

The WHIZARD NLO Tutorial

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1 Motivation

The top quark is one of the most important particles of the standard model, and can very well be considered to be the most important fermion in high-energy physics. As the heaviest known elementary particle, it is very well suited to probe the Higgs sector of the standard model or possible new physics extensions. This is featured most prominently in the dependence of the (meta-)stability of the standard model vacuum on m_t and m_H .

A future lepton collider will be able to measure top quark properties, such as its mass or its Yukawa coupling, with an unprecedented precision, making reliable and simple NLO corrections necessary.

The example of the long-standing top forward-backward asymmetry discrepancy at the Teva-

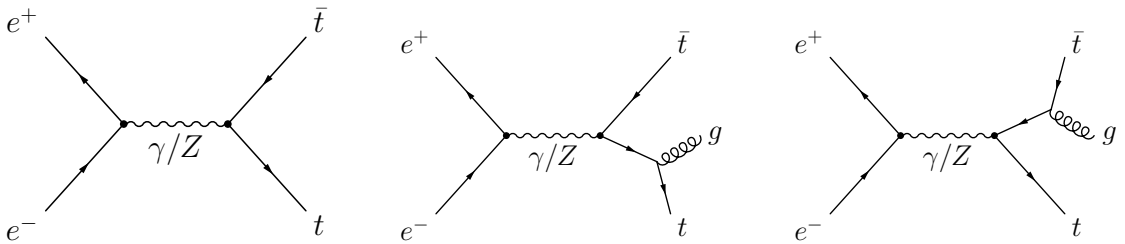


Figure 1: Tree-level Feynman diagrams for $e^+e^- \rightarrow t\bar{t}$ at $\mathcal{O}(\alpha_s^0)$ and the corresponding real-emission process $e^+e^- \rightarrow t\bar{t}g$ at $\mathcal{O}(\alpha_s^1)$.

tron collider emphasizes the importance of precisely knowing how NLO corrections influence observables. Here, the asymmetry is a pure NLO QCD effect. However, the complete description of the Tevatron results is only possible when even NNLO corrections are taken into account.

In the following, we are computing LO and NLO QCD total cross sections of the process $e^+e^- \rightarrow t\bar{t}$. There is only one contributing topology contributing at LO, as depicted in the first Feynman diagram of figure 1.

The $\mathcal{O}(\alpha_s)$ corrections to this process are made up by the virtual diagrams in figure 2 and the

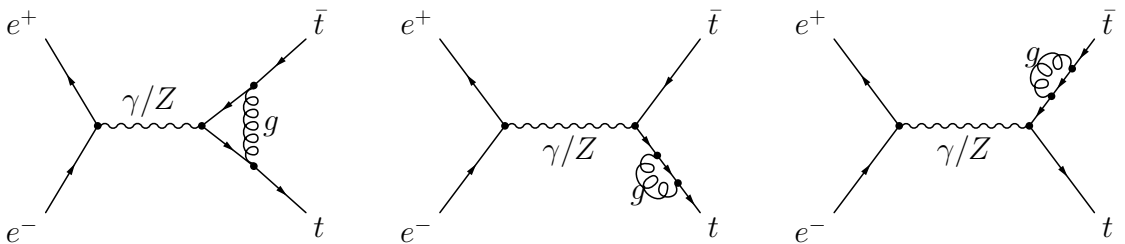


Figure 2: $\mathcal{O}(\alpha_s)$ vertex and self-energy corrections for $e^+e^- \rightarrow t\bar{t}$.

real-emission diagrams in figure 1.

The crucial point about virtual diagrams is that they are divergent integrals. The loop momentum k is not constrained, so that the singular limits $k^2 \rightarrow \infty$ (UV) and $k^2 \rightarrow 0$ (IR) are integrated over. The UV-limit¹ can be dealt with using the procedure of renormalization of the

theory's bare parameters. The IR-singularity is also present in the real-emission contribution when the gluon becomes soft. Summing up both expressions yields a cancellation of the divergent terms, leaving a finite reminder².

WHIZARD, as any other automated Monte Carlo program, deals with these divergences with a so-called subtraction procedure. Here, the IR-limits of the matrix elements are extracted generically and subtracted from the corresponding pure matrix elements such that the real and virtual components are separately finite.

2 Checking OpenLoops

Your virtual machine already contains an installation of OPENLOOPS [1], which is used by WHIZARD to compute the loop matrix elements. It is located in `~/OpenLoops`. You can test your installation by switching to `OpenLoops/examples` and executing

```
./OL_fortran.
```

OPENLOOPS then displays an intro message, loads the corresponding library and outputs a phase space point with the corresponding tree and loop matrix element, which should look like this:

```
evaluate_loop
Tree:          6.3808760213797650E-006
Loop ep^0:    -2.7830992186196647E-006
Loop ep^-1:   8.0133240158114758E-007
Loop ep^-2:  -5.4162558676905604E-007
accuracy:     2.9077914454751344E-013
```

The first number is the Born matrix element. The next three are the coefficients ε^{-2} , ε^{-1} and ε^{-1} of the Laurent expansion of the squared virtual matrix element, where $\varepsilon = 4 - 2d$ is the parameterization of the divergences in dimensional regularization.

3 SINDARIN Statements for NLO Calculations

In SINDARIN, a process is defined to be at next-to-leading order by applying the `"nlo_calculation"` attribute after the process definition, as in

```
process proc_nlo = in1, in2 => out1, out2 { nlo_calculation = "Mode" },
```

where the mode can be either `"Full"`, for a complete NLO calculation, or `"Born"`, `"Real"` or `"Virtual"` for the separate treatment of the individual components.

¹The terms UV and IR are due to the fact that large momenta correspond to small wavelengths, whereas small momenta correspond to large wavelengths. In the context of electrodynamics, these match the ultraviolet (UV) and infrared (IR) ends of the visible spectrum.

²This is the content of the Kinoshita-Lee-Nauenberg(KLN) theorem.

Below, we sum up some important SINDARIN options which can be important to set up a working NLO computation. They might be required in the following exercise. It will be your task to figure out which commands to use where!

- `alpha_power`, `alphas_power`: Orders of α_e and α_s of the tree-level process. It cannot be inferred automatically by WHIZARD.
- `?combined_nlo_integration`: Flag to integrate the sum $\sigma^{\text{Born}} + \sigma^{\text{Real}} + \sigma^{\text{Virtual}}$, as opposed to the separate mode, in which each component is integrated separately.
- `scale`: Renormalization and factorization scale μ . Note that NLO QCD corrections introduce this scale into our process, which at leading order is purely electroweak, wherefore σ^{NLO} depends on μ .
- `?alpha_s_from_mz`, `?alpha_s_from_lambda_qcd`: Use either the Z mass or Λ_{QCD} as reference scale (not renormalization scale) for the evolution of α_s .
- `?alpha_s_is_fixed`: Use a fixed α_s .
- `alphas`: WHIZARD will use this value as $\alpha_s(m_Z)$ or $\alpha_s(\Lambda_{\text{QCD}})$ and perform the evolution to $\alpha_s(\mu)$. On the other hand, if `?alpha_s_is_fixed` is set, the specified value will be used throughout the entire calculation.

For more in-depth information you can also consult the WHIZARD manual (`manual.pdf`).³ Very importantly, the top width has to be set to zero, `wtop = 0`, since we are dealing with on-shell quarks. Otherwise, the subtraction does not work, because soft factorization is performed for non-resonant particles.

4 Modifying the SINDARIN File

In the NLO directory, you find a `top.sin`, which performs a scan of the total cross section for top quark pair production at leading order. Your task is to extend it to next-to-leading order.

To learn what to do, have a look at the already existing code. First, a lineshape plot is defined in

```
plot lineshape_lo { x_min = 300 GeV x_max = 1010 GeV},
```

where `x_min` and `x_max` are the left and right limit of the plotting range. This object is like a container in which data can be stored via the `record` statement. The main loop of the SINDARIN script is

```
scan sqrts = ((350 GeV => 400 GeV /+ 10 GeV),
              (450 GeV => 1000 GeV /+ 50 GeV)) {
  integrate (proc_lo)
  record lineshape_lo (sqrts, integral (proc_lo))
}
```

We choose a finer sampling close to the top production threshold, whereas in the high-energy limit larger \sqrt{s} -steps are chosen, because here the cross section is flatter. Finally, the output file is specified in

```
compile_analysis { $out_file = "top-scan.dat" }
```

To set extend the calculation to NLO, you should ask yourself the following questions:

- What is a reasonable value for `alphas` ?
- Which renormalization scale should you use ?
- Which top width `wtop` should you use ?
- Is the width of the Z boson `wZ` influenced by NLO QCD corrections ?
- What is a suitable number of integration calls and iterations ?
- Which values do you have to set for `alpha_power` and `alphas_power` ?

To test your setup, it is advisable to first check one integration run with a definite value for \sqrt{s} . Note that you can make comments in `SINDARIN` using `#`.

In addition to the total cross sections, you should also create a graph of the K-factor. It is defined as the ratio of the NLO and the LO cross section, $K = \sigma^{\text{NLO}}/\sigma^{\text{LO}}$. Note that because arithmetic operations can be applied to `integral` objects, it can directly be written into the output file using a `record` command.

5 Scanning the Top Production Cross Section

If your scan was successful, the output file `top-scan.dat` should contain three blocks of data: one for the LO data points, another one for NLO data points, and a third one with the K-factors. To make plots, there is the script `make-plots.py`. It automatically generates graphs of the total cross section and the K-factor, if two requirements are met⁴:

- The `WHIZARD` output file has to be called `top-scan.dat` (which is the default name in `top.sin`).
- The data blocks inside `top-scan.dat` need to be in the following order: LO, NLO and K-factor.

The script produces two files, `xsection.svg` and `K-factor.svg`⁵. Because there is no X-Server installed on the virtual machine, you need to use a displaying program on your local machine. Move the files to your mounted directory. `.svg`-files can e.g. be displayed with common web browsers.

You should see the two plots depicted in figure 3. Looking at these plots, we see a very striking feature: The NLO corrections seem to diverge for $\sqrt{s} \rightarrow 4m_t!$ The K-factor goes up to two and beyond. Is the calculation wrong or is rather our understanding of the theory

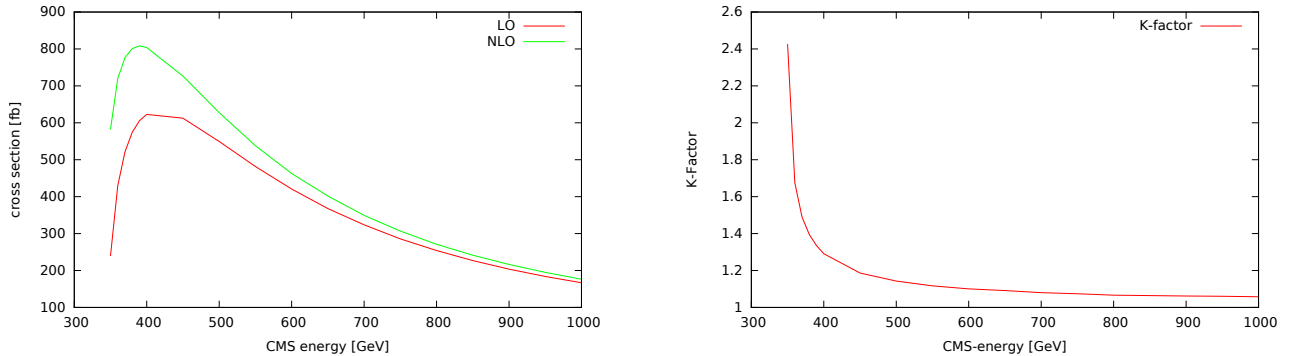


Figure 3: Total leading and next-to-leading order cross sections and their respective ratio.

incomplete?

Of course it is the latter. Let

$$\beta = \sqrt{1 - \frac{4m_t^2}{s}}$$

be the relativistic velocity of the top quarks. Relativistic, effectively massless, quarks have $\beta = 1$. Close to the threshold ($\beta \rightarrow 0$) an explicit calculation [2] shows that the NLO cross section behaves like

$$\sigma^{\text{NLO}}|_{\beta \rightarrow 0} = \sigma^{\text{LO}} \left(1 + \frac{2\pi\alpha_s}{3\beta} \right).$$

This is the divergence we observe in the K-factor. The root of the problem is a breakdown of the naive perturbative expansion in α_s . As β becomes small, the quantity $\frac{\alpha_s}{\beta}$ becomes large. Therefore, instead of performing an expansion only in α_s , one rather has to use $\frac{\alpha_s}{\beta}$ to do so.

It is important to note that any fixed-order prediction can break down in certain areas of the phase space. In our case, close to the threshold, one needs to perform a distinct calculation, which expands in $\frac{\alpha_s}{\beta}$, rather than just α_s , and is therefore more reliable. For the threshold treatment in WHIZARD, see ref. [3].

Nevertheless, away from the threshold, which is where most of the running scenarios of future lepton colliders are settled, fixed-order cross sections are reliable. We see that NLO QCD corrections are quite sizeable, but decrease with \sqrt{s} . This mirrors the fact that for large center-of-mass energies, the top quark mass becomes negligible and the K-factor approaches the value α_s/π , which is the same as for massless quarks for all \sqrt{s} .

³You cannot view it directly from the virtual machine, so that it needs to be moved to your mounted directory.

⁴Of course, you are encouraged to modify the plotting script.

⁵The uncommon choice of the file format is due to the fact that in order to keep the virtual machine simple, not all requirements to produce e.g. pdf-files are installed.

References

- [1] F Cascioli, P. Maierhöfer, and S Pozzorini. *Scattering Amplitudes with Open Loops*. Physical Review Letters, **108**.11 (2012), pp. 1–5. [1111.5206].
- [2] S. Dawson. “The Top quark, QCD, and new physics”. *Particle physics and cosmology: The quest for physics beyond the standard model(s). Proceedings, Theoretical Advanced Study Institute, TASI 2002, Boulder, USA, June 3-28, 2002*. 2003, pp. 245–299. [hep-ph/hep-ph/0303191].
- [3] J. Reuter, F. Bach, B. Chokoufe Nejad, et al. “Top Physics in WHIZARD”. *International Workshop on Future Linear Colliders (LCWS15) Whistler, B.C., Canada, November 2-6, 2015*. 2016. [hep-ph/1602.08035].