O'Mega: An Optimizing Matrix Element Generator. II: Amplitudes With Color Flow and Without Fermion Number Conservation

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Abstract

This paper describes features of the optimizing matrix element generator O'Mega that have been implemented since it was first made publically available about 10 years ago. The outstanding additions are an efficient intrinsic implementation of color flow amplitudes and support for Fermi statistics without conserved fermion number.

1 Introduction

This paper is a companion to [1], which is a minimally updated version of the preprint that was circulated first when O'Mega became public 10 years ago. It will henceforth be referred to as I.

The standard intro stuff ...

2 Color Flow Amplitudes

When O'Mega was first developed in the context of studies for a e^+e^- linear collider, the color degrees of freedom could be taken into account trivially for many interesting processes. Also it was not yet clear, which external and internal representations would produce the most efficient und useful results for practical use in Monte Carlo simulations of processes with many colored particles, e. g. at LHC.

Later, version 1 of WHIZARD [2] employed this version of O'Mega with Feynman rules for particles with an explicit color flow quantum number [3] to compute full scattering amplitudes for colored particles, including all interferences. This approach proved to very fruitful, since the resulting code is both numerically efficient and contains, in contrast to analytically color summed amplitudes, all the information required for parton showers and hadronization.

Nevertheless, this approach suffered from two major drawbacks: firstly the implementation of Feynman rules in the color flow basis is a tedious, repetitive and thererfore error prone task, in particular in models with many particles and vertices, such as supersymmetric extensions of the standard model. Secondly, while the final code is numerically very efficient, its generation consumed a lot of memory and time. The former was caused by an explosion in the number of particles and vertices and the corresponding lookup tables since each particle must be duplicated for all possible color flows. The resulting memory accesses slowed the program down, together with the dramatically increased number of combinations that had to be tried

when building the DAG – most of which would not contribute to the final result.

Both problems were addressed successfully by handling color in O'Mega explicitely and not as just another quantum number.

2.1 The `Colorize` Functor

As mentioned above, deriving the Fenman rules in the color flow basis is a repetitive task. It is therefore perfectly suited for implementation in a computer program. Each `O'Mega` model is represented by an `ocaml` module with functions that are used to inquire the available particles, their quantum numbers and the ways in which they couple at the vertices. Thus a functor `Colorize` can be implemented, that maps a module representing a model with given color representations to a new module representing a new model with Feynman rules in the color flow representation. As long as all matter fields live in the fundamental representation or its conjugate, there are only renormalizable couplings and there are no ϵ_{ijk}-style couplings, the Feynman rules for the color flows can be inferred from the representations alone. In the general case [4], which will be implemented later, additional annotation will be required.

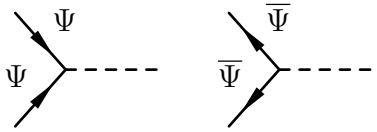
2.2 Adding color to amplitudes

3 The Great Line-up: Fermion-number violating vertices

3.1 The problem

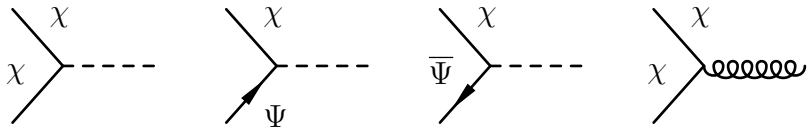
The determination of the relative sign of different parts of an S-matrix element or quantum field-theoretical amplitude due to the sign changes from anticommuting fermion fields according to Wick's theorem has been described in the first paper [1]. However, such an approach is strictly only applicable to completely vector-like theories like QED or QCD with gauge fields and matter fields that only interact via gauge interactions. As soon as matter interactions via Yukawa-type couplings are involved, it crucially depends on the content of matter representations whether such a description based on the conservation of fermion number is sufficient or not. In the Standard Model, there are accidentally only couplings which respect fermion number because of the strong constraints from the quantum numbers. In principle, it is the non-existence of a scalar particle carrying either lepton or color

quantum numbers which disallows fermion-number violating vertices at the renormalizable level. Consider the case of a charge -1/3 scalar particle in the fundamental color representation, which could couple together with an up-type and down-type quark to a singlet. This interaction term (which appears e.g. in R-parity violating supersymmetry) has to be described by one of the following two vertices:



These vertices are historically known as "clashing-arrow" vertices. Note that one could try to revert one of the arrows by using a charge-conjugation matrix, which converts a spinor into its charge-conjugated spinor. Such Feynman rules always contain an explicit charge-conjugation matrix and can e.g. be found in [5]. But they are quite error-prone when using them in operations like crossing, and the explicit presence of the charge-conjugation matrix makes the determination of the relative signs between different parts of amplitudes cumbersome.

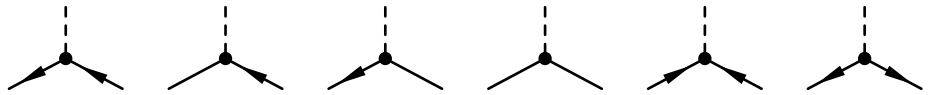
A different kind of vertices appears with the presence of neutral fermions which could be described by a four-component bispinor with left- and right-handed components related by complex conjugation, known as a Majorana spinor. Note that such a particle does not need to be completely uncharged, but could be in a real representation of a non-Abelian symmetry like the gluino in supersymmetry. Sometimes, Majorana fermions are denoted by lines with arrows pointing in both directions, especially in the context of Majorana mass terms, thereby denoting the fact that a Majorana mass term does *not* change chirality, but is vector-like. We consider Majorana fermions to be characterized as being there own antiparticles, and hence denote them by a line without any arrow. This leads to vertices of the type:



Consistent Feynman rules for fermion-number violating vertices, including both clashing arrows and Majorana fermions, have been constructed in [6].

3.2 The solution and algorithm

To summarize, there are now six types of vertices involving fermions:



(1)

The dashed line indicates an arbitrary boson. Note that the Majorana fermion is not restricted to be a spin 1/2-particle, but could also be a spin 3/2 gravitino, wich couples to a spin 1/2 fermion and a vector. Since there is no longer a well-defined direction along the fermion lines (there may be parts of lines having no direction at all due to propagating Majorana fermions, or the line's direction changes by one of the rightmost vertices) the method presented in [1] is no longer feasible. One possibility is to artificially assign a direction to a Majorana line and define different vertices for each case in which the arrows direct to or from the vertex [5]. This would give the correct analytical expressions for the Feynman diagrams, but the signs between different diagrams contributing to the same amplitude have to be derived by means of Wick's theorem, which is no good solution for implementing such rules in a computer program. Instead we follow the rather generic approach of [6]. The concept of *fermion number conservation* is abandoned, and we only use the assumption of *fermion conservation* which is the fact that half-integer Lorentz representations can only be generated or destroyed in pairs (and fermion lines must run uninterrupted through diagrams). The disentanglement of vertex operators to calculate relative signs by Wick's theorem is done by the standard external states for bispinors, propagators and vertices as well as their charge-conjugated versions:

$$\mathcal{C}\overline{v}_\sigma^T(p) = u_\sigma(p) \tag{2a}$$

$$\mathcal{C}\overline{u}_\sigma^T(p) = v_\sigma(p) \tag{2b}$$

$$S'_F \equiv \mathcal{C}S_F^T(p)\mathcal{C}^{-1} = S_F(-p) = \frac{\mathrm{i}}{-\not{p} - m} \tag{2c}$$

$$\Gamma' \equiv \mathcal{C}\Gamma^T\mathcal{C}^{-1} = \begin{cases} +\Gamma & \text{for } \Gamma \equiv \mathbb{I}, \gamma^5\gamma^\mu, \gamma^\mu \\ -\Gamma & \text{for } \Gamma \equiv \gamma^\mu, \sigma^{\mu\nu} \end{cases}, \tag{2d}$$

where $\mathcal{C}$ is the charge conjugation matrix.

The basic idea for disentangling contractions of fermionic field operators in any non-standard ordering (for more details cf. [6]) is to replace an interaction term bilinear in fermionic field operators by its transpose: $\overline{\Psi}\Gamma\Psi = \left(\overline{\Psi}\Gamma\Psi\right)^T$. This does not change anything as this operator is just a

complex number. Performing the transposition and replacing operators and gamma matrices by their charge-conjugated versions,

$$\ldots\ \overline{\Psi}\Gamma\Psi\ \ldots \qquad \longrightarrow \qquad (-1)\cdot\ \ldots\ \Psi^T\,\mathcal{C}^{-1}\left(\mathcal{C}\Gamma^T\mathcal{C}^{-1}\right)\mathcal{C}\ \overline{\Psi}^T\ldots \quad , \tag{3}$$

disentangles the contraction on the left-hand side. The sign comes from anticommuting the field operators. Such entangled contractions can in fact appear as for Majorana fermions all four contractions between field operators and their conjugates are non-vanishing, and also due to the appearance of explicitly charged conjugated fermions as e.g. in a chargino–fermion–sfermion vertex. Note that for (explicitly charged) Dirac fermions such contractions only happen in two cases: i) for the contraction of the creation and annihilation operators inside field operators with the asymptotic external states, which produces a global sign which is irrelevant (but becomes crucial when comparing different S-matrix elements in Slavnov-Taylor identities), and ii) in a closed fermion loop.

The analytical expression for a complete fermion line is just a sandwich of mutual products of vertex factors and propagators between two external bispinor states. Transposing such a complete fermion line using the expressions in Eqs. (2) yields ($w \in \{u, v\}$, $w^c \in \{u^c = v, v^c = u\}$ with $w^c \equiv \mathcal{C}\overline{w}^T$):

$$\overline{w}_1\Gamma^{(1)}S_F^{(1)}\Gamma^{(2)}\ldots\Gamma^{(n)}w_2 = (-1)\cdot\overline{w_2^c}\,\Gamma'^{(n)}\ldots S_F'^{(1)}\Gamma'^{(1)}w_1^c, \tag{4}$$

Note that such an analytical expression does not contain any anticommuting objects any more. Hence, the sign does not come from fermion anticommutation but from the antisymmetry of the charge conjugation matrix: $w^T = \overline{w^c}\,\mathcal{C}^T = -\overline{w^c}\,\mathcal{C}$.

Both ways of evaluating a fermion line produce the same result as the sign in Eq. (4) is cancelled by the one from fermion anticommutation in Eq. (3). Hence, it does not matter in which way to evaluate a fermion line, as conjugated spinors can always be transferred via transposition into spinors and vice versa. So the key ingredient of the rules in [6] is to just choose arbitrarily a direction to evaluate a fermion line, which is no longer determined by where there is the conjugated spinor and where the spinor as for a theory with purely Dirac spinors and non-clashing arrow vertices only. Whenever the arbitrarily chosen evaluation direction is opposite to the fermion arrow, one writes down the charge-conjugated (primed) expressions from Eq. (2). Thereby, the disentanglement of fermionic contractions is automatically guaranteed, and there is no need to explicitly use Wick's theorem. The relative signs of different diagrams can then be evaluated by the same method of permutations of pairs of external fermions connected by lines with respect to a

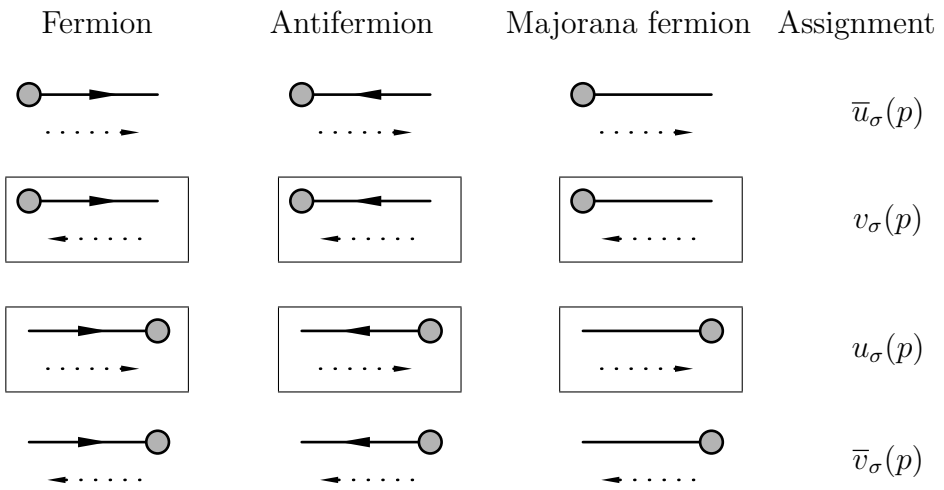


Figure 1: Feynman rules for external fermions for theories with fermion-number violating vertices. Evaluation directions are denoted by dotted arrows. The time flow is from left to right. According to the implementation in O'Mega using an inward pointing evaluation direction only the rules inside the boxes are used.

reference order as in [1]. However, now the fermion pairs are not of the form `(conjugated spinor, spinor)`, but simply `(endpoint, starting point)` of the corresponding fermion line. For more technical details cf. [6, 7].

3.3 The implementation

Now, after it is clear how to determine relative signs and calculate expressions for fermion-number violating vertices, this algorithm has to be implemented in a compatible way into O'Mega. The difficulty is that there are no complete Feynman diagrams present, but only subamplitudes representing 1POWs. The simple solution is to just assign an inward pointing evaluation direction for external fermions and to follow open lines keeping that evaluation direction until they are fused with a second line (either to yield a bosonic 1POW or a complete amplitude in a final `keystone`). Hence, external particles are always assigned a bispinor (i.e. u or v), but never a conjugated bispinor, $\overline{u}$ or $\overline{v}$. Fig. 1 summarizes the Feynman rules of [6] for external fermions, where the grey blob signifies the residual amplitude. As pointed out above, the evaluation direction is chosen such that it always points from the external fermion inward into the rest of the amplitude. Hence, only the

cases inside the boxes are used, i.e. we never assign conjugated spinors to external fermions. In detail, incoming (with respect to the time axis) fermions, antifermions and Majorana fermions are always assigned a u spinor, while the corresponding outgoing particles are denoted by a v spinor.

Consequently, in the implementation of the Feynman rules for fermions with fermion-number violating vertices, no conjugated spinors appear at all. External spinors u and v get left-multiplied by gamma matrices from vertices as well as from propagators, yielding again a spinor, and not a conjugated spinor. Finally, when two lines are fused into a closed fermion line (not in the sense of a loop, but running from one external fermion to another), we simply have to use the bilinear product $\Psi_1^T \mathcal{C}\Gamma\Psi_2$ between two spinors (containing an explicit charge-conjugation matrix), instead of the ordinary bilinear product, $\overline{\Psi}_1\Gamma\Psi_2$ between a conjugated spinor and a spinor.

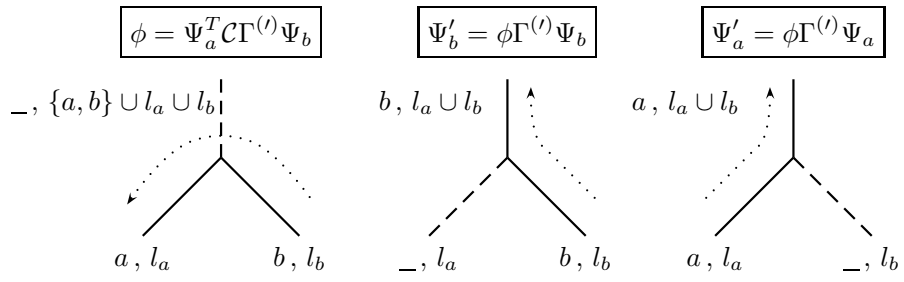
This bilinear product takes into account the fact, that when closing two open fermion lines in a final fusion, the evaluation direction of one of the two lines has to be reverted. This means, that finally a unique evaluation direction for the complete fermion line is chosen as soon as after the final fusion the complete course of the fermion line within the subamplitude is known. There are two choices, to revert either the fermion line from the left leg or the right leg of the fusion:

(5)

So when closing a fermion line in a fusion, we finally choose the evaluation direction to go from the end of the right leg through the fusion to the external end of the left leg, thereby reverting the original evaluation direction of the left leg. In principle, one would have to take a conjugated spinor $\overline{\Psi}$ for the fermion wavefunction of the left leg, which would correspond to $\Psi^{cT}\mathcal{C}$. This seems to contradict our choice $\Psi^T\mathcal{C}\Gamma\Psi$ for the final bilinear product, which is performed with $\Psi^T\mathcal{C}$ instead of $\Psi^{cT}\mathcal{C}$. However, the fermion wavefunction of the left leg had been calculated by O'Mega with evaluation direction pointing inward into the subamplitude, i.e. opposite to the one adopted after the final fusion. Consider the fermion wavefunction of the left leg to be just an external incoming fermion, which according to Fig. 1 yields simply a spinor u. But for the reversed evaluation direction, we in fact had to assign a conjugated spinor $\overline{v} = u^T\mathcal{C}$. Hence, taking $\Psi^T_{\text{left}}\mathcal{C}\Gamma\Psi_{\text{right}}$ with $\Psi_{\text{left}} \equiv u$ as the analytical expression for the fermion line is completely correct. The presence of propagators and vertex factors for the left leg does not change this picture and will be discussed below. The intuitive argument is that the spinorial

wavefunction of the left leg just gets charge-conjugated by the operation of "reversal of evaluation of direction", irrespective of the fact whether it is simply an external wavefunction or composed of a product of such with propagator and vertex factors.

In the same way as we did in the first part of the paper [1], we could summarize how to determine sign factors of subamplitudes when performing fusions of wavefunctions into so-called fusion rules for fermions with fermion-number violating vertices:



(6)

Here, a and b are the labels for the fermion from the left or right leg, respectively. l_a and l_b are the closed fermions lines contained in the 1POWs from the left and right leg, respectively. Because the evaluation direction within O'Mega always goes from right to left, a fermion pair in a fusion is always collected as a pair (a, b). The box on top of the graph shows the analytical expression that is to be calculated by O'Mega. Γ is the gamma matrix expression at the vertex, while Γ' according to 2d is the charge-conjugated vertex expression that has to be inserted whenever the evaluation direction is opposite to the fermion flow at the corresponding line. As the canonical evaluation directions in O'Mega are from bottom to top for open lines in fusions and from right to left in lines being closed at fusions, Fig. 2 collects all cases where the vertex expression is unchanged (Γ) and where is has to be charge-conjugated (Γ'). Part of the fermi statistics signs for fermion-number violating vertices is encoded in these disentanglements, as explained in [6]. As the fermion lines for fermion-number violating vertices are labeled not by pairs `(conjugated external spinor, external spinor)` but simply by `(endpoint, starting point)` for the evaluation direction for fermion lines, the remaining sign factor for an amplitude can be calculated now again (as in the case for purely fermion-number conserving interactions) by determining the number of transpositions needed to bring the entries in the pair to a canonical reference order. Note that a change of evaluation direction for a single line would transform the pair (a, b) into (b, a) resulting in an additional sign with respect to the reference order. This is exactly the sign from an additional anticommutation of Fermi field operators in Eq. 3 and

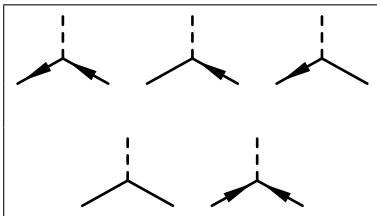
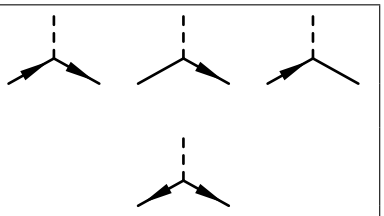
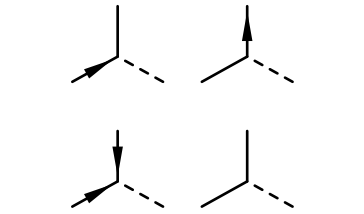
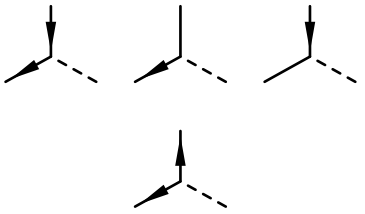
$\mathbf{\Gamma}$	$\mathbf{\Gamma'}$
	
$\_$, $\{a,b\} \cup l_a \cup l_b$	$\_$, $\{a,b\} \cup l_a \cup l_b$
	
a , $l_a \cup l_b$	a , $l_a \cup l_b$

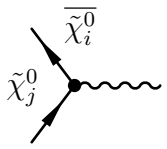
Figure 2: Fusion rules for fermions with fermion-number violating interactions. For the fusions where a fermion is produced, there are also mirror diagrams. a, b are the labels for the left fermion and the right fermion, respectively, and l_a and l_b are the corresponding closed fermion lines contained in the subamplitudes. For the conventions concerning the fusions with clashing arrows and with two Majorana fermions cf. the text.

is recovered in the relative sign between left and right hand side of Eq. 4 coming from the antisymmetry of the charge-conjugation matrix.

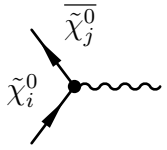
The additional signs are encoded in the vertex factors. So, in the two cases, when (i) sandwiching the vertex factor the left- and right-handed spinorial expressions in the closing of a line or (ii) the left-multiplication of the spinorial expression by the vertex factor when continuing an open fermion line in a fusion from the bottom to the top, there could appear additional signs. Namely, when the evaluation directions as shown in Fig. 2 point opposite to the (first) fermion arrow at the vertex there is a sign change in the case of vectorial and tensorial couplings. For the vertices in the first lines of the boxes in Fig. 2 it is clear how to define the vertices and how to implement them into O'Mega. Care, however, has to be taken in the case of the vertices in the second lines, containg either clashing arrows or two Majorana fermions. In the case of clashing arrows, we adopt the convention to define vertices in the *O'Mega* model files having the charge-conjugated particle on the left-hand side, i.e. define the vertex as $\overline{\Psi_1^c}\Gamma\Psi_2$ instead of $\overline{\Psi_1}\Gamma\Psi_2^c$; if we had defined them the other way round, the diagrams with clashing arrows would have to be exchanged between the two columns in Fig. **??**, as evaluation direction would point then the other way round for these vertices. For vertices with one Majorana and one Dirac line (e.g. the electron–selectron–neutralino vertex) there are no ambiguities, because the Dirac fermion automatically provides a unique direction. The case for vertices with two Majorana fermions is more complicated: If the two Majorana fermions are identical, the coupling has to be scalar, pseudoscalar or axial-vectorial, hence there is no issue with signs and ordering conventions. Consider now the case where the Majorana fermions are indeed different, e.g. in the vertex of two different MSSM neutralinos and the Z boson. Here one has to decide whether to define the vertex in the Lagrangian as

$$\overline{\tilde{\chi}_i^0}\left(g_V + g_A\gamma^5\right) \not{Z} \tilde{\chi}_j^0 \qquad \text{or} \qquad \overline{\tilde{\chi}_j^0}\left(-g_V + g_A\gamma^5\right) \not{Z} \tilde{\chi}_i^0 \quad , \quad i \neq j \quad ,$$

as well as in the model file. By adopting one of the two options, one arbitrarily assigns a fermion arrow to the vertex:



$\boxed{\overline{\tilde{\chi}_i^0}\left(g_V + g_A\gamma^5\right) \not{Z} \tilde{\chi}_j^0}$

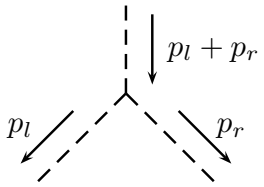


$\boxed{\overline{\tilde{\chi}_j^0}\left(-g_V + g_A\gamma^5\right) \not{Z} \tilde{\chi}_i^0}$

That pseudo-assignment of arrows means that when contracting the field operators of that interaction vertex with external states or other interaction

vertices, then we had to write down a conjugated spinor for the neutralino i and a spinor for the neutralino j on the left hand side, and vice versa for the right hand side. In Fig. 2 it is assumed that the vertex for two Majorana fermions is always defined within the O'Mega model files in such a way that in the fusion process of two fermions the left one is the conjugated spinor while in the case of the fermion line being continued, the fermion on top (i.e. the one fused to from the children) is the conjugated. Henceforth no primed vertex factors have to be used for vertices with two Majorana fermions. (In practice, there is a unique representation in O'Mega for such vertices, so when the fusion does not match that representation, then there *do* appear signs in front of the vertex factors. E.g. when we denote that neutralino neutral current by the left possiblity above, but the second neutralino appears as a left leg at a fusion and the first as a right leg, then the vector coupling constant has to be endowed with an explicit extra minus sign).

After having discussed signs from vertex factors, we have to account for possible signs from propagators for the implementation with fermion-number violating vertices. In O'Mega, the momentum flow is always outgoing, i.e. from the inside of an amplitude outwards. Because the subamplitudes joined together in fusions are made up from external states with outgoing momenta, momentum flow in fusions is always from top to bottom (for all vertices):

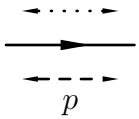


(7)

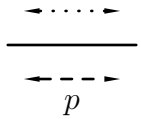
After a fusion has taken place, a fermionic wavefunction (if it is not already an external wavefunction e.g. in a decay amplitudes) is multiplied by a propagator. Thus, every wavefunction appearing as a child (left or right leg) in a fusion is either a wavefunction of an external fermion or has already been multiplied by a propagator. An exception occurs if the wavefunction is the final keystone of a subamplitude; then to only one of the fermionic wavefunctions the propagator has to be assigned. Hence, a propagator is inserted in all fusion cases where the fermion line is not closed but runs through to the top.

It turns out that only one type of propagator is need when handling both Dirac and Majorana fermions. The prescription in 2c shows that whenever the evaluation direction is opposite to the arrow of a Dirac line, one has to use the primed propagator, which means to revert the sign of the momentum

within the propagator. Of course, a momentum flow opposite to the fermion arrow has the same effect. Collecting signs for fermion propagators yields:



$$= \frac{\mathrm{i}}{\xi \not{p} - m}, \qquad (8)$$



$$= \frac{\mathrm{i}}{\zeta \not{p} - m}$$

where the sign factor ξ is $+1$ if the evaluation direction and the momentum flow are both parallel or both antiparallel to the fermion arrow, and -1 otherwise. For Majorana fermions, ζ is $+1$ if evaluation direction and momentum flow coincide, and -1 otherwise. The following table proofs that this in all cases leads to a negative sign for the propagator momentum within O'Mega:

fermion type	fermion arrow	mom.	eval.	sign
Dirac fermion	$\uparrow$	$\downarrow$	$\uparrow$	negative
Dirac antifermion	$\downarrow$	$\downarrow$	$\uparrow$	negative
Majorana fermion	-	$\downarrow$	$\uparrow$	negative

So the universally used fermion propagator for all types of fermions – fermions, antifermions and Majorana fermions – is

$$\boxed{S_{F,O'Mega} = \frac{\mathrm{i}}{-\not{p} - m}} \qquad (9)$$

Finally, we are able to proof our conjecture that is always correct to use the charge-conjuagated spinor in the bilinear product $\Psi_1^T \mathcal{C} \Gamma \Psi_2$, even if the spinor expression from the left leg of the fusion consists of a product of vertex and propagator factors. Let

$$\Psi = S_F^{(1)} \Gamma^{(1)} S_F^{(2)} \ldots \Gamma^{(n)} w, \qquad w \in \{u, v\} \; . \qquad (10)$$

be the spinor for the left leg as calculated by O'Mega just before it is fused. When performing the fusion, O'Mega uses the expression $\Psi^T \mathcal{C}$ for the left leg spinor in the product. Inserting Eq. (10) gives: is equal to

$$\begin{aligned} \Psi^T \mathcal{C} &= w^T {\Gamma^{(n)}}^T \ldots {S_F^{(2)}}^T {\Gamma^{(1)}}^T {S_F^{(1)}}^T \left(-\mathcal{C}^{-1}\right) \\ &= w^T \left(-\mathcal{C}\right) \mathcal{C} {\Gamma^{(n)}}^T \mathcal{C}^{-1} \ldots \mathcal{C} {S_F^{(2)}}^T \mathcal{C}^{-1} \mathcal{C} {\Gamma^{(1)}}^T \mathcal{C}^{-1} \mathcal{C} {S_F^{(1)}}^T \left(-\mathcal{C}^{-1}\right) \\ &= \overline{w^c} \, \Gamma^{(n)\prime} \ldots S_F^{(2)\prime} \Gamma^{(1)\prime} S_F^{(1)\prime} \end{aligned} \qquad (11)$$

We have already explained that the assignment of the wavefunction for the external fermion is correct. Now we get the primed expressions for all vertex factors and propagators. But that is what we want, since, indeed, our evaluation direction for the whole fermion line – after the fermion line has been closed by the fusion – has been reverted. This closes the proof of the correctness of the implementation of the algorithm for fermion-number violating vertices.

3.4 Cross checks and tests

The implementation of the formalism for models with fermion-number violating vertices has been extensively tested and verified. First of all, the algorithm and its implementation outlined above can also be applied to models with fermion number conservation in all vertices, which include QED, QCD, and the Standard Model. It has been verified that using the algorithm described in part I [1] and the one here yield numerically identical results. The methods described here have been successfully used to construct and test Ward and Slavnov-Taylor identities in supersymmetric gauge field theories [8]. All cross checks of O'Mega and WHIZARD [2] with other codes for models containing fermion-number violating interactions, e.g. in the MSSM and the NMSSM [9, 10], have proved the validity of the algorithm and its implementation. A very stringent and extensive test has been performed in the procedure of validating the interface of WHIZARD and O'Mega to the program FeynRules [11].

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