

Circe Version 1.02 β : Beam Spectra for Simulating Linear Collider Physics*

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IKDA 96/13-rev
hep-ph/9607454-rev

prelude.nw 66 2002-03-28 17:00:10Z ohl
circe.nw 67 2002-03-28 17:13:06Z ohl
minuit.nw 58 1999-04-15 08:42:06Z ohl
postlude.nw 42 1996-08-08 10:38:00Z ohl
July 1996 (expanded September 1996)

Abstract

I describe parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. Emphasis is put on simplicity and reproducibility of the parameterizations, supporting reproducible physics simulations. The parameterizations are implemented in a library of distribution functions and event generators.

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*Supported by Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Germany.

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Program Summary:

- **Title of program:** Circe, Version 1.02 β (September 1996)
- **Program obtainable** by anonymous ftp from the host `crunch.ikp.physik.th-darmstadt.de` in the directory `pub/ohl/circe`.
- **Licensing provisions:** Free software under the GNU General Public License.
- **Programming language used:** Fortran77
- **Number of program lines in distributed program, including test data, etc.:** ≈ 1100 (excluding comments)
- **Computer/Operating System:** Any with a Fortran77 programming environment.
- **Memory required to execute with typical data:** Negligible on the scale of typical applications calling the library.
- **Typical running time:** A small fraction (typically a few percent) of the running time of applications calling the library.
- **Purpose of program:** Provide simple and reproducible, yet realistic, parameterizations of the e^{\pm} - and γ -beam spectra for linear colliders.
- **Nature of physical problem:** The intricate beam dynamics in the interaction region of a high luminosity linear collider at $\sqrt{s} = 500\text{GeV}$ result in non-trivial energy spectra of the scattering electrons, positrons and photons. Physics simulations require simple and reproducible, yet realistic, parameterizations of these spectra.
- **Method of solution:** Parameterization, curve fitting, Monte Carlo event generation.
- **Keywords:** Event generation, beamstrahlung, linear colliders.

1 Introduction

Despite the enormous quantitative success of the electro-weak standard model up to energies of 200GeV, neither the nature of electro-weak symmetry breaking (EWSB) nor the origin of mass are understood.

From theoretical considerations, we know that clues to the answer of these open questions are hidden in the energy range below $\Lambda_{\text{EWSB}} = 4\pi v \approx 3.1\text{TeV}$. Either we will discover a Higgs particle in this energy range or signatures for a strongly interacting EWSB sector will be found. Experiments at CERN's Large Hadron Collider (LHC) will shed a first light on this regime in the next decade. In the past it has been very fruitful to complement experiments at high energy hadron colliders with experiments at e^+e^- -colliders. The simpler initial state allows more precise measurements with smaller theoretical errors. Lucid expositions of the physics opportunities of high energy e^+e^- colliders with references to the literature can be found in [1].

However, the power emitted by circular storage rings in form of synchrotron radiation scales like $(E/m)^4/R^2$ with the energy and mass of the particle and the radius of the ring. This cost becomes prohibitive after LEP2 and a Linear Collider (LC) has to be built instead.

Unfortunately, the “interesting” hard cross sections scale like $1/s$ with the square of the center of mass energy and a LC will have to operate at extremely high luminosities in excess of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. To achieve such luminosities, the bunches of electrons and positrons have to be very dense. Under these conditions, the electrons undergo acceleration from strong electromagnetic forces from the positron bunch (and vice versa). The resulting synchrotron radiation is called *beamstrahlung* [2] and has a strong effect on the energy spectrum $D(x_1, x_2)$ of the colliding particles. This changes the observable e^+e^- cross sections

$$\frac{d\sigma_0^{e^+e^-}}{d\Omega}(s) \rightarrow \frac{d\sigma^{e^+e^-}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^+e^-}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^+e^-}}{d\Omega'}(x_1 x_2 s) \quad (1a)$$

and produces luminosity for $e^\pm\gamma$ and $\gamma\gamma$ collisions:

$$\frac{d\sigma^{e^\pm\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{e^\pm\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^\pm\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1b)$$

$$\frac{d\sigma^{\gamma\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2 D_{\gamma\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{\gamma\gamma}}{d\Omega'}(x_1 x_2 s) \quad (1c)$$

Therefore, simulations of the physics expected at a LC need to know the spectra of the e^\pm and γ beams precisely.

Microscopic simulations of the beam dynamics are available (e.g. ABEL[3], CAIN[4] and Guinea-Pig[5]) and their predictions are compatible with each other. But they require too much computer time and memory for direct use in physics programs. Circe provides a fast and simple parameterization of the results from these simulations. Furthermore, even if the computational cost of the simulations would be negligible, the input parameters for microscopic simulations are not convenient for particle physics applications. Due to the highly

	SBAND	TESLA	XBAND	SBAND	TESLA	XBAND
E/GeV	250	250	250	500	500	500
$N_{\text{particles}}/10^{10}$	1.1	3.63	0.65	2.9	1.8	0.95
$\epsilon_x/10^{-6}\text{mrad}$	5	14	5	10	14	5
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.08	0.1	0.06	0.1
β_x^*/mm	10.98	24.95	8.00	32	25	10.00
β_y^*/mm	0.45	0.70	0.13	0.8	0.7	0.12
σ_x/nm	335	845	286	571.87	598.08	226
σ_y/nm	15.1	18.9	4.52	9.04	6.55	3.57
$\sigma_z/\mu\text{m}$	300	700	100	500	500	125
f_{rep}	50	5	180	50	5	180
n_{bunch}	333	1135	90	125	2270	90

Table 1: Accelerator parameters for three typical designs at $\sqrt{s} = 500\text{GeV}$ and $\sqrt{s} = 1\text{TeV}$. The resulting distributions are shown in figure 1. The design efforts are currently concentrated on a 350GeV-800GeV LC. Therefore the Tesla parameters for 1TeV are slightly out of date.

non-linear beam dynamics, the optimization of LC designs is a subtle art [6], that is best practiced by the experts. Furthermore, particle physics applications need benchmarking and easily reproducible parameterizations are required for this purpose.

The parameterizations in **Circe** are not based on approximate solutions (cf. [7]) of the beamstrahlung dynamics. Instead, they provide a “phenomenological” description of the results from full simulations. The parameterizations are as simple as possible while remaining consistent with basic physical principles:

1. *positivity*: the distribution functions $D(x_1, x_2)$ *must not* be negative in the physical region $[0, 1] \times [0, 1]$.
2. *integrability*: the definite integral of the distribution functions over the physical region $[0, 1] \times [0, 1]$ *must* exist, even though the distributions can have singularities.

This paper is organized as follows: I start in section 2 with a discussion of the input for the microscopic simulations. In section 3 I describe the usage of the **Circe** library and in section 4 I discuss some technical details of the implementation. After discussing the parameterizations available in version 1.02 β in section 5, I conclude in section 8.

2 Parameters

The microscopic simulation program **Guinea-Pig** [5] used for the current version of the parameterizations in **Circe** simulates the passage of electrons through a

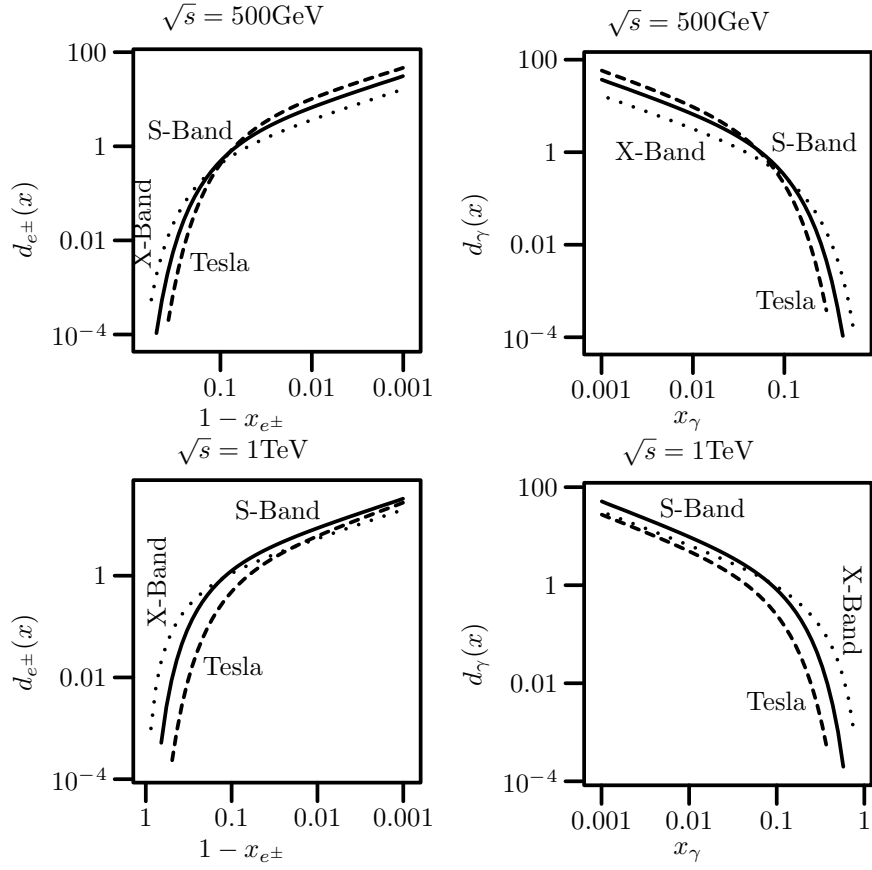


Figure 1: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions at $\sqrt{s} = 500 \text{ GeV}$ and $\sqrt{s} = 1 \text{ TeV}$ in a doubly logarithmic plot. The accelerator parameters are taken from table 1.

	TESLA	TESLA	TESLA
E/GeV	175	250	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	3.63
$\epsilon_x/10^{-6}\text{mrad}$	14	14	14
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.1
β_x^*/mm	25.00	24.95	15.00
β_y^*/mm	0.70	0.70	0.70
σ_x/nm	1010.94	845	668.67
σ_y/nm	22.6	18.9	9.46
$\sigma_z/\mu\text{m}$	700	700	700
f_{rep}	5	5	5
n_{bunch}	1135	1135	1135

Table 2: Accelerator parameters for the Tesla design at three planned [8] energies. The resulting distributions are shown in figure 2.

	High- \mathcal{L}	Low- \mathcal{L}	Low- ϵ_y
E/GeV	400	400	400
$N_{\text{particles}}/10^{10}$	3.63	3.63	1.800
$\epsilon_x/10^{-6}\text{mrad}$	14	14	12
$\epsilon_y/10^{-6}\text{mrad}$	0.1	0.25	0.025
β_x^*/mm	15.00	25.00	25.00
β_y^*/mm	0.70	0.70	0.50
σ_x/nm	668.67	700.00	
σ_y/nm	9.46		
$\sigma_z/\mu\text{m}$	700	700	500
f_{rep}	5	5	3
n_{bunch}	1135	1135	2260

Table 3: Variant accelerator parameters for the Tesla design at 800 GeV.

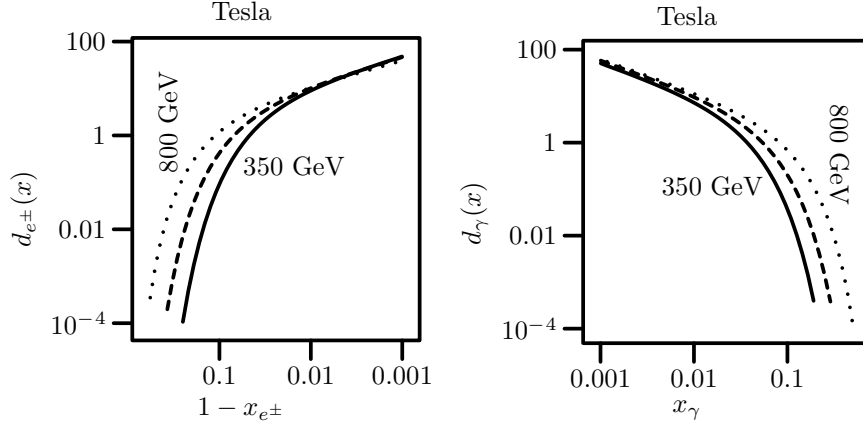


Figure 2: Version 1, revision 1996 09 02 of the factorized e^\pm - and γ -distributions for Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 2.

	TESLA	TESLA
E/GeV	250	400
$N_{\text{particles}}/10^{10}$	2	1.40
$\epsilon_x/10^{-6}\text{m rad}$	10	8
$\epsilon_y/10^{-6}\text{m rad}$	0.03	0.01
β_x^*/mm	15.00	15.00
β_y^*/mm	0.40	0.30
σ_x/nm	553	391
σ_y/nm	5	2
$\sigma_z/\mu\text{m}$	400	300
f_{rep}	5	3
n_{bunch}	2820	4500

Table 4: Accelerator parameters for a high luminosity Tesla design at two planned [8] energies. The resulting distributions are shown in figure 3.

Figure 3: Version 5, revision 1998 05 05 of the factorized e^\pm - and γ -distributions for a high luminosity Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 4.

Figure 4: *Experimental:* Version 1, revision 0 of the factorized e^- - and γ -distributions for Tesla- e^-e^- in a doubly logarithmic plot. The accelerator parameters are taken from table 2 and have *not* been endorsed for use in an e^-e^- -machine yet!.

bunch of electrons (and vice versa). It takes the following accelerator parameters as input:

E : the energy of the particles before the beam-beam interaction.

$N_{\text{particles}}$: the number of particles per bunch.

$\epsilon_{x,y}$: the normalized horizontal and vertical emittances.

$\beta_{x,y}^*$: the horizontal and vertical beta functions.

$\sigma_{x,y,z}$: the horizontal, vertical and longitudinal beam size. A Gaussian shape is used for the charge distribution in the bunches.

f_{rep} : the repetition rate.

n_{bunch} : the number of bunches per train.

The transversal beam sizes, beta functions and normalized emittances for relativistic particles are related by

$$\beta_{x,y}^* = \frac{\sigma_{x,y}^2}{\epsilon_{x,y}} \frac{E}{m_e} \quad (2)$$

The parameters used in the most recent revision of the parameterizations are collected in tables 1 and 2. The resulting factorized electron/positron and photon distributions in version 1 of the parameterizations are depicted in figures 1 and 2.

The most important purpose of **Circe** is to map the manifold of possible beam spectra for the NLC to a *finite* number of *reproducible* parameterizations. The distributions

$$D_{p_1 p_2}^{\alpha \nu \rho}(x_1, x_2; \sqrt{s}) \quad (3)$$

provided by **Circe** are indexed by three integers

α : the *accelerator design class*: currently there are three options: S-band [9], Tesla [8], X-band [10, 11]. More variety will be added later, in particular the e^-e^- mode and the $e^-\gamma$ and $\gamma\gamma$ laser backscattering modes of these designs.

ν : the *version of the parameterization*: over the years, the form of the parameterizations can change, either because better approximations are found or because new simulation programs become available. All versions will remain available in order to be able to reproduce calculations.

ρ : the *revision date for the parameterization*: a particular parameterization can contain bugs, which will be fixed in subsequent revisions. While only the most recent revision should be used for new calculations, old revisions will remain available in order to be able to reproduce calculations.

The continuous parameter \sqrt{s} in (3) is misleading, because accelerator parameters have been optimized for discrete values of the energy. Therefore the distributions are not available for all values of \sqrt{s} .

The usage of the distributions in application programs is discussed in section 3.1. **Circe** provides for each of the distributions a non-uniform random variate generator, that generates energy fractions according to the distributions. The usage of these generators is discussed in section 3.2.

3 Usage

3.1 Distributions

A generic interface to all distributions $D_{p_1 p_2}(x_1, x_2)$ is given by the **circe** function

10a *<API documentation 10a>* \equiv

```
double precision circe, d, x1, x2
integer p1, p2
d = circe (x1, x2, p1, p2)
```

Uses **circe** 28b.

where the energy fractions are specified by $x_{1,2}$ and the particles $p_{1,2}$ are identified by their standard Monte Carlo codes:[13]

10b *<Particle codes 10b>* \equiv

```
integer ELECTR, POSITR, PHOTON
parameter (ELECTR = 11)
parameter (POSITR = -11)
parameter (PHOTON = 22)
```

Defines:

ELECTR, used in chunks 19c, 28b, 66a, and 71.

PHOTON, used in chunks 28b, 66a, 71, and 78a.

POSITR, used in chunks 19d and 71b.

The distributions can have integrable singularities at the end points, therefore the calling functions *must not* evaluate them at the endpoints 0 and 1. This is usually not a problem, since standard mapping techniques (cf. (10) below) will have to be used to take care of the singularity anyway. Nevertheless, all applications should favor open quadrature formulae (i.e. formulae not involving the endpoints) over closed formulae. The distributions are guaranteed to vanish unless $0 < x_{1,2} < 1$, with two exceptions. Firstly, the value -1 allows to pick up the integral of the continuum contribution:

$$D_{p_1 p_2}(-1, x_2) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 D_{p_1 p_2}(x_1, x_2) \quad (4a)$$

$$D_{p_1 p_2}(x_1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4b)$$

$$D_{p_1 p_2}(-1, -1) = \lim_{\epsilon \rightarrow +0} \int_{\epsilon}^{1-\epsilon} dx_1 dx_2 D_{p_1 p_2}(x_1, x_2) \quad (4c)$$

The other exception is that the strength of δ -function contributions at the endpoint can be picked up from the value at this endpoint:

$$D_{e^+e^-}(x_1, x_2) = D_{e^+e^-}(1, 1)\delta(1 - x_1)\delta(1 - x_2) + \text{smooth and single } \delta \quad (5a)$$

$$D_{e^\pm\gamma}(x_1, x_2) = D_{e^\pm\gamma}(1, x_2)\delta(1 - x_1) + \text{smooth} \quad (5b)$$

$$D_{\gamma e^\pm}(x_1, x_2) = D_{\gamma e^\pm}(x_1, 1)\delta(1 - x_2) + \text{smooth} \quad (5c)$$

The use of these special values is demonstrated in an example in section 3.1.1 below.

The distributions are normalized such that

$$\lim_{\epsilon \rightarrow +0} \int_{-\epsilon}^{1+\epsilon} dx_1 dx_2 D_{e^+e^-}(x_1, x_2) = 1. \quad (6)$$

and the nominal e^+e^- -luminosity of the currently active accelerator design can be retrieved from the database with the subroutine `circel`. The value is given in units of

$$\text{fb}^{-1}v^{-1} = 10^{32} \text{cm}^{-2} \text{sec}^{-1} \quad (7)$$

where $v = 10^7 \text{sec} \approx \text{year}/\pi$ is an “effective year” of running with about 30% up-time.

11a `<API documentation 10a>+≡`
`double precision lumi`
`call circel (lumi)`

Uses `circel` 37d.

A particular parameterization is selected by the `circes` function:

11b `<API documentation 10a>+≡`
`double precision x1m, x2m, roots`
`integer acc, ver, rev, chat`
`call circes (x1m, x2m, roots, acc, ver, rev, chat)`

Uses `circes` 29a.

The parameter `roots` corresponds to the nominal center of mass energy \sqrt{s}/GeV of the collider. Currently $\sqrt{s} = 350\text{GeV}, 500\text{GeV}, 800\text{GeV}, 1\text{TeV}$ (i.e. 350D0, 500D0, 800D0 and 1000D0) are supported. Application programs can *not* assume that energy values are interpolated. For convenience, e.g. in top threshold scans around 350GeV, a small interval around the supported values will be accepted as synonymous with the central value, but a warning will be printed. Section 5 should be consulted for the discrete values supported by a particular version of the parameterizations. Negative values of `roots` will keep the currently active value for \sqrt{s} .

The parameters `x1m` and `x2m` will set thresholds $x_{1,\min}$ and $x_{2,\min}$ for the event generation in the routines described in section 3.2.

The parameter `acc` selects the accelerator design. Currently the following accelerator codes are recognized:

11c `<Accelerator codes 11c>≡`
`integer SBAND, TESLA, XBAND`
`parameter (SBAND = 1, TESLA = 2, XBAND = 3)`
`integer JLCNLC`
`parameter (JLCNLC = 3)`

```

integer SBNDEE, TESLEE, XBNDEE
parameter (SBNDEE = 4, TESLEE = 5, XBNDEE = 6)
integer NLCH
parameter (NLCH = 7)
integer NACC
parameter (NACC = 7)

```

Defines:

NACC, used in chunks 31c, 32a, 36, 37, 40, 42, 43, 47d, 49d, 51c, 55a, 57d, and 61c.
SBAND, used in chunks 32b, 36, 39–43, 45b, 46a, 81c, 94b, and 95b.
TESLA, used in chunks 30a, 32b, 36, 39c, 41–43, 45–53, 55b, 56a, 58, 61d, 63a, 81, 94b, 95b, and 97a.
XBAND, used in chunks 36, 39–43, 45b, 46a, 53c, 94b, and 95b.

The **ver** parameter is used to determine the version as follows:

ver > 0 : a frozen version which is documented in section 5. For example, version 1 is a family of factorized Beta distributions: $D(x_1, x_2) \propto x_1^{a_1} (1 - x_1)^{b_1} x_2^{a_2} (1 - x_2)^{b_2}$.

ver = 0 : the latest experimental version, which is usually not documented and can change at any time without announcement.

ver < 0 : keep the currently active version.

The **rev** parameter is used to determine the revision of a version as follows:

rev > 0 : a frozen revision which is documented in section 5. The integer **rev** is constructed from the date as follows: $\text{rev} = 10^4 \cdot \text{year} + 10^2 \cdot \text{month} + \text{day}$, where the year is greater than 1995. Since Fortran77 ignores whitespace, it can be written like 1996 07 11 for readability. If there is no exact match, the most recent revision before the specified date is chosen.

rev = 0 : the most recent revision.

rev < 0 : keep the currently active revision.

Finally, the parameter **chat** controls the “chattiness” of **circe**. If it is 0, only error messages are printed. If it is 1, the parameters in use are printed whenever they change. Higher values of **chat** can produce even more diagnostics.

In addition to the generic interface **circe**, there are specialized functions for particular particle distributions. Obviously

$$D_{e^\pm\gamma}^{\alpha\nu\rho}(x_1, x_2, s) = D_{\gamma e^\pm}^{\alpha\nu\rho}(x_2, x_1, s) \quad (8)$$

and there are three independent functions $D_{e^-e^+}$, $D_{e^- \gamma}$ and $D_{\gamma\gamma}$ for the e^+e^- colliders with reasonable mnemonics:

12 *<API documentation 10a>+≡*

```

double precision circee, circeg, circgg
d = circee (x1, x2)
d = circeg (x1, x2)
d = circgg (x1, x2)

```

Uses **circee** 37e, **circeg** 38b, and **circgg** 39a.

Calling the latter three functions is marginally faster in the current implementation, but this can change in the future.

3.1.1 Example

For clarification, let me give a simple example. Imagine we want to calculate the integrated production cross section

$$\sigma_X(s) = \int dx_1 dx_2 \sigma_{e^+e^- \rightarrow X}(x_1 x_2 s) D_{e^+e^-}(x_1, x_2, s) \quad (9)$$

Since the distributions are singular in the $x_{1,2} \rightarrow 1$ limit, we have to map away this singularity with

$$x \rightarrow t = (1 - x)^{1/\eta} \quad (10a)$$

Therefore

$$\int_0^1 dx f(x) = \int_0^1 dt \eta t^{\eta-1} f(1 - t^\eta) \quad (10b)$$

with η sufficiently large to give the integrand a finite limit at $x \rightarrow 1$. If f diverges like a power $f(x) \propto 1/(1 - x)^\beta$, this means $\eta > 1/(1 - \beta)$.

As a specific example, let us “measure” a one particle s -channel exchange cross section

$$\sigma(s) \propto \frac{1}{s} \quad (11)$$

```
13a <sample.f 13a>≡
      double precision function sigma (s)
      implicit none
      double precision s
      sigma = 1d0 / s
      end
```

I will present the example code in a bottom-up fashion, which should be intuitive and is described in some more detail in appendix A. Assuming the existence of a one- and a two-dimensional Gaussian integration function `gauss1` and `gauss2`,¹ we can perform the integral as follows:

```
13b <Gauss integration 13b>≡
      s = sigma (1d0) * circee (1d0, 1d0)
      $ + gauss1 (d1, 0d0, 1d0, EPS)
      $ + gauss1 (d2, 0d0, 1d0, EPS)
      $ + gauss2 (d12, 0d0, 1d0, 0d0, 1d0, EPS)
      write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
      1000 format (1X, A22, 1X, F6.2, '%')
```

Uses `circee` 37e, `gauss1` 80a, and `gauss2` 80c.

Note how the four combinations of continuum and δ -peak are integrated separately, where you have to use three auxiliary functions `d1`, `d2` and `d12`. The continuum contribution, including the Jacobian:

```
13c <sample.f 13a>+≡
      double precision function d12 (t1, t2)
      implicit none
      double precision t1, t2, x1, x2, sigma, circee
```

¹They are provided in the example program `sample.f`.

```

(EPS & PWR 14c)
x1 = 1d0 - t1**PWR
x2 = 1d0 - t2**PWR
d12 = PWR*PWR * (t1*t2)**(PWR-1d0)
$      * sigma (x1*x2) * circee (x1, x2)
end

```

Uses circee 37e.

the first product of continuum and δ -peak:

```

14a (sample.f 13a)+≡
double precision function d1 (t1)
implicit none
double precision t1, x1, sigma, circee
(EPS & PWR 14c)
x1 = 1d0 - t1**PWR
d1 = PWR * t1**(PWR-1d0) * sigma (x1) * circee (x1, 1d0)
end

```

Uses circee 37e.

and the second one:

```

14b (sample.f 13a)+≡
double precision function d2 (t2)
implicit none
double precision t2, x2, sigma, circee
(EPS & PWR 14c)
x2 = 1d0 - t2**PWR
d2 = PWR * t2**(PWR-1d0) * sigma (x2) * circee (1d0, x2)
end

```

Uses circee 37e.

Below you will see that the power of the singularity of the e^+e^- distributions at $x \rightarrow 1$ is $\approx -2/3$. To be on the safe side, we choose the power η in (10) as 5. It is kept in the parameter PWR, while EPS is the desired accuracy of the Gaussian integration:

```

14c (EPS & PWR 14c)≡
double precision EPS, PWR
parameter (EPS = 1d-6, PWR = 5d0)

```

The Gauss integration of the non-singular version converges to the correct value only if the final bin is integrated separately:

```

14d (Second Gauss integration 14d)≡
s = gauss2 (d12a, 0d0, 1d0-KIREPS, 0d0, 1d0-KIREPS, EPS)
$ + gauss2 (d12a, 0d0, 1d0-KIREPS, 1d0-KIREPS, 1d0, EPS)
$ + gauss2 (d12a, 1d0-KIREPS, 1d0, 0d0, 1d0-KIREPS, EPS)
$ + gauss2 (d12a, 1d0-KIREPS, 1d0, 1d0-KIREPS, 1d0, EPS)
write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0

```

Uses gauss2 80c.

```

14e (EPS & PWR 14c)+≡
double precision KIREPS
parameter (KIREPS = 1D-6)

```

15a *<sample.f 13a>+≡*

```
double precision function d12a (x1, x2)
implicit none
double precision x1, x2, sigma, kirkee
d12a = sigma (x1*x2) * kirkee (x1, x2)
end
```

Uses kirkee 66b.

These code fragments can now be used in a main program that loops over energies and accelerator designs

15b *<sample.f 13a>+≡*

```
program sample
implicit none
<Accelerator codes 11c>
<EPS & PWR 14c>
<Other variables in sample 17a>
integer acc, ver, i
double precision roots(9)
data roots / 90D0, 170D0, 250D0, 350D0, 500D0,
$           800D0, 1000D0, 1200D0, 1500D0 /
do 10 acc = JLCNLC, NLCH, NLCH-JLCNLC
  do 11 ver = 9, 9
    do 12 i = 1, 9
      call circes (0d0, 0d0, roots(i), acc, ver, 20020328, 1)
      <Gauss integration 13b>
      <Second Gauss integration 14d>
      <Monte Carlo integration 18a>
14      continue
12      continue
13      continue
11      continue
10      continue
end
```

Uses circes 29a.

with the following result

15c *<Sample output 15c>≡*

```
circe:message: starting up ...
circe:message: $Id: prelude.nw 66 2002-03-28 17:00:10Z ohl $
circe:message: updating 'roots' to 90.0
circe:message: updating 'ver' to 7
circe:message: updating 'rev' to 20000501
delta(sigma) (Gauss) = 0.11%
delta(sigma) (MC)    = 0.11%
                    +/- 0.00%
circe:message: updating 'roots' to 170.0
circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 0.38%
delta(sigma) (MC)    = 0.38%
                    +/- 0.01%
circe:message: updating 'roots' to 350.0
```

```

circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 1.67%
delta(sigma) (MC)    = 1.66%
                    +/- 0.03%
circe:message: updating 'roots' to 500.0
circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 3.66%
delta(sigma) (MC)    = 3.58%
                    +/- 0.07%
circe:message: updating 'roots' to 800.0
circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 5.21%
delta(sigma) (MC)    = 5.19%
                    +/- 0.11%
circe:message: updating 'roots' to 1000.0
circe:message: updating 'ver' to 7
circe:message: energy 1000.0GeV too high, using spectrum for 800.0GeV
delta(sigma) (Gauss) = 5.21%
delta(sigma) (MC)    = 5.19%
                    +/- 0.11%
circe:message: updating 'roots' to 90.0
circe:message: updating 'acc' to JLCNLC
circe:message: updating 'ver' to 7
circe:message: energy 90.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe:message: updating 'roots' to 170.0
circe:message: updating 'ver' to 7
circe:message: energy 170.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.68%
                    +/- 0.11%
circe:message: updating 'roots' to 350.0
circe:message: updating 'ver' to 7
circe:message: energy 350.0GeV too low, using spectrum for 500.0GeV
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe:message: updating 'roots' to 500.0
circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 4.74%
delta(sigma) (MC)    = 4.75%
                    +/- 0.11%
circe:message: updating 'roots' to 800.0
circe:message: updating 'ver' to 7
circe:message: energy 800.0GeV interpolated between 500.0 and 1000.0GeV
delta(sigma) (Gauss) = 8.37%
delta(sigma) (MC)    = 8.39%
                    +/- 0.21%

```

```

circe:message: updating 'roots' to 1000.0
circe:message: updating 'ver' to 7
delta(sigma) (Gauss) = 15.39%
delta(sigma) (MC)    = 14.68%
                    +/- 0.33%

```

Uses `circe` 28b.

We almost forgot to declare the variables in the main program

```

17a  <Other variables in sample 17a>≡
      double precision s
      double precision gauss1, gauss2, circee, sigma, d1, d2, d12, d12a
      external d1, d2, d12, d12a

```

Uses `circee` 37e, `gauss1` 80a, and `gauss2` 80c.

This concludes the integration example. It should have made it obvious how to proceed in a realistic application.

In section 3.2.1 below, I will describe a Monte Carlo method for calculating such integrals efficiently.

3.2 Generators

The function `circe` and its companions are opaque to the user. Since they will in general contain singularities, applications will *not* be able to generate corresponding samples of random numbers efficiently. To fill this gap, four random number generators are provided. The subroutine `girce` will generate particle types $p_{1,2}$ and energy fractions $x_{1,2}$ in one step, according to the selected distribution.² Particle p_1 will be either a positron or a photon and p_2 will be either an electron or a photon. The energy fractions are guaranteed to be above the currently active thresholds: $x_i \geq x_{i,\min}$. This can be used to cut on soft events—the photon distributions are rather soft—which might not be interesting in most simulations.

```

17b  <API documentation 10a>+≡
      call girce (x1, x2, p1, p2, rng)

```

Uses `girce` 71a.

The output parameters of `girce` are identical to the input parameters of `circe`, with the exception of `rng`. The latter is a subroutine with a single double precision argument, which will be assigned a uniform deviate from the interval $[0, 1]$ after each call:

```

17c  <API documentation 10a>+≡
      subroutine rng (r)
      double precision r
      r = <uniform deviate on [0, 1]> (never defined)
      end

```

Typically, it will be just a wrapper around the standard random number generator of the application program. For studies with a definite initial state, three generator functions are available.

```

17d  <API documentation 10a>+≡

```

²The implementation of the flavor selection with non-vanishing thresholds $x_{1,\min}$ and $x_{2,\min}$ is moderately inefficient at the moment. It can be improved by a factor of two.

```

      call gircee (x1, x2, rng)
      call girceg (x1, x2, rng)
      call gircgg (x1, x2, rng)

```

Uses gircee 72c, girceg 73a, and gircgg 73d.

3.2.1 Example

Returning to the example from section 3.2.1, I present a concise Monte Carlo algorithm for calculating the same integral:

```

18a  <Monte Carlo integration 18a>≡
      s = 0d0
      s2 = 0d0
      do 100 n = 1, NEVENT
        call gircee (x1, x2, random)
        w = sigma (x1*x2)
        s = s + w
        s2 = s2 + w*w
100  continue
      s = s / dble(NEVENT)
      s2 = s2 / dble(NEVENT)
      write (*, 1000) 'delta(sigma) (MC)      =', (s-1d0)*100d0
      write (*, 1000) '                        +/-',
$          sqrt((s2-s*s)/dble(NEVENT))*100d0

```

Uses gircee 72c.

```

18b  <Other variables in sample 17a>+≡
      double precision w, s2, x1, x2
      external random
      integer NEVENT, n
      parameter (NEVENT = 10000)

```

Here is a simple linear congruential random number generator for the sample program. Real applications will use their more sophisticated generators instead.

```

18c  <sample.f 13a>+≡
      subroutine random (r)
      implicit none
      double precision r
      integer m, a, c
      parameter (M = 259200, A = 7141, C = 54773)
      integer n
      save n
      data n /0/
      n = mod(n*a+c,m)
      r = dble (n) / dble (m)
      end

```

If the cross section is slowly varying on the range where the $x_{1,2}$ distributions are non-zero, this algorithm is very efficient.

However, if this condition is not met, the explicit form of the parameterizations in section 5 should be consulted and appropriate mapping techniques should be applied. The typical example for this problem is a narrow resonance just below the nominal beam energy.

3.2.2 Event Generators

For Monte Carlo event generators that use the standard `/hepevt/` common block [14], the addition of the `Circe` library is trivial. During the initialization of the event generator, the `circes` subroutine is called to set up `Circe`'s internal state. For example:

19a $\langle \text{Initialize event generator 19a} \rangle \equiv$
`call circes (0d0, 0d0, roots, acc, ver, 1996 07 11, 1)`

Uses `circes` 29a.

During event generation, before setting up the e^+e^- initial state, the `gircee` subroutine is called with the event generator's random number generator:

19b $\langle \text{Event generation 19b} \rangle \equiv$
`call gircee (x1, x2, random)`

Uses `gircee` 72c.

The resulting energy fractions x_1 and x_2 are now available for defining the initial state electron

19c $\langle \text{Event generation 19b} \rangle + \equiv$
`isthep(1) = 101`
`idhep(1) = ELECTR`
`phep(1,1) = 0d0`
`phep(2,1) = 0d0`
`phep(3,1) = x1 * ebeam`
`phep(4,1) = x1 * ebeam`
`phep(5,1) = 0d0`

Uses `ELECTR` 10b.

and positron.

19d $\langle \text{Event generation 19b} \rangle + \equiv$
`isthep(2) = 102`
`idhep(2) = POSITR`
`phep(1,2) = 0d0`
`phep(2,2) = 0d0`
`phep(3,2) = - x2 * ebeam`
`phep(4,2) = x2 * ebeam`
`phep(5,2) = 0d0`

Uses `POSITR` 10b.

Using `Circe` with other event generators should be straightforward as well.

4 Technical Notes

The structure of `Circe` is extremely simple (cf. figure 5) and is mainly a book-keeping exercise. All that needs to be done is to maintain a database of available parameterizations and to evaluate the corresponding functions. The only non trivial algorithms are used for the efficient generation of random deviates.

I have avoided the use of initialized `common` blocks (i.e. `block data` subroutines), because the Fortran77 standard does not provide a *portable* way of ensuring that `block data` subroutines are actually executed at loading time. Instead, the `/circom/` common block is tagged by a “magic number” to check

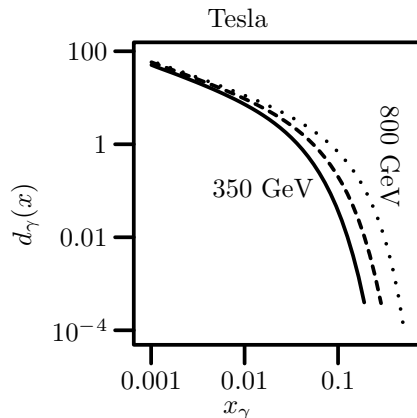


Figure 5: Architecture of **Circe**: `circes()` selects energy and accelerator and loads the parameterization. The function `circe()` calculates the values of the selected distribution function at the given energy fractions. The subroutine `girc()` generates energy fractions using a specified random number generator in accordance with the selected distribution.

for initialization and its members are filled by the `circes` subroutine when necessary.

A more flexible method would be to replace the `data` statements by reading external files. This option causes portability problems, however, because I would have to make sure that the names of the external files are valid in all file systems of the target operating systems. More significantly, splitting the implementation into several parts forces the user to keep all files up to date. This can be a problem, because Fortran source files and data input files will typically be kept in different parts of the file system.

The option of implementing **Circe** statelessly, i.e. with pure function calls and without `common` blocks, has been dismissed. While it would have been more straightforward on the side of the library, it would have placed the burden of maintaining state (accelerator, energy, etc.) on the application program, thereby complicating them considerably. Keeping an explicit state in **Circe** has the additional benefit of allowing to precompute certain internal variables, resulting in a more efficient implementation.

5 Parameterizations

Version 1.02 β of **Circe** supports just one version of the parameterizations. Future versions will provide additional parameterizations.

5.1 Version 1

The first version of the parameterization uses a simple factorized *ansatz*

$$D_{p_1 p_2}^{\alpha 1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha 1 \rho}(x_1) d_{p_2}^{\alpha 1 \rho}(x_2) \quad (12a)$$

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$31.38^{+0.22}_{-0.22}$	$106.25^{+0.71}_{-0.71}$	$95.24^{+0.73}_{-0.73}$	$36.39^{+0.29}_{-0.29}$
$\int d_{e\pm}$	$0.4812^{+0.0041}_{-0.0041}$	$0.5723^{+0.0046}_{-0.0045}$	$0.3512^{+0.0048}_{-0.0048}$	$0.3487^{+0.0040}_{-0.0040}$
$x_{e\pm}^\alpha$	$11.1534^{+0.0770}_{-0.0761}$	$15.2837^{+0.0923}_{-0.0914}$	$27.1032^{+0.3071}_{-0.3019}$	$6.9853^{+0.0733}_{-0.0718}$
$(1 - x_{e\pm})^\alpha$	$-0.6302^{+0.0013}_{-0.0012}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.6453^{+0.0017}_{-0.0017}$	$-0.6444^{+0.0017}_{-0.0017}$
$\int d_\gamma$	$0.6237^{+0.0033}_{-0.0033}$	$0.7381^{+0.0036}_{-0.0036}$	$0.3502^{+0.0034}_{-0.0034}$	$0.4149^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6911^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6947^{+0.0011}_{-0.0011}$	$-0.6876^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$14.9355^{+0.0761}_{-0.0754}$	$24.1647^{+0.1124}_{-0.1116}$	$33.6576^{+0.3021}_{-0.2983}$	$8.3227^{+0.0659}_{-0.0649}$

Table 5: Version 1, revision 1997 04 16 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$119.00^{+0.83}_{-0.83}$	214.33^{+0***}_{-0***}	212.22^{+0***}_{-0***}	$118.99^{+0.91}_{-0.91}$
$\int d_{e\pm}$	$0.5604^{+0.0040}_{-0.0039}$	$0.6686^{+0.0040}_{-0.0040}$	$0.4448^{+0.0043}_{-0.0043}$	$0.5001^{+0.0038}_{-0.0038}$
$x_{e\pm}^\alpha$	$4.2170^{+0.0258}_{-0.0255}$	$5.5438^{+0.0241}_{-0.0239}$	$9.6341^{+0.0814}_{-0.0803}$	$2.6184^{+0.0192}_{-0.0190}$
$(1 - x_{e\pm})^\alpha$	$-0.6118^{+0.0013}_{-0.0013}$	$-0.5847^{+0.0011}_{-0.0011}$	$-0.6359^{+0.0014}_{-0.0014}$	$-0.6158^{+0.0015}_{-0.0015}$
$\int d_\gamma$	$0.7455^{+0.0032}_{-0.0032}$	$1.0112^{+0.0033}_{-0.0033}$	$0.4771^{+0.0031}_{-0.0031}$	$0.6741^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6870^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0004}_{-0.0004}$	$-0.6936^{+0.0008}_{-0.0008}$	$-0.6834^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$6.7145^{+0.0310}_{-0.0308}$	$9.9992^{+0.0342}_{-0.0340}$	$13.1607^{+0.0896}_{-0.0886}$	$3.8589^{+0.0215}_{-0.0213}$

Table 6: Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.

	350 GeV	500 GeV	800 GeV	1600 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$97.45^{+0.67}_{-0.67}$	$106.25^{+0.71}_{-0.71}$	170.86^{+0***}_{-0***}	340.86^{+0***}_{-0***}
$\int d_{e\pm}$	$0.6093^{+0.0049}_{-0.0049}$	$0.5723^{+0.0046}_{-0.0045}$	$0.6398^{+0.0042}_{-0.0041}$	$0.5094^{+0.0040}_{-0.0040}$
$x_{e\pm}^\alpha$	$17.6137^{+0.1065}_{-0.1055}$	$15.2837^{+0.0923}_{-0.0914}$	$7.6221^{+0.0365}_{-0.0361}$	$5.0550^{+0.0353}_{-0.0349}$
$(1 - x_{e\pm})^\alpha$	$-0.6061^{+0.0011}_{-0.0011}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.5944^{+0.0011}_{-0.0011}$	$-0.6187^{+0.0013}_{-0.0013}$
$\int d_\gamma$	$0.7729^{+0.0039}_{-0.0039}$	$0.7381^{+0.0036}_{-0.0036}$	$0.9178^{+0.0034}_{-0.0034}$	$0.5875^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6949^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0005}_{-0.0005}$	$-0.6892^{+0.0007}_{-0.0007}$
$(1 - x_\gamma)^\alpha$	$28.9399^{+0.1370}_{-0.1361}$	$24.1647^{+0.1124}_{-0.1116}$	$13.1167^{+0.0497}_{-0.0495}$	$7.5514^{+0.0428}_{-0.0424}$

Table 7: Version 1, revision 1997 04 17 of the beam spectra for TESLA.

	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$339.80^{+0.83}_{-0.83}$	$359.36^{+0.93}_{-0.93}$
$\int d_{e^\pm}$	$0.5019^{+0.0016}_{-0.0016}$	$0.4125^{+0.0016}_{-0.0016}$
$x_{e^\pm}^\alpha$	$12.2867^{+0.0318}_{-0.0316}$	$13.3242^{+0.0442}_{-0.0440}$
$(1 - x_{e^\pm})^\alpha$	$-0.6276^{+0.0005}_{-0.0005}$	$-0.6401^{+0.0005}_{-0.0005}$
$\int d_\gamma$	$0.5114^{+0.0012}_{-0.0012}$	$0.3708^{+0.0011}_{-0.0011}$
x_γ^α	$-0.6912^{+0.0003}_{-0.0003}$	$-0.6924^{+0.0004}_{-0.0004}$
$(1 - x_\gamma)^\alpha$	$17.0673^{+0.0375}_{-0.0375}$	$16.8145^{+0.0482}_{-0.0480}$

Table 8: Version 5, revision 1998 05 05 of the beam spectra for high luminosity TESLA.

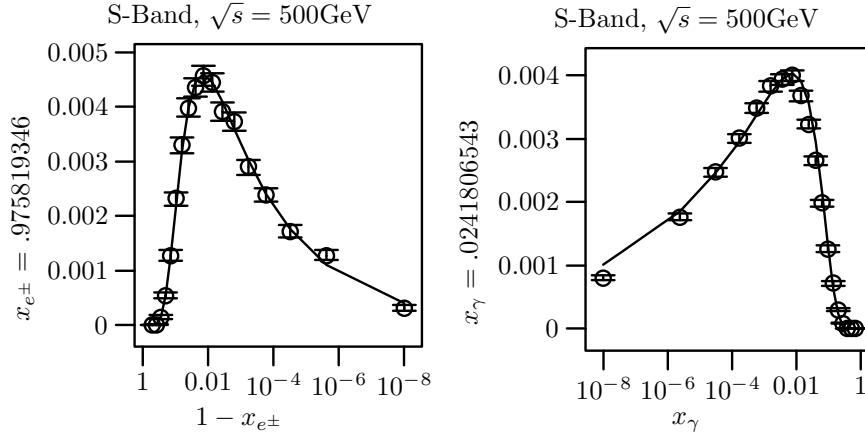


Figure 6: Fit of the e^\pm - and γ -distributions for the S-Band design at $\sqrt{s} = 500\text{GeV}$. The open circles with error bars are the result of the Guinea-Pig simulation. The full line is the fit.

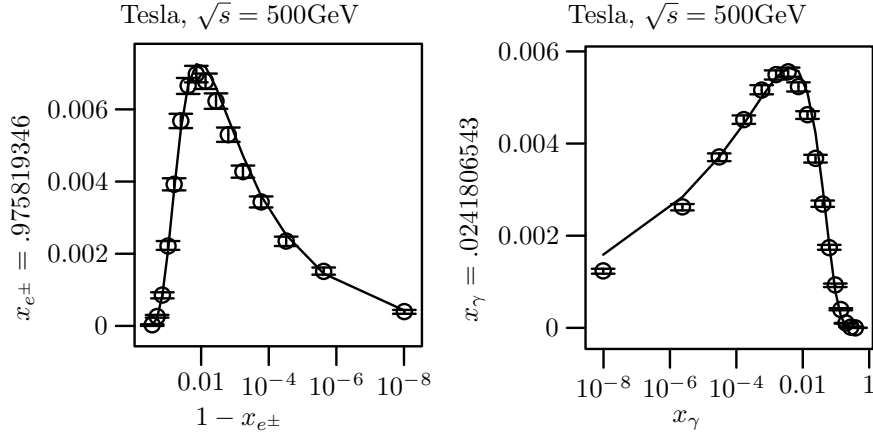


Figure 7: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 500\text{GeV}$.

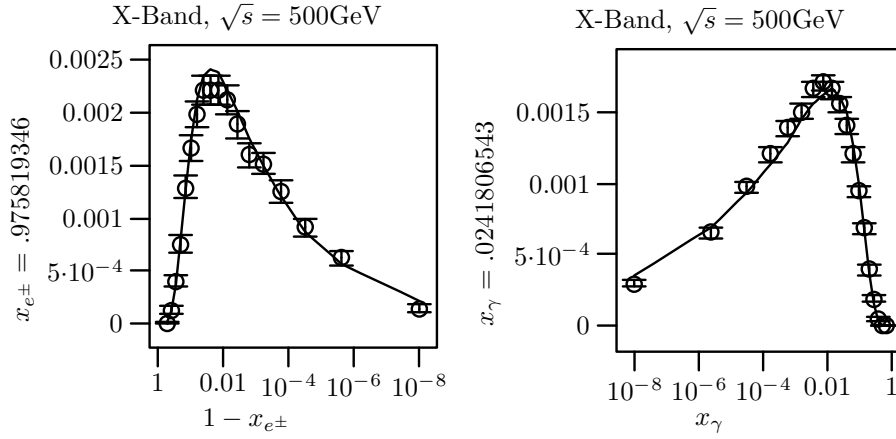


Figure 8: Fit of the e^\pm - and γ -distributions for the X-Band design at $\sqrt{s} = 500\text{GeV}$.

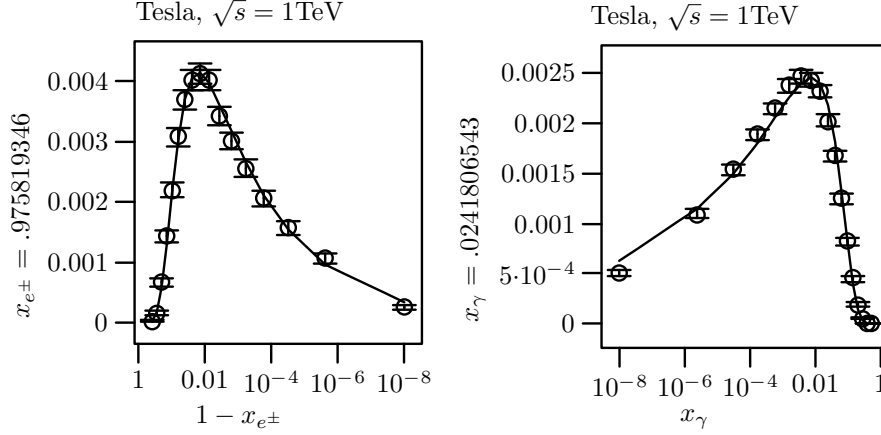


Figure 9: Fit of the e^\pm - and γ -distributions for the Tesla design at $\sqrt{s} = 1\text{TeV}$.

	SBNDEE	TESLEE	XBNDDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$9.29^{+0.06}_{-0.06}$	$21.62^{+0.17}_{-0.17}$	$13.97^{+0.10}_{-0.10}$
$\int d_{e^\pm}$	$.6513^{+0.0059}_{-0.0059}$	$.7282^{+0.0083}_{-0.0082}$	$.5270^{+0.0049}_{-0.0049}$
$x_{e^\pm}^\alpha$	$10.3040^{+0.0601}_{-0.0593}$	$14.8578^{+0.1047}_{-0.1034}$	$5.8897^{+0.0455}_{-0.0448}$
$(1 - x_{e^\pm})^\alpha$	$-.5946^{+0.0015}_{-0.0015}$	$-.5842^{+0.0018}_{-0.0018}$	$-.6169^{+0.0016}_{-0.0015}$
$\int d_\gamma$	$.4727^{+0.0035}_{-0.0035}$	$.5300^{+0.0046}_{-0.0046}$	$.3746^{+0.0029}_{-0.0029}$
x_γ^α	$-.6974^{+0.0009}_{-0.0009}$	$-.7039^{+0.0009}_{-0.0009}$	$-.6892^{+0.0010}_{-0.0010}$
$(1 - x_\gamma)^\alpha$	$20.6447^{+0.1513}_{-0.1497}$	$36.1286^{+0.3027}_{-0.2991}$	$10.0872^{+0.0822}_{-0.0815}$

Table 9: *Experimental* Version 1, revision 0 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

where the distributions are simple Beta distributions:

$$d_{e^\pm}^{\alpha 1\rho}(x) = a_0^{\alpha\rho} \delta(1-x) + a_1^{\alpha\rho} x^{a_2^{\alpha\rho}} (1-x)^{a_3^{\alpha\rho}} \quad (12b)$$

$$d_\gamma^{\alpha 1\rho}(x) = a_4^{\alpha\rho} x^{a_5^{\alpha\rho}} (1-x)^{a_6^{\alpha\rho}} \quad (12c)$$

This form of the distributions is motivated by the observation [2] that the e^\pm distributions diverge like a power for $x \rightarrow 1$ and vanish at $x \rightarrow 0$. The behavior of the γ distributions is similar with the borders exchanged.

5.1.1 Fitting

The parameters a_i in (12) have been obtained by a least-square fit of (12) to histograms of simulation results from **Guinea-Pig**. Some care has to be taken when fitting singular distributions to histogrammed data. Obviously equidistant bins are not a good idea, because most bins will be almost empty (cf. figures 1

	SBNDDEE	TESLEE	XBNDDEE
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$45.59^{+0.34}_{-0.34}$	$25.47^{+0.20}_{-0.20}$	$41.06^{+0.28}_{-0.28}$
$\int d_{e\pm}$	$.7892^{+0.0075}_{-0.0074}$	$.6271^{+0.0066}_{-0.0065}$	$.7203^{+0.0058}_{-0.0058}$
$x_{e\pm}^\alpha$	$5.4407^{+0.0285}_{-0.0281}$	$8.7504^{+0.0669}_{-0.0658}$	$2.7415^{+0.0121}_{-0.0119}$
$(1 - x_{e\pm})^\alpha$	$-.5285^{+0.0020}_{-0.0020}$	$-.6058^{+0.0017}_{-0.0017}$	$-.5049^{+0.0020}_{-0.0020}$
$\int d_\gamma$	$.6403^{+0.0040}_{-0.0040}$	$.4278^{+0.0038}_{-0.0038}$	$.6222^{+0.0032}_{-0.0032}$
x_γ^α	$-.6960^{+0.0008}_{-0.0008}$	$-.6982^{+0.0010}_{-0.0010}$	$-.6795^{+0.0008}_{-0.0008}$
$(1 - x_\gamma)^\alpha$	$12.4803^{+0.0839}_{-0.0831}$	$18.5260^{+0.1674}_{-0.1655}$	$4.7506^{+0.0262}_{-0.0260}$

Table 10: *Experimental* Version 1, revision 0 of the beam spectra at 1 TeV.

	350 GeV	500 GeV	800 GeV
$\mathcal{L}/\text{fb}^{-1}v^{-1}$	$15.18^{+0.13}_{-0.13}$	$21.62^{+0.17}_{-0.17}$	$43.98^{+0.38}_{-0.38}$
$\int d_{e\pm}$	$.6691^{+0.0083}_{-0.0083}$	$.7282^{+0.0083}_{-0.0082}$	$.7701^{+0.0090}_{-0.0089}$
$x_{e\pm}^\alpha$	$25.2753^{+0.2040}_{-0.2007}$	$14.8578^{+0.1047}_{-0.1034}$	$8.1905^{+0.0543}_{-0.0535}$
$(1 - x_{e\pm})^\alpha$	$-.5994^{+0.0017}_{-0.0017}$	$-.5842^{+0.0018}_{-0.0018}$	$-.5575^{+0.0021}_{-0.0021}$
$\int d_\gamma$	$.4464^{+0.0047}_{-0.0047}$	$.5300^{+0.0046}_{-0.0046}$	$.5839^{+0.0047}_{-0.0047}$
x_γ^α	$-.7040^{+0.0011}_{-0.0011}$	$-.7039^{+0.0009}_{-0.0009}$	$-.7046^{+0.0009}_{-0.0009}$
$(1 - x_\gamma)^\alpha$	$60.1882^{+0.5882}_{-0.5797}$	$36.1286^{+0.3027}_{-0.2991}$	$19.3944^{+0.1681}_{-0.1660}$

Table 11: *Experimental* Version 1, revision 0 of the beam spectra for TESLEE.

and 2) and consequently a lot of information will be wasted. One solution to this problem is the use of logarithmic bins. This, however, maps the compact region $[0, 1] \times [0, 1]$ to $[-\infty, 0] \times [-\infty, 0]$, which is inconvenient because of the missing lower bounds.

The more appropriate solution is to use two maps

$$\begin{aligned}\phi : [0, 1] &\rightarrow [0, 1] \\ x &\mapsto y = x^{1/\eta}\end{aligned}\tag{13}$$

where $x = x_\gamma$ or $x = 1 - x_{e^\pm}$, and to bin the result equidistantly. If η is chosen properly (cf. (10)), the bin contents will then fall off at the singularity. The fits in tables 5, 6, and 7 have been performed with $\eta = 5$ and the resulting bin contents can be read off from figures 6–9.

Using this procedure for binning the results of the simulations, the popular fitting package MINUIT [15] converges quickly in all cases considered. The resulting parameters are given in tables 5, 6, and 7. Plots of the corresponding distributions have been shown in figures 1 and 2. It is obvious that an *ansatz* like (12) is able to distinguish among the accelerator designs. Thus it can provide a solid basis for physics studies.

In figures 6–9 I give a graphical impression of the quality of the fit, which appears to be as good as one could reasonably expect for a simple *ansatz* like (12). Note that the histograms have non-equidistant bins and that the resulting Jacobians have not been removed. Therefore the bin contents falls off at the singularities, as discussed above.

The errors used for the least-square fit had to be taken from a Monte Carlo (MC) study. **Guinea-Pig** only provides the \sqrt{n} from Poissonian statistics for each bin, but the error accumulation during tracking the particles through phase space is not available. The MC studies shows that the latter error dominates the former, but appears to be reasonably Gaussian. A complete MC study of all parameter sets is computationally expensive (more than a week of processor time on a fast SGI). From an exemplary MC study of a few parameter sets, it appears that the errors can be described reasonably well by rescaling the Poissonian error in each bin with appropriate factors for electrons/positrons and photons and for continuum and delta. This procedure has been adopted.

The $\chi^2/\text{d.o.f.}$'s of the fits are less than $\mathcal{O}(10)$. The simple *ansatz* (12) is therefore very satisfactory. In fact, trying to improve the ad-hoc factorized Beta distributions by the better motivated approximations from [7] or [16], it turns out [17] that (12) provides a significantly better fit of the results of the simulations. The price to pay is that the parameters in (12) have no direct physical interpretation.

5.1.2 Generators

For this version of the parameterizations we need a fast generator of Beta distributions:

$$\beta^{a,b}(x) \propto x^{a-1}(1-x)^{b-1}\tag{14}$$

This problem has been studied extensively and we can use a published algorithm [18] that is guaranteed to be very fast for all a, b such that $0 < a \leq 1 \leq b$, which turns out to be always the case (cf. tables 5, 6, and 7).

5.2 Future Versions

There are two ways in which the parameterizations can be improved:

more complicated functions: the factorized fits can only be improved marginally by adding more positive semi-definite factors to (12). More improvement is possible by using sums of functions, but in this case, the best fits violate the positivity requirement and have to be discarded.

correlations: the parameterization in section 5.1 is factorized. While this is a good approximation, the simulations nevertheless show correlations among x_1 and x_2 . These correlations can be included in a future version.

interpolation: the parameterization in section 5.1 is based on fitting the simulation results by simple functions. Again, this appears to be a good approximation. But such fits can not uncover any fine structure of the distributions. Therefore it will be worthwhile to study interpolations of the simulation results in the future. A proper interpolation of results with statistical errors is however far from trivial: straightforward polynomial or spline interpolations will be oscillatory and violate the positivity requirement. Smoothing algorithms have to be investigated in depth before such a parameterization can be released.

other simulations: besides [5], other simulation codes are invited to contribute their results for inclusion in the `Circe` library.

6 Implementation of circe

```
27a <circe1.f 27a>≡
    c circe1.f -- canonical beam spectra for linear collider physics
    c $Id: circe.nw 67 2002-03-28 17:13:06Z ohl $
    <Copyleft notice 27b>
    <Subroutines 28b>
Uses circe 28b.
```

The following is usually not needed for scientific programs. Nobody is going to hijack such code. But let us include it anyway to spread the gospel of free software:

```
27b <Copyleft notice 27b>≡
    c Copyright (C) 1996-2010 by Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
    c
    c Circe is free software; you can redistribute it and/or modify it
    c under the terms of the GNU General Public License as published by
    c the Free Software Foundation; either version 2, or (at your option)
    c any later version.
    c
    c Circe is distributed in the hope that it will be useful, but
    c WITHOUT ANY WARRANTY; without even the implied warranty of
    c MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
    c GNU General Public License for more details.
```

```

c
c  You should have received a copy of the GNU General Public License
c  along with this program; if not, write to the Free Software
c  Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.

```

Now we can move on to the implementation.

6.1 Symbolic Constants

The file `circe.h` contains symbolic names for various magic constants used by Circe:

```

28a  <circe.h 28a>≡
      c circe.h -- canonical beam spectra for linear collider physics
      c $Id: circe.nw 67 2002-03-28 17:13:06Z ohl $
      <Copyleft notice 27b>
      <Particle codes 10b>
      <Accelerator codes 11c>
      Uses circe 28b.

```

6.2 Distributions

6.2.1 Version 1

We start with a convenience function which dispatches over the valid particle types. The hardest part is of course to avoid typos in such trivial functions ...

```

28b  <Subroutines 28b>≡
      double precision function circe (x1, x2, p1, p2)
      implicit none
      double precision x1, x2
      integer p1, p2
      double precision circee, circeg, circgg
      <Particle codes 10b>
      </circom/ 29b>
      <Initialization check 29g>
      circe = -1.0
      if (abs(p1) .eq. ELECTR) then
        if (abs(p2) .eq. ELECTR) then
          circe = circee (x1, x2)
        elseif (p2 .eq. PHOTON) then
          circe = circeg (x1, x2)
        endif
      elseif (p1 .eq. PHOTON) then
        if (abs(p2) .eq. ELECTR) then
          circe = circeg (x2, x1)
        elseif (p2 .eq. PHOTON) then
          circe = circgg (x1, x2)
        endif
      endif
      end

```

Defines:

`circe`, used in chunks 28b, 10a, 28, 15c, 28, 27a, 28a, 30a, 78a, 82, 97b, and 101e.

Uses `ELECTR` 10b, `PHOTON` 10b, `circee` 37e, `circeg` 38b, and `circgg` 39a.

```
29a  <Subroutines 28b>+≡
      subroutine circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
      implicit none
      double precision xx1m, xx2m, xroots
      double precision beta
      integer xacc, xver, xrev, xchat
      <Accelerator codes 11c>
      </circom/ 29b>
      <Local variables for circes 30c>
      <Initializations for circes 32b>
      if (magic .ne. 1904 06 16) then
        magic = 1904 06 16
        <Initialize /circom/ 30a>
      endif
      <Update /circom/ 30b>
      <formats for circes 34e>
      end
```

Defines:

`circes`, used in chunks 29a, 11b, 15b, 29a, 19a, 29, 78a, and 81b.

Uses `beta` 93a.

```
29b  </circom/ 29b>≡
      <parameter part of /circom/ 29f>
      <8-byte aligned part of /circom/ 29c>
      <4-byte aligned part of /circom/ 29d>
      save /circom/

29c  <8-byte aligned part of /circom/ 29c>≡
      double precision x1m, x2m, roots
      common /circom/ x1m, x2m, roots

29d  <4-byte aligned part of /circom/ 29d>≡
      integer acc, ver, rev, chat
      common /circom/ acc, ver, rev, chat
```

Instead of using fragile block data subroutines, we use a magic number to tag `/circom/` as initialized:

```
29e  <4-byte aligned part of /circom/ 29d>+≡
      integer magic
      common /circom/ magic

29f  <parameter part of /circom/ 29f>≡
      integer MAGIC0
      parameter (MAGIC0 = 1904 06 16)
```

Since negative values are no updated, we can call `circes` with all negative variables to ensure initialization:

```
29g  <Initialization check 29g>≡
      if (magic .ne. MAGIC0) then
        call circes (-1d0, -1d0, -1d0, -1, -1, -1, -1)
```

```

endif
Uses circes 29a.
30a <Initialize /circom/ 30a>≡
    x1m = 0d0
    x2m = 0d0
    roots = 500D0
    acc = TESLA
    ver = 0
    rev = 0
    chat = 1
    if (xchat .ne. 0) then
        call circem ('MESSAGE', 'starting up ...')
        call circem ('MESSAGE',
$            '$Id: circe.nw 67 2002-03-28 17:13:06Z ohl $')
    endif
Uses TESLA 11c and circe 28b.
30b <Update /circom/ 30b>≡
    if ((xchat .ge. 0) .and. (xchat .ne. chat)) then
        chat = xchat
        if (chat .ge. 1) then
            write (msgbuf, 1000) 'chat', chat
1000        format ('updating ', A, ''' to ', I2)
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (chat .ge. 2) then
            write (msgbuf, 1100) 'chat', chat
1100        format ('keeping ', A, ''' at ', I2)
            call circem ('MESSAGE', msgbuf)
        endif
    endif
30c <Local variables for circes 30c>≡
    character*60 msgbuf
30d <Update /circom/ 30b>+≡
    if ((xx1m .ge. 0d0) .and. (xx1m .ne. x1m)) then
        x1m = xx1m
        if (chat .ge. 1) then
            write (msgbuf, 1001) 'x1min', x1m
1001        format ('updating ', A, ''' to ', E12.4)
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (chat .ge. 2) then
            write (msgbuf, 1101) 'x1min', x1m
1101        format ('keeping ', A, ''' at ', E12.4)
            call circem ('MESSAGE', msgbuf)
        endif
    endif
endif

```

```

31a  <Update /circom/ 30b>+≡
      if ((xx2m .ge. 0d0) .and. (xx2m .ne. x2m)) then
        x2m = xx2m
        if (chat .ge. 1) then
          write (msgbuf, 1001) 'x2min', x2m
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1101) 'x2min', x2m
          call circem ('MESSAGE', msgbuf)
        endif
      endif

31b  <Update /circom/ 30b>+≡
      if ((xroots .ge. 0d0) .and.(xroots .ne. roots)) then
        roots = xroots
        if (chat .ge. 1) then
          write (msgbuf, 1002) 'roots', roots
1002      format ('updating ', A, ''' to ', F6.1)
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1102) 'roots', roots
1102      format ('keeping ', A, ''' at ', F6.1)
          call circem ('MESSAGE', msgbuf)
        endif
      endif

31c  <Update /circom/ 30b>+≡
      if ((xacc .ge. 0) .and.(xacc .ne. acc)) then
        if ((xacc .ge. 1) .and. (xacc .le. NACC)) then
          acc = xacc
          if (chat .ge. 1) then
            write (msgbuf, 1003) 'acc', accnam(acc)
1003      format ('updating ', A, ''' to ', A)
            call circem ('MESSAGE', msgbuf)
          endif
        else
          write (msgbuf, 1203) xacc
1203      format ('invalid 'acc': ', I8)
          call circem ('ERROR', msgbuf)
          write (msgbuf, 1103) 'acc', accnam(acc)
1103      format ('keeping ', A, ''' at ', A)
          call circem ('MESSAGE', msgbuf)
        endif
      else
        if (chat .ge. 2) then
          write (msgbuf, 1003) 'acc', accnam(acc)
          call circem ('MESSAGE', msgbuf)
        endif
      endif

```

```

        endif
    endif
    if ((acc .eq. SBNDEE) .or. (acc .eq. TESLEE)
    $    .or. (acc .eq. XBNDEE)) then
        <Warn that no parameter set has been endorsed for e-e- yet 32c>
    endif
    Uses NACC 11c.

32a <Local variables for circes 30c>+≡
    character*6 accnam(NACC)
    Uses NACC 11c.

32b <Initializations for circes 32b>≡
    data accnam(SBAND)  /'SBAND'/
    data accnam(TESLA)  /'TESLA'/
    data accnam(JLCNLC) /'JLCNLC'/
    data accnam(SBNDEE) /'SBNDEE'/
    data accnam(TESLEE) /'TESLEE'/
    data accnam(XBNDEE) /'XBNDEE'/
    data accnam(NLCH)  /'NLC H'/
    Uses SBAND 11c and TESLA 11c.

32c <Warn that no parameter set has been endorsed for e-e- yet 32c>≡
    call circem ('WARNING', '*****')
    call circem ('WARNING', '* The accelerator parameters have *')
    call circem ('WARNING', '* not been endorsed for use in      *')
    call circem ('WARNING', '* an e-e- collider yet!!!          *')
    call circem ('WARNING', '*****')

32d <Update /circom/ 30b>+≡
    if (xver .ge. 0) then
        ver = xver
        if (chat .ge. 1) then
            write (msgbuf, 1000) 'ver', ver
            call circem ('MESSAGE', msgbuf)
        endif
    else
        if (chat .ge. 2) then
            write (msgbuf, 1100) 'ver', ver
            call circem ('MESSAGE', msgbuf)
        endif
    endif

32e <Update /circom/ 30b>+≡
    if ((xrev .ge. 0) .and.(xrev .ne. rev)) then
        rev = xrev
        if (chat .ge. 1) then
            write (msgbuf, 1004) 'rev', rev
            1004    format ('updating ', A, ''' to ', I8)
            call circem ('MESSAGE', msgbuf)
        endif
    else

```

```

        if (chat .ge. 2) then
            write (msgbuf, 1104) 'rev', rev
1104         format ('keeping ', A, ''' at ', I8)
            call circem ('MESSAGE', msgbuf)
        endif
    endif

```

Versions 3 and 4 are identical to version 1, except for TESLA at 800 GeV.

```

33a  <Update /circom/ 30b>+≡
        ver34 = 0
        if ((ver .eq. 1) .or. (ver .eq. 0)) then
            <Update version 1 derived parameters in /circom/ 34a>
            elseif ((ver .eq. 3) .or. (ver .eq. 4)) then
                ver34 = ver
                ver = 1
            <Update version 3 and 4 derived parameters in /circom/ 46c>
            elseif (ver .eq. 5) then
                ver = 1
            <Update version 5 derived parameters in /circom/ 48b>
            elseif (ver .eq. 6) then
                ver = 1
            <Update version 6 derived parameters in /circom/ 50b>
            elseif (ver .eq. 7) then
                ver = 1
            <Update version 7 derived parameters in /circom/ 52b>
            elseif (ver .eq. 8) then
                ver = 1
            <Update version 8 derived parameters in /circom/ 56c>
            elseif (ver .eq. 9) then
                ver = 1
            <Update version 9 derived parameters in /circom/ 58e>
            <else handle invalid versions 33c>

33b  <Local variables for circes 30c>+≡
        integer ver34

33c  <else handle invalid versions 33c>≡
        elseif (ver .eq. 2) then
            <Version 2 has been retired 46b>
            elseif (ver .gt. 9) then
                call circem ('PANIC', 'versions >9 not available yet')
                return
            else
                call circem ('PANIC', 'version must be positive')
                return
            endif

33d  <4-byte aligned part of /circom/ 29d>+≡
        integer e, r, ehi, elo
        common /circom/ e, r, ehi, elo

```

```

34a  <Update version 1 derived parameters in /circom/ 34a>≡
      if (rev .eq. 0) then
        r = 0
      elseif (rev .ge. 1997 04 17) then
        r = 5
      elseif (rev .ge. 1996 09 02) then
        r = 4
      elseif (rev .ge. 1996 07 29) then
        r = 3
      elseif (rev .ge. 1996 07 11) then
        r = 2
      elseif (rev .ge. 1996 04 01) then
        r = 1
      elseif (rev .lt. 1996 04 01) then
        call circem ('ERROR',
$         'no revision of version 1 before 96/04/01 available')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
      endif
      if (chat .ge. 2) then
        write (msgbuf, 2000) rev, r
2000    format ('mapping date ', I8, ' to revision index ', I2)
        call circem ('MESSAGE', msgbuf)
      endif

34b  <Log revision mapping 34b>≡
      if (chat .ge. 2) then
        write (msgbuf, 2000) rev, r
        call circem ('MESSAGE', msgbuf)
      endif

34c  <Update version 1 derived parameters in /circom/ 34a>+≡
      <Map roots to e 34d>

34d  <Map roots to e 34d>≡
      if (roots .eq. 350d0) then
        e = GEV350
      elseif ((roots .ge. 340d0) .and. (roots .le. 370d0)) then
        write (msgbuf, 2001) roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350

34e  <formats for circes 34e>≡
      2001 format ('treating energy ', F6.1, 'GeV as ', F6.1, 'GeV')

34f  <Map roots to e 34d>+≡
      elseif (roots .eq. 500d0) then
        e = GEV500
      elseif ((roots .ge. 480d0) .and. (roots .le. 520d0)) then
        write (msgbuf, 2001) roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500

```

```

elseif (roots .eq. 800d0) then
  e = GEV800
elseif ((roots .ge. 750d0) .and. (roots .le. 850d0)) then
  write (msgbuf, 2001) roots, 800d0
  call circem ('MESSAGE', msgbuf)
  e = GEV800
elseif (roots .eq. 1000d0) then
  e = TEV1
elseif ((roots .ge. 900d0) .and. (roots .le. 1100d0)) then
  write (msgbuf, 2001) roots, 1000d0
  call circem ('MESSAGE', msgbuf)
  e = TEV1
elseif (roots .eq. 1600d0) then
  e = TEV16
elseif ((roots .ge. 1500d0) .and. (roots .le. 1700d0)) then
  write (msgbuf, 2001) roots, 1600d0
  call circem ('MESSAGE', msgbuf)
  e = TEV16
35a  <Map roots to e 34d>+≡
      else
        call circem ('ERROR',
$          'only ROOTS = 350, 500, 800, 1000 and 1600GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
      endif
35b  <Update version 1 derived parameters in /circom/ 34a>+≡
      if (xallum(e,acc,r) .lt. 0d0) then
        write (msgbuf, 2002) roots, accnam(acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
      endif
      <Log energy mapping 35d>
35c  <formats for circes 34e>+≡
      2002 format ('energy ', F6.1, ' not available for ', A6,
$          ' in revision ', I2)
35d  <Log energy mapping 35d>≡
      if (chat .ge. 2) then
        if (e .ge. GEV090) then
          write (msgbuf, 2003) roots, e
          call circem ('MESSAGE', msgbuf)
        else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
          write (msgbuf, 2013) roots, elo, ehi
          call circem ('MESSAGE', msgbuf)
        end if
      endif

```

36a *<formats for circes 34e>+≡*

```
2003 format ('mapping energy ', F6.1, ' to energy index ', I2)
2013 format ('mapping energy ', F6.1, ' to energy indices ',
$           I2, ' and ', I2)
```

The energies 250 GeV, 1.2 TeV and 1.5 TeV were entered late into the game by teh SLAC people.

36b *<Local variables for circes 30c>+≡*

```
integer GEV090, GEV170, GEV350, GEV500, GEV800, TEV1, TEV16
integer GEV250, TEV12, TEV15
parameter (GEV090 = -1, GEV170 = 0, GEV350 = 1, GEV500 = 2,
$          GEV800 = 3, TEV1 = 4, TEV16 = 5,
$          GEV250 = 6, TEV12 = 7, TEV15 = 8)
integer A1NEGY, A1NREV
parameter (A1NEGY = 5, A1NREV = 5)
integer i
```

36c *<8-byte aligned part of /circom/ 29c>+≡*

```
double precision lumi
common /circom/ lumi
double precision a1(0:7)
common /circom/ a1
```

36d *<Update version 1 derived parameters in /circom/ 34a>+≡*

```
lumi = xallum (e,acc,r)
do 10 i = 0, 7
    a1(i) = xa1(i,e,acc,r)
10 continue
```

36e *<Local variables for circes 30c>+≡*

```
real xallum(A1NEGY,NACC,0:A1NREV)
real xa1(0:7,A1NEGY,NACC,0:A1NREV)
```

Uses NACC 11c.

Revision 1. The mother of all revisions.

36f *<Initializations for circes 32b>+≡*

```
data xallum(GEV500,SBAND,1) / 5.212299E+01 /
data (xa1(i,GEV500,SBAND,1),i=0,7) /
$ .39192E+00, .66026E+00, .11828E+02, -.62543E+00,
$ .52292E+00, -.69245E+00, .14983E+02, .65421E+00 /
data xallum(GEV500,TESLA,1) / 6.066178E+01 /
data (xa1(i,GEV500,TESLA,1),i=0,7) /
$ .30196E+00, .12249E+01, .21423E+02, -.57848E+00,
$ .68766E+00, -.69788E+00, .23121E+02, .78399E+00 /
data xallum(GEV500,XBAND,1) / 5.884699E+01 /
data (xa1(i,GEV500,XBAND,1),i=0,7) /
$ .48594E+00, .52435E+00, .83585E+01, -.61347E+00,
$ .30703E+00, -.68804E+00, .84109E+01, .44312E+00 /
```

Uses SBAND 11c, TESLA 11c, and XBAND 11c.

36g *<Initializations for circes 32b>+≡*

```
data xallum(TEV1,SBAND,1) / 1.534650E+02 /
data (xa1(i,TEV1,SBAND,1),i=0,7) /
```

```

$ .24399E+00, .87464E+00, .66751E+01, -.56808E+00,
$ .59295E+00, -.68921E+00, .94232E+01, .83351E+00 /
data xaillum(TEV1, TESLA, 1) / 1.253381E+03 /
data (xa1(i, TEV1, TESLA, 1), i=0, 7) /
$ .39843E+00, .70097E+00, .11602E+02, -.61061E+00,
$ .40737E+00, -.69319E+00, .14800E+02, .51382E+00 /
data xaillum(TEV1, XBAND, 1) / 1.901783E+02 /
data (xa1(i, TEV1, XBAND, 1), i=0, 7) /
$ .32211E+00, .61798E+00, .28298E+01, -.54644E+00,
$ .45674E+00, -.67301E+00, .41703E+01, .74536E+00 /

```

Uses SBAND 11c, TESLA 11c, and XBAND 11c.

Unavailable

```

37a <Initializations for circes 32b>+≡
      data (xaillum(GEV350, i, 1), i=1, NACC) / NACC*-1d0 /
      data (xaillum(GEV800, i, 1), i=1, NACC) / NACC*-1d0 /

```

Uses NACC 11c.

Unavailable as well

```

37b <Initializations for circes 32b>+≡
      data (xaillum(GEV500, i, 1), i=SBNDDEE, NACC) / 4*-1d0 /
      data (xaillum(TEV1, i, 1), i=SBNDDEE, NACC) / 4*-1d0 /

```

Uses NACC 11c.

No 1.6TeV parameters in this revision

```

37c <Initializations for circes 32b>+≡
      data (xaillum(TEV16, i, 1), i=1, NACC) / 7*-1d0 /

```

Uses NACC 11c.

```

37d <Subroutines 28b>+≡
      subroutine circel (l)
      implicit none
      double precision l
      </circom/ 29b>
      l = lumi
      end

```

Defines:

circel, used in chunks 37d and 11a.

```

37e <Subroutines 28b>+≡
      double precision function circee (x1, x2)
      implicit none
      double precision x1, x2
      </circom/ 29b>
      double precision d1, d2
      <Initialization check 29g>
      circee = -1.0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
      <Calculate version 1 of the e+e- distribution 38a>
      <else handle invalid versions 33c>
      end

```

Defines:

circee, used in chunks 12–14, 17a, 28b, and 38a.

The first version of the parametrization is factorized

$$D_{p_1 p_2}^{\alpha 1 \rho}(x_1, x_2, s) = d_{p_1}^{\alpha 1 \rho}(x_1) d_{p_2}^{\alpha 1 \rho}(x_2) \quad (15)$$

where the distributions are

$$d_{e^\pm}^{\alpha 1 \rho}(x) = a_0^{\alpha \rho} \delta(1-x) + a_1^{\alpha \rho} x^{a_2^{\alpha \rho}} (1-x)^{a_3^{\alpha \rho}} \quad (16)$$

$$d_\gamma(x) = a_4^{\alpha \rho} x^{a_5^{\alpha \rho}} (1-x)^{a_6^{\alpha \rho}} \quad (17)$$

```
38a  <Calculate version 1 of the e+e- distribution 38a>≡
      if (x1 .eq. 1d0) then
        d1 = a1(0)
      elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
      elseif (x1 .eq. -1d0) then
        d1 = 1d0 - a1(0)
      else
        d1 = 0d0
      endif
      if (x2 .eq. 1d0) then
        d2 = a1(0)
      elseif (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = a1(1) * x2**a1(2) * (1d0 - x2)**a1(3)
      elseif (x2 .eq. -1d0) then
        d2 = 1d0 - a1(0)
      else
        d2 = 0d0
      endif
      circee = d1 * d2
```

Uses circee 37e.

```
38b  <Subroutines 28b>+≡
      double precision function circeg (x1, x2)
      implicit none
      double precision x1, x2
    </circom/ 29b>
      double precision d1, d2
    <Initialization check 29g>
      circeg = -1.0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
    <Calculate version 1 of the e±γ distribution 38c>
    <else handle invalid versions 33c>
      end
```

Defines:

circeg, used in chunks 12, 28b, and 38c.

```
38c  <Calculate version 1 of the e±γ distribution 38c>≡
      if (x1 .eq. 1d0) then
        d1 = a1(0)
      elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
```

```

elseif (x1 .eq. -1d0) then
  d1 = 1d0 - a1(0)
else
  d1 = 0d0
endif
if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
  d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
elseif (x2 .eq. -1d0) then
  d2 = a1(7)
else
  d2 = 0d0
endif
circceg = d1 * d2

```

Uses circceg 38b.

```

39a  <Subroutines 28b>+≡
      double precision function circgg (x1, x2)
      implicit none
      double precision x1, x2
</circom/ 29b>
      double precision d1, d2
<Initialization check 29g>
      circgg = -1.0
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
<Calculate version 1 of the  $\gamma\gamma$  distribution 39b>
      <else handle invalid versions 33c>
      end

```

Defines:

circgg, used in chunks 12, 28b, 39b, and 71.

```

39b  <Calculate version 1 of the  $\gamma\gamma$  distribution 39b>≡
      if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
        d1 = a1(4) * x1**a1(5) * (1d0 - x1)**a1(6)
      elseif (x1 .eq. -1d0) then
        d1 = a1(7)
      else
        d1 = 0d0
      endif
      if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
        d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
      elseif (x2 .eq. -1d0) then
        d2 = a1(7)
      else
        d2 = 0d0
      endif
      circgg = d1 * d2

```

Uses circgg 39a.

Revision 2. New Tesla parameters, including 350 GeV and 800 GeV.

```

39c  <Initializations for circses 32b>+≡
      data xaillum(GEV500,SBAND,2) /    .31057E+02 /

```

```

data (xa1(i,GEV500,SBAND,2),i=0,7) /
$ .38504E+00, .79723E+00, .14191E+02, -.60456E+00,
$ .53411E+00, -.68873E+00, .15105E+02, .65151E+00 /
data xallum(TEV1,SBAND,2) / .24297E+03 /
data (xa1(i,TEV1,SBAND,2),i=0,7) /
$ .24374E+00, .89466E+00, .70242E+01, -.56754E+00,
$ .60910E+00, -.68682E+00, .96083E+01, .83985E+00 /
data xallum(GEV350,TESLA,2) / .73369E+02 /
data (xa1(i,GEV350,TESLA,2),i=0,7) /
$ .36083E+00, .12819E+01, .37880E+02, -.59492E+00,
$ .69109E+00, -.69379E+00, .40061E+02, .65036E+00 /
data xallum(GEV500,TESLA,2) / .10493E+03 /
data (xa1(i,GEV500,TESLA,2),i=0,7) /
$ .29569E+00, .11854E+01, .21282E+02, -.58553E+00,
$ .71341E+00, -.69279E+00, .24061E+02, .77709E+00 /
data xallum(GEV800,TESLA,2) / .28010E+03 /
data (xa1(i,GEV800,TESLA,2),i=0,7) /
$ .22745E+00, .11265E+01, .10483E+02, -.55711E+00,
$ .69579E+00, -.69068E+00, .13093E+02, .89605E+00 /
data xallum(TEV1,TESLA,2) / .10992E+03 /
data (xa1(i,TEV1,TESLA,2),i=0,7) /
$ .40969E+00, .66105E+00, .11972E+02, -.62041E+00,
$ .40463E+00, -.69354E+00, .14669E+02, .51281E+00 /
data xallum(GEV500,XBAND,2) / .35689E+02 /
data (xa1(i,GEV500,XBAND,2),i=0,7) /
$ .48960E+00, .46815E+00, .75249E+01, -.62769E+00,
$ .30341E+00, -.68754E+00, .85545E+01, .43453E+00 /
data xallum(TEV1,XBAND,2) / .11724E+03 /
data (xa1(i,TEV1,XBAND,2),i=0,7) /
$ .31939E+00, .62415E+00, .30763E+01, -.55314E+00,
$ .45634E+00, -.67089E+00, .41529E+01, .73807E+00 /

```

Uses SBAND 11c, TESLA 11c, and XBAND 11c.

Unavailable

40a $\langle \text{Initializations for circles 32b} \rangle + \equiv$

```

data xallum(GEV350,SBAND,2) / -1d0 /
data xallum(GEV350,XBAND,2) / -1d0 /
data xallum(GEV800,SBAND,2) / -1d0 /
data xallum(GEV800,XBAND,2) / -1d0 /

```

Uses SBAND 11c and XBAND 11c.

Unavailable as well

40b $\langle \text{Initializations for circles 32b} \rangle + \equiv$

```

data (xallum(GEV350,i,2),i=SBNDDEE,NACC) / 4*-1d0 /
data (xallum(GEV500,i,2),i=SBNDDEE,NACC) / 4*-1d0 /
data (xallum(GEV800,i,2),i=SBNDDEE,NACC) / 4*-1d0 /
data (xallum(TEV1,i,2),i=SBNDDEE,NACC) / 4*-1d0 /

```

Uses NACC 11c.

No 1.6TeV parameters in this revision

40c $\langle \text{Initializations for circles 32b} \rangle + \equiv$

```
data (xa1lum(TEV16,i,2),i=1,NACC) / 7*-1d0 /
```

Uses NACC 11c.

Revision 3. Features:

- improved error estimates.
- cleaner fitting procedure, including delta function pieces.

41a *<Initializations for circes 32b>+≡*

```
data xa1lum(GEV500,SBAND, 3) / .31469E+02 /
data (xa1(i,GEV500,SBAND, 3),i=0,7) /
$ .38299E+00, .72035E+00, .12618E+02, -.61611E+00,
$ .51971E+00, -.68960E+00, .15066E+02, .63784E+00 /
data xa1lum(TEV1, SBAND, 3) / .24566E+03 /
data (xa1(i,TEV1, SBAND, 3),i=0,7) /
$ .24013E+00, .95763E+00, .69085E+01, -.55151E+00,
$ .59497E+00, -.68622E+00, .94494E+01, .82158E+00 /
data xa1lum(GEV350,TESLA, 3) / .74700E+02 /
data (xa1(i,GEV350,TESLA, 3),i=0,7) /
$ .34689E+00, .12484E+01, .33720E+02, -.59523E+00,
$ .66266E+00, -.69524E+00, .38488E+02, .63775E+00 /
data xa1lum(GEV500,TESLA, 3) / .10608E+03 /
data (xa1(i,GEV500,TESLA, 3),i=0,7) /
$ .28282E+00, .11700E+01, .19258E+02, -.58390E+00,
$ .68777E+00, -.69402E+00, .23638E+02, .75929E+00 /
data xa1lum(GEV800,TESLA, 3) / .28911E+03 /
data (xa1(i,GEV800,TESLA, 3),i=0,7) /
$ .21018E+00, .12039E+01, .96763E+01, -.54024E+00,
$ .67220E+00, -.69083E+00, .12733E+02, .87355E+00 /
data xa1lum(TEV1, TESLA, 3) / .10936E+03 /
data (xa1(i,TEV1, TESLA, 3),i=0,7) /
$ .41040E+00, .68099E+00, .11610E+02, -.61237E+00,
$ .40155E+00, -.69073E+00, .14698E+02, .49989E+00 /
data xa1lum(GEV500,XBAND, 3) / .36145E+02 /
data (xa1(i,GEV500,XBAND, 3),i=0,7) /
$ .51285E+00, .45812E+00, .75135E+01, -.62247E+00,
$ .30444E+00, -.68530E+00, .85519E+01, .43062E+00 /
data xa1lum(TEV1, XBAND, 3) / .11799E+03 /
data (xa1(i,TEV1, XBAND, 3),i=0,7) /
$ .31241E+00, .61241E+00, .29938E+01, -.55848E+00,
$ .44801E+00, -.67116E+00, .41119E+01, .72753E+00 /
```

Uses SBAND 11c, TESLA 11c, and XBAND 11c.

Still unavailable

41b *<Initializations for circes 32b>+≡*

```
data xa1lum(GEV350,SBAND,3) / -1d0 /
data xa1lum(GEV350,XBAND,3) / -1d0 /
data xa1lum(GEV800,SBAND,3) / -1d0 /
data xa1lum(GEV800,XBAND,3) / -1d0 /
```

Uses SBAND 11c and XBAND 11c.

Unavailable as well

```
42a <Initializations for circes 32b>+≡
      data (xaillum(GEV350,i,3),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xaillum(GEV500,i,3),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xaillum(GEV800,i,3),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xaillum(TEV1,i,3),i=SBNDDEE,NACC) / 4*-1d0 /
```

Uses NACC 11c.

No 1.6TeV parameters in this revision

```
42b <Initializations for circes 32b>+≡
      data (xaillum(TEV16,i,3),i=1,NACC) / 7*-1d0 /
```

Uses NACC 11c.

Revision 4. Features:

- a bug in Guinea-Pig's synchrotron radiation spectrum has been fixed.

```
42c <Initializations for circes 32b>+≡
      data xaillum(GEV500,SBAND, 4) / .31528E+02 /
      data (xa1(i,GEV500,SBAND, 4),i=0,7) /
$ .38169E+00, .73949E+00, .12543E+02, -.61112E+00,
$ .51256E+00, -.69009E+00, .14892E+02, .63314E+00 /
      data xaillum(TEV1, SBAND, 4) / .24613E+03 /
      data (xa1(i,TEV1, SBAND, 4),i=0,7) /
$ .24256E+00, .94117E+00, .66775E+01, -.55160E+00,
$ .57484E+00, -.68891E+00, .92271E+01, .81162E+00 /
      data xaillum(GEV350,TESLA, 4) / .74549E+02 /
      data (xa1(i,GEV350,TESLA, 4),i=0,7) /
$ .34120E+00, .12230E+01, .32932E+02, -.59850E+00,
$ .65947E+00, -.69574E+00, .38116E+02, .63879E+00 /
      data xaillum(GEV500,TESLA, 4) / .10668E+03 /
      data (xa1(i,GEV500,TESLA, 4),i=0,7) /
$ .28082E+00, .11074E+01, .18399E+02, -.59118E+00,
$ .68880E+00, -.69375E+00, .23463E+02, .76073E+00 /
      data xaillum(GEV800,TESLA, 4) / .29006E+03 /
      data (xa1(i,GEV800,TESLA, 4),i=0,7) /
$ .21272E+00, .11443E+01, .92564E+01, -.54657E+00,
$ .66799E+00, -.69137E+00, .12498E+02, .87571E+00 /
      data xaillum(TEV1, TESLA, 4) / .11009E+03 /
      data (xa1(i,TEV1, TESLA, 4),i=0,7) /
$ .41058E+00, .64745E+00, .11271E+02, -.61996E+00,
$ .39801E+00, -.69150E+00, .14560E+02, .49924E+00 /
      data xaillum(GEV500,XBAND, 4) / .36179E+02 /
      data (xa1(i,GEV500,XBAND, 4),i=0,7) /
$ .51155E+00, .43313E+00, .70446E+01, -.63003E+00,
$ .29449E+00, -.68747E+00, .83489E+01, .42458E+00 /
      data xaillum(TEV1, XBAND, 4) / .11748E+03 /
      data (xa1(i,TEV1, XBAND, 4),i=0,7) /
$ .32917E+00, .54322E+00, .28493E+01, -.57959E+00,
$ .39266E+00, -.68217E+00, .38475E+01, .68478E+00 /
```

Uses SBAND 11c, TESLA 11c, and XBAND 11c.

Still unavailable

```
43a <Initializations for circes 32b>+≡
      data xallum(GEV350,SBAND,4) / -1d0 /
      data xallum(GEV350,XBAND,4) / -1d0 /
      data xallum(GEV800,SBAND,4) / -1d0 /
      data xallum(GEV800,XBAND,4) / -1d0 /
```

Uses SBAND 11c and XBAND 11c.

Unavailable as well

```
43b <Initializations for circes 32b>+≡
      data (xallum(GEV350,i,4),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(GEV500,i,4),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(GEV800,i,4),i=SBNDDEE,NACC) / 4*-1d0 /
      data (xallum(TEV1,i,4),i=SBNDDEE,NACC) / 4*-1d0 /
```

Uses NACC 11c.

No 1.6TeV parameters in this revision

```
43c <Initializations for circes 32b>+≡
      data (xallum(TEV16,i,4),i=1,NACC) / 7*-1d0 /
```

Uses NACC 11c.

Revision 5. Features:

- a bug in Guinea-Pig has been fixed.
- updated parameter sets

```
43d <Initializations for circes 32b>+≡
      data xallum(GEV350,SBAND, 5) / 0.21897E+02 /
      data (xa1(i,GEV350,SBAND, 5),i=0,7) /
$    0.57183E+00, 0.53877E+00, 0.19422E+02, -0.63064E+00,
$    0.49112E+00, -0.69109E+00, 0.24331E+02, 0.52718E+00 /
      data xallum(GEV500,SBAND, 5) / 0.31383E+02 /
      data (xa1(i,GEV500,SBAND, 5),i=0,7) /
$    0.51882E+00, 0.49915E+00, 0.11153E+02, -0.63017E+00,
$    0.50217E+00, -0.69113E+00, 0.14935E+02, 0.62373E+00 /
      data xallum(GEV800,SBAND, 5) / 0.95091E+02 /
      data (xa1(i,GEV800,SBAND, 5),i=0,7) /
$    0.47137E+00, 0.46150E+00, 0.56562E+01, -0.61758E+00,
$    0.46863E+00, -0.68897E+00, 0.85876E+01, 0.67577E+00 /
      data xallum(TEV1,SBAND, 5) / 0.11900E+03 /
      data (xa1(i,TEV1,SBAND, 5),i=0,7) /
$    0.43956E+00, 0.45471E+00, 0.42170E+01, -0.61180E+00,
$    0.48711E+00, -0.68696E+00, 0.67145E+01, 0.74551E+00 /
      data xallum(TEV16,SBAND, 5) / 0.11900E+03 /
      data (xa1(i,TEV16,SBAND, 5),i=0,7) /
$    0.43956E+00, 0.45471E+00, 0.42170E+01, -0.61180E+00,
$    0.48711E+00, -0.68696E+00, 0.67145E+01, 0.74551E+00 /
      data xallum(GEV350,TESLA, 5) / 0.97452E+02 /
      data (xa1(i,GEV350,TESLA, 5),i=0,7) /
$    0.39071E+00, 0.84996E+00, 0.17614E+02, -0.60609E+00,
$    0.73920E+00, -0.69490E+00, 0.28940E+02, 0.77286E+00 /
```

```

data xallum(GEV500, TESLA, 5) / 0.10625E+03 /
data (xa1(i, GEV500, TESLA, 5), i=0, 7) /
$ 0.42770E+00, 0.71457E+00, 0.15284E+02, -0.61664E+00,
$ 0.68166E+00, -0.69208E+00, 0.24165E+02, 0.73806E+00 /
data xallum(GEV800, TESLA, 5) / 0.17086E+03 /
data (xa1(i, GEV800, TESLA, 5), i=0, 7) /
$ 0.36025E+00, 0.69118E+00, 0.76221E+01, -0.59440E+00,
$ 0.71269E+00, -0.69077E+00, 0.13117E+02, 0.91780E+00 /
data xallum(TEV1, TESLA, 5) / 0.21433E+03 /
data (xa1(i, TEV1, TESLA, 5), i=0, 7) /
$ 0.33145E+00, 0.67075E+00, 0.55438E+01, -0.58468E+00,
$ 0.72503E+00, -0.69084E+00, 0.99992E+01, 0.10112E+01 /
data xallum(TEV16, TESLA, 5) / 0.34086E+03 /
data (xa1(i, TEV16, TESLA, 5), i=0, 7) /
$ 0.49058E+00, 0.42609E+00, 0.50550E+01, -0.61867E+00,
$ 0.39225E+00, -0.68916E+00, 0.75514E+01, 0.58754E+00 /
data xallum(GEV350, XBAND, 5) / 0.31901E+02 /
data (xa1(i, GEV350, XBAND, 5), i=0, 7) /
$ 0.65349E+00, 0.31752E+00, 0.94342E+01, -0.64291E+00,
$ 0.30364E+00, -0.68989E+00, 0.11446E+02, 0.40486E+00 /
data xallum(GEV500, XBAND, 5) / 0.36386E+02 /
data (xa1(i, GEV500, XBAND, 5), i=0, 7) /
$ 0.65132E+00, 0.28728E+00, 0.69853E+01, -0.64440E+00,
$ 0.28736E+00, -0.68758E+00, 0.83227E+01, 0.41492E+00 /
data xallum(GEV800, XBAND, 5) / 0.10854E+03 /
data (xa1(i, GEV800, XBAND, 5), i=0, 7) /
$ 0.49478E+00, 0.36221E+00, 0.30116E+01, -0.61548E+00,
$ 0.39890E+00, -0.68418E+00, 0.45183E+01, 0.67243E+00 /
data xallum(TEV1, XBAND, 5) / 0.11899E+03 /
data (xa1(i, TEV1, XBAND, 5), i=0, 7) /
$ 0.49992E+00, 0.34299E+00, 0.26184E+01, -0.61584E+00,
$ 0.38450E+00, -0.68342E+00, 0.38589E+01, 0.67408E+00 /
data xallum(TEV16, XBAND, 5) / 0.13675E+03 /
data (xa1(i, TEV16, XBAND, 5), i=0, 7) /
$ 0.50580E+00, 0.30760E+00, 0.18339E+01, -0.61421E+00,
$ 0.35233E+00, -0.68315E+00, 0.26708E+01, 0.67918E+00 /

```

Uses SBAND 11c, TESLA 11c, and XBAND 11c.

Revision 0. Features:

- e^-e^- mode

44 *(Initializations for circes 32b)+≡*

```

data xallum(GEV500, SBNDDEE, 0) / .92914E+01 /
data (xa1(i, GEV500, SBNDDEE, 0), i=0, 7) /
$ .34866E+00, .78710E+00, .10304E+02, -.59464E+00,
$ .40234E+00, -.69741E+00, .20645E+02, .47274E+00 /
data xallum(TEV1, SBNDDEE, 0) / .45586E+02 /
data (xa1(i, TEV1, SBNDDEE, 0), i=0, 7) /
$ .21084E+00, .99168E+00, .54407E+01, -.52851E+00,
$ .47493E+00, -.69595E+00, .12480E+02, .64027E+00 /

```

```

data xallum(GEV350, TESLEE, 0) / .15175E+02 /
data (xa1(i, GEV350, TESLEE, 0), i=0, 7) /
$ .33093E+00, .11137E+01, .25275E+02, -.59942E+00,
$ .49623E+00, -.70403E+00, .60188E+02, .44637E+00 /
data xallum(GEV500, TESLEE, 0) / .21622E+02 /
data (xa1(i, GEV500, TESLEE, 0), i=0, 7) /
$ .27175E+00, .10697E+01, .14858E+02, -.58418E+00,
$ .50824E+00, -.70387E+00, .36129E+02, .53002E+00 /
data xallum(GEV800, TESLEE, 0) / .43979E+02 /
data (xa1(i, GEV800, TESLEE, 0), i=0, 7) /
$ .22994E+00, .10129E+01, .81905E+01, -.55751E+00,
$ .46551E+00, -.70461E+00, .19394E+02, .58387E+00 /
data xallum(TEV1, TESLEE, 0) / .25465E+02 /
data (xa1(i, TEV1, TESLEE, 0), i=0, 7) /
$ .37294E+00, .67522E+00, .87504E+01, -.60576E+00,
$ .35095E+00, -.69821E+00, .18526E+02, .42784E+00 /
data xallum(GEV500, XBNDDEE, 0) / .13970E+02 /
data (xa1(i, GEV500, XBNDDEE, 0), i=0, 7) /
$ .47296E+00, .46800E+00, .58897E+01, -.61689E+00,
$ .27181E+00, -.68923E+00, .10087E+02, .37462E+00 /
data xallum(TEV1, XBNDDEE, 0) / .41056E+02 /
data (xa1(i, TEV1, XBNDDEE, 0), i=0, 7) /
$ .27965E+00, .74816E+00, .27415E+01, -.50491E+00,
$ .38320E+00, -.67945E+00, .47506E+01, .62218E+00 /

```

Still unavailable

45a *(Initializations for circes 32b)+≡*

```

data xallum(GEV350, SBNDDEE, 0) / -1d0 /
data xallum(GEV350, XBNDDEE, 0) / -1d0 /
data xallum(GEV800, SBNDDEE, 0) / -1d0 /
data xallum(GEV800, XBNDDEE, 0) / -1d0 /

```

45b *(Initializations for circes 32b)+≡*

```

data xallum(GEV500, SBAND, 0) / .31528E+02 /
data (xa1(i, GEV500, SBAND, 0), i=0, 7) /
$ .38169E+00, .73949E+00, .12543E+02, -.61112E+00,
$ .51256E+00, -.69009E+00, .14892E+02, .63314E+00 /
data xallum(TEV1, SBAND, 0) / .24613E+03 /
data (xa1(i, TEV1, SBAND, 0), i=0, 7) /
$ .24256E+00, .94117E+00, .66775E+01, -.55160E+00,
$ .57484E+00, -.68891E+00, .92271E+01, .81162E+00 /
data xallum(GEV350, TESLA, 0) / .74549E+02 /
data (xa1(i, GEV350, TESLA, 0), i=0, 7) /
$ .34120E+00, .12230E+01, .32932E+02, -.59850E+00,
$ .65947E+00, -.69574E+00, .38116E+02, .63879E+00 /
data xallum(GEV500, TESLA, 0) / .10668E+03 /
data (xa1(i, GEV500, TESLA, 0), i=0, 7) /
$ .28082E+00, .11074E+01, .18399E+02, -.59118E+00,
$ .68880E+00, -.69375E+00, .23463E+02, .76073E+00 /
data xallum(GEV800, TESLA, 0) / .29006E+03 /
data (xa1(i, GEV800, TESLA, 0), i=0, 7) /

```

```

$ .21272E+00, .11443E+01, .92564E+01, -.54657E+00,
$ .66799E+00, -.69137E+00, .12498E+02, .87571E+00 /
data xallum(TEV1, TESLA, 0) / .11009E+03 /
data (xa1(i,TEV1, TESLA, 0),i=0,7) /
$ .41058E+00, .64745E+00, .11271E+02, -.61996E+00,
$ .39801E+00, -.69150E+00, .14560E+02, .49924E+00 /
data xallum(GEV500,XBAND, 0) / .36179E+02 /
data (xa1(i,GEV500,XBAND, 0),i=0,7) /
$ .51155E+00, .43313E+00, .70446E+01, -.63003E+00,
$ .29449E+00, -.68747E+00, .83489E+01, .42458E+00 /
data xallum(TEV1, XBAND, 0) / .11748E+03 /
data (xa1(i,TEV1, XBAND, 0),i=0,7) /
$ .32917E+00, .54322E+00, .28493E+01, -.57959E+00,
$ .39266E+00, -.68217E+00, .38475E+01, .68478E+00 /

```

Uses SBAND 11c, TESLA 11c, and XBAND 11c.

Still unavailable

46a *⟨Initializations for circes 32b⟩*+≡

```

data xallum(GEV350,SBAND,0) / -1d0 /
data xallum(GEV350,XBAND,0) / -1d0 /
data xallum(GEV800,SBAND,0) / -1d0 /
data xallum(GEV800,XBAND,0) / -1d0 /

```

Uses SBAND 11c and XBAND 11c.

6.2.2 Version 2

46b *⟨Version 2 has been retired 46b⟩*≡

```

call circem ('PANIC', '*****')
call circem ('PANIC', '* version 2 has been retired, *')
call circem ('PANIC', '* please use version 1 instead! *')
call circem ('PANIC', '*****')
return

```

6.2.3 Versions 3 and 4

46c *⟨Update version 3 and 4 derived parameters in /circom/ 46c⟩*≡

```

if (rev .eq. 0) then
  r = 0
elseif (rev .ge. 1997 04 17) then
  r = 5
  if (ver34 .eq. 3) then
    call circem ('WARNING', 'version 3 retired after 97/04/17')
    call circem ('MESSAGE', 'falling back to version 4')
  endif
elseif (rev .ge. 1996 10 22) then
  r = ver34
  if ((roots .ne. 800d0) .or. (acc .ne. TESLA)) then
    call circem ('ERROR', 'versions 3 and 4 before 97/04/17')
    call circem ('ERROR', 'apply to TESLA at 800 GeV only')
    call circem ('MESSAGE', 'falling back to TESLA at 800GeV')
  endif
endif

```

```

        acc = TESLA
        e = GEV800
    endif
    elseif (rev .lt. 1996 10 22) then
        call circem ('ERROR',
$         'no revision of versions 3 and 4 available before 96/10/22')
        call circem ('MESSAGE', 'falling back to default')
        r = 5
    endif
    <Log revision mapping 34b>
    Uses TESLA 11c.
47a <Update version 3 and 4 derived parameters in /circom/ 46c>+≡
    <Map roots to e 34d>
        if (xa3lum(e,acc,r) .lt. 0d0) then
            write (msgbuf, 2002) roots, accnam(acc), r
            call circem ('ERROR', msgbuf)
            call circem ('MESSAGE', 'falling back to 500GeV')
            e = GEV500
        endif
    <Log energy mapping 35d>
47b <Local variables for circes 30c>+≡
        integer A3NEGY, A3NREV
        parameter (A3NEGY = 5, A3NREV = 5)
47c <Update version 3 and 4 derived parameters in /circom/ 46c>+≡
        lumi = xa3lum (e,acc,r)
        do 20 i = 0, 7
            a1(i) = xa3(i,e,acc,r)
        20 continue
47d <Local variables for circes 30c>+≡
        real xa3lum(A3NEGY,NACC,0:A3NREV)
        real xa3(0:7,A3NEGY,NACC,0:A3NREV)
    Uses NACC 11c.
Revisions 3 & 4. The mother of all revisions.
47e <Initializations for circes 32b>+≡
        data xa3lum(GEV800,TESLA, 3) / .17196E+03 /
        data (xa3(i,GEV800,TESLA, 3),i=0,7) /
$         .21633E+00, .11333E+01, .95928E+01, -.55095E+00,
$         .73044E+00, -.69101E+00, .12868E+02, .94737E+00 /
        data xa3lum(GEV800,TESLA, 4) / .16408E+03 /
        data (xa3(i,GEV800,TESLA, 4),i=0,7) /
$         .41828E+00, .72418E+00, .14137E+02, -.61189E+00,
$         .36697E+00, -.69205E+00, .17713E+02, .43583E+00 /
    Uses TESLA 11c.
Revision 5.
47f <Initializations for circes 32b>+≡
        data xa3lum(GEV350,TESLA, 5) / 0.66447E+02 /
        data (xa3(i,GEV350,TESLA, 5),i=0,7) /

```

```

$ 0.69418E+00, 0.50553E+00, 0.48430E+02, -0.63911E+00,
$ 0.34074E+00, -0.69533E+00, 0.55502E+02, 0.29397E+00 /
data xa3lum(GEV500, TESLA, 5) / 0.95241E+02 /
data (xa3(i, GEV500, TESLA, 5), i=0, 7) /
$ 0.64882E+00, 0.45462E+00, 0.27103E+02, -0.64535E+00,
$ 0.35101E+00, -0.69467E+00, 0.33658E+02, 0.35024E+00 /
data xa3lum(GEV800, TESLA, 5) / 0.16974E+03 /
data (xa3(i, GEV800, TESLA, 5), i=0, 7) /
$ 0.58706E+00, 0.43771E+00, 0.13422E+02, -0.63804E+00,
$ 0.35541E+00, -0.69467E+00, 0.17528E+02, 0.43051E+00 /
data xa3lum(TEV1, TESLA, 5) / 0.21222E+03 /
data (xa3(i, TEV1, TESLA, 5), i=0, 7) /
$ 0.55525E+00, 0.42577E+00, 0.96341E+01, -0.63587E+00,
$ 0.36448E+00, -0.69365E+00, 0.13161E+02, 0.47715E+00 /
data xa3lum(TEV16, TESLA, 5) / 0.34086E+03 /
data (xa3(i, TEV16, TESLA, 5), i=0, 7) /
$ 0.49058E+00, 0.42609E+00, 0.50550E+01, -0.61867E+00,
$ 0.39225E+00, -0.68916E+00, 0.75514E+01, 0.58754E+00 /

```

Uses TESLA 11c.

Revision 0. Currently identical to revision 5.

48a *<Initializations for circes 32b>+≡*

```

data xa3lum(GEV350, TESLA, 0) / 0.66447E+02 /
data (xa3(i, GEV350, TESLA, 0), i=0, 7) /
$ 0.69418E+00, 0.50553E+00, 0.48430E+02, -0.63911E+00,
$ 0.34074E+00, -0.69533E+00, 0.55502E+02, 0.29397E+00 /
data xa3lum(GEV500, TESLA, 0) / 0.95241E+02 /
data (xa3(i, GEV500, TESLA, 0), i=0, 7) /
$ 0.64882E+00, 0.45462E+00, 0.27103E+02, -0.64535E+00,
$ 0.35101E+00, -0.69467E+00, 0.33658E+02, 0.35024E+00 /
data xa3lum(GEV800, TESLA, 0) / 0.16974E+03 /
data (xa3(i, GEV800, TESLA, 0), i=0, 7) /
$ 0.58706E+00, 0.43771E+00, 0.13422E+02, -0.63804E+00,
$ 0.35541E+00, -0.69467E+00, 0.17528E+02, 0.43051E+00 /
data xa3lum(TEV1, TESLA, 0) / 0.21222E+03 /
data (xa3(i, TEV1, TESLA, 0), i=0, 7) /
$ 0.55525E+00, 0.42577E+00, 0.96341E+01, -0.63587E+00,
$ 0.36448E+00, -0.69365E+00, 0.13161E+02, 0.47715E+00 /
data xa3lum(TEV16, TESLA, 0) / 0.34086E+03 /
data (xa3(i, TEV16, TESLA, 0), i=0, 7) /
$ 0.49058E+00, 0.42609E+00, 0.50550E+01, -0.61867E+00,
$ 0.39225E+00, -0.68916E+00, 0.75514E+01, 0.58754E+00 /

```

Uses TESLA 11c.

6.2.4 Version 5

48b *<Update version 5 derived parameters in /circom/ 48b>≡*

```

if (rev .eq. 0) then
  r = 0
elseif (rev .ge. 1998 05 05) then

```

```

        r = 1
    elseif (rev .lt. 1998 05 05) then
        call circem ('ERROR',
$      'no revision of version 5 available before 98/05/05')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 34b>
49a <Update version 5 derived parameters in /circom/ 48b>+≡
        if (acc .ne. TESLA) then
            call circem ('ERROR', 'versions 5 applies to TESLA only')
            acc = TESLA
        end if
    <Map roots to e 34d>
        if (xa5lum(e,acc,r) .lt. 0d0) then
            write (msgbuf, 2002) roots, accnam(acc), r
            call circem ('ERROR', msgbuf)
            call circem ('MESSAGE', 'falling back to 500GeV')
            e = GEV500
        endif
    <Log energy mapping 35d>
    Uses TESLA 11c.
49b <Local variables for circes 30c>+≡
        integer A5NEGY, A5NREV
        parameter (A5NEGY = 5, A5NREV = 1)
49c <Update version 5 derived parameters in /circom/ 48b>+≡
        lumi = xa5lum (e,acc,r)
        do 30 i = 0, 7
            a1(i) = xa5(i,e,acc,r)
        30 continue
49d <Local variables for circes 30c>+≡
        real xa5lum(A5NEGY,NACC,0:A5NREV)
        real xa5(0:7,A5NEGY,NACC,0:A5NREV)
    Uses NACC 11c.
Revision 1. The mother of all revisions. Note that  $3.3980 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.4099 \cdot 10^{34} \text{ m}^{-2} \cdot 2820.5 \text{ s}^{-1}$  and  $3.5936 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1} = 2.6619 \cdot 10^{34} \text{ m}^{-2} \cdot 4500.3 \text{ s}^{-1}$ . This unit conversion is missing in all earlier versions, unfortunately.
49e <Initializations for circes 32b>+≡
        data xa5lum(GEV350,TESLA, 1) / -1.0 /
        data xa5lum(GEV500,TESLA, 1) / 0.33980E+03 /
        data (xa5(i,GEV500,TESLA, 1),i=0,7) /
$      0.49808E+00, 0.54613E+00, 0.12287E+02, -0.62756E+00,
$      0.42817E+00, -0.69120E+00, 0.17067E+02, 0.51143E+00 /
        data xa5lum(GEV800,TESLA, 1) / 0.35936E+03 /
        data (xa5(i,GEV800,TESLA, 1),i=0,7) /
$      0.58751E+00, 0.43128E+00, 0.13324E+02, -0.64006E+00,
$      0.30682E+00, -0.69235E+00, 0.16815E+02, 0.37078E+00 /

```

```

data xa5lum(TEV1, TESLA, 1) / -1.0 /
data xa5lum(TEV16, TESLA, 1) / -1.0 /

```

Uses TESLA 11c.

Revision 0. Currently identical to revision 1.

```

50a <Initializations for circes 32b>+≡
      data xa5lum(GEV350,TESLA, 0) / -1.0 /
      data xa5lum(GEV500,TESLA, 0) / 0.33980E+03 /
      data (xa5(i,GEV500,TESLA, 0),i=0,7) /
$    0.49808E+00, 0.54613E+00, 0.12287E+02, -0.62756E+00,
$    0.42817E+00, -0.69120E+00, 0.17067E+02, 0.51143E+00 /
      data xa5lum(GEV800,TESLA, 0) / 0.35936E+03 /
      data (xa5(i,GEV800,TESLA, 0),i=0,7) /
$    0.58751E+00, 0.43128E+00, 0.13324E+02, -0.64006E+00,
$    0.30682E+00, -0.69235E+00, 0.16815E+02, 0.37078E+00 /
      data xa5lum(TEV1, TESLA, 0) / -1.0 /
      data xa5lum(TEV16, TESLA, 0) / -1.0 /

```

Uses TESLA 11c.

6.2.5 Version 6

```

50b <Update version 6 derived parameters in /circom/ 50b>≡
      if (rev .eq. 0) then
        r = 0
      elseif (rev .ge. 1999 04 15) then
        r = 1
      elseif (rev .lt. 1999 04 15) then
        call circem ('ERROR',
$      'no revision of version 6 available before 1999/04/15')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
      endif
      <Log revision mapping 34b>

50c <Update version 6 derived parameters in /circom/ 50b>+≡
      if (acc .ne. TESLA) then
        call circem ('ERROR', 'versions 6 applies to TESLA only')
        acc = TESLA
      end if
      <Map roots to e at low energies 50d>
      if (xa6lum(e,acc,r) .lt. 0d0) then
        write (msgbuf, 2002) roots, accnam(acc), r
        call circem ('ERROR', msgbuf)
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
      endif
      <Log energy mapping 35d>

      Uses TESLA 11c.

50d <Map roots to e at low energies 50d>≡
      if (roots .eq. 90d0) then

```

```

        e = GEV090
    elseif ((roots .ge. 85d0) .and. (roots .le. 95d0)) then
        write (msgbuf, 2001) roots, 90d0
        call circem ('MESSAGE', msgbuf)
        e = GEV090
    elseif (roots .eq. 170d0) then
        e = GEV170
    elseif ((roots .ge. 160d0) .and. (roots .le. 180d0)) then
        write (msgbuf, 2001) roots, 170d0
        call circem ('MESSAGE', msgbuf)
        e = GEV170
    elseif (roots .eq. 350d0) then
        e = GEV350
    elseif ((roots .ge. 340d0) .and. (roots .le. 370d0)) then
        write (msgbuf, 2001) roots, 350d0
        call circem ('MESSAGE', msgbuf)
        e = GEV350
    elseif (roots .eq. 500d0) then
        e = GEV500
    elseif ((roots .ge. 480d0) .and. (roots .le. 520d0)) then
        write (msgbuf, 2001) roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    else
        call circem ('ERROR',
$           'only ROOTS = 90, 170, 350, and 500GeV available')
        call circem ('MESSAGE', 'falling back to 500GeV')
        e = GEV500
    endif

51a  <Local variables for circes 30c>+≡
        integer A6NEGY, A6NREV
        parameter (A6NEGY = 2, A6NREV = 1)

51b  <Update version 6 derived parameters in /circom/ 50b>+≡
        lumi = xa6lum (e,acc,r)
        do 40 i = 0, 7
            a1(i) = xa6(i,e,acc,r)
        40 continue

51c  <Local variables for circes 30c>+≡
        real xa6lum(GEV090:A6NEGY,NACC,0:A6NREV)
        real xa6(0:7,GEV090:A6NEGY,NACC,0:A6NREV)

Uses NACC 11c.

Revision 1. The mother of all revisions.

51d  <Initializations for circes 32b>+≡
        data xa6lum(GEV090,TESLA, 1) / 0.62408E+02 /
        data (xa6(i,GEV090,TESLA, 1),i=0,7) /
$      0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$      0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
        data xa6lum(GEV170,TESLA, 1) / 0.11532E+02 /

```

```

data (xa6(i,GEV170,TESLA, 1),i=0,7) /
$ 0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$ 0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
data xa6lum(GEV350,TESLA, 1) / 0.24641E+03 /
data (xa6(i,GEV350,TESLA, 1),i=0,7) /
$ 0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$ 0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /
data xa6lum(GEV500,TESLA, 1) / 0.30340E+03 /
data (xa6(i,GEV500,TESLA, 1),i=0,7) /
$ 0.52744E+00, 0.52573E+00, 0.13895E+02, -0.63145E+00,
$ 0.40824E+00, -0.69150E+00, 0.18645E+02, 0.47585E+00 /

```

Uses TESLA 11c.

Revision 0. Currently identical to revision 1.

```

52a <Initializations for circes 32b>+≡
data xa6lum(GEV090,TESLA, 0) / 0.62408E+02 /
data (xa6(i,GEV090,TESLA, 0),i=0,7) /
$ 0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$ 0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
data xa6lum(GEV170,TESLA, 0) / 0.11532E+02 /
data (xa6(i,GEV170,TESLA, 0),i=0,7) /
$ 0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$ 0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
data xa6lum(GEV350,TESLA, 0) / 0.24641E+03 /
data (xa6(i,GEV350,TESLA, 0),i=0,7) /
$ 0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$ 0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /
data xa6lum(GEV500,TESLA, 0) / 0.30340E+03 /
data (xa6(i,GEV500,TESLA, 0),i=0,7) /
$ 0.52744E+00, 0.52573E+00, 0.13895E+02, -0.63145E+00,
$ 0.40824E+00, -0.69150E+00, 0.18645E+02, 0.47585E+00 /

```

Uses TESLA 11c.

6.2.6 Version 7

```

52b <Update version 7 derived parameters in /circom/ 52b>≡
if (rev .eq. 0) then
  r = 0
elseif (rev .ge. 2000 04 26) then
  r = 1
elseif (rev .lt. 2000 04 26) then
  call circem ('ERROR',
$ 'no revision of version 7 available before 2000/04/26')
  call circem ('MESSAGE', 'falling back to default')
  r = 1
endif
<Log revision mapping 34b>

52c <Update version 7 derived parameters in /circom/ 52b>+≡
if (acc .ne. TESLA .and. acc .ne. JLCNLC) then
  call circem ('ERROR',

```

```

$                                'version 7 applies to TESLA and JLCNLC only')
  call circem ('ERROR', 'falling back to TESLA')
  acc = TESLA
end if
<Linearly interpolate energies 53c>
<Log energy mapping 35d>
Uses TESLA 11c.

53a <formats for circes 34e>+≡
    2004 format ('energy ', F6.1, 'GeV too low, using spectrum for ',
      *          F6.1, 'GeV')
    2005 format ('energy ', F6.1, 'GeV too high, using spectrum for ',
      *          F6.1, 'GeV')
    2006 format ('energy ', F6.1, 'GeV interpolated between ',
      *          F6.1, ' and ', F6.1, 'GeV')

53b <Local variables for circes 30c>+≡
    double precision eloval, ehival
    double precision DELTAE
    parameter (DELTAE = 0.5d0)

The rules are as follows: XBAND has 500 GeV and 1 TeV, TESLA has 500 GeV and
800 TeV. Low energy TESLA will be added.

53c <Linearly interpolate energies 53c>≡
    e = GEV090 - 1
    elo = e
    ehi = e
    if (acc .eq. TESLA) then
      if (roots .lt. 90d0 - DELTAE) then
        write (msgbuf, 2004) roots, 90d0
        call circem ('MESSAGE', msgbuf)
        e = GEV090
      elseif (abs (roots-90d0) .le. DELTAE) then
        e = GEV090
      elseif (roots .lt. 170d0 - DELTAE) then
        write (msgbuf, 2005) roots, 170d0
        call circem ('MESSAGE', msgbuf)
        e = GEV170
      elseif (abs (roots-170d0) .le. DELTAE) then
        e = GEV170
      elseif (roots .lt. 350d0-DELTAE) then
        write (msgbuf, 2006) roots, 170d0, 350d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV170
        ehi = GEV350
        eloal = 170d0
        ehival = 350d0
      elseif (abs (roots-350d0) .le. DELTAE) then
        e = GEV350
      elseif (roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2006) roots, 350d0, 500d0

```

```

        call circem ('MESSAGE', msgbuf)
        elo = GEV350
        ehi = GEV500
        eloal = 350d0
        ehival = 500d0
    elseif (abs (roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (roots .lt. 800d0 - DELTAE) then
        write (msgbuf, 2006) roots, 500d0, 800d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = GEV800
        eloal = 500d0
        ehival = 800d0
    elseif (abs (roots-800d0) .le. DELTAE) then
        e = GEV800
    else
        write (msgbuf, 2005) roots, 800d0
        call circem ('MESSAGE', msgbuf)
        e = GEV800
    endif
elseif (acc .eq. XBAND) then
    if (roots .lt. 500d0 - DELTAE) then
        write (msgbuf, 2004) roots, 500d0
        call circem ('MESSAGE', msgbuf)
        e = GEV500
    elseif (abs (roots-500d0) .le. DELTAE) then
        e = GEV500
    elseif (roots .lt. 1000d0 - DELTAE) then
        write (msgbuf, 2006) roots, 500d0, 1000d0
        call circem ('MESSAGE', msgbuf)
        elo = GEV500
        ehi = TEV1
        eloal = 500d0
        ehival = 1000d0
    elseif (abs (roots-1000d0) .le. DELTAE) then
        e = TEV1
    else
        write (msgbuf, 2005) roots, 1000d0
        call circem ('MESSAGE', msgbuf)
        e = TEV1
    endif
endif
endif

```

Uses TESLA 11c and XBAND 11c.

54a \langle Local variables for circes 30c $\rangle + \equiv$

```

integer A7NEGY, A7NREV
parameter (A7NEGY = TEV1, A7NREV = 1)

```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

```

54b  <Update version 7 derived parameters in /circom/ 52b>+≡
      if (e .ge. GEV090) then
        lumi = xa7lum(e,acc,r)
        do 50 i = 0, 7
          a1(i) = xa7(i,e,acc,r)
50    continue
      elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
        lumi = ((roots-eloal)*xa7lum(ehi,acc,r)
$         + (ehival-roots)*xa7lum(elo,acc,r)) / (ehival - eloal)
        do 51 i = 1, 6
          a1(i) = ((roots-eloal)*xa7(i,ehi,acc,r)
$         + (ehival-roots)*xa7(i,elo,acc,r)) / (ehival - eloal)
51    continue
        a1(0) = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
        a1(7) = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
      endif

```

Uses beta 93a.

```

55a  <Local variables for circes 30c>+≡
      real xa7lum(GEV090:A7NEGY,NACC,0:A7NREV)
      real xa7(0:7,GEV090:A7NEGY,NACC,0:A7NREV)

```

Uses NACC 11c.

Revision 1. The mother of all revisions.

```

55b  <Initializations for circes 32b>+≡
      data xa7lum(GEV090,TESLA,1) / 0.62408E+02 /
      data (xa7(i,GEV090,TESLA,1),i=0,7) /
$    0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$    0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
      data xa7lum(GEV170,TESLA,1) / 0.11532E+02 /
      data (xa7(i,GEV170,TESLA,1),i=0,7) /
$    0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$    0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
      data xa7lum(GEV350,TESLA,1) / 0.24641E+03 /
      data (xa7(i,GEV350,TESLA,1),i=0,7) /
$    0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$    0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /
      data xa7lum(GEV500,TESLA,1) / 0.34704E+03 /
      data (xa7(i,GEV500,TESLA,1),i=0,7) /
$    0.51288E+00, 0.49025E+00, 0.99716E+01, -0.62850E+00,
$    0.41048E+00, -0.69065E+00, 0.13922E+02, 0.51902E+00 /
      data xa7lum(GEV800,TESLA,1) / 0.57719E+03 /
      data (xa7(i,GEV800,TESLA,1),i=0,7) /
$    0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00,
$    0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /
      data xa7lum(TEV1, TESLA,1) / -1.0 /

```

Uses TESLA 11c.

```

55c  <Initializations for circes 32b>+≡
      data xa7lum(GEV090,JLCNLC,1) / -1.0 /
      data xa7lum(GEV170,JLCNLC,1) / -1.0 /

```

```

data xa7lum(GEV350,JLCNLC,1) / -1.0 /
data xa7lum(GEV500,JLCNLC,1) / 0.63039E+02 /
data (xa7(i,GEV500,JLCNLC,1),i=0,7) /
$ 0.58967E+00, 0.34035E+00, 0.63631E+01, -0.63683E+00,
$ 0.33383E+00, -0.68803E+00, 0.81005E+01, 0.48702E+00 /
data xa7lum(TEV1,JLCNLC,1) / 0.12812E+03 /
data (xa7(i,TEV1,JLCNLC,1),i=0,7) /
$ 0.50222E+00, 0.33773E+00, 0.25681E+01, -0.61711E+00,
$ 0.36826E+00, -0.68335E+00, 0.36746E+01, 0.65393E+00 /

```

Revision 0.

56a *<Initializations for circes 32b>+≡*

```

data xa7lum(GEV090,TESLA,0) / 0.62408E+02 /
data (xa7(i,GEV090,TESLA,0),i=0,7) /
$ 0.72637E+00, 0.75534E+00, 0.18180E+03, -0.63426E+00,
$ 0.36829E+00, -0.69653E+00, 0.18908E+03, 0.22157E+00 /
data xa7lum(GEV170,TESLA,0) / 0.11532E+02 /
data (xa7(i,GEV170,TESLA,0),i=0,7) /
$ 0.65232E+00, 0.67249E+00, 0.66862E+02, -0.63315E+00,
$ 0.38470E+00, -0.69477E+00, 0.75120E+02, 0.30162E+00 /
data xa7lum(GEV350,TESLA,0) / 0.24641E+03 /
data (xa7(i,GEV350,TESLA,0),i=0,7) /
$ 0.54610E+00, 0.59105E+00, 0.20297E+02, -0.62747E+00,
$ 0.41588E+00, -0.69188E+00, 0.26345E+02, 0.43818E+00 /
data xa7lum(GEV500,TESLA,0) / 0.34704E+03 /
data (xa7(i,GEV500,TESLA,0),i=0,7) /
$ 0.51288E+00, 0.49025E+00, 0.99716E+01, -0.62850E+00,
$ 0.41048E+00, -0.69065E+00, 0.13922E+02, 0.51902E+00 /
data xa7lum(GEV800,TESLA,0) / 0.57719E+03 /
data (xa7(i,GEV800,TESLA,0),i=0,7) /
$ 0.52490E+00, 0.42573E+00, 0.69069E+01, -0.62649E+00,
$ 0.32380E+00, -0.68958E+00, 0.93819E+01, 0.45671E+00 /
data xa7lum(TEV1, TESLA,0) / -1.0 /

```

Uses TESLA 11c.

56b *<Initializations for circes 32b>+≡*

```

data xa7lum(GEV090,JLCNLC,0) / -1.0 /
data xa7lum(GEV170,JLCNLC,0) / -1.0 /
data xa7lum(GEV350,JLCNLC,0) / -1.0 /
data xa7lum(GEV500,JLCNLC,0) / 0.63039E+02 /
data (xa7(i,GEV500,JLCNLC,0),i=0,7) /
$ 0.58967E+00, 0.34035E+00, 0.63631E+01, -0.63683E+00,
$ 0.33383E+00, -0.68803E+00, 0.81005E+01, 0.48702E+00 /
data xa7lum(TEV1,JLCNLC,0) / 0.12812E+03 /
data (xa7(i,TEV1,JLCNLC,0),i=0,7) /
$ 0.50222E+00, 0.33773E+00, 0.25681E+01, -0.61711E+00,
$ 0.36826E+00, -0.68335E+00, 0.36746E+01, 0.65393E+00 /

```

6.2.7 Version 8

56c *<Update version 8 derived parameters in /circom/ 56c>≡*

```

        if (rev .eq. 0) then
            r = 0
        elseif (rev .ge. 2001 06 17) then
            r = 1
        elseif (rev .lt. 2001 06 17) then
            call circem ('ERROR',
$           'no revision of version 8 available before 2001/06/17')
            call circem ('MESSAGE', 'falling back to default')
            r = 1
        endif
        <Log revision mapping 34b>
57a <Update version 8 derived parameters in /circom/ 56c>+≡
        if (acc .eq. NLCH) then
            acc = JLCNLC
        end if
        if (acc .ne. JLCNLC) then
            call circem ('ERROR',
$           'version 8 applies to JLCNLC (NLC H) only')
            call circem ('ERROR', 'falling back to JLCNLC')
            acc = JLCNLC
        end if
        <Linearly interpolate energies 53c>
        <Log energy mapping 35d>
57b <Local variables for circes 30c>+≡
        integer A8NEGY, A8NREV
        parameter (A8NEGY = TEV1, A8NREV = 1)

        Note that ew must not interpolate a1(0) and a1(7) because they depend non-
        linearly on the other parameters!
57c <Update version 8 derived parameters in /circom/ 56c>+≡
        if (e .ge. GEV090) then
            lumi = xa8lum(e,acc,r)
            do 60 i = 0, 7
                a1(i) = xa8(i,e,acc,r)
60        continue
            elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
                lumi = ((roots-eloval)*xa8lum(ehi,acc,r)
$                 + (ehival-roots)*xa8lum(elo,acc,r)) / (ehival - eloal)
                do 61 i = 1, 6
                    a1(i) = ((roots-eloal)*xa8(i,ehi,acc,r)
$                     + (ehival-roots)*xa8(i,elo,acc,r)) / (ehival - eloal)
61        continue
                a1(0) = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
                a1(7) = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
            endif
        Uses beta 93a.
57d <Local variables for circes 30c>+≡
        real xa8lum(GEV090:A8NEGY,NACC,0:A8NREV)
        real xa8(0:7,GEV090:A8NEGY,NACC,0:A8NREV)

```

Uses NACC 11c.

Revision 1. The mother of all revisions.

```
58a  <Initializations for circes 32b>+≡
      data xa8lum(GEV090, TESLA, 1) / -1.0 /
      data xa8lum(GEV170, TESLA, 1) / -1.0 /
      data xa8lum(GEV350, TESLA, 1) / -1.0 /
      data xa8lum(GEV500, TESLA, 1) / -1.0 /
      data xa8lum(GEV800, TESLA, 1) / -1.0 /
      data xa8lum(TEV1,  TESLA, 1) / -1.0 /
```

Uses TESLA 11c.

```
58b  <Initializations for circes 32b>+≡
      data xa8lum(GEV090, JLCNLC, 1) / -1.0 /
      data xa8lum(GEV170, JLCNLC, 1) / -1.0 /
      data xa8lum(GEV350, JLCNLC, 1) / -1.0 /
      data xa8lum(GEV500, JLCNLC, 1) / 0.239924E+03 /
      data (xa8(i, GEV500, JLCNLC, 1), i=0, 7) /
$    0.57025E+00, 0.34004E+00, 0.52864E+01, -0.63405E+00,
$    0.31627E+00, -0.68722E+00, 0.69629E+01, 0.47973E+00 /
      data xa8lum(TEV1, JLCNLC, 1) / 0.40858E+03 /
      data (xa8(i, TEV1, JLCNLC, 1), i=0, 7) /
$    0.52344E+00, 0.31536E+00, 0.25244E+01, -0.62215E+00,
$    0.31935E+00, -0.68424E+00, 0.35877E+01, 0.57315E+00 /
```

Revision 0.

```
58c  <Initializations for circes 32b>+≡
      data xa8lum(GEV090, TESLA, 0) / -1.0 /
      data xa8lum(GEV170, TESLA, 0) / -1.0 /
      data xa8lum(GEV350, TESLA, 0) / -1.0 /
      data xa8lum(GEV500, TESLA, 0) / -1.0 /
      data xa8lum(GEV800, TESLA, 0) / -1.0 /
      data xa8lum(TEV1,  TESLA, 0) / -1.0 /
```

Uses TESLA 11c.

```
58d  <Initializations for circes 32b>+≡
      data xa8lum(GEV090, JLCNLC, 0) / -1.0 /
      data xa8lum(GEV170, JLCNLC, 0) / -1.0 /
      data xa8lum(GEV350, JLCNLC, 0) / -1.0 /
      data xa8lum(GEV500, JLCNLC, 0) / 0.239924E+03 /
      data (xa8(i, GEV500, JLCNLC, 0), i=0, 7) /
$    0.57025E+00, 0.34004E+00, 0.52864E+01, -0.63405E+00,
$    0.31627E+00, -0.68722E+00, 0.69629E+01, 0.47973E+00 /
      data xa8lum(TEV1, JLCNLC, 0) / 0.40858E+03 /
      data (xa8(i, TEV1, JLCNLC, 0), i=0, 7) /
$    0.52344E+00, 0.31536E+00, 0.25244E+01, -0.62215E+00,
$    0.31935E+00, -0.68424E+00, 0.35877E+01, 0.57315E+00 /
```

6.2.8 Version 9

```
58e  <Update version 9 derived parameters in /circom/ 58e>≡
      if (rev .eq. 0) then
```

```

        r = 0
    elseif (rev .ge. 2002 03 28) then
        r = 1
    elseif (rev .lt. 2002 03 28) then
        call circem ('ERROR',
$      'no revision of version 9 available before 2002/03/28')
        call circem ('MESSAGE', 'falling back to default')
        r = 1
    endif
    <Log revision mapping 34b>
59a  <Update version 9 derived parameters in /circom/ 58e>+=
        if (acc .ne. JLCNLC .and. acc .ne. NLCH) then
            call circem ('ERROR',
$              'version 9 applies to JLCNLC and NLCH only')
            call circem ('ERROR', 'falling back to JLCNLC')
            acc = JLCNLC
        end if
        if (acc .eq. JLCNLC) then
            <Linearly interpolate energies for JLC/NLC 2002 59b>
        else if (acc .eq. NLCH) then
            <Linearly interpolate energies for NLC H 2002 60>
        end if
        <Log energy mapping 35d>
59b  <Linearly interpolate energies for JLC/NLC 2002 59b>=
        e = GEV090 - 1
        elo = e
        ehi = e
        if (roots .lt. 250d0 - DELTAE) then
            write (msgbuf, 2004) roots, 250d0
            call circem ('MESSAGE', msgbuf)
            e = GEV250
        elseif (abs (roots-250d0) .le. DELTAE) then
            e = GEV250
        elseif (roots .lt. 500d0 - DELTAE) then
            write (msgbuf, 2006) roots, 250d0, 500d0
            call circem ('MESSAGE', msgbuf)
            elo = GEV250
            ehi = GEV500
            eloval = 250d0
            ehival = 500d0
        elseif (abs (roots-500d0) .le. DELTAE) then
            e = GEV500
        elseif (roots .lt. 800d0 - DELTAE) then
            write (msgbuf, 2006) roots, 500d0, 800d0
            call circem ('MESSAGE', msgbuf)
            elo = GEV500
            ehi = GEV800
            eloval = 500d0
            ehival = 800d0

```

```

elseif (abs (roots-800d0) .le. DELTAE) then
  e = GEV800
elseif (roots .lt. 1000d0 - DELTAE) then
  write (msgbuf, 2006) roots, 800d0, 1000d0
  call circem ('MESSAGE', msgbuf)
  elo = GEV800
  ehi = TEV1
  eloal = 800d0
  ehival = 1000d0
elseif (abs (roots-1000d0) .le. DELTAE) then
  e = TEV1
elseif (roots .lt. 1200d0 - DELTAE) then
  write (msgbuf, 2006) roots, 1000d0, 1200d0
  call circem ('MESSAGE', msgbuf)
  elo = TEV1
  ehi = TEV12
  eloal = 1000d0
  ehival = 1200d0
elseif (abs (roots-1200d0) .le. DELTAE) then
  e = TEV12
elseif (roots .lt. 1500d0 - DELTAE) then
  write (msgbuf, 2006) roots, 1200d0, 1500d0
  call circem ('MESSAGE', msgbuf)
  elo = TEV12
  ehi = TEV15
  eloal = 1200d0
  ehival = 1500d0
elseif (abs (roots-1500d0) .le. DELTAE) then
  e = TEV15
else
  write (msgbuf, 2005) roots, 1500d0
  call circem ('MESSAGE', msgbuf)
  e = TEV15
endif

```

60 *<Linearly interpolate energies for NLC H 2002 60>*≡

```

e = GEV090 - 1
elo = e
ehi = e
if (roots .lt. 500d0 - DELTAE) then
  write (msgbuf, 2004) roots, 500d0
  call circem ('MESSAGE', msgbuf)
  e = GEV500
elseif (abs (roots-500d0) .le. DELTAE) then
  e = GEV500
elseif (roots .lt. 1000d0 - DELTAE) then
  write (msgbuf, 2006) roots, 500d0, 1000d0
  call circem ('MESSAGE', msgbuf)
  elo = GEV500
  ehi = TEV1

```

```

        eloval = 500d0
        ehival = 1000d0
    elseif (abs (roots-1000d0) .le. DELTAE) then
        e = TEV1
    elseif (roots .lt. 1500d0 - DELTAE) then
        write (msgbuf, 2006) roots, 1000d0, 1500d0
        call circem ('MESSAGE', msgbuf)
        elo = TEV1
        ehi = TEV15
        eloval = 1000d0
        ehival = 1500d0
    elseif (abs (roots-1500d0) .le. DELTAE) then
        e = TEV15
    else
        write (msgbuf, 2005) roots, 1500d0
        call circem ('MESSAGE', msgbuf)
        e = TEV15
    endif

```

61a *<Local variables for circes 30c>+≡*

```

        integer A9NEGY, A9NREV
        parameter (A9NEGY = TEV15, A9NREV = 1)

```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

61b *<Update version 9 derived parameters in /circom/ 58e>+≡*

```

        if (e .ge. GEV090) then
            lumi = xa9lum(e,acc,r)
            do 70 i = 0, 7
                a1(i) = xa9(i,e,acc,r)
70        continue
        elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
            lumi = ((roots-eloval)*xa9lum(ehi,acc,r)
$           + (ehival-roots)*xa9lum(elo,acc,r)) / (ehival - eloal)
            do 71 i = 1, 6
                a1(i) = ((roots-eloal)*xa9(i,ehi,acc,r)
$           + (ehival-roots)*xa9(i,elo,acc,r)) / (ehival - eloal)
71        continue
            a1(0) = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
            a1(7) = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
        endif

```

Uses beta 93a.

61c *<Local variables for circes 30c>+≡*

```

        real xa9lum(GEV090:A9NEGY,NACC,0:A9NREV)
        real xa9(0:7,GEV090:A9NEGY,NACC,0:A9NREV)

```

Uses NACC 11c.

Revision 1. The mother of all revisions.

61d *<Initializations for circes 32b>+≡*

```

        data xa9lum(GEV090,TESLA,1) / -1.0 /
        data xa9lum(GEV170,TESLA,1) / -1.0 /

```

```

data xa9lum(GEV350, TESLA, 1) / -1.0 /
data xa9lum(GEV500, TESLA, 1) / -1.0 /
data xa9lum(GEV800, TESLA, 1) / -1.0 /
data xa9lum(TEV1, TESLA, 1) / -1.0 /
data xa9lum(TEV12, TESLA, 1) / -1.0 /
data xa9lum(TEV15, TESLA, 1) / -1.0 /
data xa9lum(TEV16, TESLA, 1) / -1.0 /

```

Uses TESLA 11c.

62a *(Initializations for circes 32b)+≡*

```

data xa9lum(GEV090, JLCNLC, 1) / -1.0 /
data xa9lum(GEV170, JLCNLC, 1) / -1.0 /
data xa9lum(GEV250, JLCNLC, 1) / 109.886976 /
data (xa9(i, GEV250, JLCNLC, 1), i=0, 7) /
$ 0.65598E+00, 0.34993E+00, 0.13766E+02, -0.64698E+00,
$ 0.29984E+00, -0.69053E+00, 0.16444E+02, 0.36060E+00 /
data xa9lum(GEV350, JLCNLC, 1) / -1.0 /
data xa9lum(GEV500, JLCNLC, 1) / 220.806144 /
data (xa9(i, GEV500, JLCNLC, 1), i=0, 7) /
$ 0.57022E+00, 0.33782E+00, 0.52811E+01, -0.63540E+00,
$ 0.32035E+00, -0.68776E+00, 0.69552E+01, 0.48751E+00 /
data xa9lum(GEV800, JLCNLC, 1) / 304.63488 /
data (xa9(i, GEV800, JLCNLC, 1), i=0, 7) /
$ 0.54839E+00, 0.31823E+00, 0.33071E+01, -0.62671E+00,
$ 0.31655E+00, -0.68468E+00, 0.45325E+01, 0.53449E+00 /
data xa9lum(TEV1, JLCNLC, 1) / 319.95648 /
data (xa9(i, TEV1, JLCNLC, 1), i=0, 7) /
$ 0.56047E+00, 0.29479E+00, 0.28820E+01, -0.62856E+00,
$ 0.29827E+00, -0.68423E+00, 0.39138E+01, 0.52297E+00 /
data xa9lum(TEV12, JLCNLC, 1) / 349.90848 /
data (xa9(i, TEV12, JLCNLC, 1), i=0, 7) /
$ 0.56102E+00, 0.28503E+00, 0.24804E+01, -0.62563E+00,
$ 0.29002E+00, -0.68376E+00, 0.33854E+01, 0.52736E+00 /
data xa9lum(TEV15, JLCNLC, 1) / 363.15648 /
data (xa9(i, TEV15, JLCNLC, 1), i=0, 7) /
$ 0.57644E+00, 0.26570E+00, 0.22007E+01, -0.62566E+00,
$ 0.27102E+00, -0.68283E+00, 0.29719E+01, 0.50764E+00 /
data xa9lum(TEV16, JLCNLC, 1) / -1.0 /

```

62b *(Initializations for circes 32b)+≡*

```

data xa9lum(GEV090, NLCH, 1) / -1.0 /
data xa9lum(GEV170, NLCH, 1) / -1.0 /
data xa9lum(GEV250, NLCH, 1) / -1.0 /
data xa9lum(GEV350, NLCH, 1) / -1.0 /
data xa9lum(GEV500, NLCH, 1) / 371.4624 /
data (xa9(i, GEV500, NLCH, 1), i=0, 7) /
$ 0.33933E+00, 0.55165E+00, 0.29138E+01, -0.57341E+00,
$ 0.54323E+00, -0.68590E+00, 0.51786E+01, 0.88956E+00 /
data xa9lum(GEV800, NLCH, 1) / -1.0 /
data xa9lum(TEV1, NLCH, 1) / 516.41856 /
data (xa9(i, TEV1, NLCH, 1), i=0, 7) /

```

```

$ 0.35478E+00, 0.46474E+00, 0.17666E+01, -0.56949E+00,
$ 0.49269E+00, -0.68384E+00, 0.31781E+01, 0.91121E+00 /
data xa9lum(TEV12, NLCH, 1) / -1.0 /
data xa9lum(TEV15, NLCH, 1) / 575.06688 /
data (xa9(i,TEV15, NLCH, 1),i=0,7) /
$ 0.38183E+00, 0.40310E+00, 0.13704E+01, -0.57742E+00,
$ 0.44548E+00, -0.68341E+00, 0.24956E+01, 0.87448E+00 /
data xa9lum(TEV16, NLCH, 1) / -1.0 /

```

Revision 0.

63a *<Initializations for circes 32b>+≡*

```

data xa9lum(GEV090, TESLA, 0) / -1.0 /
data xa9lum(GEV170, TESLA, 0) / -1.0 /
data xa9lum(GEV350, TESLA, 0) / -1.0 /
data xa9lum(GEV500, TESLA, 0) / -1.0 /
data xa9lum(GEV800, TESLA, 0) / -1.0 /
data xa9lum(TEV1, TESLA, 0) / -1.0 /
data xa9lum(TEV12, TESLA, 0) / -1.0 /
data xa9lum(TEV15, TESLA, 0) / -1.0 /
data xa9lum(TEV16, TESLA, 0) / -1.0 /

```

Uses TESLA 11c.

63b *<Initializations for circes 32b>+≡*

```

data xa9lum(GEV090, JLCNLC, 0) / -1.0 /
data xa9lum(GEV170, JLCNLC, 0) / -1.0 /
data xa9lum(GEV250, JLCNLC, 0) / 109.886976 /
data (xa9(i,GEV250, JLCNLC, 0),i=0,7) /
$ 0.65598E+00, 0.34993E+00, 0.13766E+02, -0.64698E+00,
$ 0.29984E+00, -0.69053E+00, 0.16444E+02, 0.36060E+00 /
data xa9lum(GEV350, JLCNLC, 0) / -1.0 /
data xa9lum(GEV500, JLCNLC, 0) / 220.806144 /
data (xa9(i,GEV500, JLCNLC, 0),i=0,7) /
$ 0.57022E+00, 0.33782E+00, 0.52811E+01, -0.63540E+00,
$ 0.32035E+00, -0.68776E+00, 0.69552E+01, 0.48751E+00 /
data xa9lum(GEV800, JLCNLC, 0) / 304.63488 /
data (xa9(i,GEV800, JLCNLC, 0),i=0,7) /
$ 0.54839E+00, 0.31823E+00, 0.33071E+01, -0.62671E+00,
$ 0.31655E+00, -0.68468E+00, 0.45325E+01, 0.53449E+00 /
data xa9lum(TEV1, JLCNLC, 0) / 319.95648 /
data (xa9(i,TEV1, JLCNLC, 0),i=0,7) /
$ 0.56047E+00, 0.29479E+00, 0.28820E+01, -0.62856E+00,
$ 0.29827E+00, -0.68423E+00, 0.39138E+01, 0.52297E+00 /
data xa9lum(TEV12, JLCNLC, 0) / 349.90848 /
data (xa9(i,TEV12, JLCNLC, 0),i=0,7) /
$ 0.56102E+00, 0.28503E+00, 0.24804E+01, -0.62563E+00,
$ 0.29002E+00, -0.68376E+00, 0.33854E+01, 0.52736E+00 /
data xa9lum(TEV15, JLCNLC, 0) / 363.15648 /
data (xa9(i,TEV15, JLCNLC, 0),i=0,7) /
$ 0.57644E+00, 0.26570E+00, 0.22007E+01, -0.62566E+00,
$ 0.27102E+00, -0.68283E+00, 0.29719E+01, 0.50764E+00 /
data xa9lum(TEV16, JLCNLC, 0) / -1.0 /

```

```

64a  <Initializations for circes 32b>+≡
      data xa9lum(GEV090,NLCH,  0) / -1.0 /
      data xa9lum(GEV170,NLCH,  0) / -1.0 /
      data xa9lum(GEV250,NLCH,  0) / -1.0 /
      data xa9lum(GEV350,NLCH,  0) / -1.0 /
      data xa9lum(GEV500,NLCH,  0) / 371.4624 /
      data (xa9(i,GEV500,NLCH,  0),i=0,7) /
$    0.33933E+00,  0.55165E+00,  0.29138E+01, -0.57341E+00,
$    0.54323E+00, -0.68590E+00,  0.51786E+01,  0.88956E+00 /
      data xa9lum(GEV800,NLCH,  0) / -1.0 /
      data xa9lum(TEV1,  NLCH,  0) / 516.41856 /
      data (xa9(i,TEV1,  NLCH,  0),i=0,7) /
$    0.35478E+00,  0.46474E+00,  0.17666E+01, -0.56949E+00,
$    0.49269E+00, -0.68384E+00,  0.31781E+01,  0.91121E+00 /
      data xa9lum(TEV12, NLCH,  0) / -1.0 /
      data xa9lum(TEV15, NLCH,  0) / 575.06688 /
      data (xa9(i,TEV15, NLCH,  0),i=0,7) /
$    0.38183E+00,  0.40310E+00,  0.13704E+01, -0.57742E+00,
$    0.44548E+00, -0.68341E+00,  0.24956E+01,  0.87448E+00 /
      data xa9lum(TEV16, NLCH,  0) / -1.0 /

```

6.3 Special Functions

```

64b  <Subroutines 28b>+≡
      double precision function beta (a, b)
      implicit none
      double precision a, b
      double precision dlogam
      beta = exp (dlogam(a) + dlogam(b) - dlogam(a+b))
      end

```

Uses beta 93a.

```

64c  <Subroutines 28b>+≡
      CERNLIB C304
      DOUBLE PRECISION FUNCTION DLOGAM(X)
      IMPLICIT NONE
      DOUBLE PRECISION P1(7),Q1(7),P2(7),Q2(7),P3(7),Q3(7),C(5),XL(5)
      DOUBLE PRECISION X,Y,ZERO,ONE,TWO,HALF,AP,AQ
      INTEGER I
      DATA ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
      DATA XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
      DATA P1
1/+3.84287 36567 460D+0, +5.27068 93753 010D+1,
2 +5.55840 45723 515D+1, -2.15135 13573 726D+2,
3 -2.45872 61722 292D+2, -5.75008 93603 041D+1,
4 -2.33590 98949 513D+0/
      DATA Q1
1/+1.00000 00000 000D+0, +3.37330 47907 071D+1,
2 +1.93877 84034 377D+2, +3.08829 54973 424D+2,
3 +1.50068 39064 891D+2, +2.01068 51344 334D+1,

```

```

4 +4.57174 20282 503D-1/
  DATA P2
1/+4.87402 01396 839D+0, +2.48845 25168 574D+2,
2 +2.17973 66058 896D+3, +3.79751 24011 525D+3,
3 -1.97780 70769 842D+3, -3.69298 34005 591D+3,
4 -5.60177 73537 804D+2/
  DATA Q2
1/+1.00000 00000 000D+0, +9.50999 17418 209D+1,
2 +1.56120 45277 929D+3, +7.23400 87928 948D+3,
3 +1.04595 76594 059D+4, +4.16994 15153 200D+3,
4 +2.76785 83623 804D+2/
  DATA P3
1/-6.88062 40094 594D+3, -4.30699 69819 571D+5,
2 -4.75045 94653 440D+6, -2.94234 45930 322D+6,
3 +3.63218 04931 543D+7, -3.35677 82814 546D+6,
4 -2.48043 69488 286D+7/
  DATA Q3
1/+1.00000 00000 000D+0, -1.42168 29839 651D+3,
2 -1.55528 90280 854D+5, -3.41525 17108 011D+6,
3 -2.09696 23255 804D+7, -3.45441 75093 344D+7,
4 -9.16055 82863 713D+6/
  DATA C
1/ 1.12249 21356 561D-1, 7.95916 92961 204D-2,
1 -1.70877 94611 020D-3, 9.18938 53320 467D-1,
2 1.34699 05627 879D+0/
  IF(X .LE. XL(1)) THEN
    print *, 'ERROR: DLOGAM non positive argument: ', X
    DLOGAM=ZERO
  ENDIF
  IF(X .LE. XL(2)) THEN
    Y=X+ONE
    AP=P1(1)
    AQ=Q1(1)
    DO 2 I = 2,7
      AP=P1(I)+Y*AP
2      AQ=Q1(I)+Y*AQ
    Y=-LOG(X)+X*AP/AQ
  ELSEIF(X .LE. XL(3)) THEN
    AP=P1(1)
    AQ=Q1(1)
    DO 3 I = 2,7
      AP=P1(I)+X*AP
3      AQ=Q1(I)+X*AQ
    Y=(X-ONE)*AP/AQ
  ELSEIF(X .LE. XL(4)) THEN
    AP=P2(1)
    AQ=Q2(1)
    DO 4 I = 2,7
      AP=P2(I)+X*AP
4      AQ=Q2(I)+X*AQ

```

```

        Y=(X-TWO)*AP/AQ
    ELSEIF(X .LE. XL(5)) THEN
        AP=P3(1)
        AQ=Q3(1)
        DO 5 I = 2,7
            AP=P3(I)+X*AP
5        AQ=Q3(I)+X*AQ
        Y=AP/AQ
    ELSE
        Y=ONE/X**2
        Y=(X-HALF)*LOG(X)-X+C(4)+(C(1)+Y*(C(2)+Y*C(3)))/
1        ((C(5)+Y)*X)
    ENDIF
    DLOGAM=Y
    END

```

6.4 Non-Singular Distributions

66a *<Subroutines 28b>+≡*

```

    double precision function kirke (x1, x2, p1, p2)
    implicit none
    double precision x1, x2
    integer p1, p2
    double precision kirkee, kirkeg, kirkgg
    <Particle codes 10b>
    </circom/ 29b>
    <Initialization check 29g>
    kirke = -1.0
    if (abs(p1) .eq. ELECTR) then
        if (abs(p2) .eq. ELECTR) then
            kirke = kirkee (x1, x2)
        elseif (p2 .eq. PHOTON) then
            kirke = kirkeg (x1, x2)
        endif
    elseif (p1 .eq. PHOTON) then
        if (abs(p2) .eq. ELECTR) then
            kirke = kirkeg (x2, x1)
        elseif (p2 .eq. PHOTON) then
            kirke = kirkgg (x1, x2)
        endif
    endif
    end

```

Defines:

kirke, never used.

Uses ELECTR 10b, PHOTON 10b, kirkee 66b, kirkeg 69a, and kirkgg 69b.

66b *<Subroutines 28b>+≡*

```

    double precision function kirkee (x1, x2)
    implicit none
    double precision x1, x2

```

```

</circom/ 29b>
double precision d1, d2
<Initialization check 29g>
kirkee = -1.0
if ((ver .eq. 1) .or. (ver .eq. 0)) then
  <Calculate version 1 of the non-singular  $e^+e^-$  distribution 68a>
  <else handle invalid versions 33c>
end

```

Defines:

kirkee, used in chunks 15a, 66a, and 68a.

```

67a <parameter part of /circom/ 29f>+≡
    double precision KIREPS
    parameter (KIREPS = 1D-6)

67b <8-byte aligned part of /circom/ 29c>+≡
    double precision elect0, gamma0
    common /circom/ elect0, gamma0

```

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{1-\epsilon}^{1-} dx x^{a_2^{\alpha \rho}} (1-x)^{a_3^{\alpha \rho}} \quad (18)$$

Approximately

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{1-\epsilon}^{1-} dx (1-x)^{a_3^{\alpha \rho}} = a_0^{\alpha \rho} + a_1^{\alpha \rho} \int_{0+}^{\epsilon} d\xi \xi^{a_3^{\alpha \rho}} \quad (19)$$

and therefore

$$\int_{1-\epsilon}^{1+} dx d_{e^\pm}^{\alpha_1 \rho}(x) = a_0^{\alpha \rho} + a_1^{\alpha \rho} \frac{1 - \epsilon^{a_3^{\alpha \rho} + 1}}{a_3^{\alpha \rho} + 1} \quad (20)$$

This simple approximation is good enough

```

67c <Update /circom/ 30b>+≡
    elect0 = a1(0) + a1(1) * KIREPS**(a1(3)+1) / (a1(3)+1)
    elect0 = elect0 / KIREPS
    gamma0 = a1(4) * KIREPS**(a1(5)+1) / (a1(5)+1)
    gamma0 = gamma0 / KIREPS

```

but we can also use incomplete Beta functions for the exact result:

```

67d <Alternative: Update /circom/ 67d>≡
    elect0 = a1(0) + a1(1) * beta (a1(2)+1, a1(3)+1)
    $
    * (1d0 - betinc (a1(2)+1, a1(3)+1, 1d0 - KIREPS))
    elect0 = elect0 / KIREPS
    gamma0 = a1(7) + a1(4) * beta (a1(5)+1, a1(6)+1)
    $
    * betinc (a1(5)+1, a1(6)+1, KIREPS)
    gamma0 = gamma0 / KIREPS

```

Uses beta 93a.

```

67e <Alternative: Local variables for circes 67e>≡
    double precision betinc
    external betinc

```

68a \langle Calculate version 1 of the non-singular e^+e^- distribution 68a $\rangle \equiv$

```

    if (x1 .gt. 1d0) then
        d1 = 0d0
    elseif (x1 .ge. (1d0 - KIREPS)) then
        d1 = elect0
    elseif (x1 .ge. 0d0) then
        d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
    else
        d1 = 0d0
    endif
    if (x2 .gt. 1d0) then
        d2 = 0d0
    elseif (x2 .ge. (1d0 - KIREPS)) then
        d2 = elect0
    elseif (x2 .ge. 0d0) then
        d2 = a1(1) * x2**a1(2) * (1d0 - x2)**a1(3)
    else
        d2 = 0d0
    endif
    kirkee = d1 * d2

```

Uses kirkee 66b.

68b \langle Calculate version 1 of the non-singular $e^\pm\gamma$ distribution 68b $\rangle \equiv$

```

    if (x1 .gt. 1d0) then
        d1 = 0d0
    elseif (x1 .ge. (1d0 - KIREPS)) then
        d1 = elect0
    elseif (x1 .ge. 0d0) then
        d1 = a1(1) * x1**a1(2) * (1d0 - x1)**a1(3)
    else
        d1 = 0d0
    endif
    if (x2 .gt. 1d0) then
        d2 = 0d0
    elseif (x2 .gt. KIREPS) then
        d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
    elseif (x2 .ge. 0d0) then
        d2 = gamma0
    else
        d2 = 0d0
    endif
    kirkeg = d1 * d2

```

Uses kirkeg 69a.

68c \langle Calculate version 1 of the non-singular $\gamma\gamma$ distribution 68c $\rangle \equiv$

```

    if (x1 .gt. 1d0) then
        d1 = 0d0
    elseif (x1 .gt. KIREPS) then
        d1 = a1(4) * x1**a1(5) * (1d0 - x1)**a1(6)
    elseif (x1 .ge. 0d0) then
        d1 = gamma0
    else
        d1 = 0d0
    endif

```

```

else
    d1 = 0d0
endif
if (x2 .gt. 1d0) then
    d2 = 0d0
elseif (x2 .gt. KIREPS) then
    d2 = a1(4) * x2**a1(5) * (1d0 - x2)**a1(6)
elseif (x2 .ge. 0d0) then
    d2 = gamma0
else
    d2 = 0d0
endif
kirkgg = d1 * d2

```

Uses kirkgg 69b.

69a *<Subroutines 28b>+≡*

```

double precision function kirkeg (x1, x2)
implicit none
double precision x1, x2
</circom/ 29b>
double precision d1, d2
<Initialization check 29g>
kirkeg = -1.0
if ((ver .eq. 1) .or. (ver .eq. 0)) then
    <Calculate version 1 of the non-singular  $e^{\pm\gamma}$  distribution 68b>
    <else handle invalid versions 33c>
end

```

Defines:

kirkeg, used in chunks 66a and 68b.

69b *<Subroutines 28b>+≡*

```

double precision function kirkgg (x1, x2)
implicit none
double precision x1, x2
</circom/ 29b>
double precision d1, d2
<Initialization check 29g>
kirkgg = -1.0
if ((ver .eq. 1) .or. (ver .eq. 0)) then
    <Calculate version 1 of the non-singular  $\gamma\gamma$  distribution 68c>
    <else handle invalid versions 33c>
end

```

Defines:

kirkgg, used in chunks 66a and 68c.

69c *<Alternative: Subroutines 69c>≡*

```

double precision function betinc (a, b, x)
implicit none
double precision x, a, b
double precision bt, betacf, dlogam
external betacf, dlogam
if (x .lt. 0d0 .or. x .gt. 1d0) then

```

```

        betinc = 0d0
    else
        if (x .eq. 0d0 .or. x .eq. 1d0) then
            bt = 0d0
        else
            bt = exp(dlogam(a+b)-dlogam(a)-dlogam(b)
$           + a*log(x) + b*log(1d0-x))
        endif
        if (x .lt. (a+1d0)/(a+b+2d0)) then
            betinc = bt*betacf (a, b, x) / a
        else
            betinc = 1d0 - bt*betacf (b, a, 1d0-x) / b
        endif
    endif
end
end

```

70 *<Alternative: Subroutines 69c>+≡*

```

double precision function betacf (a, b, x)
implicit none
double precision x, a, b
integer ITMAX
double precision EPS
parameter (ITMAX = 100, EPS = 3D-7)
double precision am, bm, curr, prev, qab, qap, qam, bz,
$   ap, bp, app, bpp, em, tem, d
integer m
am = 1d0
bm = 1d0
curr = 1d0
qab = a + b
qap = a + 1d0
qam = a - 1d0
bz = 1d0 - qab * x / qap
do 10 m = 1, ITMAX
    em = m
    tem = 2*em
    d = em * (b - m) * x / ((qam + tem) * (a + tem))
    ap = curr + d*am
    bp = bz + d*bm
    d = - (a + em) * (qab + em) * x / ((a + tem) * (qap + tem))
    app = ap + d * curr
    bpp = bp + d * bz
    prev = curr
    am = ap / bpp
    bm = bp / bpp
    curr = app / bpp
    bz = 1d0
    if (abs (curr - prev) .lt. EPS * abs (curr)) then
        betacf = curr
        return
    endif
enddo

```

```

        endif
10 continue
    print *, 'betacf: failed to converge'
    betacf = 0d0
end

```

6.5 Generators

6.5.1 Version 1

Beta distributions have the practical advantage that they have been popular among mathematicians.[?]

```

71a  <Subroutines 28b>+≡
      subroutine girce (x1, x2, p1, p2, rng)
      implicit none
      double precision x1, x2
      integer p1, p2
      external rng
      </circom/ 29b>
      double precision u, w, circgg
      <Particle codes 10b>
      <Initialization check 29g>
      <x1m, x2m kludge, part 1 72a>
      <Select particles p1 and p2 71b>
      if (abs(p1) .eq. ELECTR) then
        if (abs(p2) .eq. ELECTR) then
          call gircee (x1, x2, rng)
        elseif (p2 .eq. PHOTON) then
          call girceg (x1, x2, rng)
        endif
      elseif (p1 .eq. PHOTON) then
        if (abs(p2) .eq. ELECTR) then
          call girceg (x2, x1, rng)
        elseif (p2 .eq. PHOTON) then
          call gircgg (x1, x2, rng)
        endif
      endif
      <x1m, x2m kludge, part 2 72b>
end

```

Defines:

girce, used in chunks 71a, 17b, and 71.

Uses ELECTR 10b, PHOTON 10b, circgg 39a, gircee 72c, girceg 73a, and gircgg 73d.

```

71b  <Select particles p1 and p2 71b>≡
      w = 1d0 / (1d0 + circgg (-1d0, -1d0))
      call rng (u)
      if (u*u .le. w) then
        p1 = POSITR
      else
        p1 = PHOTON
      endif

```

```

endif
call rng (u)
if (u*u .le. w) then
  p2 = ELECTR
else
  p2 = PHOTON
endif

```

Uses ELECTR 10b, PHOTON 10b, POSITR 10b, and circgg 39a.

The flavor selection is incorrect, because the relative weights depend on the minimum energy fractions. We resort to a moderately inefficient kludge, because we don't have the distribution functions available yet. We'll have to implement incomplete Beta functions and other horrible things for this. Fortunately, the efficiency can not drop below the relative contribution of e^+e^- .

72a *<x1m, x2m kludge, part 1 72a>*≡
 99 continue

Crude rejection:

72b *<x1m, x2m kludge, part 2 72b>*≡
 if ((x1 .lt. x1m) .or. (x2 .lt. x2m)) goto 99

72c *<Subroutines 28b>*+≡
 subroutine gircee (x1, x2, rng)
 implicit none
 double precision x1, x2
 external rng
 </circom/ 29b>
 double precision u, girceb
 <Initialization check 29g>
 if ((ver .eq. 1) .or. (ver .eq. 0)) then
 <Generate version 1 of the e^+e^- distribution 72d>
 <else handle invalid versions 33c>
 end

Defines:

gircee, used in chunks 17d, 18a, 72c, 19b, and 71a.

Uses girceb 74a.

For version 1 of the parametrizations we rely on girceb, a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (21)$$

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1}(1-\xi)^{b-1} \quad (22)$$

72d *<Generate version 1 of the e^+e^- distribution 72d>*≡
 call rng (u)
 if (u .le. a1(0)) then
 x1 = 1d0
 else
 x1 = 1d0 - girceb (0d0, 1d0-x1m, a1(3)+1d0, a1(2)+1d0, rng)
 endif
 call rng (u)

```

      if (u .le. a1(0)) then
        x2 = 1d0
      else
        x2 = 1d0 - girceb (0d0, 1d0-x2m, a1(3)+1d0, a1(2)+1d0, rng)
      endif

```

Uses girceb 74a.

73a *<Subroutines 28b>+≡*

```

      subroutine girceg (x1, x2, rng)
      implicit none
      double precision x1, x2
      external rng
      </circom/ 29b>
      double precision u, girceb
      <Initialization check 29g>
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
        <Generate version 1 of the  $e^\pm\gamma$  distribution 73b>
      <else handle invalid versions 33c>
      end

```

Defines:

girceg, used in chunks 17d and 71a.

Uses girceb 74a.

73b *<Generate version 1 of the $e^\pm\gamma$ distribution 73b>≡*

```

      call rng (u)
      if (u .le. a1(0)) then
        x1 = 1d0
      else
        x1 = 1d0 - girceb (0d0, 1d0-x1m, a1(3)+1d0, a1(2)+1d0, rng)
      endif
      x2 = girceb (x2m, 1d0, a1(5)+1d0, a1(6)+1d0, rng)

```

Uses girceb 74a.

73c *<Subroutines 28b>+≡*

```

      subroutine gircgg (x1, x2, rng)
      implicit none
      double precision x1, x2
      external rng
      </circom/ 29b>
      double precision girceb
      <Initialization check 29g>
      if ((ver .eq. 1) .or. (ver .eq. 0)) then
        <Generate version 1 of the  $\gamma\gamma$  distribution 73d>
      <else handle invalid versions 33c>
      end

```

Uses girceb 74a and gircgg 73d.

73d *<Generate version 1 of the $\gamma\gamma$ distribution 73d>≡*

```

      x1 = girceb (x1m, 1d0, a1(5)+1d0, a1(6)+1d0, rng)
      x2 = girceb (x2m, 1d0, a1(5)+1d0, a1(6)+1d0, rng)

```

Defines:

gircgg, used in chunks 17d, 71a, and 73c.

Uses girceb 74a.

6.5.2 Version 2

Retired.

6.5.3 Version 3 and 4

Identical to version 1.

6.6 Utilities

For version 1 of the parametrizations we need a fast generator of β -distributions:

$$\beta_{x_{\min}, x_{\max}}^{a,b}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max} - x)\Theta(x - x_{\min})}{I(x_{\min}, a, b) - I(x_{\max}, a, b)} \quad (23)$$

with the *incomplete Beta-function* I :

$$I(x, a, b) = \int_x^1 d\xi \xi^{a-1} (1-\xi)^{b-1} \quad (24)$$

$$B(a, b) = I(0, a, b) \quad (25)$$

This problem has been studied extensively [?] and we can use an algorithm [18] that is very fast for $0 < a \leq 1 \leq b$, which turns out to be the case in our application.

```

74a  <Subroutines 28b>+≡
      double precision function girceb (xmin, xmax, a, b, rng)
      implicit none
      double precision xmin, xmax, a, b
      external rng
      double precision t, p, u, umin, umax, x, w
      <Check a and b 74b>
      <Set up girceb parameters 75a>
10   continue
      <Generate a trial x and calculate its weight w 75b>
      call rng (u)
      if (w .le. u) goto 10
      girceb = x
      end

Defines:
      girceb, used in chunks 72c, 74a, 72-74, and 76c.

In fact, this algorithm works for  $0 < a \leq 1 \leq b$  only:

74b  <Check a and b 74b>≡
      if ((a .gt. 1d0) .or. (b .lt. 1d0)) then
        girceb = -1d0
        call circem ('ERROR', 'beta-distribution expects a<=1<=b')
        return
      endif

```

Uses girceb 74a.

The trick is to split the interval $[0, 1]$ into two parts $[0, t]$ and $[t, 1]$. In these intervals we obviously have

$$x^{a-1}(1-x)^{b-1} \leq \begin{cases} x^{a-1} & \text{for } x \leq t \\ t^{a-1}(1-x)^{b-1} & \text{for } x \geq t \end{cases} \quad (26)$$

because we have assumed that $0 < a \leq 1 \leq b$. The integrals of the two dominating distributions are t^a/a and $t^{a-1}(1-t)^b/b$ respectively and therefore the probability for picking a random number from the first interval is

$$P(x \leq t) = \frac{bt}{bt + a(1-t)^b} \quad (27)$$

We postpone the discussion of the choice of t until later:

```
75a  <Set up girceb parameters 75a>≡
      <Set up best value for t 77a>
      p = b*t / (b*t + a * (1d0 - t)**b)
```

The dominating distributions can be generated by simple mappings

$$\phi : [0, 1] \rightarrow [0, 1] \quad (28)$$

$$u \mapsto \begin{cases} t \left(\frac{u}{p}\right)^{\frac{1}{a}} & < t \text{ for } u < p \\ t & = t \text{ for } u = p \\ 1 - (1-t) \left(\frac{1-u}{1-p}\right)^{\frac{1}{b}} & > t \text{ for } u > p \end{cases} \quad (29)$$

The beauty of the algorithm is that we can use a single uniform deviate u for both intervals:

```
75b  <Generate a trial x and calculate its weight w 75b>≡
      call rng (u)
      u = umin + (umax - umin) * u
      if (u .le. p) then
        x = t * (u/p)**(1d0/a)
        w = (1d0 - x)**(b-1d0)
      else
        x = 1d0 - (1d0 - t) * ((1d0 - u)/(1d0 - p))**(1d0/b)
        w = (x/t)**(a-1d0)
      endif
```

The weights that are derived by dividing the distribution by the dominating distributions are already normalized correctly:

$$w : [0, 1] \rightarrow [0, 1] \quad (30)$$

$$x \mapsto \begin{cases} (1-x)^{b-1} & \in [(1-t)^{b-1}, 1] \text{ for } x \leq t \\ \left(\frac{x}{t}\right)^{a-1} & \in [t^{1-a}, 1] \text{ for } x \geq t \end{cases} \quad (31)$$

To derive $u_{\min, \max}$ from $x_{\min, \max}$ we can use ϕ^{-1} :

$$\phi^{-1} : [0, 1] \rightarrow [0, 1] \quad (32)$$

$$x \mapsto \begin{cases} p \left(\frac{x}{t}\right)^a & < p \text{ for } x < t \\ p & = p \text{ for } x = t \\ 1 - (1 - p) \left(\frac{1-x}{1-t}\right)^b & > p \text{ for } x > t \end{cases} \quad (33)$$

We start with u_{\min} . For efficiency, we handle the most common cases (small x_{\min}) first:

```
76a <Set up girceb parameters 75a>+≡
      if (xmin .le. 0d0) then
        umin = 0d0
      elseif (xmin .lt. t) then
        umin = p * (xmin/t)**a
      elseif (xmin .eq. t) then
        umin = p
      elseif (xmin .lt. 1d0) then
        umin = 1d0 - (1d0 - p) * ((1d0 - xmin)/(1d0 - t))**b
      else
        umin = 1d0
      endif
```

Same procedure for u_{\max} ; again, handle the most common cases (large x_{\max}) first:

```
76b <Set up girceb parameters 75a>+≡
      if (xmax .ge. 1d0) then
        umax = 1d0
      elseif (xmax .gt. t) then
        umax = 1d0 - (1d0 - p) * ((1d0 - xmax)/(1d0 - t))**b
      elseif (xmax .eq. t) then
        umax = p
      elseif (xmax .gt. 0d0) then
        umax = p * (xmax/t)**a
      else
        umax = 0d0
      endif
```

Check for absurd cases.

```
76c <Set up girceb parameters 75a>+≡
      if (umax .lt. umin) then
        girceb = -1d0
        return
      endif
```

Uses girceb 74a.

It remains to choose the best value for t . The rejection efficiency ϵ of the algorithm is given by the ratio of the dominating distribution and the distribution

$$\frac{1}{\epsilon(t)} = \frac{B(a, b)}{ab} (bt^a + at^{a-1}(1-t)^b). \quad (34)$$

It is maximized for

$$bt - bt(1-t)^{b-1} + (a-1)(1-t)^b = 0 \quad (35)$$

This equation has a solution which can be determined numerically. While this determination is far too expensive compared to a moderate loss in efficiency, we could perform it once after fitting the coefficients a , b . Nevertheless, it has been shown,[18] that

$$t = \frac{1 - a}{b + 1 - a} \quad (36)$$

results in non-vanishing efficiency for all values $1 < a \leq 1 \leq b$. Empirically we have found efficiencies of at least 80% for this choice, which is enough for our needs.

```

77a  <Set up best value for t 77a>≡
      t = (1d0 - a) / (b + 1d0 - a)

77b  <Subroutines 28b>+≡
      subroutine circem (errlvl, errmsg)
      implicit none
      character*(*) errlvl, errmsg
      </circum/ 29b>
      integer errcnt
      save errcnt
      data errcnt /0/
      if (errlvl .eq. 'MESSAGE') then
        print *, 'circe:message: ', errmsg
      elseif (errlvl .eq. 'WARNING') then
        if (errcnt .lt. 100) then
          errcnt = errcnt + 1
          print *, 'circe:warning: ', errmsg
        elseif (errcnt .eq. 100) then
          errcnt = errcnt + 1
          print *, 'circe:message: more than 100 messages'
          print *, 'circe:message: turning warnings off'
        endif
      elseif (errlvl .eq. 'ERROR') then
        if (errcnt .lt. 200) then
          errcnt = errcnt + 1
          print *, 'circe:error: ', errmsg
        elseif (errcnt .eq. 200) then
          errcnt = errcnt + 1
          print *, 'circe:message: more than 200 messages'
          print *, 'circe:message: turning error messages off'
        endif
      elseif (errlvl .eq. 'PANIC') then
        if (errcnt .lt. 300) then
          errcnt = errcnt + 1
          print *, 'circe:panic: ', errmsg
        elseif (errcnt .eq. 300) then
          errcnt = errcnt + 1
          print *, 'circe:message: more than 300 messages'
          print *, 'circe:message: turning panic messages off'
        endif
      endif
      else

```

```

        print *, 'circe:panic:    invalid error code ', errlvl
    endif
end

```

6.7 Examples

6.7.1 Distributions

```

78a  <cplot.f 78a>≡
      program cplot
      implicit none
      <Particle codes 10b>
      double precision xmin, xmax, y, roots
      integer xory, nstep, p1, p2, acc, ver, rev
      double precision x, logx, d, circe
      read *, xory, xmin, xmax, nstep, y, p1, p2, roots, acc, ver, rev
      call circes (0d0, 0d0, roots, acc, ver, rev, 0)
      do 10 logx = log (xmin), log (xmax), log (xmax/xmin) / nstep
         x = exp (logx)
         d = 0d0
         if (xory .eq. 1) then
            if (p1 .eq. PHOTON) then
               d = circe (x, y, p1, p2)
            else
               d = circe (1d0 - x, y, p1, p2)
            endif
         elseif (xory .eq. 2) then
            if (p1 .eq. PHOTON) then
               d = circe (y, x, p1, p2)
            else
               d = circe (y, 1d0 - x, p1, p2)
            endif
         endif
         if (d .gt. 1d-4) print *, x, d
      10 continue
      end

```

Uses PHOTON 10b, circe 28b, and circes 29a.

6.7.2 Library functions

If Fortran77 only had first class functions, then the following cruft would not be necessary. OK, here's the outline of the adaptive Gauss integration routine from CERNLIB:

```

78b  <Part one of Gaussian integration 78b>≡
      double precision f, a, b, eps
      external f
      double precision Z1, HF, CST
      parameter (Z1 = 1, HF = Z1/2, CST = 5*Z1/1000)

```

```

integer i
double precision h, const, aa, bb, c1, c2, s8, s16, u
<Gaussian weights 81a>
h = 0
if (b .eq. a) go to 99
const = CST / dabs(b-a)
bb = a
1 continue
  aa = bb
  bb = b
2 continue
  c1 = HF*(bb+aa)
  c2 = HF*(bb-aa)
  s8 = 0
  do 3 i = 1, 4
    u = c2*x(i)

```

Here are now the first two function calls that we have to fill in later in various ways:

```

79a <Function call stub 79a>≡
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))

```

Continuing

```

79b <Part two of Gaussian integration 79b>≡
      3 continue
      s16 = 0
      do 4 i = 5, 12
        u = c2*x(i)

```

And here are the other two function calls:

```

79c <Function call stub 79a>+≡
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))

```

Terminating:

```

79d <Part three of Gaussian integration 79d>≡
      4 continue
      s16 = c2*s16
      if (dabs(s16-c2*s8) .le. eps*(1+dabs(s16))) then
        h = h + s16
        if (bb .ne. b) go to 1
      else
        bb = c1
        if (1 + const*dabs(c2) .ne. 1) go to 2
        h = 0
        print *, 'gauss: too high accuracy required'
        go to 99
      end if
99 continue

```

This one is still reasonably straightforward

$$\text{gauss1} : (f, a, b) \mapsto \int_a^b dx f(x) \quad (37)$$

```

80a  <sample.f 13a>+≡
      double precision function gauss1 (f, a, b, eps)
      implicit none
      <Part one of Gaussian integration 78b>
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
      <Part two of Gaussian integration 79b>
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
      <Part three of Gaussian integration 79d>
      gauss1 = h
      end

```

Defines:

`gauss1`, used in chunks 80a, 13b, 17a, and 80.

But this almost identical repeat

$$\text{gaussx} : (f, a, b) \mapsto \left(y \mapsto \int_a^b dx f(y, x) \right) \quad (38)$$

would not be necessary in a modern programming language with currying:

```

80b  <sample.f 13a>+≡
      double precision function gaussx (f, y, a, b, eps)
      implicit none
      double precision y
      <Part one of Gaussian integration 78b>
      s8 = s8 + w(i) * (f (y, c1+u) + f (y, c1-u))
      <Part two of Gaussian integration 79b>
      s16 = s16 + w(i) * (f (y, c1+u) + f (y, c1-u))
      <Part three of Gaussian integration 79d>
      gaussx = h
      end

```

Defines:

`gaussx`, used in chunk 80.

Fortunately, this is the last one we need

$$\begin{aligned} \text{gauss2} : (f, a, b, a_1, b_1) &\mapsto \int_a^b dx \int_{a_1}^{b_1} dy f(x, y) \\ &= \text{gauss1}(\text{gaussx}(f, a, b), a_1, b_1) \end{aligned} \quad (39)$$

```

80c  <sample.f 13a>+≡
      double precision function gauss2 (f, a, b, a1, b1, eps)
      implicit none
      double precision a1, b1, gaussx
      <Part one of Gaussian integration 78b>
      s8 = s8 + w(i) * (gaussx (f, c1+u, a1, b1, eps)
$                               + gaussx (f, c1-u, a1, b1, eps))
      <Part two of Gaussian integration 79b>
      s16 = s16 + w(i) * (gaussx (f, c1+u, a1, b1, eps)
$                               + gaussx (f, c1-u, a1, b1, eps))
      <Part three of Gaussian integration 79d>
      gauss2 = h

```

end

Defines:

`gauss2`, used in chunks 80c, 13b, 14d, 17a, and 80c.

Uses `gaussx` 80b.

81a \langle *Gaussian weights* 81a $\rangle \equiv$

```
double precision w(12), x(12)
data x( 1) /9.6028985649753623d-1/, w( 1) /1.0122853629037626d-1/
data x( 2) /7.9666647741362674d-1/, w( 2) /2.2238103445337447d-1/
data x( 3) /5.2553240991632899d-1/, w( 3) /3.1370664587788729d-1/
data x( 4) /1.8343464249564980d-1/, w( 4) /3.6268378337836198d-1/
data x( 5) /9.8940093499164993d-1/, w( 5) /2.7152459411754095d-2/
data x( 6) /9.4457502307323258d-1/, w( 6) /6.2253523938647893d-2/
data x( 7) /8.6563120238783174d-1/, w( 7) /9.5158511682492785d-2/
data x( 8) /7.5540440835500303d-1/, w( 8) /1.2462897125553387d-1/
data x( 9) /6.1787624440264375d-1/, w( 9) /1.4959598881657673d-1/
data x(10) /4.5801677765722739d-1/, w(10) /1.6915651939500254d-1/
data x(11) /2.8160355077925891d-1/, w(11) /1.8260341504492359d-1/
data x(12) /9.5012509837637440d-2/, w(12) /1.8945061045506850d-1/
```

6.7.3 Generators

6.8 Dumping Parameters

81b \langle params.f 81b $\rangle \equiv$

```
program params
implicit none
 $\langle$ Accelerator codes 11c $\rangle$ 
integer acc, ver, i
double precision roots(7)
data roots / 90D0, 170D0, 350D0, 500D0, 800D0, 1000D0, 1500D0 /
do 10 ver = 7, 8
  print *, 'VERSION ', ver
  do 11 acc = TESLA, XBNDEE
    do 12 i = 1, 7
      print *, '=====
      call circes (0d0, 0d0, roots(i), acc, ver, 20020307, 0)
      call dump ()
12      continue
11    continue
10  continue
end
```

Uses TESLA 11c and circes 29a.

81c \langle params.f 81b $\rangle + \equiv$

```
subroutine dump
implicit none
 $\langle$ /circom/ 29b $\rangle$ 
 $\langle$ Accelerator codes 11c $\rangle$ 
character*9 name
```

```

      if (acc .eq. SBAND) then
        name = 'SBAND'
      else if (acc .eq. TESLA) then
        name = 'TESLA'
      else if (acc .eq. JLCNLC) then
        name = 'JLCNLC'
      else if (acc .eq. SBNDEE) then
        name = 'SBAND/EE'
      else if (acc .eq. TESLEE) then
        name = 'TESLA/EE'
      else if (acc .eq. XBNDEE) then
        name = 'JLCNLC/EE'
      end if
      write (*, 1000) name, roots
      write (*, 1001) 'e+ e-', lumi
      write (*, 1002) 'e+/e-', a1(0)
      write (*, 1003) 'e+/e-', 1 - a1(0)
      write (*, 1004) 'e+/e-', a1(1), a1(2), a1(3)
      write (*, 1003) 'gamma', a1(7)
      write (*, 1004) 'gamma', a1(4), a1(5), a1(6)
1000 format (A9, ' @ ', F5.0, ' GeV')
1001 format (4X, A7, ' lumi          = ', F7.2,
$          ' * 1032 cm-2 sec-1')
1002 format (4X, A7, ' delta strength = ', F9.5)
1003 format (4X, A7, ' integral(cont.) = ', F9.5)
1004 format (4X, A7, ' distribution   = ',
$          F9.5, ' * x{', F9.5, '}' * (1-x){', F9.5, '}'')
      end

```

Uses SBAND 11c and TESLA 11c.

7 Fitting

7.1 Version 1: Factorized Beta Distributions

```

82 <fit_v1.f90 82>≡
   c fit_v1.f90 -- fitting for circe
   <Copyleft notice 27b>
   program fit
   implicit none
   external fct
   integer i, rcode
   <Declare NPARAM 83a>
   <Declare parameters 83b>
   <Declare arguments 83c>
   <Initialize parameters for fit_v1.f90 83e>
   call mninit (5, 6, 7)
   <Load parameters 83d>

```

```

call mnseti ('CIRCE: fit version 1      ')
argv(1) = 1
call mnexcm (fct, 'SET PRINTOUT        ', argv, 1, rcode, 0d0)
argv(1) = 1
call mnexcm (fct, 'CALL FCT            ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'MIGRAD              ', argv, 0, rcode, 0d0)
call mnexcm (fct, 'MINOS               ', argv, 0, rcode, 0d0)
argv(1) = 3
call mnexcm (fct, 'CALL FCT            ', argv, 1, rcode, 0d0)
call mnexcm (fct, 'STOP                ', argv, 0, rcode, 0d0)
end

```

Defines:

fit, used in chunks 97d and 101d.

Uses circe 28b and fct 83f.

83a *<Declare NPARAM 83a>*≡
integer NPARAM
parameter (NPARAM = 6)

Defines:

NPARAM, used in chunks 83 and 89e.

83b *<Declare parameters 83b>*≡
integer pnum(NPARAM)
character*10 pname(NPARAM)
double precision pstart(NPARAM), pstep(NPARAM)

Uses NPARAM 83a.

83c *<Declare arguments 83c>*≡
integer ARGC
parameter (ARGC = 10)
double precision argv(ARGC)

83d *<Load parameters 83d>*≡
do 10 i = 1, NPARAM
call mnparm (pnum(i), pname(i), pstart(i), pstep(i),
\$ 0d0, 0d0, rcode)
if (rcode .ne. 0) then
print *, 'fit: MINUIT won''t accept parameter ', pnum(i)
stop
endif
10 continue

Uses NPARAM 83a.

83e *<Initialize parameters for fit_v1.f90 83e>*≡
data pnum / 1, 2, 3, 4, 5, 6 /
data pname / '1_e', 'x_e', '1-x_e', '1_g', 'x_g', '1-x_g' /
data pstart / -1.00, 20.00, 0.20, -1.00, 0.20, 20.00 /
data pstep / 0.01, 0.01, 0.01, 0.01, 0.01, 0.01 /

83f *<fit_v1.f90 82>*+≡
subroutine fct (nx, df, f, a, mode, g)
implicit none
integer nx, mode

```

double precision f, df(*), a(*), g
  <Local variables for fct (v1) 84c>
external scale
if (mode .eq. 1) then
  <Read input data (v1) 84a>
else if (mode .eq. 2) then
  <Calculate  $\nabla f$  87c>
endif
  <Calculate f (v1) 87d>
999 continue
if (mode .eq. 3) then
  <Write output (v1) 89d>
endif
end

Defines:
  fct, used in chunks 82 and 97.
Uses scale 87b.

84a <Read input data (v1) 84a>≡
  <Read data from file 84b>
  <Fixup errors 85e>
  <Normalize 86b>

84b <Read data from file 84b>≡
  call gethst ('ee', NDATA, xee, fee, dfee, see, tee, pwr)
  call gethst ('eg', NDATA, xeg, feg, dfeg, seg, teg, pwr)
  call gethst ('ge', NDATA, xge, fge, dfge, sge, tge, pwr)
  call gethst ('gg', NDATA, xgg, fgg, dfgg, sgg, tgg, pwr)

Uses gethst 84d.

84c <Local variables for fct (v1) 84c>≡
  integer NDATA
  parameter (NDATA = 20)
  double precision see, tee, dtee, xee(2,0:NDATA+1,0:NDATA+1),
$   fee(0:NDATA+1,0:NDATA+1), dfee(0:NDATA+1,0:NDATA+1)
  double precision seg, teg, dteg, xeg(2,0:NDATA+1,0:NDATA+1),
$   feg(0:NDATA+1,0:NDATA+1), dfeg(0:NDATA+1,0:NDATA+1)
  double precision sge, tge, dtge, xge(2,0:NDATA+1,0:NDATA+1),
$   fge(0:NDATA+1,0:NDATA+1), dfge(0:NDATA+1,0:NDATA+1)
  double precision sgg, tgg, dtgg, xgg(2,0:NDATA+1,0:NDATA+1),
$   fgg(0:NDATA+1,0:NDATA+1), dfgg(0:NDATA+1,0:NDATA+1)
  double precision pwr

84d <fit_v1.f90 82>+≡
  subroutine gethst (tag, ndata, x, f, df, s, t, pwr)
  implicit none
  character*(2) tag
  integer ndata
  double precision s, t, pwr, x(2,0:ndata+1,0:ndata+1)
  double precision f(0:ndata+1,0:ndata+1), df(0:ndata+1,0:ndata+1)
  integer i, j
  open (10, file = 'lumidiff-'//tag//'.dat')

```

```

read (10, *) pwr
s = 0d0
<Read continuum, summing in s 85a>
t = s
<Read single  $\delta$ , summing in t 85b>
<Read double  $\delta$ , summing in t 85d>
close (10)
end

```

Defines:

gethst, used in chunk 84b.

```

85a <Read continuum, summing in s 85a>≡
      do 10 i = 1, ndata
        do 11 j = 1, ndata
          read (10, *) x(1,i,j), x(2,i,j), f(i,j), df(i,j)
          s = s + f(i,j)
11      continue
10      continue

85b <Read single  $\delta$ , summing in t 85b>≡
      do 20 i = 1, ndata
        read (10, *) x(1,i,0), f(i,0), df(i,0),
$          f(i,ndata+1), df(i,ndata+1)
        x(1,i,ndata+1) = x(1,i,0)
        t = t + f(i,0) + f(i,ndata+1)
20      continue

85c <Read single  $\delta$ , summing in t 85b>+≡
      do 21 i = 1, ndata
        read (10, *) x(2,0,i), f(0,i), df(0,i),
$          f(ndata+1,i), df(ndata+1,i)
        x(2,ndata+1,i) = x(2,0,i)
        t = t + f(0,i) + f(ndata+1,i)
21      continue

85d <Read double  $\delta$ , summing in t 85d>≡
      read (10, *) f(0,0), df(0,0), f(0,ndata+1), df(0,ndata+1)
      t = t + f(0,0) + f(0,ndata+1)
      read (10, *) f(ndata+1,0), df(ndata+1,0),
$          f(ndata+1,ndata+1), df(ndata+1,ndata+1)
      t = t + f(ndata+1,0) + f(ndata+1,ndata+1)

```

Guinea-Pig does not provide the full error. A Monte Carlo study shows that it is a reasonable approximation to rescale the bin error by suitable factors. These factors are different for each distribution and the factors for the δ -pieces are bigger than those for the continuum parts. The follows factors are for the **slow** parameter set.

```

85e <Fixup errors 85e>≡
      call fixerr (NDATA, dfec, 20d0, 30d0, 40d0)
      call fixerr (NDATA, dfeg, 15d0, 20d0, 0d0)
      call fixerr (NDATA, dfge, 15d0, 20d0, 0d0)
      call fixerr (NDATA, dfgg, 10d0, 0d0, 0d0)

```

Uses fixerr 86a.

```

86a  <fit_v1.f90 82>+≡
      subroutine fixerr (ndata, df, c, sd, dd)
      implicit none
      integer ndata
      double precision df(0:ndata+1,0:ndata+1), c, sd, dd
      integer i, j
      do 1 i = 1, NDATA
        do 2 j = 1, NDATA
          df(i,j) = c * df(i,j)
2      continue
1    continue
      do 3 i = 1, NDATA
        df(0,i) = sd * df(0,i)
        df(i,0) = sd * df(i,0)
        df(ndata+1,i) = sd * df(ndata+1,i)
        df(i,ndata+1) = sd * df(i,ndata+1)
3      continue
      df(0,0) = dd * df(0,0)
      df(ndata+1,0) = dd * df(ndata+1,0)
      df(0,ndata+1) = dd * df(0,ndata+1)
      df(ndata+1,ndata+1) = dd * df(ndata+1,ndata+1)
      end

```

Defines:

`fixerr`, used in chunk 85e.

The error on the integrated luminosity is obtained from adding the error in channels in quadrature.

```

86b  <Normalize 86b>≡
      dtee = sumsqu (NDATA, dfee)
      dteg = sumsqu (NDATA, dfeg)
      dtge = sumsqu (NDATA, dfge)
      dtgg = sumsqu (NDATA, dfgg)

```

Uses `sumsqu` 86d.

```

86c  <Local variables for fct (v1) 84c>+≡
      double precision sumsqu
      external sumsqu

```

Uses `sumsqu` 86d.

```

86d  <fit_v1.f90 82>+≡
      double precision function sumsqu (ndata, f)
      implicit none
      integer ndata
      double precision f(0:ndata+1,0:ndata+1)
      integer i, j
      double precision s2
      s2 = 0
      do 100 i = 0, NDATA+1
        do 101 j = 0, NDATA+1
          s2 = s2 + f(i,j)*f(i,j)
101      continue

```

```

100 continue
    sumsqu = sqrt (s2)
end

```

Defines:

sumsqu, used in chunk 86.

```

87a <Normalize 86b>+≡
    call scale (NDATA, 1d0/tee, fee)
    call scale (NDATA, 1d0/tee, dfec)
    call scale (NDATA, 1d0/tee, feg)
    call scale (NDATA, 1d0/tee, dfeg)
    call scale (NDATA, 1d0/tee, fge)
    call scale (NDATA, 1d0/tee, dfge)
    call scale (NDATA, 1d0/tee, fgg)
    call scale (NDATA, 1d0/tee, dfgg)

```

Uses scale 87b.

```

87b <fit_v1.f90 82>+≡
    subroutine scale (ndata, s, f)
    implicit none
    integer ndata
    double precision s, f(0:ndata+1,0:ndata+1)
    integer i, j
    do 100 i = 0, NDATA+1
        do 101 j = 0, NDATA+1
            f(i,j) = s * f(i,j)
101        continue
100    continue
end

```

Defines:

scale, used in chunks 83f and 87a.

```

87c <Calculate  $\nabla f$  87c>≡
    print *, 'ERROR:  $\nabla f$  n.a.'
stop

```

Log-likelihood won't fly, because we can't normalize the likelihood function for an unbounded parameter range. Let's use good ole least-squares instead.

```

87d <Calculate  $f(v1)$  87d>≡
    f = 0d0
    do 10 i = 1, NDATA
        do 11 j = 1, NDATA
            if (dfec(i,j) .gt. 0d0) then
                f = f + ((phie(xee(1,i,j),a) * phie(xee(2,i,j),a)
$                    - fec(i,j)) / dfec(i,j))**2
            endif
            if (dfeg(i,j) .gt. 0d0) then
                f = f + ((phie(xeg(1,i,j),a) * phig(xeg(2,i,j),a)
$                    - feg(i,j)) / dfeg(i,j))**2
            endif
            if (dfge(i,j) .gt. 0d0) then
                f = f + ((phig(xge(1,i,j),a) * phie(xge(2,i,j),a)

```

```

$          - fge(i,j)) / dfge(i,j))**2
    endif
    if (dfgg(i,j) .gt. 0d0) then
        f = f + ((phig(xgg(1,i,j),a) * phig(xgg(2,i,j),a)
$          - fgg(i,j)) / dfgg(i,j))**2
    endif
11      continue
10      continue
Uses phie 89b and phig 89c.
88a  <Local variables for fct (v1) 84c>+≡
      integer i, j
      double precision phie, phig, delta
      external phie, phig
Uses phie 89b and phig 89c.
88b  <Calculate f (v1) 87d>+≡
      if ((a(2) .le. -1d0) .or. (a(3) .le. -1d0/pwr)) then
          print *, 'warning: discarding out-of-range a2/3: ', a(2), a(3)
          <Give up on f 88c>
      endif
      delta = 1d0 - exp(a(1)) * beta(a(2)+1d0,a(3)+1d0/pwr)
$          * dble(NDATA) / pwr
      if (delta .lt. 0d0) then
          print *, 'warning: delta forced to 0 from ', delta
          delta = 0d0
      endif
Uses beta 93a.
88c  <Give up on f 88c>≡
      f = 1d100
      goto 999
88d  <Calculate f (v1) 87d>+≡
      do 12 i = 1, NDATA
          if (dfee(ndata+1,i) .gt. 0d0) then
              f = f + ((delta*phie(xee(2,ndata+1,i),a)
$              - fee(ndata+1,i)) / dfce(ndata+1,i))**2
          endif
          if (dfeg(ndata+1,i) .gt. 0d0) then
              f = f + ((delta*phig(xeg(2,ndata+1,i),a)
$              - feg(ndata+1,i)) / dfeg(ndata+1,i))**2
          endif
          if (dfce(i,ndata+1) .gt. 0d0) then
              f = f + ((delta*phie(xee(1,i,ndata+1),a)
$              - fee(i,ndata+1)) / dfce(i,ndata+1))**2
          endif
          if (dfge(i,ndata+1) .gt. 0d0) then
              f = f + ((delta*phig(xge(1,i,ndata+1),a)
$              - fge(i,ndata+1)) / dfge(i,ndata+1))**2
          endif
12      continue

```

Uses phie 89b and phig 89c.

```
89a  <Calculate f (v1) 87d>+≡
      if (dfee(ndata+1,ndata+1) .gt. 0d0) then
        f = f + ((delta*delta
$          - fee(ndata+1,ndata+1)) / dfee(ndata+1,ndata+1))**2
      endif
```

```
89b  <fit_v1.f90 82>+≡
      double precision function phie (x, a)
      implicit none
      double precision x, a(6)
      phie = exp (a(1) + a(2)*log(x) + a(3)*log(1d0-x))
      end
```

Defines:

phie, used in chunks 87, 88, and 91d.

```
89c  <fit_v1.f90 82>+≡
      double precision function phig (x, a)
      implicit none
      double precision x, a(6)
      phig = exp (a(4) + a(5)*log(x) + a(6)*log(1d0-x))
      end
```

Defines:

phig, used in chunks 87, 88, and 91d.

```
89d  <Write output (v1) 89d>≡
      a1(1) = exp(a(1)) * dble(NDATA) / pwr
      a1(2) = a(2)
      a1(3) = a(3) - 1d0 + 1d0/pwr
      a1(4) = exp(a(4)) * dble(NDATA) / pwr
      a1(5) = a(5) - 1d0 + 1d0/pwr
      a1(6) = a(6)
      open (10, file = 'Parameters')
      write (10, 1000) REV, tee / 1D32
1000 format ( '      data xa5lum(@ENERGY@,@ACC@,', I2, ') / ',
$          E12.5, ' /')
      write (10, 1001) REV,
$      1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0),
$      a1(1), a1(2), a1(3), a1(4), a1(5), a1(6),
$      a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
1001 format ( '      data (xa5(i,@ENERGY@,@ACC@,', I2 ,'),i=0,7) /', /,
$          '      $ ', 4(E12.5,', '), /,
$          '      $ ', 3(E12.5,', '), E12.5, ' /')
      close (10)
```

Uses beta 93a.

```
89e  <Local variables for fct (v1) 84c>+≡
      <Declare NPARAM 83a>
      double precision beta, a1(NPARAM)
      integer REV
      parameter (REV = 1)
```

Uses NPARAM 83a and beta 93a.

The average elektron energy in the continuum can be calculated analytically:

$$\begin{aligned}\langle E_{e^\pm} \rangle_{\text{cont}} &= E_{\text{beam}} \langle x_{e^\pm} \rangle_{\text{cont}} = E_{\text{beam}} \frac{\int dx x^{a_2} (1-x)^{a_3} x}{B(a_2, a_3)} \\ &= E_{\text{beam}} \frac{B(a_2+1, a_3)}{B(a_2, a_3)} = E_{\text{beam}} \frac{a_2+1}{a_2+a_3+2}\end{aligned}\quad (40)$$

```
90a  <Write output (v1) 89d>+≡
      delta = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
      print *, '< x_e > = ',
      $      delta + (1d0-delta)*(a1(2)+1d0)/(a1(2)+a1(3)+2d0)
```

Uses beta 93a.

similarly:

$$\langle E_\gamma \rangle = E_{\text{beam}} \frac{a_5+1}{a_5+a_6+2}\quad (41)$$

```
90b  <Write output (v1) 89d>+≡
      print *, '< x_g > = ',
      $      (a1(5)+1d0)/(a1(5)+a1(6)+2d0)
```

Count the degrees of freedom in ndof:

```
90c  <Write output (v1) 89d>+≡
      ndof = 0
      do 40 i = 0, ndata+1
        do 41 j = 0, ndata+1
          if (dfec(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfeg(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfge(i,j) .gt. 0d0) ndof = ndof + 1
          if (dfgg(i,j) .gt. 0d0) ndof = ndof + 1
        41 continue
      40 continue
      print *, 'CHI2 = ', f / ndof
```

```
90d  <Local variables for fct (v1) 84c>+≡
      integer ndof
```

The error on the luminosity is just the (possibly rescaled) counting error:

```
90e  <Write output (v1) 89d>+≡
      open (10, file = 'Errors.tex')
      write (10, 1099) tee / 1d32, dtee / 1d32, dtee / 1d32
      1099 format ('$', F8.2, '_{-', F4.2, '}^{+', F4.2, '}$')
```

After retrieving the error from MINUIT, we have to take care of the mapping of the parameters

$$a'_{1/4} = e^{a_{1/4}} B(a_{2/5}+1, a_{3/6}+1) N_{\text{bins}} \eta^{-1} \implies \delta a'_{1/4} = a'_{1/4} \delta a_{1/4}\quad (42)$$

ignoring the errors in the integral (i.e. the Beta function).

```
90f  <Write output (v1) 89d>+≡
      call mnerrs (1, eplus, eminus, epara, corr)
      ab = a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
      write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
      1100 format ('$', F8.4, '_{-', F6.4, '}^{+', F6.4, '}$')
```

Uses beta 93a.

91a *<Local variables for fct (v1) 84c>+≡*
double precision ab

The other mappings are even more trivial:

$$a'_{2/6} = a_{2/6} - 1 + \eta^{-1} \implies \delta a'_{2/6} = \delta a_{2/6} \quad a'_{3/5} = a_{3/5} - 1 + \eta^{-1} \implies \delta a'_{3/5} = \delta a_{3/5} \quad (43)$$

91b *<Write output (v1) 89d>+≡*
do 110 i = 2, 3
call mnerrs (i, eplus, eminus, epara, corr)
write (10, 1100) a1(i), abs (eminus), abs (eplus)
110 continue
call mnerrs (4, eplus, eminus, epara, corr)
ab = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
do 111 i = 5, 6
call mnerrs (i, eplus, eminus, epara, corr)
write (10, 1100) a1(i), abs (eminus), abs (eplus)
111 continue
close (10)

Uses beta 93a.

91c *<Local variables for fct (v1) 84c>+≡*
double precision eplus, eminus, epara, corr
integer n

91d *<Write output (v1) 89d>+≡*
do 30 n = 1, 10
call pslice ('ee','x',n,NDATA,xee,fee,dfee,phie,phie,a)
call pslice ('eg','x',n,NDATA,xeg,feg,dfeg,phie,phig,a)
call pslice ('ge','x',n,NDATA,xge,fge,dfge,phig,phie,a)
call pslice ('gg','x',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
call pslice ('ee','y',n,NDATA,xee,fee,dfee,phie,phie,a)
call pslice ('eg','y',n,NDATA,xeg,feg,dfeg,phie,phig,a)
call pslice ('ge','y',n,NDATA,xge,fge,dfge,phig,phie,a)
call pslice ('gg','y',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
30 continue
call pslice ('ee','x',21,NDATA,xee,fee,dfee,phie,phie,a)
call pslice ('eg','x',21,NDATA,xeg,feg,dfeg,phie,phig,a)
call pslice ('ee','y',21,NDATA,xee,fee,dfee,phie,phie,a)
call pslice ('ge','y',21,NDATA,xge,fge,dfge,phig,phie,a)

Uses phie 89b, phig 89c, and pslice 92.

UNIX Fortran compiler want backslashes escaped:

91e *<Write output (v1) 89d>+≡*
open (10, file = 'Slices.mp4')
write (10,*) 'picture eslice[], gslice[];'
do 31 n = 1, NDATA
write (10,*) 'eslice[' , n, ']' := ' ,
\$ 'btex \$x_{e^{\backslash pm}} = ' , xee(1,n,1), '\$ etex;'
write (10,*) 'gslice[' , n, ']' := ' ,

```

$          'btex $x_\gamma = ', xgg(1,n,1), '$ etex;'
31  continue
    close (10)

92  <fit_v1.f90 82>+≡
    subroutine pslice (pp, xy, n, ndata, x, f, df, phi1, phi2, a)
    implicit none
    character*2 pp
    character*1 xy
    integer n, ndata
    double precision x(2,0:ndata+1,0:ndata+1)
    double precision f(0:ndata+1,0:ndata+1), df(0:ndata+1,0:ndata+1)
    double precision a(6)
    double precision z
    double precision phi1, phi2, d, delta, pwr, beta
    external phi1, phi2, beta
    integer i
    character*2 digits
    write (digits, '(I2.2)') n
    open (10, file = 'lumidiff-'//pp//xy//digits//'.dat')
    open (11, file = 'lumidiff-'//pp//xy//digits//'.fit')
    open (12, file = 'lumidiff-'//pp//xy//digits//'.chi')
    if (n .eq. ndata+1) then
        pwr = 5d0
        delta = 1d0 - exp(a(1))*beta(a(2)+1d0,a(3)+1d0/pwr)
$          * dble(NDATA) / pwr
    else
        delta = 0
    endif
    if (xy .eq. 'x') then
        do 10 i = 1, ndata
            if (df(n,i) .gt. 0d0) then
                if (pp(2:2) .eq. 'g') then
                    z = x(2,n,i)
                else
                    z = 1d0 - x(2,n,i)
                endif
                if (n .eq. ndata+1) then
                    d = delta*phi2(x(2,n,i),a)
                else
                    d = phi1(x(1,n,i),a)*phi2(x(2,n,i),a)
                endif
                write (10,*) z, f(n,i), df(n,i)
                write (11,*) z, d
                write (12,*) z, (f(n,i) - d) / df(n,i)
            endif
10      continue
        else if (xy .eq. 'y') then
            do 11 i = 1, ndata
                if (df(i,n) .gt. 0d0) then

```

```

        if (pp(1:1) .eq. 'g') then
            z = x(1,i,n)
        else
            z = 1d0 - x(1,i,n)
        endif
        if (n .eq. ndata+1) then
            d = phi1(x(1,i,n),a)*delta
        else
            d = phi1(x(1,i,n),a)*phi2(x(2,i,n),a)
        endif
        write (10,*) z, f(i,n), df(i,n)
        write (11,*) z, d
        write (12,*) z, (f(i,n) - d) / df(i,n)
    endif
11      continue
    endif
    close (10)
    close (11)
    close (12)
end

```

Defines:

pslice, used in chunk 91d.

Uses beta 93a.

```

93a <fit_v1.f90 82>+≡
    double precision function beta (a, b)
    implicit none
    double precision a, b
    double precision dlgamma
    beta = exp (dlgamma(a) + dlgamma(b) - dlgamma(a+b))
end

```

Defines:

beta, used in chunks 29a, 54b, 57c, 61b, 64b, 67d, 88-92, and 95a.

```

93b <fit_v1 93b>≡
    #! /bin/sh
    # mode=${2-slow}
    mode=${2-fast}
    root='pwd'
    indir=${root}/${3-input}
    tmpdir=${root}/tmp
    outdir=${root}/output
    acc="${1-sband350 sband500 sband800 sband1000 sband1600
        tesla350 tesla500 tesla800 tesla1000 tesla1600
        tesla350-low tesla500-low tesla800-low tesla1000-low tesla1600-low
        xband350 xband500 xband800 xband1000 xband1600}"

```

```

93c <fit_v1 93b>+≡
    xmkdir () {
        for d in "$@"; do
            mkdir $d 2>/dev/null || true
        done
    }

```

```

}
rm -fr ${tmpdir}
mkdir ${outdir} ${tmpdir}
94a <fit_v1 93b>+=
cd ${tmpdir}
cat /dev/null >${outdir}/Params.f90
for a in $acc; do
  case "$a" in
    *1600*) energy=TEV16;;
    *1000*) energy=TEV1;;
    *800*) energy=GEV800;;
    *500*) energy=GEV500;;
    *3[56]0*) energy=GEV350;;
    *170*) energy=GEV170;;
    *90*) energy=GEV090;;
    *) energy=GEV500;;
  esac
  cp ${indir}/${a}_${mode}/lumidiff-???.dat .
  ${root}/fit_v1.bin
  rm -fr ${outdir}/${a}_${mode}
  mkdir ${outdir}/${a}_${mode}
  cp Slices.mp4 ${outdir}
  cp Errors.tex lumidiff-??x[0-9][0-9].??? ${outdir}/${a}_${mode}
  sed -e "s/@ENERGY@/$energy/g" \
    -e "s/@ACC@/'echo $a | tr a-z A-Z | tr -cd A-Z'/g" Parameters \
    >>${outdir}/Params.f90
done
cd ${root}
rm -fr ${tmpdir}
94b <fit_v1 93b>+=
cat >${outdir}/Params.tex <<'END'
\begin{table}
\begin{center}
\renewcommand{\arraystretch}{1.3}
\begin{tabular}{|c|c|c|c|c|}\hline
& \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA}' & \texttt{XBAND} \\
\hline\hline
\end{tabular}
\end{center}
END
Uses SBAND 11c, TESLA 11c, and XBAND 11c.
94c <fit_v1 93b>+=
line () {
  for a in $acc; do
    case $a in
      *350* | *800* | *1000* | *1600*)
        ;;
      *) echo -n ' & '
        sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
        ;;
    esac
  done
}

```

```

        esac
    done
    echo '\\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\text{upsilon}^{-1}$'; line 1
echo '$\int d_{\text{e}^{\text{pm}}}$'; line 2
echo '$x_{\text{e}^{\text{pm}}}^{\alpha}$'; line 3
echo '$(1-x_{\text{e}^{\text{pm}}})^{\alpha}$'; line 4
echo '$\int d_{\text{gamma}}$'; line 5
echo '$x_{\text{gamma}}^{\alpha}$'; line 6
echo '$(1-x_{\text{gamma}})^{\alpha}$'; line 7
) >>${outdir}/Params.tex
95a <fit_v1 93b>+=
    cat >>${outdir}/Params.tex <<'END'
        \end{tabular}
    \end{center}
    \caption{\label{tab:param}%
        Version 1, revision 1997 04 16 of the beam spectra at 500 GeV.
        The rows correspond to the luminosity per effective year, the
        integral over the continuum and the powers in the factorized Beta
        distributions~(\ref{eq:beta}).}
    \end{table}
END
Uses beta 93a.
95b <fit_v1 93b>+=
    cat >>${outdir}/Params.tex <<'END'
        \begin{table}
        \begin{center}
            \renewcommand{\arraystretch}{1.3}
            \begin{tabular}{|c||c|c|c|c|}\hline
                & \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA}' & \texttt{XBAND} \\
                \\\hline\hline
            \end{tabular}
        \end{center}
        END
Uses SBAND 11c, TESLA 11c, and XBAND 11c.
95c <fit_v1 93b>+=
    line () {
        for a in $acc; do
            case $a in
                *1000*)
                    echo -n ' & '
                    sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                    ;;
            esac
        done
        echo '\\\\hline'
    }
(echo '$\mathcal{L}/\text{fb}^{-1}\text{upsilon}^{-1}$'; line 1
echo '$\int d_{\text{e}^{\text{pm}}}$'; line 2

```

```

        echo '$x_{e\pm}^{\alpha}$'; line 3
        echo '$(1-x_{e\pm})^{\alpha}$'; line 4
        echo '$\int d\gamma$'; line 5
        echo '$x_{\gamma}^{\alpha}$'; line 6
        echo '$(1-x_{\gamma})^{\alpha}$'; line 7
    ) >>${outdir}/Params.tex

96a (fit_v1 93b)+=
    cat >>${outdir}/Params.tex <<'END'
        \end{tabular}
        \end{center}
        \caption{\label{tab:param/TeV}%
            Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.}
        \end{table}
    END

96b (fit_v1 93b)+=
    cat >>${outdir}/Params.tex <<'END'
        \begin{table}
        \begin{center}
            \renewcommand{\arraystretch}{1.3}
            \begin{tabular}{|c||c|c|c|c|}\hline
                & 350 GeV & 500 GeV & 800 GeV & 1600 GeV \\
                \\\hline\hline
            \end{tabular}
        \end{center}
    END

96c (fit_v1 93b)+=
    line () {
        for a in $acc; do
            case $a in
                tesla*-low)
                    ;;
                tesla1000)
                    ;;
                tesla*)
                    echo -n ' & '
                    sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
                    ;;
            esac
        done
        echo '\\\\hline'
    }
    (echo '$\mathcal{L}/\text{fb}^{-1}\epsilon^{-1}$'; line 1
    echo '$\int d_{e\pm}$'; line 2
    echo '$x_{e\pm}^{\alpha}$'; line 3
    echo '$(1-x_{e\pm})^{\alpha}$'; line 4
    echo '$\int d\gamma$'; line 5
    echo '$x_{\gamma}^{\alpha}$'; line 6
    echo '$(1-x_{\gamma})^{\alpha}$'; line 7
    ) >>${outdir}/Params.tex

```

```

97a  <fit_v1 93b>+=
      cat >>${outdir}/Params.tex <<'END'
      \end{tabular}
      \end{center}
      \caption{\label{tab:param/Tesla}%
        Version 1, revision 1997 04 17 of the beam spectra for TESLA.}
      \end{table}
      END
      exit 0
Uses TESLA 11c.

```

7.2 Experimental

7.2.1 Quasi One Dimensional

```

97b  <minuit1.f90 97b>≡
      c minuit1.f90 -- fitting for circe
      <Copyleft notice 27b>
Uses circe 28b.

```

We're utilizing the familiar “MINUIT” package [15].

```

97c  <minuit1.f90 97b>+=
      <Minuit main program 97d>
      <Function to minimize 97e>
97d  <Minuit main program 97d>≡
      program fit
      implicit none
      external fct
      call minuit (fct, 0d0)
      end

```

Uses fct 83f and fit 82.

```

97e  <Function to minimize 97e>≡
      subroutine fct (nx, df, f, a, mode, g)
      implicit none
      integer nx, mode
      double precision f, df(*), a(*), g
      <Local variables for fct 98b>
      if (mode .eq. 1) then
        <Read input data 98a>
      else if (mode .eq. 2) then
        <Calculate ∇f 87c>
      endif
      <Calculate f 98c>
      if (mode .eq. 3) then
        <Write output 98d>
      endif
      end

```

Uses fct 83f.

```

98a  <Read input data 98a>≡
      open (10, file = 'minuit.data')
      do 10 i = 1, NDATA
        do 11 j = 1, NDATA
          read (10, *) xi(1,i,j), xi(2,i,j), fi(i,j), dfi(i,j)
          fi(i,j) = fi(i,j)/1d30
          dfi(i,j) = dfi(i,j)/1d30
11      continue
10      continue
      close (10)

98b  <Local variables for fct 98b>≡
      integer NDATA
      parameter (NDATA = 20)
      double precision xi(2,NDATA,NDATA),
$      fi(NDATA,NDATA), dfi(NDATA,NDATA)
      integer i, j, n
      double precision phi, chi, chi2

98c  <Calculate f 98c>≡
      f = 0d0
      do 110 i = 1, NDATA
        do 111 j = 1, NDATA
          if (dfi(i,j).gt.0d0) then
            f = f + ((phi(xi(1,i,j),xi(2,i,j),a)
$              - fi(i,j)) / dfi(i,j))**2
          endif
111      continue
110      continue

98d  <Write output 98d>≡
      chi2 = 0d0
      n = 0
      open (10, file = 'minuit.fit')
      do 210 i = 1, NDATA
        do 211 j = 1, NDATA
          if (dfi(i,j).gt.0d0) then
            chi = (phi(xi(1,i,j),xi(2,i,j),a)-fi(i,j))/dfi(i,j)
            write (10,*) xi(1,i,j), xi(2,i,j),
$              1d30 * phi(xi(1,i,j),xi(2,i,j),a),
$              1d30 * fi(i,j),
$              chi
            chi2 = chi2 + chi**2
            n = n + 1
          else
            write (10,*) xi(1,i,j), xi(2,i,j),
$              1d30 * phi(xi(1,i,j),xi(2,i,j),a),
$              1d30 * fi(i,j)
          endif
211      continue
210      continue

```

```

        close (10)
        print *, 'CHI2 = ', chi2/n
99a  <minuit1.f90 97b>+=
        double precision function phi (e1, e2, a)
        implicit none
        double precision e1, e2, a(17)
        double precision y1, y2
        y1 = e1 / 250d0
        y2 = e2 / 250d0
        phi = exp (
$      + a( 1) * 1d0
$      + a( 2) * log(y1)
$      + a( 3) * log(1d0-y1)
$      + a( 4) * log(-log(y1))
$      + a( 5) * log(-log(1d0-y1))
$      + a( 6) * y1
$      + a( 7) * log(y1)**2
$      + a( 8) * log(1d0-y1)**2
$      + a( 9) * log(-log(y1))**2
$      + a(10) * log(-log(1d0-y1))**2
$      + a(11) * y1**2
$      + a(12) / log(y1)
$      + a(13) / log(1d0-y1)
$      + a(14) / log(-log(y1))
$      + a(15) / log(-log(1d0-y1))
$      + a(16) / y1
$      + a(17) / (1d0-y1)
$      + a( 2) * log(y2)
$      + a( 3) * log(1d0-y2)
$      + a( 4) * log(-log(y2))
$      + a( 5) * log(-log(1d0-y2))
$      + a( 6) * y2
$      + a( 7) * log(y2)**2
$      + a( 8) * log(1d0-y2)**2
$      + a( 9) * log(-log(y2))**2
$      + a(10) * log(-log(1d0-y2))**2
$      + a(11) * y2**2
$      + a(12) / log(y2)
$      + a(13) / log(1d0-y2)
$      + a(14) / log(-log(y2))
$      + a(15) / log(-log(1d0-y2))
$      + a(16) / y2
$      + a(17) / (1d0-y2)
$    )
        end
99b  <minuit1.sh 99b>≡
        #! /bin/sh
        minuit_bin='pwd'/minuit1.bin
        <Process arguments 100a>

```

```

(
  <Define parameters 100d>
  <Fix parameters 101a>
  <Fix strategy 101b>
  <Run Minuit 101c>
) | eval "$minuit_bin $filter"
<Maybe plot results 101d>
exit 0

100a <Process arguments 100a>≡
  tmp="$IFS"
  IFS=:
  args=":$*:"
  IFS="$tmp"

100b <Process arguments 100a>+≡
  filter="| \
    awk '/STATUS=(CONVERGED|CALL LIMIT|FAILED)/ { p=1; print }; \
    /@.* \.00000 *fixed/ { next }; \
    /EDM=|CHI2|@/ && p { print }' "

100c <Process arguments 100a>+≡
  case "$args" in
    *:v:*) filter=;;
    *)
  esac

100d <Define parameters 100d>≡
  cat <<END
  set title
  CIRCE
  parameters
  1 '@ 1          ' 0.00 0.01
  2 '@ lx         ' 0.20 0.01
  3 '@ l(1-x)     ' 0.20 0.01
  4 '@ llx        ' 0.00 0.01
  5 '@ ll(1-x)    ' 0.00 0.01
  6 '@ x          ' 0.00 0.01
  7 '@ lx^2       ' 0.00 0.01
  8 '@ l(1-x)^2   ' 0.00 0.01
  9 '@ llx^2      ' 0.00 0.01
  10 '@ ll(1-x)^2 ' 0.00 0.01
  11 '@ x^2       ' 0.00 0.01
  12 '@ 1/lx      ' 0.00 0.01
  13 '@ 1/l(1-x)  ' 0.00 0.01
  14 '@ 1/llx     ' 0.00 0.01
  15 '@ 1/ll(1-x)' 0.00 0.01
  16 '@ 1/x       ' 0.00 0.01
  17 '@ 1/(1-x)   ' 0.00 0.01

  END

```

```

101a  <Fix parameters 101a>≡
      for p in 1 2 3 4 5 6 7 8 9 10 \
              11 12 13 14 15 16 17; do
      case "$args" in
        *:$p=*) val='echo "$args" | sed 's/.*:'"$p"'=\\([0-9.-]*\\):.*\\/\\1/'';
                  echo set parameter $p $val;
                  echo fix $p;;
        *:$p:*) ;;
        *) echo fix $p;;
      esac
      done

101b  <Fix strategy 101b>≡
      case "$args" in
        *:S0:*) echo set strategy 0;;
        *:S1:*) echo set strategy 1;;
        *:S2:*) echo set strategy 2;;
      esac

101c  <Run Minuit 101c>≡
      cat <<END
      migrat 10000 0.01
      stop
      END

101d  <Maybe plot results 101d>≡
      case "$args" in
        *:p:*) awk '$5 != "" { print $1, $2, $5 }' minuit.fit > chi2
                awk '$5 != "" { print $1, $5 }' minuit.fit > chix
                awk '$5 != "" { print $2, $5 }' minuit.fit > chiy
                gnuplot -geometry -0+0 plot2 >/dev/null 2>&1
      esac

      Uses fit 82.

```

7.2.2 Quasi Two Dimensional

```

101e  <minuit2.f90 101e>≡
      c minuit2.f90 -- fitting for circe
      <Copyleft notice 27b>
      Uses circe 28b.

101f  <minuit2.f90 101e>+≡
      <Minuit main program 97d>
      <Function to minimize 97e>

101g  <minuit2.f90 101e>+≡
      double precision function phi (e1, e2, a)
      implicit none
      double precision e1, e2, a(33)
      double precision y1, y2
      y1 = e1 / 250d0
      y2 = e2 / 250d0

```

```

phi = exp (
$   + a( 1) * 1d0
$   + a( 2) * log(y1)
$   + a( 3) * log(1d0-y1)
$   + a( 4) * log(-log(y1))
$   + a( 5) * log(-log(1d0-y1))
$   + a( 6) * y1
$   + a( 7) * log(y1)**2
$   + a( 8) * log(1d0-y1)**2
$   + a( 9) * log(-log(y1))**2
$   + a(10) * log(-log(1d0-y1))**2
$   + a(11) * y1**2
$   + a(12) / log(y1)
$   + a(13) / log(1d0-y1)
$   + a(14) / log(-log(y1))
$   + a(15) / log(-log(1d0-y1))
$   + a(16) / y1
$   + a(17) / (1d0-y1)
$   + a(18) * log(y2)
$   + a(19) * log(1d0-y2)
$   + a(20) * log(-log(y2))
$   + a(21) * log(-log(1d0-y2))
$   + a(22) * y2
$   + a(23) * log(y2)**2
$   + a(24) * log(1d0-y2)**2
$   + a(25) * log(-log(y2))**2
$   + a(26) * log(-log(1d0-y2))**2
$   + a(27) * y2**2
$   + a(28) / log(y2)
$   + a(29) / log(1d0-y2)
$   + a(30) / log(-log(y2))
$   + a(31) / log(-log(1d0-y2))
$   + a(32) / y2
$   + a(33) / (1d0-y2)
$   )
end

```

```

102a <minuit2.sh 102a>≡
    #! /bin/sh
    minuit_bin='pwd'/minuit2.bin
    <Process arguments 100a>
    (
        <Define parameters (2dim) 102b>
        <Fix parameters (2dim) 103>
        <Fix strategy 101b>
        <Run Minuit 101c>
    ) | eval "$minuit_bin $filter"
    <Maybe plot results 101d>
    exit 0

```

```

102b <Define parameters (2dim) 102b>≡

```

```

cat <<END
set title
CIRCE
parameters
1  '@ 1          ' 0.00 0.01
2  '@ 1x         ' 0.20 0.01
3  '@ 1(1-x)     ' 0.20 0.01
4  '@ 11x        ' 0.00 0.01
5  '@ 11(1-x)    ' 0.00 0.01
6  '@ x          ' 0.00 0.01
7  '@ 1x^2       ' 0.00 0.01
8  '@ 1(1-x)^2   ' 0.00 0.01
9  '@ 11x^2      ' 0.00 0.01
10 '@ 11(1-x)^2  ' 0.00 0.01
11 '@ x^2        ' 0.00 0.01
12 '@ 1/1x       ' 0.00 0.01
13 '@ 1/1(1-x)   ' 0.00 0.01
14 '@ 1/11x      ' 0.00 0.01
15 '@ 1/11(1-x)' 0.00 0.01
16 '@ 1/x        ' 0.00 0.01
17 '@ 1/(1-x)    ' 0.00 0.01
18 '@ 1y         ' 0.20 0.01
19 '@ 1(1-y)     ' 0.20 0.01
20 '@ 11y        ' 0.00 0.01
21 '@ 11(1-y)    ' 0.00 0.01
22 '@ y          ' 0.00 0.01
23 '@ 1y^2       ' 0.00 0.01
24 '@ 1(1-y)^2   ' 0.00 0.01
25 '@ 11y^2      ' 0.00 0.01
26 '@ 11(1-y)^2  ' 0.00 0.01
27 '@ y^2        ' 0.00 0.01
28 '@ 1/1y       ' 0.00 0.01
29 '@ 1/1(1-y)   ' 0.00 0.01
30 '@ 1/11y      ' 0.00 0.01
31 '@ 1/11(1-y)' 0.00 0.01
32 '@ 1/y        ' 0.00 0.01
33 '@ 1/(1-y)    ' 0.00 0.01

```

END

```

103  <Fix parameters (2dim) 103>≡
      for p in 1 2 3 4 5 6 7 8 9 10 \
          11 12 13 14 15 16 17 18 19 20 \
          21 22 23 24 25 26 27 28 29 30 \
          31 32 33; do
      case "$args" in
        *:$p:*) val='echo "$args" | sed 's/.*:.'"$p"'=\\([0-9.-]*\\):.*\\/1/'';
                echo set parameter $p $val;
                echo fix $p;;
        *:$p:*) ;;

```

```

*) echo fix $p;;
esac
done

```

7.3 Version 2

8 Conclusions

I have presented a library of simple parameterizations of realistic e^\pm - and γ -beam spectra at future linear e^+e^- -colliders. The library can be used for integration and event generation. Emphasis is put on simplicity and reproducibility of the parameterizations for supporting reproducible physics simulations.

Acknowledgements

Daniel Schulte made his simulation code **Guinea-Pig** available and answered questions. Harald Anlauf and Torbjörn Sjöstrand have contributed useful suggestions. The Tesla group at DESY/Zeuthen made error estimates feasible by donating time on the multi-headed number cruncher **Hydra**. The 1996 ECFA/Desy Linear Collider Workshop got me started and provided support. Thanks to all of them.

Identifiers

ELECTR: [10b](#), [19c](#), [28b](#), [66a](#), [71a](#), [71b](#)
NACC: [11c](#), [31c](#), [32a](#), [36e](#), [37a](#), [37b](#), [37c](#), [40b](#), [40c](#), [42a](#), [42b](#), [43b](#), [43c](#), [47d](#), [49d](#), [51c](#), [55a](#), [57d](#), [61c](#)
NPARAM: [83a](#), [83b](#), [83d](#), [89e](#)
PHOTON: [10b](#), [28b](#), [66a](#), [71a](#), [71b](#), [78a](#)
POSITR: [10b](#), [19d](#), [71b](#)
SBAND: [11c](#), [32b](#), [36f](#), [36g](#), [39c](#), [40a](#), [41a](#), [41b](#), [42c](#), [43a](#), [43d](#), [45b](#), [46a](#), [81c](#), [94b](#), [95b](#)
TESLA: [11c](#), [30a](#), [32b](#), [36f](#), [36g](#), [39c](#), [41a](#), [42c](#), [43d](#), [45b](#), [46c](#), [47e](#), [47f](#), [48a](#), [49a](#), [49e](#), [50a](#), [50c](#), [51d](#), [52a](#), [52c](#), [53c](#), [55b](#), [56a](#), [58a](#), [58c](#), [61d](#), [63a](#), [81b](#), [81c](#), [94b](#), [95b](#), [97a](#)
XBAND: [11c](#), [36f](#), [36g](#), [39c](#), [40a](#), [41a](#), [41b](#), [42c](#), [43a](#), [43d](#), [45b](#), [46a](#), [53c](#), [94b](#), [95b](#)
beta: [29a](#), [54b](#), [57c](#), [61b](#), [64b](#), [67d](#), [88b](#), [89d](#), [89e](#), [90a](#), [90f](#), [91b](#), [92](#), [93a](#), [95a](#)
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circeg: [12](#), [28b](#), [38b](#), [38c](#)
circel: [37d](#), [11a](#), [37d](#)
circes: [29a](#), [11b](#), [15b](#), [29a](#), [19a](#), [29a](#), [29a](#), [29g](#), [78a](#), [81b](#)
circgg: [12](#), [28b](#), [39a](#), [39b](#), [71a](#), [71b](#)
fct: [82](#), [83f](#), [97d](#), [97e](#)
fit: [82](#), [97d](#), [101d](#)
fixerr: [85e](#), [86a](#)

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 gauss2: 80c, 13b, 14d, 17a, 80c, 80c
 gaussx: 80b, 80b, 80b, 80c
 gethst: 84b, 84d
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A Literate Programming

A.1 Paradigm

I have presented the sample code in this paper using the *literate programming* paradigm. This paradigm has been introduced by Donald Knuth [19] and his programs `TEX` [20] and `METAFONT` [21] provide excellent examples of the virtues of literate programming. Knuth summarized his intention as follows ([19], p. 99)

“Let us change our traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a *computer* what to do, let us concentrate rather on explaining to *human beings* what we want a computer to do.”

Usually, literate programming uses two utility programs to produce two kinds of files from the source

`tangle` produces the computer program that is acceptable to an “illiterate” (Fortran, C, etc.) compiler. This process consists of stripping documentation and reordering code. Therefore it frees the author from having to present the code in the particular order enforced by a compiler for purely technical reasons. Instead, the author can present the code in the order that is most comprehensible.

`weave` produces a documents that describes the program. Extensive cross referencing of the code sections is usually provided, which has been suppressed in this paper. If a powerful typesetting system (such a `TEX`) is used, the document can present the algorithms in clear mathematical notation alongside the code. These features improve readability and maintainability of scientific code immensely.

A.2 Practice

`Circe` uses the `noweb` [22] system. This system has the advantage to work with any traditional programming language and support the essential features described in section A.1 with minimal effort. `noweb`’s `tangle` program only reorders the code sections, but does not reformat them. Therefore its output can be used just like any other “illiterate” program.

The examples above should be almost self-explaining, but in order to avoid any ambiguities, I give another example:

110a `<Literate programming example 110a>≡`
 `<Code that has to be at the top 110c>`
 `<Other code 110b>`

I can start the presentation with the first line of the “other code”:

110b `<Other code 110b>≡`
 `line 1 of the other code`

If appropriate, the first line of the code that has to appear *before* the other code can be presented later:

110c `<Code that has to be at the top 110c>≡`
 `line 1 of the code at the top`

Now I can augment the sections:

```
111a  <Other code 110b>+≡  
      line 2 of the other code  
111b  <Code that has to be at the top 110c>+≡  
      line 2 of the code at the top
```

The complete “program” will be presented to the compiler as

```
line 1 of the code at the top  
line 2 of the code at the top  
line 1 of the other code  
line 2 of the other code
```

The examples in section 3.1.1 show that this reordering is particularly useful for declaring variables when they are first used (rather than at the beginning) and for zooming in on code inside of loops.

B Fortran Name Space

In addition to the ten procedures and one `common` block discussed in section 3

- `circe`, `circee`, `circeg`, `circgg`,
- `girce`, `gircee`, `girceg`, `gircgg`,
- `circes`, `circel`, `/circom/`,

there are two more globally visible functions which are used internally:

- `circem`: error message handler,
- `girceb`: efficient Beta distribution generator.

Even if the `/circom/` is globally visible, application programs *must not* manipulate it directly. The `circes`, subroutine is provided for this purpose and updates some internal parameters as well.

With features from the current Fortran standard (Fortran90), I could have kept the last two functions and the `common` block private. But since Fortran90 has only been adopted by a small fraction of the high energy physics community, I have decided to remain in the confines of Fortran77 (except for the ubiquitous `implicit none`).

Application programs wishing to remain compatible with future versions of `Circe` must not use `common` blocks or procedures starting with `circe` or `girce`.

C Updates

Information about updates can be obtained

- on the World Wide Web:
<http://crunch.ikp.physik.th-darmstadt.de/nlc/beam.html>
- by internet FTP:

host: crunch.ikp.physik.th-darmstadt.de

user: anonymous

password: your email address

directory: pub/ohl/circe

- from mailing lists:

circe-announce@crunch.ikp.physik.th-darmstadt.de

circe-bugs@crunch.ikp.physik.th-darmstadt.de

circe-discuss@crunch.ikp.physik.th-darmstadt.de

Subscriptions are available from

majordomo@crunch.ikp.physik.th-darmstadt.de

Contributions of results from other simulation programs and updated accelerator designs are welcome at

Thorsten.Ohl@Physik.TH-Darmstadt.de