
    ! histograms.f90 --
    <Copyleft notice 1>
    module histograms
        use kinds
        use utils, only: find_free_unit
        implicit none
        private
        <Declaration of histograms procedures 295b>
        <Interfaces of histograms procedures 295c>
        <Variables in histograms 295e>
        <Declaration of histograms types 294b>
    contains
        <Implementation of histograms procedures 295f>
    end module histograms

294b <Declaration of histograms types 294b>≡ (294a) 295a>
    type, public :: histogram
    private
        integer :: n_bins
        real(kind=default) :: x_min, x_max
        real(kind=default), dimension(:), pointer :: bins => null ()
        real(kind=default), dimension(:), pointer :: bins2 => null ()

```

```

        real(kind=default), dimension(:), pointer :: bins3 => null ()
    end type histogram
295a <Declaration of histograms types 294b>+≡ (294a) <294b
    type, public :: histogram2
        private
        integer, dimension(2) :: n_bins
        real(kind=default), dimension(2) :: x_min, x_max
        real(kind=default), dimension(:, :), pointer :: bins => null ()
        real(kind=default), dimension(:, :), pointer :: bins2 => null ()
    end type histogram2
295b <Declaration of histograms procedures 295b>≡ (294a) 295d>
    public :: create_histogram
    public :: fill_histogram
    public :: delete_histogram
    public :: write_histogram
295c <Interfaces of histograms procedures 295c>≡ (294a) 300b>
    interface create_histogram
        module procedure create_histogram1, create_histogram2
    end interface
    interface fill_histogram
        module procedure fill_histogram1, fill_histogram2s, fill_histogram2v
    end interface
    interface delete_histogram
        module procedure delete_histogram1, delete_histogram2
    end interface
    interface write_histogram
        module procedure write_histogram1, write_histogram2
        module procedure write_histogram1_unit
    end interface
295d <Declaration of histograms procedures 295b>+≡ (294a) <295b 299a>
    private :: create_histogram1, create_histogram2
    private :: fill_histogram1, fill_histogram2s, fill_histogram2v
    private :: delete_histogram1, delete_histogram2
    private :: write_histogram1, write_histogram2
295e <Variables in histograms 295e>≡ (294a)
    integer, parameter, private :: N_BINS_DEFAULT = 10
295f <Implementation of histograms procedures 295f>≡ (294a) 296a>
    elemental subroutine create_histogram1 (h, x_min, x_max, nb)
        type(histogram), intent(out) :: h
        real(kind=default), intent(in) :: x_min, x_max
        integer, intent(in), optional :: nb

```



```

    if (present (nb)) then
        h%n_bins = nb
    else
        h%n_bins = N_BINS_DEFAULT
    end if
    h%x_min = x_min
    h%x_max = x_max
    allocate (h%bins(0:h%n_bins+1), h%bins2(0:h%n_bins+1))
    h%bins = 0
    h%bins2 = 0
    allocate (h%bins3(0:h%n_bins+1))
    h%bins3 = 0
end subroutine create_histogram1
296a  <Implementation of histograms procedures 295f>+≡ (294a) <295f 296b>
pure subroutine create_histogram2 (h, x_min, x_max, nb)
    type(histogram2), intent(out) :: h
    real(kind=default), dimension(:), intent(in) :: x_min, x_max
    integer, intent(in), dimension(:), optional :: nb
    if (present (nb)) then
        h%n_bins = nb
    else
        h%n_bins = N_BINS_DEFAULT
    end if
    h%x_min = x_min
    h%x_max = x_max
    allocate (h%bins(0:h%n_bins(1)+1,0:h%n_bins(1)+1), &
              h%bins2(0:h%n_bins(2)+1,0:h%n_bins(2)+1))
    h%bins = 0
    h%bins2 = 0
end subroutine create_histogram2
296b  <Implementation of histograms procedures 295f>+≡ (294a) <296a 297a>
elemental subroutine fill_histogram1 (h, x, weight, excess)
    type(histogram), intent(inout) :: h
    real(kind=default), intent(in) :: x
    real(kind=default), intent(in), optional :: weight
    real(kind=default), intent(in), optional :: excess
    integer :: i
    if (x < h%x_min) then
        i = 0
    else if (x > h%x_max) then
        i = h%n_bins + 1
    else
        i = 1 + h%n_bins * (x - h%x_min) / (h%x_max - h%x_min)

```

```

!WK! i = min (max (i, 0), h%n_bins + 1)
end if
if (present (weight)) then
    h%bins(i) = h%bins(i) + weight
    h%bins2(i) = h%bins2(i) + weight*weight
else
    h%bins(i) = h%bins(i) + 1
    h%bins2(i) = h%bins2(i) + 1
end if
if (present (excess)) h%bins3(i) = h%bins3(i) + excess
end subroutine fill_histogram1
297a <Implementation of histograms procedures 295f>+≡ (294a) <296b 297b>
elemental subroutine fill_histogram2s (h, x1, x2, weight)
    type(histogram2), intent(inout) :: h
    real(kind=default), intent(in) :: x1, x2
    real(kind=default), intent(in), optional :: weight
    call fill_histogram2v (h, (/ x1, x2 /), weight)
end subroutine fill_histogram2s
297b <Implementation of histograms procedures 295f>+≡ (294a) <297a 297c>
pure subroutine fill_histogram2v (h, x, weight)
    type(histogram2), intent(inout) :: h
    real(kind=default), dimension(:), intent(in) :: x
    real(kind=default), intent(in), optional :: weight
    integer, dimension(2) :: i
    i = 1 + h%n_bins * (x - h%x_min) / (h%x_max - h%x_min)
    i = min (max (i, 0), h%n_bins + 1)
    if (present (weight)) then
        h%bins(i(1),i(2)) = h%bins(i(1),i(2)) + weight
        h%bins2(i(1),i(2)) = h%bins2(i(1),i(2)) + weight*weight
    else
        h%bins(i(1),i(2)) = h%bins(i(1),i(2)) + 1
        h%bins2(i(1),i(2)) = h%bins2(i(1),i(2)) + 1
    end if
end subroutine fill_histogram2v
297c <Implementation of histograms procedures 295f>+≡ (294a) <297b 297d>
elemental subroutine delete_histogram1 (h)
    type(histogram), intent(inout) :: h
    deallocate (h%bins, h%bins2)
    deallocate (h%bins3)
end subroutine delete_histogram1
297d <Implementation of histograms procedures 295f>+≡ (294a) <297c 298>
elemental subroutine delete_histogram2 (h)

```

```

    type(histogram2), intent(inout) :: h
    deallocate (h%bins, h%bins2)
end subroutine delete_histogram2
298 <Implementation of histograms procedures 295f>+≡ (294a) <297d 299b>
subroutine write_histogram1 (h, name, over)
    type(histogram), intent(in) :: h
    character(len=*), intent(in), optional :: name
    logical, intent(in), optional :: over
    integer :: i, iounit
    if (present (name)) then
        call find_free_unit (iounit)
        if (iounit > 0) then
            open (unit = iounit, action = "write", status = "replace", &
                file = name)
            if (present (over)) then
                if (over) then
                    write (unit = iounit, fmt = *) &
                        "underflow", h%bins(0), sqrt (h%bins2(0))
                end if
            end if
            do i = 1, h%n_bins
                write (unit = iounit, fmt = *) &
                    midpoint (h, i), h%bins(i), sqrt (h%bins2(i))
            end do
            if (present (over)) then
                if (over) then
                    write (unit = iounit, fmt = *) &
                        "overflow", h%bins(h%n_bins+1), &
                        sqrt (h%bins2(h%n_bins+1))
                end if
            end if
            close (unit = iounit)
        else
            print *, "write_histogram: Can't find a free unit!"
        end if
    else
        if (present (over)) then
            if (over) then
                print *, "underflow", h%bins(0), sqrt (h%bins2(0))
            end if
        end if
        do i = 1, h%n_bins
            print *, midpoint (h, i), h%bins(i), sqrt (h%bins2(i))
        end do
    end if
end subroutine write_histogram1

```

```

end do
if (present (over)) then
  if (over) then
    print *, "overflow", h%bins(h%n_bins+1), &
      sqrt (h%bins2(h%n_bins+1))
  end if
end if
end if
end subroutine write_histogram1

```

299a *<Declaration of histograms procedures 295b>+≡ (294a) <295d 300a>*
 !WK! public :: write_histogram1_unit



I don't like the format statement with the line number. Use a character constant instead (after we have merged with WHIZARD's branch).

299b *<Implementation of histograms procedures 295f>+≡ (294a) <298 300d>*
 subroutine write_histogram1_unit (h, iounit, over, show_excess)
 type(histogram), intent(in) :: h
 integer, intent(in) :: iounit
 logical, intent(in), optional :: over, show_excess
 integer :: i
 logical :: show_exc
 show_exc = .false.; if (present(show_excess)) show_exc = show_excess
 if (present (over)) then
 if (over) then
 if (show_exc) then
 write (unit = iounit, fmt = 1) &
 "underflow", h%bins(0), sqrt (h%bins2(0)), h%bins3(0)
 else
 write (unit = iounit, fmt = 1) &
 "underflow", h%bins(0), sqrt (h%bins2(0))
 end if
 end if
 end if
 do i = 1, h%n_bins
 if (show_exc) then
 write (unit = iounit, fmt = 1) &
 midpoint (h, i), h%bins(i), sqrt (h%bins2(i)), h%bins3(i)
 else
 write (unit = iounit, fmt = 1) &
 midpoint (h, i), h%bins(i), sqrt (h%bins2(i))
 end if
 end do
 end do

```

    if (present (over)) then
      if (over) then
        if (show_exc) then
          write (unit = iounit, fmt = 1) &
            "overflow", h%bins(h%n_bins+1), &
            sqrt (h%bins2(h%n_bins+1)), &
            h%bins3(h%n_bins+1)
        else
          write (unit = iounit, fmt = 1) &
            "overflow", h%bins(h%n_bins+1), &
            sqrt (h%bins2(h%n_bins+1))
        end if
      end if
    end if
    1 format (1x,4(G16.9,2x))
  end subroutine write_histogram1_unit
300a  <Declaration of histograms procedures 295b>+≡ (294a) <299a 300c>
      private :: midpoint
300b  <Interfaces of histograms procedures 295c>+≡ (294a) <295c>
      interface midpoint
        module procedure midpoint1, midpoint2
      end interface
300c  <Declaration of histograms procedures 295b>+≡ (294a) <300a>
      private :: midpoint1, midpoint2
300d  <Implementation of histograms procedures 295f>+≡ (294a) <299b 300e>
      elemental function midpoint1 (h, bin) result (x)
        type(histogram), intent(in) :: h
        integer, intent(in) :: bin
        real(kind=default) :: x
        x = h%x_min + (h%x_max - h%x_min) * (bin - 0.5) / h%n_bins
      end function midpoint1
300e  <Implementation of histograms procedures 295f>+≡ (294a) <300d 300f>
      elemental function midpoint2 (h, bin, d) result (x)
        type(histogram2), intent(in) :: h
        integer, intent(in) :: bin, d
        real(kind=default) :: x
        x = h%x_min(d) + (h%x_max(d) - h%x_min(d)) * (bin - 0.5) / h%n_bins(d)
      end function midpoint2
300f  <Implementation of histograms procedures 295f>+≡ (294a) <300e>
      subroutine write_histogram2 (h, name, over)
        type(histogram2), intent(in) :: h

```

```

character(len=*), intent(in), optional :: name
logical, intent(in), optional :: over
integer :: i1, i2, iounit
if (present (name)) then
  call find_free_unit (iounit)
  if (iounit > 0) then
    open (unit = iounit, action = "write", status = "replace", &
          file = name)
    if (present (over)) then
      if (over) then
        write (unit = iounit, fmt = *) &
          "double underflow", h%bins(0,0), sqrt (h%bins2(0,0))
        do i2 = 1, h%n_bins(2)
          write (unit = iounit, fmt = *) &
            "x1 underflow", midpoint (h, i2, 2), &
            h%bins(0,i2), sqrt (h%bins2(0,i2))
        end do
        do i1 = 1, h%n_bins(1)
          write (unit = iounit, fmt = *) &
            "x2 underflow", midpoint (h, i1, 1), &
            h%bins(i1,0), sqrt (h%bins2(i1,0))
        end do
      end if
    end if
    do i1 = 1, h%n_bins(1)
      do i2 = 1, h%n_bins(2)
        write (unit = iounit, fmt = *) &
          midpoint (h, i1, 1), midpoint (h, i2, 2), &
          h%bins(i1,i2), sqrt (h%bins2(i1,i2))
      end do
    end do
    if (present (over)) then
      if (over) then
        do i2 = 1, h%n_bins(2)
          write (unit = iounit, fmt = *) &
            "x1 overflow", midpoint (h, i2, 2), &
            h%bins(h%n_bins(1)+1,i2), &
            sqrt (h%bins2(h%n_bins(1)+1,i2))
        end do
        do i1 = 1, h%n_bins(1)
          write (unit = iounit, fmt = *) &
            "x2 overflow", midpoint (h, i1, 1), &
            h%bins(i1,h%n_bins(2)+1), &

```

```

        sqrt (h%bins2(i1,h%n_bins(2)+1))
    end do
    write (unit = iounit, fmt = *) "double overflow", &
        h%bins(h%n_bins(1)+1,h%n_bins(2)+1), &
        sqrt (h%bins2(h%n_bins(1)+1,h%n_bins(2)+1))
    end if
end if
close (unit = iounit)
else
    print *, "write_histogram: Can't find a free unit!"
end if
else
    if (present (over)) then
        if (over) then
            print *, "double underflow", h%bins(0,0), sqrt (h%bins2(0,0))
            do i2 = 1, h%n_bins(2)
                print *, "x1 underflow", midpoint (h, i2, 2), &
                    h%bins(0,i2), sqrt (h%bins2(0,i2))
            end do
            do i1 = 1, h%n_bins(1)
                print *, "x2 underflow", midpoint (h, i1, 1), &
                    h%bins(i1,0), sqrt (h%bins2(i1,0))
            end do
        end if
    end if
    do i1 = 1, h%n_bins(1)
        do i2 = 1, h%n_bins(2)
            print *, midpoint (h, i1, 1), midpoint (h, i2, 2), &
                h%bins(i1,i2), sqrt (h%bins2(i1,i2))
        end do
    end do
    if (present (over)) then
        if (over) then
            do i2 = 1, h%n_bins(2)
                print *, "x1 overflow", midpoint (h, i2, 2), &
                    h%bins(h%n_bins(1)+1,i2), &
                    sqrt (h%bins2(h%n_bins(1)+1,i2))
            end do
            do i1 = 1, h%n_bins(1)
                print *, "x2 overflow", midpoint (h, i1, 1), &
                    h%bins(i1,h%n_bins(2)+1), &
                    sqrt (h%bins2(i1,h%n_bins(2)+1))
            end do
        end if
    end if
end if

```

```

        print *, "double overflow", &
            h%bins(h%n_bins(1)+1,h%n_bins(2)+1), &
            sqrt (h%bins2(h%n_bins(1)+1,h%n_bins(2)+1))
    end if
end if
end if
end subroutine write_histogram2

```


—G—

MISCELLANEOUS UTILITIES

```

304a  <utils.f90 304a>≡
      ! utils.f90 --
      <Copyleft notice 1>
      module utils
        use kinds
        implicit none
        private
        <Declaration of utils procedures 304b>
        <Parameters in utils 311c>
        <Variables in utils 312b>
        <Interfaces of utils procedures 304c>
        contains
        <Implementation of utils procedures 305c>
      end module utils

```

G.1 Memory Management

```

304b  <Declaration of utils procedures 304b>≡                                     (304a) 306d>
      public :: create_array_pointer
      private :: create_integer_array_pointer
      private :: create_real_array_pointer
      private :: create_integer_array2_pointer
      private :: create_real_array2_pointer

304c  <Interfaces of utils procedures 304c>≡                                     (304a) 306e>
      interface create_array_pointer
        module procedure &
          create_integer_array_pointer, &
          create_real_array_pointer, &
          create_integer_array2_pointer, &
          create_real_array2_pointer

```

```

end interface

305a  <Body of create_*_array_pointer 305a>≡ (305c 306a)
  if (associated (lhs)) then
    if (size (lhs) /= n) then
      deallocate (lhs)
      if (present (lb)) then
        allocate (lhs(lb:n+lb-1))
      else
        allocate (lhs(n))
      end if
    end if
  else
    if (present (lb)) then
      allocate (lhs(lb:n+lb-1))
    else
      allocate (lhs(n))
    end if
  end if
  lhs = 0

305b  <Body of create_*_array2_pointer 305b>≡ (306)
  if (associated (lhs)) then
    if (any (ubound (lhs) /= n)) then
      deallocate (lhs)
      if (present (lb)) then
        allocate (lhs(lb(1):n(1)+lb(1)-1,lb(2):n(2)+lb(2)-1))
      else
        allocate (lhs(n(1),n(2)))
      end if
    end if
  else
    if (present (lb)) then
      allocate (lhs(lb(1):n(1)+lb(1)-1,lb(2):n(2)+lb(2)-1))
    else
      allocate (lhs(n(1),n(2)))
    end if
  end if
  lhs = 0

305c  <Implementation of utils procedures 305c>≡ (304a) 306a>
  pure subroutine create_integer_array_pointer (lhs, n, lb)
    integer, dimension(:), pointer :: lhs
    integer, intent(in) :: n
    integer, intent(in), optional :: lb

```

```

    <Body of create_*_array_pointer 305a>
end subroutine create_integer_array_pointer

306a <Implementation of utils procedures 305c>+≡ (304a) <305c 306b>
    pure subroutine create_real_array_pointer (lhs, n, lb)
        real(kind=default), dimension(:), pointer :: lhs
        integer, intent(in) :: n
        integer, intent(in), optional :: lb
        <Body of create_*_array_pointer 305a>
    end subroutine create_real_array_pointer

306b <Implementation of utils procedures 305c>+≡ (304a) <306a 306c>
    pure subroutine create_integer_array2_pointer (lhs, n, lb)
        integer, dimension(:, :), pointer :: lhs
        integer, dimension(:), intent(in) :: n
        integer, dimension(:), intent(in), optional :: lb
        <Body of create_*_array2_pointer 305b>
    end subroutine create_integer_array2_pointer

306c <Implementation of utils procedures 305c>+≡ (304a) <306b 307a>
    pure subroutine create_real_array2_pointer (lhs, n, lb)
        real(kind=default), dimension(:, :), pointer :: lhs
        integer, dimension(:), intent(in) :: n
        integer, dimension(:), intent(in), optional :: lb
        <Body of create_*_array2_pointer 305b>
    end subroutine create_real_array2_pointer

Copy an allocatable array component of a derived type, reshaping the target
if necessary. The target can be disassociated, but its association must not
be undefined.

306d <Declaration of utils procedures 304b>+≡ (304a) <304b 307e>
    public :: copy_array_pointer
    private :: copy_integer_array_pointer
    private :: copy_real_array_pointer
    private :: copy_integer_array2_pointer
    private :: copy_real_array2_pointer

306e <Interfaces of utils procedures 304c>+≡ (304a) <304c 308a>
    interface copy_array_pointer
        module procedure &
            copy_integer_array_pointer, &
            copy_real_array_pointer, &
            copy_integer_array2_pointer, &
            copy_real_array2_pointer
    end interface

```

```

307a  <Implementation of utils procedures 305c>+≡ (304a) <306c 307b>
      pure subroutine copy_integer_array_pointer (lhs, rhs, lb)
        integer, dimension(:), pointer :: lhs
        integer, dimension(:), intent(in) :: rhs
        integer, intent(in), optional :: lb
        call create_integer_array_pointer (lhs, size (rhs), lb)
        lhs = rhs
      end subroutine copy_integer_array_pointer

307b  <Implementation of utils procedures 305c>+≡ (304a) <307a 307c>
      pure subroutine copy_real_array_pointer (lhs, rhs, lb)
        real(kind=default), dimension(:), pointer :: lhs
        real(kind=default), dimension(:), intent(in) :: rhs
        integer, intent(in), optional :: lb
        call create_real_array_pointer (lhs, size (rhs), lb)
        lhs = rhs
      end subroutine copy_real_array_pointer

307c  <Implementation of utils procedures 305c>+≡ (304a) <307b 307d>
      pure subroutine copy_integer_array2_pointer (lhs, rhs, lb)
        integer, dimension(:, :), pointer :: lhs
        integer, dimension(:, :), intent(in) :: rhs
        integer, dimension(:), intent(in), optional :: lb
        call create_integer_array2_pointer &
           (lhs, (/ size (rhs, dim=1), size (rhs, dim=2) /), lb)
        lhs = rhs
      end subroutine copy_integer_array2_pointer

307d  <Implementation of utils procedures 305c>+≡ (304a) <307c 308b>
      pure subroutine copy_real_array2_pointer (lhs, rhs, lb)
        real(kind=default), dimension(:, :), pointer :: lhs
        real(kind=default), dimension(:, :), intent(in) :: rhs
        integer, dimension(:), intent(in), optional :: lb
        call create_real_array2_pointer &
           (lhs, (/ size (rhs, dim=1), size (rhs, dim=2) /), lb)
        lhs = rhs
      end subroutine copy_real_array2_pointer

```

G.2 Sorting

```

307e  <Declaration of utils procedures 304b>+≡ (304a) <306d 309d>
      public :: swap
      private :: swap_integer, swap_real

```

308a *<Interfaces of utils procedures 304c>+≡* (304a) <306e 310a>

```
interface swap
  module procedure swap_integer, swap_real
end interface
```

308b *<Implementation of utils procedures 305c>+≡* (304a) <307d 308c>

```
elemental subroutine swap_integer (a, b)
  integer, intent(inout) :: a, b
  integer :: tmp
  tmp = a
  a = b
  b = tmp
end subroutine swap_integer
```

308c *<Implementation of utils procedures 305c>+≡* (304a) <308b 308d>

```
elemental subroutine swap_real (a, b)
  real(kind=default), intent(inout) :: a, b
  real(kind=default) :: tmp
  tmp = a
  a = b
  b = tmp
end subroutine swap_real
```

Straight insertion:

308d *<Implementation of utils procedures 305c>+≡* (304a) <308c 309b>

```
pure subroutine sort_real (key, reverse)
  real(kind=default), dimension(:), intent(inout) :: key
  logical, intent(in), optional :: reverse
  logical :: rev
  integer :: i, j
  <Set rev to reverse or .false. 308e>
  do i = 1, size (key) - 1
    <Set j to minloc(key) 309a>
    if (j /= i) then
      call swap (key(i), key(j))
    end if
  end do
end subroutine sort_real
```

308e *<Set rev to reverse or .false. 308e>≡* (308 309)

```
if (present (reverse)) then
  rev = reverse
else
  rev = .false.
end if
```

```

309a  <Set j to minloc(key) 309a>≡ (308 309)
      if (rev) then
        j = sum (maxloc (key(i:))) + i - 1
      else
        j = sum (minloc (key(i:))) + i - 1
      end if

309b  <Implementation of utils procedures 305c>+≡ (304a) <308d 309c>
      pure subroutine sort_real_and_real_array (key, table, reverse)
        real(kind=default), dimension(:), intent(inout) :: key
        real(kind=default), dimension(:, :), intent(inout) :: table
        logical, intent(in), optional :: reverse
        logical :: rev
        integer :: i, j
        <Set rev to reverse or .false. 308e>
        do i = 1, size (key) - 1
          <Set j to minloc(key) 309a>
          if (j /= i) then
            call swap (key(i), key(j))
            call swap (table(:, i), table(:, j))
          end if
        end do
      end subroutine sort_real_and_real_array

309c  <Implementation of utils procedures 305c>+≡ (304a) <309b 310c>
      pure subroutine sort_real_and_integer (key, table, reverse)
        real(kind=default), dimension(:), intent(inout) :: key
        integer, dimension(:), intent(inout) :: table
        logical, intent(in), optional :: reverse
        logical :: rev
        integer :: i, j
        <Set rev to reverse or .false. 308e>
        do i = 1, size (key) - 1
          <Set j to minloc(key) 309a>
          if (j /= i) then
            call swap (key(i), key(j))
            call swap (table(i), table(j))
          end if
        end do
      end subroutine sort_real_and_integer

309d  <Declaration of utils procedures 304b>+≡ (304a) <307e 310b>
      public :: sort
      private :: sort_real, sort_real_and_real_array, sort_real_and_integer

```

310a \langle Interfaces of `utils` procedures 304c $\rangle + \equiv$ (304a) \triangleleft 308a \triangleright

```

interface sort
  module procedure &
    sort_real, sort_real_and_real_array, &
    sort_real_and_integer
end interface

```

G.3 Mathematics

310b \langle Declaration of `utils` procedures 304b $\rangle + \equiv$ (304a) \triangleleft 309d 310d \triangleright

```

public :: outer_product

```

Admittedly, one has to get used to this notation for the tensor product:

310c \langle Implementation of `utils` procedures 305c $\rangle + \equiv$ (304a) \triangleleft 309c 310e \triangleright

```

pure function outer_product (x, y) result (xy)
  real(kind=default), dimension(:), intent(in) :: x, y
  real(kind=default), dimension(size(x),size(y)) :: xy
  xy = spread (x, dim=2, ncopies=size(y)) &
    * spread (y, dim=1, ncopies=size(x))
end function outer_product

```

Greatest common divisor and least common multiple

310d \langle Declaration of `utils` procedures 304b $\rangle + \equiv$ (304a) \triangleleft 310b 312a \triangleright

```

public :: factorize, gcd, lcm
private :: gcd_internal

```

For our purposes, a straightforward implementation of Euclid's algorithm suffices:

310e \langle Implementation of `utils` procedures 305c $\rangle + \equiv$ (304a) \triangleleft 310c 310f \triangleright

```

pure recursive function gcd_internal (m, n) result (gcd_m_n)
  integer, intent(in) :: m, n
  integer :: gcd_m_n
  if (n <= 0) then
    gcd_m_n = m
  else
    gcd_m_n = gcd_internal (n, modulo (m, n))
  end if
end function gcd_internal

```

Wrap an elemental procedure around the recursive procedure:

310f \langle Implementation of `utils` procedures 305c $\rangle + \equiv$ (304a) \triangleleft 310e 311a \triangleright

```

elemental function gcd (m, n) result (gcd_m_n)
  integer, intent(in) :: m, n
  integer :: gcd_m_n

```

```

gcd_m_n = gcd_internal (m, n)
end function gcd

```

As long as $m \cdot n$ does not overflow, we can use $\text{gcd}(m, n) \text{lcm}(m, n) = mn$:

311a *<Implementation of utils procedures 305c>+≡ (304a) <310f 311b>*

```

elemental function lcm (m, n) result (lcm_m_n)
  integer, intent(in) :: m, n
  integer :: lcm_m_n
  lcm_m_n = (m * n) / gcd (m, n)
end function lcm

```

A very simple minded factorization procedure, that is not fool proof at all. It maintains $n == \text{product}(\text{factors}(1:i))$, however, and will work in all cases of practical relevance.

311b *<Implementation of utils procedures 305c>+≡ (304a) <311a 312c>*

```

pure subroutine factorize (n, factors, i)
  integer, intent(in) :: n
  integer, dimension(:), intent(out) :: factors
  integer, intent(out) :: i
  integer :: nn, p
  nn = n
  i = 0
  do p = 1, size (PRIMES)
    try: do
      if (modulo (nn, PRIMES(p)) == 0) then
        i = i + 1
        factors(i) = PRIMES(p)
        nn = nn / PRIMES(p)
        if (i >= size (factors)) then
          factors(i) = nn
          return
        end if
      else
        exit try
      end if
    end do try
    if (nn == 1) then
      return
    end if
  end do
end subroutine factorize

```

311c *<Parameters in utils 311c>≡ (304a)*

```

integer, dimension(13), parameter, private :: &
  PRIMES = (/ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 /)

```


G.4 I/O

312a *<Declaration of utils procedures 304b>+≡* (304a) <310d
 public :: **find_free_unit**

312b *<Variables in utils 312b>≡* (304a)
 integer, parameter, private :: **MIN_UNIT** = 11, **MAX_UNIT** = 99

312c *<Implementation of utils procedures 305c>+≡* (304a) <311b
 subroutine find_free_unit (u, iostat)
 integer, intent(out) :: u
 integer, intent(out), optional :: iostat
 logical :: exists, is_open
 integer :: **i**, status
 do **i** = **MIN_UNIT**, **MAX_UNIT**
 inquire (**unit** = **i**, exist = exists, opened = is_open, &
 iostat = status)
 if (status == 0) then
 if (exists .and. .not. is_open) then
 u = **i**
 if (present (iostat)) then
 iostat = 0
 end if
 return
 end if
 end if
 end do
 if (present (iostat)) then
 iostat = -1
 end if
 u = -1
 end **subroutine find_free_unit**

—H—

LINEAR ALGEBRA

```

313a  <linalg.f90 313a>≡
      ! linalg.f90 --
      <Copleft notice 1>
      module linalg
        use kinds
        use utils
        implicit none
        private
        <Declaration of linalg procedures 313b>
      contains
        <Implementation of linalg procedures 314>
      end module linalg

```

H.1 LU Decomposition

```

313b  <Declaration of linalg procedures 313b>≡                                     (313a) 315e>
      public :: lu_decompose

```

$$A = LU \tag{H.1a}$$

In more detail

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ l_{21} & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ l_{n1} & l_{n2} & \dots & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & u_{nn} \end{pmatrix} \tag{H.1b}$$

Rewriting (H.1) in block matrix notation

$$\begin{pmatrix} a_{11} & a_{1\cdot} \\ a_{\cdot 1} & A \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ l_{\cdot 1} & L \end{pmatrix} \begin{pmatrix} u_{11} & u_{1\cdot} \\ 0 & U \end{pmatrix} = \begin{pmatrix} u_{11} & u_{1\cdot} \\ l_{\cdot 1} u_{11} & l_{\cdot 1} \otimes u_{1\cdot} + LU \end{pmatrix} \tag{H.2}$$

we can solve it easily

$$u_{11} = a_{11} \quad (\text{H.3a})$$

$$u_{1\cdot} = a_{1\cdot} \quad (\text{H.3b})$$

$$l_{\cdot 1} = \frac{a_{\cdot 1}}{a_{11}} \quad (\text{H.3c})$$

$$LU = A - \frac{a_{\cdot 1} \otimes a_{1\cdot}}{a_{11}} \quad (\text{H.3d})$$

and (H.3c) and (H.3d) define a simple iterative algorithm if we work from the outside in. It just remains to add pivoting.

314 *Implementation of linalg procedures 314* \equiv (313a) 316 \triangleright

```

pure subroutine lu_decompose (a, pivots, eps, l, u)
  real(kind=default), dimension(:,:), intent(inout) :: a
  integer, dimension(:), intent(out), optional :: pivots
  real(kind=default), intent(out), optional :: eps
  real(kind=default), dimension(:,:), intent(out), optional :: l, u
  real(kind=default), dimension(size(a,dim=1)) :: vv
  integer, dimension(size(a,dim=1)) :: p
  integer :: j, pivot
  (eps = 1 315a)
  vv = maxval (abs (a), dim=2)
  if (any (vv == 0.0)) then
    a = 0.0
    (pivots = 0 and eps = 0 315c)
    return
  end if
  vv = 1.0 / vv
  do j = 1, size (a, dim=1)
    pivot = j - 1 + sum (maxloc (vv(j:) * abs (a(j:,j))))
    if (j /= pivot) then
      call swap (a(pivot,:), a(j,:))
      (eps = - eps 315b)
      vv(pivot) = vv(j)
    end if
    p(j) = pivot
    if (a(j,j) == 0.0) then
      a(j,j) = tiny (a(j,j))
    end if
    a(j+1:,j) = a(j+1:,j) / a(j,j)
    a(j+1:,j+1:) &
      = a(j+1:,j+1:) - outer_product (a(j+1:,j), a(j,j+1:))
  end do

```

```

    <Return optional arguments in lu_decompose 315d>
end subroutine lu_decompose

315a <eps = 1 315a>≡ (314)
    if (present (eps)) then
        eps = 1.0
    end if

315b <eps = - eps 315b>≡ (314)
    if (present (eps)) then
        eps = - eps
    end if

315c <pivots = 0 and eps = 0 315c>≡ (314)
    if (present (pivots)) then
        pivots = 0
    end if
    if (present (eps)) then
        eps = 0
    end if

315d <Return optional arguments in lu_decompose 315d>≡ (314)
    if (present (pivots)) then
        pivots = p
    end if
    if (present (l)) then
        do j = 1, size (a, dim=1)
            l(1:j-1,j) = 0.0
            l(j,j) = 1.0
            l(j+1:,j) = a(j+1:,j)
        end do
        do j = size (a, dim=1), 1, -1
            call swap (l(j,:), l(p(j),:))
        end do
    end if
    if (present (u)) then
        do j = 1, size (a, dim=1)
            u(1:j,j) = a(1:j,j)
            u(j+1:,j) = 0.0
        end do
    end if

```

H.2 Determinant

315e <Declaration of linalg procedures 313b>+≡ (313a) <313b 317a>

```
public :: determinant
```

This is a subroutine to comply with F's rules, otherwise, we would code it as a function.

316 \langle Implementation of linalg procedures 314 $\rangle + \equiv$ (313a) \langle 314 317b \rangle

```
pure subroutine determinant (a, det)
  real(kind=default), dimension(:, :), intent(in) :: a
  real(kind=default), intent(out) :: det
  real(kind=default), dimension(size(a,dim=1),size(a,dim=2)) :: lu
  integer :: i
  lu = a
  call lu_decompose (lu, eps = det)
  do i = 1, size (a, dim = 1)
    det = det * lu(i,i)
  end do
end subroutine determinant
```

H.3 Diagonalization

The code is an implementation of the algorithm presented in [17, 18], but independent from the code presented in [19] to avoid legal problems.

A Jacobi rotation around the angle ϕ in row p and column q

$$P(\phi; p, q) = \begin{pmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & \cos \phi & \cdots & \sin \phi & & \\ & & \vdots & 1 & \vdots & & \\ & & -\sin \phi & \cdots & \cos \phi & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{pmatrix} \quad (\text{H.4})$$

results in

$$A' = P^T(\phi; p, q) \cdot A \cdot P(\phi; p, q) = \begin{pmatrix} & & A'_{1p} & & A'_{1q} & & \\ & & \vdots & & \vdots & & \\ A'_{p1} & \cdots & A'_{pq} & \cdots & A'_{pq} & \cdots & A'_{pn} \\ & & \vdots & & \vdots & & \\ A'_{q1} & \cdots & A'_{qp} & \cdots & A'_{qq} & \cdots & A'_{qn} \\ & & \vdots & & \vdots & & \\ & & A'_{np} & & A'_{nq} & & \end{pmatrix} \quad (\text{H.5})$$

317a *<Declaration of linalg procedures 313b>+≡* (313a) <315e 320b>
public :: diagonalize_real_symmetric

317b *<Implementation of linalg procedures 314>+≡* (313a) <316 320a>
pure subroutine diagonalize_real_symmetric (a, eval, evec, num_rot)
real(kind=default), dimension(:, :), intent(in) :: a
real(kind=default), dimension(:), intent(out) :: eval
real(kind=default), dimension(:, :), intent(out) :: evec
integer, intent(out), optional :: num_rot
real(kind=default), dimension(size(a,dim=1),size(a,dim=2)) :: aa
real(kind=default) :: off_diagonal_norm, threshold, &
c, g, h, s, t, tau, cot_2phi
logical, dimension(size(eval),size(eval)) :: upper_triangle
integer, dimension(size(eval)) :: one_to_ndim
integer :: p, q, ndim, j, sweep
integer, parameter :: MAX_SWEEPS = 50
ndim = size (eval)
one_to_ndim = (/ (j, j=1,ndim) /)
upper_triangle = &
spread (one_to_ndim, dim=1, ncopies=ndim) &
> spread (one_to_ndim, dim=2, ncopies=ndim)
aa = a
call unit (evec)
<Initialize num_rot 320d>
sweeps: do sweep = 1, MAX_SWEEPS
off_diagonal_norm = sum (abs (aa), mask=upper_triangle)
if (off_diagonal_norm == 0.0) then
eval = diag (aa)
return
end if
if (sweep < 4) then
threshold = 0.2 * off_diagonal_norm / ndim**2
else
threshold = 0.0
end if
do p = 1, ndim - 1
do q = p + 1, ndim
<Perform the Jacobi rotation resulting in $A'_{pq} = 0$ 318a>
end do
end do
end do sweeps
if (present (num_rot)) then
num_rot = -1
end if

```

!!! print *, "linalg::diagonalize_real_symmetric: exceeded sweep count"
end subroutine diagonalize_real_symmetric

318a  <Perform the Jacobi rotation resulting in  $A'_{pq} = 0$  318a>≡ (317b)
      g = 100 * abs (aa (p,q))
      if ((sweep > 4) &
          .and. (g <= min (spacing (aa(p,p)), spacing (aa(q,q))))) then
          aa(p,q) = 0.0
      else if (abs (aa(p,q)) > threshold) then
          <Determine  $\phi$  for the Jacobi rotation  $P(\phi; p, q)$  with  $A'_{pq} = 0$  318b>
          < $A' = P^T(\phi; p, q) \cdot A \cdot P(\phi; p, q)$  319b>
          < $V' = V \cdot P(\phi; p, q)$  320c>
          <Update num_rot 320e>
      end if

```

We want

$$A'_{pq} = (c^2 - s^2)A_{pq} + sc(A_{pp} - A_{qq}) = 0 \quad (\text{H.6})$$

and therefore

$$\cot 2\phi = \frac{1 - \tan^2 \phi}{2 \tan \phi} = \frac{\cos^2 \phi - \sin^2 \phi}{2 \sin \phi \cos \phi} = \frac{A_{pp} - A_{qq}}{2A_{pq}} \quad (\text{H.7})$$

i.e. with $t = \tan \phi = s/c$

$$t^2 + 2t \cot 2\phi - 1 = 0 \quad (\text{H.8})$$

This quadratic equation has the roots

$$t = -\cot 2\phi \pm \sqrt{1 + \cot^2 2\phi} = \frac{\epsilon(\cot 2\phi)}{|\cot 2\phi| \pm \epsilon(\cot 2\phi)\sqrt{1 + \cot^2 2\phi}} \quad (\text{H.9})$$

and the smaller in magnitude of these is

$$t = \frac{\epsilon(\cot 2\phi)}{|\cot 2\phi| + \sqrt{1 + \cot^2 2\phi}} \quad (\text{H.10})$$

and since $|t| \leq 1$, it corresponds to $|\phi| \leq \pi/4$. For very large $\cot 2\phi$ we will use

$$t = \frac{1}{2 \cot 2\phi} = \frac{A_{pq}}{A_{pp} - A_{qq}} \quad (\text{H.11})$$

$$h = A_{qq} - A_{pp} \quad (\text{H.12})$$

```

318b  <Determine  $\phi$  for the Jacobi rotation  $P(\phi; p, q)$  with  $A'_{pq} = 0$  318b>≡ (318a) 319a>
      h = aa(q,q) - aa(p,p)
      if (g <= spacing (h)) then

```

```

    t = aa(p,q) / h
else
    cot_2phi = 0.5 * h / aa(p,q)
    t = sign (1.0_default, cot_2phi) &
        / (abs (cot_2phi) + sqrt (1.0 + cot_2phi**2))
end if

```

Trivia

$$\cos^2 \phi = \frac{\cos^2 \phi}{\cos^2 \phi + \sin^2 \phi} = \frac{1}{1 + \tan^2 \phi} \quad (\text{H.13a})$$

$$\sin \phi = \tan \phi \cos \phi \quad (\text{H.13b})$$

$$\tau \sin \phi = \frac{\sin^2}{1 + \cos \phi} = \frac{1 - \cos^2}{1 + \cos \phi} = 1 - \cos \phi \quad (\text{H.13c})$$

319a $\langle \text{Determine } \phi \text{ for the Jacobi rotation } P(\phi; p, q) \text{ with } A'_{pq} = 0 \text{ } \mathbf{318b} \rangle + \equiv \quad (\mathbf{318a}) \triangleleft \mathbf{318b}$

```

c = 1.0 / sqrt (1.0 + t**2)
s = t * c
tau = s / (1.0 + c)

```

$$\begin{aligned} A'_{pp} &= c^2 A_{pp} + s^2 A_{qq} - 2sc A_{pq} = A_{pp} - t A_{pq} \\ A'_{qq} &= s^2 A_{pp} + c^2 A_{qq} + 2sc A_{pq} = A_{qq} + t A_{pq} \\ A'_{pq} &= (c^2 - s^2) A_{pq} + sc(A_{pp} - A_{qq}) \end{aligned} \quad (\text{H.14})$$

319b $\langle A' = P^T(\phi; p, q) \cdot A \cdot P(\phi; p, q) \text{ } \mathbf{319b} \rangle \equiv \quad (\mathbf{318a}) \text{ } \mathbf{319c} \triangleright$

```

aa(p,p) = aa(p,p) - t * aa(p,q)
aa(q,q) = aa(q,q) + t * aa(p,q)
aa(p,q) = 0.0

```

$$\begin{aligned} r \neq p < q \neq r : A'_{rp} &= c A_{rp} - s A_{rq} \\ A'_{rq} &= s A_{rp} + c A_{rq} \end{aligned} \quad (\text{H.15})$$

Here's how we cover the upper triangular region using array notation:

$$\begin{pmatrix} & \mathbf{a(1:p-1,p)} & & \mathbf{a(1:p-1,q)} & \\ \cdots & A_{pq} & \mathbf{a(p,p+1:q-1)} & A_{pq} & \mathbf{a(p,q+1:ndim)} \\ & \vdots & & \mathbf{a(p+1:q-1,q)} & \\ \cdots & A_{qp} & \cdots & A_{qq} & \mathbf{a(q,q+1:ndim)} \\ & \vdots & & \vdots & \end{pmatrix} \quad (\text{H.16})$$

319c $\langle A' = P^T(\phi; p, q) \cdot A \cdot P(\phi; p, q) \text{ } \mathbf{319b} \rangle + \equiv \quad (\mathbf{318a}) \triangleleft \mathbf{319b}$

```

call jacobi_rotation (s, tau, aa(1:p-1,p), aa(1:p-1,q))
call jacobi_rotation (s, tau, aa(p,p+1:q-1), aa(p+1:q-1,q))
call jacobi_rotation (s, tau, aa(p,q+1:ndim), aa(q,q+1:ndim))

```


Using (H.13c), we can write the rotation as a perturbation:

$$\begin{aligned} V'_p &= cV_p - sV_q = V_p - s(V_q + \tau V_p) \\ V'_q &= sV_p + cV_q = V_q + s(V_p - \tau V_q) \end{aligned} \quad (\text{H.17})$$

```

320a  <Implementation of linalg procedures 314>+≡ (313a) <317b 320f>
      pure subroutine jacobi_rotation (s, tau, vp, vq)
        real(kind=default), intent(in) :: s, tau
        real(kind=default), dimension(:), intent(inout) :: vp, vq
        real(kind=default), dimension(size(vp)) :: vp_tmp
        vp_tmp = vp
        vp = vp - s * (vq + tau * vp)
        vq = vq + s * (vp_tmp - tau * vq)
      end subroutine jacobi_rotation

320b  <Declaration of linalg procedures 313b>+≡ (313a) <317a 321a>
      private :: jacobi_rotation

320c  <V' = V · P(φ; p, q) 320c>≡ (318a)
      call jacobi_rotation (s, tau, evec(:,p), evec(:,q))

320d  <Initialize num_rot 320d>≡ (317b)
      if (present (num_rot)) then
        num_rot = 0
      end if

320e  <Update num_rot 320e>≡ (318a)
      if (present (num_rot)) then
        num_rot = num_rot + 1
      end if

320f  <Implementation of linalg procedures 314>+≡ (313a) <320a 320g>
      pure subroutine unit (u)
        real(kind=default), dimension(:, :), intent(out) :: u
        integer :: i
        u = 0.0
        do i = 1, min (size (u, dim = 1), size (u, dim = 2))
          u(i,i) = 1.0
        end do
      end subroutine unit

320g  <Implementation of linalg procedures 314>+≡ (313a) <320f>
      pure function diag (a) result (d)
        real(kind=default), dimension(:, :), intent(in) :: a
        real(kind=default), dimension(min(size(a,dim=1),size(a,dim=2))) :: d
        integer :: i
        do i = 1, min (size (a, dim = 1), size (a, dim = 2))

```

```

        d(i) = a(i,i)
    end do
end function diag
321a  ⟨Declaration of linalg procedures 313b⟩+≡ (313a) <320b
      public :: unit, diag

```

H.4 Test

```

321b  ⟨la_sample.f90 321b⟩≡
      ! la_sample.f90 --
      ⟨Copyleft notice 1⟩
      program la_sample
      use kinds
      use utils
      use tao_random_numbers
      use linalg
      implicit none
      integer, parameter :: N = 200
      real(kind=default), dimension(N,N) :: a, evec, a0, l, u, NAG_bug
      real(kind=default), dimension(N) :: b, eval
      real(kind=default) :: d
      integer :: i
      call system_clock (i)
      call tao_random_seed (i)
      print *, i
      do i = 1, N
          call tao_random_number (a(:,i))
      end do
      NAG_bug = (a + transpose (a)) / 2
      a = NAG_bug
      a0 = a
      call lu_decompose (a, l=l, u=u)
      a = matmul (l, u)
      print *, maxval (abs(a-a0))
      call determinant (a, d)
      print *, d
      call diagonalize_real_symmetric (a, eval, evec)
      print *, product (eval)
      stop
      call sort (eval, evec)
      do i = 1, N
          b = matmul (a, evec(:,i)) - eval(i) * evec(:,i)

```

```

        write (unit = *, fmt = "(A,I3, 2(A,E11.4))") &
            "eval #", i, " = ", eval(i), ", |(A-lambda)V|_infty = ", &
            maxval (abs(b)) / maxval (abs(evec(:,i)))
    end do
end program la_sample

```

—I—

PRODUCTS

```
323 <products.f90 323>≡
! products.f90 --
<Copyleft notice 1>
module products
  use kinds
  implicit none
  private
  public :: dot, sp, spc
contains
  pure function dot (p, q) result (pq)
    real(kind=default), dimension(0:), intent(in) :: p, q
    real(kind=default) :: pq
    pq = p(0)*q(0) - dot_product (p(1:), q(1:))
  end function dot
  pure function sp (p, q) result (sppq)
    real(kind=default), dimension(0:), intent(in) :: p, q
    complex(kind=default) :: sppq
    sppq = cmplx (p(2), p(3), kind=default) * sqrt ((q(0)-q(1))/(p(0)-p(1))) &
      - cmplx (q(2), q(3), kind=default) * sqrt ((p(0)-p(1))/(q(0)-q(1)))
  end function sp
  pure function spc (p, q) result (spcpq)
    real(kind=default), dimension(0:), intent(in) :: p, q
    complex(kind=default) :: spcpq
    spcpq = conjg (sp (p, q))
  end function spc
end module products
```

—J—

KINEMATICS

```

324a  <kinematics.f90 324a>≡                                     329d>
      ! kinematics.f90 --
      <Copyleft notice 1>
      module kinematics
        use kinds
        use constants
        use products, only: dot
        use specfun, only: gamma
        implicit none
        private
        <Declaration of kinematics procedures 324b>
        <Interfaces of kinematics procedures 324c>
        <Declaration of kinematics types 326f>
        contains
        <Implementation of kinematics procedures 325a>
      end module kinematics

```

J.1 Lorentz Transformations

```

324b  <Declaration of kinematics procedures 324b>≡                (324a) 326c>
      public :: boost_velocity
      private :: boost_one_velocity, boost_many_velocity
      public :: boost_momentum
      private :: boost_one_momentum, boost_many_momentum

324c  <Interfaces of kinematics procedures 324c>≡                (324a) 326e>
      interface boost_velocity
        module procedure boost_one_velocity, boost_many_velocity
      end interface
      interface boost_momentum
        module procedure boost_one_momentum, boost_many_momentum

```

end interface

Boost a four vector p to the inertial frame moving with the velocity β :

$$p'_0 = \gamma (p_0 - \vec{\beta} \vec{p}) \quad (\text{J.1a})$$

$$\vec{p}' = \gamma (\vec{p}_{\parallel} - \vec{\beta} p_0) + \vec{p}_{\perp} \quad (\text{J.1b})$$

with $\gamma = 1/\sqrt{1 - \vec{\beta}^2}$, $\vec{p}_{\parallel} = \vec{\beta}(\vec{\beta} \vec{p})/\vec{\beta}^2$ and $\vec{p}_{\perp} = \vec{p} - \vec{p}_{\parallel}$. Using $1/\vec{\beta}^2 = \gamma^2/(\gamma + 1) \cdot 1/(\gamma - 1)$ and $\vec{b} = \gamma \vec{\beta}$ this can be rewritten as

$$p'_0 = \gamma p_0 - \vec{b} \vec{p} \quad (\text{J.2a})$$

$$\vec{p}' = \vec{p} + \left(\frac{\vec{b} \vec{p}}{\gamma + 1} - p_0 \right) \vec{b} \quad (\text{J.2b})$$

325a \langle Implementation of kinematics procedures 325a $\rangle \equiv$ (324a) 325b \rangle

```
pure function boost_one_velocity (p, beta) result (p_prime)
  real(kind=default), dimension(0:), intent(in) :: p
  real(kind=default), dimension(1:), intent(in) :: beta
  real(kind=default), dimension(0:3) :: p_prime
  real(kind=default), dimension(1:3) :: b
  real(kind=default) :: gamma, b_dot_p
  gamma = 1.0 / sqrt (1.0 - dot_product (beta, beta))
  b = gamma * beta
  b_dot_p = dot_product (b, p(1:3))
  p_prime(0) = gamma * p(0) - b_dot_p
  p_prime(1:3) = p(1:3) + (b_dot_p / (1.0 + gamma) - p(0)) * b
end function boost_one_velocity
```

325b \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \langle 325a 325c \rangle

```
pure function boost_many_velocity (p, beta) result (p_prime)
  real(kind=default), dimension(:,0:), intent(in) :: p
  real(kind=default), dimension(1:), intent(in) :: beta
  real(kind=default), dimension(size(p,dim=1),0:3) :: p_prime
  integer :: i
  do i = 1, size (p, dim=1)
    p_prime(i,:) = boost_one_velocity (p(i,:), beta)
  end do
end function boost_many_velocity
```

Boost a four vector p to the rest frame of the four vector q . The velocity is $\vec{\beta} = \vec{q}/|q_0|$:

325c \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \langle 325b 326a \rangle

```

pure function boost_one_momentum (p, q) result (p_prime)
  real(kind=default), dimension(0:), intent(in) :: p, q
  real(kind=default), dimension(0:3) :: p_prime
  p_prime = boost_velocity (p, q(1:3) / abs (q(0)))
end function boost_one_momentum

```

326a \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \langle 325c 326b \rangle

```

pure function boost_many_momentum (p, q) result (p_prime)
  real(kind=default), dimension(:,0:), intent(in) :: p
  real(kind=default), dimension(0:), intent(in) :: q
  real(kind=default), dimension(size(p,dim=1),0:3) :: p_prime
  p_prime = boost_many_velocity (p, q(1:3) / abs (q(0)))
end function boost_many_momentum

```

J.2 Massive Phase Space

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca = (a - b - c)^2 - 4bc \quad (\text{J.3})$$

and permutations

326b \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \langle 326a 327a \rangle

```

pure function lambda (a, b, c) result (lam)
  real(kind=default), intent(in) :: a, b, c
  real(kind=default) :: lam
  lam = a**2 + b**2 + c**2 - 2*(a*b + b*c + c*a)
end function lambda

```

326c \langle Declaration of kinematics procedures 324b $\rangle + \equiv$ (324a) \langle 324b 326d \rangle

```

public :: lambda

```

326d \langle Declaration of kinematics procedures 324b $\rangle + \equiv$ (324a) \langle 326c 328a \rangle

```

public :: two_to_three
private :: two_to_three_massive, two_to_three_massless

```

326e \langle Interfaces of kinematics procedures 324c $\rangle + \equiv$ (324a) \langle 324c 328b \rangle

```

interface two_to_three
  module procedure two_to_three_massive, two_to_three_massless
end interface

```

326f \langle Declaration of kinematics types 326f $\rangle \equiv$ (324a)

```

type, public :: LIPS3
  real(kind=default), dimension(3,0:3) :: p
  real(kind=default) :: jacobian
end type LIPS3

```

$$dLIPS_3 = \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^4(p_1 + p_2 + p_3 - p_a - p_b) \quad (J.4)$$

The jacobian is given by

$$dLIPS_3 = \frac{1}{(2\pi)^5} \int d\phi dt_1 ds_2 d\Omega_3^{[23]} \frac{1}{32\sqrt{ss_2}} \frac{|p_3^{[23]}|}{|p_a^{[ab]}|} \quad (J.5)$$

where $\vec{p}_i^{[jk]}$ denotes the momentum of particle i in the center of mass system of particles j and k .

327a \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \langle 326b 327b \rangle

```

pure function two_to_three_massive &
  (s, t1, s2, phi, cos_theta3, phi3, ma, mb, m1, m2, m3) result (p)
  real(kind=default), intent(in) :: &
    s, t1, s2, phi, cos_theta3, phi3, ma, mb, m1, m2, m3
  type(LIPS3) :: p
  real(kind=default), dimension(0:3) :: p23
  real(kind=default) :: Ea, pa_abs, E1, p1_abs, p3_abs, cos_theta
  pa_abs = sqrt (lambda (s, ma**2, mb**2) / (4 * s))
  Ea = sqrt (ma**2 + pa_abs**2)
  p1_abs = sqrt (lambda (s, m1**2, s2) / (4 * s))
  E1 = sqrt (m1**2 + p1_abs**2)
  p3_abs = sqrt (lambda (s2, m2**2, m3**2) / (4 * s2))
  p%jacobian = &
    1.0 / (2*PI)**5 * (p3_abs / pa_abs) / (32 * sqrt (s * s2))
  cos_theta = (t1 - ma**2 - m1**2 + 2*Ea*E1) / (2*pa_abs*p1_abs)
  p%p(1,1:3) = polar_to_cartesian (p1_abs, cos_theta, phi)
  p%p(1,0) = on_shell (p%p(1,:), m1)
  p23(1:3) = - p%p(1,1:3)
  p23(0) = on_shell (p23, sqrt (s2))
  p%p(3:2:-1,:) = one_to_two (p23, cos_theta3, phi3, m3, m2)
end function two_to_three_massive

```

A specialized version for massless particles can be faster, because the kinematics is simpler:

327b \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \langle 327a 328c \rangle

```

pure function two_to_three_massless (s, t1, s2, phi, cos_theta3, phi3) &
  result (p)
  real(kind=default), intent(in) :: s, t1, s2, phi, cos_theta3, phi3
  type(LIPS3) :: p
  real(kind=default), dimension(0:3) :: p23
  real(kind=default) :: pa_abs, p1_abs, p3_abs, cos_theta
  pa_abs = sqrt (s) / 2
  p1_abs = (s - s2) / (2 * sqrt (s))

```



```

    p3_abs = sqrt (s2) / 2
    p%jacobian = 1.0 / ((2*PI)**5 * 32 * s)
    cos_theta = 1 + t1 / (2*pa_abs*p1_abs)
    p%p(1,0) = p1_abs
    p%p(1,1:3) = polar_to_cartesian (p1_abs, cos_theta, phi)
    p23(1:3) = - p%p(1,1:3)
    p23(0) = on_shell (p23, sqrt (s2))
    p%p(3:2:-1,:) = one_to_two (p23, cos_theta3, phi3)
end function two_to_three_massless

328a <Declaration of kinematics procedures 324b>+≡ (324a) <326d 329a>
    public :: one_to_two
    private :: one_to_two_massive, one_to_two_massless

328b <Interfaces of kinematics procedures 324c>+≡ (324a) <326e
    interface one_to_two
        module procedure one_to_two_massive, one_to_two_massless
    end interface

328c <Implementation of kinematics procedures 325a>+≡ (324a) <327b 328d>
    pure function one_to_two_massive (p12, cos_theta, phi, m1, m2) result (p)
        real(kind=default), dimension(0:), intent(in) :: p12
        real(kind=default), intent(in) :: cos_theta, phi, m1, m2
        real(kind=default), dimension(2,0:3) :: p
        real(kind=default) :: s, p1_abs
        s = dot (p12, p12)
        p1_abs = sqrt (lambda (s, m1**2, m2**2) / (4 * s))
        p(1,1:3) = polar_to_cartesian (p1_abs, cos_theta, phi)
        p(2,1:3) = - p(1,1:3)
        p(1,0) = on_shell (p(1,:), m1)
        p(2,0) = on_shell (p(2,:), m2)
        p = boost_momentum (p, - p12)
    end function one_to_two_massive

328d <Implementation of kinematics procedures 325a>+≡ (324a) <328c 329b>
    pure function one_to_two_massless (p12, cos_theta, phi) result (p)
        real(kind=default), dimension(0:), intent(in) :: p12
        real(kind=default), intent(in) :: cos_theta, phi
        real(kind=default), dimension(2,0:3) :: p
        real(kind=default) :: p1_abs
        p1_abs = sqrt (dot (p12, p12)) / 2
        p(1,0) = p1_abs
        p(1,1:3) = polar_to_cartesian (p1_abs, cos_theta, phi)
        p(2,0) = p1_abs
        p(2,1:3) = - p(1,1:3)
        p = boost_momentum (p, - p12)

```

```

end function one_to_two_massless

329a <Declaration of kinematics procedures 324b>+≡ (324a) <328a 332c>
public :: polar_to_cartesian, on_shell

329b <Implementation of kinematics procedures 325a>+≡ (324a) <328d 329c>
pure function polar_to_cartesian (v_abs, cos_theta, phi) result (v)
  real(kind=default), intent(in) :: v_abs, cos_theta, phi
  real(kind=default), dimension(3) :: v
  real(kind=default) :: sin_phi, cos_phi, sin_theta
  sin_theta = sqrt (1.0 - cos_theta**2)
  cos_phi = cos (phi)
  sin_phi = sin (phi)
  v = (/ sin_theta * cos_phi, sin_theta * sin_phi, cos_theta /) * v_abs
end function polar_to_cartesian

329c <Implementation of kinematics procedures 325a>+≡ (324a) <329b 332d>
pure function on_shell (p, m) result (E)
  real(kind=default), dimension(0:), intent(in) :: p
  real(kind=default), intent(in) :: m
  real(kind=default) :: E
  E = sqrt (m**2 + dot_product (p(1:3), p(1:3)))
end function on_shell

```

J.3 Massive 3-Particle Phase Space Revisited

$$\begin{array}{ccccc}
U_1 & \xrightarrow{\xi_1} & P_1 & \xrightarrow{\phi_1} & M \\
\pi_U \downarrow & & \downarrow \pi_P & & \parallel \\
U_2 & \xrightarrow{\xi_2} & P_2 & \xrightarrow{\phi_2} & M
\end{array} \tag{J.6}$$

$$\begin{array}{ccccc}
U_1 & \xrightarrow{\xi} & P_1 & \xrightarrow{\phi} & M \\
\pi_U \downarrow & & \downarrow \pi_P & & \downarrow \pi \\
U_2 & \xrightarrow{\xi} & P_2 & \xrightarrow{\phi} & M
\end{array} \tag{J.7}$$

```

329d <kinematics.f90 324a>+≡ <324a
module phase_space
  use kinds
  use constants
  use kinematics !NODEP!
  use tao_random_numbers
  implicit none
  private

```

```

    <Declaration of phase_space procedures 331b>
    <Interfaces of phase_space procedures 331c>
    <Declaration of phase_space types 330a>
contains
    <Implementation of phase_space procedures 331d>
end module phase_space

                                LIPS3_unit : [0, 1]5                                (J.8)

330a <Declaration of phase_space types 330a>≡                                (329d) 330b>
    type, public :: LIPS3_unit
        real(kind=default), dimension(5) :: x
        real(kind=default) :: s
        real(kind=default), dimension(2) :: mass_in
        real(kind=default), dimension(3) :: mass_out
        real(kind=default) :: jacobian
    end type LIPS3_unit

330b <Declaration of phase_space types 330a>+≡                                (329d) <330a 330c>
    type, public :: LIPS3_unit_massless
        real(kind=default), dimension(5) :: x
        real(kind=default) :: s
        real(kind=default) :: jacobian
    end type LIPS3_unit_massless

                                LIPS3_s2_t1_angles : (s2, t1, ϕ, cos θ3, ϕ3)                                (J.9)

330c <Declaration of phase_space types 330a>+≡                                (329d) <330b 330d>
    type, public :: LIPS3_s2_t1_angles
        real(kind=default) :: s2, t1, phi, cos_theta3, phi3
        real(kind=default) :: s
        real(kind=default), dimension(2) :: mass_in
        real(kind=default), dimension(3) :: mass_out
        real(kind=default) :: jacobian
    end type LIPS3_s2_t1_angles

330d <Declaration of phase_space types 330a>+≡                                (329d) <330c 330e>
    type, public :: LIPS3_s2_t1_angles_massless
        real(kind=default) :: s2, t1, phi, cos_theta3, phi3
        real(kind=default) :: s
        real(kind=default) :: jacobian
    end type LIPS3_s2_t1_angles_massless

                                LIPS3_momenta : (p1, p2, p3)                                (J.10)

330e <Declaration of phase_space types 330a>+≡                                (329d) <330d 331a>
    type, public :: LIPS3_momenta
        real(kind=default), dimension(0:3,3) :: p

```

```

    real(kind=default) :: s
    real(kind=default), dimension(2) :: mass_in
    real(kind=default), dimension(3) :: mass_out
    real(kind=default) :: jacobian
end type LIPS3_momenta

331a <Declaration of phase_space types 330a>+≡ (329d) <330e>
type, public :: LIPS3_momenta_massless
    real(kind=default), dimension(0:3,3) :: p
    real(kind=default) :: s
    real(kind=default) :: jacobian
end type LIPS3_momenta_massless

331b <Declaration of phase_space procedures 331b>≡ (329d) 331f>
public :: random_LIPS3
private :: random_LIPS3_unit, random_LIPS3_unit_massless

331c <Interfaces of phase_space procedures 331c>≡ (329d)
interface random_LIPS3
    module procedure random_LIPS3_unit, random_LIPS3_unit_massless
end interface

331d <Implementation of phase_space procedures 331d>≡ (329d) 331e>
pure subroutine random_LIPS3_unit (rng, lips)
    type(tao_random_state), intent(inout) :: rng
    type(LIPS3_unit), intent(inout) :: lips
    call tao_random_number (rng, lips%x)
    lips%jacobian = 1
end subroutine random_LIPS3_unit

331e <Implementation of phase_space procedures 331d>+≡ (329d) <331d 332a>
pure subroutine random_LIPS3_unit_massless (rng, lips)
    type(tao_random_state), intent(inout) :: rng
    type(LIPS3_unit_massless), intent(inout) :: lips
    call tao_random_number (rng, lips%x)
    lips%jacobian = 1
end subroutine random_LIPS3_unit_massless

331f <Declaration of phase_space procedures 331b>+≡ (329d) <331b>
private :: LIPS3_unit_to_s2_t1_angles, LIPS3_unit_to_s2_t1_angles_m0

331g <(Unused) Interfaces of phase_space procedures 331g>≡
interface assignment(=)
    module procedure &
        LIPS3_unit_to_s2_t1_angles, LIPS3_unit_to_s2_t1_angles_m0
end interface

```

332a \langle Implementation of phase_space procedures 331d $\rangle + \equiv$ (329d) \langle 331e 332b \rangle
 pure subroutine LIPS3_unit_to_s2_t1_angles (s2_t1_angles, unit)
 type(LIPS3_s2_t1_angles), intent(out) :: s2_t1_angles
 type(LIPS3_unit), intent(in) :: unit
 end subroutine LIPS3_unit_to_s2_t1_angles

332b \langle Implementation of phase_space procedures 331d $\rangle + \equiv$ (329d) \langle 332a
 pure subroutine LIPS3_unit_to_s2_t1_angles_m0 (s2_t1_angles, unit)
 type(LIPS3_s2_t1_angles_massless), intent(out) :: s2_t1_angles
 type(LIPS3_unit_massless), intent(in) :: unit
 end subroutine LIPS3_unit_to_s2_t1_angles_m0

J.4 Massless n -Particle Phase Space: RAMBO

332c \langle Declaration of kinematics procedures 324b $\rangle + \equiv$ (324a) \langle 329a 333b \rangle
 public :: massless_isotropic_decay

The massless RAMBO algorithm [26]:

332d \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \langle 329c 333c \rangle
 pure function massless_isotropic_decay (roots, ran) result (p)
 real (kind=default), intent(in) :: roots
 real (kind=default), dimension(:, :), intent(in) :: ran
 real (kind=default), dimension(size(ran, dim=1), 0:3) :: p
 real (kind=default), dimension(size(ran, dim=1), 0:3) :: q
 real (kind=default), dimension(0:3) :: qsum
 real (kind=default) :: cos_theta, sin_theta, phi, qabs, x, r, z
 integer :: k
 \langle Generate isotropic null vectors 332e \rangle
 \langle Boost and rescale the vectors 333a \rangle
 end function massless_isotropic_decay

Generate a xe^{-x} distribution for $q(\mathbf{k}, 0)$

332e \langle Generate isotropic null vectors 332e $\rangle \equiv$ (332d)
 do k = 1, size (p, dim = 1)
 q(k, 0) = - log (ran(k, 1) * ran(k, 2))
 cos_theta = 2 * ran(k, 3) - 1
 sin_theta = sqrt (1 - cos_theta**2)
 phi = 2 * PI * ran(k, 4)
 q(k, 1) = q(k, 0) * sin_theta * cos (phi)
 q(k, 2) = q(k, 0) * sin_theta * sin (phi)
 q(k, 3) = q(k, 0) * cos_theta
 enddo

The proof that the Jacobian of the transformation vanishes can be found in [26]. The transformation is really a Lorentz boost (as can be seen easily).

333a \langle Boost and rescale the vectors 333a $\rangle \equiv$ (332d)

```
qsum = sum (q, dim = 1)
qabs = sqrt (dot (qsum, qsum))
x = roots / qabs
do k = 1, size (p, dim = 1)
  r = dot (q(k,:), qsum) / qabs
  z = (q(k,0) + r) / (qsum(0) + qabs)
  p(k,1:3) = x * (q(k,1:3) - qsum(1:3) * z)
  p(k,0) = x * r
enddo
```

333b \langle Declaration of kinematics procedures 324b $\rangle + \equiv$ (324a) \triangleleft 332c

```
public :: phase_space_volume
```

$$V_n(s) = \frac{1}{8\pi} \frac{n-1}{(\Gamma(n))^2} \left(\frac{s}{16\pi^2} \right)^{n-2} \quad (\text{J.11})$$

333c \langle Implementation of kinematics procedures 325a $\rangle + \equiv$ (324a) \triangleleft 332d

```
pure function phase_space_volume (n, roots) result (volume)
  integer, intent(in) :: n
  real (kind=default), intent(in) :: roots
  real (kind=default) :: volume
  real (kind=default) :: nd
  nd = n
  volume = (nd - 1) / (8*PI * (gamma (nd))**2) * (roots / (4*PI))**(2*n-4)
end function phase_space_volume
```

J.5 Tests

333d \langle ktest.f90 333d $\rangle \equiv$

```
program ktest
  use kinds
  use constants
  use products
  use kinematics
  use tao_random_numbers
  implicit none
  real(kind=default) :: &
    ma, mb, m1, m2, m3, s, t1, s2, phi, cos_theta3, phi3
  real(kind=default) :: t1_min, t1_max
  real(kind=default), dimension(5) :: r
  type(LIPS3) :: p
```

```

integer :: i
character(len=*), parameter :: fmt = "(A,4(1X,E12.5))"
ma = 1.0
mb = 1.0
m1 = 10.0
m2 = 20.0
m3 = 30.0
s = 100.0 ** 2
do i = 1, 10
  call tao_random_number (r)
  s2 = (r(1) * (sqrt (s) - m1) + (1 - r(1)) * (m2 + m3)) ** 2
  t1_max = ma**2 + m1**2 - ((s + ma**2 - mb**2) * (s - s2 + m1**2) &
    + sqrt (lambda (s, ma**2, mb**2) * lambda (s, s2, m1**2))) / (2*s)
  t1_min = ma**2 + m1**2 - ((s + ma**2 - mb**2) * (s - s2 + m1**2) &
    - sqrt (lambda (s, ma**2, mb**2) * lambda (s, s2, m1**2))) / (2*s)
  t1 = r(2) * t1_max + (1 - r(2)) * t1_min
  phi = 2*PI * r(3)
  cos_theta3 = 2 * r(4) - 1
  phi3 = 2*PI * r(5)
  p = two_to_three (s, t1, s2, phi, cos_theta3, phi3, ma, mb, m1, m2, m3)
  print fmt, "p1      = ", p%p(1,:)
  print fmt, "p2      = ", p%p(2,:)
  print fmt, "p3      = ", p%p(3,:)
  print fmt, "p1,2,3^2 = ", dot (p%p(1,:), p%p(1,)), &
    dot (p%p(2,:), p%p(2,)), dot (p%p(3,:), p%p(3,))
  print fmt, "sum(p)   = ", p%p(1,:) + p%p(2,:) + p%p(3,:)
  print fmt, "|J|     = ", p%jacobian
end do
end program ktest

```



Trivial check for typos, should be removed from the finalized program!

334 \langle Trivial ktest.f90 334 $\rangle \equiv$

```

program ktest
  use kinds
  use constants
  use products
  use kinematics
  use tao_random_numbers
  implicit none
  real(kind=default), dimension(0:3) :: p, q, p_prime, p0
  real(kind=default) :: m
  character(len=*), parameter :: fmt = "(A,4(1X,E12.5))"
  integer :: i

```

```

do i = 1, 5
  if (i == 1) then
    p = (/ 1.0_double, 0.0_double, 0.0_double, 0.0_double /)
    m = 1.0
  else
    call tao_random_number (p)
    m = sqrt (PI)
  end if
  call tao_random_number (q(1:3))
  q(0) = sqrt (m**2 + dot_product (q(1:3), q(1:3)))
  p_prime = boost_momentum (p, q)
  print fmt, "p    = ", p
  print fmt, "q    = ", q
  print fmt, "p'   = ", p_prime
  print fmt, "p^2  = ", dot (p, p)
  print fmt, "p'^2 = ", dot (p_prime, p_prime)
  if (dot (p, p) > 0.0) then
    p0 = boost_momentum (p, p)
    print fmt, "p0   = ", p0
    print fmt, "p0^2 = ", dot (p0, p0)
  end if
end do
end program ktest

```


—K—

COORDINATES

```

336  <coordinates.f90 336>≡
      ! coordinates.f90 --
      <Copyleft notice 1>
      module coordinates
        use kinds
        use constants, only: PI
        use specfun, only: gamma
        implicit none
        private
        <Declaration of coordinates procedures 337a>
      contains
        <Implementation of coordinates procedures 337b>
      end module coordinates

```

K.1 Angular Spherical Coordinates

$$\begin{aligned}
 x_n &= r \cos \theta_{n-2} \\
 x_{n-1} &= r \sin \theta_{n-2} \cos \theta_{n-3} \\
 &\dots \\
 x_3 &= r \sin \theta_{n-2} \sin \theta_{n-3} \cdots \cos \theta_1 \\
 x_2 &= r \sin \theta_{n-2} \sin \theta_{n-3} \cdots \sin \theta_1 \cos \phi \\
 x_1 &= r \sin \theta_{n-2} \sin \theta_{n-3} \cdots \sin \theta_1 \sin \phi
 \end{aligned} \tag{K.1}$$

and

$$J = r^{n-1} \prod_{i=1}^{n-2} (\sin \theta_i)^i \tag{K.2}$$

We can minimize the number of multiplications by computing the products

$$P_j = \prod_{i=j}^{n-2} \sin \theta_i \quad (\text{K.3})$$

Then

$$\begin{aligned} x_n &= r \cos \theta_{n-2} \\ x_{n-1} &= r P_{n-2} \cos \theta_{n-3} \\ &\dots \\ x_3 &= r P_2 \cos \theta_1 \\ x_2 &= r P_1 \cos \phi \\ x_1 &= r P_1 \sin \phi \end{aligned} \quad (\text{K.4})$$

and

$$J = r^{n-1} \prod_{i=1}^{n-2} P_i \quad (\text{K.5})$$

Note that $\theta_i \in [0, \pi]$ and $\phi \in [0, 2\pi]$ or $\phi \in [-\pi, \pi]$. Therefore $\sin \theta_i \geq 0$ and

$$\sin \theta_i = \sqrt{1 - \cos^2 \theta_i} \quad (\text{K.6})$$

which is not true for ϕ . Since `sqrt` is typically much faster than `sin` and `cos`, we use (K.6) where ever possible.

```

337a <Declaration of coordinates procedures 337a>≡ (336) 338c>
      public :: spherical_to_cartesian_2, &
                spherical_to_cartesian, spherical_to_cartesian_j

337b <Implementation of coordinates procedures 337b>≡ (336) 338a>
      pure subroutine spherical_to_cartesian_2 (r, phi, theta, x, jacobian)
        real(kind=default), intent(in) :: r, phi
        real(kind=default), dimension(:), intent(in) :: theta
        real(kind=default), dimension(:), intent(out), optional :: x
        real(kind=default), intent(out), optional :: jacobian
        real(kind=default), dimension(size(theta)) :: cos_theta
        real(kind=default), dimension(size(theta)+1) :: product_sin_theta
        integer :: n, i
        n = size (theta) + 2
        cos_theta = cos (theta)
        product_sin_theta(n-1) = 1.0_default
        do i = n - 2, 1, -1
          product_sin_theta(i) = &
            product_sin_theta(i+1) * sqrt (1 - cos_theta(i)**2)
        end do

```

```

if (present (x)) then
  x(1) = r * product_sin_theta(1) * sin (phi)
  x(2) = r * product_sin_theta(1) * cos (phi)
  x(3:) = r * product_sin_theta(2:n-1) * cos_theta
end if
if (present (jacobian)) then
  jacobian = r**(n-1) * product (product_sin_theta)
end if
end subroutine spherical_to_cartesian_2

```



Note that call inside of a function breaks F-compatibility. Here it would be easy to fix, but the inverse can not be coded as a function, unless a type for spherical coordinates is introduced, where `theta` could not be assumed shape ...

- 338a *<Implementation of coordinates procedures 337b>+≡ (336) <337b 338b>*
 pure function `spherical_to_cartesian` (r, phi, theta) result (x)
 real(kind=default), intent(in) :: r, phi
 real(kind=default), dimension(:), intent(in) :: theta
 real(kind=default), dimension(size(theta)+2) :: x
 call `spherical_to_cartesian_2` (r, phi, theta, x = x)
 end function `spherical_to_cartesian`
- 338b *<Implementation of coordinates procedures 337b>+≡ (336) <338a 338d>*
 pure function `spherical_to_cartesian_j` (r, phi, theta) &
 result (jacobian)
 real(kind=default), intent(in) :: r, phi
 real(kind=default), dimension(:), intent(in) :: theta
 real(kind=default) :: jacobian
 call `spherical_to_cartesian_2` (r, phi, theta, jacobian = jacobian)
 end function `spherical_to_cartesian_j`
- 338c *<Declaration of coordinates procedures 337a>+≡ (336) <337a 340c>*
 public :: `cartesian_to_spherical_2`, &
 `cartesian_to_spherical`, `cartesian_to_spherical_j`
- 338d *<Implementation of coordinates procedures 337b>+≡ (336) <338b 340a>*
 pure subroutine `cartesian_to_spherical_2` (x, r, phi, theta, jacobian)
 real(kind=default), dimension(:), intent(in) :: x
 real(kind=default), intent(out), optional :: r, phi
 real(kind=default), dimension(:), intent(out), optional :: theta
 real(kind=default), intent(out), optional :: jacobian
 real(kind=default) :: local_r
 real(kind=default), dimension(size(x)-2) :: cos_theta
 real(kind=default), dimension(size(x)-1) :: product_sin_theta

```

integer :: n, i
n = size (x)
local_r = sqrt (dot_product (x, x))
if (local_r == 0) then
  if (present (r)) then
    r = 0
  end if
  if (present (phi)) then
    phi = 0
  end if
  if (present (theta)) then
    theta = 0
  end if
  if (present (jacobian)) then
    jacobian = 1
  end if
else
  product_sin_theta(n-1) = 1
  do i = n, 3, -1
    if (product_sin_theta(i-1) == 0) then
      cos_theta(i-2) = 0
    else
      cos_theta(i-2) = x(i) / product_sin_theta(i-1) / local_r
    end if
    product_sin_theta(i-2) = &
      product_sin_theta(i-1) * sqrt (1 - cos_theta(i-2)**2)
  end do
  if (present (r)) then
    r = local_r
  end if
  if (present (phi)) then
    ! Set phi = 0 for vanishing vector
    if (x(1) == 0 .and. x(2)==0) then
      phi = 0
    else
      phi = atan2 (x(1), x(2))
    end if
  end if
  if (present (theta)) then
    theta = acos (cos_theta)
  end if
  if (present (jacobian)) then
    jacobian = local_r**(1-n) / product (product_sin_theta)
  end if
end if

```

```

        end if
    end if
end subroutine cartesian_to_spherical_2
340a  <Implementation of coordinates procedures 337b>+≡ (336) <338d 340b>
pure subroutine cartesian_to_spherical (x, r, phi, theta)
    real(kind=default), dimension(:), intent(in) :: x
    real(kind=default), intent(out) :: r, phi
    real(kind=default), dimension(:), intent(out) :: theta
    call cartesian_to_spherical_2 (x, r, phi, theta)
end subroutine cartesian_to_spherical
340b  <Implementation of coordinates procedures 337b>+≡ (336) <340a 340d>
pure function cartesian_to_spherical_j (x) result (jacobian)
    real(kind=default), dimension(:), intent(in) :: x
    real(kind=default) :: jacobian
    call cartesian_to_spherical_2 (x, jacobian = jacobian)
end function cartesian_to_spherical_j

```

K.2 Trigonometric Spherical Coordinates

```

340c  <Declaration of coordinates procedures 337a>+≡ (336) <338c 341c>
public :: spherical_cos_to_cartesian_2, &
        spherical_cos_to_cartesian, spherical_cos_to_cartesian_j

```

Using the cosine, we have to drop P_1 from the Jacobian

```

340d  <Implementation of coordinates procedures 337b>+≡ (336) <340b 341a>
pure subroutine spherical_cos_to_cartesian_2 (r, phi, cos_theta, x, jacobian)
    real(kind=default), intent(in) :: r, phi
    real(kind=default), dimension(:), intent(in) :: cos_theta
    real(kind=default), dimension(:), intent(out), optional :: x
    real(kind=default), intent(out), optional :: jacobian
    real(kind=default), dimension(size(cos_theta)+1) :: product_sin_theta
    integer :: n, i
    n = size (cos_theta) + 2
    product_sin_theta(n-1) = 1.0_default
    do i = n - 2, 1, -1
        product_sin_theta(i) = &
            product_sin_theta(i+1) * sqrt (1 - cos_theta(i)**2)
    end do
    if (present (x)) then
        x(1) = r * product_sin_theta(1) * sin (phi)
        x(2) = r * product_sin_theta(1) * cos (phi)
        x(3:) = r * product_sin_theta(2:n-1) * cos_theta
    end if
end subroutine spherical_cos_to_cartesian_2

```

```

        end if
        if (present (jacobian)) then
            jacobian = r**(n-1) * product (product_sin_theta(2:))
        end if
    end subroutine spherical_cos_to_cartesian_2
341a  <Implementation of coordinates procedures 337b>+≡ (336) <340d 341b>
    pure function spherical_cos_to_cartesian (r, phi, theta) result (x)
        real(kind=default), intent(in) :: r, phi
        real(kind=default), dimension(:), intent(in) :: theta
        real(kind=default), dimension(size(theta)+2) :: x
        call spherical_cos_to_cartesian_2 (r, phi, theta, x = x)
    end function spherical_cos_to_cartesian
341b  <Implementation of coordinates procedures 337b>+≡ (336) <341a 341d>
    pure function spherical_cos_to_cartesian_j (r, phi, theta) &
        result (jacobian)
        real(kind=default), intent(in) :: r, phi
        real(kind=default), dimension(:), intent(in) :: theta
        real(kind=default) :: jacobian
        call spherical_cos_to_cartesian_2 (r, phi, theta, jacobian = jacobian)
    end function spherical_cos_to_cartesian_j
341c  <Declaration of coordinates procedures 337a>+≡ (336) <340c 343b>
    public :: cartesian_to_spherical_cos_2, &
        cartesian_to_spherical_cos, cartesian_to_spherical_cos_j
341d  <Implementation of coordinates procedures 337b>+≡ (336) <341b 342>
    pure subroutine cartesian_to_spherical_cos_2 (x, r, phi, cos_theta, jacobian)
        real(kind=default), dimension(:), intent(in) :: x
        real(kind=default), intent(out), optional :: r, phi
        real(kind=default), dimension(:), intent(out), optional :: cos_theta
        real(kind=default), intent(out), optional :: jacobian
        real(kind=default) :: local_r
        real(kind=default), dimension(size(x)-2) :: local_cos_theta
        real(kind=default), dimension(size(x)-1) :: product_sin_theta
        integer :: n, i
        n = size (x)
        local_r = sqrt (dot_product (x, x))
        if (local_r == 0) then
            if (present (r)) then
                r = 0
            end if
            if (present (phi)) then
                phi = 0
            end if
        end if
    end subroutine

```

```

    if (present (cos_theta)) then
        cos_theta = 0
    end if
    if (present (jacobian)) then
        jacobian = 1
    end if
else
    product_sin_theta(n-1) = 1
    do i = n, 3, -1
        if (product_sin_theta(i-1) == 0) then
            local_cos_theta(i-2) = 0
        else
            local_cos_theta(i-2) = x(i) / product_sin_theta(i-1) / local_r
        end if
        product_sin_theta(i-2) = &
            product_sin_theta(i-1) * sqrt (1 - local_cos_theta(i-2)**2)
    end do
    if (present (r)) then
        r = local_r
    end if
    if (present (phi)) then
        ! Set phi = 0 for vanishing vector
        if (x(1) == 0 .and. x(2)==0) then
            phi = 0
        else
            phi = atan2 (x(1), x(2))
        end if
    end if
    if (present (cos_theta)) then
        cos_theta = local_cos_theta
    end if
    if (present (jacobian)) then
        jacobian = local_r**(1-n) / product (product_sin_theta(2:))
    end if
end if
end subroutine cartesian_to_spherical_cos_2
342  <Implementation of coordinates procedures 337b>+≡ (336) <341d 343a>
    pure subroutine cartesian_to_spherical_cos (x, r, phi, cos_theta)
        real(kind=default), dimension(:), intent(in) :: x
        real(kind=default), intent(out) :: r, phi
        real(kind=default), dimension(:), intent(out), optional :: cos_theta
        call cartesian_to_spherical_cos_2 (x, r, phi, cos_theta)
    end subroutine cartesian_to_spherical_cos

```

343a \langle Implementation of coordinates procedures 337b $\rangle + \equiv$ (336) \triangleleft 342 343c \triangleright

```

pure function cartesian_to_spherical_cos_j (x) result (jacobian)
  real(kind=default), dimension(:), intent(in) :: x
  real(kind=default) :: jacobian
  call cartesian_to_spherical_cos_2 (x, jacobian = jacobian)
end function cartesian_to_spherical_cos_j

```

K.3 Surface of a Sphere

343b \langle Declaration of coordinates procedures 337a $\rangle + \equiv$ (336) \triangleleft 341c

```

public :: surface

```

$$\int d\Omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)} = S_n \quad (\text{K.7})$$

343c \langle Implementation of coordinates procedures 337b $\rangle + \equiv$ (336) \triangleleft 343a

```

pure function surface (n) result (vol)
  integer, intent(in) :: n
  real(kind=default) :: vol
  real(kind=default) :: n_by_2
  n_by_2 = 0.5_default * n
  vol = 2 * PI**n_by_2 / gamma (n_by_2)
end function surface

```


—L—

IDIOMATIC FORTRAN90 INTERFACE FOR MPI

```

344a  <mpi90.f90 344a>≡
      ! mpi90.f90 --
      <Copyleft notice 1>
      module mpi90
        use kinds
        use mpi
        implicit none
        private
        <Declaration of mpi90 procedures 344b>
        <Interfaces of mpi90 procedures 347c>
        <Parameters in mpi90 (never defined)>
        <Variables in mpi90 (never defined)>
        <Declaration of mpi90 types 349b>
      contains
        <Implementation of mpi90 procedures 344c>
      end module mpi90

```

L.1 Basics

```

344b  <Declaration of mpi90 procedures 344b>≡                                     (344a) 347b▷
      public :: mpi90_init
      public :: mpi90_finalize
      public :: mpi90_abort
      public :: mpi90_print_error
      public :: mpi90_size
      public :: mpi90_rank

344c  <Implementation of mpi90 procedures 344c>≡                               (344a) 345c▷
      subroutine mpi90_init (error)

```

```

integer, intent(out), optional :: error
integer :: local_error
character(len=*), parameter :: FN = "mpi90_init"
external mpi_init
call mpi_init (local_error)
<Handle local_error (no mpi90_abort) 345a>
end subroutine mpi90_init

345a <Handle local_error (no mpi90_abort) 345a>≡ (344c 345d)
  if (present (error)) then
    error = local_error
  else
    if (local_error /= MPI_SUCCESS) then
      call mpi90_print_error (local_error, FN)
      stop
    end if
  end if

345b <Handle local_error 345b>≡ (345-48 350c 352d 353b 355a)
  if (present (error)) then
    error = local_error
  else
    if (local_error /= MPI_SUCCESS) then
      call mpi90_print_error (local_error, FN)
      call mpi90_abort (local_error)
      stop
    end if
  end if

345c <Implementation of mpi90 procedures 344c>+≡ (344a) <344c 345d>
  subroutine mpi90_finalize (error)
    integer, intent(out), optional :: error
    integer :: local_error
    character(len=*), parameter :: FN = "mpi90_finalize"
    external mpi_finalize
    call mpi_finalize (local_error)
    <Handle local_error 345b>
  end subroutine mpi90_finalize

345d <Implementation of mpi90 procedures 344c>+≡ (344a) <345c 346a>
  subroutine mpi90_abort (code, domain, error)
    integer, intent(in), optional :: code, domain
    integer, intent(out), optional :: error
    character(len=*), parameter :: FN = "mpi90_abort"
    integer :: local_domain, local_code, local_error
    external mpi_abort

```

```

    if (present (code)) then
        local_code = code
    else
        local_code = MPI_ERR_UNKNOWN
    end if
    <Set default for domain 346b>
    call mpi_abort (local_domain, local_code, local_error)
    <Handle local_error (no mpi90_abort) 345a>
end subroutine mpi90_abort

346a <Implementation of mpi90 procedures 344c>+≡ (344a) <345d 346c>
subroutine mpi90_print_error (error, msg)
    integer, intent(in) :: error
    character(len=*), optional :: msg
    character(len=*), parameter :: FN = "mpi90_print_error"
    integer :: msg_len, local_error
    external mpi_error_string
    call mpi_error_string (error, msg, msg_len, local_error)
    if (local_error /= MPI_SUCCESS) then
        print *, "PANIC: even MPI_ERROR_STRING() failed!!!"
        call mpi90_abort (local_error)
    else if (present (msg)) then
        print *, trim (msg), ": ", trim (msg(msg_len+1:))
    else
        print *, "mpi90: ", trim (msg(msg_len+1:))
    end if
end subroutine mpi90_print_error

346b <Set default for domain 346b>≡ (345-49 355a) 353f>
    if (present (domain)) then
        local_domain = domain
    else
        local_domain = MPI_COMM_WORLD
    end if

346c <Implementation of mpi90 procedures 344c>+≡ (344a) <346a 347a>
subroutine mpi90_size (sz, domain, error)
    integer, intent(out) :: sz
    integer, intent(in), optional :: domain
    integer, intent(out), optional :: error
    character(len=*), parameter :: FN = "mpi90_size"
    integer :: local_domain, local_error
    external mpi_comm_size
    <Set default for domain 346b>
    call mpi_comm_size (local_domain, sz, local_error)

```

```

    <Handle local_error 345b>
end subroutine mpi90_size
347a <Implementation of mpi90 procedures 344c>+≡ (344a) <346c 347d>
    subroutine mpi90_rank (rank, domain, error)
        integer, intent(out) :: rank
        integer, intent(in), optional :: domain
        integer, intent(out), optional :: error
        character(len=*), parameter :: FN = "mpi90_rank"
        integer :: local_domain, local_error
        external mpi_comm_rank
        <Set default for domain 346b>
        call mpi_comm_rank (local_domain, rank, local_error)
        <Handle local_error 345b>
    end subroutine mpi90_rank

```

L.2 Point to Point

```

347b <Declaration of mpi90 procedures 344b>+≡ (344a) <344b 350d>
    public :: mpi90_send
    public :: mpi90_receive
    public :: mpi90_receive_pointer
347c <Interfaces of mpi90 procedures 347c>≡ (344a) 349d>
    interface mpi90_send
        module procedure &
            mpi90_send_integer, mpi90_send_double, &
            mpi90_send_integer_array, mpi90_send_double_array, &
            mpi90_send_integer_array2, mpi90_send_double_array2
    end interface
347d <Implementation of mpi90 procedures 344c>+≡ (344a) <347a 347e>
    subroutine mpi90_send_integer (value, target, tag, domain, error)
        integer, intent(in) :: value
        integer, intent(in) :: target, tag
        integer, intent(in), optional :: domain
        integer, intent(out), optional :: error
        call mpi90_send_integer_array ((/ value /), target, tag, domain, error)
    end subroutine mpi90_send_integer
347e <Implementation of mpi90 procedures 344c>+≡ (344a) <347d 348a>
    subroutine mpi90_send_double (value, target, tag, domain, error)
        real(kind=default), intent(in) :: value
        integer, intent(in) :: target, tag

```

```

integer, intent(in), optional :: domain
integer, intent(out), optional :: error
call mpi90_send_double_array (/ value /), target, tag, domain, error)
end subroutine mpi90_send_double

348a <Implementation of mpi90 procedures 344c>+≡ (344a) <347e 348c>
subroutine mpi90_send_integer_array (buffer, target, tag, domain, error)
integer, dimension(:), intent(in) :: buffer
integer, intent(in) :: target, tag
integer, intent(in), optional :: domain
integer, intent(out), optional :: error
character(len=*), parameter :: FN = "mpi90_send_integer_array"
integer, parameter :: datatype = MPI_INTEGER
<Body of mpi90_send.*_array 348b>
end subroutine mpi90_send_integer_array

348b <Body of mpi90_send.*_array 348b>≡ (348)
integer :: local_domain, local_error
external mpi_send
<Set default for domain 346b>
call mpi_send (buffer, size (buffer), datatype, target, tag, &
               local_domain, local_error)
<Handle local_error 345b>

348c <Implementation of mpi90 procedures 344c>+≡ (344a) <348a 348d>
subroutine mpi90_send_double_array (buffer, target, tag, domain, error)
real(kind=default), dimension(:), intent(in) :: buffer
integer, intent(in) :: target, tag
integer, intent(in), optional :: domain
integer, intent(out), optional :: error
character(len=*), parameter :: FN = "mpi90_send_double_array"
integer, parameter :: datatype = MPI_DOUBLE_PRECISION
<Body of mpi90_send.*_array 348b>
end subroutine mpi90_send_double_array

348d <Implementation of mpi90 procedures 344c>+≡ (344a) <348c 349a>
subroutine mpi90_send_integer_array2 (value, target, tag, domain, error)
integer, dimension(:, :), intent(in) :: value
integer, intent(in) :: target, tag
integer, intent(in), optional :: domain
integer, intent(out), optional :: error
integer, dimension(size(value)) :: buffer
buffer = reshape (value, shape(buffer))
call mpi90_send_integer_array (buffer, target, tag, domain, error)
end subroutine mpi90_send_integer_array2

```

349a *<Implementation of mpi90 procedures 344c>+≡* (344a) *<348d 349c>*
`subroutine mpi90_send_double_array2 (value, target, tag, domain, error)`
`real(kind=default), dimension(:, :), intent(in) :: value`
`integer, intent(in) :: target, tag`
`integer, intent(in), optional :: domain`
`integer, intent(out), optional :: error`
`real(kind=default), dimension(size(value)) :: buffer`
`buffer = reshape (value, shape(buffer))`
`call mpi90_send_double_array (buffer, target, tag, domain, error)`
`end subroutine mpi90_send_double_array2`

349b *<Declaration of mpi90 types 349b>≡* (344a)
`type, public :: mpi90_status`
`integer :: count, source, tag, error`
`end type mpi90_status`

349c *<Implementation of mpi90 procedures 344c>+≡* (344a) *<349a 350a>*
`subroutine mpi90_receive_integer (value, source, tag, domain, status, error)`
`integer, intent(out) :: value`
`integer, intent(in), optional :: source, tag, domain`
`type(mpi90_status), intent(out), optional :: status`
`integer, intent(out), optional :: error`
`integer, dimension(1) :: buffer`
`call mpi90_receive_integer_array (buffer, source, tag, domain, status, error)`
`value = buffer(1)`
`end subroutine mpi90_receive_integer`

349d *<Interfaces of mpi90 procedures 347c>+≡* (344a) *<347c 352a>*
`interface mpi90_receive`
`module procedure &`
`mpi90_receive_integer, mpi90_receive_double, &`
`mpi90_receive_integer_array, mpi90_receive_double_array, &`
`mpi90_receive_integer_array2, mpi90_receive_double_array2`
`end interface`

349e *<Set defaults for source, tag and domain 349e>≡* (350c 352c)
`if (present (source)) then`
`local_source = source`
`else`
`local_source = MPI_ANY_SOURCE`
`end if`
`if (present (tag)) then`
`local_tag = tag`
`else`
`local_tag = MPI_ANY_TAG`
`end if`

<Set default for domain 346b>

- 350a *<Implementation of mpi90 procedures 344c>+≡* (344a) *<349c 350b>*
 subroutine mpi90_receive_double (value, source, tag, domain, status, error)
 real(kind=default), intent(out) :: value
 integer, intent(in), optional :: source, tag, domain
 type(mpi90_status), intent(out), optional :: status
 integer, intent(out), optional :: error
 real(kind=default), dimension(1) :: buffer
 call mpi90_receive_double_array (buffer, source, tag, domain, status, error)
 value = buffer(1)
 end subroutine mpi90_receive_double
- 350b *<Implementation of mpi90 procedures 344c>+≡* (344a) *<350a 350e>*
 subroutine mpi90_receive_integer_array &
 (buffer, source, tag, domain, status, error)
 integer, dimension(:), intent(out) :: buffer
 integer, intent(in), optional :: source, tag, domain
 type(mpi90_status), intent(out), optional :: status
 integer, intent(out), optional :: error
 character(len=*), parameter :: FN = "mpi90_receive_integer_array"
 integer, parameter :: datatype = MPI_INTEGER
 <Body of mpi90_receive_.array 350c>*
 end subroutine mpi90_receive_integer_array
- 350c *<Body of mpi90_receive_*.array 350c>≡* (350b 351a)
 integer :: local_source, local_tag, local_domain, local_error
 integer, dimension(MPI_STATUS_SIZE) :: local_status
 external mpi_receive, mpi_get_count
 <Set defaults for source, tag and domain 349e>
 call mpi_recv (buffer, size (buffer), datatype, local_source, local_tag, &
 local_domain, local_status, local_error)
 <Handle local_error 345b>
 if (present (status)) then
 call **decode_status** (status, local_status, datatype)
 end if
- 350d *<Declaration of mpi90 procedures 344b>+≡* (344a) *<347b 353d>*
 private :: **decode_status**



Can we ignore ierror???

- 350e *<Implementation of mpi90 procedures 344c>+≡* (344a) *<350b 351a>*
 subroutine decode_status (status, mpi_status, datatype)
 type(mpi90_status), intent(out) :: status
 integer, dimension(:), intent(in) :: mpi_status

```

integer, intent(in), optional :: datatype
integer :: ierror
if (present (datatype)) then
    call mpi_get_count (mpi_status, datatype, status%count, ierror)
else
    status%count = 0
end if
status%source = mpi_status(MPI_SOURCE)
status%tag = mpi_status(MPI_TAG)
status%error = mpi_status(MPI_ERROR)
end subroutine decode_status

```

351a *<Implementation of mpi90 procedures 344c>+≡* (344a) *<350e 351b>*

```

subroutine mpi90_receive_double_array &
    (buffer, source, tag, domain, status, error)
real(kind=default), dimension(:), intent(out) :: buffer
integer, intent(in), optional :: source, tag, domain
type(mpi90_status), intent(out), optional :: status
integer, intent(out), optional :: error
character(len=*), parameter :: FN = "mpi90_receive_double_array"
integer, parameter :: datatype = MPI_DOUBLE_PRECISION
<Body of mpi90_receive.*_array 350c>
end subroutine mpi90_receive_double_array

```

351b *<Implementation of mpi90 procedures 344c>+≡* (344a) *<351a 351c>*

```

subroutine mpi90_receive_integer_array2 &
    (value, source, tag, domain, status, error)
integer, dimension(:, :), intent(out) :: value
integer, intent(in), optional :: source, tag, domain
type(mpi90_status), intent(out), optional :: status
integer, intent(out), optional :: error
integer, dimension(size(value)) :: buffer
call mpi90_receive_integer_array &
    (buffer, source, tag, domain, status, error)
value = reshape (buffer, shape(value))
end subroutine mpi90_receive_integer_array2

```

351c *<Implementation of mpi90 procedures 344c>+≡* (344a) *<351b 352b>*

```

subroutine mpi90_receive_double_array2 &
    (value, source, tag, domain, status, error)
real(kind=default), dimension(:, :), intent(out) :: value
integer, intent(in), optional :: source, tag, domain
type(mpi90_status), intent(out), optional :: status
integer, intent(out), optional :: error
real(kind=default), dimension(size(value)) :: buffer


```



```

        call mpi90_receive_double_array &
            (buffer, source, tag, domain, status, error)
        value = reshape (buffer, shape(value))
    end subroutine mpi90_receive_double_array2
352a  <Interfaces of mpi90 procedures 347c>+≡ (344a) <349d 353e>
        interface mpi90_receive_pointer
            module procedure &
                mpi90_receive_integer_pointer, mpi90_receive_double_pointer
        end interface
352b  <Implementation of mpi90 procedures 344c>+≡ (344a) <351c 353c>
        subroutine mpi90_receive_integer_pointer &
            (buffer, source, tag, domain, status, error)
            integer, dimension(:), pointer :: buffer
            integer, intent(in), optional :: source, tag, domain
            type(mpi90_status), intent(out), optional :: status
            integer, intent(out), optional :: error
            character(len=*), parameter :: FN = "mpi90_receive_integer_pointer"
            integer, parameter :: datatype = MPI_INTEGER
            <Body of mpi90_receive_*_pointer 352c>
        end subroutine mpi90_receive_integer_pointer
352c  <Body of mpi90_receive_*_pointer 352c>≡ (352b 353c) 352d>
        integer :: local_source, local_tag, local_domain, local_error, buffer_size
        integer, dimension(MPI_STATUS_SIZE) :: local_status
        integer :: ierror
        external mpi_receive, mpi_get_count
        <Set defaults for source, tag and domain 349e>
352d  <Body of mpi90_receive_*_pointer 352c>+≡ (352b 353c) <352c 352e>
        call mpi_probe (local_source, local_tag, local_domain, &
            local_status, local_error)
        <Handle local_error 345b>

```

 Can we ignore ierror???

```

352e  <Body of mpi90_receive_*_pointer 352c>+≡ (352b 353c) <352d 353a>
        call mpi_get_count (local_status, datatype, buffer_size, ierror)
        if (associated (buffer)) then
            if (size (buffer) /= buffer_size) then
                deallocate (buffer)
                allocate (buffer(buffer_size))
            end if
        else
            allocate (buffer(buffer_size))
        end if
    
```

353a *<Body of mpi90_receive_*_pointer 352c>+≡* (352b 353c) *<352e 353b>*
 call mpi_recv (buffer, size (buffer), datatype, local_source, local_tag, &
 local_domain, local_status, local_error)

353b *<Body of mpi90_receive_*_pointer 352c>+≡* (352b 353c) *<353a*
 <Handle local_error 345b>
 if (present (status)) then
 call decode_status (status, local_status, datatype)
 end if

353c *<Implementation of mpi90 procedures 344c>+≡* (344a) *<352b 354a>*
 subroutine mpi90_receive_double_pointer &
 (buffer, source, tag, domain, status, error)
 real(kind=default), dimension(:), pointer :: buffer
 integer, intent(in), optional :: source, tag, domain
 type(mpi90_status), intent(out), optional :: status
 integer, intent(out), optional :: error
 character(len=*), parameter :: FN = "mpi90_receive_double_pointer"
 integer, parameter :: datatype = MPI_DOUBLE_PRECISION
 *<Body of mpi90_receive_*_pointer 352c>*
 end subroutine mpi90_receive_double_pointer

L.3 Collective Communication

353d *<Declaration of mpi90 procedures 344b>+≡* (344a) *<350d*
 public :: mpi90_broadcast

353e *<Interfaces of mpi90 procedures 347c>+≡* (344a) *<352a*
 interface mpi90_broadcast
 module procedure &
 mpi90_broadcast_integer, mpi90_broadcast_integer_array, &
 mpi90_broadcast_integer_array2, mpi90_broadcast_integer_array3, &
 mpi90_broadcast_double, mpi90_broadcast_double_array, &
 mpi90_broadcast_double_array2, mpi90_broadcast_double_array3, &
 mpi90_broadcast_logical, mpi90_broadcast_logical_array, &
 mpi90_broadcast_logical_array2, mpi90_broadcast_logical_array3
 end interface

353f *<Set default for domain 346b>+≡* (345–49 355a) *<346b*
 if (present (domain)) then
 local_domain = domain
 else
 local_domain = MPI_COMM_WORLD
 end if

```

354a  <Implementation of mpi90 procedures 344c>+≡ (344a) <353c 354b>
      subroutine mpi90_broadcast_integer (value, root, domain, error)
        integer, intent(inout) :: value
        integer, intent(in) :: root
        integer, intent(in), optional :: domain
        integer, intent(out), optional :: error
        integer, dimension(1) :: buffer
        buffer(1) = value
        call mpi90_broadcast_integer_array (buffer, root, domain, error)
        value = buffer(1)
      end subroutine mpi90_broadcast_integer

354b  <Implementation of mpi90 procedures 344c>+≡ (344a) <354a 354c>
      subroutine mpi90_broadcast_double (value, root, domain, error)
        real(kind=default), intent(inout) :: value
        integer, intent(in) :: root
        integer, intent(in), optional :: domain
        integer, intent(out), optional :: error
        real(kind=default), dimension(1) :: buffer
        buffer(1) = value
        call mpi90_broadcast_double_array (buffer, root, domain, error)
        value = buffer(1)
      end subroutine mpi90_broadcast_double

354c  <Implementation of mpi90 procedures 344c>+≡ (344a) <354b 354d>
      subroutine mpi90_broadcast_logical (value, root, domain, error)
        logical, intent(inout) :: value
        integer, intent(in) :: root
        integer, intent(in), optional :: domain
        integer, intent(out), optional :: error
        logical, dimension(1) :: buffer
        buffer(1) = value
        call mpi90_broadcast_logical_array (buffer, root, domain, error)
        value = buffer(1)
      end subroutine mpi90_broadcast_logical

354d  <Implementation of mpi90 procedures 344c>+≡ (344a) <354c 355b>
      subroutine mpi90_broadcast_integer_array (buffer, root, domain, error)
        integer, dimension(:), intent(inout) :: buffer
        integer, intent(in) :: root
        integer, intent(in), optional :: domain
        integer, intent(out), optional :: error
        character(len=*), parameter :: FN = "mpi90_broadcast_integer_array"
        integer, parameter :: datatype = MPI_INTEGER
        <Body of mpi90_broadcast.*_array 355a>

```

```

        end subroutine mpi90_broadcast_integer_array

355a  <Body of mpi90_broadcast_*.array 355a>≡ (354 355)
        integer :: local_domain, local_error
        external mpi_bcast
        <Set default for domain 346b>
        call mpi_bcast (buffer, size (buffer), datatype, root, &
                        local_domain, local_error)
        <Handle local_error 345b>

355b  <Implementation of mpi90 procedures 344c>+≡ (344a) <354d 355c>
        subroutine mpi90_broadcast_double_array (buffer, root, domain, error)
            real(kind=default), dimension(:), intent(inout) :: buffer
            integer, intent(in) :: root
            integer, intent(in), optional :: domain
            integer, intent(out), optional :: error
            integer, parameter :: datatype = MPI_DOUBLE_PRECISION
            character(len=*), parameter :: FN = "mpi90_broadcast_double_array"
            <Body of mpi90_broadcast_*.array 355a>
        end subroutine mpi90_broadcast_double_array

355c  <Implementation of mpi90 procedures 344c>+≡ (344a) <355b 355d>
        subroutine mpi90_broadcast_logical_array (buffer, root, domain, error)
            logical, dimension(:), intent(inout) :: buffer
            integer, intent(in) :: root
            integer, intent(in), optional :: domain
            integer, intent(out), optional :: error
            integer, parameter :: datatype = MPI_LOGICAL
            character(len=*), parameter :: FN = "mpi90_broadcast_logical_array"
            <Body of mpi90_broadcast_*.array 355a>
        end subroutine mpi90_broadcast_logical_array

355d  <Implementation of mpi90 procedures 344c>+≡ (344a) <355c 355e>
        subroutine mpi90_broadcast_integer_array2 (value, root, domain, error)
            integer, dimension(:, :), intent(inout) :: value
            integer, intent(in) :: root
            integer, intent(in), optional :: domain
            integer, intent(out), optional :: error
            integer, dimension(size(value)) :: buffer
            buffer = reshape (value, shape(buffer))
            call mpi90_broadcast_integer_array (buffer, root, domain, error)
            value = reshape (buffer, shape(value))
        end subroutine mpi90_broadcast_integer_array2

355e  <Implementation of mpi90 procedures 344c>+≡ (344a) <355d 356a>
        subroutine mpi90_broadcast_double_array2 (value, root, domain, error)

```

```

real(kind=default), dimension(:,:), intent(inout) :: value
integer, intent(in) :: root
integer, intent(in), optional :: domain
integer, intent(out), optional :: error
real(kind=default), dimension(size(value)) :: buffer
buffer = reshape (value, shape(buffer))
call mpi90_broadcast_double_array (buffer, root, domain, error)
value = reshape (buffer, shape(value))
end subroutine mpi90_broadcast_double_array2
356a <Implementation of mpi90 procedures 344c>+≡ (344a) <355e 356b>
subroutine mpi90_broadcast_logical_array2 (value, root, domain, error)
logical, dimension(:,:), intent(inout) :: value
integer, intent(in) :: root
integer, intent(in), optional :: domain
integer, intent(out), optional :: error
logical, dimension(size(value)) :: buffer
buffer = reshape (value, shape(buffer))
call mpi90_broadcast_logical_array (buffer, root, domain, error)
value = reshape (buffer, shape(value))
end subroutine mpi90_broadcast_logical_array2
356b <Implementation of mpi90 procedures 344c>+≡ (344a) <356a 356c>
subroutine mpi90_broadcast_integer_array3 (value, root, domain, error)
integer, dimension(:,:,:), intent(inout) :: value
integer, intent(in) :: root
integer, intent(in), optional :: domain
integer, intent(out), optional :: error
integer, dimension(size(value)) :: buffer
buffer = reshape (value, shape(buffer))
call mpi90_broadcast_integer_array (buffer, root, domain, error)
value = reshape (buffer, shape(value))
end subroutine mpi90_broadcast_integer_array3
356c <Implementation of mpi90 procedures 344c>+≡ (344a) <356b 357>
subroutine mpi90_broadcast_double_array3 (value, root, domain, error)
real(kind=default), dimension(:,:,:), intent(inout) :: value
integer, intent(in) :: root
integer, intent(in), optional :: domain
integer, intent(out), optional :: error
real(kind=default), dimension(size(value)) :: buffer
buffer = reshape (value, shape(buffer))
call mpi90_broadcast_double_array (buffer, root, domain, error)
value = reshape (buffer, shape(value))
end subroutine mpi90_broadcast_double_array3

```

357 *⟨Implementation of mpi90 procedures 344c⟩+≡* (344a) *◁356c*

```

subroutine mpi90_broadcast_logical_array3 (value, root, domain, error)
  logical, dimension(:,:,:), intent(inout) :: value
  integer, intent(in) :: root
  integer, intent(in), optional :: domain
  integer, intent(out), optional :: error
  logical, dimension(size(value)) :: buffer
  buffer = reshape (value, shape(buffer))
  call mpi90_broadcast_logical_array (buffer, root, domain, error)
  value = reshape (buffer, shape(value))
end subroutine mpi90_broadcast_logical_array3

```

—M— IDEAS

M.1 Toolbox for Interactive Optimization

Idea: Provide a OpenGL interface to visualize the grid optimization.

Motivation: Would help multi channel developers.

Implementation: Coding is straightforward, but interface design is hard.

M.2 Partially Non-Factorized Importance Sampling

Idea: Allow non-factorized grid optimization in two- or three-dimensional subspaces.

Motivation: Handle nastiest subspaces. Non-factorized approaches are impossible in higher than three dimensions (and probably only realistic in two dimensions), but there are cases that are best handled by including non-factorized optimization in two dimensions.

Implementation: The problem is that the present `vamp_sample_grid0` can't accomodate this, because other auxiliary information has to be collected, but a generalization is straightforward. Work has to start from an extended `divisions` module.

M.3 Correlated Importance Sampling (?)

Idea: Is it possible to include *some* correlations in a mainly factorized context?

Motivation: Would be nice ...

Implementation: First, I have to think about the maths ...

M.4 Align Coordinate System (i.e. the grid) with Singularities (or the hot region)

Idea: Solve **vegas**' nastiest problem by finding the direction(s) along which singularities are aligned.

Motivation: Automatically choose proper coordinate system in generator generators and separate wild and smooth directions.

Implementation: Diagonalize the covariance matrix $\text{cov}(x_i x_j)$ to find better axes. Caveats:

- damp rotations (rotate only if eigenvalues are spread out sufficiently).
- be careful about blow up of the integration volume, which is $V' = V d^{d/2}$ in the worst case for hypercubes and can be even worse for stretched cubes. (An adaptive grid can help, since we will have more smooth directions!)

Maybe try non-linear transformations as well.

M.5 Automagic Multi Channel

Idea: Find and extract one singularity after the other.

Motivation: Obvious.

Implementation: Either use multiple of **vegas**' $p(x)$ for importance sampling. Or find hot region(s) and split the integration region (à la signal/background).

—N—

CROSS REFERENCES

N.1 Identifiers

abs_evec: [128a](#), [128b](#), [128c](#), [129a](#), [130a](#), [130b](#)
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